

University of Dhaka
Department of Computer Science and Engineering
3rd Year 2nd Semester Final Examination, 2022
CSE-3201: Operating Systems (3 Credits)

Time: 3 hours

Total Marks: 70

Answer any five (5) out of the following seven (7) questions. Marks are given in the right margin.

- 1 (a)** Suppose "Intelligent Machines" plan to develop an operating system for smart home management. Suggest which methodology (between Microkernel and Monolithic Kernel) should be preferred and why. [4]

- (b)** A process has address space (all in Hex) from 0x10000 to 0xFFFFF. "TEXT" section starts from lower address location (0x10000), "DATA" section starts from 0x30000, "BSS" starts from 0x40000, and "STACK" starts from 0xFFFFF (higher to lower). The stack size is 1KB. PSP (process stack) points to the stack top, where the OS keeps 64 bytes for storing 16 CPU registers of the process. Determine the addresses of the variables declared in the following program: [6]

```
1. const uint32_t MAXNUMBER = 20;
2. static uint32_t coffee_pot = 10;
3. uint16_t content;
4. void make_coffee(void)
5. {
6.     double water = 200; //cc
7.     double coffee = 10; //cc
8.     double milk = 50; //cc
9.     double sugar = 20; //grams
10.    uint32_t i = 0;
11.    double coffee_list[10];
12.    if (coffee_pot > MAXNUMBER) return;
13.    while(i < coffee_pot)
14.    {
15.        mixing(coffee_list[i], water, coffee, milk);
16.        i++;
17.    }
18. }
```

- (c)** Determine the process state (currently running) for the following events: [4]
- I. An 'EXTR' interrupt occurs
 - II. Process experiencing page fault
 - III. Context switching
 - IV. Unsuccessful to enter a critical section

- 2 (a)** How can you speed up paging using TLB? [4]

[6]

- (b) Support a server system with R_{max} resources with six network ports, eight hard disks, six printers, and five USB ports. The maximum requirement (P_{need}) and current allocation matrix A of five processes is given as:

$$R_{max} = [6 \ 8 \ 6 \ 5], P_{need} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 5 & 2 & 2 \\ 2 & 3 & 2 & 3 \\ 4 & 5 & 2 & 2 \\ 3 & 4 & 2 & 3 \end{bmatrix}, A = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 2 & 3 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Evaluate and determine whether the system is in a safe/unsafe state. Next, if the process p_5 requests additional two network ports, two hard disks, one printer, and two USB ports, does the OS allocate resources immediately to process p_5 ?

- (c) Consider the following segment of code of ten threads. Determine the critical sections, and rewrite the code segment to mitigate the synchronization problem (if any). Your solution must be deadlock-free and meet the four synchronization requirements. [4]

```

1. static uint32_t hit_count = 0; //global shared variable
2. void show_hit_count(void)
3. {
4.     uint32_t value = count; hit_count
5.     while(1)
6.     {
7.         uint32_t s = getHttpHitRequest();
8.         Value+= s;
9.         if(s == 0) break;
10.    }
11.    count = value;
12.    kprintf("Current HTTP hit count %d\n", count);
13. }
```

- 3 (a) Assume you have the following processes to execute with one processor, with the processes arriving at the following times and having the following CPU burst times and priorities: [8]

Process	Arrival Time	CPU Burst Time	Priority		
A	0	8	3	15	7
B	3	4	1	4	6
C	5	7	4	17	10
D	8	3	2	3	0

For each of the following scheduling algorithms: (I) Shortest Job First (SJF), Preemptive and (II) Priority, Preemptive:

- (i) Draw a Gantt chart.
- (ii) Calculate average waiting time.
- (iii) Calculate turnaround time of each process.

- (b) Suppose "Intelligent Machines" plan to design an operating system for managing shopping mall surveillance, access control, and possible vulnerability detection. The shopping mall has four groups of users: (i) management, (ii) computer emergency response team (CERT), (iii) shop owners, and (iv) customers. The surveillance of each camera must be visual at a one-minute time interval; customers are advised to enter the shopping mall in a first come, first serve manner, and management barely looks at the system and is dependent on the CESR. Shop owner requests are services in a round-robin fashion. However, the management decides to assign priority {1, 2, 3, 4} to the OS groups {CERT, Management, ShopOwner, Customers}. Suggest a reasonable process scheduling policy for the OS (hint: use multi-level ready queue). [6]

- 4 (a) An operating system implements a variable-size dynamic partition memory (RAM) allocation scheme for the processes. The maximum partition size is 64KB, and the Logical address is 20-bits allocating. The most significant 4-bits indicates the partition number (need translation), and the limit register contains the actual memory allocation size. The allocated partition tuple is $\{Process, StartingAddress, Limit\}$. The three process allocation information are $\{P_1, 1000, 200\}$, $\{P_2, 2000, 300\}$, and $\{P_3, 2400, 500\}$. Determine the physical or absolute address for the following logical addresses: [5]

$$\begin{bmatrix} P_1 \rightarrow & 150 & 250 & 130 \\ P_2 \rightarrow & 160 & 230 & 350 & 500 \\ P_3 \rightarrow & 200 & 400 & 350 & 501 \end{bmatrix}$$

- (b) Let a system has four 512-byte memory frame. A process has ten pages. Determine the number of page faults for the following references using optimal and LRU page replacement algorithms: [5]

1, 2, 4, 5, 8, 3, 4, 10, 2, 3, 6, 7, 8, 2, 6, 8, 9

- (c) Determine the working-set model (5-unit time) for processes with page references given below: [4]

$$\begin{aligned} P_1 - &> 1, 3, 4, 2, 1, 4, 5, 6, 1, 5, 2, 4, 7, 3, 8, 2, 1 \\ P_2 - &> 10, 10, 4, 4, 1, 4, 4, 6, 6, 1, 2, 4, 7, 7, 7, 2, 1 \\ P_3 - &> 6, 7, 8, 9, 6, 4, 5, 3, 4, 1, 1, 1, 5, 4, 4, 4, 3 \\ P_4 - &> 8, 8, 8, 5, 4, 4, 5, 3, 3, 4, 1, 1, 5, 2, 3, 4, 3 \end{aligned}$$

If the system has ten frames, does the system go to thrashing? In this scenario, how is the operating system rescued from thrashing?

- 5 (a) It has been proven beyond doubt that caching can radically increase the efficiency of a system. Why are then modern system manufacturers still using multiple layers of memory rather than caching everything? If the choice is up to you, would you do it? Explain. [4]

- (b) To speed up data transfer among networked machines, keyboard machines, and machine-display devices, suggest suitable I/O buffering with practical senses. [5]

- (c) From the OS developer's point of view, what are the advantages of designing a device-independent driver? How do device-independent drivers interact with the actual device functionalities? [5]

- 6 (a) Let a file system uses block groups that contain super block, *inode* table, and data blocks. The average file block access time is 2ms. Determine the read and write operations for direct, single indirect, and double indirect blocks. [4]

- (b) Suggest a virtual file system structure where the user can open, share, and access files, and the operating system can handle various categories of file systems such as EXT4, XFS, VFAT, NTFS, and so on. [6]

- (c) Compare, discuss, and put your comments on disk space utilization and system performance based on the file system block size. [4]

- 7 (a) Write an advantage and a disadvantage of each of RAID levels 0 through 6. [7]

- (b) Calculate the seek time for SSTF, Elevator, and C-SCAN algorithms for the following track read requests submitted by processes running in a server. Please assume that the head reading track 78 and previously transferred data from track 34. [7]

55, 45, 58, 39, 25, 100, 46, 180, 199, 65, 35, 25

25, 25, 35, 35, 45, 46, 55, 58, 65, 100, 180, 199

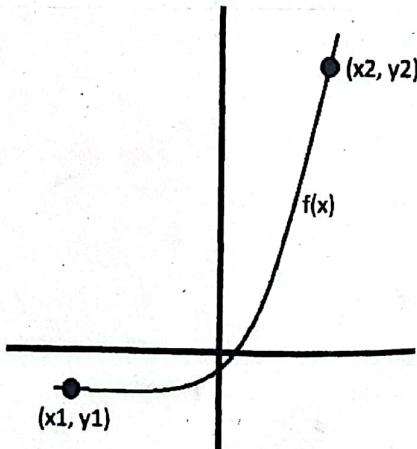
University of Dhaka
Department of Computer Science and Engineering
3rd Year 2nd Semester Final Examination, 2022
CSE-3202: Numerical Methods (3 Credits)

Time: 3 hours

Total Marks: 70

Answer any five (5) out of the following seven (7) questions. Marks are given in the right margin.

- 1 (a)** Let $f(x) = x^3 - 7x - 1$ be a function. Find the solution for $f(x) = 0$ using Newton Raphson method. Report the values of $x_i, f(x_i), f'(x_i), x_{i+1}$, for each iteration, up to four iterations. Assume $x_0 = 3$. [4]
- (b)** What is the difference between machine-epsilon and precision? Explain this using IEEE single precision number. [3]
- (c)** Suppose you have a new programming language that cannot perform division operation. Now you have to implement a division function. You can use addition, subtraction, and multiplication operations and loops. Write the pseudocode of division(x, y) function that calculates x/y . [7]
- 2 (a)** Let $f(x) = x^3 - 7x - 1$ be a function. Find the solution for $f(x) = 0$ using Secant method. Report the values of $x_{i-1}, f(x_{i-1}), x_i, f(x_i), x_{i+1}$, for each iteration, up to five iterations. Assume $x_0 = 3$ and $x_1 = 5$. [7]
- (b)** Suppose you have to find the root of a function $f(x)$ using Secant method. There is exactly one root between $x = a$ and $x = b$. Now, you have to modify the Secant method such that the algorithm converges to the root between a and b . You must use the secant line to find the next approximation like the original method. Write the pseudocode of your proposed algorithm. [7]
- 3 (a)** Use the bisection method to find the root of the nonlinear equation $x^3 = 20$. Use initial lower and upper guesses of 1 and 4, respectively. [7]
 - Conduct three iterations to estimate the root of the equation.
 - Find the absolute relative approximate error at the end of each iteration.
 - Find the number of significant digits that are at least correct at the end of each iteration.
- (b)** Explain which method will perform better between Bisection method and False Position method for the following function $f(x)$ and initial points choice of x_1 and x_2 . [7]



- 4 (a) Solve the given set of equations by using Jacobi's method and Gauss-Seidel method with matrix operations. Report values of x , y , and z up to four iterations. Assume initial guesses are $x, y, z = 0, 0, 0$. [8]

$$x + y + z = 4$$

$$x + 4y + 3z = 8$$

$$x + 6y + 2z = 6$$

- (b) Find a root for the equation $2e^x \sin(x) = 3$ using the False Position method and correct it to three decimal places with three iterations. [6]

- 5 (a) Let $f(x) = x^3 - 7x - 1$. Find the approximations of $f'(x)$ using forward, backward and central difference along with the relative true error value. [$h = 0.001, 0.01, 0.1$] $x = -1$ [7]

- (b) Explain which among the forward, backward and central difference method approximates better in terms of error. [7]

- 6 (a) Find the quadratic spline interpolation for the following set of points. You may only write down the equations to be solved to find the parameters rather than solving for them. [7]

x	y
1	12
2	15
3	13

- (b) Find the Taylor series of $f(x) = \ln x$ centered at $x = 1$. [7]

- 7 (a) Using Newton's forward interpolation formula, find the cubic (interpolation) polynomial. [7]

x	0	1	2	3
$f(x)$	1	2	1	10

- (b) (I) Which interpolation formulas are used for unequal intervals? (II) Using interpolation, find the value of $f(x)$ when $x = 15$ [7]

x	3	7	11	19
$f(x)$	42	43	47	60

University of Dhaka
Department of Computer Science and Engineering
3rd Year 2nd Semester Final Examination, 2022
CSE-3204: Formal Language, Automata and Computability (3 Credits)

Time: 3 hours

Total Marks: 70

Answer any five (5) out of the following seven (7) questions. Marks are given in the right margin.

- 1(a)** Prove or disprove each of the following statements: [3+3]
- If L_1 is context free and L_2 is not context free then $L_1 L_2$ is not context free.
 - If L_1 is context free and L_2 is not context free then $L_1 L_2$ is context free.
- (b)** For each language L below choose the most restrictive class it belongs to and give justification for [4+4] your answer. i.e. if you think it is regular, then give the regular expression; if you think it is context free or TM-recognizable, give the working principle of the CFG or TM.
- $L = \{xw \mid x, w \in \{a, b\}^* \text{ and } |x| = |w|\}$
(Choose among regular, context free or TM-recognizable)
 - $L = \{a^i b^j c^k d^l \mid i + j + k + l \text{ is a multiple of } 13\}$
(Choose among regular, context free or TM-recognizable)

- 2(a)** Convert the following grammar in CNF and use CYK algorithm to determine whether $ababb$ is a [4+4] string in the language of the grammar (S is the start symbol).

$$\begin{aligned} S &\rightarrow ASA \mid aB \\ A &\rightarrow B \mid S \\ B &\rightarrow b \mid \epsilon \end{aligned}$$

- (b)** Formally prove that all strings that the following grammar generates have even length. (S is the start [6] symbol)

$$\begin{aligned} S &\rightarrow SB \mid aa \\ B &\rightarrow bSBb \mid ab \end{aligned}$$

- 3 (a)** Consider the following language L over $\Sigma = \{0, 1, \#\}$. [6]

$$L = \{x\#y \mid x, y \in \{0, 1\}^+ \text{ and } y \text{ is the binary number equal to } x + 1\}$$

Hence, $1110\#1111 \in L$, and $111\#110 \notin L$.

Prove that L is not regular.

- (b)** Design a PDA to accept the following language by empty stack. [4]
- $$\{0^n 1^m 0^m \mid m, n \geq 1\}.$$

- (c)** Convert the following PDA to a CFG : [4]

$$P = (\{p, q\}, \{0, 1\}, \{X, Z_0\}, \delta, q, Z_0)$$

1. $\delta(q, 1, Z_0) = \{(q, XZ_0)\}$
2. $\delta(q, 1, X) = \{(q, XX)\}$
3. $\delta(q, 0, X) = \{(p, X)\}$
4. $\delta(q, \epsilon, X) = \{(q, \epsilon)\}$
5. $\delta(p, 1, X) = \{(p, \epsilon)\}$
6. $\delta(p, 0, Z_0) = \{(q, Z_0)\}$

4 (a) Let, $\Sigma = \{T, F, p, q, \sim, \wedge, \vee\}$. The set of well-formed Boolean formulas (WFF) is a language over Σ defined inductively as follows: [5+5]

- T, F, p and q are elements of WFF
 - If x is in WFF, then so is $\sim x$
 - If x and y are in WFF, then $x \wedge y$ and $x \vee y$ are in WFF
- i. Design a DFA with at most 3 states that recognizes WFF.
 - ii. Construct a regular expression for the language WFF.

(b) Let $L = \{w \in \{a, b\}^*: \text{the first, middle, and the last character of } w \text{ are identical}\}$ [4]
Give a context free grammar that generates L .

5 (a) Let $L = \{w \in \{a, b\}^*: \text{the first, middle, and the last character of } w \text{ are identical}\}$ [5+2]
Show the transition diagram of a PDA by final state that accepts L . In words describe the working principle of your given PDA.

(b) Give state diagram for a minimized DFA for the following language: [7]
 $\{w \mid w \text{ has an even number of } a's \text{ and each } a \text{ is followed by at least one } b\}$

6 (a) Design a Turing machine for the language L , where: [7]

$$L = \{w \mid w \text{ is a binary string (contains only 0s and 1s) and palindrome}\}$$

(b) Using the CYK algorithm, test whether the string $w = dabba$ belongs to the following grammar: [7]

$$\begin{aligned} S &\rightarrow AB|CD|d \\ A &\rightarrow BC|DC|d \\ D &\rightarrow AD|CD|b \\ C &\rightarrow DB|CB|b \\ B &\rightarrow a|BS|AB \end{aligned}$$

7 (a) Give transition diagram of a DFA that will accept all binary numbers starting with 1, and remainder is always 11 (or three) when divisor is 101 (or five). [6]

(b) Design a Turing Machine (describe its working principle and also give its transition table) for the following language: [8]

$$L = \{a^n b^n c^n \mid n \geq 1\}$$

University of Dhaka
Department of Computer Science and Engineering
3rd Year 2nd Semester Final Examination, 2022
CSE-3203: Design and Analysis of Algorithms - II (3 Credits)

Time: 3 hours

Total Marks: 70

Answer any five (5) out of the following seven (7) questions. Marks are given in the right margin.

- 1 (a)** Describe the differences between Linear Probing and Quadratic Probing with proper examples. [4]

(b)

[2]

idx	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
val		11	101		22	50		34	

You are given the above hash table D. To insert a number n in H, first, its index (idx) is calculated based on the following equation. Identify which of the following entries will create multiple entries for the same idx.

$$\text{idx} = n \% 9$$

- i) 600 ii) 666 iii) 682 iv) 701

- (c)** You are given the string S = "aaaaacbafadfcbb". You are calculating the hash values H for each three-length consecutive sequence from S based on the following equation. Here if B=7 and M=13 are used, identify if there are any two different sequences that may have similar hash values. [8]

$$H_{(i,i+2)} = (\text{ASCII}(S[i]) * B^2 + \text{ASCII}(S[i+1]) * B + \text{ASCII}(S[i+2])) \% M$$

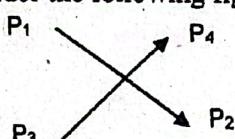
- 2 (a)** We call a pattern P nonoverlappable if $P_k \supseteq P_q$ implies $k = 0$ or $k = q$. Give an example of a pattern of the above definition. Describe the state transition diagram of the string-matching automaton for a nonoverlappable pattern. [4]

- (b)** Draw a standard trie for the following set of strings: {abab, baba, ccccc, bbaaaa, caa, bbaacc, cbcc, cbca}. [4]

- (c)** Given a string X of length n and a string Y of length m, describe an $O(n + m)$ - time algorithm for finding the longest prefix of X that is a suffix of Y. [6]

- 3 (a)** Prove that if $p_1 \times p_2$ is positive, then vector p_1 is clockwise from vector p_2 with respect to the origin (0,0) and that if this cross product is negative, then p_1 is counter clockwise from p_2 . [3]

- (b)** Consider the following figure. [6]



Compute the sign (>0 or <0) of following expression.

- i. $(p_1 - p_3) \times (p_4 - p_3)$ —
- ii. $(p_3 - p_1) \times (p_2 - p_1)$ —
- iii. $(p_4 - p_1) \times (p_2 - p_1)$ —
- iv. $(p_2 - p_3) \times (p_4 - p_3)$ +

- (c)** Write down the pseudocode of Graham Scan algorithm to generate a convex hull from a given set of points. Show two cases (with diagram) i) include a point as a vertex of a convex hull and ii) exclude a point from the vertex set of a convex hull. [5]

4 (a) Compute the value d, x, y of $(899, 493)$ using extended Euclid algorithm. [4]

(b) Find the Euler's totient function $\phi(7000)$. [3]

(c) Consider an RSA key set with $p = 17$, $q = 11$, $n = 187$, and $e = 7$. Determine the public and the private key. What is the encryption of the message $M = 88$? Also prove that the decryption of cipher text is 88. [7]

5 (a) Distinguish between backtracking and branch and bound. [3]

(b) Given a set of non-negative integers and a value sum, the task is to print the subset of the given set whose sum is equal to the given sum. Draw the state-space tree of a given set $S = \{5, 10, 12, 13, 15, 18\}$ and a value sum $m = 30$ to solve the problem using backtracking. Also, define the bounding functions. [5]

(c) We can get optimal solution of 0/1 knapsack using dynamic programming if items weights are integers. For general cases, we can use branch and bound, or back tracking. Prove that back tracking do better than branch and bound by showing their corresponding state-space tree. Use the following data. Knapsack capacity is 10kg. [6]

Item	Weight (kg)	Profit (Taka)
A	2	40
B	3.14	50
C	1.98	100
D	5	95
E	3	30

6 (a) Discuss the steps that are used to solve a NP Problem. [4]

(b) Discuss the idea of a polynomial reduction algorithm to convert problem A to another problem B. [3]

(c) You are given two circles A (A_x, A_y, A_r) and B(B_x, B_y, B_r). For each circle, the first value denotes the x coordinate, the second value denotes the y coordinate and the last value denotes the radius. Write, [7]

- an algorithm that will find if the circles A and B intersect or not
- an algorithm that will find if one circle is inside another circle.

7 (a) Mention the features of the online algorithm. Define the competitive ratio. [3]

(b) Given a set of items in a list and accessing of an item cost is proportional to the distance from the head of the list i.e. a linked list and a request sequence of accesses, the problem is to come up with a strategy of reordering the list so that the total cost of access can be minimized. We can define the problem formally as follows.

Input: Initial list, $I = (1, 2, 3, \dots, n)$ and sequence of requests i_1, i_2, i_3, \dots

Action: Search item i_j

Goal: minimize the total cost

Strategy: move the access item forward

Answer the following questions.

i) Calculate the competitive ration for the strategy to move the item by two position [4]

forward

ii) Prove that no algorithm has competitive ration better than 2. [7]

University of Dhaka
Department of Computer Science and Engineering
3rd Year 2nd Semester Final Examination, 2022
STAT-3205: Introduction to Probability and Statistics (3 Credits)

Time: 3 hours

Total Marks: 70

Answer any five (5) out of the following seven (7) questions. Marks are given in the right margin.

- 1 (a)** Define 'descriptive statistics' and 'inferential statistics' with example. What are the necessary steps for achieving the objective of inferential statistics? [3]

- (b)** We collected 50 pennies and recorded their ages, by calculating $AGE = CURRENT\ YEAR - YEAR\ ON\ PENNY$.

41	9	0	4	3	0	3	8	21	3
2	10	4	0	14	0	25	12	24	19
3	1	14	7	2	4	4	5	1	20
14	9	3	5	3	0	8	17	16	0
0	7	3	5	23	7	28	17	9	2

- i) Use the range approximation to approximate the standard deviation of these 50 measurements. [1]
 - ii) Construct a relative frequency histogram to describe the distribution of penny ages. What can you say about the shape of this distribution? [4]
 - iii) Find the mode and specify modality. [1]
 - iv) Calculate the z-score for the smallest and largest observations. Is either of these observations unusually large or unusually small? [2]
- (c)** With example differentiate among arithmetic mean, trimmed mean and winsorized mean. [3]

- 2 (a)** State Tchebysheff's Theorem. What are the advantages of Tchebysheff's Theorem with comparison to the Empirical rule? [3]

- (b)** Given the following data set:

25, 22, 26, 39, 23, 27, 26, 28, 5, 12, 18, 25, 10, 30, 8, 11

- i) Find the five-number summary and the IQR. [2]
- ii) Construct a box plot for these data and identify any outliers using the boxplot. [3]

- (c)** The data below show the two midterm scores (out of 100 marks) for six students in an introductory statistics class. [6]

Student	Midterm 1	Midterm 2
1	70	88
2	58	52
3	85	84
4	70	80
5	40	36
6	85	48

- i) Construct a scatterplot for the data. Are there any clusters or outliers?
- ii) Calculate r , the correlation coefficient between the two midterm scores. Comment on r .
- iii) Calculate the regression line for predicting a student's score on the second midterm exam based on the first midterm score of 85.

- 3 (a)** "Mutually exclusive events are not independent". Justify your answer in connection to probability. [2]

- (b)** Suppose that $P(A) = 0.3$ and $P(B) = 0.5$. If events A and B are mutually exclusive, find these probabilities:

i. $P(A \cup B)$ ii. $P(A \cap B)$

- (c) A certain manufactured item is visually inspected by two different inspectors. When a defective item comes through the line, the probability that it gets by the first inspector is 0.1. Of those that get past the first inspector, the second inspector will "miss" 5 out of 10. What fraction of the defective items gets by both inspectors? [3]

- (d) Over a long period of time, it has been observed that a professional basketball player can make a free throw on a given trial with probability equal to 0.8. Suppose he shoots four free throws.
- What is the probability that he will make exactly two free throws?
 - What is the probability that he will make at least one free throw?

- (e) How many times should a coin be tossed to obtain a probability equal to or greater than 0.9 of observing at least one head? [3]

- 4 (a)** Two cold tablets are unintentionally placed in a box containing two aspirin tablets. The four tablets are identical in appearance. One tablet is selected at random from the box and is swallowed by the first patient. A tablet is then selected at random from the three remaining tablets and is swallowed by the second patient. Define the following events as specific collections of simple events:

- The sample space S
- The event A that the first patient obtained a cold tablet
- The event B that exactly one of the two patients obtained a cold tablet
- The event C that neither patient obtained a cold tablet

- (b)** A key ring contains four office keys that are identical in appearance, but only one will open your office door. Suppose you randomly select one key and try it. If it does not fit, you randomly select one of the three remaining keys. If it does not fit, you randomly select one of the last two. Each different sequence that could occur in selecting the keys represents one of a set of equi-probable simple events.

- List the simple events in S and assign probabilities to the simple events.
- Let x equal the number of keys that you try before you find the one that opens the door ($x = 1, 2, 3, 4$). Then assign the appropriate value of x to each simple event. Calculate the values of $p(x)$ and display them in a table.
- Construct a probability histogram for $p(x)$.

- (c)** State and explain the 'Law of Total Probability' with example. [5]

- 5 (a)** An electronics store sells a particular model of a laptop computer. There are only four laptops in stock, and the manager wonders what today's demand for this particular model will be. She learns from the marketing department that the probability distribution for x , the daily demand for the laptop, is as shown in the table. [4]

x	0	1	2	3	4	5
$p(x)$	0.10	0.40	0.20	0.15	0.10	0.05

- Find the mean, variance, and standard deviation of x .
 - Is it likely that five or more customers will want to buy the laptop today?
- (b)** Let x be a hypergeometric random variable with $N = 15$, $n = 3$, and $M = 4$. [4]
- Calculate $p(1)$ and $p(3)$.
 - Construct the probability histogram for x .
 - Calculate $\mu = E(x)$ and σ .
- (c)** What do you mean by the central tendency of data? Describe the relationship between mean and median depending on the shape of the distribution. [3]
- (d)** A candy dish contains five blue and three red candies. A child reaches up and selects three candies without looking. [3]
- What is the probability that there are two blue and one red candies in the selection?
 - What is the probability that the candies are all red?

- 6 (a)** Write down the characteristics of standardized normal random variable, z based on its probability distribution curve. [2]

- (b) You arrive into a building and are about to take an elevator to lift to your floor. Assume that the elevator arrives uniformly between 0 and 40 seconds after you press the button. [3]
- Calculate the probability that elevator takes less than 15 seconds to arrive.
 - Find the mean and variance of the uniform random variable.
- (c) The length of life (in days) of an alkaline battery has an exponential distribution with an average life of 1 year, so that $\lambda = 1/365$. [3]
- What is the probability that an alkaline battery will fail before 180 days?
 - If a device requires two batteries, what is the probability that both batteries last beyond 1 year?
- (d) A psychological introvert-extrovert test produced scores that had a normal distribution with mean and standard deviation 75 and 12, respectively. If we wish to designate the highest 15% as extroverts, what would be the proper score to choose as the cutoff point? [4]
- (e) When it is appropriate to use the normal approximation to binomial probabilities? [2]
- 7 (a) A population consists of $N = 5$ items—two of which are considered “successes” (S_1 and S_2) and three of which are considered “failures” (F_1 , F_2 , and F_3). A random sample of $n = 2$ items is selected, without replacement. [4]
- List the possible samples that can be selected and find the proportion of successes in the sample.
 - Find the sampling distribution for the sample proportion and display it as a table and as a probability histogram.
- (b) A soda filling machine is supposed to fill cans of soda with 12 fluid ounces. Suppose that the fills are actually normally distributed with a mean of 12.1 oz and a standard deviation of 0.2 oz. What is the probability that the average fill for a 6-pack of soda is less than 12 oz? [3]
- (c) Packages of food whose average weight is 450 grams with a standard deviation of 17 grams are shipped in boxes of 24 packages. If the package weights are approximately normally distributed, what is the probability that a box of 24 packages will weigh more than 11 kilograms? [3]
- (d) A gas-detector system uses two devices, A and B. If gas is present, the probability that it will be detected by device A is 0.89, by device B is 0.98, and by both devices is 0.94. ~~0.64~~ ~~0.88~~ [4]
- If gas is present, find the probability that the gas will be detected by device A or device B, or both devices.
 - Find the probability that the gas will not be detected.