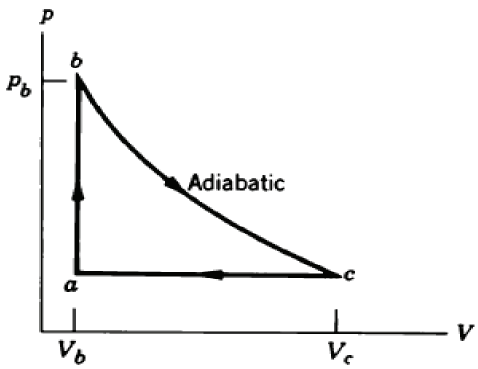
In-course Examination

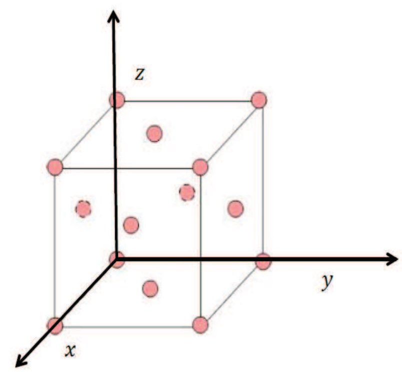
Physics, CSE-1104

March 27,2019

Answer ALL questions | Time: 1 hour 30 minutes | [Marks 25]

1. Two moles of a monatomic ideal gas are caused to go through the cycle shown in the left figure below. Process bc is a reversible adiabatic expansion. Also $p_b = 10.4$ atm, $V_b = 1.22$ m^3 and $V_c = 9.13$ m^3. Calculate: (a) the heat added to the gas, (b) the heat leaving the gas, (c) the net work done by the gas, (d) the change of entropy in processes ca and ab, and (e) the efficiency of the cycle. [1+1+1+(1+1)+1=6]





2. **Maxwell-Boltzmann Distribution:** The Maxwell-Boltzmann speed distribution formula is given by:

$$N_v(v) = 4\pi N\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_BT}$$

   (a) Draw the above distribution with respect to speed of the molecules for different temperatures. Identify the most probable speed, average speed and the RMS speed in the figure. [1+1]

   (b) From the speed distribution formula, derive the energy distribution formula assuming only kinetic energy as the internal energy of an ideal gas. [1]

   (c) Hence derive the mean energy, the most probable energy and the RMS energy of an ideal gas in thermal equilibrium at temperature T. [1+1+1]
   You may find the following integrals useful: $\int_0^\infty u^{3/2}e^{-u}\mathrm{d}u = (3/4)\sqrt{\pi}$, $\int_0^\infty u^{5/2}e^{-u}\mathrm{d}u = (15/8)\sqrt{\pi}$

   (d) Is the most probable energy equal to $(1/2)mv_{mp}^2$, where m is the mass of a gas molecule and v_{mp} is the most probable speed? Explain. [1]

3. **FCC Lattice:** Consider a cubic lattice with the edges of the conventional unit cell along the $x-$, $y-$ and $z-$axis and the length of an edge equal to a.

   (a) Draw the (100), (101) and (111) planes of the lattice within the unit cell. [0.5+0.5+0.5]

   (b) Draw all the members of the family of planes belonging to $\{100\}$. How many planes will be in this family? Write the Miller indices of all of them. [1.5]

   (c) How many members are in the families of planes $\{110\}$ and $\{111\}$? [0.5+0.5]

   (d) How many members are in the family of directions $< 100 >$, $< 110 >$ and $< 111 >$? Write the indices of all the members. [0.5+0.5+0.5]

   (e) The planar density of atoms is defined as the number of atoms per unit area in a particular plane. Considering the FCC lattice, find the planar density of atoms in the (100), (110) and (111) planes. [0.5+0.5+0.5]

   (f) Considering the FCC lattice, calculate the packing fraction, if the atoms at the lattice points are considered as identical spheres. [2]

   (g) Find the reciprocal lattice vectors of the FCC lattice. [2]

   (h) Identify in a clear figure the nearest neighbours of a particular atom in the FCC lattice. Find the distance to the nearest neighbours and the next-to-nearest neighbours. [0.5+(0.25+0.25)]

(a) Starting from the first law of thermodynamics, show that

i. $nC_V dT = nRT(dP/P) - nRdT$ [2]

ii. $nC_V dT = -nRT(dV/V)$ [2]

(b) Derive equations: (a) relating T and P **using i. above** and (b) relating T and V **using ii. above**, for the reversible adiabatic process (compression or expansion) of an ideal gas with constant heat capacities. Hence find the relation between (c) P and V for the adiabatic process. [1+1+1]

5. n moles of a diatomic ideal gas are taken through the cycle with the molecules rotating but not oscillating, where $V_{23} = 3.00V_1$.

(a) What are the values of p_2/p_1, p_3/p_1 and T_3/T_1 ? [$\frac{1}{2} \times 3 = 1.5$]

(b) For path $1 \to 2$, what are (i) W/nRT_1, (ii) Q/nRT_1, (iii) $\Delta E_{int}/nRT_1$ and (iv) $\Delta S/nR$? [$\frac{1}{2} \times 4 = 2$]

(c) For path $3 \to 1$, what are (i) W/nRT_1, (ii) Q/nRT_1, (iii) $\Delta E_{int}/nRT_1$ and (iv) $\Delta S/nR$? [$\frac{1}{2} \times 4 = 2$]

(d) Find the average speed, rms speed and the most probable speed of the gas at state 1 in terms of p_1 and V_1. [$\frac{1}{2} \times 3 = 1.5$]

6. Consider the crystal structure of Sphalerite or Zinc Blend (ZnS) as shown in the second figure above. The larger spheres represent S atoms and the smaller ones represent Zn atoms.

(a) Identify the type of the Bravais lattice. [1]

(b) Draw the three primitive lattice vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ and write them in terms of the Cartesian unit vectors $\hat{x}$, $\hat{y}$ and $\hat{z}$. Taking the length of the side of the cube as a, find the volume of the primitive unit cell. [$1 + 1 = 2$]

(c) Mark the basis of the crystal and find the position vectors of the atoms in the basis. [$1 + 0.5 + 0.5 = 2$]

(d) Find the coordination number of the Zn and S atoms. [$0.5 + 0.5 = 1$]

(e) If Zn and S atoms are replaced by carbon atoms, the above becomes the structure of diamond (c.f. third figure above). Assuming C atoms as hard spheres, find the packing fraction of the diamond structure. [3]

(f) Find the Miller indices of a plane passing through three C atoms in the middle of the xy-, zx- and zy- planes (as shown in the figure). [2]