<u>Syllabus for Test 9</u>: L21 and L22 and L 23 (partial)

L21: General definition of wave motion, periodic and solitary waves; Properties of progressive waves: time period, frequency, wavelength, crest, cycle, amplitude of a wave; Transverse and longitudinal waves; Phase of a wave, wave-front, rays; Superposition principle, standing waves, nodes/anti-nodes, resonant frequencies, Energy transport by a wave, intensity of a wave; General description of wave phenomena: reflection and refraction of waves, refractive index, diffraction and interference of waves.

L22: Review of general properties of wave motion and general wave phenomena; Polarization of waves, unpolarized, plane-polarized, elliptically or circularly polarized lights; Propagation of waves, propagation of a general pulse, general equation of a wave; Transverse waves on a string, one-dimensional differential equation for transverse waves on a stretched string, general solution to the one-dimensional differential equation for wave motion; Reflection and transmission of waves on a stretched string, cases on boundary conditions.

L23: Review of one-dimensional differential equation of wave motion and its general solution; Review of reflection and transmission of waves; Criteria of phase change upon reflection at a boundary.

1.
A wave passes by a swimmer who moves up and down. The time it takes for the swimmer to move from the trough to the crest of the wave would be equal to:

the period of the wave
1/4 the period of the wave
1/2 the period of the wave
twice the period of the wave
none of the mentioned

Ans: 1/2 the period of the wave

2.
A bird floating on the water surface goes up and down while a wave passes by. The amplitude of a water wave is 0.2 m and its frequency is 2 Hz. How much distance will the bird move UP AND DOWN with every wave and how many times will it do this every second?

A distance of 0.4 m twice per second.
A distance of 0.2 m twice per second.
A distance of 0.4 m once per second.
A distance of 0.2 m once per second.
A distance of 0.4 m four times per second.

Ans: A distance of 0.4 m twice per second.

3.
Which of the following is NOT a characteristic of a general wave in a material medium?

amplitude
time period
speed
energy
power

Ans: Time period is a property of a PERIODIC wave only. For a single pulse, we cannot define a time period.

4.
Two wave pulses are travelling down a very long rope in the same direction. One is sent earlier and the other at a later time. The first wave is smaller than the second one i.e. the bigger one. Which of the following is true?

The bigger wave will catch the smaller one since the bigger wave has more speed
The bigger wave will never catch the smaller one since the smaller one moves faster
The bigger wave will catch the smaller one and be reflected back from the smaller
The bigger wave will move past the smaller one and the smaller one will move backward
The bigger wave will never catch the smaller one as wave speeds are independent of size

Ans: The bigger wave will never catch the smaller one as wave speeds are independent of size

5.
The amplitude of a sound wave at distance r spreading in air from a point source is A. Then amplitude at a distance 2r will be

2A
A
A/2
A/4
A/√2

Answer: A/2 . Explanation: For a wave coming from a point source and spreading as a spherical wave has its intensity decreasing inversely proportional to the square of the distance from the source (inverse square law). Since the intensity varies as square of the amplitude, we get $A^2 \propto \frac{1}{r^2} \Rightarrow A \propto \frac{1}{r}$, Hence as $r \rightarrow 2r \Rightarrow A \rightarrow A/2$

6.
Consider a transverse wave on a stretched string whose equation is given y = 2 sin (0.01x + 30t). The wave moves on the string from one end to another end is 0.5 sec. If x and y are in cm and t is in sec, then the length of the string is

6 m
9 m
12 m
15 m
30 m

Ans: 15 m. Explanation: Speed of the wave is $v = \omega/k$, The equation of the travelling wave is: $y = A\sin(kx \pm \omega t)$. Comparing with $y = 2\sin(0.01x + 30t)$, we get $k = 0.01$ /cm and $\omega = 30$ Hz. This gives : $v = 30/0.01\text{cm/s} = 3000$ cm/s = 30 m/s. Hence in 0.5 second, the wave travels a distance 15 m.

7.
Consider two points on a wave whose phase difference is pi/3. The frequency of the wave is 50 Hz and its speed is 330 m/s. What is the distance between the two points?

2.2 m
1.1 m
2.7 m
0.7 m
3.3 m

Ans: 1.1 m. Explanation: $\Delta x/\lambda = \Delta\phi/2\pi \Rightarrow \Delta x = (\lambda/3)\pi/(2\pi) \Rightarrow \Delta x = \lambda/6 = 330\text{m}/(50 \times 6)$=1.1 m

8.
Choose the equation that CANNOT represent a plane progressive wave

y = a sin[(2π/λ) (vt±x)]
y = a log (kx) + b log (vt/λ)
y = a sin (ωt ± kx)
y = a sin[2π (t/T±x/λ)]
y = a exp [i (vt±x)/ λ]

Answer: y = a log (kx) + b log (vt/λ) = log [(kx)^a (vt/λ)^b], which does not represent a function of the form f(x ±vt). All other functions are of the above form.

9.
A wave pulse is moving with a speed 1 m/sec in the positive x-direction. The equation of the wave at time t = 0 is given as: $y = f(x) = 1/(1+x^2)$. Its equation at time t = 1 second can be given as:

$y = 1/(1+(1+x)^2)$
$y = 1/(1+(1-x)^2)$
$y = 1/(1+(1+x^2))$
$y = 1/(1+(1/(1+x^2)))$
$y = 1/(1+(1-x^2))$

Ans: $y = 1/(1+(1-x)^2)$. Explanation: The general equation of a progressive wave motion is of the form: $f(x \pm vt)$. For $v = 1$ m/s, the equation becomes, $f(x \pm t)$. At time $t = 1$ second, the equation becomes: $f(x \pm 1)$. Hence, the only option is $y = 1/(1+(1-x)^2)$.

10.
Which of the following is a wave equation (alpha, beta and gamma are all real constants)?

del y/del t = del^2 y/del x^2 + y
del^2 y/del t^2 = alpha del y/del x
del^2 y/del t^2 = alpha^2 del^2 y/del x^2
del y/del t = gamma del^2 y/del x^2
del^2 y/del t^2 = -beta^2 del^2 y/del x^2

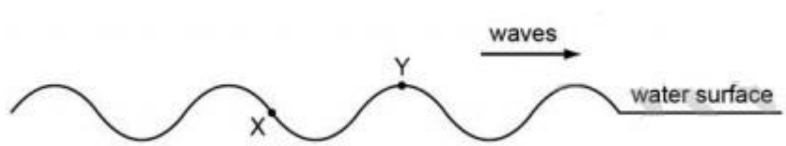
Ans: del^2 y/del t^2 = alpha^2 del^2 y/del x^2

11.
The equation of a stationary wave is given by $y = 3\cos(\pi x/8)\sin(15\pi t)$ where x and y are in cm and t is in seconds. The distance between the consecutive nodes is (in cm)

8
12
14
16
20

Ans: 8. Explanation: This stationary wave is in the form $y = 2A\cos(2\pi x/\lambda)\sin(2\pi vt/\lambda)$ so that $2\pi/\lambda = \pi/8$ from which λ = 16cm. The distance between consecutive nodes is λ/2 = 8cm.

12.
Consider a wave on water in a ripple tank. X and Y are two points on the surface of water. A source oscillating at a constant frequency begins to generate waves which then travel past X and Y, causing them to oscillate. What is the phase difference between the points X and Y?



45 degrees
135 degrees
180 degrees
270 degrees
315 degrees

Ans: 270 degrees. Explanation: delta phi /(2pi) = Delta x/lambda
=> Delta phi = 2pi (3/4)lambda/lambda =(3/2) pi =270 degrees

13.
Consider a string that is hanging from a rigid support due to its own weight. A transverse wave pulse is set up at the bottom. The velocity v of the pulse related to the distance covered by it is given as:

v ∝ √x
v ∝ x
v ∝ 1/x
v ∝ 1/√x
None of the mentioned.

Ans: v ∝ √x .
Explanation: Tension in the string varies with the position on the string. At the end, at x=0, there is no tension (nothing to pull the string downward) and at x=L, at the highest point there is maximum tension.

Tension on the string = weight of the mass of the portion of the string below the point.
Hence, $T = (x\rho)g$. Hence, $v = \sqrt{T/\rho} \propto \sqrt{x}$

14.
Which of the following phenomenon cannot take place with sound wave in air?

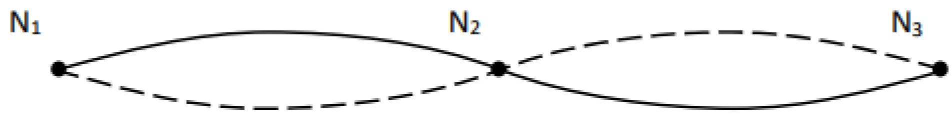
Reflection
Interference
Diffraction
Polarization
Superposition

Answer: Polarization. Polarization is a property of transverse wave and sound wave in air is longitudinal wave.

15.
When a wave refracts from one medium into another which of the following quantity does not change:

Speed of the wave
Frequency of the wave
Wavelength of the wave
Amplitude of the wave
Power carried by the wave

Ans: Frequency of the wave.



16.
Consider a standing wave on a string having three nodes as shown: Which statement is CORRECT?

All points on the string vibrate in phase.
All points on the string vibrate with the same amplitude.
Points equidistant from N2 vibrate with the same frequency and in phase.
Points equidistant from N2 vibrate with the same frequency and the same amplitude.
All points on the string vibrate with the same potential energy.

Ans: Points equidistant from N2 vibrate with the same frequency and the same amplitude.

17.
A guitar string resonates at a fundamental frequency of 300 Hz. Which of the following frequencies can set the string into resonance?

30 Hz
100 Hz
400 Hz
500 Hz
600 Hz

Ans: 600 Hz. Explanation: The frequencies at which standing waves occur are called the fundamental frequency and its harmonics. These are given by: $f_n = v/\lambda_n = v/(2l/n) = nv/2l = nf_1$ where the fundamental frequency is: $f_1 = v/2l$.

18.
A wave is diffracted as it passes through an opening in a barrier. The amount of diffraction that the wave undergoes depends on

amplitude and frequency of the incident wave.
wavelength and speed of the incident wave.
wavelength of the incident wave and the size of the opening.
amplitude of the incident wave and the size of the opening.
none of the others mentioned.

Ans: wavelength of the incident wave and the size of the opening. Amount of diffraction depends on the size of the opening relative to the wavelength.

19.
Two waves represented by y = a sin (ωt –kx) and y = a cos (ωt –kx) are superposed. The resultant wave will have an amplitude:

$a/2$
$\sqrt{2}\,a$
$2a$
$a/\sqrt{2}$
$2\sqrt{2}a$

Ans: √2 a. Explanation: The net displacement is: $y_{net} = y_1 + y_2 = a\sin(\omega t - kx) + a\ \cos(\omega t - kx)$
i.e. $y_{net} = a\sin(\omega t - kx\ ) + a\sin(\omega t - kx + \pi/2) \Rightarrow y_{net} = 2a\sin\left(\omega t - kx + \frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$
i.e. $y_{net} = \sqrt{2}a\ \sin\left(\omega t - kx + \frac{\pi}{4}\right)$

20.
Consider two waves of displacement in a medium. They have the same amplitude and frequency and arrive at a point simultaneously. The resultant amplitude is same as amplitude of each wave. What is the initial phase difference of the two waves?

pi/4
pi/2
pi/3
2 pi/3
3 pi/4

Ans: 2 pi/3. Explanation: Same frequency implies the wavelength is also the same. The net displacement is: $y_{net} = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx - \omega t + \delta)$ where δ is the (initial) phase difference between the waves. According to the requirement: $y_{net} = A\sin(kx - \omega t) + A\sin(kx - \omega t + \delta) = A\sin(kx - \omega t + \phi)$. Now using the addition of two sine functions, we get

$$y_{net} = y_1 + y_2 = A\sin(kx - \omega t)\ + A\sin(kx - \omega t + \delta) = A[\sin(\alpha) + \sin(\alpha + \delta)]$$
$$\Rightarrow y_{net} = A\ 2\sin\left[\frac{(2\alpha + \delta)}{2}\right]\cos\left[\frac{(\alpha + \delta - \alpha)}{2}\right] = 2A\sin\left(kx - \omega t + \frac{\delta}{2}\right)\cos\left(\frac{\delta}{2}\right)$$

Hence we get, $y_{net} = 2A\sin(kx - \omega t + \phi)\ \left(\frac{1}{2}\right)\ \Rightarrow \phi = \delta/2$, and $\cos\left(\frac{\delta}{2}\right) = \frac{1}{2} = \cos(60°)$ which gives $\delta = 120°$

21.
Consider a progressive sinusoidal wave in a stretched string. It has a speed of 2 m/s and a frequency of 100 Hz. What is the phase difference between two points 25 mm apart?

pi
5 pi/2
3 pi/2
pi/4
pi/2

Ans: 5π/2. Explanation: v = nu λ = 2 m/s => λ=(2/100) m = 2 cm, Now $\Delta x/\lambda\ = \Delta\phi/2\pi$ => $\Delta\phi = 2\ \pi\ \Delta x/\lambda \Rightarrow \Delta\phi = 2\pi\ \left(\frac{25}{20}\right) = 2.5\ \pi$

22.
Two sinusoidal waves of the same nature coming from two independent sources meet at a point in free space. They will:

superpose at the point
interfere at the point
create diffraction pattern at that point
create standing wave at that point
will just pass through the point without creating any effect

Answer: Superpose at the point. Explanation: For interference to occur, the sources cannot be independent, they have to be coherent. For diffraction to occur, there must be some obstacle or slit which is not present in free space. For standing waves to occur, they must have the same frequency and must come from opposite direction. They will always superpose and hence will not pass independently without creating any effect.