University of Dhaka
Department of Computer Science & Engineering
3rd Year 1st Semester 2018
In Course Exam
Course: CSE 3202 Numerical Methods

Time: 1.5 Hour

Full Marks: 30

(Answer all of the questions).

1. A group took a trip on a bus, at $3 per child and $3.20 per adult for a total of $118.40. They took the train back at $3.50 per child and $3.60 per adult for a total of $135.20. How many children, and how many adults? To solve this, you need to first formulate the systems of linear equations. Then, solve this system of linear equations by inversion of matrix method. You can choose either any one of the method of elementary row operations or using minor matrix, co-factor matrix and adjugate matrix. However, you need to explicitly show the each steps you followed to perform the matrix inversion. [5 Marks]

2. Use false position method to find the root of the following equation up to 4th iterations in the range mentioned (where x is in radian, you only need to find out the root after 4th iteration, error approximation is not mandatory) [5 Marks]

$$x\tan x - 3 = 0; \quad x_l = 0 \text{ and } x_u = 4$$

3. The upward velocity of a rocket is given at three different times in following table. The velocity data is approximated by a polynomial as $v(t) = b_1 t^2 + b_2 t + b_3$. Find the values of b_1, b_2 and b_3 using the naive Gauss elimination method. (To avoid round off error, use six place after decimal point) [5 Marks]

Time (s)	Velocity (m/s)
5	106.8
8	177.2
12	279.2

4. Use Newton Raphson method to find the root and approximation error at the end of each iteration of the following equation up to 6 iterations in the range mentioned

$$x^2 e^{-x} - 0.5 = 0, \quad x_i = 0.2$$

Write down three pitfalls of using Newton-Raphson Method. [3.5+1.5 Marks]

5. Differentiate between round-off error and truncation error with suitable examples. [2 Marks]

6. Describe the statements (or rules) for the notion of significant digits. What are the significant digits for 4.2231 and 422.0231? Which rule(s) are applicable for the identification of significant digit for 4.2231 and 422.0231? [3 Marks]

Department of Computer Science and Engineering
3rd Year 2nd Semester B. Sc. Final Examination, 2018
CSE-3202: Numerical Methods

Total Marks: 60 Time: 3 Hours

(Answer any four (4) of the following questions)

1. a) For an arbitrary system of equations, what are the possibilities of finding the solution? [2]
   b) Write a program in C to find the roots of a system of linear equations using Basic Gauss Elimination Method in a following way. [8]
      1) The A and B is taken from a text file.
      2) If the pivot element is zero, then the whole line will be swapped with the bottom line.
      3) The program consists of three functions
         a) `read_data()`
         b) `forward_elimination()`
         c) `back_substitution()`
   c) Solve the following system of equations using Gauss Elimination Method. [5]

$$2x_0 + x_1 + x_2 + -2x_3 = 0$$
$$4x_0 + 2x_2 + x_3 = 8$$
$$3x_0 + 2x_1 + 3x_2 = 7$$
$$x_0 + 3x_1 + 2x_2 = 3$$

2. a) Derive the False Position method to obtain the root of a function f(x) = 0. What are the disadvantages of using Newton-Raphson method to find out the root of any function? [5]
   b) Use Secant method to find the root and approximation error at the end of each iteration of the following equation up to 4 iterations in the mentioned range. (To avoid round off error, use four places after decimal point) [5]

$$2x^3+7x^2-14x+5=0, \quad x_{-1} = -5.5 \text{ and } x_0 = -4.5$$

   c) Use bisection method to find the root of the following equation in the mentioned range up to 4th iterations. (To avoid round off error, use six places after decimal point) [5]

$$x\tan(x) - 3 = 0;\ x_l = 0 \text{ and } x_u = 4$$

3. a) Find the inverse of the following matrix using Elementary Row Operations. [6]

$$\begin{bmatrix} 3 & 4 & 7 & 9 \\ 5 & 4 & -1 & 4 \\ 8 & 7 & 8 & 5 \\ 4 & 3 & 0 & 9 \end{bmatrix}$$

   b) Apply Jacobi-iteration method to solve the following system. Continue upto four iterations with the initial guesses a=0, b=0 and c=0. (To avoid round off error, use four places after decimal point) [6]

$$5a - 2b + 3c = -1$$
$$-3a + 9b + c = 2$$
$$2a - b - 7c = 3$$

   c) Describe the pros and cons for Cremar's rule to solve a linear system. [3]

4. There is strong evidence that the first level of processing what we see is done in the retina. It involves detecting something called edges or positions of transitions from dark to bright or bright to dark points in images. These points usually coincide with boundaries of objects. To model the edges, derivatives of functions such as

$$f(x) = \begin{cases} 1-e^{-ax}, & x \geq 0 \\ e^{ax}-1, & x \leq 0 \end{cases}$$

need to be found.

   a) Use the backward divided difference approximation of the first derivative of $f(x)$ to calculate the functions derivative at $x = 0.1$ for $a = 0.24$. Use a step size of $\Delta x = 0.05$. Also, calculate the absolute relative true error, when the true value is 0.23431. [6]

b) Use the central divided difference approximation of the first derivative of $f(x)$ to calculate the functions derivative at $x = 0.1$ for $a = 0.12$. Use a step size of $\Delta x = 0.05$. Also, calculate the absolute relative true error, when the true value is 0.23431. [2]

c) Does the estimate for 4(b) of the derivative increase or decrease with respect to the estimation of 4(a)? Justify the reason behind your claim. [8]

5. a) The upward velocity of a rocket is given as a function of time in Table 1. Find the velocity at t=16 seconds using the Newton Divided Difference method for cubic interpolation.

Table 1 Velocity as a function of time

t(s)	Velocity (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

b) A robot arm with a rapid laser scanner is doing a quick quality check on holes drilled in a 15"×10" rectangular plate. The centers of the holes in the plate describe the path the arm needs to take, and the hole centers are located on a Cartesian coordinate system (with the origin at the bottom left corner of the plate) given by the specifications in Table 2. [5]

Table 2 The coordinates of the holes on the plate

x (in.)	y (in.)
2.00	7.2
4.25	7.1
5.25	6.0
7.81	5.0
9.20	3.5
10.60	5.0

If the laser is traversing from $x = 2.00$ to $x = 4.25$ to $x = 5.25$ in a quadratic path, what is the value of y at $x = 4.00$ using the direct method of interpolation and a second order polynomial? Find the absolute relative approximate error for the second order polynomial approximation.

c) Mention some advantages of spline method for interpolation. [2]

6. a) Human vision has the remarkable ability to infer 3D shapes from 2D images. The intriguing question is: can we replicate some of these abilities on a computer? Yes, it can be done and to do this, integration of vector fields is required. The following integral needs to be integrated. [9]

$$I = \int_0^{100} f(x)dx$$

where

$$f(x) = 0, \quad 0 < x < 30$$

$$= -9.1688\times10^{-6}x^3 + 2.7961\times10^{-3}x^2 - 2.8487\times10^{-1}x + 9.6778, \quad 30 \le x \le 172$$

$$= 0, \quad 172 < x < 200$$

i. Use four segment Simpson's 1/3 Rule to find the value of the integral

ii. Find the true error, E_t, for part (i), when the true value is 60.973.

iii. Find the absolute relative true error for part (i).

b) Derive the multiple segment trapezoidal rule for the following integral [6]

$$I = \int_a^b f(x)dx$$