## University of Dhaka
Department of Computer Science and Engineering
1st Year 2nd Semester B.Sc. Final Examination, 2014
CSE-1201: Discrete Mathematics

Total Marks: 60   **Time: 2.5 Hours**

[Answer any Four (4) of the following Questions]

1. a) What are **contingency** and **contradiction**? Determine whether $(\neg p \to q) \to r$ and $\neg p \to (q \to r)$ are logically equivalent or not. **4**

    b) Express the negations of the following proposition using predicates and quantifiers: "There is a student in this class who has taken every CSE course offered at this school". **3**

    c) Use rule of inference to show that the hypotheses **5**
       i) "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,"
       ii) "If the sailing race is held , then the trophy will be awarded," and
       iii) "The trophy was not awarded"
       Imply the conclusion "**It rained**".

    d) Let A={1,2} and B={a,b,c}. Find B×A. **3**

2. a) Define **abelian group** and **ring** with appropriate examples. **3**
    b) Find cardinality of $P(P(\{\{\emptyset\}, \emptyset, \{\emptyset, a\}\}))$. **3**
    c) Let A, B and C are sets. Show that $(A - B) - C = (A - C) - (B - C)$. **3**
    d) Let $f(x) = floor(x^2/2)$ and $g(x) = ceiling(3x^2-1)$. Find the following: **3**
       i) (fg)(5.3)
       ii) (g+f)(5.4)
       iii) gof(5.9)

    e) Determine whether each of following functions having domain {a, b, c, d} and co-domain {a, b, c, d, e} is **one-to-one/onto**. Also find their range: **3**
       i) $f(a) = b, f(b) = a, f(c) = c, f(d) = d$
       ii) $f(a) = b, f(b) = a, f(c) = d, f(d) = c$
       iii) $f(a) = d, f(b) = b, f(c) = c, f(d) = e, f(b) = a$

3. a) Solve each linear congruence equation: **6**
       i) $3x \equiv 2 \pmod 7$ ii) $4x \equiv 6 \pmod{10}$ iii) $48x \equiv 284 \pmod{356}$

    b) Applying CRT find the smallest positive integer x such that when x is divided by 3 it yields a remainder 1, when x is divided by 5 it yields a remainder 2, and when x is divided by 8 it yields a remainder 6. **3**

    c) Using Mathematical induction prove that **3**
    $$\sum_{i=0}^{n}(-\frac{1}{2})^i = \frac{2^{n+1}+(-1)^n}{3.2^n}, \text{ wherever } n \text{ is nonnegative integer.}$$

    d) Using mathematical induction prove that $n^3 - n$ is divisible by 3 when n is a positive integer. **3**

4. a) Let R be the relation {(1, 2), (1, 3), (2, 3), (2, 4), (3, 2), (4, 3), (4, 2) }, and let S be the relation {(2, 1), (3, 1), (3, 2), (4, 2), (2, 4), (3, 4)}. Find SoR. **4**

    b) For the Hasse diagram below, answer the following questions: **4**
       i) Find the maximal elements.
       ii) Find the minimal elements.
       iii) Is there a greatest element?
       iv) Is there a least element?
       v) Find all upper bounds of {a, b, c}.
       vi) Find the least upper bound of {a, b, c}, if it exists.
       vii) Find all lower bounds of {l, j, k}.
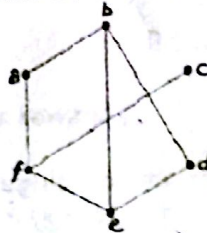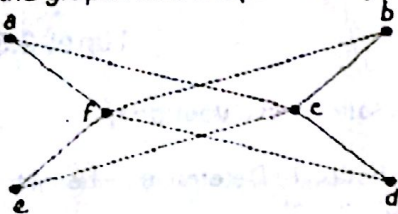   viii) Find the greatest lower bound of {l, j, k}, if it exists.

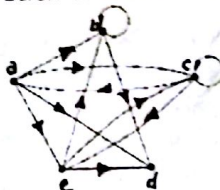3
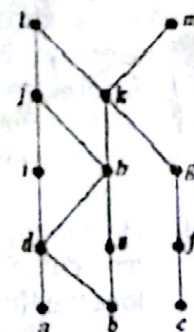
c) Draw the following graphs
   i) $K_{4,2}$
   ii) W_5
   iii) Q_3
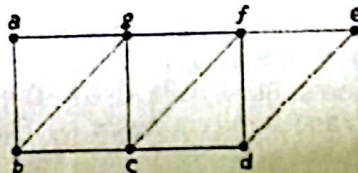
d) Are the graphs below bipartite? If yes, show the partitions. Otherwise state why not.   4
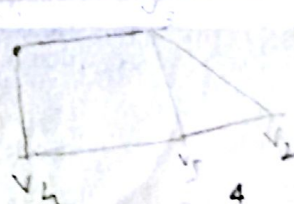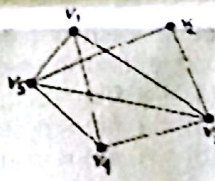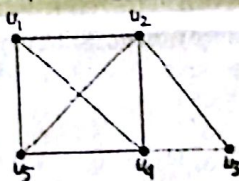

5. a) Find in degree and out degree of each of vertices of the graph below   4
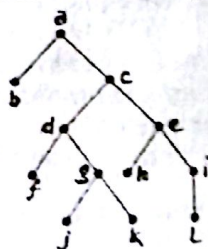

b) Consider the graph below. Does it contain any Euler path/circuit. If yes, show the   3
   path.

c) Determine whether the pair of graphs below are isomorphic to each other or not.   4

d) Find in-order and post-order traversal of the tree given below: 4

6. a) Seven women and nine men are on the faculty in the mathematics department at a   3
   school. How many ways are there to select a committee of five members of the
   department if at least one woman and one man must be on the committee?

b) Find the number of mathematics students in a college taking at least one of the   3
   languages French, German and Russian given the following data:
   62 study French, 45 study German, 40 study Russian, 20 study French and
   German, 23 French and Russian, 15 study German and Russian, 5 study all three
   languages.

c) State generalized pigeonhole principle. At least how many people were born in the   3
   same month among 150 people?

d) How many different bit strings of length six are there?   1

e) How many ways 4 boys and 3 girls can sit in a row? How many ways 4 boys and 3   5
   girls can sit around a circular table? How many ways can they sit if the girls donot sit
   together (not even two girls)?

University of Dhaka
Department of Computer Science and Engineering
1st Year 2nd Semester B.Sc. Final Examination, 2014
MATH 1223: Linear Algebra

Total Marks: 60　　　　　　　　　　　　　　　　　　　**Time: 2.5 Hours**

[Answer any four (4) of the following Questions]

1. a) Define the terms Euclidean inner product, norm and distance in $\Re^n$ and C^n. Find the Euclidean norms of u and v ; inner product $u \cdot v$ and distance between u and v where $u = (2 - 3i, 1 + i, 3 - 7i)$, $v = (4 + 5i, -3i, 5 - 4i)$.　　　**7**

   b) When a matrix invertible? If A is an invertible matrix, then prove that A^T is also invertible and $(A^T)^{-1} = (A^{-1})^T$. Also show that the product of a matrix and its transpose is symmetric.

2. a) Determine the value of k such that the following system of equations x, y and z has:　　　**8**

   i)　　a unique solution　　　　　　　　$x - 3z = -3$
   ii)　　no solution　　　　　　　　　　$2x + ky - z = -2$
   iii)　more than one solutions　　　　$x + 2y + kz = 1$

   b) Solve the following system of equations using Gauss-Jordan Elimination.　　　**7**

   $$2x - 2y - 4z = -2$$
   $$3x - 3y - 6z = -3$$
   $$-2x + 3y + z = 7$$

3. a) Define linear dependence and independence of a set of vectors in a vector space. Determine whether the vectors $(2, -1, 0, 3)$, $(1, 2, 5, -1)$ and $(7, -1, 5, 8)$ in $\Re^4$ are linearly dependent or independent.　　　**8**

   b) Define liner combination of vectors. Write the matrix $E = \begin{pmatrix} 3 & -1 \\ 1 & -2 \end{pmatrix}$ as a linear combination of the matrices $A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$　　　**7**

4. a) Express the following system of equations in matrix form and then solve it by inverse matrix technique:　　　**7**

   $$3x_1 - x_2 + x_3 = -5$$
   $$-x_1 - x_2 = 1$$
   $$x_1 + x_3 = -4$$

   b) State dimension theorem. Find the rank and nullity of the matrix　　　**8**

   $$A = \begin{pmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{pmatrix}$$ and verify the dimension theorem.

5. a) Define basis and dimension of a vector space. Let S and T be the following subspaces of $\Re^4$:　　　**7**

   $$S = \{(x, y, z, t) : y + z + t = 0\}$$
   $$T = \{(x, y, z, t) : x + y = 0, z - 2t = 0\}$$

   Find basis and dimension of $S \cap T$

b) Define the kernel and image of a linear transformation. Let $T : \Re^4 \to \Re^3$ be a linear transformation defined by

$$T(x, y, s, t) = (x - y + s + t, \; x + 2s - t, \; x + y + 3s - 3t)$$

Find a basis and dimension of the image of T and kernel of T.

6. a) Define eigenvalues and eigenvectors of a linear transformation. Given

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix}$$

Find the eigenvalues and eigenvectors of A. And also find the matrix P that diagonalizes A.

b) State Cayley-Hamilton theorem. Verify this theorem for

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$$

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1st Year 2nd Semester B.Sc. Final Examination, 2014
STAT-1224: Introduction to Statistics

Total Marks: 60   **Time: 2.5 Hours**

[Answer any Four (4) of the following Questions]

1. a) A group of 100 students is selected among the first year honours from Dhaka University for the purpose of estimating average/mean height. The average/mean height of the 100 students is calculated as 1.65 meters. Identify the following with respect to the above: **4**
   Population, Sample, Parameter, and Estimator.

   b) Classify the following variables as categorical/discrete/continuous. Also mention their scale of measurement. **5**
   Economic status (poor, middle, rich), Marital status (single, married, widowed, divorced and separated), Calendar time (5 AM, 7 AM etc), Age (in years) and Family size.

   c) Discuss different types of classification. Distinguish between classification and tabulation. **6**

2. a) What do you mean by frequency distribution and relative frequency distribution? **3**

   b) The tensile strength of 30 samples of rubber was measured and the results, in suitable units, were as follows: **8**

174	160	141	153	161	159	163	186	179	167
156	159	171	156	142	169	160	171	188	151
172	181	152	178	151	177	180	186	169	168

   i) Construct a relative frequency distribution.
   ii) Construct a stem and leaf plot.
   iii) Draw a frequency curve.

   c) The accompanying table shows the percentage of Bangladeshi expatriates in 2010. **4**

Expatriates	Percent
Unskilled	42.6
Skilled	35.1
Semi-skilled	15.5
Others	6.8

   Display the data by a suitable graph.

3. a) Define Arithmetic Mean (AM), Geometric Mean (GM), and Harmonic Mean (HM) with useful examples. A variable X takes on n values which are in geometric progression, show that $AM \times HM = GM^2$ **8**

   b) The following table shows the frequency distribution of marks obtained by some students of CSE department. Calculate Median, Mode and 7th decile for the marks of the students **7**

Marks	47.5-52.5	52.5-57.5	57.5-62.5	62.5-67.5	67.5-72.5	72.5-77.5	77.5-82.5
# of students	4	9	18	24	31	16	5

4. a) Define interquartile range, standard deviation and coefficient of variation. Prove that the variance is independent of origin but dependant on the scale. **6**

   b) The rainfall (mm) for one week period in two different area are given as **6**

Area A:	103	110	102	108	115	104	120
Area B:	107	113	109	114	112	108	115

   Rainfall of which area is more consistent? Why?

7 c) In a study of the gasoline mileage of model year 2004 automobiles, the mean miles 3
per gallon was 27.5 and median was 26.8. The smallest value in the study was 12.70,
and the largest was 50.20. The first and third quartiles were 17.95 and 35.45
respectively. Construct a box and whisker plot and comment.

5. a) The following data shows the daily pocket money spent by some students of CSE 6
department.

Money (in Tk)	40-50	50-60	60-70	70-80	80-90	90-100	100-110
# of students	3	7	20	35	40	32	27

Calculate four raw moments and hence find four central moments. Also find the
coefficient of skewness (β_1) and coefficient of kurtosis (β_2) and also comment on the
shape of the distribution.

b) Show that the correlation coefficient (r) lies between -1 and +1. 4

c) The following data gives number of trees and yields of orchards in a village 5

No. of trees	55	32	28	48	24	44	18	35	60
Yields(in kg)	600	350	300	440	300	500	220	400	550

i) Draw a scatter diagram
ii) Find the coefficient of correlation between no. of trees and yields. Comment on
your findings.

6. a) Define regression analysis. State the utilities of studying regression analysis. 3

b) In the study of Systolic Blood Pressure (SBP) as a function of age, height and weight, 7
the latter were converted to a Quetlet index using the formula $QUI = 100 \times$
$weight/height^2$. A sample of 6 individual gives the following information

Age	41	43	45	48	49	52
SBP	122	120	135	132	130	148
QUI	3.25	2.79	2.88	3.02	3.10	3.77

Note that for X=(1 Age QUI) & Y=SBP, we have the following results

$$(X'X)^{-1} = \begin{pmatrix} 27.532358 & -0.455837 & -1.992100 \\ -0.455837 & 0.018253 & -0.124370 \\ -1.992100 & -0.124370 & 2.473553 \end{pmatrix} \text{ and } X'Y = \begin{pmatrix} 787.0 \\ 36639.0 \\ 2479.7 \end{pmatrix}$$

i) Fit a linear regression model of SBP on Age and QUI and comment
ii) Predict the SPB of individual having age 50, weight 150 and height 70
iii) Find the percentage of variation in the SBP that could be explained by both the
Age and QUI.

c) One hundred eighty (180) managers from various levels were randomly selected and 5
interviewed regarding the concern about environmental issues. The response of each
person was tallied into one of the categories: no concern, and some concern. The
result were

Level of Management	Environmental concern	
	No concern	Some concern
Top Management	15	25
Middle Management	20	40
Group leader	20	52

Determine whether there is an association between management level and
environmental concern.