

Answer any Five(5) of the following questions ($5 \times 14 = 70$)

1. (a) Draw a block diagram of CPU (Central Processing Unit) with Memory system. Point out the main works for each of the units. [5]
- (b) What kind of software a computer virus is? Give supportive characteristics. [3]
- (c) What is the way of measurement to compute the power of a computer? [3]
- (d) A HDD (Hard Disk Drive) contains of 50 disks. Both the sides of a disk can be used as memory unit where the sides are divided into 120 tracks. In addition, every track is divided into 20 sectors of same size. Find out the capacity of the HDD if the sector size is 1024KB. [3]
2. (a) Define RAM (Random Access Memory). Compare different types of RAM architecture. [4]
- (b) In a programming language, both the integer and float need 4-Bytes to store values. How can the range of values in float be bigger than integer. Present mathematical explanation on this regard. [4]
- (c) Suppose the initial state of a RAM for 3 variables A , B and C is given below.

Variable	A	B	C
Memory Address	12348	56980	97824
Value	10	-27	69

A block of instructions executed as follows:

1. $A = B * 10 + 100$
2. $C += 3$
3. $B = A + C$
4. $\text{if}(B < 100)$
5. $C = -1$
6. else
7. $C = 1$

Now answer the following questions

[$2*3=6$]

- i. Find the value of the Bytes with memory address 12349 after executing first instruction.
- ii. Show the states of the RAM after executing all the instructions.
3. (a) Which of the following processes require real-time processing? [4]
- i. Weather forecast
 - ii. Multiplayer online game
 - iii. Messenger App
 - iv. Drones
- (b) Explain the role of ROM during bootstrapping or booting. [3]

(c) If each time slice in a multiprogramming system is 50 milliseconds and each context switch requires at most two microseconds, how many processes can the machine service in a single second? [2]

(d) Suppose the following solutions have been proposed for removing the deadlock that occurs on a single-lane bridge when two cars meet. Identify which of the three conditions for deadlock is removed by each solution. [5]

i. Do not let a car onto the bridge until the bridge is empty.

ii. If cars meet, make one of them back up.

iii. Add a second lane to the bridge.

4. (a) Suppose the memory cells at addresses 20 through 28 in the machine described in Appendix A contain the following bit patterns: [6]

Address	Contents
20	12
21	20
22	32
23	30
24	B0
25	21
26	24
27	C0
28	00

Assume that the machine starts with its program counter containing 20.

i. What bit patterns will be in registers 0, 1, and 2 when the machine halts?

ii. What bit pattern will be in the memory cell at address 30 when the machine halts?

iii. What bit pattern will be in the memory cell at address B0 when the machine halts?

(b) Is the communication medium from CPU to a color Monitor parallel or serial? Explain your answer. [2]

(c) Explain why would a designer choose a CISC architecture when a RISC architecture can perform essentially the same instructions? [3]

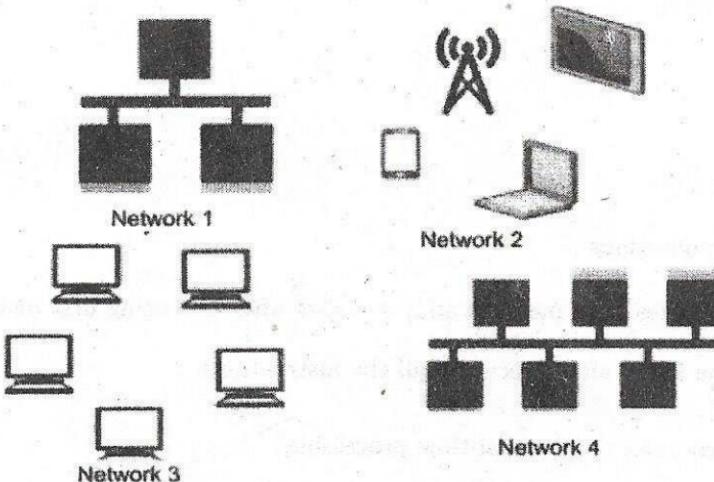
(d) Decode the following machine language instructions and write them in plain english using Appendix A:

(i) BADE (ii) 35A7

[3]

5. (a) Connect the following networks using the most appropriate devices

[6]



- i. Network 1 and network 2.
- ii. Network 3, Network 4
- iii. Network 1, Network 2 and Network 4
- iv. Network 1, Network 3 and Network 4

(b) Though IP addresses are convenient for Internet devices to locate a particular machine or network. Humans are more comfortable to use mnemonic names (domain mnemonic addresses). What is the service that is used to find IP address when the mnemonic address is given. Explain with a suitable example how it works. [4]

(c) In a content delivery network (CDN) e.g. Netflix, which type of transmission is more suitable? [4]

i. unicast

ii. N-unicast

iii. Multicast or is there another type of transmission that is more suitable for the task?

6. (a) Write an algorithm to make a list of all positive integers. Explain your answer. [3]

(b) Identify some points of vagueness in our informal definition of an algorithm stated below: Informally, an algorithm is a set of steps that defines how a task is performed. [4]

(c) Given, List: 19, 37, 53, 71, 96, 137, 289, 374, 559, 797, 979.

i. Which search algorithm (sequential or binary) will find the number 137 more quickly? [1.5]

ii. Which will find the number 19 more quickly? [1.5]

iii. Which will detect the absence of the number 99 more quickly? [2]

iv. How many numbers will be interrogated when searching for the number 96 when using the sequential search? How many will be interrogated when using the binary search? [2]

7. (a) List: 20, 56, 12, 78, 45, 10.

Sort the above list of numbers using [3+3]

i. Insertion Sort

ii. Bubble Sort

(b) Count the number of comparisons made in each type of sorting algorithm in question 7(a) and comment which algorithm (if any) is preferable over the other. [4]

(c) Suppose a software package is so expensive that you would never purchase it. Is it ethical to copy it for your own use? After all, you are not cheating the supplier out of a sale because you would not have bought the package anyway. [4]

Appendix A (The Machines Architecture)

The machine has 16 general-purpose registers numbered 0 through F (in hexadecimal). Each register is one byte (eight bits) long. For identifying registers within instructions, each register is assigned the unique four-bit pattern that represents its register number. Thus register 0 is identified by 0000 (hexadecimal 0), and register 4 is identified by 0100 (hexadecimal 4).

There are 256 cells in the machines main memory. Each cell is assigned a unique address consisting of an integer in the range of 0 to 255. An address can therefore be represented by a pattern of eight bits ranging from 00000000 to 11111111 (or a hexadecimal value in the range of 00 to FF).

The Machines Language

Each machine instruction is two bytes long. The first 4 bits provide the op-code; the last 12 bits make up the operand field. The table that follows lists the instructions in hexadecimal notation together with a short description of each. The letters R, S, and T are used in place of hexadecimal digits in those fields representing a register identifier that varies depending on the particular application of the instruction. The letters X and Y are used in lieu of hexadecimal digits in variable fields not representing a register.

Opcode	Operand	Description
1	RXY	LOAD the register R with the bit pattern found in the memory cell whose address is XY.
2	RXY	LOAD the register R with the bit pattern XY.
3	RXY	STORE the bit pattern found in register R in the memory cell whose address is XY.
4	0RS	MOVE the bit pattern found in register R to register S.
5	RST	ADD the bit patterns in registers S and T as though they were twos complement representations and leave the result in register R.
6	RST	ADD the bit patterns in registers S and T as though they represented values in floating-point notation and leave the floating-point result in register R.
7	RST	OR the bit patterns in registers S and T and place the result in register R.
8	RST	AND the bit patterns in registers S and T and place the result in register R.
9	RST	EXCLUSIVE OR the bit patterns in registers S and T and place the result in register R.
A	R0X	ROTATE the bit pattern in register R one bit to the right X times. Each time place the bit that started at the low-order end at the high-order end.
B	RXY	JUMP to the instruction located in the memory cell at address XY if the bit pattern in register R is equal to the bit pattern in register number 0. Otherwise, continue with the normal sequence of execution. (The jump is implemented by copying XY into the program counter during the execute phase.)
C	0 0 0	HALT execution.

University of Dhaka
Department of Computer Science and Engineering
1st Year 1st Semester B. Sc. Final Examination 2018
CSE 1103: Electrical Circuits

Duration: 3 hours

Full Marks: 70

(Answer any five of the following seven questions)

1. a) A resistor of 10Ω has charge flowing through it at the rate of 300 coulombs per minute (C/min). How much power is dissipated? 2
- b) i. What is the total efficiency of three systems in cascade with efficiencies of 98%, 87%, and 21%? 4
- ii. If the system with the least efficiency (21%) were removed and replaced by one with an efficiency of 90%, what would be the percentage increase in total efficiency?
- c) A motor is rated to deliver 2 hp. 6
- i. If it runs on 110 V and is 90% efficient, how many watts does it draw from the power line?
- ii. What is the input current?
- iii. What is the input current if the motor is only 70% efficient?
- d) A $20\text{ k}\Omega$ resistor has a rating of 100 W. What are the maximum current and the maximum voltage that can be applied to the resistor? 2
2. a) Design a voltage divider circuit that will permit the use of an 8 V, 50 mA bulb in an automobile with a 12 V electrical system. 2
- b) Find the voltage across each resistor in Fig. 2(b), if $R_1 = 2R_3$ and $R_2 = 7R_3$. 3

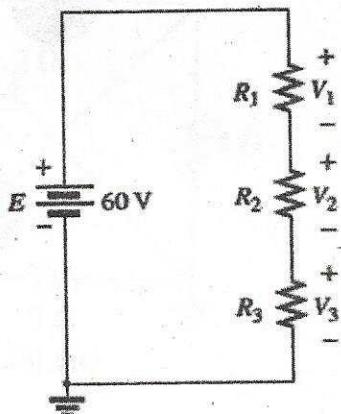


Fig. 2(b)

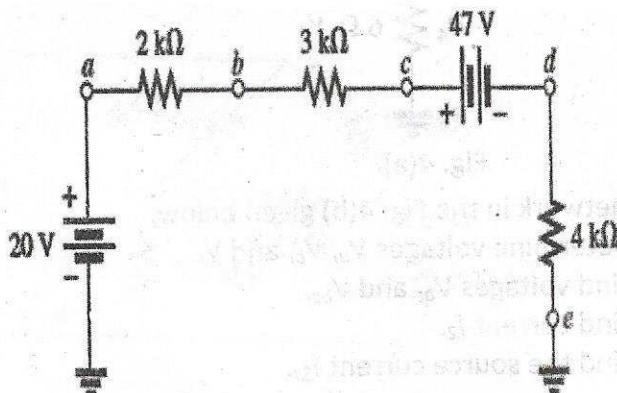


Fig. 2(c)

- c) For the network in Fig. 2(c), determine the voltages:
- V_a, V_b, V_c, V_d, V_e
 - V_{ab}, V_{dc}, V_{cb}
 - V_{ac}, V_{db}
- d) Show that, "if two elements are in parallel, the voltage across them must be the same. However, if the voltage across two neighboring elements is the same, the two elements may or may not be in parallel."
- 3
3. a) What happens when two batteries of different voltages are placed in parallel? 2
- b) For the circuit in Fig. 3(b) given below:
- Determine currents I_1 and I_2 2
 - Calculate the value of resistor R_2
 - Find the power delivered by each source
- 2+1+

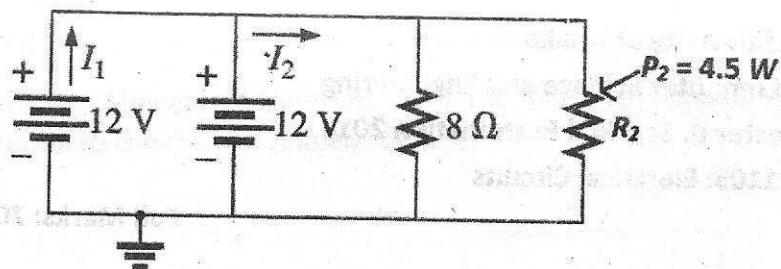


Fig. 3(b)

- c) For the network in Fig. 3(c), determine
 i. The short-circuit currents I_1 and I_2
 ii. The voltages V_1 and V_2
 iii. The source current I_s .

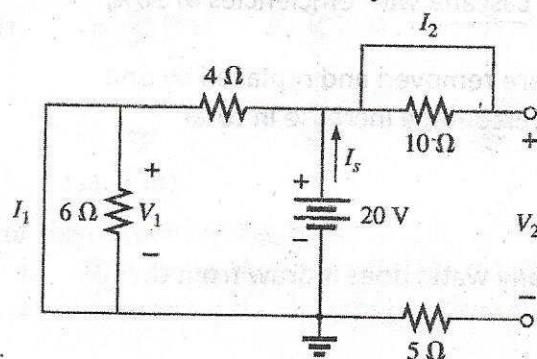


Fig. 3(c)

- d) Describe in brief about finding the unknown quantities of a circuit using 'reduce and return' approach.

4. a) For the network in Fig. 4(a), determine the voltages V_1 and V_2 and the current I .

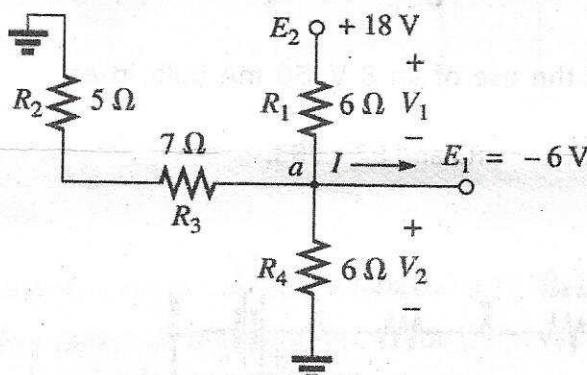


Fig. 4(a)

- b) For the network in the Fig. 4(b) given below,
 i. Determine voltages V_a , V_b , and V_c .
 ii. Find voltages V_{ac} and V_{bc} .
 iii. Find current I_2 .
 iv. Find the source current I_{s3} .

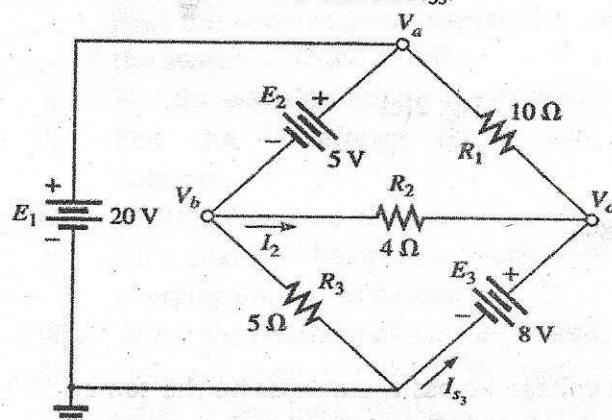


Fig. 4(b)

- c) Determine R_1 , R_2 , and R_3 for the voltage divider supply in Fig. 4(c). Can 2W resistors be used in the design?

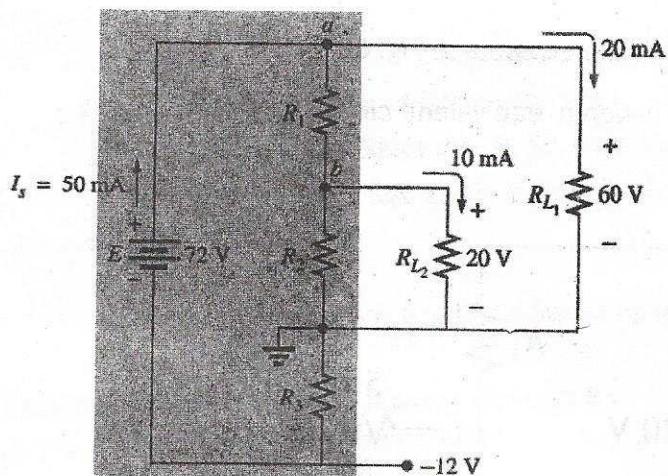


Fig. 4(c)

5. a) For the networks in the Fig. 5(a), determine the current I_2 using branch-current analysis, and then find the voltage V_{ab} . 4

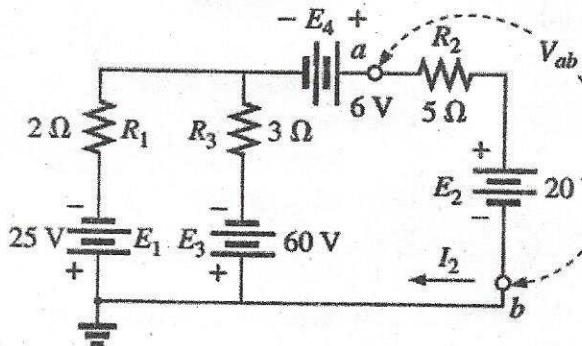


Fig. 5(a)

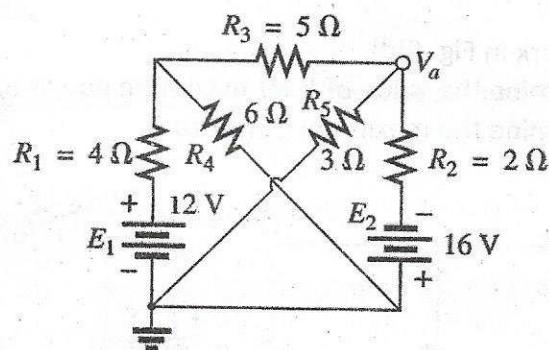


Fig. 5(b)

- b) Using format approach of mesh analysis, determine the current through the 5Ω resistor for each network in the Fig. 5(b). Then determine the voltage V_a . 4
- c) Determine the nodal voltages for the networks in Fig. 5(c). 3
- d) Using a Δ -Y or Y- Δ conversion, find the current I in the network of Fig. 5(d). 3

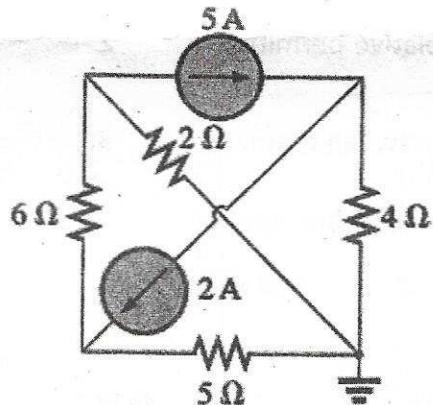


Fig. 5(c)

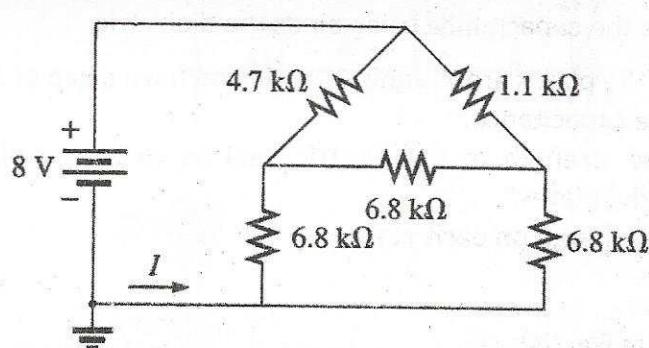


Fig. 5(d)

6. a) When do we call a bridge network balanced? Show that, the following bridge network of

4

Fig. 6(a) is balanced when $\frac{R_1}{R_3} = \frac{R_2}{R_4}$.

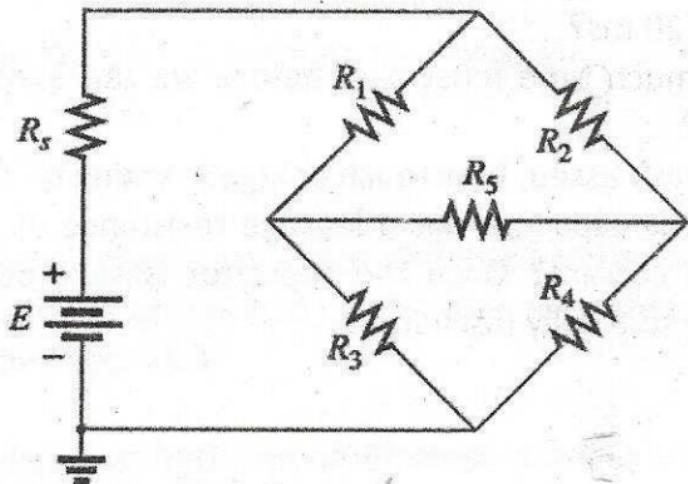


Fig. 6(a)

- b) Using superposition, find the current I through the 24V source Fig. 6(b). 3
- c) State the Thévenin's theorem. Find the Thévenin equivalent circuit for the network external to the resistor R in Fig. 6(c). 4

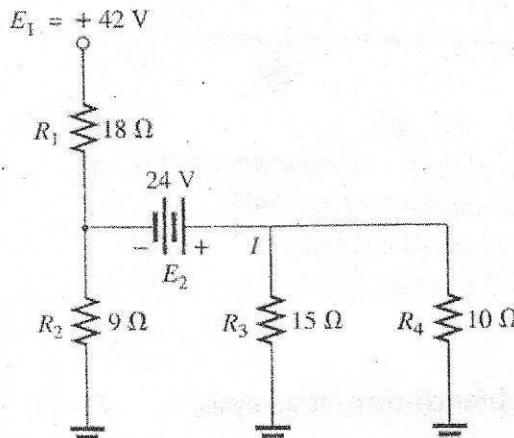


Fig. 6 (b)

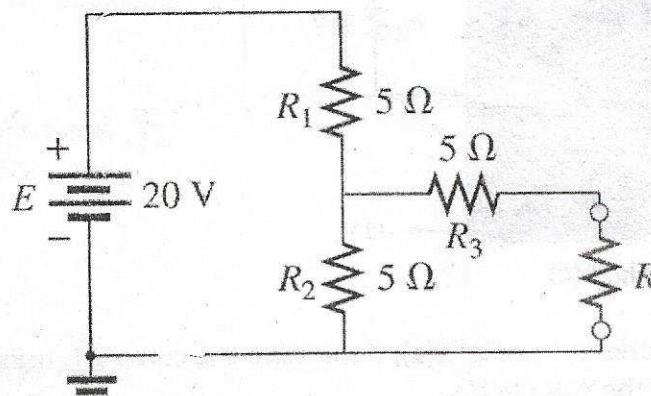


Fig. 6 (c)

- d) For the network in Fig. 6(d),
- Determine the value of R for maximum power to R .
 - Determine the maximum power to R .

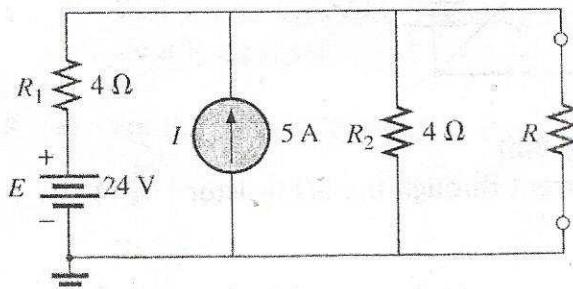


Fig. 6(d)

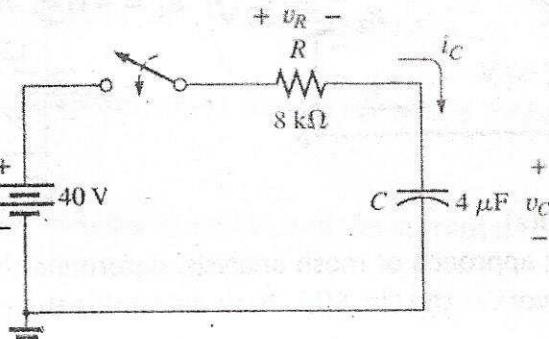


Fig. 7 (c)

7. a) Show that, the capacitance of a capacitor with a dielectric having a relative permittivity of ϵ_r is ϵ_r times the capacitance using air as the dielectric. 2

- b) In a capacitor, the plates are 2" long, 2" wide and have a gap of 1/32" between them.
- Find the capacitance.
 - Find the strength of the electric field between the plates if 48 V are applied across the plates.
 - Find the charge on each plate.

- c) For the circuit in Fig 7(c):

- Find the mathematical expression for the transient behavior of v_C , i_C and v_R if the switch is closed at $t=0$ s. 1+1+
- Plot the waveform of v_C versus the time. 2
- Plot the waveforms of i_C and v_R versus the time constant of the network.
- What is the value of v_C at $t = 20$ ms?
- On a practical basis, how much time must pass before we can assume that the charging phase has passed?
- When the charging phase has passed, how much charge is sitting on the plates?
- Define leakage current. If the capacitor has a leakage resistance of 10,000 MΩ, what is the initial leakage current? Once the capacitor is separated from the circuit, how long will it take to totally discharge?

University of Dhaka
Department of Computer Science and Engineering
1st Year 1st Semester B. Sc. Final Examination, 2018

CSE 1102: Discrete Mathematics

Total Marks: 70

Time: 3 Hours

(Answer any five (5) of the following Questions)

1. / a) What is propositional logic? What is the difference between an implication and a conjunction? 4
Prove that $a \rightarrow b$ is equivalent to $\neg a \vee b$.
- b) Let A = "Aldo is Italian" and B = "Bob is English". Formalize the following sentences using 5 propositional logic
- "Aldo isn't Italian"
 - "Aldo is Italian while Bob is English"
 - "If Aldo is Italian then Bob is not English"
 - "Aldo is Italian or if Aldo isn't Italian then Bob is English"
 - "Either Aldo is Italian and Bob is English, or neither Aldo is Italian nor Bob is English"
- c) You are walking in a labyrinth and all of a sudden you find yourself in front of three possible 5 roads: the road on your left is paved with gold; the one in front of you is paved with marble, while the one on your right is made of small stones. Each street is protected by a guardian. You talk to the guardians and this is what they tell you:
- The guardian of the gold street: "This road will bring you straight to the center. Moreover, if the stones take you to the center, then also the marble takes you to the center."
 - The guardian of the marble street: "Neither the gold nor the stones will take you to the center."
 - The guardian of the stone street: "Follow the gold and you'll reach the center, follow the marble and you will be lost."

Given that you know that all the guardians are liars, can you choose a road being sure that it will lead you to the center of the labyrinth? If this is the case, which road do you choose? Provide a propositional language and a set of axioms that formalize the problem and show whether you can choose a road being sure it will lead to the center.

2. a) Find the set A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$ and $A \cap B = \{3, 6, 9\}$. 2
- b) Suppose that A is the multiset that has its elements the types of computer equipment needed by one department of a university where the multiplicities are the number of pieces of each type needed, and B is the analogous multiset for a second department of the university. For instance, A could be the multiset {107.Personal computer, 44.routers, 6.servers} and B could be the multiset {14.personal computers, 6.routers, 2.mainframes}.
i) What combination of A and B represents the equipment the university should buy assuming both departments use the same equipment?
- c) Give an explicit formula for a function from the set of integers to the set of positive integers that is 6
a) one-to-one, but not onto.
b) onto but not one-to-one
c) one-to-one and onto
d) neither one-to-one nor onto
- d) What are the values of these sums where $S = \{1, 3, 5, 7\}$? 3
i) $\sum_{j \in S} j$
ii) $\sum_{j \in S} j^2$
3. a) Let $P(n)$ be the statement that $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4}n((n+1)/2)^2$ for the positive integer n . 6

- i. What is the statement $P(1)$?
 - ii. Show that $P(1)$ is true, completing the basis step of proof.
 - iii. What is the inductive hypothesis?
 - iv. What do you need to prove in the inductive step?
 - v. Complete the inductive step.
 - vi. Explain why these steps show that this formula is true whenever n is a positive integer.
- b) What is wrong with this “proof”? 3

Theorem For every positive integer n , $\sum_{i=1}^n i = (n + \frac{1}{2})^2 / 2$.

Basis Step: This formula is true for $n = 1$.

Inductive Step: Suppose that $\sum_{i=1}^n i = (n + \frac{1}{2})^2 / 2$

Then $\sum_{i=1}^{n+1} i = (\sum_{i=1}^n i) + (n + 1)$.

$$\begin{aligned} \text{By inductive hypothesis, } \sum_{i=1}^{n+1} i &= (n + \frac{1}{2})^2 / 2 + n + 1 = (n^2 + n + \frac{1}{4}) / 2 + n + 1 \\ &= (n^2 + 3n + \frac{9}{4}) / 2 = (n + \frac{3}{2})^2 / 2 = [(n + 1) + \frac{1}{2}]^2 / 2 \end{aligned}$$

Completing the inductive step.

- c) i) Prove that $3^n < n!$ if n is an integer greater than 6. 3
- ii) Prove that for every positive integer n ,

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n + 1) = n(n + 1)(n + 2) / 3$$

2

4. a) How many license plates can be made using either three letters followed by three digits or four letters followed by two digits? 2
- b) i. State the generalized pigeonhole principle. 3
- ii. Explain how the generalized pigeon principles can be used to show that among 91 integers, there are at least ten that end with the same digit.
- c) Prove that $\sum_{k=0}^n \binom{n}{k} = 2^n$ where n is a nonnegative integer using i) algebraic method, and ii) combinatorial method. 5
- d) Prove that if n is a positive integer and r is an integer with $1 \leq r \leq n$, then there are 4
- $$P(n, r) = n(n - 1)(n - 2) \cdots (n - r + 1).$$

5. a) What is a relation on a set? 1
- b) Give an example of a relation on the set {1, 2, 3, 4} that is 5
- reflexive, symmetric, and not transitive.
 - not reflexive, symmetric, and transitive.
 - reflexive, antisymmetric, and not transitive
 - reflexive, symmetric and transitive
 - reflexive, antisymmetric and transitive
- c) A relation is represented by a matrix. How do you determine whether the relation is 3
- reflexive, ii) symmetric, and iii) antisymmetric?
- d) i) Define equivalence relation. Prove that the “divides” relation on the set of integers is not an equivalence relation. 3
- ii) What are the equivalence classes of 0 and 1 for congruence modulo 4? 2
6. a) Show that if a, b, c , and d are integers such that $a|c$ and $b|d$, then $ab|cd$. 3
- b) Show that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, where a, b, c, d , and m are integers with $m \geq 2$, then $a - c \equiv b - d \pmod{m}$. 4
- c) Prove that if n is a composite integer, then n has a prime divisor less than or equal to \sqrt{n} . 2
- d) Show that if a and b are positive integers, then $ab = \gcd(a, b) \cdot \text{lcm}(a, b)$. 5
7. a) What is the difference between a graph and tree? 3
- b) Define Euler and Hamiltonian path in a graph with proper examples. 5
- c) Given a graph consisting of N nodes, how many different graphs can you produce? A graph is different from another graph, is there is an edge in a particular graph that missing in another. 6

Answer any Five(5) of the following questions ($5 \times 14 = 70$)

1. (a) Define internal energy of a system and mechanical equivalent of heat. [1+1]
- (b) In a constant-volume gas thermometer, the pressure at 20.0°C is 0.980 atm. [1+1]
- What is the pressure at 45.0°C?
 - What is the temperature when the pressure is 0.5 atm?
- (c) Write down the statement of the first law of thermodynamics. Starting from the first law of thermodynamics, show that for an ideal gas undergoing a reversible adiabatic compression or expansion: $nC_v dT = nRT(dP/P) - nRdT$. [2+2]
- (d) How much work is required to compress 5.00 mol of air at 20.0°C and 1.00 atm to one tenth of the original volume
- by an isothermal process?
 - by an adiabatic process?
 - What is the final pressure in each of these two cases?

Take air to be a diatomic gas with $C_v = 5R/2$

[2+2+2]

2. (a) Define entropy of a system. Prove that, the change of entropy for an ideal gas

$$\Delta S = S_f - S_i = nR \ln\left(\frac{V_f}{V_i}\right) + nC_v \ln\left(\frac{T_f}{T_i}\right)$$

where the symbols have their usual meanings.

[2+4]

- (b) Suppose 1.0 mole of nitrogen gas is confined to the left side of a container connected by a stop-cock to an evacuated container of the same volume. We open the stopcock, and the volume of the gas doubles.
- What is the work done in the process? Explain. [2]
 - What is the change in entropy of the gas for this irreversible process? [2]
- (c) Draw and explain the Carnot cycle in PV and ST diagrams. [2+2]
3. (a) How are Miller indices determined? [2.5]
- (b) Prove that the reciprocal lattice for the reciprocal lattice is the original lattice. [3.5]
- (c) X-ray of wavelength $\lambda = 1.5\text{\AA}$ is diffracted from an FCC lattice. If the (111) diffraction peak appears at $2\theta = 37^\circ$, what is the lattice constant for this sample? [4]
- (d) Write a short note on (i) Twin boundaries (ii) Stacking Fault. [2+2]

4. (a) Define with appropriate figures crystalline, polycrystalline and amorphous materials. [1+1+1]
- (b) Find the coordination number and packing fraction of face-centered cubic lattice. [1+4]
- (c) Explain reciprocal lattice vectors in three dimensions. [3]

- (d) Consider a two-dimensional(triangular) lattice described by the two primitive vectors:

$$\vec{a} = \alpha(1, 0) = \alpha\hat{x}, \vec{b} = \alpha(1/2, \sqrt{3}/2) = (\alpha/2)\hat{x} + (\alpha\sqrt{3}/2)\hat{y}$$

Find the two primitive lattice vectors \vec{a}^* and \vec{b}^* describing the reciprocal lattice. [3]

5. (a) Set up the differential equation of motion of a forced harmonic oscillator with external sinusoidal forcing, damping force proportional to the velocity and restoring force proportional to the displacement. Solve the equation for the steady-state solution. [2+4]
- (b) For the above forced harmonic oscillator, explain amplitude resonance and find the condition for amplitude resonance. [2+2]
- (c) Explain Lissajous figures. Two vibration at right angles to one another are described by the equations:

$$x = 10\cos(5\pi t), y = 10\cos(10\pi t + \pi/3)$$

Construct the Lissajous figure of the combined motion. [2+2]

6. (a) Define interference of waves. [2]
- (b) In an Young's double-slit experiment, with the slits acting as point sources, the separation of the slits is d . Show that for coherent light coming from the slits, the intensity pattern on a screen far away from the slits is given by:

$$I = I_{max} \cos^2\left(\frac{ds\sin\theta}{\lambda}\right)$$

$$\frac{j\pi}{\lambda} + \frac{w^2\pi}{\lambda} = 0$$

where the symbols have their usual meanings. [4]

- (c) Consider a plane polarized light of intensity I_0 incident on a polarizer with the angle between the polarizer's transmission axis and the plane of polarization of the incident light being θ . Show that the intensity of the transmitted light is $I = I_0 \cos^2\theta$ [3]
- (d) A red light beam with wavelength $\lambda = 0.625\mu m$ travels through glass ($n = 1.46$) a distance of 1mm. A second beam, parallel to the first one and originally in phase with it, travels the same distance through sapphire ($n = 1.77$). [1+1+1]
- i. How many wavelengths are there of each beam inside the material?
 - ii. What is the phase difference in the beams when they come out?
 - iii. How thick should the glass be so that the beams are exactly out of phase at the exit(destructive interference)?

- (e) Semiconductor, such as silicon is used to build solar cells. They are coated with a transparent thin film, whose index of refraction is 1.45, in order to minimize reflected light. If the index of reflection of silicon is 3.5, what is the minimum width of the coating that will produce the least reflection at a wavelength of 552nm? [2]
7. (a) Sketch a diagram showing clearly how coherent sources are produced in a bi-prism. [2]
- (b) Show that the distance between two consecutive bright fringes formed in Young's experiment is given by, $\beta = \frac{\lambda D}{d}$. [6]
- (c) Why the center of Newton's ring is dark? [2]
- (d) Light of wavelength 5500 Å from a narrow slit is incident on a double slit. The overall separation of 5 fringes on a screen 200cm away is 1cm. Calculate (i) the slit separation and (ii) the fringe width. [4]

University of Dhaka
 Department of Computer Science and Engineering
 1st Year 1st Semester B.Sc. Final Examination, 2018

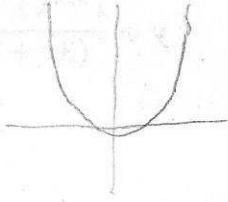
MATH 1105: Differential and Integral Calculus

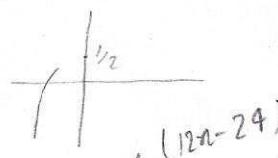
Full Marks: 70

Time: 3 Hours

(Answer any Five (5) of the following Questions)

1. a) Sketch the graph of the following functions. Also find their domain and range. 10
 - i. $f(x) = (x - 5)^2 + 2$
 - ii. $f(x) = e^{-x^2}$
 - iii. $y = \frac{x^2 - 1}{x^2 + 1}$
 - iv. $f(x) = \frac{|x-3|}{x-3} + 2$
- b) Determine whether $f(x) = \ln x$ is a one-one function. If it is one-one then find $f^{-1}(x)$. Also draw the graph $f(x)$ and $f^{-1}(x)$ in the same axes. Finally, comment on the their symmetry. 4
2. a) Consider the following function:

$$f(x) = \begin{cases} 1/(x+2), & x < -2 \\ x^2 - 5, & -2 < x \leq 3 \\ \sqrt{x+13}, & x > 3 \end{cases}$$

 - i. Evaluate the limits: $\lim_{x \rightarrow -2} f(x)$ and $\lim_{x \rightarrow 3} f(x)$ 4
 - ii. Is $f(x)$ continuous and differentiable at $x = 2$ and $x = 3$? 6
 - b) Define vertical asymptotes. What does it say about a function? 4
3. a) According to Fermat's theorem, if a function f has a local maximum or minimum value at point c , then $f'(c) = 0$. Based on this, find out whether the following functions have maximum or minimum values in the interval $(-\infty, \infty)$: 4
 - i. $f(x) = x^3$
 - ii. $f(x) = |x|$
- b) Discuss the curve $f(x) = x^4 - 4x^3$ with respect to concavity, points of inflection, and local maxima and minima. 6
- c) Find the local maximum and minimum values of the function

$$g(x) = x + 2 \sin x \quad 0 \leq x \leq 2\pi$$

 4
4. a) Find the local linear approximation of $f(x) = \sin x$ at $x_0 = 0$. 3
- b) Use the local linear approximation obtained in part (a) to approximate $\sin 2^\circ$, and compare your approximation to the result produced directly by your calculator. 3
- c) Apply L'Hospital's rule to evaluate :
 - i. $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$
 - ii. $\lim_{x \rightarrow 0} (1 + \sin x)^{1/x}$
- d) Find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$. 2
5. a) State Fundamental Theorem of Calculus (I and II). Use the theorem to evaluate

$$\frac{d}{dx} \left(\int_1^x \frac{t^5 \ln 3t}{t^2 + 1} dt \right).$$

b) Evaluate the following integrals

10

i. $\int \frac{2x+4}{x^3 - 2x^2} dx$

ii. $\int \frac{e^x}{\sqrt{1 + e^x + e^{2x}}} dx$

iii. $\int_0^1 \tan^{-1} x dx$

iv. $\int_0^2 |1 - x^2| dx$

6. a) What is the difference between the following terms: Definite integral, Indefinite integral and Anti-derivative. 3

b) Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$. 5

c) Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 2$ and $x = 0$ is revolved about the y -axis. 6

7. a) Use cylindrical shells to find the volume of the solid generated when the region R under $y = x^2$ over the interval $[0, 2]$ is revolved about the line $y = -1$. 5

b) Prove that the equation $x^3 + x - 1 = 0$ has exactly one real root. 4

c) Differentiate: 5

i. $y = \frac{x^{3/4}\sqrt{x^2 + 1}}{(3x + 2)^5}$

ii.

$$g(t) = \left(\frac{t-2}{2t+1}\right)^9$$