<u>Syllabus for Test 11</u>: L 25 (partial), L26, L27 (partial).

L25 (partial): Diffraction of Light, Definition, Diffraction of waves, Types of Diffraction, Fresnel and Fraunhofer diffractions and difference between them; Arago ort Poisson spot, single-slit diffraction, condition for minima in single-slit diffraction, intensity pattern of single-slit diffraction, positions of minima and maxima of intensity pattern, Numerical examples of single-slit diffraction pattern: ratio of intensities of diffraction peaks, angular and linear separations between minima and maxima, width of dark and bright fringes; Intensity distribution of double-slit diffraction pattern, interference factor and diffraction factor, comparison of single-slit and double-slit diffraction patterns.

L26: Review of:
Definition of diffraction of waves, types of diffraction and their differences, Arago or Poisson spot, single-slit diffraction pattern, conditions for minima and maxima in single-slit diffraction pattern, intensity distribution of single-slit diffraction pattern, Positions of maxima and minima in single-list diffraction pattern; Double-slit diffraction pattern, combination of the effects of interference and diffraction, interference factor and diffraction factor, comparison of single-slit and double-slit diffraction patterns.

Double-slit diffraction pattern: Limiting cases to (a) double-slit interference pattern, (b) single-slit diffraction pattern, Positions of maxima and minima in double-slit diffraction pattern; Missing orders, Numerical examples of double-slit diffraction pattern: Missing order, good missing order. Multiple-slit diffraction pattern, interference factor and diffraction factor; Intensity distribution of N-slit diffraction pattern, Resultant intensity as a modulation of N-slit interference pattern and a single-slit diffraction pattern; Special cases of N-slit diffraction pattern: (a) N=1 i.e. single-slit diffraction pattern, (b) N=1 i.e. double-slit diffraction pattern; Types of maxima and minima: Interference maxima and minima, diffraction maxima and minima; Positions of interference maxima and minima, Condition of combined minima; Different types of maxima and minima: (a) Principal maxima, (b) Secondary minima between two principal maxima, (c) Secondary maxima between two principal maxima; Condition for maxima: (a) Condition for principal maxima, order of principal maxima, finiteness of the order of principal maxima, (b) Conditions for central and non-central principal maxima, (c) Condition for secondary/subsidiary maxima; Number of subsidiary minima and subsidiary maxima, Positions of subsidiary maxima; Diffraction grating, grating constant, Numerical examples of diffraction grating.

Polarization of light: Definition, polarization of transverse waves; Methods of polarization: Polarization by selective absorption, transmission axis, polarizer.

L27 (partial): Linearly polarized light, Methods of polarization: Polarization by selective absorption, transmission axis, polarizer, Malus' law, crossed polarizers, Numerical examples of Malus' law; Polarization by reflection, Brewster's law, Brewster angle, Polarization by double-refraction, Optic axis, Nicol prism, O- and e-rays, Retarders, full-wave plate, half-wave plate, quarter-wave plate; Circular polarization; Right- and left-circular polarizations, polarization by scattering.

1. Consider single-slit diffraction pattern with a= slit-width, λ=wavelength of light and θ= diffraction angle. The equation: $\Delta\varphi=(2\pi a/\lambda)\sin(\theta)$ gives the phase difference between:

the rays emitted from the middle and the top edge of the slit
the extreme rays coming from the edges of the slit
the rays emitted from the middle and a point a/4 inside from the top edge of the slit
the rays emitted from the middle and a point a/4 inside from the bottom edge of the slit
the rays emitted from two points a/4 distance inside from the top and bottom edges

Ans: the extreme rays coming from the edges of the slit

Explanation: The extreme rays implies the rays that are diffracted from the edges of the slit. The phase difference between two rays originating from two points in the slit separated by a distance Δy is equal to: $\Delta\varphi = (2\pi/\lambda)\,\Delta y \sin(\theta)$, where θ is the angle of diffraction.
This gives for Δy=a, the phase difference between the extreme rays as: $\Delta\varphi\_extreme = (2\pi a/\lambda)\sin(\theta)$

2. In a single-slit diffraction pattern, two rays from two extreme edges of the slit, reach the position of the second minimum (at one side of the central maximum). The phase difference between the two rays is:

π
2π
3π
4π
5π

Ans: 4π.

Explanation: At the position of the second minimum, $\sin(\theta\_dark) = 2\,\lambda/a$.
Now the extreme rays implies the rays that are diffracted from the edges of the slit. The phase difference between two rays originating from two points in the slit separated by a distance Δy is equal to: $\Delta\varphi = (2\pi/\lambda)\,\Delta y \sin(\theta)$.
This gives for Δy=a, the phase difference between the extreme rays is: $\Delta\varphi\_extreme = (2\pi a/\lambda)\sin(\theta)$
Hence, $\Delta\varphi = (2\pi a/\lambda)\sin(\theta\_dark) = (2\pi a/\lambda)\,2\,\lambda/a = 4\pi$.

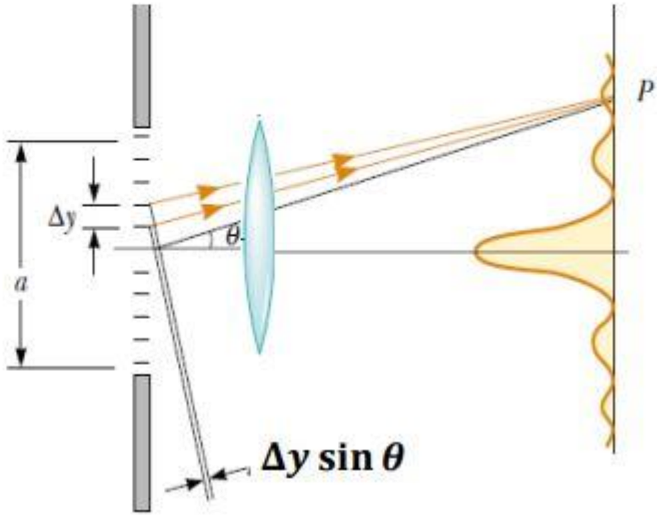
3. A point source of monochromatic light is placed in front of an opaque disk at a suitable distance and a screen is placed behind the disk. At the center of the disk there is a bright spot instead of a dark shadow. The appearance of the bright spot is a proof of:

Light is electromagnetic in nature
Light is a wave
Light is a transverse type of wave
Light can be polarized
Light gets scattered by the disk.

Ans: Light is a wave.

Explanation: The central spot is due to Fresnel type of diffraction and is called the Arago or Poisson spot. The bright and dark rings outside are due to diffraction at an edge. Arago or Poisson spot is a proof that light is a wave and it could not be explained by Newton's corpuscular theory.

4. A student wishes to obtain a good single-slit diffraction pattern by using slits of different widths that are provided. Let a= slit-width and D = perpendicular distance between the slit and the screen. A converging lens is placed between the slit and the screen very near the slit. The slit width a could be:



$\lambda/\sqrt{2}$
10λ
λ^2/D
$\lambda/2$
D^2/λ

Ans: 10λ.

Explanation: For diffraction pattern to be observed, the slit-width must be greater than the wavelength. If, however the slit-width is too large, the wavetrains reaching different points on the slit would have originated from different photons i.e we can not have a coherent source. In that case we would see geometrically bright region at the center with very small Fresnel type of diffraction at the edge of the bright region.

Diffraction pattern is best viewed for slit-width ~ wavelength and greater than λ.

5. Consider a single-slit diffraction pattern produced by a long narrow slit of width a, illuminated by monochromatic light. What changes are observed in the diffraction pattern if the whole apparatus is immersed in pure water?

The intensity at the minimum increases giving only partially dark fringes
Width of central maximum increases
Width of central maximum decreases
Peak intensity of the central maxima decreases
Frequency of light decreases

Ans: Width of central maximum decreases.

Explanation: As the whole apparatus is now immersed in water, the wavelength of the light will change. Therefore, as the refractive index of water is greater than the air, the wavelength of light will decrease. The new wavelength in the liquid is: $\lambda' = \lambda/n = \lambda/1.33$

Width of central maxima = $2\lambda/a$, where a is the slit-width.

Therefore, as the wavelength decreases, the width of the central maxima decreases.

As the water is pure, there will be no absorption of electromagnetic waves and hence intensity will not decrease.

6. Consider a single-slit diffraction pattern produced by a long narrow slit illuminated by monochromatic light. Let the intensity of the central maximum be I_0. If ONLY the slit-width a is decreased with NO other parameters changed, then:

I_0 decreases and the pattern expands away from the center
I_0 increases and the pattern contracts toward the center
I_0 does not change but the pattern expands away from the center
I_0 does not change but the pattern contracts toward the center
I_0 decreases and the pattern contracts toward the center

Ans: I_0 decreases and the pattern expands away from the center.

Explanation: Intensity pattern of single-slit diffraction pattern is:

$$I = I_0 \sin^2(\beta)/(\beta^2) = I_0 (\sin(\beta)/\beta)^2$$

where $\beta = (\pi a/\lambda) \sin(\theta)$ and $I_0 = E_0^2$, where E_0 is the amplitude of the electric field at the point on the screen.
However, (1) $E_0 = E_{0\text{-source}}/R$, where, $E_{0\text{-source}}$ is the electric flied at a point on the slit, R= distance from the source point to the point on the screen. (**This is called the FAR FIELD.**)
(2) E_0 at screen, depends on the number of small zones of width Δy and as the slit width is decreased, the number of such zones decreases.

Intensity of the central maximum is I_0. This is because, at the central maximum, $\theta=0$ giving $\beta = (\pi a/\lambda)\sin(\theta) = 0$ and $\lim_{\theta \to 0} (\sin(\beta)/\beta)^2 = \lim_{\beta \to 0} (\sin(\beta)/\beta)^2 = 1$. If NO other parameters than the slit-width a are changed, i.e. everything else kept constant, then I_0 will decrease as a decreases, since the amount of light coming to the slit will decrease.
The positions of the first minimum on either side are given by:

$$\sin(\theta_{dark}) = \pm \lambda/a$$

which gives the angular separation between the first minima on either sides as:

$$\Delta \theta_{dark} \approx \Delta [\sin(\theta_{dark})] = (\lambda/a) - (-\lambda/a) = 2\lambda/a$$

Hence, as a is DECREASED, the pattern expands away from the center.

7. A diffraction pattern is formed by shining a laser light (monochromatic) through a narrow slit. The pattern is projected onto a wall that is 5 m away from the slit. The width of the central maximum is 2 cm. If the slit-width is halved, the width of the central maximum will become:

1 cm
4 cm
10 cm
5 cm
0.5 cm

Ans: 4 cm. **Explanation**: The condition for the minima in single-slit diffraction pattern is:

$$\sin(\theta_{dark}) = m\lambda/a$$

For central maxima, m= ±1. The angular width of the central maxima is thus:
$\Delta \theta = (\theta_{dark\_m=1}) - (\theta_{dark\_m=-1}) \approx \sin(\theta_{dark\_m=1}) - \sin(\theta_{dark\_m=-1}) = (1-(-1))\, \lambda/a = 2\lambda/a$
Again, linear separation of the first order minima (on both sides) on the screen is (with L = perpendicular distance from the slit to the screen): $\Delta y = \Delta\theta\, L = 2\lambda L/a$
Here, $a \to a/2$ gives $\Delta y \to 2\,\Delta y$ i.e the width of the central maximum doubles.

8. Consider the double-slit diffraction pattern where the width of the slit is a and the separation between the slits is d and the wavelength of the light is λ. The linear separation of two adjacent interference dark fringes on a screen at a perpendicular distance D from the slits is:

λa/D
λd/D
λD/d
Dd/λ
λD/a

Ans: λD/d.

Explanation: The intensity distribution of a double-slit diffraction pattern is given by:

$$I = 4 I_0 [\sin^2 (\beta) / \beta^2] \cos^2 (\gamma)$$

where, $\beta = \pi a \sin(\theta)/\lambda$ and $\gamma = \pi d \sin(\theta)/\lambda$, where a = width of the slit and d = separation between the slits. The first factor i.e. $[\sin^2 (\beta) / \beta^2]$ is called the diffraction factor while the second factor i.e. $\cos^2 (\gamma)$ is called the interference factor.

The condition of interference minima:
$\cos(\gamma) = 0 \Rightarrow \gamma = \pi d \sin(\theta)/\lambda = (n+1/2)\pi$, where n=0,1,2,3,... = ORDER of the interference peak

This gives: $d \sin(\theta_{min-int}) = (n+1/2) \lambda \Rightarrow \theta_{min-int} \approx (n+1/2) (\lambda/d)$

The angular width of the interference fringe is thus: $\Delta \theta_{min-int} \approx [\Delta(n+1/2)] (\lambda/d) = \lambda/d$

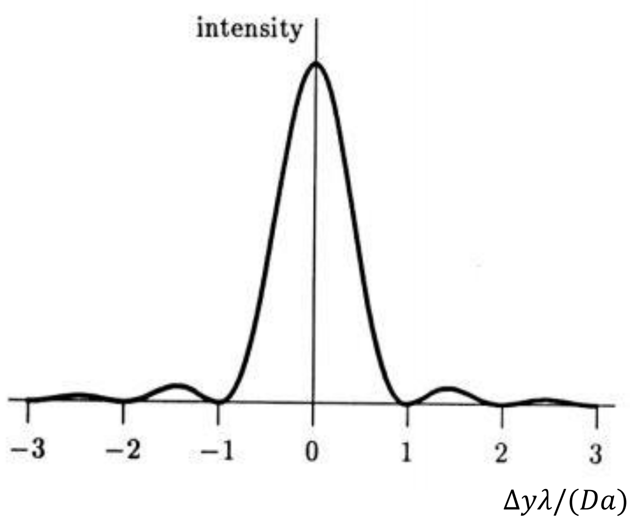
The linear separation is found from the position of the interference minimum on the screen:

$$y = D \tan(\theta_{min-int}) \approx D \sin(\theta_{min-int}) \approx D \theta_{min-int}$$

Hence the linear width of the interference fringe on the screen a perpendicular distance D away is:

$$\Delta y \approx D \Delta \theta_{min-int} \approx D \lambda/d = \lambda D/d \quad \text{(Proved)}$$

9. Consider light of wavelength λ normally incident on some plane optical device. An intensity pattern is observed on a screen D distance away from a convex lens placed near the device. Let θ be the angle measured from the normal to the device at the center of the device and Δy be the linear separation of dark fringes on the screen. The intensity pattern is given in the figure. The device could be:



a single-slit of width a
a single-slit of width 2a
two narrow slits with separation a
two narrow slits of separation 2a
a diffraction grating of slit separation a

Ans: a single-slit of width a.

10. As more slits with the same spacing and width, are added to a diffraction grating, the lines corresponding to the principal maxima:

spread farther apart
move closer together
become wider
become narrower
do not change in position or width

Ans: become narrower. **Explanation**: For diffraction by a grating, the condition for principal maxima is given by: $d \sin(\theta) = m \lambda$

The angular separation of the lines corresponding to the PRINCIPAL MAXIMA in a diffraction pattern of the multiple-slit diffraction is given by: $\Delta \theta\_max \approx \Delta [\sin(\theta\_max)] = (\Delta m) \lambda/d = \lambda/d$ which is independent of the number of slits. Hence the lines DO NOT spread farther of move close together.

The intensity pattern of multiple-slit diffraction is given by:

$$I = I\_0 \ [\sin\text{^}2 (\beta)/\beta\text{^}2] \ \{ \sin\text{^}2 (N\gamma) / \sin\text{^}2 (\gamma) \}$$

where the first factor $[\sin\text{^}2 (\beta)/\beta\text{^}2]$ is called the diffraction factor while the second factor given by $\{ \sin\text{^}2 (N\gamma) / \sin\text{^}2 (\gamma) \}$ is called the interference factor. Here, $\beta = \pi a \sin(\theta)/\lambda$ and $\gamma = \pi d \sin(\theta)/\lambda$.

The condition for INTERFERENCE minima is: $[\sin (N\gamma) / \sin (\gamma)] = 0$ which gives $\sin(N\gamma)=0$ but with $\sin (\gamma) \neq 0$. This gives: $N\gamma = p\pi$ with $p \neq 0, N, 2N, 3N, \ldots$ giving:

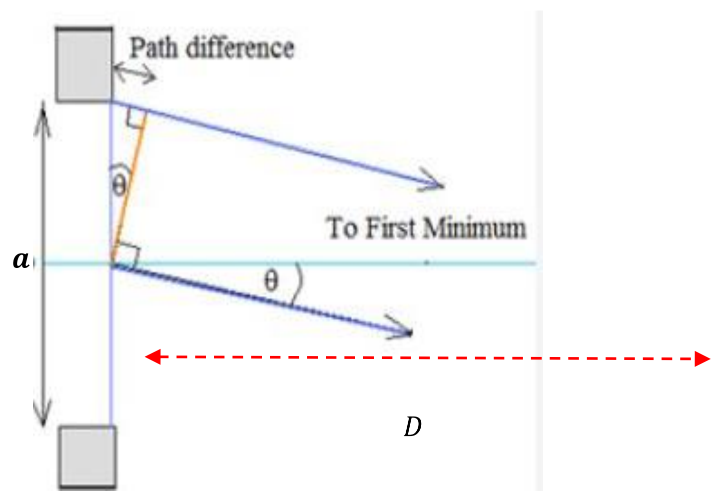
$$N \pi d \sin(\theta\_min\text{-}int)/\lambda = p\pi \Rightarrow \sin(\theta\_min\text{-}int) = p (\lambda/d) (1/N)$$

Width of the interference fringes is thus: $\Delta (\theta\_min\text{-}int) \approx \Delta [\sin(\theta\_min\text{-}int)] = (\Delta p) \lambda/(Nd) = \lambda/(Nd)$

But, d = slit-separation and N = total number of slits. Hence, Nd = total width of the grating.

Hence as more slits are added, N increases. This makes the width of the interference fringes (the width of the principal maxima lines) DECREASE. Hence the lines BECOME SHARPER.

11. The diagram shows rays from two Huygen's wavelets that meet at the first minimum. The perpendicular distance from the slit to the screen is D. The path difference between the two rays shown is:



$\lambda/2$
λ
2λ
$\lambda a/D$
$2\lambda a/D$

Ans: $\lambda/2$.

Explanation: At the first minimum, the wavelets will cancel each other. Hence the path difference is $\lambda/2$.

12. An unpolarized beam of light with intensity I_0 is incident on an ideal polarizing sheet and passes through onto another ideal polarizing sheet. The axes of polarization of the two plane polarizer makes an angle of φ with each other. The intensity of the emerging light from the last polarizer is $I_0/4$. The angle φ is:

$\sin^{-1}(1/2)$
$\sin^{-1}(\sqrt{3}/2)$
$\cos^{-1}(1/2)$
$\sin^{-1}(1/\sqrt{2})$
$\cos^{-1}(\sqrt{3}/2)$

Ans: $\sin^{-1}(1/\sqrt{2})$. **Explanation**: Unpolarized light may be thought of having electric field in two perpendicular directions. One of the directions may be chosen parallel to the transmission axis of the first polarizer.
Hence, after the first polarizer, only half of the intensity will pass through. The light incident on the first polarizer is unpolarized, so the angle is irrelevant – the intensity is reduced by a factor of 2. Thus $I_1 = I_0/2$

{ Alternative argument :
This can be found alternatively by averaging the intensities due to all polarizations in the incident light over all angles. Since in unpolarized light, the randomly changing polarizations are uncorrelated, taking average over time is equivalent to averaging over all angles:
$I_1 = I_0 \langle\cos^2(\theta)\rangle = I_0 \left[\int_0^{2\pi} (\cos^2(\theta))\, d\theta \right] / \left[\int_0^{2\pi} (\cos^2(\theta))\, d\theta\right] = I_0 (1/2) \left[\int_0^{2\pi} (1+\cos(2\theta))\, d\theta \right]/(2\pi) = I_0 (1/4\pi) \left[2\pi + \{\sin(2\theta)\}_{(\theta=2\pi)} - \{\sin(2\theta)\}_{(\theta=0)} \right] = (1/2)$
}

After passing through the second polarizer, we get, from Malus law: $I=I_2 = I_1 \cos^2(\varphi)$, where φ is the angle between the transmission axes of the two polarizers.
Hence, $I_2 = (I_0/2) \cos^2(\varphi) = I_0/4$ which gives, $\cos^2(\varphi)=(1/2) \Rightarrow \cos(\varphi)=(1/\sqrt{2})=\cos(45°) = \sin(45°)$ Hence $\varphi = \sin^{-1}(1/\sqrt{2})$.

13. Unpolarized light is incident on a block of glass with an index of refraction 1.52. The **angle of incidence** for which the refracted light is linearly polarized is:

$\sin^{-1}(1/1.52)$
$\tan^{-1}(1.52)$
$\tan^{-1}(1/1.52)$
any angle between 0° and 90°
no angle between 0° and 90°

Ans: no angle between 0° and 90°.

Explanation: For refraction of **unpolarized** light from air to glass, the transmitted or refracted wave always has both polarized components of the electric field, parallel and perpendicular to the plane of incidence.

14. Three ideal plane polarizing sheets are placed in a sequence where the polarization axes of the first and the third sheets are perpendicular to each other. Initially unpolarized light falls on the first sheet. At what angle of the transmission axis of the second intermediate polarizing sheet, making with the transmission axis of the FIRST polarizing sheet, the final transmitted intensity would be the maximum?

0°
30°
45°
60°
Not possible; no light would pass through the total system.

Ans: 45° . **Explanation**: The initial light is unpolarized. After the first polarizer the transmitted intensity is: I_1 = I_0/2

Let the angle between the transmission axes of the FIRST and the SECOND polarizers be θ. Then the transmitted intensity after the second polarizer is: I_2 = I_1 cos^2(θ) = (I_0/2) cos^2(θ)

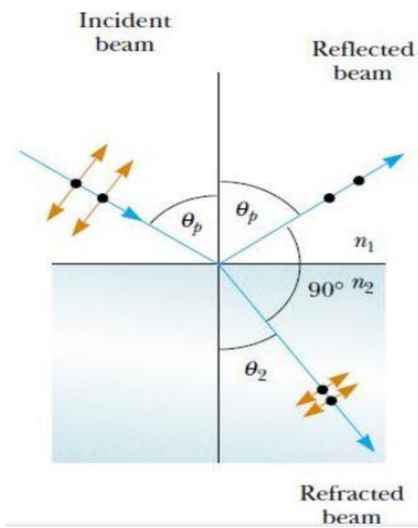
Light of this intensity is incident on the THIRD polarizer. Now, the angle between the transmission axes of the second and the third polarizers is: 90° -θ.

Hence the final emergent intensity is: I_3 = I_2 cos^2(90° -θ) = [(I_0/2) cos^2(θ)] sin^2 (θ)
=> I_3 = (I_0/2) cos^2(θ) sin^2(θ) = (I_0/2)(1/4) sin^2 (2θ) = (I_0/8) sin^2 (2θ)

To get the maximum emergent polarization we need to maximize sin^2 (2θ) , by varying θ. This is done simply by differentiation:
d/dθ [sin^2 (2θ)]_{θ=θ_max} =0 => 2 sin(2θ_max) cos(2θ_max) =0 => sin(4θ_max) =0 => 4θ_max=π => θ_max=π/4

Thus the angle that the transmission axis of the second polarizer makes with that of the FIRST (and of course of the THIRD) polarizer is 45° or π/4.

15. Linearly polarized light is incident on a block of glass from air with the angle of incidence equal to the Brewster angle. The electric field in the incident light is perpendicular to the plane of incidence. As a result:



there is no reflected wave
there is no refracted wave
both reflected and transmitted waves are linearly polarized
both reflected and transmitted waves are partially polarized
both reflected and transmitted waves are unpolarized

Ans: both reflected and transmitted waves are linearly polarized.

Explanation: Both the reflected and transmitted waves are linearly polarized with the electric field confined in a plane perpendicular to the plane of incidence.

16. The phase difference introduced between the o- and e-waves upon emerging from a retarder (such as a quarter-wave plate or half-wave plate) depends on:

thickness of the plate
difference of refractive indices for the o- and e-waves
frequency of light
the difference of the speeds of light for the o- and e-waves
all of the others mentioned

Ans: all of the others mentioned.

Explanation: The phase difference introduced by the retarder is given by:

$$\Delta\varphi = (2\pi/\lambda_0)\, d\, (|n_O - n_E|) = (2\pi f_0/c)\, d\, (|n_O - n_E|)$$

Hence $\Delta\varphi$ depends on : (a) thickness of the plate d, (b) difference of refractive indices for the o- and e-waves i.e. $|n_O - n_E|$, (c) frequency of light f_0 and (d) the difference of the speeds of light for the o- and e-waves i.e. $c|(1/c_O) - (1/c_E)| = |n_O - n_E|$.

17. What should be the phase difference between the two plane-polarized light waves, vibrating at right angles to each other, to produce circularly polarized light?

$\pi/6$
$\pi/2$
$\pi/4$
$\pi/3$
π

Ans: $\pi/2$.

Explanation: The circularly polarized light is produced when the phase difference between the two rays is $\pi/2$. In that case, the equation of the polarized light turns out to be $x^2 + y^2 = a^2$, which is the equation of a circle.

18. Unpolarized light ray is incident on a birefringent crystal (such as calcite or quartz) surface such that the waves travel along the optic axis of the crystal. For the ordinary- and the extra-ordinary rays (i.e. the o- and the e-rays):

the speed of the ordinary ray i.e. o-ray is greater than that of the extra-ordinary i.e. e-ray
the speed of the extra-ordinary ray i.e. e-ray is greater than that of the ordinary i.e. o-ray
the speed of both the ordinary and extra-ordinary rays is the same
the polarizations of both the rays is the same
both rays are unpolarized

Ans: the speed of both the ordinary and extra-ordinary rays is the same.

Explanation: In certain crystalline structures (such as calcite and quartz), the speed of light is not the same in all directions. However, there is one direction, called the optic axis, along which the ordinary and extraordinary rays have the same speed. Hence along the optic axis $n_O = n_E$.

19. A linear polarizing sheet is given to a student and he/she is asked to distinguish between a circularly polarized light and an unpolarized light of the **same intensity**. The student fails to distinguish because:

the intensity of the two incident lights is the same
the intensities of the emergent light is the same for both lights
linear polarizer only allows linearly polarized incident light to pass
student fails to rotate the linear polarizer at the frequency of the light to get any transmission
linear polarizer only passes the electric field and does not pass the magnetic field through it

Ans: the intensities of the emergent light is the same for both lights.

Explanation: For unpolarized and circularly polarized light of the same intensity of I_0, the intensity of the emergent light is I_0/2. Hence the student cannot distinguish between circularly polarized light from the unpolarized light.
Even if the incident intensities are different, as the student does not know which light is incident at a time, and which has greater intensity, he/she will just see a difference of intensities but will not be able to distinguish between the polarizations of the incident lights.

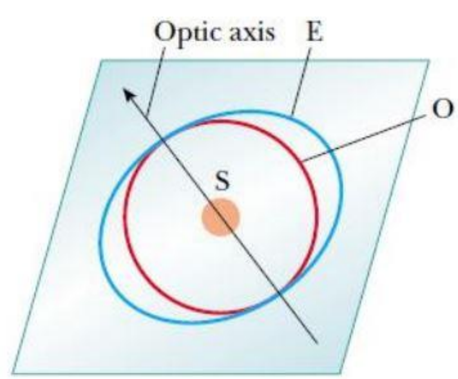
Furthermore, the student cannot rotate the polarizer at the very high frequency of visible light to follow the electric field of the circularly polarized light. For that he has to first identify the direction of the electric field and then instantly start rotation at the required angular frequency. This is impossible as instant angular acceleration requires infinite torque.

20. A circularly polarized light is incident on a quarter-wave plate. The transmitted light is:

unpolarized
circularly polarized in the same sense as the incident light
circularly polarized in the opposite sense as the incident light
linearly polarized
elliptically polarized

Ans: linearly polarized. **Explanation**: A linearly polarized light passing through a quarter-wave plate becomes a circularly polarized light. Now consider the time-reversed situation. Then we will see a linearly polarized light falling from the final side on the quarter-wave plate will emerge from the initial side as a circularly polarized light. Since physics at the level is time-reversal independent, this can happen. Hence the answer.

21. The shape of the o-wavefront is:



Planar
spherical
ellipsoidal
cylindrical
parabolic.

Ans: spherical.

22. Consider a double-slit diffraction with a = width of the slits and d = separation between the slits and λ= wavelength of light. Let $\Delta\ \theta\_c$ = angular width of the central diffraction peak and let M = number of bright interference fringes within the central diffraction peak. If ONLY the separation between the slits, d is increased with NO other parameter changed, what should be observed?

$\Delta\ \theta\ \_c$ and M increases
$\Delta\ \theta\ \_c$ and M decreases
$\Delta\ \theta\ \_c$ remains constant and M increases
$\Delta\ \theta\ \_c$ remains constant and M decreases
$\Delta\ \theta\ \_c$ decreases and M remains constant

Ans: $\Delta\ \theta\_c$ remains constant and M increases.

Explanation: For double-slit diffraction pattern, the intensity distribution is given by:

$$I = 4\ I\_0\ [\ \sin^{\wedge}2\ (\beta)\ /\ \beta^{\wedge}2\ ]\ \cos^{\wedge}2\ (\gamma)$$

where, $\beta = \pi a \sin(\theta)/\lambda$ and $\gamma = \pi d \sin(\theta)/\lambda$, where a = width of the slits and d = separation between the slits.

The first factor i.e. [sin^2 (β) / β^2] is called the diffraction factor while the second factor i.e. cos^2 (γ) is called the interference factor. The diffraction factor represents the diffraction pattern produced by a single slit of width a. The interference factor represents the interference pattern produced by two point sources separated by a distance d.

The width of the central peak is found by considering the first minimum of the diffraction envelope: $\sin(\theta\_dark) = m\ \lambda/a \Rightarrow \theta\_1 \approx \sin(\theta\_1) = \lambda/a$ (for m=1)

$$\Delta\ \theta\ \_c\ = \Delta\ \theta\_1 \approx (\lambda/a) - (-\lambda/a) = 2\ \lambda/a$$

The number of interference peaks WITHIN THE CENTRAL DIFFRACTION PEAK is found from the condition of interference minima:

$$\cos(\gamma) = 0 \Rightarrow \gamma = \pi d \sin(\theta)/\lambda = (n+1/2)\pi,$$ where n=0,1,2,3,... = ORDER of the interference peak

This gives: $d \sin(\theta\_min\text{-}int) = (n+1/2)\ \lambda \Rightarrow \theta\_min\text{-}int \approx (n+1/2)\ (\lambda/d)$

Hence the number of interference peaks within the first diffraction minima (within one side of the central diffraction maxima) is found from:

$$\theta\_min\text{-}int \le \theta\_1 \text{ which gives } (n+1/2)\ (\lambda/d) \le \lambda/a \quad \Rightarrow (n+1/2) \le d/a$$

Thus the number of interference peaks within the first diffraction minima is: M = [d/a], where [x] represents the greatest integer less than or equal to x.

Hence, as the separation between the slits d is increased, with all other parameters kept constant:

A. The width of the central diffraction peaks remains constant, $\Delta\ \theta\ \_c$ being independent of d

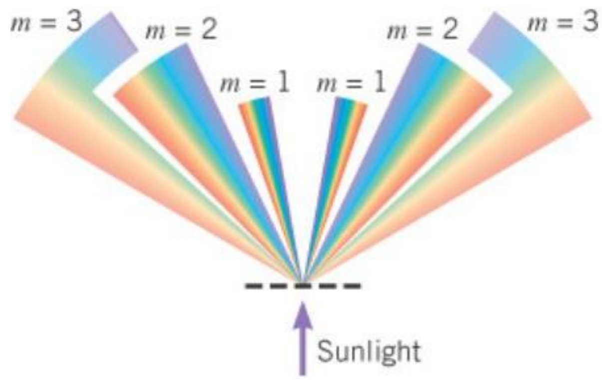
B. The number of interference maxima within the central diffraction peak $M \propto d$ increases with increasing d

23. The velocity of light in water is 2.2 × 10^8 m/s. The polarizing angle of incidence in degrees is:

36.25°
42.83°
47.17°
53.74°
65.36°

Ans: 53.74°. **Explanation**: Refractive index of water = Speed of light in space/Speed of light in water = 3/2.2 = 1.3636. Hence using Brewster's law, angle of incidence = tan^(-1)[μ] = tan^(-1)[1.3636] = 53.74°

24. How shall an N-slit diffraction pattern change when white light is used instead of a monochromatic light?



The pattern will no longer be visible
A colored pattern will be observed with a white central peak and red light near the central peak
A colored pattern will be observed with a white central peak and violet light near the central peak
A colored pattern will be observed with red and violet colored lights at two extremes
No colored pattern will be observed and only bright and dark fringes will change positions

Ans: A colored pattern will be observed with a white central peak and violet light near the central peak. **Explanation**: When white light is used instead of monochromatic light, then the central maximum remains white as all the wavelengths meet there in the same phase. The first minimum and second maximum will be formed by violet color due to its shortest wavelength while the last is due to the red color as it has the longest wavelength. Thus, a colored pattern is observed.
However, after the first few colored bands, the clarity of the band is lost, due to overlapping.

25. What is the half-angular width of the central bright maximum in the Fraunhofer diffraction pattern of a single slit of width 1400 nm when the slit is illuminated by monochromatic light of wavelength 700 nm?

20°
30°
40°
45°
60°

Ans: 30° **Explanation**: We know, for first minimum, sin θ =λ/a. This gives the half-angular width of the central maximum at θ. Hence, we get given a=1400 nm, λ=700 nm, we get sin θ = 700/1400 = 0.5 => θ=30°.

26. Consider a double-slit diffraction with a = width of the slit and d = separation between the slits and λ= wavelength of light. Let $\Delta \theta\_c$ = angular width of the central peak and let M = number of bright fringes within the central peak. If ONLY the slit-width a is increased with NO other parameters changed, what should be observed?

$\Delta \theta\_c$ and M increases
$\Delta \theta\_c$ and M decreases
$\Delta \theta\_c$ remains constant and M increases
$\Delta \theta\_c$ decreases and M remains constant
$\Delta \theta\_c$ remains constant and M decreases

Ans: $\Delta \theta\_c$ and M decreases.
Explanation: For double-slit diffraction pattern, the intensity distribution is given by:

$$I = 4 I\_0 [\sin^2 (\beta) / \beta^2] \cos^2 (\gamma)$$

where, $\beta = \pi a \sin(\theta)/\lambda$ and $\gamma = \pi d \sin(\theta)/\lambda$, where a = width of the slits and d = separation between the slits.

The first factor i.e. $[\sin^2 (\beta) / \beta^2]$ is called the diffraction factor while the second factor i.e. $\cos^2 (\gamma)$ is called the interference factor. The diffraction factor represents the diffraction pattern produced by a single slit of width a. The interference factor represents the interference pattern produced by two point sources separated by a distance d.

The width of the central peak is found by considering the first minimum of the diffraction envelope: $\sin(\theta\_dark) = m \lambda/a \Rightarrow \theta\_1 \approx \sin(\theta\_1) = \lambda/a$ (for m=1)

$$\Delta \theta\_c = \Delta \theta\_1 \approx (\lambda/a) - (-\lambda/a) = 2 \lambda/a$$

The number of interference peaks WITHIN THE CENTRAL DIFFRACTION PEAK is found from the condition of interference minima:

$$\cos(\gamma) = 0 \Rightarrow \gamma = \pi d \sin(\theta)/\lambda = (n+1/2)\pi, \text{ where } n=0,1,2,3,... = \text{ORDER of the interference peak}$$

This gives: $d \sin(\theta\_min\text{-}int) = (n+1/2) \lambda \Rightarrow \theta\_min\text{-}int \approx (n+1/2) (\lambda/d)$

Hence the number of interference peaks within the first diffraction minima (within one side of the central diffraction maxima) is found from:

$$\theta\_min\text{-}int \le \theta\_1 \text{ which gives } (n+1/2) (\lambda/d) \le \lambda/a \Rightarrow (n+1/2) \le d/a$$

Thus the number of interference peaks within the first diffraction minima is: M = [d/a], where [x] represents the greatest integer less than or equal to x.

Hence, as the slit-width a is increased, with all other parameters kept constant:

A. The width of the central diffraction peaks $\Delta \theta\_c \propto 1/a$, decreases
B. The number of interference maxima within the central diffraction peak $M \propto 1/a$, decreases

27. Consider a quarter-wave plate retarder on which light of wavelength 500 nm is incident in free space. The refractive indices of the e- and o-waves are, μe = 1.553 and μo = 1.544, respectively. What should be the minimum approximate thickness of the quarter-wave plate retarder?

13888 nm
14384 nm
14880 nm
15333 nm
16666 nm

Ans: 13888 nm. Explanation: The phase difference introduced by the retarder is given by:

$$\Delta\varphi = (2\pi/\lambda\_0)\ d\ (|n\_O - n\_E|) = (2\pi f\_0/c)\ d\ (|n\_O - n\_E|)$$

For quarter wave plate, $\Delta\varphi = \pi/2$ giving :

$$\Delta\varphi = \pi/2 = (2\pi/\lambda\_0)\ d\ (|n\_O - n\_E|)$$

$$\Rightarrow \quad d = (\lambda\_0/4)\ (1/|n\_O - n\_E|) = (500\ \text{nm}/4)\ 1/(0.009) = (500/0.036)\ \text{nm}$$

28. The magnitude of the Ex and Ey components of the electric field may be the same in which type of polarization?

linear and elliptic polarizations
linear and circular polarizations
circular and elliptic polarizations
circular polarization only
elliptic polarization only

Ans: linear and circular polarizations.

Explanation: In circular polarization, the x- and y-component of the electric fields are obviously the same in magnitude but out of phase by π/2. Linear polarization may have same magnitude of the x- and y-component of the electric fields but in that case the phase difference between the components would be either 0° or 180°.