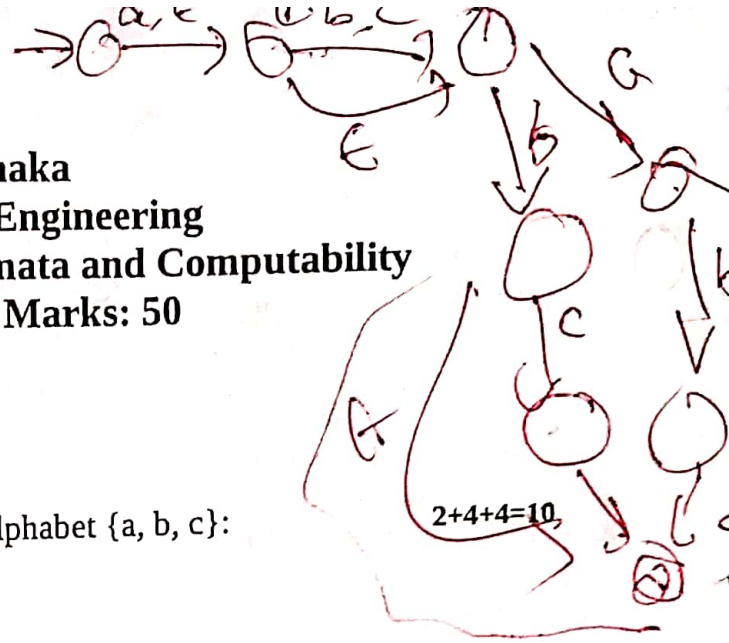
MidTerm
University of Dhaka
Computer Science and Engineering
CSE3204: Formal Language, Automata and Computability
Time: 1 hour 30 mins, Marks: 50



Answer all the following questions:

1. Consider the following regular expression R over alphabet {a, b, c}: 2+4+4=10

$$R = (a \mid \varepsilon)\ (b \mid c)^*\ (bc \mid ab)^*$$

Answer the following questions:

i. List all the strings of length ≤ 2 generated by expression R.

ii. Design the non-deterministic automaton A of expression R.

iii. Design the deterministic automaton of expression R, obtained from the previous one by means of the subset construction.

2. Given L = {$a^n b^n$: n≥§ 0}, describe in English the complement of L. 2

3. The answer to these problems should be choosing True or False; no other explanation is necessary. 2

i. All finite languages are regular.

ii. If L is regular and, $L' \subseteq L$ then L' is regular.

4. Suppose LΣ* = L, for an alphabet Σ . What can we say about possible strings in L? 2

5. Let L_1 and L_2 be languages over the same alphabet. Identify all possible scenarios, based on the strings in L_1 and L_2, in which we have $L_1L_2 = L_2$.

6. *Write down the regular expression for the following languages:* 2.5x4=10

i. The language of strings of 0's and 1's without two consecutive 1's.

ii. The language of strings of length at least three who have a 1 in the third-to-last position.

iii. The language of strings of length at least two who have a 1 in position k, for every k≥1.

iv. The language of strings of that contain at least two 1's or two 0's.

7. *Consider the three regular expressions* $r1=aa; r2=aa(a+b); r3=(b+ab)aa(b+ba)$. Give a FA for $L((r1)(r2+\overline{r3}))$. 5

8. Design DFA for the following languages: 2.5x3=7.5

i. The language of strings that the sum of their digits is a multiple of 4, where the alphabet $\Sigma = \{0,1,2,..,9\}$

ii. The language of strings of the form 0^i1^j where i is even and $j \geq 2$.

iii. The language of strings of length at least m that have a 0 in the position m from the end. No more than $m+1$ states and $2m+1$ transitions.

9. Convert the nondeterministic automaton given below to an equivalent deterministic one using the subset construction. Omit inaccessible states. Draw the graph of the resulting DFA. 5

	a	b
→ q_0	$\{q_1\}$	$\{q_2\}$
q_1	$\{q_0, q_1\}$	$\{q_0\}$
q_2 F	$\emptyset$	$\{q_1, q_2\}$

10. Answer the following question :

i. Describe in formal statement what the $^+$ operation over the following alphabets produces: $\Sigma = \{0,1\}$ **1**

ii. A string is infinite when its length is infinite. Let Σ be an arbitrary alphabet. **2**

a) Does Σ^* contain any infinite string?

b) If Σ would be an infinite alphabet (which it actually may not be), would the answer still be the same?

iii. Let Σ be an alphabet, and let ε be the empty string over Σ.

a) Is ε in Σ ? **1**

b) Does it hold that $\varepsilon\varepsilon\varepsilon = \varepsilon$? does it hold for ε^i where $i \geq 2$? **1**

c) Let x and y be two strings over Σ. Is the concatenation of x and y always the same as the concatenation of y and x ? **1.5**

University of Dhaka

Department of Computer Science and Engineering

3rd Year 2nd Semester B. Sc. Final Examination, 2018

CSE-3204: Formal Language, Automata and Computability

Time: 3 Hours

Total Marks: 60

(Answer any four (4) of the following questions)

1. a) 1) For each of the following language over $\sum = \{a, b\}$, give a DFA [4]
   i. $L_1 = \{xbyaz : x, y, z \in \sum^*\}$
   ii. ii) $L_2 = \{w : w \in \sum^*$, where w does not have aa as a substring. [4]
   2) Give a DFA for $L_1 \cup \overline{L_2}$. [3]

   b) Design a NFA for the language of strings of length at least two whose last two symbols are the same. Use no more than four states and six transitions. [4]

   c) Convert the NFA of question 1(b) to an equivalent DFA. [5]

2. a) Find a regular expression that denotes all bit strings whose value when interpreted as a binary integer is at least 40. [6]

   b) Language L over alphabet $\{a, b, c\}$ is defined by the following 3 conditions (every string of L must satisfy all of them) :
   i. strings start with letter a
   ii. strings end with letter c
   iii. strings may not contain any of the following four two-letter factors:
   $ba\ bb\ ca\ cc$
   Answer the following:
   1. Construct the automaton of L.
   2. Write the RE of L by using union, concatenation and star (or cross) operators.

   c) The following table consists of two rows and each row contains two regular expressions; the former is in column L_1, the latter in column L_2. Answer the following: [4]

   "In each row, list by increasing length, the three shortest strings belonging to the language defined by set difference $L_1 \setminus L_2$ (or $L_1 - L_2$)."

L_1	L_2	Write three shortest strings of $L_1 \setminus L_2$
$a((b\|bb)a)^+$	$(ab)^*ba$	
$a^*b^*a^*$	$(ab\|bb\|a)^*$	

3. a) Design a Turing Machine (TM) to accept the language $L = \{a^n | n > 0$ and n is divisible by 2$\}$. Identify the components of the Turing Machine. [5]

   b) Design a Turing Machine (TM) to accept the language $L = \{a^p b^q c^r | 0 < p < r < q\}$. [6]

   c) Consider two regular languages *L1* and *L2*. Prove that the problem of finding the existence of the string 'abacus' in both *L1* and *L2* is a decidable problem. [2]

   d) Consider two regular languages *L1* and *L2*. Here, *L2* is undecidable. Find whether *L1* is decidable if *L1* is reducible to the complement of *L2*. [2]

4. a) Construct the following grammar into Chomsky Normal Form: [4]

$$S \rightarrow ABb \mid a$$
$$A \rightarrow aaA \mid B$$
$$B \rightarrow bAb \mid c$$

   b) Using pumping lemma prove: [7]
   i. $L = \{w_1 w_2 : w_1, w_2 \in (a+b)^*, w_1 \neq w_2\}$ is not Context Free.
   ii. $L = \{a^n b^m c^k : n > m > k > 0\}$ is not Regular.

   c) Use CYK algorithm to determine $w \in L(G)$, where $w = abbc$. Following are the production of grammar G. [4]

$$S \rightarrow AB$$
$$A \rightarrow CC \mid a \mid c$$
$$B \rightarrow BC \mid b$$
$$C \rightarrow CB \mid BA \mid c$$

5. a) Design CFG for the following language:

i. $L = \{a^n b^m a^m b^n : m, n \geq 0\}$ over alphabet $\{a, b\}$

ii. $L = \{l^i \# l^j \# l^{i+j} : i, j \geq 0\}$ over alphabet $\{l, \#\}$

b) Given the language $L = \{a^n b^{n+1} : n \geq 0\}$, find a CFG for $\bar{L}$, the complement of L. You must explain your answer. [4]

c) Consider the following CFG G: [5]

$$S \rightarrow AB \mid aaB$$
$$A \rightarrow a \mid Aa$$
$$B \rightarrow b$$

Show that G is ambiguous for any particular string (you have the freedom to choose a string) and also remove the ambiguity from G.

6. a) Define the language accepted by a PDA. [2]

b) What language is accepted by PDA $P = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b, Z\}, \delta, \{q_0\}, Z, \{q_2\})$ with the following transitions? [2]

$$\delta(q_0, a, Z) = \{(q_1, a), (q_2, \lambda)\}$$
$$\delta(q_1, b, a) = \{(q_1, b)\}$$
$$\delta(q_1, b, b) = \{(q_1, b)\}$$
$$\delta(q_1, a, b) = \{(q_2, \lambda)\}$$

c) Construct NPDA that accepts the following language: [11]

i. $L = \{a^n b^m : 0 \leq n \leq m \leq 3n\}$

ii. $L = \{w \in \{a, b\}^* : 2n_a(w) \leq n_b(w) \leq 3n_a(w)\}$, where n_a is the number of occurrences of a in w and n_b is the number of occurrences of b in w.