Syllabus for Test 10: L23 (partial), L24 and L 25 (partial)

L23: Sinusoidal wave, general form of a sinusoidal wave; wavelength, frequency, time period, wave number, angular frequency and phase constant or epoch angle of a sinusoidal wave. Energy transport in a wave, instantaneous velocity of a particle or element of the medium carrying a wave; kinetic energy, potential energy and total energy per wavelength in the medium carrying a wave; Power of wave motion, Intensity of a wave, intensity of sound wave; Energy density in an electromagnetic wave, Power and intensity of an electromagnetic wave.

Interference of waves, Young's double-slit experiment, interference by the method of division of wave front, production of coherent sources in Young's double-slit experiment, Conditions of constructive and destructive interferences in Young's double-slit experiment, Intensity Distribution of the double-Slit Interference Pattern, Numerical problems on Young's double-slit experiment.

L24: Review of:

Energy transport in a wave, energy stored per wavelength in the medium, Power and intensity of wave motion, intensity of sound, intensity of electromagnetic wave, Definition of interference, coherent sources, interference by the division of wave front, Young's double-slit experiment, conditions of maxima and minima in terms of angular and linear position on the screen, Intensity distribution of Young's double-slit experiment.

Interference in thin films, definition of a thin film, examples of interference in thin films; 180° phase change upon reflection from thin film, conditions for constructive and destructive interferences in thin films, different types of thin films and corresponding conditions for constructive and destructive interferences; Numerical problems on interference in thin films, Reflective and anti-reflective coatings using interference phenomena; Newton's ring, equations for dark and bright rings in Newton's ring setup determination of wavelength of light using Newton's ring experiment.

L25: Review of:

Interference in thin films, ; 180° phase change upon reflection from thin film, conditions for constructive and destructive interferences in thin films, different types of thin films and corresponding conditions for constructive and destructive interferences; Newton's ring, equations for dark and bright rings in Newton's ring setup determination of wavelength of light using Newton's ring experiment.

Michelson interferometer, Michelson-Morley Experiment, LIGO (The rest of the materials of L25 i.e. topics of diffraction will put in the next test.)

1.

A source of frequency f sends waves of wavelength λ , travelling with a speed v in some medium. If the frequency is changed from f to 2f, then the new wavelength and the new speed are (respectively):

 2λ , v $\frac{\lambda}{2}$, v λ , 2v λ , v/2 λ /2, 2v

Ans: $\lambda/2$, v. Explanation: Since in a medium the speed of a sinusoidal wave is constant (the speed depends on the property of the medium only), the wavelength is inversely proportional to the frequency. Hence the answer.

2. When a sound wave travels from one place to another, what does it transport?

Molecules of gases in air

Density variation

Mass

None of the mentioned

All of the choices mentioned

Ans: Density variation.

A transverse wave on a stretched string is represented by the equation (in CGS or cm-gm-second units): $y = 4 \sin(\pi/6) \sin(3x - 15t)$. It has:

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amplitude = 4 cm
wavelength = 4 \pi/3 cm
velocity = 5 \text{ cm/s}
time period = \pi/15 seconds
wave number = \pi/3 \text{ cm}^{-1}
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Ans: velocity = 5 cm/s. Explanation: We have the general equation of the wave motion as: $y = A \sin(kx - \omega t + \delta)$ which gives upon comparison: $k = 2\pi/\lambda = 3/\text{cm} \Rightarrow \lambda = 2\pi/3$ cm; $\omega = 15$ Hz; $T = 2\pi/\omega = 2\pi/15$ seconds; $v = \lambda/T = [(2\pi/3)/(2\pi/15)]$ cm/s = 15/3 cm/s = 5 cm/s; $A = 4 \sin(\pi/6)$ cm; $A = 4 \sin(\pi/6)$ cm;

4.

Two identical but separate strings, with the same tension, carry sinusoidal waves with the same frequency. Wave A has an amplitude that is twice that of wave B, and transmits energy at a rate that is ______ of wave B.

half twice one-fourth four times eight times

Ans: four times. Explanation: The power transmitted by a wave on a string (mechanical wave) is proportional to the square of the amplitude and square of the frequency: $P \propto f^2 A^2$. Hence, as the amplitude doubles, the power (energy/time) transmitted becomes four times larger.

5.

Equation of a progressive wave in a stretched string is given by (in CGS or cm-gm-second units): $y = 4 \sin[\pi(t/5 - x/9) + \pi/6]$ Then which of the following is correct?

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v = 5 \text{ cm/s}

\lambda = 18 \text{ cm}

T = 5 \text{ seconds}

k = \pi / \text{cm}

\omega = 5/\pi \text{ Hz}
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Ans: $\lambda = 18$ cm. Explanation: We have the general equation of the wave motion as: $y = A \sin(kx - \omega t + \delta)$ which gives upon comparison: $k = 2\pi/\lambda = \pi/(9 \text{ cm}) => \lambda = 18 \text{ cm}$; Also, $\omega = \pi/5 \text{ Hz}$ and $T = 2\pi/\omega = 10 \text{ seconds}$ and $v = \lambda/T = 18/10 \text{ cm/s}$

6.

The phenomenon of interference of light is an evidence that:

the speed of light is very large light is a transverse wave light is electromagnetic in character light is a wave phenomenon light obeys conservation of energy

Ans: light is a wave phenomenon. Explanation: Interference can occur in any type of wave. Hence interference of light is an evidence of its wave nature.

7

If f(x,t) represents a sinusoidal wave with wavelength λ and time period T then which one of the following is true in general?

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f(x,t) = f(x+\lambda/2,t+T/2)
f(x,t) = f(x+3\lambda,t+2T)
f(x,t) = f(x+\lambda,t+T/2)
f(x,t) = f(x+\lambda/4,t+T/4)
f(x,t) = f(x+2\lambda,t+T/2)
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Ans: $f(x,t) = f(x+3\lambda,t+2T)$. Explanation: A sinusoidal wave is periodic both in space and time with spatial period λ and temporal period T. Any integer multiple of full respective periods added/subtracted with the position and/or time would not change the value of the quantity representing the wave:

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f(x,t) = f(x+\lambda,t+T) = f(x+2\lambda,t+T) = f(x+m\lambda,t) = f(x,t+nT), etc.
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8

Fully constructive interference between two sinusoidal waves of the same frequency occurs only if they:

travel in opposite directions and are in phase

travel in opposite directions and are 180 degrees out of phase

travel in the same direction and are in phase

travel in the same direction and are 180 degrees out of phase travel in the same direction and are 90 degrees out of phase

Ans: travel in the same direction and are in phase. Explanation: If the waves travel in opposite directions, they create standing waves. For fully constructive interference, the waves must be in phase.

9.

Two sinusoidal waves are travelling in a medium have the same frequency and the same amplitude A. They travel in the same direction. If the waves differ in phase by 70 degrees, the amplitude of the resultant wave is:

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A cos(70 degrees)
2A cos(70 degrees)
A cos(35 degrees)
2A cos(35 degrees)
2A cos(110 degrees)
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Ans: 2A $\cos(35 \text{ degrees})$. Explanation: The amplitude of the resultant wave is $C = \operatorname{sqrt}(A^2 + A^2 + 2A^2 \cos(\phi)) = A \operatorname{sqrt}(2+2\cos(\phi)) = A \operatorname{sqrt}(2[1+\cos(\phi)]) = A \operatorname{sqrt}(4\cos^2(\phi/2)) = 2A \cos(\phi/2)$, where ϕ is the phase difference between the waves.

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NOTE:
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\begin{split} y1 &= A \sin(kx - \omega t) \text{ , } y2 = B \sin(kx - \omega t + \phi) \text{ gives:} \\ y &= y1 + y2 = A \sin(kx - \omega t) + B \left[\sin(kx - \omega t) \cos(\phi) + \cos(kx - \omega t) \sin(\phi)\right] \\ &= \sin(kx - \omega t) \left[A + B \cos(\phi)\right] + \cos(kx - \omega t) \left[B \sin(\phi)\right] \\ &= C \left[\sin(kx - \omega t) \cos(\delta) + \cos(kx - \omega t) \sin(\delta)\right] \\ &= C \sin(kx - \omega t + \delta) \end{split} where, C \cos(\delta) = A + B \cos(\phi) and C \sin(\delta) = B \sin(\phi) giving, C = \text{sqrt}(A^2 + B^2 + 2AB \cos(\phi)) and \tan(\delta) = B \sin(\phi)/(A + B \cos(\phi))
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In Young's double-slit experiment, the waves emitted from the two slits are in phase. The waves travel to a point in the screen producing a second order maximum. The path difference between the waves travelled from the two sources is (λ is the wavelength of light):

 $\lambda/2$

 $\frac{\lambda}{3\lambda/2}$

 $\frac{2\lambda}{2}$

5λ/2

Ans: 2λ . Explanation: The condition for a maximum is: path difference = $d \sin(\theta_b) = m\lambda$, where m is the order number. For second order maximum, we get path difference = 2λ .

11.

In Young's double-slit experiment, let d=center-to-center slit-spacing, D=perpendicular distance from the slits to the screen, a=slit width of both the slits and λ =wavelength of light. The number of bright fringes per unit width on the screen is:

 λ/Dd

d/λD

D/\d

a/\lambdad

λ/aD

Ans: $d/\lambda D$. Explanation: In Young's double-slit experiment, the fringe width of either bright or dark fringes is $l=\lambda(D/d)$. Hence, number of fringes per unit width is: $1/l=d/\lambda D$.

12.

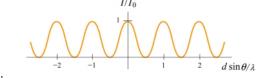
In a Young's double-slit experiment, the slit separation is doubled keeping the other distances fixed. This results in:

an increase in fringe intensity. a decrease in fringe intensity.

a halving of the fringe spacing.

a doubling of the fringe spacing.

both i.e. increase in fringe intensity and a doubling of the fringe spacing.



Ans: a halving of the fringe spacing. Explanation: In Young's double-slit experiment, the fringe width of either bright or dark fringes is $l=(D/d) \lambda$, where d=center-to-center slit-spacing, D=perpendicular distance from the slits to the screen, a=slit width of both the slits and λ =wavelength of light.

Intensity is, for $L \gg d$, $I = I_{max} \cos^2(\pi dy/\lambda L)$, as $d \to 2d$, we get $y \to y/2$ i.e. the fringes get closer together. Note that, the maximum intensity remains as I_{max} and the maximum value of the function $\cos^2(\pi dy/\lambda L)$ is +1.

<u>Alternatively</u>: Fringe width is $\Delta y = \lambda D/d$ which gives as $d \to 2d$, the fringe width gets halved: $\Delta y \to \Delta y/2$ Hence as only the slit separation is doubled, the fringe width is halved.

13.

The phase difference between two waves that give rise to a dark spot in a Young's double-slit experiment is (where m = integer):

zero

 $2\pi m + \pi/8$

 $2\pi m + \pi/4$

 $2\pi m + \pi/2$

 $2\pi m + \pi$

Ans: $2\pi m + \pi$; Explanation: For dark spot, the phase difference has to be an odd multiple of π .

To observe interference in thin films with a light of wavelength λ , the thickness of the film

should be much smaller than λ should be much larger than λ

should be of the order of λ

should be of the order of nanometer

no relation exists between the thickness and wavelength $\boldsymbol{\lambda}$

Ans: should be of the order of λ

15.

A thin film of a material having a refractive index of 1.5 and thickness 1 cm is placed in the path of light. What is the path difference observed between a ray coming directly in air and a ray coming through the film?

0.002 m

0.003 m

0.004 m

0.005 m

0.006 m

Ans: 0.005 m. Explanation: Path difference = (μt) - t = (1.5-1)t = 0.5t = 0.5 cm = 0.005 m

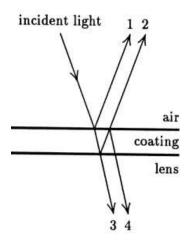
16.

A thin layer of transparent material (as a thin film) is added on the lens surfaces of binoculars or microscopes. This is done often for:

constructive interference between rays 1 and 2 constructive interference between rays 3 and 4 making the speed of light in the coating to be less than that in the lens. destructive interference between rays 3 and 4 destructive interference between rays 1 and 2

Ans: destructive interference between rays 1 and 2.

Explanation: This is done for putting an anti-reflection coating so that little or no light of a particular wavelength gets reflected by the lens. This will allow most of the light of that wavelength (apart from some absorption) to pass through the coating+lens material into the binocular or microscope's interior.



17.

In Newton's Ring experiment, the diameter of bright rings is proportional to

square root of odd natural numbers

natural numbers even natural numbers square root of natural numbers square root of even natural numbers

Ans: square root of odd natural numbers. Explanation: The formula for the radius of the bright rings in Newton's ring experiment is $r_n = \operatorname{sqrt}((2n+1) \lambda R/2)$, where R is the radius of curvature of the convex surface of the plano-convex lens.

To reduce reflected light, a glass lens (ng=1.6) is coated with a thin film of refractive index nf=1.3. Let λ' denote the wavelength in the thin film material. What is the minimum width of the thin film for which the reflected light will be a MINIMUM?

much less than $\lambda'/4$

$\lambda'/4$

 $\lambda'/2$

λ'

 $3\lambda'/4$

Ans: λ'/4

Explanation: One ray is reflected from the top surface of the which undergoes a phase change of π upon reflection.

A second ray is reflected at the interface between the film and the glass lens and again goes a phase change of π upon reflection.

Phase difference between the two rays that emerge back i.e. are reflected, from the upper and lower surfaces of the film is: $(2t/\lambda')2\pi$

For destructive interference and for minimum thickness of the film, we get: $(2t/\lambda')2\pi = \pi = t = t = t = t$

19.

In Newton's ring experiment, the diameter of the 10th ring changes from 1.40 to 1.27 cm when a liquid is introduced between the lens and glass plate. What is the refractive index of the liquid?

1.05

1.15

1.25 1.35

1.40

Ans: 1.25

Explanation: We know, $Dn^2 = 4n\lambda R/\mu$; Without liquid in air, $\mu = 1$, $D10^2$

= $40\lambda R$; With liquid, D' $10^2 = 40\lambda R/\mu$

Dividing both we get, $\mu = D10^2/(D'10^2) = \{1.40/1.27\}^2 = 1.25$

20.

In Michelson interferometer semi-silvered mirror M0 is used to obtain

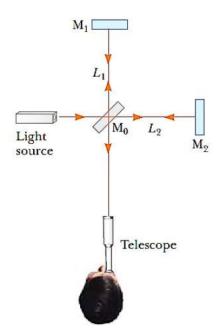
thin film interference

coherent sources

monochromatic light colored fringe

phase change upon reflection

Ans: coherent sources. Explanation: A ray of light from a monochromatic source is split into two rays by mirror M0, which is inclined at 45° to the incident light beam. This creates coherent sources having phase relationship between the rays travelling along the two arms.



The displacement of a string carrying a travelling sinusoidal wave is given by: $y(x,t) = A \sin(kx - \omega t + \varphi)$ At time t=0, the point at x=0 has a velocity of v0 and a displacement y0. The phase constant φ is given by $\tan^{(-1)}[\varphi] =$

v0/(\omega y0) (\omega y0)/v0 sqrt((\omega y0)/v0) (\omega y0)^2/v0^2 sqrt(v0/(\omega y0))

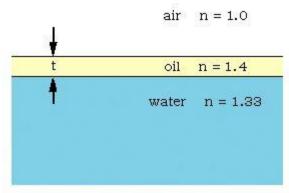
Ans: $(\omega \ y0)/v0$. Explanation: Wave velocity $= v = \lambda/T = (2\pi/k) \ (\omega/2\pi) = \omega/k$. At time t=0 and point x=0, we get $y0 = A \sin(\varphi)$ The speed of a general point on the string is: $v = A\omega \cos(kx - \omega t + \varphi)$ which at time t=0 and point x=0 is: $v = A\omega \cos(\varphi)$; Hence, $\tan(\varphi) = [A \sin(\varphi)]/[A \cos(\varphi)] = y0/(v0/\omega) = (\omega \ y0)/v0$.

22.

A thin layer of colorless oil is spread over water in a container (μ = 1.4) with air above the oil layer. If the light of wavelength 640 nm is absent in the reflected light, what is the minimum thickness of oil layer?

107.1 nm 112.8 nm 198.3 nm 214.3 nm 225.6 nm

Ans: 214.3 nm



Explanation: Oil, although floats on water, has a greater refractive index than water. The ray reflected from the oil surface undergoes a phase change of π upon reflection.

Another ray, reflected at the interface of oil and water does NOT undergo any phase change upon reflection. Phase difference between the two rays that are reflected, from the upper and lower surfaces of the oil is: $\pi + (2t/\lambda')2\pi$, where t= thickness of the oil film and λ' = wavelength of light in oil.

For destructive interference $\pi + (2t/\lambda')2\pi = (2m+1)\pi = 2m\pi + \pi => (2t/\lambda')2\pi = 2m\pi$ For minimum thickness m=1 => $2t/\lambda' = 1$ => $t_min = \lambda'/2 = \lambda/(2\mu) = 600$ nm/(2.8) = 214.3 nm