University of Dhaka
Department of Computer Science and Engineering
3rd Year 2nd Semester B.Sc.(Hons.) Final Examination, 2017
CSE 3203: Finite Language, Automata and Computation

Full Marks: 60 **Time: 3 Hours**

(Answer any Four (4) of the following Questions)

1. a) Prove or disprove each of the following statements: 2+2
   i. If L_1 is context free and L_2 is not context free then L_1L_2 is not context free.
   ii. If L_1 is context free and L_2 is not context free then L_1L_2 is context free.

   b) For each language L below choose the most restrictive class it belongs to and give justification for your answer. 3+3+5
   i. $L = \{xw \mid x, w \in \{a, b\}^* \text{ and } |x| = |w|\}$
   (Choose among regular, context free or TM-decidable)
   ii. $L = \{a^i b^j c^k d^l \mid i + j + k + l \text{ is a multiple of } 13\}$
   (Choose among regular, context free or TM-decidable)
   iii. $L = \left\{ \langle w, M_1, M_2, \cdots, M_k \rangle \;\middle|\; \begin{array}{c} w \text{ is a string} \\ k \text{ is an odd number larger than 2} \\ \text{each } M_i \text{ is a TM and the majority} \\ \text{of the } M_i\text{'s accpets } w \end{array} \right\}$
   (Choose among TM-Decidable, TM-recognizable, not TM-recognizable.

2. a) Convert the following grammar in CNF and use CYK algorithm to determine whether $ababb$ is a string in the language of the grammar (S is the start symbol). 4+4
   $S \rightarrow ASA \mid aB$
   $A \rightarrow B \mid S$
   $B \rightarrow b \mid \epsilon$

   b) Formally prove that all strings that the following grammar generates have even length. 7
   (S is the start symbol)
   $S \rightarrow SB \mid aa$
   $B \rightarrow bSBb \mid ab$

3. a) Consider the following language L over $\Sigma = \{0, 1, \#\}$. 6
   $L = \{x\#y \mid x, y \in \{0,1\}^+ \text{ and } y \text{ is the binary number equal to } x + 1\}$
   Hence, 1110#1111 $\in L$, and 111#110 $\notin L$.
   Prove that L is not regular.

   b) Give informal description of a PDA that recognizes the following language: 4
   "The set of strings over the language {a, b} with more a's than b's

   c) Give a context free grammar that generates the following language. Also, give informal description of the language generated by each variable you have used in the grammar. 5
   $\{w\#x \mid w^R \text{ is a substring of } x \text{ for } w, x \in \{0, 1\}^*\}$

4. a) Define ambiguous grammar. Show that the following two grammars are ambiguous. 2+
   i. S ->ep|SS|(S) 1.5+
   ii. S → aFbS | aFbSeS | ϵ 1.5
   F → f
   Rewrite the grammars so that they are not ambiguous.

   b) Design CFG in the following cases: 2+2
   i. All binary strings that are palindrome, such as: 11, 101, 0110 etc.
   ii. $\{a^m b^n : m \geq n\}$

   c) Identify the following languages as regular, non-regular, state the reasons. 3
   i. $\{\omega = xy : x, y \in \{a, b\}^*\}$ and y contains x as a substring}
   ii. $\{\omega = x^i : x \in \{a, b\}^* \text{ and } i \geq 0\}$
   iii. $\{\omega = \{a, b\}^* : \omega$ has more a than b$\}$

d) Write regular expressions for the following languages. $\Sigma = \{0, 1\}^*$ 3

i. The set of strings whose number of 1's is divisible by six.

ii. All strings that do not contain the substring 11.

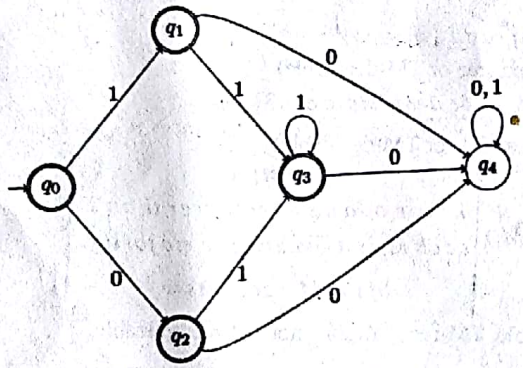
5. a) Let, $\Sigma = \{T,\ F,\ p,\ q,\ \sim,\ \wedge,\ \vee\}$. The set of well formed Boolean formulas (WFF) is a language over Σ defined inductively as follows: 4+4

- T, F, p *and* q are elements of WFF
- If x is in WFF, then so is $\sim x$
- If x *and* y are in WFF, then $x \wedge y$ and $x \vee y$ are in WFF

i. Design a DFA with at most 3 states that recognizes WFF.

ii. Construct a regular expression for the language WFF.

b) Consider the following DFA, M 2.5+4.5



i. The language $Suffix(M, q)$ is the collection of all words accepted if q were the initial state of DFA M. For each state of M, describe $Suffix(M, q)$ using regular expression.

ii. For any language L and any string x, let $suffix$ language of L with respect to x is defined as $suffix(L, x) = \{y \in \Sigma^* \mid xy \in L\}$; i.e. $suffix(L, x)$ is the collection of strings y which when prefixed by string x, result in a string in L. For each of the following values of x, describe $suffix(L, x)$.

a. $x = \epsilon$

b. $x = 0$

c. $x = 10$

6. a) Write a formal definition of pushdown automation. 2

b) Using pumping lemma show that the languages consisting of all strings with equal number of 0's and 1's (not in any particular order) is not a regular language. 4

c) Compare a nondeterministic finite automation and pushdown automation. 3

d) Show the contents of the stack in each step for the following PDA, when the input is "011110". 6