Syllabus:
1. Mean free path
2. Adiabatic Processes for an Ideal Gas, inadequacies of the first law, the second law
3. Heat engines and heat pumps/refrigerators, COP of heat pump/refrigerator

1
Pressure of an ideal gas is doubled while its volume becomes one-third in a process. Its mean free path was initially l. What will be the changed mean free path?

l/2
l/3 A
3l/2
l
2l/3

Solution: l= 1/(sqrt(2) pi d^2 n_V), here n_V = number density of molecules = (number of molecules)/volume. n_V = N/V implies

$$l = \frac{V}{N\sqrt{2}\pi d^2}$$

Again n_V depends on temperature inversely, because we have, n_V = N/V = P/kT . Hence l varies as T

$$l = \frac{kT}{P\sqrt{2}\pi d^2}$$

From the first relation: $V \rightarrow V/3$ implies $l \rightarrow l/3$

From the second relation, $P \rightarrow 2P$; $V \rightarrow V/3$; implies $T \rightarrow 2T/3$ at constant number of molecules.
Hence we get: $l \rightarrow l\frac{2}{3}\frac{1}{2} = l/3$ as expected.

2
Certain Ideal gas is found to be obey PV^(5/3)=Constant during an adiabatic process. If such a gas at initial temperature T is adiabatically compressed to half the initial volume. Its mean free path was initially λ. What will be the changed mean free path?

λ /2 A
λ (1/2)^(5/3)
2^(5/3) λ
2 λ
λ /2^(2/3)

Solution: $V \rightarrow V/2$ implies $\lambda \rightarrow \lambda/2$; no other complications are needed.

3

Consider two gases under identical conditions of pressure and volume but the first gas having molar mass twice that of the second gas. Their molecular diameters are 20 nm and 10 nm, respectively. The ratio of the mean free paths l_1 : l_2 of the molecules of two gases is:

1:2

1:4 A

2:1

4:1

1:sqrt(2)

Solution: We have

$$l = \frac{V}{N\sqrt{2}\pi d^2}$$

Hence, free path $l \propto 1/d^2$. Hence, $l_1 : l_2 = d_2^2 : d_1^2 = (10/20)^2 = 1 : 4$

4

Consider two gases of the same volume having the same root-mean square speed. The molar mass of the first gas is twice that of the second gas. Their molecular diameters are 20 nm and 10 nm, respectively. The mass of sample of the first gas is twice that of sample of the second gas. The ratio of the mean free paths l_1 : l_2 of the molecules of two gases is:

1:2

1:4 A

2:1

4:1

1:sqrt(2)

Solution: We have $l = \frac{V}{N\sqrt{2}\pi d^2}$. Hence, free path $l \propto 1/d^2$, given all other quantities concerned are the same. Hence, $l_1 : l_2 = d_2^2 : d_1^2 = (10/20)^2 = 1 : 4$

Since the volume is the same, RMS speed does not have any role to play.

<u>Alternatively</u>, we have $v_{rms}^2 = 3kT/m = 3RT/M$, Hence from the formula of the mean free path: $l = \frac{kT}{P\sqrt{2}\pi d^2}$ we get,

$$l = v_{rms}^2 \left(\frac{m}{3}\right) \frac{1}{P\sqrt{2}\pi d^2}$$

As $m_1 N_A = 2\ (m_2 N_A)$ and $Q_1 = 2Q_2 => N_1 m_1 = 2(N_2 m_2)$ where Q_i, $i = 1,2$ are the mass of the samples of the gases, we get $N_1 = N_2$ i.e. the number of molecules of the first gas is the same as that of the molecules of the second gas. Hence $n_{V-1} = n_{V-2}$ i.e the number densities are the same.

Hence, we get free path $l \propto 1/d^2$ and $l_1 : l_2 = d_2^2 : d_1^2 = (10/20)^2 = 1 : 4$

5

Which of the following is a reversed heat engine?

Heat pump A

Refrigerator A

Carnot engine

Heat reservoir

None of the others

Solution: Any of the first two answers is okay.

6

A heat engine that in each cycle does positive work and does not lose energy as heat, would violate:

the zeroth law of thermodynamics

the first law of thermodynamics

the second law of thermodynamics A

the third law of thermodynamics

both the first law and second law of thermodynamics

Solution: Using the Kelvin-Planck statement: It is impossible to convert all of heat taken in into work.

7

A heat engine that in each cycle does positive work and loses more heat energy than it takes, would violate:

the zeroth law of thermodynamics

the first law of thermodynamics A

the second law of thermodynamics

the third law of thermodynamics

both the first law and second law of thermodynamics

Solution: Basically

(i) The engine takes in heat (as it is a heat engine)

(ii) Does positive work, i.e. converts (at least some amount) heat into work

(iii) It rejects more heat than it takes in.

Hence, it rejects (at least) some amount of heat to the low temperature reservoir. And hence does not violate the second law. But it rejects more heat than it takes in. This will violate energy conservation. Hence only the first law is violated.

8.
The food compartment of a refrigerator is maintained at 4°C by removing heat from it at a rate of 3600 kJ/hr. If the required power input to the refrigerator is 2kW, the coefficient of performance of the refrigerator is:
Figure:

5
4
3
2 A
1

Explanation: COP = (3600/2)(1/(60*60)) = 2.

9
Coefficient of performance of an ideal refrigerator working between temperature T1 and T2 (T1 > T2) is

$\beta = T2/(T1 - T2)$ A
$\beta = T2/(T1 + T2)$
$\beta = T1/(T1 - T2)$
$\beta = T1/(T1 + T2)$
$\beta = \mathrm{sqrt}(T1\ T2)/(T1 - T2)$

Solution: $COP = Q_c/W = Q_c/(Q_h - Q_c) = 1/(Q_h/Q_c - 1)$; But for a refrigerator, working as the reverse of a Carnot engine, we have $Q_h/Q_c = T_1/T_2$.

Hence, we get
$COP = 1/(T_1/T_2 - 1) = T_2/(T_1 - T_2)$

10
A Carnot engine works between freezing point and the boiling point of water at the standard atmospheric pressure. Its efficiency will be

15.35 %
26.81 % A
53.36 %
71.23 %
85.42 %

Solution: Efficiency = $1 - T_L/T_H = 1 - (273.15/373.16) = 26.80\ \%$ Approximately.

11

Air in a cylinder is suddenly compressed by a piston which is then maintained at the same position. After sometime:

Pressure will increase
Pressure remains the same
Pressure will decrease
Pressure may increase or decrease
Cannot be answered, insufficient information. A

Solution: When the air in the cylinder is compressed, its pressure increases so also its temperature. The process may happen in two situations:

(i) The cylinder's wall are insulating. In this case this heat cannot flow out. The increased temperature and pressure will remain the same at the increased values.

(ii) The cylinder's wall or at least one of them is diathermic. In this case heat will flow out and the temperature will decrease. Since the volume remains the same, $P \propto T$ and hence the pressure will decrease.

In any case, the pressure **will never increase further**.

Hence, without information about the property of the material of the cylinder's wall, we cannot conclude anything. Answer: Cannot be answered, insufficient information.