But... the formula $s_{CFR} = \sqrt{\frac{CFR(1-CFR)}{N}}$ where N = D + R (i.e. Deaths plus Recoveries) breaks down if CFR=0 (0%) or 1 (100%).

In these cases, we find s_{CFR}=0 regardless of the value of N.

Furthermore, the **normal approximation** doesn't work well if N is relatively small or if CFR equals or is near 0 or 1.

For example:
If N=20 and CFR = 6/20 = 0.30 then s_{CFR} = 0.102 and the 95% CI is 0.099 to 0.501 (based on the normal approximation)
But
If N=20 and CFR = 1/20 = 0.05 then s_{CFR} = 0.049 and the 95% CI is **-0.046** to 0.146 (based on the normal approximation)

This clearly doesn't make sense because CFR can't be negative and furthermore if even just 1 person died, then the CFR can't plausibly be 0.

An alternative is a **likelihood-based binomial 95% confidence interval**

If N=20 and **CFR = 0/20 = 0.0,** what is the upper limit of the 95% CI?

The answer (obtained numerically) is 0.092. **Making the 95% CI 0.000 to 0.092.**

This is because the log likelihood of the data if CFR=0.092 is 20*ln(1-0.092) = -1.92
And the log likelihood of the data if CFR=0 is 20*ln(1-0) = 0
Making 2 times the difference equal to 2(0 - -1.92) = 3.84 (which is the 95th centile of the chi-squared distribution with 1 degree of freedom).

How does this work if CFR doesn't equal 0 or 1?

Similarly!

If N=20 and **CFR = 1/20 = 0.05**, what are the limits of the 95% CI?

The answers (obtained numerically) are 0.0029 and 0.202. **Making the 95% CI 0.0029 to 0.202.**

This is because the log likelihood of the data if CFR=0.202 is 1*ln(0.202)+19*ln(1-0.202) = -5.89 And the log likelihood of the data if CFR=0.0029 is 1*ln(0.0029)+19*ln(1-0.0029) = -5.89
And the log likelihood of the data if CFR=0.05 is 1*ln(.05)+19*ln(1-0.5) = -3.97
Making 2 times the difference equal to 2(-3.97 - -5.89) = 3.84 (which is the 95th centile of the chi-squared distribution with 1 degree of freedom).

Another alternative is the **exact binomial 95% confidence interval.**

Here the equations for the lower and upper bounds of the 95% confidence interval (p_{LB} and p_{UB}, respectively) are:

$$\sum_{k=0}^{r} \binom{N}{k} p_{UB}^{k} (1 - p_{UB})^{(N-k)} = 0.025$$

$$\sum_{k=r}^{N} \binom{N}{k} p_{LB}^{r} \left(1 - p_{LB}\right)^{(N-k)} = 0.025$$

where r is the number of deaths and N is the sample size.

For N=20 and CFR = 0/20 = 0.0, the exact 95% CI is **(0.0, 0.1684)**

For N=20 and CFR = 1/20 = 0.05, the exact 95% CI is **(0.0013, 0.2487)**