HYPOTHESIS TESTS

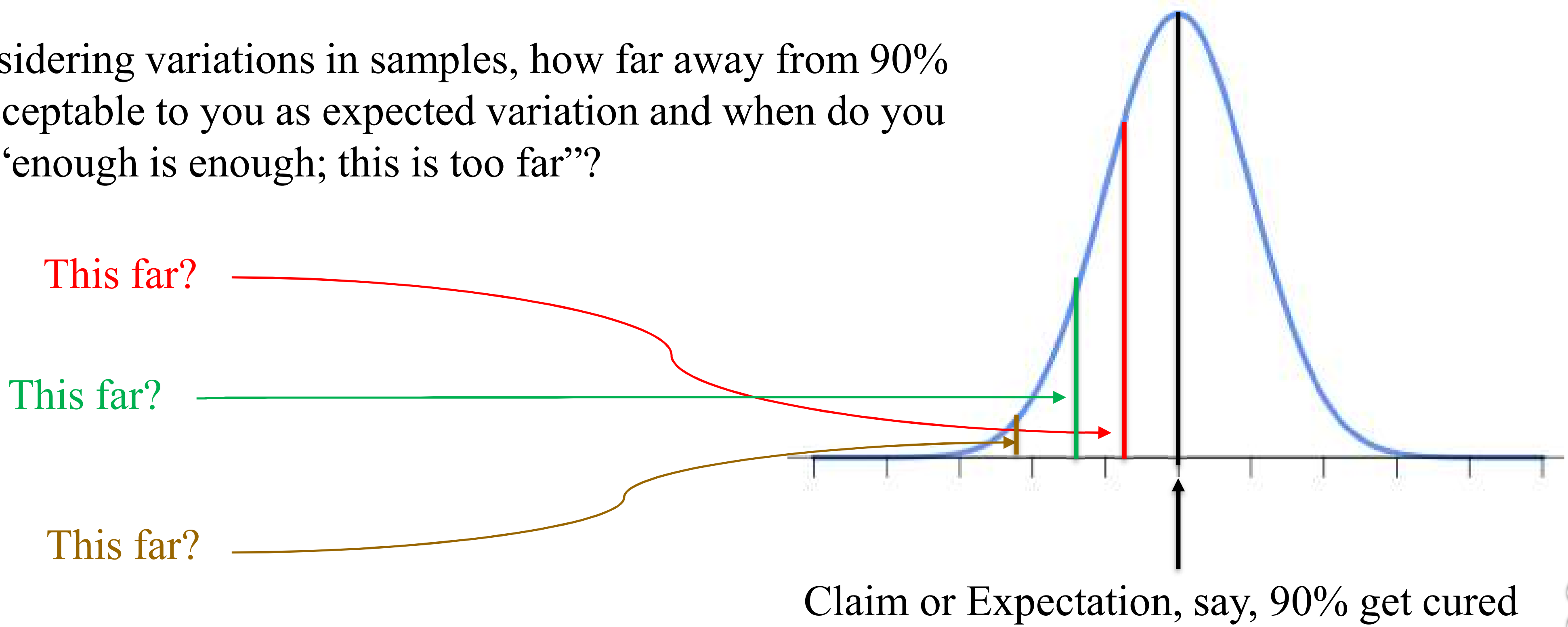
Hypothesis tests give a way of using samples to test whether or not statistical claims are likely to be true or not.

Dr. Unsnora prescribes SnoreCull to 15 of her patients and records whether it cured them or not after 2 weeks. She found that 11 were cured and 4 were not.

If the drug maker claimed that 90% get cured, 13.5 or 14 patients should have been cured. Is the company making false claims or is the doctor's sampling biased?

Hypothesis Testing Process

Considering variations in samples, how far away from 90% is acceptable to you as expected variation and when do you say “enough is enough; this is too far”?



Step 1: Decide on the hypothesis

SnoreCull cures 90% of the patients within 2 weeks.

This is called Null Hypothesis and is represented by H_0.

In this case, H_0: $p = 0.9$

If Null Hypothesis is rejected based on evidence, an Alternate Hypothesis, H_1, needs to be accepted. **We always start with the assumption that Null Hypothesis is true.**

In this case, H_1: $p < 0.9$

Examples of Hypotheses

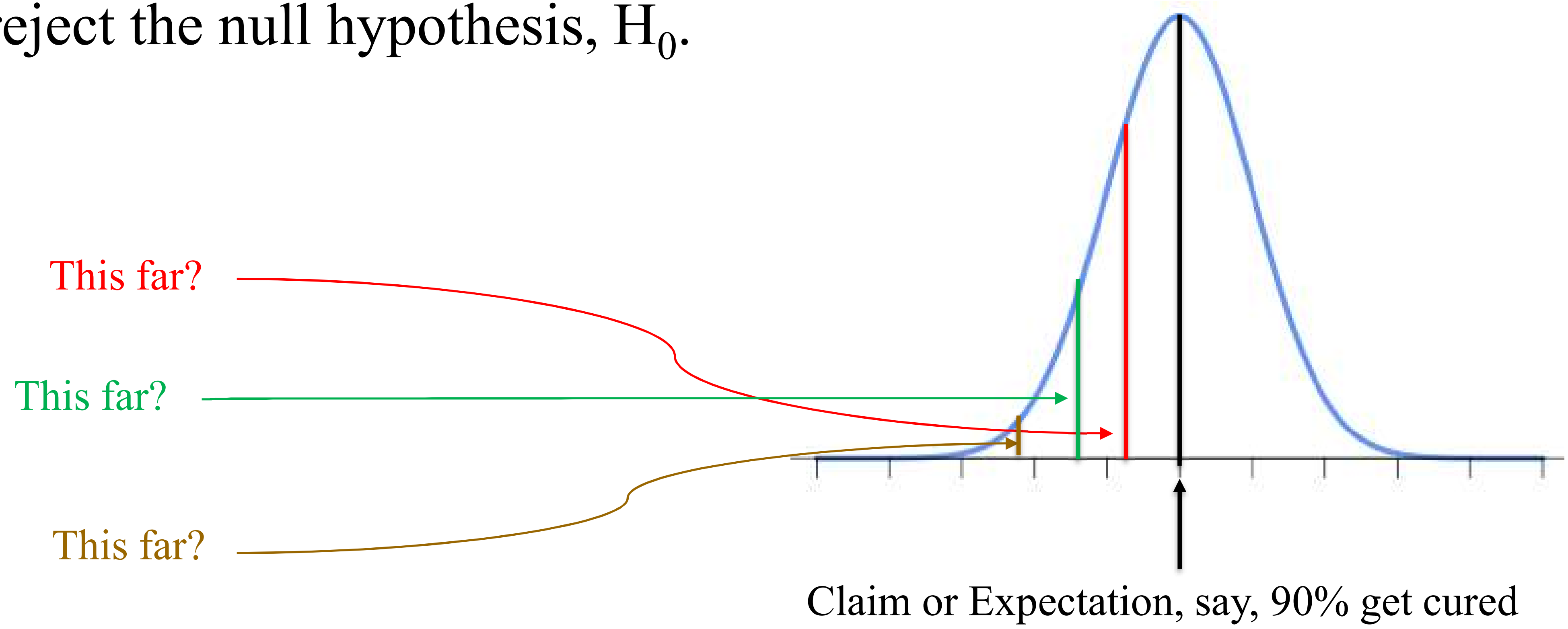
- Two hypotheses in competition:
  - H_0: The NULL hypothesis, usually the most conservative.
  - H_1 or H_A: The ALTERNATIVE hypothesis, the one we are actually interested in.
- Examples of NULL Hypothesis:
  - The coin is fair
  - The new drug is no better (or worse) than the placebo
- Examples of ALTERNATIVE hypothesis:
  - The coin is biased (either towards heads or tails)
  - The coin is biased towards heads
  - The coin has a probability 0.6 of landing on tails
  - The drug is better than the placebo

Step 2: Choose your statistic

$X \sim B(15, 0.9)$

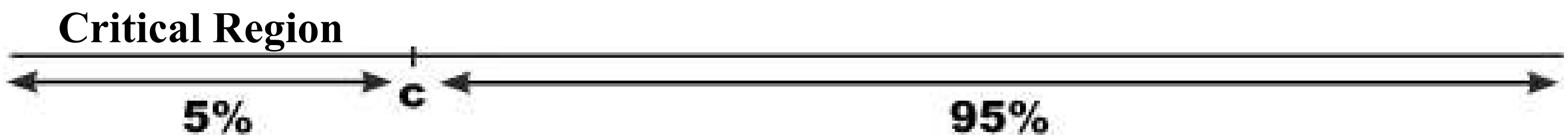
Step 3: Specify the Significance Level

First, we must decide on the Significance Level, α. It is a measure of how unlikely you want the results of the sample to be before you reject the null hypothesis, H_0.



Step 4: Determine the critical region

If X represents the number of snorers cured, the critical region is defined as $P(X < c) < \alpha$ where $\alpha = 5\%$.



Recall that in a 95% CI, there is a 5% chance that the sample will not contain the population mean. Hence if the sample falls in the critical region, the null hypothesis that 90% snorers are cured, is rejected.

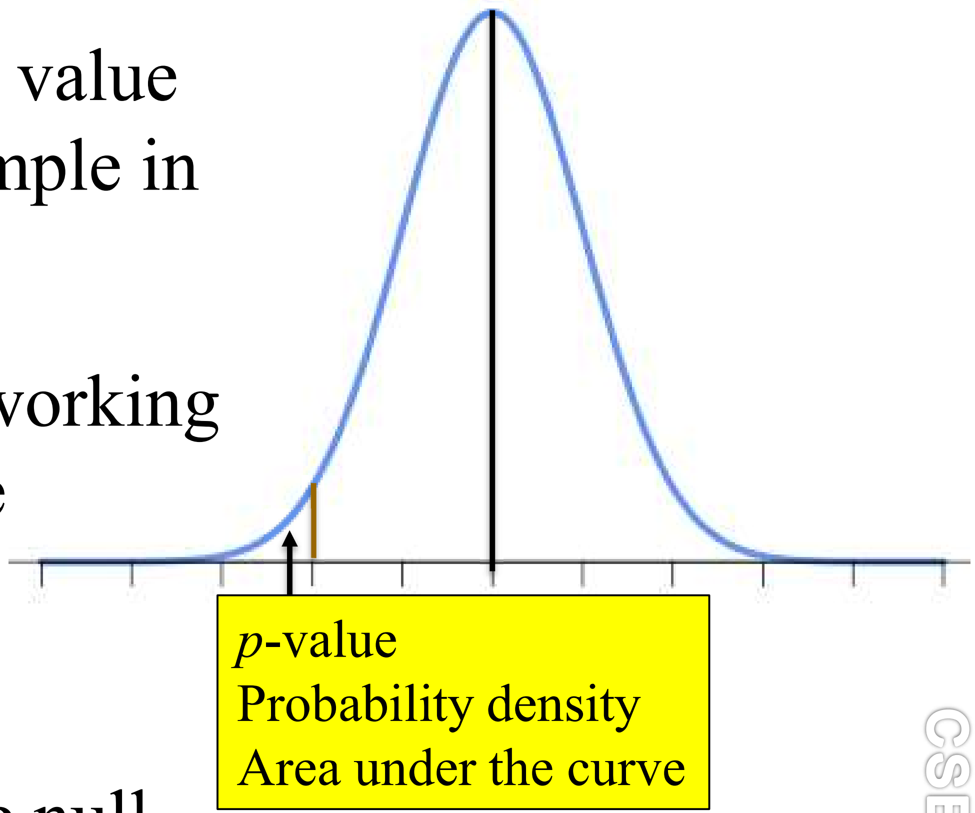
That is the reason 5% or 0.05 is called the Significance Level. In a 99% CI, 0.01 is the Significance Level.

Step 5: Find the *p*-value

p-value is the probability of getting a value up to and including the one in the sample in the direction of the critical region.

It is a way of taking the sample and working out whether the result falls within the critical region of the hypothesis test.

Essentially, this is the value used to determine whether or not to reject the null hypothesis.



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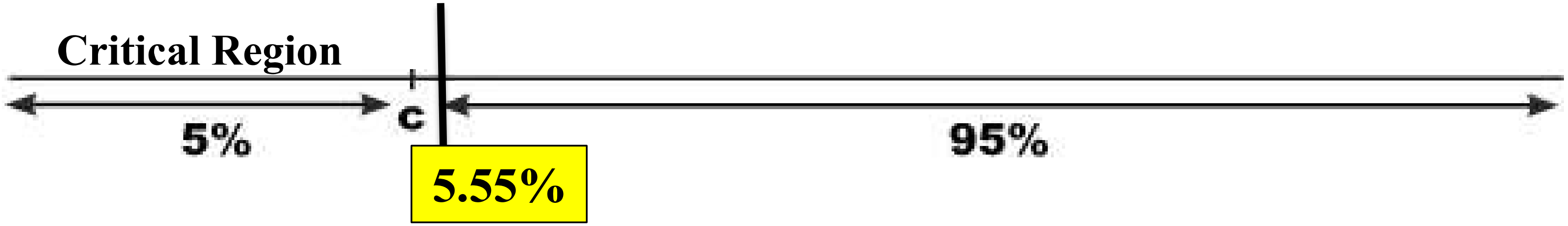
Step 5: Find the *p*-value

In the SnoreCull test done by Dr. Unsnora, 11 people were cured. This means our *p*-value is $P(X \leq 11)$, where X is the distribution of the number of people cured in the sample.

If $P(X \leq 11) < 0.05$ (Significance Level), it indicates that 11 is inside the critical region, and hence H_0 can be rejected.

Given that $X \sim B(15, 0.9)$, $P(X \leq 11) = 1 - P(X \geq 12) = 0.0555$

Step 6: Is the sample result in the critical region?



Step 7: Make your decision

There isn't sufficient evidence to reject the null hypothesis and so, the claims of the company are accepted.

Dr. Unsnora is not convinced and did another test with 100 people where 80 got cured and 20 didn't. What is your decision going to be now?

What are the null and alternate hypotheses?

H_0: $p = 0.9$

H_1: $p < 0.9$

What is the test statistic?

$X \sim B(100, 0.9)$ Oh! Dear

What probability distribution can be used to approximate the Binomial distribution?

Since np>5 and nq>5, Central Limit Theorem can be applied to sampling proportions.

What is the probability of 80% or fewer getting cured?

CONTINUITY CORRECTION FACTOR

$$z = \frac{\hat{p} + \frac{0.5}{n} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.805 - 0.9}{\sqrt{\frac{0.9 * 0.1}{100}}} = \frac{0.805 - 0.9}{\sqrt{0.0009}} = -3.17$$

p-value = P(Z < -3.17) = 0.0008

What is your decision?

Since the p-value (0.0008) is less than the Significance Level of 0.05, the null hypothesis can be rejected.

Attention Check

In hypothesis testing, do you assume the null hypothesis to be true or false?

True.

If there is sufficient evidence against the null hypothesis, do you accept it or reject it?

Reject it.

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