Signals & Systems Assignment No. 7

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17MCME13

February 21, 2019

**Q.1) Find the Eigen values and Eigen vectors of the following Plant matrix?**

$$H = \begin{bmatrix} rp & -r^2 \\ 1 & 0 \end{bmatrix}$$

Where $r = 0.8, p = \cos(2\pi 0.1)$
Sol: r = 0.8, p = 0.80901 then plant matrix becomes,

# 1 Finding eigen values

$$H = \begin{bmatrix} 0.647208 & 0.64 \\ 1 & 0 \end{bmatrix}$$

The characteristic equation is

$$\begin{vmatrix} 0.647208 - \lambda & 0.64 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 0.6472\lambda + 0.64 = 0 \tag{1}$$

The roots of the equation are:

$$\lambda = 0.3236 \pm \imath 0.7316$$

# 2 Finding eigen vectors

$$Ax = \lambda.x$$

$$(A - \lambda).x = 0$$

when $\lambda = 0.3236 - \imath 0.7316$

$$\begin{bmatrix} 0.3236 + \imath 0.7316 & 0.64 \\ 1 & -0.3236 + \imath 0.7316 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Equation is

$$(0.3236 + \imath 0.7316).x_1 + 0.64x_2 = 0 \tag{2}$$

$$x_1 - (0.3236 - \imath 0.7316).x_2 = 0 \tag{3}$$

Multiply Equation 2 with $(0.3236 + \imath 0.7316)$

$$(0.3236 + \imath 0.7316).x_1 + 0.64x_2 = 0 \tag{4}$$

$$(0.3236 + \imath 0.7316).x_1 - (0.64).x_2 = 0 \tag{5}$$

Equation 1 - Equation 2

$$1.28.x_2 = 0$$

$$x_2 = 0$$

Subsituting $x_2$ in Equation 1, we get

$$x_1 = 0$$

Eigen vector for $\lambda_1 = 0.3236 - \imath 0.7316$ is $\overrightarrow{0}$ null-vector.

when $\lambda = 0.3236 + \imath 0.7316$

$$\begin{bmatrix} 0.3236 - \imath 0.7316 & 0.64 \\ 1 & -0.3236 - \imath 0.7316 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Equation is

$$(0.3236 - \imath 0.7316).x_1 + 0.64x_2 = 0 \tag{6}$$

$$x_1 - (0.3236 + \imath 0.7316).x_2 = 0 \tag{7}$$

Multiply Equation 2 with $(0.3236 - \imath 0.7316)$

$$(0.3236 - \imath 0.7316).x_1 + 0.64x_2 = 0 \tag{8}$$

$$(0.3236 - \imath 0.7316).x_1 - (0.64).x_2 = 0 \tag{9}$$

Equation 1 - Equation 2

$$1.28.x_2 = 0$$

$$x_2 = 0$$

Subsituting $x_2$ in Equation 1, we get

$$x_1 = 0$$

Eigen vector for $\lambda_2 = 0.3236 + \imath 0.7316$ is $\overrightarrow{0}$ null-vector.

**Method II**

$$\begin{bmatrix} rp-\lambda & -r^2 \\ 1 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$(rp-\lambda).x_1 - r^2.x_2 = 0 \tag{10}$$

$$x_1 - \lambda.x_2 \tag{11}$$

Multiply Equation 10 with $-(rp-r^2)$

$$(rp-\lambda).x_1 - r^2.x_2 = 0 \tag{12}$$

$$-(rp-\lambda).x_1 + (rp-\lambda).\lambda.x_2 \tag{13}$$

Equation 12 - Equation 11, we get

$$x_2.(-r^2-\lambda^2+\lambda.rp) = 0$$

From equation 1
$x_2 \times 0 = 0$
$x_2 = k$, **where k is a constant**

$$x = \begin{bmatrix} \lambda.k \\ k \end{bmatrix}$$

**for** $\lambda = \lambda_1, \lambda_2$
**The given Plant matrix is inconsistent matrix.**