Signals & Systems Assignment No. 8

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Q.1) Find all the roots of $\mathbf{1 - z^{-N} = 0}$?

Sol: Given Equation $1 - z^{-N} = 0$

$$\frac{z^N - 1}{z^N} = 0 \tag{1}$$

decompposing into two polynomials,

$$(z-1)(z^{N-1} + z^{N-2} + z^{N-3} + \cdots + z^2 + z^1 + 1) \tag{2}$$

for any real value of z

$$(z^{N-1} + z^{N-2} + z^{N-3} + \cdots + z^2 + z^1 + 1) \neq 0$$

$$z - 1 = 0$$

$$z = 1 \tag{3}$$

for any complex value of z

$$z - 1 \neq 0$$

$$(z^{N-1} + z^{N-2} + z^{N-3} + \cdots + z^2 + z^1 + 1) = 0 \tag{4}$$

By De morive's law

$$(\sin(x) + \imath\cos(x))^n = (\sin(nx) + \imath\cos(nx))$$

when $x = 2\pi/n$,

$$\left(\sin(\frac{2\pi}{n}) + \imath\cos(\frac{2\pi}{n})\right)$$

for any k

$$\left(\sin(\frac{2\pi}{n}) + \imath\cos(\frac{2\pi}{n})\right)^k = (\sin(\frac{2\pi.k}{n}) + \imath\cos(\frac{2\pi.k}{n}))$$

roots of Equation 4 are

$$(\sin(\frac{2\pi.k}{n}) + \imath\cos(\frac{2\pi.k}{n}))$$

from $k = 1, 2, 3, \cdots, n-1$

$\therefore$ Roots of the given equation are $1, (\sin(\frac{2\pi.k}{n}) + \imath\cos(\frac{2\pi.k}{n}))$
$k = 1, 2, 3, \cdots, n-1$