Signals & Systems Assignment No. 5

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1 Given Statement

Given Equation is: $h_k = 1.62h_{k-1}$ - $0.81h_{k-2} + \delta_k$
where δ_k is a delta function:

$$\delta_k = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

The General form of the equation is $h_k = r.p.h_{k-1}$ - $r^2 0.81h_{k-2} + a.\delta_k + b.\delta_{k-1}$ for the given problem r = 0.9, p = 1.8, a = 1, b = 0

2 Difference Equation to transfer function

$h_k = 1.62h_{k-1}$ - $0.81h_{k-2} + \delta_k$
$h_k = 1.62h_k.z^{-1}$ - $0.81h_k.z^{-2} + \delta_k$
h_k - $1.62h_k.z^{-1} + 0.81h_k.z^{-2} = \delta_k$
$h_k.(1$ - $1.62z^{-1} + 0.81z^{-2}) = \delta_k$

$$h_k = \frac{1}{(1 - 1.62z^{-1} + 0.81z^{-2})}\delta_k \tag{1}$$

$$h_k = H(z)\delta_k \tag{2}$$

Where $H(z)$ is a transfer function.
solving for the value of z we get, $z = 0.81 \pm \imath 0.3923$

3 Changing to polar form

Writing this in terms of $e^{r\theta}$,

$$r = \sqrt{(0.81)^2 + (0.3923)^2} = 0.9$$

$$\theta = \arctan\left(\frac{0.3923}{0.81}\right) = 25.8419^\circ$$

$$z = e^{(0.9).(25.8419^\circ)} = 0.9(\cos(25.8419^\circ) \pm \imath \sin(25.8419^\circ)) \tag{3}$$

The frequency of the wave is given by:

$$f_n = \frac{\arccos(\frac{p}{2})}{2\pi}$$

and for this problem f_n is 0.0717.
No. of samples is $\left[\frac{360^\circ}{\theta^\circ}\right]$ which is $13.9308 \approx 14$.