Sample Space

We define Ω as the set of *all* possible outcomes of a random event. $\Omega = \{\cdots\}$

Collectively Exhaustive

We define the probability of *no element* in Ω occurring as 0.

$$\mathbf{P}\left(\,\emptyset\,\right) = 0$$

Nonnegativity

For all $\mathbf{A}$ in Ω the probability of $\mathbf{A}$ is not negative:

$$\forall\,\mathbf{A}\,\in\,\Omega \quad \mathbf{P}\left(\,\mathbf{A}\,\right) \geq 0$$

Normalization

We define the sum of the probability of all the elements in Ω as being equal to 1.

$$1 = \sum_{\mathbf{A}\,\in\,\Omega} \mathbf{P}\left(\,\mathbf{A}\,\right)$$

Mutually Exclusive

We say $\mathbf{A}$ and $\mathbf{B}$ are mutually independent *if and only if*, $\mathbf{A}$ and $\mathbf{B}$ have no overlap.

$$\mathbf{A} \cap \mathbf{B} = \emptyset \iff \text{mutually exclusive random variables}$$

Additivity

The probability of $\mathbf{A}$ or $\mathbf{B}$ occurring is:

$$\begin{aligned}
\mathbf{P}\left(\,\mathbf{A} \cup \mathbf{B}\,\right) &= \mathbf{P}\left(\,\mathbf{A}\,\right) + \mathbf{P}\left(\,\mathbf{B}\,\right) - \mathbf{P}\left(\,\mathbf{A} \cap \mathbf{B}\,\right) \\
\mathbf{P}\left(\,\mathbf{A} \cup \mathbf{B}\,\right) &= \mathbf{P}\left(\,\mathbf{A}\,\right) + \mathbf{P}\left(\,\mathbf{B}\,\right) - 0 \iff \mathbf{A}\ \&\ \mathbf{B} \text{ are mutually exclusive}
\end{aligned}$$

Complement

The probability of $\mathbf{A}$ not occurring is equal to 1 minus the probability $\mathbf{A}$ occurring:

$$\mathbf{P}\left(\,\overline{\mathbf{A}}\,\right) = 1 - \mathbf{P}\left(\,\mathbf{A}\,\right)$$

Joint Probability

The probability of **A** or **B** occurring is:

$$
\begin{aligned}
&\mathbf{P}\,(\ \mathbf{A} \cap \mathbf{B}\ ) = \mathbf{P}\,(\ \mathbf{A}\ ) * \mathbf{P}\,(\ \mathbf{B} \mid \mathbf{A}\ ) \iff \mathbf{A}\ \&\ \mathbf{B} \text{ dependent} \\
&\mathbf{P}\,(\ \mathbf{A} \cap \mathbf{B}\ ) = \mathbf{P}\,(\ \mathbf{A}\ ) * \mathbf{P}\,(\ \mathbf{B}\ ) \iff \mathbf{A}\ \&\ \mathbf{B} \text{ are independent} \\
&\mathbf{P}\,(\ \mathbf{A} \cap \mathbf{B}\ ) = 0 \iff \mathbf{A}\ \&\ \mathbf{B} \text{ are mutually exclusive}
\end{aligned}
$$