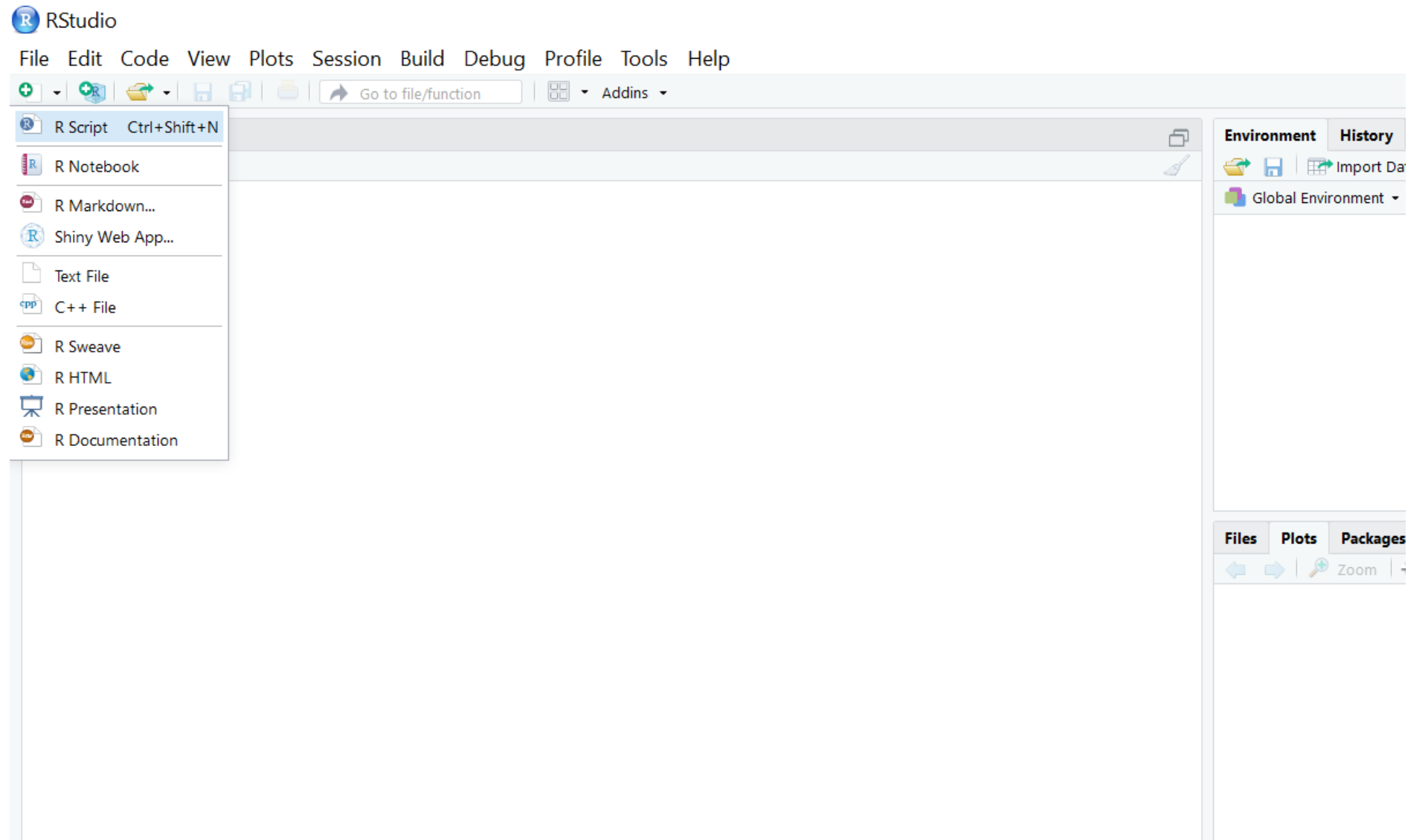
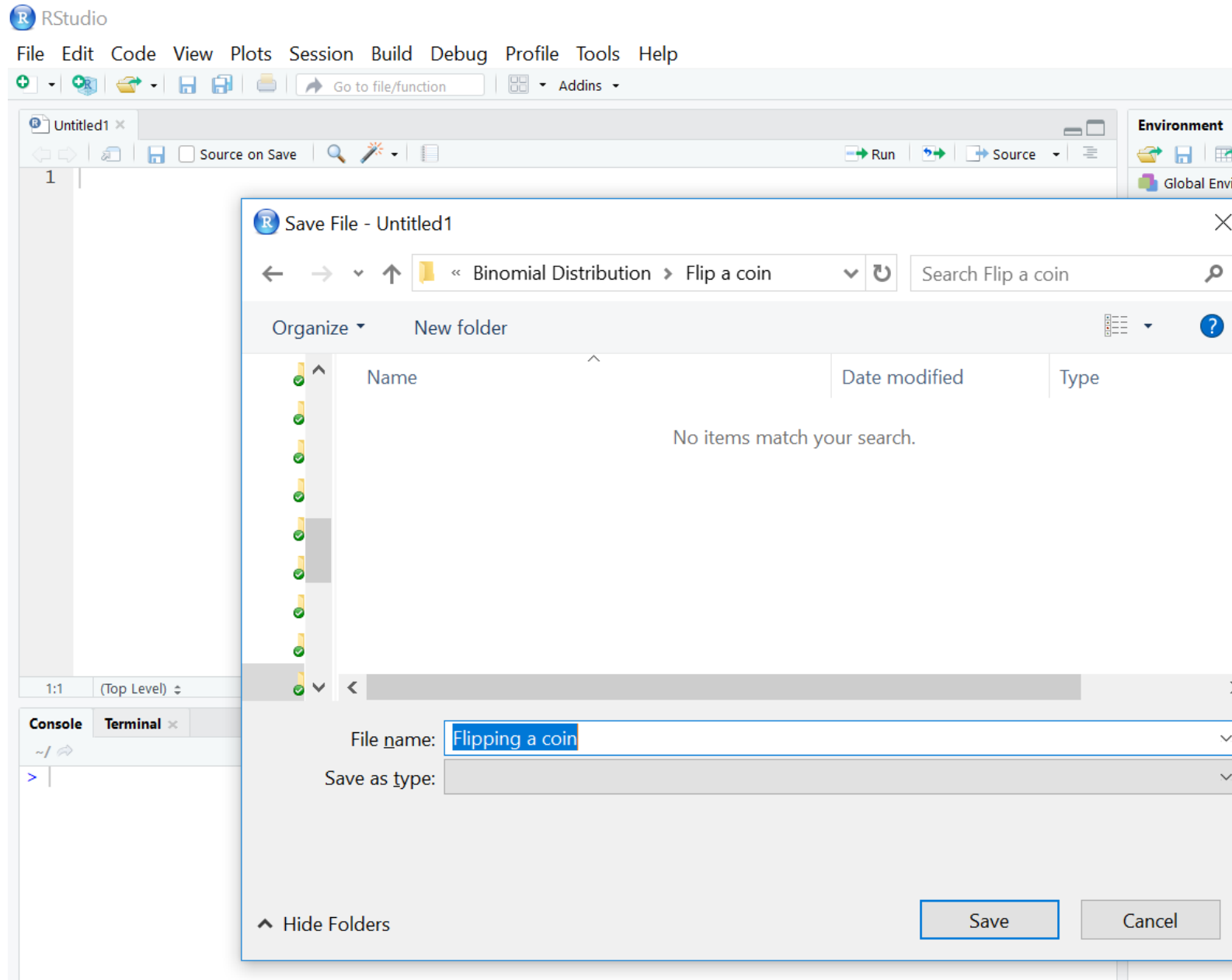


Binomial Distribution

- Open Rstudio. Click on the plus button under File on the right upper side. Then click on “R Script” to create a new R script.



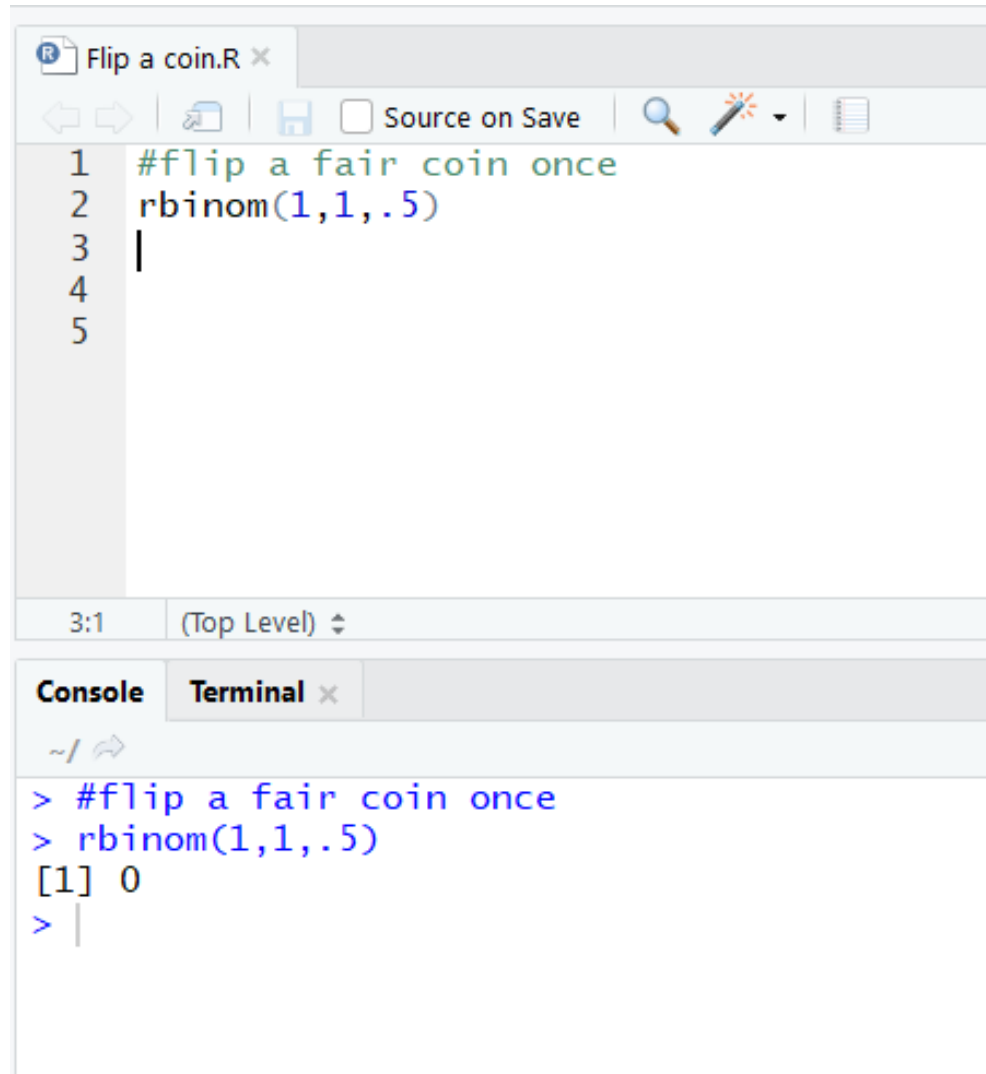
- Press ctrl+ S to save the file and rename it.



Flipping a coin in R

- Flipping a fair coin once, $P(\text{Head})=P(\text{Tail})= 0.5$
- Function: `rbinom(1,1,.5)`
- We expect the result to be 1 or 0.
- 0 represents tail and 1 represents Head.
- In R type `rbinom(1,1,.5)` and click on Run.

As you can see the result is tail (0).



The image shows a screenshot of the RStudio interface. The top pane is the script editor, titled 'Flip a coin.R'. It contains the following R code:

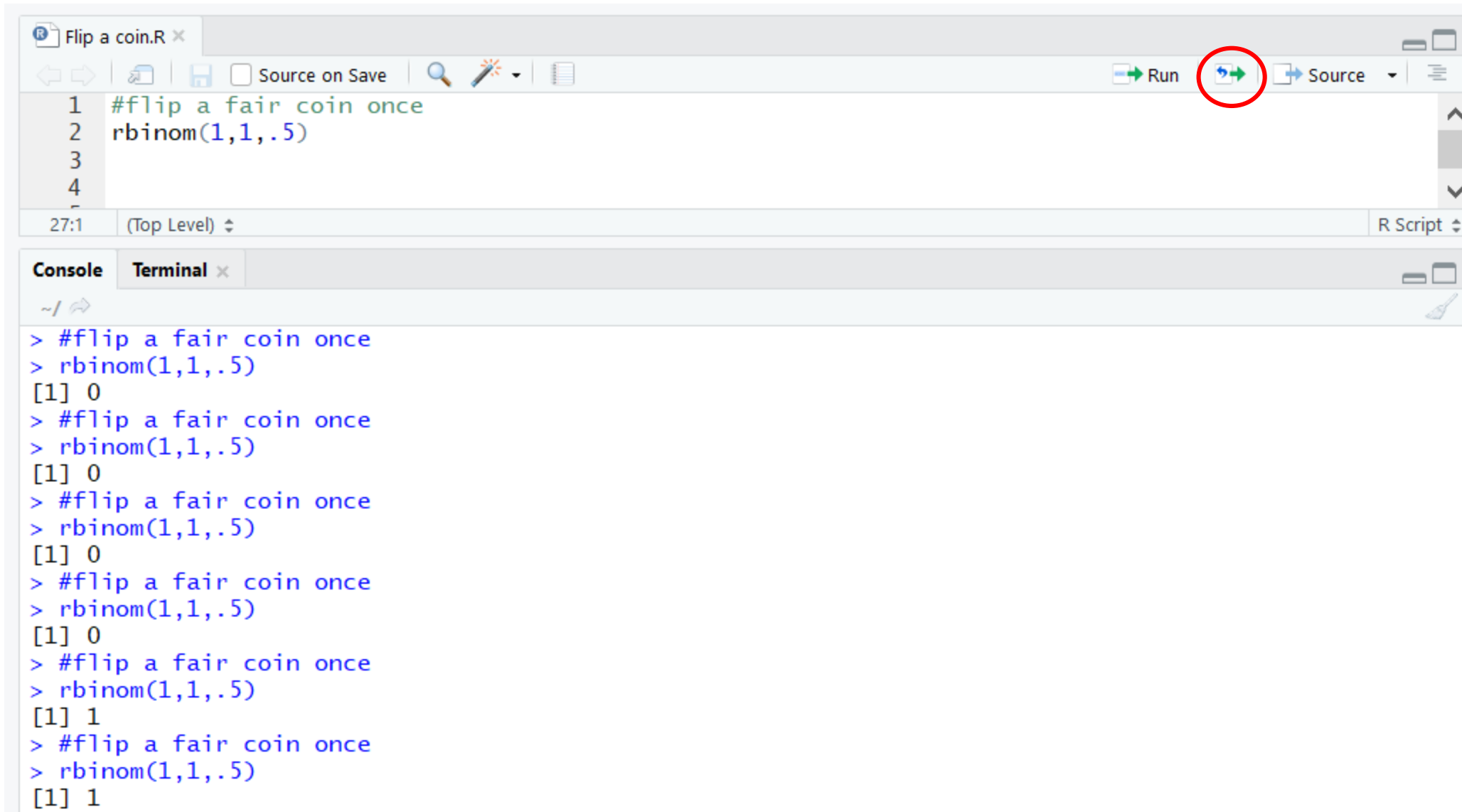
```
1 #flip a fair coin once
2 rbinom(1,1,.5)
3 |
4
5
```

Below the script editor is the console pane, which is currently active. It shows the execution of the code from the script:

```
> #flip a fair coin once
> rbinom(1,1,.5)
[1] 0
> |
```

The console output indicates that the result of the `rbinom(1,1,.5)` function call is 0, which represents a tail in a coin flip simulation.

- Click on the button between Run and Source few times to re-run rbinom(1,1,.5) few times.



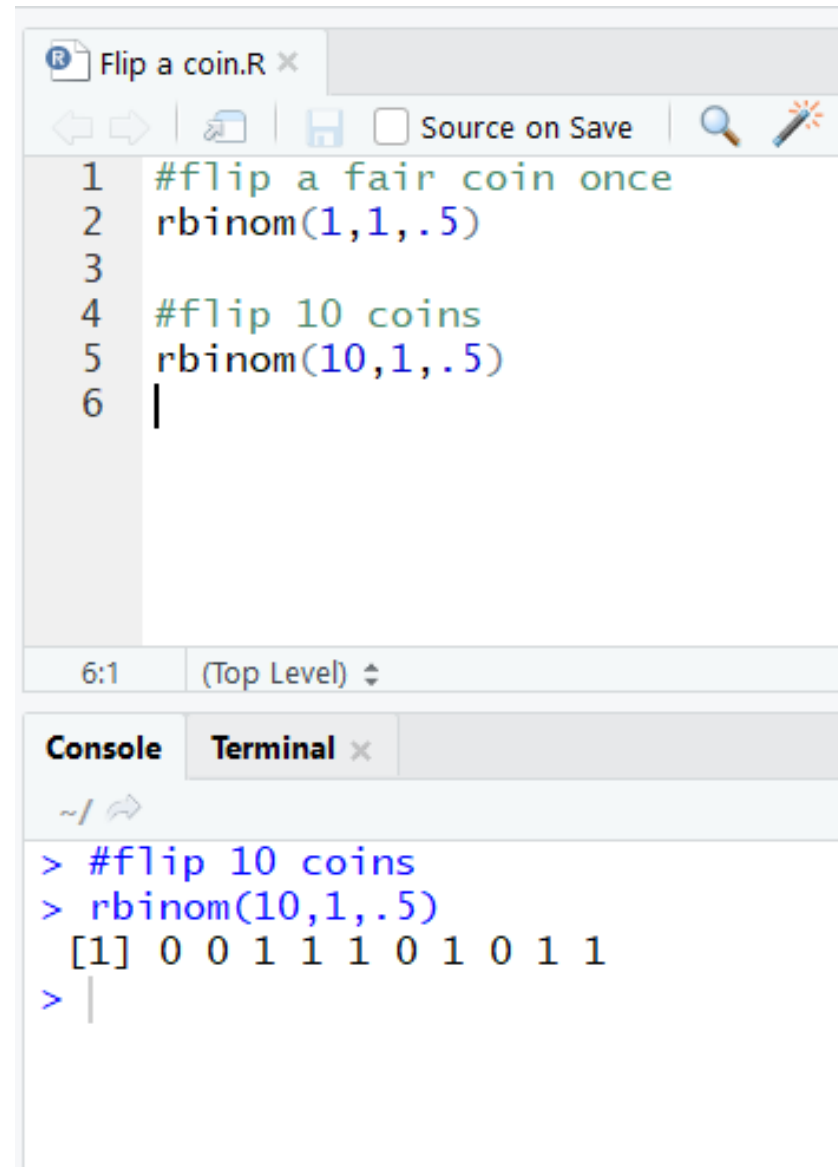
The screenshot shows the RStudio interface. The top toolbar contains buttons for 'Run' and 'Source', with a third button (a blue square with a white arrow) circled in red. The script editor shows the following code:

```
1 #flip a fair coin once
2 rbinom(1,1,.5)
3
4
```

The console shows the following output:

```
> #flip a fair coin once
> rbinom(1,1,.5)
[1] 0
> #flip a fair coin once
> rbinom(1,1,.5)
[1] 0
> #flip a fair coin once
> rbinom(1,1,.5)
[1] 0
> #flip a fair coin once
> rbinom(1,1,.5)
[1] 0
> #flip a fair coin once
> rbinom(1,1,.5)
[1] 1
> #flip a fair coin once
> rbinom(1,1,.5)
[1] 1
```

- Now using `rbinom(10,1,.5)`, flip a coin 10 times.



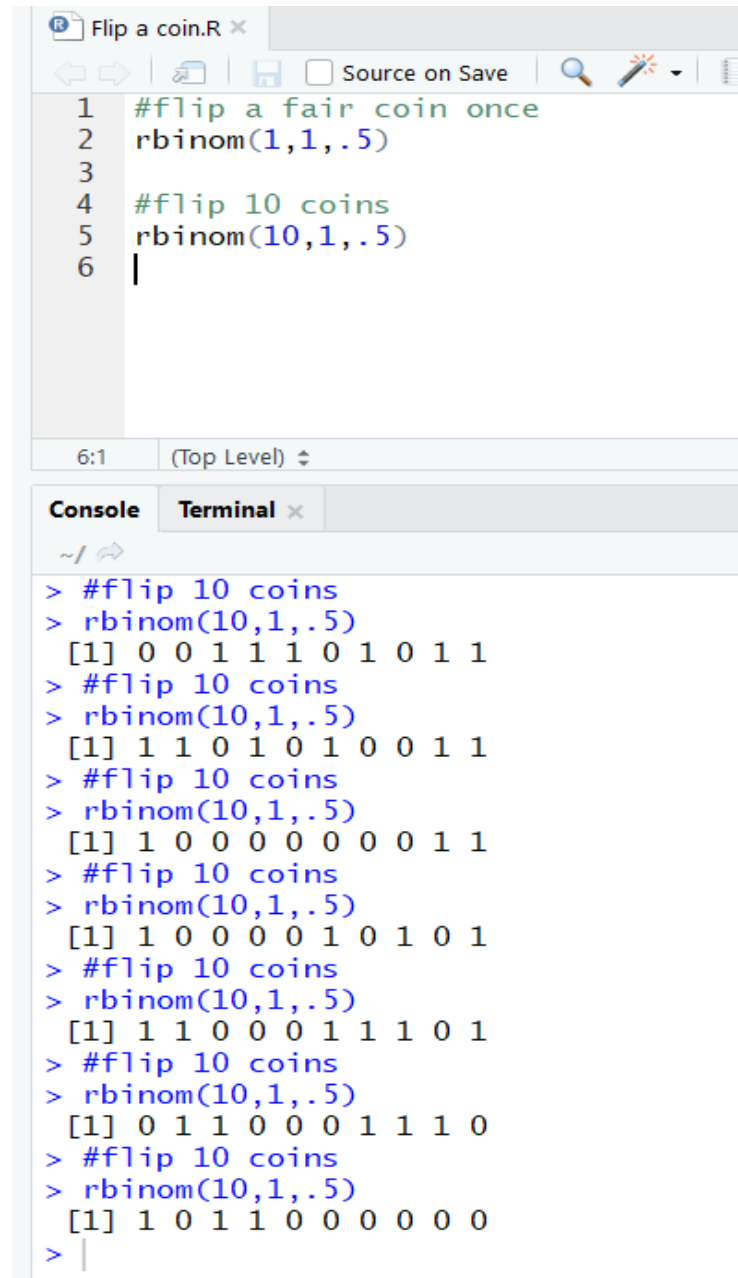
The screenshot shows an RStudio editor window titled "Flip a coin.R". The editor contains the following R code:

```
1 #flip a fair coin once
2 rbinom(1,1,.5)
3
4 #flip 10 coins
5 rbinom(10,1,.5)
6 |
```

Below the editor, the "Console" tab is active, showing the execution of the code:

```
> #flip 10 coins
> rbinom(10,1,.5)
[1] 0 0 1 1 1 0 1 0 1 1
> |
```

- Re-run the code to see different outcomes.



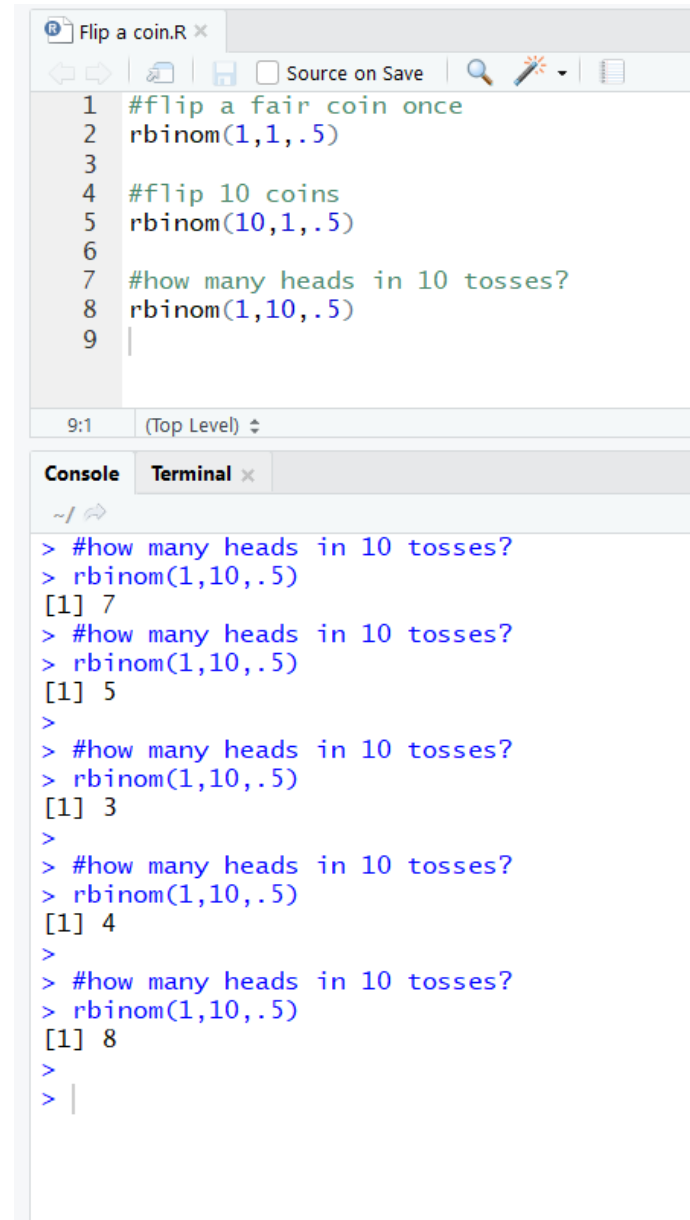
The screenshot shows an RStudio window with a script editor and a console. The script editor contains the following R code:

```
1 #flip a fair coin once
2 rbinom(1,1,.5)
3
4 #flip 10 coins
5 rbinom(10,1,.5)
6 |
```

The console shows the output of the code, which consists of 10 random binary outcomes (0 or 1) generated by the `rbinom` function. The output is displayed as a vector of length 10 for each call to `rbinom(10,1,.5)`.

```
> #flip 10 coins
> rbinom(10,1,.5)
[1] 0 0 1 1 1 0 1 0 1 1
> #flip 10 coins
> rbinom(10,1,.5)
[1] 1 1 0 1 0 1 0 0 1 1
> #flip 10 coins
> rbinom(10,1,.5)
[1] 1 0 0 0 0 0 0 0 1 1
> #flip 10 coins
> rbinom(10,1,.5)
[1] 1 0 0 0 0 1 0 1 0 1
> #flip 10 coins
> rbinom(10,1,.5)
[1] 1 1 0 0 0 1 1 1 0 1
> #flip 10 coins
> rbinom(10,1,.5)
[1] 0 1 1 0 0 0 1 1 1 0
> #flip 10 coins
> rbinom(10,1,.5)
[1] 1 0 1 1 0 0 0 0 0 0
> |
```


- `rbinom(1,10, 0.5)` will return number of heads in 10 tosses.



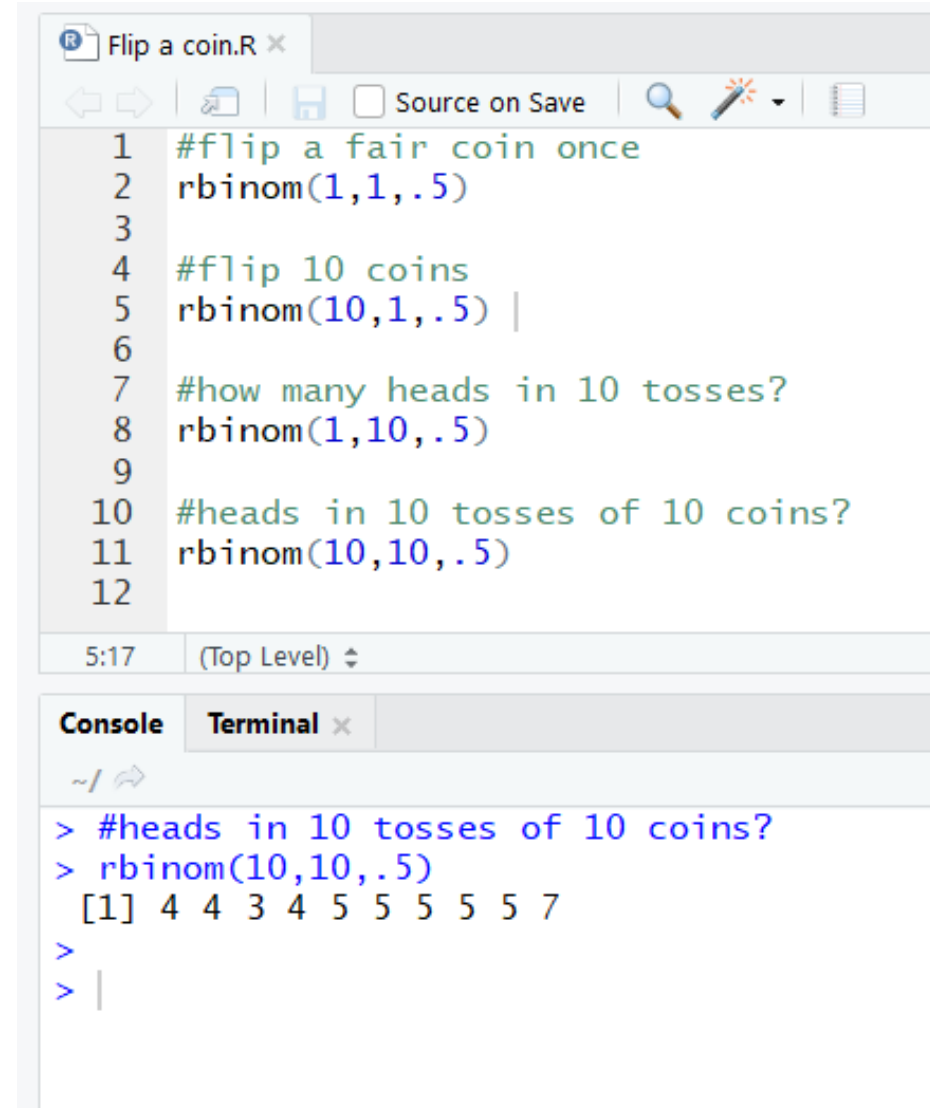
The screenshot shows an RStudio window titled "Flip a coin.R". The source editor contains the following R code:

```
1 #flip a fair coin once
2 rbinom(1,1,.5)
3
4 #flip 10 coins
5 rbinom(10,1,.5)
6
7 #how many heads in 10 tosses?
8 rbinom(1,10,.5)
9 |
```

Below the source editor is a console window with the following output:

```
> #how many heads in 10 tosses?
> rbinom(1,10,.5)
[1] 7
> #how many heads in 10 tosses?
> rbinom(1,10,.5)
[1] 5
>
> #how many heads in 10 tosses?
> rbinom(1,10,.5)
[1] 3
>
> #how many heads in 10 tosses?
> rbinom(1,10,.5)
[1] 4
>
> #how many heads in 10 tosses?
> rbinom(1,10,.5)
[1] 8
>
> |
```

- `rbinom(10,10, 0.5)` will toss 10 coins each 10 times and return number of heads out of 10 tosses for each coin.



The screenshot shows an RStudio editor window titled "Flip a coin.R". The editor contains the following R code:

```
1 #flip a fair coin once
2 rbinom(1,1,.5)
3
4 #flip 10 coins
5 rbinom(10,1,.5) |
6
7 #how many heads in 10 tosses?
8 rbinom(1,10,.5)
9
10 #heads in 10 tosses of 10 coins?
11 rbinom(10,10,.5)
12
```

Below the editor, the "Console" tab is active, showing the execution of the code:

```
> #heads in 10 tosses of 10 coins?
> rbinom(10,10,.5)
[1] 4 4 3 4 5 5 5 5 5 7
>
> |
```

- Re-run `rbinom(10,10, 0.5)`.

```
9
10 #heads in 10 tosses of 10 coins?
11 rbinom(10,10,.5)
12 |
```

12:1	(Top Level) ⬆
------	---------------

Console

Terminal x

~/ ➡

```
> #heads in 10 tosses of 10 coins?
> rbinom(10,10,.5)
[1] 4 4 3 4 5 5 5 5 5 7
>
> #heads in 10 tosses of 10 coins?
> rbinom(10,10,.5)
[1] 4 1 5 5 4 6 6 6 7 7
> #heads in 10 tosses of 10 coins?
> rbinom(10,10,.5)
[1] 5 7 7 4 8 6 3 5 4 2
> #heads in 10 tosses of 10 coins?
> rbinom(10,10,.5)
[1] 4 5 5 7 5 3 5 6 6 6
> #heads in 10 tosses of 10 coins?
> rbinom(10,10,.5)
[1] 4 7 6 6 7 5 3 6 7 8
> |
```

- `rbinom(10,10, 0.8)` will return number of heads in 10 tosses of 10 **unfair** coins.
- Each toss is a head with probability 0.8.

```
12  
13 #heads in 10 tosses of 10 unfair coins  
14 rbinom(10,10,.8)  
15 |  
16
```

15:1 (Top Level) ↕

Console Terminal x

~/

```
· #heads in 10 tosses of 10 unfair coins  
· rbinom(10,10,.8)  
[1] 7 9 9 8 9 7 10 10 10 8  
· #heads in 10 tosses of 10 unfair coins  
· rbinom(10,10,.8)  
[1] 9 8 8 8 7 6 5 7 10 9  
· #heads in 10 tosses of 10 unfair coins  
· rbinom(10,10,.8)  
[1] 9 8 9 6 7 7 6 8 7 8  
· #heads in 10 tosses of 10 unfair coins  
· rbinom(10,10,.8)  
[1] 8 5 9 8 9 10 8 7 9 9  
· #heads in 10 tosses of 10 unfair coins  
· rbinom(10,10,.8)  
[1] 7 8 9 7 7 9 8 8 8 6  
· #heads in 10 tosses of 10 unfair coins  
· rbinom(10,10,.8)  
[1] 9 8 10 6 8 7 8 9 7 8  
· |
```

What is the most common number? Why?

- `rbinom(10,10, 0.2)` will return number of heads in 10 tosses of 10 **unfair** coins.
- Each toss is a head with probability 0.2.

```
15  
16 #heads in 10 tosses of 10 unfair coins  
17 rbinom(10,10,.2)  
18 |  
19
```

18:1 (Top Level) ↕

onsole Terminal x

```
#heads in 10 tosses of 10 unfair coins  
rbinom(10,10,.2)  
[1] 2 2 1 2 3 2 1 1 1 2  
#heads in 10 tosses of 10 unfair coins  
rbinom(10,10,.2)  
[1] 3 0 3 2 0 2 2 4 1 3  
#heads in 10 tosses of 10 unfair coins  
rbinom(10,10,.2)  
[1] 1 3 4 4 2 0 2 1 0 3  
#heads in 10 tosses of 10 unfair coins  
rbinom(10,10,.2)  
[1] 5 3 4 1 3 0 4 2 1 2  
#heads in 10 tosses of 10 unfair coins  
rbinom(10,10,.2)  
[1] 2 4 1 2 0 3 1 4 1 2  
#heads in 10 tosses of 10 unfair coins  
rbinom(10,10,.2)  
[1] 0 2 4 2 3 0 4 0 0 1  
#heads in 10 tosses of 10 unfair coins  
rbinom(10,10,.2)  
[1] 1 2 0 1 2 2 4 0 1 3  
|
```

What is the most common number? Why?

More generally,

The first argument of `rbinom()` is number of experiments.

The second argument is the number of coins flips.

The third argument is the probability of a 1 (“heads”).

For example, `rbinom(100000,10,.5)` is flipping 10 fair coins, and repeating the experiment 100,000 times.

Exercise 1

- Generate 100 experiments of flipping 10 coins, each with 30% probability.
- What is the most common number? Why?

- Binomial Distribution has two parameters:
 - $X \sim \text{Binomial}(\text{size}, p)$
 - Size= number of coin flips
 - p = the probability of seeing one head in a coin flip
 - Random variable X denotes number of heads.

- We flip a fair coins 10 times. What is the probability of seeing 5 heads?
 - $X \sim \text{Binomial}(10, .5)$
 - $\Pr(X = 5) ?$

- We flip a fair coins 10 times. What is the probability of seeing 5 heads?

Simulation:

- Repeat this experiment 100,000 times: “number of draws=100,000”
- `flips <- rbinom(100000,10,.5)`
- flips contains 100000 numbers, each between 0 and 10 (number of heads).
- `mean(flips == 5)`, returns percentage of number “5” among 100000 numbers.

The result is 0.24769.

```
21  
22 # 100,000 experiments, in each experiment there are 10 coin flips, the coin is fair  
23 flips <- rbinom(100000,10,.5)  
24  
25 # percentage of 'seeing 5 heads' among 100,000 outcomes  
26 mean(flips == 5)  
27 |  
28
```

27:1 (Top Level) ↕

Console

Terminal x

~/ ↩

```
> # 100,000 experiments, in each experiment there are 10 coin flips, the coin is fair  
> flips <- rbinom(100000,10,.5)  
> # percentage of 'seeing 5 heads' among 100,000 outcomes  
> mean(flips == 5)  
[1] 0.24769  
> |
```

- `dbinom(5,10,.5)` returns probability of seeing 5 heads out of 10 tosses, for a fair coin using exact calculation.
- Note that if you re-run it, you will get the same result.
- As you can see, the result of exact calculation is **0.2460938** which is very close to the result of our simulation **0.24769**

```
17 rbinom(10,10,.2)
18
19 #flip 10 fair coins, what is the probability that we see 5 heads?
20 dbinom(5,10,.5)
21 |
22
```

21:1 (Top Level) ⬆

Console **Terminal** ✕

~/ ➡

```
> #flip 10 fair coins, what is the probability that we see 5 heads?
> dbinom(5,10,.5)
[1] 0.2460938
> #flip 10 fair coins, what is the probability that we see 5 heads?
> dbinom(5,10,.5)
[1] 0.2460938
> #flip 10 fair coins, what is the probability that we see 5 heads?
> dbinom(5,10,.5)
[1] 0.2460938
> |
```

If $X \sim \text{Binomial}(10, .5)$, then

`dbinom(k,10,.5)` returns $\Pr(X = k) = p_X(k)$

```
25 #flip 10 fair coins, what is the probability that we see 5 heads?
26 dbinom(5,10,.5)
27
28 #flip 10 fair coins, what is the probability that we see 6 heads?
29 dbinom(6,10,.5)
30
31 #flip 10 fair coins, what is the probability that we see 4 heads?
32 dbinom(7,10,.5)
33
```

33:1 (Top Level) ↕

Console

Terminal ×

~/ ↗

```
> #flip 10 fair coins, what is the probability that we see 5 heads?
> dbinom(5,10,.5)
[1] 0.2460938
> #flip 10 fair coins, what is the probability that we see 6 heads?
> dbinom(6,10,.5)
[1] 0.2050781
> #flip 10 fair coins, what is the probability that we see 4 heads?
> dbinom(7,10,.5)
[1] 0.1171875
> |
```

Exercise 2

- If you flip 10 coins each with a 30% probability of coming up heads, what is the probability exactly 2 of them are heads?
- Compare your simulation with the exact calculation.

Exercise 3

- For exercise 2,
- Part a) use 10000 experiments and report the result.
- Part b) use 100000000 experiments and report the result.
- Compare the result of part a and part b, with the exact calculation.
What is your conclusion?

If $X \sim \text{Binomial}(10, .5)$, what is the $E[X]$? using calculation $E[X] = 5$.

- Simulation: run the experiment 100,000 times.
- `flips <- rbinom(100000, 10, .5)`
- `mean(flips)`: the average number of heads

```
33  
34 #average number of heads  
35 mean(flips <- rbinom(100000, 10, .5))  
36 |  
37  
38
```

36:1 (Top Level) ↕

Console

Terminal ×

~/ ↗

```
> #average number of heads  
> mean(flips <- rbinom(100000, 10, .5))  
[1] 4.99987  
> #average number of heads  
> mean(flips <- rbinom(100000, 10, .5))  
[1] 4.99843  
> #average number of heads  
> mean(flips <- rbinom(100000, 10, .5))  
[1] 5.00363  
> #average number of heads  
> mean(flips <- rbinom(100000, 10, .5))  
[1] 4.99523  
> |
```

Result of simulation is close to 5

If $X \sim \text{Binomial}(100, .2)$, what is the $E[X]$? using calculation $E[X] = 20$.

- Simulation: run the experiment 100,000 times.
- `flips <- rbinom(100000, 100, .2)`
- `mean(flips)`: the average number of heads

```
36  
37 #average number of heads  
38 mean(flips <- rbinom(100000, 100, .2))  
39 |  
40
```

39:1 (Top Level) ↕

Console

Terminal ×

~/ ↗

```
> #average number of heads  
> mean(flips <- rbinom(100000, 100, .2))  
[1] 20.02572  
> #average number of heads  
> mean(flips <- rbinom(100000, 100, .2))  
[1] 20.01465  
> #average number of heads  
> mean(flips <- rbinom(100000, 100, .2))  
[1] 19.99774  
> #average number of heads  
> mean(flips <- rbinom(100000, 100, .2))  
[1] 20.01099  
> #average number of heads  
> mean(flips <- rbinom(100000, 100, .2))  
[1] 20.00108  
> |
```

Result of simulation is close to 20

Exercise 4

- What is the expected value of a binomial distribution where 25 coins are flipped, each having a 30% chance of heads?
- Compare your simulation with the exact calculation.

If $X \sim \text{Binomial}(10, .5)$, what is the $\text{Var}[X]$? using calculation $\text{Var}[X] = 2.5$.

- Simulation: run the experiment 100,000 times.
- `X <- rbinom(100000, 10, .5)`
- `var(X)`: the variance

```
39  
40 var(rbinom(100000, 10, .5))  
41  
42  
40:1 (Top Level) ↕  
Console Terminal ×  
~/ ➡  
> var(rbinom(100000, 10, .5))  
[1] 2.487262  
> var(rbinom(100000, 10, .5))  
[1] 2.511887  
> var(rbinom(100000, 10, .5))  
[1] 2.490653  
> var(rbinom(100000, 10, .5))  
[1] 2.511053  
> |
```

Result of simulation is close to 2.5

If $X \sim \text{Binomial}(100, .2)$, what is the $\text{Var}[X]$? using calculation $\text{Var}[X] = 16$.

- Simulation: run the experiment 100,000 times.
- `X <- rbinom(100000, 100, .2)`
- `var(X)`: the variance

```
41  
42 var(rbinom(100000, 100, .2))  
43 |  
44  
43:1 (Top Level) ⚡  
Console Terminal ×  
~/  
> var(rbinom(100000, 100, .2))  
[1] 16.04723  
> var(rbinom(100000, 100, .2))  
[1] 15.99349  
> var(rbinom(100000, 100, .2))  
[1] 16.0699  
> var(rbinom(100000, 100, .2))  
[1] 15.99154  
> var(rbinom(100000, 100, .2))  
[1] 15.96807  
> var(rbinom(100000, 100, .2))  
[1] 16.02688  
> |
```

Result of simulation is close to 16

Exercise 5

- What is the variance of a binomial distribution where 25 coins are flipped, each having a 30% chance of heads?
- Compare your simulation with the exact calculation.