CSc 217 – Probability and Statistics for Computer Science

Examination №2

by

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Administered on

2023 – 11 – 07

Student: _____

Write both your first and last name on the line above

Examination Instructions

- Read each question carefully, preferably twice.
- If you do not understand a question, raise your hand and ask for clarification.
- There are no "trick questions."
  The most straight-forward interpretation of the problem statement is likely the correct interpretation.
- Show as much of your work and thought process as possible.
  Partial credit will be given for partially correct answers.
- I am looking to *give* credit for correctness, not *deduct* credit for mistakes.
  It is *always better* to be more verbose than to be terse, since you will receive full credit as long some of your solution correctly conveys the answer.

Examination Scoring

Final Score = **Question 1**
+ **Question 2**
+ **Question 3**
+ Best 2 out of 3
(**Question 4**, **Question 5**, **Question 6**)

Question 1:

Definitions (20 Points)

Describe to the best of your ability the definition of each probability theory concept. You may use the English language, mathematical notation, or some combination of both.

1 (a): (5 Points)

" Probability Mass Function (PMF) of a random variable $\mathbf{X}$ "

1 (b): (5 Points)

" PDF Total Probability Theorem "

1 (c): (5 Points)

" Expectation of a *discrete* random variable $\mathbf{X}$ "

1 (d): (5 Points)

" Standard Deviation of a random variable $\mathbf{X}$ "

Question 2:

True or False **(20 Points)**

Decide whether each statement is `True` or `False`,
based on the probability mass functions for random variables $\mathbf{X}$ and $\mathbf{Y}$:

$$\rho_{\mathbf{X}} = \begin{cases} 1 & \mapsto & 4/10 \\ 2 & \mapsto & 3/10 \\ 3 & \mapsto & 2/10 \\ 4 & \mapsto & 1/10 \end{cases} \qquad \rho_{\mathbf{Y}} = \begin{cases} 1 & \mapsto & 1/30 \\ 2 & \mapsto & 4/30 \\ 3 & \mapsto & 9/30 \\ 4 & \mapsto & 16/30 \end{cases}$$

2 (a): (4 Points)

"`True` $\oplus$ `False` : $2 = \mathbb{E}[\mathbf{X}]$"

2 (b): (4 Points)

"`True` $\oplus$ `False` : $\pi = \mathbb{E}[\pi \times \mathbf{X} - \pi]$"

2 (c): (4 Points)

"`True` $\oplus$ `False` : $\pi \geq \mathbb{E}[\mathbf{Y}]$"

2 (d): (4 Points)

"`True` $\oplus$ `False` : $\pi \geq \mathtt{Var}(\,1/10 \times \mathbf{Y} + 5040\,)$"

2 (e): (4 Points)

"`True` $\oplus$ `False` : $\mathbb{E}[3 \times \mathbf{X} + 2] = \mathbb{E}[3 \times \mathbf{Y} - 2]$"

Question 3:

Support or Falsify (20 Points)

Consider each statement and regarding the following *joint* probability mass function $\rho_{\mathbf{X},\mathbf{Y}}$ and either:

$$\rho_{\mathbf{X},\mathbf{Y}} = \begin{cases} 3 \mid {}^{1}/_{68} & {}^{2}/_{68} & {}^{25}/_{68} \\ 2 \mid {}^{2}/_{68} & {}^{4}/_{68} & {}^{16}/_{68} \\ 1 \mid {}^{3}/_{68} & {}^{6}/_{68} & {}^{9}/_{68} \\ \hline \mathbf{Y}/\mathbf{X} \mid 1 & 2 & 3 \end{cases}$$

(Support): State that it is `True`**. Explain the reason why you believe that is the case. If you feel capable of providing a proof or a sketch/outline of a proof, please do so**

(Falsify): State that it is `False`**. Explain the reason why you believe that is the case. If you have a counterexample which shows that the statement is false, please provide it.**

3 (a): (5 Points)

"`Support` $\oplus$ `Falsify` : $\mathbb{E}\left[\rho_{\mathbf{X}}\right] < \mathbb{E}\left[\rho_{\mathbf{Y}}\right]$"

3 (b): (5 Points)

"`Support` $\oplus$ `Falsify` : $\rho_{\mathbf{X}|\mathbf{Y}}(x \mid 2) < \rho_{\mathbf{Y}|\mathbf{X}}(y \mid 2)$"

3 (c): (5 Points)

"`Support` $\oplus$ `Falsify` : ${}^{1}/_{2} < P(\mathbf{X} < \mathbf{Y})$"

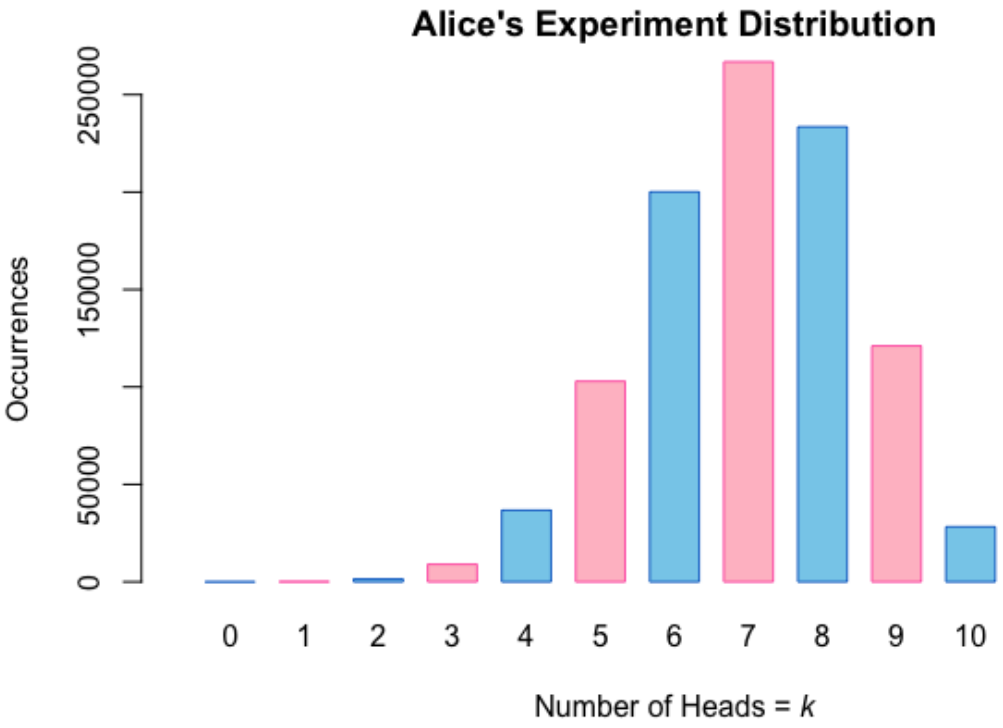
3 (d): (5 Points)

"`Support` $\oplus$ `Falsify` : ${}^{1}/_{2} < P(\mathbf{X} = 3)$"

Question 4:

Problem 1 (20 Points)

Alice flips a coin n times. She counts k heads occurring from the n coin flips. In order to determine the probability $P(\mathrm{H}) = q$ that a coin flip produces H, Alice repeats the experiment $1,000,000$ times. The plot of outcomes from all of Alice's experiments is shown below.



4 (a): (10 Points)

"How many coins did Alice flip in each experiment; i.e. what is the value of n?"

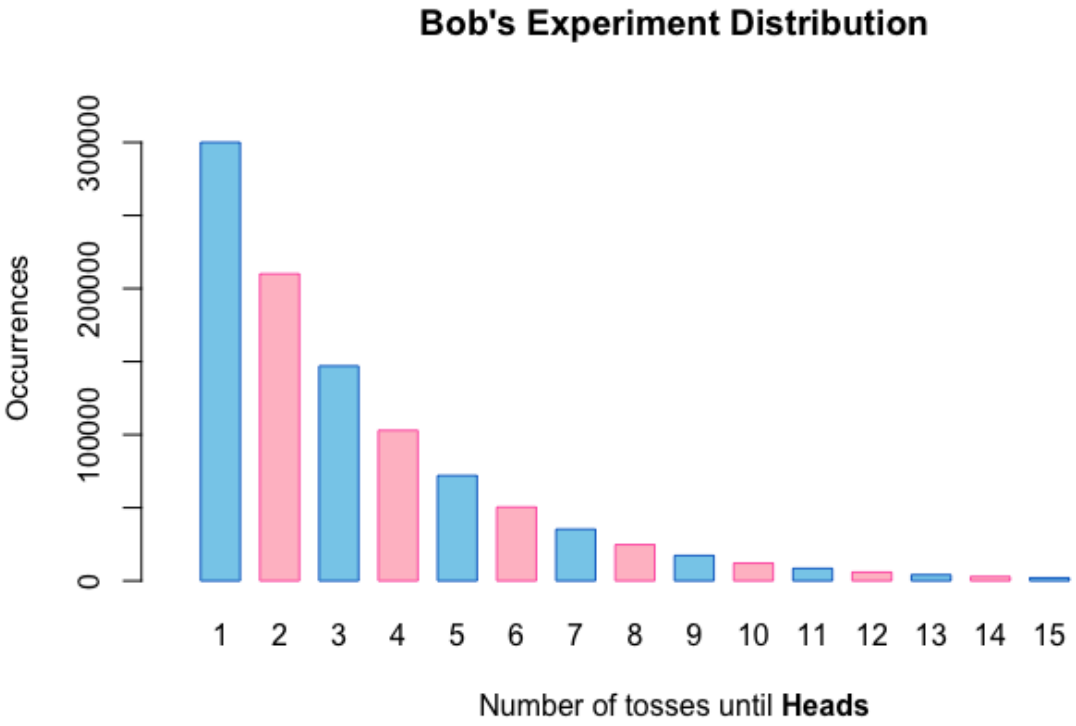
4 (b): (10 Points)

"What is the probability of getting heads Alice flips her coin; i.e. what is $P(\mathrm{H}) = q$?"

Question 5:

Problem 2 (20 Points)

Bob flips a coin as many times as necessary until a heads $\mathtt{H}$ occurs. He notes the number of flips required for $\mathtt{H}$ to occur. In order to determine the probability $P(\mathtt{H}) = q$ that a coin flip produces $\mathtt{H}$, Bob repeats the experiment $1,000,000$ times. The plot of outcomes from all of Bob's experiments is shown below.



5 (a): (10 Points)

"What is the probability of heads when flipping Bob's coin; i.e. what is $P(\mathtt{H}) = q$?"

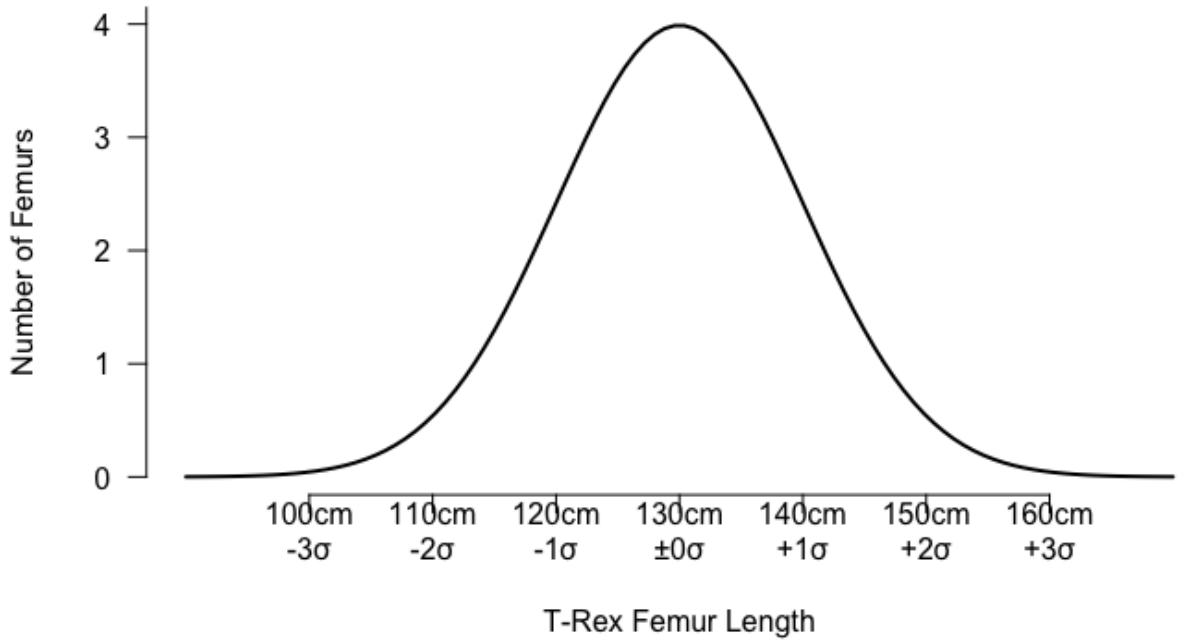
5 (b): (10 Points)

"On average, how many coin flips are required for a $\mathtt{H}$ to occur; i.e. $\mathbb{E}[\mathbf{X}] = q$?"

Question 6:

Problem 3 (20 Points)

Ross is a paleontologist with the Museum of Prehistoric History and just unearthed a mass grave of $100!$ T-Rex fossils on his fieldwork expedition! Ross diligently takes measurements of all fossils recovered. Later Ross looks over his measurements of all 100 T-Rex femur lengths and plots the number of femurs with the same length as shown below.



6 (a): (10 Points)

" What is the average femur length Ross found? "

6 (b): (10 Points)

" What is the standard deviation of the femur lengths Ross found? "