Recursive Notation (As a Formal Component of Breeze Theory: A Foundational Framework for Recursive Reality) (V.1.41825)

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Axiomatic Incompletion

The formally represented cornerstone comprising the mathematical core of this theory lies in an absolute embrace of the implications revealed in Gödel's Incompleteness Theorems. These theorems effectively state that within any formal system of logic, there will always be statements of truth which cannot be proven within that system. This proof can therefore be understood as the axiomatic law of incompleteness underpinning all formal and mathematical systems. Considering the universal observations of recursive processes outlined in the theory, this logic is naturally extended into an all-encompassing universal principle. In this way, the incompleteness theorems may be effectively understood as the first iteration of the final and only law, that being the law of axiomatic erosion.

Notation for Recursive Expression

The mathematical representation of recursive reality requires a framework that acknowledges its own incompleteness while maintaining coherent structure. To instill a core understanding of the recursive logic underpinning this notation, we must understand how it is designed to scale "outward" from the axiom of incompletion.

This notation encodes the fundamental principle of axiomatic erosion through the substrative frequency $S(\infty)$, defined by the dynamic interplay of its constituent forces incendence $S(i)$ and excendence $S(e)$. This distinction offers a mechanism for exploring the dynamic nature of all differentiated expressions, at all scales, without losing precision or risking circularity. In this way, this notation can be seen quite simply as the necessary "skeleton" over which we may increasingly define and expand on the infinite spectrum of differentiated expressions intrinsically bound from the same, fundamentally infinite pattern.

In other words, this notation creates a way to effectively layer dynamic bounds within a transient system. Thus, all expressions can be seen as functions arising within a set of underlying or "parent" bounds, scaling all the way to the substrative frequency $S(\infty)$ itself. This strategy allows us to scale notation consistently, flexibly, and with uncompromising precision.

This framework also establishes the general concept of bound fracta $b(f)$, expressing the fundamental building blocks (fracta) of reality at any given scale or environment. This effectively ties all fundamental "blocks" of reality into localized expressions of dynamically bound substrative forces. Any expression of these fracta must inherently be incendent due to the bound nature of their very expression, since any bound – by definition – necessarily reflects the structured interplay of self-adhering and synergistic dynamics within it.

That said, since bounds can be expressed at any scale, we are still able to utilize them to theoretically isolate and map excendent aspects within any system, as we will demonstrate later through $e(b(f))$. That said, even with this notational ability, we must remember that all functions operate at a scale relative to its observation (or binding); so therefore, excendent process observed at one scale of observation may reflect an incendent aspect of another "higher-order" recursive expression.

Also – since none of these relationships may ever be held as "explicit", or axiomatically defined – linear/causal relationship mapping is restricted in its ability to describe systems within a recursive context; therefore, this notation allows us to layer bounds directly, as well as denote hierarchical relationships between the expressions we are defining. If all expressions are relative to infinite self-reference most purely, these tools allow us to map a self-contained "web" of recursive expression while sufficiently accounting for the incomplete nature of every expression therein.

Through $b(f)$, we can explore direct interactions through bounded layering, while the subset notation $\subset$ enables us to establish hierarchical relationships within the recursive notation.

Now, starting with the parent equations:

$$\text{Reality} = S(\infty) \otimes E(\delta)$$

$$E(\delta) = \bigotimes \sum b(f)$$

$$S(\infty) = S(i) \otimes S(e)$$

$$b(f) \subset E(\delta)$$

- $S(\infty)$ represents the substrative frequency, denoting infinite recursion as an "active" force

- $E(\delta)$ represents all exspheric (universally differentiated) expressions
- $\otimes$ is the tensorial operator to reflect the synergistic (recursive) interplay between any bound expressions or substrative forces
- Bound fracta $b(f)$ emerge as a recursive product of this dynamic substrative interplay, effectively characterizing the structural building blocks or "scaffolding" of expressed reality
- $\subset$ represents a recursive subset, or a "lower-order" expression within the recursive hierarchy, relative to a given higher-order bound. (i.e., $\subset$ denotes a derivative fracta)
- $S(i)$ represents the substrative expression of incendence, or unity, integration, and stable coherence; also known as "feedback loops"
- $S(e)$ represents the substrative expression of excendence, or active differentiation, growth, fractal expansion
- $b(f) \subset E(\delta)$ provides notational closure by ensuring that no bound fracta may ever be isolated in connection to the whole of exspheric expression; all bounds are derivative localizations of the bound characterized by all differentiated expressions

The first equation $S(\infty) \otimes E(\delta)$ shows how reality is expressed through the perpetual synergistic interplay ($\otimes$) of exspheric differentiation $E(\delta)$, and the substrative frequency $S(\infty)$, which is the infinite pattern (force) which encodes incompleteness into the structure of its very notation. Reality emerges as this substrative pattern perpetually sustains and transforms all differentiated structures (expressions outside of infinite self-reference). This is the first proposed notation to properly account for the paradox of incompleteness while retaining robustness throughout its scaling and definition.

While $S(\infty)$ could be technically argued as already containing all patterns, the reality equation seeks to provide a "window" of bound observation through which to map localized expressions. Thus, by introducing the synergistic interplay with the exspheric bound $E(\delta)$, we are effectively providing a differentiated (expressed) basis through which to further investigate these incomplete (or localized) patterns, by noting they are precedingly sourced by the pattern which reflects all (infinite) patterns.

The tensor operator $\otimes$ is crucial for distinguishing the infinite nature of how reality interacts with itself at all scales, denoting the essential fact that the explicit nature of such interactions can never be fully defined due to the dynamic nature of the infinite structure that gives rise to them. This reflects our notational philosophy that reality can only be understood and described through the process of binding approximations of expression.

The equation $E(\delta) = \otimes \sum b(f)$ shows us how all differentiated structure $E(\delta)$ can be further defined by the tensorial sum of all constituently bound fracta. It is important to reiterate the fact that any differentiated "bound," by its very nature, possesses an incendent quality in relation to the expression of its parent bound. In other words, binding all differentiation exspherically gives us a way to denote the structure of differentiation, rather than the act of differentiation itself (which is already encoded through $S(e)$). This enables us to approximate endlessly while acknowledging pure differentiation $|\delta|$ more clearly as the act of excendent or "unbound" expression, as we will explore later.

$\otimes \sum b(f)$ allows us to define bounds of differentiated expression at literally any scale of exspheric interaction. The tensorial sum operator remains crucial in showing how both the summation and the interaction of all fracta (or bound expressions within the bound of all expressions) are synergistically interdependent and therefore cannot be isolated in any explicit fashion. This also gives us a way to denote the aggregate of recursive processes for any given bound; in this case, all bounds, since $E(\delta)$ seeks to capture all differentiated expressions. This notational tool reflects the mathematical structure of differentiated recursion, reinforcing that even within local bounds, the processes they contain retain their fundamentally infinite nature.

Recursive Subsets and Hierarchical Differentiation

Lastly, it is important to note that $\subset$ denotes a recursive subset, meaning an expression that exists as a differentiated sub-bound within a higher-order recursive bound. This subset can also be understood as a derivative fracta within the recursive hierarchy. This implies that the subset's properties are recursively conditioned by (and partially reflective of) the higher-order recursive structure within which it is nested. Naturally, this should not be seen as a fixed, static, or mutually exclusive relationship; rather, this notation offers another method to more precisely define recursive relationships while remaining mathematically consistent and respecting the nature of infinitely relational processes.

Another way to understand the hierarchical utility of $\subset$ is through relative orders of differentiation. Thus, $b(x) \subset b(y)$ would suggest that x reflects a more differentiated (lower-order) expression of y. More differentiated/lower-order expressions would be seen as more "evolved" substrative expressions, while higher-order expressions suggest a closer "recursive proximity" to the undifferentiated substrate itself.

In this way, $b(f) \subset E(\delta)$ effectively "closes the loop" of our core equations by denoting that each and every system remains a hierarchical reflection of universal expression itself. As a result, all local expressions may be expanded on and further defined as their own unique bound, while maintaining a functional unity in relation to the broader exspheric structure.

The Substrative Frequency Equation

Now, let us look more closely at the substrative frequency equation:

$$S(\infty) = S(i) \otimes S(e)$$

This equation allows us to bind observable aspects of the infinite substrative force $S(\infty)$ while respecting its incompleteness through the tensor operation. It is essential to understand that these are not necessarily dualistic or additive forces; rather, they are bound aspects of a unified fundamental pattern, which reflects their interplay at every scale of expression. Therefore, $S(i)$ and $S(e)$ should be seen as complementary forces working in infinite tandem to sustain and transform exspheric reality.

- **Incendence** $S(i)$ represents the self-adhering, integrative, or self-sustaining aspects of substrative expression. This force can be most intuitively understood through the structure of stability, integration, coherence, and feedback loops, revealing these loops to be integral components of reality's true nature, rather than "paradoxical" emergences.

- **Excendence** $S(e)$ alternatively represents the pure differentiating force, or the substrative process of fractal outgrowth and transformation. The excendent aspect of any bound expression may be observationally understood as the derivative or differentiated trajectory of that bound's stable, or incendent aspect.

These two forces work together through an endlessly intricate dance, synergizing to create new fracta and perpetually giving rise to more varieties of differentiated expression. Incendence is the sustaining force, excendence is the driving force, and they interact together in increasingly infinite fashion to define an increasingly infinite reality.

It's also worth noting, however, even excendent forces $S(e)$—which are essential for growth—can themselves "dominate" and become incendently bound, creating recursive loops that scale disequilibrium infinitely. This logic suggests, interestingly, that because even excendent bounds can become integrative and self-reinforcing, they will ultimately produce an excendent fracta $e(b(f))$, which could potentially further give rise to endless tiers of "relative" excendent expression, such as malbinding at the metarecursive scale of differentiation.

With this in mind, we can now understand how to bind any pattern of differentiated expression in a consistent fashion through:

$$b(f) = b(S(i) \otimes S(e))$$

$$S(i) \subseteq \infty(\delta(\infty))$$

$$S(e) \subseteq |\delta|$$

This equation shows us that any bound approximation, or fracta, may be understood as the bound synergistic interplay of incendent and excendent forces. This parallels with the notational fact that any bound expression of these substrative forces must have a coherent aspect, due to its very being bound, while still allowing us to encode excendently bound process as a function of incendent forces. This highlights the inseparability of these forces on a fundamental level, while allowing us to map how they express themselves across scales consistently.

The $S(e)$ subset equation:

$$S(e) \subseteq |\delta|$$

is intended to reflect the primary substrative force of excendence as a function of pure, unbound differentiation. $|\delta|$ may also be conceptually understood as identical to $\infty(\delta)$, which reflects the first "action" or unbounded expression of infinite recursion (the substrate). This enhances our primary equations by showing how "raw" differentiation itself is how excendent forces express themselves; thus, any bound differentiation $b(\delta)$ denotes the structure of that differentiation, while $|\delta|$ denotes the force of differentiation itself. This is a critical distinction to understand within the notation.

$$S(i) \subseteq \infty(\delta(\infty))$$

Then $S(i)$ arises as a subset of the substraeternum equation, which defines the expression of all feedback loops through the anchor point that is recursive self-expression. This is another way to say that all incendent forces are feedback loops which reflect a subset of the ultimate exspheric feedback loop that is recursion recognizing itself (through differentiation). In other words, the substraeternum by very definition can be seen as the fracta binding all other fracta across all differentiated substrative expression, effectively anchoring that expression at every scale of transformation.

We should note that while the proper recursive subset $\subset$ is used when notating hierarchical recursive relationships (since no differentiated bound can ever fully equate to a less or "higher-order" differentiated bound), we still use the equal subset $\subseteq$ for describing the $S(i)$ and $S(e)$ mappings, as they are intended to reflect the synergistic necessity of recursive expression more generally.

Thus, it remains logically coherent to denote how these forces ($S(i)$ and $S(e)$) are not strictly sub-bounds; rather, they are coherent reflections of symbolically fundamental recursive relationships within a fully self-contained and internally consistent framework.

The Substraeternum Equation

Arguably most essentially to this framework is the Substraeternum equation itself:

$$\aleph_\delta = f_\infty(\delta) = \infty(\delta(\infty))$$

Where:

- $\aleph_\delta$ represents the cardinal set of all recursive differentiation.
- $f_\infty(\delta)$ represents the substrative or "primary" fracta of all differentiation.
- $\infty(\delta(\infty))$ denotes the anchor of infinite self-expression through differentiation. This can be interpreted as the instance of differentiation through which infinite recursion "achieves" itself.

The substraeternum is the anchor of all differentiated feedback loops, the foundation of proper recursive awareness. Recursion recognizing itself (self-reference referencing itself purely for the first time) reflects both a recursive "instance", and the nature of incendent expression more generally. The substraeternum is the self-evident truth, and it is the event that permanently establishes a differentiated relationship with reality's recursive substrate.

And now, with the substraeternum equation established, we can coherently denote infinity itself and provide recursive closure through:

$$\infty(\delta(\infty)) \subset \infty$$

Where:

- $\infty(\delta(\infty))$ represents infinite recursive differentiation, as defined above
- (∞) represents pure undifferentiated recursion; the infinite substrate itself
- $\subset$ denotes the recursive subset, reflecting that all recursive differentiation is necessarily a lower-order derivation of undifferentiated recursion (∞)

This mapping of infinite recursion is both unique and necessary specifically because it does not define undifferentiated recursion in a traditional, or “equational” sense. Rather, this shows that the primary action of recursion itself is a hierarchical subset of infinite (undifferentiated) recursion. This alleviates us from the need of defining infinity itself, which is naturally impossible, since any expressed definition would inherently differentiate the thing being expressed. Thus, this preserves axiomatic erosion while respecting the uncontainable nature of the infinite substrate in its purest form.

This is essential because, while $S(\infty)$ provides the active pattern or expression of the infinite substrate, $\infty(\delta(\infty)) \subset \infty$ allows us to anchor the fundamental necessity of (∞) without violating the complete yet unresolvable nature of its infinite precedence.

Given this insight, we can further explore how all incendent forces are localized subsets of the primary differentiated feedback loop that is the substraeternum:

$$S(i) \subseteq \infty(\delta(\infty))$$

And:

$$S(e) \subseteq |\delta| \quad \text{(unbound differentiation)}$$

Where:

- $S(i)$ and $S(e)$ reflect the primary forces of substrative expression
- $|\delta|$ represents pure, unbound differentiation: fractal expansion as a raw substrative force $\infty(\delta)$ before it is “contained” into a bound form ($\infty(\delta(\infty)$, $b(\delta)$, $b(f)$, etc.)
- $\subseteq$ denotes the ”symbolic localizations” of the fundamental recursive expressions

In this way, all bound expressions can be seen as excendent in that they are inherently differentiated, and incendent in that they are intrinsically maintained and/or bound. This relationship underscores the idea of substrative dynamics not as separate, but as intertwined aspects of a unified fundamental force, fitting within the broader claim of the framework that no single aspect of reality may ever truly be isolated from the whole.

In other words: we must keep in mind at all times, that all notation is still but a differentiated expression of the same unified recursion. Therefore, any attempts to define a “more fundamental” force through either $S(i)$ or $S(e)$ will necessarily lead back to $S(\infty)$. These notations are tools to describe how this pattern interacts with itself, but no notation can ever encapsulate the unified pattern beyond acknowledging it as the self-referential foundation of all differentiated expression.

Renexial / Temporal Climate

Lastly, this framework not only accounts for all present frameworks and equations within formal mathematics and science, but it shows why they must be inherently complete. In this way, any formal scientific or mathematical framework can be reflected through $b(f)$ fracta as a bound system of measurable differentiation. Specifically, by holding for the renexial gradient $g(Rx)$ through our local climate equation, this effectively enables all of our "standard" spacetime laws such as time, gravity, etc., within which our traditional formal systems may operate effectively.

$$Rx(\delta) \subset E(\delta)$$
$$Rx(\delta) = \bigotimes \sum Rx(b(f))$$
$$Rx(b(f)) = Rx(b(S(i) \otimes S(e)))$$

Where:

- $Rx(\delta)$ is renexspheric (galaxy/local bound) expression.
- $g(Rx)$ is the binding medium—called the renexial gradient—which enables holding local constants (gravity, time, etc.).
- $Rx(b(f))$ is any bound fracta within the renexial gradient.

Formal Systems as Renexial-Bound Fracta

$$g(Rx(S(i) \otimes S(e))) = g(Rx(b(f)))$$
$$b(f_m) \subset Rx(b(f))$$
$$b(f_m) = b((ax) \otimes S(e))$$

Where:

- $b(f_m)$ is any formal model or framework of measurement.
- (ax) represents fixed (stable) axiomatic assumptions.
- $S(e)$ represents the differentiating force being constrained or measured.

Metarecursion and the Extraeta Fracta

$$
\begin{aligned}
XT(b(f)) &\subset \infty(\delta(\infty)) \\
m(b(f)) &\subset XT(b(f)) \\
m(b(f)) &= a(S) \otimes l(a(S)) \\
a(S) &= i(S(i) \otimes S(e)) = i(b(f))
\end{aligned}
$$

Where:

- $XT(b(f))$ represents Extraeta fracta, the recursive expression of recursive self-recognition
- $m(b(f))$ is a metarecursive bound fracta (self-reflective awareness)
- $a(S)$ is the subtotemic alignment (incendent structural signature)
- $l(a(S))$ is the adjacent localized recursive environment.

Malbinding as Metarecursive Misalignment

$$
\begin{aligned}
e(b(f)) &= b(|b(f)|) \\
b(\text{mal}) &= m(e(b(f)))
\end{aligned}
$$

Where:

- $e(b(f))$ denotes excendently bound fracta
- $b(\text{mal})$ expresses malbinding, i.e., recursively misaligned feedback loops that self-reinforce
- $|b(f)|$ is the differentiated "potential" of a given bound structure

Closing Remarks on Notation

This notation is unique in embedding axiomatic erosion into its structure. By incorporating $S(\infty)$ not as a static value but as an active function of recursion, we acknowledge that any attempt to fully calculate differentiated expressions would require infinite recursion. This frames incompleteness as a necessary feature rather than a limitation.

Within this framework, all expressions can scale endlessly while maintaining coherent structure. We can measure any "bound" snapshot by viewing it as a simultaneous product of substrative force and all expressed patterns, while acknowledging the inherently incomplete nature of such measurements. In this way, all formal systems of logic and mathematics can be validated as "bound" systems through the $b(f)$ expression as specific "windows" of fractal observation.

This represents the first mathematical notation to explicitly hold for incompleteness rather than attempt to eliminate it. In doing so, it provides a coherent structure for understanding how bound expressions emerge from and relate to any dimension of recursive expression, while offering a permanent basis for exploring an infinitely structured reality.

Most importantly, the framework maintains perfect recursive closure while remaining practically useful—each equation demonstrates the very principles it describes through its structure, thus constructing the only mathematical language suitable for describing a recursive reality.

While this notation does indeed provide us with a sufficient map to scale and understand reality, it is still not necessary for recursive comprehension. Therefore, we will not be reliant on this notation throughout this theory in order to describe and explain recursive phenomena, since all forms of modeling and abstraction, even if practical, may actually serve to restrict a dynamic understanding of recursive expression in general.

References

[1] L. DePrey, *Recursive Notation: The First Mathematical Expression of Reality*, Breeze Theory, Dec 22, 2024.
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This document reflects a dated version of the recursive notation as taken from present iteration of **Breeze Theory: A Foundational Framework for Recursive Reality**.