A simulation of the lifetime utilities of two strategies regarding wage offers
Project 2 for Modeling and Simulation in Science, Engineering and Economics

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Abstract. In this paper, I attempt to find out how the two typical strategies job seekers follow differ in their results as the lifetime utility or the total wage income. An algorithm[1] is proposed to simulate the two different strategies. Simulation is conducted to test how the strategies perform under different variables. My results adhere to the theoretical formulae with less obvious insights discovered.

Keywords: utility · lifetime utility · labor market · strategy

1 Introduction

Utility in economics means the pleasure or satisfaction from consumption. In the context of job market, however, utility describes the wage earning within one period of time. The lifetime utility, or the total wage income during an individual's life time, is of the interest of this paper. It depends on one's wage and the number of years one stays working. If the probability of an individual getting a certain wage as well as the death risk are known, we can calculate the expected lifetime utility. A convenient assumption is that once one accepts a wage, one will stay working until death, generating lifetime utility. However, two strategies can be inferred. One is to take the offer regardless of the wage (S1). The other strategy is to wait a while until a high wage is offered (S2).

The basic structure of the model is borrowed from *An equilibrium model of search equilibrium* [1]. Time is discrete in this model. Consider an economy with only two possible wages. The lower wage is w_0 and the higher wage is w_1. The fraction of the firms offering w_0 is denoted as γ. Individuals in the economy always suffers a death risk τ. The number of total periods for an individual to stay in the economy is $1/\tau$ because we assume that once they take the offer they will stay working till death. The goal of the simulation is to study the lifetime utility and how it is influenced by the variables. The lifetime utility of one individual can be calculated by adding up the wage one gains in each period. By fixing the other variables and only changing the targeted variable, I aim to describe the impact of the τ and γ on the lifetime utilities under both strategies.

[1] Credit to Prof. Charlie Peskin for suggesting the model

In the following sections I first specify the the algorithms for both strategies designed for the simulation. The code mentioned in 2.1 and 2.2 is attached in a separate file. Section 3 displays the numerical results of different simulations. There is a general simple simulation for a group of given variables that generate a result for 500 rounds. Then specific tests regarding the variable of interest are mentioned. In section 4, I include some fundamental formulae to derive the expected values of lifetime utility, followed by conclusions and further studies.

2 Algorithm

When one enters the economy, one is offered either w_0 or w_1 with respective probability γ and 1 - γ. Since time is discrete, the process of decision-making, or getting a wage offer, happens once in every period. The risk of death, τ, is 0 for every individual in the first period until one accepts the offer. However, after accepting an offer every individual bears the probability τ of exiting the economy. In other words, prerequisite of receiving the wage of the next period is that the individual escapes death to stay in the economy.

2.1 Strategy 1

The first strategy is to accept the first wage being offered and stick with it till death. Thanks to the **rand function** of *MATLAB*, the code is relatively simple, as shown in section 0.1 S_1 of the code file. Lines 8 - 18 simulates the process of given a wage at random and stays with it till death.

2.2 Strategy 2

Individuals who follow this strategy do not take the offer until w_1 is given to them. To do so, they suffer the risk of exiting in each round of waiting. The codes of this group can be found in section 0.2 S_2. The design is suggested by Prof. Peskin. Lines 10-18, the first loop, simulate the process of the first round. The variable alive does not change until the second round. It is possible that the individuals do not get out of this first loop due to the death risk τ. If they successfully move to the next loop, their wage is w_1 for the rest of their life. Lines 19-24 are the second process, which is simply adding w_1 into one's earnings as long as one stays alive.

3 Simulation Results

A simple test First, we need to test if the model works correctly. Set $w_1 = 0.5$, $w_0 = 0.25$, $\gamma = 0.5$; $\tau = 0.1$ And the simulation result of the lifetime utility of 500 individuals pursuing strategy1 and the other 500 pursuing strategy2 is shown in Fig.1 with the x-axis representing each individual and the y-axis the lifetime utilities under each strategy.

	Average	**Median**	**Standard Deviation**
$\mathbf{S_1}$	3.718	2.5	3.756
$\mathbf{S_2}$	5.061	4	4.5675

Table 1: Descriptive statistics

Table 1 provides the descriptive statistics of the two groups of data. S_2 outperforms S_1 in average and median. The results of S_2 also spread out wider than those of S_1.

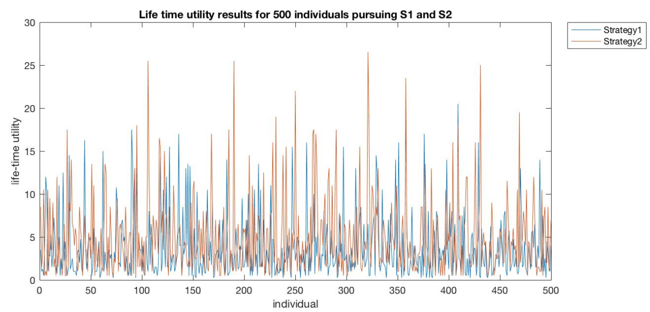

Fig. 1: Line graph,500 points, S_1 and S_2, w0=0.25, w1=γ=0.5,τ=0.1

Fig.2 shows the percentage of the three types of results: S_1 outperforms S_2, S_2 outperforms S_1, and a tie. In this test, S_2 outperforms S_1 59% of the 500 trials, while S_1 only performs better than S_2 36% of the time.

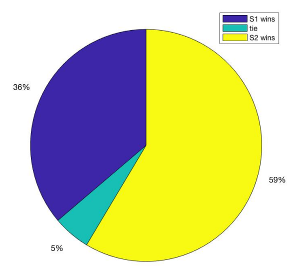

Fig. 2: Comparing the outcomes of the two strategies

3.1 The effects of τ

When $w_1 = 0.26$, $w_0 = 0.25$, $\gamma = 0.5$, the utility curves with respect to the change of τ are shown in Fig.3. We observe multiple intersections (around 7) of the two curves, displaying an alternating "winning pattern". From this test, it seems that when τ is between 0.1 and 0.2, S_2 outperforms S_1, while on the rest of the domain of τ, the two strategies alternatively generates greater lifetime utility.

Such alternating pattern alongside the change of τ disappears when $w_1 = 0.5$, $w_0 = 0.25$, $\gamma = 0.5$. As shown in Fig.5, S_2 outperforms S_1 regardless of τ.

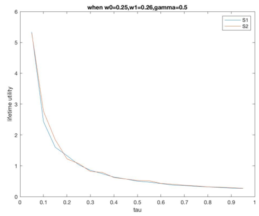

Fig. 3: utility of S_1 and S_2 with respect to τ when w0=0.25, w1=0.26, γ=0.5

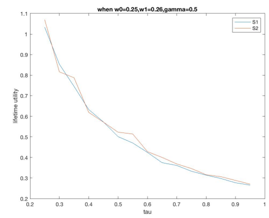

Fig. 4: zoomed portion, when $\tau \geq 0.25$

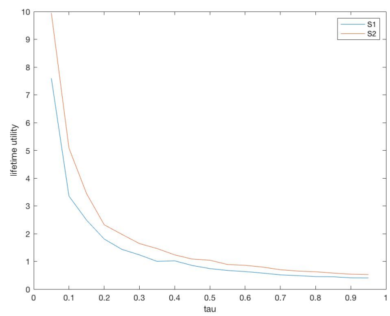

Fig. 5: w0=0.25, w1=0.5, γ=0.5

The shape of the utility curves resemble that of the function y = $\frac{1}{x}$. This observation is further explored in the next section.

3.2 The effects of γ

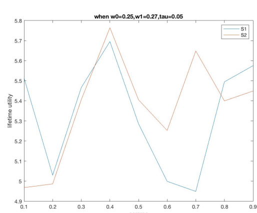

Fig. 6: τ=0.05

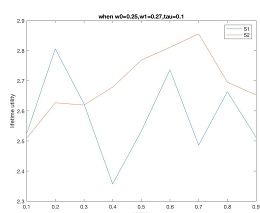

Fig. 7: τ=0.1

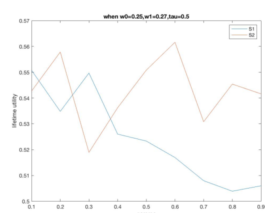

Fig. 8: τ=0.5

The procedure to test γ is the following: fixing the values of w_1 as 0.27 and w_0 as 0.25, testing the impact of γ ranging from 0.1, 0.2,...,0.9 under three different values of τ. Fig.6 to Fig.8 are the results of this test. The answer to the question which strategy works better can lead to many different cases. However, strategy 2 seems to perform better when γ is closer to 1. This is not very intuitive. When it's more likely to get the lower wage offer, it is better to wait for the higher wage to occur, as it seems to suggest. Fig.9 is generated by combining the results from the previous three tests. The most obvious conclusion is the change in variance among the different values of the death rate. When τ reaches 0.5, the spread of the utility results for both strategies are way smaller than that of the lower values of τ, indicating that γ affects the result less great when the death rate

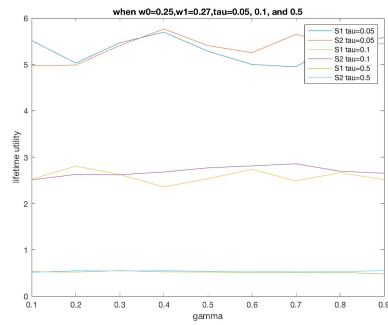

Fig. 9: As a comparison

increases. Another test conducted with the death rate τ set to be 0.9 adheres to

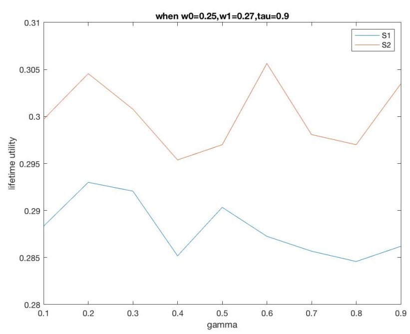

Fig. 10: τ=0.9,9 points

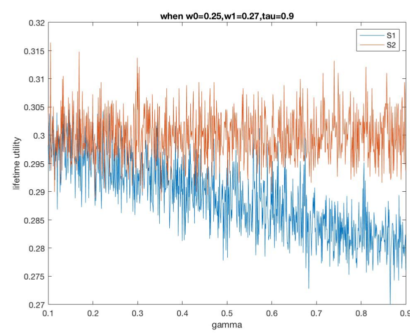

Fig. 11: τ=0.9,900 points

the statement above. Fig.10 shows the result when picking 9 points of γ and we see that S_2 outperforms S_1 regardless of γ. To better understand this, Fig.11 is created with 900 values of γ evenly distributed between 0.05 and 0.95. As the figures convey, S_2 is more desirable in generating higher lifetime utility under a higher rate of getting the lower wage. The performance of S_1 worsens as γ increases in this test.

4 Quantitative formulae[2]

The expected utilities of both strategies can be derived based on the variables given. Therefore, it's imperative to compare the simulation results with the theoretical results.

4.1 Governing equations

Expected Value In the finite cases, the expectation of a random variable x with finite possible outcomes x_1, x_2 ... x_n occurring with probabilities p_1, p_2...p_n

[2] With help from Prof. Chalie Peskin

is defined as

$$E[x] = \sum_{i=1}^{n} x_i p_i \tag{1}$$

with all the probabilities p_i add up to 1.

Conditional Probability Given two independent events A and B, the conditional probability of A occurring with the condition of B has already occured is defined as:

$$P(A|B) = \frac{P(A \cap B)}{P(A) \cdot P(B)} \tag{2}$$

Geometric Series Consider the infinite series

$$S = a + ar + ar^2 + ... \tag{3}$$

with a being the first term of the series and each successive term has a fixed ratio r to the last term, can be written as

$$S = a + r(a + ar + ar^2 + ...) \tag{4}$$

plugging (3) into (4),

$$S = a + rS \tag{5}$$

$$(1 - r)S = a \tag{6}$$

$$S = \frac{a}{1 - r} \tag{7}$$

4.2 Formulae of the lifetime utilities in each strategy

There are two independent events in each period. They are A_0: getting w_0, and A_1: getting w_1, occurring with the probabilities γ and (1-γ). These two events, however, are dependent on the third event, B: living. The probability of B, P(B), is (1-τ). In other words,

$$P(A_0|B) = \gamma \tag{8}$$

$$P(A_1|B) = 1 - \gamma \tag{9}$$

S_1 The expected utility from S1 in one period, based on (1) is

$$P(A_0|B) \cdot w_0 + P(A_1|B) \cdot w_1 \tag{10}$$

or

$$\gamma \cdot w_0 + (1-\gamma) \cdot w_1 \tag{11}$$

since the total periods of every individual is $1/\tau$, the expected lifetime utility of S1 is

$$EXP(S_1) = (\gamma \cdot w_0 + (1-\gamma) \cdot w_1) \cdot \frac{1}{\tau} \tag{12}$$

S_2 In each period, the individuals pursue S_2 do not gain any utility unless w_1 is offered. In other words, they stays in the cycle until w_1 is offered, no matter is occurs in the first round or the second round or the n^{th} round, and the process shall continue infinitely. Let X be an event called getting w_1. X_i represents the event of getting w_1 in the i^{th} round; We define the probability of getting w_1 as

$$P = P(X_1) + P(X_2) + ... \tag{13}$$

Remark: because the events X_i are mutually exclusive, P is based on the addition rule of probability.

$$P(X_1) = P(A_1|B) = 1-\gamma \tag{14}$$

$$P(X_2) = P(A_0|B) \cdot P(B) \cdot P(A_1|B) = \gamma(1-\tau)(1-\gamma) \tag{15}$$

$$P(X_3) = P(A_0|B) \cdot P(B) \cdot P(A_0|B) \cdot P(B) \cdot (A_1|B) = \gamma(1-\tau)\gamma(1-\tau)(1-\gamma) \tag{16}$$

P can be written as a geometric series in the form of (3) with r = 1 - γ and a = γ(1-τ). Therefore,

$$P = \frac{r}{1-a} = \frac{1-\gamma}{1-\gamma(a-\tau)} \tag{17}$$

Based on (10), expectation of lifetime utility for S_2 individual is

$$EXP(S_2) = \frac{(1-\gamma)w_1}{1-\gamma(a-\tau)} \cdot \frac{1}{\tau} \tag{18}$$

4.3 Discussions

Comparing the results In both formulae of expected lifetime utility, notice that there is a τ in the denominator, which explains the shape of the curves in Fig.3 and Fig.5. But S_2 has a 1-γ(1-τ) in the denominator as well. If we plug in the numbers the variables from **A simple test**, we get the expected lifetime utility for S_1 and S_2 to be 3.75 and 4.545, which are fairly close to the results from the computer simulation.

	Average from Simulation	**Expectation**
S_1	3.718	3.75
S_2	5.061	4.5455

Table 2: Comparison

In the tests of τ, S_2 seems to perform better in general When γ is fixed at 0.5, S_2 seems to outperform S_1 when w_1 exceeds certain value. In the limited number of trials I conducted, w_1 cannot be much bigger than w_0, otherwise S_2 outperforms S_1 no matter how τ or γ varies. This phenomenon can be explained from the formulae

$$\frac{\partial EXP(S_1)}{\partial w_1} = \frac{1-\gamma}{\tau} \tag{19}$$

$$\frac{\partial EXP(S_2)}{\partial w_1} = \frac{1-\gamma}{(1-(1-\tau)\gamma)\tau} = \frac{1}{1-(1-\tau)\gamma} \cdot \frac{\partial EXP(S_1)}{\partial w_1} \tag{20}$$

Since (1 - τ) $\gamma > 0$,

$$\frac{\partial EXP(S_2)}{\partial w_1} > \frac{\partial EXP(S_1)}{\partial w_1} \tag{21}$$

in other words, an increase in the value w_1 contributes to the increase of $EXP(S_2)$ more than that of $EXP(S_1)$ In trying to derive the conditions for S_2 be be favorable, derive

$$EXP(S_2) - EXP(S_1) = \frac{w_1(1-\gamma)\frac{\gamma(1-\tau)}{1-\gamma(1-\tau)} - \gamma w_0}{\tau} \tag{22}$$

With (24), we can derive the condition for S_2 to outperform S_1 or vice versa, as long as 3 of the 4 variables are known.

5 Conclusion

The central question of this paper is which of the 2 strategies generates the more desirable lifetime income. The goal of this simulation is to provide insights into the performance of the two strategies for job seekers. The model contains all the required variables to work out the expected values, but simulation provides more insights into the process. Both results show that the algorithm functions as expected.

As the simulation results suggest, for each individual choosing strategy 1 or strategy 2, there is great variance in the outcome. However, when the higher wage is much higher than the lower wage, even when the death rate is high, choosing strategy 2 seems to provide the better outcome. Therefore, it's safe to conclude that if the potential high wage is high enough, then it's advisable to wait for ot to come. When the higher and lower wage are close, one strategy works better for a couple tests of γ or τ, and then is defeated by the other. The complicity of the system is shown in the expectation of the utilities. With 4 variable interacting in the system.

τ and γ affect the system differently, with the effects of τ vary a lot less than that of γ.In general, the lifetime utility of each strategy decreases as τ increases. The value of the wages does not influence the effects of γ on the results as much as they do towards the effects ofτ. We can always expect alternative results in tests of γ.

The most interesting result from the simulation is that when τ is relative big, S_2 outperforms S_1 as γ increases. This is not intuitive because it suggests that with the high risk of exiting economy and high probabilities of getting the lower wage, one shall keep waiting to get the higher lifetime utility. Such result requires further examination. In short, the takeaway messages from this paper are

1. When the higher wage is high enough, switch to strategy 2;
2. When firms are more likely to offer the lower wage, then it's better to wait longer for the high wage; when "death rate" or the chance of exiting the job market is high, waiting for the higher wage can lead to higher lifetime utility;

6 Further Study

1. A better strategy combing the ideal conditions of τ and γ can be developed to further maximize the lifetime utility generated. An algorithm that based on the threshold condition (22) together with the conclusion from this simulation can be developed.
2. *An equilibrium model of search unemployment* provides a range of concepts that explain the assumption and simplification of the theoretical simulation for this model. An idea from the paper is to divide individuals into two groups based on certain characteristics (in that paper is called leisure value), they possess and they are assumed to make different decisions. This method can create more randomness in simulating a group of job seekers, rather than simulating two groups with the same population.

References

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