$$\hat{M}_a = \sum_{m=1}^{L} e^{2\pi i \frac{m}{L} \cdot a} \hat{S}_m^z$$

$$f_a(n) = \langle n | \hat{M}_a | n \rangle = \langle n | \sum_{m=1}^{L} e^{2\pi i \frac{m}{L} a} \hat{S}_m^z | n \rangle$$

$$= \sum_{m=1}^{L} e^{2\pi i \frac{m}{L} a} \langle n | \hat{S}_m^z | n \rangle$$

Average over States:

$$[f_a(n)] = \frac{1}{N} \sum_{n=1}^{N} f_a(n)$$

$$= \sum_{m=1}^{L} e^{2\pi i \frac{m}{L} a} \frac{1}{N} \sum_{n=1}^{N} \langle n | \hat{S}_m^z | n \rangle$$

$$=: \sum_{m=1}^{L} e^{2\pi i \frac{m}{L} a} \underbrace{[\langle S_m^z \rangle]}$$

= constant for every subspace, where $S^z_{total} = const$