Expectation value of M_1 (from Fig 2 of http://arxiv.org/abs/1010.1992)

$$\hat{M}_1 = \sum_{j=1}^{L} \hat{S}_j^z \exp\left(i \frac{2\pi j}{L}\right)$$

$$\rho_0 = (1 + \varepsilon \hat{M}_1^+)/Z$$ small pertubation around infinite temperature (all states equally probable)

=> Spin polarization in this mode

$$\langle \hat{M}_1 \rangle_0 = \sum_n \langle n | \rho_0 \hat{M}_1 | n \rangle$$

$$= \underbrace{\frac{1}{Z} \sum_n \langle n | \hat{M}_1 | n \rangle}_{} + \frac{\varepsilon}{Z} \sum_n \langle n | \hat{M}_1^+ \hat{M}_1 | n \rangle$$

$$= \frac{1}{Z} \sum_n \langle n | \sum_j \hat{S}_j^z \exp\left(i \frac{2\pi j}{L}\right) | n \rangle$$

$$= \frac{1}{Z} \sum_j \exp\left(i \frac{2\pi j}{L}\right) \underbrace{\sum_n \langle n | \hat{S}_j^z | n \rangle}_{= \langle \hat{S}_j^z \rangle = \mathrm{tr}(\mathbb{1} \hat{S}_j^z) = 0} = 0$$

$$= \frac{\varepsilon}{Z} \sum_n \langle n | \hat{M}_1^+ \hat{M}_1 | n \rangle$$

If ETH is true, the time average $\rho_\infty := \lim_{t \to \infty} \frac{1}{t} \int_0^t \rho(t)\, dt$ will be diagonal in the eigenbasis of H

$$\Rightarrow \quad \rho_\infty = \sum_n (\rho_\infty)_{n,n} = \sum_n |n\rangle\langle n| / Z$$

$$\Rightarrow \langle M_1 \rangle_\infty = \sum_n \langle n | \rho_\infty \hat{M}_1 | n \rangle$$

$$= \frac{1}{Z} \sum_{n,n'} \underbrace{\langle n | n' \rangle}_{\delta_{nn'}} \langle n' | \hat{M}_1 | n \rangle = \frac{1}{Z} \sum_n \langle n | \hat{M}_1 | n \rangle = 0$$

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If $\varrho_\infty = (\mathbb{1} + \varepsilon \hat{M}_1) \sum_n |n\rangle\langle n| / Z$

$$\Rightarrow \langle M_1 \rangle_\infty = \underbrace{\sum_n \langle n | \sum_{n'} |n'\rangle\langle n'| \frac{1}{Z} \hat{M}_1 | n \rangle}_{=0} + \sum_n \langle n | \varepsilon \hat{M}_1 \sum_{n'} |n'\rangle\langle n'| \frac{1}{Z} \hat{M}_1 | n \rangle$$

$$\overset{?}{=} \frac{\varepsilon}{Z} \sum_n \langle n | \hat{M}_1 | n \rangle \langle n | \hat{M}_1^+ | n \rangle \quad ?$$