250227 Weekly Lab meeting

Weekly Lab Meeting

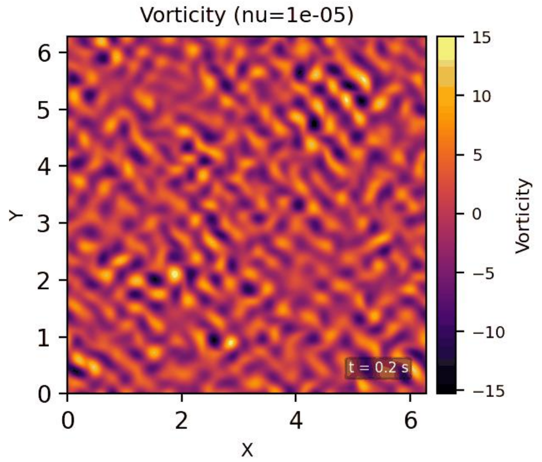
Juseong Kim

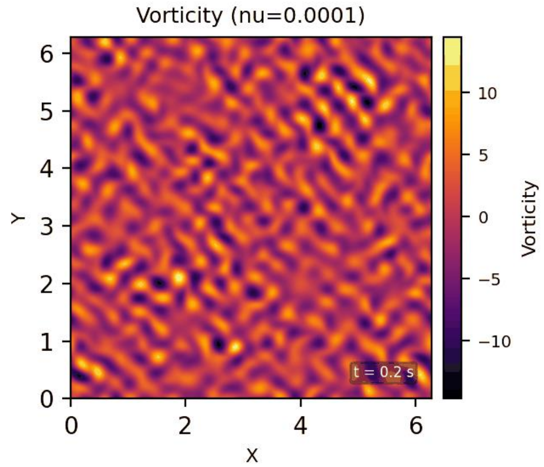
Flow Physics & Computational Engineering Innovation Lab

Contents

01 Fitting

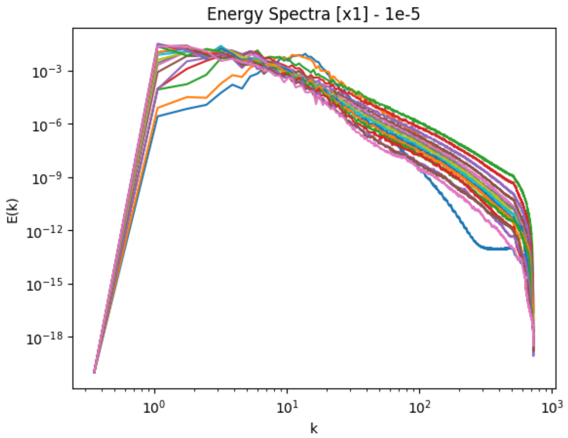
Assignment -1

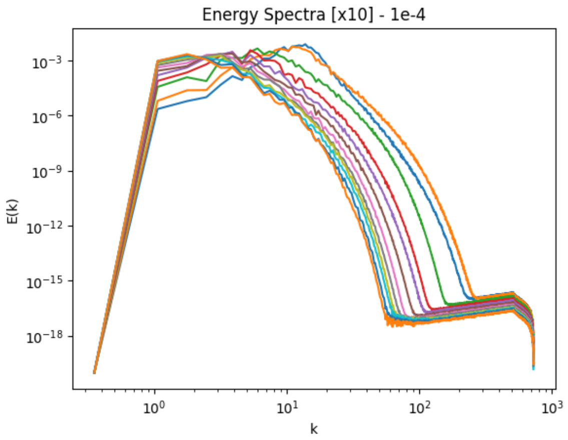




1. Turbulence Simulation

OpenFOAM으로 IC와 BC,
초기 속도장을 설정한 후
동점성 계수(ν)를
논문값(1e-5) 대비
x2, x0.5, x10 하여
시뮬레이션 진행





2. Data Analysis

시뮬레이션 결과로 나온
데이터들의 에너지 스펙트럼,
통계적 특성들을 분석

i) ν에 따른 스펙트럼의 기울기
ii) Vorticty PDF 분석

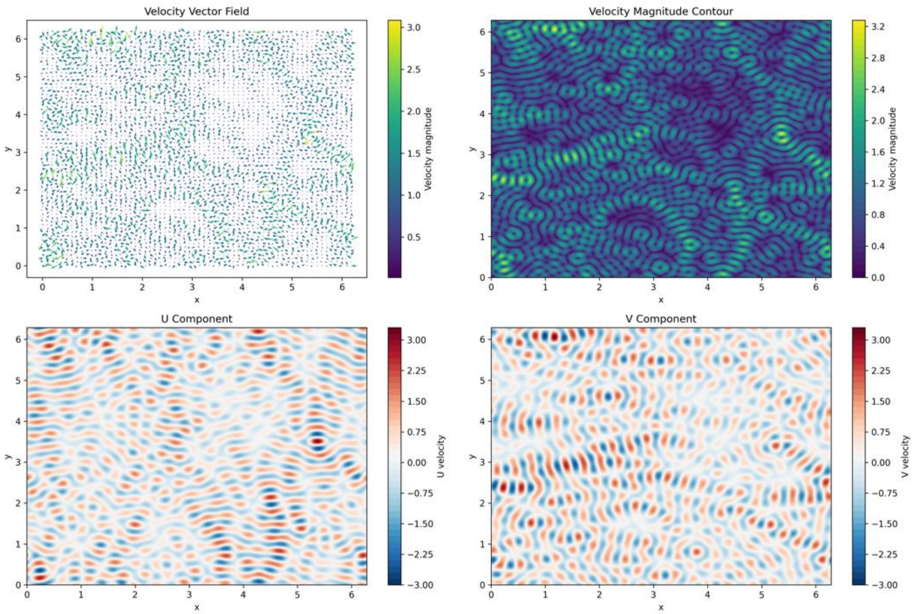
Omar Sallam <OpenFoam implementation of two-dimensional isotropic homogenous turbulence decay problem>

Assignment -2

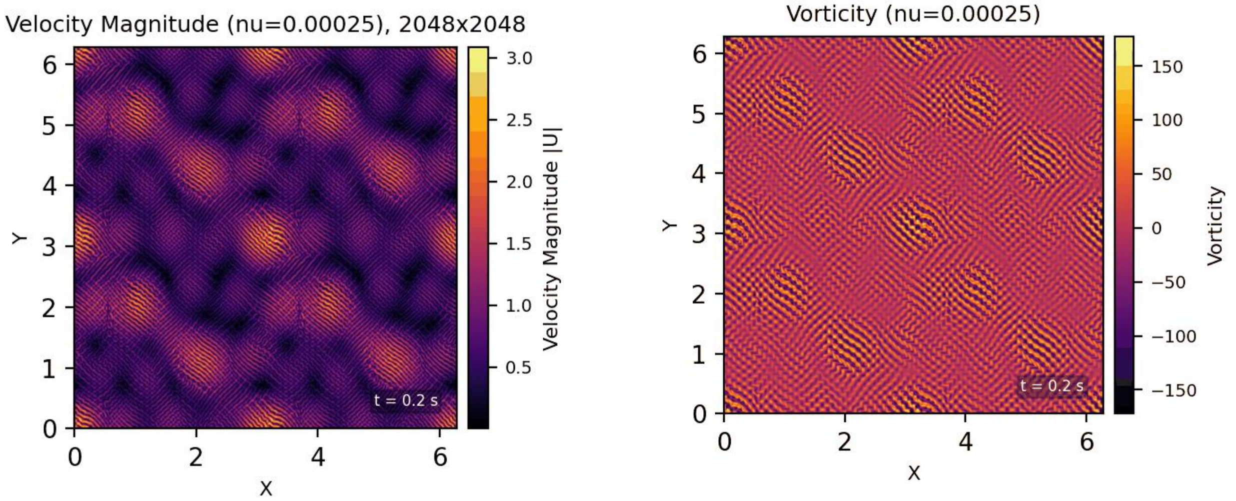
To mimic such a procedure in our simulations, we performed 50 two-dimensional simulations of the Navier-Stokes equations on a regular grid of 2048^2 points. Initial conditions were a random superposition of harmonic modes between wave numbers $k = 18$ and 22, with the spectrum peaking at $k_0 = 20$. Viscosity was $\nu = 2.5 \times 10^{-4}$ in all these runs and the box had length 2π. The initial rms velocity $U_{\rm rms}$ in all runs was 1, corresponding to a turnover time of $\tau_{\rm NL} = L_0/U_{\rm rms} = 2\pi/20 \approx 0.3$ $(L_0 = 2\pi/k_0)$. As a result, the runs differed only by their random initial phases.

Validation model

P. D. Mininni <Inverse cascade behavior in freely decaying two-dimensional fluid turbulence>



Generating Initial Velocity field using python



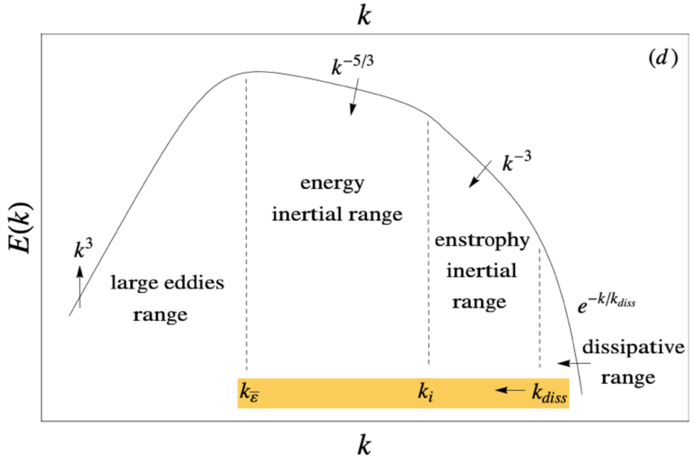
Valid Simulation Result

3. Code validation

OpenFOAM과 데이터 분석 코드의 유효성을 검증하기 위해 다른 논문의 난류 모델을 시뮬레이션 한 후 데이터 분석 결과를 비교

01 Fitting

Fitting Range



[Leonardo Campanelli]() <Dimensional analysis of two-dimensional turbulence>

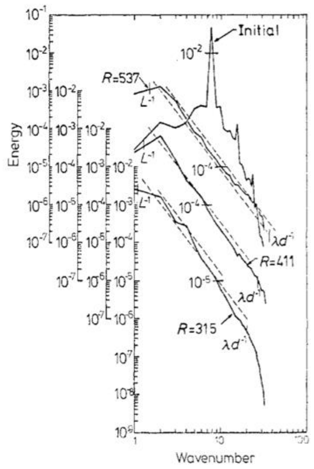


Figure 12. Energy spectrum for three 2D Navier–Stokes decay runs (from Lilly 1971), averaged over 200 time steps, and labelled by Reynolds number. Broken lines have slope −3.

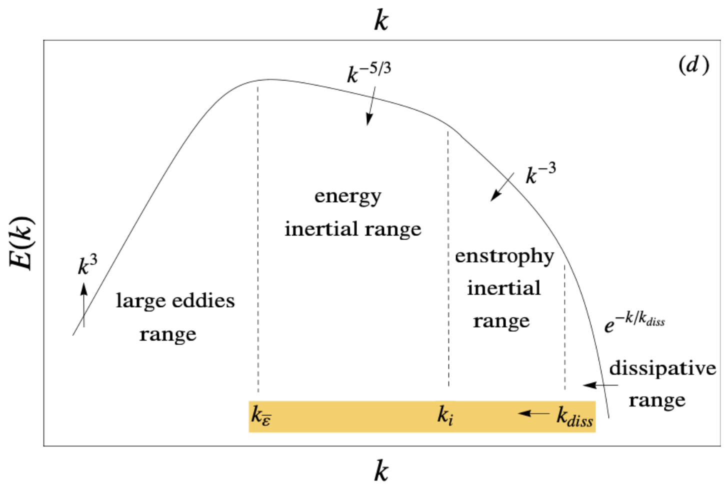
R H Kraichnan <Two-dimensional turbulence>

Energy spectrum은 4개의 구간으로 나누어져 있으며 각각 Large eddies, Energy inertial, Enstrophy inertial, Dissipative range로 구성된다.

이 때 inertial range 구간의 기울기 경향성이 이론값인
Energy inertial range에서 -5/3, (kolmogorov)
Enstrophy inertial range에서 -4 (Kraichnan)
를 만족하는지 확인한다.

01 Fitting

Selection of Range -1



$$E(k,t) = \nu^{3/2} t^{-1/2} \psi(k\sqrt{\nu t}), \quad (1)$$

where ψ is an arbitrary function of its argument. The only scale in the model is the dissipation length $L_{diss}(t) = \sqrt{\nu t}$, to which it corresponds the wavenumber $k_{diss}(t) = 1/L_{diss}$.

Inertial Range Fitting 구간 선정($k_{\bar{\varepsilon}}$ < k < k_{diss})

Left point. $k_{\bar{\varepsilon}}$: E(k)가 max일 때 k 값이라 가정

Right point $k_{diss} = 1/\sqrt{\nu t}$

Leonardo Campanelli <Dimensional analysis of two-dimensional turbulence>

Selection of Range -2

8 A. Alexakis, L. Biferale / Physics Reports 767–769 (2018) 1–101

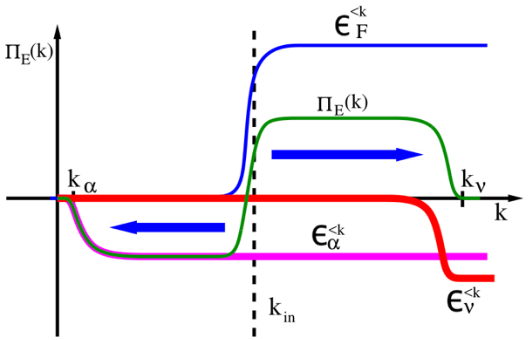


Fig. 3. Qualitative sketch of the stationary scale-by-scale energy balance for the energy flux, $\Pi_E(k)$. Notice that the assumption of scale separation, $k_\alpha \ll k_{in} \ll k_\nu$ predicts the existence of two *inertial ranges* where the energy flux is constant and due only to the non-linear triadic interactions.

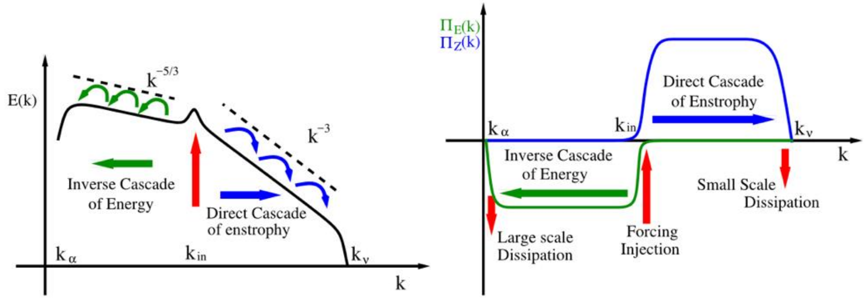


Fig. 6. Log–log sketch of the energy spectrum (left) and of energy and enstrophy fluxes (right) for the 2D Batchelor–Kraichnan theory (59)–(58).

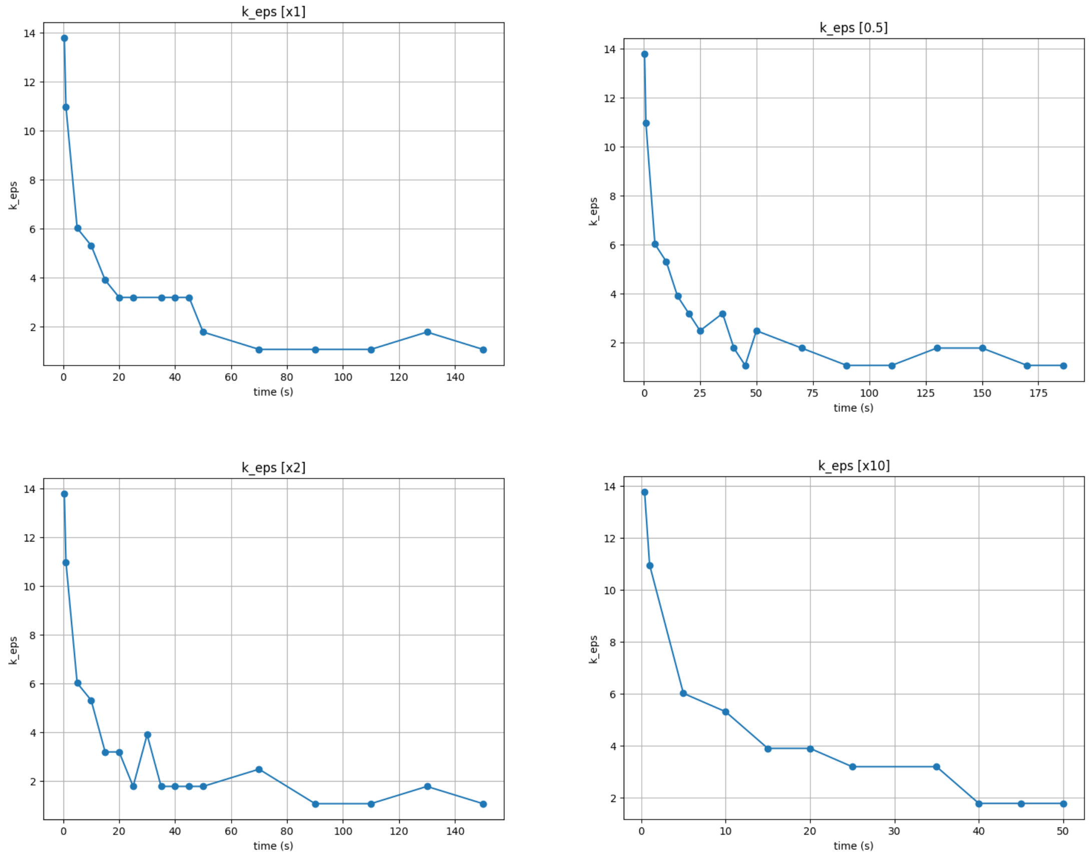
A.Alexakis <Cascades and transitions in turbulent flows>

Inertial Range Fitting 구간 선정($k_{\bar{\varepsilon}}$ < k < k_{diss})

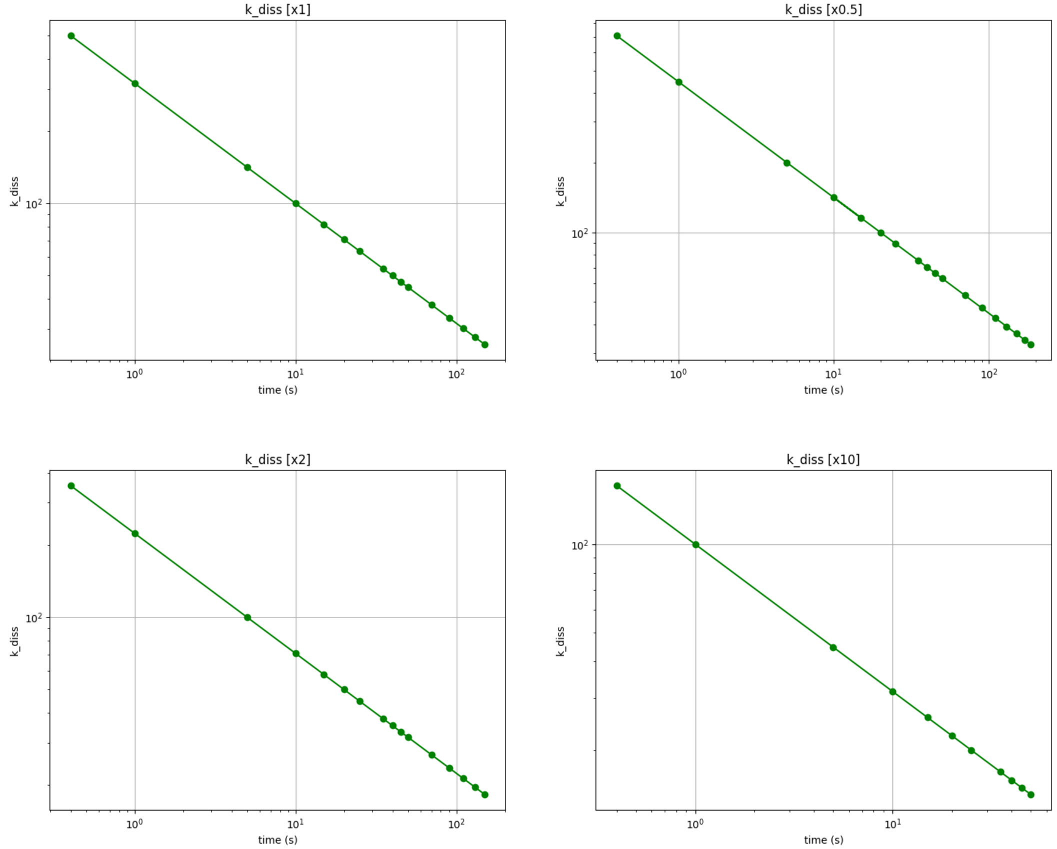
Middle point k_i: Energy Flux가 양수가 되는 지점

Selection of Range -3

Left point $k_{\bar{\varepsilon}}(\max of\ E(k))\ Over\ time$



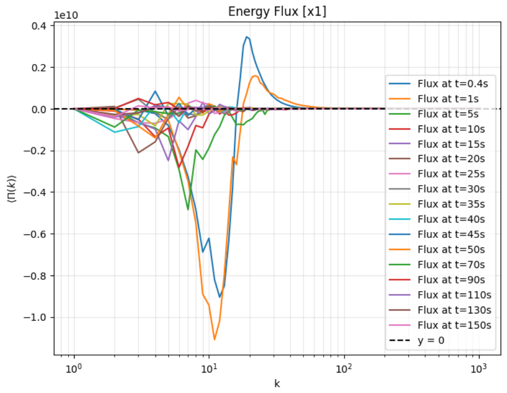
Right point $k_{diss}(1/\sqrt{\nu t})\ Over\ time$



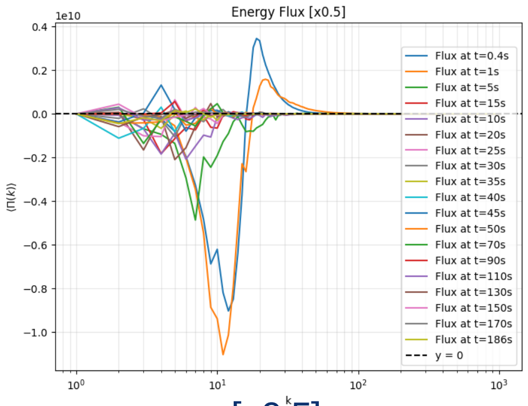
모든 동점성 계수 case에서 시간이 지남에 따라
스펙트럼의 peak point에 해당하는 k가 점차 감소

Selection of Range -4

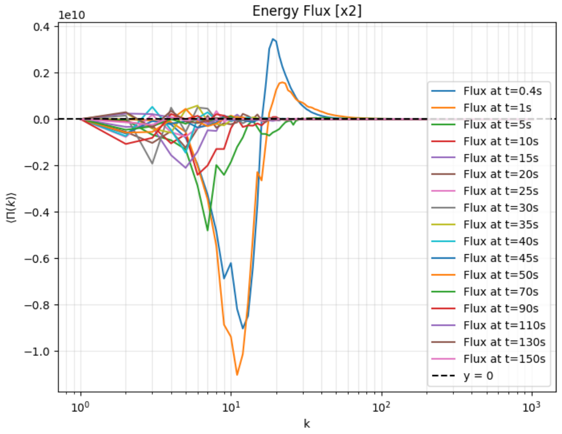
Middle point k_i



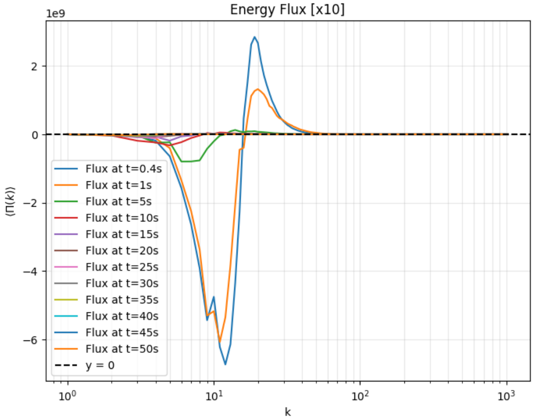
[x1]



[x0.5]



[x2]

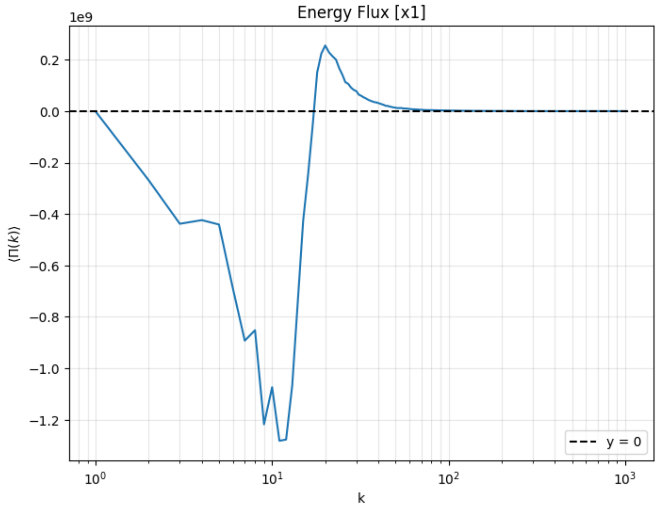


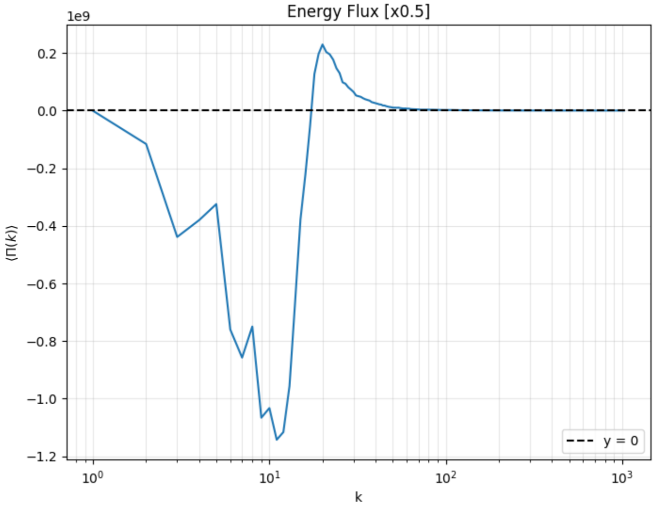
[x10]

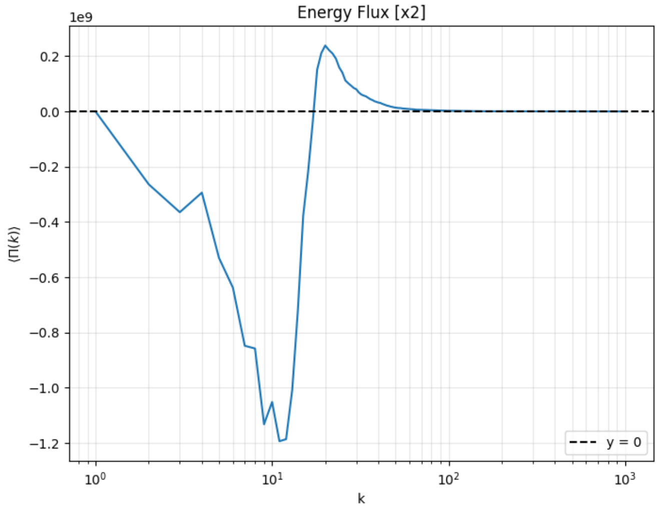
[x10] is clearly distinct by time periods
>> Inverse cascade 구간을 정확하게 확인 가능

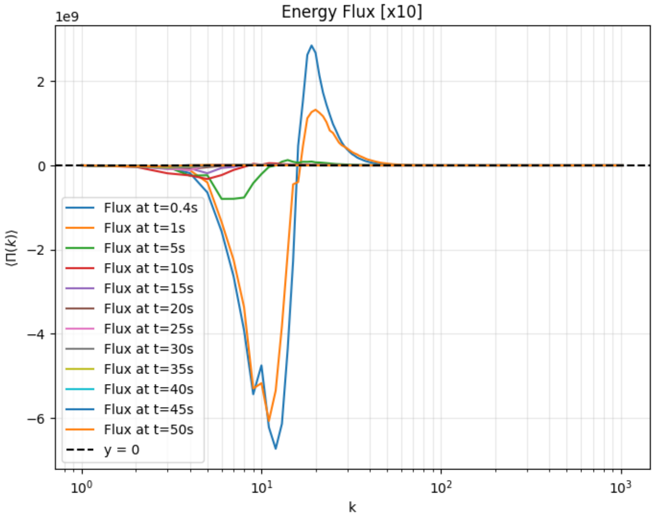
[x1], [x0.5], [x2]
>> 진동으로 인해 inverse cascade 구간의 식별이 어려워
flux를 시간 평균하여 k_i 도출

Selection of Range -5







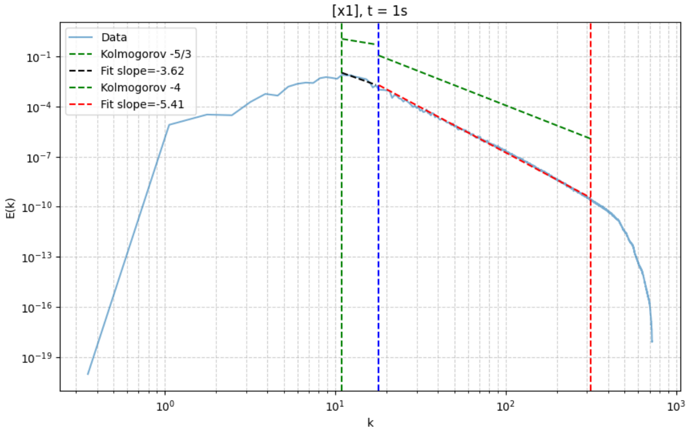


Middle point k_i

[x1]: 18 (Averaged)
[x0.5]: 18 (Averaged)
[x2]: 18 (Averaged)

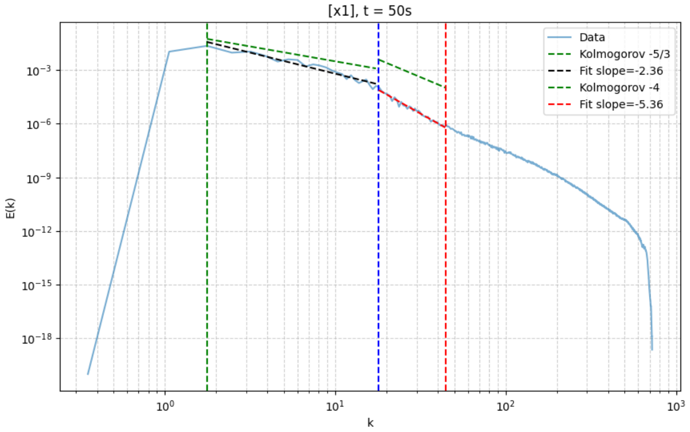
[x10]
16 at t= 0.4s
17 at t= 1s
13 at t= 5s
9 at t= 10s
8 at t= 15s
7 at t= 20s
5 at t= 25s
7 at t= 30s
5 at t= 35s
5 at t= 40s
4 at t= 45s
4 at t= 50s

Fitting Result – [x1] each time step



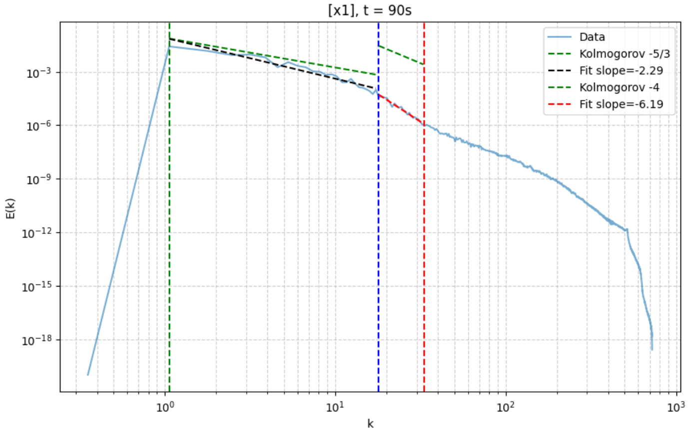
Energy inertial range Slope: -3.616
Deviation from -5/3: 116.95%
R^2: 0.829

Enstophy inertial range Slope: -5.411
Deviation from -4: 35.28%
R^2: 0.998



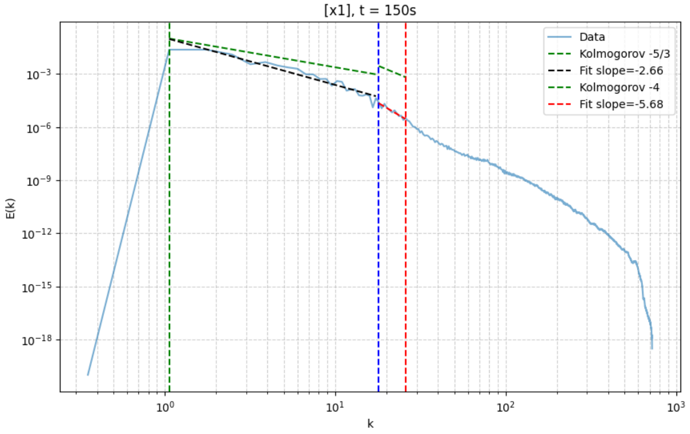
Energy inertial range Slope: -2.357
Deviation from -5/3: 41.42%
R^2: 0.935

Enstophy inertial range Slope: -5.358
Deviation from -4: 33.96%
R^2 : 0.980



Energy inertial range Slope: -2.292
Deviation from -5/3: 37.52%
R^2 for Inverse Cascade: 0.931

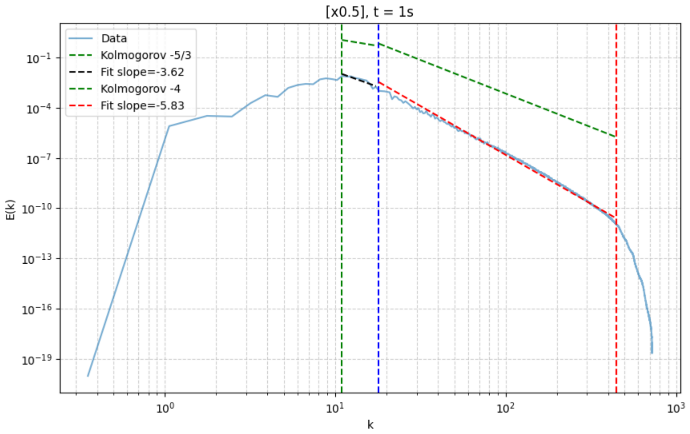
Enstophy inertial range Slope: -6.186
Deviation from -4: 54.65%
R^2: 0.972



Energy inertial range Slope: -2.662
Deviation from -5/3: 59.72%
R^2 for Inverse Cascade: 0.920

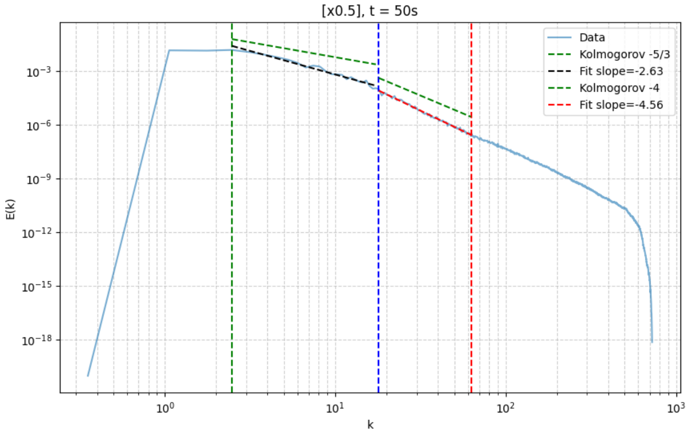
Enstophy inertial range Slope: -5.675
Deviation from -4: 41.88%
R^2: 0.891

Fitting Result – [x0.5] each time step



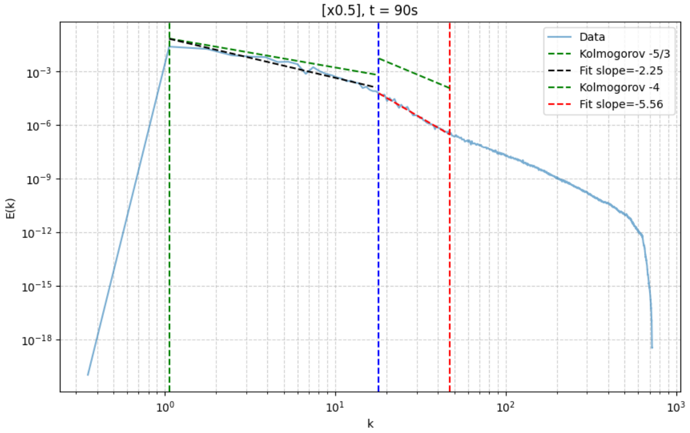
Energy inertial range Slope: -3.618
Deviation from -5/3: 117.09%
R^2 for Inverse Cascade: 0.829

Enstophy inertial range Slope: -5.829
Deviation from -4: 45.72%
R^2: 0.994



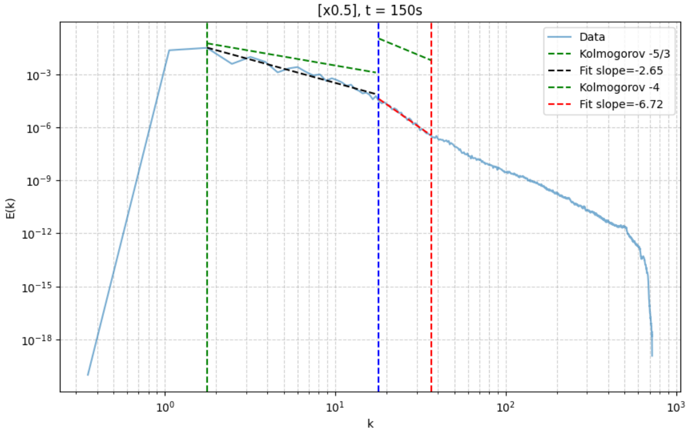
Energy inertial range Slope: -2.629
Deviation from -5/3: 57.75%
R^2 for Inverse Cascade: 0.970

Enstophy inertial range Slope: -4.561
Deviation from -4: 14.02%
R^2: 0.993



Energy inertial range Slope: -2.245
Deviation from -5/3: 34.70.%
R^2: 0.938

Enstophy inertial range Slope: -5,556
Deviation from -4: 38.91%
R^2: 0.986

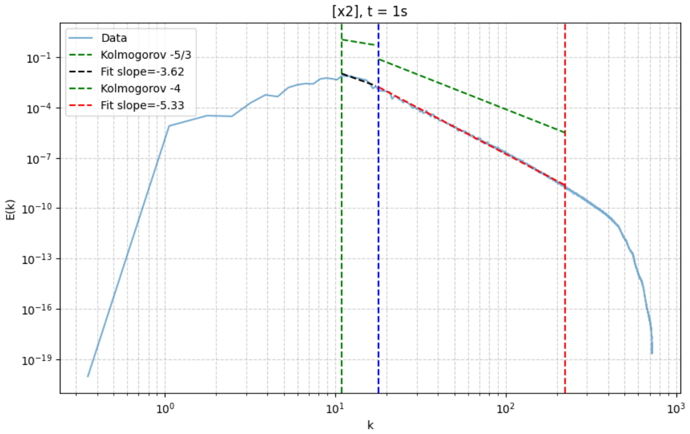


Energy inertial range Slope: -2.650
Deviation from -5/3: 58.99%
R^2: 0.930

Enstophy inertial range Slope: -6.718
Deviation from -4: 67.96%
R^2: 0.986

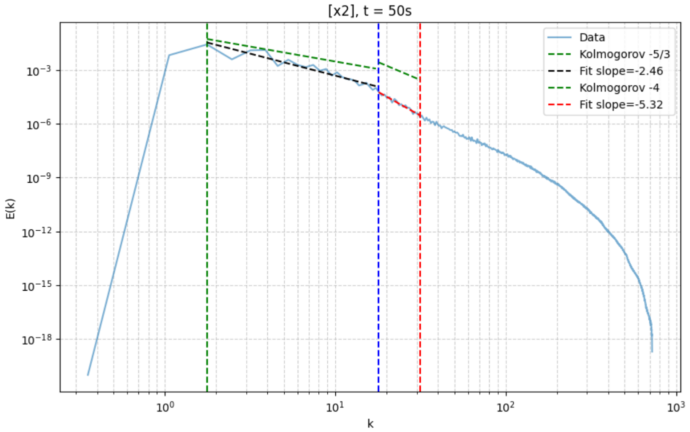
01 Fitting

Fitting Result – [x2] each time step



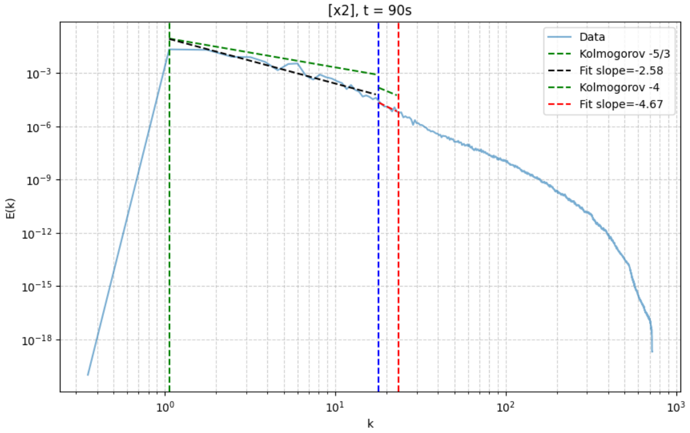
Energy inertial range Slope: -3.618
Deviation from -5/3: 117.09%
R^2: 0.829

Enstophy inertial range Slope: -5.33
Deviation from -4: 33.25%
R^2: 0.998



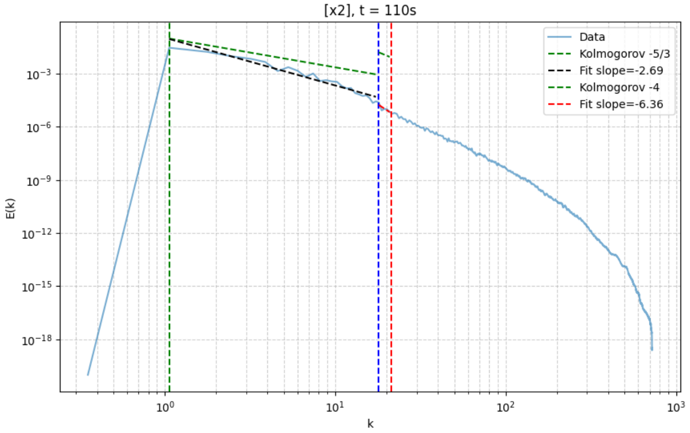
Energy inertial range Slope: -2.456
Deviation from -5/3: 47.34%
R^2: 0.910

Enstophy inertial range Slope: -5.325
Deviation from -4: 33.12%
R^2: 0.959



Energy inertial range Slope: -2.579
Deviation from -5/3: 54.68%
R^2 for Inverse Cascade: 0.915

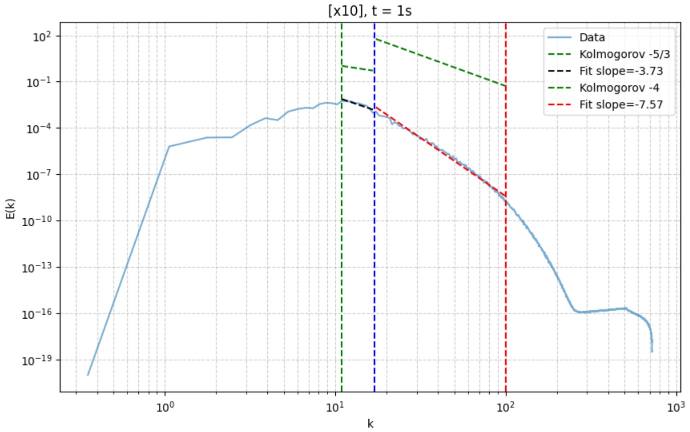
Enstophy inertial range Slope: -4.666
Deviation from -4: 16.66%
R^2: 0.785



Energy inertial range Slope: -2.689
Deviation from -5/3: 61.33%
R^2 for Inverse Cascade: 0.945

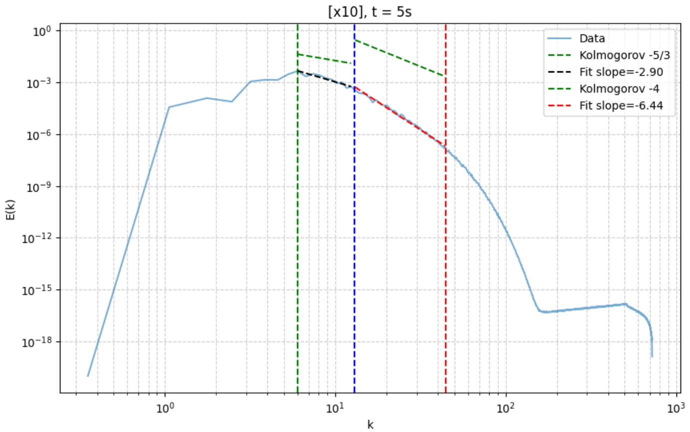
Enstophy inertial range Slope: -6.362
Deviation from -4: 59.05%
R^2: 0.605

Fitting Result – [x10] each time step



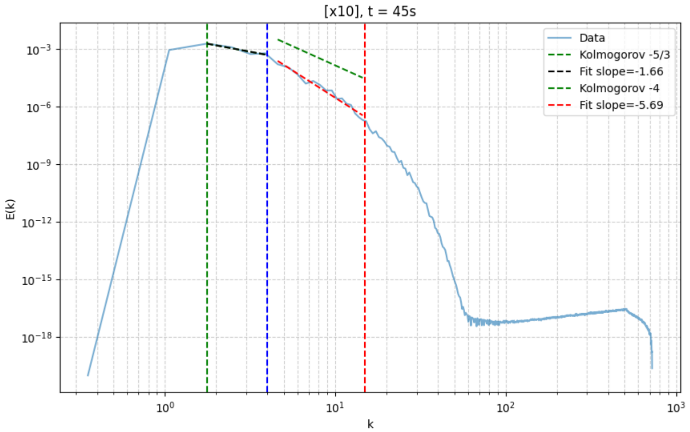
Energy inertial range Slope: -3.734
Deviation from -5/3: 124.02%
R^2 for Inverse Cascade: 0.816

Enstophy inertial range Slope: -7.568
Deviation from -4: 89.19%
R^2: 0.990



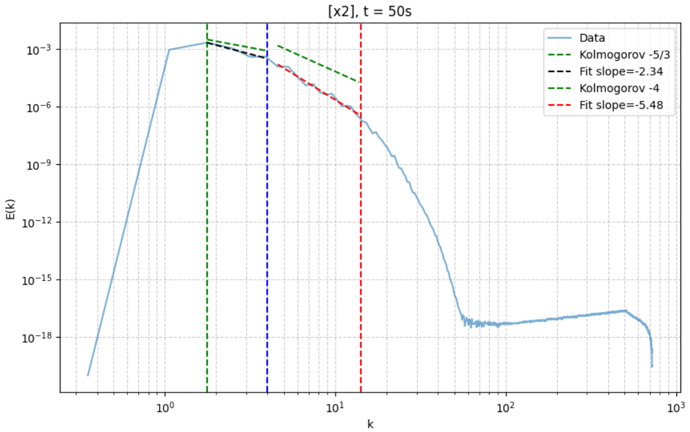
Energy inertial range Slope: -2.897
Deviation from -5/3: 73.82%
R^2 for Inverse Cascade: 0.888

Enstophy inertial range Slope: -6443
Deviation from -4: 61.07%
R^2: 0.991



Energy inertial range Slope: -1.661
Deviation from -5/3: 0.34%
R^2 for Inverse Cascade: 0.892

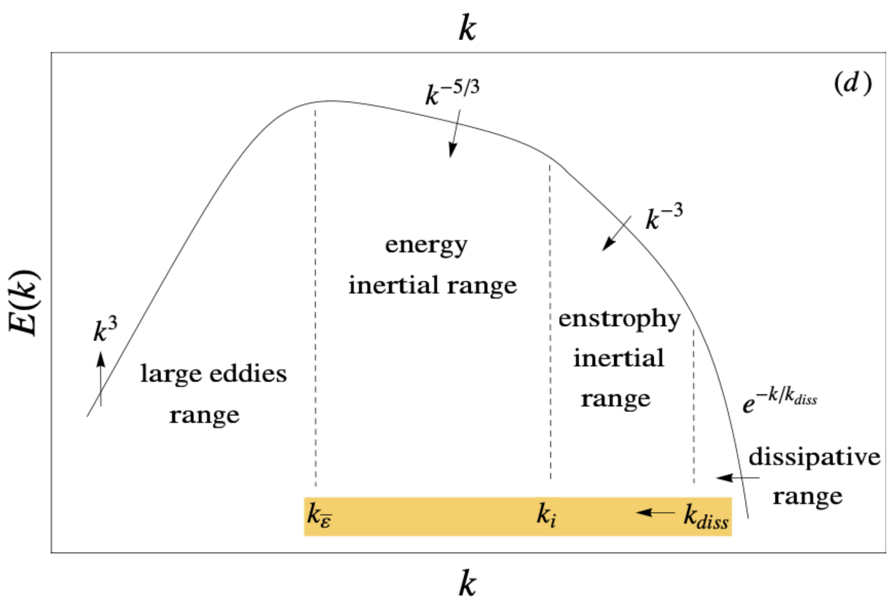
Enstophy inertial range Slope: -5.688
Deviation from -4: 42.20%
R^2: 0.964



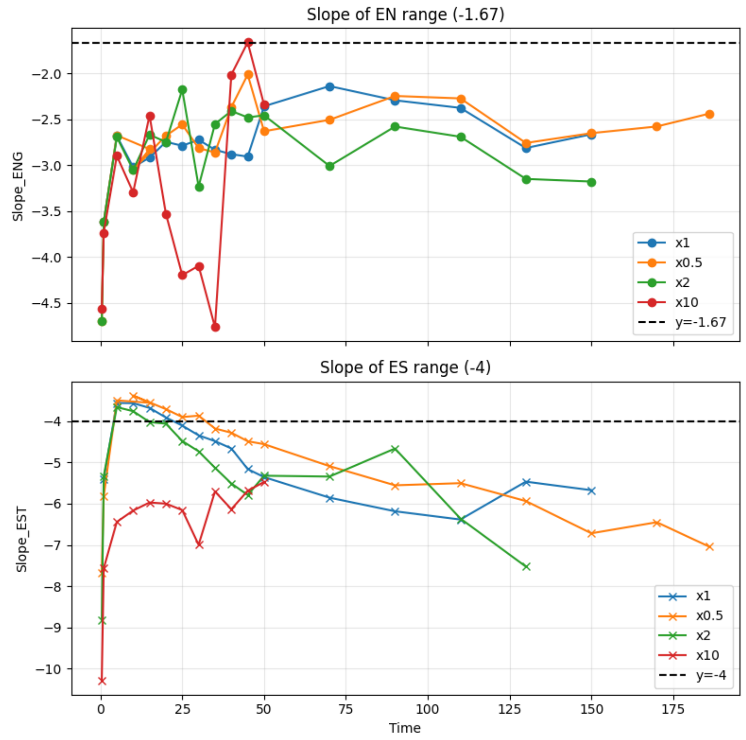
Energy inertial range Slope: -2.341
Deviation from -5/3: 40.43%
R^2 for Inverse Cascade: 0.922

Enstophy inertial range Slope: -5.476
Deviation from -4: 36.88%
R^2: 0.974

Fitting Summary



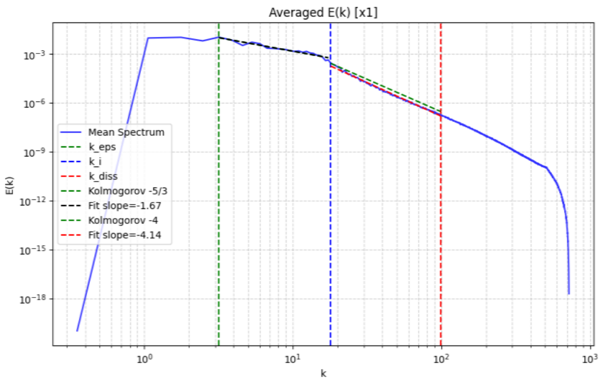
이론값

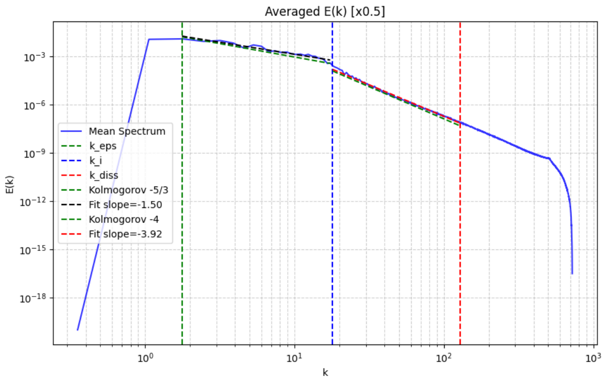


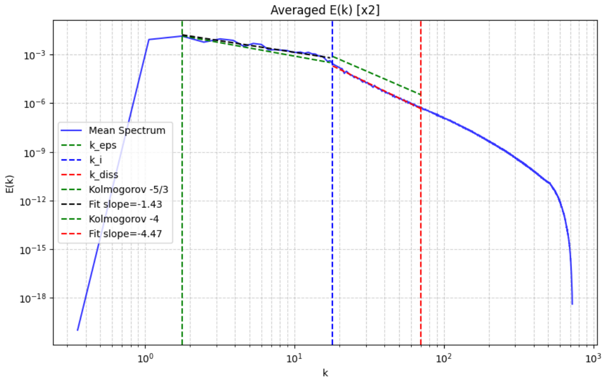
fitting result (time – slope)
Energy Inertial Range와 Enstrophy Inertial Range에서
각각의 이론값(검은 점선)과 완벽히 부합하지 않음,

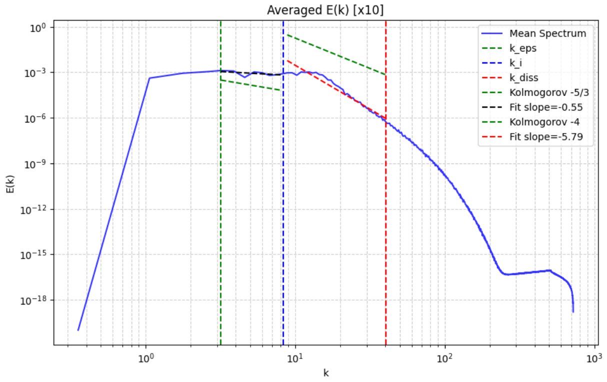
Fitting Summary

Averaged Spectra Fitting Result



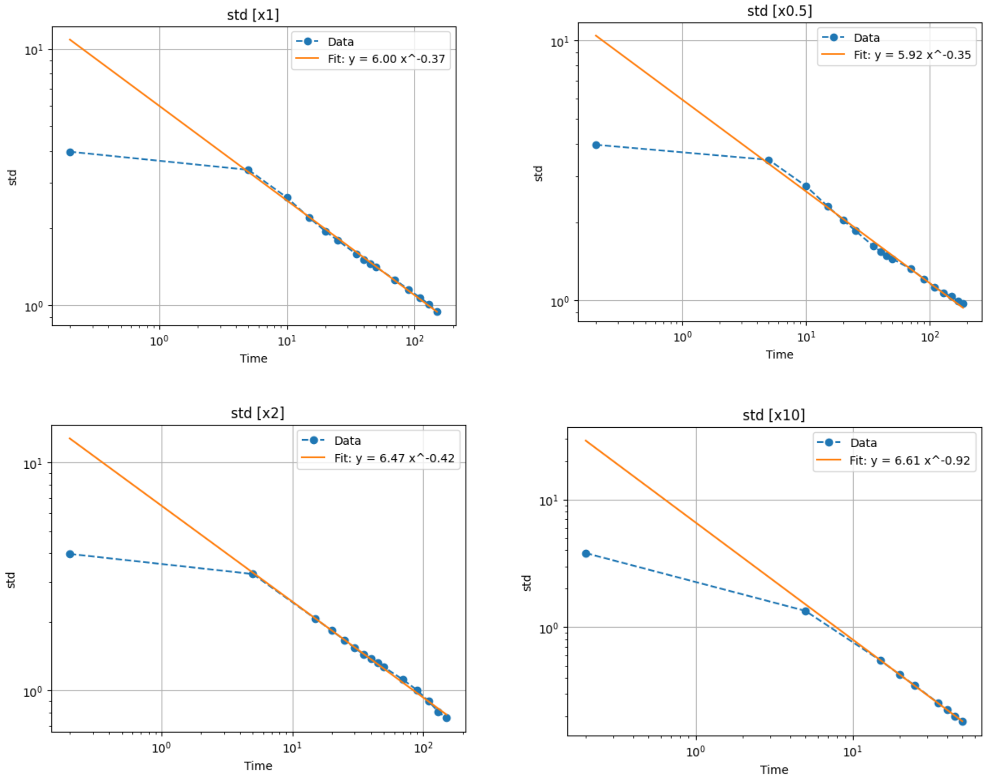




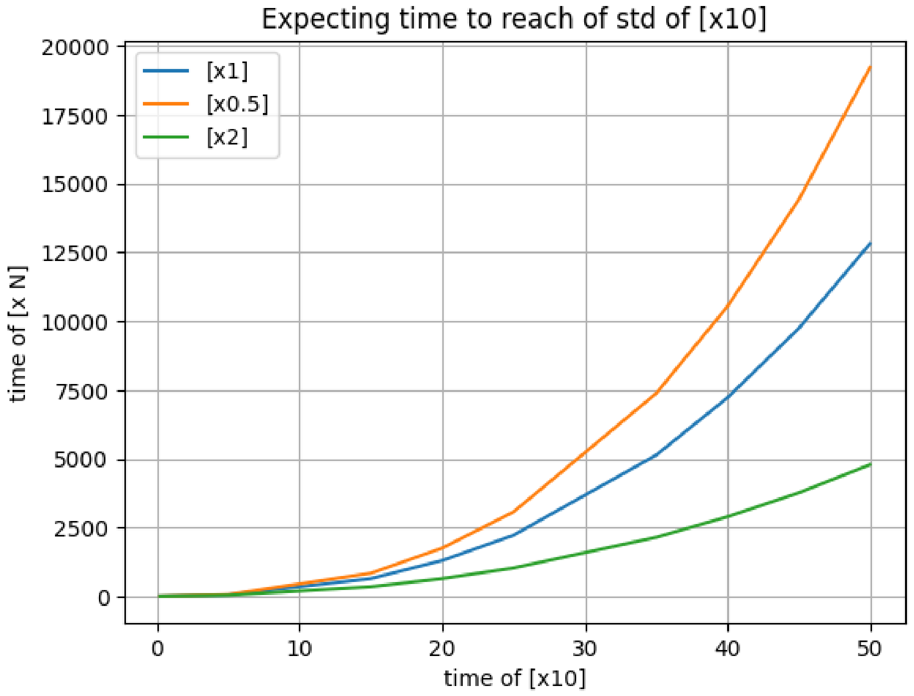


Energy Inertial Range 이론값: -1.67	Enstrophy Inertial Range 이론값: -4
[x1]: -1.67	[x1]: -4.14
[x0.5]: -1.50	[x0.5]: -3.92
[x2]: 1.43	[x2]: -4.47
[x10]: -0.55	[x10]: -5.79

Std of PDF(log scale) for regression



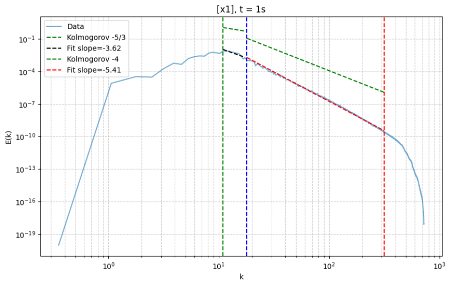
Std를 log scale로 표현하여 선형 구간에서의 회귀 분석 진행
점도가 높을수록 회귀선의 기울기 증가
[x10]: -0.92 | [x2]: -0.42 | [x1]: -0.37 | [x0.5]: -0.35

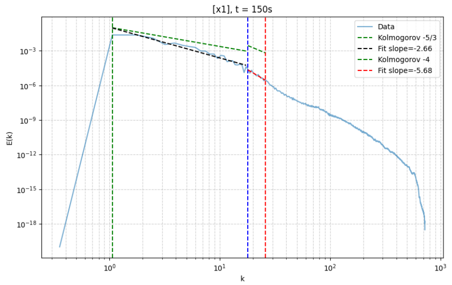


(점도별) [x10]에서의 표준편차(std)에 도달하는 데 걸리는 시간

[x10]에서 50초일 때의 std에 도달하는데 [x2]는 약 5000s 소요

Background





$$E(k,t) = \nu^{3/2} t^{-1/2} \psi(k\sqrt{\nu t}), \qquad (1)$$

where ψ is an arbitrary function of its argument. The only scale in the model is the dissipation length $L_{diss}(t) = \sqrt{\nu t}$, to which it corresponds the wavenumber $k_{diss}(t) = 1/L_{diss}$.

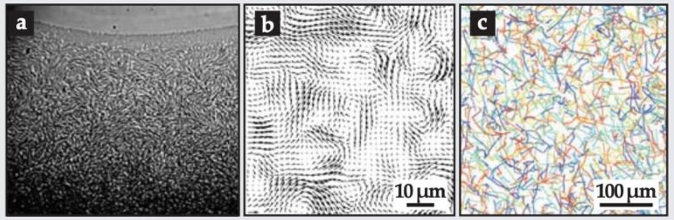


Figure 4. Suspensions of swimming bacteria exhibit transient, recurring states of collective motion known as bacterial turbulence. **(a)** In the turbulent state, densely suspended *Bacillus subtilis* microbes adopt local, but not long-range, orientational order. **(b)** A snapshot of their instantaneous velocities shows a pattern of vortices and jets. **(c)** For a suspension of *Escherichia coli*, a map of swimmers' trajectories over an eight-second period reflects the chaos and disorder that prevails at longer time scales. (Panels a and b are adapted from ref. 10; panel c is adapted from ref. 17.)

Eric Lauga <Dance of the micro swimmers>

구간 선정 후 Fitting하는 방식이 아닌
이론값(-5/3, -4)에 부합하는 기울기를 가진 구간을 추적

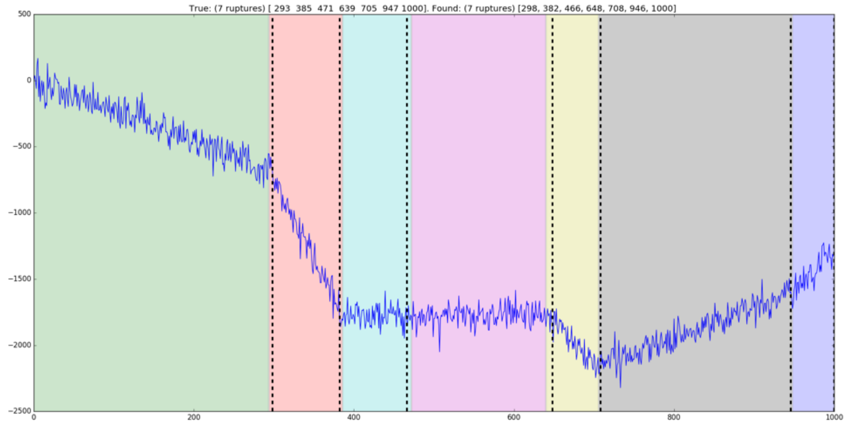
Features

| Change Point Detection:

Utilize advanced algorithms (such as the PELT algorithm in ruptures, open code, with a custom cost function) to segment the spectrum into distinct scaling regions.

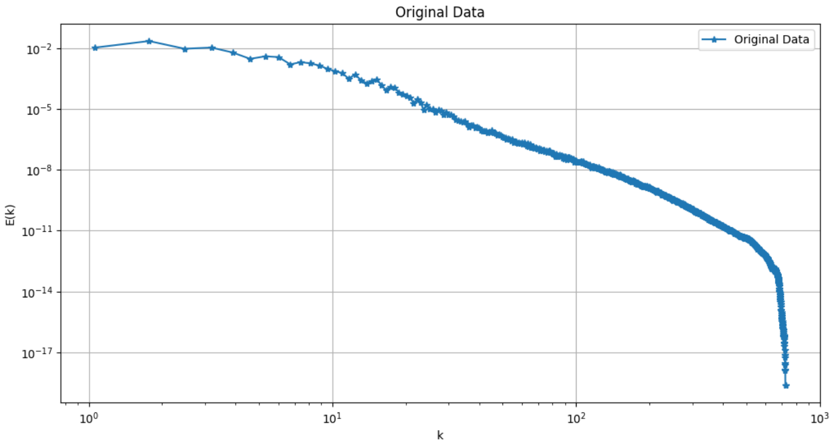
| Segment Fitting:

Apply linear regression on each segment to estimate power-law exponents and perform statistical tests to compare against theoretical models.

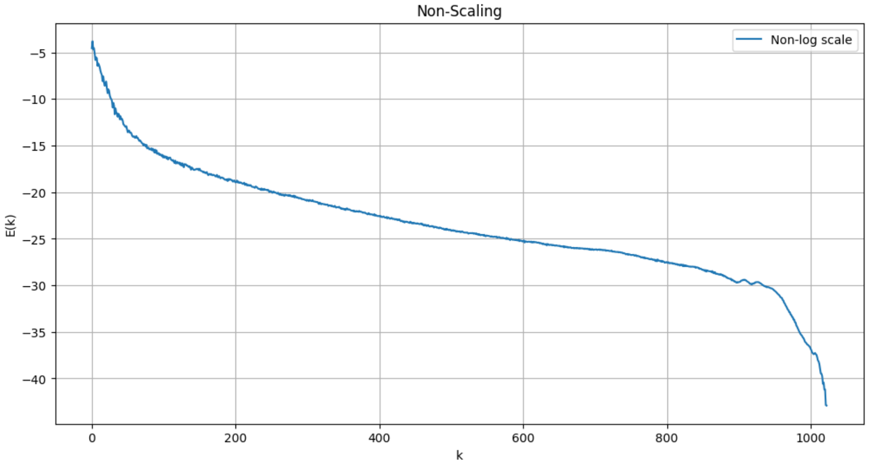


C. Truong <Selective review of offline change point detection methods. Signal Processing>

Preprocess for segmentation

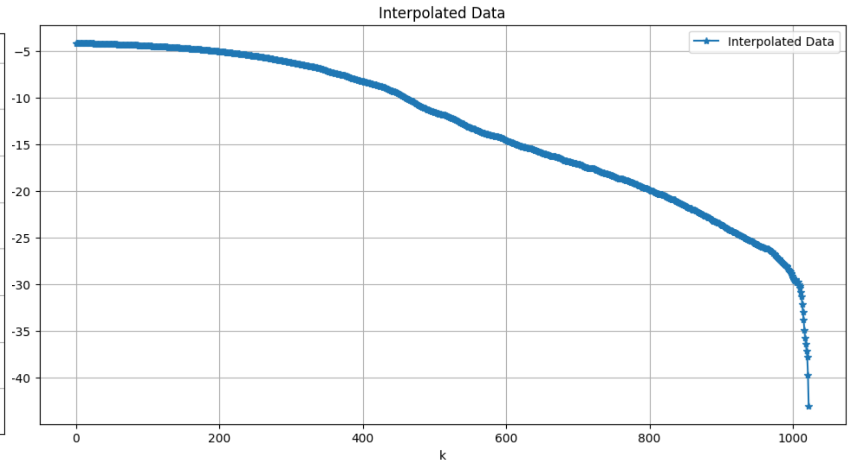


1. Original X-Y Log scale plot



2. Log(E(k))

x축이 log scale이 아니므로 PELT의 알고리즘에 그대로 사용할 경우 segmentation이 적절하게 이루어지지 않을 가능성이 존재



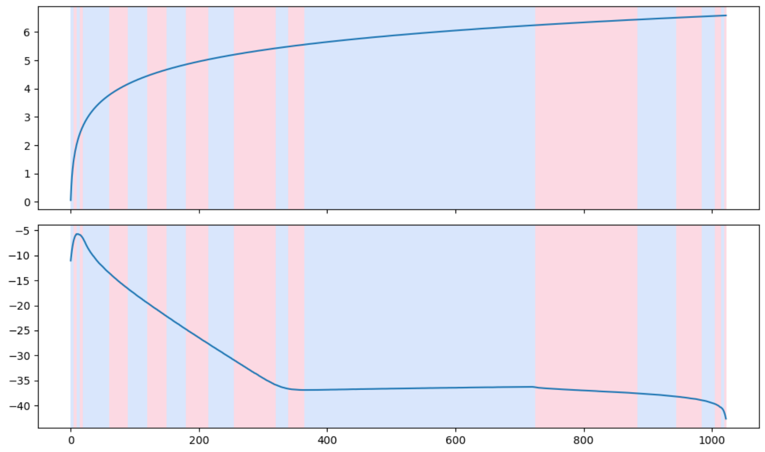
3. Smoothing(Savitzky-Golay Filter)

4. Interpolation

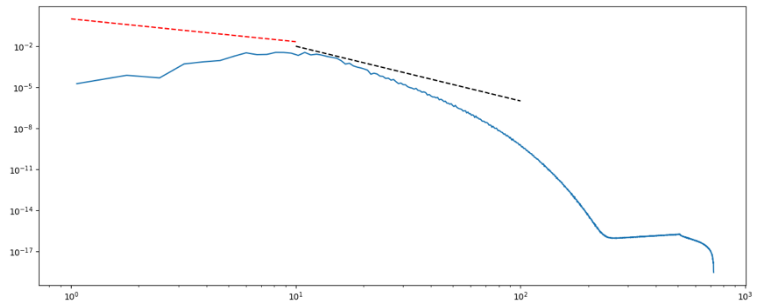
ln(k) = linspace(min(ln(k)), max(ln(k)), N)

ln(k) – ln(E(k)) interpolation

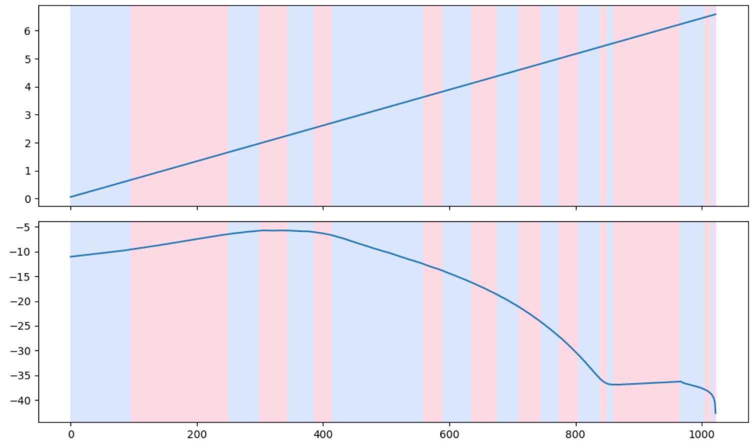
Preprocess for segmentation



Non - Interpolation



Original



Interpolation

Custom Cost Function

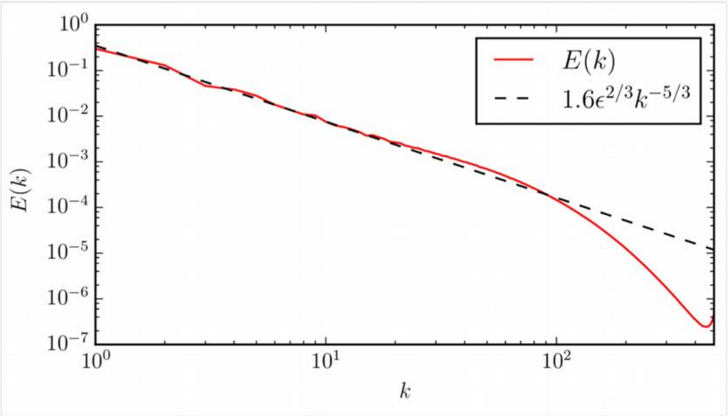


Figure 1: Radial kinetic energy spectrum, averaged in time between t=0 and 10.056.

Three dimensional Energy Spectrum

$$E(k) \sim k^n$$

1. E(k) Log scale => $y_i \approx m\, x_i + b$

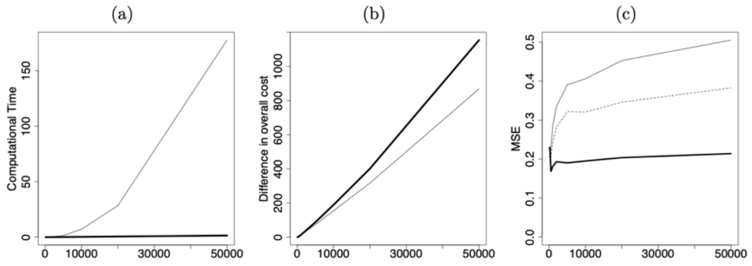
2. LSM, SSE=> $$\min_{m,b} \sum_{i=s}^{e-1} \left(y_i - \left(m\, x_i + b\right)\right)^2$$

$$\text{Cost}(s,e) = \sum_{i=s}^{e-1} \left(y_i - \left(m\, x_i + b\right)\right)^2$$

3. PELT(Pruned Exact Linear Time) => $$\sum_{i=1}^{m+1} \left[C\left(y(\tau_{i-1}+1):\tau_i\right) \right] + \beta f(m)$$

C: cost Function
Bf(m): penalty

Figure 2: (a) Average Computational Time (in seconds) for a change in variance (thin: OP, thick: PELT). (b) Average difference in cost between PELT and BS for subBS (thin), optimal BS (thick)) (c) MSE for PELT (thick), optimal BS (thin) and subBS (dotted).



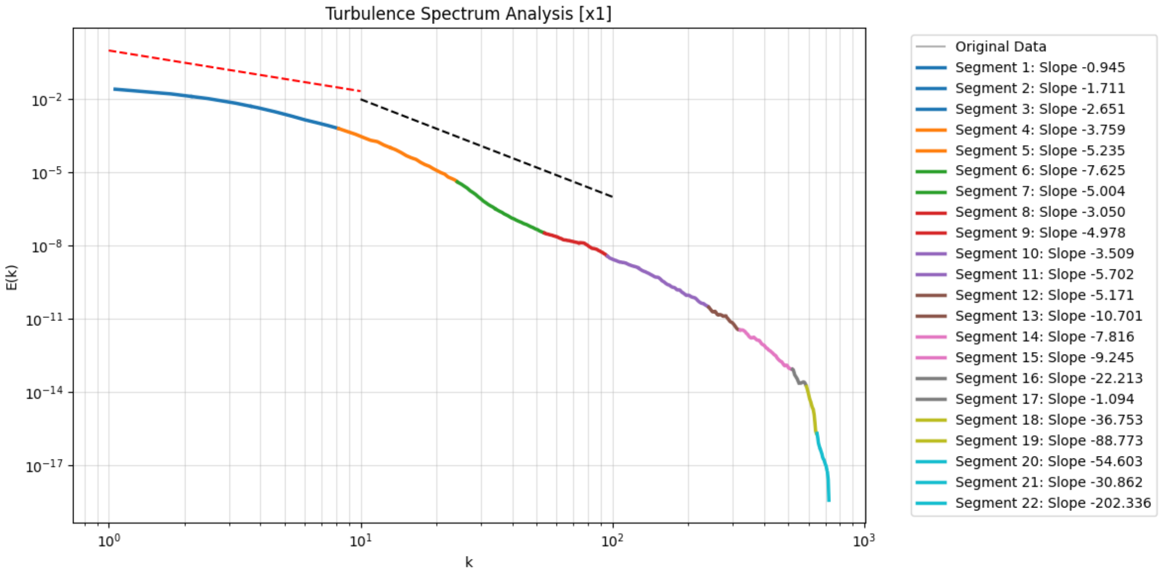
Killick, R. <Optimal detection of changepoints with a linear computational cost>

4. scoring(t-test) => $$t_{\text{stat}} = \frac{m - target}{SE_m}$$

p_value = 2 * (1 - stats.t.cdf(np.abs(t_stat), df=n-2))

Result Summary

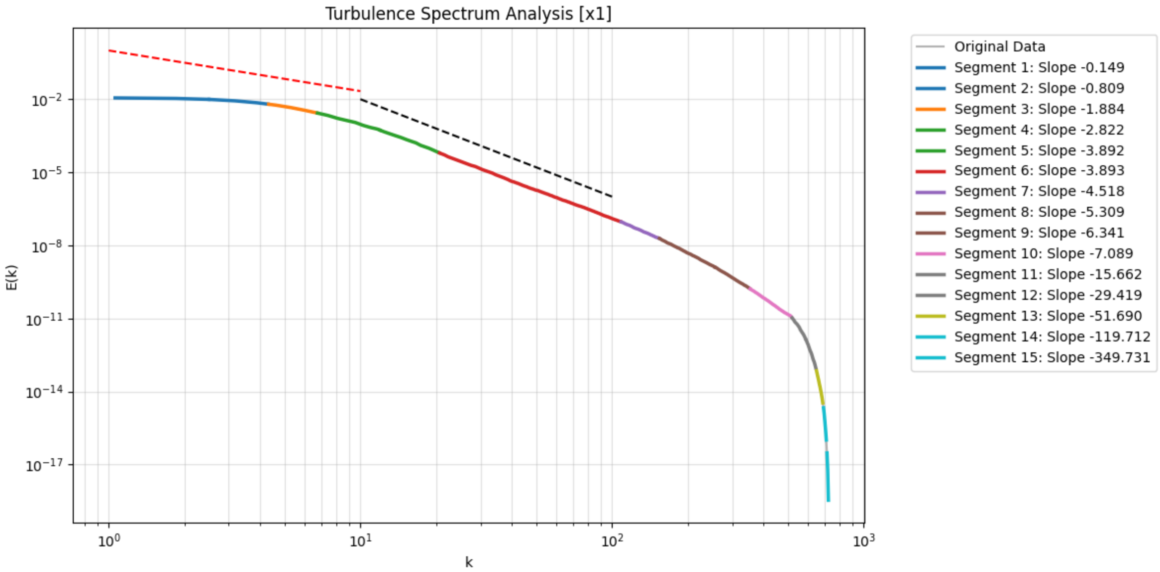
t=150s



Energy inertial range:
Segment 2: k from 2.14 to 3.66

Enstrophy inertial range:
Segment 4~11: k from 8.18 to 240

Averaged across Time



Energy inertial range:
Segment 3: k from 4.32 to 6.71

Enstrophy inertial range:
Segment 5~8: k from 12 to 256

250227 Weekly Lab meeting

Thank you for your attention!

Code