

250214 Weekly Lab meeting

Weekly Lab Meeting

Juseong Kim

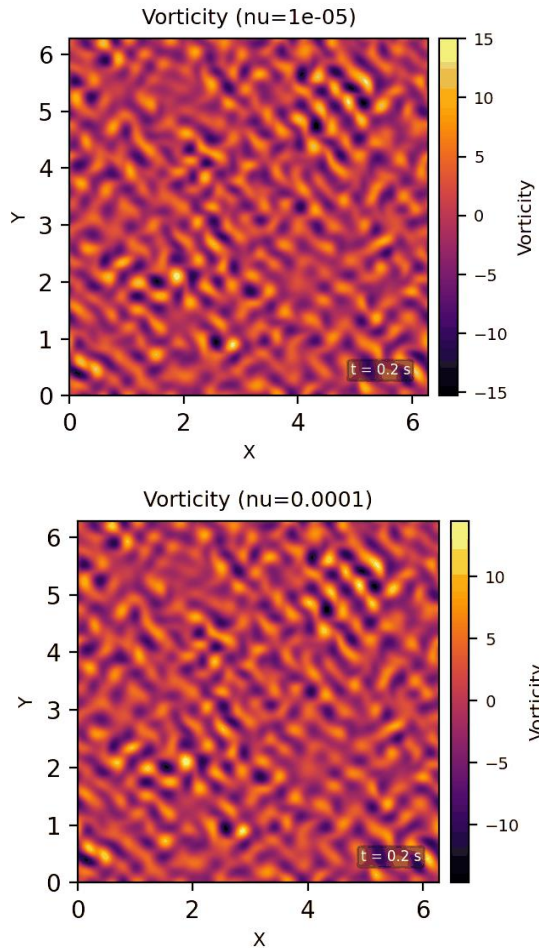


Flow Physics & Computational
Engineering Innovation Lab

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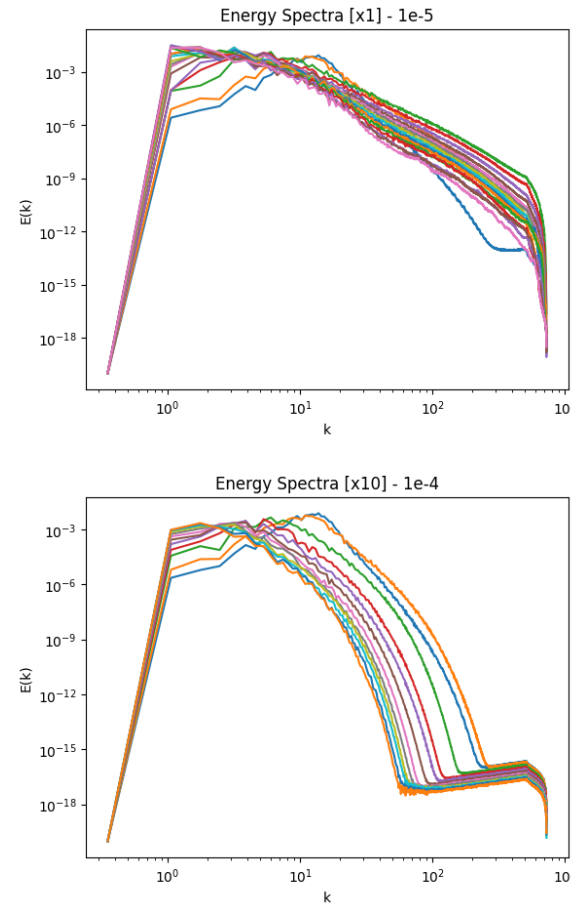
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Assignment -1



1. Turbulence Simulation

OpenFOAM으로 IC와 BC,
초기 속도장을 설정한 후
동점성 계수(ν)를
논문값($1e-5$) 대비
 $\times 2$, $\times 0.5$, $\times 10$ 하여
시뮬레이션 진행



2. Data Analysis

시뮬레이션 결과로 나온
데이터들의 에너지 스펙트럼,
통계적 특성들을 분석

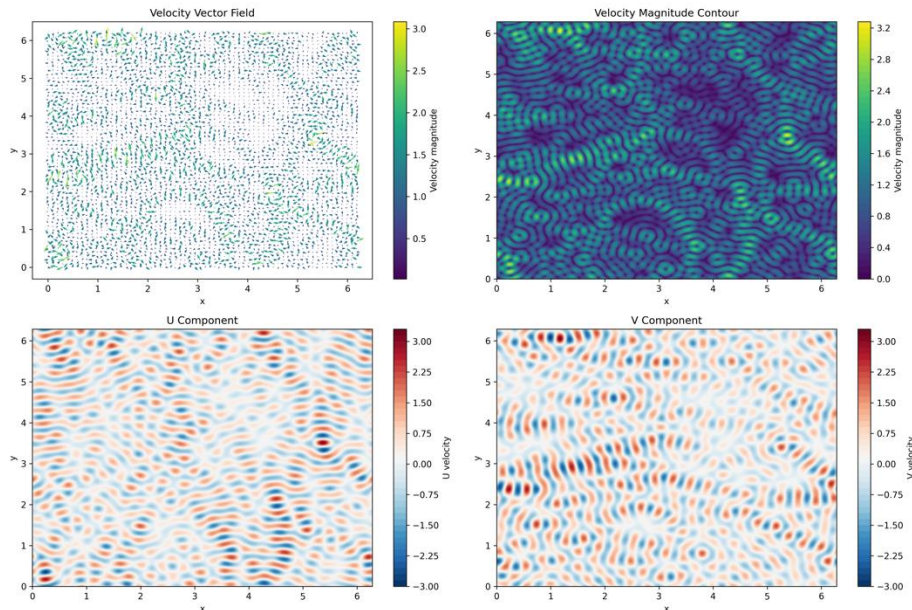
- i) ν 에 따른 스펙트럼의 기울기
- ii) Vorticity PDF 분석

Assignment -2

To mimic such a procedure in our simulations, we performed 50 two-dimensional simulations of the Navier-Stokes equations on a regular grid of 2048^2 points. Initial conditions were a random superposition of harmonic modes between wave numbers $k = 18$ and 22 , with the spectrum peaking at $k_0 = 20$. Viscosity was $\nu = 2.5 \times 10^{-4}$ in all these runs and the box had length 2π . The initial rms velocity U_{rms} in all runs was 1, corresponding to a turnover time of $\tau_{NL} = L_0/U_{rms} = 2\pi/20 \approx 0.3$ ($L_0 = 2\pi/k_0$). As a result, the runs differed only by their random initial phases.

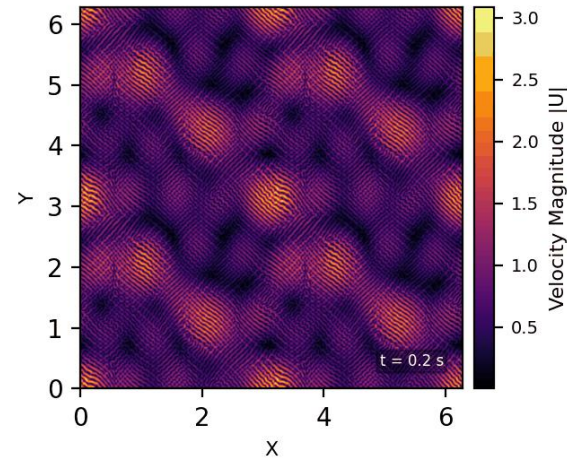
Validation model

P. D. Mininni <Inverse cascade behavior in freely decaying two-dimensional fluid turbulence>

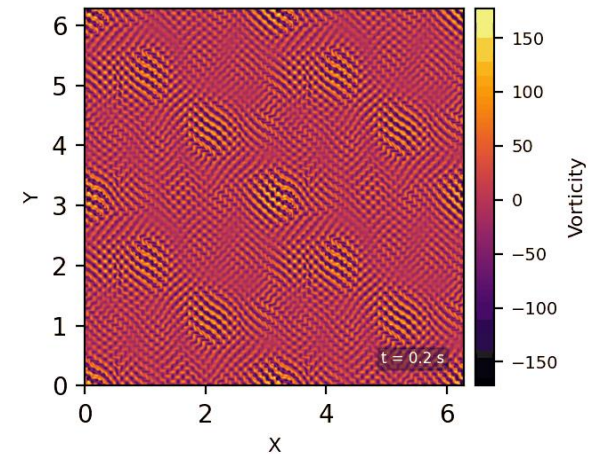


Generating Initial Velocity field using python

Velocity Magnitude (nu=0.00025), 2048x2048



Vorticity (nu=0.00025)

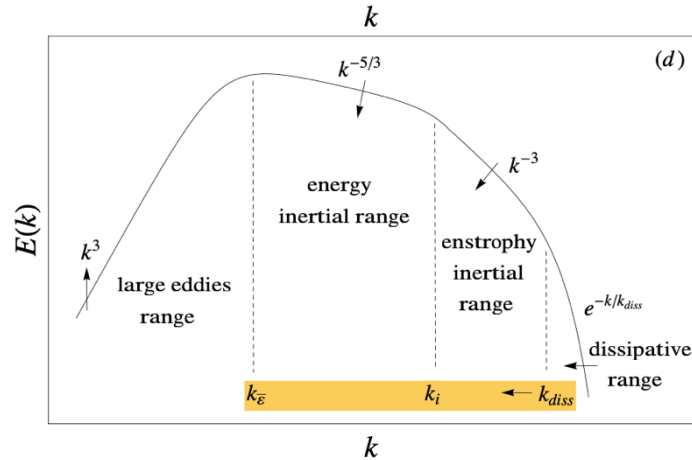


Valid Simulation Result

3. Code validation

OpenFOAM과 데이터 분석 코드의 유효성을 검증하기 위해 다른 논문의 난류 모델을 시뮬레이션 한 후 데이터 분석 결과를 비교

Fitting Range



[Leonardo Campanelli](#) <Dimensional analysis of two-dimensional turbulence>

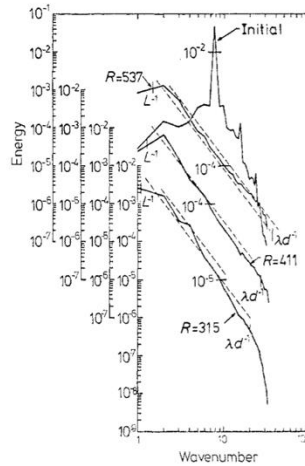


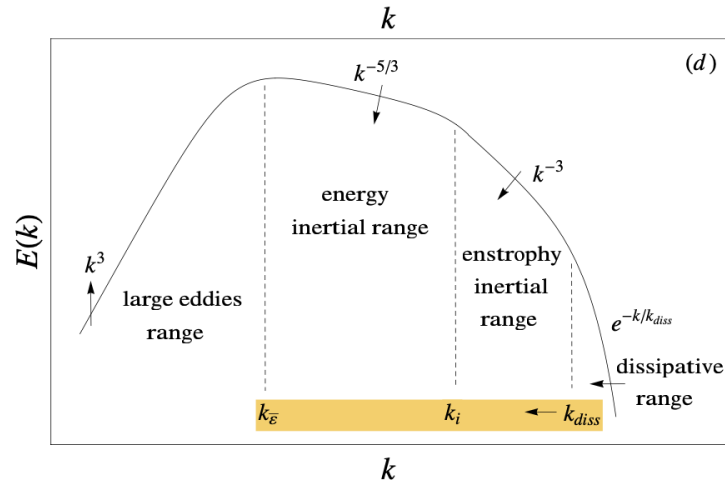
Figure 12. Energy spectrum for three 2D Navier-Stokes decay runs (from Lilly 1971), averaged over 200 time steps, and labelled by Reynolds number. Broken lines have slope -3 .

R H Kraichnan <Two-dimensional turbulence>

Energy spectrum은 4개의 구간으로 나누어져 있으며 각각 Large eddies, Energy inertial, Enstrophy inertial, Dissipative range로 구성된다.

이 때 inertial range 구간의 기울기 경향성이 이론값인 Energy inertial range에서 $-5/3$, (kolmogorov) Enstrophy inertial range에서 -4 (Kraichnan)를 만족하는지 확인한다.

Selection of Range -1



Inertial Range Fitting 구간 선정($k_{\bar{\epsilon}} < k < k_{diss}$)

Left point. $k_{\bar{\epsilon}}$: $E(k)$ 가 max일 때 k 값이라 가정

Right point $k_{diss} = 1/\sqrt{\nu t}$

$$E(k, t) = \nu^{3/2} t^{-1/2} \psi(k\sqrt{\nu t}), \quad (1)$$

where ψ is an arbitrary function of its argument.

The only scale in the model is the dissipation length $L_{diss}(t) = \sqrt{\nu t}$, to which it corresponds the wavenumber $k_{diss}(t) = 1/L_{diss}$.

Selection of Range -2

8

A. Alexakis, L. Biferale / Physics Reports 767–769 (2018) 1–101

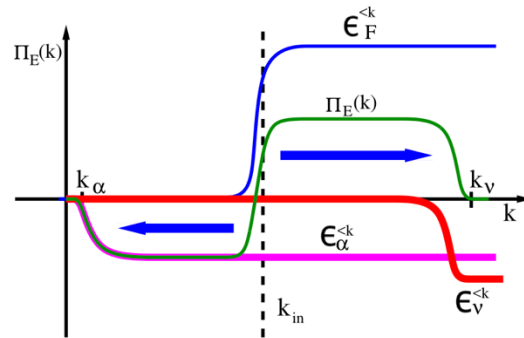


Fig. 3. Qualitative sketch of the stationary scale-by-scale energy balance for the energy flux, $\Pi_E(k)$. Notice that the assumption of scale separation, $k_{\alpha} \ll k_{in} \ll k_v$ predicts the existence of two *inertial ranges* where the energy flux is constant and due only to the non-linear triadic interactions.

Inertial Range Fitting 구간 선정 ($k_{\bar{\epsilon}} < k < k_{diss}$)

Middle point k_i : Energy Flux가 양수가 되는 지점

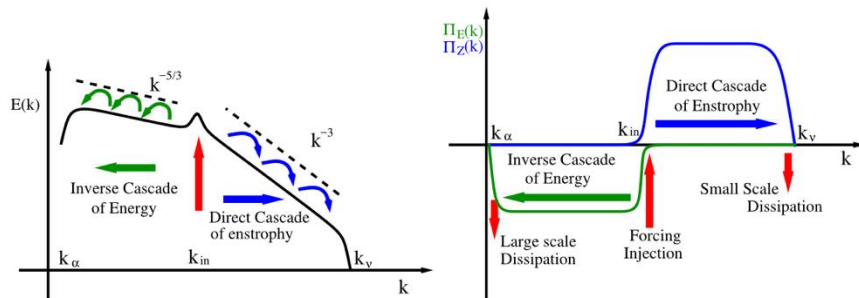
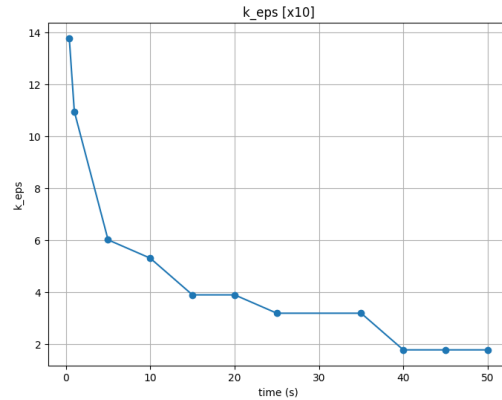
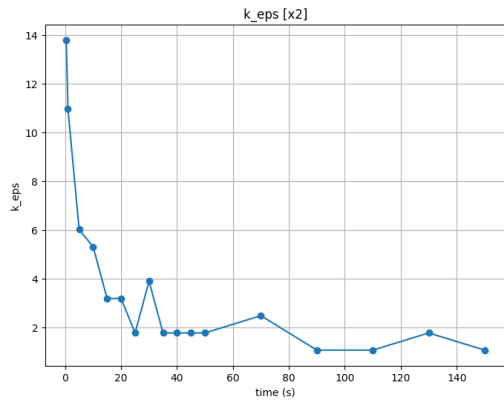
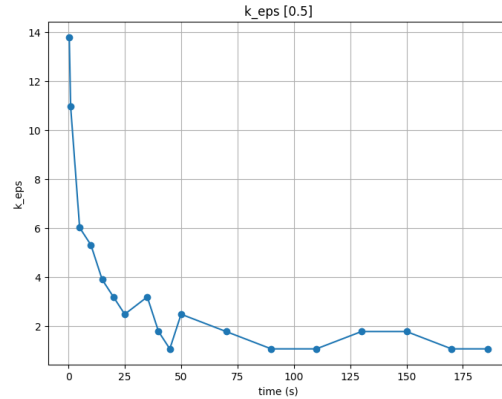
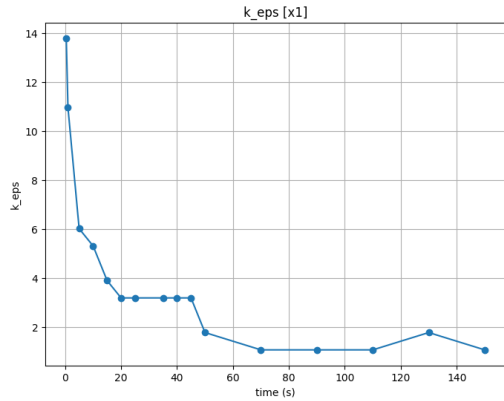


Fig. 6. Log-log sketch of the energy spectrum (left) and of energy and enstrophy fluxes (right) for the 2D Batchelor–Kraichnan theory (59)–(58).

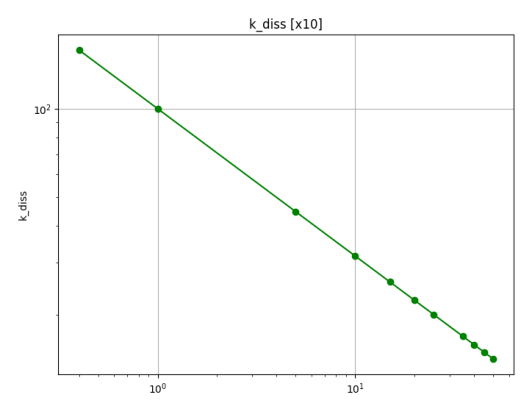
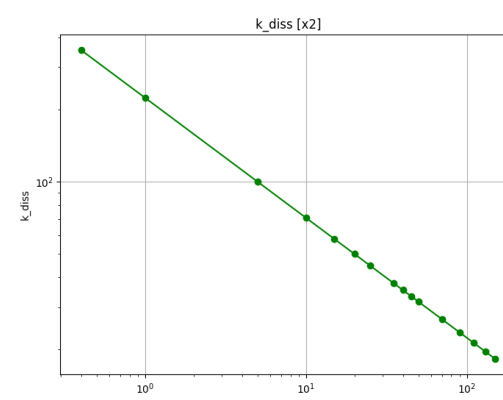
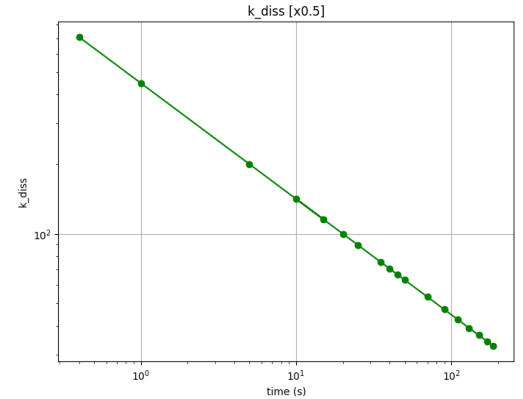
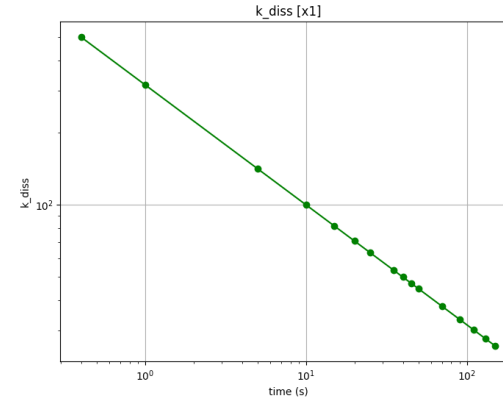
A.Alexakis <Cascades and transitions in turbulent flows>

Selection of Range -3

Left point $k_{\bar{\epsilon}}$ (max of $E(k)$) Over time

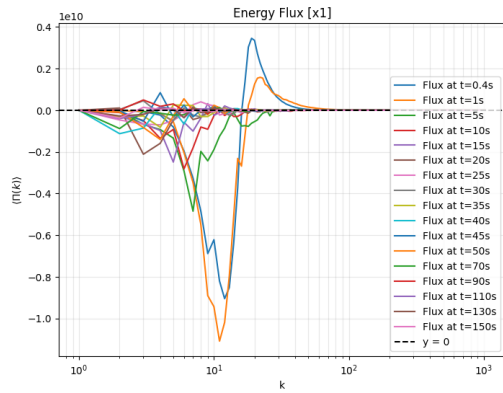


Right point $k_{diss}(1/\sqrt{vt})$ Over time

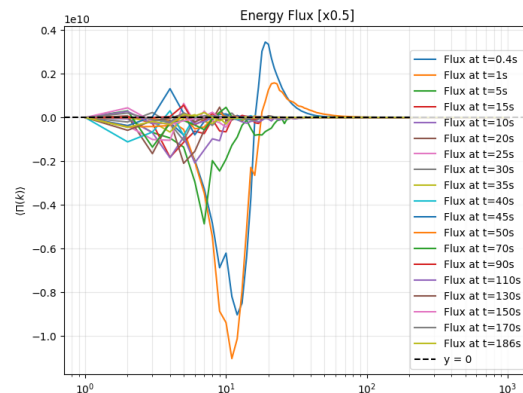


모든 동점성 계수 case에서 시간이 지남에 따라
스펙트럼의 peak point에 해당하는 k 가 점차 감소

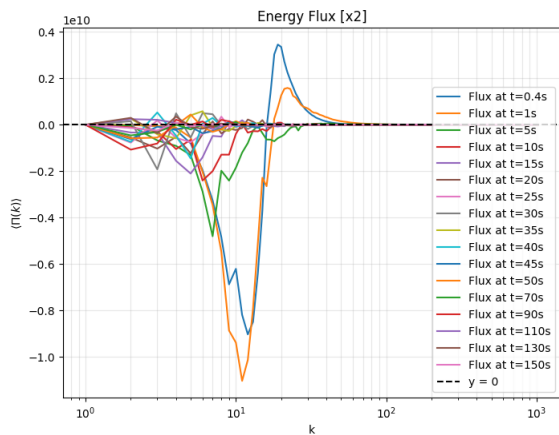
Selection of Range -4

Middle point k_i 

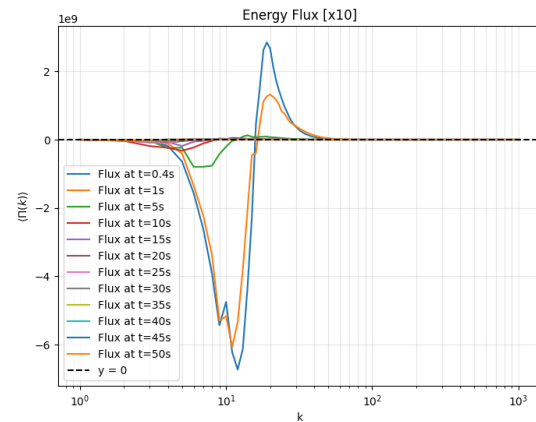
[x1]



[x0.5]



[x2]

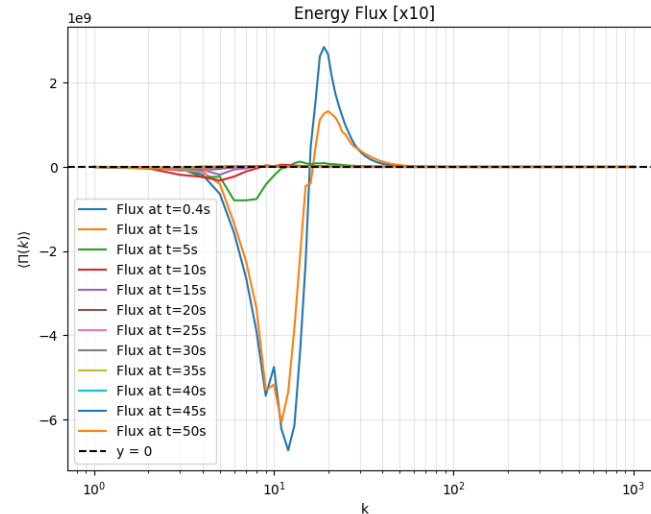
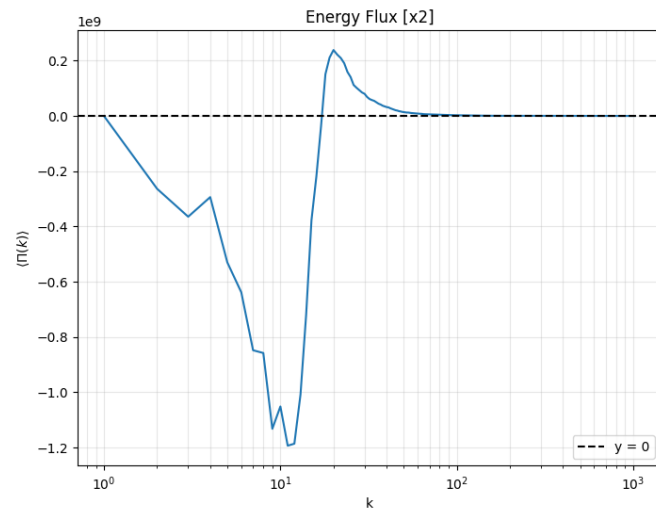
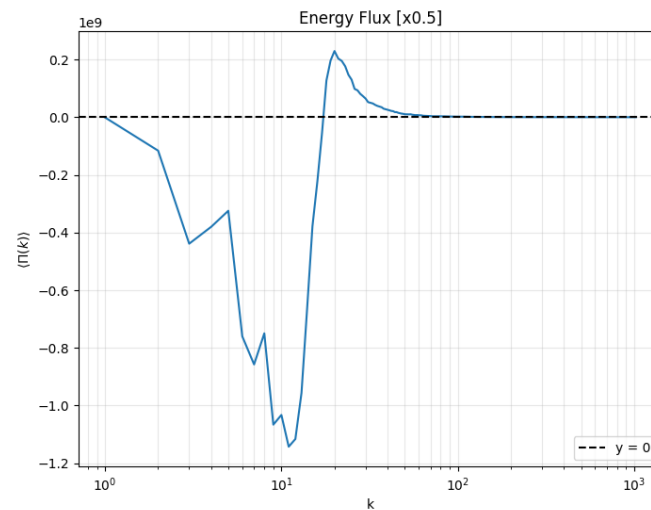
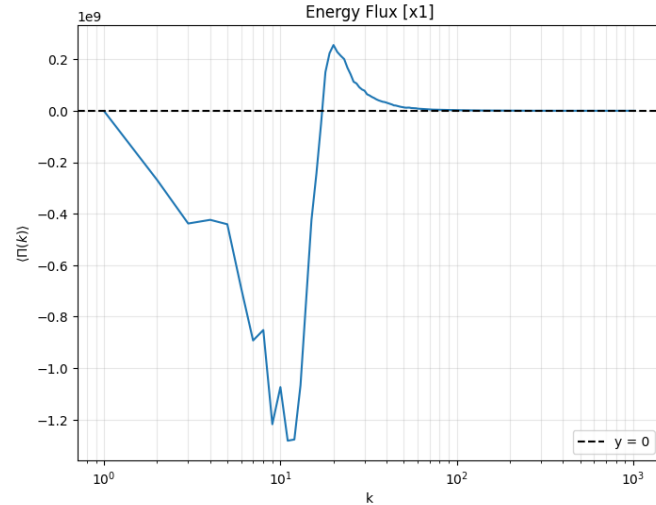


[x10]

[x10] is clearly distinct by time periods
 >> Inverse cascade 구간을 정확하게 확인 가능

[x1], [x0.5], [x2]
 >> 진동으로 인해 inverse cascade 구간의 식별이 어려워
 flux를 시간 평균하여 k_i 도출

Selection of Range -5



Middle point k_i

[x1]: 18 (Averaged)

[x0.5]: 18 (Averaged)

[x2]: 18 (Averaged)

[x10]

16 at t= 0.4s

17 at t= 1s

13 at t= 5s

9 at t= 10s

8 at t= 15s

7 at t= 20s

5 at t= 25s

7 at t= 30s

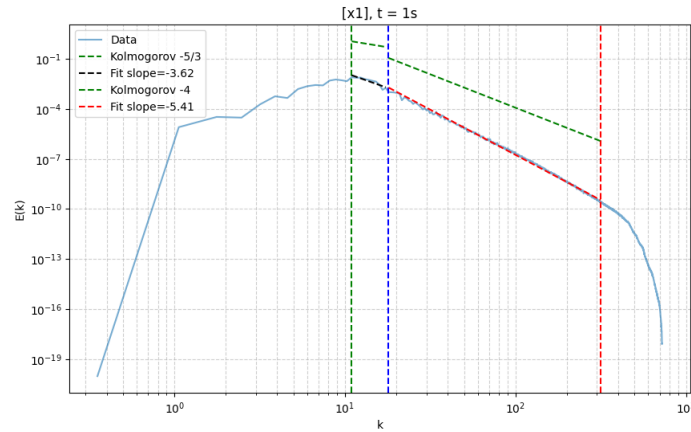
5 at t= 35s

5 at t= 40s

4 at t= 45s

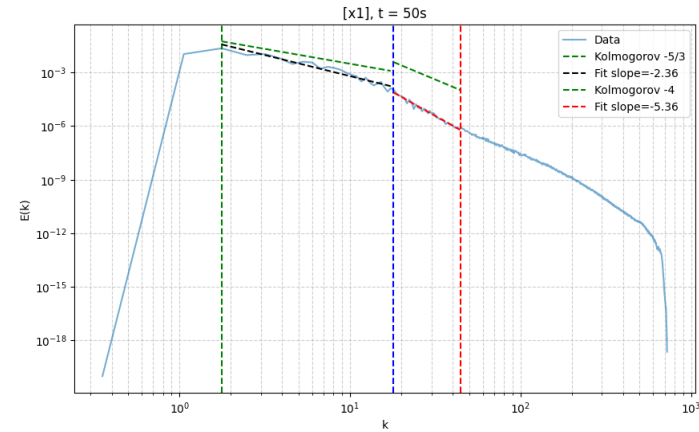
4 at t= 50s

Fitting Result – [x1] each time step



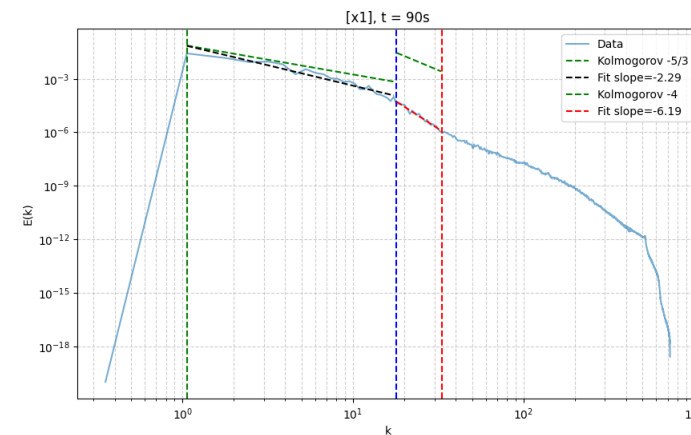
Energy inertial range Slope: -3.616
Deviation from -5/3: 116.95%
 R^2 : 0.829

Enstrophy inertial range Slope: -5.411
Deviation from -4: 35.28%
 R^2 : 0.998



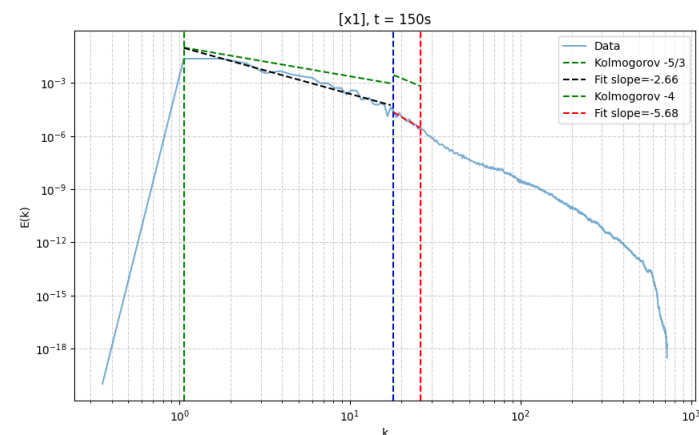
Energy inertial range Slope: -2.357
Deviation from -5/3: 41.42%
 R^2 : 0.935

Enstrophy inertial range Slope: -5.358
Deviation from -4: 33.96%
 R^2 : 0.980



Energy inertial range Slope: -2.292
Deviation from -5/3: 37.52%
 R^2 for Inverse Cascade: 0.931

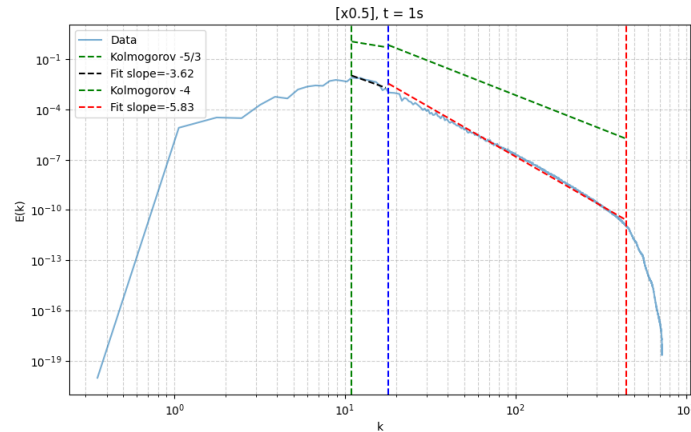
Enstrophy inertial range Slope: -6.186
Deviation from -4: 54.65%
 R^2 : 0.972



Energy inertial range Slope: -2.662
Deviation from -5/3: 59.72%
 R^2 for Inverse Cascade: 0.920

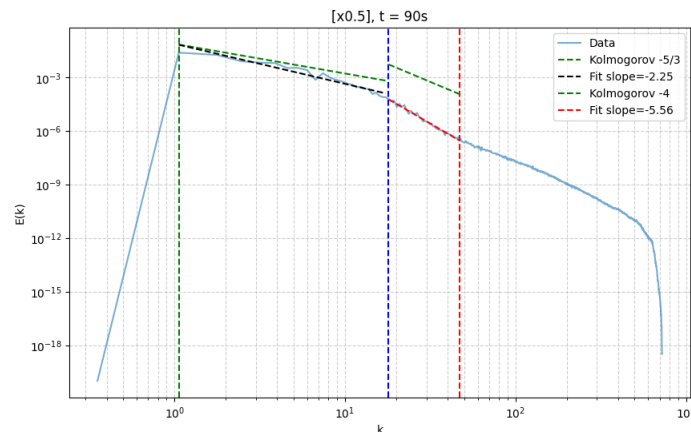
Enstrophy inertial range Slope: -5.675
Deviation from -4: 41.88%
 R^2 : 0.891

Fitting Result – [x0.5] each time step



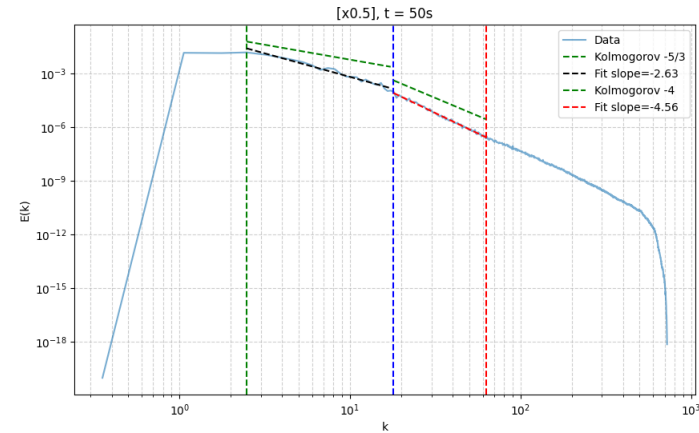
Energy inertial range Slope: -3.618
Deviation from -5/3: 117.09%
 R^2 for Inverse Cascade: 0.829

Enstrophy inertial range Slope: -5.829
Deviation from -4: 45.72%
 R^2 : 0.994



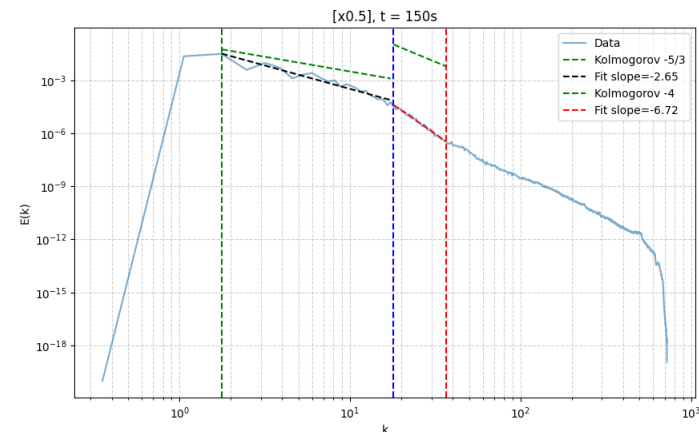
Energy inertial range Slope: -2.245
Deviation from -5/3: 34.70.%
 R^2 : 0.938

Enstrophy inertial range Slope: -5.556
Deviation from -4: 38.91%
 R^2 : 0.986



Energy inertial range Slope: -2.629
Deviation from -5/3: 57.75%
 R^2 for Inverse Cascade: 0.970

Enstrophy inertial range Slope: -4.561
Deviation from -4: 14.02%
 R^2 : 0.993

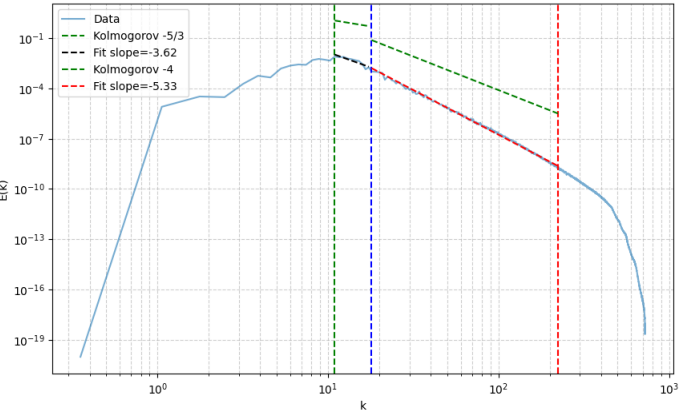


Energy inertial range Slope: -2.650
Deviation from -5/3: 58.99%
 R^2 : 0.930

Enstrophy inertial range Slope: -6.718
Deviation from -4: 67.96%
 R^2 : 0.986

Fitting Result – [x2] each time step

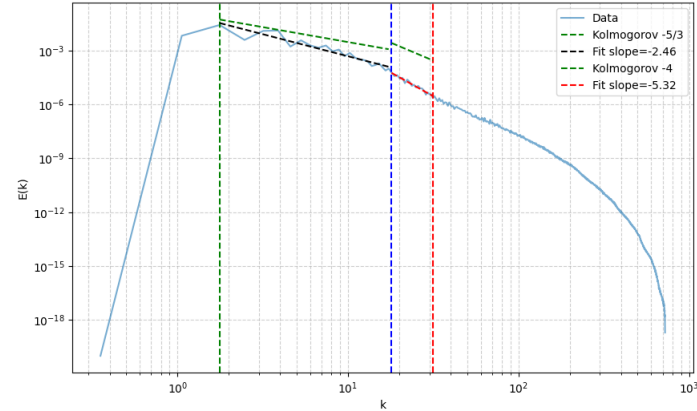
[x2], t = 1s



Energy inertial range Slope: -3.618
Deviation from -5/3: 117.09%
 R^2 : 0.829

Enstrophy inertial range Slope: -5.33
Deviation from -4: 33.25%
 R^2 : 0.998

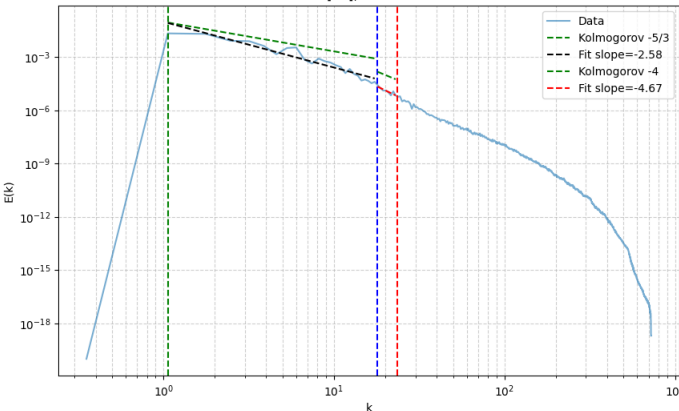
[x2], t = 50s



Energy inertial range Slope: -2.456
Deviation from -5/3: 47.34%
 R^2 : 0.910

Enstrophy inertial range Slope: -5.325
Deviation from -4: 33.12%
 R^2 : 0.959

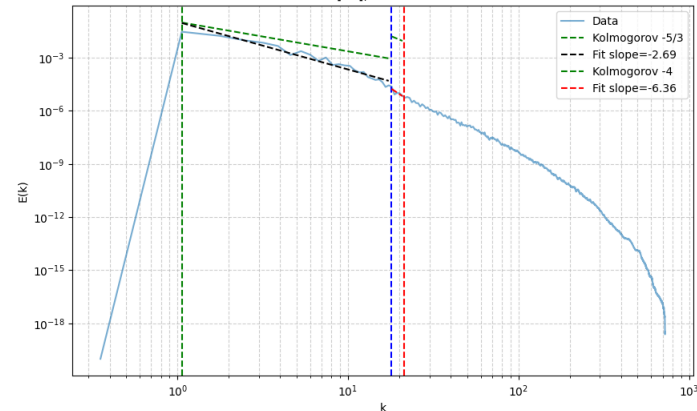
[x2], t = 90s



Energy inertial range Slope: -2.579
Deviation from -5/3: 54.68%
 R^2 for Inverse Cascade: 0.915

Enstrophy inertial range Slope: -4.666
Deviation from -4: 16.66%
 R^2 : 0.785

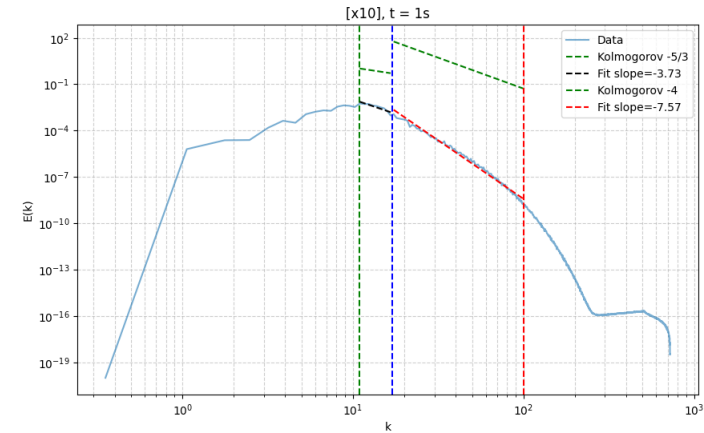
[x2], t = 110s



Energy inertial range Slope: -2.689
Deviation from -5/3: 61.33%
 R^2 for Inverse Cascade: 0.945

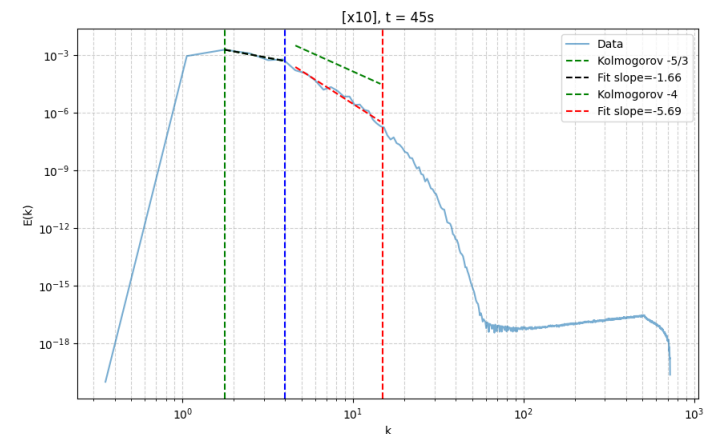
Enstrophy inertial range Slope: -6.362
Deviation from -4: 59.05%
 R^2 : 0.605

Fitting Result – [x10] each time step



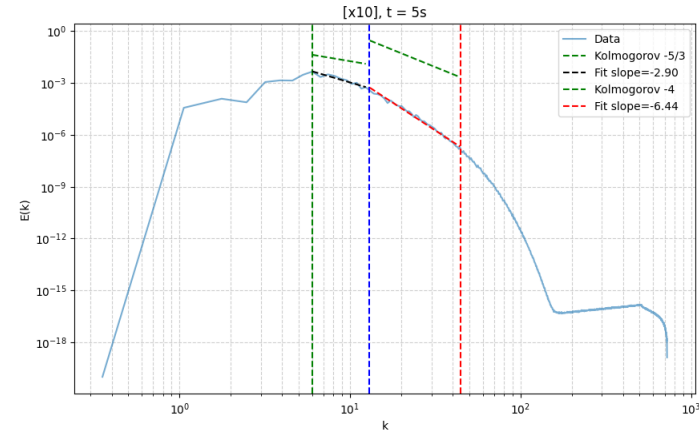
Energy inertial range Slope: -3.734
Deviation from -5/3: 124.02%
 R^2 for Inverse Cascade: 0.816

Enstrophy inertial range Slope: -7.568
Deviation from -4: 89.19%
 R^2 : 0.990



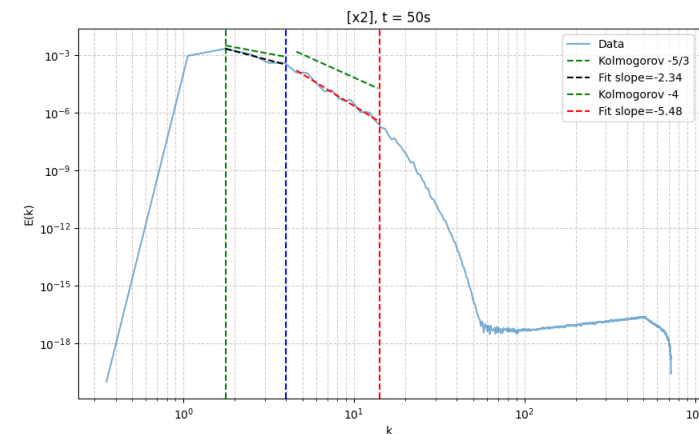
Energy inertial range Slope: -1.661
Deviation from -5/3: 0.34%
 R^2 for Inverse Cascade: 0.892

Enstrophy inertial range Slope: -5.688
Deviation from -4: 42.20%
 R^2 : 0.964



Energy inertial range Slope: -2.897
Deviation from -5/3: 73.82%
 R^2 for Inverse Cascade: 0.888

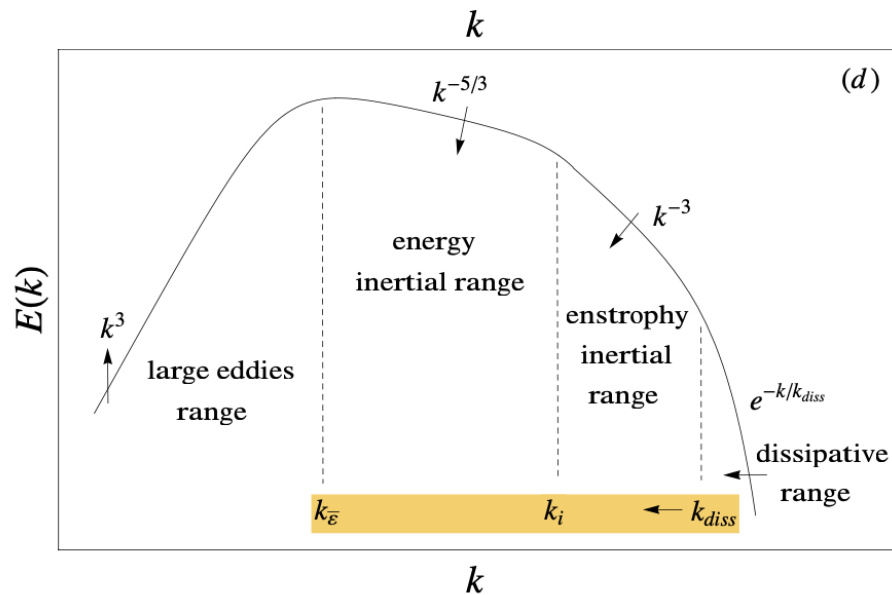
Enstrophy inertial range Slope: -6.443
Deviation from -4: 61.07%
 R^2 : 0.991



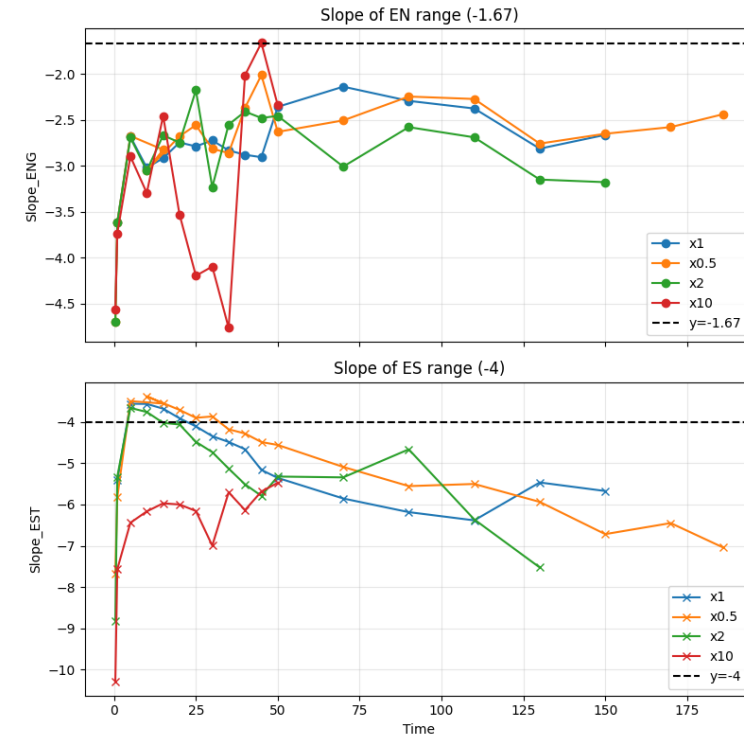
Energy inertial range Slope: -2.341
Deviation from -5/3: 40.43%
 R^2 for Inverse Cascade: 0.922

Enstrophy inertial range Slope: -5.476
Deviation from -4: 36.88%
 R^2 : 0.974

Fitting Summary



이론값

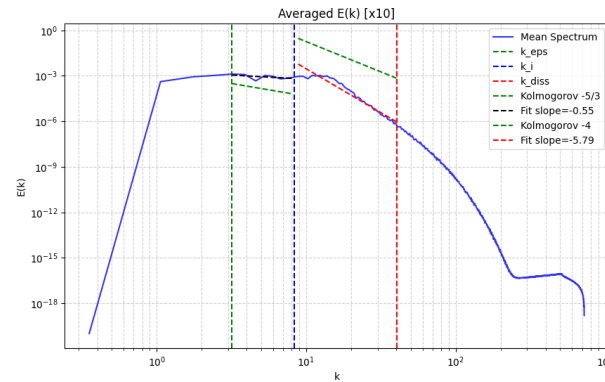
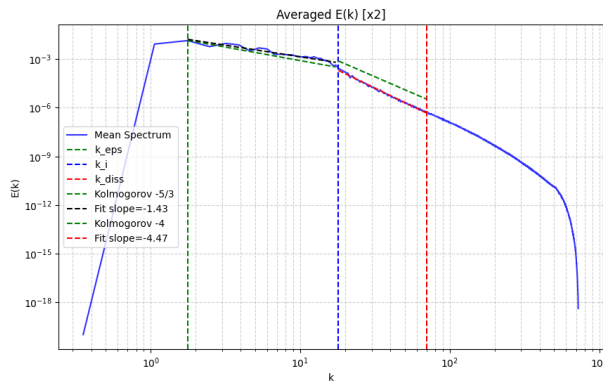
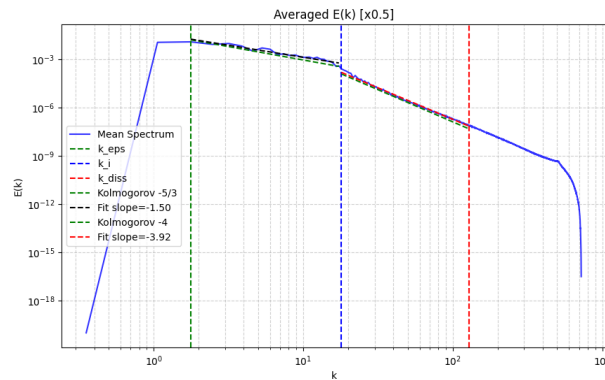
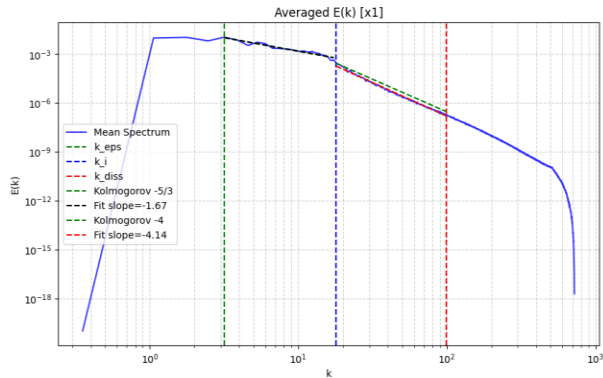


fitting result (time – slope)

Energy Inertial Range와 Enstrophy Inertial Range에서
각각의 이론값(검은 점선)과 완벽히 부합하지 않음,

Fitting Summary

Averaged Spectra Fitting Result



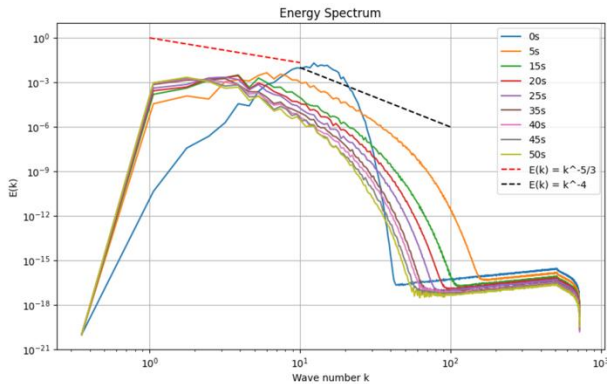
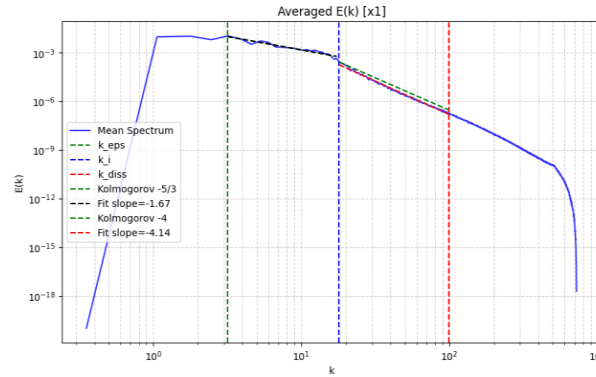
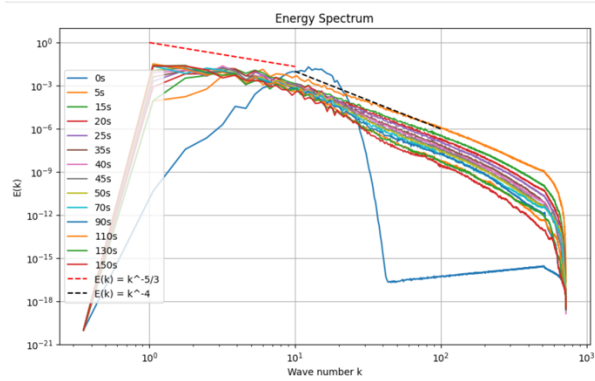
Energy Inertial Range
이론값: -1.67

Enstrophy Inertial Range
이론값: -4

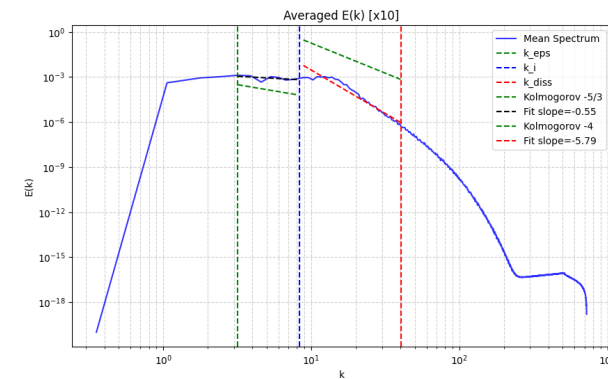
[x1]: -1.67
[x0.5]: -1.50
[x2]: 1.43
[x10]: -0.55

[x1]: -4.14
[x0.5]: -3.92
[x2]: -4.47
[x10]: -5.79

Fitting Summary



[x1]

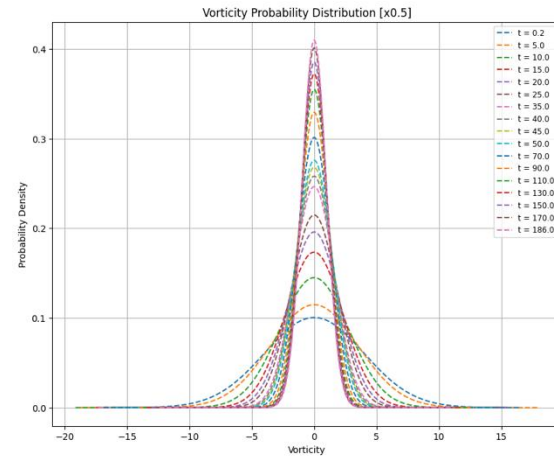
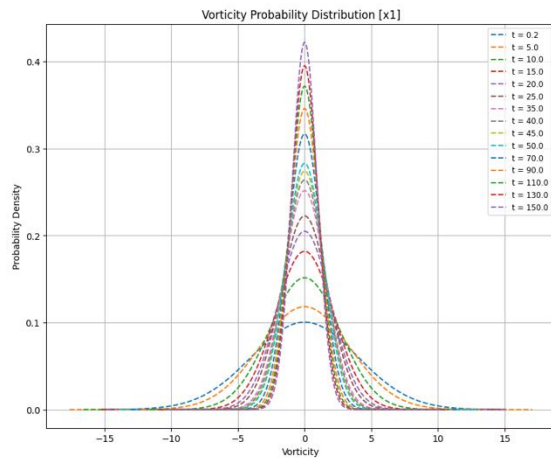


[x10]

[x10]

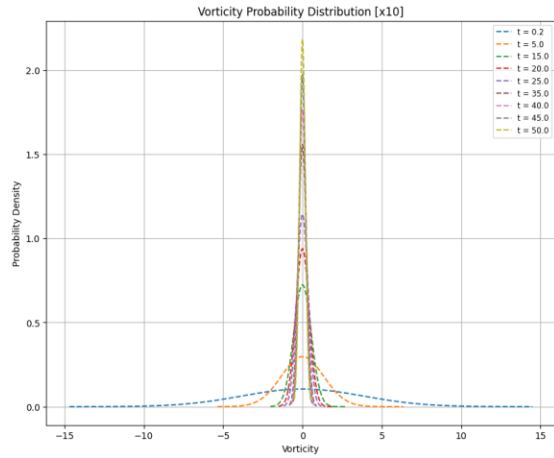
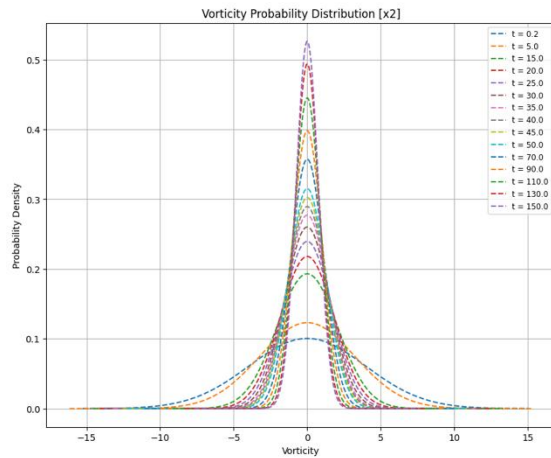
>>시간에 따른 Spectra의 변동성이 심한 모델은 짧은 시간 범위에서 평균을 계산 해야 더 정확한 값을 얻을 수 있을 것으로 추정

Vorticity PDF

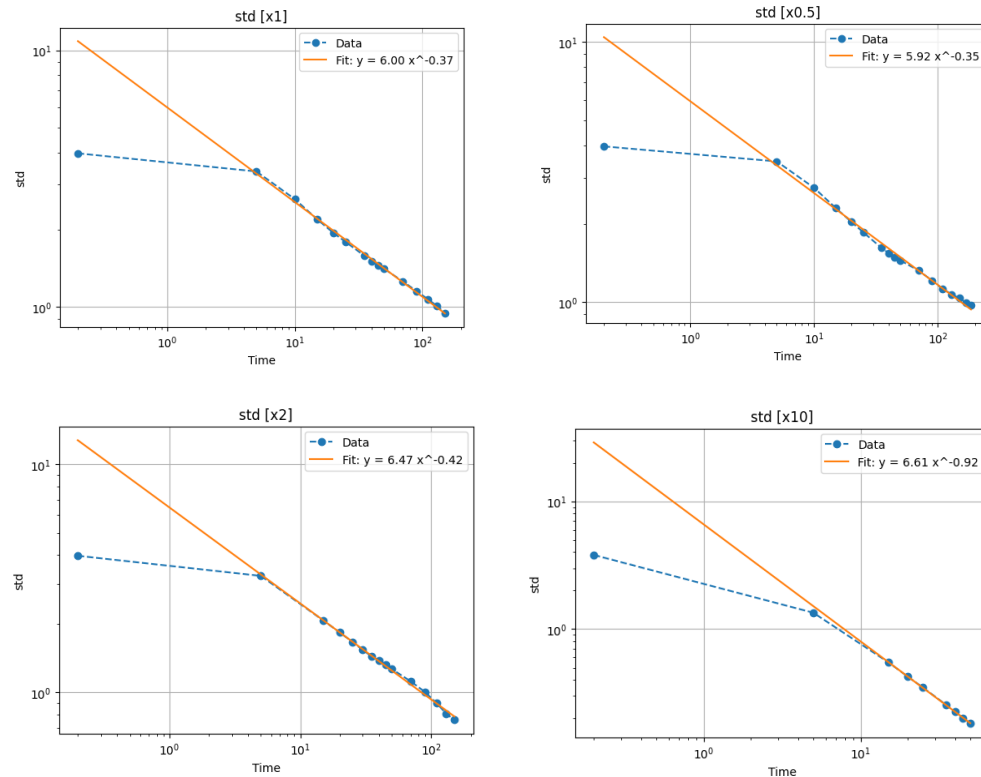


시간이 지남에 따라 소산이 진행되면서 꼬리가 짧아짐(std 감소)

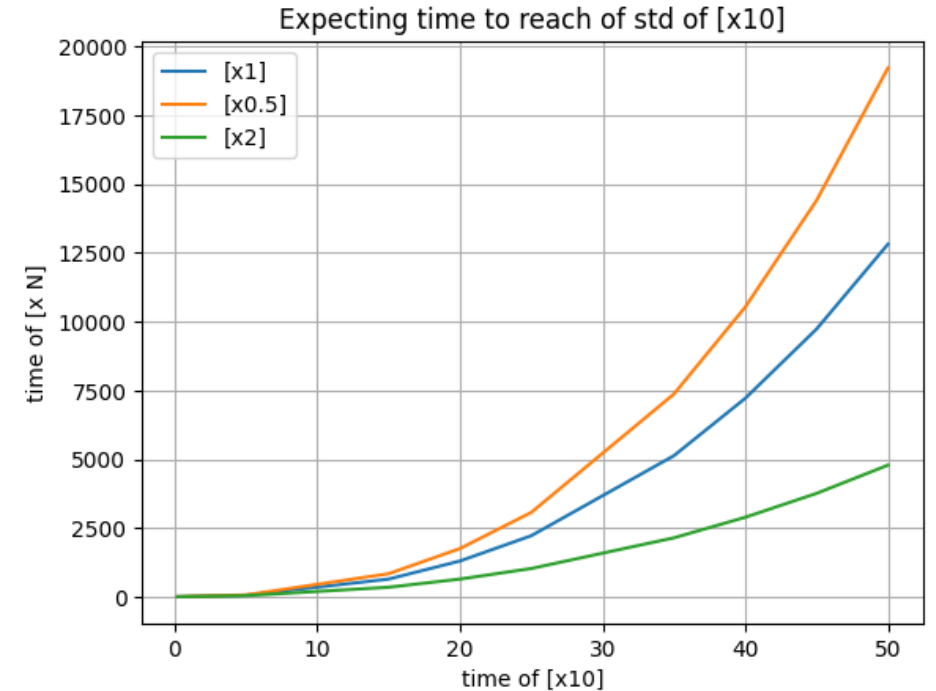
Vorticity PDF를 통해 점도 별 소산 정도를 예측



Std of PDF(log scale) for regression



Std를 log scale로 표현하여 선형 구간에서의 회귀 분석 진행
 점도가 높을수록 회귀선의 기울기 증가
 [x10]: -0.92 | [x2]: -0.42 | [x1]: -0.37 | [x0.5]: -0.35



(점도별) [x10]에서의 표준편차(std)에 도달하는 데 걸리는 시간

[x10]에서 50초일 때의 std에 도달하는데 [x2]는 약 2500s 소요

250214 Winter Weekly Lab meeting

Thank you for your attention!



Flow Physics & Computational
Engineering Innovation Lab