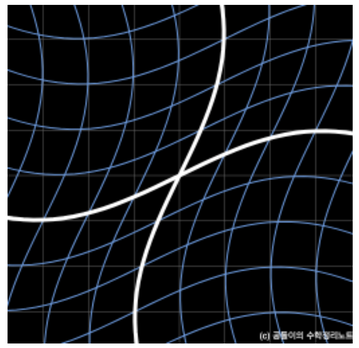
Jacobian Matrix

- Linearlization of non-linear transformation
- definition

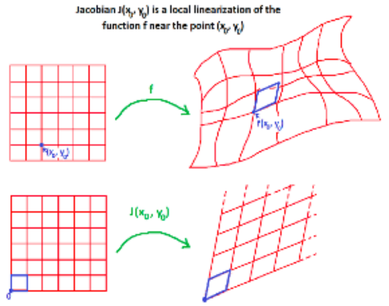
$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

- example of non-linear transformation

$$f(x, y) = \begin{bmatrix} x + \sin(\frac{y}{2}) \\ y + \sin(\frac{x}{2}) \end{bmatrix}$$



- *임의의 점(내가 원하는 점)의 기울기만 취한다. = linearlization



- 하려는 것

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = J \begin{bmatrix} du \\ dv \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} du \\ dv \end{bmatrix}$$

- 세부 계산

$$dx = a \times du + b \times dv$$
$$dy = c \times du + d \times dv$$

- 따라서 J 행렬은

$$J = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$$

Comparision with Formula : AE / Flow ~

- 고차원 변수 x, 잠재 변수 z
- $z = f(x)$: f는 고차원에서 저차원
- $x = g(z)$: g는 저차원에서 고차원
- $x = g(f(x))$: Auto-Encoder
- f^{-1} => x : flow-based generative model