Change of variable

- 먼저 single variable에 대해서

$$\int p(x)dx = \int \pi(x)dz = 1 \qquad (1)$$

$$p(x) = \pi(z)|\frac{dz}{dx}| \qquad (2)$$

$$= \pi(f^{-1}(x))|\frac{df^{-1}}{dx}| \qquad (3)$$

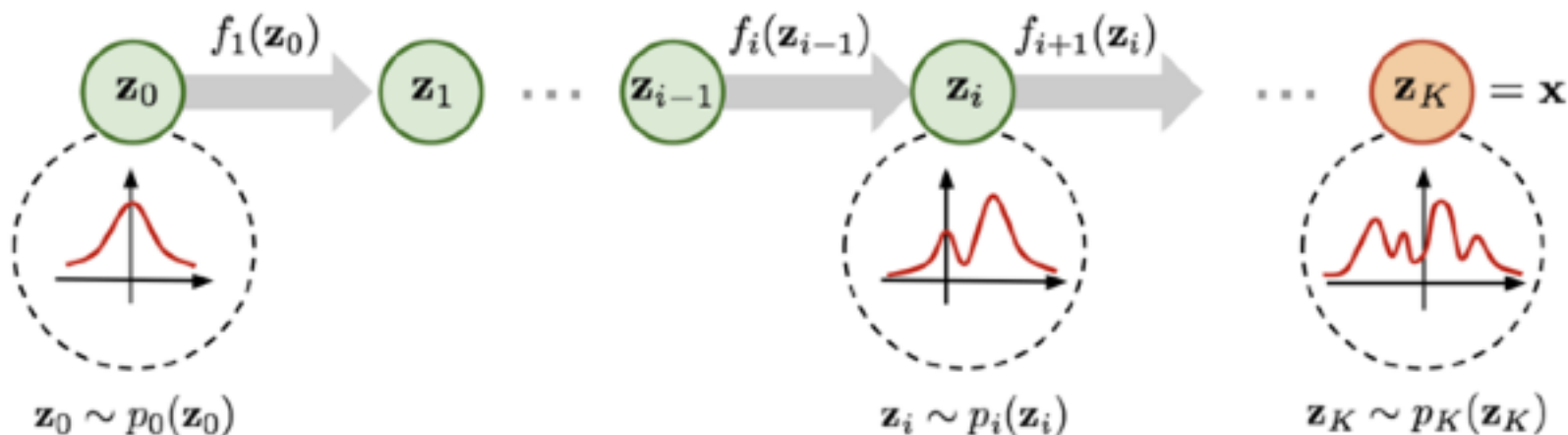$$= \pi(f^{-1}(x))|(f^{-1})'(x)| \qquad (4)$$

- 이를 multivariable에 나타내면

$$p(\boldsymbol{X}) = \pi(\boldsymbol{Z})|\det\frac{d\boldsymbol{Z}}{d\boldsymbol{X}} = \pi(f^{-1}(\boldsymbol{X}))|\det\frac{df^{-1}}{d\boldsymbol{X}}|$$

Normalizing flows

- Flow = 조금씩 변화시켜 나가는 것.

$$\mathbf{z}_{i-1} \sim p_{i-1}(\mathbf{z}_{i-1})$$

$$\mathbf{z}_i = f_i(\mathbf{z}_{i-1}), \text{ thus } \mathbf{z}_{i-1} = f_i^{-1}(\mathbf{z}_i)$$

$$p_i(\mathbf{z}_i) = p_{i-1}(f_i^{-1}(\mathbf{z}_i)) \left| \det \frac{df_i^{-1}}{d\mathbf{z}_i} \right|$$