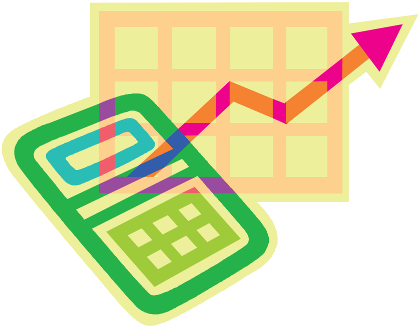
# Lecture 3 –
# Two Determinants of Investment Decisions:
# Return and Risk

# Recap from Last Lecture

- Financial Markets and The Economy
  - Why do we need financial markets?
- Securities
  - Money Market,
  - Capital Market (Bond & Equity)
  - Derivatives
- Indexes

# Outline of Today's Lecture

- **Arithmetic Average Returns**
- Expected vs. Realized Return & Risk
- Value at Risk (VaR)
- Risky, Riskfree Prospects & The Risk Premium
- Risk Preferences & Returns Utility



# Holding-Period Return

$$r_t = \frac{P_t - P_{t-1} + Div_t}{P_{t-1}}$$

$P_t$ is the security price at time $t$

$P_{t-1}$ is the security price at time $t-1$

$Div_t$ is income from time $t$ and $t-1$

- Example: $P_0 = 50,\ P_1 = 53,\ P_2 = 54$
  $D_1 = D_2 = 2$

$$r_1 = \frac{53 - 50 + 2}{50} = 10\%,\ r_2 = \frac{54 - 53 + 2}{53} = 5.66\%$$

# Arithmetic Average Returns

- Simple average of returns earned over time:

$$r_A = \sum_{t=1}^{n} \frac{r_t}{n} = \frac{\left(r_1 + r_2 + \ldots + r_{n-1} + r_n\right)}{n}$$

$$= \frac{10\% + 5.66\%}{2} = 7.83\%$$

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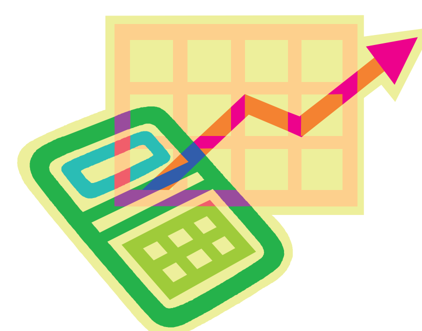
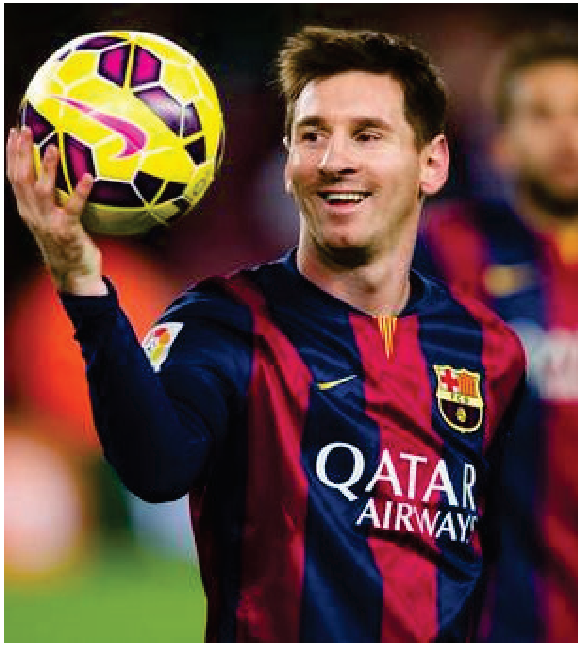


# Value of Soccer Players


Ronaldo (€133m)




Messi (€220m)

- Past performance: matches, goals, dribbles, etc.
- Characteristics: age, position, contract duration, etc.

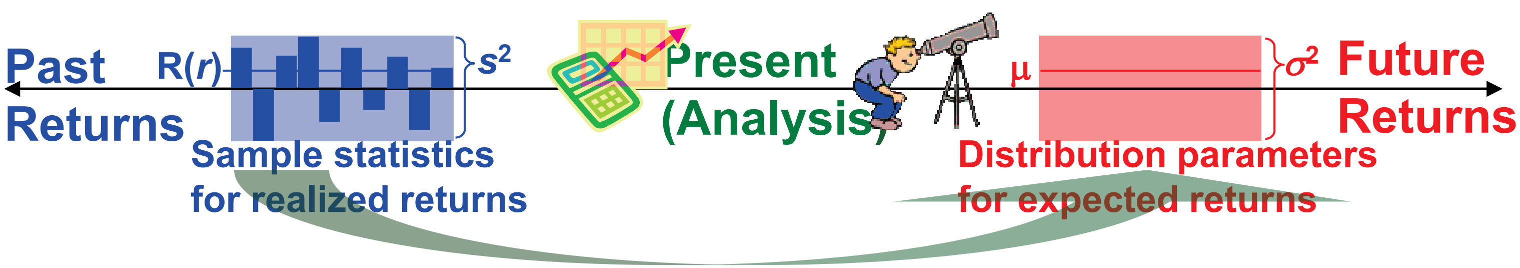
# Expected vs. Realized Returns

**Expected Returns, $\mu$, and Expected Risk, $\sigma^2$:**

- *Forward*-looking, what *should* happen on average
- Assumes knowledge of the returns distribution

**Realized Returns, R($r$), and Realized Risk, $s^2$:**

- *Backward*-looking, what *did* happen on average
- Analyses sample returns to learn the distribution
- Usually used to extrapolate what expected returns and risk are



# Expected Return & Risk: $\mu$ & $\sigma^2$

- $E[r]$ or $\mu = \Sigma p(s) r(s)$ over all possible states, *s*

  where *p(s)* is the probability of state *s,* and

  *r(s)* is the return when state *s* occurs

- $\sigma^2 = \Sigma p(s)[r(s) - \mu]^2$ over all possible states, *s*

  where $\sigma^2$ is the variance and

  $\sigma = (\sigma^2)^{1/2}$ is the standard deviation

# Realized Return & Risk: R($r$) & $s^2$

- Average realized return, R($r$) = $r_A$

  where $r_A$ is the *arithmetic* average return

- $s^2 = \Sigma[r_t - r_A]^2 \div (n\text{-}1)$ over all sample returns, $r_t$

  where $s^2$ is the *sample* variance,

  $s = (s^2)^{1/2}$ is the *sample* standard deviation,

  and $n$ is the number of returns in the sample

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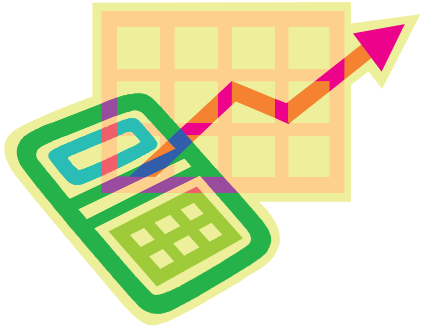
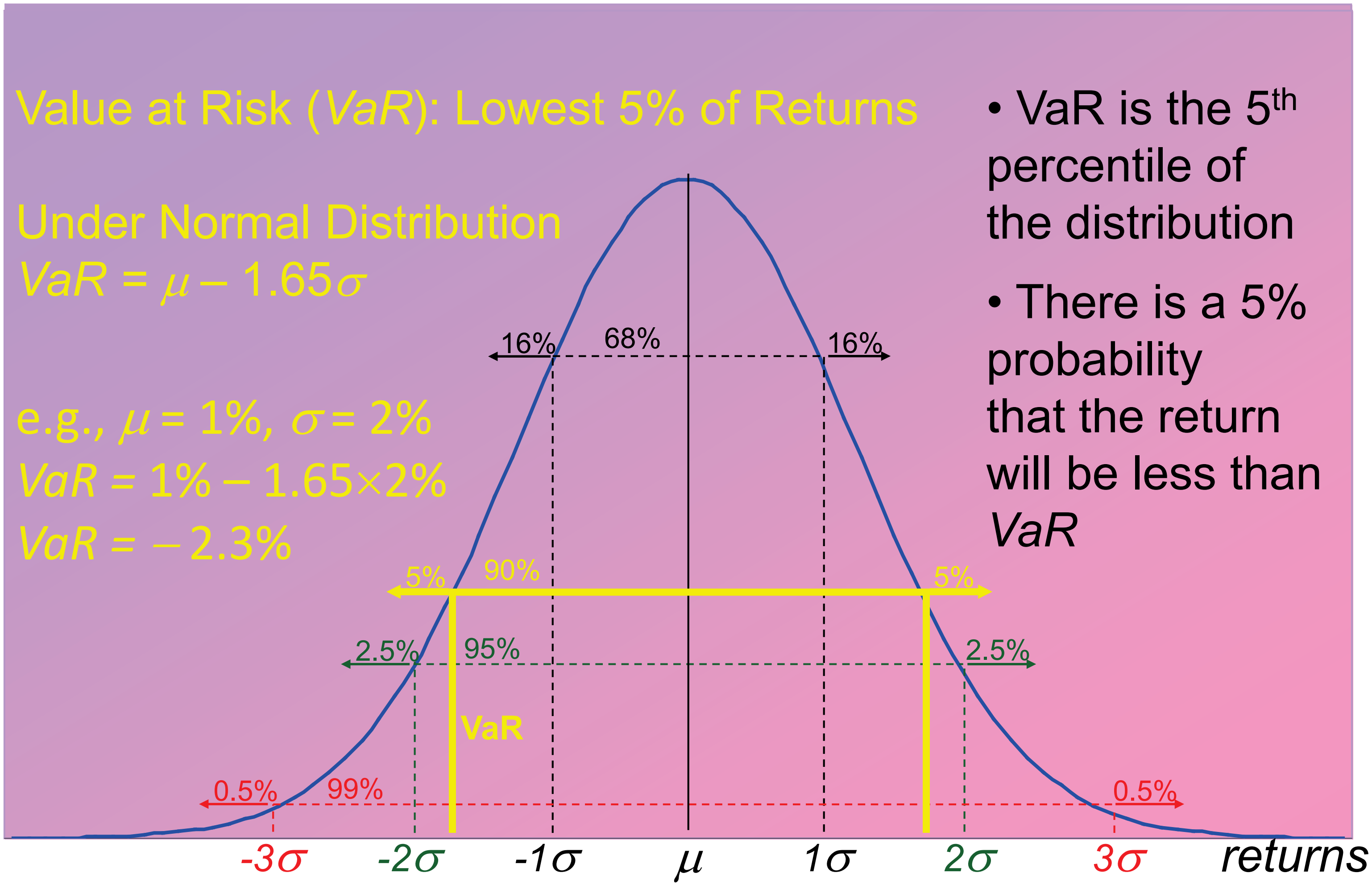


# Another Risk Measure: *Value at Risk*



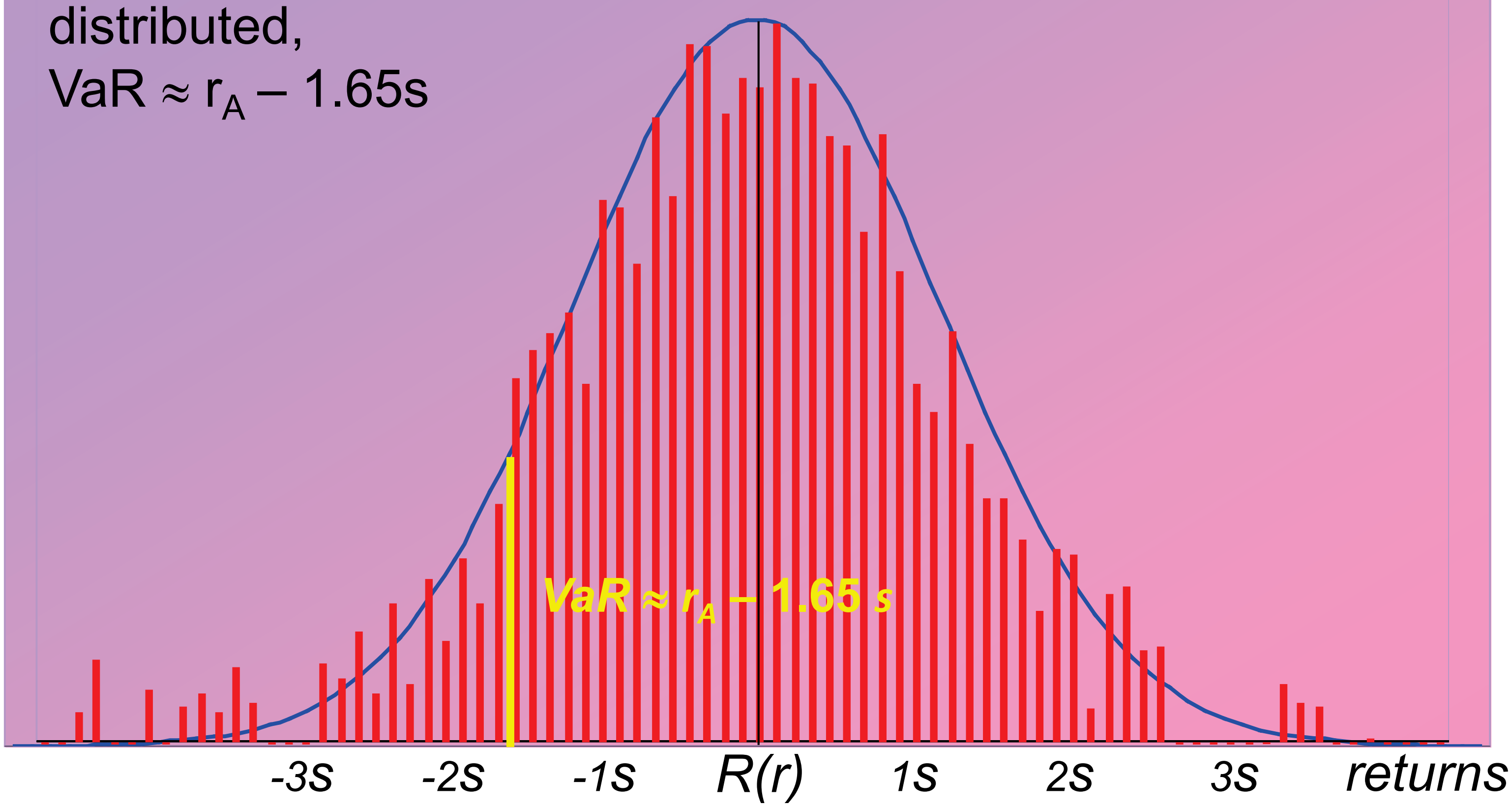
# Realized Returns: $r \approx N(R(r), s^2)$

## *Value at Risk*

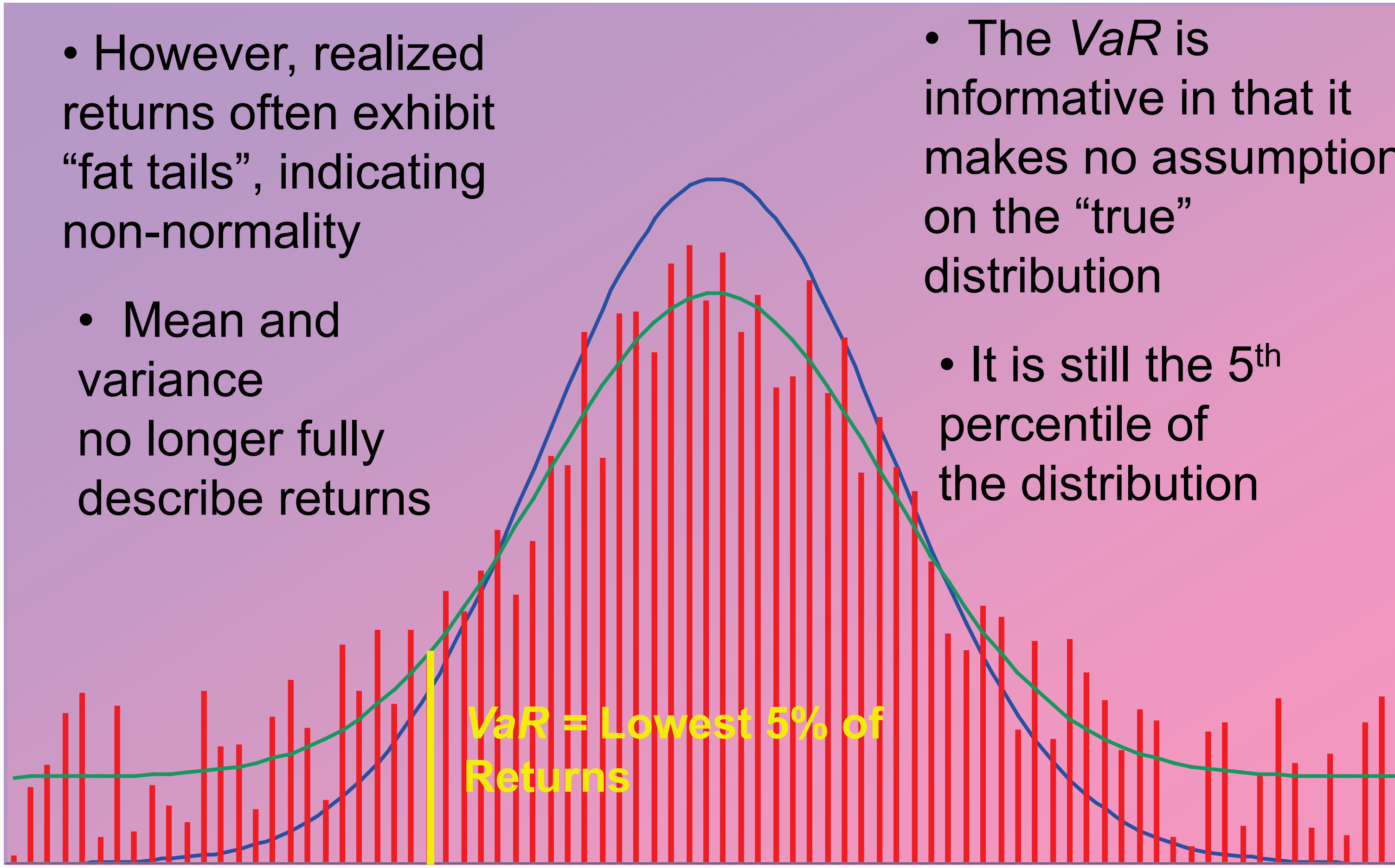
- If realized returns are approximately normally distributed,
  $VaR \approx r_A - 1.65s$

$r_A$: arithmetic mean of returns

*s*: standard deviation of returns



# Non-Normal Returns
## *Value at Risk*




- However, realized returns often exhibit “fat tails”, indicating non-normality
- Mean and variance no longer fully describe returns
- The *VaR* is informative in that it makes no assumption on the “true” distribution
- It is still the 5$^{th}$ percentile of the distribution

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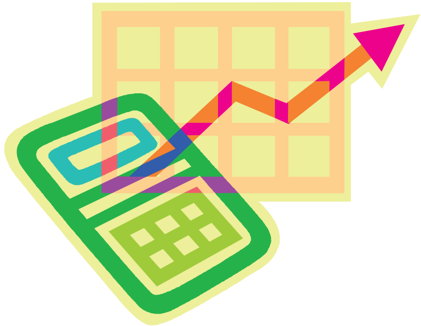
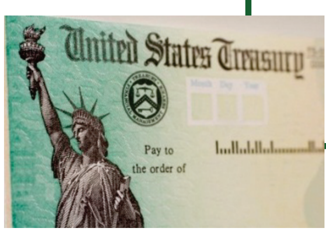


# Risky & Riskfree Prospects

| Investment $W$ | State $s$ | Probability $p(s)$ | Outcome $o(s)$ | Return $r(s)$ | Risk $\sigma$ |
|---|---|---|---|---|---|
| | $g$ | 60% | \$150 | 50% | |
| \$100 | | | E[$r$] = | 22% | 34% |
| | $p$ | 40% | \$80 | -20% | |
| Riskfree Prospect (Government debt) | | | \$105 | 5% | 0% |



# Simple Prospects & Risk Premium

| | $E[r]$ | $\sigma$ |
|---|---|---|
| Risky Prospect | 22% | 34% |
| | – | |
| Riskfree Prospect | 5% | 0% |
| | = | |
| Risk Premium | 17% | $\equiv E[r] - R_F$ |


$\equiv$ Return (Reward) for Risk Taken

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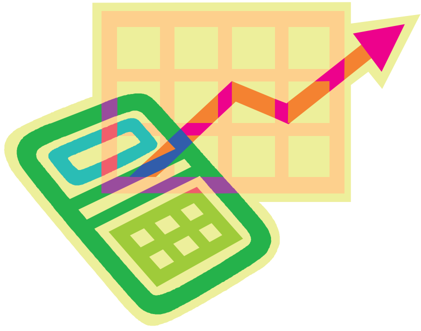

# Risk Preferences & Returns Utility

- ***Utility Score*** (or ***Certainty Equivalent, CE***):
- Combines risk aversion, expected returns & risk
- Puts a preference score on securities & portfolios

- Helps to predict how an investor will invest
- Helps advise investors on what they should do

- Formula: $U = \mathrm{E}[r] - \frac{1}{2} \times A \times \sigma^2 = CE$

  Where A measures investor risk aversion

# Utility Scores, Certainty Equivalent

- The (hypothetical) riskfree return that would make you indifferent between investing in a riskfree prospect and a risky one.

- Hence: “Certainty Equivalent”

# How Do We Find Risk Aversion?


- Example: Deal or No Deal

1. Contestant claims a case to begin the game, value unknown → E(r) & $\sigma_r^2$

2. Contestant then chooses cases to be removed from play. The amount inside each choice is immediately revealed → Change E(r) & $\sigma_r^2$

3. Throughout the game, the banker offers the contestant an amount of money to quit the game:


- "Deal", accepting the offer presented and ending the game,
- "No Deal", rejecting the offer and continuing the game.

→ Certainty Equivalent!

- Post, van den Assem, Baltussen & Thaler (American Economic Review 2008)

# Utility Scores & Investment Choice

- Suppose E[$r$] = 22%, $\sigma$ = 34%, and $R_F$ = 5%
- What is the utility score for the risky prospect?

  $U = 22\% - ½ \times A \times 34\%^2$, depends on $A$:

If $A = 1$, $U = 16\% > R_F \rightarrow$ **Risky** prospect

If $A = 2$, $U = 10\% > R_F \rightarrow$ **Risky** prospect

If $A = 3$, $U = \;\; 4\% < R_F \rightarrow$ **Riskfree** prospect


**Moral: Different investors, different choices**