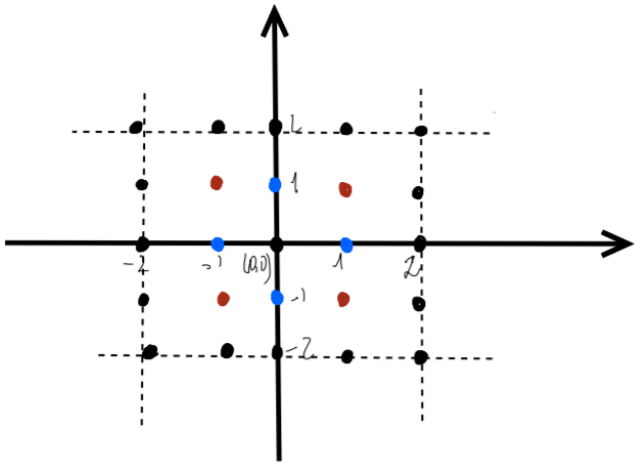
Optiver application - Quantitative Researcher

Réda Arab

1 First question



Let us consider the following objects :

- $e :=$ the average number of steps from the start (0,0) needed to reach the interesting area
- $b :=$ the average number of steps from a blue point (i.e all the points which can be reached after the first move) needed to reach the interesting area
- $r :=$ the average number of steps from a red point (i.e a part of the points which can be reached after the second move)needed to reach the interesting area

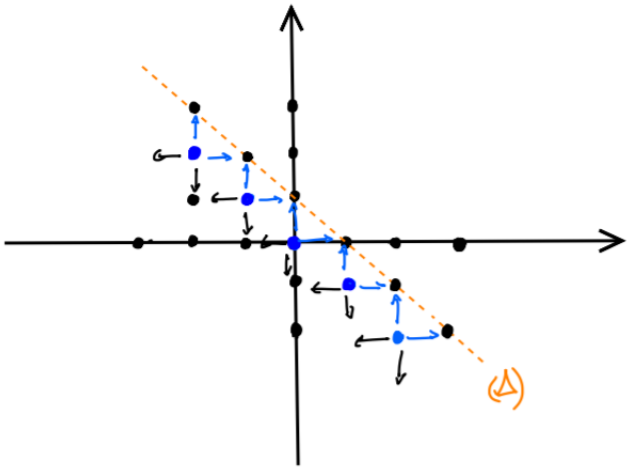
By conditioning, we have the following system :

$$(\mathrm{S}) : \begin{cases} e = 1 + b \\ b = \frac{1}{4}(e+1) + \frac{1}{2}(r+1) + \frac{1}{4} + 1 \\ r = \frac{1}{2}(b+1) + \frac{1}{2} \end{cases}$$

Solving this system gives us : $\boxed{e = \frac{9}{2}}$

Remark : A python simulation in the notebook attached gives us the same result.

2 Second question



Considering the following line : $(\Delta) := \{(x, y) \in \mathbb{Z}^2, 1 - x - y = 0\}$

Denoting the position of the ant at step n as (X_n, Y_n), our problem is to know the average number necessary to reach (Δ) from a starting point which is $(X_0, Y_0) = (0, 0)$.

As we can see in the graph, it can be interesting to consider the distance to the line. And, as $\phi(x, y) = 1 - x - y$ is a linear form, we know that outside the line, ϕ is strictly positive or negative. Therefore, let us consider the quantity $Z_n = 1 - X_n - Y_n$.

We have :

$$Z_{n+1} = \begin{cases} Z_n + 1 & \text{with probability } 0.5 \\ Z_n - 1 & \text{with probability } 0.5 \end{cases}$$

We are in the case of a random walk in 1D starting from 1 and we want to know how many steps are necessary to reach 0. This is $\boxed{+\infty}$.