Research project on random projections in LP

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Outline

Introduction

Introduction

$$\mathcal{P} \equiv min\{c^T x | Ax = b \wedge x \geq 0 \wedge x \in X\} \tag{1}$$

$$T\mathcal{P} \equiv min\{c^T x | TAx = Tb \wedge x \geq 0 \wedge x \in X\} \tag{2}$$

$$TD \equiv max\{(yT^T) Tb | (yT^T) TA \leq c\} \tag{3}$$

Introduction

- If the original dual is feasible, the projected dual is too
- We derive from the Johnson-Lindenstauss lemma approximation guarantees on the projected dual objective function
- If $\bar{u} \in \arg \text{opt}(\text{TD})$, let $\tilde{y} = \bar{u}T$

Methodology

Model

Sets	
$I = \{0, \ldots, n-1\}$	constraints in P
$J = \{0, \ldots, m-1\}$	variables in P
Parameters	
$c\{J\}$	cost vector in P
$A\{I, J\}$	matrix of constraints in P
$b\{I\}$	constraints RHS in P
$\tilde{y}\{I\}$	projected solution of TD

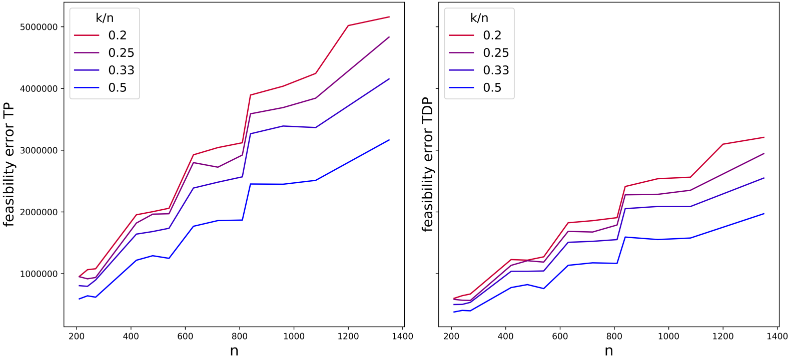
Model TDP

Variables	
$\tilde{x}\{J\}$	estimation of x
$s_p^+\{J\}$	positive deviation from primal KKT
$s_p^-\{J\}$	negative deviation from primal KKT
$s_d^+\{I\}$	positive deviation from dual KKT
$s_d^-\{I\}$	negative deviation from dual KKT

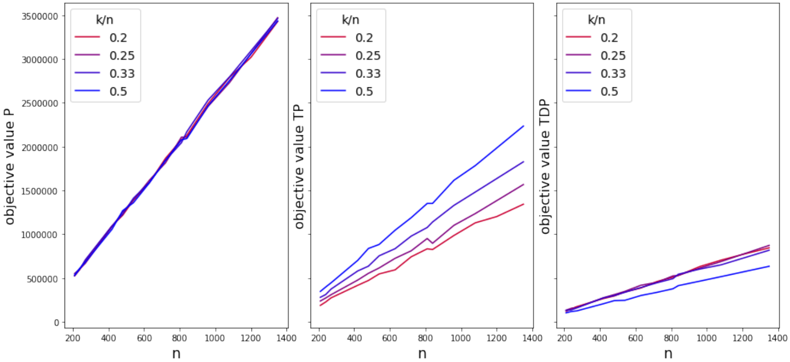
$$\begin{aligned}
\underset{s_p^+, s_p^-, s_d^+, s_d^-}{\text{minimize}} \quad & \sum_{j \in J} (s_p^+)_j + (s_p^-)_j + \sum_{i \in I} (s_d^+)_i + (s_p^-)_i \\
\text{subject to} \quad & \left(\sum_{i \in I} \tilde{y}_i A_{i,j} - c_j \right) \tilde{x}_j + (s_p^+)_j - (s_p^-)_j = 0, \; j \in J \\
& \tilde{y}_i \left(b_i - \sum_{j \in J} A_{i,j} \tilde{x}_j - \right) + (s_d^+)_i - (s_d^-)_i = 0, \; i \in I
\end{aligned}$$

Numerical results

Feasibility error



Objective value



Conclusion

Thank you for your attention !