CHAPTER 3: LINEAR EQUATIONS AND INEQUALITIES

Chapter Objectives

By the end of this chapter, the student should be able to
   ✓ Solve linear equations (simple, dual-side variables, infinitely many solutions or no solution, rational coefficients)
   ✓ Solve linear inequalities
   ✓ Solve literal equations with several variables for one of the variables

Contents

SECTION 3.1: LINEAR EQUATIONS

A. VERIFYING SOLUTIONS

A **linear equation** is made up of two expressions that are equal to each other. A linear equation may have one or two variables in it, where each variable is raised to the power of 1. No variable in a linear equation can have a power greater than 1.

Linear equation: $2y = 3x + 1$
(each variable in the equation is raised to the power of 1)

Not a linear equation: $y^2 = 3x + 1$
(y is raised to the power of 2, therefore this is not linear)

The solution to an equation is the value, or values, that make the equation **true**. Given a solution, we plug the value(s) into the respective variable(s) and then simplify both sides. The equation is **true** if both sides of the equation equal each other.

	MEDIA LESSON
	Is it a solution? (Duration 5:00)

View the video lesson, take notes and complete the problems below

A solution to an equation is the _____ for the _____ that makes the equation

_____. To test a possible solution, _____ the _____ with the

_____.

Example. Is $a = 3$ the solution to $4a - 18 = 2a$? Explain your answer.

_____

YOU TRY

a) Verify that $x = -3$ is a solution to the algebraic equation $5x - 2 = 8x + 7$.

b) Is $m = -1$ a solution to the algebraic equation $m + 9 = 3m + 5$?

c) Is $a = 5$ a solution to the algebraic equation $-4(a + 1) = 6(1 - a)$?

B. ONE-STEP EQUATIONS

The Addition Property of Equality
If $a = b$, then for any number c, $$a + c = b + c$$ That is, if we are given an equation, then we are allowed to add the same number to both sides of the equation to get an equivalent statement.

	MEDIA LESSON
	Addition Principle (Duration 5:00)

View the video lesson, take notes and complete the problems below

To clear a negative we _____ it to _____.

Example (follow the structure in the video and fill in the diagram below)

$$x - 9 = 4$$

The Multiplication Property of Equality

If $a = b$, then for any number c,
$$a \cdot c = b \cdot c$$
That is, if we are given an equation, then we are allowed to multiply by the same number on both sides of the equation to get an equivalent statement.

We use these two properties to help us solve an equation. To solve an equation means to "undo" all the operations of the equation, leaving the variable by itself on one side. This is known as **isolating the variable.**

	MEDIA LESSON
	Multiplication (Division) Principle (Duration 5:00)

View the video lesson, take notes and complete the problems below

To clear multiplication we _____ both sides by the _____.

Example (follow the structure in the video and fill in the diagram below)

$$-8x = 72$$

❖ **NOTE:** When using the Multiplication Property of Equality on an equation like
$$-x = 4$$
It is easier to think of the negative in front of the variable as a -1 being multiplied by x, that is
$$-1 \cdot x = 4$$
We then multiply both sides by -1 to isolate the variable.
$$(-1) \cdot -1 \cdot x = 4 \cdot (-1)$$
$$\Rightarrow 1 \cdot x = -4$$
$$\Rightarrow x = -4$$
When using the Multiplication Property of Equality on an equation where the coefficient is a number other than 1
$$3x = 3$$
We take the coefficient's **reciprocal** then multiply both sides of the equation by that reciprocal. This will isolate the variable, that is
$$\left(\frac{1}{3}\right) \cdot 3 \cdot x = 3 \cdot \left(\frac{1}{3}\right)$$
$$\Rightarrow 1 \cdot x = \frac{3}{3}$$
$$\Rightarrow x = 1$$

YOU TRY

Solve.

a) $x + 7 = 18$

b) $r - 4 = 5$

c) $-4 + b = 45$

d) $3 = 19 + m$

e) $-3y = -42$

f) $-5 = -x$

C. TWO-STEP EQUATIONS

Steps to solve a linear two-step equation.
1. Apply the Addition Property of Equality.   2. Apply the Multiplication Property of Equality to isolate the variable.   3. Check by substituting your answer into the original equation.

	MEDIA LESSON [Basic Two Step (Duration 4:59)](#)

View the video lesson, take notes and complete the problems below

Simplifying we use order of operations and we _____ before we _____. Solving

we work in reverse so we will _____ first and then _____ second.

Example (follow the structure in the video and fill in the diagram below)

$$-9 = -5 - 2x$$

YOU TRY

Solve for the variable in each of the following equations. Check your answers.

a) Solve: $2b - 4 = 12$ Check:

b) Solve: $4 + 3r = 5$ Check:

c) Solve: $3 = 19 - 2m$ Check:

d) Solve: $11 - y = 32$ Check:

D. GENERAL EQUATIONS

We will now look at some more general linear equations, that is, equations that require more than two steps to solve. These equations may have more than one of the same variable on each side of the equal sign

$$x - 5 = 4x + 7$$

and/or may contain parentheses

$$3(4n - 2) = 5(n + 3)$$

	MEDIA LESSON
	General Equations (Duration 5:00)

View the video lesson, take notes and complete the problems below

Move variables to one side by

_____.

Sometimes we may have to _____ first. Simplify by _____ and

_____ on each side.

Example (follow the structure in the video and fill in the diagram below)

$$2x + 7 = -5x - 3$$

Use the following steps to solve a general equation.

1. Simplify each side of the equation. Remove parentheses if necessary. Combine like terms.
2. Add terms on each side of the equation so that all terms containing the variable are on one side of the equal sign and all constant terms are on the other side.
3. Simplify each side of the equation by combining like terms.
4. Apply the Multiplication Property of Equality to isolate the variable.
5. Check by substituting the solution into the original equation.

YOU TRY

Solve for the variable in each of the following equations. Check your answers.

a) Solve: $x - 5 = 4x + 7$ Check:

b) Solve: $3(4n - 2) = 5(n + 3)$ Check:

c)   Solve: $4 - (2y - 1) = 2(5y + 9) + y$ Check:

E. SOLVING EQUATIONS WITH FRACTIONS

When solving linear equations with fractions, it is vital to remember the Multiplication Property of Equality. Previously, we've only dealt with coefficients that were integers. Now we will be looking at coefficients that are rational numbers.

$$\frac{5x}{6} = -5$$

We can manipulate the left side of this equation as such

$$\frac{5}{6} \cdot x = -5$$

Looking at it this way, we can then use the Multiplication Property of Equality and multiply both sides of the equation by the coefficient's reciprocal

$$\left(\frac{6}{5}\right) \cdot \frac{5}{6} \cdot x = -5 \cdot \left(\frac{6}{5}\right)$$
$$\Rightarrow 1 \cdot x = -6$$
$$\Rightarrow x = -6$$

Another way to solve this type of equation is to clear the fractions in the equation by multiplying by the LCD.

	MEDIA LESSON
	Distributing with Fractions (Duration 5:00)

View the video lesson, take notes and complete the problems below

Important: Always _____first and _____ second.

Solve the equation below by multiplying the equation by the LCD.

$$\frac{2}{3}(x + 4) = 5\left(\frac{5}{6}x - \frac{7}{15}\right)$$

YOU TRY

a) Solve: $\frac{x}{6} = -5$ Check:

b) Solve: $\frac{3}{4}a = 8$ Check:

c) Solve: $0 = -\frac{5}{4}\left(x - \frac{6}{5}\right)$ Check:

EXERCISES

Solve for the variable in each of the following equations. Reduce, simplify, and check your answers. Show all steps, and box your answer.

1) $8x - 2 = 30$

2) $5 - x = 3$

3) $-\frac{1}{2}x - 4 = 8$

4) $\frac{2}{3}x + 3 = 15$

5) $4x - 8 = -x + 7$

6) $\frac{3}{4}x - \frac{1}{2} = \frac{9}{8}x + \frac{3}{2}$

7) $6x - 4(-2x + 8) = 10$

8) $-2(4x - 2) = -2(x - 8)$

9) $(2x - 7) - (4x + 8) = 4(x + 6)$

10) $2(4x + 3) = 8x + 1$

11) $5(x + 6) - x = 4(x + 7) + 2$

12) $\frac{3}{4} - \frac{5}{4}m = \frac{113}{24}$

13) $-\frac{8}{3} - \frac{1}{2}x = -\frac{4}{3}x - \frac{2}{3}\left(-\frac{13}{4}x + 1\right)$

SECTION 3.2: LINEAR INEQUALITIES

A. GRAPHING LINEAR INEQUALITIES

An **algebraic inequality** is a mathematical sentence connecting an expression to a value, variable, or another expression with an inequality sign. Below is a table of inequalities we will be using

Symbol	In Words	Examples
$<$	less than	$1 < 2$ "1 is less than 2"
$>$	greater than	$4 > 3$ "4 is greater than 3"
$\leq$	less than or equal to	$0 \leq 5$ "0 is less than 5"
$\geq$	greater than or equal to	$-1 \geq -1$ "-1 is equal to -1"
$\neq$	not equal	$3 \neq 4$ "3 is not equal to 4"

A **solution** to an inequality is a value that makes the inequality true. For example, a solution to the inequality

$$x < 1$$

may be 0 since 0 is indeed less than 1. However, 2 cannot possibly be a solution since 2 is not less than 1.
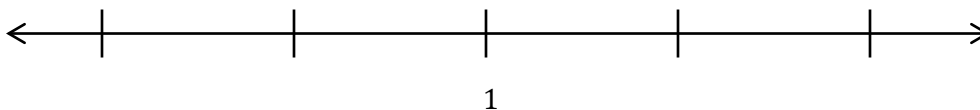
NOTE: The inequality symbols $<$ and $>$ can be quite easy to interpret, however, the inequalities symbols $\leq$ and $\geq$ on the other hand, can be tricky. For example,
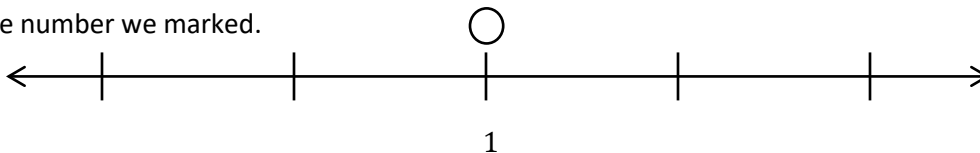$$x \leq 1$$
is read as "x is less than **or** equal to 1." The keyword here is the word "or." The word "or" tells us that our solution can be less than 1 **or** equal to 1. So 0 is a solution to this inequality since 0 is less than 1. As it turns out, 1 is also a solution to this inequality. The solution 1 is not less than 1 but it is equivalent to 1, thus 1 is a solution.
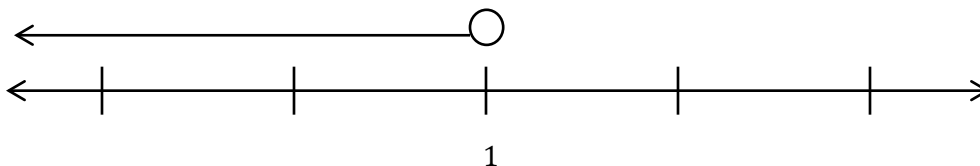Notice that this reasoning does <u>not</u> work with strict inequalities.

To graph an inequality, let us look at $x < 1$. We first draw a number line and mark the number in our inequality on the line.

1

We then draw an open circle or closed circle (depending on the inequality symbol) on the number line, above the number we marked.

1

The final step is to draw a line in the direction of the solutions.

1

Remember: We use an open circle ∘ with the symbols $<$ and $>$, and a closed circle · with the symbols $\leq$ or $\geq$

	MEDIA LESSON
[QR code]	Interval Notation (Duration 3:04)

View the video lesson, take notes and complete the problems below

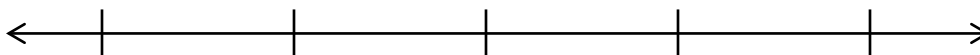Interval notation is used to _____ a graph with _____ numbers.

Interval notation will always be read _____ to _____.

$$(\qquad , \qquad)$$

We use _____ parentheses for less/greater than, and _____ for less/greater than or equal to.

The symbols $-\infty$ and ∞ will always use _____ parentheses.

Example, graph the interval $(-\infty, -1)$ on the number line below.

YOU TRY

a) Determine whether the number 4 is a solution to the following inequalities.

$x > 1$	$x < 1$	$x \leq 9$	$x > 4$	$x \geq 4$

b) Graph the following inequalities in the box below. Write the solution using interval notation.

Inequality	Graph	Interval Notation
$x > 2$		
$x \geq 2$		
$x < -2$		
$x \leq -2$		

B. SOLVING LINEAR INEQUALITIES

A linear inequality has the form

$$ax + b < c$$

where a, b, and c are real numbers. This definition is the same for $\leq, \geq$, or $>$.

To solve linear inequalities we use the following properties (in the following properties we use the $<$ symbol. Keep in mind that these properties work with the other inequality symbols too):

The Addition Property of Inequalities
For real numbers a, b, and c, if $a < b$, then $a + c < b + c$.

The Multiplication Property of Inequalities
For real numbers a, b, and $c > 0$, if $a < b$, then $a \cdot c < b \cdot c$ If $c < 0$, then $a \cdot c > b \cdot c$ When we are multiplying or dividing by a <u>negative number</u>, we reverse the sign of the inequality.

Steps to solve a general equation.
1. Simplify each side of the inequality. Remove parentheses if necessary. Collect like terms. 2. Add terms on each side of the inequality so that all terms containing the variable are on one side and all constant terms are on the other side. 3. Simplify each side of the inequality by combining like terms. 4. Apply the Multiplication Property of Inequalities to isolate the variable. 5. Check by substituting the solution (endpoint and a value from the solution set) into the original inequality.

	MEDIA LESSON Solving (Duration 5:00)

View the video lesson, take notes and complete the problems below

Solving inequalities is just like _____.

The only exception is if you _____ or _____ by a _____, you must _____.

Example. Solve the inequality below using the video as a guide. Write the solution in interval notation.

$$7 - 5x \leq 17$$

YOU TRY

Solve the inequality, check your answer, and graph the solution on a number line. Give the solution in interval notation.

a) $3x > x + 6$
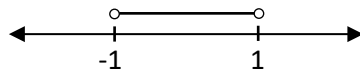
b) $3 - 5a \leq 2(a + 5)$

c) $-5(x + 2) \geq -3(x + 4)$

C. TRIPARTITE INEQUALITIES

We have been dealing with inequalities where the variable (or expression containing the variable) is on the left or the right side of a number. We will now look at a special type of inequality called a **tripartite inequality**, where the expression containing the variable is between two numbers, for example
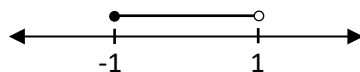
$$-1 < x < 1$$

The solution for this inequality in interval notation is $(-1, 1)$. Graphing the solution we get

Below is another example.

$$-1 \leq x < 1$$

The solution for this inequality in interval notation is $[-1, 1)$. Graphing the solution we get

	MEDIA LESSON
	Tripartite (Duration 5:00)

View the video lesson, take notes and complete the problems below

A tripartite inequality is a _____ part inequality. We use a tripartite inequality when our variable is

_____ two numbers

When solving these type of inequalities we will

_____.

When graphing, we will graph the inequality _____ the numbers.

Example. Solve in the inequality below. Write the solution in interval notation.

$$5 < 5 - 4x \leq 13$$

YOU TRY

a) Which of the following values are in the solution set for $-3 \leq n < 5$?

$n = -5$ $\qquad$ $n = -3$ $\qquad$ $n = 0$ $\qquad$ $n = 4.9$ $\qquad$ $n = 5$ $\qquad$ $n = 12$

b) Write a compound inequality to represent the following situation. Clearly indicate what the variable represents.

A number is greater than or equal to 5 but les than 8

EXERCISES

1) Which of the following values are in the solution set for $x < 3$?

$x = 0$ $\qquad$ $x = -1$ $\qquad$ $x = -5$ $\qquad$ $x = 3$ $\qquad$ $x = 5$ $\qquad$ $x = -\dfrac{5}{3}$

2) Which of the following values are in the solution set for $x \geq -1$?

$x = 0$ $\qquad$ $x = -1$ $\qquad$ $x = -5$ $\qquad$ $x = 3$ $\qquad$ $x = 5$ $\qquad$ $x = -\dfrac{5}{3}$

3) Which of the following values are in the interval $[-2, \infty)$?

$x = 0$ $\qquad$ $x = -1$ $\qquad$ $x = -5$ $\qquad$ $x = 3$ $\qquad$ $x = 5$ $\qquad$ $x = -\dfrac{5}{3}$

4) Which of the following values are in the interval $(-\infty, -1)$?

$x = 0$ $\qquad$ $x = -1$ $\qquad$ $x = -5$ $\qquad$ $x = 3$ $\qquad$ $x = 5$ $\qquad$ $x = -\dfrac{5}{3}$

5) Which of the following values are in the interval $(-1, 5]$?

$x = 0$ $\qquad$ $x = -1$ $\qquad$ $x = -5$ $\qquad$ $x = 3$ $\qquad$ $x = 5$ $\qquad$ $x = -\dfrac{5}{3}$

6) Which of the following values are in the interval $-5 < x \leq 3$?

$x = 0$ $\qquad$ $x = -1$ $\qquad$ $x = -5$ $\qquad$ $x = 3$ $\qquad$ $x = 5$ $\qquad$ $x = -\dfrac{5}{3}$

For questions 7-14, solve the inequality, check your answer, and graph the solution on a number line. Give the solution in interval notation.

7) $7 - 4x \geq -5$

8) $4x \leq 2x + 12$

9) $14m + 8 > 6m - 8$

10) $5(-2a - 8) \leq -9a + 4$

11) $6x + 13 < 5(2x - 3)$

12) $3 \leq 9 + x \leq 7$

13) $5 \geq \dfrac{x}{5} + 1$

14) $-4 < 8 - 3m \leq 11$

15) Translate the statement into a compound inequality.

A number n is greater than 0 and less than or equal to 8

SECTION 3.3: LITERAL EQUATIONS

A. SOLVING FOR A VARIABLE

In this section will be constructing linear equations from sentences. Use the steps below as a guide when approaching each problem.

Steps for Writing and Solving Equations
1) Read and understand the problem. Underline the givens and circle the goal.
2) Form a strategy to solve the problem.
3) Choose a variable to represent the unknown quantity.
4) Read every word in the problem, and translate the given information into an algebraic equation.
5) Solve the equation.
6) Write your answer in a complete sentence.

Example:

The cost of leasing a new Ford mustang is $2,311 for a down payment and processing fee plus $276 per month. For how many months can you lease this car with $10,000?

Solution:

Step 1. The cost of leasing a new Ford mustang is $2,311 for a down payment and processing fee plus $276 per month. For how many months can you lease this car with $10,000?

Step 2. Since $2,311 is a down payment, this number must be constant, in other words, this number does not change no matter how many months go by.

The $276 does change as the months go by. In the first month we pay $276 on top of the down payment. In the second month we pay $276 + $276 plus the down payment, and so on. Take note that the down payment is a one time payment not a monthly payment like the $276.

Step 3. We will let m be our variable to represent the number of months we can lease the car.

Step 4. By the information that is given, the linear equation is $276m + 2311 = 10,000$.

Step 5. Solving for m we get 27.8 months. So how many months can we lease the car? If we say we can lease the car for 28 months (rounding up) then we would go over the $10,000 limit. So we must round down and say 27 months.

Step 6. We can lease this car for 27 months.

YOU TRY

a) You have just bought a new Sony 55" 3D television set for $1,600. The value of the television set decreases by $250 per year. How long before the television set is worth half of its original value?

EXERCISES

For each of the following, *underline* the Givens and *circle* the Goal of the problem. Form a Strategy, Solve, and Check. Show all work, and write your answers in complete sentences.

1) John is a door to door vacuum salesman. His weekly salary, S, is $200 plus $50 for each vacuum he sells. This can be written as $S = 200 + 50v$, where v is the number of vacuums sold. If John earns $1000 for a week's work, how many vacuums did he sell?

2) Paul is planning to sell bottled water at the local Lollapalooza. He buys 2 crates of water (2000 bottles) for $360 and plans on selling the bottles for $1.50 each. Paul's profits, P in dollars, from selling b bottles of water is given by the formula $P = 1.5b - 360$. How many bottles does Paul need to sell in order to break even?

3) A new Sony 55" 3D television costs $2,499. You are going to pay $600 as a down payment, and pay the rest in equal monthly installments for one year. Write an equation to represent this situation, and use it to determine how much you should pay each month. Clearly indicate what the variable in your equation represents. Solve the equation, and write your answer in a complete sentence.

4) Your yard is a mess, and you decide to hire a landscaper. The Greenhouse charges a $20 consultation fee plus $11 per hour for the actual work. Garden Pros does not charge a consulting fee, but charges $15 per hour for the actual work. Write an equation that will help you determine the number of hours at which the two companies charge the same. Clearly indicate what the variable represents. Solve the equation, and write your answer in a complete sentence.

5) Let p represent the marked price of an item at Toys R Us. Emma's aunt gave her a $50 gift card to Toys R Us for her birthday. If sales tax is currently 9%, set up an equation to express how much she can spend using her gift card. Solve the equation, and interpret your answer in a complete sentence.

6) Carlos recently hired a roofer to do some necessary work. On the final bill, Carlos was charged a total of $1105, where $435 was listed for parts and the rest for labor. If the hourly rate for labor was $67, how many hours of labor was needed to complete the job?

CHAPTER REVIEW

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the workbook. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the text or in the media lesson.	
Linear Equation	
Solution to a Linear Equation	
Addition Property Of Equality	
Multiplication Property of Equality	
Algebraic Inequality	
$<$	
$\leq$	
$\geq$	
$>$	
Addition Property of Inequalities	
Multiplication Property of Inequalities	
$\circ$	
$\cdot$	