

1. Calculate the total number of energy states per unit volume, in silicon, between the lowest level in the conduction band and a level kT above this level, at $T = 300$ K. The effective mass of the electron in the conduction band is 1.08 times that of a free-electron.

The number of available states between E_c and $(E_c + kT)$ is given by

$$\begin{aligned}
 &= \frac{(8 \pi m_c)^{3/2}}{3 h^3} (E - E_c)^{3/2} && \text{Given } (E - E_c) = kT = 1.38 \times 10^{-23} \text{ J/K} \times 300 \\
 &= \frac{(8 \times \pi \times 1.08 \times 9.11 \times 10^{-31})^{3/2}}{3 (6.63 \times 10^{-34})^3} (1.38 \times 10^{-23} \times 300)^{3/2} \\
 &= 2.11 \times 10^{25} / m^3
 \end{aligned}$$

2 Calculate the probability that an energy level (a) kT (b) $3 kT$ (c) $10 kT$ above the fermi-level is occupied by an electron.

Probability that an energy level E is occupied is given by $f(E) = \frac{1}{e^{\frac{E - E_F}{kT}} + 1}$

For $(E - E_F) = kT$, $f(E) = \frac{1}{e^{\frac{kT}{kT}} + 1} = \frac{1}{e + 1} = 0.268$

For $(E - E_F) = 3kT$, $f(E) = \frac{1}{(e^3 + 1)} = 0.047$

For $(E - E_F) = 10kT$, $f(E) = \frac{1}{(e^{10} + 1)} = 4.5 \times 10^{-5}$

3 The fermi-level in a semiconductor is 0.35 eV above the valence band. What is the probability of non-occupation of an energy state **at the top** of the valence band, at (i) 300 K (ii) 400 K ?

The probability that an energy state in the valence band is not occupied is

$$(i) T=300K$$

$$1-f(E) = \frac{1}{\left(e^{\frac{E_V - E_F}{kT}} + 1\right)} = \frac{1}{\left(e^{-\frac{0.35}{0.0259}} + 1\right)} = 1.353 \times 10^{-6}$$

Alternate method: for $E_F - E_V > kT$

$$1-f(E) \cong e^{\frac{E_V - E_F}{kT}} = 1.353 \times 10^{-6}$$

$$(ii) T=400K \quad 1-f(E) \cong e^{\frac{E_V - E_F}{kT}} = 3.9 \times 10^{-5}$$

5 For copper at 1000K (a) find the energy at which the probability $P(E)$ that a conduction electron state will be occupied is 90%. (b) For this energy, what is the $n(E)$, the distribution in energy of the available state? (c) for the same energy what is $n_0(E)$ the distribution in energy of the occupied states? The Fermi energy is 7.06eV.

$$\text{The fermi factor } f(E) = \frac{1}{\left(e^{\frac{E - E_F}{kT}} + 1\right)} = 0.90$$

$$e^{\frac{E - E_F}{kT}} = \left[\frac{1}{0.90} - 1 \right] = 0.11$$

$$E = E_F + kT (\ln 0.11)$$

$$= 7.06 - 0.19 = 6.87 \text{ eV}$$

$$\begin{aligned}
\text{Density of available state } n(E) &= \frac{\pi m_c^{3/2}}{h^3} 2^{7/2} (E)^{1/2} \\
&= \frac{\pi (9.11 \times 10^{-31})^{3/2}}{(6.63 \times 10^{-34})^3} 2^{7/2} (6.87 \times 1.6 \times 10^{-19})^{1/2} \\
&= (1.061 \times 10^{56}) (6.87 \times 1.6 \times 10^{-19})^{1/2} \\
&= 1.11 \times 10^{47} \text{ m}^{-3} \text{ J}^{-1} \\
&= (1.11 \times 10^{47}) \times (1.6 \times 10^{-19}) \text{ m}^{-3} \text{ eV}^{-1} \\
&= 1.78 \times 10^{28} \text{ m}^{-3} \text{ eV}^{-1}
\end{aligned}$$

The density of occupied states is =
(The density of states at an energy E) \times (probability of
occupation of the state E)

$$\text{i.e } n_o(E) = n(E) f(E)$$

$$\begin{aligned}
&= (1.78 \times 10^{28}) \times (0.90) \\
&= 1.60 \times 10^{28} \text{ m}^{-3} \text{ eV}^{-1}
\end{aligned}$$

7 The effective mass of hole and electron in GaAs are respectively 0.48 and 0.067 times the free electron mass. The band gap energy is 1.43 eV. How much above is its fermi-level from the top of the valence band at 300 K?

Fermi energy in an Intrinsic semiconductor is $E_F = \frac{E_c + E_v}{2} + \frac{3}{4} k T \ln \left(\frac{m_v}{m_c} \right)$

Write $\frac{E_c + E_v}{2} = E_v + \frac{E_c - E_v}{2} = E_v + \frac{E_g}{2}$

$$\begin{aligned} \therefore E_F &= E_v + \frac{1.43}{2} + \frac{3}{4} (0.0259) \ln \left(\frac{0.48}{0.067} \right) \\ &= E_v + (0.715 + 0.0383) \\ &= E_v + 0.75 \text{ eV} \end{aligned}$$

\therefore The fermi level is 0.75eV above the top of the VB

9 The effective mass of the conduction electron in Si is 0.31 times the free electron mass. Find the conduction electron density at 300 K, assuming that the Fermi level lies exactly at the centre of the energy band gap (= 1.11 eV).

Electron concentration in CB is = $n = 2 \left[\frac{2\pi m_c k T}{h^2} \right]^{3/2} \exp \frac{-(E_c - E_F)}{k T}$

$$\begin{aligned} n &= 2 \left[\frac{2\pi(0.31 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23})}{(6.63 \times 10^{-34})^2} \right]^{3/2} T^{3/2} \exp \frac{-(E_c - E_F)}{k T} \\ &= \left[4.81 \times 10^{21} (0.31)^{3/2} (300)^{3/2} \right] \left[e^{-\frac{1.11/2}{0.0259}} \right] \\ &= \left[4.31 \times 10^{21} \right] \left[4.94 \times 10^{-10} \right] = 2.13 \times 10^{15} m^{-3} \end{aligned}$$

10 In intrinsic GaAs, the electron and hole mobilities are 0.85 and 0.04 m² V⁻¹s⁻¹ respectively and the effective masses of electron and hole respectively are 0.068 and 0.50 times the electron mass. The energy band gap is 1.43 eV. Determine the carrier density and conductivity at 300K.

Intrinsic carrier concentration is given by

$$\begin{aligned} n_i &= \left[\frac{2^{5/2} (\pi k T)^{3/2} (m_v m_c)^{3/4}}{h^3} \right] \left[e^{-\frac{E_g}{2kT}} \right] \\ &= [1.98 \times 10^{24}] [1.025 \times 10^{-12}] \\ &= 2 \times 10^{12} \text{ m}^{-3} \end{aligned}$$

14 A flat copper ribbon 0.330mm thick carries a steady current 50.0A and is located in a uniform 1.30-T magnetic field directed perpendicular to the plane of the ribbon. If a Hall voltage of 9.60 μV is measured across the ribbon. What is the charge density of the free electrons?

Charge carrier density n is given by

$$\begin{aligned} n &= \frac{1}{R_H e} = \frac{B I}{V_H t e} \\ &= \frac{1.3 \times 50}{(9.6 \times 10^{-6}) \times (0.330 \times 10^{-3}) (1.6 \times 10^{-19})} \\ &= 1.28 \times 10^{29} / \text{m}^3 \end{aligned}$$