Course: Fundamentals Of Linear algebra, Calculus and Numerical Methods

Course Code: MA211TA

<table>
<tr><td colspan="3">PART A</td></tr>
<tr><td>1.1</td><td>A is a 4×4 matrix. The third row is a combination of the first and second rows. The fourth row is combination of second and third rows, then rank of A = _____.</td><td>1</td></tr>
<tr><td>1.2</td><td>The Eigen values of $A = \begin{bmatrix} 0 & 5 \\ 3 & 0 \end{bmatrix}$ are _____.</td><td>1</td></tr>
<tr><td>1.3</td><td>If Eigen values of A are 1, 3, 5 then $trace(A^2) =$ _____</td><td>1</td></tr>
<tr><td>1.4</td><td>The system $kx + 3y = 6;\ 3x + ky = -6$ is inconsistent for k =_____.</td><td>1</td></tr>
<tr><td>1.5</td><td>If $u = xy^2z^3$, then the total differential du = _____.</td><td>2</td></tr>
<tr><td>1.6</td><td>If $u = x(1-y)$ and $v = x^2y$, then $\frac{\partial(u,v)}{\partial(x,y)}$ = _____.</td><td>2</td></tr>
<tr><td>1.7</td><td>The Maclaurin series expansion for $\log(1+x)$ is _____.</td><td>2</td></tr>
<tr><td>1.8</td><td>Two sound waves are represented as polar curves intersecting orthogonally, in the usual notation at the point of intersection $\tan(\theta_1) = a$ and $\tan(\theta_2) = -\frac{a}{4}$. The value of a is_____.</td><td>2</td></tr>
<tr><td>1.9</td><td>Evaluate: $\int_0^1 \int_0^{1-x} dy\, dx$</td><td>2</td></tr>
<tr><td>1.10</td><td>Express the integral $\int_0^2 \int_0^{\sqrt{4-x^2}} dydx$ as equivalent integral by changing to polar coordinates.</td><td>2</td></tr>
<tr><td>1.11</td><td>Construct a forward difference table for the following values of x, y.
<table>
<tr><td>x</td><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td></tr>
<tr><td>y</td><td>6</td><td>10</td><td>13</td><td>17</td><td>23</td></tr>
</table></td><td>2</td></tr>
<tr><td>1.12</td><td>The velocity of a train which starts from rest is given by the table.
<table>
<tr><td>Minutes</td><td>0</td><td>2</td><td>4</td><td>6</td><td>8</td></tr>
<tr><td>Miles/hour</td><td>0</td><td>10</td><td>18</td><td>25</td><td>29</td></tr>
</table>
The approximate distance travelled in 8 minutes by using Simpson's $1/3^{rd}$ rule is _____.</td><td>2</td></tr>
<tr><td colspan="3">PART – B</td></tr>
<tr><td>2a</td><td>Examine the consistency of the following system of equations and solve, if consistent.
$2x - y - z = 2$
$x + 2y + z = 2$</td><td>6</td></tr>
</table>

	$4x - 7y - 5z = 2$	
2b	Solve the following system of equations by Gauss Siedel Method. $x + 2y + z = 0$ $3x + y - z = 0$ $x - y + 4z = 3$ Carry out 3 iterations by taking the initial approximation as (1,1,1).	5
2c	Compute the largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by applying Rayleigh's power method taking $X^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ as the initial vector. Carry out 4 iteration.	5
3a	Show that the two curves $r^2 = a^2 cos2\theta$ and $r = a(1 + cos\theta)$ intersect at an angle $3\sin^{-1}\left(\frac{3}{4}\right)^{\frac{1}{4}}$.	8
3b	Expand $f(x) = (x+2)^4 + 5(x+2)^3 + 6(x+2)^2 + 7(x+2) + 8$ in ascending powers of $(x-1)$.	8
4a	Obtain the radius of curvature at any point $P(x, y)$ on the curve $x = a\cos^3\theta, y = \alpha\sin^3\theta;\ \ \alpha > 0.$	8
4b	Find the centre of curvature and circle of curvature of the curve $y = x^3 - 6x^2 + 3x + 1$ at a point (1, -1).	8
5a	The two-dimensional Laplace equation in polar coordinates is given by $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. Any function $u(r, \theta)$ satisfying this equation called a harmonic function. Show that $u = e^{a\theta}\cos(a\log r)$ is a harmonic function.	8
5b	Evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$, Given $x = u,\ y = u\tan v,\ z = w$	8
	OR	
6a	If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.	8
6b	Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum. Solve by Lagrange method of undetermined multipliers.	8
7a	Compute the area enclosed by the parabola $y^2 = 4ax$ and the lines $x + y = 3a,\ y = 0$ in the first quadrant.	8

<table>
<tr><td></td><td></td><td></td></tr>
<tr><td>7b</td><td>By changing the order of integration, evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xy\,dy\,dx$</td><td>8</td></tr>
<tr><td></td><td>OR</td><td></td></tr>
<tr><td>8a</td><td>Evaluate: $\int\int\int x^2yz\,dx\,dy\,dz$ over the region bounded by the planes $x=0, y=0, z=0\ and\ x+y+z=1$.</td><td>8</td></tr>
<tr><td>8b</td><td>Evaluate $\iint_R r\sqrt{a^2-r^2}\ \ dr\,d\theta$, over the upper half of the circle $r = acos\theta$.</td><td>8</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>9a</td><td>Find the number of persons getting wages less than Rs. 15 from the following data:
<table>
<tr><td>Wages in Rs.</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td></tr>
<tr><td>No. of persons</td><td>9</td><td>30</td><td>35</td><td>42</td></tr>
</table></td><td>8</td></tr>
<tr><td>9b</td><td>A body moving with velocity ‘v’ at any time ‘t’ satisfies the data
<table>
<tr><td>t</td><td>0</td><td>1</td><td>3</td><td>4</td></tr>
<tr><td>v</td><td>21</td><td>15</td><td>12</td><td>10</td></tr>
</table>
Obtain the distance travelled in 4 seconds.</td><td>8</td></tr>
<tr><td></td><td>OR</td><td></td></tr>
<tr><td>10a</td><td>A rod is rotating in a plane. The following table gives the angle A (in radians) through which the rod has turned for various values of time T (seconds). Calculate the angular velocity and angular acceleration of the rod at T = 0.6 second.
<table>
<tr><td>T</td><td>0</td><td>0.2</td><td>0.4</td><td>0.6</td><td>0.8</td><td>1</td></tr>
<tr><td>A</td><td>0</td><td>0.12</td><td>0.49</td><td>1.12</td><td>2.02</td><td>3.20</td></tr>
</table></td><td>8</td></tr>
<tr><td>10b</td><td>Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson’s 1/3 rule, 3/8 rule and Weddle’s rule.</td><td>8</td></tr>
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