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Numerical Examples in De-Broglie Wavelength & Uncertainty principle

Problem #1: What is the wavelength in meters of a proton traveling at 255,000,000 m/s (which is 85% of the speed of light)? (Assume the mass of the proton to be 1.673×10^{-27} kg.)

Solution: 1) Calculate the kinetic energy of the proton:

v_p, m_p $\lambda = ?$

KE = $(1/2)mv^2$

$x = (1/2)(1.673 \times 10^{-27}\text{ kg})(2.55 \times 10^{8}\text{ m/s})^2$

$x = 5.43934 \times 10^{-11}$ J

2) Use the de Broglie equation:

$\lambda = h/p$

$\lambda = h/\sqrt{(2Em)}$

$x = 6.626 \times 10^{-34}\text{ J s} / \sqrt{[(2)(5.43934 \times 10^{-11}\text{ J})(1.673 \times 10^{-27}\text{ kg})]}$

$x = 1.55 \times 10^{-15}$ m

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{1.673 \times 10^{-27} \times 2.55 \times 10^{8}} = 1.55 \times 10^{-15}$$

This wavelength is comparable to the radius of the nuclei of atoms, which range from 1×10^{-15} m to 10×10^{-15} m (or 1 to 10 fm).

doubt

Problem #2: Calculate the wavelength (in nanometers) of a H atom (mass = 1.674×10^{-27} kg) moving at 698 cm/s

$v = 6.98$ m/s

Solution: 1) Convert cm/s to m/s:

698 cm/s = 6.98 m/s

2) Calculate the kinetic energy of the proton:

KE = $(1/2)mv^2$

$x = (1/2)(1.674 \times 10^{-27}\text{ kg})(6.98\text{ m/s})^2$

$x = 5.84226 \times 10^{-27}$ J

3) Use the de Broglie equation:

$\lambda = h/p$

$\lambda = h/\sqrt{(2Em)}$

$x = 6.626 \times 10^{-34}\text{ J s} / \sqrt{[(2)(5.84226 \times 10^{-27}\text{ J})(1.674 \times 10^{-27}\text{ kg})]}$

$x = 1.50 \times 10^{-7}$ m

4) Convert m to nm:

1.50×10^{-7} m = 150. Nm

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{1.674 \times 10^{-27} \times 6.98} = 0.5671 \times 10^{-7} \times \frac{10^2}{10^2} = \underline{\underline{56.71\text{ nm}}}$$

Problem #3: An atom of helium has a de Broglie wavelength of 4.30×10^{-12} meter. What is its velocity?

Solution: 1) Use the de Broglie equation to determine the energy (not momentum) of the atom [note the appearence of the mass (in kg) of a He atom]:

$\lambda = h/p$

$\lambda = h/\sqrt{(2Em)}$

$4.30 * 10^{-12}$ m = $6.626 * 10^{-34}$ J s / $\sqrt{[(2) * (6.646632348 * 10^{-27} \text{ kg})]}$

$4.30 * 10^{-12} * \sqrt{[(2) * (6.646632348 \times 10^{-27})]} = 6.626 \times 10^{-34}]$

$\sqrt{[(2) * (6.646632348 * 10^{-27})]} = 6.626 \times 10^{-34} / 4.30 \times 10^{-12}$

I divided the right side and then squared both sides.

$(2)\ (x)\ (6.646632348 \times 10^{-27}) = 2.374466 \times 10^{-44}$

$x = 1.786217333 \times 10^{-18}$ J

2) Use the kinetic energy equation to get the velocity:

$KE = (1/2)mv^2$

$1.786217333 * 10^{-18} = (1/2)\ (6.646632348 * 10^{-27})\ v^2$

$v^2 = 5.3748 * 10^8$

$v = 2.32 * 10^4$ m/s

$\lambda = \frac{h}{mv}$

$4.3 \times 10^{-12} = \frac{\cancel{6.6} \times 10^{-34}}{\cancel{6.6} \times 10^{-27} \times v}$

$v = 0.23 \times 10^5$

$v = 2.3 \times 10^4$ m/s

Problem #4: Determine the wavelength of an electron accelerated by a 100V potential difference.

Solution: Try this!!! (If you can't solve this… come see me !!!)

$\lambda e = \sqrt{\frac{150}{V}}$

$= 1.22$

Problem #5: *Calculate de Broglie wavelength associated with electron having 10 keV kinetic energy, where me = 9.1 × 10^{-31} kg and h = 6.6 × 10^{-34} Js.*

Solution: Try this!!! (If you can't solve this… come see me !!!)

$\lambda e = \frac{h}{\sqrt{2 m_e E}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10^4 \times 1.6 \times 10^{-19}}}$

$= \frac{6.6 \times 10^{-34}}{\sqrt{29.12 \times 10^{-46}}}$

$= \frac{6.6}{5.3} \ \frac{\times 10^{-34}}{10^{-23}}$

$= 1.24 \times 10^{-11}$

Problem #6: The average period of that elapses between excitation of an atom and the time it emits radiation is 10^{-8} sec. Find the uncertainty in the frequency of light emitted.

$\Delta\nu$ Δt

Solution:

$$\Delta E \Delta t = h \Rightarrow \Delta E = \frac{6.63 \times 10^{-34}}{2\pi \times 10^{-8}} = 1.054 \times 10^{-26} \text{ J}$$

$$\Delta E = h\Delta\nu \Rightarrow \Delta\nu = \frac{\Delta E}{h} = \cancel{1.59 \times 10^7 \text{ Hz}}$$

$\Delta E \Delta t \geq \frac{h}{4\pi}$

$\cancel{h} \Delta\nu \Delta t \geq \frac{\cancel{h}}{4\pi}$

$\Delta\nu = \frac{1}{4 \times 3.14 \times 10^{-8}}$

$= \frac{10^2 \times 10^6}{4\pi}$

$\Delta\nu = 7.96 \times 10^6$ Hz.

Problem #7: *Find the uncertainty in the kinetic energy of a proton confined to a nucleus size of* 10^{-14} *m. (mass of proton =* 1.66 × 10^{-27} *kg).*

$KE = \frac{1}{2} \frac{m^2 v^2}{m}$

$KE = \frac{p^2}{2m}$

$\Delta p \Delta x = \frac{h}{4\pi}$

$\sqrt{2m\Delta E}\ \Delta x = \frac{h}{4\pi}$

$\Delta E = \left(\frac{h}{4\pi \Delta x}\right)^2 \frac{1}{2m}$

Solution:

$$\Delta P_x = \frac{h}{4\pi\Delta x} = 5.27 \times 10^{-19} \text{ kg m / s}$$

$= \frac{(5.2 \times 10^{-19})^2}{2 \times 1.66 \times 10^{-27}}$

$$\Delta E_k = \frac{\Delta p_x^2}{2m} = 2.365 \times 10^{-11} \text{ J}$$

$\Delta E = 8.14 \times 10^{-11}$

Problem #8: The uncertainty in the momentum Δp of a ball traveling at 20m/s is 1×10−6 of its momentum. Calculate the uncertainty in position Δx? Mass of the ball is given as 0.5kg. h = 6.626×10^{-34}Js

Solution: We know that,
P = m×v = 0.5×20 = 10kgm/s
$\Delta p = 10 \times 1 \times 10^{-6}$
$\Delta p = 10^{-5}$
Heisenberg Uncertainty principle formula is given as,

$$\Delta \times \Delta p \geq \frac{h}{4\pi}$$

$\Delta p = 10^{-6} p = 10^{-6} \times 0.5 \times 20$

$\Delta p = 10^{-5}$

$\Delta x = \frac{h}{4\pi \Delta p} = \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 10^{-5}}$

$\Delta x = 0.52 \times 10^{-29}$ m

Heisenberg Uncertainty principle formula is given as,

$$\Delta \times \Delta p \geq \frac{h}{4\pi}$$

$$\Delta \times \geq \frac{h}{4\pi\Delta p}$$

$$\Delta \times \geq \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 10^{-5}} = 0.527 \times 10^{-29} m$$

$\Delta x = \frac{h}{4\pi\Delta p} = \frac{}{4 \times 3.14 \times 10^{-5}}$

$\Delta x = 0.52 \times 10^{-29}$ m

Problem #9: An electron in a molecule travels at a speed of 40m/s. The uncertainty in the momentum Δp of the electron is 10^{-6} of its momentum. Compute the uncertainty in position Δx if the mass of an electron is 9.1×10^{-31}kg?

$\Delta p = 10^{-6} p = 10^{-6} \times 9.1 \times 10^{-31} \times 40$

Solution: Heisenberg Uncertainty principle formula is given as,

$$\Delta \times \Delta p \geq \frac{h}{4\pi}$$

$$\Delta \times \geq \frac{h}{4\pi\Delta p}$$

$$\Delta \times \geq \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 364 \times 10^{-37}} = 1.44m$$

$\Delta x = \frac{h}{4\pi\Delta p} = \frac{6.6 \times 10^{-34}}{4\pi \times 9.1 \times 40 \times 10^{-37}}$

$\Delta x = 1.44$m

$\Delta v = \frac{600 \times 0.005}{100} = 3 \times 10^{-2}$

Problem #10: In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%. Certainty with which the position of the electron can be located is: (h=6.6 x 10^{-34} kg m^2 s^{-1}; mass of electron, e_m = 9.1 x 10^{-31} kg)

Solution: $\Delta x = \frac{h}{4\pi m \Delta v} = \frac{6.6 \times 10^{-34}}{4\pi \times 9.1 \times 10^{-31} \times 3 \times 10^{-2}}$

$$\boxed{\Delta x = \frac{h}{n\pi} \times \frac{1}{m\Delta v} = \frac{6.6 \times 10^{-34} kg\, m^2\, s^{-1}}{4 \times 3.14 \times 9.1 \times 10^{-31}\, kg \times 3 \times 10^{-2} m\, s^{-1}} = 1.93 \times 10^{-3}\ m}$$

$= 0.0192 \times 10^{-1}$

$= 1.92 \times 10^{-3}$

Problem #10: How fast does a proton have to be moving in order to have the same de-Broglie wavelength as an electron that is moving with a speed of 4.50 x 10^6 m/s?

$\frac{h}{m_e v_e} = \frac{h}{m_p v_p}$

$\frac{1}{2}mv^2 = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{2hc}{mv^2}$

Problem #11: The kinetic energy of a particle is equal to the energy of a photon. The particle moves at 5% of the speed of light. Find the ratio of the photon wavelength to the de-Broglie wavelength

(11) $\frac{\frac{2hc}{mv^2}}{\frac{h}{mv}} = \frac{2c}{v} = \frac{2c}{\frac{5c}{100}} = 40$

Problem #12: A particle has de-Broglie wavelength of 2.7 *10^{-10}m. Then its K.E doubles. What is the particles new wavelength, ignoring relativistic effects?

$\lambda = \frac{h}{\sqrt{2mK}}$; $\lambda' = \frac{h}{\sqrt{2m(2K)}} = \frac{\lambda}{\sqrt{2}} = \frac{2.7 \times 10^{-10}}{1.41} = 1.9 \times 10^{-10}$

Answer: 1.9 x 10^{-10} m

Problem #13: Consider a line is 2.5 m long. A moving object is somewhere along this line, but its position is not known. Find the minimum uncertainty in the momentum and velocity of the object if the object is an golf ball= 0.045 kg and an electron

$\Delta p \Delta x \geq \frac{h}{4\pi}$

Answer: momentum: 2.1 x 10^{-35} Kgm/s; $velocity_{golfball}$: 4.7 x 10^{-34} m/s; $velocity_{electron}$: 2.3 x 10^{-5} m/s

Problem #14: A proton has kinetic energy E=100keV which is equal to energy of a photon. Let λ_1 be the de-Broglie wavelength of the proton and λ_2 be the wavelength of the photon. The ratio λ_1/λ_2 is proportional to?

$\lambda_1 = \frac{h}{\sqrt{2mE}}$

(14) $E = \frac{hc}{\lambda_2}$

$\lambda_2 = \frac{hc}{E}$

$\frac{\lambda_1}{\lambda_2} = \frac{\frac{h}{\sqrt{2mE}}}{\frac{hc}{E}}$

ratio $= \frac{\sqrt{E}}{c\sqrt{2m}}$

Problem #15: Calculate the de Broglie wavelength of an electron moving with a speed of 10^5 m/s and also that of an electron moving with a speed of 0.99 × 10^8 m/s. Be careful in your choice of formulae in the second case as it is relativistic. Hint: $p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$

Problem #16: An enclosure filled with helium is heated to 400K. A beam of He-atoms emerges out of the enclosure. Calculate the de-Broglie wavelength corresponding to He atoms. Mass of He is 1.67 * 10^{-27} kg

Problem #17: An electron beam is accelerated from rest through a potential difference of 200V. Calculate the associated wavelength.

Problem #18: Calculate the de-Broglie wavelength of neutron of energy 12.8 MeV, mass neutron= 1.67 *10^{-27} kg

Problem #19: The above beam is passed through a diffraction grating of spacing 3Å. At what angle of deviation from the incident direction will be the first maximum observed. **(Try this, its tricky !!!)**

(Don't Solve this Yet!!!) Problem #20: A particle with mass m is in an infinite square well potential with walls at x = −L/2 and x = L/2.

- Write the wave functions for the states n = 1, n = 2 and n = 3

Answer:

$$\psi_1(x) = \sqrt{\frac{2}{L}} \cos\left(\frac{\pi x}{L}\right)$$

Answer:

$$\psi_1(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$$

$$\psi_2(x) = -\sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$$

$$\psi_3(x) = -\sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right)$$

Problem #21:

A proton is confined in an infinite square well of width 10 fm. (The nuclear potential that binds protons and neutrons in the nucleus of an atom is often approximated by an infinite square well potential.)

•Calculate the energy and wavelength of the photon emitted when the proton undergoes a transition from the first excited state ($n = 2$) to the ground state ($n = 1$).

•In what region of the electromagnetic spectrum does this wavelength belong?

Solution:

Text Eq. (5.17) gives the energy E_n of a particle of mass m in the nth energy state of an infinite square well potential with width L:

$$E_n = \frac{n^2 h^2}{8mL^2} \tag{22}$$

The energy E and wavelength λ of a photon emitted as the particle makes a transition from the $n = 2$ state to the $n = 1$ state are

$$E = E_2 - E_1 = \frac{3h^2}{8mL^2} \tag{23}$$

$$\lambda = \frac{hc}{E}. \tag{24}$$

For a proton (m=938 MeV/c^2), $E = 6.15\,\mathrm{MeV}$ and $\lambda = 202\,\mathrm{fm}$. The wavelength is in the gamma ray region of the spectrum.

Problem #22:

A particle is in the nth energy state $\psi_n(x)$ of an infinite square well potential with width L.

•Determine the probability $P_n(1/a)$ that the particle is confined to the first $1/a$ of the width of the well.

•Comment on the n-dependence of $P_n(1/a)$.

Solution:

The wave function $\psi_n(x)$ for a particle in the nth energy state in an infinite square box with walls at $x = 0$ and $x = L$ is

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right). \tag{29}$$

The probability $P_n(1/a)$ that the electron is between $x = 0$ and $x = L/a$ in the state $\psi_n(x)$ is

$$P_n\left(\frac{1}{a}\right) = \int_0^{L/a} |\psi_n(x)|^2\,dx = \frac{2}{L}\int_0^{L/a} \sin^2\left(\frac{n\pi x}{L}\right) dx = \frac{1}{a} - \frac{\sin(2n\pi/a)}{2n\pi} \tag{30}$$

$P_n(1/a)$ is the probability that the particle in the state $\psi_n(x)$ is confined to the first $1/a$ of the width of the well. The sinusoidal n-dependent term decreases as n increases and vanishes in the limit of large n:

$$P_n\left(\frac{1}{a}\right) \to \frac{1}{a} \quad \text{as} \quad n \to \infty \tag{31}$$

$P_n(1/a) = 1/a$ is the classical result. The above analysis is consistent with the correspondence principle, which may be stated symbolically as

$$\text{quantum physics} \to \text{classical physics} \quad \text{as} \quad n \to \infty \tag{32}$$

where n is a typical quantum number of the system.

Problem #23:

A 1.00 g marble is constrained to roll inside a tube of length $L = 1.00\,\mathrm{cm}$. The tube is capped at both ends.

•Modelling this as a one-dimensional infinite square well, determine the value of the quantum number n if the marble is initially given an energy of 1.00 mJ.

•Calculate the exitation energy required to promote the marble to the next available energy state.

Solution:

The allowed energy values E_n for a particle of mass m in a one-dimensional infinite square well potential of width L are given by Eq. (22) from which

$$n = 4.27 \times 10^{28} \tag{33}$$

when $E_n = 1.00\,\mathrm{mJ}$.

The excitation energy E required to promote the marble to the next available energy state is

$$E = E_{n+1} - E_n = \frac{(2n+1)h^2}{8mL^2} = 4.69 \times 10^{-32}\ \mathrm{J}. \tag{34}$$

This example illustrates the large quantum numbers and small energy differences associated with the behavior of macroscopic objects.