RV College of Engineering®

HANDBOOK OF MATHEMATICS

FOR

FIRST YEAR B.E. PROGRAM

RV College of Engineering®

CONTENTS

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TRIGONOMETRY

1. Basic Functions

- $sin\,\theta = \frac{Opposite\ Side}{Hypotenuse}$
- $cos\,\theta = \frac{Adjacent\ Side}{Hypotenuse}$
- $tan\,\theta = \frac{Opposite\ Side}{Adjacent\ Side} = \frac{\sin\theta}{\cos\theta}$
- $sec\,\theta = \frac{Hypotenuse}{Adjacent\ Side} = \frac{1}{\cos\theta}$
- $cosec\,\theta = \frac{Hypotenuse}{Opposite\ Side} = \frac{1}{\sin\theta}$
- $cot\,\theta = \frac{Adjacent\ Side}{Opposite\ Side} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

2. Identities

- $\sin(-x) = -\sin x$
- $\cos(-x) = \cos x$
- $\tan(-x) = -\tan x$
- $\sin\left(\frac{\pi}{2} - x\right) = \cos x$
- $\cos\left(\frac{\pi}{2} - x\right) = \sin x$
- $\tan\left(\frac{\pi}{2} - x\right) = \cot x$
- $\sin\left(\frac{\pi}{2} + x\right) = \cos x$
- $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$
- $\tan\left(\frac{\pi}{2} + x\right) = -\cot x$
- $\sin(\pi - x) = \sin x$
- $\cos(\pi - x) = -\cos x$
- $\tan(\pi - x) = -\tan x$
- $\sin(\pi + x) = -\sin x$
- $\cos(\pi + x) = -\cos x$
- $\tan(\pi + x) = \tan x$
- $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$
- $\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$
- $\tan\left(\frac{3\pi}{2} - x\right) = \cot x$
- $\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$
- $\cos\left(\frac{3\pi}{2} + x\right) = \sin x$
- $\tan\left(\frac{3\pi}{2} + x\right) = -\cot x$
- $\sin(x + y) = \sin x\cos y + \cos x\sin y$
- $\cos(x + y) = \cos x\cos y - \sin x\sin y$
- $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$
- $\sin(2x) = 2\sin x\cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$
  $= 2\cos^2 x - 1$
- $\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$
- $\cos^2 x + \sin^2 x = 1$
- $\sec^2 x - \tan^2 x = 1$
- $\mathrm{cosec}^2\, x - \cot^2 x = 1$
- $\sin 3x = 3\sin x - 4\sin^3 x$
- $\cos 3x = 4\cos^3 x - 3\cos x$
- $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$
- $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

BASIC CALCULUS

1. Differentiation

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
a	0	a^x	$a^x \log_e a$
$x^n, n \neq -1$	nx^{n-1}	e^{ax}	ae^{ax}
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	$\log_e x$	$\frac{1}{x}$
$\sin x$	$\cos x$	$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos x$	$-\sin x$	$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan x$	$\sec^2 x$	$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\operatorname{cosec} x$	$-\cot x \operatorname{cosec} x$	$\operatorname{cosec}^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\sec x$	$\tan x \sec x$	$\sec^{-1} x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\cot x$	$-\operatorname{cosec}^2 x$	$\cot^{-1} x$	$-\frac{1}{1+x^2}$
$\sinh x = \frac{e^x - e^{-x}}{2}$	$\cosh x$	$\sinh^{-1} x$	$\frac{1}{\sqrt{x^2+1}}$
$\cosh x = \frac{e^x + e^{-x}}{2}$	$\sinh x$	$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh x$	$\operatorname{sech}^2 x$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\operatorname{cosech} x$	$-\coth x \operatorname{cosech} x$	$\operatorname{cosech}^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2+1}}$
$\operatorname{sech} x$	$-\tanh x \operatorname{sech} x$	$\operatorname{sech}^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{1-x^2}}$
$\coth x$	$-\operatorname{cosech}^2 x$	$\coth^{-1} x$	$\frac{1}{1-x^2}$

2. Rules of differentiation
    - $\frac{d}{dx}(fg) = gf' + fg'$
    - $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf'-fg'}{g^2}$
    - $\frac{d}{dx}\big(f(t)\big) = \frac{d}{dt}\big(f(t)\big)\frac{dt}{dx}$

3. Integration

$\boldsymbol{f(x)}$	$\boldsymbol{\int f(x)dx}$	$\boldsymbol{f(x)}$	$\boldsymbol{\int f(x)dx}$
x^n	$\frac{x^{n+1}}{n+1}$	$\frac{1}{x}$	$\log_e x$
e^{ax}	$\frac{e^{ax}}{a}$	$\log_e x$	$x(\log_e x - 1)$
a^x	$\frac{a^x}{\log_e a}$	$\operatorname{cosec} x$	$\log_e(\operatorname{cosec} x - \cot x)$
$\sin x$	$-\cos x$	$\sec x$	$\log_e(\sec x + \tan x)$
$\cos x$	$\sin x$	$\cot x$	$\log_e \sin x$
$\tan x$	$\log_e \sec x$	$\sec^2 x$	$\tan x$
$\sinh x$	$\cosh x$	$\operatorname{cosec}^2 x$	$-\cot x$
$\cosh x$	$\sinh x$	$\tanh x$	$\log_e \cosh x$
$\frac{1}{\sqrt{a^2-x^2}}$	$sin^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{a^2+x^2}}$	$sinh^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}tan^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{x^2-a^2}}$	$cosh^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\log_e\left(\frac{a+x}{a-x}\right)$	$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\log_e\left(\frac{x-a}{x+a}\right)$
$\sqrt{a^2-x^2}$	$\frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2sin^{-1}\left(\frac{x}{a}\right)\right]$	$e^{ax}\sin bx$	$\frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$
$u(x)v(x)$	$u\int v\,dx - \int\left[\frac{du}{dx}\left[\int v\,dx\right]dx\right]$	$e^{ax}\cos bx$	$\frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

DIFFERENTIAL CALCULUS

1. Transformations for polar coordinates to Cartesian coordinates: $x = r\cos\theta, y = r\sin\theta$.
2. Transformations for Cartesian coordinates to polar coordinates: $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$, $r \geq 0, 0 \leq \theta \leq 2\pi$.
3. The angle between the radius vector and tangent for a polar curve $r = f(\theta)$: $\tan\phi = r\frac{d\theta}{dr}$
4. The radius of curvature:
   - Cartesian curve $y = f(x)$: $\rho = \frac{[1+(y')^2]^{3/2}}{y''}$
   - Parametric curve $x = x(t), y = y(t)$: $\rho = \frac{[(x')^2+(y')^2]^{3/2}}{x'y''-x''y'}$
   - Polar curve $r = f(\theta)$: $\rho = \frac{[r^2+(r')^2]^{3/2}}{r^2+2(r')^2-rr''}$
5. Centre of curvature: $\bar{x} = x - \frac{y'[1+(y')^2]}{y''}$ and $\bar{y} = y + \frac{[1+(y')^2]}{y''}$
6. Taylor series expansion:
$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$
7. Maclaurin series expansion:
$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

PARTIAL DIFFERENTIATION

1. Let $z = f(x, y)$ be a function of two variables x and y.
   - The first order partial derivative of z with respect to x, denoted by $\frac{\partial z}{\partial x}$ or $\frac{\partial f}{\partial x}$or z_x or f_x or $\boldsymbol{p}$ is defined as $\frac{\partial z}{\partial x} = \lim_{\delta x\to 0}\frac{f(x+\delta x,y)-f(x,y)}{\delta x}$ provided the limit exists.
   - The first order partial derivative of z with respect to y, denoted by $\frac{\partial z}{\partial y}$or $\frac{\partial f}{\partial y}$or z_y or f_y or **q** is defined as$\frac{\partial z}{\partial y} = \lim_{\delta y\to 0}\frac{f(x,y+\delta y)-f(x,y)}{\delta y}$ provided the limit exists.
2. Notations of second order partial derivatives:
   - $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x^2}$ or $\frac{\partial^2 f}{\partial x^2}$or z_{xx} or f_{xx} or **r**
   - $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial x\partial y}$or z_{xy} or f_{xy}or **s**
   - $\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial y^2}or\frac{\partial^2 f}{\partial y^2}$or z_{yy} or f_{yy} or **t**
   - $\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial y\partial x}$or z_{yx} or f_{yx} or **s**

3. Total differential: Let $z = f(x, y)$ be a differentiable function of two variables, x and y then total differential (or exact differential) is defined by $dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$.
4. Total derivative: Further, if $z = f(x, y)$, where $x = x(t)$,$y = y(t)$, then total derivative of z is given by $\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$.
5. Differentiation of implicit functions: For $f(x, y) = 0$, $\frac{dy}{dx} = -\frac{\left(\frac{\partial f}{\partial x}\right)}{\frac{\partial f}{\partial y}}$.
6. Differentiation of composite functions (chain rule):
   Let z be function of x and y and that $x = \varphi(u, v)$ and $y = \phi(u, v)$ are functions of u and v then, $\frac{\partial z}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}$ and $\frac{\partial z}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v}$.
7. Jacobian: If u and v are functions of variables x and y, then the determinant $\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$ is called the Jacobian of u, v with respect to x, y and denoted by $\frac{\partial(u,v)}{\partial(x,y)}$.
8. If u, v are functions of r, s and r, s are functions of x, y, then $\frac{\partial(u,v)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(r,s)}\frac{\partial(r,s)}{\partial(x,y)}$.

MULTIPLE INTEGRAL

1. Area of a region $R = \iint_R dA$
2. Volume of a Solid S= $\iiint_S dx\, dy\, dz$
3. Change of variables: From Cartesian xy plane to
   - uv-plane $\iint_{R_{xy}} f(x, y)\, dx\, dy = \iint_{R_{uv}} f\big(\phi(u, v), \psi(u, v)\big)\, |J|\, du\, dv$
   - polar coordinates $\iint_{R_{xy}} f(x, y)\, dx\, dy = \iint_{R_{r\theta}} f(r cos\theta, r sin\theta)\, r\, dr\, d\theta$
4. Mass of two-dimenssional object with surface density $f(x, y)$: $M = \iint_R f(x, y) dxdy$
5. The center of gravity: $\overline{x} = \frac{1}{M}\iint_R x f(x, y) dxdy$ and $\overline{y} = \frac{1}{M}\iint_R y f(x, y) dxdy$
6. Mass of a solid S, with density $f(x, y, z)$: $M = \iiint_S f(x, y, z) dxdydz$
7. The center of gravity: $\overline{x} = \frac{1}{M}\iiint_S x f(x, y, z) dxdydz$, $\overline{y} = \frac{1}{M}\iiint_S y f(x, y, z) dxdydz$ and $\overline{z} = \frac{1}{M}\iiint_S z f(x, y, z) dxdydz$

ORDINARY DIFFERENTIAL EQUATIONS

1. **Auxiliary/Characteristic Equation:** The equation $F(m) = 0$ is known as the Auxiliary equation of $F(D)y = g(x)$.

2. **Solution of a Homogeneous ODE with constant coefficients:** For the differential equation $(a_0D^2 + a_1D + a_n)y = 0$, if m_1 and m_2 are the roots of auxiliary equation, then solution is given by following cases

- If roots are real and distinct, then $y = c_1e^{m_1x} + c_2e^{m_2x}$.
- If $m_1 = m_2$ are real, then $y = c_1e^{m_1x} + c_2xe^{m_1x}$.
- If roots are complex say $\alpha \pm i\beta$, then $y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$.

3. **Non-homogeneous Linear ODE with constant coefficients:** The general solution of $F(D)y = g(x)$ is given by $y = y_c + y_p$, where y_c is the solution of the associated homogeneous equation $F(D)y = 0$ and $y_p = \frac{1}{F(D)}g(x)$ is called the particular integral.

4. **Rules for finding particular integral:**

- If $g(x) = ke^{ax}$, then $y_p = k\frac{1}{F(D)}e^{ax} = k\frac{1}{F(a)}e^{ax}$, provided $F(a) \neq 0$.
  - If $F(a) = 0$ then $y_p = k\frac{x}{[F'(D)]_{D=a}}e^{ax}$, provided $F'(a) \neq 0$.
- If $g(x) = \sin(ax + b)$ or $\cos(ax + b)$, then
  - $y_p = \frac{1}{F(D^2)}\sin(ax + b)$ or $\frac{1}{F(D^2)}\cos(ax + b)$
    $= \frac{1}{F(-a^2)}\sin(ax + b)$ or $\frac{1}{F(-a^2)}\cos(ax + b)$, provided $F(-a^2) \neq 0$
  - If $F(-a^2) = 0$, $y_p = \frac{x}{F'(-a^2)}\sin(ax + b)$ or $\frac{x}{F'(-a^2)}\cos(ax + b)$, provided $F'(-a^2) \neq 0$
- If $g(x) = x^m$, then $y_p = \frac{1}{F(D)}x^m = [F(D)]^{-1}x^m$. Expanding the right hand side as a binomial series, the particular integral can be obtained. The following series expansions are useful:

$$(1 + D)^{-1} = 1 - D + D^2 - D^3 + \cdots$$

$$(1 + D)^{-2} = 1 - 2D + 3D^2 - 4D^3 + \cdots$$

- If $g(x) = e^{ax}V(x)$, then $y_p = \frac{1}{F(D)}e^{ax}V(x) = e^{ax}\frac{1}{F(D+a)}V(x)$

5. **Cauchy-Euler equation:** The linear ODE of the form $(a_0x^nD^n + a_1x^{n-1}D^{n-1} + a_2x^{n-2}D^{n-2} + \cdots + a_{n-1}xD + a_n)y = g(x)$, where $a_0, a_1, \cdots a_n$ are constants, is known as 'Cauchy-Euler' or equidimensional equation.

   This equation can be reduced to ODE with constant coefficients by changing the independent variable as follows –

   Take $x = e^z$, then $xDy = D_1y$,
   $x^2D^2y = D_1(D_1 - 1)y$,
   $x^3D^3y = D_1(D_1 - 1)(D_1 - 2)y$
   where $D_1 = \frac{d}{dz}$

6. **Wronskian:** For two functions $y_1(x)$ and $y_2(x)$, the Wronkian is defined by $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

7. **Method of Variation of Parameters:**

   For the second order ODE of the form $y'' + P(x)y' + Q(x)y = g(x)$. Let $y = c_1y_1 + c_2y_2$ be solution of the equation with $g(x) = 0$, the general solution is given by

   $y = A(x)y_1 + B(x)y_2$, where $A(x) = -\int \frac{y_2 g(x)}{W} dx + c_1$ and $B(x) = \int \frac{y_1 g(x)}{W} dx + c_2$, *and* $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$

PARTIAL DIFFERENTIAL EQUATIONS

1. **Lagrange's linear equation:** The first order linear partial differential equation of the form $P\frac{\partial u}{\partial x} + Q\frac{\partial u}{\partial y} = R$, where P, Q and R are functions of x, y, z is known as Lagrange's Linear equation.
2. **Subsidiary/Auxiliary Equation:** The equation $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ is known as the subsidiary/ auxiliary equation of as Lagrange's Linear equation $P\frac{\partial u}{\partial x} + Q\frac{\partial u}{\partial y} = R$.
3. **One-Dimensional Wave Equation:** $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$, where $c^2 = \frac{T}{\rho}$ the phase speed, T is the tension, and ρ density of the string.
4. **One-Dimensional Heat Equation:** $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$, where $c^2 = \frac{\kappa}{s\rho}$ the thermal diffusivity, κ thermal conductivity, s specific heat and ρ density of the material of the body.
5. **Two-Dimensional Laplace equation:** $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

NUMERICAL METHODS

1. Forward difference:
   - $\Delta f(x) = f(x+h) - f(x)$
   - $\Delta^n y_i = \Delta^{n-1} y_{i+1} - \Delta^{n-1} y_i$

2. Backward difference:
   - $\nabla f(x) = f(x) - f(x-h)$
   - $\nabla^n y_i = \nabla^{n-1} y_i - \nabla^{n-1} y_{i-1}$

3. Relation between forward and backward difference: $\Delta^n y_r = \nabla^n y_{n+r}$
4. $\Delta^n f(x) = a_0 n(n-1)(n-2)\ldots 1.h^n = a_0\, n!\; h^n$, where $f(x)$ is a polynomial of degree n.
5. Newton-Gregory Forward Interpolation Formula:

   $y_p = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots\ldots + \frac{p(p-1)\ldots\ldots(p-n+1)}{n!}\Delta^n y_0$

   where $x = x_0 + ph$
6. Newton-Gregory Backward Interpolation Formula:

   $y_p = f(x) = y_n + \text{p}\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \ldots\ldots + \frac{p(p+1)\ldots\ldots(p+n-1)}{n!}\nabla^n y_n$

   where $x = x_n + ph$
7. Lagrange's Interpolation Formula:

$$y = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}y_1 + \cdots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}y_n$$

8. Numerical Differentiation:

9. $\left(\frac{dy}{dx}\right)_{x=x_0+ph} = \frac{1}{h}\left[\Delta y_0 + \frac{(2p-1)}{2!}\Delta^2 y_0 + \frac{(3p^2-6p+2)}{3!}\Delta^3 y_0 + \frac{(4p^3-18p^2+22p-6)}{4!}\Delta^4 y_0 + \cdots\right]$

10. $\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right]$

11. $\left(\frac{dy}{dx}\right)_{x=x_n+ph} = \frac{1}{h}\left[\nabla y_n + \frac{(2p+1)}{2!}\nabla^2 y_n + \frac{(3p^2+6p+2)}{3!}\nabla^3 y_n + \ldots\right]$

12. $\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \cdots\right]$

13. $\left(\frac{d^2y}{dx^2}\right)_{x=x_0+ph} = \frac{1}{h^2}\left[\Delta^2 y_0 + (p-1)\Delta^3 y_0 + \frac{(6p^2-18p+11)}{12}\Delta^4 y_0 + \cdots\right]$

14. $\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \cdots\right]$

15. $\left(\frac{d^2y}{dx^2}\right)_{x=x_n+ph} = \frac{1}{h^2}\left[\nabla^2 y_n + (p+1)\nabla^3 y_n + \frac{(6p+18p+11)}{12}\nabla^4 y_n + \cdots\right]$

16. $\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \frac{137}{180}\nabla^6 y_n + \cdots\right]$

17. Regula - Falsi method: $x_{n+1} = \frac{x_{n-1}f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$

18. Newton Raphson Method: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

19. Runge - Kutta fourth order method:

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\,; \quad n = 0,1,2,3\,...$$

where, $k_1 = hf(x_n, y_n)$, $k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$, $k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$,

$k_4 = hf(x_n + h, y_n + k_3)$

20. Milne's Predictor Formula:

$$y_{n+1}^{(p)} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]; \qquad n = 0,1,2,3\,...\,....$$

21. Milne's Corrector Formula:

$$y_{n+1}^{(c)} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]; \qquad n = 3, 4, 5\,...\,...$$

22. Newton – Cote's Quadrature formula:

$$I = nh\left(y_0 + \frac{n}{2}(\Delta y_0) + \frac{1}{2!}\left(\frac{n^2}{3} - \frac{n}{2}\right)(\Delta^2 y_0) + \frac{1}{3!}\left(\frac{n^2}{4} - n^2 + n\right)(\Delta^3 y_0) + \cdots\right)$$

23. Simpson's 1/3rd rule:

$$I = \frac{h}{3}\left((y_0 + y_n) + 4\,(y_1 + y_3 + \cdots + y_{n-1}) + 2\,(y_2 + y_4 + \cdots + y_{n-2})\right)$$

24. Simpson's 3/8th rule:

$$I = \frac{3h}{8}\left((y_0 + y_n) + 3\,(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-1}) + 2\,(y_3 + y_6 + \cdots + y_{n-3})\right)$$

25. Weddle's rule:

$$I = \frac{3h}{10}\left[(y_0 + y_n) + 5(y_1 + y_5 + \cdots + y_{n-5} + y_{n-1}) + (y_2 + y_4 + \cdots + y_{n-4} + y_{n-2}) + 2(y_6 + y_{12} + \cdots + y_{n-6}) + 6(y_3 + y_9 + \cdots + y_{n-3})\right]$$

VECTOR CALCULUS

1. For $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$
   - $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta = a_1b_1 + a_2b_2 + a_3b_3$,
   - $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \ \hat{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
2. Vector Differential Operator: $\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$.
3. Gradient of a scalar point function: $\nabla\phi = \frac{\partial\phi}{\partial x}\hat{\imath} + \frac{\partial\phi}{\partial y}\hat{\jmath} + \frac{\partial\phi}{\partial z}\hat{k}$.
4. Divergence of a vector point function: $\nabla \cdot \vec{f} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$, where $\vec{f} = f_1\hat{\imath} + f_2\hat{\jmath} + f_3\hat{k}$.
5. Curl of vector function: $\nabla \times \vec{f} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$, where $\vec{f} = f_1\hat{\imath} + f_2\hat{\jmath} + f_3\hat{k}$.
6. Laplacian of a scalar field: $\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$
7. Cylindrical coordinate system: $x = r\,cos\theta, y = r\,sin\theta, z = z$
8. Spherical coordinate system: $x = rsin\theta cos\phi, y = rsin\theta sin\phi, z = rcos\theta$
9. Expression for gradient:
   - In cylindrical polar coordinates: $\nabla\psi = \frac{\partial\psi}{\partial r}e_r + \frac{1}{r}\frac{\partial\psi}{\partial\theta}e_\theta + \frac{\partial\psi}{\partial z}e_z$
   - In spherical polar coordinates: $\nabla\psi = \frac{\partial\psi}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\hat{e}_\theta + \frac{1}{rsin\theta}\frac{\partial\psi}{\partial\phi}\hat{e}_\phi$
10. Expression for divergence:
    - In cylindrical polar coordinates: $div(\vec{f}) = \frac{1}{r}\left[\frac{\partial}{\partial r}(rf_1) + \frac{\partial}{\partial\theta}(f_2) + \frac{\partial}{\partial z}(rf_3)\right]$,

      where $\vec{f} = f_1\hat{e}_r + f_2\hat{e}_\theta + f_3\,\hat{e}_z$
    - In spherical polar coordinates:

      $div(\vec{f}) = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(r^2\sin\theta\ f_1) + \frac{\partial}{\partial\theta}(r\sin\theta\, f_2) + \frac{\partial}{\partial\phi}(rf_3)\right]$,

      where $\vec{f} = f_1\hat{e}_r + f_2\hat{e}_\theta + f_3\,\hat{e}_\phi$
11. Expression for Laplacian:
    - In cylindrical polar coordinates: $\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} + \frac{\partial^2\phi}{\partial z^2}$
    - In spherical polar coordinates: $\nabla^2\phi = \frac{\partial^2\psi}{\partial r^2} + \frac{2}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{cot\theta}{r^2}\frac{\partial\psi}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$

12. Expression for Curl:

- In cylindrical polar coordinates: $curl\,\vec{f} = \frac{1}{r}\begin{vmatrix} \hat{e}_r & r\hat{e}_\theta & \hat{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ f_1 & rf_2 & f_3 \end{vmatrix}$,

  where $\vec{f} = f_1\hat{e}_r + f_2\hat{e}_\theta + f_3\,\hat{e}_z$

- In spherical polar coordinates**:** $curl\,\vec{f} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{e}_r & r\hat{e}_\theta & r\,sin\theta\hat{e}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ f_1 & rf_2 & r\,sin\theta f_3 \end{vmatrix}$,

  where $\vec{f} = f_1\hat{e}_r + f_2\hat{e}_\theta + f_3\,\hat{e}_\phi$

13. Green's Theorem: If R is a closed region in XY-plane, bounded by a simply closed curve C and if $P(x, y)$ and $Q(x, y)$, $\frac{\partial}{\partial x}Q(x, y)$, $\frac{\partial}{\partial y}P(x, y)$ are continuous functions at every point in R, then

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

14. Stokes Theorem: If S be an open surface bounded by a simple closed curve C and $\vec{F}$ be any vector point function having continuous first order partial derivatives, then

$$\oint_C \vec{F}\cdot d\vec{r} = \iint_S curl\,\vec{F}\cdot\hat{n}\,dS = \iint_S curl\,\vec{F}\cdot\,d\vec{S}$$

where $\hat{n}$ is the outward drawn unit normal at any point to S.

15. Gauss Divergence Theorem: If V is the volume bounded by a closed surface S and $\vec{F}$ is a vector point function having continuous derivatives, then

$$\iint_S \vec{F}\cdot\hat{n}\,dS = \iiint_V \nabla\cdot\vec{F}\,dV,$$

where $\hat{n}$ is the outward unit normal drawn to S.

LAPLACE TRANSFORMS

1. Gamma function
   - $\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx, (n > 0)$
   - $\Gamma(1) = 1$
   - $\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2n-1} dx$
   - $\Gamma(n+1) = n\Gamma(n)$
   - $\Gamma(n) = \frac{\Gamma(n+1)}{n}, (n < 0)$
   - $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

2. Beta Function
   - $\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$
   - $\beta(m,n) = \beta(n,m)$
   - $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$
   - $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
   - $\beta(m,n) = \int_0^\infty \frac{t^{m-1}}{(1+t)^{m+n}} dt$

3. Laplace transform of $f(t)$: $L\{f(t)\} = \int_0^\infty e^{-st} f(t) dt$
4. Transform of elementary functions:

- $L(e^{at}) = \frac{1}{s-a}, \quad s > a$
- $L(\sinh at) = L\left(\frac{e^{at}-e^{-at}}{2}\right) = \frac{a}{s^2-a^2}, s > |a|$
- $L(\sin\ at) = \frac{a}{s^2+a^2}, \quad s > 0$
- $L(\cosh\ at) = \frac{s}{s^2-a^2}, \quad s > |a|$
- $L(\cos a t) = \frac{s}{s^2+a^2}, \quad s > 0$
- $L(t^n) = \frac{\Gamma(n+1)}{s^{n+1}}$
- $L[H(t-a)] = \frac{e^{-as}}{s}$, where H is Heaviside unit step function

5. Properties of Laplace transform:
   - $L[a f(t) + b g(t)] = a L[f(t)] + b L[g(t)]$.
   - If $L[f(t)] = F(s)$, then $L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$, where a is a positive constant.
   - Let a be any real constant then $L[e^{at} f(t)] = F(s-a)$
   - If $L[f(t)] = F(s)$, then $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s), n = 1, 2, 3, \ldots$
   - If $L[f(t)] = F(s)$, then $L\left(\frac{f(t)}{t}\right) = \int_s^\infty F(s) ds$.
   - If $L[f(t)] = F(s)$, then $L\{f^n(t)\} = s^n L\{f(t)\} - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots\ldots - f^{n-1}(0)$
   - If $L[f(t)] = F(s)$, then $L \int_0^t f(t) dt = \frac{1}{s} F(s)$

- Let $f(t)$ be a periodic function of period T then $L\{f(t)\} = \frac{1}{1-e^{-ST}}\int_0^T e^{-st} f(t)dt$.
- If $L\{f(t)\} = F(s)$, then $L[f(t-a)H(t-a)] = e^{-as}F(s)$
- $f(t)$ be a continuous function at $t = a$, then $\int_0^\infty f(t)\delta(t-a)dt = f(a)$, where $\delta(t-a)$ is unit impulse function.

6. Inverse Laplace transform of $F(s)$ using Convolution theorem: If $L^{-1}[F(s)] = f(t)$ and $L^{-1}[G(s)] = g(t)$, then $L^{-1}[F(s)G(s)] = \int_0^t f(u)g(t-u)du = f(t) * g(t)$.

NUMBER THEORY

1. The number of all positive divisors of a, denoted by $T(a)$, where $a = p_1^{a_1} p_2^{a_2} \cdots p_n^{a_n}$
$$T(a) = (1+a_1)(1+a_2)\cdots(1+a_n)$$
2. The sum of all positive divisors of a, denoted by $S(a)$,
$$S(a) = \left(\frac{p_1^{a_1+1}-1}{p_1-1}\right)\left(\frac{p_2^{a_2+1}-1}{p_2-1}\right)\cdots\left(\frac{p_n^{a_n+1}-1}{p_n-1}\right)$$
3. Euler's theorem: if $(a,m) = 1$, then $a^{\phi(m)} \equiv 1 \ (mod\ m)$.
4. If p is a prime number, then $\phi(p) = p - 1$
5. If p is a prime number and $k > 0$, then $\phi(p^k) = p^k - p^{k-1}$
6. If the integer $n > 1$ has the prime factorization, $n = p_1^{k_1} \times p_2^{k_2} \times \cdots \times p_r^{k_r}$, then
$$\phi(n) = n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\cdots\left(1-\frac{1}{p_r}\right)$$
7. Cipher text: $c = m^e (mod\ n)$, where m is the message.
8. Decryption: $m = c^d (mod\ n)$, where d is the private key.

STATISTICS

1. Moments for ungrouped data:
   - The r^{th} moment about origin: $\mu_r' = \frac{1}{n}\sum_{i=1}^n x_i^r$, where $r = 1, 2, 3 \ldots$, $x_1, x_2 \cdots x_n$ are n observations
   - The r^{th} central moment: $\mu_r = \frac{1}{n}\sum_{i=1}^n (x_i - \bar{x})^r$, where $r = 1, 2, 3 \cdots$, $\bar{x}$ is mean
2. Moments for grouped data:
   - The r^{th} moment about origin: $\mu_r' = \frac{1}{N}\sum_{i=1}^n f_i x_i^r$, $r = 1, 2, 3 \cdots$, where observations $x_1, x_2, \ldots, x_n$ are the mid points of the class-intervals and $f_1, f_2, \ldots, f_n$ are their corresponding frequencies and N = $\sum_{i=1}^n f_i$
   - The r^{th} central moment: $\mu_r = \frac{1}{N}\sum_{i=1}^n f_i (x_i - \bar{x})^r$, $r = 1, 2, 3 \cdots$

- The r^{th} moment about any point A: $\mu_r' = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A)^r,\ r = 1, 2, 3 \cdots$

3. Relation between raw (Moments about origin or any point) and Central Moments:
   - $\mu_r = \mu_r' - {}^rC_1\, \mu_{r-1}'\, \mu_1' + {}^rC_2\, \mu_{r-2}'\, {\mu'}_1^2 - \ldots + (-1)^r {\mu_1'}^r$, r = 1, 2, 3 …
   - $\mu_r' = \mu_r + {}^rC_1 \mu_{r-1}\, \mu_1' + {}^rC_2 \mu_{r-2} {\mu_1'}^2 - \ldots + {\mu_1'}^r$

4. Measures of Kurtosis: $\beta_2 = \frac{\mu_4}{\mu_2^2}$

5. Measures of Skewness: Karl Pearson's coefficient of Skewness**:** $S_k = \frac{\sqrt{\beta_1}\,(\beta_2+3)}{2(5\beta_2 - 6\beta_1 - 9)}$, where $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

6. Fitting of a straight line: $y = a + bx$ for the data $(x_1, y_1), (x_2, y_2), \cdots (x_n, y_n)$

   The normal equations for estimating the values of a and b are

   $$\sum y = n\mathrm{a} + b\sum x,$$
   $$\sum xy = a\sum x + b\sum x^2.$$

7. Fitting of a second-degree equation (quadratic): $y = a + bx + cx^2$

   The normal equations for estimating the values of a , b , c are

   $$\sum y = n\mathrm{a} + b\sum x + c\sum x^2,$$
   $$\sum xy = \mathrm{a}\sum x + b\sum x^2 + c\sum x^3,$$
   $$\sum x^2 y = \mathrm{a}\sum x^2 + b\sum x^3 + c\sum x^4.$$

8. Correlation Coefficient (Karl Pearson correlation coefficient):
   - $r = \frac{\sum(x-\bar{x})(y-\bar{y})}{n\sigma_x\sigma_y}$, where $\sigma_x^2 = \frac{\sum(x-\overline{x})^2}{n}$ variance of the x series, $\sigma_y^2 = \frac{\sum(y-\bar{y})^2}{n}$ variance of the y series,
   - $\overline{x} = \frac{\sum x}{n}$ → Mean of the x series $\overline{y} = \frac{\sum y}{n}$ → mean of the y series.
   - $r = \frac{n\sum \mathrm{xy} - (\sum x)(\sum y)}{\sqrt{\{n\sum x^2 - (\sum x)^2\}\{n\sum y^2 - (\sum y)^2\}}}$

9. Regression line of y on x: $y - \bar{y} = b_{yx}(y - \overline{y})$, where $b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{\sum(x-\overline{x})(y-\overline{y})}{\sum(x-\overline{x})^2} = \frac{n\sum \mathrm{xy} - \sum x\sum y}{n\sum x^2 - (\sum x)^2}$

10. Regression line of x on y: $x - \overline{x} = b_{xy}(y - \overline{y})$ where $b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{\sum(x-\overline{x})(y-\overline{y})}{\sum(y-\overline{y})^2} = \frac{n\sum xy - \sum x\sum y}{n\sum y^2 - (\sum y)^2}$