VECTOR CALCULUS, LAPLACE TRANSFORM & NUMERICAL METHODS (MA221TA)

UNIT-I

VECTOR DIFFERENTIATION

TUTORIAL SHEET-1

1. If $\vec{f}$ is a vector function with constant magnitude then $\vec{f}.\frac{d\vec{f}}{dt} =$ _______ ans: 0

2. The displacement of a particle moving along a path is given by $x = (1 - t^3)$, $y = (1 + t^2)$, $z = (2t - 5)$ the magnitude of velocity vector at t = 1 second is _____ ans: √17

3. The temperature at a point (x,y,z) in space is given by T(x,y,z) = $x^2 + y^2 - z$. A mosquito located at (1,1,2) desires to fly in such a direction that it gets cooled faster. Which of the following direction it should fly?
Ans: a) 2i + 2j – k

4. For the curves whose equations are given below, find the unit tangent vectors:

(i) $x = t^2 + 1,\ y = 4t - 3,\ z = 2(t^2 - 3t)$ at $t = 0$.

(ii) $\vec{r} = (a\cos 3t)\ i + (a\sin 3t)\ j + (4at)\ k$ at $t = \frac{\pi}{4}$

ans: (i) $\hat{t} = \frac{(2\hat{j} - 3\hat{k})}{\sqrt{13}}$ (ii) $\frac{1}{5\sqrt{2}}\left[-3\hat{i} - 3\hat{j} + 4\sqrt{2}\,\hat{k}\right]$

5. A particle moves along the curve $\vec{r} = 2t^2\ i + (t^2 - 4t)\ j + (3t - 5)\ k$. Find the component of velocity and acceleration in the direction of vector $c = i - 3j + 2k$ at $t = 1$.

ans: $\frac{16}{\sqrt{14}}$ & $\frac{-2}{\sqrt{14}}$

6. A person on a hang glider is spiralling upward due to rapidly rising air on a path having position vector $r(t) = (3cost)i + (3sint)j + t^2\ k$. Find (a) the velocity and acceleration vectors (b) the glider's speed at any time *t*.
Ans: $= v - (3sint)i + (3cost)j + 2t\ k$; $a = (-3cost)i + (-3sint)j + 2k$;
$|v| = \sqrt{9 + 4t^2}$

7. A Particle move along the curve $x = e^{-t}, y = 2\cos 3t\,, z = 2\sin 3t$, where t is the time. Determine its velocity and acceleration vectors and also the magnitude of velocity and acceleration at t=0.
Ans: $\sqrt{37}, \sqrt{13}$

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UNIT-I

VECTOR DIFFERENTIATION

TUTORIAL SHEET-2

1. If $\phi(x,y,z) = xy^2z^3 - x^3y^2z$, then $|\nabla\phi|$ at (1, -1, 1) = ____________. ans: $2\sqrt{2}$

2. The maximum directional derivative of $\phi(x,y,z) = x^2y + yz^2 - xz^3$ at $(-1,2,1)$ is ____.

   ans: $\sqrt{78}$

3. If $\phi(x,y,z) = x^2 + \sin y + z$ then $\nabla\phi$ at $\left(0, \frac{\pi}{2}, 1\right)$ is ____________. Ans: $\hat{k}$

4. Find the unit normal vector to the surface $\phi(x,y,z) = x^2y + y^2z + z^2x = 5$ at the point (1, -1, 2).

   ans: $\frac{1}{\sqrt{38}}(2i - 3j + 5k)$

5. Show that the surfaces $4x^2 - z^3 = 4$ and $5x^2 - 2yz = 7x$ intersect orthogonally at the point $(1,-1,-2)$.

6. Find the constants a and b so that the surface $3x^2 - 2y^2 - 3z^2 + 8 = 0$ is orthogonal to the surface $ax^2 + y^2 = bz$ at the point (-1, 2, 1). Ans: $a = 4/9,\ b = 40/9$

7. Find a and b such that the surfaces $ax^2 - bxyz = (a+2)x$ and $4x^{2y} + z^3 = 4$ cut orthogonally at (1,-1,2). Ans: $a = 5/2, b = 1$

8. Find the directional derivative of $\phi(x,y,z) = xyz - xy^2z^3$ at (1, 2, -1) in the direction of $i - j - 3k$. Ans: $29/\sqrt{11}$

9. Find the directional derivation of $x^2y^2z^2$ at the point (1,-1,1) in the direction of the tangent to the curve $x = e^t, y = \sin 2t + 1, z = 1 - \cos t \ \ at\ t = 0$
Ans: i+2j, $\frac{6}{\sqrt{5}}$

VECTOR CALCULUS, LAPLACE TRANSFORM & NUMERICAL METHODS (MA221TA)

UNIT-I

VECTOR DIFFERENTIATION

TUTORIAL SHEET-3

1. If $\phi = 3x^2y - y^3z^2$, grad ϕ at the point (1, -2, -1) is ______ Ans: - (12i+5j+8k)
2. If $\vec{f} = \tan^{-1}(y/x)$ then div (grad f) is equal to _____ Ans: 0
3.

1. If $\vec{f} = 3x^2 i + 5xy^2 j + xyz^3 k$ then find $div\,\vec{f}$ at (1, 2, 3) . ans: 80

2. Find If $\vec{f} = (y^2 + z^2 - x^2)i + (z^2 + x^2 - y^2)j + (x^2 + y^2 - z^2)k$ then find $div\,\vec{f}, curl\,\vec{f}$

Ans: $div\,\vec{f} = -2(x + y + z), curl\,\vec{f} = 2\{(y - z)i + (z - x)j + (x - y)k\}$

3. Show that the vector field $\vec{f} = (x + 3y)\hat{\imath} + (y - 3z)\hat{\jmath} + (x - 2z)\hat{k}$ is solenoidal.

4. Determine the constant a such that the vector field

$\vec{f} = (x + 3y)i + (y - 2z)j + (x - az)k$ is solenoidal. ans: 2

5. If $\vec{f} = x^2 i + y^2 j + z^2 k$ and $\vec{g} = yzi + zxj + xyk$ then show that $f \times g$ is solenoidal.

6. If $\vec{f} = (2x + 3y + az)i + (bx + 2y + 3z)j + (2x + cy + 3z)k$ is irrotational vector Field, then find the constants a, b, c . Ans: $a = 2, b = 3, c = 3$

7. If $\phi = x^2 y + 2xy + z^2$ then show that $\nabla\phi$ is irrotational.

8. If $\phi = x^2 - y^2$ then show that ϕ satisfies the Laplacian equation.

9. If $\phi = 2x^2 yz^3$ then find $\nabla^2\phi$ at (1, 1, 1). Ans: 16

10. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$ then show that $r^n\,\vec{r}$ is irrotational for all values of n and solenoidal for $n = -3$.

11. Show that $\vec{f} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational. Find the function ϕ such that $\vec{f} = grad\ \emptyset$. Ans: $\emptyset = 3x^2y + xz^3 - yz$.

VECTOR CALCULUS, LAPLACE TRANSFORM & NUMERICAL METHODS (MA221TA) UNIT-I

<u>VECTOR DIFFERENTIATION</u>

<u>TUTORIAL SHEET-4</u>

I Find gradient and Laplacian of

1. $\Psi = r^2 sin2\theta sin\emptyset$ in spherical coordinates $(r, \theta, \emptyset)$

2. $f = \rho^2 + 2\rho cos\emptyset - e^z sin\emptyset$ in cylindrical coordinates $(\rho, \emptyset, z)$

II Find divergence of

1. $\vec{f} = r^2\, \widehat{e_r} - 2cos^2\emptyset\, \widehat{e_\theta} + \frac{\emptyset}{r^2+1}\widehat{e_\emptyset}$ in spherical coordinates $(r, \theta, \emptyset)$.

2. $\vec{A} = z[sin\theta\widehat{e_r} + cos\theta\, \widehat{e_\theta}] - rcos\theta\widehat{e_z}$ in cylindrical coordinates (r, θ, z). Hence interpret the result.

III Find curl of

1. $\vec{f} = \rho z sin2\theta\widehat{e_\rho} + \rho z cos2\theta\, \widehat{e_\theta} - \frac{\rho^2 sin^2\theta}{2}\widehat{e_z}$ in cylindrical coordinates (ρ, θ, z). Hence interpret the result.

2. $\vec{A} = \frac{\mathbf{2cos\theta}}{\boldsymbol{\rho^3}}\widehat{e_\rho} - \frac{\boldsymbol{sin\theta}}{\boldsymbol{\rho^3}}\, \widehat{e_\theta}$ in spherical coordinates $(\rho, \theta, \emptyset)$.