

Unit 5: Numerical Methods

Interpolation:

It is the technique of finding the dependent variable y for the intermediate values of independent variable (x) of the given data.

Extrapolation:

Estimating the dependent variable for the independent variable outside the range is called extrapolation.

Interpolation techniques depend on 3 types of differences.

- (1) Forward difference,
- (2) Backward difference,
- (3) Central difference.

Forward difference:

Let $y = f(x)$ be defined for equal spaced of values of x as

$$\begin{array}{ccccccc} x_0 & x_0 + h & x_0 + 2h & \dots & x_0 + nh & \dots \\ y_0 & y_1 & y_2 & \dots & y_n & \dots \end{array}$$

then the differences $y_1 - y_0, y_2 - y_1, y_3 - y_2, \dots, y_n - y_{n-1}$ are called first order forward differences denoted by $\Delta y_0, \Delta y_1, \Delta y_2, \Delta y_{n-1}$

Second order forward differences: $\Delta^2 y_0 = \Delta(\Delta y_0)$

$$\begin{aligned} \Delta^2 y_0 &= \Delta(y_1 - y_0) = \Delta y_1 - \Delta y_0 \\ &= y_2 - y_1 - (y_1 - y_0) \\ &= y_2 - 2y_1 + y_0. \end{aligned}$$

Binomial expression

$$\Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0.$$

$$\Delta^4 y_1 = y_5 - 4y_4 + 6y_3 - 4y_2 + y_1.$$

$$\Delta^n y_0 = y_n - n y_{n-1} + \dots + (-1)^n \cdot n y_0.$$

Backward difference:

Let the f^n $y = f(x)$ given by

$$\begin{array}{ccccccc} x_0 & x_0 - h & x_0 - 2h & \dots & x_0 - nh \\ y_0 & y_1 & y_2 & \dots & y_n \end{array}$$

then the differences $y_1 - y_0, y_2 - y_1, y_n - y_{n-1}$ are called first order backward differences denoted by $\nabla y_1, \nabla y_2, \dots, \nabla y_{n-1}$

stable (or) del.

NOTE: $\Delta f(x) = f(x+h) - f(x)$.

$$\nabla f(x) \approx f(x) - f(x-h)$$

$$\nabla^2 y_2 = \nabla(\nabla y_2)$$

$$\Rightarrow \nabla(\nabla y_2 - y_1)$$

$$= \nabla y_2 - \nabla y_1$$

$$= (y_2 - y_1) - (y_1 - y_0)$$

$$\Rightarrow y_2 - 2y_1 + y_0 = \Delta^2 y_0$$

$$\boxed{\Delta^r y_k = \nabla^r y_{r+k}}$$

(Q) $\Delta \tan^{-1} x$

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta \tan^{-1} x = \tan^{-1}(x+h) - \tan^{-1}(x)$$

$$= \tan^{-1}\left(\frac{x+h-x}{1+(x+h)x}\right)$$

$$= \tan^{-1}\left(\frac{h}{1+x^2+2xh}\right)$$

(Q) $\Delta \log(2x+3)$

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta \log(2x+3) = \log(2x+3+2h) - \log(2x+3)$$

$$= \log\left(\frac{2x+3+2h}{2x+3}\right)$$

$$= \log\left(\frac{2h}{2x+3}\right)$$

(Q) ∇e^{3x+2} with $h=1$

$$\nabla f(x) = f(x) - f(x-h)$$

$$= e^{3x+2} - e^{3(x-1)+2}$$

$$= e^{3x+2} - e^{3x+2-3h}$$

$$= e^{3x+2} - \frac{e^{3x+2}}{e^{3h}}$$

$$= \frac{e^{3x+2+3h} - e^{3x+2}}{e^{3h}}$$

$$= \frac{e^{3x+5} - e^{3x+2}}{e^3}$$

Forward difference table:

Let the $f(x) = y$ be given by

$$x_0, x_0 + h = x_1, x_2, x_3, x_4$$

$$y_0, y_1, y_2, y_3, y_4$$

x

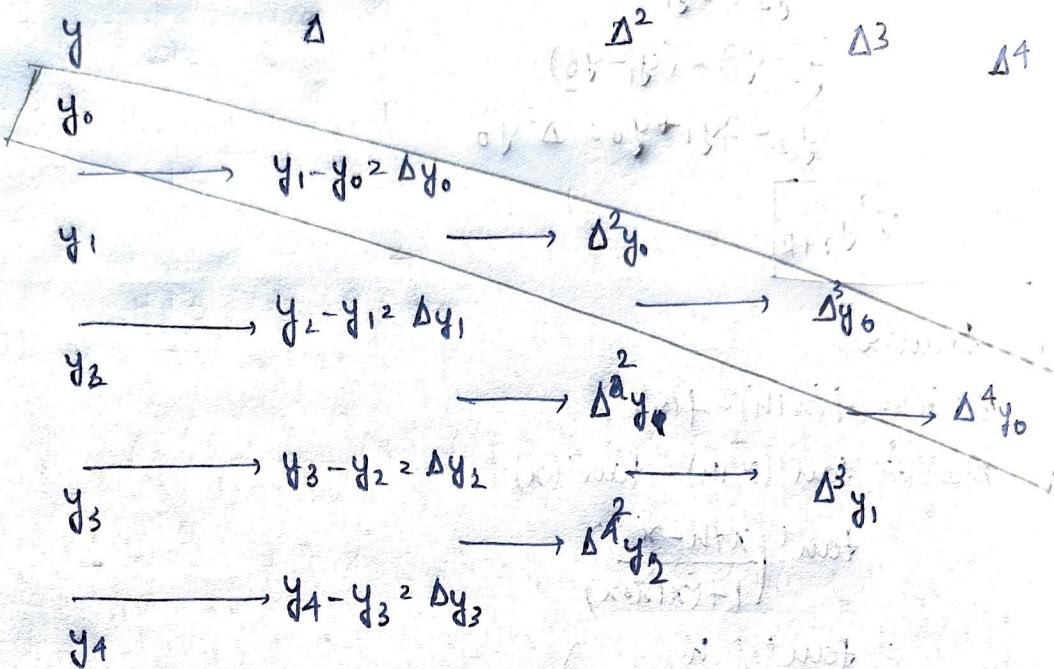
x_0

x_1

x_2

x_3

x_4



The differences $y_0, \Delta y_0, \Delta^2 y_0, \Delta^3 y_0, \Delta^4 y_0$ are called leading entities in forward difference table.

Backward difference table:

$$x_0, x_0 - h = x_1, x_2, x_3, x_4$$

$$y_0, y_1, y_2, y_3, y_4$$

x

x_0

x_1

x_2

x_3

x_4

$$y, \nabla, \nabla^2, \nabla^3, \nabla^4$$

$$y_0, y_1, y_2, y_3, y_4$$

$$\rightarrow y_1 - y_0 -> \nabla y_1$$

$$y_1, \rightarrow \nabla^2 y_2$$

$$y_2, \rightarrow \nabla^3 y_3$$

$$y_3, \rightarrow \nabla^4 y_4$$

$$y_3, \rightarrow \nabla^3 y_4$$

$$y_4, \rightarrow \nabla^2 y_4$$

$$y_4, \text{Leading entities in backward difference table}$$

$$x: 10 \quad 20 \quad 30 \quad 40$$

$$y: 66 \quad 42 \quad 85 \quad 90$$

$$\Delta^1 y_0 = 85, \Delta^2 y_0 = 85.$$

x	y	Δ	Δ^2	Δ^3
10	66		-24	
20	42	67	1	-105
30	85	43	-38	
			5	

Newton's Forward difference formula (NFIF)
 To compute y for starting table values of x , we use NFIF formula:

$$y = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!} \Delta^4 y_0$$

$$\boxed{p = \frac{x - x_0}{h}}$$

step length

Newton's Backward interpolation formula (NBIF)

To find estimate y at the end values of x , we use NBIF formula:

$$y = y_n + p \nabla y_n + \frac{p(p+1)}{2!} \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!} \nabla^4 y_n$$

$$\boxed{p = \frac{x - x_n}{h}}$$

When the value of x is in the middle \rightarrow either forward/backward

- (Q) In an experiment, output y are recorded for input x from 1.0 to 3.5 at 0.5 intervals, estimate the values of y by using appropriate interpolation formula for the inputs $x=1.2, x=2.4, x=3.8$, x is 1, 1.5, 2, 2.5, 3.

x	1.5	2.0	2.5	3.0	3.5
y	277	166	146	130	115

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1	277	-111				
1.5	166	91	-20		-87	
2.0	146	4			84	
2.5	130	1		-3		-80
3.0	115	2		1		
3.5	102	-13				

$$y \approx y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!} \Delta^4 y_0$$

$$\textcircled{1} \quad x=1.2$$

$$y_0 = 277$$

$$277 + \frac{0.4}{2!} \times -111 + \frac{0.4 \times (0.4-1)}{3!} \times 91 + \frac{0.4 \times (0.4-1)(0.4-2)}{4!} \times -87$$

$$p = \frac{x-x_0}{h} = \frac{1.2-1}{0.5} = 0.4$$

$$+ \frac{0.4 \times (0.4-1)(0.4-2)(0.4-3)}{5!} \times 84 +$$

$$+ \frac{0.4 \times (0.4-1)(0.4-2)(0.4-3)(0.4-4)}{6!} \times (-80)$$

$$127.26 - 44.4 - 10.92 + \cancel{- 27.04} + 5.568 - 3.2494 = 2.396$$

~~1. $x=1.2$~~

~~2. $x=3.4$~~

~~2. $x=3.4$~~

~~3. $x=3.5$~~

~~4. $x=1.02$~~

~~3. $x=3.8$~~

$$p = \frac{3.8-3.5}{0.5} = \frac{0.3}{0.5} = 0.6$$

$$p = \frac{3.1-3.5}{0.5} = \frac{-0.4}{0.5}$$

$$y = 102 + 0.6 \times -13 + \frac{0.6 \times (0.6+1) \times 2}{2!} + 0.6 \times (0.6+1) \times 1 +$$

$$+ \frac{0.6 \times (0.6+1) \times (0.6+2) \times (0.6+3) \times 4}{5!} +$$

$$+ \frac{0.6 \times (0.6+1) \times (0.6+2) \times (0.6+3) \times (0.6+4) \times (-80)}{6!}$$

$$5!$$

$$102 - 7.8 + 0.96 + 0.416 + 1.4975 \cancel{+} - 27.55$$

$$= 69.52$$

(Q) The details given below are monthly income of 2000 adults in a colony using interpolation estimate the no. of persons who have monthly income b/w ₹8000 - ₹10000.

(A)	Monthly income	Below 5K	5K-10K	10K-15K	15K-20K	20K-25K
	no. of persons	535	660	470	270	65
Monthly income (x)	no. of persons (y)		Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
5000	535		660			
10000	1195	660	-190			
15000	1665	470	-200	-10		
20000	1935	270	-205	-5		
25000	2000	65				

$$y(8000) = y_0 + p\Delta y_0 + \frac{p(p-1)(y_0)^2}{2!} + \frac{p(p-1)(p-2)}{3!} \Delta^2 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!} \Delta^3 y_0$$

$$p \approx \frac{x-x_0}{h} = \frac{8000 - 5000}{5000} = \frac{3000}{5000} = 0.6$$

$$= 535 + 0.6 \times 660 + \frac{0.6 \times (0.6-1) \times -190}{2!} + \frac{(0.6) \times (0.6-1) \times (0.6-2) \times -10}{3!} + \frac{0.6 \times (0.6-1) \times (0.6-2) \times (0.6-3) \times 5}{4!}$$

$$= 535 + 396 + 22.8 - 0.56 - 0.168$$

$$= 953.072$$

$$\text{Diff: } 1195 - 953 \rightarrow \text{income b/w } ₹8000 \text{ & } ₹10000$$

$$2^2 242$$

$$\begin{array}{r} 0 \\ 1195 \\ - 953 \\ \hline 242 \end{array}$$

Ans (Q) The details regarding marks secured by 280 candidates in an examination are given by the following table, using interpolation estimate the no. of people having marks between 45 and 50.

Marks	Below 30.	30-40	40-50	50-60	60-70	70-80
no. of candidates	35	49	62	74	40	20

(c) find the interpolated polynomial for the following data, hence find $y(12)$.

x (values) = 3 5 7 9 11

$x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4$

y (values) = 1 3 8 16 27

$y_0 \quad y_1 \quad y_2 \quad y_3 \quad y_4$

$$x \quad y \quad \Delta y_0 \quad \Delta^2 y_0 \quad \Delta^3 y_0 \quad \Delta^4 y_0$$

3

1 → +2

5

3 → 3

7

5 → 0.

9

3 → 0

11

11 → 0.

$$\text{NIFIF: } y = y_0 + \frac{\Delta y_0 \cdot p}{2!} + \frac{\Delta^2 y_0 \cdot p(p-1)}{3!} + \frac{p(p-1)(p-2) \Delta^3 y_0}{4!}$$

$$p = \frac{x - x_0}{2} = \frac{x - 3}{2}$$

$$y_0 = 1$$

$$y = 1 + \frac{2(x-3)}{2} + \frac{3 \cdot \frac{(x-3)}{2} \cdot \frac{(x-3-1)}{2}}{2!}$$

$$= 1 + x - 3 + \frac{3(x-3)(x-5)}{8} \rightarrow \text{collect } x^2 \text{ coefficient}$$

$$y(x)^2 = 84 + x - 3 + 3 \rightarrow 3/8 x^2 - 2x + 29/8$$

$$y(12) = 3/8 \times 144 - 2 \times 12 + 29/8$$

$$= 54 - 24 + 3.625$$

$$= 30 + 3.625 = 33.625$$

(Q) Using NBIF, find the cubic polynomial satisfying $f(-1) = -2.5$;
 $f(-2) = 1$; $f(0) = 3$; $f(2) = 29$; $f(4) = 127$; hence find $f(3)$ and $f(5)$

(A)

x	y	∇y_0	$\nabla^2 y_0$	$\nabla^3 y_0$	$\nabla^4 y_0$
-4	-2.5				
-2	1				
0	3	2			
2	29	24	2.5		
4	127	98	48	22.5	

$$P = \frac{x_n - x}{n} = \frac{4 - x}{2} = y_n = 127$$

$$\begin{aligned} & \left(\frac{x-4}{2}\right)\left(\frac{x-2}{2}\right)\left(\frac{x}{2}\right) \\ & \left(\frac{x-4}{2}\right)\left(\frac{x-2}{2}\right)\left(\frac{x}{2}\right)\left(\frac{x+2}{2}\right) \end{aligned}$$

$$y = y_n + p \nabla y_0 + \frac{p(p+1)}{2!} \nabla^2 y_0 + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_0 + \frac{p(p+1)(p+2)(p+3)}{4!} \nabla^4 y_0$$

$$= 127 + \frac{\left(\frac{x-4}{2}\right)}{2} \times 98 + \frac{\left(\frac{x-4}{2}\right)}{2} \times \left(\frac{x-4+1}{2}\right) \left(\frac{x-4+2}{2}\right) \times 48 + \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-4+1}{2}\right)}{2!} \times$$

$$+ \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-4+1}{2}\right)\left(\frac{x-4+2}{2}\right)\left(\frac{x-4+3}{2}\right)}{4!} \times 22.5$$

$$= 127 + 49(x-4) + (x-4)(x-2)(x) + 9(x-4)(x-2) + 0.0058(x-4)(x-2)$$

$$= 127 + 49x - 196 + 9x^2 - 54x + 72 + x^3 - 6x^2 + 8x + (x)(x+2)$$

$$= 0.0058x^4 - 0.348x^3 + 0.464x^2 +$$

$$= 0.0058x^4 - 0.348x^3 + 0.464x^2 + 0.116x^3 - 0.696x^2 + 0.928x$$

$$= 0.0058x^4 + 0.116x^3 + 2.768x^2 + 3.928x + 3$$

$$f(3) = 65.13$$

$$-5x + 8$$

$$f(5) = 224.09$$

(Q) n^{th} order differences of a polynomial of degree n are constant, all $(n+1)$ order differences are 0.

Numerical differentiation

Let the fn $y = f(x)$ be defined

$$x \quad x_0 \quad x_0 + h \quad x_0 + 2h \dots x_0 + nh$$

$$y \quad y_0 \quad y_1 \quad y_2 \dots y_n$$

The process of finding the derivatives of y is called numerical differentiation

Newton's forward interpolation formula to find the derivatives for the specified values of x are

$$y' = \frac{1}{h} \left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} \dots \right]$$

$$y'' = \frac{1}{h^2} \left[\frac{\Delta^2 y_0}{2} - \frac{\Delta^3 y_0}{3} + \frac{11}{12} \Delta^4 y_0 - \frac{5}{6} \Delta^5 y_0 \dots \right]$$

$$y''' = \frac{1}{h^3} \left[\frac{\Delta^3 y_0}{3} - \frac{3}{2} \Delta^4 y_0 + \frac{1}{4} \Delta^5 y_0 \dots \right]$$

Newton's backward interpolation formulae are:

$$y' = \frac{1}{h} \left[\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} \dots \right]$$

$$y'' = \frac{1}{h^2} \left[\nabla^2 y_n + \frac{\nabla^3 y_n}{2} + \frac{11}{12} \nabla^4 y_n + \frac{5}{6} \nabla^5 y_n \dots \right]$$

$$y''' = \frac{1}{h^3} \left[\nabla^3 y_n + \frac{3}{2} \nabla^4 y_n + \frac{7}{4} \nabla^5 y_n \dots \right]$$

(Q) A rod is rotating in a plane, the following table gives the angle θ in radian through which the rod has turned for various values of time t in seconds.

t	0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.12	0.19	0.12	2.02	3.2	4.67

Find angular velocity and angular acceleration at $t = 0.2, 1.0$ and 1.2

Answe

t	θ	$\Delta\theta_0$	$\Delta^2\theta_0$	$\Delta^3\theta_0$	$\Delta^4\theta_0$	$\Delta^5\theta_0$
0	0					
0.2	0.12	0.12	0.25	-0.12	1	
0.4	0.49	0.37	-0.74	3.01	2.52	-10
0.6	0.12	-0.37	2.07	2.07	-5.25	20
0.8	2.02	1.9	-0.72	-2.99	10	9.26
1.0	3.2	1.18	0.29	1.01	1	
1.2	4.67	1.47				

$$t=0.2, \frac{d\theta}{dt} = \frac{1}{h} \cdot 0.02.$$

$$y' = \frac{1}{0.2} \left[\Delta\theta_0 - \frac{\Delta^2\theta_0}{2} + \frac{\Delta^3\theta_0}{3} - \frac{\Delta^4\theta_0}{4} + \frac{\Delta^5\theta_0}{5} \right]$$

$$= \frac{1}{0.2} \left[0.37 + \frac{0.14}{2} + \frac{3.01}{3} + \frac{6}{4} + \frac{10}{5} \right]$$

$$= \frac{1}{0.2} [0.74 + 1.00 + 1.05 + 2]$$

$$= \frac{1}{0.2} [5.24] = 26.2.$$

$$y'' = \frac{1}{(0.2)^2} \left[\Delta^2\theta_0 - \Delta^3\theta_0 + \frac{1}{12} \Delta^4\theta_0 - \frac{5}{6} \Delta^5\theta_0 + \dots \right]$$

$$= \frac{1}{(0.2)^2} \left[-0.74 - 3.01 + \frac{1}{12} \times 6 - \frac{5}{6} \times 10 \right]$$

$$= \frac{1}{0.04} [-0.74 - 3.01 - 5.5 - 8.3]$$

$$= \frac{1}{0.04} x - 17.55 = -438.75.$$

$$t=1.0.$$

$$y' = \frac{1}{0.2} \left[\Delta\theta_0 + \frac{\Delta^2\theta_0}{2} + \frac{\Delta^3\theta_0}{3} + \dots \right]$$

$$= \frac{1}{0.2} \left[1.18 - \frac{0.72}{2} - \frac{2.99}{3} - \frac{6}{4} - \frac{10}{5} \right]$$

$$= \frac{1}{0.2} [1.18 - 0.36 - 0.99 - 1.5 - 2]$$

$$= \frac{1}{0.2} [-3.67] = -18.35$$

Interpolation with unequal intervals (Lagrange's interpolation)

Let $f(x)$ be defined as

$$\begin{array}{ccccccc} x & x_0 & x_1 & x_2 & \cdots & x_n \\ y & y_0 & y_1 & y_2 & \cdots & y_n \end{array} \quad (x \text{ need not be equally spaced.})$$

$$y = y_0 \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_n-x_0)(x_n-x_1)} y_n + \dots + \frac{(x-x_0)(x-x_1) \cdots (x-x_{n-1})}{(x_n-x_0)(x_n-x_1) \cdots (x_n-x_{n-1})} y_n$$

Inverse Lagrange's interpolation formula:

$$y = \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)} x_0 + \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)} x_1 + \dots + \frac{(y-y_0)(y-y_1) \cdots (y-y_{n-1})}{(y_n-y_0)(y_n-y_1) \cdots (y_n-y_{n-1})} x_n$$

(Q) Find $y(10)$ for the given data:

x	x_0	x_1	x_2	x_3	x_4
y	1	2	5	8	9
y_0	5	-4	91	24	1613
y_1	y_0	y_1	y_2	y_3	y_4

$$(A) y = \frac{(x-2)(x-5)(x-8)(x-9)}{(1-2)(1-5)(1-8)(1-9)} x_5 + \frac{(x-1)(x-2)(x-8)(x-9)}{(2-1)(2-5)(2-8)(2-9)} x_4 + \frac{(x-1)(x-2)(x-8)(x-9)}{(5-1)(5-2)(5-8)(5-9)} x_91 + \frac{(x-1)(x-2)(x-5)(x-9)}{(8-1)(8-2)(8-5)(8-9)} x_{24} + \frac{(x-1)(x-2)(x-5)(x-8)}{(9-1)(9-2)(9-5)(9-8)} x_{1613} \rightarrow \text{put } x=10$$

$$y = \underline{\underline{0.257}} \underline{\underline{8.72}} + \underline{\underline{4.57}} + 91 \bar{A} 68.57 + 5184.64 \\ = \underline{\underline{5350.56}} . 5211.7142$$

Numerical Integration: evaluate special & improper integrals

Let the $f^n = f(x)$

$$\begin{array}{cccccc} x & x_0 & x_0 + h & x_0 + 2h & \dots & x_0 + nh \\ y & y_0 & y_1 & y_2 & \dots & y_n \end{array} \left. \right] y^2 f(x)$$

The process of evaluating the integral $\int_a^b f(x) dx$ (definite integral) is called numerical integration.

Simpson's $\frac{1}{3}$ rd rule:

$$\int_a^b f(x) dx = \frac{h}{3} [y_0 + y_n + 2(y_2 + y_4 + y_6 + \dots) + 4(y_1 + y_3 + y_5 + \dots)]$$

Simpson's $\frac{3}{8}$ th rule:

$$\int_a^b f(x) dx = \frac{3h}{8} [y_0 + y_n + 2(y_3 + y_6 + y_9 + \dots) + 3(y_1 + y_2 + y_4 + y_5 + \dots)]$$

Middle's rule:

$$\int_a^b f(x) dx = \frac{3h}{10} [y_0 + 5y_1 + 3y_2 + 6y_3 + y_4 + 5y_5 + y_6 + 3y_7 + 5y_8 + 6y_9 + y_{10} + 5y_{11} + y_{12} + \dots]$$

NOTE: For Simpson's $\frac{1}{3}$ rd rule, the number of subintervals must be even.

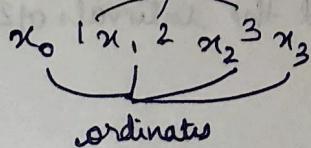
For Simpson's $\frac{3}{8}$ th rule, the number of subintervals must be multiples of 3.

For Middle's rule, the number of subintervals must be multiples of 6.

$$h = \frac{b-a}{n}$$

For trigonometric functions, always keep calculator in radian mode.

(Q) Evaluate $\int_0^{\pi/2} \sin x dx$ by using (1) Simpson's $\frac{1}{3}$ rd rule (2) Simpson's $\frac{3}{8}$ th rule (3) Middle's rule. by taking 7 ordinates



$$h = \frac{b-a}{n} = \frac{\pi/2 - 0}{6} = \frac{\pi/12}{1}$$

x	0	$\frac{\pi}{12}$	$\frac{2\pi}{12}$	$\frac{3\pi}{12}$	$\frac{4\pi}{12}$	$\frac{5\pi}{12}$	$\frac{6\pi}{12} = \frac{\pi}{2}$
$y = \sin x$	0	0.5087	0.7071	0.8408	0.9306	0.9828	1
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$(1) \int_a^b f(x) \cdot dx = \frac{\pi}{36} \left[0 + 1 + 2[0.7011 + 0.9306] + 4[0.5081 + 0.8408 + 0.9828] \right]$$

$$= \frac{\pi}{36} [1 + 3.2154 + 9.3292]$$

$$= \frac{\pi}{36} [13.6046]$$

$$= 1.1866$$

$$(2) \int_a^b f(x) \cdot dx = \frac{3\pi}{36 \times 8} \left[0 + 1 + 2[0.8408] + 3[0.5081 + 0.7011 + 0.9306 + 0.9828] \right]$$

$$\Rightarrow \frac{3\pi}{36 \times 8} [0 + 1 + 1.6816 + 9.3876]$$

$$= \frac{3\pi}{12 \times 8} [12.0692] = 1.1848$$

$$(3) \int_a^b f(x) \cdot dx = \frac{3\pi}{12 \times 10} \left[0 + 5(0.5081) + 0.7011 + 6(0.8408) + 0.9306 + 5(0.9828) + 1 \right]$$

$$= \frac{3\pi}{12 \times 10} \times 15.14 = 1.1849$$

(Q) ① $\int_0^{10} x^3 \cdot dx$ by taking 9 ordinates \rightarrow 8 intervals \rightarrow Simpson's $\frac{1}{3}$ rd rule

② $\int_0^{\pi} \frac{\sin x}{x} \cdot dx$ by taking 12 intervals; ③ $\int_0^1 \frac{dx}{1+x^2}$ by taking 6 intervals and compare with exact result.
 Simpson's $\frac{1}{3}$ rd rule, middle, Simpson's $\frac{3}{8}$ th rule

(Q) $\int_0^6 e^{-x^2} \cdot dx$ by using middle's rule.

(Q) The velocity of a train which starts from rest from a station and reaches the other station in 12 minutes is recorded at the intervals of 2 minutes.

t(min)	2	4	6	8	10
v(kmph)	20	36	60	64	42

Find the distance b/w the stations.

[n26.]

t(min)	0	$\frac{2}{60}$	$\frac{4}{60}$	$\frac{6}{60}$	$\frac{8}{60}$	$\frac{10}{60}$	$\frac{12}{60}$
v(kmph)	0	20	36	60	64	42	0

(A) Apply the rules.

(Q) A curve is described by the points mentioned below by using Simpson's rule, find the area under the curve bounded by the curve and the x-axis included b/w $x=0$ and $x=4$

x	0	0.5	1.0	1.5	2.0	2.5	3	3.5	4
y	4.6	3.8	2.8	2.0	2.5	3.2	5.8	4	4

(A) $n=8 \rightarrow$ Simpson's $\frac{1}{3}$ rd rule