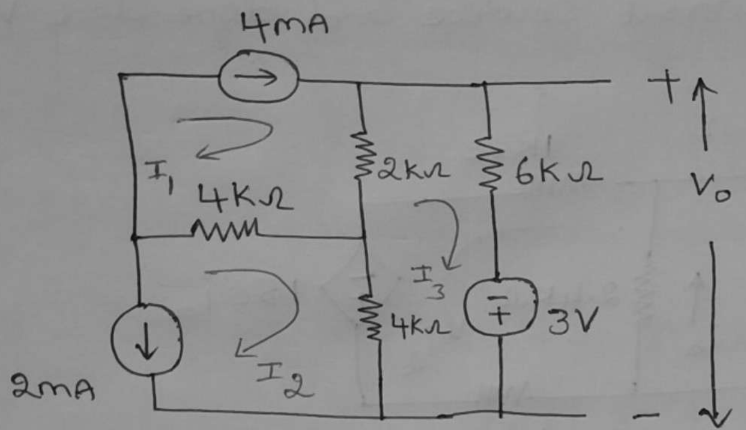
Mesh Analysis with independent current source

Ex: Find the voltage V_o in the circuit shown in figure using mesh analysis method.



solution: $V_o = 6K I_3 - 3$

Apply KVL for the mesh 3, we get

$$2K(I_3 - I_1) + 6K I_3 - 3 + 4K(I_3 - I_2) = 0$$

$$-2K I_1 - 4K I_2 + 12K I_3 = 3 \quad \text{— (1)}$$

From mesh 1, we get

$$I_1 = 4mA \quad \text{— (2)}$$

From mesh 2, we get, $I_2 = -2mA$ — (3)

substitute (2) and (3) in equⁿ (1)

$$-2K(4m) - 4K(-2m) + 12K I_3 = 3$$

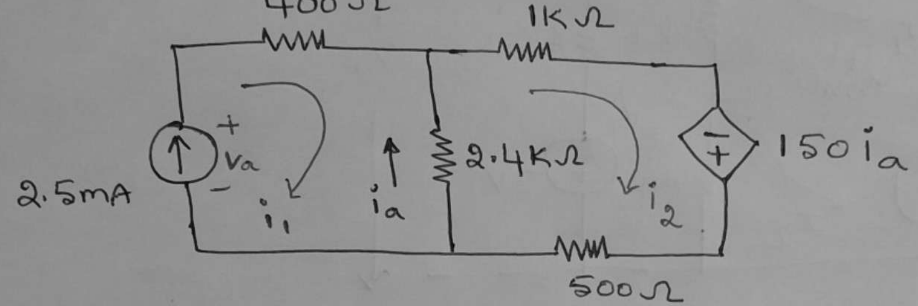
$$I_3 = 0.25mA$$

$$V_o = 6K I_3 - 3 = 6K(0.25m) - 3$$

$$\boxed{V_o = -1.5V}$$

Ex: Mesh analysis using dependent source

using mesh analysis method to solve for i_a in the circuit. Also find the power delivered by the independent current source and dependent voltage source.



Solution: $i_a = i_2 - i_1$ — ①

From mesh 1 ⇒ $\boxed{i_1 = 2.5\,mA}$

Apply KVL to mesh 2,

$$1K\, i_2 - 150 i_a + 500 i_2 + 2.4K\,(i_2 - i_1) = 0 \quad \text{— ②}$$

From equn ① $i_2 = i_a + i_1 = i_a + 2.5mA$ — ③

Sub equn ③ in equn ②

$$1K\,(i_a + 2.5mA) - 150 i_a + 500(i_a + 2.5m) + 2.4K(i_a + 2.5m - 2.5m) = 0$$

$$1K i_a + 2.5 - 150 i_a + 500 i_a + 500 \times 2.5m + 2.4K i_a = 0$$

$$\boxed{i_a = -1.0\,mA}$$

$$\boxed{i_2 = -1m + 2.5m = 1.5mA}$$

Apply KVL to mesh 1

$$400 i_1 + 2.4K\,(i_1 - i_2) - V_a = 0$$

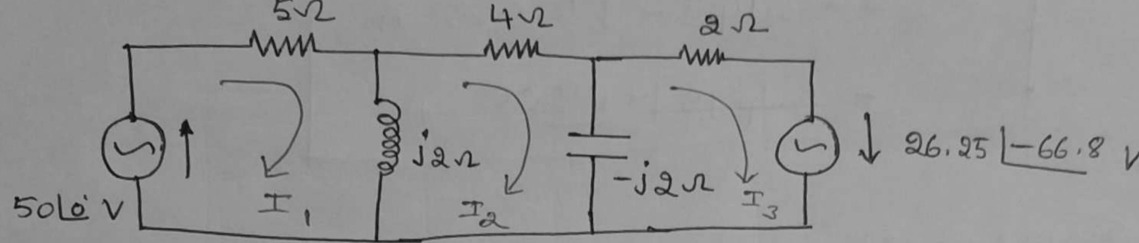
$$\boxed{V_a = 3.4V}$$

$$Power_{independent\ source} = VI = V_a i_1 = (3.4)(2.5m) = \underline{8.5mW}$$

$$Power_{dependent\ source} = VI = 150 i_a i_2 = (150)(-1m)(1.5m) = \underline{-0.225m}$$

Ex: Mesh analysis using AC source.

Find the current through 4 Ω resistor by using loop current method as shown in network.



Solution: Current through 4 Ω resistor is I_2

$$\begin{bmatrix} (5+j2) & -j2 & 0 \\ -j2 & 4 & j2 \\ 0 & j2 & (2-j2) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50\angle 0° \\ 0 \\ 26.25\angle -66.8 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} (5+j2) & -j2 & 0 \\ -j2 & 4 & j2 \\ 0 & j2 & (2-j2) \end{bmatrix}$$

In calculator write $2i$

$$\Delta = (5+j2)\{4(2-j2)+4\} + j2\left(-j2(2-j2)\right)$$

$$\Delta = 76-16j+8-8j$$

$$\Delta = 84-j24$$

$26.25\angle -66.8$
$= 10.34-j24.12$

$$\Delta_2 = \begin{bmatrix} (5+j2) & 50\angle 0° & 0 \\ -j2 & 0 & j2 \\ 0 & 26.25\angle -66.8 & (2-j2) \end{bmatrix}$$

$$\Delta_2 = (5+j2)\{-j2(26.25\angle -66.8)\} - 50\angle 0°\{-j2(2-j2)\}$$

$$\Delta_2 = -199.84-199.88j-200-200j$$

$$\Delta_2 = -399.84-399.88j$$

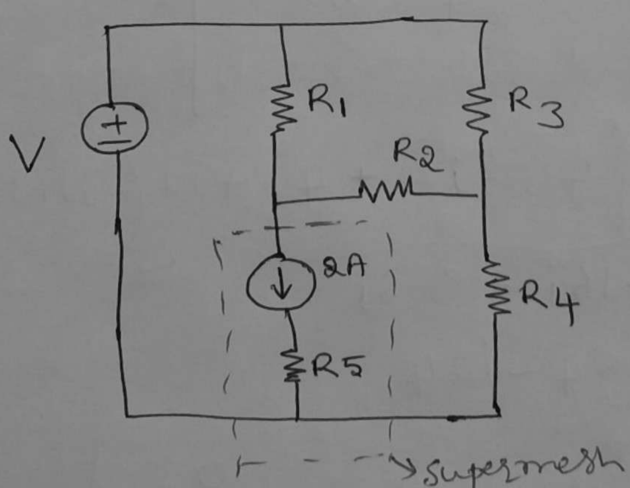
$$\therefore \quad I_2 = \frac{\Delta_2}{\Delta} = \frac{-399.84 - 399.88j}{84 - j24}$$

$$\boxed{\begin{aligned} I_2 &= -3.143 - j5.658 \text{ A} \\ &= 6.472 \angle -119.05 \text{ A} \end{aligned}}$$

Supermesh

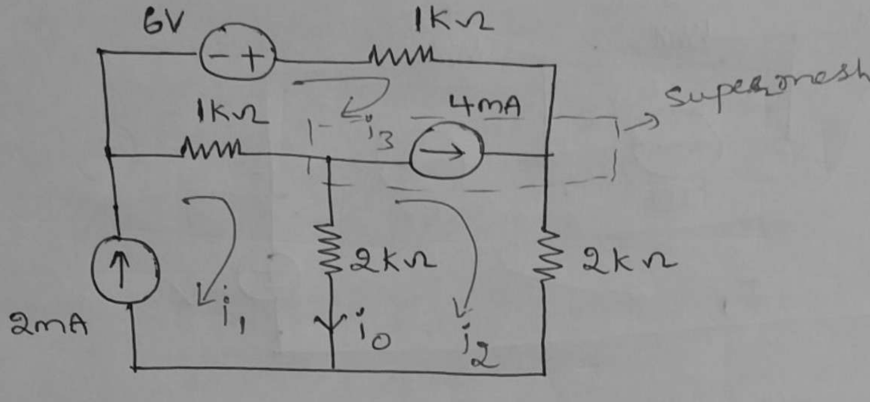
- (*) When a current source is common to two meshes, involves the concept of a supermesh.
- (*) Supermesh is created from two meshes that have a current source as a common element.
- (*) Thus reduce the number of meshes by one for each current source present.

Ex:



Example of supermesh

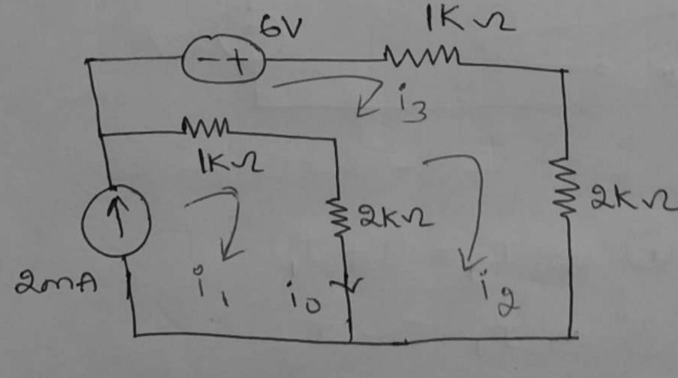
Ex: Find the current i_0 in the circuit shown in Figure using mesh analysis.



* step 1: Specify the mesh currents.
* Step 2: write supermesh constrained eqn by the current source

ie, $i_2 - i_3 = 4mA$ — (1)

* Remove the current source and redraw the circuit.



Apply KVL to mesh 1, $i_1 = 2mA$

Apply KVL to mesh 2 and 3 together,

$$-6 + 1K I_3 + 2K I_2 + 2K(I_2 - I_1) + 1K(I_3 - I_1) = 0$$

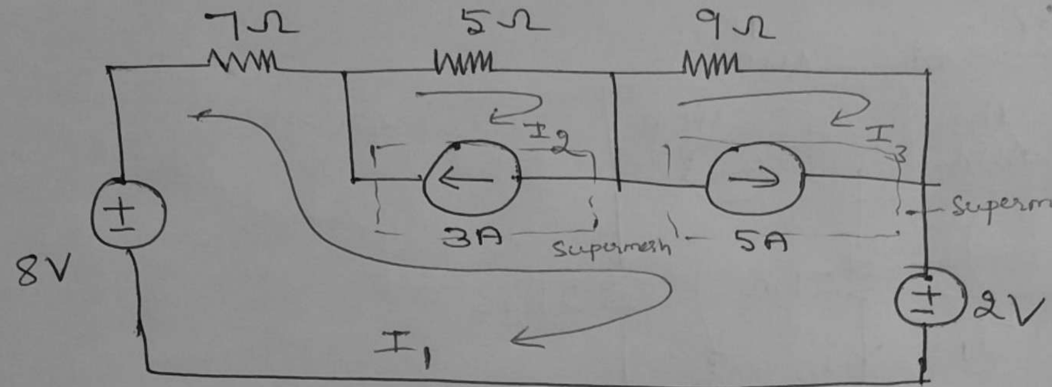
Substituting $i_1 = 2mA$, $i_3 = i_2 - 4m$ → from eqn (1)

solving $i_2 = 10/3\, mA = 3.33mA$

$$i_0 = i_1 - i_2 = 2mA - 3.33mA = -1.33mA$$

$$\boxed{i_0 = -1.33mA}$$

Ex: using mesh analysis, calculate the current I_1 shown in the figure.

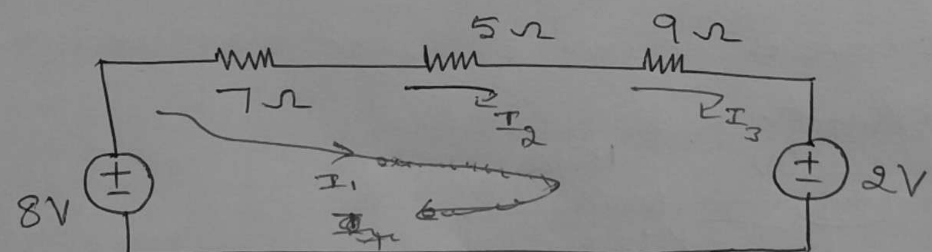


Solution: Supermesh constrained equations.

$$I_2 - I_1 = 3 \quad — ①$$

$$I_1 - I_3 = 5 \quad — ②$$

Remove the current source, redraw the circuit



$$7I_1 + 5I_2 + 9I_3 + 2 - 8 = 0 \quad — ③$$

From equ^n ① $\Rightarrow I_2 = 3 + I_1$

From equ^n ② $\Rightarrow I_1 - 5 = I_3$

Substituting in equ^n ③

$$7I_1 + 5(3 + I_1) + 9(I_1 - 5) = 6$$

$$7I_1 + 15 + 5I_1 + 9I_1 - 45 = 6$$

$$21I_1 = 36$$

$$\boxed{I_1 = \frac{36}{21} = 1.7142A}$$