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RV COLLEGE OF ENGINEERING
Autonomous Institution affiliated to VTU
II Semester B.E. October-2023 Examinations
DEPARTMENT OF MATHEMATICS
VECTOR CLACULUS, LAPLACE TRANSFORM AND NUMERICAL METHODS
(2022 SCHEME)
MODEL QUESTION PAPER
(Branches: EE, EC, EI, ET)

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, and 9 and 10.

PART-A (Objective type for one or two marks)
(True & false and match the following questions are not permitted)

1	1.1	If $\vec{r}$ is the position vector of the point (x, y, z), then curl $\vec{r}$ = ____.	1
	1.2	If ϕ is a harmonic function in cylindrical coordinates, then the Laplacian $\nabla^2\phi =$___________.	1
	1.3	The region of convergence for $L[\cosh 2t] = \frac{s}{s^2-4}$ to hold good is ___________.	1
	1.4	Given Laplace transform of the signal $f(t)$ is $\left[\frac{5s}{(s^2+9)^2}\right]$, then Laplace of $f(3t)$ is ____.	1
	1.5	If $L^{-1}[F(s)] = \sin 2t$, then $L^{-1}\left[\frac{F(s)}{s}\right]$ is _________.	1
	1.6	Find $L^{-1}\left[\frac{5e^{-3s}}{s}\right]$.	1
	1.7	The root of the equation $xlog_{10}(x) = 2$ lies in the interval _______.	1
	1.8	In Newton- Raphson method for finding a real root of an equation $f(x) = 0$, the curve $y = f(x)$ is replaced by _____.	1
	1.9	Find the value of 'c' so that the vector $\vec{f} = (x+y)\hat{\imath} + (y+2)\hat{\jmath} - cz\hat{k}$ is solenoidal.	2
	1.10	Evaluate $\int_c \vec{f}\cdot d\vec{r}$, where $\vec{f} = x^2\hat{\imath} - y\hat{\jmath}$, $\vec{r} = x\hat{\imath} + y\hat{\jmath}$ and c is the straight line $y = x$ passing through the points (1,1) and (3,3).	2
	1.11	If V is the volume bounded by a closed surface S and $\vec{F}$ is a vector point function having continuous partial derivatives, then divergence theorem converts___ to ______.	2
	1.12	$L\{t2^t\}$ = ________.	2
	1.13	Transform the function $\frac{e^{-4s}}{s^2+4}$ into time domain.	2
	1.14	The approximate solution of $\frac{dy}{dx} = 3x + y^2$ with $y(0) = 1$ at $x = 0.1$ using Taylor series up to second degree term is ________.	2

PART-B

		UNIT-I	
2	a	Obtain the directional derivative of $f(x,y,z) = xy^3 + yz^3$ at the point $(2,-1,1)$ in the direction of the vector $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.	4
	b	Show that the field $\overrightarrow{F} = 2xyz^2\hat{\imath} + (x^2z^2 + z\cos yz)\hat{\jmath} + (2x^2yz + y\cos yz)\hat{k}$ is conservative vector field. Hence determine its scalar potential.	6
	c	Compute gradient and Laplacian of the scalar field $\psi(r,\theta,z) = r + z\cos\theta$ in the cylindrical coordinates (r,θ,z).	6

		UNIT-II	
3	a	Verify Green's theorem for $\oint_C \{(2xy - x^2)dx + (x^2 + y^2)dy\}$, where C is the boundary of the region bounded by the parabolas $y = x^2$ and $y^2 = x$ described in positive direction.	8
	b	Employing the divergence theorem, evaluate $\iint_S \vec{F}\cdot\hat{n}\, ds$, where $\vec{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ and S is the surface bounded by the region $x^2+y^2 = 4$, $z = 0$ and $z = 3$.	8
		OR	
4	a	Determine the total work done by a force $\vec{F} = (2y-x^2)\hat{\imath} + 6yz\hat{\jmath} - 8xz^2\hat{k}$ from the point (0,0,0) to the point (1,1,1) along the straight line joining these points.	8
	b	Verify Stokes' theorem for $\vec{F} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$, where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.	8

		UNIT-III	
5	a	Evaluate $L\left[\frac{2sint\ sin2t}{t} + 2^t\right]$.	6
	b	Determine Laplace transform of the triangular wave given by $f(t) = \begin{cases} \frac{1}{a}t, & 0 < t < a \\ \frac{1}{a}(2a - t), & a < t < 2a \end{cases}$ with $f(t) = f(t + 2a)$.	5
	c	Using Laplace transform show that $\int_0^\infty (t\, e^{-t} sin2t\, dt) = \frac{4}{25}$.	5
		OR	
6	a	Obtain the Laplace transform of $f(t) = cosh^2 2t - 3e^{-3t} sin5t$.	6
	b	Evaluate $L\left\{\int_0^t \frac{e^{-t} sin3t}{t} dt\right\}$.	5
	c	Express $f(t) = \begin{cases} sint, & 0 < t \le \pi/2 \\ cost, & t > \pi/2 \end{cases}$ in terms of the unit step function and hence find its Laplace transform.	5

UNIT-IV			
7	a	Using convolution theorem, transform the following function in time domain: $$F(s) = \left[\frac{s}{(s^2+a^2)\ (s^2+b^2)}\right].$$	8
	b	Solve the differential equation $\frac{d^2y}{dt^2} - 3\frac{dy}{dx} + 2y = 1 - e^{2t}$ under the conditions $y(0) = 1,\ y'(0) = 0$ applying Laplace transform technique.	8
OR			
8	a	Determine the inverse Laplace transform of the following: $\frac{s+3}{s^2-4s+13}$ (ii) $\frac{e^{-3s}}{(s^2+1)\ (s^2+9)}$	8
	b	A voltage $E(t) = Ee^{-at}$ is applied at t = 0 to a circuit of inductance L and resistance R satisfying the equation $L\frac{di}{dt} + Ri = E(t)$. Show that the current at any time t is $\frac{E}{R - aL}\left[e^{-at} - e^{-\frac{Rt}{L}}\right]$.	8

UNIT-V			
9	a	Using Newton-Raphson method, find the root of the equation $3x = \sqrt{1 + \sin(x)}$ correct to 3 decimal places choosing the initial guess $x_0 = 0.5$	5
	b	Applying Runge-Kutta method of 4th order, solve the initial value problem $y' = \frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$ at $x = 0.2$.	5
	c	Use Milne predictor-corrector method to find the solution of the differential equation $\frac{dy}{dx} = x^2 - y$ at $x = 0.4$ given that $y(0) = 1, y(0.1) = 0.9051$, $y(0.2) = 0.8212,\ y(0.3) = 0.7491$.	6
OR			
10	a	Find a real root of $xe^x = \cos(x)$ correct to 4 decimal places by using Regula - Falsi method that lies between 0 and 1. Perform four iterations.	5
	b	Apply Taylor series method to obtain $y(0.1)$ considering up to fourth degree term if $y(x)$ satisfies the equation $\frac{dy}{dx} = x - y^2;\ \ y(0) = 1$.	5
	c	Use the Runge-Kutta method of fourth order with $h = 0.1$ to find approximate solution of the initial value problem $\frac{dy}{dx} + 2y = x^3e^{-2x};\ y(0) = 1$ at $x = 0.1$.	6

Signature of Scrutinizer: Signature of Chairman

Name: Name: