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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

I Semester B. E. Supplementary Examinations Aug-2024

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS

(Common to EC, EE, EI, ET)

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of mathematics Handbook is permitted. Do not write anything on handbook.

PART-A

			M	BT	CO
1	1.1	Rank of singular matrix of order 4 is ________.	01	2	1
	1.2	In solving $n \times n$ non-homogeneous system of equations $AX = B$, using Gauss-Jordan method, the coefficient matrix A is reduced to ________.	01	1	1
	1.3	The radius of curvature for straight line $y = mx + c$ is ________.	01	1	1
	1.4	Transform the circle $x^2 + y^2 - 2x = 0$ in polar form.	01	2	2
	1.5	Simpson's three-eight rule is used when number of subintervals is multiple of ________.	01	1	1
	1.6	The value of $\Delta^4[(x-2)(2x-3)(3x-4)]$ is ________.	01	2	2
	1.7	Eigenvalues of the matrix A = $\begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}$ are ________.	02	1	2
	1.8	Find the angle between the radius vector and tangent to the curve $r = ae^{\theta \cot \alpha}$.	02	1	1
	1.9	If $w(x, y) = y^2 \sin(x)$, then at the point $(\pi, 1)$, $\frac{\partial^2 w}{\partial x \partial y} =$ ________.	02	1	1
	1.10	Given $z = xy^2 + x^3 y$ where x and y are functions of t with $x(1) = 1$, $y(1) = 2$, $x'(1) = 3$ and $y'(1) = 4$. The value of $\frac{dz}{dt}$ at $t = 1$ is ________.	02	2	2
	1.11	Evaluate $\int_1^3 \int_1^2 x dx\, dy$.	02	3	2
	1.12	Transform the integral to polar form $\int_0^2 \int_0^{\sqrt{4-x^2}} dydx$.	02	3	3
	1.13	Construct the difference table for the following data points: x: 2, 4, 6, 8 y: 3, 8, 12, 7	02	1	1

PART-B

			M	BT	CO
2	a	Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & 1 \\ 3 & 1 & 0 \\ 1 & 1 & -2 \end{bmatrix}$.	05	2	2
	b	Estimate the values of p and q for which the system of linear equations: $x + y + z = 1$, $2x + y + 4z = 2$, $4x + y + pz = q$ has i) a unique solution ii) no solution iii) an infinite number of solutions.	05	3	2

	c	The eigenvalues given the displacement of an atom or a molecule from its equilibrium position and the direction of displacement is given by eigenvectors. Use the Rayleigh's power method to identify the dominant eigenvalue and corresponding eigenvector of the matrix. $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ with initial vector $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Carry out four iterations.	06	3	3
3	a	Determine the angle of intersection of the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$.	08	2	2
	b	Find the centre of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$.	08	2	2
		OR			
4	a	Obtain the radius of curvature at any point of the cycloid $x = a(t + \sin t),\ y = a(1 - \cos t)$.	08	2	2
	b	Using Maclaurin series expand $e^x \cos x$ in powers of x up to fourth degree terms.	08	3	3
5	a	The two-dimensional Laplace equation in polar coordinates is given by $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. Any function $u(r, \theta)$ satisfying this equation called a harmonic function. Show that $u = e^{a\theta}\cos(a\ \log r)$ is a harmonic function.	08	3	2
	b	Compute $J\left(\frac{u,v,w}{x,y,z}\right)$ when $u = 3x + 2y - z,\ v = x - 2y + z$ and $w = x(x + 2y - z)$. Interpret the result.	08	2	3
		OR			
6	a	If z is a function of x and y and if $x = e^u \sin v,\ y = e^u \cos v$, prove that i) $\frac{\partial z}{\partial u} = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ ii) $\frac{\partial z}{\partial x} = e^{-u}\left(\sin v\frac{\partial z}{\partial u} + \cos v\frac{\partial z}{\partial v}\right)$.	08	3	3
	b	A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point (x, y, z) on the probe's surface is $T(x, y, z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's surface, using Lagrange's method of multipliers.	08	4	4
7	a	Evaluate $\iint_R xy\ dxdy$ where R is the triangular region bounded by the x-axis, y-axis and the line $x + y = 2$.	08	3	2
	b	Determine the centroid of the rectangular lamina bounded by $x = 0, x = 4, y = 0, y = 3$ when the density is xy at the point (x, y).	08	4	4
		OR			
8	a	Change the order of integration and hence evaluate $\int_0^2 \int_{x/2}^{\sqrt{x/2}} (x^2 + y^2)\ dydx$.	08	3	3
	b	Determine the volume of the tetrahedron $x \geq 0, y \geq 0, z \geq 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} \leq 1$.	08	4	4

9 a The following data defines the sea-level concentration of dissolved oxygen for fresh water as a function of temperature:

T (°C)	0	8	16	24	32
O_2 (mg/L)	14.621	11.843	9.870	8.418	7.305

Use appropriate Newton's interpolation formula to calculate the amount of oxygen when temperature is 10 °C and 35 °C. 08 3 3

b For the given data:

x	2	2.5	3	3.5	4	4.5	5
y	1.3863	1.4351	1.4816	1.5260	1.5686	1.6094	1.6486

Compute $\int_2^5 y\,dx$ using:

i) Simpson's one-third rule
ii) Simpson's three-eighth rule and
iii) Weddle's rule. 08 2 2

OR

10 a The following table gives the viscosity of oil as a function of temperature. Use Lagrange's interpolation formula to find viscosity of oil at a temperature of 120° and 140°.

$Temperature$	110	130	160	190
$Viscosity$	10.8	8.1	5.5	4.8

08 2 2

b A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned for various values of the time t (in seconds).

t	0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.12	0.49	1.12	2.02	3.2	4.67

Calculate the angular velocity and the angular acceleration at $t =$ 0.4 and 0.8 using numerical differentiation. 08 3 3