* Photoelectric Effect:

→ If light shines on a metal, there is a freq. called Threshold freq, above which e^- are ejected from the metal

→ Below threshold freq. (ν_0), no e^- are ejected

- Above it, the no of e^- ejected depends on the intensity of light

$$E = h\nu = W_0 + \frac{1}{2}mv^2 \rightarrow KE.$$

W_0 → work function of metal.

↳ energy used to free e^- from the atoms of the metal surface.

$\boxed{\nu_0}$ → the min freq. which can cause photoelectric emission. Below which no emission of e^- takes place.

$$E_0 = h\nu_0 = W_0.$$

Compton Effect: (conclusive evidence of particle nature of em-radiation)

↳ phenomena of $E\downarrow$ ($\lambda\uparrow$) of compton scattered photon.

$\nu\downarrow$

part of energy of photon → transferred to recoiling e^-

Amt by which lights λ changes → compton shift.

Matter Waves: (probability waves) → waves represent the prob. of finding a particle in space.

$\boxed{\lambda = \frac{h}{b} = \frac{h}{p} = \frac{h}{mv}}$ $h = 6.63 \times 10^{-34}$. [de broglie λ]

→ waves associated with moving particles.

→ $m\downarrow \lambda\uparrow$ (lighter the particle) $v\downarrow \lambda\uparrow$ (lesser the vel.)

→ {$p = 0 / v = 0$, $\lambda = \infty$ indeterminate}

→ wave & particle nature are not exhibited simulta.

→ particle – localised mass.

wave – spread out disturbance.

λ of p

$\Rightarrow E = h\nu = \frac{hc}{\lambda}$ (planks theory)

$E = mc^2$ (Einsteins mass-energy relation)

$\Rightarrow \frac{hc}{\lambda} = mc^2 \Rightarrow \lambda = \frac{h}{mc} = \frac{h}{p}$

here, $p \to$ momentum ; $\lambda \to$ wavelength of matter waves.
$h \to$ planks const.

$E = \frac{1}{2} m v^2$ (KE of e^-) {de-broglie λ of an e^-}

$E = \frac{1}{2m} (mv)^2$

$mv = \sqrt{2mE}$

$\lambda = \frac{h}{p} = \frac{h}{mv}$ (de broglie wavelength)

$\lambda = \frac{h}{\sqrt{2mE}}$

$\lambda = \frac{h}{\sqrt{2meV}}$ $[E = eV]$ electrical work done

$\lambda = \frac{12.28}{\sqrt{V}}$ Å

λ_b of a particle with charge 'q' accelerated through a pot. diff V $\rightarrow$ $\lambda = \frac{h}{\sqrt{2mqV}}$

Matter waves	EM waves
(8) (non-em waves).	
→ produced by charged/uncharged particles in motion	→ produced only by a moving charged particle
→ In a isotropic medium, λ of matter waves changes with ~~charg~~ velocity of the particle	→ In an isotropic medium, the λ of em waves remains constant.

→ Phase velocity:

the vel. with which planes of constant phase moves.

(angular freq.) $\omega = 2\pi\nu$

$$v_p = \frac{\omega}{k} = \nu\lambda \qquad k = \frac{2\pi}{\lambda}$$

→ Group velocity:

the vel. with which the envelope of waves (wave packet) formed from the superimposition of two or more progressive waves ~~move~~ with slightly diff. wavelengths.

$$v_g = \frac{\partial\omega}{\partial k}$$

→ Relation b/w v_p & v_g

$$v_p = \frac{\omega}{k} \Rightarrow \omega = v_p k \quad \text{(1)}$$

(from (1))

$$v_g = \frac{\partial\omega}{\partial k} = \frac{\partial(v_p k)}{\partial k} = v_p + k\frac{\partial v_p}{\partial k}$$

$$v_g = v_p + k\frac{\partial v_p}{\partial\lambda}\frac{\partial\lambda}{\partial k} \quad \text{(2)}$$ (multiply & divide by $\partial\lambda$)

$$k = \frac{2\pi}{\lambda}\,;\quad \frac{\partial k}{\partial\lambda} = \frac{-2k}{\lambda^2} \Rightarrow \frac{\partial\lambda}{\partial k} = \frac{-\lambda^2}{2\pi} \quad \text{(3)}$$

subs. (3) in (2).

$$v_g = v_p + \frac{2\pi}{\lambda}\cdot\frac{\partial v_p}{\partial k}\cdot\frac{-\lambda^2}{2\pi}$$

$$\boxed{v_g = v_p - \lambda\frac{\partial v_p}{\partial k}} \rightarrow \text{dispersive medium.}$$

in non-dispersive medium,

$$\frac{\partial v_p}{\partial k} = 0 \Rightarrow v_g = v_p$$

→ **Relation b/w v_g and $v_{particle}$:**

$$E = h\nu = \frac{h\omega}{2\pi} \Rightarrow \omega = \frac{2\pi E}{h}$$

$$p = \frac{h}{\lambda} = \frac{hk}{2\pi} \Rightarrow k = \frac{2\pi p}{h}$$

using plancks theory and de broglie wavelength

— (1)

$$v_g = \frac{\partial \omega}{\partial k} = \frac{\partial E}{\partial p} \quad (\text{using } (1))$$

$$v_g = \frac{\partial (p^2/2m)}{\partial p} = \frac{1}{2m} 2p = \frac{p}{m}$$

$$v_g = \frac{\cancel{m} v_{particle}}{\cancel{m}} \Rightarrow v_g = v_{particle}$$

Relation b/w v_p, v_g, $v_{particle}$:

$$v_p = \frac{\omega}{k} = \frac{E}{p}$$

$$v_p = \frac{\cancel{m}c^2}{\cancel{m} v_{particle}}$$

$$v_p = \frac{c^2}{v_g} \quad (\because v_{particle} = v_g)$$

$$\boxed{v_p \cdot v_g = c^2}$$

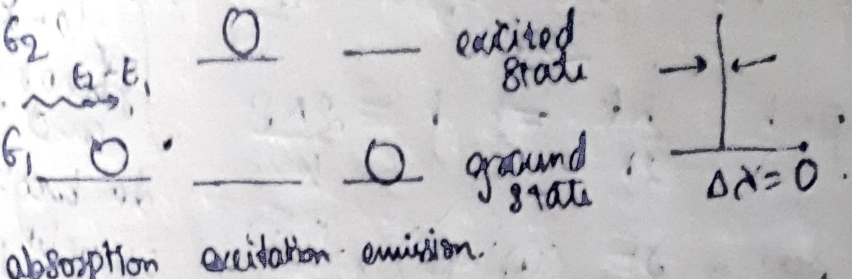
→ v_p can never be greater than or equal to vel. of light (c)

HUP:

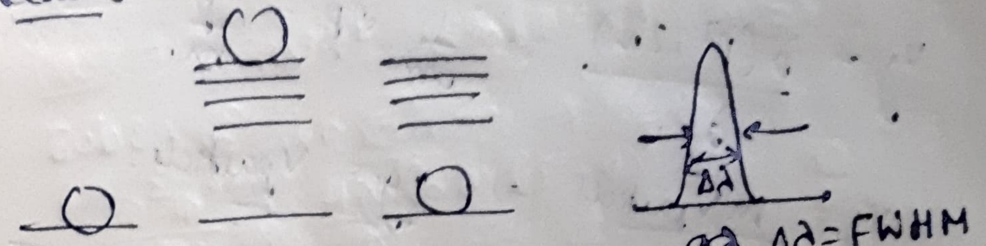
→ states that it is impossible to specify precisely and simultaneously certain pairs of physical quan. like position and momentum that describe the behaviour of an atomic system.

→ $\Delta p \cdot \Delta x \geq \frac{h}{4\pi}$; $\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$; $\Delta J \cdot \Delta \theta \geq \frac{h}{4\pi}$

→ **Ideal:**



Reality:



Broadening of spectral lines:

→ $E = h\nu = \frac{hc}{\lambda}$ (Energy of emitted photon) ①

$\Delta E = \frac{hc}{\lambda^2}\Delta\lambda$

$\Delta E \cdot \Delta t \geq \frac{h}{4\pi} \Rightarrow \Delta E \geq \frac{h}{4\pi\Delta t}$ → ② (from HUP)

↳ uncertainity of energy of emitted photon corresponding to finite lifetime Δt of the excited state.

→ equating ① & ②:

$\frac{hc}{\lambda^2}\Delta\lambda \geq \frac{h}{4\pi\Delta t}$

$\boxed{\Delta\lambda \geq \frac{\lambda^2}{4\pi c\Delta t}} \Rightarrow \boxed{\Delta\lambda \propto \frac{1}{\Delta t}}$

for a finite lifetime of the excited state, emitted photon λ will have a spread of wavelengths around the mean value λ.

SWE: → a partial differential eqn.

→ fundamental eqn for describing quantum mechanical behavior.

TISE: (Time - Independent 1D SWE).

→ let $\psi(x,t)$ be the wavefunction of the matter waves associated with a particle of mass 'm' and velocity 'v'.

→ the DE of the wave motion —

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \rightarrow (1)$$

the soln of the above DE is —

$$\psi(x,t) = \psi_0(x)\, e^{-i\omega t}$$

differentiate the above twice w.r.t t.

$$\frac{\partial\psi}{\partial t} = -i\omega\, \psi_0(x)\, e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2} = i^2\omega^2\, \psi_0(x)\, e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\, \psi(x,t) \rightarrow (2)$$

~~wkt, ω~~ = subs (2) in (1)

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{\omega^2}{v^2}\psi(x,t) \rightarrow (3)$$

wkt, $\frac{\omega^2}{v^2} = k^2 = \left(\frac{2\pi}{\lambda}\right)^2 = \frac{4\pi^2}{\lambda^2} \rightarrow (4)$

subs (4) in (3)

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{4\pi^2}{\lambda^2}\psi$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{4\pi}{\lambda^2}\psi = 0$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{4\pi m^2 v^2}{h^2}\psi = 0 \rightarrow (5)$$

$\left[\lambda = \frac{h}{mv}\right]$ from debroglie wavelength wkt, $TE = KE + PE$

$E = \frac{1}{2}mv^2 + V$

$\frac{mv^2}{2} = E - V$

$\frac{m^2v^2}{2m} = E - V$

$m^2v^2 = 2m(E-V) \rightarrow (6)$

$n^2 = \frac{E8ma^2}{h^2}$

$\frac{10^{-19} \times 10^{-31} \times 10^{-18}}{10^{-34} \times}$

~~32.3024~~

258.41

subs (6) in (5) $\rightarrow$

$\frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi}{h^2}(2m(E-V))\psi = 0$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi m}{h^2}(E-V)\psi = 0}$$

ψ is a function of x alone ~~in~~ independent of time.

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physical significance of wave function

→ $\psi(x,t)$ is the solⁿ of the ~~SWE~~.
  ↳ describes the behavior of a moving particle
→ ψ cannot be measured directly by any physical exp.
→ ψ (probability amp)
  ↳ gives a measure of the probability of finding a particle at a particular pos.
→ has no physical meaning as it is complex and non observable.

$\rightarrow P(x,t) = \psi\psi^* = |\psi(x,t)|^2 = |\psi|^2$

$\downarrow$

probability density

$\leftrightarrow$ ~~measure~~ of probability per unit volume of the particle being at a point.

ψ^* : complex conjugate of the wave funcⁿ

Normalization of wave function $\Rightarrow$

normalization condition $\Leftrightarrow$ $\cancel{\int_x \psi^2 dx = 1}$
(1D)

$\Rightarrow \int_x \psi^2 \, dx = 1$

If ψ is the wave function associated with the particle then $|\psi^2| \, d\tau$ is called the probability of finding the particle in a volume $d\tau$.

$\rightarrow$ If we are certain of definitely locating the particle then as per statistical rule, $\int_0^T |\psi|^2 d\tau = 1$

Properties of wave func.

ψ (if conditions are met) $\rightarrow$ eigen fn

$\rightarrow$ If the wave fn follows all the below

mer

consider a particle of mass 'm' moving in the +ve x-direction from $x=0$ to $x=a$. The potential energy is infinite beyond the boundaries ($V=\infty$) and $V=0$ inside the region (i.e. potential is constant)

Case I: outside the region $V=\infty$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0$$

$\psi = 0$ for all points outside the box.

$\therefore |\psi|^2 = 0 \Rightarrow$ particle cannot be found.

Case II: inside the region $V=0$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}E\psi = 0$$

$$E\psi = \left(\frac{-h^2}{8\pi^2 m}\frac{d^2}{dx^2}\right)\psi$$

$$E\psi = \hat{H}\psi$$

↳ Hamiltonian operator.

$$\frac{8\pi^2 m E}{h^2} = k^2$$

$$\frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0$$

→ Soln: $\psi = A\cos kx + B\sin kx$

Condition 1: at $x=0$, $\psi=0$

$0 = A\cos 0 + B\sin 0$

$A=0$

condition 2: at $x=a$, $\psi=0$

$0 = (A)(\cos ka) + B\sin ka$

$B\sin ka = 0$

$ka = n\pi$

$k = \frac{n\pi}{a}$

$\psi = B\sin\frac{n\pi}{a}x$

→ WKT $k^2 = \frac{8\pi^2 mE}{h^2}$

$\frac{n^2\cancel{\pi^2}}{a^2} = \frac{8\cancel{\pi^2}mE}{h^2}$

$$\boxed{E = \frac{n^2h^2}{8ma^2}}$$

→ $\psi = B\sin\frac{n\pi}{a}x$

probability of finding the particle

$\int_0^a |\psi|^2 dx = 1$

$\int_0^a B^2\sin^2\frac{n\pi}{a}x\,dx = 1$

$\frac{B^2}{2}\int_0^a 1-\cos\frac{2n\pi x}{a}\,dx = 1$

$\frac{B^2}{2}\left[x - \sin\frac{2n\pi x}{a}\cdot\frac{a}{2n\pi}\right]_0^a \to 1$

$\frac{B^2}{2}\left[a - 0 - (0-0)\right] = 1$

$\frac{B^2a}{2} = 1 \Rightarrow B = \sqrt{\frac{2}{a}}$ $\psi = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$