RV College of Engineering®

DEPARTMENT OF MATHEMATICS

MANUAL FOR EXPERIENTIAL LEARNING USING MATLAB

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS (MA211TA)

I - SEMESTER

B.E. Programs: EC, EE, ET

Rashtreeya Sikshana Samithi Trust
RV COLLEGE OF ENGINEERING®
(Autonomous Institution Affiliated to VTU, Belagavi)

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Contents

Introduction

MATLAB (MATrix LABoratory)

MATLAB is a software package for high-performance numerical computation and visualization. It provides an interactive environment with hundreds of built-in functions for technical computation, graphics, and animation. Best of all, it also offers easy extensibility with its own high-level programming language. The name MATLAB stands for MATrix LABoratory. It has powerful built-in routines that enable a very wide variety of computations. It possesses easy-to-use graphics commands that make the visualization of results immediately available. Specific applications are executed using tool boxes, which are a collection of routines that are designed to do common things. There are toolboxes for signal processing, symbolic computation, control theory, simulation, optimization and several other fields of applied science and engineering.

How to access MATLAB R2019a

The next two sections outline instructions on obtaining access to MATLAB. You will first need to associate to the RVCE MATLAB Campus-Wide License and then install MATLAB.
Associate to the Campus-Wide License:

1. Open the RVCE MATLAB Portal or simply scan the QR Code
2. Click on Sign-In to get started.
3. Sign in using your MathWorks Account with your RVCE email address.
4. If you do not have a MathWorks Account with your RVCE email address, click on Create Account.
5. Complete the steps to create your MathWorks Account (If you have any trouble creating an account, contact MathWorks Customer Service via info@mathworks.in or + 91-80-6632-6000).
6. Once logged in on the Portal, you would automatically be associated to the RVCE MATLAB Campus-Wide License.

Installing MATLAB

Detailed installation instructions can be found in the link below:
`https://www.mathworks.com/matlabcentral/answers/98886`
For any installation issues, contact the MathWorks Install Support team at: Info@mathworks.in or call + 91-80-6632-6000

To Download the Software:

1. Click the download button for the current release. (Users can also download previous releases here).
2. Choose a supported platform and download the installer.
3. Run the installer.
4. In the installer, select Log in with a MathWorks Account and follow the online instructions.
5. When prompted to do so, select the Academic Total Headcount license labeled Individual.
6. Select the products you want to download and install.
7. After downloading and installing your products, keep the Activate MATLAB checkbox selected and click Next.
8. Select "Activate automatically using the internet".
9. Log into your MathWorks account
10. Select the Academic Total Headcount license labeled Individual.
11. Click "finish" to complete the activation process.

Will MATLAB run on all computers?

Supported Hardware and Operating Systems

- **Operating System Requirement:** MATLAB R2016a is supported by 64-bit Windows, Mac OS X and Linux operating systems. These include Windows 10, Windows 8.1, Windows 8, Windows 7 SP1, Mac OS X 10.11 (El Capitan), Mac OS X 10.10 (Yosemite), Ubuntu 14.04 LTS through 15.10, Red Hat Enterprise Linux 6 and 7, SUSE Linux Enterprise Desktop 11 and 12, and Debian 7.x, 8.x.
- **Processor requirement:** Any Intel or AMD x86-64 processor
- **RAM requirement:** Minimum 2GB. However 4GB RAM is recommended.
- **Disk Space requirement:** 2GB for MATLAB only. However, a disk space of 4-6 GB is recommended.

How to start MATLAB?

One can enter MATLAB by double-clicking on the MATLAB shortcut icon on Windows desktop. When MATLAB is started, a special window called the MATLAB desktop appears which consists of following sub-windows:
Command Window- This is the main part of the window where commands can be entered. It is indicated by the command prompt (>>).

Current Folder- The files created in MATLAB are saved in current directory. The saved files can be viewed and accessed for further use.

Workspace- The workspace shows all the variables created and/or imported from files and displays type and size of the variables.

Command History- The commands typed in the command window automatically get recorded and stored in command history day wise. These can be retrieved any time for execution. The command history gets cleaned only when the command ‘Clear Command History’ is used.

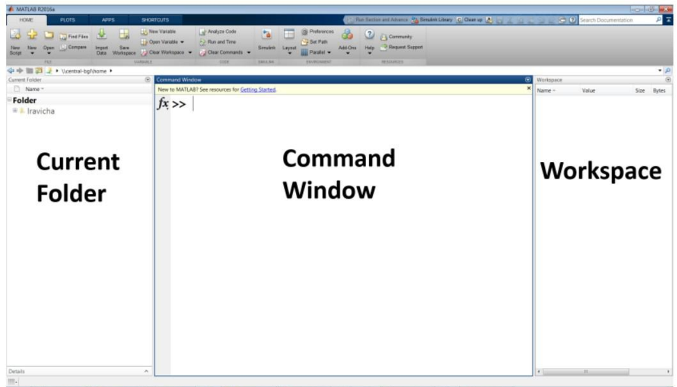


Figure : MATLAB Desktop

How to open and save files?

In the toolbar of MATLAB window, the options ‘HOME’ or ‘EDITOR/LIVE EDITOR’ possess the icon  which is to ‘open’ a file with ‘.m’ or ‘.mlx’ extension. The particular file gets loaded in editor window once it is clicked on. All DOS commands are also applicable. A click on the icon  saves the file.

Symbolic Math Toolbox

Symbolic Math Toolbox provides functions for solving and plotting symbolic math equations. One can create, run and share symbolic math code using the MATLAB Live Editor. The toolbox provides libraries of functions in common mathematical areas such as calculus, linear algebra, algebraic and ordinary differential equations, equation simplification and equation manipulation.

Symbolic Math Toolbox lets analytically perform differentiation, integration, simplification, transforms, and equation solving. Computations can be performed either analytically or using variable precision arithmetic, with the results displayed in mathematical typeset. It is possible to share the symbolic work as live scripts with other MATLAB users or convert them to HTML or PDF for publication. MATLAB functions, Simulink function blocks, and Simscape equations can be generated directly from symbolic expressions.

Modules

1 Arithmetic Operations and Elementary Math Built-In Functions

Topic learning outcomes:

Student will be able to:

1. Understand how to use MATLAB as a calculator to carry out arithmetic operations such as addition, subtraction, multiplication, division and exponentiation.
2. Use format commands to control floating point output display.
3. Execute arithmetic expressions involving various operations.
4. Demonstrate elementary math functions such as algebraic, trigonometric, logarithmic, exponential and complex valued functions using symbolic math.
5. Create and work with arrays.
6. Calculate the value of functions at different points.

Arithmetic Operators and Special Characters

Operations	Symbol	Examples	Operations	Symbol	Examples
Addition	$+$	$4+3=7$	Right division	$/$	$1/2$
Subtraction	$-$	$5-3=2$	Left division	$\backslash$	$4\backslash 8=8/4=2$
Multiplication	$*$	$6*7=42$	Exponentiation	$\hat{}$	$5^3=125$

Some mathematical symbols:

1. pi - π 2. Inf - ∞ 3. i (or j) - $\sqrt(-1)$

Syntax and description:

- `format short`- Fixed-decimal format with 4 digits after the decimal point.
- `format long` - Fixed-decimal format with 15 digits after the decimal point.

Example 1.1. *Calculate* $\frac{5}{4}\cdot 7\cdot 6^2+\frac{3^7}{(9^3-652)}$.

```
(5/4)*7*6^2+(3^7/(9^3-652))
ans =  343.4026
```

Example 1.2. *Evaluate* $(2+3i)(4-i)+\frac{(5-2i)}{(3-7i)}$.

```
(2+3*i)*(4-i)+((5-2*i)/(3-7*i))
ans =  11.5000 +10.5000i
```

Elementary Math Built-In Function

Syntax and description:

```
sin       - Sine
sind      - Sine of argument in degrees
sinh      - Hyperbolic sine
asin      - Inverse sine
asind     - Inverse sine, result in degrees
asinh     - Inverse hyperbolic sine
cos       - Cosine
tan       - Tangent
sec       - Secant
csc       - Cosecant
cot       - Cotangent
exp       - Exponential
log       - Natural logarithm
log10     - Common (base 10) logarithm
sqrt      - Square root
abs        - Absolute value
imag      - Complex imaginary part
real      - Complex real part
```

Note: MATLAB users need not have to remember the syntax. Each MATLAB function has supporting documentation that includes description of function inputs, outputs, calling syntax and examples. For this click the help button  on the toolbar which opens **Help** browser. Enter search terms in **Search Documentation** box.

Variables

Variables are generally denoted symbolically by individual characters (like $'a'$ or $'x'$). Symbolic Math Toolbox introduces a special data type - symbolic objects. This data type includes symbolic numbers, symbolic variables, symbolic expressions and symbolic functions. It also includes vectors, matrices and multidimensional arrays of symbolic numbers, variables, expressions, and functions. Using symbolic objects in computations indicates that MATLAB must perform these computations analytically instead of numerically. Symbolic computations are exact and are not prone to round-off errors.

Syntax and description:

- `syms var1 ... varN` - creates symbolic variables `var1 ... varN`. Separate variables by spaces.

Note: Semicolon (;) at the end of the command suppress the screen output.

Example 1.3. *Assign $a = 2$, $b = 3$ and find addition of a and b, difference of a and b, product of a and b, division of b by a, sine of a, exponential of a, product of tangent of a and b.*

```
a=2;
b=3;
a+b
a-b
a*b
b/a
sin(a)
exp(a)
b*tan(a)
```

```
ans =  5
ans =   -1
ans =     6
ans =    1.5000
ans =    0.9093
ans =    7.3891
ans =   -6.5551
```

Example 1.4. *Find real and imaginary part of $x + iy$.*

```
syms x y real            % defines x, y as symbollic variables
f = x+i*y;
imag(f)
real(f)
```

```
ans =y
ans =x
```

Example 1.5. *Find real and imaginary part of function $\frac{x+iy}{x+2iy} + \sin(x + i\frac{y}{2})$.*

```
syms x y real
g=(x+i*y)/(x+2*i*y)+sin(x+i*y/2);
imag(g)
real(g)
```

```
ans =sinh(y/2)*cos(x) - (x*y)/(x^2 + 4*y^2)
ans =x^2/(x^2 + 4*y^2) + (2*y^2)/(x^2 + 4*y^2) + cosh(y/2)*sin(x)
```

Example 1.6. *Find imaginary and real part of function* $e^{(x+iy)}$.

```
syms x y real
h=exp(x+i*y);
imag(h)
real(h)
```

```
ans =exp(x)*sin(y)
ans =exp(x)*cos(y)
```

Arrays

An important aspect in programming is that of an array (or matrix). This is just an ordered sequence of numbers (known as elements).

Syntax and description:

- `[x1 x2 x3 ....]  or [x1,x2,x3,...]` - returns a row vector, assigning discrete values.
- `[x1;x2;x3 ....]` - returns column vector, assigning discrete values.
- `variable name = [x1:  spacing :  x2]` - generates continuous values with equal spacing.
- `variable name = [x1 :  x2]` - generates continuous values with default spacing 1.
- `variable name = linspace(x1, x2)` - returns a row vector of 100 evenly spaced points between x1 and x2.
- `variable name = linspace(x1, x2, n)` – generates n points. The spacing between the points is (x2-x1)/(n-1).

Array Operations

Array operations work on corresponding elements of arrays with equal dimensions. Each element in the first operand gets matched up with the element in the same location in the second operand. The following table provides a summary of arithmetic array operators in MATLAB.

Arithmetic array operators

Operator	Purpose	Description
$+$	Addition	$A+B$ adds A and B
$+$	Unary plus	$+A$ returns A
$-$	Subtraction	$A-B$ subtracts B from A
$-$	Unary minus	$-A$ negates the elements of A
$.*$	Element-wise multiplication	$A.*B$ is the element-by-element product of A and B
$.\hat{}$	Element-wise power	$A.\hat{}B$ is the matrix with elements $A(i,j)$ to the $B(i,j)$ power
$./$	Right array division	$A./B$ is the matrix with elements $A(i,j)/B(i,j)$
$.'$	Array transpose	$A.'$ is the array transpose of A.

Example 1.7. *Express the following numbers as row and column vectors : 1, 22, -0.4.*

```
[1 22 -0.4]
[1;22;-0.4]
```

```
ans =    1.0000   22.0000   -0.4000
ans =
1.0000
22.0000
-0.4000
```

Example 1.8. *Assign discrete values 5, 2, 1, 6, 7 to a variable x.*

```
x=[5 2 1 6 7]
```

```
x =5     2     1     6     7
```

Example 1.9. *Assign continuous integral values from 1-5 to a variable x.*

```
x=[1:5]
```

```
x =     1     2     3     4     5
```

Example 1.10. *Generate numbers between -3 & 9 with spacing 2.*

```
y=[-3:2:9]
```

```
y =-3    -1     1     3     5     7     9
```

Example 1.11. *Generate 5 values between 0 to 25 with equal divisions.*

```
z=linspace(0,25,5)
```

```
z =0    6.2500   12.5000   18.7500   25.0000
```

Example 1.12. *Find the value of* $\frac{x^2-1}{2x+3}$ *at the points 1, 5, -7, 1/2.*

```
x=[1,5,-7,1/2];           % Assigns the values 1, 5, -7, 1/2 to x as row vector.
y=((x.^2-1)./(2.*x+3))    % Assigns the given expression to a variable 'y'.
```

```
y = 0    1.8462   -4.3636   -0.1875
```

Note: Here '.' is used before the operators to perform element-wise operation.

Exercise:

To solve the following use MATLAB codes.

1. Evaluate the following:

(a) $\frac{3^5}{2^5-1}$.

(b) $3\frac{(\sqrt{7/2}-1)}{(2/3-4)^2} - 2$.

(c) $\frac{1+2i}{2-3i}$.

(d) $(2+7i)(3-5i) + \frac{7}{2}i - \frac{6}{7}i$.

2. Multiply 3 by 8, divide this product by the difference between 13 & 7.
3. Define the variable x and z as $x = 9.6$ and $z = 8.1$, then evaluate $\frac{443z}{2x^3} + \frac{e^{-xz}}{(x+z)}$.
4. . A triangle has sides a = 18cm, b = 35cm and c = 50cm. Define a, b and c as variables and calculate the angle γ by using the law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$.
5. Find the values of $\sin x$ and $\cos x$ for $x = 0(\frac{\pi}{6})2\pi$.
6. Find the distance between the points (2,4,6) and (1,2,3).
7. Calculate the radius 'r' of a sphere that has a volume of 350 inch^3.
8. Find the values of z on the unit sphere in the first octant, corresponding to $x = 0 : 0.1 : 1$ and $y = 0.5$.
9. Find the values of Y on the cone $z = x^2 + y^2$ in the first octant, corresponding to $x = 0 : 0.1 : 1$ and $z = 2$.

2 Elementary Linear Algebra

Topic learning outcomes:

Student will be able to:

1. Create and work with matrices.
2. Find the eigenvalues and eigenvectors of a square matrix.
3. Examine the consistency of system of linear equations.
4. Solve the system of linear equations.

Matrices in MATLAB

A matrix is a two-dimensional array of real or complex numbers. Linear algebra defines many matrix operations that are directly supported by MATLAB. Linear algebra includes matrix arithmetic, linear equations, eigenvalues, eigenvectors and etc.

Introduction to matrices and commands connected to the matrices

Informally, the terms matrix and array are often used interchangeably. More precisely, a matrix is a two-dimensional rectangular array of real or complex numbers that represents a linear transformation. The linear algebraic operations defined on matrices have found applications in a wide variety of technical fields. (The optional Symbolic Math Toolbox extends the capabilities of MATLAB to operations on various types of nonnumeric matrices.) In MATLAB, every variable is treated as a matrix, i.e., if a variable a is stored the value 10, then it is treated as 1×1 matrix.
The following is the list of symbols used to define a matrix:

- Square brackets - mark the beginning and the end of the matrix,
- Commas - separate the values in different columns,
- Semicolons - separate the values of different rows.

The following syntax can be used to define a matrix, where blank spaces are optional (but make the line easier to read) and "$\cdots$" denotes intermediate values: $A = [a11, a12, \cdots, a1n; a21, a22, \cdots, a2n; \cdots; an1, an2, \cdots, ann]$. A simpler syntax is available, which does not require using the comma and semicolon characters. When creating a matrix, the blank space separates the columns while the new line separates the rows, as in the following syntax:
$A = [a11 \quad a12 \quad \cdots \quad a1n$
$a21 \quad a22 \quad \cdots \quad a2n$
$\cdots$
$an1 \quad an2 \quad \cdots \quad ann]$.
This allows lightening considerably the management of matrices. Several MATLAB commands allow creating matrices from a given size, i.e. from a given number of rows and columns.

Matrix operators and element wise operators

$+$	Addition	.*	Element wise multiplication
$-$	Subtraction	./	Element wise right division
$*$	Multiplication	.\	Element wise left division
/	Right division	.∧	Element wise power
\	Left division	.'	Transpose (but not conjugate)
^ or **	power, i.e. x^y	'	Transpose and conjugate

Syntax and description

$size(A)$	returns an order of matrix
$det(A)$	returns the determinant of square matrix A
$inv(A)$	computes the inverse of square matrix A
$rank(A)$	returns the rank of matrix A
$eig(A)$	returns a column vector containing the eigenvalues of square matrix A
$[V, D] = eig(A)$	returns diagonal matrix D of eigenvalues and matrix V whose columns are the corresponding right eigenvectors
$rref(A)$	produces the reduced row echelon form of A
$linsolve(A, B)$	solves the linear system $AX = B$

Example 2.1. *Create two matrices*

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 4 & -1 & 1 \\ 1 & 6 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} -8 & 2 & 1 \\ 4 & -1 & 3 \\ 4 & 7 & 9 \end{bmatrix}$$

and addition of A and B, difference of A and B, product of A and B, determinant of A, inverse of A, rank of A, eigenvalues and eigenvectors of A, row reduced echelon form of A and B.

$A = [1, 2, 1; 4, -1, 1; 1, 6, 9]$
$B = [-8, 2, 1; 4, -1, 3; 4, 7, 9]$
$C = A + B$
$E = A - B$
$F = A * B$
$G = B * A$
$det(A)$
$inv(A)$
$rank(A)$
$eig(A)$
$[V, D] = eig(A)$
$size(A)$
$rref(A)$
$rref(B)$

$A =$
$$\begin{matrix} 1 & 2 & 1 \\ 4 & -1 & 1 \\ 1 & 6 & 9 \end{matrix}$$

$B =$
$$\begin{matrix} -8 & 2 & 1 \\ 4 & -1 & 3 \\ 4 & 7 & 9 \end{matrix}$$

$C =$
$$\begin{matrix} -7 & 4 & 2 \\ 8 & -2 & 4 \\ 5 & 13 & 18 \end{matrix}$$

$D =$
$$\begin{matrix} 9 & 0 & 0 \\ 0 & 0 & -2 \\ -3 & -1 & 0 \end{matrix}$$

$E =$
$$\begin{matrix} 4 & 7 & 16 \\ -32 & 16 & 10 \\ 52 & 59 & 100 \end{matrix}$$

$F =$
$$\begin{matrix} 1 & -12 & 3 \\ 3 & 27 & 30 \\ 41 & 55 & 92 \end{matrix}$$

ans $= -60$

ans $=$
$$\begin{matrix} 0.2500 & 0.2000 & -0.0500 \\ 0.5833 & -0.1333 & -0.0500 \\ -0.4167 & 0.0667 & 0.1500 \end{matrix}$$

ans $= 3$

ans $=$
$$\begin{matrix} -3.0000 \\ 2.0000 \\ 10.0000 \end{matrix}$$

$V =$
$$\begin{matrix} 0.3277 & -0.5774 & 0.1400 \\ -0.8557 & -0.5774 & 0.1400 \\ 0.4005 & 0.5774 & 0.9802 \end{matrix}$$

$D =$
$$\begin{matrix} -3.0000 & 0 & 0 \\ 0 & 2.0000 & 0 \\ 0 & 0 & 10.0000 \end{matrix}$$

ans $=$

3 3

ans =

1	0	0
0	1	0
0	0	1

ans =

1	0	0
0	1	0
0	0	1

Example 2.2. *Check for consistency and obtain the solution if consistent for the following system of equations* $x + y + z = 6, x + 2y + 3z = 10, x + 2y + 5z = 2$.

$A = [1, 1, 1; 1, 2, 3; 1, 2, 5]$
$B = [6; 10; 2]$
$[A, B]$
$rank(A)$
$rank([A, B])$
$linsolve(A, B)$

$A =$

1	1	1
1	2	3
1	2	5

$B =$
6
10
2

ans =

1	1	1	6
1	2	3	10
1	2	5	2

ans = 3
ans = 3
ans =
-2
12
-4

Exercise:

1. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$

2. Find the rank of the matrix $M = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$

3. Find the inverse of the matrix $S = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

4. Verify the following system of equations for consistency and obtain the solution if consistent:

   (a) $x + y - 2z = 5, x - 2y + z = -2, -2x + y + z = 4$

   (b) $x - z = 1, 2x + y + z = 2, y - z = 3, x + y + z = 4, 2y - z = 0$

5. Verify the following system of equations for consistency and obtain the solution if consistent:

   (a) $x - z = 1, 3x + 2y + z = 1, 2x - y + z = 1$

   (b) $x + y + z = 1, x + 3y - 2z = 1, 5x + 7y + 6z = 5$

6. Find the eigenvalues and the eigenvectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

7. Find the eigenvalues and the corresponding eigenvectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

3 Plotting of Standard Cartesian Curves and Polar Curves

Topic learning outcomes:

Student will be able to:

1. Plot two dimensional Cartesian and polar curves.
2. Set the line style, marker symbol, color, label axes with text strings and title the graph with a text string in graphs.
3. Plot multiple curves in one graph.

Cartesian curves

Syntax and description:

- `line([x1 x2],[y1 y2])` - creates a straight line passing through the points `(x1,y1)` and `(x2,y2)`.
- `plot(X,Y)` – creates a 2-D line plot of the data in `Y` versus the corresponding values in `X`.
- `plot(X,Y,LineSpec)` – sets the line style, marker symbol, and color.
- `plot(X1,Y1,...,Xn,Yn)` – plots multiple `X`, `Y` pairs using the same axes for all lines.
- `plot(X1,Y1,LineSpec1,...,Xn,Yn,LineSpecn)` – sets the line style, marker type, and color for each line. You can mix `X`, `Y`, `LineSpec triplets with` `X`, `Y` pairs. For example, `plot(X1,Y1,X2,Y2,LineSpec2,X3,Y3)`.
- `ezplot(f,[min,max])` or `fimplicit(f,[min,max])` - plots `f` over the specified range. If `f` is a univariate expression or function, then `[min,max]` specifies the range for that variable. This is the range along the abscissa (horizontal axis). If `f` is an equation or function of two variables, then `[min,max]` specifies the range for both variables, that is the ranges along both the abscissa and the ordinate.
- `ezplot(f,[xmin,xmax,ymin,ymax])` or `fimplicit(f,[xmin,xmax,ymin,ymax])` - plots `f` over the specified ranges along the abscissa and the ordinate.

Line Style:

‘ - ’ (default solid line) | ‘ – ’ (dashed line) | ‘ : ’ (dotted line) | ‘ -.’ (dashed-dotted line) | ‘ none ’ (no line)

Marker Symbol:

Symbol	Description	Symbol	Description
o	Circle	$+$	Plus sign
$*$	Asterisk	.	Point
x	Cross	$square$ or s	Square
$diamond$ or d	Diamond	ˆ	Upward-pointing triangle
v	Downward-pointing triangle	$>$	Right-pointing triangle
$<$	Left-pointing triangle	$pentagramorp$	Five-pointed star (pentagram)
$hexagramorh$	Six-pointed star (hexagram)	$none$	No markers

Colors and their codes for curve:

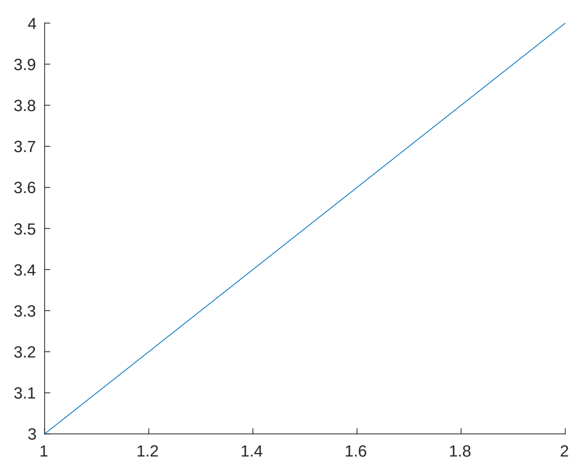
Code	Color	Code	Color
w	White	k	Black
b	Blue	r	Red
c	Cyan	g	Green
m	Magenta	y	Yellow

Commands for labeling along the x-axis and y-axis, adding title and grid lines

- `xlabel('name')` - generate labels along x-axis.
- `ylabel('name')` - generate labels along y-axis.
- `title('name')` - allows to put a title on the graph.
- `grid on` - allow to put the grid lines on the graph.
- `stem(y)` - shade the region between x-axis and the curve.

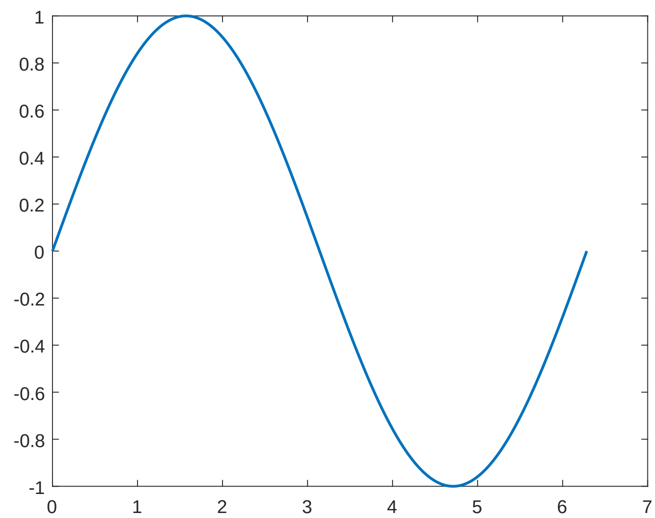
Example 3.1. *Plot the straight passing through the points* $(1, 3)$ *and* $(2, 4)$.

```
lin([1 2],[3 4])
```



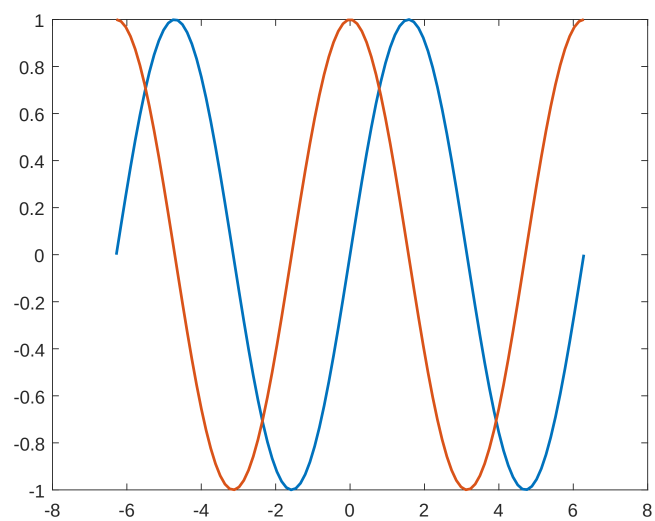
Example 3.2. *Define x as a vector of linearly spaced values between* 0 *and* 2π. *Use an increment of π/100 between the values. Plot* y *as sine values of* x.

```
x = 0:pi/100:2*pi;
y = sin(x);
plot(x,y)
```



Example 3.3. *Define x as 100 linearly spaced values between* -2π *and* 2π*. Assign y1 and y2 as sine and cosine values of x. Create a line-plot of both sets of data.*

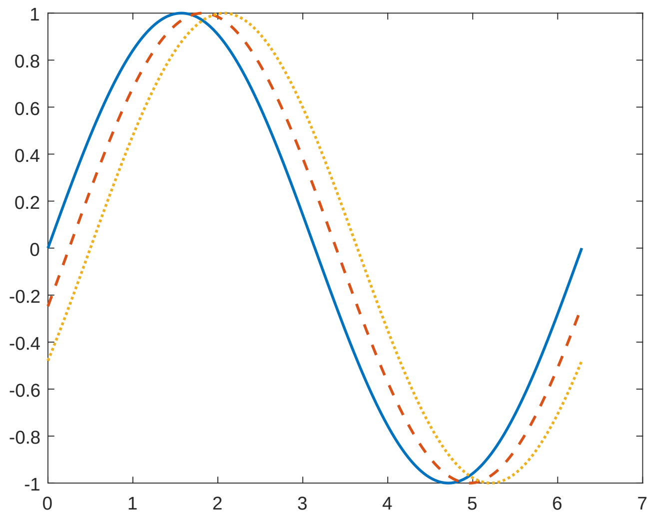
```
x = linspace(-2*pi,2*pi);
y1 = sin(x);
y2 = cos(x);
plot(x,y1,x,y2)
```



Example 3.4. *Plot three sine curves with a small phase shift between each line. Use the default line style for the first line. Specify a dashed line style for the second line and a dotted line style for the third line.*

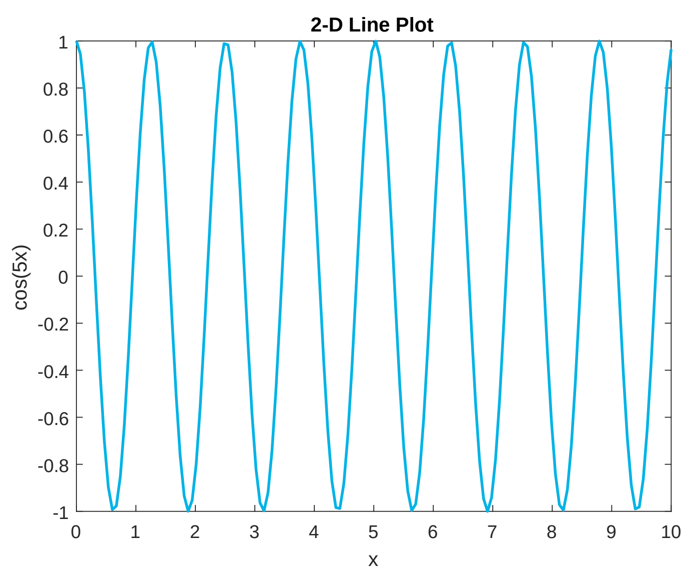
```
x = 0:pi/100:2*pi;
y1 = sin(x);
y2 = sin(x-0.25);
```

```
y3 = sin(x-0.5);
plot(x,y1,x,y2,'--',x,y3,':')
```



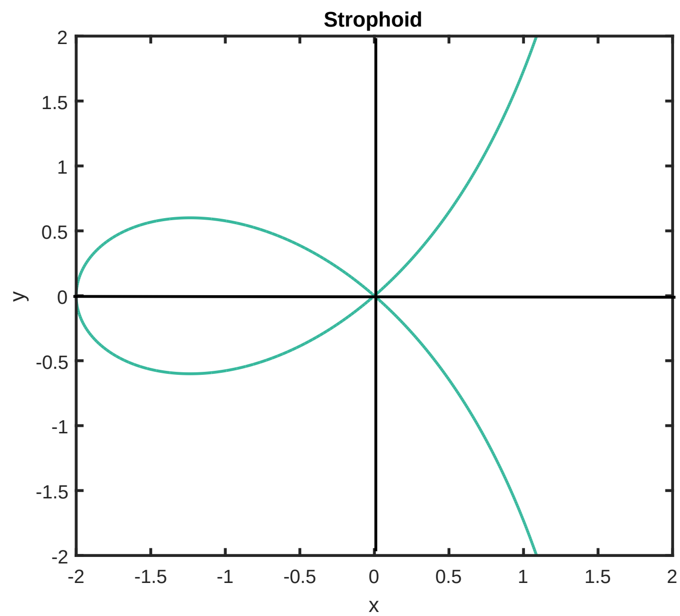
Example 3.5. *Create a 2-D line plot of the cosine curve. Change the line color to a shade of blue-green using an RGB color value. Add a title and axis labels to the graph using the* `title`*,* `xlabel` *and* `ylabel` *commands.*

```
x = linspace(0,10,150);
y = cos(5*x);plot(x,y,'Color',[0,0.7,0.9])
title('2-D Line Plot')
xlabel('x')
ylabel('cos(5x)')
```



Example 3.6. *Plot the graph of strophoid given by* $y^2(2-x) = x^2(2+x)$ *for -2<x<2.*

```
syms x y
fimplicit(y^2*(2-x)-x^2*(2+x),[-2,2])  % To plot implicit functions
title('Strophoid')
xlabel('x')
ylabel('y')
grid on
```



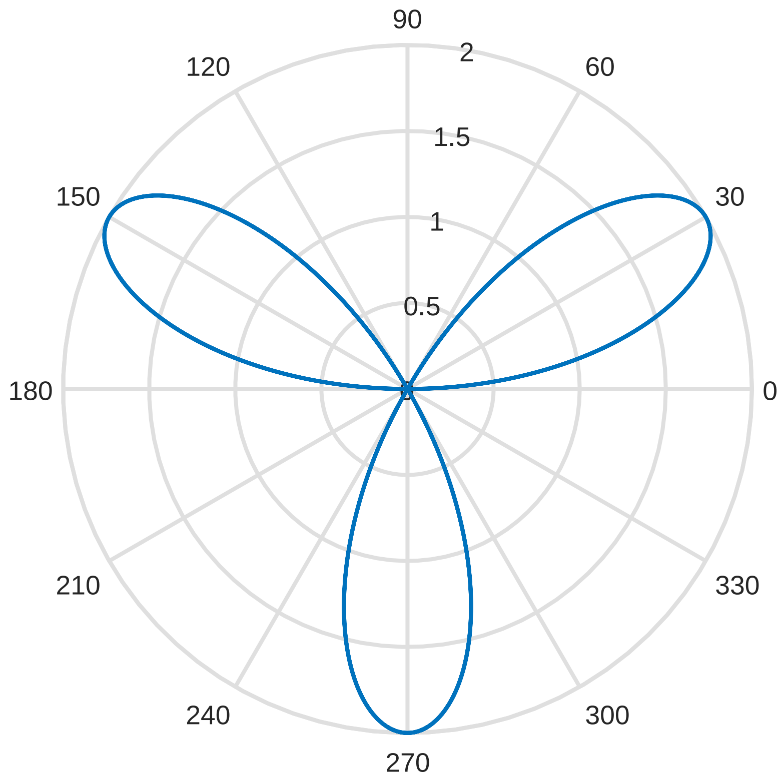
Polar curves

Syntax and description:

- `polarplot(theta,rho)` – plots a line in polar coordinates, with `theta` indicating the angle in radians and `rho` indicating the radius value for each point. The inputs must be vectors with equal length or matrices with equal size. If the inputs are matrices, then `polarplot` plots columns of `rho` versus columns of `theta`. Alternatively, one of the inputs can be a vector and the other a matrix as long as the vector is the same length as one dimension of the matrix.
- `polarplot(theta,rho,LineSpec)` – sets the line style, marker symbol and color for the line.

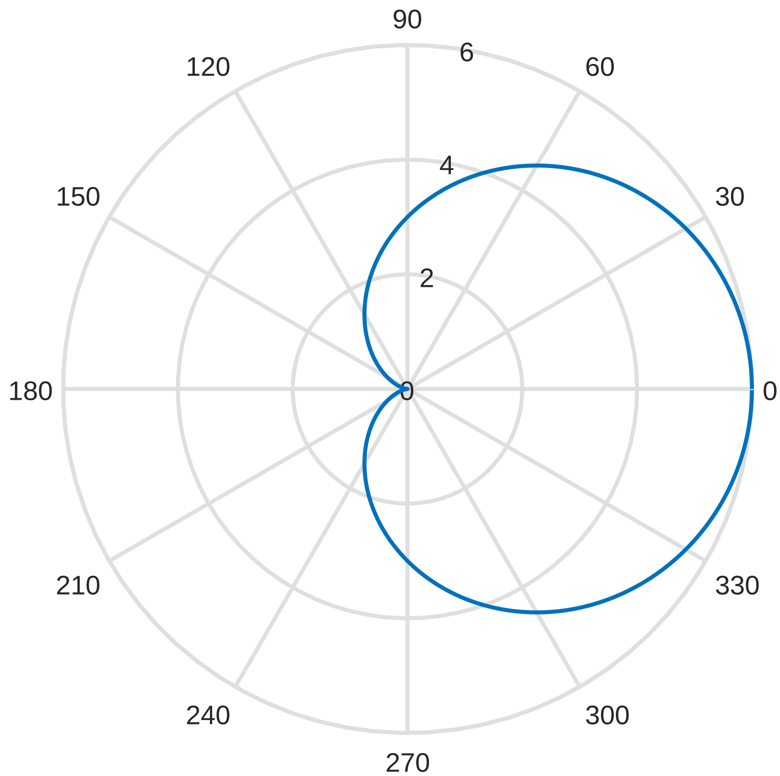
Example 3.7. *Plot the curve $r = 2\sin 3\theta$(three leaved rose).*

```
syms r theta
theta = 0:0.01:2*pi;
r = 2*sin(3*theta);
polarplot(theta,r)
```



Example 3.8. *Plot the graph of Cardiod given by* $r = 3(1 + \cos\theta)$.

```
theta = 0:0.01:2*pi;
r = 3*(1+cos(theta));
polarplot(theta,r)
```



Exercise:

1. Draw the graph of the curve $y = x^3 + 2x + 3$.
2. Generate the plot of the curve $y = e^{-2x}\sin(2x)$.
3. An array speaker emits audio waves with peak amplitude of 3db and transmits in two directions. Plot the curve using MATLAB.
   Hint: Equation of the curve in polar form is: $r = 3\cos(2\theta)$.
4. Plot the graph of the curve $r^2 = 4\cos(2\theta)$.

4 Differentiation

Topic learning outcomes:

Student will be able to:

1. Differentiate symbolic expression or functions of one or several variables with respect to one or more independent variables up to required order.
2. Use this to solve application problems such as obtaining velocity and acceleration from displacement function.

Differentiate symbolic expression or function

Syntax and description:

- `diff(F)` differentiates `F` with respect to the variable determined by `symvar(F,1)`.
- `diff(F,var)` differentiates `F` with respect to the variable `var`.
- `diff(F,n)` computes the n^{th} derivative of F with respect to the variable determined by `symvar`.
- `diff(F,var,n)` computes the n^{th} derivative of F with respect to the variable `var`.
- `diff(F,var1,...varN)` differentiates `F` with respect to the variables `var1,...,varN`.

Example 4.1. *Find the first derivative of* $\frac{\sqrt{(x^2+1)}}{x}$.

```
syms x
y1=diff((x^2+1)^(1/2)/x,x)


y1 =1/(x^2 + 1)^(1/2) - (x^2 + 1)^(1/2)/x^2
```

Example 4.2. *Find second order derivative of* $\frac{1}{(sinx+cosx)}$.

```
syms x
diff(1/(sin(x)+cos(x)),x,2)


ans =1/(cos(x) + sin(x)) + (2*(cos(x) - sin(x))^2)/(cos(x) + sin(x))^3
```

Example 4.3. *If* $f(x) = ye^x + x^2y - log(xy)$, *find* $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial x \partial y}$ *and* $\frac{\partial^2 f}{\partial x^2}$.

```
syms x y
f=y*exp(x)+x^2*y-log(x*y);
diff(f,x)
diff(f,y)
diff(f,x,y)
diff(f,x,x)
```

```
ans =
2*x*y + y*exp(x) - 1/x
ans =
exp(x) + x^2 - 1/y
ans =
2*x + exp(x)
ans =
2*y + y*exp(x) + 1/x^2
```

Example 4.4. *Find $\frac{\partial f}{\partial x}$ at x = 2 , y = 3 if $f(x) = x^2y$.*

```
syms x y
f=x^2*y;
z=diff(f,x);
subs(z,{x,y},{2,3})
```

```
ans =12
```

Example 4.5. *If the motion of a particle is $s = ae^t + be^{-t}$. Show that the acceleration is always equal to displacement s.*

```
syms t a b
diff(a*exp(t)+b*exp(-t),t,2)
```

```
ans =a*exp(t) + b*exp(-t)
```

Therefore the acceleration is always equal to displacement.

Exercise:

1. If s is the distance traversed in meters by a particle in time t sec and $s = 4t^3 - 6t^2 + t - 7$, find the velocity and acceleration when $t = 2$ sec.

2. If $y = \cos^4(\log x) + e^{ax}\sin(bx)$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

3. Show that the function $f = cosx\ coshy$ satisfies 2-dimensional Laplace's equation: $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

4. If $u = e^{a\theta}\cos(a\log r)$, show that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$.

5 Three-Dimensional Plots

Topic learning outcomes:

Student will be able to:

1. Plot 3-D parametric curves.
2. Plot 3-D contours, surfaces, multiple surfaces.
3. Specify the properties: line properties, color data, surface height, axes label, title and marker symbol.
4. Use plots to visualize numerical data.

Syntax and description:

- `fplot3(xt,yt,zt)` - plots the parametric curve `xt` = $x(t)$, `yt` = $y(t)$ and `zt` = $z(t)$ over the default interval $-5 < t < 5$.
- `fplot3(xt,yt,zt,[tmin tmax])` - plots `xt` = $x(t)$, `yt` = $y(t)$ and `zt` = $z(t)$ over the interval `tmin` $< t <$ `tmax`.
- `fplot3(_,LineSpec)` - uses `LineSpec` to set the line style, marker symbol and line color.
- `fplot3(_,Name,Value)` - specifies line properties using one or more `Name,Value` pair arguments. Use this option with any of the input argument combinations in the previous syntaxes. `Name,Value` pair settings apply to all the lines plotted. To set options for individual lines, use the objects returned by `fplot3`.
- `plot3(X1,Y1,Z1,...)`, where `X1`, `Y1` and `Z1` are vectors or matrices - plots one or more lines in three-dimensional space through the points whose coordinates are the elements of `X1`, `Y1` and `Z1`.
- `fsurf(f)` - creates a surface plot of the symbolic expression *f(x,y)* over the default interval [-5 5] for *x* and *y*.
- `fsurf(f, [xmin xmax ymin ymax], LineSpec)` - uses `LineSpec` to set the line style, marker symbol and face color.
- `fsurf(_, Name,Value)` - specifies line properties using one or more `Name,Value` pair arguments. Use this option with any of the input argument combinations in the previous syntaxes.
- `surf(Z)` - creates a three-dimensional shaded surface from the `z` components in matrix `Z`, using *x* = $1:n$ and *y* = $1:m$, where $[m,n]$ = size(`Z`). The height `Z` is a single-valued function defined over a geometrically rectangular grid. `Z` specifies the color data, as well as surface height, so color is proportional to surface height.
- `surf(X,Y,Z)` - uses `Z` for the color data and surface height. `X` and `Y` are vectors or matrices defining the *x* and *y* components of a surface. If `X` and `Y` are vectors, length(`X`) = *n* and length(`Y`) = *m*, where $[m,n]$ = size(`Z`). In this case, the vertices of the surface faces are `(X(j), Y(i), Z(i,j))` triples. To create `X` and `Y` matrices for arbitrary domains, use the `meshgrid` function.
- `surfc(Z)` - creates a contour plot under the three-dimensional shaded surface from the z components in matrix `Z` using *x* = $1:n$ and *y* = $1:m$, where $[m,n]$ = size(`Z`). The height `Z` is a single-valued function defined over a geometrically rectangular grid. `Z` specifies the color data as well as surface height. Therefore color is proportional to surface height.
- `sphere` - generates a sphere consisting of 20-by-20 faces.
- `[X,Y,Z] = sphere(...)` - returns the coordinates of the n-by-n sphere in three matrices that are $(n+1)$-by-$(n+1)$ in size. You draw the sphere with `surf(X,Y,Z)` or `mesh(X,Y,Z)`.
- `[X,Y,Z] = cylinder` - returns the *x*-, *y*- and *z*- coordinates of a cylinder with a radius equal to one. The cylinder has 20 equally spaced points around its circumference.

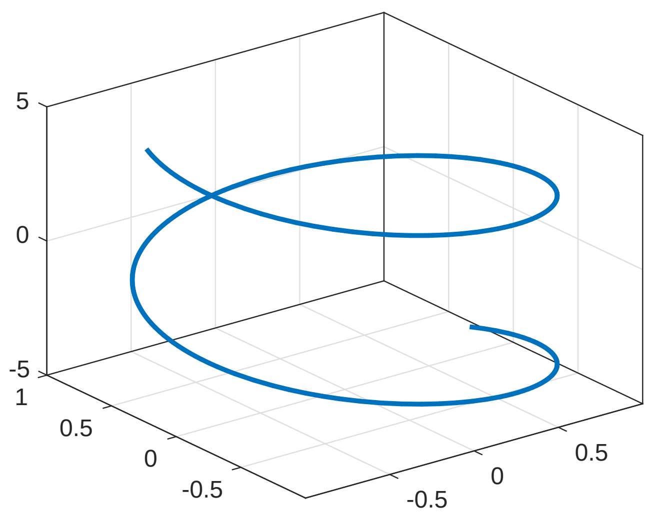
- `[X,Y,Z] = cylinder(r)` - returns the *x*-, *y*- and *z*- coordinates of a cylinder using *r* to define a profile curve. `cylinder` treats each element in *r* as a radius at equally spaced heights along the unit height of the cylinder. The cylinder has 20 equally spaced points around its circumference.
- `[X,Y,Z] = cylinder(r,n)` - returns the *x*-, *y*- and *z*- coordinates of a cylinder based on the profile curve defined by vector *r*. The cylinder has *n* equally spaced points around its circumference.
- `[x,y,z] = ellipsoid(xc,yc,zc,xr,yr,zr,n)` - generates a surface mesh described by three $n+1$-by-$n+1$ matrices, enabling `surf(x,y,z)` to plot an ellipsoid with center (xc, yc, zc) and semi-axis lengths (xr, yr, zr).
- `[x,y,z] = ellipsoid(xc,yc,zc,xr,yr,zr)` - uses $n = 20$.

Note:

`fmesh(f)` also creates a mesh(surface) plot of the symbolic expression $f(x, y)$ over the default interval $[-5, 5]$ for x and y.

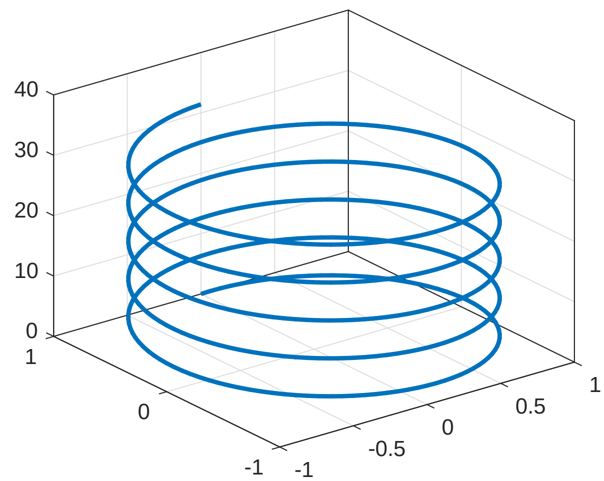
Example 5.1. *Plot the 3-D parametric curve* $x = \sin(t)$*,* $y = \cos(t)$*,* $z = t$ *over the default parameter range* $[-5, 5]$.

```
syms t
xt = sin(t);
yt = cos(t);
zt = t;
fplot3(xt,yt,t)
```



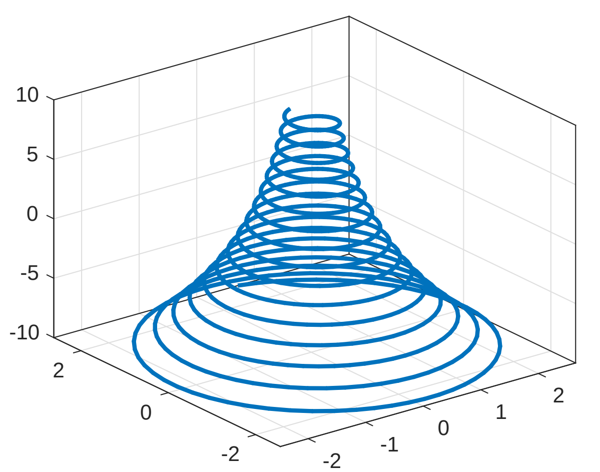
Example 5.2. *Define t as values between 0 and 10π. Define st and ct as vectors of sine and cosine values. Plot a 3-D helix.*

```
t = 0:pi/50:10*pi;
st = sin(t);
ct = cos(t);
figure
plot3(st,ct,t)
```



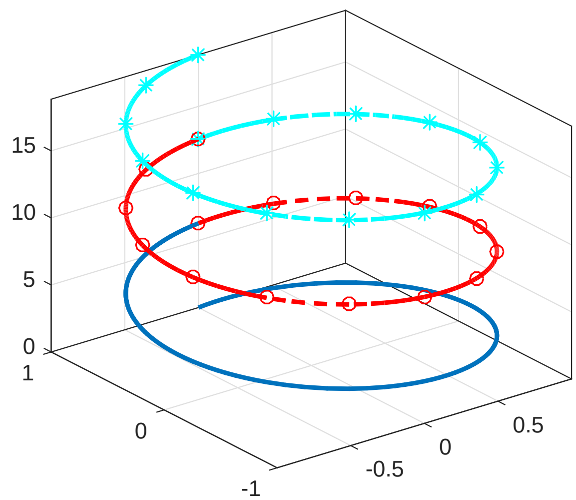
Example 5.3. *Plot the parametric curve $x = e^{-t/10}\sin(5t)$, $y = e^{-t/10}\cos(5t)$, $z = t$ over the parameter range [-10, 10].*

```
syms t
xt = exp(-t/10).*sin(5*t);
yt = exp(-t/10).*cos(5*t);
zt = t;
figure
fplot3(xt,yt,zt,[-10 10])
```



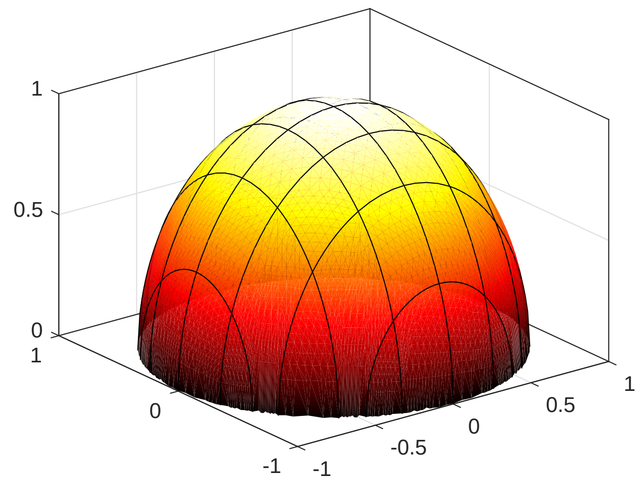
Example 5.4. *Plot the helix given by the parametric equation $x = \sin(t)$, $y = \cos(t)$, $z = t$ three times over different intervals of the parameter. For the first curve, use a* `Linewidth` *of 2. For the second, specify a dashed red line style with circle markers. For the third, specify a cyan, dash-dot line style with asterisk markers.*

```
syms t
figure
fplot3(sin(t), cos(t), t, [0 2*pi], 'LineWidth', 2)
hold on
fplot3(sin(t), cos(t), t, [2*pi 4*pi], '--or')
fplot3(sin(t), cos(t), t, [4*pi 6*pi], '-.*c')
```



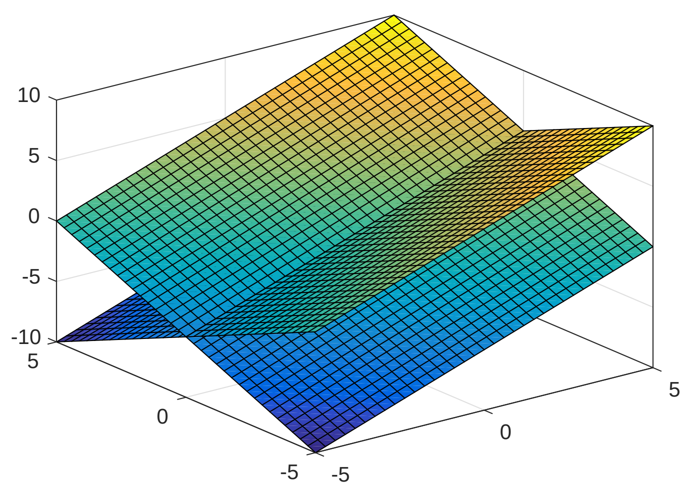
Example 5.5. *Plot the unit sphere $x^2 + y^2 + z^2 = 1$, $x, y, z > 0$ using the colors in preset* `hot` *of colormap.*

```
syms x y
figure
fsurf(sqrt(1-x.^2-y.^2))
colormap hot
```



Example 5.6. *Plot the planes $z = x + y$ and $z = x - y$ in the same graph.*

```
syms x y
figure
fsurf([x+y x-y])
```

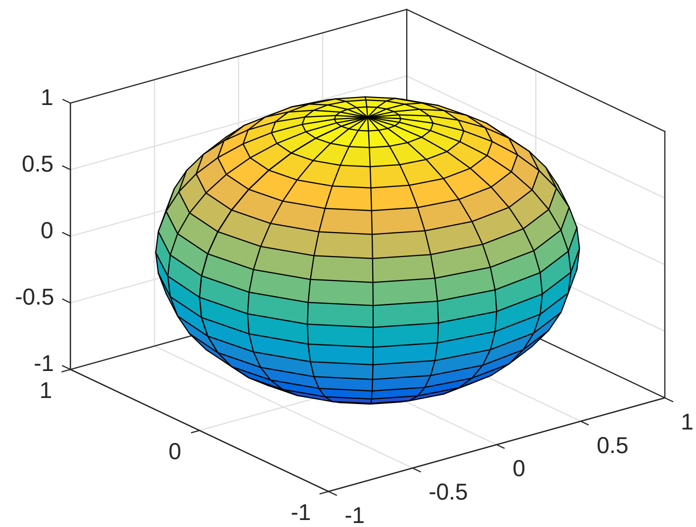


Note:

This can also be plotted using `hold on`.

Example 5.7. *Plot the unit sphere centered at origin.*

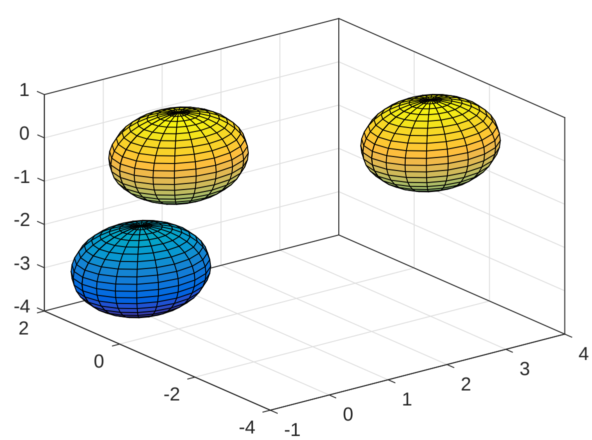
```
figure
sphere
```



Example 5.8. *Define x, y and z as coordinates of the points on the unit sphere centered at origin. Plot a sphere centered at the origin. Plot two more spheres centered at (3,-2,0) and (0,1,-3).*

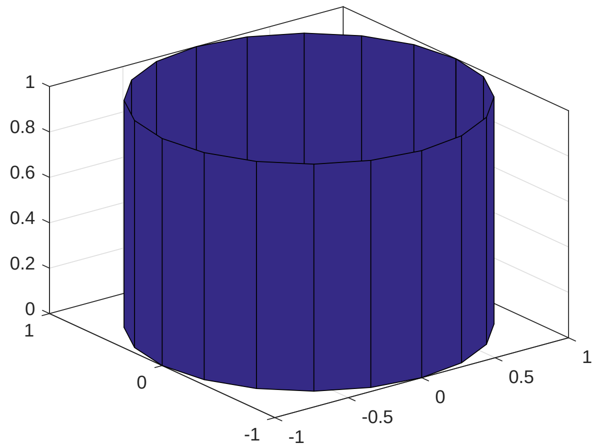
```
[x,y,z] = sphere;
figure
surf(x,y,z)

hold on
surf(x+3,y-2,z) % centered at (3,-2,0)
surf(x,y+1,z-3) % centered at (0,1,-3)
```



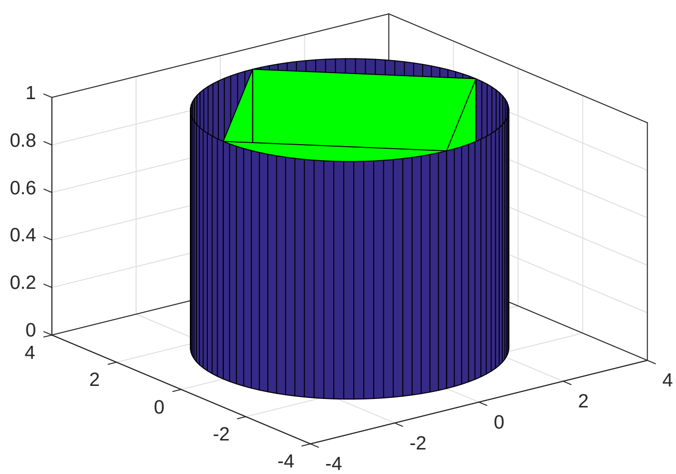
Example 5.9. *Display unit cylinder.*

```
figure
cylinder
```



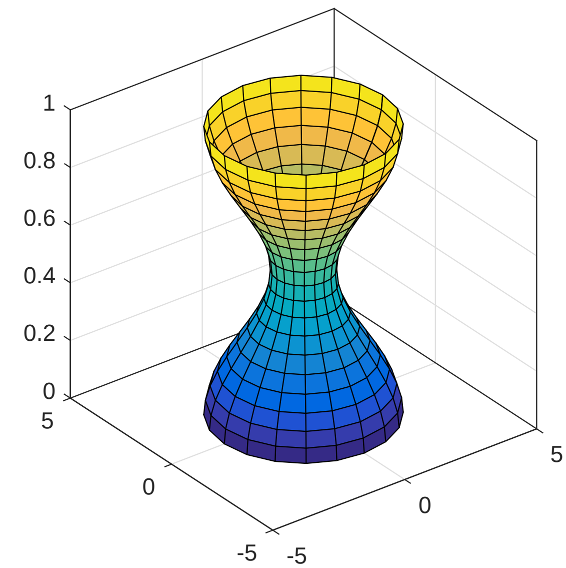
Example 5.10. *Plot a square box inside cylinder of radius 3.*

```
figure
cylinder(3,100)
hold on
[x,y,z] = cylinder(3,4);
surf(x,y,z,'FaceColor','Green')
```



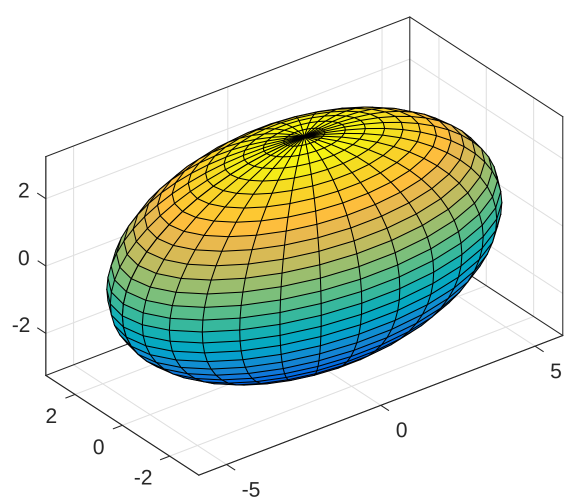
Example 5.11. *Generate a cylinder defined by the profile function* $2 + \cos(t)$.

```
t = 0:pi/10:2*pi;
figure
[X,Y,Z] = cylinder(2+cos(t));
surf(X,Y,Z)
axis square
```



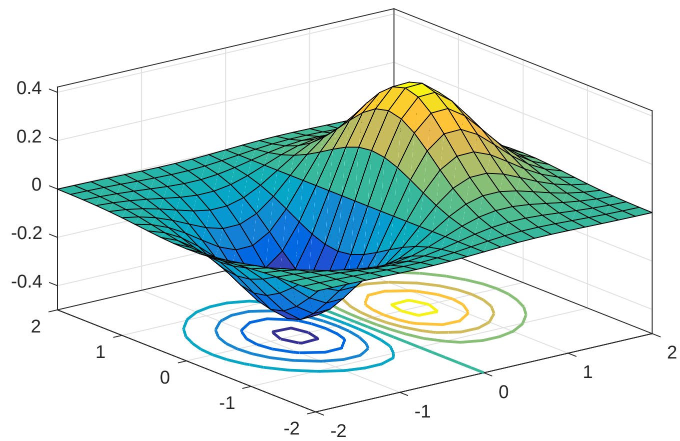
Example 5.12. *Generate data for an ellipsoid with the center at (0,0,0) and semi-axis lengths* $(5.9, 3.25, 3.25)$. *Use* `surf` *to plot the ellipsoid.*

```
[x, y, z] = ellipsoid(0,0,0,5.9,3.25,3.25,30);
figure
surf(x, y, z)
axis equal
```



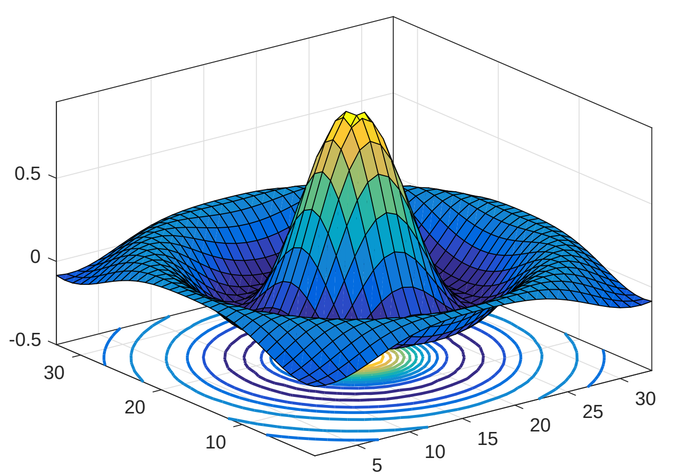
Example 5.13. *Plot the function* $z = xe^{-x^2-y^2}$ *on the domain* $-2 \leq x \leq 2$ *and* $-2 \leq y \leq 2$ *. Use* `meshgrid` *to define* x *and* y*. Then define* z *and create a surface plot.*

```
[X,Y] = meshgrid(-2:0.2:2,-2:0.2:2);
Z = X.*exp(-X.^2 - Y.^2);
figure
surfc(X,Y,Z)
```



Example 5.14. *Create a plot of the sinc function* $z = \frac{\sin(r)}{r}$.

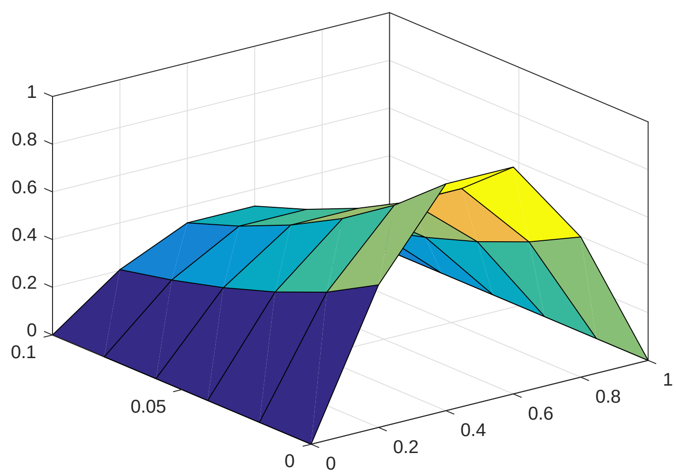
```
[X,Y] = meshgrid(-8:.5:8);
R = sqrt(X.^2 + Y.^2);
Z = sin(R)./R;
figure
surfc(Z)
```



Example 5.15. *Solution of heat conduction equation* $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ *under the conditions* $u(0,t) = u(1,t) = 0$ *and* $u(x,0) = \sin \pi x$, $0 \leq x \leq 1$ *is given in the following table using numerical method. Plot the solution* `u(x,t)`.

	x	0	0.2	0.4	0.6	0.8	1
t		0	1	2	3	4	5
0	0	0	0.5878	0.9511	0.9511	0.5878	0
0.02	1	0	0.4756	0.7695	0.7695	0.4756	0
0.04	2	0	0.3848	0.6225	0.6225	0.3848	0
0.06	3	0	0.3113	0.5036	0.5036	0.3113	0
0.08	4	0	0.2518	0.4074	0.4074	0.2518	0
0.1	5	0	0.2037	0.3296	0.3296	0.2037	0

```
x=0:0.2:1;
y=0:0.02:0.1;
z=[0,.5878,.9511,.9511,.5878,0;0,.4756,.7695,.7695,.4756,0;...
0,.3848,.6225,.6225,.3848,0;0,.3113,.5036,.5036,.3113,0;...
0,.2518,.4074,.4074,.2518,0;0,.2037,.3296,.3296,.2037,0];
figure
surf(x,y,z)
```



Exercise:

1. The position of a moving particle as a function of time is given by $x = (2+4\cos(t))\cos(t)$, $y = (2+4\cos(t))\sin(t)$, $z = t^2$. Plot the position of the particle for $0 \le t \le 20$.

2. Draw the surface $z = \sin^2 x + \sin^2 y;\ |x| \le \pi/2,\ |y| \le \pi/2.$

3. Plot of the function $z = \frac{-5}{1+x^2+y^2}$, $|x| \le 3$, $|y| \le 3$.

4. The transverse displacement *u* of a point at a distance *x* from one end and at any time *t* of a vibrating string satisfies the equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$, with boundary conditions $u = 0$ at $x = 0, t > 0$ and initial conditions $u = x(4-x)$ and $\frac{\partial u}{\partial t} = 0, 0 \le x \le 4$. The transverse displacement u is given for one half period of vibration, taking $h = 1, k = 1/2$ in the following table:

$j \setminus i$	0	1	2	3	4
0	0	3	4	3	0
1	0	2	3	2	0
2	0	0	0	0	0
3	0	-2	-3	-2	0
4	0	-3	-4	-3	0

Plot $u(x, t)$.

6 Integration

Topic learning outcomes:

Student will be able to:

1. Evaluate definite and indefinite integrals.
2. Use this to solve application problems such as finding area.
3. Evaluate double and triple integrals.
4. Use them to determine area of plane figures, volumes of solids, etc.

Syntax and description:

- `int(expr,var)` – computes the indefinite integral of `expr` with respect to the symbolic scalar variable `var`. Specifying the variable `var` is optional. If you do not specify it, `int` uses the default variable determined by `symvar`. If `expr` is a constant, then the default variable is `x`.
- `int(expr,var,a,b)` – computes the definite integral of `expr` with respect to `var` from `a` to `b`. If you do not specify it, `int` uses the default variable determined by `symvar`. If `expr` is a constant, then the default variable is `x`.

Note:`int(expr,var,[a,b])`, `int(expr,var,[a b])`, and `int(expr,var,[a;b])` are equivalent to `int(expr,var,a,b)`.

Example 6.1. *Integrate* $\cos^3 x$ *with respect to x.*

```
syms x
int(cos(x)^3,x)
```

```
ans =sin(x) - sin(x)^3/3
```

Example 6.2. *Find the area bounded by the curve* $y = x^2 + x + 2$*,* x*-axis and the ordinates* $x = 1, x = 2$.

```
syms x y
y=x^2+x+2;
h=int(y,x,1,2)
```

```
h =
35/6
```

Multiple Integrals

Syntax and description:

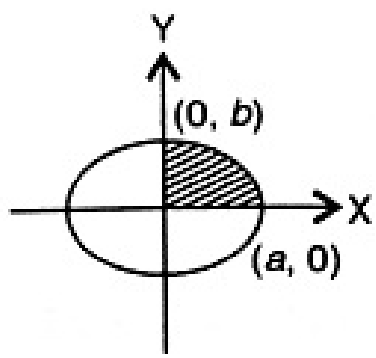
- `int(int(expr, var1,a,b),var2,c,d)` - computes double integral, i.e., integral of `expr` with respect to `var1` from `a` to `b` and then integral of resulting expression with respect to `var2` from `c` to `d`.
- `int(int(int(expr, var1,a,b),var2,c,d),var3,e,f)` - computes triple integral, i.e., integrates the `expr` with respect to `var1` from `a` to `b`, `var2` from `c` to `d` and then `var3` from `e` to `f`.

Example 6.3. *Evaluate* $\int_0^a \int_0^{y^2/a} e^{x/y}\, dx\, dy$.

```
syms x y a
int(int(exp(x/y),x,0,(y^2/a)),y,0,a)
```

```
ans = a^2/2
```

Example 6.4. *Evaluate* $\int\int_D x^3 y\, dx\, dy$ *where* D *is the region bounded by the ellipse* $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ *in the first quadrant.*



Hint: $\int\int_D x^3 y\, dx\, dy = \int_0^a \int_0^{\frac{b\sqrt{a^2-x^2}}{a}} x^3 y\, dy\, dx$.

```
syms x y a b
I = int(int(x^3*y,y,0,(b/a)*sqrt(a^2-x^2)),x,0,a)
```

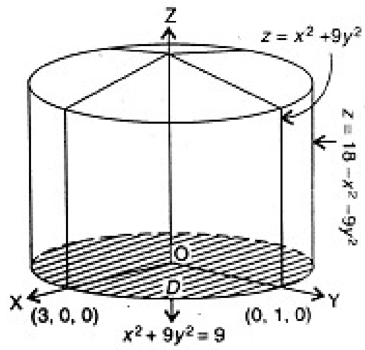
```
I =(a^4*b^2)/24
```

Example 6.5. *Find the area bounded by the curves* $y^2 = x^3$ *and* $x^2 = y^3$.
Hint: *Area*=$\int_0^1 \int_{x^{3/2}}^{x^{2/3}} dy\, dx$.

```
syms x y
I = int(int(1,y,x^(3/2),x^(2/3)),x,0,1)
```

```
I =1/5
```

Example 6.6. *Find the volume bounded by the elliptic paraboloids $z = x^2 + 9y^2$ and $z = 18 - x^2 - 9y^2$.*
Hint: *The two surfaces intersect on the elliptic cylinder $z = x^2 + 9y^2$ and $z = 18 - x^2 - 9y^2$, i.e., $x^2 + 9y^2 = 9$. The projection of this volume onto xy-plane is the plane region D enclosed by ellipse as shown below:*



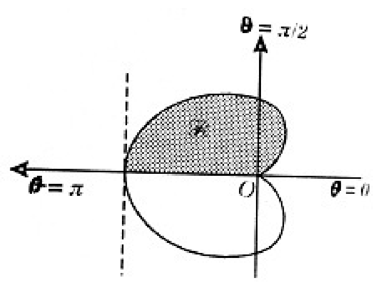
Volume bounded by elliptic paraboloids is $V = \int_{-3}^{3}\int_{-\sqrt{\frac{9-x^2}{9}}}^{\sqrt{\frac{9-x^2}{9}}}\int_{x^2+9y^2}^{18-x^2-9y^2} dz\, dy\, dx.$

```
syms x y z
int(int(int(1,z,x^2+9*y^2,18-x^2-9*y^2),y,-sqrt((9-x^2)/9),...
sqrt((9-x^2)/9)),x,-3,3)
```

```
ans =

27*pi
```

Example 6.7. *Evaluate $\int\int_R r\sin\theta\, dr\, d\theta$, where R is the region bounded by the cardioid $r = a(1-\cos\theta)$ above the initial line.*



Hint: $\int\int_R r\sin\theta\, dr d\,\theta = \int_0^{\pi}\int_0^{a(1-\cos\theta)} r\sin\theta\, dr\, d\theta.$

```
syms r theta a
int(int(r*sin(theta),r,0,a*(1-cos(theta))),theta,0,pi)
```

```
ans =(4*a^2)/3
```

Example 6.8. *Evaluate* $I = \int_0^{\pi/2} \int_0^{a\sin\phi} \int_0^{\frac{(a^2-r^2)}{a}} r\, dz\, dr\, d\phi.$

```
syms r phi z a
int(int(int(r,z,0,(a^2-r^2)/a),r,0,a*sin(phi)),phi,0,(pi/2))
```

```
ans =(5*pi*a^3)/64
```

Exercise:

1. Evaluate $\int \frac{3x+5}{x^2-6x+12} dx$.
2. Evaluate $\int \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x}$.
3. Find the area of the region bounded by $y = 2x - x^2$ and $x-$ axis.
4. Calculate area enclosed by the curve $y = \sin(2x)$, x-axis and the lines $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$.
5. Determine the area bounded by the curves $xy = 2$, $4y = x^2$ and $y = 4$.
6. Find the mass, centroid and moments of inertia relative to x-axis, y-axis and origin of the plane region R having mass density $x + y$ and bounded by the parabola $x = y - y^2$ and the straight line $x + y = 0$.
7. Compute the volume of the ellipsoid of semi axes a, b, c.
8. Find the value of $\int\int\int z\, dx\, dy\, dz$ over the hemisphere $x^2 + y^2 + z^2 \leq a^2, z \geq 0$ by changing to spherical coordinate system.

7 Higher Order Ordinary Differential Equations

Topic learning outcomes:

Student will be able to:

1. Solve linear differential equations of second and higher order with constant coefficients.
2. Solve linear differential equations of second and higher order with variable coefficients.
3. Obtain solution of initial and boundary value problems.

Syntax and description:

- `S = dsolve(eqn)` - solves the ordinary differential equation `eqn`. Here `eqn` is a symbolic equation containing `diff` to indicate derivatives. Alternatively, you can use a string with the letter `D` indicating derivatives.
- `S = dsolve(eqn,cond)` - solves the ordinary differential equation `eqn` with the initial or boundary condition `cond`.

Example 7.1. *Solve* $2x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - y = 0$.

```
syms y(x)
dsolve(2*x^2*diff(y, 2) + 3*x*diff(y) - y == 0)
```

```
ans =

C1/(3*x) + C2*x^(1/2)
```

Example 7.2. *Solve the boundary value problem* $y'' + 6y' + 9y = 2e^{-3x}$, $y(0) = 0$ *and* $y(1) = 1$.

```
syms y(x)
dsolve(diff(y, 2)+6*diff(y) +9*y == 2*exp(-3*x),y(0)==0,y(1)==1)
```

```
ans =

x^2*exp(-3*x) + x*exp(-3*x)*(exp(3) - 1)
```

Example 7.3. *Obtain the solution of the differential equation* $(D^4 - 1)y = \cos 2x \cosh x$.

```
syms y(x)
simplify(dsolve(diff(y, 4)-y == cos(2*x)*cosh(x)))
```

```
ans =

C5*cos(x) - (cos(2*x)*exp(x))/160 + C8*exp(x) - (3*sin(2*x)*exp(x))/160 ...
+ C6*sin(x) - (cos(2*x)*exp(-x))/160 + C7*exp(-x) + (3*sin(2*x)*exp(-x))/160
```

Example 7.4. *A 32 lb weight is suspended from a spring having constant 4 lb/ft. Find position of weight at any time, if a force* $16 \sin 2t$ *is applied and damping force is negligible. Assume that initially the weight is at rest in the equilibrium position.*

Hint: The differential equation describing this phenomenon is

$$\frac{d^2x}{dt^2} + 4x = 16 \sin 2t; x(0) = 0, x'(0) = 0.$$

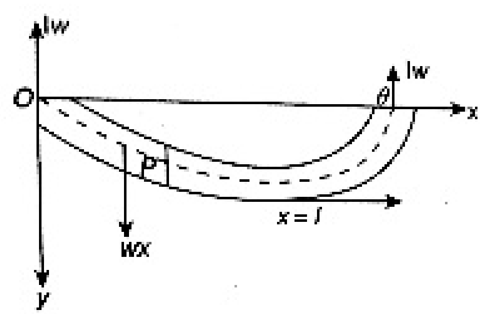
```
syms x(t)
Dx=diff(x);
simplify(dsolve(diff(x, 2)+4*x == 16*sin(2*t),x(0)==0,Dx(0)==0))
```

```
ans =

2*sin(2*t) - 4*t*cos(2*t)
```

Example 7.5. *A horizontal beam of length* $2lft$ *is freely supported at both ends. Find the maximum deflection if the load is* w *per unit length.*

Hint:



Let P be any point (x, y) on the elastic curve. The total weight supported is $2lw$. Hence the upward thrust of the support at each end is lw. Consider the segment OP of the beam. The forces acting in this region are upward thrust lw at zero and load wx acting at the midpoint of OP then the differential equation describing this phenomena is

$$El\frac{d^2y}{dx^2} = \frac{wx^2}{2} - wlx, \text{ under the boundary conditions } y(0) = 0, y(2l) = 0.$$

```
syms y(x) E l w
dsolve(E*l*diff(y, 2) == (w*x^2/2)-w*l*x, y(0) == 0,y(2*l) == 0)
max_deflection=subs(ans,x,l)
```

```
ans =

(x*(8*w*l^3 - 4*w*l*x^2 + w*x^3))/(24*E*l)
max_deflection =

(5*l^3*w)/(24*E)
```

Exercise:

1. Find the transient and steady state solutions of the mechanical system corresponding to the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = \sin 2t - 2\cos 2t$; $x(0) = 0$, $\frac{dx}{dt}(0) = 0$.

2. In the case of a stretched elastic spring which has one end fixed and a particle of mass m attached at the other end, the equation of motion is $m\frac{d^2x}{dt^2} = -\frac{mg}{e}(x - l)$ where l is the natural length of the string and e is the elongation due to weight mg. Find x under the condition that at $t = 0$, $x = a$ and $\frac{dx}{dt} = 0$.

3. The deflection of a strut of length l with one end $(x = 0)$ built-in and the other supported subjected to end thrust p satisfies the equation $\frac{d^2y}{dx^2} + a^2y = \frac{a^2R}{p}(l - x), y = 0, \frac{dy}{dx} = 0$ at $x = 0$. Find the deflection.

4. An electric circuit consists of an inductance of 0.05 Henrys, a resistance of 5 Ohms and a condenser of capacitance 4×10^{-4} Farad and a constant emf of 110 Volts. Governing differential equation is $\frac{d^2Q}{dt^2} + 100\frac{dQ}{dt} + 50000Q = 2200$, under the conditions $Q(0) = I(0) = 0$. Determine charge $Q(t)$.

2 Interpolation

Topic learning outcomes:

Student will be able to:

1. Interpolate from equally and unequally spaced domain points.
2. Extrapolate for evaluating points that lie outside the domain.
3. Use this concept to solve real life and engineering problems.

Syntax and description:

- `yq = interp1(x,y,xq)` - returns interpolated values of a 1-D function at specific query points using linear interpolation. `x` contains the sample points and `y` contains the corresponding values, $y(x)$. `xq` contains the coordinates of the query points.
- `yq = interp1(x,y,xq,method)` - specifies an alternative interpolation method: `'nearest'`, `'next'`, `'previous'`, `'linear'`,`'spline'`,`'pchip'`, or `'cubic'`. The default method is `'linear'`.
- `yq = interp1(x,y,xq,method,extrapolation)` - specifies a strategy for evaluating points that lie outside the domain of `x`. Set `extrapolation` to `'extrap'` when you want to use the `method` algorithm for extrapolation. Alternatively, you can specify a scalar value, in which case, `interp1` returns that value for all points outside the domain of `x`.

Interpolation method, specified as a value from the table below.

Method	Description
'linear'	Linear interpolation.
'nearest'	Nearest neighbor interpolation.
'next'	Next neighbor interpolation.
'previous'	Previous neighbor interpolation.
'pchip'	Shape-preserving piecewise cubic interpolation.
'cubic'	Same as 'pchip'.
'spline'	Spline interpolation using not-a-knot end conditions.

Example 2.1. *Using interpolation process find the value of* $\sin 48^o$.

Hint: Define the sample points x and corresponding sample values v.

```
x = 0:5:90;
v = sind(x);
xq = 48;
vq1 = interp1(x,v,xq)

vq1 =

    0.7425
```

Example 2.2. *The following table gives the distances (in miles) of the visible horizon for the given heights (in feet) above the earth's surface.*

x :	200	250	300	350	400
$y = f(x)$:	15.04	16.81	18.42	19.9	21.27

Find the value of y at $x = 218$ and 389 ft.

```
x = 200:50:400;
v = [15.04 16.81 18.42 19.9 21.27];
xq = [218 389];
vq1 = interp1(x,v,xq)
vq3 = interp1(x,v,xq,'spline')
```

```
vq1 =

   15.6772   20.9686


vq3 =

   15.6977   20.9771
```

Example 2.3. *The area A of a circle for different diameters d is given in the following table:*

d :	80	85	90	95	100
A :	5026	5674	6362	7088	7854

Calculate the area of a circle of diameter 105 units.

```
x=[80    85    90    95    100];
y= [5026    5674    6362    7088    7854];
xq=105;
yq1=interp1(x,y,xq,'pchip')
vq3 = interp1(x,y,xq,'spline')
```

```
yq1 =

   8.6589e+03


vq3 =

        8663
```

Example 2.4. *The following table gives the viscosity of oil as a function of temperature. Find the viscosity of oil at a temperature of* 140^o.

Temp(degree):	110	130	160	190
Viscosity:	10.8	8.1	5.5	4.8

```
x=[110    130    160    190];
y=[10.8    8.1    5.5    4.8];
xq= 140;
yq1 = interp1(x,y,xq,'spline')
```

```
yq1 =

    7.0333
```

Exercise:

1. Estimate the population in 1895 and 1925 from the following statistics:

Year x :	1891	1901	1911	1921	1931
Population y :	46	66	81	93	101

2. Using interpolation find the value of $e^{1.85}$.

3. The pressure p of wind corresponding to velocity v is given in following table. Estimate p when v = 25, 35, 45.

Year v :	10	20	30	40
Population p :	1.1	2	4.4	7.9

4. A rod is rotating in a plane. The following table gives the angle θ (radians) through which the rod has turned for various values of the time t second.

t :	0	0.2	0.6	0.8	1.2
θ :	0	0.12	1.12	2.02	4.67

Calculate the angle θ at t = 0.4, 1, 1.3.

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MISSION

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