vector differentiation

Tut-1

1] 0.

2] $x=1-t^3,\ y=1+t^2,\ z=2t-5$

$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\bar{r}=(1-t^3)i+(1+t^2)j+(2t-5)k$

$\vec{v}=\frac{d\vec{r}}{dt}=-3t^2\hat{i}+2t\hat{j}+2\hat{k}$

at $t=1$, $\vec{v}=-3\hat{i}+2\hat{j}+2\hat{k}$

$|\vec{v}|=\sqrt{-3^2+2^2+2^2}$

$=\sqrt{9+4+4}$

$|\vec{v}|=\sqrt{17}$

3] $T(x,y,z)=x^2+y^2-z$

$\cancel{\frac{d\bar{T}}{dt}}\ \nabla T=\frac{\partial \bar{T}}{\partial x}i+\frac{\partial \bar{T}}{\partial y}j+\frac{\partial \bar{T}}{\partial z}k$

$\nabla T=2x\hat{i}+2y\hat{j}-\hat{k}$

at $(1,1,2)$, $\nabla T=2\hat{i}+2\hat{j}-\hat{k}$

To cool faster, it should go in opp dirn.

$\Rightarrow -2\hat{i}-2\hat{j}+\hat{k}$

4] (i) $x=t^2+1,\ y=4t-3,\ z=2(t^2-3t)$

$\bar{r}=(t^2+1)i+(4t-3)j+2(t^2-3t)k$

$\vec{t}=\frac{d\vec{r}}{dt}=2t\hat{i}+4\hat{j}+2(2t-3)\hat{k}$

at $t=0$, $\vec{t}=0\hat{i}+4\hat{j}-6\hat{k}$

$|\vec{t}|=\sqrt{0^2+4^2+6^2}=\sqrt{52}=2\sqrt{13}$

$\hat{t}=\frac{\vec{t}}{|\vec{t}|}=\frac{4\hat{j}-6\hat{k}}{2\sqrt{13}}=\frac{2\hat{j}-3\hat{k}}{\sqrt{13}}$

(ii) $\vec{r}=(a\cos 3t)\hat{i}+(a\sin 3t)\hat{j}+(4at)\hat{k}$

$\vec{t}=\frac{d\vec{r}}{dt}=(-3a\sin 3t)\hat{i}+(3a\cos 3t)j+(4a)\hat{k}$

at $t=\pi/4$, $\vec{t}=\frac{-3a}{\sqrt{2}}\hat{i}-\frac{3a}{\sqrt{2}}\hat{j}+(4a)\hat{k}$

$|\vec{t}|=\sqrt{\left(\frac{-3a}{\sqrt{2}}\right)^2+\left(\frac{-3a}{\sqrt{2}}\right)^2+(4a)^2}=\frac{50a^2}{2}=5a$

$\hat{t}=\frac{\vec{t}}{|\vec{t}|}=\frac{1}{5\sqrt{2}}(-3\hat{i}-3\hat{j}+4\sqrt{2}\hat{k})$

5] $\vec{r}=2t^2\hat{i}+(t^2-4t)\hat{j}+(3t-5)\hat{k}$

$\vec{v}=\frac{d\vec{r}}{dt}=(4t)\hat{i}+(2t-4)\hat{j}+3\hat{k}$

at $t=1$, $\vec{v}=4\hat{i}-2\hat{j}+3\hat{k}$

$\vec{a}=\frac{d\vec{v}}{dt}=4\hat{i}+2\hat{j}+0\hat{k}$ even at $t=1$.

$\rightarrow c=i-3j+2k$ $\qquad \hat{c}=\frac{\vec{c}}{|\vec{c}|}=\frac{i-3j+2k}{\sqrt{14}}$.

$|\vec{c}|=\sqrt{1^2+-3^2+2^2}$

$=\sqrt{1+9+4}$

$|\vec{c}|=\sqrt{14}$

$\rightarrow \vec{v}\cdot\hat{c}=(4i-2j+3k)\cdot\frac{(i-3j+2k)}{\sqrt{14}}$

$=\frac{4+6+6}{\sqrt{14}}=\frac{16}{\sqrt{14}}$

$\rightarrow \vec{a}\cdot\hat{c}=(4i+2j)\cdot\frac{(i-3j+2k)}{\sqrt{14}}$

$=\frac{4-6}{\sqrt{14}}=\frac{-2}{\sqrt{14}}$

6] $r(t)=(3\cos t)i+(3\sin t)j+t^2k$

$\vec{v}(t)=\frac{d\vec{r}(t)}{dt}=(-3\sin t)\hat{i}+(3\cos t)\hat{j}+2t\hat{k}$

$\vec{a}(t)=\frac{d\vec{v}(t)}{dt}=(-3\cos t)\hat{i}-(3\sin t)\hat{j}+2\hat{k}$

$|\vec{v}|=\sqrt{(-3\sin t)^2+(3\cos t)^2+(2t)^2}$

speed. $=\sqrt{9\sin^2 t+9\cos^2 t+4t^2}$

$|\vec{v}|=\sqrt{9+4t^2}$

7] $\vec{r}=$ ~~e^t i~~ $xi+yj+zk=$

$\vec{r}=e^{-t}i+(2\cos 3t)j+(2\sin 3t)k$

$\vec{v}=\frac{d\vec{r}}{dt}=-e^{-t}i-(6\sin 3t)j+(6\cos 3t)k$

at $t=0$, $\vec{v}=-\hat{i}+0\hat{j}+6\hat{k}$

$|\vec{v}|=\sqrt{-1^2+0^2+6^2}=\sqrt{37}$

$\vec{a}=\frac{d\vec{v}}{dt}=e^{-t}i-(18\cos 3t)j-(18\sin 3t)\hat{k}$

at $t=0$, $\vec{a}=\hat{i}-18\hat{j}+0\hat{k}$

$|\vec{a}|=\sqrt{1^2+(-18)^2+0^2}=\sqrt{325}$

Tut - 2

1] $\phi(x,y,z) = xy^2z^3 - x^3y^2z$

$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$

$\frac{\partial\phi}{\partial x} = y^2z^3 - 3x^2y^2z \xrightarrow{\text{at } (1,-1,1)} 1-3 = -2$

$\frac{\partial\phi}{\partial y} = 2xyz^3 - 2x^3yz \xrightarrow{\text{at } (1,-1,1)} -2+2 = 0$

$\frac{\partial\phi}{\partial z} = 3xy^2z^2 - x^3y^2 \xrightarrow{\text{at } (1,-1,1)} = 3-1 = 2$

$\nabla\phi = -2\hat{i} + 0\hat{j} + 2\hat{k}$

$|\nabla\phi| = \sqrt{-2^2 + 0^2 + 2^2}$

$|\nabla\phi| = 2\sqrt{2}$

2] $\phi = x^2y + yz^2 - xz^3$

$\frac{\partial\phi}{\partial x} = 2xy - z^3$

at $(-1,2,1)$, $\frac{\partial\phi}{\partial x} = -4-1 = -5$

$\frac{\partial\phi}{\partial y} = x^2 + z^2$

at $(-1,2,1)$, $\frac{\partial\phi}{\partial y} = 1+1 = 2$

$\frac{\partial\phi}{\partial z} = 2yz - 3xz^2$

at $(-1,2,1)$, $\frac{\partial\phi}{\partial z} = 4+3 = 7$

$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$

$\nabla\phi = -5i + 2j + 7k$

$|\nabla\phi| = \sqrt{-5^2 + 2^2 + 7^2}$

$|\nabla\phi| = \sqrt{49}$.

↳ max directional derivative.

3] $\phi = x^2 + \sin y + z$

$\frac{\partial\phi}{\partial x} = 2x$, $\frac{\partial\phi}{\partial y} = \cos y$, $\frac{\partial\phi}{\partial z} = 1$.

at $(0, \pi/2, 1)$, $\frac{\partial\phi}{\partial x} = 0$, $\frac{\partial\phi}{\partial y} = 0$, $\frac{\partial\phi}{\partial z} = 1$

$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k = 0i + 0j + 1\hat{k}$

$\Rightarrow \nabla\phi = \hat{k}$

4] $\phi = x^2y + y^2z + z^2x$

$\frac{\partial\phi}{\partial x} = 2xy + z^2$, $\frac{\partial\phi}{\partial y} = x^2 + 2yz$, $\frac{\partial\phi}{\partial z} = y^2 + 2xz$

at $(1,-1,2)$; $\frac{\partial\phi}{\partial x} = -2+4 = 2$, $\frac{\partial\phi}{\partial y} = 1-4 = -3$, $\frac{\partial\phi}{\partial z} = 1+4 = 5$

$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k = 2i - 3j + 5k$

$|\nabla\phi| = \sqrt{6^2 + -3^2 + 5^2} = \sqrt{4 + 9 + 25} = \sqrt{38}$

$\hat{n} = \longrightarrow$ $\hat{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{2i - 3j + 5k}{\sqrt{38}}$

↳ unit normal vector

5] $\phi_1 = 4x^2 - z^3 - 4$

$\frac{\partial\phi_1}{\partial x} = 8x$, $\frac{\partial\phi_1}{\partial y} = 0$, $\frac{\partial\phi_1}{\partial z} = -3z^2$.

at $(1,-1,-2)$, $\frac{\partial\phi_1}{\partial x} = 8$, $\frac{\partial\phi_1}{\partial y} = 0$, $\frac{\partial\phi_1}{\partial z} = -12$

$\nabla\phi_1 = \frac{\partial\phi_1}{\partial x}i + \frac{\partial\phi_1}{\partial y}j + \frac{\partial\phi_1}{\partial z}k = 8i - 12k$

$|\nabla\phi_1| = \sqrt{8^2 + -12^2} = \sqrt{208}$.

$\rightarrow \phi_2 = 5x^2 - 2yz - 7x$

$\frac{\partial\phi_2}{\partial x} = 10x - 7$, $\frac{\partial\phi_2}{\partial y} = -2z$, $\frac{\partial\phi_2}{\partial z} = -2y$

at $(1,-1,-2)$, $\frac{\partial\phi_2}{\partial x} = 3$, $\frac{\partial\phi_2}{\partial y} = 4$, $\frac{\partial\phi_2}{\partial z} = 2$

$\nabla\phi_2 = \frac{\partial\phi_2}{\partial x}i + \frac{\partial\phi_2}{\partial y}j + \frac{\partial\phi_2}{\partial z}k = 3i + 4j + 2k$

$|\nabla\phi_2| = \sqrt{3^2 + 4^2 + 2^2} = \sqrt{29}$

$\hat{n}_1 = \frac{\nabla\phi_1}{|\nabla\phi_1|} = \frac{8i - 12k}{\sqrt{208}}$; $\hat{n}_2 = \frac{\nabla\phi_2}{|\nabla\phi_2|} = \frac{3i + 4j + 2k}{\sqrt{29}}$

$\rightarrow \hat{n}_1 \cdot \hat{n}_2 = \frac{8i - 12k}{\sqrt{208}} \cdot \frac{3i + 4j + 2k}{\sqrt{29}} = \frac{24 - 24}{\sqrt{208}\sqrt{29}} = 0$.

$\Rightarrow \cos\theta = \hat{n}_1 \cdot \hat{n}_2 = 0 \Rightarrow \theta = \pi/2$

thus the surfaces intersect orthogonally.

6] $\phi_1 = 3x^2 - 2y^2 - 3z^2 + 8$

$\frac{\partial\phi_1}{\partial x} = 6x$, $\frac{\partial\phi_1}{\partial y} = -4y$, $\frac{\partial\phi_1}{\partial z} = -6z$

at $(-1,2,1)$, $\frac{\partial\phi_1}{\partial x} = -6$, $\frac{\partial\phi_1}{\partial y} = -8$, $\frac{\partial\phi_1}{\partial z} = -6$

$\nabla\phi_1 = \frac{\partial\phi_1}{\partial x} i + \frac{\partial\phi_1}{\partial y} j + \frac{\partial\phi_1}{\partial z} k = -6i - 8j - 6k$

$\rightarrow \phi_2 = ax^2 + y^2 - bz$

$\frac{\partial\phi_2}{\partial x} = 2ax$, $\frac{\partial\phi_2}{\partial y} = 2y$, $\frac{\partial\phi_2}{\partial z} = -b$

at $(-1,2,1)$, $\frac{\partial\phi_2}{\partial x} = -2a$, $\frac{\partial\phi_2}{\partial y} = 4$, $\frac{\partial\phi_2}{\partial z} = -b$

$\nabla\phi_2 = \frac{\partial\phi_2}{\partial x} i + \frac{\partial\phi_2}{\partial y} j + \frac{\partial\phi_2}{\partial z} k = -2ai + 4j - b\hat{k}$

$\rightarrow$ To intersect orthogonally, $\theta = \pi/2$

$\cos\theta = 0 \Rightarrow \hat{n}_1 \cdot \hat{n}_2 = 0$.

$\Rightarrow \nabla\phi_1 \cdot \nabla\phi_2 = 0$.

$\rightarrow \nabla\phi_1 \cdot \nabla\phi_2 = (-6i - 8j - 6k) \cdot (-2ai + 4j - b) = 0$

$\cancel{12}^{6}a - \cancel{32}^{16} + \cancel{6}^{3}b = 0$

$6a + 3b = 16 \rightarrow (1)$.

$\rightarrow$ ~~put~~ subs $(-1,2,1)$ in $ax^2 + y^2 = bz$.

$a + 4 = b \rightarrow (2)$.

$\rightarrow$ subs (2) in (1).

$6a + 3(a+4) = 16$.

$9a + 12 = 16$

$\underline{a = 4/9}$.

; $b = a + 4$

$= \frac{4}{9} + 4$

$\underline{b = 40/9}$

7] $\phi_1 = ax^2 - bxyz - (a+2)x$

$\frac{\partial\phi_1}{\partial x} = 2ax - byz - (a+2)$

$\frac{\partial\phi_1}{\partial y} = -bxz$, $\frac{\partial\phi_1}{\partial z} = -bxy$

at $(1,-1,2)$, $\frac{\partial\phi_1}{\partial x} = 2a + 2b - a - 2$

$= a + 2b - 2$

$\frac{\partial\phi_1}{\partial y} = -2b$, $\frac{\partial\phi_1}{\partial z} = +b$.

$\nabla\phi_1 = \frac{\partial\phi_1}{\partial x} i + \frac{\partial\phi_1}{\partial y} j + \frac{\partial\phi_1}{\partial z} k$

$\nabla\phi_1 = (a + 2b - 2)i - 2bj + b\hat{k}$

$\rightarrow \phi_2 = 4x^{2y} + z^3 - 4$

$\frac{d}{dx}(a^x) = a^x \log a$

$\frac{\partial\phi_2}{\partial x} = 4 \cdot 2y x^{2y-1} = 8yx^{2y-1}$, $\frac{\partial\phi_2}{\partial y} = 4 \cdot x^{2y} \log x \cdot 2 = 8x^{2y}\log x$

$\frac{\partial\phi_2}{\partial z} = 3z^2$;

at $(1,-1,2)$, $\frac{\partial\phi_2}{\partial x} = -8 \cdot (1)^{-3} = -8$, $\frac{\partial\phi_2}{\partial y} = 0$, $\frac{\partial\phi_2}{\partial z} = 12$

$\nabla\phi_2 = \frac{\partial\phi_2}{\partial x} i + \frac{\partial\phi_2}{\partial y} j + \frac{\partial\phi_2}{\partial z} k = -8i + 12k$

$\rightarrow$ to intersect orthogonally, $\theta = \pi/2$

$\Rightarrow \cos\theta = 0$, $\Rightarrow \hat{n}_1 \cdot \hat{n}_2 = 0$.

$\Rightarrow \nabla\phi_1 \cdot \nabla\phi_2 = 0$

$\rightarrow ((a + 2b - 2)i - 2bj + bk) \cdot (-8i + 12k) = 0$

$-8a - 16b + 16 + 12b = 0$.

$8a + 4b = 16 \Rightarrow 2a + b = 4 \rightarrow (1)$

$\rightarrow$ subs $(1,-1,2)$ in $ax^2 - bxyz - (a+2)x$

$\cancel{a} + 2b - \cancel{a} - 2 = 0$

$\underline{b = 1} \rightarrow (2)$

$\rightarrow$ subs $b = 1$ in (1):

$2a + b = 4 \Rightarrow \underline{a = 3/2}$

8] $\phi = xyz - xy^2z^3$.

$\frac{\partial\phi}{\partial x} = yz - y^2z^3$, $\frac{\partial\phi}{\partial y} = xz - 2xyz^3$

$\frac{\partial\phi}{\partial z} = xy - 3xy^2z^2$

at $(1,2,-1)$,

$\frac{\partial\phi}{\partial x} = -2 + 4 = 2$, $\frac{\partial\phi}{\partial y} = -1 + 4 = +3$, $\frac{\partial\phi}{\partial z} = 2 - 12 = -10$.

$\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k = 2i + 3j - 10k$

$\rightarrow \vec{d} = i - j - 3k$, $|\vec{d}| = \sqrt{1^2 + -1^2 + -3^2}$

$= \sqrt{1 + 1 + 9}$

$= \sqrt{11}$

$\hat{d} = \frac{\vec{d}}{|\vec{d}|} = \frac{i - j - 3k}{\sqrt{11}}$

$\rightarrow \nabla\phi \cdot \hat{d} = (2i + 3j - 10k)\frac{(i - j - 3k)}{\sqrt{11}}$

directional derivative

$\nabla\phi \cdot \hat{d} = \frac{2 - 3 + 30}{\sqrt{11}} = \frac{29}{\sqrt{11}}$ //

$\textcircled{4}$

9] $\phi = x^2y^2z^2$;

~~$\vec{r} = xi + yj + zk$~~

~~$\vec{t} = \frac{d\vec{r}}{dt} = e^t i + (\sin 2t + 1)j + (1-\cos t)k$~~

~~at $t=0$, $\vec{t} = i + \partial j + 0k$~~

~~$|\vec{r}| = \sqrt{1+1^2} = \sqrt{2}$~~

~~$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{i+j}{\sqrt{2}}$.~~

$\frac{\partial\phi}{\partial x} = 2xy^2z^2$, $\frac{\partial\phi}{\partial y} = 2yx^2z^2$; $\frac{\partial\phi}{\partial z} = 2zx^2y^2$

at $(1,-1,1)$, $\frac{\partial\phi}{\partial x} = 2$, $\frac{\partial\phi}{\partial y} = -2$, $\frac{\partial\phi}{\partial z} = 2$

$\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k = 2i - 2j + 2k$

~~$\rightarrow \nabla\phi \cdot \hat{r} = (2i-2j+2k) \cdot \frac{(i+j)}{\sqrt{2}} = 2-2$~~;

$\rightarrow \vec{r} = e^t i + (\sin 2t + 1)j + (1-\cos t)k$.

$\vec{t} = \frac{d\vec{r}}{dt} = e^t i + (2\cos 2t)j + (\sin t)k$

at $t=0$, $\vec{t} = i + 2j + 0k$; $\hat{t} = \frac{\vec{t}}{|\vec{t}|} = \frac{i+2j}{\sqrt{5}}$

$|\vec{t}| = \sqrt{1+2^2} = \sqrt{5}$.

$\rightarrow \nabla\phi \cdot \hat{t}$

$= (2i - 2j + 2k) \cdot \frac{(i+2j)}{\sqrt{5}}$

$= \frac{2-4}{\sqrt{5}} \Rightarrow \nabla\phi \cdot \hat{t} = \frac{-2}{\sqrt{5}}$

directional derivative of $x^2y^2z^2$

Tut-3.

1] $\phi = 3x^2y - y^3z^2$.

$\frac{\partial\phi}{\partial x} = 6xy$, $\frac{\partial\phi}{\partial y} = 3x^2 - 3y^2z^2$, $\frac{\partial\phi}{\partial z} = 2y^3z$

at $(1,-2,-1)$, $\frac{\partial\phi}{\partial x} = -12$, $\frac{\partial\phi}{\partial y} = 3-12 = -9$, $\frac{\partial\phi}{\partial z} = -16$

(gradϕ)

$\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k = -12\hat{i} - 9\hat{j} - 16\hat{k}$

2] $f = \tan^{-1}(y/x)$.

$\frac{\partial f}{\partial x} = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{-y}{x^2} = \frac{-y}{x^2+y^2}$

$\frac{\partial f}{\partial y} = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x} = \frac{x}{x^2+y^2}$

$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j = \frac{-y}{x^2+y^2} i + \frac{x}{x^2+y^2} j$

(grad f)

$\text{div}(\nabla f) = \frac{\partial}{\partial x}\left(\frac{-y}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)$

$= -\frac{(0(x^2+y^2) - y(2x))}{(x^2+y^2)^2} + \frac{(0(x^2+y^2) - x(2y))}{(x^2+y^2)^2}$

$= \frac{-(-2xy) + (-2xy)}{(x^2+y^2)^2} = 0$.

$\Rightarrow \text{div}(\nabla f) = 0$.

$\text{div}(\text{grad } f) = \underline{0}$.

1] $\bar{f} = 3x^2 i + 5xy^2 j + xyz^3 k$.

$\text{div}\,\bar{f} = \frac{\partial}{\partial x}(3x^2) + \frac{\partial}{\partial y}(5xy^2) + \frac{\partial}{\partial z}(xyz^3)$

$= 6x + 10xy + 3xyz^2$

at $(1,2,3)$, $\text{div}\,\bar{f} = 6 + 20 + 54 = \underline{80}$

2] $\bar{f} = (y^2+z^2-x^2)i + (z^2+x^2-y^2)j + (x^2+y^2-z^2)k$

$[\bar{f} = f_1 i + f_2 j + f_3 k]$.

$\frac{\partial}{\partial x}(f_1) = \frac{\partial}{\partial x}(y^2+z^2-x^2) = -2x$

$\frac{\partial}{\partial y}(f_2) = \frac{\partial}{\partial y}(z^2+x^2-y^2) = -2y$

$\frac{\partial}{\partial z}(f_3) = \frac{\partial}{\partial y}(x^2+y^2-z^2) = -2z$.

$\rightarrow \text{div}\,\bar{f} = \frac{\partial}{\partial x}f_1 + \frac{\partial}{\partial y}f_2 + \frac{\partial}{\partial z}f_3 = -2x-2y-2z$

$= -2(x+y+z)$

$\rightarrow \text{curl}\,\bar{f} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2+z^2-x^2 & z^2+x^2-y^2 & x^2+y^2-z^2 \end{vmatrix}$

$= \left[\frac{\partial}{\partial y}(x^2+y^2-z^2) - \frac{\partial}{\partial z}(z^2+x^2-y^2)\right]i - \left[\frac{\partial}{\partial x}(x^2+y^2-z^2) - \frac{\partial}{\partial z}(y^2+z^2-x^2)\right]j + \left[\frac{\partial}{\partial x}(z^2+x^2-y^2) - \frac{\partial}{\partial y}(y^2+z^2-x^2)\right]k$

$= (2y-2z)\hat{i} - (2x-2z)\hat{j} + (2x-2y)\hat{k}$

$= 2((y-z)\hat{i} + (z-x)\hat{j} + (x-y)\hat{k})$

3] $\bar{f} = \underset{f_1}{(x+3y)}i + \underset{f_2}{(y-3z)}j + \underset{f_3}{(x-2z)}k$

$\frac{\partial f_1}{\partial x} = \frac{\partial}{\partial x}(x+3y) = 1$

$\frac{\partial f_2}{\partial y} = \frac{\partial}{\partial y}(y-3z) = 1$; $\frac{\partial f_3}{\partial z} = \frac{\partial}{\partial z}(x-2z) = -2$

$\rightarrow \operatorname{div}\bar{f} = 0$ (for solenoidal)

$\rightarrow \nabla\cdot\bar{f} = \frac{\partial}{\partial x}f_1 + \frac{\partial}{\partial y}f_2 + \frac{\partial}{\partial z}f_3$ ~~$-i+j-2k$~~

$\nabla\cdot\bar{f} = 1+1-2 = 0$ $\therefore$ solenoidal.

4] $\bar{f} = \underset{f_1}{(x+3y)}i + \underset{f_2}{(y-2z)}j + \underset{f_3}{(x-az)}k$

$\frac{\partial f_1}{\partial x} = \frac{\partial}{\partial x}(x+3y) = 1$, $\frac{\partial f_2}{\partial y} = \frac{\partial}{\partial y}(y-2z) = 1$

$\frac{\partial f_3}{\partial z} = \frac{\partial}{\partial z}(x-az) = -a$.

$\rightarrow$ solenoidal $\Rightarrow \nabla\cdot\bar{f} = 0$.

$\nabla\cdot\bar{f} = \frac{\partial}{\partial x}f_1 + \frac{\partial}{\partial y}f_2 + \frac{\partial}{\partial z}f_3 = 0$.

$1+1-a = 0 \Rightarrow \underline{\underline{a=2}}$

5] $\bar{f} = x^2 i + y^2 j + z^2 k$

$\bar{g} = yz\,i + zx\,j + xy\,k$.

$$f\times g = \begin{vmatrix} i & j & k \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$$

let,

$h = \underset{h_1}{(xy^3 - xz^3)}i - \underset{h_2}{(yx^3 - yz^3)}j + \underset{h_3}{(zx^3 - zy^3)}k$

$\frac{\partial h_1}{\partial x} = \frac{\partial}{\partial x}(xy^3 - xz^3) = y^3 - z^3$; $\frac{\partial h_3}{\partial z} = \frac{\partial}{\partial z}(zx^3 - zy^3) = x^3 - y^3$.

$\frac{\partial h_2}{\partial y} = \frac{\partial}{\partial y}(-yx^3 + yz^3) = z^3 - x^3$.

$\rightarrow \operatorname{div}(h) = \frac{\partial h_1}{\partial x} + \frac{\partial h_2}{\partial y} + \frac{\partial h_3}{\partial z}$ $\therefore f\times g$ is solenoidal

$= y^3 - z^3 + z^3 - x^3 + x^3 - y^3 = \underline{\underline{0}}$

6] $\bar{f} = (2x+3y+az)i + (bx+2y+3z)j + (2x+cy+3z)k$

irrotational $\Rightarrow$ curl $f = 0$

$$\nabla\times\bar{f} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x+3y+az & bx+2y+3z & 2x+cy+3z \end{vmatrix}$$

$= \left[\frac{\partial}{\partial y}(2x+cy+3z) - \frac{\partial}{\partial z}(bx+2y+3z)\right]i - \left[\frac{\partial}{\partial x}(2x+cy+3z) - \frac{\partial}{\partial z}(2x+3y+az)\right]j$

$+ \left[\frac{\partial}{\partial x}(bx+2y+3z) - \frac{\partial}{\partial y}(2x+3y+az)\right]k = (c-3)i - (2-a)j + (b-3)k = 0$

$\Rightarrow c=3,\ a=2,\ b=3$.

7] $\phi = x^2y + 2xy + z^2$

$\frac{\partial\phi}{\partial x} = 2xy + 2y$, $\frac{\partial\phi}{\partial y} = x^2 + 2x$, $\frac{\partial\phi}{\partial z} = 2z$

$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k = (2xy+2y)i + (x^2+2x)j + (2z)k$

$$\operatorname{curl}(\nabla\phi) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy+2y & x^2+2x & 2z \end{vmatrix}$$

$= \left[\frac{\partial}{\partial y}(2z) - \frac{\partial}{\partial z}(x^2+2x)\right]i - \left[\frac{\partial}{\partial x}(2z) - \frac{\partial}{\partial z}(2xy+2y)\right]j$

$+ \left[\frac{\partial}{\partial x}(x^2+2x) - \frac{\partial}{\partial y}(2xy+2y)\right]k$

$= (0-0)i - (0-0)j + (\cancel{2x} + \cancel{2} - \cancel{2x} - \cancel{2})k$

$\Rightarrow \operatorname{curl}(\nabla\phi) = \nabla\times\nabla\phi = 0 \rightarrow$ irrotational.

8] $\phi = x^2 - y^2$

$\frac{\partial\phi}{\partial x} = 2x$; $\frac{\partial^2\phi}{\partial x^2} = 2$; $\frac{\partial\phi}{\partial y} = -2y$; $\frac{\partial^2\phi}{\partial y^2} = -2$

$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 2 - 2 = 0$

$\therefore$ laplacian eqn.

9] $\phi = 2x^2yz^3$

$\frac{\partial\phi}{\partial x} = 4xyz^3$, $\frac{\partial\phi}{\partial y} = 2x^2z^3$, $\frac{\partial\phi}{\partial z} = 6x^2yz^2$

$\frac{\partial^2\phi}{\partial x^2} = 4yz^3$, $\frac{\partial^2\phi}{\partial y^2} = 0$; $\frac{\partial^2\phi}{\partial z^2} = 12x^2yz$

at $(1,1,1)$, $\frac{\partial^2\phi}{\partial x^2} = 4$, $\frac{\partial^2\phi}{\partial y^2} = 0$, $\frac{\partial^2\phi}{\partial z^2} = 12$

$\rightarrow \nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 4+0+12 = \underline{\underline{16}}$

10] $\bar{r} = xi + yj + zk$

$|\bar{r}| = \sqrt{x^2+y^2+z^2}$

$r^2 = x^2+y^2+z^2$

$r^2 = \Sigma x^2$

$2r\frac{dr}{dx} = 2x$; $2r\frac{dr}{dy} = 2y$; $2r\frac{dr}{dz} = 2z$

$\frac{dr}{dx} = \frac{x}{r}$. $\quad \frac{dr}{dy} = \frac{y}{r}$ $\quad \frac{dr}{dz} = \frac{z}{r}$.

⑥

10) $r^n\vec{r} \to$ irrotational.

$\Rightarrow \text{curl}(r^n\vec{r}) = 0$

$$\nabla \times r^n\vec{r} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ r^n x & r^n y & r^n z \end{vmatrix} = 0$$

$$\Rightarrow \left[\frac{\partial}{\partial y} r^n z - \frac{\partial}{\partial z} r^n y\right] i - \left[\frac{\partial}{\partial x} r^n z - \frac{\partial}{\partial z} r^n x\right] j - \left[\frac{\partial}{\partial x} r^n y - \frac{\partial}{\partial y} r^n x\right] k = 0$$

$$\Rightarrow \left[\cancel{n r^{n-1} z \cdot \frac{y}{r}} - \cancel{n r^{n-1}} y \cdot \frac{z}{r}\right] i - \left[\cancel{n r^{n-1}} z \cdot \frac{x}{r} - n r^{n-1} \cancel{x \cdot \frac{z}{r}}\right] j - \left[\cancel{n r^{n-1}} y \frac{x}{r} - n \cancel{r^{n-1}} x \cdot \frac{y}{r}\right] k$$

$\text{curl}(r^n\vec{r}) = \nabla \times r^n\vec{r} = 0$

for any value of n.

$\to r^n\vec{r} \to$ solenoidal, $\Rightarrow \text{div}(r^n\vec{r}) = 0$.

$$\nabla \cdot r^n\vec{r} = \frac{\partial}{\partial x}(r^n x) + \frac{\partial}{\partial y}(r^n y) + \frac{\partial}{\partial z}(r^n z) = \Sigma \frac{\partial}{\partial x}(r^n x)$$

$$\Rightarrow \Sigma\left(r^n + n r^{n-1} \cdot \frac{dr}{dx} \cdot x\right) = 0$$

$$\Rightarrow \Sigma\left(r^n + x \cdot n \cdot r^{n-1} \cdot \frac{x}{r}\right) = 0$$

$$\Rightarrow \Sigma\left(r^n + n r^{n-2} x^2\right) = 0$$

$$\Rightarrow \Sigma r^n + \Sigma n r^{n-2} x^2 = 0$$

$$\Rightarrow 3r^n + n r^{n-2} \Sigma x^2 = 0$$

$$\Rightarrow 3r^n + n r^{n-2}(r^2) = 0.$$

$$3r^n + n r^n = 0$$

$$r^n(3+n) = 0$$

at $\underline{\underline{n = -3}}$ $r^n\vec{r}$ is solenoidal

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11) $\vec{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$

$$\nabla \times \bar{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 6xy+z^3 & 3x^2-z & 3xz^2-y \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(3xz^2-y) - \frac{\partial}{\partial z}(3x^2-z)\right] i - \left[\frac{\partial}{\partial x}(3xz^2-y) - \frac{\partial}{\partial z}(6xy+z^3)\right] j + \left[\frac{\partial}{\partial x}(3x^2-z) - \frac{\partial}{\partial y}(6xy+z^3)\right] k$$

$= (-1+1)i - (3z^2 - 3z^2)j + (6x - 6x)k = 0.$

$\text{curl } f = \nabla \times \bar{F} = 0 \Rightarrow$ irrotational.

$\to \vec{F} = \text{grad } \phi = \frac{\partial}{\partial x}\phi i + \frac{\partial}{\partial y}\phi j + \frac{\partial \phi}{\partial z} k$

$\Rightarrow \frac{\partial \phi}{\partial x} = 6xy + z^3$

$\phi_x = \int 6xy + z^3 dx = \frac{\cancel{6}^3 x^2}{\cancel{2}} y + z^3 x + C_x$

$\frac{\partial \phi}{\partial y} = 3x^2 - z.$

$\phi_y = \int 3x^2 - z = 3x^2 y - zy + C_y$

$\frac{\partial \phi}{\partial z} = 3xz^2 - y$

$\phi_z = \int 3xz^2 - y = \frac{\cancel{3}xz^3}{\cancel{3}} - yz + C_z$

we choose C_x, C_y, C_z such that no terms are repeated in final ϕ.

$\phi = 3x^2y + xz^3 - yz + C.$

Tut-4.

I. 1) $\psi = r^2 \sin 2\theta \sin\phi$. (r, θ, ϕ)

$\frac{\partial \psi}{\partial r} = 2r \sin 2\theta \sin\phi$; $\frac{\partial^2 \psi}{\partial r^2} = 2\sin 2\theta \sin\phi$

$\frac{\partial \psi}{\partial \theta} = 2r^2 \cos 2\theta \sin\phi$; $\frac{\partial^2 \psi}{\partial \theta^2} = -4r^2 \sin 2\theta \sin\phi$

$\frac{\partial \psi}{\partial \phi} = r^2 \sin 2\theta \cos\phi$; $\frac{\partial^2 \psi}{\partial \phi^2} = -r^2 \sin 2\theta \sin\phi$

$$\to \nabla\psi = \frac{\partial \psi}{\partial r}\hat{e_r} + \frac{1}{r}\frac{\partial \psi}{\partial \theta}\hat{e_\theta} + \frac{1}{r\sin\theta}\frac{\partial \psi}{\partial \phi}\hat{e_\phi}$$

$$= 2r\sin 2\theta \sin\phi\, \hat{e_r} + \frac{1}{\cancel{r}} 2r^{\cancel{2}} \cos 2\theta \sin\phi\, \hat{e_\theta} + \frac{1}{\cancel{r}\sin\theta} r^{\cancel{2}} \sin 2\theta \cos\phi\, \hat{e_\phi}$$

$$\nabla\psi = 2r\sin 2\theta \sin\phi\, \hat{e_r} + 2r\cos 2\theta \sin\phi\, \hat{e_\theta} + 2r\cos\theta \cos\phi\, \hat{e_\phi}$$

$$\to \nabla^2\psi = \frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial \psi}{\partial \theta} + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 \psi}{\partial \phi^2}$$

$$= 2\sin 2\theta \sin\phi + \frac{2}{\cancel{r}} \cdot 2\cancel{r}\sin 2\theta \sin\phi + \frac{1}{\cancel{r^2}}(-4\cancel{r^2}\sin 2\theta \sin\phi) + \frac{\cot\theta}{\cancel{r^2}}(2\cancel{r^2}\cos 2\theta \sin\phi) + \frac{1}{\cancel{r^2}\sin^2\theta}(-\cancel{r^2}\sin 2\theta \sin\phi)$$

$$-\frac{2\sin\theta\cos\theta \sin\phi}{\sin^2\theta}$$

$$= 2\sin 2\theta \sin\phi + 4\cancel{\sin 2\theta}\sin\phi - 4\cancel{\sin 2\theta}\sin\phi + 2\cos 2\theta \sin\phi \cot\theta - 2\cot\theta \sin\phi$$

$$= 2\sin\phi(\sin 2\theta + \cos 2\theta \cot\theta - \cot\theta)$$

$$= 2\sin\phi(\sin 2\theta - \cot\theta(1 - \cos 2\theta))$$

$$= 2\sin\phi(\sin 2\theta - \cot\theta \cdot 2\sin^2\theta)$$

$$\frac{\cos\theta}{\sin\theta} \cdot 2\sin^2\theta$$

$$= 2\sin\phi(\sin 2\theta - 2\cos\theta\sin\theta) = 0 //$$

$f = \rho^2 + 2\rho\cos\phi - e^2\sin\phi \quad (\rho,\phi,z)$

$\frac{\partial f}{\partial \rho} = 2\rho + 2\cos\phi \quad ; \frac{\partial^2 f}{\partial \rho^2} = 2$

$\frac{\partial f}{\partial \phi} = -2\rho\sin\phi - e^2\cos\phi \quad ; \frac{\partial^2 f}{\partial \phi^2} = -2\rho\cos\phi + e^2\sin\phi$

$\frac{\partial f}{\partial z} = -e^2\sin\phi \quad ; \frac{\partial^2 f}{\partial z^2} = -e^2\sin\phi$

$\rightarrow \nabla f = \frac{\partial f}{\partial \rho} e_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \phi} e_\phi + \frac{\partial f}{\partial z} e_z$

$\nabla f = (2\rho + 2\cos\phi) e_\rho + \frac{1}{\rho}(-2\rho\sin\phi - e^2\cos\phi) e_\phi + -(e^2\sin\phi) e_z$

$\rightarrow \nabla^2 f = \frac{\partial^2 f}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial f}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$

$= 2 + \frac{1}{\rho}(2\rho + 2\cos\phi) + \frac{1}{\rho^2}(-2\rho\cos\phi + e^2\sin\phi) + (-e^2\sin\phi)$

$= 2 + 2 + \cancel{\frac{2\cos\phi}{\rho}} - \cancel{\frac{2\cos\phi}{\rho}} + \cancel{e^2\sin\phi} - \cancel{e^2\sin\phi}$

$\underline{\underline{\nabla^2 f = 4}}$

II 1] $\bar{f} = r^2 \hat{e}_r - 2\cos^2\phi \hat{e}_\theta + \frac{\phi}{r^2+1}\hat{e}_\phi \quad (r,\theta,\phi)$

$f_1 \quad f_2 \quad f_3$.

$\mathrm{div}(\bar{f}) = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(r^2\sin\theta f_1) + \frac{\partial}{\partial \theta}(r\sin\theta f_2) + \partial/\partial\phi (r f_3)\right]$

$= \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(r^4\sin\theta) + \frac{\partial}{\partial \theta}(-2r\sin\theta\cos^2\phi) + \frac{\partial}{\partial \phi}\left(\frac{r\phi}{r^2+1}\right)\right]$

$= \frac{1}{r^2\sin\theta}\left[4r^3\sin\theta - 2r\cos\theta\cos^2\phi + \frac{r}{r^2+1}\right]$

$= 4r - \frac{2}{r}\cos^2\phi\cot\theta + \frac{\operatorname{cosec}\theta}{r(r^2+1)}$

$\bar{f}$ is not solenoid.

II 2] $\bar{A} = z[\sin\theta \hat{e}_r + \cos\theta \hat{e}_\theta] - r\cos\theta \hat{e}_z$

$f_1 = z\sin\theta \quad f_2 = z\cos\theta \quad f_3 = -r\cos\theta$

$\mathrm{div}(\bar{f}) = \frac{1}{r}\left[\frac{\partial}{\partial r}(r f_1) + \frac{\partial}{\partial \theta}(f_2) + \frac{\partial}{\partial z}(r f_3)\right]$

$= \frac{1}{r}\left[\frac{\partial}{\partial r}(rz\sin\theta) + \frac{\partial}{\partial \theta}(z\cos\theta) + \frac{\partial}{\partial z}(-r^2\cos\theta)\right]$

$= \frac{1}{r}\left[z\cancel{\sin\theta} + (-z\cancel{\sin\theta}) + 0\right]$

$\mathrm{div}(\bar{f}) = 0$.

$\Rightarrow f$ is solenoidal.

2] $\bar{A} = \frac{2\cos\theta}{\rho^3}\hat{e}_\rho - \frac{\sin\theta}{\rho^3}\hat{e}_\theta \quad (\rho,\theta,\phi)$

$f_1 \quad f_2 \quad f_3 = 0$.

$\mathrm{curl}\,\bar{f} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{e}_\rho & \rho\hat{e}_\theta & \rho\sin\theta\hat{e}_\phi \\ \partial/\partial\rho & \partial/\partial\theta & \partial/\partial\phi \\ \frac{2\cos\theta}{\rho^3} & -\frac{\sin\theta}{\rho^3} & 0 \end{vmatrix}$

$= \frac{1}{\rho^2\sin\theta}\left\{\left[\frac{\partial}{\partial\theta}(0) - \frac{\partial}{\partial\phi}\left(-\frac{\sin\theta}{\rho^3}\right)\right]\hat{e}_\rho - \left[\frac{\partial}{\partial\rho}(0) - \frac{\partial}{\partial\phi}\left(\frac{2\cos\theta}{\rho^3}\right)\right]\rho\hat{e}_\theta + \left[\frac{\partial}{\partial\rho}\left(\frac{-\sin\theta}{\rho^3}\right) - \frac{\partial}{\partial\theta}\left(\frac{2\cos\theta}{\rho^3}\right)\right]\rho\sin\theta\hat{e}_\phi\right.$

$= \frac{1}{\rho^2\sin\theta}\left\{(0-0)\hat{e}_\rho - (0-0)\rho\hat{e}_\theta + \left(\frac{3\sin\theta}{\rho^4} + \frac{2\sin\theta}{\rho^3}\right)\rho\sin\theta\hat{e}_\phi\right.$

$= \frac{1}{\rho^2\sin\theta} \times \frac{2\sin\theta}{\rho^3}\rho\sin\theta\hat{e}_\phi$

$\frac{1}{\rho^2\sin\theta}\left(\frac{3\sin\theta}{\rho^4} + \frac{2\sin\theta}{\rho^3}\right)\rho\sin\theta\hat{e}_\phi = \frac{\sin\theta}{\rho^4}\left(\frac{3}{\rho}+2\right) \neq 0$.

f is irrotational.

III 1] $\bar{f} = \rho z\sin 2\theta \hat{e}_\rho + \rho z\cos 2\theta \hat{e}_\theta - \frac{\rho^2\sin^2\theta}{2}\hat{e}_z$

$f_1 \quad f_2 \quad f_3$

$\mathrm{curl}\,\bar{f} = \frac{1}{\rho}\begin{vmatrix} \hat{e}_\rho & \hat{e}_\theta & \hat{e}_z \\ \partial/\partial\rho & \partial/\partial\theta & \partial/\partial z \\ \rho z\sin 2\theta & \rho z\cos 2\theta & \frac{\rho^2\sin^2\theta}{2} \end{vmatrix}$

$= \frac{1}{\rho}\left\{\left[\frac{\partial}{\partial\theta}\left(\rho^2\frac{\sin^2\theta}{2}\right) - \frac{\partial}{\partial z}(\rho^2 z\cos 2\theta)\right]\hat{e}_\rho - \left[\frac{\partial}{\partial\rho}\left(\rho^2\frac{\sin^2\theta}{2}\right) - \frac{\partial}{\partial z}(\rho z\sin 2\theta)\right]\hat{e}_\theta + \left[\frac{\partial}{\partial\rho}(\rho^2 z\cos 2\theta) - \frac{\partial}{\partial\theta}(\rho z\sin 2\theta)\right]\hat{e}_z\right.$

$= \frac{1}{\rho}\left\{\left(\frac{\rho^2 \cdot 2\sin\theta\cos\theta}{2} - \rho^2\cos 2\theta\right)\hat{e}_\rho - \left(\frac{2\rho^2\sin^2\theta}{2} - \rho\sin 2\theta\right)\hat{e}_\theta + (2\rho z\cos 2\theta - 2\rho z\cos 2\theta)\hat{e}_z\right\}$

$= \frac{1}{\rho}\left(\rho^2\sin\theta\cos\theta - \rho\cos 2\theta\right)\hat{e}_\rho - (2\rho\sin^2\theta - \sin 2\theta)\hat{e}_\theta \neq 0$ k.

$\neq 0 \quad \therefore$ not ~~irrotational~~

III 1] $\bar{f} = \underbrace{\rho z \sin 2\theta}_{f_1} \hat{e_\rho} + \underbrace{\rho z \cos 2\theta}_{f_2} \hat{e_\theta} - \underbrace{\rho^2 \frac{\sin^2\theta}{2}}_{f_3} \hat{e_z}$

$$\text{curl}\,\bar{f} = \frac{1}{\rho}\begin{vmatrix} \hat{e_\rho} & \rho\hat{e_\theta} & \hat{e_z} \\ \partial/\partial\rho & \partial/\partial\theta & \partial/\partial z \\ f_1 & \rho f_2 & f_3 \end{vmatrix}$$

$$= \frac{1}{\rho}\left\{\left[\frac{\partial}{\partial\theta}\rho^2\frac{\sin^2\theta}{2} - \frac{\partial}{\partial z}(\rho^2 z\cos 2\theta)\right]\hat{e_\rho} - \left[\frac{\partial}{\partial\rho}\left(\rho^2\frac{\sin^2\theta}{2}\right) - \frac{\partial}{\partial z}(\rho z\sin 2\theta)\right]\rho\hat{e_\theta}\right.$$

$$\left. + \left[\frac{\partial}{\partial\rho}(\rho^2 z\cos 2\theta) - \frac{\partial}{\partial\theta}(\rho z\sin 2\theta)\right]\hat{e_z}\right\}$$

$$= \frac{1}{\rho}\left\{\left(\cancel{2}\rho^2\frac{\sin\theta\cos\theta}{\cancel{2}} - \rho^2\cos 2\theta\right)\hat{e_\rho} - \left(\cancel{2}\rho\frac{\sin^2\theta}{\cancel{2}} - \rho\sin 2\theta\right)\rho\hat{e_\theta}\right.$$

$$\left. + \left(2\rho z\cancel{\cos 2\theta} - 2\rho z\cancel{\cos 2\theta}\right)\hat{e_z}\right\}$$

$\text{curl}\,\bar{f} = (\rho\sin\theta\cos\theta - \rho\cos 2\theta)\hat{e_\rho} - (\rho\sin^2\theta - \rho\sin 2\theta)\hat{e_\theta}$ $\therefore$ not irrotational

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2] $\bar{A} = \underbrace{\frac{2\cos\theta}{\rho^3}}_{f_1}\hat{e_\rho} - \underbrace{\frac{\sin\theta}{\rho^3}}_{f_2}\hat{e_\theta}$ $f_3 = 0.$

$$\text{curl}\,\bar{f} = \frac{1}{\rho^2\sin\theta}\begin{vmatrix} \hat{e_\rho} & \rho\hat{e_\theta} & \rho\sin\theta\,\hat{e_\phi} \\ \partial/\partial\rho & \partial/\partial\theta & \partial/\partial\phi \\ f_1 & \rho f_2 & \underbrace{\rho\sin\theta f_3}_{=0} \end{vmatrix}$$

$$= \frac{1}{\rho^2\sin\theta}\left\{\left[\frac{\partial}{\partial\theta}(\rho\sin\theta)(0) - \frac{\partial}{\partial\phi}\rho\cdot\frac{-\sin\theta}{\rho^3}\right]\hat{e_\rho} - \left[\frac{\partial}{\partial\rho}(0) - \frac{\partial}{\partial\phi}\cdot\frac{2\cos\theta}{\rho^3}\right]\rho\hat{e_\theta}\right.$$

$$\left. + \left[\frac{\partial}{\partial\rho}\rho\frac{-\sin\theta}{\rho^{2}} - \frac{\partial}{\partial\theta}\left(\frac{2\cos\theta}{\rho^3}\right)\right]\rho\sin\theta\,\hat{e_\phi}\right\}$$

$$= \frac{1}{\rho^2\sin\theta}\left\{\cancel{(0-0)\hat{e_\rho}} - \cancel{(0-0)}\rho\hat{e_\theta} + \left(\frac{2\sin\theta}{\rho^3} + \frac{2\sin\theta}{\rho^3}\right)\rho\sin\theta\,\hat{e_\phi}\right\}$$

$$= \frac{1}{\rho^2\cancel{\sin\theta}}\left(\frac{2\sin\theta}{\rho^3} + \frac{2\sin\theta}{\rho^3}\right)\rho\cancel{\sin\theta}\,\hat{e_\phi} \Rightarrow \underline{\frac{4\sin\theta}{\rho^4}\hat{e_\phi}} = \text{curl}\,\bar{f}$$

$$= \cancel{\frac{2\sin\theta}{\rho^4} + \frac{2\sin 2\theta}{\rho^4} = \frac{2}{\rho^4}(\sin\theta + \sin 2\theta)}$$