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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)
I Semester B. E. Examinations May-2023

(Common to EC, EE, EI, ET)

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of scientific calculator is permitted.
4. Use of Handbook of Mathematics is permitted.

PART-A

1 1.1 Given $A = \begin{bmatrix} 0 & 2 & 4 \\ 3 & 5 & 13 \\ 4 & 4 & 12 \end{bmatrix}$, then the rank of A is __________. 02

1.2 Sum and product of the eigenvalues of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is __________ and __________. 02

1.3 The angle between the radius vector and tangent to the curve $r = e^{\theta}$ is __________. 02

1.4 The Maclaurin series expansion for $\sin hx$ is __________. 02

1.5 If $u = \frac{2x}{y}$ and $v = \frac{2y}{x}$, then $\frac{\partial(u,v)}{\partial(x,y)} =$ __________. 02

1.6 Given $f_{xx} = 6x, f_{xy} = 0$ and $f_{yy} = 6y$, then the nature of stationary point at $(-1,2)$ is __________. 02

1.7 Evaluate $\int_0^2 \int_1^3 y\, dy\, dx$ 02

1.8 Sketch the domain of the integral $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\, dy\, dx$. 02

1.9 Construct the table of differences for the data below.

x	0	10	20	30	40
y	1	1.5	2.2	3.1	4.6

Evaluate $\Delta^2 f(20)$ and $\Delta^3 f(10)$. 02

1.10 If $y_0 = 0,\ y_1 = 2,\ y_2 = 2.5,\ y_3 = 2.3, y_4 = 2,\ y_5 = 1.7, y_6 = 1.5$, then compute the area using Weddle's rule. 02

PART-B

2 a In control theory, the rank of a matrix is used to determine whether a linear system is controllable or observable. To analyze one such system, reduce the given matrix into the row echelon form and hence obtain rank of a matrix. $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 4 & 3 \\ 6 & 8 & 7 \end{bmatrix}$ 06

b Eigenvalue analysis is used to design the car stereo system. Apply Rayleigh's power method to compute the largest eigenvalue and the corresponding eigenvector for the matrix representing the position of car stereo given by $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ taking initial vector as $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Carry out 6 iterations. 10

3 a Determine the angle between the radius vector and tangent for the curve $r = a(1 + \sin\theta)$ at any point θ. Hence find the slope at $\theta = \frac{\pi}{2}$. 08

b Find the centre of curvature and circle of curvature on the curve $y = e^{3x}$ at a point where the curve crosses the y −axis. 08

OR

4 a Obtain the radius of curvature at any point $P(x, y)$ on the curve $y = a\cosh\left(\frac{x}{a}\right)$; $a > 0$. 08

b Expand $y = \log_e(\sec x)$ as a series in powers of $'x'$. Hence obtain the series expansion for $\tan x$. 08

5 a If $u = \log_e(x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right)$, show that u satisfies the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. 08

b Evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ if $u = \frac{yz}{x}$, $v = \frac{zx}{y}$ and $w = \frac{xy}{z}$. 08

OR

6 a If $u = f(x - y, y - z, z - x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. 08

b Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$. 08

7 a Evaluate $\iint_R dx\,dy$ where R is the region bounded by the lines $= x$, $x + y = 4$, $y = 1$ and $y = 0$. 08

b Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a\sin\theta} \int_0^{\frac{a^2 - r^2}{a}} r\,dz\,dr\,d\theta$. 08

OR

8 a Find the area bounded by the curves $xy = 2, 4y = x^2, y = 4$ and the co-ordinate axes using double integrals. 08

b Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\,dy$ by changing to polar co-ordinates. 08

9 a A survey conducted in a locality reveals the following information as classified below.

Income (per month) Rs.	Under 10k	(10-20)k	(20-30)k	(30-40)k	(40-50)k
Number of persons	20	45	115	210	115

Estimate the probable number of persons in the income group $20k - 25k$. 08

b Applying Lagrange's formula in appropriate form fit an interpolating polynomial of the form $x = f(y)$ for the given data and hence find $x(5)$ and $y(5)$.

x	2	10	17
y	1	3	4

08

OR

10 a Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}th$ rule. Hence deduce the value of $\ln 2$. 08

b Find the radius of curvature of a Cartesian curve $y = f(x)$ when $x = 23$ given the following data using numerical differentiation.

x	19	20	21	22	23
y	91	100.25	110	120.25	131

08