VECTOR CALCULUS LAPLACE TRANSFORM AND NUMERICAL METHODS (MA221TA)

Practice Problems

UNIT-I

VECTOR CALCULUS

Q.No.	Objective type Questions
1	The unit normal to the surface $\Phi(x,y,z) = yx^2z + 4xz^2$ at the point $(1,-2,-1)$ is ______.
2	The directional derivative of $\Phi(x,y,z) = x^2yz^3$ at $(2,1,-1)$ is maximum along _____ direction.
3	If the vector $\vec{F} = (2x^2y^2 + z^2)\hat{\imath} + (3xy^3 - x^2z)\hat{\jmath} + (\gamma xy^2z + xy)\hat{k}$ is incompressible, then the value of the constant 'γ' is _________.
4	Find the value 'a' if the vector $\vec{F} = (axy - z^2)\hat{\imath} + (x^2 + 2yz)\hat{\jmath} + (y^2 - axz)\hat{k}$ is irrotational.
5	div$\left(\frac{\vec{r}}{r}\right)$ = _________.
6	If $\Phi(x,y,z) = 2xz - y^2$ find grad Φ at the point $(1,3,2)$
7	Find the directional derivative of the function $\Phi = x^2 - y^2 + 2z^2$ at the point P$(1,2,3)$ in the direction of the line PQ where Q =$(5,0,4)$.
8	The minimum directional derivative is given by _____.
9	$\Delta\Phi(r) =$ ___________________________.
10	In $\nabla^2 r^n = n(n+1)r^{n-2}$, where n is a non-zero constant and r^n is harmonic if and only if n = ____ Where $r \neq 0$.
	Essay type Questions
1	If $\vec{F} = (2xy^2 + yz)\hat{\imath} + (2x^2y + xz + 2yz^2)\hat{\jmath} + (2zy^2 + xy)\hat{k}$ is the velocity vector of a fluid, show that the velocity vector field is irrotational and determine a scalar potential function $\phi(x,y,z)$ such that $\vec{F} = \nabla\phi$
2	The temperature at a point (x,y,z) in space is $T(x,y,z) = x^2 + y^2 - z$, A mosquito located at the point (4, 4, 2) desires to fly in such a direction that it gets cooled faster. Find the direction in which it should fly.

3	Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$.
4	Find the directional derivative of $\phi=x^2y^2+z^2y^2+x^2z^2$ at $(1,1,-2)$ in the direction of tangent to the curve $x=e^{-t},\ y=2sint+1,\ z=t-cost$ at $t=0$.
5	Compute the curl and divergence of $\vec{A}=(2r+kcos\phi)\widehat{e_r}-ksin\theta\widehat{e_\theta}+rcos\theta\widehat{e_\phi}$, where k is a constant in spherical coordinates.
6	Find the values of constant γ and μ so that the surfaces $\gamma x^2-\mu yz=(\gamma+2)x$ and $4x^2y+z^3=4$ intersect orthogonally at the point$(1,-1,2)$.
7	Compute the values of a and b so that the vector field $\vec{F}=(y^2cosx+z^3)\hat{\imath}+(aysinx-4)\hat{\jmath}+bxz^2\hat{k}$ is a conservative field, hence find the scalar potential function ϕ such that $\vec{F}=\nabla\phi$.
8	Show that $r^2\vec{r}$ is an irrotational vector for any value of α, but is solenoidal if $\alpha+3=0$ Where $\vec{r}=x\,\hat{\imath}+y\hat{\jmath}+z\hat{k}$ and $\lvert\vec{r}\rvert=r$.
9	If$\Phi=x^2yz+xyz+4xz^2$, find div(grad Φ).
10	Find the values of the constants a, b and c so that the directional derivative of If $\Phi=ay^2x+byz+cx^3z^2$ at $(1,2,-1)$ has maximum of magnitude 64 in a direction parallel to the z-axis.

UNIT-II

VECTOR INTEGRATION

Objective type Questions	
1	The work done by the force $\vec{F}=5xy\hat{\imath}+2y\hat{\jmath}$, in displacing a particle from $x=1$ to $x=2$ along $y=t^3, x=t$ is ________.
2	The line integral $\frac{1}{2}\int_C(Mdx+Ndy)$ represents the area of region bounded by C if $M=$ ____ and $N=$ ______.
3	If $\vec{F}$ represents the velocity of a fluid in a region and on a smooth surface S then, $\int_S \vec{F}.\hat{n}dS$ physically represents __________.
4	If $\vec{F}$ represents a conservative force field over a closed path C, then $\int_C \vec{F}.d\vec{r}=$__________.
5	The Stokes' theorem converts surface integral over an orientable surface S bounded by closed curve C to _____________ integral.
Essay type Questions	
1	If $\vec{F}=(5xy-6x^2)\hat{\imath}+(2y-4x)\hat{\jmath}$, evaluate $\int_C \vec{F}.d\vec{r}$ where C is the curve $y=x^3$ from (1,1) to (2,8).
2	Find the work done by the force $\vec{F}=(3x^2+6y)\hat{\imath}-14yz\hat{\jmath}+20xz^2\hat{k}$, displacing the particle from (0,0,0) to (1,1,1) along the path given by $C: x=t, y=t^2, z=t^3$
3	Evaluate $\int_S \vec{F}.\hat{n}dS$, where S is the part of the surface of the sphere $x^2+y^2+z^2=a^2$ in the first octant, and $\vec{F}=yz\hat{\imath}+zx\hat{\jmath}+xy\hat{k}$.

4	Evaluate $\int_S \vec{F}.\hat{n}dS$, where S is the part of the surface of the plane $2x+y+2z=6$ which lies in the positive octant, and $\vec{F}= 4x\hat{\imath}+y\hat{\jmath}+z\hat{k}$.
5	Verify Green's theorem for $\int_C (xy+y^2)dx+x^2dy$, where C is the path bounded by the line $y=x$ and the parabola $y=x^2$
6	Evaluate $\int_C (3x^2-8y^2)dx+(4y-6xy)dy$, where C is the boundary of the region enclosed by the lines $x=0, y=0, x+y=1$
7	Verify Stokes' theorem for the line integral of $\vec{f}=(x^2+y^2)\hat{\imath}-2xy\hat{\jmath}$, taken around the rectangle bounded by the lines $x=\pm a, y=0, y=b$.
8	Evaluate $\int_C \vec{F}.d\vec{r}$ using Stokes' theorem where $\overrightarrow{F}= y\hat{\imath}+z\hat{\jmath}+x\hat{k}$, where C is the rim for the upper part of the sphere $x^2+y^2+z^2=a^2$
9	Verify divergence theorem for $\overrightarrow{F}=(2x-z)\hat{\imath}+x^2y\hat{\jmath}-xz^2\hat{k}$ for the region bounded by the planes $x=0, x=1, y=0, y=1, z=0, z=1$.
10	Evaluate $\int_S \vec{F}.\hat{n}dS$, where S is the part of the surface of the cylindrical region bounded by $x^2+y^2=9$, $z=0$ and $z=2$, $\overrightarrow{F}=y\hat{\imath}+x\hat{\jmath}+z^2\hat{k}$.

. UNIT-3

LAPLACE TRANSFORM

Objective type Questions	
1	$L[\sqrt{t}] =$ ________.
2	Given $L\left[\frac{\sin t}{t}\right]=\cot^{-1}s$, then $L\left[\frac{\sin 3t}{t}\right]$=___________.
3.	Laplace transform of the signal $e^{-\frac{t}{2}}\cos 3t$ is ______.
4	Given $L[t\cos t]=\frac{s^2-1}{(s^2+1)^2}$, then $\int_0^{\infty} te^{-3t}\cos t\, dt$=________.
5	Sketch the graph of the wave form periodically defined by $f(t)=\begin{cases} 5, & 0<t<a \\ -5, & a<t<2a \end{cases}, \quad f(t+2a)=f(t)$
6	$L[\![\sin t\, u(t-\pi)]\!] =$ ______
7	$L[t^2\delta(t-2)]$=______.
8	$L[2^{-t}+t\sin 2t] =$ ______.
Essay type Questions	
1	Find the Laplace transform the following functions: $\cos 3t \sin^2 t$ b) $\cos h^2 3\, t \ \cos 5t \cos 2t$

2	Obtain the Laplace transform of the signal $e^{-4t}\int_0^t t\sin 5t\,dt + \frac{\cos 2t-\cos 3t}{t}$
3	Determine the Laplace transform of the following periodic function: $f(t) = \begin{matrix} t, & 0 \le t \le 4 \\ 8-t, & 4 \le t \le 8, \end{matrix}$ $f(t+8) = f(t)$. Also sketch the graph of the function.
4	Express the following function in terms of Heaviside unit step function and hence evaluate its Laplace transform. $$f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \sin t, & t > \pi \end{cases}$$
5	Evaluate $\int_0^\infty \frac{e^{-2t}\sin 3t}{t}\,dt$.
6	Find the Laplace transform of the full wave rectifier $f(t) = E\sin\omega t, 0 < t < \omega$ having the period $\frac{\pi}{\omega}$.
7	Express the following function in terms of Heaviside unit step function and hence evaluate its Laplace transform. $$f(t) = \begin{cases} 1, 0 < t < 1, \\ t, 1 < t \le 2 \\ t^2, t > 2. \end{cases}$$
8	Find the Laplace transform of the following: $(t^2-6t+9)^2 e^{-(t-3)} u(t-3)$ b) $e^{-t}\frac{\delta(t-3)}{t}$
9	Find the Laplace transform of the function: $t^2 e^{-2t}\sin 2t + \sin(3t-1)$.
10	Express the following function in terms of Heaviside unit step function and hence evaluate its Laplace transform. $$f(t) = \begin{cases} \cos t, 0 < t < \pi, \\ \cos 2t, \pi < t \le 2\pi \\ \cos 3t, \quad t > 2\pi. \end{cases}$$

UNIT-4

INVERSE LAPLACE TRANSFORM

Objective type Questions	
1	Find the inverse Laplace transforms of the following: (i) $\frac{1}{2s-5}$ (ii) $\frac{s+b}{s^2+a^2}$ (iii) $\frac{2s-5}{4s^2+25}+\frac{4s-9}{9-s^2}$
2	Find inverse Laplace transform of $\frac{3}{S(S^2+a^2)}$.
3	$L^{-1}\left[\frac{3S+5\sqrt{2}}{S^2+8}\right] =$ ________.
4	$L^{-1}\left[\frac{1}{(3s-1)^3}\right] =$ ________
5	$L^{-1}\left[\frac{1}{s^2+5s+7}\right] =$ ________

Essay type Questions	
1	Find $L^{-1}\left\{\frac{4s+5}{(s+1)^2(s+2)}\right\}$
2	Find $L^{-1}\left(\frac{s}{(2s-1)(s+2)(s+3)}\right)$
3	Find $L^{-1}\left\{s\log\left(\frac{s+1}{s-1}\right)+2\right\}$.
4	Using inverse Laplace transform convert the frequency domain signal $\tan^{-1}\left(\frac{2}{s}\right)$ in time domain.
5	Find $L^{-1}\left(\frac{1}{(s+1)(s^2+1)}\right)$ using Convolution Theorem.
6	Find $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right), a \neq b$ using Convolution Theorem.
7	Find $L^{-1}\left(\frac{1}{(s^2+a^2)^2}\right)$ using Convolution Theorem
8	Find $L^{-1}\left(\frac{1}{s^2(s+1)^2}\right)$ using Convolution Theorem
9	Find $L^{-1}\left(\frac{s}{(s^2+4)^2}\right)$ using Convolution Theorem
10	Find inverse Laplace transform of $\frac{Se^{-\frac{s}{2}}+\pi e^{-s}}{S^2+\pi^2}$ in terms of unit step function
11	Solve using Laplace transform method $y''(t)+y(t) = H(t-1)$, given that $y(0) = 0$ & $y'(0) = 1$.
12	Employ Laplace Transform method to solve $y'' - 5y' + 6y = 5e^{2t}$ given $y(0) = 2$, $y'(0) = 1$.
13	Solve by the method of Laplace transform, the equation $(D^3+2D^2-D-2)y = 0$, given $y(0) = y'(0) = 0$ *and* $y''(0) = 6$.

UNIT-5

NUMERICAL METHODS

Objective type Questions	
1	A real root of the equation $2^x - x = 3$ lies in the interval _______
2	Given $e^{-x} - \sin(x) = 0$, $f(0) = 1$ & $f(1) = -0.4736$ first approximate value of x by the method of chord is ________
3	First approximate to the root of the equation $x = 3\cos(x - \frac{\pi}{4})$ by Newton – Raphson method near $x = 1$ is_______

4	The value of y'' for the initial value problem $\frac{dy}{dx} = 1 - 2xy$ given that $y(0) = 0$.
5	Given $\frac{dy}{dx} = x + y$ given that $y = 1.2$ when $x = 1$ $h = 0.2$, $k_1 = 0.2, k_2 = 0.24, k_3 = 0.244$ then $k_4 = 3.122$ & k =____ & $y_1 =$ _____using Runge- Kutta fourth order method
6	Given $\frac{dy}{dx} = 3x + y^2$ given that $y = 1.2$ when $x = 1, h = 0.2$, $k_1 = 0.244$, $k_2 = 0.2798, k_3 = 0.2845$ & $k_4 =$ _____and k = __ using Runge- Kutta fourth order method.
7	Given $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ given that $y(0) = 1$ $h = 0.2$, $k_1 = 0.2$, $k_2 = 0.1967, k_3 = 0.196$ & $k_4 =$ __ using Runge- Kutta method.
8	The Runge-Kutta method for solution of $y' = \frac{y^2-x^2}{y^2+x^2}$, $y(0) = 1$ at $x = 0.2$, $h = 0.2$ yields $k_1 = 0.2$, $k_2 = 0.19672$, then values of $k_3 =$ _______ and $k_4 =$ _________.
9	The Milne's predicted solution for the differential equation $xy' = 2y, x \neq 0$ at the point x = 2 given that $y(1) = 2, y(1.25) = 3.13, y(1.5) = 4.5, y(1.75) = 6.13$ is _________.
10	Given differential equation $\frac{dy}{dx} = x^2(1 + y)$and $y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979, y_4^{(p)} = 2.5738$ by Milne's method at $x = 1.4, y_4^{(c)} =$ _____ .
	Essay type Questions
1	Find a positive real root of the equation $x\log_{10}(x) = 1.2$ in $[2.6, 3]$ by the method of false position up to four places of decimals. Perform four iterations. Compute the real root of equation $x^2 - \ln(x) = 12$ which lies between (3,4) correct to four decimal places using Regula- falsi method.
2	Find approximate root of the equation $x = 3\cos(x - \frac{\pi}{4})$ by Newton – Raphson method near $x = 1$
3	Apply iteration method of Newton – Raphson to find the approximate root of the equation $x\sin(x) = -\cos(x)$ which is near $x_0 = \pi$ correct to five decimal places.
4	Employ Taylor,s series method to find an approximate value of y when x = 0.1 for the differential equation $y' = xy^2$ given that y = 1 when x = 0 up to third degree.
5	Obtain a numerical solution of the differential equation $\frac{dy}{dx}$ = 3 (1+ x) – y using the Taylor series method, given the initial conditions that x = 1 when y = 4. Compute y (1.2) and y (1.4).
6	Apply Runge- Kutta fourth order method to find an approximate value of y when x = 1.1 given that $\frac{dy}{dx} = 3x + y^2$ given that $y = 1.2$ when $x = 1$.
7	Employ Runge- Kutta 4 th order method to find an approximate value of y = 0.1 for the initial value problem $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ given that $y(0) = 1$, $h = 0.2$.
8	Apply Milne's to find the solution of initial value problem $y' + y^2 = x$ at $x = 0.8$, 1.0 given $y(0) = 0, y(0.2) = 0.020, y(0.4) = 0.0795$ and $y(0.6) = 0.1762$.
9	Apply Milne's to find the solution of initial value problem $f(x, y) = x^2(1 + y)$, $y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979$ at x = 1.4 &1.5.