inverse laplace transform_tutorials

03 May 2025 10:12 AM

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Department of Mathematics

VECTOR CALCULUS, LAPLACE TRANSFORM AND NUMERICAL METHODS (MA221TA)

UNIT-IV: INVERSE LAPLACE TRANSFORM

TUTORIAL SHEET-1

I. Objective type questions:

1) Laplace transform of the signal is found to be $\frac{1}{(s+3)^3}$, then the corresponding signal in time domain is _____ **Ans:** $\frac{e^{-3t}t^2}{2}$

2) The frequency response of a system is found to be $\frac{4}{4s^2-3}$, find the corresponding time response. **Ans:** $\frac{2}{\sqrt{3}}\sinh\frac{\sqrt{3}}{2}t$

3) $L^{-1}\left(\frac{1}{s^{3/2}}\right)$= _________. **Ans:** $2\sqrt{\frac{t}{\pi}}$

4) $L^{-1}\left(\frac{1+e^{-3s}}{s^2}\right)$= _________. **Ans:** $t+(t-3)u(t-3)$

5) $L^{-1}\left(\frac{1}{s^2}-\frac{48}{s^5}\right)$= _________. **Ans:** $t-2t^4$

6) $L^{-1}\left\{\frac{(s+2)^2}{s^3}\right\}$= _________. **Ans:** $1+2t^2+4t$

II. Find the inverse Laplace transform of the following signals:

i) $\frac{s+2}{s^2+36}+\frac{4s-1}{s^2+25}+\frac{(s+3)^3}{s^6}$

Ans: $\cos 6t+\frac{1}{3}\sin 6t+4\cos 5t-\frac{1}{5}\sin 5t+\frac{t^2}{2}+\frac{t^3}{2}+\frac{9t^4}{8}+\frac{9t^5}{40}$

ii) $\frac{2s+1}{s^2+3s+1}$ **Ans:** $e^{-t/2}\left\{\cos\left(\frac{\sqrt{5}}{2}t\right)+1/\sqrt{3}\sin\left(\frac{\sqrt{5}}{2}t\right)\right\}$ XXX ¿?

iii) $\frac{7s+4}{4s^2+4s+9}$ **Ans:** $\frac{e^{-t/2}}{4}\left[7\cos(\sqrt{2}t)+1/2\sqrt{2}\sin(\sqrt{2}t)\right]$

iv) $\frac{2(s^2+2a^2)e^{-2s}}{s^4+4a^4}$ **Ans:** $\frac{2}{a}\{\sin a(t-2)\cosh a(t-2)\}u(t-2)$

v) $\frac{5s+3}{(s-1)(s^2+2s+5)}$ **Ans:** $e^t+e^{-t}\left[\frac{3}{2}\sin 2t-\cos 2t\right]$

vi) $\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$ **Ans:** $\frac{e^{-t}}{3}[\sin t+\sin 2t]$

vii) $\frac{1}{s}\left(\cos\frac{1}{s}\right)$ **Ans:** $1-\frac{t^2}{(2!)^2}+\frac{t^4}{(4!)^2}-\cdots$

viii) $\frac{e^{4-3s}}{(s+4)^{5/2}}$ **Ans:** $\frac{4}{3\sqrt{\pi}}e^{-4(t-4)}(t-3)^{\frac{3}{2}}u(t-3)$

1 | Page

1] $L^{-1}\left(\frac{1}{(s+3)^3}\right)=e^{-3t}L^{-1}\left(\frac{1}{s^3}\right)$
$=e^{-3t}\frac{t^2}{2}$

2] $L^{-1}\left(\frac{4}{4s^2-3}\right)=L^{-1}\left(\frac{1}{s^2-3/4}\right)$
$=\frac{2}{\sqrt{3}}\sinh\frac{\sqrt{3}}{2}$

3] $L^{-1}\left(\frac{1}{s^{3/2}}\right)=L^{-1}\left(\frac{1}{s^{1/2+1}}\right)$

$n+1=3/2$, $n=1/2$; $\left\{t^{1/2}=\frac{\Gamma(1/2+1)}{s^{1/2+1}}\right\}$

$=L^{-1}\left(\frac{t^{1/2}}{\Gamma(1/2+1)}\right)=2\sqrt{\frac{t}{\pi}}$

4] $L^{-1}\left(\frac{1+e^{-3s}}{s^2}\right)=L^{-1}\left(\frac{1}{s^2}\right)+L^{-1}\left(\frac{e^{-3s}}{s^2}\right)$
$=t+L^{-1}\left(e^{-3s}F(s)\right)$
$=t+f(t-3)u(t-3)$
$=t+(t-3)u(t-3)$

$\left\{F(s)=\frac{1}{s^2};\ f(t)=t;\ f(t-3)=t-3\right\}$

5] $L^{-1}\left(\frac{1}{s^2}-\frac{48}{s^5}\right)$
$=L^{-1}\left(\frac{1}{s^2}\right)-L^{-1}\left(\frac{48}{s^5}\right)$
$=t-\frac{48t^4}{4!}$
$=t-2t^4$

6] $L^{-1}\left(\frac{(s+2)^2}{s^3}\right)=L^{-1}\left(\frac{s^2+4s+4}{s^3}\right)$
$=L^{-1}\left(\frac{1}{s}\right)+L^{-1}\left(\frac{4}{s^2}\right)+L^{-1}\left(\frac{4}{s^3}\right)$
$=1+4t+\frac{4t^2}{2!}=1+4t+2t^2$

i) $L^{-1}\left(\frac{s+2}{s^2+36}+\frac{4s-1}{s^2+25}+\frac{(s+3)^3}{s^6}\right)$ $\quad (s+3)^3=s^3+27+3(s^2)(3)+3(s)(3^2)$

$=L^{-1}\left(\frac{s}{s^2+36}\right)+L^{-1}\left(\frac{2}{s^2+36}\right)+L^{-1}\left(\frac{4s}{s^2+25}\right)-L^{-1}\left(\frac{1}{s^2+25}\right)+L^{-1}\left(\frac{1}{s^3}\right)+L^{-1}\left(\frac{27}{s^6}\right)+L^{-1}\left(\frac{9}{s^4}\right)+L^{-1}\left(\frac{27}{s^5}\right)$

$=\cos 6t+\frac{2\sin 6t}{6}+4\cos 5t-\frac{\sin 5t}{5}+\frac{t^2}{2}+\frac{27t^5}{5!}+\frac{9t^3}{3!}+\frac{27t^4}{4!}$

$+\frac{9t^5}{40}+\frac{3t^3}{2}+\frac{9t^4}{8}$

ii) $L^{-1}\left(\frac{2s+1}{s^2+3s+1}\right)$ $\quad \left\{s^2+3s+1 = s^2+2(s)\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+1\right\}$

$L^{-1}\left(\frac{2(s+3/2)-3+1}{(s+3/2)^2-\left(\frac{\sqrt{5}}{2}\right)^2}\right)$

$=L^{-1}\left(\frac{2(s+3/2)}{(s+3/2)^2-(\sqrt{5}/2)^2}\right)-L^{-1}\left(\frac{2}{(s+3/2)^2-(\sqrt{5}/2)^2}\right)$

$=e^{-\frac{3t}{2}}L^{-1}\left(\frac{2s}{s^2-(\sqrt{5}/2)^2}\right)-e^{-\frac{3t}{2}}L^{-1}\left(\frac{2}{s^2-(\sqrt{5}/2)^2}\right)$

$=e^{-\frac{3t}{2}}\left(2\cosh\frac{\sqrt{5}t}{2}-2\sinh\frac{\sqrt{5}}{2}t\times\frac{2}{\sqrt{5}}\right)$

$=2e^{-3t/2}\left(\cosh\frac{\sqrt{5}}{2}t-\frac{2}{\sqrt{5}}\sinh\frac{\sqrt{5}t}{2}\right)$

iii) $L^{-1}\left(\frac{7s+4}{4s^2+4s+9}\right)$ $\quad 4s^2+4s+9$
$=4\left(s^2+s+9/4\right)$
$=4\left[(s)^2+2(s)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+\frac{9}{4}\right]$
$=4\left[(s+1/2)^2+\frac{-1+9}{4}\right]$
$=4\left[(s+1/2)^2+2\right]$ $\quad \frac{-7}{2}+\frac{8}{2}=\frac{1}{2}$

$-\frac{7}{2}+4$

$=L^{-1}\left(\frac{7(s+1/2)-7/2+4}{4\left[(s+1/2)^2+2\right]}\right)$

$=L^{-1}\left(\frac{7(s+1/2)}{4\left[(s+1/2)^2+(\sqrt{2})^2\right]}\right)+L^{-1}\left(\frac{1/2}{4\left[(s+1/2)^2+(\sqrt{2})^2\right]}\right)$

$=\frac{7}{4}e^{-\frac{t}{2}}L^{-1}\left(\frac{s}{s^2+(\sqrt{2})^2}\right)+\frac{1}{8}e^{-t/2}L^{-1}\left(\frac{1}{s^2+(\sqrt{2})^2}\right)$

$=\frac{7}{4}e^{-t/2}\cos\sqrt{2}t+\frac{1}{8}e^{-t/2}\sin\sqrt{2}t\times\frac{1}{\sqrt{2}}$

$=\frac{e^{-t/2}}{4}\left(7\cos\sqrt{2}t+\frac{\sin\sqrt{2}t}{2\sqrt{2}}\right)$

vi) $\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$ $\rightarrow s^2+2s+2$
$=s^2+2(s)(1)+1^2-1^2+2$; $\quad s^2+2s+5$, $(s+1)^2+4$
$=(s+1)^2+1$

$\rightarrow$ let $(s+1)=x \Rightarrow s=x-1$

$\Rightarrow L^{-1}\left(\frac{(x-1)^2+2(x-1)+3}{(x^2+1)(x^2+4)}\right)$

$\rightarrow (x-1)^2+2(x-1)+3=x^2+1-\cancel{2x}+\cancel{2x}-2+3$
$=x^2+2$

$\rightarrow \frac{x^2+2}{(x^2+1)(x^2+4)}=\frac{A}{x^2+1}+\frac{B}{x^2+4}$

$x^2+2=A(x^2+4)+B(x^2+1)$
$x^2+2=(A+B)x^2+4A+B$
$\Rightarrow A+B=1 \Rightarrow B=1-A$
$4A+B=2$ $\quad B=2/3$
$4A+1-A=2$
$3A=1$
$A=1/3$

$\rightarrow L^{-1}\left(\frac{1/3}{x^2+1}\right)+L^{-1}\left(\frac{2/3}{x^2+4}\right)$

$=L^{-1}\left(\frac{1/3}{(s+1)^2+1}\right)+L^{-1}\left(\frac{2/3}{(s+1)^2+4}\right)$

$=e^{-t}L^{-1}\left(\frac{1/3}{s^2+1^2}\right)+e^{-t}L^{-1}\left(\frac{2/3}{s^2+2^2}\right)$

$=e^{-t}\left(\frac{\sin t}{3}+\frac{\sin 2t}{\cancel{2}}\times\frac{\cancel{2}}{3}\right)$

$=\frac{e^{-t}}{3}\left(\sin t+\sin 2t\right)$

iv) $\frac{2(s^2+2a^2)e^{-2s}}{s^4+4a^4}$

$L^{-1}\left(e^{-as}F(a)\right)=f(t-a)u(t-a)$
$a=2$
$=f(t+2)u(t-2)$ $\quad (s^2)^2+(2a^2)^2$
$\rightarrow F(a)=\frac{2(s^2+2a^2)}{s^4+4a^4}$ $\quad =(s^2+2a^2)^2-2(s^2)(2a^2)$

$=\frac{2(s^2+2a^2)}{(s^2)^2+(2a^2)^2}$ $\quad \langle$lengthy asf $\rightarrow$ at Night$\rangle$

$L^{-1}(F(a))=$

v) $\frac{5s+3}{(s-1)(s^2+2s+5)}$ $\quad s^2+2s+5$, s^2+

$\rightarrow \frac{5s+3}{(s-1)(s^2+2s+5)}=\frac{A}{(s-1)}+\frac{Bs+C}{(s^2+2s+5)}$

$5s+3=A(s^2+2s+5)+(Bs+C)(s-1)$
at $s=1 \Rightarrow 8=8A \Rightarrow A=1$
at $s=0 \Rightarrow 3=5A-C \Rightarrow C=2$
at $s=2 \Rightarrow 13=A(4+4+5)+(2B+C)(2-1)$
$13=13+(2B+2)$
$2B=-2 \Rightarrow B=-1$

$\rightarrow s^2+2s+5$
$=s^2+2(s)(1)+1^2-1^2+5$
$=(s+1)^2+4$

$\rightarrow L^{-1}\left(\frac{1}{s-1}\right)-L^{-1}\left(\frac{-s+2-1+1}{(s+1)^2+2^2}\right)$

$=e^t L^{-1}\left(\frac{1}{s}\right)-L^{-1}\left(\frac{s+1}{(s+1)^2+2^2}\right)+L^{-1}\left(\frac{3}{(s+1)^2+2^2}\right)$

$=e^t\cdot(1)-e^{-t}L^{-1}\left(\frac{s}{s^2+2^2}\right)+e^{-t}L^{-1}\left(\frac{3}{s^2+2^2}\right)$

$=e^t-e^{-t}\cos 2t+e^{-t}\frac{3\sin 2t}{2}$

$=e^t+e^{-t}\left(\frac{3\sin 2t}{2}-\cos 2t\right)$

(viii) $\frac{e^{4-3s}}{(s+4)^{5/2}}$

$L^{-1}\left(\frac{e^4\cdot e^{-3s}}{(s+4)^{5/2}}\right)$

$=L^{-1}\left(\frac{e^4\cdot e^{-3(s+4)+12}}{(s+4)^{5/2}}\right)$

$=L^{-1}\left(\frac{e^{16}e^{-3(s+4)}}{(s+4)^{5/2}}\right)$

$=e^{-4t}L^{-1}\left(\frac{e^{16}e^{-3s}}{s^{5/2}}\right)$

$=e^{-4t}e^{16}L^{-1}\left(e^{-3s}s^{-5/2}\right)$

$=e^{-4t}e^{16}\cdot f(t-3)u(t-3)$

$=e^{16-4t}\frac{4(t-3)^{3/2}}{3\sqrt{\pi}}u(t-3)$

$L^{-1}\left(e^{-as}F(a)\right)=f(t-a)H(t-a)$
$\rightarrow a=3$
$\rightarrow F(a)=s^{-5/2}$
$L^{-1}(F(a))=L^{-1}\left(s^{-5/2}\right)$

$\rightarrow L^{-1}\left(\frac{1}{s^{5/2}}\right)=L^{-1}\left(\frac{1}{s^{3/2+1}}\right)$

$=\frac{t^{3/2}}{\Gamma(3/2+1)}$ $\quad \frac{\Gamma(3/2+1)}{s^{3/2+1}}=t^{3/2}$

$=\frac{t^{3/2}}{\frac{3\sqrt{\pi}}{4}}$ $\quad \Gamma(3/2+1)=3/2\,\Gamma(1/2+1)$
$=3/2\cdot 1/2\,\Gamma(1/2)$
$=\frac{3\sqrt{\pi}}{4}$

$f(t)=\frac{4t^{3/2}}{3\sqrt{\pi}}$

$f(t-3)=\frac{4(t-3)^{3/2}}{3\sqrt{\pi}}$

Ans: $\frac{4}{3\sqrt{\pi}}e^{-4(t-4)}(t-3)^{\frac{3}{2}}u(t-3)$

$-4t+16$

(vii) $\frac{1}{s}\left(\cos\frac{1}{s}\right)$ $\quad L\int_0^t f(t)dt=\frac{1}{s}F(s)$

$\int_0^t f(t)dt=L^{-1}\left(\frac{1}{s}F(s)\right)$

$L^{-1}\left(\frac{1}{s}\left(\cos\frac{1}{s}\right)\right)$

$\rightarrow$ here $F(s)=\cos\left(\frac{1}{s}\right)\rightarrow$ expansion ~~ew~~ later

$s\log(s+1)$
$\log(s+1)$ $\quad \frac{s}{s+1}$

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1] $L^{-1}\left(\log\frac{(s^2+1)}{s(s-1)}\right)$

$F(s)=\log(s^2+1)-\log s(s-1)$

2] $s\log\left(\frac{s+1}{s-1}\right)$

$F(s)=s\log(s+1)-s\log(s-1)$

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VECTOR CALCULUS, LAPLACE TRANSFORM AND NUMERICAL METHODS (MA221TA)

UNIT-IV: INVERSE LAPLACE TRANSFORM

TUTORIAL SHEET-II

Find the inverse Laplace transform of the following signals.

1. $\log\left[\frac{s^2+1}{s(s-1)}\right]$ **Ans:** $\frac{1+e^{-t}-2\cos t}{t}$
2. $s\log\left(\frac{s+1}{s-1}\right)$ **Ans:** $\frac{2}{t^2}(\sinh t - t\cosh t)$
3. $\tan^{-1}\left(\frac{2}{s^2}\right)$ **Ans:** $\frac{2\sin t\sinh t}{t}$
4. $\log\left(1+\frac{a^2}{s^2}\right)$ **Ans:** $2\left(\frac{1-\cos at}{t}\right)$
5. $\frac{e^{-3s}}{s}-\frac{e^{-s}}{s^2}$ **Ans:** $f(t)=\begin{cases}0 & t<1\\ 1-t & 1<t<3\\ 4-t & t\geq 3\end{cases}$
6. $\frac{e^{-3s}}{(s+1)^3}$ **Ans:** $u(t-3)\left(\frac{1}{2}e^{-(t-3)}(t-3)^2\right)$
7. $\frac{s+1}{(s^2+2s+2)^2}$ **Ans:** $\frac{t}{2}e^{-t}\sin t$
8. $\frac{1}{s(s^2+2s+2)}$ **Ans:** $\frac{1}{2}[1-e^{-t}(\sin t+\cos t)]$

Applying convolution theorem, find the inverse transform of the following functions.

1) $\frac{1}{(s^2+6)^2}$ **Ans:** $\frac{1}{12}\left(\frac{\sin\sqrt{6}t}{\sqrt{6}}+t\cos\sqrt{6}t\right)$
2) $\frac{s}{(s^2+6)^2}$ **Ans:** $\frac{t}{2\sqrt{6}}\sin\sqrt{6}t$
3) $\frac{1}{s^2(s^2+6)}$ **Ans:** $\frac{1}{6}\left(t-\frac{\sin\sqrt{6}t}{\sqrt{6}}\right)$
4) $\frac{3s+1}{(s-2)(s^2+1)}$ **Ans:** $\frac{1}{5}(7e^{2t}-7\cos t+\sin t)$
5) $\frac{1}{(s^2+4)(s+1)^2}$ **Ans:** $\frac{e^{-t}}{50}[10e^{-t}-3\sin 2t-4\cos 2t]$

Verify convolution theorem for the following functions.

1) $f(t)=\sin t,\ g(t)=e^{-t}$
2) $f(t)=t,\ g(t)=te^{-t}$
3) $f(t)=\sin at,\ g(t)=\cos at$
4) $f(t)=t,\ g(t)=\cos t$

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Department of Mathematics

VECTOR CALCULUS, LAPLACE TRANSFORM AND NUMERICAL METHODS (MA221TA)

UNIT-IV: INVERSE LAPLACE TRANSFORM

TUTORIAL SHEET-III

Solve the following differential equations using Laplace transform method.

1. $y''-3y'+2y=4t+e^{3t}, y(0)=1, y'(0)=1.$
   Ans: $y=3+2t-\frac{5}{2}e^{t}+\frac{1}{2}e^{3t}$
2. $\frac{d^3y}{dt^3}+\frac{d^2y}{dt^2}-4\frac{dy}{dt}-4y=t+1, y(0)=y'(0)=0$
   Ans: $y=\frac{5}{8}(\sinh 2t-2t)$
3. $y''+y=f(t), y(0)=1, y'(0)=0$, where $f(t)=\begin{cases}3, & 0\leq t\leq 4\\ 2t-5, & t>4\end{cases}$
   Ans: $y=3-2\cos t+2[(t-4)-\sin(t-4)]u(t-4)$
4. $\frac{d^2x}{dt^2}+9x=\cos 2t\ x(0)=1, x\left(\frac{\pi}{2}\right)=-1$
   Ans: $x=\frac{4}{5}\cos 3t+\frac{4}{5}\sin 3t+\frac{1}{5}\cos 2t$
5. The current i flowing in an electric circuit is governed by the differential equation $\frac{di}{dt}+i=E(t)$, e is a positive constant in $0<t<1$ and $E(t)=0$ for $t>1$. The circuit carries no current at time $t=0$. Find the current at any time $t>0$.
   Ans: $i=\begin{cases}E(1-e^{-t}), & 0<t\leq 1\\ E(e-1)e^{-t}, & t>1.\end{cases}$
6. $y'+y-2\int_0^t y\,dt=\frac{t^2}{2}, y(0)=1, y'(0)=-2$
   Ans: $y(t)=\frac{1}{3}e^{t}+\frac{11}{12}e^{-2t}-\frac{t}{2}-\frac{1}{4}$
7. $(D^2+1)y=\sin t\sin 2t, y(0)=1, y'(0)=0$
   Ans: $y(t)=\frac{15}{16}\cos t+\frac{t}{4}\sin t+\frac{1}{16}\cos 3t$

1] $L^{-1}\left(\log\frac{(s^2+1)}{s(s-1)}\right)$

$F(s)=\log(s^2+1)-\log s(s-1)$

$=\log(s^2+1)-\log(s(s-1))$

$-\frac{d}{dt}F(s)=-\frac{d}{dt}\left(\log(s^2+1)\right)-\left(-\frac{d}{dt}\left(\log(s(s-1))\right)\right)$

$=\frac{-2s}{s^2+1}+\frac{2s-1}{s(s-1)}$

$\frac{2s-1}{s(s-1)}=\frac{A}{s}+\frac{B}{s-1}$

$2s-1=A(s-1)+B(s)$

at $s=0$, $-1=A(-1)\Rightarrow A=1$

at $s=1$, $1=B(1)\Rightarrow B=1$

$tf(t)=L^{-1}\left(-\frac{d}{dt}(F(s))\right)$

$tf(t)=L^{-1}\left(\frac{-2s}{s^2+1}\right)+L^{-1}\left(\frac{1}{s}+\frac{1}{s-1}\right)$

$tf(t)=-2\cos t+1+e^t$

$f(t)=\frac{1+e^t-2\cos t}{t}$

3] $F(s)=\tan^{-1}\left(\frac{2}{s^2}\right)$

$-\frac{d}{dt}F(s)=-\frac{d}{dt}\tan^{-1}\left(\frac{2}{s^2}\right)$

$=-\frac{1}{1+\left(\frac{2}{s^2}\right)^2}\cdot 2\left(\frac{-2}{s^3}\right)$

$=\frac{s^4}{s^4+2^2}\cdot\frac{4}{s^3}$

$-\frac{d}{dt}F(s)=\frac{4s}{s^4+2^2}=\frac{4s}{s^4+4}$

$\rightarrow s^4+4=(s^2)^2+2(s^2)(2)+2^2-4s^2$

$=(s^2+2)^2-(2s)^2$

$s^2+4=(s^2+2+2s)(s^2+2-2s)$

$\rightarrow \frac{4s}{(s^2+2+2s)(s^2+2-2s)}=\frac{As+B}{s^2+2+2s}+\frac{Cs+D}{s^2+2-2s}$

$4s=(As+B)(s^2+2-2s)+(Cs+D)(s^2+2+2s)$

$4s=As^3+2As-2As^2+Bs^2+2B-\cancel{2Bs}+Cs^3+2Cs+2Cs^2+Ds^2+2D+2Ds$

$\rightarrow A+C=0\Rightarrow C=-A\ /\ A=-C$

$-2A+B+2C+D=0\Longrightarrow 2C-\cancel{D}+2C+\cancel{D}=0\Rightarrow C=0$ $\Rightarrow A=0$

$2A+2C-2B+2D=4\Rightarrow 4D=4\Rightarrow D=1$ $B=-1$

$2B+2D=0\Rightarrow D=-B\ /\ B=-D$

$\rightarrow \frac{4s}{s^4+4}=\frac{1}{s^2+2-2s}-\frac{1}{s^2+2+2s}$

$-\frac{d}{dt}F(s)=\frac{4s}{s^4+4}=\frac{1}{(s-1)^2+1}-\frac{1}{(s+1)^2+1}$

$\rightarrow tf(t)=L^{-1}\left(-\frac{d}{dt}F(s)\right)$

$=L^{-1}\left(\frac{1}{(s-1)^2+1^2}\right)-L^{-1}\left(\frac{1}{(s+1)^2+1^2}\right)$

$tf(t)=e^tL^{-1}\left(\frac{1}{s^2+1^2}\right)-e^{-t}L^{-1}\left(\frac{1}{s^2+1^2}\right)$

$f(t)=\frac{\sin t(e^t-e^{-t})}{t}$

$f(t)=\frac{2\sin t\sinh t}{t}$

$\log(s+1)$ $s+1$

2] $s\log\left(\frac{s+1}{s-1}\right)$

$F(s)=s\log(s+1)-s\log(s-1)$

$-F'(s)=-\left[\log(s+1)+\frac{s}{s+1}-\log(s-1)-\frac{s}{s-1}\right]$

$-\left[\log(s+1)-\log(s-1)+\frac{s+1-1}{s+1}-\frac{s-1+1}{s-1}\right]$

$-\left[\log(s+1)-\log(s-1)+\cancel{1}-\frac{1}{s+1}-\cancel{1}-\frac{1}{s-1}\right]$

$-\log(s+1)+\log(s-1)+\frac{1}{s+1}+\frac{1}{s-1}$

$F''(s)=\frac{-1}{s+1}+\frac{1}{s-1}-\frac{1}{(s+1)^2}-\frac{1}{(s-1)^2}$

$t^2f(t)=L^{-1}(F''(s))$

$t^2f(t)=-e^{-t}+e^t-e^{-t}t-e^tt$

$t^2f(t)=2\frac{(e^t-e^{-t})}{2}-t\,2\frac{(e^t+e^{-t})}{2}$

$f(t)=\frac{2}{t^2}(\sinh t-t\cosh t)$

4] $\log\left(1+\frac{a^2}{s^2}\right)$

$\rightarrow F(s)=\log(s^2+a^2)-\log(s^2)$

$-\frac{d}{dt}F(s)=-\frac{d}{dt}\left[\log(s^2+a^2)-\log(s^2)\right]$

$=\frac{-2s}{s^2+a^2}+\frac{2s}{s^2}$

$-\frac{d}{dt}F(s)=\frac{2}{s}-\frac{2s}{s^2+a^2}$

$\rightarrow tf(t)=L^{-1}\left(-\frac{d}{dt}F(s)\right)$

$=L^{-1}\left(\frac{2}{s}\right)-L^{-1}\left(\frac{2s}{s^2+a^2}\right)$

$tf(t)=2-2\cos at$

$f(t)=\frac{2(1-\cos at)}{t}$

6] $\frac{e^{-3s}}{s}-\frac{e^{-s}}{s^2}$

$f(t-a)H(t-a)$, $a=3$, $F(s)=\frac{1}{s}$, $L^{-1}(F(s))=1$, $f(t)=1$, $f(t-3)=1$

$g(t-a)H(t-a)$, $a=1$, $G(s)=\frac{1}{s^2}$, $L^{-1}(G(s))=t$, $g(t)=t$, $g(t-1)=t-1$

$\rightarrow L^{-1}(e^{-3s}f(t))-L^{-1}(e^{-s}g(t))$

$=f(t-3)u(t-3)-g(t-1)\mu(t-1)$

$=\mu(t-3)-(t-1)\mu(t-1)$

7] $\frac{e^{-3s}}{(s+1)^3}$

$f(t-a)H(t-a)$, $a=3$

$\rightarrow F(s)=\frac{1}{(s+1)^3}$

$L^{-1}(f(s))=L^{-1}\left(\frac{1}{(s+1)^3}\right)$

$=e^{-t}L^{-1}\left(\frac{1}{s^3}\right)$

$f(t)=e^{-t}\frac{t^2}{2}$

$f(t-3)=\frac{(t-3)^2}{2}e^{-(t-3)}$

$\rightarrow L^{-1}(e^{-3s}F(s))=f(t-3)\mu(t-3)$

$=\frac{(t-3)^2}{2}e^{-(t-3)}\mu(t-3)$

7] 7. $\frac{s+1}{(s^2+2s+2)^2}$

s^2+2s+2

$s^2+2(s)(1)+1+1$

$(s+1)^2+1$

$L^{-1}\left(\frac{s+1}{((s+1)^2+1)^2}\right)=e^{-t}L^{-1}\left(\frac{s}{(s^2+1)^2}\right)$

$L^{-1}\left(\frac{1}{s^2+1}\right)=\sin t$

$L^{-1}\left(\frac{2s}{(s^2+1)^2}\right)=t\sin t$

$=e^{-t}\cdot\frac{1}{2}L^{-1}\left(\frac{2s}{(s^2+1)^2}\right)$

$=\frac{e^{-t}t\sin t}{2}$

8] $\frac{1}{s(s^2+2s+2)}$ $\frac{s^2+2s+2}{(s+1)^2+1}$

$F(s)=\frac{1}{(s+1)^2+1}$

$f(t)=L^{-1}(F(s))$

$=e^{-t}L^{-1}\left(\frac{1}{s^2+1}\right)$

$f(t)=e^{-t}\sin t$

$\rightarrow L^{-1}\left(\frac{F(s)}{s}\right)=\int_0^t f(t)$

ILATE

$I=\int_0^t e^{-t}\sin t$

$=-\sin t\,e^{-t}-\int\cos t(-e^{-t})dt$

$=-\sin t\,e^{-t}+\left[-\cos t\,e^{-t}-\int(+\sin t)(e^{-t})\right]$

$I=-\sin t\,e^{-t}-\cos t\,e^{-t}-I$

$2I=-e^{-t}(\sin t+\cos t)$

$I=-\frac{e^{-t}}{2}(\sin t+\cos t)\Big|_0^t$

$I=\frac{1}{2}\left[1-e^{-t}(\sin t+\cos t)\right]$