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RV COLLEGE OF ENGINEERING
Autonomous Institution affiliated to VTU
I Semester B.E. February -2024 Examinations
DEPARTMENT OF MATHEMATICS
FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS
(2022 SCHEME)
(Non-Integrated Course)

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, and 9 and 10.

PART-A (Objective type for one or two marks)
(True & false and match the following questions are not permitted)

1	1.1	Given $A=\begin{bmatrix}1 & 2 & 3\\ 0 & -1 & 5\\ 0 & 0 & 3\end{bmatrix}$, then the eigen values of the matrix A^{-2} are ____________.	1
	1.2	The reduced system of set of linear equations is $\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ 3\\ 0\end{bmatrix}$. Then the solution of the system is ____________.	1
	1.4	Angle between radius vector and tangent to the curve $r=e^{-\theta}$ is ____________.	2
	1.5	If rank of the matrix $A=\begin{bmatrix}1 & 5 & 4\\ 0 & 3 & 2\\ k & 13 & 10\end{bmatrix}$ is 2, then the value of $k=$_______.	2
	1.6	The Maclaurin series expansion for $\cosh x$ is _______.	2
	1.7	If $x=r\cos\theta$, $y=r\sin\theta$, then $J\left(\frac{x,y}{r,\theta}\right)=$ __________.	2
	1.8	If $w(x,y)=x^2y$ then $\frac{\partial^2 w}{\partial x\partial y}$ at the point (-1, 1)=__________.	2
	1.9	Evaluate $\int_0^1\int_2^3 xy\,dy\,dx$.	2
	1.10	Sketch the domain of integral $\int_2^5\int_1^3 x(x^2+y^2)dydx$.	2
	1.11	If $f(0)=0$, $f\left(\frac{1}{4}\right)=2.45$, $f\left(\frac{2}{4}\right)=3.97$, $f\left(\frac{3}{4}\right)=5.58$ and $f(1)=5.78$, then $\int_0^1 f(x)dx=$_______.	2
	1.12	Given $y(2)=-2$, $y(4)=4$, $y(6)=10$, $y(8)=12$, $y(10)=14$, then $\nabla^3 y(8)=$______.	2

PART-B

		UNIT-I	
2	a	Investigate for what values of λ and μ the system of linear equations $2x+3y+5z=9,\ 7x+3y-2z=8, 2x+3y+\lambda z=\mu$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.	5
	b	The currents I_1, I_2, I_3 in an electrical network follow the linear equations $8I_1-3I_2+2I_3=20, 4I_1+11I_2-I_3=33, 2I_1+I_2+4I_3=12$. Compute I_1, I_2, I_3 using Gauss-Seidel iteration method. Perform four iterations.	5
	c	Eigen values and eigen vectors are used to calculate the theoretical limit of how much information can be carried via a communication channel. Identify the dominant eigen value and corresponding eigen vectors of the matrix $A=\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking the initial approximation $\begin{bmatrix} 1 \\ 0.8 \\ 0.8 \end{bmatrix}$. Perform 5 iterations.	6

		UNIT-II	
3	a	Find the slope of the tangent to the parabola $\frac{2a}{r}=1-\cos\theta$ at $\theta=\frac{2\pi}{3}$.	8
	b	Obtain the circle for curvature of the curve $\sqrt{x}+\sqrt{y}=4$ at the point where it cuts the line passing through the origin making an angle 45^0 with x-axis.	8
		OR	
4	a	Show that the radius of curvature at any point (r,θ) on the Cardiod $r=a(1-\cos\theta)$ varies as $\sqrt{r}$.	8
	b	Expand $f(x)=\tan^{-1}x$ in ascending powers of x upto the term containing x^5, hence obtain the expansion of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.	8

		UNIT-III	
5	a	Prove that $v=e^{a\theta}\cos(a\log r)$ is the solution of the Laplace equation $v_{rr}+\frac{1}{r}v_r+\frac{1}{r^2}v_{\theta\theta}=0.$	8
	b	If $u=x^2+y^2+z^2, v=x+y+z$ and $w=xy+yz+zx$, prove that $J(u,v,w)$ vanishes identically. Also find the relation between the given functions.	8
		OR	
6	a	(i) If $z=f(y-3x)+g(y+2x)+\sin x-y\cos x$, show that $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-6\frac{\partial^2 z}{\partial y^2}=y\cos x.$ (ii) Find $\frac{du}{dt}$ given $u=\sin\left(\frac{x}{y}\right)$ where $x=e^t, y=t^2$.	8
	b	The temperature T at any point (x,y,z) in space is $T=600xyz^2$. Find the highest temperature on the unit sphere using Lagrange multiplier.	8

UNIT-IV			
7	a	Evaluate $\iint_R xydxdy \;\; dx\,dy$ where R is the triangular bounded by the axes of coordinates and the line $\frac{x}{2}+\frac{y}{3}=1$.	8
	b	Show that $\int_0^{\frac{\pi}{2}}\int_0^{a\sin\theta}\int_0^{\frac{a^2-r^2}{a}} r\;dz\,drd\theta=\frac{5\pi a^3}{64}$.	8
OR			
8	a	Using double integral find the area enclosed by the curve $r=a(1+\cos\theta)$ and lying above the initial line.	8
	b	Change the order of integration and evaluate $\int_1^0\int_x^{\sqrt{2-x^2}}\frac{x}{\sqrt{x^2+y^2}}\;dy\,dx$.	8

UNIT-V			
9	a	In an experiment, the values of the output y were recorded for input x from 1.0 to 3.5 at intervals of 0.5. Estimate by using appropriate interpolation formulae, the values of y for input $(i)\; x=1.2,\;(ii)\; x=3.4\;\;(iii)\; x=3.8$.	8
	b	Fit a cubic polynomial for the following data and hence find $f(5)$ and $f(6)$.	8
OR			
10	a	Numerical integration is used to simulate devices such as semiconductors and antennas. Estimate the value of the integral $\int_0^2 e^{-x^2}dx$ using Simpson's 1/3, Simpson's 3/8 and Weddle's rules, by dividing the interval [0,2] into six equal sub intervals.	8
	b	The following data defines the sea-level concentration of dissolved oxygen for fresh water as a function of temperature:	8

Table for 9 (a):

x	1.0	1.5	2.0	2.5	3.0	3.5
y	177	166	146	130	115	102

Table for 9 (b):

x	1	2	3	7
$f(x)$	2	4	8	128

Table for 10 (b):

T (℃)	0	8	15	25	32
$O_2\ (mg/L)$	14.621	11.843	9.870	8.418	7.305

Calculate the amount of oxygen when temperature 10℃ and 35℃.

Signature of Scrutinizer: Signature of Chairman

Name: Name: