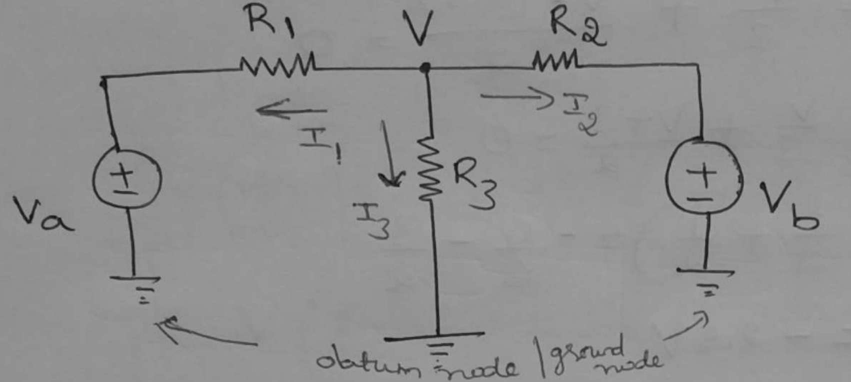
Node voltage analysis



Applying KCL at V

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V - V_a}{R_1} + \frac{V - V_b}{R_2} + \frac{V}{R_3} = 0$$

No of node equations = N − 1

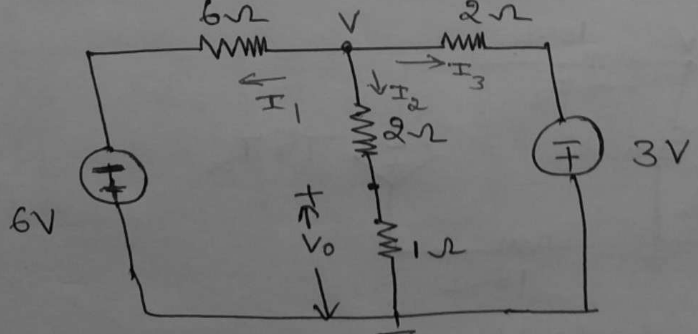
N = No of principle nodes

In the above figure no of principle node = 2

ie V and ground node | datum node | Reference node

∴ No of eqns = 2 − 1 = 1

Ex: use nodal analysis to find V_0 in the circuit in figure



No of nodes including datum node = 2

∴ No of equation = 2 − 1 = 1

KCL at V

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V-(-6)}{6} + \frac{V}{3} + \frac{V-(-3)}{2} = 0$$

$$\frac{V+6}{6} + \frac{V}{3} + \frac{V+3}{2} = 0$$

$$V\left(\frac{1}{6} + \frac{1}{3} + \frac{1}{2}\right) = -\frac{6}{6} - \frac{3}{2}$$

$$V = -2.5V.$$

To find V_o,

$$V_o = I_2 \, 1\Omega$$

$$I_2 = \frac{V}{3} = -\frac{2.5}{3} = -0.833A$$

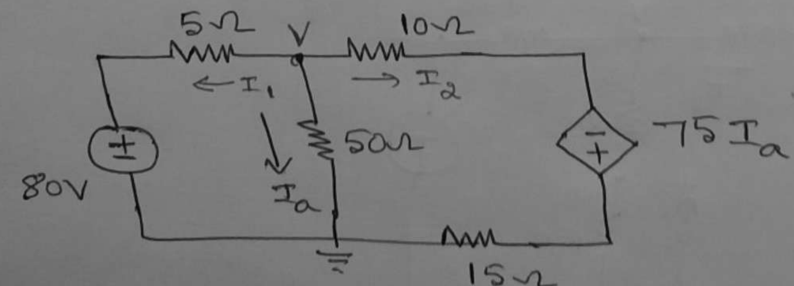
$$\boxed{\therefore V_o = -0.833V}$$

(or) using voltage division rule

$$\boxed{V_o = \frac{V(1)}{2+1} = -\frac{2.5}{3} = -0.833V}$$

Ex: of nodal analysis using dependent source

Find the power delivered by the dependent V/g source in the n/w shown in the figure.



Solution: KCL at node V,

$$I_1 + I_a + I_2 = 0$$

$$\frac{V-80}{5} + \frac{V}{50} + \frac{V+75I_a}{25} = 0$$

$$V\left(\frac{1}{5}+\frac{1}{50}+\frac{1}{25}\right)-\frac{80}{5}+\frac{75\,I_a}{25}=0$$

But $I_a = \dfrac{V}{50}$

$$V(0.26)+\frac{75}{25}\left(\frac{V}{50}\right)=16$$

$$V(0.32)=16$$

$$V = 50\,V$$

$$I_a = \frac{50}{50} = 1\,A$$

$$\text{Power}_{75 I_a} = VI = 75 I_a I_2$$

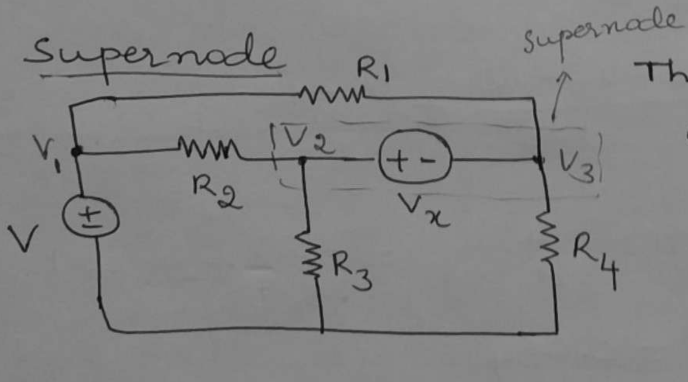
$$P = 75(1)(I_2)$$

$$I_2 = \frac{V+75I_a}{25} = \frac{50+75(1)}{25} = 5$$

$$P = 75(1)(5) = 375\,W$$

$$\boxed{P = 375\,W}$$

Supernode



The nodes V_2 and V_3 are connected directly though a voltage source without any circuit element.

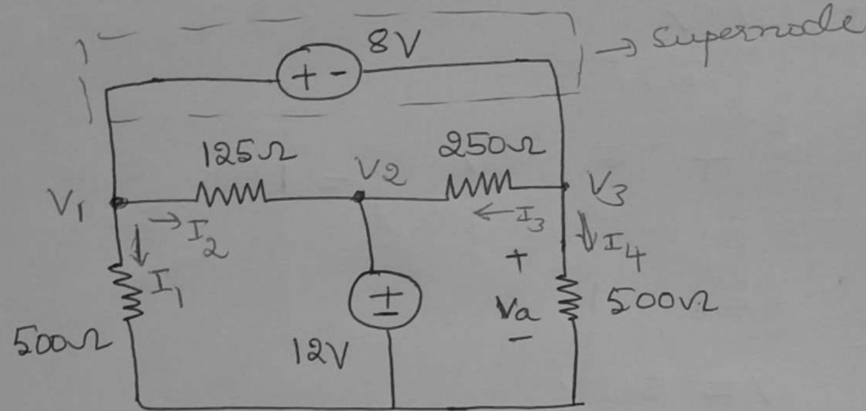
A voltage source which connects the two node directly is called supernode

Step 1 : write supernode constraint equation ie

$$V_x = V_2 - V_3$$

Step 2 : Apply KCL at V_2 and V_3 simultaneously.

Ex: using nodal analysis, find V_a for the circuit shown in figure.



Solution: The constraint equation is,

$$V_1 - V_3 = 8 \quad \text{— (1)}$$

Apply KCL at the supernode, ie V_1 and V_3

$$I_1 + I_2 + I_3 + I_4 = 0$$

$$\frac{V_1}{500} + \frac{V_1 - V_2}{125} + \frac{V_3 - V_2}{250} + \frac{V_3}{500} = 0$$

From equn (1) $V_1 = 8 + V_3$

At node $V_2 = 12$

$$\frac{8+V_3}{500} + \frac{8+V_3-12}{125} + \frac{V_3-12}{250} + \frac{V_3}{500} = 0$$

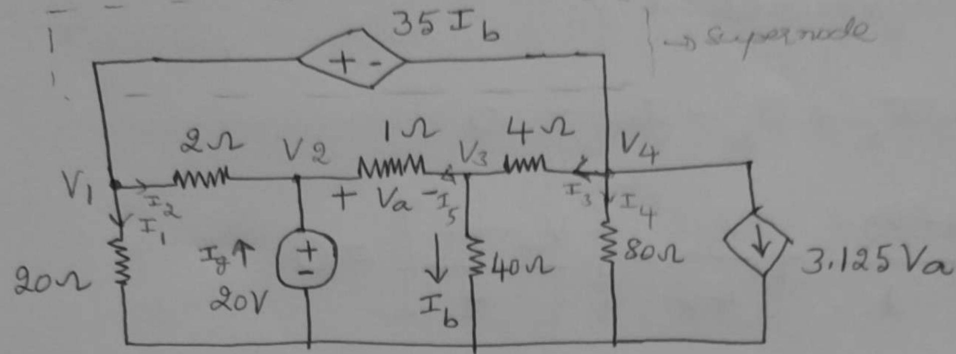
$$V_3\left(\frac{1}{500} + \frac{1}{125} + \frac{1}{250} + \frac{1}{500}\right) = -\frac{8}{500} + \frac{4}{125} + \frac{12}{250}$$

$$V_3(0.016) = 0.064$$

$$V_3 = 4V$$

$$V_a = V_3 = 4V$$

Ex: use the node Voltage method to find the power developed by the 20V Source in the circuit shown in fig



Solution: Constraint equations

$$V_1 - V_4 = 35 I_b \quad —①$$

$$I_b = \frac{V_3}{40}, \quad V_2 = 20V$$

KCL at supernode V_1 and V_4

$$I_1 + I_2 + I_3 + I_4 + 3.125 V_a = 0$$

$$\frac{V_1}{20} + \frac{V_1 - V_2}{2} + \frac{V_4 - V_3}{4} + \frac{V_4}{80} + 3.125 V_a = 0 \quad —②$$

But $V_a = V_2 - V_3 = 20 - V_3$ — ③

From equation①, $V_1 = 35 I_b + V_4 = 35 \cdot \frac{V_3}{40} + V_4$

$$V_1 = 0.875 V_3 + V_4 \quad —④$$

Sub equⁿ ③ and ④ in equⁿ ②

$$\frac{0.875V_3 + V_4}{20} + \frac{0.875V_3 + V_4}{2} - \frac{20}{2} + \frac{V_4}{4} - \frac{V_3}{4} + \frac{V_4}{80} + 3.125(20 - V_3) = 0$$

$$V_3\left(\frac{0.875}{20} + \frac{0.875}{2} - \frac{1}{4} - 3.125\right) + V_4\left(\frac{1}{20} + \frac{1}{2} + \frac{1}{4} + \frac{1}{80}\right) = 10 - 3.125(20)$$

$V_3(-2.893) + V_4(0.8125) = -52.5$ — (5)

Applying KCL at V_3

$I_5 + I_6 - I_3 = 0$

$\frac{V_3 - V_2}{1} + \frac{V_3}{40} - \left\{\frac{V_4 - V_3}{4}\right\} = 0$

$V_3 - 20 + \frac{V_3}{40} - \frac{V_4}{4} + \frac{V_3}{4} = 0$

$V_3\left(1 + \frac{1}{40} + \frac{1}{4}\right) - V_4\left(\frac{1}{4}\right) = 20$

$V_3(1.275) - V_4(0.25) = 20$ — (6)

solve equ^n (5) and (6)

$$\begin{bmatrix} -2.893 & 0.8125 \\ 1.275 & -0.25 \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -52.5 \\ 20 \end{bmatrix}$$

$\Delta = -0.312$, $\Delta_3 = -3.125$, $V_3 = \frac{\Delta_3}{\Delta} = 10V$

$\Delta_4 = 9.0775$, $V_4 = \frac{\Delta_4}{\Delta} = -29.1V$

$\therefore V_1 = 0.875V_3 + V_4 = 0.875(10) - 29.1V$

$V_1 = -20.35V$

$Power_{20V} = VI = 20I_g$

To find I_g, KCL at $V_2 \Rightarrow I_g + I_2 + I_5 = 0$

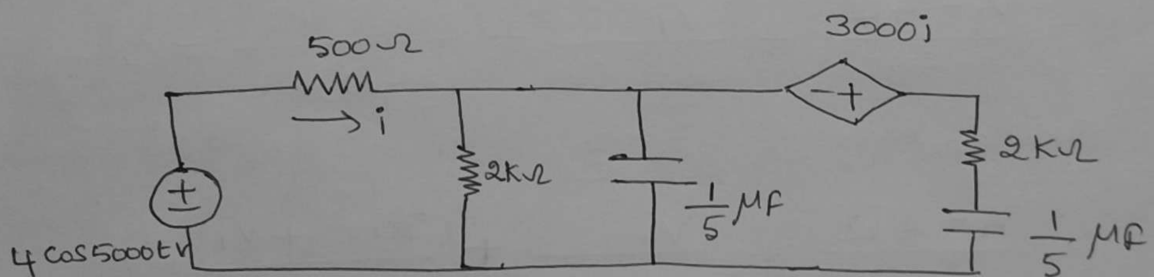
$I_g = -I_2 - I_5 = -\left[\frac{V_1 - V_2}{2}\right] - \left[\frac{V_3 - V_2}{1}\right]$

$I_g = -\left[\frac{-20.35 - 20}{2}\right] - \left[\frac{10 - 20}{1}\right] = 30.175A$

$\boxed{P_{20V} = (20)(30.175) = 603.5W}$

Type of the circuit	Impedance Z
purely resistive →	$Z = R$
purely inductive →	$Z = j\omega L = jX_L$
purely capacitive →	$Z = \frac{-j}{\omega C} = -jX_C = \frac{1}{j\omega C}$
RL →	$Z = R + j\omega L = R + jX_L$
RC →	$Z = R - \frac{j}{\omega C} = R - jX_C$
RLC →	$Z = R + j\omega L - \frac{j}{\omega C} = R + j(X_L - X_C)$

Ex: using nodal technique, find the current i in the circuit shown in figure.



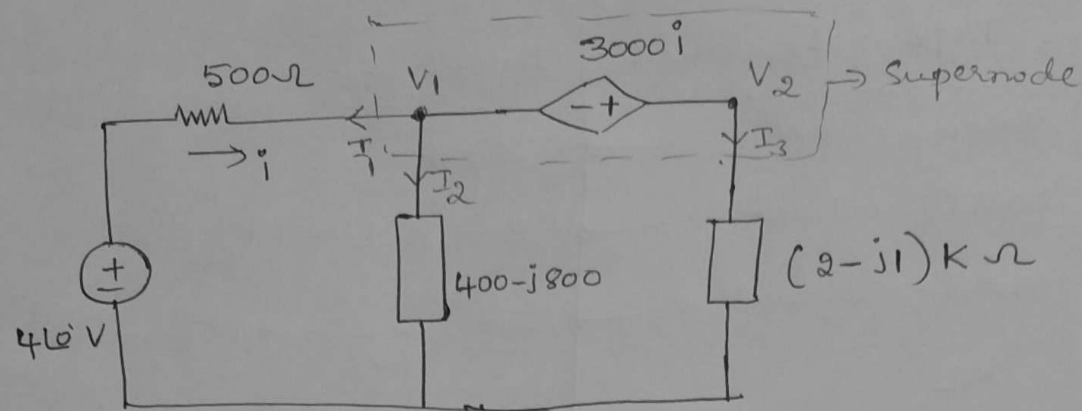
The reactance of the capacitor $= \frac{1}{j\omega C}$

$$= \frac{1}{j(5000)(\frac{1}{5} \times 10^{-6})}$$

$$= -j1k\Omega$$

The parallel combinations of $2k\Omega$ and $-j1k\Omega$ is

$$Z_p = \frac{2k(-jk)}{2k - jk} = 400 - j800$$



Constraint equation,

$$V_2 - V_1 = 3000i$$

$$V_2 = 3000i + V_1 \quad \text{—} ①$$

Kcl at Super node

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_1 - 4\angle 0}{500} + \frac{V_1}{400-j800} + \frac{V_2}{(2-j1)K} = 0 \quad \text{—} ②$$

Sub equⁿ ① in equⁿ ②

$$\frac{V_1 - 4\angle 0^\circ}{500} + \frac{V_1}{400-j800} + \frac{3000i + V_1}{(2-j1)K} = 0$$

$$i = \frac{4\angle 0^\circ - V_1}{500}$$

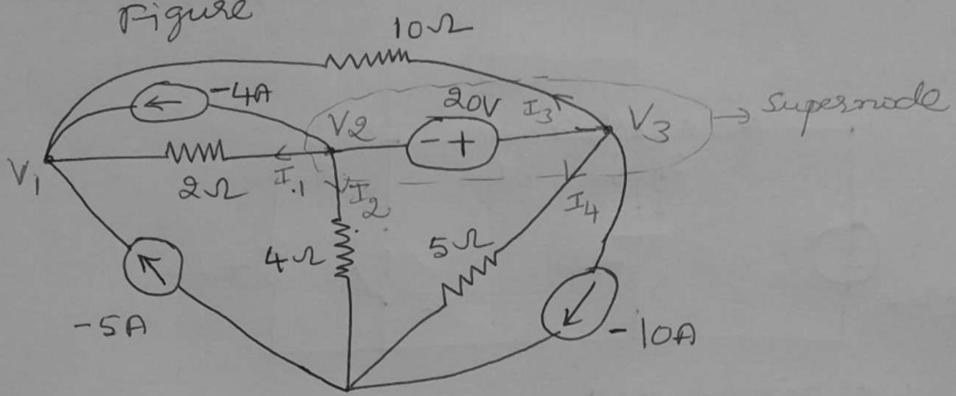
$$\therefore \frac{V_1}{500} - \frac{4\angle 0^\circ}{500} + \frac{V_1}{400-j800} + \frac{3000\left(\frac{4\angle 0^\circ - V_1}{500}\right)}{(2-j1)K} + \frac{V_1}{(2-j1)K} = 0$$

By solving $V_1 = (-7201.04 - j9596.2)\,mV$

$$\therefore i = \frac{4\angle 0 - V_1}{500} = \boxed{24\angle 53.1\ mA} \Rightarrow 24\angle{-126.86}\ mA$$

In time domain, $\boxed{i = 24\cos(5000t + 53.1)\ mA}$

Ex: Find the node voltages in the circuit shown in Figure



constrain equations,

$$V_3 - V_2 = 20 \quad —(1)$$

Kcl at supernodes V_2 and V_3

$$I_1 + I_2 + I_3 + I_4 - 4 - 10 = 0$$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{4} + \frac{V_3 - V_1}{10} + \frac{V_3}{5} - 14 = 0$$

$$V_1\left(-\frac{1}{2} - \frac{1}{10}\right) + V_2\left(\frac{1}{2} + \frac{1}{4}\right) + V_3\left(\frac{1}{10} + \frac{1}{5}\right) = 14$$

$$-0.6V_1 + 0.75V_2 + 0.3V_3 = 14 \quad —(2)$$

Kcl at node 1

$$I_1 + I_3 - 4 - 5 = 0$$

$$\frac{V_2 - V_1}{2} + \frac{V_3 - V_1}{10} = 9$$

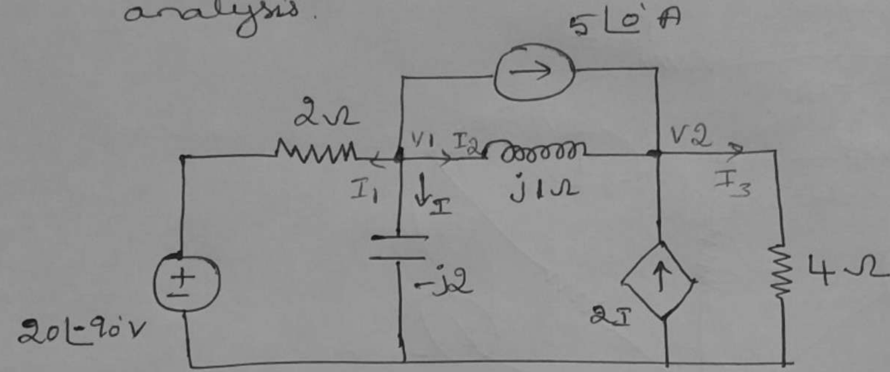
$$V_1\left(-\frac{1}{2} - \frac{1}{10}\right) + V_2\left(\frac{1}{2}\right) + V_3\left(\frac{1}{10}\right) = 9$$

$$-0.6V_1 + 0.5V_2 + 0.1V_3 = 9 \quad —(3)$$

solving equ^n (1), (2) and (3) we get

$$\boxed{V_1 = -9.44V, \quad V_2 = 2.226V, \quad V_3 = 22.226V}$$

Ex: Solve for current I in the circuit using nodal analysis.



KCl at node 1

$$\frac{V_1 - 20\angle{-90}}{2} + \frac{V_1}{-j2} + 5\angle 0 + \frac{V_1 - V_2}{j1} = 0$$

$$(0.5 - j0.5)V_1 + jV_2 = -5 - j10 \quad -(1)$$

KCl at node 2

$$I_2 + 2I + 5\angle 0° - I_3 = 0$$

$$\frac{V_1 - V_2}{j1} + 2\left[\frac{V_1}{-j2}\right] + 5\angle 0° - \frac{V_2}{4} = 0$$

$$V_1\left(\frac{1}{j1} - \frac{2}{j2}\right) - V_2\left(\frac{1}{j1} + \frac{1}{4}\right) = -5\angle 0°$$

$$V_2 = \frac{5}{0.25 - j} \quad -(2)$$

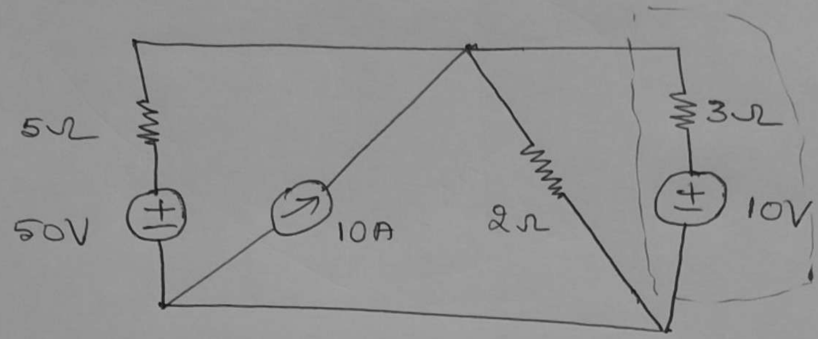
Sub equn (2) in equn (1), we get.

$$V_1 = 15.81\angle{-46.5} \text{ V}$$

$$\therefore I = \frac{V_1}{-j2} = \frac{15.81\angle{-46.5}}{2\angle{-90°}} = 5.73 + j5.441$$

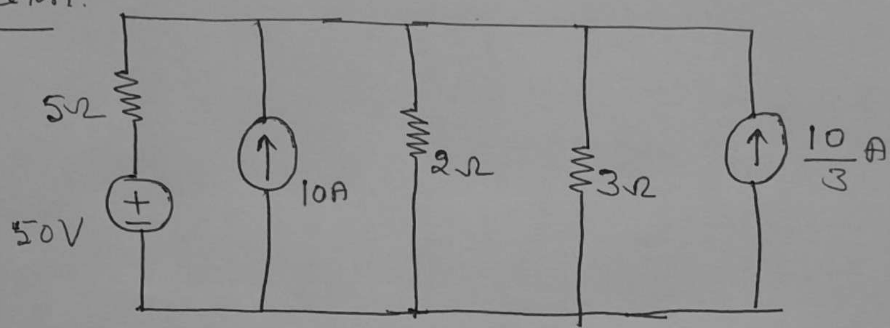
$$\boxed{I = 7.90\angle 43.5 \text{ A}}$$

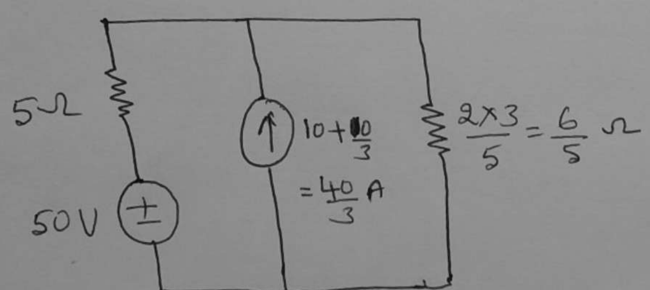
Ex: using source transformation find the power delivered by the 50V voltage source in the circuit shown in figure.

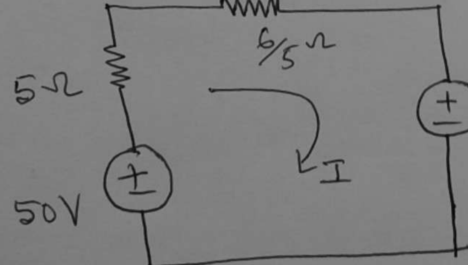


→ convert to current source

Solution.







$V = \frac{40}{3} \times \frac{6}{5} = 16V$

Apply KVL to mesh $\Rightarrow 5I + \frac{6}{5}I + 16 - 50 = 0$

$$6.2I = 34$$

$$I = 5.48A$$

$\therefore$ $\text{power}_{50V} = VI = 50I = (50)(5.48)$

$$\boxed{P = 274W}$$