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Department of Mathematics

VECTOR CALCULUS, LAPLACE TRANSFORM & NUMERICAL METHODS (MA221TA)

UNIT-II

VECTOR INTEGRATION

TUTORIAL SHEET

1. Find the total work done by the force represented by $\vec{F} = 3xy\hat{\imath} - y\hat{\jmath} + 2zx\hat{k}$ in moving a particle round the circle $x^2 + y^2 = 4, x = 2\cos\theta, y = 2\sin\theta$ & $z = 0, 0 \le \theta \le 2\pi$.
2. Evaluate $\int_C y^2 dx - 2x^2 dy$ along the parabola y = x^2 from (0,0) to (2,4).
3. Evaluate the line integral $\int_C (x^2 + xy)dx + (x^2 + y^2)dy$, where $C{:}\, square{:}\, x = \pm 1, \quad y = \pm 1 \quad Ans{:}\, 0$
4. Verify Green's theorem for $\int_C (e^{-x}\sin y)dx + (e^{-x}\cos y)dy$, where C is the rectangle, whose vertices are $(0,0), (\pi, 0), \left(\pi, \frac{\pi}{2}\right)$ *and* $\left(0, \frac{\pi}{2}\right)$. Ans: $[2(e^{-\pi} - 1)]$
5. Using Green's theorem, evaluate $\oint_C (x^2 - \cosh y\, dx + (y + sinx)dy)$ where C is the boundary of the rectangle $0 \le x \le \pi,\ 0 \le y \le 1$. $Ans{:}\, \pi(\cosh 1 - 1)$
6. Using the Green's theorem ,find the area enclosed between the parabolas y^2 = 4ax and x^2 = 4ay
7. If S is the surface of the sphere $x^2 + y^2 + z^2 = d^2$ and $\vec{A} = ax\hat{\imath} + by\hat{\jmath} + cz\hat{k}$, evaluate $\iint_S \vec{A} \cdot \hat{n}\, ds$. $Ans{:}\, \frac{2\pi d^3}{3}(a + b + c)$
8. If $\vec{F} = 2y\hat{\imath} - 3\hat{\jmath} + x^2\hat{k}$ and S is the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4$ and $z = 6$, show that $\iint_S \vec{F} \cdot \hat{n}\, ds = 132$.
9. Find the surface integral over the parallelepiped $x = 0, y = 0, x = 1, y = 2, z = 3$ when $\vec{A} = 2xy\hat{\imath} + yz^2\hat{\jmath} + xz\hat{k} \quad Ans{:}\, 33$.
10. Using divergence theorem, evaluate $\iint_S \vec{r}.\hat{n}\, ds$ where S is the surface of the sphere$x^2 + y^2 + z^2 = 9$. $Ans{:}\, 108\pi$
11. Verify divergence theorem for $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\,\hat{k}$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.
12. Using divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}\, ds$ over the entire surface S of the region above xy plane bounded by the cone $x^2 + y^2 = z^2$ the plane $z = 4$ where $\vec{F} = 4xz\hat{\imath} - xyz^2\hat{\jmath} + 3z\,\hat{k} \quad Ans{:}\, 704\pi$
13. Verify Stokes's theorem where $\vec{A} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\,\hat{k}$ and S: upper half of the surface of the sphere $x^2 + y^2 + z^2 = 1$ $Ans{:}\, \pi$
14. Evaluate $\oint_C xy\, dx + xy^2\, dy$ by Stoke's theorem where C is the square in the xy plane with vertices (1,0) (−1,0) (0,1) (0, −1).
15. Evaluate $\oint_C 4z\, dx - 2x\, dy + 2x\, dz$ by Stoke's theorem where C is the ellipse $x^2 + y^2 = 1,\ z = y + 1$. $Ans{:}\, -4\pi$