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# NUMERICAL METHODS

## Solution of algebraic and transcendental equations :-

Algebraic equation is an equation obtained from multiplying, adding, subtracting or dividing two polynomials and equating it to zero.

Ex :- $x^2 - x + 1 = 0$

Transcendental equation is an equation that contains algebraic, logarithmic, trigonometric or exponential functions is called transcendental equation.

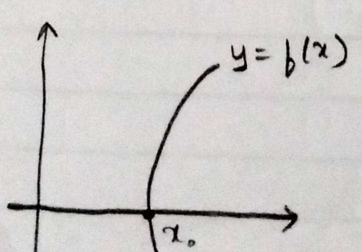
Trigonometric functions are also called as Circular Equations.

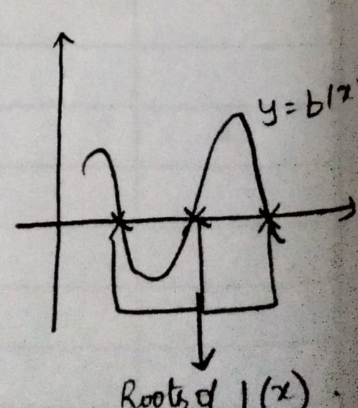
Ex :- $\sin x + x = 0$

$\tan x + \sec x = 1$.

- **Root of an Equation :-**

The value of $x$ which satisfies the given equation is called root of the equation. Geometrically, if $x_0$ is root of the equation, $f(x) = 0$, then the graph of $y = f(x)$ meets $x$ axis at $x = x_0$



- **Direct method :-**

  It is the method which gives the exact solutions, all solutions at a time.

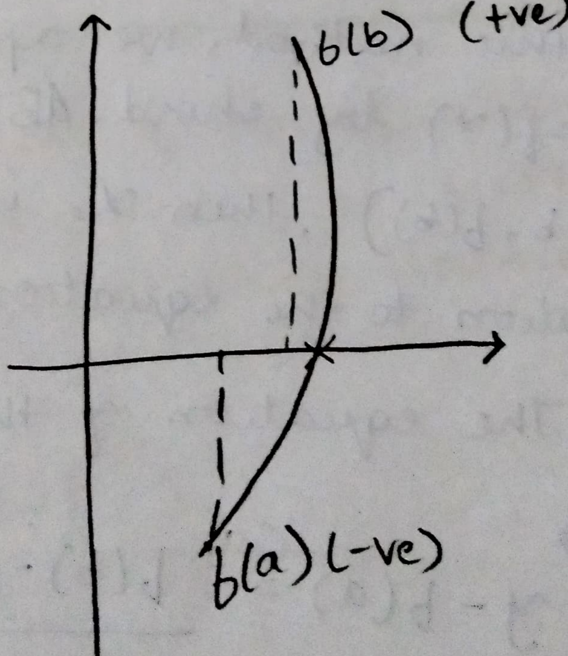
- **Iterative method :-**

  This method gives an approximate solution of the equation starting from one or two initial solutions / approximations to improve the solution by iterating the process.

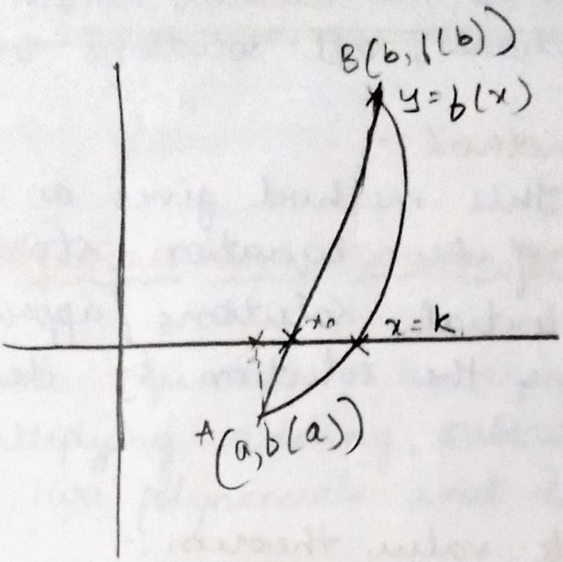
- **Intermediate value theorem :-**

  If $f(x)$ is continuos in the interval $[a, b]$ such that $f(a)$, $f(b)$ are of opposite sides $[f(a) f(b) < 0]$, there exists is atleast one real root between a & b

We get only one solution.



## REGULA FALSI METHOD (or) METHOD OF FALSE POSITION :-




Let $f(x)$ is continuous function in $(a,b)$, such that $f(a)$, $f(b)$ are opposite signs. Let the graph of $f(x)$ meet $x$ axis at $x = k$. Therefore $k$ is the exact root of $f(x)$.

In this method, we approximate, the curve $y = f(x)$ by chord AB where $A(a, f(a))$ and $B(b, f(b))$, then $x_0$ is the initial approximation to the equation $f(x) = 0$.

The equation of the chord AB,

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

The chord AB meets $x$ axis at $(x_0, 0)$. Substituting this point in the above equation,

$$0 - f(a) = \frac{f(b) - f(a)}{b - a}(x_0 - a)$$

$\Rightarrow \quad -bf(a) + \cancel{af(a)} = x_0 \cdot f(b) - x_0 \cdot f(a) - af(b) + \cancel{af(a)}$

$\Rightarrow \quad -bf(a) = x_0 \cdot f(b) - x_0 \cdot f(a) - af(b)$

$\Rightarrow \quad af(b) - bf(a) = x_0 \cdot (f(b) - f(a))$

$$x_0 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$\therefore \quad * \boxed{x = \frac{af(b) - bf(a)}{f(b) - f(a)}} *$$

1] Find the real root of the equation, $x^3 + 2x - 5 = 0$, by method of false position, correct to three decimal places.

$\rightarrow \quad x^3 + 2x - 5 = 0$

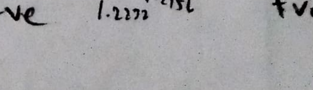
$f(x) = x^3 + 2x - 5.$

$f(0) = -5 < 0$

$f(1) = -2 < 0$

$f(2) = 7 > 0$.

$\therefore$ The real root lies in between 1 & 2.




$a = 1, \quad b = 2$

$f(a) = -2$

$f(b) = 7$

$$x_0 = \frac{af(b) - bf(a)}{f(b) - f(a)} \Rightarrow \frac{1(7) - 2(-2)}{7+2} = \frac{11}{9}$$

$= 1.2222$

$f(x_0) = (1.2222)^3 + 2(1.2222) - 5$

$= 1.8257 + 2.4444 - 5$

$= -0.7299$

$a = 1.2222$, & $b = 2$.

$$x_1 = \frac{(1.2222)(7) - 2(-0.7299)}{7 + 0.7299}$$

$$= \frac{8.5554 + 1.4598}{7.7299}$$

$x_1 = 1.2956$

$f(x_1) = (1.2956)^3 + 2(1.2956) - 5$

$= 2.1748 + 2.5912 - 5$

$= -0.2340$

$a = 1.2956$ $\quad$ $b = 2$

$f(a) = -0.2340$ $\quad$ $f(b) = 7$.

$$x_2 = \frac{(1.2956)(7) - 2(-0.2340)}{7 - (-0.2340)}$$

$$= \frac{9.0692 + 0.4680}{7 + 0.2340}$$

$$= \frac{9.5372}{7.2340}$$

$x_2 = 1.3184$

$f(x_2) = 2.2916 + 2.6368 - 5$

$= -0.0716$

$a = 1.3184$ $\quad$ $b = 2$

$f(a) = -0.0716$ $\quad$ $f(b) = 7$.

$$\Rightarrow \quad x_3 = \frac{1.3184(7) - 2(-0.0716)}{7 + 0.0716}$$

$$= \frac{9.2288 + 0.1432}{7.0716}$$

$$x_3 = \underline{\underline{1.3253}}$$

$$f(x_3) = 2.3278 + 2.6506 - 5$$

$$= \underline{\underline{-0.0216}}.$$

$a = 1.3253$ $\quad$ $b = 2$

$f(a) = -0.0216$ $\quad$ $f(b) = 7$

$$x_4 = \frac{(1.3253)(7) - 2(-0.0216)}{7 + 0.0216}$$

$$= \frac{9.2771 + 0.0432}{7.0216}$$

$$= \frac{9.3203}{7.0216} \Rightarrow \underline{\underline{1.3274}}.$$

$$f(x_4) = 2.3389 + 2.6548 - 5$$

$$= \underline{\underline{-0.0063}}.$$

$a = 1.3274$. $\quad$ $b = 2$

$f(a) = -0.0063$ $\quad$ $f(b) = 7$.

$$x_5 = \frac{(1.3274)7 - 2(-0.0063)}{7 - (-0.0063)}$$

$$= \frac{9.3044}{7.0063} = 1.3280.$$

$$f(x_5) = -0.0020$$

$$a = 1.3280 \qquad b = 2$$

$$f(a) = -0.0020 \qquad f(b) = 7.$$

$$x_6 = \frac{(1.3280)7 - 2(-0.002)}{7-(-0.002)}$$

$$= \frac{9.3}{7.002} = \underline{\underline{1.3282}}.$$

$$f(x_6) = -0.0005.$$

$$a = 1.3282 \qquad b = 2$$

$$f(a) = -0.0005 \qquad f(b) = 7.$$

$$x_7 = \frac{(1.3282)(7) - 2(-0.0005)}{7-(-0.0005)}$$

$$= \frac{9.2984}{7.0005} = \underline{\underline{1.3282}}.$$

$\therefore$ Root is $\underline{\underline{1.3282}}$.

2] Find the real root of $x^3 + x + 1 = 0$.

$\rightarrow$ $f(x) = x^3 + x + 1 = 0$

$f(0) = 1 > 0$.

$f(-1) = -1 < 0$.

Since $f(0) > 0$, $f(-1) < 0$, a real root lies between $0$ & $-1$.

| a | | b |
| --- | --- | --- |
| $-1$ (−1) | 1 <br> $-0.5$ <br> (0.3750) | 1 0 <br> 0.1059 |

$a = -1 \qquad b = 0$

$b(a) = -1 \qquad b(b) = 1$

$$x_0 = \frac{-1(1) - 0(-1)}{2} = -0.5$$

$$b(x_0) = -0.125 - 0.5 + 1$$
$$= \underline{\underline{0.3750}}.$$

$a = -1 \qquad b = -0.5$

$b(a) = -1 \qquad b(b) = 0.3750.$

$$x_1 = \frac{-1(0.3750) - (-0.5)(-1)}{0.3750 + 1}$$

$$= \frac{-0.3750 - 0.5}{1.3750}$$

$$x_1 = \underline{\underline{-0.6364}}.$$

$$b(x_1) = -0.2577 - 0.6364 + 1$$
$$= \underline{\underline{0.1059}}.$$

$a = -1 \qquad b = -0.6364$

$b(a) = -1 \qquad b(b) = 0.1059$

$$x_2 = \frac{(-1)(0.1059) - (-0.6364)(-1)}{0.1059 + 1}$$

$$= \frac{-0.1059 - 0.6364}{1.1059}$$

$$= \frac{-0.7423}{1.1059} = \underline{\underline{-0.6712}}.$$

$$b(x_2) = -0.3024 - 0.6712 + 1$$
$$= \underline{\underline{0.0264}}.$$

$a = -1$ $\quad$ $b = -0.6712$

$f(a) = -1$ $\quad$ $f(b) = 0.0264.$

$$x_3 = \frac{(-1)(0.0264) - (-0.6712)(-1)}{0.0264 + 1}$$

$$= \frac{-0.0264 - 0.6712}{1.0264}$$

$$= \frac{-0.6976}{1.0264}$$

$$= \underline{\underline{-0.6797}}$$

$$f(x_3) = -0.3140 - 0.6797 + 1$$

$$= \underline{\underline{0.0063}}.$$

$a = -1$ $\quad$ $b = -0.6797$

$f(a) = -1$ $\quad$ $f(b) = 0.0063$

$$x_4 = \frac{(-1)(0.0063) - (-0.6797)(-1)}{0.0063 + 1}$$

$$= \frac{-0.0063 - 0.6791}{1.0063}$$

$$= \frac{-0.6860}{1.0063}$$

$$= \underline{\underline{-0.6817}}.$$

$$f(x_4) = -0.3168 - 0.6817 + 1$$

$$= \underline{\underline{0.0015}}.$$

$a = -1$ $\quad$ $b = -0.6817$

$f(a) = -1$ $\quad$ $f(b) = 0.0015$

$$x_5 = \frac{(-1)(0.0015) - (-0.6817)(-1)}{0.0015+1}$$

$$= \frac{-0.0015 - 0.6817}{1.0015}$$

$$= \frac{-0.6832}{1.0015}$$

$$= \underline{-0.6822}.$$

$$f(x_5) = -0.3175 - 0.6822 + 1$$

$$= \underline{0.0003}.$$

$a = -1$ $\qquad$ $b = -0.6822$

$f(a) = -1$ $\qquad$ $f(b) = 0.0003$

$$x_6 = \frac{(-1)(0.0003) - (-0.6822)(-1)}{0.0003+1}$$

$$= \frac{-0.0003 - 0.6822}{1.0003}$$

$$= \frac{-0.6825}{1.0003}$$

$$x_6 = \underline{-0.6823}.$$

$$f(x_6) = -0.3176 - 0.6823 + 1$$

$$= \underline{0.0001}.$$

3] Find the real root of the equation $x^2 - \log x = 12$. in $(3, 4)$

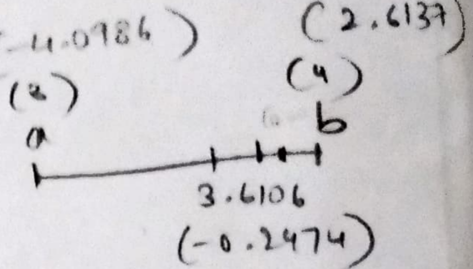
$f(x) = x^2 - \log x - 12$. $[\log = \log_e]$

$f(3) = 9 - 1.0986 - 12$

$= -4.0986$

$f(4) = 16 - 1.3863 - 12$

$= 2.6137$




| $a = A$ | $b = B$ | $f(a) = C$ | $f(b) = D$ | $x = (AD - BC) \div (D - C)$ | $y = x^2 - \log x - 12$ |
|---|---|---|---|---|---|
| 3 | 4 | -4.0986 | 2.6137 | 3.6106 | -0.2474 |
| 3.6106 | 4 | -0.2474 | 2.6134 | 3.6443 | -0.0124 |
| 3.6443 | 4 | -0.0124 | 2.6134 | 3.6460 | -0.0005 |
| 3.6460 | 4 | -0.0005 | 2.6134 | 3.6461 | 0.0002 |
| 3.6460 | 3.6461 | -0.0005 | 0.0002 | 3.6461 | 0.0002 |

Root = 3.6461

4] Find the real root of the equation $xe^x = \cos x$

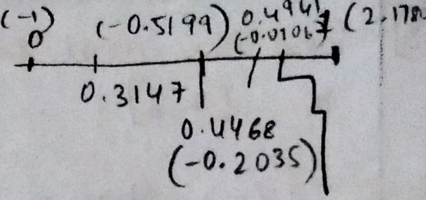
$\rightarrow f(x) = xe^{x} - \cos x$

$f(0) = 0 - 1 = -1$

$f(1) = 2.7183 - 0.5403$

$= 2.1780$.

~~f(-x)>4~~




| $a = A$ | $b = B$ | $f(a) = C$ | $f(b) = D$ | $X = \frac{(AD - BC)}{D - C}$ | $Y = xe^{x} - \cos x$ |
|---|---|---|---|---|---|
| 0 | 1 | -1 | 2.1780 | 0.3147 | -0.5199 |
| 0.3147 | 1 | -0.5199 | 2.1780 | 0.4468 | -0.2035 |
| 0.4468 | 1 | -0.2035 | 2.1780 | 0.4941 | -0.0706 |
| 0.4941 | 1 | -0.0706 | 2.1780 | 0.5100 | -0.0235 |
| 0.5100 | 1 | -0.0235 | 2.1780 | 0.5152 | -0.0077 |
| 0.5152 | 1 | -0.0077 | 2.1780 | 0.5169 | -0.0026 |
| 0.5169 | 1 | -0.0026 | 2.1780 | 0.5175 | -0.0009 |
| 0.5175 | 1 | -0.0009 | 2.1780 | 0.5177 | -0.0002 |
| 0.5177 | 1 | -0.0002 | 2.1780 | 0.5177 | 0 |

Real root, $x = \underline{\underline{0.5177}}$.

5] Find the real root of the equation $\tan x + \tanh x = 0$ in $[2, 3]$

$\Rightarrow$ $f(x) = \tan x + \tanh x$

$f(2) = -1.2210$

$f(3) = 0.8525$

| 2 | 2.5889 | 3 |
|---|---|---|
| (-1.2210) | (0.3719) | (0.8525) |

| a = A | b = B | f(a) = C | f(b) = D | X = $\frac{(AD-BC)}{D-C}$ | Y = $\tan x + \tanh x$ |
|---|---|---|---|---|---|
| 2 | 3 | -1.2210 | 0.8525 | 2.5889 | 0.3719 |
| 2 | 2.5889 | -1.2210 | 0.3719 | 2.4514 | 0.1596 |
| 2 | 2.4514 | -1.2210 | 0.1596 | 2.3992 | 0.0662 |
| 2 | 2.3992 | -1.2210 | 0.0662 | 2.3787 | 0.0269 |
| 2 | 2.3787 | -1.2210 | 0.0269 | 2.3705 | 0.0110 |
| 2 | 2.3705 | -1.2210 | 0.0110 | 2.3672 | 0.0043 |
| 2 | 2.3672 | -1.2210 | 0.0043 | 2.3659 | 0.0018 |
| 2 | 2.3659 | -1.2210 | 0.0018 | 2.3654 | 0.0007 |
| 2 | 2.3654 | -1.2210 | 0.0007 | 2.3652 | 0.0003 |
| 2 | 2.3652 | -1.2210 | 0.0003 | 2.3651 | 0.0002 |
| 2 | 2.3651 | -1.2210 | 0.0002 | 2.3650 | 0.000 |

Real root, $x = \underline{\underline{2.3650}}$.

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6] Find an approximate root of the equation $x^4 - x - 10 = 0$ by Regula Falsi method correct to 5 decimal places :-

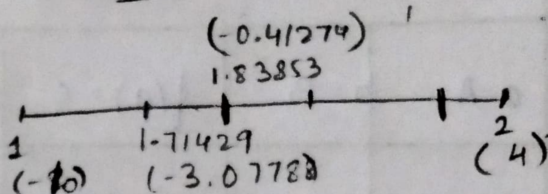
→ $f(x) = x^4 - x - 10 = 0$ $\qquad$ $f(1) = -\cancel{8}10$.

$f(0) = -10 < 0$, $= -10$

$f(2) = 16 - 2 - 10 > 0 = \underline{4}$

$\cancel{f(3)/7}$ $\cancel{8}$

∴ Interval $= (1, 2)$.




| $a = A$ | $b = B$ | $f(a) = C$ | $f(b) = D$ | $X = \dfrac{(AD - BC)}{D - C}$ | $Y = x^4 - x - 10$ |
|---|---|---|---|---|---|
| 1 | 2 | -10 | 4 | 1.71429 | -3.07788 |
| 1.71429 | 2 | -3.07788 | 4 | 1.83853 | -0.41274 |
| 1.83853 | 2 | -0.41274 | 4 | 1.85363 | -0.04785 |
| 1.85363 | 2 | -0.04785 | 4 | 1.85536 | -0.00551 |
| 1.85536 | 2 | -0.00551 | 4 | 1.85556 | -0.00063 |
| 1.85556 | 2 | -0.00063 | 4 | 1.85558 | -0.00004 |
| 1.85558 | 2 | -0.00004 | 4 | 1.85558 | -0.0000 |

∴ Real root, $x = \underline{1.85558}$.

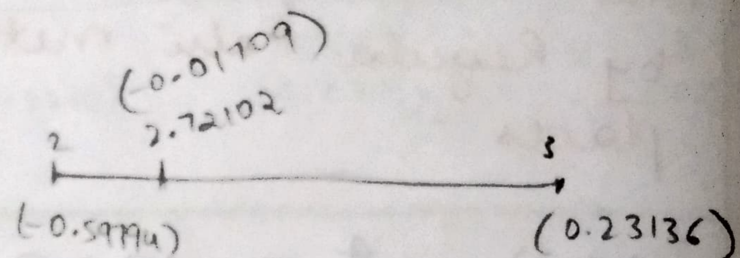
7] Find the real root of $x \log_{10} x = 1.2$. Correct to 5 decimal places:-

$\Rightarrow f(x) = x \log_{10} x - 1.2.$

$f(1) = -1.2.$

$f(2) = -0.59794$

$f(3) = 0.23136.$




Interval $(2, 3)$.

| $a = A$ | $b = B$ | $f(a) = c$ | $f(b) = D$ | X. | $Y = x\log_{10}^{x} - 1.2$ |
|---|---|---|---|---|---|
| 2 | 3 | -0.59794 | 0.23136 | 2.72102 | -0.01709 |
| 2.72102 | 3 | -0.01709 | 0.23136 | 2.74021 | -0.00038 |
| 2.74021 | 3 | -0.00038 | 0.23136 | 2.74064 | -0.00001 |
| 2.74064 | 3 | -0.00001 | 0.23136 | 2.74065 | 0.00000 |

Real root, $x = 2.74065$.