numericals_infinite well[1]

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Numerical Problems Infinite Potential Well and Energy Eigen Values

1.) An electron is trapped in an infinite potential well of width 0.01m; find the principal quantum number for which energy is 1 eV. $m_e = 9.1 \times 10^{-31}$ kg
2.) A proton is confined in an infinite square well of width 10 fm. Calculate the energy and wavelength of the photon emitted, when the proton undergoes a transition from the first excited state to the ground state.
3.) A particle is in the nth energy state of an infinite square well potential with width L
a.) Determine the probability that the particle is confined to the first (1/a) of the width of the well
4.) The wave function for a certain particle is $\psi = A \cos^3 x$ for -, Find the value of A.
5.) The normalized wave function of a particle is $\psi = A$. Calculate the energy Eigen value of the particle.
6.) An electron is moving freely with energy 2 eV. Calculate its de-Broglie Wavelength
7.) An electron is trapped completely in a 1-D well of length 1 Angstrom. How much energy must be supplied to excite the electron from the first excited state to the 3rd excited state?
8.) A quantum particle confined to a 1-dimensional box of width 'a' is in its first excited state. What is the probability of finding the particle over an interval of (a/2) marked symmetrically at the center of the box?
9.) An electron is trapped in a 1-D potential well of infinite depth and width 1×10^{-10} m. What is the probability of finding the electron in the region from $x_1 = 0.09 \times 10^{-10}$m to $x_2 = 0.11 \times 10^{-10}$ m in the ground state?
10.) Find the probability that a particle trapped in an infinite well of width L can be found between o.45L and 0.55L for the ground and first excited states?

n=2

$$\psi = B \sin \frac{n\pi}{a} x$$

$$\int_0^a |\psi^2| \, dx = 1$$

$$B = \sqrt{\frac{2}{a}}$$

$$\psi = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x$$

$n = 2$

$$\int_{a/6}^{a/3} |\psi|^2 \, dx$$

$$E = \frac{n^2 h^2}{8 m a^2}$$

① $a = 0.01$m

② $E = \frac{2^2 h^2}{8 m a}$

$a = 10 \times 10^{-12}$ m

∞ m a –

① $a = 0.01\,m$

$E = 1eV = 1.6 \times 10^{-19}\,V$

$m_e = 9.1 \times 10^{-31}$

③ $\int_0^{1/a} |\psi|^2 dx$

$\int_0^{1/a} B^2 \sin^2 \frac{n\pi}{L} x\, dx$

$\frac{B^2}{2} \int 1 - \cos \frac{2n\pi x}{L}\, dx$

$\frac{B^2}{2} \left[x - \frac{L}{2n\pi} \sin \frac{2n\pi x}{L} \right]_0^{1/a}$

$\frac{B^2}{2} \left[\frac{1}{a} - \frac{L}{2n\pi} \sin \frac{2n\pi}{aL} - 0 \right]$ ($\sin 0 = 0$)

$\frac{B^2}{2a} = 1$

$B = \sqrt{2a}$

⑧ $\psi = \sqrt{\frac{2}{a}} \sin \left(\frac{2\pi}{a}\right) x$

$P = \int_0^{a/2} |\psi|^2 dx$

$= \frac{2}{a} \int_0^{a/2} \sin^2 \frac{2\pi}{a} x\, dx$

$= \frac{2}{a} \times \frac{1}{2} \int 1 - \cos \frac{4\pi x}{a}\, dx$

$= \frac{1}{a} \cdot \left[x - \frac{\sin \frac{4\pi x}{a}}{\frac{4\pi}{a}} \right]_0^{a/2}$

$= \frac{1}{a} \left[\frac{a}{2} - \frac{\sin 4\pi \frac{a}{2}}{\frac{4\pi}{a}} - 0 \right]$ ($\sin 2\pi = 0$)

$= \frac{1}{a} \left[\frac{a}{2} - \sin 2\pi \frac{a}{4\pi} \right]$

$P = \frac{1}{2}$

⑩ $P = \frac{2}{a} \int_{0.5a}^{0.65a} \sin \frac{2\pi}{L} x\, dx$ $n = 2$

∞ m a

$a = 10 \times 10^{-12}\,m$

④ $\int_0^a |\psi|^2 dx = 1$

$\int_0^a A^2 \cos^6 x\, dx = 1$

$A^2 \int_0^a (\cos^2 x)^3 = 1$

⑤ $\lambda = \frac{h}{\sqrt{2 m_e E}}$ $m_e = 9.1 \times 10^{-31}$, $E = 2 \times 1.6 \times 10^{-19}$

⑦ $E_4 - E_2$

$= \frac{4^2 h^2}{8 m a^2} - \frac{2^2 h^2}{8 m a^2}$

$a = 1\,\text{Å} = 10^{-10}\,m$

⑨ $P = \frac{2}{a} \int_{x_1}^{x_2} \sin^2 \frac{\pi}{a} x\, dx$

$= \frac{2}{a} \times \frac{1}{2} \int 1 - \cos \frac{2\pi x}{a}\, dx$

$= \frac{1}{a} \left[x - \sin \frac{2\pi x}{a} \cdot \frac{a}{2\pi} \right]_{x_1}^{x_2}$

$= \frac{1}{a} \left[0.11\,\text{Å} - 0.09\,\text{Å} \right]$

$= \frac{0.02 \times 10^{-10}}{1 \times 10^{-10}}$

$P = 0.02$

(10) $P = \frac{2}{a} \int_{0.45L} \sin \frac{2\pi x}{L} \, dx$

$\frac{2}{L} \int 1 - \cos \frac{4\pi x}{L} \, dx$

$\frac{2}{L} \left[x - \sin \frac{4\pi x}{L} \cdot \frac{L}{4\pi} \right]_{0.45L}^{0.55L}$

$= \frac{2}{L} \left[0.55L - 0.45L \right]$

$= 2 (0.10)$

$P = 0.20$