RV Educational Institutions®
RV College of Engineering®

DEPARTMENT OF PHYSICS

SECOND SEMESTER BE PROGRAMS (CS, CD, CY, IS, AIML, & BT)
ACADEMIC YEAR 2022-2023

Date	10th July 2023	Maximum Marks	50
Course Code	22PHY22C	Duration	90 min
Course	QUANTUM PHYSICS FOR ENGINEERS	**CIE-I (Test)**	

Instruction: *Answer all questions.*

Qn	Questions	M	BTL	CO
1(a)	An excited state of an atom with lifetime of 10^{-8}s shows more broadening in its emitted spectral line than a state with lifetime of 10^{-3}s. Justify this statement by deriving a relation for spectral line broadening.	4	2	2
1(b)	Show that the de Broglie wavelength of an electron accelerated through a potential difference V is inversely proportional to $\sqrt{V}$.	4	1	1
1(c)	What is i) Born interpretation of wave function and ii) Normalization of wave function.	2	1	1
2(a)	Making use of the concept of matter waves setup one dimensional time independent Schrodinger wave equation.	6	2	2
2(b)	At time $t = 0$ a particle is represented by the wave function $$\Psi(x,0) = \begin{cases} \sqrt{(3/b)}(x/a), & 0 \le x \le a, \\ \sqrt{(3/b)}(b-x)/(b-a), & a \le x \le b, \\ 0, & \text{otherwise,} \end{cases}$$ where a and b are (positive) real constants representing two locations on the x-axis. What is the probability of finding the particle i) to the left of a and ii) to the right of a?	4	4	3
3(a)	For a particle in an one dimensional potential well of infinite depth, solve time independent Schrodinger wave equation and obtain normalized wave functions for first three allowed states.	6	1	1
3(b)	A particle in an one dimensional potential well of infinite depth makes two consecutive transitions from energy levels $E_{n+1} \to E_n \to E_{n-1}$ releasing energy of 3.384 eV and 2.632 eV respectively. Determine n.	4	4	3
4(a)	Obtain an expression for energy density of photons in terms of Einstein's coefficients.	7	1	1
4(b)	A laser operating at temperature of 300 K and wavelength of 680 nm is at thermal equilibrium. Determine the ratio of Einstein's coefficients A and B.	3	3	3
5(a)	With energy band diagram explain the construction and working of semiconductor laser.	7	1	1
5(b)	Two levels of an atomic system at thermal equilibrium has energy difference of 1.8 eV. If the system is at temperature 300 K, determine the ratio of population of these two energy levels.	3	3	3

BTL-Blooms Taxonomy Level, CO-Course Outcomes, M-Marks

Marks	Particulars	CO1	CO2	CO3	L1	L2	L3	L4
Distribution	Max Marks	26	10	14	26	10	6	8

RV Educational Institutions
RV College of Engineering
Autonomous Institution Affiliated to Visvesvaraya Technological University, Belagavi
Approved by AICTE, New Delhi

DEPARTMENT OF PHYSICS

Course: Quantum Physics for Engineers	CIE: II First semester 2022-2023	Maximum marks: 50
Course code: 22PHY22C	Physics Cycle: Computer Science Stream	Time: 90 Minutes Date: 21/8/2023

Instructions to candidates:

i. ***Answer all the questions.***

1. *Physical constants:* h = 6.625 x 10^{-34}Js, m_e = 9.1 x 10^{-31}kg, k_b= 1.38x10^{-23}J/K.

Q. No		M	BT	CO
1a.	For a pure Si semiconductor derive a general expression for electron concentration in the conduction band.	7	L2	2
b.	A fiber surrounded by air has a numerical aperture of 0.369. Will light entering the fiber at an angle of incidence of 25° remain in the fiber, or will it escape? Why?	3	L3	3
2a.	Describe in detail the basics of point-to-point communication and describe the role of repeater.	7	L2	1
b.	A bar of n-type Germanium bar of dimensions (1cm x 0.1cm x 0.1 cm) in the order of length, width and thickness is placed in a magnetic field of 0.2 T. If the drift velocity of the electrons is 4 cm/s calculate the Hall voltage produced in the bar. Assume the magnetic field to be along the direction of width.	3	L1	3
3a.	With a neat figure derive the expression for numerical aperture of an optical fibre and the fractional index change.	7	L3	2
b	With a neat sketch explain the structure and working of GRIN fiber.	3	L1	1
4a.	Define Hall Effect and with a neat figure arrive at an expression for hall coefficient for a pentavalent doped semiconductor.	7	L3	2
b.	For Silicon at 30°C, calculate the number of states per unit energy per unit volume at an energy 26meV above the bottom of the conduction band ($m_e^* = 1.18\ m_e$)	3	L3	3
5a.	With a neat figure describe the variation of Fermi level with respect to temperature for an intrinsic semiconductor doped with a trivalent impurity.	7	L1	1
b.	Define fermi factor, fermi energy level and sketch the variation of fermi level when T$\neq$ 0K.	3	L2	1

COs	CO 1	CO 2	CO 3
Marks	20	21	9

*****

RV College of Engineering®

Autonomous Institution Affiliated to Visvesvaraya Technological University, Belagavi | Approved by AICTE, New Delhi, Accredited

DEPARTMENT OF PHYSICS

Course: Quantum Physics for Engineers	Improvement CIE	Marks: 50
Course code: 22PHY22C	Second semester 2022-2023 Physics Cycle: Computer Science Cluster	Time: 90 Minutes Date: 05.09.23

Instructions: Answer all questions.

Q No	PART B – Test	M	BT	CO
1a	Solve the problem of a particle in an infinite well to arrive at the un-normalized eigen function and eigen values.	7	3	1
1b	The 1st excited state wave function of a particle in an infinite well is given by $\psi = B\sin(10^9 \pi x)$. Calculate B and energy of the state.	3	4	3
2a	Using Heisenberg's uncertainty principle, explain the broadening of atomic spectral lines. Hence derive an expression for the minimum line broadening.	7	2	1
2b	Calculate the difference in energy levels, given that the broadening of the emission line spectrum between them is 100Å and lifetime of the higher energy level is 100 μs.	3	3	3
3a	State de-Broglie's hypothesis. Use the expression relating momentum of a particle to the wavelength of its equivalent wave to arrive at the expression for the energy of a particle of mass m in the ground state of an infinite well of width a.	7	2	2
3b	An infinite well between 0 to a is shifted to the new position -0.5a to 0.5a. Will the eigenvalues and eigenfunctions change for this new configuration? Explain why.	3	4	3
4a	State the condition of unitarity of a matrix. Show that the Pauli Matrices σ_x, σ_y, σ_z are unitary matrices.	7	2	1
4b	Prove that the matrix given below is a unitary matrix. All intermediate steps need to be shown explicitly. $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{pmatrix}$	3	3	3
5a	With a neat labelled diagram involving a single photon source, two beam splitters, two mirrors and two detectors, prove that a single photon simultaneously travels through both the paths at the same time and does not choose any one of the paths in a random fashion.	7	3	2
5b	Calculate the inner product between two vectors $\|\psi>$ and $\|\chi>$, given that for a particular basis, the two kets can be written as : $\|\psi> = 0.707\|\varphi1> + 0.707\|\varphi2>$ and $\|\chi> = 0.5\|\varphi1> + 0.866\|\varphi2>$.	3	4	4

COs	CO 1	CO 2	CO 3
Marks	21	14	15

RV Educational Institutions®
RV College of Engineering®

DEPARTMENT OF PHYSICS

SECOND SEMESTER BE PROGRAMS (CS, CD, CY, IS, AIML, & BT)
ACADEMIC YEAR 2022-2023

Dated	10th July 2023	Maximum Marks	50
Course Code	22PHY22C	Duration	90 min
Course	QUANTUM PHYSICS FOR ENGINEERS	**CIE-I (Test) - Long Scheme**	

1(a) *An excited state of an atom with lifetime of 10^{-8}s shows more broadening in its emitted spectral line than a state with lifetime of 10^{-3}s. Justify this statement by deriving a relation for spectral line broadening.* ④

The energy of the emitted photon is given by,

$$E = h\nu = \frac{hc}{\lambda} \tag{1}$$

Where h is Planck's constant, ν is the frequency, c is the velocity and λ is the wavelength of the emitted radiation. Differentiating this equation with respect to λ, we get

$$\Delta E = -hc\frac{\Delta\lambda}{\lambda^2} \quad ①$$

Considering only the magnitude of the difference,

$$|\Delta E| = hc\frac{\Delta\lambda}{\lambda^2} \tag{2}$$

According to Heisenberg's uncertainty principle, the finite lifetime Δt of the excited state means there will be an uncertainty of ΔE in the energy of the state. Hence the emitted photon energy will also have an uncertainty of ΔE in its energy and is related by,

$$\Delta E\,\Delta t \geq \hbar/2 \qquad \text{Or,} \qquad \Delta E \geq \frac{h}{4\pi\Delta t} \quad ①$$

Substituting for ΔE from (2) we get,

$$hc\frac{\Delta\lambda}{\lambda^2} \geq \frac{h}{4\pi\Delta t} \qquad \text{Or} \qquad \boxed{\Delta\lambda \geq \frac{1}{4\pi c}\frac{\lambda^2}{\Delta t}} \quad ①$$

This shows that for a finite lifetime of the excited state, the measured value of the emitted photon wavelength will have a spread of wavelengths around the mean value λ. This demands that for a very narrow spread, the lifetime of the excited state must be very high. Justifying that if Δt is less $\Delta\lambda$ will be more. ①

1(b) *Show that the de Broglie wavelength of an electron accelerated through a potential difference V is inversely proportional to $\sqrt{V}$.* ④

Kinetic energy of a particle, with mass m, moving with non-relativistic velocity v is given by,

$$E = \frac{1}{2}mv^2$$

The equation can be rearranged as,

$$m^2v^2 = 2mE$$

where $mv = p$ is the momentum of the particle. Hence it can be rewritten as,

$$p = \sqrt{2mE} \quad ①$$

Therefore de Broglie wavelength of the particle can be expressed in terms of kinetic energy as,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \qquad ①$$

Consider an electron of mass m and charge $-e$ accelerated from rest through an electric potential V. The work done on the electron, which is equal to eV, is then equal to the kinetic energy E gained by the electron.

$$E = eV \qquad ①$$

de Broglie wavelength of this electron in terms of the accelerating potential is,

$$\boxed{\lambda = \frac{h}{\sqrt{2meV}}} \qquad ①$$

1(c) *What is i) Born interpretation of wave function and ii) Normalization of wave function.* ②

i) Probability of finding the particle in the position range $[a, b]$ is $P_{ab} = \int_a^b |\Psi(x,t)|^2\, dx$ ①

ii) Total probability of finding the particle anywhere is 1, $\int_{-\infty}^{+\infty} |\Psi(x,t)|^2\, dx = 1$ ①

2(a) *Making use of the concept of matter waves setup one dimensional time independent Schrodinger wave equation.* ⑥

Let $\Psi(x,t)$ be the wave function for the matter wave associated with a moving particle. The corresponding wave equation is as follows,

$$\frac{\partial^2\Psi}{\partial x^2} = \frac{1}{v_p^2}\frac{\partial^2\Psi}{\partial t^2} \tag{1}$$

where v_p is the phase velocity of the associated matter waves. The solution of eq. (1) is,

$$\Psi(x,t) = \psi(x)e^{-i\omega t} \qquad ① \tag{2}$$

where ω is the angular frequency.

Differentiating eq. (2) partially twice with respect to t, we get,

$$\frac{\partial\Psi}{\partial t} = (-i\omega)\psi(x)e^{-i\omega t} \qquad \text{and} \qquad \frac{\partial^2\Psi}{\partial t^2} = (-i\omega)^2\psi(x)e^{-i\omega t} = -\omega^2\Psi \tag{3}$$

Substituting eq. (3) in eq. 1

$$\frac{\partial^2\Psi}{\partial x^2} = -\frac{\omega^2}{v_p^2}\Psi$$

We have, for matter waves, phase velocity $v_p = \lambda\nu$ and angular frequency $\omega = 2\pi\nu$, where ν is the frequency.

$$\frac{\omega^2}{v_p^2} = \left(\frac{2\pi f}{\lambda f}\right)^2 = \left(\frac{2\pi}{\lambda}\right)^2 = \frac{4\pi^2}{\lambda^2}$$

Substituting this and in the above equation we get,

$$\frac{\partial^2\Psi}{\partial x^2} = -\frac{4\pi^2}{\lambda^2}\Psi \qquad ②$$

Substituting Ψ from eq. (2) into it we get,

$$\frac{\partial^2[\psi(x)e^{-i\omega t}]}{\partial x^2} = -\frac{4\pi^2}{\lambda^2}[\psi(x)e^{-i\omega t}]$$

$$\cancel{e^{-i\omega t}}\frac{d^2\psi(x)}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi(x)\cancel{e^{-i\omega t}}$$

$$\frac{d^2\psi(x)}{dx^2} + \frac{4\pi^2}{\lambda^2}\psi(x) = 0$$

In the above equations you can note that partial differential is changed into ordinary differential because ψ is a function of x alone. Substituting de Broglie's wavelength $\lambda = h/(mv)$ for the matter waves of a particle of mass m moving with velocity v into it, we get,

$$\frac{d^2\psi(x)}{dx^2} + 4\pi^2\frac{m^2v^2}{h^2}\psi(x) = 0 \tag{4}$$

If E is the total energy and V is the potential energy of the particle respectively, then the kinetic energy of the particle is,

$$E_k = \frac{1}{2}mv^2 = E - V, \qquad \therefore \quad m^2v^2 = 2m(E-V) \qquad 2$$

Substituting this into eqn. (4), we get,

$$\boxed{\frac{d^2\psi(x)}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi(x) = 0} \qquad 1 \tag{5}$$

Here ψ is a function of x alone and is independent of time. We use this equation for the cases where $V \equiv V(x)$ is a function of x alone so that the whole equation is independent of time. This equation is called one dimensional time-independent Schrödinger equation.

2(b) *At time $t = 0$ a particle is represented by the wave function*

$$\Psi(x,0) = \begin{cases} \sqrt{(3/b)}(x/a), & 0 \le x \le a, \\ \sqrt{(3/b)}(b-x)/(b-a), & a \le x \le b, \\ 0, & \text{otherwise,} \end{cases}$$

where a and b are (positive) real constants representing two locations on the x-axis. What is the probability of finding the particle i) to the left of a and ii) to the right of a? 4

$P_{\text{left of }a} = \int_0^a \left[\sqrt{(3/b)}(x/a)\right]\left[\sqrt{(3/b)}(x/a)\right] dx = \int_0^a (3/(ba^2))x^2\, dx = a/b$ 2

$P_{\text{right of }a} = 1 - P_{\text{left of }a} = 1 - a/b = (b-a)/b$ 2

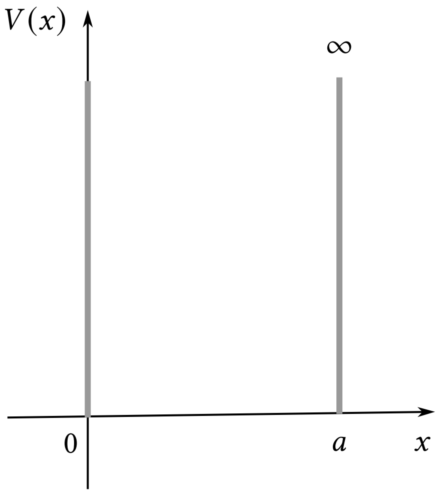
Or, $P_{\text{right of }a} = \int_a^b \left[\sqrt{(3/b)}(b-x)/(b-a)\right]\left[\sqrt{(3/b)}(b-x)/(b-a)\right] dx = \int_a^b \frac{3}{b(b-a)^2}(b-x)^2\, dx$

$= \left[\frac{3}{b(b-a)^2}\frac{(b-x)^3}{(-1)3}\right]_a^b = \frac{(b-a)^3}{b(b-a)^2} = (b-a)/b = 1 - a/b$

3(a) *For a particle in an one dimensional potential well of infinite depth, solve time independent Schrodinger wave equation and obtain normalized wave functions for first three allowed states.* 6

Suppose the potential is,

$$V(x) = \begin{cases} 0, & 0 \le x \le a, \\ \infty, & \text{otherwise} \end{cases}$$



A particle in this potential is completely free, except at the two ends ($x = 0$ and $x = a$), where an infinite force prevents it from escaping. A classical model would be a cart on a frictionless horizontal air track, with perfectly elastic bumpers—it just keeps bouncing back and forth forever. (This potential is artificial, of course, but I urge you to treat it with respect. Despite its simplicity—or rather, precisely because of its simplicity—it serves as a wonderfully accessible test case for all the fancy machinery that comes later.)

We begin by time independent Schrödinger equation which reads,

$$\frac{d^2\psi(x)}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi(x) = 0 \tag{1}$$

Outside the potential well $(0 \le x \le a)$ it reads

$$\frac{d^2\psi(x)}{dx^2} + \frac{8\pi^2 m}{h^2}(E - \infty)\psi(x) = 0$$

This equation holds good only if $\psi(x) = 0$ for all points outside the well. The probability of finding the particle there is zero.

Inside the potential well, where $V = 0$, the time-independent Schrödinger equation (1) reads,

$$\frac{d^2\psi(x)}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi(x) = 0$$

Or

$$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0, \qquad \text{where } k^2 = \frac{8\pi^2 m}{h^2}E \qquad ② \tag{2}$$

Solution: Equation (2) is the classical *simple harmonic oscillator* equation; the general solution is,

$$\psi(x) = A\sin(kx) + B\cos(kx) \tag{3}$$

where A and B are arbitrary constants. Typically, these constants are fixed by the *boundary conditions* of the problem.

Boundary conditions: What are the appropriate boundary conditions for $\psi(x)$? Ordinarily, both ψ and $d\psi/dx$ are continuous, but where the potential goes to infinity only the first of these applies. Continuity of $\psi(x)$ requires that

$$\psi(0) = \psi(a) = 0$$

so as to join onto the solution outside the well. What does this tell us about A and B?

Condition 1: At $x = 0$, $\psi = 0$

Substituting it in equation (3), we get

$$\psi(0) = A\sin 0 + B\cos 0 = B$$

so $B = 0$ as $\psi(0) = 0$, and hence equation (3) will become,

$$\psi(x) = A\sin(kx) \qquad ① \tag{4}$$

Condition 2: At $x = a$, $\psi = 0$

Substituting it in equation 4, we get

$$\psi(a) = A\sin(ka)$$

Since $\psi(a) = 0$, $A\sin(ka) = 0$, so either $A = 0$ (in which case we're left with the trivial—non-normalizable—solution $\psi(x) = 0$), or else $\sin(ka) = 0$ which means that

$$ka = 0,\ \pm\pi,\ \pm 2\pi\ \pm 3\pi,\ \ldots$$

But $k = 0$ is no good (again, that would imply $\psi(x) = 0$, which means the particle is not inside the well), and the negative solutions give nothing new, since $\sin(-\theta) = -\sin(\theta)$ and we can absorb the minus sign into A. So the *distinct* solutions are

$$k = \frac{n\pi}{a}, \qquad \text{with } n = 1, 2, 3, \ldots \qquad ① \tag{5}$$

Normalization of wave function: Actually, it's the wave function $\Psi(x,t)$ that must be normalized, but in view of $\Psi(x,t) = \psi(x)e^{-i\omega t}$ we will get $\Psi^*\Psi = \psi^*\psi$. This entails the normalization of $\psi(x)$.

To find A, we normalize ψ from equation 4 with normalization condition:

$$\int_{-\infty}^{+\infty} \psi^*\psi\, dx = \int_{-\infty}^{+\infty} |\psi(x)|^2\, dx = 1$$

Since $\psi(x) = 0$ outside the well, it is sufficient to integrate between 0 to a. With $\psi(x) = A\sin(kx)$ inside the well we have,

$$\int_0^a |\psi(x)|^2\, dx = \int_0^a |A|^2 \sin^2(kx)\, dx = 1$$

With the identity, $\sin^2\theta = (1-\cos(2\theta))/2$ the above integral can be written as,

$$\frac{|A|^2}{2}\int_0^a [1-\cos(2kx)]\ dx = 1 \qquad \text{or,} \qquad \frac{|A|^2}{2}\left[x - \frac{1}{2k}\sin(2kx)\right]_0^a = 1$$

By substituting limits and k from equation (5) we get,

$$\frac{|A|^2}{2}\left[a - \cancel{\frac{a}{2n\pi}\sin(2n\pi)}^{\,0}\right] = 1 \qquad \therefore \quad |A|^2 = \frac{2}{a}$$

This only determines the magnitude of A, but it is simplest to pick the positive real root and the phase of A carries no physical significance anyway. Hence,

$$A = \sqrt{\frac{2}{a}}$$

Inside the well, then, the normalized solutions (also called eigenfunctions) are,

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)}$$

The eigenfunctions for $n = 1, 2, 3$ are as follows. $\psi_1 = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi}{a}x\right)$, $\psi_2 = \sqrt{\frac{2}{a}}\sin\left(\frac{2\pi}{a}x\right)$, $\psi_3 = \sqrt{\frac{2}{a}}\sin\left(\frac{3\pi}{a}x\right)$ ②

3(b) *A particle in an one dimensional potential well of infinite depth makes two consecutive transitions from energy levels $E_{n+1} \longrightarrow E_n \longrightarrow E_{n-1}$ releasing energy of 3.384 eV and 2.632 eV respectively. Determine n.* ④

$E_{n+1} - E_n = \frac{(n+1)^2h^2}{8ma^2} - \frac{n^2h^2}{8ma^2} = (2n+1)\frac{h^2}{8ma^2}$ ①

$E_n - E_{n-1} = \frac{n^2h^2}{8ma^2} - \frac{(n-1)^2h^2}{8ma^2} = (2n-1)\frac{h^2}{8ma^2}$ ①

$\frac{E_{n+1}-E_n}{E_n-E_{n-1}} = \frac{2n+1}{2n-1} = \frac{3.384}{2.632} = 1.286 = f$ (say) ①

$2n+1 = 2fn - f$ and $2fn - 2n = f + 1$

$n = \frac{f+1}{2f-2} = \frac{1.286+1}{2\times1.286-2} = 3.9965 \approx 4$ ①

4(a) *Obtain an expression for energy density of photons in terms of Einstein's coefficients.* ⑦

Consider an atomic system interacting with radiation field of energy density U_ν. Let E_1 and E_2, be the two energy states of atomic system ($E_2 > E_1$). Let us consider atoms are to be in thermal equilibrium with radiation field, which means that the energy density U_ν is constant in spite of the interaction that is taking place between itself and the incident radiation. This is possible only if the number of photons absorbed by the system per second is equal to the number of photons it emits per second by both the stimulated and spontaneous emission processes. We know that

- The rate of induced absorption = $B_{12}U_\nu N_1$
- The rate of spontaneous emission = $A_{21}N_2$
- The rate of stimulated emission = $B_{21}U_\nu N_2$

N_1 and N_2 are the number of atoms in the energy state E_1 and E_2 respectively. B_{12}, A_{21} and B_{21} are the Einstein coefficients for induced absorption, spontaneous emission and stimulated emission respectively. ①
At thermal equilibrium, *Rate of induced absorption = Rate of spontaneous emission + Rate of stimulated emission*

$$\therefore \;\; B_{12}N_1U_\nu = A_{21}N_2 + B_{21}N_2U_\nu \qquad \text{or} \qquad U_\nu(B_{12}N_1 - B_{21}N_2) = A_{21}N_2$$

$$\text{or,} \qquad U_\nu = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12N_1}}{B_{21}N_2} - 1}\right] \quad ② \tag{1}$$

In a state of thermal equilibrium, the populations of energy levels E_2 and E_1 are fixed by the Boltzmann factor. The population ratio is given by,

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/(kT)} \qquad \text{with } \; E_2 - E_1 = h\nu \;\text{ it can written as,} \qquad \frac{N_1}{N_2} = e^{h\nu/(kT)} \quad ①$$

Substituting it into eqn. (1) we get,

$$U_\nu = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}}e^{h\nu/(kT)} - 1}\right] \quad ① \tag{2}$$

According to Planck's law of black body radiation, the equation for U_ν is,

$$U_\nu = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{h\nu/(kT)} - 1}\right] \tag{3}$$

Now comparing the eqns. (2) and (3) term by term on the basis of positional identity we have

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \qquad \text{and} \qquad \frac{B_{12}}{B_{21}} = 1 \quad \text{or} \quad B_{12} = B_{21}$$

This implies that the probability of induced absorption is equal to the probability of stimulated emission. Due to this identity the subscripts could be dropped, and A_{21} and B_{21} can be simply represented as A and B and eqn. (2) can be rewritten. Therefore, at thermal equilibrium the equation for energy density is,

$$\boxed{U_\nu = \frac{A}{B}\left[\frac{1}{e^{h\nu/(kT)} - 1}\right]} \quad ②$$

4(b) *A laser operating at temperature of 300 K and wavelength of 680 nm is at thermal equilibrium. Determine the ratio of Einstein's coefficients A and B.* ③

$\frac{A}{B} = \frac{8\pi h\nu^3}{c^3} = \frac{8\pi h}{\lambda^3}$

$\frac{A}{B} = \frac{8\times3.142\times6.626\times10^{-34}\text{Js}}{(680\times10^{-9}\text{m})^3}$ ①

Simplification ①

$= 5.296 \times 10^{-14}\text{Pa s}$ ①

5(a) *With energy band diagram explain the construction and working of semiconductor laser.* ⑦

Schematic diagram ①
Construction explanation ①
Energy band diagram ②
Working explanation ③

5(b) *Two levels of an atomic system at thermal equilibrium has energy difference of 1.8 eV. If the system is at temperature 300 K, determine the ratio of population of these two energy levels.* ③

$\frac{N_2}{N_1} = e^{-h\nu/(kT)} = e^{-\Delta E/(kT)} = e^{-1.8\text{eV}/(1.381\times10^{-23}\text{J K}^{-1}\times300\text{K})}$ ①

$= e^{-1.8\times1.602\times10^{-19}\text{J}/(1.381\times10^{-23}\text{J K}^{-1}\times300\text{K})}$ ①

$= 5.772 \times 10^{-31}$ Or $\frac{N_1}{N_2} = 0.173 \times 10^{31}$ ①

DEPARTMENT OF PHYSICS

SECOND SEMESTER BE PROGRAMS (CS, CD, CY, IS, AIML, & BT)
ACADEMIC YEAR 2022-2023

Dated	21st August 2023	Maximum Marks	50
Course Code	22PHY22C	Duration	90 min
Course	QUANTUM PHYSICS FOR ENGINEERS	**CIE-II (Test) - Long Scheme**	

1(a) *For a pure silicon semiconductor derive a general expression for electron concentration in the conduction band.* ⑦

The number of electrons in the conduction band is given by

$$n = \int_{E_C}^{\text{top of the band}} g_c(E) f(E)\, dE$$

where E_C is the bottom most energy level of the conduction band.

As $f(E)$ rapidly approaches zero for higher energies, we can write

$$n = \int_{E_C}^{\infty} g_c(E) f(E)\, dE \tag{1}$$

Density of states for energies $E \geq E_C$ is given by,

$$g_c(E) = \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E - E_C)^{1/2}\, dE \tag{2}$$

Any energy above E_C is the conduction band electron's kinetic energy ($= E - E_C$) relative to E_C. Electrons may gain energy by getting accelerated in an electric field and may lose energy through collisions with imperfections in the crystal.

Fermi factor is given by,

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \tag{3}$$

Substituting eqns. (2) and (3) in eqn. (1),

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} \int_{E_c}^{\infty} \frac{(E - E_C)^{1/2}}{e^{(E-E_F)/kT} + 1}\, dE$$

At sufficiently large E with $E > E_F$,

$$E - E_F \gg kT \quad \text{makes} \quad e^{(E-E_F)/kT} \gg 1$$

Hence we can approximate,

$$e^{(E-E_F)/kT} + 1 \approx e^{(E-E_F)/kT}$$

Therefore the above integral can be written as,

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} \int_{E_c}^{\infty} (E - E_C)^{1/2} e^{-(E-E_F)/kT}\, dE$$

Adding and subtracting E_c to the exponential term gives,

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{(E_F - E_C)/kT} \int_{E_c}^{\infty} (E - E_C)^{1/2} e^{-(E-E_C)/kT}\, dE$$

To solve the integral, let $E - E_C = x$, then $dE = dx$ and

Lower limit: when $E = E_C$, $x = E_C - E_C = 0$

Upper limit: when $E = \infty$, $x = \infty - E_C = \infty$

Then the integral part will become, by letting $1/kT = a$,

$$\int_{E_C}^{\infty} (E - E_C)^{1/2} e^{-(E-E_C)/kT}\, dE = \int_0^{\infty} x^{1/2} e^{-ax}\, dx$$

This form of integrals are known as a gamma functions. From literature,

$$\int_0^{\infty} x^{1/2} e^{-ax}\, dx = \frac{\sqrt{\pi}}{2a\sqrt{a}}$$

Substituting it back into the equation for n we get,

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{(E_F - E_C)/kT} \left[\frac{\sqrt{\pi}}{2}(kT)^{3/2}\right]$$

By rearranging,

$$\boxed{n = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} e^{-(E_C - E_F)/kT}}$$

Or,

$$\boxed{n = N_C e^{-(E_C - E_F)/kT}} \qquad \text{with, } \boxed{N_C = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}}$$

N_C is called the *effective density of states* of the conduction band.

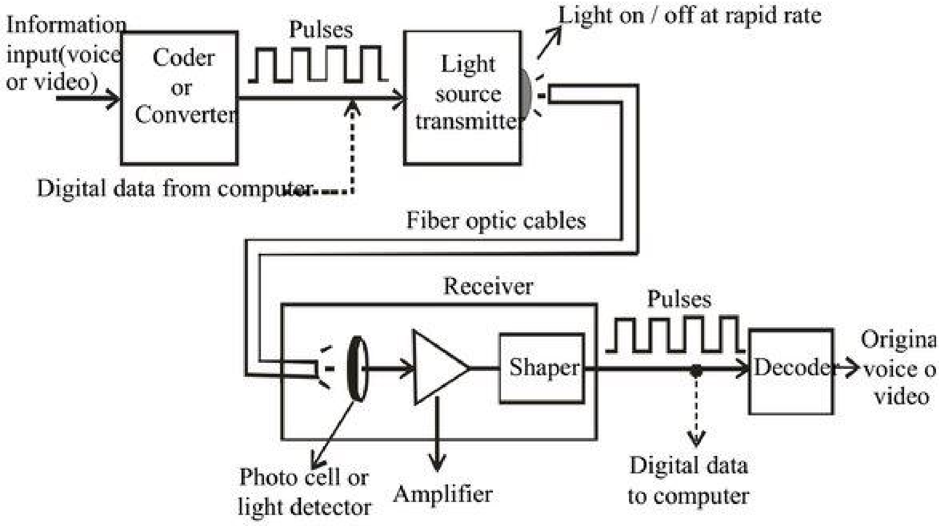
1(b) *A fiber surrounded by air has a numerical aperture of 0.369. Will the light entering the fiber at an angle of incidence of 25° remain in the fiber or will it escape? Why?* ③

From the given numerical aperture NA = 0.369 acceptance angle can be calculated as,

$$\theta_0 = \sin^{-1}(\text{NA}) = \sin^{-1} 0.369 = 21.654^\circ$$

Given angle of incidence $\theta_i = 25^\circ$ is greater than the calculated acceptance angle $\theta_0 = 21.654^\circ$. Hence the ray will escape out into the cladding.

2(a) *Describe in detail the basics of point-to-point communication and describe the role of repeater.* ⑦



Point-to-point Communication The use of optical fibres in the field of communication has revolutionized the modern world. An optical fibre acts as the channel of communication (like electrical wires), but transmits the information in the form of optical waves. A simple p to p communication system using optical fibres is illustrated in the figure. The main components of p to p communication is:

1) An optical transmitter, i.e., the light source to transmit the signals/pulses

2) The communication medium (channel) i.e., optical fibre

3) An optical receiver, usually a photo cell or a light detector, to convert light pulses back into electrical signal.

The information in the form of voice or video to be transmitted will be in an analog electric signal format. This analog signal at first converted into digital electric (binary) signals in the form of electrical pulses using a Coder or converter and fed into the optical transmitter which converts digital electric signals into optic signals. An optical fibre can receive and transmit signals only in the form of optical pulses. The function of the light source is to work as an efficient transducer to convert the input electrical signals into suitable light pulses. An LED or laser is used as the light source for this purpose. Laser is more efficient because of its monochromatic and coherent nature. Hence semiconductor lasers are used for their compact size and higher efficiency.

The electrical signal is fed to the semiconductor laser system, and gets modulated to generate an equivalent digital sequence of pulses, which turn the laser on and off. This forms a series of optical pulses representing the input information, which is coupled into the optical fibre cable at an incidence angle less than that of acceptance cone half angle of the fibre.

Next the light pulses inside the fibre undergo total internal reflection and reach the other end of the cable. Good quality optical fibres with less attenuation to be chosen to receive good signals at the receiver end.

The final step in the communication system is to receive the optical signals at the end of the optical fibre and convert them into equivalent electrical signals. Semiconductor photodiodes are used as optical receivers. A typical optical receiver is made of a reverse biased junction, in which the received light pulses create electron-hole charge carriers. These carriers, in turn, create an electric field and induce a photocurrent in the external circuit in the form of electrical digital pulses. These digital pulses are amplified and re-gain their original form using suitable amplifier and shaper. The electrical digital pulses are further decoded into an analogues electrical signal and converted into the usable form like audio or video etc.,

As the signal propagates through the fibre it is subjected to two types of degradation. Namely attenuation and delay distortion. Attenuation is the reduction in the strength of the signal because power loss due to absorption and scattering of photons. Delay distortion is the reduction in the quality of the signal because of the spreading of pulses with time. These effects cause continuous degradation of the signal as the light propagates and may reach a limiting stage beyond which it may not be retrieve information from the light signal. At this stage repeater is needed in the transmission path.

Reciever	Amplifier	Transmitter

An optical repeater consists of a receiver and a transmitter arranged adjacently. The receiver section converts the optical signal into corresponding electrical signal. Further the electrical signal is amplified and recast in the original form and it is sent into an optical transmitter section where the electrical signal is again converted back to optical signal and then fed into an optical fibre.

Finally at the receiving end the optical signal from the fibre is fed into a photo detector where the signal is converted to pulses of electric current which is then fed to decoder which converts the sequence of binary data stream into an analog signal which will be the same information which was there at the transmitting end.

2(b) *A bar of n-type germanium of dimension* $1cm \times 0.1cm \times 0.1cm$ *in the order of length, width and thickness is placed in a magnetic field of* 0.2 T. *If the drift velocity of the electrons is* 4cm/s *calculate the Hall voltage produced in the bar. Assume the magnetic field to be along the direction of width.* ③

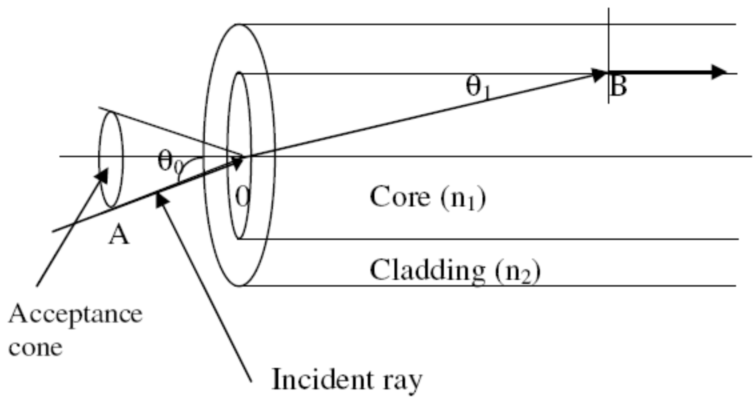
Expression for Hall voltage produced in a n-type semiconductor is, in which current $I = neAv_d$ is flowing is,

$$V_H = -\frac{BI}{net} = -\frac{BneAv_d}{net} = -\frac{Bnewtv_d}{net} = -Bwv_d$$

Substituting $B = 0.2$T, $w = 0.1$cm and $v_d = 4$ cm/s we get,

$$V_H = -0.2\text{T} \times 0.1\text{cm} \times 4\,\text{cm/s} = -8 \times 10^{-6}\text{V}$$

3(a) *With a neat figure derive the expression for numerical aperture of an optical fibre and the fractional index change?* ⑦



Refractive index of surrounding = n_0
Refractive index of core = n_1
Refractive index of cladding = n_2

$$n_0 < n_2 < n_1$$

$90^\circ - \theta_1$ is the critical angle of incidence for *total internal reflection (TIR)* to happen. Hence, OB further grazes along core and cladding interface. Incident angles $< \theta_0$ will further undergo TIR but $> \theta_0$ will simply refract into the cladding. Rotating OA forms a cone called *acceptance cone*. Only rays entering this cone will undergo TIR others refract into the cladding.

θ_0 is called the waveguide acceptance angle or the acceptance cone half angle which is the maximum angle from the axis of optical fibre at which light ray may enter the fibre and propagates in core by TIR.

$\sin\theta_0$ is called the numerical aperture (NA). It determines the light gathering aability of the fibre and purely depends on the refractive index of core, cladding and the surrounding.

$$\theta_0 = \theta_0(n_0, n_1, n_2)$$

By applying Snell's law at O,

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$
$$\therefore \quad \sin\theta_0 = \frac{n_1}{n_0}\sin\theta_1 \tag{1}$$

By applying Snell's law at B,

$$n_1 \sin(90^\circ - \theta_1) = n_2 \sin 90^\circ$$
$$\therefore \quad \cos\theta_1 = \frac{n_2}{n_1} \tag{2}$$

Using $\sin\theta_1 = \sqrt{1-\cos^2\theta_1}$ and eqn. (2) in (1),

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{1-\cos^2\theta_1} = \frac{n_1}{n_0}\sqrt{1-\frac{n_2^2}{n_1^2}}$$
$$\therefore \quad \sin\theta_0 = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2}$$

If the surrounding medium is air, then $n_0 \approx 1$. Therefore numerical aperture (NA) is,

$$\therefore \quad \text{NA} = \sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

The condition for incident rays to be propagated in the fibre is,

$$\begin{aligned} &\theta_i < \theta_0 \\ \text{or} \quad &\sin\theta_i < \sin\theta_0 \\ \text{or} \quad &\sin\theta_i < \text{NA} \end{aligned}$$

Fraction index change (Δ) is defined as, the ratio of difference in refractive indices of the core and the cladding to the refractive index of the core.

$$\Delta = \frac{\text{Refractive index of core} - \text{Refractive index of cladding}}{\text{Refractive index of core}} = \frac{n_1 - n_2}{n_1}$$

It is also known as relative core clad index difference. That is,

$$\Delta = 1 - \frac{n_2}{n_1}$$

3(b) *With a neat sketch explain the structure and working of GRIN fibre.* ③

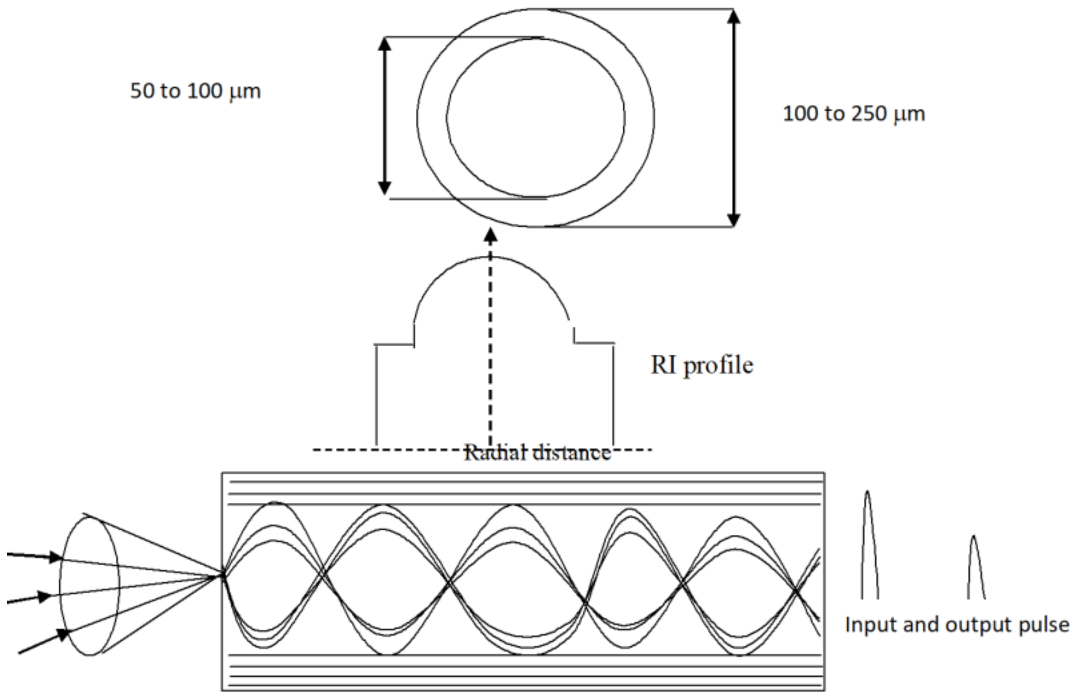
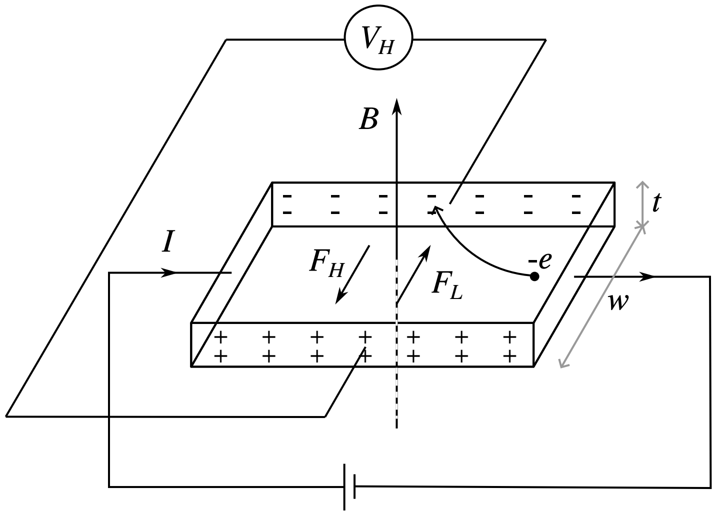
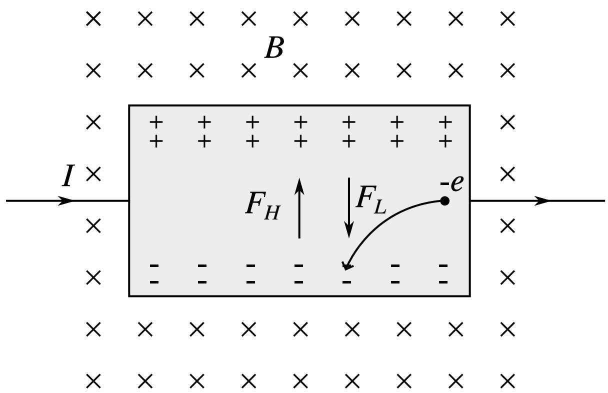


Fig. Graded index multimode fibre

- Refractive index of the core varies across the core diameter (radially graded) as shown in figure, while the refractive index of the cladding is fixed.
- Many modes can be transmitted without intermodal dispersion.
- The rays move in a curved path through the core.
- Light travels at lower speed in the high-index region – fastest components of the ray take the longer path and the slower components take the shorter path in the core.
- Hence the travel time of the different modes will be almost same.
- Losses are minimum. Very little pulse broadening.
- Suitable for large bandwidth, medium distance and medium bit rate communication systems.
- Either a laser or LED source can be used.
- Joining the optical fibres is relatively easy.

4(a) *Define Hall effect and with a neat figure arrive at an expression for hall coefficient for a pentavalent doped semiconductor.* ⑦





Consider a rectangular plate of n-type semiconductor (n-type is pentavalent doped) having width w and thickness t. When a potential difference is applied across its ends a current I flows through it opposite to the direction of flow of electrons. The current passing through the semiconductor is given by,

$$I = -neAv_d \qquad \text{or} \qquad v_d = \frac{-I}{neA} = \frac{-I}{newt} \tag{1}$$

where n is concentration of electrons, $A = wt$ is area of cross section of end face, e is electronic charge, v_d is drift velocity of electrons.

If magnetic field is applied perpendicular to the current flow the Lorentz force deflects the electrons to sideways. The magnitude of this force on an electron is,

$$F_L = eBv_d \tag{2}$$

As electrons pile up on one side an equivalent amount of positive charges are left on the other opposite side. As a result an electric field E_H is developed between these separated unlike charges. This transverse electric field E_H is known as Hall field. The magnitude of the corresponding electrostatic force on an electron will be,

$$F_H = eE_H = e\frac{V_H}{w} \tag{3}$$

where V_H is the corresponding voltage for the field E_H and it is called Hall voltage.

A condition of equilibrium is reached when the force F_H due to transverse electric field balances the Lorentz force F_L.

$$F_L = F_H$$

Substituting the forces from eqns. (2) and (3),

$$e\frac{V_H}{w} = eBv_d$$

Substituting v_d from eqn. (1) into it and rearranging we get expression for **Hall voltage** as,

$$\boxed{V_H = -\frac{BI}{net}} \tag{4}$$

Reciprocal of carrier charge density is called Hall coefficient R_H. In case of electrons,

$$\boxed{R_H = \frac{-1}{ne}}$$

With this eqn. (4) can be written as,

$$\boxed{V_H = R_H \frac{BI}{t}} \qquad \text{or} \qquad \boxed{R_H = V_H \frac{t}{BI}}$$

The Hall voltage can be measured with a voltmeter. **For conductors and *n*-type semiconductors Hall voltage is** conventionally taken as **negative**.

4(b) *For silicon at 30°C, calculate the number of states per unit energy per unit volume at an energy 26meV above the bottom of the conduction band ($m_e^* = 1.18 m_e$).* ③

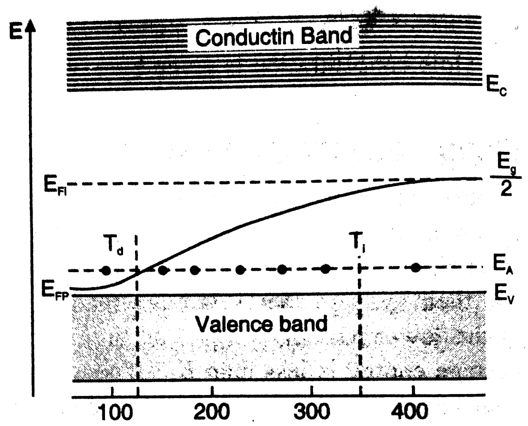
Density of states for energies $E \geq E_C$ is given by,

$$g_c(E) = \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E - E_C)^{1/2}\, dE$$

Substituting $E - E_C = 26\text{meV} = 26 \times 10^{-3} \times 1.602 \times 10^{-19}\text{J}$, $m_e^* = 1.18 m_e = 1.18 \times 9.109 \times 10^{-31}\text{kg}$, $h = 6.626 \times 10^{-34}\text{Js}$ into it, we get,

$$\begin{aligned}\frac{g_c(E)}{dE} &= \frac{4 \times 3.1416}{(6.626 \times 10^{-34}\text{Js})^3}(2 \times 1.18 \times 9.109 \times 10^{-31}\text{kg})^{3/2}(26 \times 10^{-3} \times 1.602 \times 10^{-19}\text{J})^{1/2} \\ &= 8.788 \times 10^{45}\,\text{J}^{-1}\text{m}^{-3}\end{aligned}$$

5(a) *With a neat figure describe the variation of Fermi level with respect to temperature for an intrinsic semiconductor doped with a trivalent impurity.* ⑦



In case of p-type semiconductor the Fermi level E_{Fp} rises with increasing temperature from below the acceptor level to intrinsic level E_{Fi} as shown in the figure.

This can be analyzed by the following general expression for Fermi energy of p-type semiconductor.

$$E_{Fp} = \frac{E_A + E_V}{2} + \frac{kT}{2}\ln\frac{N_V}{N_a} \quad \text{and} \quad E_{Fi} = \frac{E_C + E_V}{2}$$

where E_A is acceptor energy level, E_V is top most energy level in valence band, T is temperature, N_a is negative acceptor ions density and

$$N_V = 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2}$$

1. As the valence band is the source of electrons and the acceptor levels are the recipients for them, the Fermi level must lie between the top of the valence band and the acceptor levels. Also, for $T = 0\,\mathrm{K}$ the above expression becomes,
$$E_{Fp} = \frac{E_A + E_V}{2}$$
Hence the Fermi level lies midway between the acceptor levels and the top of the valence band. In the low temperature region, holes in the valence band are only due to the transitions of electrons from the valence band to the acceptor levels.

2. As the temperature increases the acceptor levels gradually get filled and the Fermi level moves upward. At the temperature of saturation T_d (beginning of depleted region), the Fermi level coincides with the acceptor level E_A. Thus,
$$E_{FP} = E_A \qquad \text{at } T = T_d$$

3. As the temperature grows above T_d, the Fermi level shifts upward in an approximately linear fashion.

4. At temperature T_i intrinsic behaviour sets in. At higher temperatures, the p-type semiconductor loses its extrinsic character and behaves as an intrinsic semiconductor. In the intrinsic region, the hole concentration in the valence band increases exponentially and the Fermi level approaches the intrinsic value. Thus,
$$E_{Fp} = E_{Fi} = \frac{E_C + E_V}{2} \qquad \text{at } T = T_i$$

5(b) *Define Fermi factor, Fermi energy and sketch the variation of Fermi level (factor?) when T ≠ 0K.* ③

Fermi factor $f(E)$ represents the probability of finding a particle with energy E, or in the language of statistical mechanics, the probability that a state with energy E is occupied at the absolute temperature T.

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where E_F is called the Fermi energy and k is the Boltzmann's constant.

Fermi energy:

- Fermi energy is the highest occupied energy level at absolute zero.
- The probability of finding an electron with an energy equal to the Fermi energy is exactly 1/2 at any temperature.
- Also, Fermi energy is the *average energy* possessed by the conduction electrons in conductors at temperatures above 0 K.

DEPARTMENT OF PHYSICS

SECOND SEMESTER BE PROGRAMS (CS, CD, CY, IS, AIML, & BT)

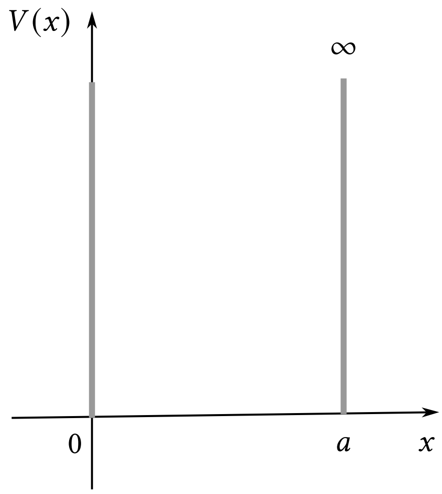
ACADEMIC YEAR 2022-2023

Dated	05th September 2023	Maximum Marks	50
Course Code	22PHY22C	Duration	90 min
Course	Quantum Physics For Engineers	**CIE-III (Test) - Long Scheme**	

1(a) *Solve the problem of a particle in an infinite well to arrive at the un-normalized eigen functions and eigen values.* ⑦

Suppose the potential is,

$$V(x) = \begin{cases} 0, & 0 \le x \le a, \\ \infty, & \text{otherwise} \end{cases}$$



A particle in this potential is completely free, except at the two ends ($x = 0$ and $x = a$), where an infinite force prevents it from escaping. A classical model would be a cart on a frictionless horizontal air track, with perfectly elastic bumpers—it just keeps bouncing back and forth forever. (This potential is artificial, of course, but I urge you to treat it with respect. Despite its simplicity—or rather, precisely because of its simplicity—it serves as a wonderfully accessible test case for all the fancy machinery that comes later.)

We begin by time independent Schrödinger equation which reads,

$$\frac{d^2\psi(x)}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi(x) = 0 \tag{1}$$

Outside the potential well ($0 \le x \le a$) it reads

$$\frac{d^2\psi(x)}{dx^2} + \frac{8\pi^2 m}{h^2}(E - \infty)\psi(x) = 0$$

This equation holds good only if $\psi(x) = 0$ for all points outside the well. The probability of finding the particle there is zero.

Inside the potential well, where $V = 0$, the time-independent Schrödinger equation (1) reads,

$$\frac{d^2\psi(x)}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi(x) = 0$$

Or

$$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0, \qquad \text{where } k^2 = \frac{8\pi^2 m}{h^2}E \tag{2}$$

Solution: Equation (2) is the classical *simple harmonic oscillator* equation; the general solution is,

$$\psi(x) = A\sin(kx) + B\cos(kx) \tag{3}$$

where A and B are arbitrary constants. Typically, these constants are fixed by the *boundary conditions* of the problem.

Boundary conditions: What are the appropriate boundary conditions for $\psi(x)$? Ordinarily, both ψ and $d\psi/dx$ are continuous, but where the potential goes to infinity only the first of these applies.

Continuity of $\psi(x)$ requires that

$$\psi(0) = \psi(a) = 0$$

so as to join onto the solution outside the well. What does this tell us about A and B?

Condition 1: At $x = 0$, $\psi = 0$

Substituting it in equation (3), we get

$$\psi(0) = A\sin 0 + B\cos 0 = B$$

so $B = 0$ as $\psi(0) = 0$, and hence equation (3) will become,

$$\boxed{\psi(x) = A\sin(kx)} \tag{4}$$

Condition 2: At $x = a$, $\psi = 0$

Substituting it in equation 4, we get

$$\psi(a) = A\sin(ka)$$

Since $\psi(a) = 0$,

$$A\sin(ka) = 0$$

so either $A = 0$ (in which case we're left with the trivial—non-normalizable—solution $\psi(x) = 0$), or else $\sin(ka) = 0$ which means that

$$ka = 0,\ \pm\pi,\ \pm 2\pi\ \pm 3\pi,\ \ldots$$

But $k = 0$ is no good (again, that would imply $\psi(x) = 0$, which means the particle is not inside the well), and the negative solutions give nothing new, since $\sin(-\theta) = -\sin(\theta)$ and we can absorb the minus sign into A. So the *distinct* solutions are

$$k = \frac{n\pi}{a}, \qquad \text{with } n = 1, 2, 3, \ldots \tag{5}$$

Curiously, the boundary condition at $x = a$ does not determine the constant A, but rather the constant k, and hence the possible values of E by substituting k^2 into equation (2). We get,

$$\frac{n^2\pi^2}{a^2} = \frac{8\pi^2 m}{h^2}E$$

or

$$\boxed{E = \frac{n^2h^2}{8ma^2}} \qquad \text{with } n = 1, 2, 3, \ldots \tag{6}$$

In radical contrast to the classical case, a quantum particle in the infinite square well cannot have just any old energy—it has to be one of these special ("allowed") values.

1(b) *The 1st excited state wave function of a particle in an infinite well is given by $\psi = B\sin(10^9\pi x)$. Calculate B and energy of the state.* ③

For first excited state $n = 2$ and

$$\psi_2 = \sqrt{\frac{2}{a}}\sin\left(\frac{2\pi}{a}x\right)$$

Comparing it with the given wave function,

$$\psi = B \sin\left((10^9 \mathrm{m}^{-1})\pi x\right)$$

we identify,

$$(10^9 \mathrm{m}^{-1})\pi x = \frac{2\pi}{a} x \qquad \implies a = \frac{2}{10^9 \mathrm{m}^{-1}} = 2 \times 10^{-9} \mathrm{m}$$

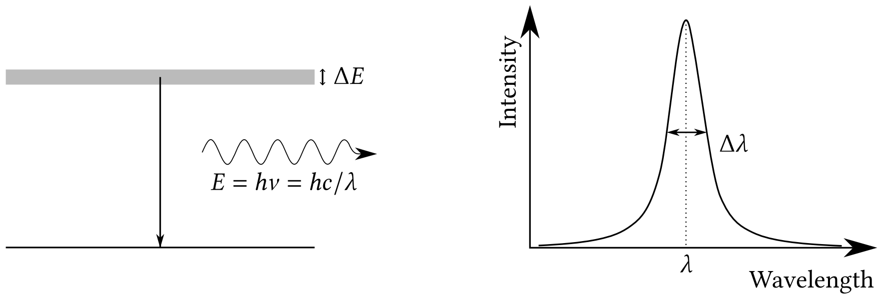
and using this a,

$$B = \sqrt{\frac{2}{a}} = \sqrt{\frac{2}{2 \times 10^{-9} \mathrm{m}}} = 10^{9/2} \mathrm{m}^{-1/2}$$

Assuming the particle as an electron with $m = 9.109 \times 10^{-31}$kg the energy of the state is given by,

$$E = \frac{n^2 h^2}{8ma^2} = \frac{2^2 \times (6.626 \times 10^{-34}\,\mathrm{J\,s})^2}{8 \times 9.109 \times 10^{-31}\mathrm{kg} \times (2 \times 10^{-9}\mathrm{m})^2} = 6.025 \times 10^{-20}\mathrm{J} = 0.376\mathrm{eV}$$

2(a) *Using Heisenberg's uncertainty principle, explain the broadening of atomic spectral lines. Hence derive an expression for the minimum line broadening.* ⑦



The energy of the emitted photon is given by,

$$E = h\nu = \frac{hc}{\lambda} \tag{1}$$

Where h is Planck's constant, ν is the frequency, c is the velocity and λ is the wavelength of the emitted radiation. Differentiating this equation with respect to λ, we get,

$$\Delta E = -hc\frac{\Delta\lambda}{\lambda^2}$$

Considering only the magnitude of the difference,

$$|\Delta E| = hc\frac{\Delta\lambda}{\lambda^2} \tag{2}$$

According to Heisenberg's uncertainty principle, the finite lifetime Δt of the excited state means there will be an uncertainty of ΔE in the energy of the state. Hence the emitted photon energy will also have an uncertainty of ΔE in its energy and is related by,

$$\Delta E\, \Delta t \geq \hbar/2$$

Or,

$$\Delta E \geq \frac{h}{4\pi\Delta t}$$

Substituting for ΔE from (2) we get,

$$hc\frac{\Delta\lambda}{\lambda^2} \geq \frac{h}{4\pi\Delta t}$$

Or

$$\boxed{\Delta\lambda \geq \frac{1}{4\pi c}\frac{\lambda^2}{\Delta t}}$$

This shows that for a finite lifetime of the excited state, the measured value of the emitted photon wavelength will have a spread of wavelengths around the mean value λ. This demands that for a very narrow spread, the lifetime of the excited state must be very high. Such excited levels are called metastable states whose lifetimes will be of the order of 10^{-3}s. This concept is used in the production of laser light, which will be highly monochromatic due to the involvement of a metastable state.

2(b) *Calculate the difference in energy levels, given that the broadening of the emission line spectrum between them is* 100 Å *and lifetime of the higher energy level is* 100 µs. ③

Broadening of the spectral line is given by,

$$\Delta\lambda \geq \frac{1}{4\pi c}\frac{\lambda^2}{\Delta t}$$

We need to find out λ and hence $E = h\nu = hc/\lambda$ from it. It is given that $\Delta\lambda = 100$Å and $\Delta t = 100\,\mu$s.

$$\therefore \quad \lambda = \sqrt{4\pi c\,\Delta\lambda\,\Delta t} = \sqrt{4 \times 3.142 \times 3 \times 10^8\text{ms}^{-1} \times 100 \times 10^{-10}\text{m} \times 100 \times 10^{-6}\text{s}} = 0.061\,38\text{ m}$$

$$\text{and} \quad E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34}\text{ J s} \times 3 \times 10^8\text{ms}^{-1}}{0.061\,38\text{ m}} = 3.2364 \times 10^{-24}\text{J} = 20.2 \times 10^{-6}\text{eV}$$

3(a) *State de-Broglie's hypothesis. Use the expression relating momentum of a particle to the wavelength of its equivalent wave to arrive at the expression for the energy of a particle of mass m in the ground state of an infinite well of width a.* ⑦

According to the de Broglie's hypothesis, any material particle of mass m moving with velocity v and momentum p also shows wave particle duality similar to photons. The wavelength of such matter waves is given by

$$\boxed{\lambda = \frac{h}{mv} = \frac{h}{p}}$$

This is called the de Broglie wavelength.

For particle in ground state of an infinite square well, $\lambda = 2a$ (wave function is a half sine wave) where a is the width of the well. Using de Broglie's wavelength, its momentum is given by,

$$p = \frac{h}{\lambda}$$

Therefore, energy of this particle is,

$$E = \frac{p^2}{2m} = \frac{1}{2m}\frac{h^2}{\lambda^2} = \frac{h^2}{2m(2a)^2} = \frac{h^2}{8ma^2}$$

3(b) *An infinite well between* 0 *to* a *is shifted to the new position* $-0.5a$ *to* $0.5a$. *Will the eigenvalues and eigenfunctions change for this new configuration? Explain why.* ③

Energy of the particle in the potential well is inversely dependent on square of the well width.

$$E = \frac{n^2 h^2}{8ma^2}$$

As the well width $= 0.5a - (-0.5a) = a$ does n't change due to shifting the energy eigenvalues remain same.

However the wave functions will change to account for the new symmetry of the well.

4(a) *State the condition of unitarity of a matrix. Show that the Pauli Matrices* σ_x, σ_y, σ_z *are unitary matrices.* ⑦

A matrix **U** is said to be unitary if it satisfies,

$$\mathbf{U}\mathbf{U}^\dagger = \mathbf{U}^\dagger\mathbf{U} = \mathbf{I}$$

To show that the Pauli's matrices are unitary matrices:

$$\sigma_x\sigma_x^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad \sigma_x^\dagger\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_y\sigma_y^\dagger = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad \sigma_y^\dagger\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_z\sigma_z^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad \sigma_z^\dagger\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

4(b) *Prove that the matrix given below is a unitary matrix. All intermediate steps need to be shown explicitly.* ③

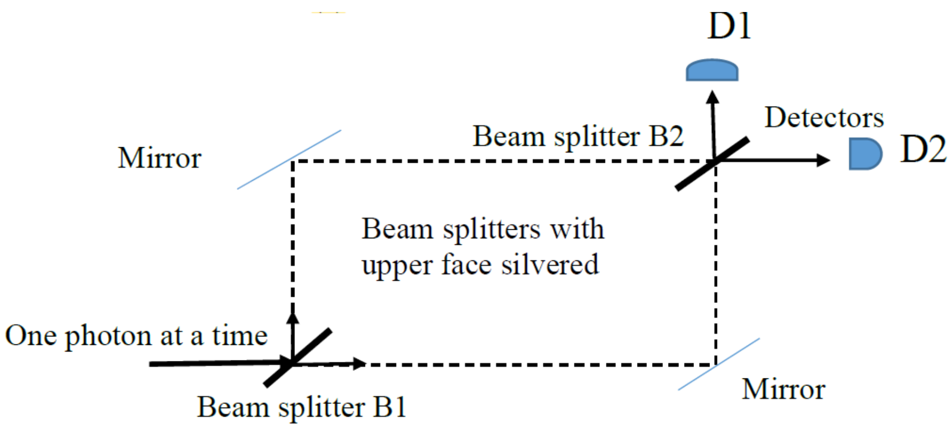
$$\mathbf{U} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{pmatrix}$$

$$\mathbf{U} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \qquad \text{and} \qquad \mathbf{U}^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$$

$$\therefore \quad \mathbf{U}\mathbf{U}^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1+1 & -i+i \\ i-i & -i^2-i^2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$$

$$\therefore \quad \mathbf{U}^\dagger\mathbf{U} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1-i^2 & 1+i^2 \\ 1+i^2 & 1-i^2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$$

5(a) *With a neat labeled diagram involving a single photon source, two beam splitters, two mirrors and two detectors, prove that a single photon simultaneously travels through both the paths at the same time and does not choose any one of the paths in a random fashion.* ⑦



5(b) Calculate the inner product between two vectors $|\psi\rangle$ and $|\chi\rangle$, given that for a particular basis, the two kets can be written as : $|\psi\rangle = 0.707|\phi_1\rangle + 0.707|\phi_2\rangle$ and $|\chi\rangle = 0.5|\phi_1\rangle + 0.866|\phi_2\rangle$. ③

In matrix form $|\psi\rangle$ and $|\chi\rangle$ can be written as,

$$|\psi\rangle = \begin{pmatrix} 0.707 \\ 0.707 \end{pmatrix} \qquad \text{and} \qquad |\chi\rangle = \begin{pmatrix} 0.5 \\ 0.866 \end{pmatrix}$$

Therefore their inner product is,

$$\langle\psi|\chi\rangle = \begin{pmatrix} 0.707 & 0.707 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.866 \end{pmatrix} = 0.9658$$

USN 1 R V 2 2 A I 0 0 7

RV COLLEGE OF ENGINEERING®
(An Autonomous Institution affiliated to VTU)
II Semester B. E. Examinations October– 2023
COMMON TO AI, BT, CS, CY, CD, IS

QUANTUM PHYSICS FOR ENGINEERS

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 & 11 are compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10, and 11 lab components (compulsory).
3. Handbook of Physics is allowed.

PART A

1	1.1	According to Heisenberg's uncertainity principle, if an electron resides in a state with very high life-time as it de-excites to the ground state, the broadening in the wavelength of the photons emitted by the atom will ______.	01
	1.2	At $T > 0K$, for energy level $E = E_F$, the Fermi factor is __________.	01
	1.3	Define bit in classical computing.	01
	1.4	Write an expression for the rate of stimulated absorption in a Laser.	01
	1.5	The free electron density of aluminum is $18.1 \times 10^{28} m^{-3}$. Calculate the Fermi Energy at $0K$.	02
	1.6	Mention any two properties of wave function.	02
	1.7	An electron moves with a speed of $4.70 \times 10^6 m/s$. What is its de Broglie wavelength?	02

PART B

2	a	For a particle in a one dimensional potential well of infinite depth, solve time independent Schrodinger wave equation and obtain an expression for the normalized wave function. Hence write the normalized wave function for the lowest 3 energy eigen states.	10
	b	The first excited state wave function of a particle in an infinite well is $\psi = B\sin(10^9 \pi x)$ Calculate B. An electron is confined to move between two rigid wall separated by $20A°$.Find the de Broglie wavelength and the corresponding energy eigen values for the first two allowed energy states.	04
3	a	Prove that Pauli matrices σ_x, σ_y and σ_z are unitary matrices.	06
	b	Elucidate the difference between classical and quantum computing. Consider $\lvert\Psi\rangle$ and $\lvert\phi\rangle$ are the two wavefunctions. Prove that $\langle\Psi\vert\phi\rangle = \langle\phi\vert\Psi\rangle^*$	08
		OR	
4	a	With a neat labeled diagram, prove that a single photon simultaneously travels through both the paths at the same time and does not choose any one of the paths in a random way.	10
	b	Discuss $CNOT$ gate and its operation on four different input states.	04

5	a	With a neat labeled block diagram, explain point to point communication system using optical fibers. Explain the role of repeater.	06
	b	Distinguish spontaneous and stimulated emission and identify which plays a role in the operation of laser. Two levels of an atomic system at thermal equilibrium has energy difference of $1.8eV$. If the system is at temperature $300K$ determine the ratio of population of these two energy levels.	08
		OR	
6	a	With energy band diagram, explain construction and working of semiconductor diode laser.	07
	b	Explain Graded index multimode fiber with a neat sketch of ray propagation and refractive index profile diagram. A laser operating at temperature of $300K$ and wavelength of 680nm is at thermal equilibrium. Determine the ratio of Einstein coefficients.	07
7	a	What is Fermi factor in metals? Discuss the temperature dependence of Fermi factor in metals at $T = 0K$ and $T > 0K$	04
	b	Draw energy band diagram for the n-type semiconductor, and explain the effect of doping concentration on the band gap. At what temperature, can we expect an 8% probability of occupancy by electrons in an energy level which is 2% above the Fermi level? Given $E_F = 7eV$.	10
		OR	
8	a	What is Hall Effect? With a neat diagram, explain the Hall effect setup and arrive at the expression for Hall coefficient of an n-type semiconductor.	06
	b	Elucidate the difference between classical and quantum free electron theory An intrinsic semiconductor has an energy gap of $0.4eV$. Calculate the ratio of probability of occupation of the lowest level in the conduction band at $300K$.	08
9	a	Explain DC and AC Josephson effect with relevant diagram. With graph, explain the dependence of resistivity on temperature of a superconductor with that of a normal conductor.	10
	b	The transition temperature for Pb is $7.2K$. However, at $5K$ it loses the superconducting property if subjected to magnetic field of 3.3×10^4A/m. Find the maximum value of H which will allow the metal to retain its superconductivity at $0\ K$.	04
		OR	
10	a	Classify superconductors based on the penetration of the magnetic field into the superconductor with the help of $M - H$ graphs.	06
	b	Discuss the principle and working of $DC\ SQUID$. Mention its applications. The critical magnetic field at $5K$ is $2x10^3$A/m in a superconductor ring of radius $0.02m$. Find the value of critical current.	08
11	a	Write the condition for diffraction of light to be observed. Write the experimental procedure to determine the tan θ and wavelength of given Laser beam using a diffraction grating. From the data given below determine the tan θ and wavelength of Laser beam. *Diffraction order*: 1, 2, 3, 4, 5, 6 *Distance* $2Xn$ *(cm)*: 2, 4.1, 6.1, 8, 10.2, 12.3 Distance between the grating & screen $d = 72\ cm$ Grating constant = $5.08 \times 10^{-5}\ m$	10
	b	With experimental circuit diagram and model graphs, explain the procedure to draw the input and output characteristics of an npn transistor in common emitter mode. Write the expression for current gain in CE mode.	10