CH: Quantum Mechanics

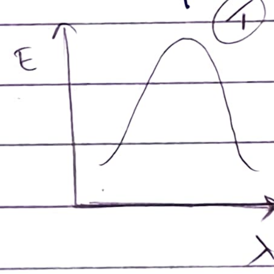
→ Classical mechanics (CM) (eqns / formulas) fails to explain microscopic bodies.
↳ used for macroscopic bodies

1. Black body radiation
2. photoelectric effect
3. Compton effect
4. Zeeman effect etc.

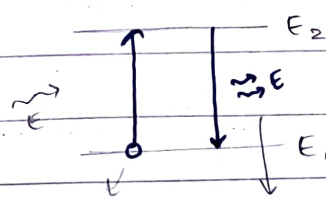
→ CM gives a deterministic approach.

→ Insufficiency of C.M lead to the growth of Q.M (due to dual nature)
↓
probabilistic approach.

Black body Radiation → 1. Kirchhoff
2. Hertz
3. Planck
↓
Planck's hypothesis (QM)
↓
couldn't prove it properly.



peak changed with change in temp couldn't be explained using CM



absorbs energy

radiates energy and comes back to ground state

there are definite energy levels & when energy is given e^- move into an excited state

→ When studying light (Radiation)

Wave nature — spread out disturbance
1. Amplitude
2. Frequency
3. Wavelength
4. Intensity → C.I / D.I

particle nature → concentrated mass
1. mass
2. position / momentum

⇒ Black body radiation etc

→ · Interference
· Diffraction
· Polarization

note: Dual Behavior is not observed (particle & Wave)

De Broglie Waves ⇒ (Matter Waves / Pilot Waves)

Matter Wave: The waves associated with matter in motion or moving particle or material particle are called as matter waves / de broglie waves / Pilot waves.

(Matter corelated to light
∴ Hypothesis → proof)

1) De Broglie wavelength ⇒ $E = h\nu$ → Planck's hypothesis
$E = mc^2$ → Einstein

$h\nu = mc^2 \Rightarrow \frac{hc}{\lambda} = mc^2 \Rightarrow \boxed{\lambda = \frac{h}{mc} = \frac{h}{mv} = \frac{h}{P}}$

momentum of particle

2) Energy associated with the particle $\Rightarrow$

If 'E' is the kinetic energy of the particle then $E = \frac{1}{2} mv^2$

$$E = \frac{1}{2} mv^2$$

$$E = \frac{1}{2m} m^2 v^2 = \frac{1}{2m} p^2$$

$$\boxed{P = \sqrt{2mE}}$$

3)

De Broglie wavelength associated with an $e^- \Rightarrow$

consider an e^- 'e' accelerated by a potential 'V'

$$\Rightarrow \frac{1}{2} mv^2 = eV$$

$$\Rightarrow m^2 v^2 = 2meV$$

$$\Rightarrow p^2 = 2meV \Rightarrow p = \sqrt{2meV}$$

$$\Rightarrow \lambda = \frac{h}{p} \Rightarrow \boxed{\lambda = \frac{h}{\sqrt{2meV}}} \Rightarrow \frac{12.28}{\sqrt{V}} \mathring{A}$$

1.6×10^{-19}

9.1×10^{-31} kg

eqn used in application of -SEM-
(Scanning electron microscope)

envelope: envelope of a wave packet shows the region where a particle is likely to be found

Properties / characteristics of matter waves:

- lighter the particle → $\lambda \uparrow$
- lesser the velocity → $\lambda \uparrow$

matter waves	Electromagnetic waves.
· produced by charged / uncharged particles in motion	· Produced only by a moving charged particle
· In an isotropic medium (material that has a high degree of spatial symmetry) wavelength of a matter wave changes with the velocity of the particle	· in an isotropic medium the wavelength of an electro magnetic wave remains constant.

Wave packet → probability amplitude

wave packet / wave grp → probability of finding a particle

when waves are superimposed on each other & move as a tight unit → have slightly diff wavelengths.

When 2 or more waves are superimposed on each other (diff slightly in wavelengths) → resultant pattern emerges in shape of variation in amplitude.

speed at which a certain / specific component of a wave travels. ex crest → a wave phase

phase velocity → the velocity with which planes of constant phase moves

$V_p = \frac{\omega}{k} = u\lambda$ ω → ang. freq.

$k \to \frac{2\pi}{\lambda}$ → wave vector

the speed at which the entire wave disturbance / entire envelope travels

Group velocity → the velocity with which wave packets formed due to superposition of two or more progressive waves is called group velocity

$V_g = \frac{\Delta \omega}{\Delta k} = \frac{\partial \omega}{\partial k}$ | V_g is diff from individual V_p's

Relation between V_g & V_p

$V_p = \frac{\omega}{k}$ $\quad V_g = \frac{\partial \omega}{\partial k}$

$\Downarrow$

$kV_p = \omega$

$\times$ & $\div$ $\partial\lambda$

$V_g = \frac{\partial (V_p k)}{\partial k} = V_p + k\frac{\partial V_p}{\partial k} \Rightarrow V_p + k\frac{\partial V_p}{\partial \lambda}\cdot\frac{\partial \lambda}{\partial k}$

then $k = \frac{2\pi}{\lambda} \Rightarrow \frac{\partial \lambda}{\partial k} = \frac{-\lambda^2}{2\pi}$ substitute

$V_g = V_p + \left(\frac{2\pi}{\lambda}\right)\frac{\partial V_p}{\partial \lambda}\left(\frac{-\lambda^2}{2\pi}\right)$

$V_g = V_p - \lambda\frac{\partial V_p}{\partial \lambda}$

Q) calculate the frequency & wavelength of a photon given it's energy as 75eV

$\because E = h\nu$

$75\times1.6\times10^{-19} = 6.625\times10^{-34}\times\nu$

$\nu: 0.0019 \times 10^{15} \Rightarrow 18.13\times10^{15}$ Hz

$\lambda = \frac{c}{\nu} = \frac{3\times10^{8}}{18.13\times10^{15}} \Rightarrow 1.6547\times10^{-8}$

Q) A proton is moving with a speed of 2×10^8 m/s. find the wavelength of the matter wave associated with it.

$V = 2 \times 10^8$

$\lambda = ?$

$m_p = 1.67 \times 10^{-27}$ kg

$$\lambda = \frac{h}{mV} = \frac{6.625 \times 10^{-34}}{3.34 \times 10^{-19}} \Rightarrow 1.98 \times 10^{-15} \text{ m}$$

Q) the de-broglie wavelength associated with an e^- is $0.1 A^\circ$ find the potential difference by which the e^- is accelerated

$\lambda = 0.1 A^\circ$

$V = ?$

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{\cancel{12.28} A^\circ}{\sqrt{\cancel{V}}} = \cancel{0.1} \cancel{A}^\circ$$

$$\frac{6.625 \times 10^{-34}}{\sqrt{29.12 \times 10^{-50} \times V}} = 0.1 \times 10^{-10}$$

$V = 15.05$ kV

Q) calculate the de-broglie wavelength of an α particle accelerated through a potential difference of 200V

$\lambda_\alpha = ?$ $V = 200V$

$$\lambda_\alpha = \frac{h}{\sqrt{2 m_\alpha e_\alpha V}}$$

$m_\alpha = 4 \times m_p$, $e_\alpha = 2 \times e$

$$2 m_\alpha e_\alpha V = \frac{h^2}{\lambda_\alpha^2}$$

$$\frac{\cancel{1068.81}}{32.69} = 0.202$$

$\lambda_\alpha = 7.16 \times 10^{-13}$ m

Q) find the de broglie wavelength of an e^- ~~of~~ in the first Bohr's orbit of the hydrogen atom.

$\lambda_e = ?$

$E_n = \frac{-13.6}{n^2} eV = +13.6 \times 1.6 \times 10^{-19}$

$= 21.76 \times 10^{-19}$

$n = 1$

$\lambda_e = \frac{h}{\sqrt{2mE}}$

$\downarrow$

9.1×10^{-31}

396.032×10^{-50}

1.99

$\lambda_e = 3.3 A^\circ$

Q) calculate the ratio of de-broglie waves associated with a proton & an e^- having $KE = 20 MeV$

$\frac{\lambda_p}{\lambda_e} = \frac{\cancel{h}}{\sqrt{\cancel{2} m_p E_p}} \times \frac{\sqrt{\cancel{2} m_e E_e}}{\cancel{h}} = \frac{\sqrt{m_e E_e}}{\sqrt{m_p E_p}}$

Heisenberg's Uncertainity Principle ⇒

→ It is impossible to specify precisely & simultaneously certain pairs of physical quantities like position & momentum that describe the behaviour of an atomic System

→ ①

$\Delta P \cdot \Delta x \geq h/4\pi$

ΔP → uncertainity in momentum

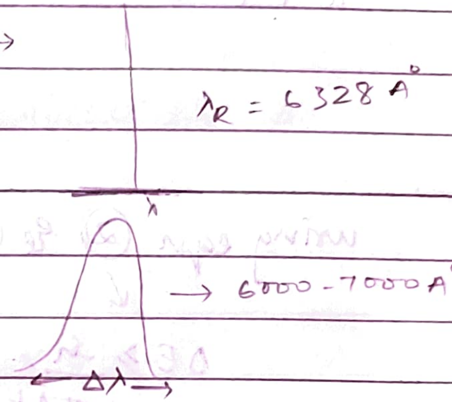
Δx → uncertainity in position

→ ②

$\Delta E \cdot \Delta t \geq h/4\pi$

ΔE → Δ energy

Δt → Δ time



$\Delta J \cdot \Delta\theta \geq h/4\pi$

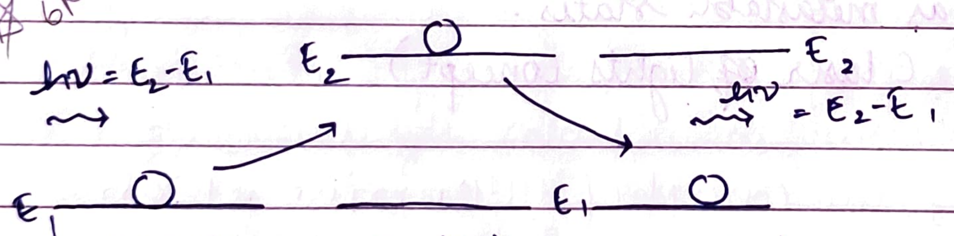
ΔJ → Δ ang. momentum

$\Delta\theta$ → Δ ang. displacement.

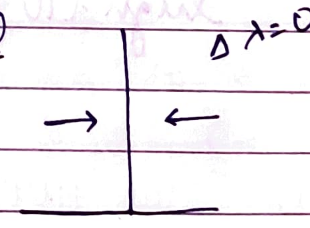
(Broadning of spectral lines)

Application of HUP :- Broadening of Spectral lines ⇒

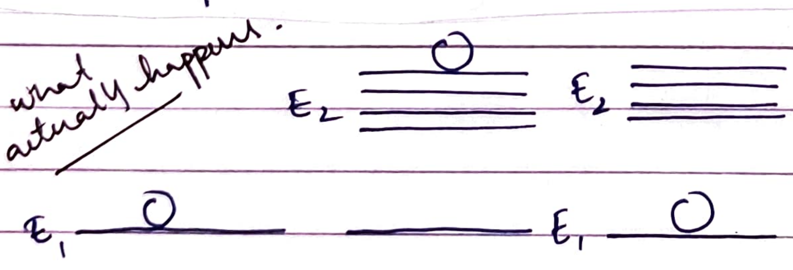
☆ 6M



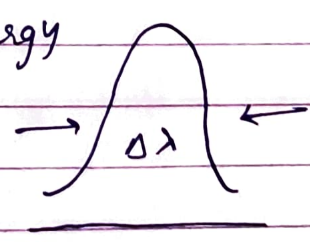
Ideal



what actually happens.



→ discreet energy levels.



full wave half maxima

$\Delta\lambda = FWHM$

$\Delta t \rightarrow$ life time of atom in the excited state
$\Rightarrow 10^{-8} - 10^{-9}s$ //

$$E = h\nu = \frac{hc}{\lambda}$$

differentiating E w.r.t λ $\Rightarrow$ $\Delta E = \frac{-hc\Delta\lambda}{\lambda^2}$

$$|\Delta E| = \frac{hc\Delta\lambda}{\lambda^2} \rightarrow (3)$$

using eqn (2) & (3) $\Rightarrow$ $\frac{\cancel{h}c\Delta\lambda}{\lambda^2} = \frac{\cancel{h}}{4\pi\Delta t}$

$$\Delta E \geq \frac{h}{4\pi\Delta t}$$

$$\Rightarrow \boxed{\Delta\lambda = \frac{\lambda^2}{4\pi c\Delta t}}$$

conclusion $\longrightarrow$ $\Delta\lambda \propto \frac{1}{\Delta t}$, for narrow $\Delta\lambda$, the life time of the excited state must be very high ($10^{-3}s$) known as metastable states.
(laser ~~of~~ lights concept)

(ex: State HUP, applying the concept of HUP arrive at an expression for broadening of wavelength w.r.t time & write its conclusion)
[concept of HUP $\rightarrow$ 1M
conclusion $\rightarrow$ 1M
derivation $\rightarrow$ rest]

Q) if the uncertainity in the position of an e^- $\Delta x = 4 \times 10^{-10}$ m Calculate the ΔP.

$$\Delta P \cdot \Delta x \geq \frac{h}{4\pi}$$

$$\Delta P = \frac{6.625 \times 10^{-34} \times 10^{10}}{4 \times 3.14 \times 4} = 1.31 \times 10^{-25} \text{ kgm/s}$$

Q) an e^- has the speed of 600 m/s with an accuracy of 0.005% Calculate the uncertainity with which we can locate the Δx.

$$\Delta P \cdot \Delta x \geq \frac{h}{4\pi}$$

$$\Delta P = m\left(\frac{\Delta V}{100}\right) \times V$$

$$= 9.1 \times 10^{-31} \times \frac{0.005}{1\cancel{00}} \times 6\cancel{00}$$

$$= 2.73 \times 10^{-32}$$

$$\Delta x = \frac{6.625 \times 10^{-34}}{4 \times 3.14 \times 2.73 \times 10^{-3}} = 1.93 \times 10^{-3} \text{ m}$$

Q) the uncertainity in the location of a particle is equal to its de brogliewavelength Calculate its uncertainity in its velocity.

$$\Delta x = \lambda = h/P \Rightarrow \Delta x \cdot \Delta P \geq \frac{h}{4\pi}$$

ΔV

$$\Delta P = m\Delta V$$

$$\frac{h}{P} \cdot \Delta P \geq \frac{h}{4\pi}$$

$$\frac{\cancel{m}\Delta V}{\cancel{m}V} = \frac{1}{4\pi} \Rightarrow \boxed{\Delta V = \frac{V}{4\pi}}$$

Q) the position & momentum of ~~10k e-volt~~ 10keV e^- are simultaneously measured, if the position is located with $1A^\circ$, what is the percentage of uncertainity in its wavelength

$E = 10keV = 10 \times 10^{3} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-15}$

$\Delta x = 1A^\circ = 10^{-10}$

$\% \ \frac{\Delta p}{p} \times 100$

$p = \sqrt{2mE} = \cancel{1.71 \times 10^{-23} \ kgm/s} \quad 5.39 \times 10^{-23}$

$\Delta p = \frac{h}{4\pi \Delta x} = 5.27 \times 10^{-25} \ kgm/s$

$\% = 0.97\%$

Q) If an excited state of a Hydrogen atom has a life time of $\Delta t = 2.5 \times 10^{-14} s$, what is the minimum error with which energy of state can measured.

$\Delta E \cdot \Delta t \geq \frac{h}{4\pi} \qquad \Delta E = \frac{h}{4\pi \Delta t} = \frac{6.625 \times 10^{-34}}{4 \times 3.14 \times 2.5 \times 10^{-14}}$

$\Delta E = 2.1 \times 10^{-21} \ J$

Q) an excited atom has an average lifetime of $\Delta t = 10^{-8} s$. during this period it emits a photon & returns to the ground state, what is the min uncertainity in the freq. of this photon

$E = h\nu \quad \Delta E = h\Delta\nu \Rightarrow \cancel{h}\Delta\nu \times \Delta t = \frac{\cancel{h}}{4\pi} \Rightarrow \Delta\nu = \frac{1}{4\pi \Delta t}$

$= 7.96 MHz$

Schrodinger's Wave Equation →

time independent (following this)

time dependent

Wave mechanics - using de-broglie eqn.. Schrodinger developed a mathematical theory called wave mechanics which was successful in explaining the behavior of atomic systems & their interaction with ~~sele~~ electromagnetic radiation & other particles

in classical mechanics we have an eqn $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 y}{\partial t^2}$ this eqn governs a wave whose variable quantity is 'y' that propagates in the 'x' direction with a speed 'v'

ex: in water waves the quantity that varies periodically is the height of water surface, in sound wave - pressure, light waves - electric and magnetic fields

In quantum mechanics, the quantity whose variation in position w.r.t time (x, t) of a matter wave is described as the wave function(ψ) it gives the probable value of position & time $\psi(x, t)$

PTO →

6M

Setting up of schrodinger wave eqn ⇒

Time independent one dimensional Schrodinger wave eqn (TISE)

from classical mechanics (eqn) → $\frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2} \cdot \frac{\partial^2 y}{\partial t^2}$

y → varying physical quantity.

let $\psi(x,t)$ be the wave function of matter wave associate with a particle of mass 'm' moving with velocity 'v'. The differential eqn of the wave motion is as follows:

→ replacing y with ψ

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \longrightarrow (1)$$

the solution of eqn (1) as a periodic displacement of time 't'

$$\psi(x,t) = \psi_0(x) e^{-i\omega t} \longrightarrow (2)$$

differentiating eqn (2) partially twice w.r.t 't' we get

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0(x) e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = i^2 \omega^2 \psi_0(x) e^{-i\omega t} = -\omega^2 \psi_0(x) e^{-i\omega t} = -\omega^2 \psi(x)$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi \longrightarrow (3)$$

substitute eqn (3) in (1)

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = -\frac{\omega^2 \psi}{v^2} \longrightarrow (4)$$

$$\Rightarrow \frac{\omega^2}{v^2} = \underbrace{k^2}_{\text{wave number}} = \left(\frac{2\pi}{\lambda}\right)^2 = \frac{4\pi^2}{\lambda^2}$$

Substituting in eqn (4) $\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = -\frac{4\pi^2}{\lambda^2}\psi \longrightarrow (5)$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2 \psi}{\lambda^2} = 0 \longrightarrow (6)$$

from de-broglie wavelength $\Rightarrow \lambda = \frac{h}{mv} \rightarrow \lambda^2 = \frac{h^2}{m^2v^2}$

$$\frac{\partial^2 \psi}{\partial x^2} + 4\pi^2 \times \frac{m^2v^2}{h^2}\psi = 0 \longrightarrow (7)$$

WKT, Total energy = KE + PE

$\therefore$ KE = TE − PE
(E) (V)

$\longrightarrow$ the KE of the particle is E_k

$$E_K = \frac{1}{2}mv^2 = E - V$$

$$\therefore m^2v^2 = 2m(E-V)$$

Substitute in eqn (7)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2 \times 2m(E-V)}{h^2}\psi = 0 \Rightarrow \boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0}$$

ψ is a function of X alone & is independent of time.

$\psi(x,t) \rightarrow$ describes behavior of moving particle

Physical significance of the wave function $\Rightarrow$

- the wave function has no physical significance, as the wave function is imaginary (as 'i' is an imaginary term ie it has a negative term in the equation). $e^{-i\omega t}$

- the physical significance of ψ is realised through it's probabilistic nature which is depicted in terms of probability density

- in classical mechanics, the square of the wave amplitude associated with EM radiation is interpreted as a measure of radiation intensity. (no of photons per second per unit Volume)

- Similar interpretation is given for de broglie waves.

- let us consider a single particle ie ψ is the wave function associated with the particle then $|\psi|^2$ is the probability per unit volume, found in the given region.

- let us consider a volume 'τ' inside which a particle is known to be present. but where inside τ the particle is located is not known. then the probability of finding the particle in a certain volume $d(\tau)$ of τ is given by

$$|\psi|^2 d\tau = 1$$

$|\psi|^2$ → probability density; $d\tau$ → probability

$=1 \rightarrow$ certain of locating particle

$=0 \rightarrow$ not certain at locating particle.

- this interpretation was given by Max Bohr - 1926

· $|\psi^2|$ is also written as $\psi\psi^*$ where ψ^* is the complex conjugate of the wave function

Normalization of wave function ⇒

normalization condition ⇒ $\int_x \psi^2 dx = 1$
(1D)

If ψ is the wave function associated with the particle then $|\psi^2| d\tau$ is called the probability of finding the particle in a volume $d\tau$. If we are certain of definitely locating the particle then as per statistical rule.

$$\int_0^\tau |\psi|^2 d\tau = 1$$

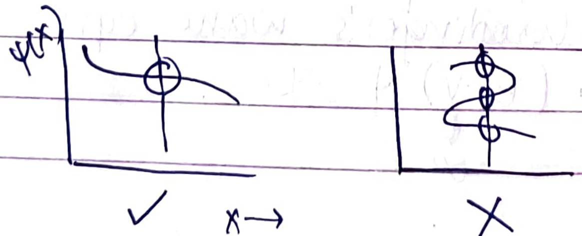
the value '1' for probability means that we are certain.

Properties of wave function ⇒

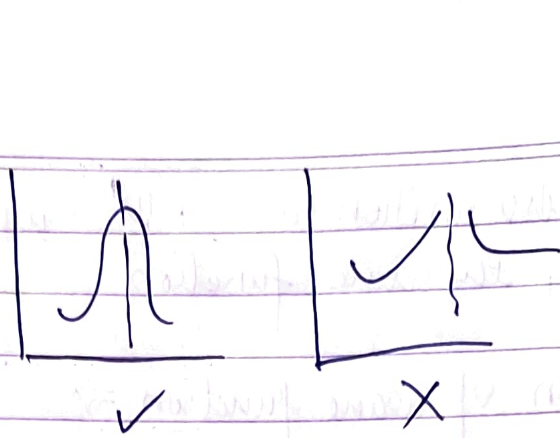
ψ (if conditions are met) ⟶ eigen fn.

If the wave function satisfies all the below mentioned properties then the wave function is called as <u>acceptable</u> wave function or an <u>eigen</u> function.

1) the wave function must be continous and single valued everywhere.



2) ψ is finite everywhere.



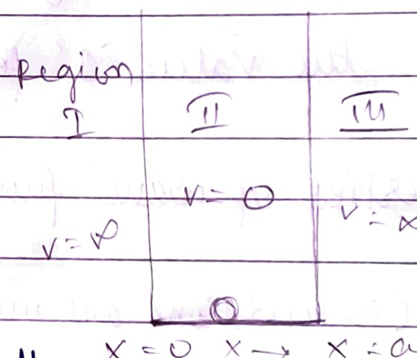
3) the first derivation of ψ w.r.t x should be continous & single valued everywhere, $\because$ it is related to the momentum of the particle which should be finite

4) It must satisfy the condition of Normalization.

Application of Schrodinger's wave Eqn.

1) Particle in one-Dimensional potential well of infinite depth (particle in a box)

Consider a particle of mass 'm' moving in the +ve 'x' direction moving in the region b/w $x=0$ to $x=a$. Outside this region, the potential energy is infinity and within the region $V=0$ (constant potential). the PE b/w two walls is constant and no forces acting on the particle. Such a configuration of potential in space is called 'infinite potential well'.



Case I: If the particle is present outside the well, $V=\infty$. substituting in schrodinger's wave eqn.

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E-\underset{\infty}{\underset{\downarrow}{V}})\psi = 0$$

If $\psi = 0$ for all points outside the box
i.e. $|\psi|^2 = 0$ $\Rightarrow$ particle cannot be found outside the box

Case 2: If the particle is present inside the well, $V = 0$.
substituting in schrodinger's wave eqn.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E\psi) = 0 \longrightarrow (1)$$

rearranging the terms $\Rightarrow$ $E\psi = \left(\frac{-h^2}{8\pi^2 m}\frac{d^2}{dx^2}\right)\psi$

$$\boxed{\hat{H}\psi = E\psi}$$

the above eqn is of the form $\hat{H}\psi = E\psi$
where $\hat{H} \rightarrow$ Hamitonian operator (QM operator)
$E \rightarrow$ Energy Eigen value

the constants in the above eqn are replaced by k^2 and the eqn is written as.

$$k^2 = \frac{8\pi^2 m E}{h^2} \longrightarrow (3)$$

$$\frac{\partial^2 \psi}{\partial x^2} + k^2\psi = 0 \longrightarrow (2)$$

the solution of eqn (2) is

$$\psi = A\cos kx + B\sin kx$$

where A and B are constants which depends on the ~~boud~~ boundary conditions of the well i.e, $x = 0$ and $x = a$

Applying Boundary conditions.

Condition 1: the soln of the above eqn at $x=0, \psi=0$

as per condition (1)

$$\psi = A\cos kx + B\sin kx$$

$$0 = A\cos 0 + (B\sin 0)$$

$$\Rightarrow A = 0$$

Condition 2: at $x=a, \psi=0$

$$0 = A\cos ka + B\sin ka$$

$$0 = 0 + B\sin ka.$$

$$\sin ka = 0$$

$$ka = n\pi$$

$$k = \frac{n\pi}{a}$$

substituting value of k in eqn (3)

$$\cancel{\frac{\partial^2\psi}{\partial x^2}} \left(\frac{n\pi}{a}\right)^2 = \frac{8\pi^2 mE}{h^2}$$

$$\frac{n^2\cancel{\pi^2}}{a^2} = \frac{8\cancel{\pi^2} mE}{h^2} \Rightarrow \boxed{E = \frac{n^2h^2}{8ma^2}}$$

$n \geq 1$

in HUP

E_2 —— excited state

$n=2$

E_1 (ground state)

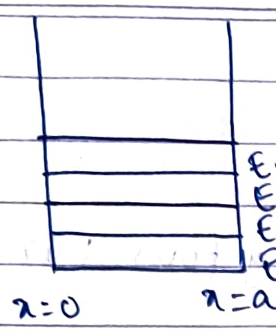
$n=1$

$n=1$

$$E_1 = \frac{h^2}{8ma^2}$$

$n=2$

$$E_2 = \frac{4h^2}{8ma^2}$$



$E_4 = 16E_1$ → 2nd excited state

$E_3 = 9E_1$ → 1st excited state

$E_2 = 4E_1$

$E_1 = \dfrac{h^2}{8ma^2}$ = Ground state

(Zero point energy)

Normalization of wave function

from condition ② we get $\psi = B\sin kx = B\sin \dfrac{n\pi}{a}x$

(particle inside the well)

the probability of finding the particle is

$$\int_0^a |\psi|^2 dx = 1 \Rightarrow \int_0^a B^2 \sin^2 \frac{n\pi}{a} x \, dx = 1$$

W.K.T $\sin^2\theta = \dfrac{1}{2}(1-\cos 2\theta)$

$$B^2 \int_0^a \frac{1}{2}\left(1 - \cos \frac{2n\pi x}{a}\right) dx = 1 \Rightarrow \frac{B^2}{2}\left[\int_0^a dx - \int_0^a \cos \frac{2n\pi x}{a} dx\right] = 1$$

$$\frac{B^2}{2}\left[x - \frac{a}{2n\pi}\sin\left(\frac{2n\pi x}{a}\right)\right]_0^a = 1 \Rightarrow \frac{B^2}{2}\left[a - \frac{a}{2n\pi}\sin(2n\pi) - 0\right] = 1$$

$$\Rightarrow \frac{B^2 a}{2} = 1 \Rightarrow B = \sqrt{\frac{2}{a}}$$

Eigen Functions and Eigen Values:

Case 1: $n=1$

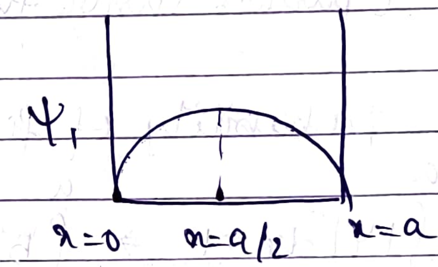
Ground state and the particle is normally found in this state.

for $n=1$, Eigen function is $\boxed{\psi_1 = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi}{a}\right)x}$

$\psi_1 = 0$ at $x=0$ and $x = a$'

$\therefore$ ψ_1 has maximum value at $x = a/2$

$$\psi_1 = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi}{a}\right)\frac{a}{2} = \sqrt{\frac{2}{a}}$$



$(\psi_1)^2 = 2/a \rightarrow$ maximum.

$|\psi_1|^2 \rightarrow$ probability density

at Ground state the particle cannot be found at the walls of the box and the probability and the probability of finding the particle is maximum at the central region

$$E_1 = \frac{h^2}{8ma^2}$$

(for CIE draw the graph as well)

Case 2: $n=2$; The first Excited state.

Eigen function $\Rightarrow$ $\psi_2 = \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi}{a}\right) x$

at $x = 0, a/2, a$ $\quad \psi = 0 \quad |\psi|^2 = 0$

$x = a/4 \quad \psi = \sqrt{\frac{2}{a}} \quad |\psi|^2 = \frac{2}{a}$

$x = 3a/4 \quad \psi = -\sqrt{\frac{2}{a}} \quad |\psi|^2 = \frac{2}{a}$.

In the 1st excited state the particle cannot be observed either at the walls or at the center.

Energy $E_2 = 4E_1$

Case 3: $n=3$, 2nd excited state

$\psi_3 = \sqrt{\frac{2}{a}} \sin\left(\frac{3\pi}{a}\right) x$

at $x = 0, a/3, 2a/3, a \quad \psi = 0 \quad |\psi|^2 = 0$

$x = a/6, 5a/6 \quad \psi = \sqrt{2/a} \quad |\psi|^2 = 2/a$

$x = a/2 \quad \psi = -\sqrt{2/a} \quad |\psi|^2 = 2/a$

$|\psi_3|^2 \neq 0$ particle can be found at maximum that is $x = a/6, 5a/6, a/2$

Q) find the lowest energy of an electron confined to move in a 1D box of length ~~#~~ 1Å. express the results in eV.

$E = 37.4$

Q) an e^- is confined to move b/w two rigid walls separated by 1nm. Find the debroglie wavelength representing the first two energy states of the e^- and the corresponding energies.

$$E_1 = \frac{43.8906 \times 10^{-68}}{8 \times 9.1 \times 10^{-31} \times 10^{-18}}$$

$$= \frac{43.8906 \times 10^{-19}}{8} = 0.5907 \times 10^{-19}.$$

$$\lambda_1 = \frac{h}{\sqrt{2mE_1}}$$

Q) Calculate the energy required for an electron required to jump from grd state to 2nd excited state in a potential well of width 'L'

$$E_3 - E_1 = \frac{h^2}{mL^2}$$

Q) A particle is confined to 1D well of width 'a' and is known to be present in it's 2^{nd} excited state. What is the probability that it would be present b/w $a/3$ and $2a/3$