n umerical methods_tutorials

01 June 2025 09:35 PM

n umerical methods_...

5) $f(x) = x^4 - x - 10 = 0$

$x_0 = 1 \quad x_1 = 2$

$f(x_0) = -10 \quad f(x_1) = 4$
(-ve) (+ve)

$x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \dfrac{(1)(4) - (2)(-10)}{4 - (-10)}$

$= 1.71429$

$f(x_2) = -3.07788$ (-ve)

$f(2) = 4$ (+ve)

$x_0 = 1.71429 \quad x_1 = 2$

$f(x_0) = -3.07788 \quad f(x_1) = 4$

$x_3 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \dfrac{(1.71429)(4) - (-3.07788)(2)}{4 - (-3.07788)}$

$= 1.83853$

$f(x_3) = -0.41278$

$x_0 = 1.83853 \quad x_1 = 2$

$f(x_0) = -0.41278 \quad f(x_1) = 4$

$x_4 = \dfrac{(1.83853)(4) - (-0.41278)(2)}{4 - (-0.41278)}$

$= 1.85364$

$f(x_4) = -0.047775$

$x_0 = 1.85364 \quad x_1 = 2$

$f(x_0) = -0.047775 \quad f(x_1) = 4$

$x_5 = 1.85536$

$f(x_5) = -5.4192 \times 10^{-3}$

$x_6 = 1.85556$

$f(x_6) = -6.2537 \times 10^{-4}$

$x_7 = 1.85558$

$f(x_7) = -6.8314 \times 10^{-5}$

6) $x - \dfrac{\cos x + 1}{3} = 0$

as $\dfrac{\cos x + 1}{3}$ is a root

$\rightarrow f(a) \cdot f(b) < 0$

$f(0) = 0 - \dfrac{\cos 0 + 1}{3} = -2/3 = -0.6667$ (-ve)

$f(1) = 1 - \dfrac{\cos 1 + 1}{3} = 0.4866$ (+ve)

$\rightarrow x_0 = 0 \quad x_1 = 1$

$f(x_0) = -0.6667 \quad f(x_1) = 0.4866$

$x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \dfrac{0(0.4866) - (1)(-0.6667)}{0.4866 - (0.6667)}$

$x_2 = 0.5781$

$f(x_2) = -0.0344$

$x_3 = 0.6060$

$f(x_3) = -1.359 \times 10^{-5}$

$x_4 = 0.6070$

$f(x_4) = -5.1093 \times 10^{-5}$

$x_5 = \underline{0.6071}$

$f(x_5) = -4.7125 \times 10^{-6}$

7) $2x - \ln(x) - 6 = 0$

$f(1) = -4$

$f(2) = -2.693$

$f(3) = -1.099$ (-ve)

$f(4) = 0.614$ (+ve)

$x_0 = 3 \quad x_1 = 4$

$f(x_0) = -1.0986 \quad f(x_1) = 0.6137$

$x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \dfrac{3(0.614) - 4(-1.097)}{0.614 - (-1.099)}$

$= 3.642$

$f(x_2) = -9.234 \times 10^{-3}$

$x_3 = 3.647$

$f(x_3) = -6.12 \times 10^{-5}$

$x_4 = \underline{\underline{3.647}}$

8) $f(x) = x \sin x + \cos x = 0$

$f(2) = 1.402$ (+ve)

$f(3) = -0.567$ (-ve)

$x_0 = 2 \quad x_1 = 3$

$f(x_0) = 1.402 \quad f(x_1) = -0.567$

$x_2 = 2.712$

$f(x_2) = 0.220$

$x_3 = 2.793$

$f(x_3) = 0.0151$

$x_4 = 2.798$

$f(x_4) = 8.886 \times 10^{-4}$

$x_5 = \underline{\underline{2.798}}$

1) $f(x) = x^3 - 21x + 3500 = 0 \Rightarrow f'(x) = 3x^2 - 21$

$f(0) = 3500$

$f(1) = 3480$

$\left.\begin{array}{l} f(-15) = 440 \text{ (+ve)} \\ f(-16) = -260 \text{ (-ve)} \end{array}\right\}$ root lies here

$x_0 = -15$??

-16 -15 0

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$= -15 - \dfrac{440}{654}$

$\left[f'(-15) = 3(-15)^2 - 21 \right]$

2) $f(x) = x \sin x + \cos x = 0$

$f(\pi) = -1$ (-ve)

$f(0) = 1$ (+ve)

0 $\frac{\pi}{2}$

$x_0 = \pi \quad f(x_0) = -1$

$f'(x) = \cancel{\sin x} + x\cos x - \cancel{\sin x}$

$= x \cos x$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$= -15 - \frac{440}{654}$

$= -15.6727$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$\left[f'(-15) = 3(-15)^2 - 21 \right]$

$\rightarrow f(x_1) = -20.61385$ (-20.6732)

$f'(x_1) = 715.90$

$x_2 = -15.64391$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$f(x_2) = -0.04202$ (-0.03918)

$f'(x_2) = 713.19576$ (713.19539)

$x_3 = -15.64385$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$f(x_0) = -1$

$f'(x_0) = -\pi = -3.14159$

$= \pi - \frac{(-1)}{(-\pi)}$

$x_1 = 2.82328$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$f(x_1) = -0.06618$

$f'(x_1) = -2.68145$

$x_2 = 2.79860$

$x_3 = 2.79838$

4) $c = 4e^{-2t} + te^{-0.1t}$

$c = 0.5 \quad t = ?$

$f(c) = 4e^{-2t} + e^{-0.1t}$

$f'(t) = -8e^{-2t} - 0.1e^{-0.1t}$

$f(0) = 5$

$f(1) = 1.44618$

$f(2) = 0.89199$

at $t_0 = 0$

$t_1 = t_0 - \frac{f(t_0)}{f'(t_0)}$

$f(0) = 5 - 0.5 = 4.5$

$f'(0) = -8.1$

$t_1 = 0 - \left(\frac{4.5}{-8.1}\right)$

$= 0.55556$

$t_2 = t_1 - \frac{f(t_1)}{f'(t_1)}$

$f(t_1) = 1.76272$

$f'(t_1) = -2.72812$

$t_2 = 1.20169$

$t_3 = t_2 - \frac{f(t_2)}{f'(t_2)}$

$f(t_2) = 0.74842$

$f'(t_2) = -0.81197$

$t_3 = 2.12342$

$t_4 = t_3 - \frac{f(t_3)}{f'(t_3)}$

$f(t_3) = 0.36593$

$f'(t_3) = -0.19534$

$t_4 = 3.99672$

$t_5 = t_4 - \frac{f(t_4)}{f'(t_4)}$

$f(t_4) = 0.17189$

$f'(t_4) = -0.06976$

$t_5 = 6.466074$

$t_6 = t_5 - \frac{f(t_5)}{f'(t_5)}$

$f(t_5) = 0.02383$

$f'(t_5) = -0.05240$

$t_6 = 6.90574$

$t_7 = t_6 + \frac{f(t_6)}{f'(t_3)}$

$f(t_6) = 0.00129$

$f'(t_6) = -0.05014$

$t_7 = 6.93147$

3) $x = 1/N$

$f(x) = Nx - 1 = 0$

$f'(x) = N$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f(x_n) = Nx - 1$

$f'(x_n) = N$

$x_{n+1} = x_n - \frac{(Nx_n - 1)}{N}$

$= \frac{Nx_n - Nx_n + 1}{N}$

$= \frac{1}{N}$

$N = 31$

$31x - 1 = 0$

$x = 0.03226$

5) $I = 10e^{-t}\sin(2\pi t)$ $\quad -10\left[-e^{-t}\sin 2\pi t + 2\pi e^{-t}\cos 2\pi t\right)$

at 2A, $2 = 10e^{-t}\sin(2\pi t)$

$f(t) = 2 - 10e^{-t}\sin(2\pi t) = 0$

$f'(t) = 10e^{-t}\sin(2\pi t) - 20\pi e^{-t}\cos(2\pi t)$

let $t_0 = 0$

$f(0) = 2$

$f'(0) = -62.83$

$t_1 = t_0 - \frac{f(t_0)}{f'(t_0)}$

$t_1 = 0.03183$

$t_2 = t_1 - \frac{f(t_1)}{f'(t_1)}$

$f(t_1) = 1.9662$

$f'(t_1) = -60.8292$

$t_2 = \cancel{0.03314}$ 0.06415

$t_3 = t_2 - \frac{f(t_2)}{f'(t_2)}$

$f(t_2) = \cancel{0.00015}$ 1.9340

$f'(t_2) = \cancel{-57.47092}$ -58.8600

$t_3 = \cancel{0.03314}$ 0.09701

6) $f(x) = x^2 - \ln(x) - 12) = 0$

$f'(x) = 2x - \frac{1}{x}$

$f(3) = -4.09861$ (-ve)

$f(4) = 2.61371$ (+ve)

$\rightarrow$ $x_0 = 3$ $f(x_0) = -4.09861$

$f'(x_0) = 5.66667$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 3 - \left(\frac{-4.09861}{5.6667}\right)$

$x_1 = 3.72328$

$f(x_1) = 0.54821$

$f'(x_1) = 7.17798$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 3.64691$

$f(x_2) = 0.00607$

$f'(x_2) = 7.01962$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$x_3 = 3.64605$

1) $y' = y\sin x + \cos x$

$y'' = y'\sin x + y\cos x - \sin x$

$y''' = y''\sin x + 2y'\cos x - y\sin x - \cos x$

$y^{iv} = y'''\sin x + 3y''\cos x + 2y'(-\sin x)$

$- y'\sin x - y'\cos x + \sin x$

y' at $x_0 = 0$ $y_0 = 0$

$y' = 1$

$y'' = 0$

$y''' = 0$

$y^{iv} = 0$

$y^{v} = 1$

2) $xy' = x - y$

$y' = \frac{x-y}{x} = 1 - y/x$

$y'' = -y'x^{-1} + y\left(\frac{1}{x^2}\right)$

$= \frac{-xy' + y}{x^2}$

$y''' = \frac{x^3y'' + 2x^2y' - 2xy}{x^4}$

at $x_0 = 2$ $y_0 = 2$

$y' = 0$ $y''' = -1/4$

$y'' = 1/2$

$\rightarrow$ $y = y_0 + \frac{(x-x_0)}{1!}y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0'''$

$y = 2 + (x-2)(0) + \frac{(x-2)^2}{2} \times \frac{1}{2} + \frac{(x-2)^3}{3!} \times \frac{-1}{4}$

$y^{iv} = 0$
$y^{v} = 1$

TSM formula

$$y = y_0 + \frac{(x - x_0)}{1!} y_0' + \frac{(x - x_0)^2}{2!} y_0'' \ldots$$

$$= 0 + \frac{x(1)}{1} + \frac{x^2(0)}{2!} + \frac{x^3(1)}{6} + \frac{x^4(0)}{4!} + \frac{x^5(1)}{120}$$

$$y = x + \frac{x^3}{6} + \frac{x^5}{120}$$

$$y(0.1) = 0.1 + \frac{(0.1)^3}{6} + \frac{(0.1)^5}{120}$$

$$= 0.10017$$

$$y = 2 + (x-2)(0) + \frac{(x-2)^2}{2} \times \frac{1}{2} + \frac{(x-2)^3}{6} \times \frac{-1}{4}$$

$$y = 2 + \frac{(x-2)^2}{4} - \frac{(x-2)^3}{24}$$

$$y(2.1) = 2 + \frac{(2.1-2)^2}{4} - \frac{(2.1-2)^3}{24}$$

$$= 2.0024$$

3) $y' = 2x - y$ $\quad y(1) = 3 \quad h = 0.1$

$x_1 = x_0 + h \quad x_0 = 1$

$y_1 = y_0 + k \quad y_0 = 3$

$$K = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_0, y_0) = 0.1 \times (2(1) - 3) = -0.1$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = -0.085$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = -0.08575$$

$$k_4 = hf(x_0 + h, y_0 + k_3) = -0.07143$$

$$K = \frac{1}{6}\left[-0.1 + 2(-0.085) + 2(-0.08575) + (-0.07143)\right]$$

$$K = -0.08549$$

$$y_{1.1} = y_0 + k = 3 - 0.08549$$

$$= 2.91451$$

4) $y' - x^2 y = x$ $\quad y(0) = 1$

$y' = x + x^2 y \quad x_0 = 0$

$y_0 = 1$

$x_1 = x_0 + h \quad \langle h = 0.1 \rangle$

$y_1 = y_0 + h$

$\rightarrow k_1 = hf(x_0, y_0) = 0$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.00525$

$x_1 = 0.1 \quad y_1 = 1.0053$

$$k_1' = hf(x_1, y_1) = 0.0110$$

$$k_2' = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right) = 0.0173$$

$$k_3' = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right) = 0.0173$$

$$k_4' = hf(x_1 + h, y_1 + k_3) = 0.0241$$

$$k = \frac{1}{6}\left[k_1' + 2k_2' + 2k_3' + k_4'\right]$$

$k_1 = h f(x_0, y_0) = 0$

$k_2 = h f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}) = 0.00525$

$k_3 = h f(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}) = 0.00525$

$k_4 = h f(x_0 + h, y_0 + k) = 0.01101$

$k = \frac{1}{6}\left[0 + 2(0.00525) + 2(0.00525) + 0.01101\right]$

$k = 0.0053$

$y(0.1) = y_0 + k$

$= 1 + 0.0053$

$y(0.1) = 1.0053$

$k = \frac{1}{6}\left[k_1' + 2k_2' + 2k_3' + k_4'\right]$

$k' = 0.174$

$y_2 = y_1 + k'$

$= 1.0035 + 0.174$

$y(0.2) = 1.0227$

5) $y' = x^2 + y^2$

$y'' = 2x + 2yy'$

$y''' = 2 + 2(y')^2 + 2yy''$

$y^{iv} = 4y'.y'' + 2y'y'' + 2y.y'''$

at $x_0 = 0$ $y_0 = 1$

$y' = 1$ $y''' = 8$

$y'' = 2$ $y^{iv} = 28$

x	y	$f(x,y) = dy/dx$
$x_0 = 0$	$y_0 = 1$	$f_0 = 1$
$x_1 = 0.1$	$y_1 = 1.11145$	$f_1 = 1.24532$
$x_2 = 0.2$	$y_2 = 1.25253$	$f_2 = 1.60883$
$x_3 = 0.3$	$y_3 = 1.43545$	$f_3 = 2.15052$
$x_4 = 0.4$	$y_4^{(p)} = 1.69105$	$f_4^{(p)} = 3.01965$
	$y_4^{(c)} = 1.69355$	$f_4^{(c)} = 3.02811$
	$(y_4^{(c)})' = 1.69383$	$f_4^{(c)'} = 3.02986$

TSM

$y = y_0 + \frac{(x-x_0)}{1!}y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0'''$

$= 1 + (x)(1) + \frac{x^2}{2}(2) + \frac{x^3}{6}(8) + \frac{x^4}{24}(28)$

$y = 1 + x + x^2 + \frac{4}{3}x^3 + \frac{7}{6}x^4$

$y(0.1) = 1.11145$

$y(0.2) = 1.25253$

$y(0.3) = 1.43545$

$y_4^{(p)} = y_0 + \frac{4}{3}h(2f_1 - f_2 + 2f_3)$

$= 1.69105$

$y_4^{(c)} = y_2 + \frac{h}{3}(f_2 + 4f_3 + f_4^{(p)})$

$= 1.69355$

$(y_4^{(c)})' = y_2 + \frac{h}{3}(f_2 + 4f_3 + f_4^{(c)})$

$= 1.69383$