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RV College of Engineering

Department of Mathematics

VECTOR CALCULUS, LAPLACE TRANSFORM & NUMERICAL METHODS
(MA221TA)
UNIT-V

Numerical Methods

TUTORIAL SHEET-1

1. If f(x) is a continuous function such that f(a).f(b) < 0, then the equation f(x) = 0, has atleast 1 roots in the interval [a, b].
2. In the method of false position for finding the root of an equation f(x) = 0, in the interval [a, b] the curve f(x) is replaced by chord AB
3. In Newton-Raphson method for finding the root of an equation f(x) = 0, in the interval [a, b] the curve f(x) is replaced by tangent
4. If f(x) is a continuous function such that f(a) and f(b) have same sign, then the equation f(x) = 0, has 1 roots in the interval [a, b].
5. Find the positive real root of the equation $x^4 - x - 10 = 0$, which lies between 1 and 2 by method of false position correct to five places of decimal. (Answer: 1.855584)
6. Find an approximate real root of the equation $= \frac{\cos(x)+1}{3}$, correct to four places of decimal using Regula falsi method. (Answer: 0.6071)
7. Find a positive real root of the equation $2x - \ln(x) = 6$, correct to three places of decimal by method of chords. (Answer: 3.257)
8. Using method of false position find a positive real root of the equation $x\sin(x) + \cos(x) = 0$ which lies between 2 and 3 correct three places of decimal. (Answer: 2.798)

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TUTORIAL SHEET-2

1. Find the negative real root of equation $x^3 - 21x + 3500 = 0$, correct to four places of decimal using Newton-Raphson method. (Answer: -15.64385)
2. Find a positive real root of equation $x\sin(x) + \cos(x) = 0$ near $x = \pi$ using method of tangents. (Answer: 2.798)
3. Using Newton-Raphson method find the reciprocal of a non-zero positive number 'N', hence find (1/31).
4. The bacteria concentration in a reservoir varies as $c = 4e^{-2t} + e^{-0.1t}$, using Newton Raphson method, calculate the time required for bacteria concentration to be 0.5 (Answer: 6.889)
5. The current 'I' in an electric circuit is given by $I = 10e^{-t}\sin(2\pi t)$, 't' in seconds, using Newton Raphson method find the value of 't' for I = 2A. (Answer: 0.0333)
6. Determine the root of the equation $x^2 - \ln(x) - 12 = 0$, in (3, 4) using Newton's method. (Answer: 3.646044)

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5) $f(x) = x^4 - x - 10 = 0$

$x_0 = 1 \quad x_1 = 2$

$f(x_0) = -10$ (-ve) $\quad f(x_1) = 4$ (+ve)

$x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \dfrac{(1)(4) - (2)(-10)}{4 - (-10)} = 1.71429$

$f(x_2) = -3.07788$ (-ve)

$f(2) = 4$ (+ve)

$x_0 = 1.71429 \quad x_1 = 2$

$f(x_0) = -3.07788 \quad f(x_1) = 4$

$x_3 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \dfrac{(1.71429)(4) - (-3.07788)(2)}{4 - (-3.07788)}$

$= 1.83853$

$f(x_3) = -0.41278$

$x_0 = 1.83853 \quad x_1 = 2$

$f(x_0) = -0.41278 \quad f(x_1) = 4$

$x_4 = \dfrac{(1.83853)(4) - (-0.41278)(2)}{4 - (-0.41278)}$

$= 1.85364$

$f(x_4) = -0.047775$

$x_0 = 1.85364 \quad x_1 = 2$

$f(x_0) = -0.047775 \quad f(x_1) = 4$

$x_5 = 1.85536$

$f(x_5) = -5.4192 \times 10^{-3}$

$x_6 = 1.85556$

$f(x_6) = -6.2537 \times 10^{-4}$

$x_7 = 1.85558$

$f(x_7) = -6.8314 \times 10^{-5}$

6) $x - \dfrac{\cos x + 1}{3} = 0$

as $\dfrac{\cos x + 1}{3}$ is a soul

$\rightarrow f(a) \cdot f(b) < 0$

$f(0) = 0 - \dfrac{\cos 0 + 1}{3} = -2/3 = -0.6667$ (-ve)

$f(1) = 1 - \dfrac{\cos 1 + 1}{3} = 0.4866$ (+ve)

$\rightarrow x_0 = 0 \quad x_1 = 1$

$f(x_0) = -0.6667 \quad f(x_1) = 0.4866$

$x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \dfrac{0(0.4866) - (1)(-0.6667)}{0.4866 - (-0.6667)}$

$x_2 = 0.5781$

$f(x_2) = -0.0344$

$x_3 = 0.6060$

$f(x_3) = -1.359 \times 10^{-5}$

$x_4 = 0.6070$

$f(x_4) = -5.1093 \times 10^{-5}$

$x_5 = 0.6071$

$f(x_5) = -4.7125 \times 10^{-6}$

7) $2x - \ln(x) - 6 = 0$

$f(1) = -4$

$f(2) = -2.693$

$f(3) = -1.099$ (-ve)

$f(4) = 0.614$ (+ve)

$x_0 = 3 \quad x_1 = 4$

$f(x_0) = -1.0986 \quad f(x_1) = 0.6137$

$x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \dfrac{3(0.614) - 4(-1.097)}{0.614 - (-1.099)}$

$= 3.642$

$f(x_2) = -9.234 \times 10^{-3}$

$x_3 = 3.647$

$f(x_3) = -6.12 \times 10^{-5}$

$x_4 = 3.647$

8) $f(x) = x\sin x + \cos x = 0$

$f(2) = 1.402$ (+ve)

$f(3) = -0.567$ (-ve)

$x_0 = 2 \quad x_1 = 3$

$f(x_0) = 1.402 \quad f(x_1) = -0.567$

$x_2 = 2.712$

$f(x_2) = 0.220$

$x_3 = 2.793$

$f(x_3) = 0.0151$

$x_4 = 2.798$

$f(x_4) = 8.886 \times 10^{-4}$

$x_5 = 2.798$

1) $f(x) = x^3 - 21x + 3500 = 0 \Rightarrow f'(x) = 3x^2 - 21$

$f(0) = 3500$

$f(1) = 3480$

$f(-15) = 440$ (+ve)

$f(-16) = -260$ (-ve)

root lies here

$-16 \quad -15 \quad 0$

$x_0 = -15$??

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$= -15 - \dfrac{440}{654}$ $\quad [f'(-15) = 3(-15)^2 - 21]$

$= -15.6727$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$f(x_1) = -20.61385$ (-20.6732)

$f'(x_1) = 715.90$

$x_2 = -15.64391$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$

$f(x_2) = -0.04202$ (-0.03918)

$f'(x_2) = 713.19576$ (713.19539)

$x_3 = -15.64385$

2) $f(x) = x\sin x + \cos x = 0$

$f(\pi) = -1$ (-ve)

$f(0) = 1$ (+ve)

$0 \quad \pi/2$

$x_0 = \pi \quad f(x_0) = -1$

$f'(x) = \cancel{\sin x} + x\cos x - \cancel{\sin x}$

$= x\cos x$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$f(x_0) = -1$

$f'(x_0) = -\pi$

$= \pi - \dfrac{(-1)}{(-\pi)}$

$x_1 = 2.82328$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$f(x_1) = -0.06618$

$f'(x_1) = -2.68145$

$x_2 = 2.79860$

$x_3 = 2.79838$

3) $x = 1/N$

$f(x) = Nx - 1 = 0$

$f'(x) = N$

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$f(x_n) = Nx_n - 1$

$f'(x_n) = N$

$x_{n+1} = x_n - \dfrac{(Nx_n - 1)}{N}$

$= \dfrac{Nx_n - Nx_n + 1}{N}$

$= \dfrac{1}{N}$

$N = 31$

$31x - 1 = 0$

$x = 0.03226$

4) $c = 4e^{-2t} + e^{-0.1t}$

$c = 0.5 \quad t = ?$

$f(c) = 4e^{-2t} + e^{-0.1t}$

$f'(t) = -8e^{-2t} - 0.1e^{-0.1t}$

$f(0) = 5$

$f(1) = 1.44618$

$f(2) = 0.89199$

at $t_0 = 0$

$t_1 = t_0 - \dfrac{f(t_0)}{f'(t_0)}$

$f(0) = 5 - 0.5 = 4.5$

$f'(0) = -8.1$

$t_1 = 0 - \left(\dfrac{4.5}{-8.1}\right)$

$= 0.55556$

$t_2 = t_1 - \dfrac{f(t_1)}{f'(t_1)}$

$f(t_1) = 1.76272$

$f'(t_1) = -2.72812$

$t_2 = 1.20169$

$t_3 = t_2 - \dfrac{f(t_2)}{f'(t_2)}$

$f(t_2) = 0.74842$

$f'(t_2) = -0.81197$

$t_3 = 2.12342$

$t_4 = t_3 - \dfrac{f(t_3)}{f'(t_3)}$

$f(t_3) = 0.36593$

$f'(t_3) = -0.19534$

$t_4 = 3.99672$

$t_5 = t_4 - \dfrac{f(t_4)}{f'(t_4)}$

$f(t_4) = 0.17889$

$f'(t_4) = -0.06976$

$t_5 = 6.466074$

$t_6 = t_5 - \dfrac{f(t_5)}{f'(t_5)}$

5) $I = 10e^{-t}\sin(2\pi t)$ $\quad -10\left[-e^{-t}\sin 2\pi t + 2\pi e^{-t}\cos 2\pi t\right]$

at 2A, $2 = 10e^{-t}\sin(2\pi t)$

$f(t) = 2 - 10e^{-t}\sin(2\pi t) = 0$

$f'(t) = 10e^{-t}\sin(2\pi t) - 20\pi e^{-t}\cos(2\pi t)$

let $t_0 = 0$

$f(0) = 2$

$f'(0) = -62.83$

$t_1 = t_0 - \dfrac{f(t_0)}{f'(t_0)}$

$t_1 = 0.03183$

$t_2 = \cancel{0.03314}$ 0.06415

$t_3 = t_2 - \dfrac{f(t_2)}{f'(t_2)}$

$f(t_2) = \cancel{0.00015}$ 1.9340

$f'(t_2) = \cancel{-57.47042}$

6) $f(x) = x^2 - \ln(x) - 12 = 0$

$f'(x) = 2x - \dfrac{1}{x}$

$f(3) = -4.09861$ (-ve)

$f(4) = 2.61371$ (+ve)

$\rightarrow x_0 = 3 \quad f(x_0) = -4.09861$

$f'(x_0) = 5.66667$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$= 3 - \left(\dfrac{-4.09861}{5.6667}\right)$

$x_1 = 3.72328$

$f(x_1) = 0.54821$

$f'(x_1) = 7.17798$

$f'(t_1) = -2.72812$

$t_2 = 1.20169$

$t_7 = t_6 + \frac{f(t_6)}{f'(t_6)}$

$f(t_6) = 0.00129$

$f'(t_6) = -0.05014$

$t_7 = 6.93147$

$t_5 = 6.466074$

$t_6 = t_5 - \frac{f(t_5)}{f'(t_5)}$

$f(t_5) = 0.02383$

$f'(t_5) = -0.05240$

$t_6 = 6.90574$

$t_1 = t_0 - \frac{f(t_0)}{f'(t_0)}$

$t_1 = 0.03183$

$t_2 = t_1 - \frac{f(t_1)}{f'(t_1)}$

$f(t_1) = 1.9662$

$f'(t_1) = -60.8292$

$f(t_2) = \cancel{0.00015}\ 1.9340$

$f'(t_2) = \cancel{-57.47092}\ -58.8600$

$t_3 = \cancel{0.03314}\ 0.09701$

$f(x_1) = 0.54821$

$f'(x_1) = 7.17798$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 3.64691$

$f(x_2) = 0.00607$

$f'(x_2) = 7.01962$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$x_3 = 3.64605$

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VECTOR CALCULUS, LAPLACE TRANSFORM & NUMERICAL METHODS (MA221TA)

UNIT-V

Numerical Methods

TUTORIAL SHEET-3

1. Obtain the Taylor series solution of the initial value problem $y' = y\sin(x) + \cos(x)$, $y(0) = 0$, up to terms containing x^5. Hence, find $y(0.1)$. (Answer: $y = x + \frac{x^3}{6} + \frac{x^5}{120} + \dots$, $y(0.1) = 0.10016675$)
2. Find the approximate Taylor series solution of $xy' = x - y$, $y(2) = 2$ for $x = 2.1$ (Answer: 2.00238)
3. Using the Runge-Kutta method evaluate $y(1.1)$, given $y' = 2x - y$, $y(1) = 3$, $h = 0.1$ (Answer: 2.9145)
4. Solve $y' - x^2 y = x$, $y(0) = 1$ for $x = 0.1, 0.2$ by using Runge-Kutta method. (Answer: 1.0053, 1.0227)
5. By using Milne method solve $y' = x^2 + y^2$, $y(0) = 1$ finding $y(0.1)$, $y(0.2)$ and $y(0.3)$ from the Taylor series method. (Answer: 1.6876)

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1) $y' = y\sin x + \cos x$

$y'' = y\sin x + y\cos x - \sin x$

$y''' = y''\sin x + 2y'\cos x - y\sin x - \cos x$

$y^{v} = y'''\sin x + 3y''\cos x + 2y'(-\sin x) - y'\sin x - y'\cos x + \sin x$

y' at $x_0 = 0$ $y_0 = 0$

$y' = 1$

$y'' = 0$

$y''' = 1$

$y^{iv} = 0$

$y^{v} = 1$

TSM formula

$y = y_0 + \frac{(x - x_0)}{1!}y_0' + \frac{(x - x_0)^2}{2!}y_0''$

$= 0 + \frac{x(1)}{1} + \frac{x^2(0)}{2!} + \frac{x^3(1)}{6} + \frac{x^4(0)}{4!} + \frac{x^5(1)}{120}$

$y = x + \frac{x^3}{6} + \frac{x^5}{120}$

$y(0.1) = 0.1 + \frac{(0.1)^3}{6} + \frac{(0.1)^5}{120}$

$= 0.10017$

2) $xy' = x - y$

$y' = \frac{x - y}{x} = 1 - y/x$

$y'' = -y'x^{-1} + y\left(\frac{1}{x^2}\right)$

$= \frac{-xy' + y}{x^2}$

$y''' = \frac{x^3y'' + 2x^2y' - 2xy}{x^4}$

at $x_0 = 2$ $y_0 = 2$

$y' = 0$ $y''' = -1/4$

$y'' = 1/2$

$y = y_0 + \frac{(x - x_0)}{1!}y_0' + \frac{(x - x_0)^2}{2!}y_0'' + \frac{(x - x_0)^3}{3!}y_0'''$

$y = 2 + (x-2)(0) + \frac{(x-2)^2}{2} \times \frac{1}{2} + \frac{(x-2)^3}{6} \times \frac{-1}{4}$

$y = 2 + \frac{(x-2)^2}{4} - \frac{(x-2)^3}{24}$

$y(2.1) = 2 + \frac{(2.1-2)^2}{4} - \frac{(2.1-2)^3}{24}$

$= 2.0024$

3) $y' = 2x - y$ $y(1) = 3$ $h = 0.1$

$x_0 = 1$

$y_0 = 3$

$x_1 = x_0 + h$

$y_1 = y_0 + k$

$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = hf(x_0, y_0) = 0.1 \times (2(1) - 3) = -0.1$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = -0.085$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = -0.08575$

$k_4 = hf(x_0 + h, y_0 + k_3) = -0.07143$

$k = \frac{1}{6}\left[-0.1 + 2(-0.085) + 2(-0.08575) + (-0.07143)\right]$

$k = -0.08549$

$y_{1.1} = y_0 + k = 3 - 0.08549$

$= 2.91451$

4) $y' - x^2y = x$ $y(0) = 1$

$y' = x + x^2y$

$x_0 = 0$

$y_0 = 1$

$\langle h = 0.1 \rangle$

$x_1 = x_0 + h$

$y_1 = y_0 + h$

$\rightarrow k_1 = hf(x_0, y_0) = 0$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.00525$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = 0.00525$

$k_4 = hf(x_0 + h, y_0 + k) = 0.01101$

$k = \frac{1}{6}\left[0 + 2(0.00525) + 2(0.00525) + 0.01101\right]$

$k = 0.0053$

$y(0.1) = y_0 + k$

$= 1 + 0.0053$

$y(0.1) = 1.0053$

$x_1 = 0.1$ $y_1 = 1.0053$

$k_1' = hf(x_1, y_1) = 0.0110$

$k_2' = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right) = 0.0173$

$k_3' = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right) = 0.0173$

$k_4' = hf(x_1 + h, y_1 + k_3) = 0.0241$

$k = \frac{1}{6}\left[k_1' + 2k_2' + 2k_3' + k_4'\right]$

$k' = 0.174$

$y_2 = y_1 + k'$

$= 1.0035 + 0.174$ $y(0.2) = 1.0227$

5) $y' = x^2 + y^2$

$y'' = 2x + 2yy'$

$y''' = 2 + 2(y')^2 + 2yy''$

$y^{iv} = 4y'y'' + 2y'y'' + 2y\,y'''$

at $x_0 = 0$ $y_0 = 1$

$y' = 1$ $y''' = 8$

$y'' = 2$ $y^{iv} = 28$

TSM

$y = y_0 + \frac{(x - x_0)}{1!}y_0' + \frac{(x - x_0)^2}{2!}y_0'' + \frac{(x - x_0)^3}{3!}y_0'''$

$= 1 + (x)(1) + \frac{x^2}{2}(2) + \frac{x^3}{6}(8) + \frac{x^4}{24}(28)$

$y = 1 + x + x^2 + \frac{4x^3}{3} + \frac{7}{6}x^4$

$y(0.1) = 1.11145$

$y(0.2) = 1.25253$

$y(0.3) = 1.43545$

$f(x, y) = dy/dx$

x	y	f
$x_0 = 0$	$y_0 = 1$	$f_0 = 1$
$x_1 = 0.1$	$y_1 = 1.11145$	$f_1 = 1.24532$
$x_2 = 0.2$	$y_2 = 1.25253$	$f_2 = 1.60883$
$x_3 = 0.3$	$y_3 = 1.43545$	$f_3 = 2.15052$
$x_4 = 0.4$	$y_4^{(p)} = 1.69105$	$f_4^{(p)} = 3.01965$
	$y_4^{(c)} = 1.69355$	$f_4^{(c)} = 3.02811$
	$(y_4^{c})' = 1.69383$	$f_4^{c'} = 3.02986$

$y_4^{(p)} = y_0 + \frac{4}{3}h(2f_1 - f_2 + 2f_3)$

$= 1.69105$

$y_4^{(c)} = y_2 + \frac{h}{3}\left(f_2 + 4f_3 + f_4^{(p)}\right)$

$= 1.69355$

$(y_4^{c})' = y_2 + \frac{h}{3}\left(f_2 + 4f_3 + f_4^{(c)}\right)$

$= 1.69383$