



R V COLLEGE OF ENGINEERING
(An autonomous institution affiliated to VTU, Belgaum)
DEPARTMENT OF MATHEMATICS
FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS & NUMERICAL
METHODS (MAT211TA)
UNIT-1: ELEMENTARY LINEAR ALGEBRA
TUTORIAL SHEET-1

I. Objective type questions:

1. If A is a 3×4 matrix then rank of A cannot exceed _____.
2. Rank of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ is _____.
3. Rank of identity matrix of order 4 is _____.
4. If the rank of the transpose matrix A is 3 then the rank of matrix A is _____.
5. Rank of singular matrix of order 5 is _____.

II. Find the rank of the following matrices

1) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$

Answer: rank of $A=2$

2) $A = \begin{bmatrix} 1 & 1 & -1 & 3 \\ 2 & -2 & 6 & 8 \\ 3 & 5 & -7 & 3 \end{bmatrix}$

Answer: rank of $A=2$

3) $A = \begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 2 & 0 & 4 & -2 \\ 12 & -3 & 9 & -3 \end{bmatrix}$

Answer: rank of $A=4$

- 4) Find the values of k such that the rank of the matrix A is 3, where

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & k \end{bmatrix}$$

Answer: $k=1$

- 5) For which value of b the rank of the matrix

$$A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix} \text{ is } 2$$

$b=2$.

- 6) Find the rank of A , B , $A+B$, BA and AB if

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$$

Answer: rank of $A=2$, rank of $B=1$, rank of $(A+B)=2$, rank of $(AB)=0$, rank of $(BA)=1$.



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UNIT-1: ELEMENTARY LINEAR ALGEBRA
TUTORIAL SHEET-2

1. Test the consistency of the following system of equations
 $2x+6y=-11$
 $6x+20y-6z=-3$
 $6y-18z=-1$
Answer: Inconsistent
2. Test the consistency of the following system and solve if the system is consistent
 $x_1 + 2x_2 + x_3 = 2$
 $3x_1 + x_2 - 2x_3 = 1$
 $4x_1 - 3x_2 - x_3 = 3$
 $2x_1 + 4x_2 + 2x_3 = 4$
Answer: Consistent. $x_1 = 1, x_2 = 0, x_3 = 1$.
3. Find the value of k such that the following system of equations possesses a non-trivial solution. Also find the solution of the system
 $4x_1 + 9x_2 + x_3 = 0$
 $kx_1 + 3x_2 + kx_3 = 0$
 $x_1 + 4x_2 + 2x_3 = 0$
Answer: $k = 1, x_1 = 2k, y = -k, z = k$.
4. Investigate the values of λ and μ so that the equations
 $2x+3y+5z=9$
 $7x+3y-2z=8$
 $2x+3y+\lambda z=\mu$
have (a) Unique solution (b) Infinite number of Solutions (c) No Solution
Answer: (a) $\lambda \neq 5$ (b) $\lambda = 5, \mu = 9$ (c) $\lambda = 5, \mu \neq 9$
5. Solve the system of equations by Gauss elimination method
 $x-2y+3z=2$
 $3x-y+4z=4$
 $2x+y-2z=5$
Answer: $x = \frac{11}{5}, y = -\frac{7}{5}, z = -1$
6. Solve the system of equations by Gauss elimination method
 $6x_1 - 2x_2 + 2x_3 + 4x_4 = 16$
 $12x_1 - 8x_2 + 6x_3 + 10x_4 = 26$
 $3x_1 - 13x_2 + 9x_3 + 3x_4 = -19$
 $-6x_1 + 4x_2 + x_3 - 18x_4 = -34$
Answer: $x_1 = 3, x_2 = 1, x_3 = -2, x_4 = 1$.



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TUTORIAL SHEET-3

1. Solve the following system of equations by Gauss –Jordan method

$$2x+y+z=10$$

$$3x+2y+3z=18$$

$$x+4y+9z=16$$

Answer: $x=7, y=-9, z=5$.

2. Find the inverse of a matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ using Gauss-Jordan method.

Answer: $A^{-1} = \begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & -\frac{4}{5} & -\frac{14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}$

3. Solve the system of equations by Gauss elimination method

$$9x+2y+4z=20$$

$$x+10y+4z=6$$

$$2x-4y+10z=-15$$

Answer: $x=2.7372, y=0.9872, z=-1.6525$

4. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$.

Answer: $\lambda = 0, 1, 2$ and $X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, X_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

5. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$.

Answer: $\lambda = 3, 6, 9$ and $X_1 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, X_2 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, X_3 = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$

6. The sum and product of the eigenvalues of the matrix $A = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}$ are

Answer: 0 and 8.

7. If two eigenvalues of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 15, then the third eigenvalue is

Answer: 0

8. If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, then the eigenvalues of A^{-1} are

Answer: 1 and $\frac{1}{3}$.



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TUTORIAL SHEET-4

1. Find the largest eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \text{ by Rayleigh power method. (Perform 5 iterations)}$$

$$\text{Answer: } AX^{(4)} = 6.941 \begin{bmatrix} 0.341 \\ 0.039 \\ 1 \end{bmatrix}$$

2. Find the largest eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \text{ by Rayleigh power method taking initial eigenvector as}$$

$$[1 \ 1 \ 1]^T. \text{ (Perform 5 iterations)}$$

$$\text{Answer: } AX^{(4)} = 6.941 \begin{bmatrix} 0.341 \\ 0.039 \\ 1 \end{bmatrix}$$