unit5_tutorials

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS (MA211TA)

UNIT 5: NUMERICAL METHODS

TUTORIAL SHEETS – 1

Objective type Questions:

1. Construct the table of differences for the data below and evaluate $\Delta^3 f(1)$ 0.4

x	0	1	2	3	4
$f(x)$	1	1.5	2.2	3.1	4.6

$a_0 n!\, h^n$ $a_0 n$ $a_0 = 3x \cdot -5x \cdot -4 = 60$

$60 \times 3! \times 1^3 = 360$

2. The value of $\Delta^3[(1+3x)(1-5x)(1-4x)]$ taking interval of differencing $h = 1$ is ______.
3. Construct the difference table of the polynomial $f(x) = x^3 + 5x - 7$ for $x = -1,0,1,2,3,4$ and hence find $\Delta y_0, \Delta y_3, \Delta^2 y_1$. and find eqn.
4. The $(n+1)^{th}$ order difference of the n^{th} degree polynomial is ___0___.

Descriptive Questions:

1. Find a polynomial $f(x)$ which takes the values given by the following table

x	0	1	2	3	4
$f(x)$	10	21	6	43	66

2. The following data defines the sea-level concentration of dissolved oxygen for fresh water as a function of temperature:

$T^0(C)$	0	8	16	24	32
O(mg/L)	14.621	11.843	9.870	8.418	7.305

Using Newton-Gregory formula, calculate the amount of oxygen when temperature 10^0C and 35^0C.

3. From the following data, estimate the number of students who obtained marks between 40 and 45 using Newton's interpolation method

Marks:	30-40	40-50	50-60	60-70	70-80
Number of Students:	31	42	51	35	31

4. Estimate the values of $f(22)$ and $f(42)$ from the following data

x	20	25	30	35	40	45
$f(x)$	354	332	291	260	231	204

①

x	$f(x)$	$\Delta f(x)$	Δ^2	Δ^3	Δ^4
0	1	0.5	0.2	0 (y_0)	0.4
1	1.5	0.7	0.2	0.4 (y_1)	
2	2.2	0.9	0.6		
3	3.1	1.5			
4	4.6				

③ x^3+5x-7

x	$f(x)$				
-1	-13	6	0	6	-5
0	-7	6	6	1	20
1	-1	12	7	21	25
2	11	19	28		
3	30	47			
4	77				

②
$y = (1+3x)(1-5x)(1-4x)$

$= (1+3x)(1-9x+20x^2)$

$= 1-9x+20x^2+3x-27x^2+60x^3$

$y = 60x^3 - 7x^2 - 6x + 1$

$y' = 180x^2 - 14x - 6$

$y'' = 360x - 14$

$\Delta^3(360x - 14) =$

$\boxed{1}$

x	$f(x)$ (y_0)	Δy_0	Δ^2	Δ^3	Δ^4
0	10	11	-26	78	-144
1	21	-15	52	-66	
2	6	37	-14		
3	43	23 (Δy_3)			
4	66				

$p = \frac{x-x_0}{h} = \frac{x-0}{1} = x$

$\rightarrow y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$

$= 10 + x(11) + \frac{x(x-1)}{\cancel{2}}(\cancel{-26})^{-12} + \frac{x(x-1)(x-2)}{\cancel{6}}(\cancel{78})^{13} + \frac{x(x-1)(x-2)(x-3)}{\cancel{24}\ 2}(\cancel{-144})^{\cancel{12}\ 6}$

$x(x^2-3x+2)$ $\quad x(x-1)(x^2-5x+6)$ $\quad x^3-6x^2+11x-6$

$= 10 + 11x + -12(x^2-x) + 13(x^3-3x^2+2x) - 6(x^4-6x^3+11x^2-6x)$

$y = -6x^4 + 49x^3 - 104x^2 + 72x + 10$

$\boxed{2}$

$T^\circ C$ (x)	y_0	Δy_0	Δ^2	Δ^3	Δ^4
0	14.621	-2.778	0.805	-0.292	0.134
8	11.843	-1.973	0.513	-0.158	
16	9.870	-1.460	0.355		
24	8.410	-1.105			
32	7.305				

$p = \frac{x-x_0}{h} = \frac{x-0}{8} = \frac{x}{8}$

$\rightarrow y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 \ldots$

$= 14.621 + \frac{x}{8}(-2.778) + \frac{\frac{x}{8}\left(\frac{x}{8}-1\right)}{2}(0.805) + \frac{\frac{x}{8}\left(\frac{x}{8}-1\right)\left(\frac{x}{8}-2\right)}{6}(-0.292) + \frac{x\left(\frac{x}{8}-1\right)\left(\frac{x}{8}-2\right)\left(\frac{x}{8}-3\right)(0.134)}{24}$

$= 14.621 + x(-0.3473) + \frac{x^2-8x}{8\times8\times2}(0.806) + x^2 - 8x(0.0063) \rightarrow$ solve.

subs, $x=10$, $y(10) = ?$

$x = 35$, $y(35) = ?$

$\boxed{3}$

x	y_0	Δy_0	$\Delta^2 y_0$		
40	31	42	9	-25	37
50	73	51	-16	12	
60	124	35	-4		
70	159	31			
80	190				

$p = \frac{x-x_0}{h} = \frac{x-40}{10}$

$\rightarrow y = y_0 + p(\Delta y_0) + \frac{p(p-1)}{2!}(\Delta^2 y_0) \ldots$

$= 31 + \frac{x-40}{10}(42) + \frac{\frac{x-40}{10}\left(\frac{x-50}{10}\right)(9)}{2} + \frac{\frac{x-40}{10}\left(\frac{x-50}{10}\right)\left(\frac{x-60}{10}\right)(-25)}{6} + \frac{\left(\frac{x-40}{10}\right)\left(\frac{x-50}{10}\right)\left(\frac{x-60}{10}\right)\left(\frac{x-70}{10}\right)(37)}{24}$

$= 31 + 4.2(x-40) + \frac{4.5}{100}(x^2-90x+2000) +$

$y(45) = 31 + 21 - \frac{9}{8} - \frac{25}{\cancel{6}\,2} \times \frac{1}{2} \times \frac{+1}{2} \times \frac{\cancel{+3}}{2} + \frac{37}{\cancel{24}\,8} \times \frac{1}{2} \times \frac{-1}{2} \times \frac{\cancel{+3}}{2} \times \frac{+5}{2}$

$\boxed{4}$

x	y_0	∇y_0	$\nabla^2 y_0$	$\nabla^3 y_0$	$\nabla^4 y_0$	$\nabla^5 y_0$
20	354	-22	-19	29	-37	45
25	332	-41	10	-8	8	
30	291	-31	2	0		
35	260	-29	2			
40	231	-27				
45	204					

$p = \frac{x_n - x}{h} = \frac{45-x}{5}$

$\rightarrow y = y_0 + p\nabla y_0 + \frac{p(p+1)}{2!}\nabla^2 y_0 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_0 \ldots$

$y = 204 + \frac{45-x}{5}(-27) + \frac{\left(\frac{45-x}{5}\right)\left(\frac{50-x}{5}\right)}{\cancel{2}}(\cancel{2}) + (\ )(0)$

$+ \frac{\left(\frac{45-x}{5}\right)\left(\frac{50-x}{5}\right)\left(\frac{55-x}{5}\right)\left(\frac{60-x}{5}\right)}{\cancel{5!}\ 120}(8) + \frac{\left(\frac{45-x}{5}\right)\left(\frac{50-x}{5}\right)\left(\frac{55-x}{5}\right)\left(\frac{60-x}{5}\right)\left(\frac{65-x}{5}\right)}{\cancel{6!}\ 720}(45)$ 23.

$y(22) = 204 - \frac{621}{5} + \frac{644}{25} + 0 - 86.1414 + 694.5154$

$= 713.934$

$y(42) = 204 - 16.2 + 0.96 + 0 - 0.599 + 2.5834$

$= 190.7444$

$= 52 - \frac{9}{8} - \frac{25}{16} - \frac{185}{128}$

$= \frac{6127}{128} = 47.867 \approx 48$

→ from 40 to 45 → $y(45) - y(40)$

$= 48 - 31$

$= 17$

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS (MA211TA)

UNIT 5: NUMERICAL METHODS

TUTORIAL SHEETS – 2

Descriptive Questions:

1. Given that $f(0) = 7,\ f(1) = 18,\ f(3) = 18,\ f(5) = -230,\ f(6) = 18$ find $f(x)$ as a polynomial in x and hence find $f(2)$.

2. The following table gives the viscosity of an oil as a function of temperature. Use Lagrange's formula to find viscosity of oil at a temperature of 140°.

Temp :	110	130	160	190
Viscosity:	10.8	8.1	5.5	4.8

3. Using Lagrange's interpolation, find the polynomial of lowest degree which agrees with the point (x, y) given in the following table. Hence find $y(2.5)$

similarly,

x	3	2	1	-1	0
$y(x)$	8	26	32	-40	14

4. The following data was collected for the distance travelled versus time:

t(sec):	0	25	50	75	100	125
y(km):	0	32	59	78	92	100

Use numerical differentiation to calculate velocity and acceleration at $t = 25$ and $t = 100$.

5. A rod is rotating in a plane. The following table gives the angle θ (radians) through which the rod has turned for various values of the time t second.

Similarly,

t:	0	0.2	0.4	0.6	0.8	1.0	1.2
θ:	0	0.12	0.49	1.12	2.02	3.20	4.67

Calculate the angular velocity and the angular acceleration of the rod, when $t = 1.0$ second.

6. Find $f'(1), f''(1)$ and $f'(3)$ from the following data:

Similarly

x	1	2	3	4	5	6
$f(x)$	3.614	4.604	5.857	7.451	9.467	11.985

7. The following table gives the temperature θ (in degree Celsius) of a cooling body at different instants of time t (*in sec*).

t:	1	3	5	7	9
θ:	85.3	74.5	67	60.5	54.3

Similarly,

Calculate θ at $t = 2$ and also find approximately the rate of cooling at $t = 9$ sec.

$\theta(2) = ?$ $\qquad \left.\frac{d\theta}{dt}\right|_{t=9} \rightarrow$ use ∇

(1) x | $f(x)$

x_0 0 | 7 y_0

x_1 1 | 18 y_1

x_2 3 | 18 y_2

x_3 5 | -230 y_3

x_4 6 | 18 y_4

$\rightarrow f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} \times y_0 + \frac{(x-x_0)(x-x_2)\dots}{}$

$= \frac{(x-1)(x-3)(x-5)(x-6)}{(0-1)(0-3)(0-5)(0-6)} \times 7 + \frac{(x-0)(x-3)(x-5)(x-6)}{(1-0)(1-3)(1-5)(1-6)} \times 18$

$+ \frac{(x-0)(x-1)(x-5)(x-6)}{(3-0)(3-1)(3-5)(3-6)} \times 18 + \frac{(x-0)(x-1)(x-3)(x-6)}{(5-0)(5-1)(5-3)(5-6)} \times (-230)$

$+ \frac{(x-0)(x-1)(x-3)(x-5)}{(6-0)(6-1)(6-3)(6-5)} \times 18$

$x^2 - 4x + 3$ $\quad x^2 - 11x + 30$ $\quad 44$ $\quad 33$

$-120 \quad -33$

$= \frac{7}{90}(x^4 - 15x^3 + 77x^2 - 153x + 90) - \frac{18}{40}(x^4$

⟶ solve or skip

(2) x | $f(x)$

110 | 10.8

130 | 8.1

160 | 5.5

190 | 4.8

$\rightarrow f(x) = \frac{(x-130)(x-160)(x-190)}{(110-130)(110-160)(110-190)} \times 10.8 + \dots$

(140) $x - 110 = 30$, $x - 130 = 10$, $x - 160 = -20$, $x - 190 = -50$

$f(140) = \frac{\cancel{10} \times -\cancel{20} \times -\cancel{50}}{-\cancel{20} \times -\cancel{50} \times -\cancel{80}} \times 10.8 + \frac{\cancel{30} \times -\cancel{20} \times -\cancel{50}}{\cancel{20} \times -\cancel{30} \times -\cancel{60}} \times 8.1 +$

$+ \frac{\cancel{30} \times 10 \times -\cancel{50}}{\cancel{50} \times \cancel{30} \times -\cancel{30}} \times 5.5 + \frac{\cancel{30} \times 10 \times -\cancel{20}}{\cancel{80} \times \cancel{60} \times \cancel{30}} \times 4.8$

$= -1.35 + 6.75 + 1.83 - 0.2$

$f(140) = 7.03$

(4)

t	y	Δy_0	Δ^2	Δ^3	Δ^4	Δ^5
0	0	32	-5	-3	6	-10
25	32	27	-8	3	-4	
50	59	19	-5	-1		
75	78	14	-6			
100	92	8				
125	100					

$v = \left.\frac{dy}{dt}\right|_{t=25} = \frac{1}{h}\left[\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \frac{1}{4}\Delta^4\right]$

$= \frac{1}{25}\left[27 - \frac{1}{2}(-8) + \frac{1}{3}(3) + \frac{1}{4}(-4)\right]$

$= \frac{1}{25}\left(27 + 4 + \cancel{1} - \cancel{1}\right)$

$v = 31/25 = 1.24$

$a = \left.\frac{d^2y}{dt^2}\right|_{t=100} = \frac{1}{h^2}\left[\nabla^2 + \nabla^3 + \frac{11}{12}\nabla^4\right]$

$= \frac{1}{25^2}\left[-5 + 3 + \frac{11}{\cancel{12}_{2}} \times \cancel{6}\right]$

$a = 0.0056$

(1) $\int_0^{\pi/2} \sqrt{\cos x}\, dx$ $\qquad f(x) = \sqrt{\cos x}$

x	0	$\pi/12$	$2\pi/12$	$3\pi/12$	$4\pi/12$	$5\pi/12$	$6\pi/12$
$f(x)$	1	0.9828	0.9306	0.8409	0.7071	0.5087	0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$\rightarrow h = \frac{b-a}{n} = \frac{\pi/2 - 0}{6} = \pi/12$

→ 1/3rd rule →

$I = \frac{h}{3}\left[y_0 + y_6 + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)\right]$

$= \frac{\pi}{36}[13.605] \Rightarrow I = 1.1873$

→ 3/8 Rule →

$I = \frac{3h}{8}\left[y_0 + y_6 + 2(y_3) + 3(y_1 + y_2 + y_4 + y_5)\right]$

$= \frac{3\pi}{8 \times 12_4}[12.0694] \Rightarrow I = 1.1849$

→ Weddles →

$I = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right]$

$= \frac{3\pi}{10 \times 12_4}[15.1406] \Rightarrow I = 1.1891$

(2) $\int_0^{\pi} e^{\sin\theta}\, d\theta = I$

$f(x)$

$h = \frac{b-a}{n} = \frac{\pi - 0}{6} = \frac{\pi}{6}$

x	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	π
$f(x)$							

⟶ solve.

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS (MA211TA)

UNIT 5: NUMERICAL METHODS

Calculate θ at $t = 2$ and also find approximately the rate of cooling at $t = 9$ sec.

TUTORIAL SHEETS – 3

Objective type Questions:

1. While applying Simpson's three-eight rule, the number of sub intervals should be taken as 3. (3-8)

2. $f(x)$ is given by

x	0	0.5	1	1.5	2
$f(x)$	0	0.25	1	2.25	4
	y_0	y_1	y_2	y_3	y_4

Then the value of $\int_0^2 f(x)\,dx$ by Simpson's three-eight rule.

3. Find $\int_0^{0.3} f(x)\,dx$ if $f(0) = 1, f(0.1) = 0.99, f(0.2) = 0.9608$ and $f(0.3) = 0.9139$ by Simpson's three-eight rule.

Descriptive Questions:

1. Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos(x)}\,dx$ by dividing the interval into six equal parts using Simpson's one-

(2) $h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

$\rightarrow \int_0^2 f(x) = \frac{3h}{8}\left(y_0 + y_4 + 2(y_3) + 3(y_1 + y_2)\right)$

$= \frac{3(0.5)}{8}\left[0 + 4 + 2(2.25) + 3(0.25 + 1)\right]$

$= 2.2969$

(3)

x	0	0.1	0.2	0.3
$f(x)$	1	0.99	0.9608	0.9129

-2.2769 $\quad f(x)$

$y_0 \quad y_1 \quad y_2 \quad y_3 \quad y_4$

3. Find $\int_0^{0.3} f(x)\,dx$ if $f(0) = 1, f(0.1) = 0.99, f(0.2) = 0.9608$ and $f(0.3) = 0.9139$ by Simpson's three-eight rule.

$\boxed{3}$

x	0	0.1	0.2	0.3
$f(x)$	1	0.99	0.9608	0.9139
	y_0	y_1	y_2	y_3

$\rightarrow$ solve.

$\rightarrow h = \dfrac{b-a}{n} = \dfrac{0.3-0}{3} = 0.1$

$\rightarrow \int_0^{0.3} f(x) = \dfrac{3h}{8}\left[y_0 + y_3 + 3(y_1 + y_2)\right]$

$= \dfrac{3(0.1)}{8}\left[1 + 0.9139 + 3(0.99 + 0.9608)\right]$

$= 0.2912$

Descriptive Questions:

1. Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos(x)}\,dx$ by dividing the interval into six equal parts using Simpson's one-three rule, Simpson's three-eight rule and Weddle's rule.

Similarly

2. Find an approximate value of $\int_0^{\pi} e^{\sin\theta}\,d\theta$, by considering seven ordinates of the interval $(0, \pi)$ using Simpson's one-three rule, Simpson's three-eight rule and Weddle's rule.

Similarly

3. Find $\int_2^5 y\,dx$ by (i) the Simpson's one-three rule (ii) Simpson's three-eight rule (iii) Weddle's rule if x and y are as given below

x	2	2.5	3	3.5	4	4.5	5
y	1.3863	1.4351	1.4816	1.5260	1.5686	1.6094	1.6486
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

4. A river is 80feet wide. The depth d in feet at a distance x feet from one bank is given by:

80ft

$d\downarrow$

$n=8$

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

$A = \int_0^{80} f(x)\,dx.$

$h = \dfrac{80-0}{8} = 10$

apply 1/3rd rule.

Find approximately the area of across section of the river.

5. A curve is drawn to pass through the points given by the following table:

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS (MA211TA)

UNIT 5: NUMERICAL METHODS

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

$n = 6$

$h = \dfrac{4-1}{6} = 0.5$

$\rightarrow$ solve.

Using Simpson's one-three rule, Simpson's three-eight rule and Weddle's rule, estimate the area bounded by the curve, the axis and the lines $x = 1$, $x = 4$.

$A = \int_1^4 f(x)\,dx$