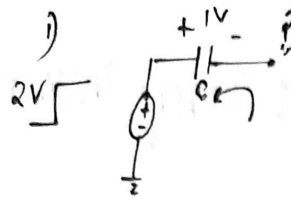
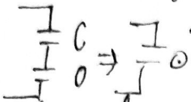
NETWORK ANALYSIS

Lecture - 9a Analysis of Capacitor ckts. with current inputs

1) 

$V_x = 1V$

Intuitively we can say that there is no path for the capacitor to get charged/discharged

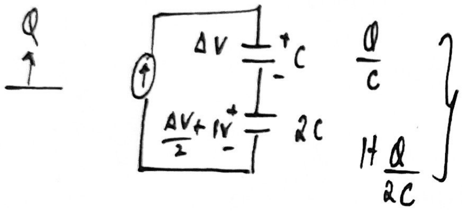


∴ Potential across the capacitor is constant.

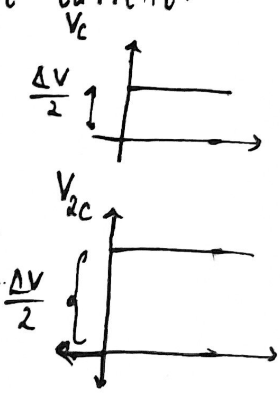
Charge transferred from the voltage source is zero.

$Q = CV$

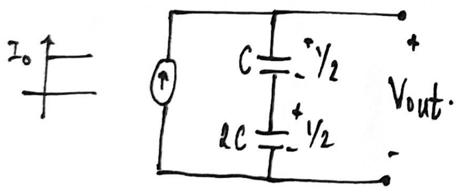
$$\boxed{Q = (0)(V) = 0}$$

2) 

Steady state voltages in the presence of an application of an impulse current.



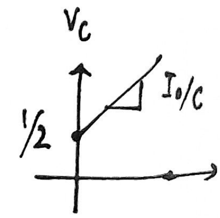
3) Behavior of Capacitor voltages on constant current application:-



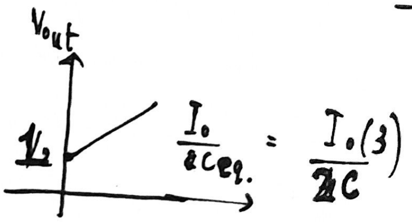
$$V(t) = V(0) + \int_0^t i(t)\,dt$$

$$V_C = \frac{1}{2} + \int_0^t \frac{I_0}{C}\,dt.$$

$$= \frac{1}{2} + \frac{I_0 t}{C}$$



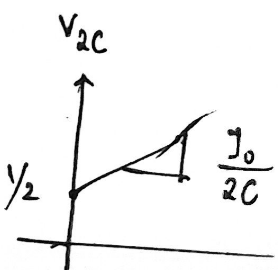
$$V_{2C} = \frac{1}{2} + \frac{I_0}{2C}t$$



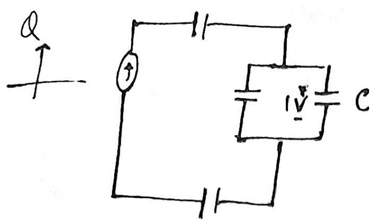
$$V_{out} = V_C + V_{2C}$$

$$= 1 + I_0 t\left(\frac{1}{C} + \frac{1}{2C}\right)$$

$$\boxed{V_{out} = 1 + \frac{I_0 t}{(C_{eq})}}$$



~~4)~~ Find steady state Voltage across capacitors.

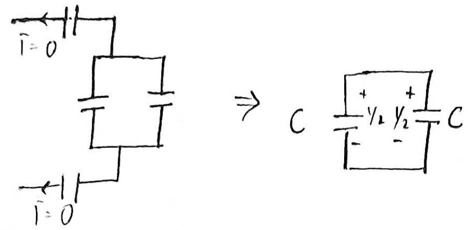


Apply superposit^{n}

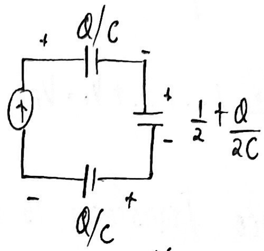
i) Find how the 1V will distribute across other caps.

ii) Then find effect of current.

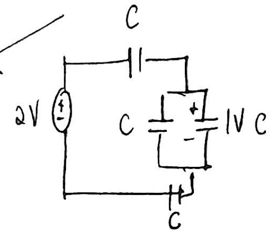
①

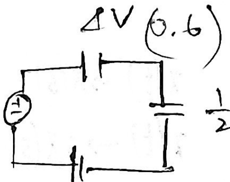


②



~~Q.~~

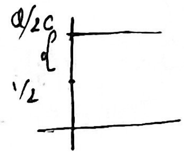




$\frac{1}{2} + \frac{\Delta V}{2}$ 0.8

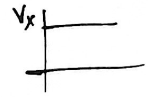
$\Delta V \cdot (0.6)$ $\frac{1}{2} + 2\Delta + \frac{\Delta V}{2} = 2.$

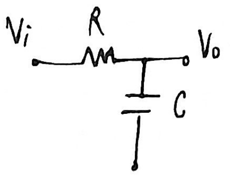
$\frac{5\Delta V}{2} = \frac{3}{2}$ $\boxed{\Delta V = 0.6}$



Lecture 10:- 1st order systems & Frequency domain analysis of capacitors.

Suppose I/P is step





$$C\frac{dV_o}{dt} = \frac{V_i - V_o}{R}$$

$$RC\frac{dV_o}{dt} + V_o = V_i$$

$$\tau\frac{dV_o}{dt} + V_o = V_i$$

$$V_i = 0$$

$$\tau\frac{dV_o}{dt} + V_o = 0.$$

$\boxed{V_o(t) = V_i e^{-t/\tau}}$ → Natural response (without I/P)

$V_{no}(t)$

$V_{fo}(t)$ → Forced

$$V_o(t) = V_{no}(t) + V_{fo}(t)$$

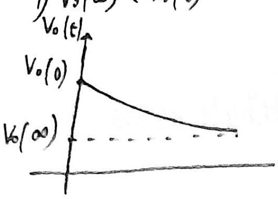
$$\tau\frac{dV_o}{dt} + V_o = V_x u(t)$$

$$V_{fo}(t) = V_o(\infty)\left(1 - e^{-t/\tau}\right)u(t)$$

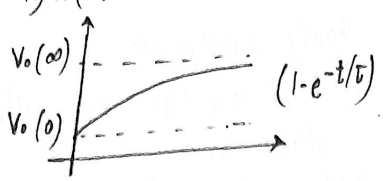
$$V_o(t) = V_o(\infty)\left(1 - e^{-t/\tau}\right) + V_o(0)e^{-t/\tau}$$

$$V_o(t) = V_o(\infty) + \left(V_o(0) - V_o(\infty)\right)e^{-t/\tau}$$

i) $V_o(\infty) < V_o(0)$

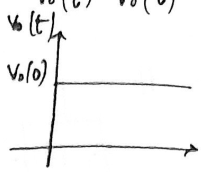


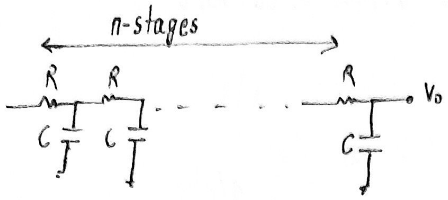
ii) $V_o(\infty) > V_o(0)$



iii) $V_o(0) = V_o(\infty)$

$V_o(t) = V_o(0)$





$$\frac{d^n V_o}{dt^n} + \dots\dots + V_o = V_i$$

Laplace Transform is used

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt.$$

$x(t) \to X(s)$

$x'(t) \to sX(s)$

$\int x(t) \to \frac{X(s)}{s}$

$\delta(t) \to 1.$

$u(t) \to 1/s$

$e^{-at} u(t) \to \frac{1}{s+a}$

$$\tau \frac{dV_o}{dt} + V_o = V_i$$

$$\tau s V_o(s) + V_o(s) = V_i(s)$$

$V_i = V_x u(t)$

$$\boxed{V_o(s) = \frac{1}{(1+s\tau)} V_i(s)}$$

$$V_o(s) = \frac{1}{(1+s\tau)} \frac{V_x}{s}$$

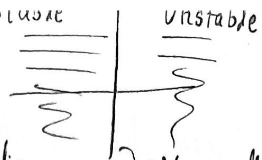
$$= V_x \left[\frac{1}{s} - \frac{\tau}{1+s\tau}\right]$$

$$= V_x \left[\frac{1}{s} - \frac{1}{s+1/\tau}\right]$$

$$= V_x \left[u(t) - e^{-t/\tau} u(t)\right]$$

$$\boxed{V_o(t) = V_x\, u(t) \left[1 - e^{-t/\tau}\right]}$$

$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{1}{1+s\tau}$ $\quad H(s)\big|_{s=\omega_p} \to \infty$ $\quad H(s)\big|_{s=\omega_z} \to \infty$



poles should lie on LHS of s-plane

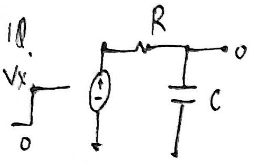
Marginally stable

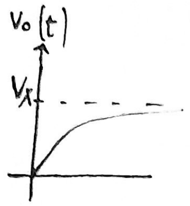
$$\tau\frac{dV_o}{dt} + V_o = \frac{1}{\omega_z}\frac{dV_i}{dt} + V_i$$

$$(s\tau+1)\,V_o(s) = (1+s/\omega_z)\,V_i(s)$$

$$\frac{V_o}{V_i} = \frac{(1+s/\omega_z)}{(s\tau+1)} \qquad s = -\omega_z \qquad s = -1/\tau$$

1Q.





$i = \frac{V_x - V_o(t)}{R}$

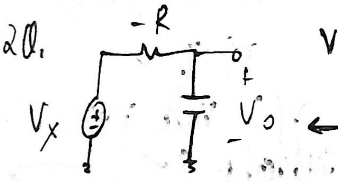
$V_o(t)$ gradually ↑es, so the slope of current ↓es. At $t=0$; Cap. behaves as short ckt.

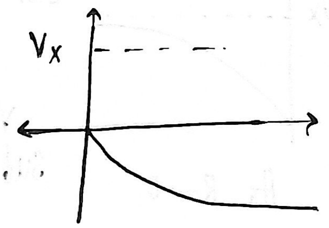
At $t=\infty$; Cap. → open ckt.

Also, $H(s) = \frac{1}{1+s\tau}$

$\therefore \boxed{s_p = -1/\tau} \to$ stable.

2Q.





$i = -\frac{(V_x - V_o)}{R}$

As V_o ↑es, $V_i \to -\infty$.

Hence, unstable $\quad s_p = 1/\tau$

$\frac{V_o}{V_i} = H(s)$

$V_o(t) = V_o(\infty)\left(1 - e^{t/\tau}\right)$

$V_o = H(s)\,V_i \to 1$

$\boxed{V_o = H(s)}$

Fourier Transform

$$x(t) = \sum c_k e^{jk\omega_0 t}$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df$$

$$H(j\omega) = A(\omega)\, e^{j\phi(\omega)}$$

D.C.

$\omega = 0 \to |Z_c(\omega)| \to \infty$

Open ckt.

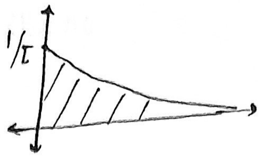
$\omega = \infty \Rightarrow |Z_c(\omega)| \to 0.$

Short ckt

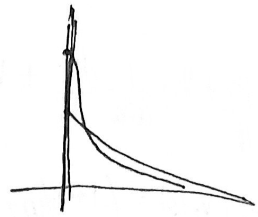
For a sudden step I/P, capacitor behaves as short ckt.

Lecture II - Energy loss in charging and discharging of capacitor ckts.

$\frac{1}{\tau} e^{-t/\tau} u(t)$



$\tau \to \tau/2$



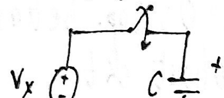
$\int_{-\infty}^{\infty} \frac{1}{\tau} e^{-t/\tau} u(t) = \frac{1}{\tau} \int_{-\infty}^{\infty} e^{-t/\tau} u(t)\,dt = 1.$

$\tau \to 0 \Rightarrow$ Impulse

$\frac{1}{\tau} e^{-t/\tau} u(t) \to \delta(t)$ as $\tau \to 0$

Charging of capacitors:-

Q.

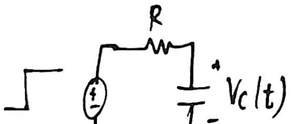


W.D. by Voltage source:-

$V_x\, Q = V_x^2 (C)$

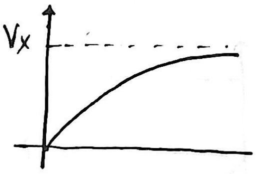
Energy stored in capacitor $= \frac{1}{2} C V_x^2$

Assume finite resistance of switch R.



$V_c(0) = 0$

$V_c(\infty) = V_x$



$V_c(t) = V_x(1 - e^{-t/\tau}) u(t)$

If $\tau = 0$

V(c) will rise suddenly ↑

$V_R(t) = V_x u(t) - V_x(1 - e^{-t/\tau}) u(t)$

$= V_x e^{-t/\tau} u(t)$

$i_R(t) = \frac{V_x e^{-t/\tau} u(t)}{R}$.

$P_R(t) = \frac{V_x^2 e^{-2t/\tau}}{R} u(t) \to$ Decays twice as faster

$P_R(t) = \frac{1}{2} \frac{V_x^2 (2C)}{RC} e^{-2t/\tau} u(t)$

$= \frac{1}{2} C V_x^2 \frac{2}{\tau} e^{-2t/\tau} u(t)$

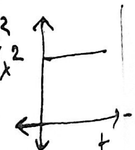
As $R \to 0$

$P_R(t) = \frac{1}{2} C V_x^2 \frac{2}{\tau} e^{-2t/\tau} u(t)$

$\boxed{P_R(t) = \frac{1}{2} C V_x^2 \delta(t)} \to$ Energy will be finite

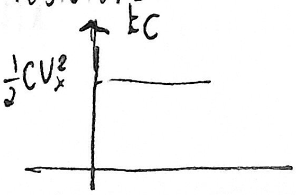
Current in the resistor is an impulse.

Impulse of power is dissipated across the resistor.



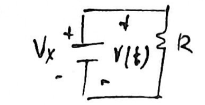
$E = \int P_R(t)\,dt$

$= \frac{1}{2} C V_x^2$

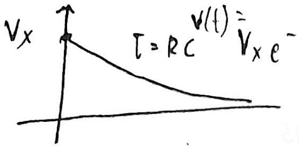


Discharging of capacitors:-

$V(0) = V_x$

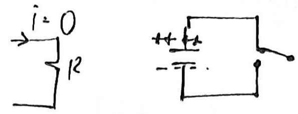


$V(\infty) = 0.$

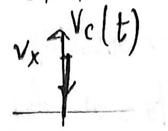


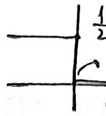
$\tau = RC \quad v(t) = V_x e^{-t/\tau} u(t)$

Steady state ⇒ Capacitor behaves as an open ckt.



If $R = 0$





$\frac{1}{2}CV_x^2$

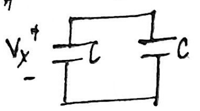
At $t = 0$ → When switch is turned ON, it $\downarrow$es to 0.

Energy is $\frac{1}{2}CV_x^2$ and suddenly it is lost to 0.

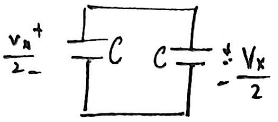
$P = \frac{dE}{dt} = \frac{1}{2}CV_x^2 \delta(t).$

Charge sharing bet^n 2 capacitors:-

$E = \frac{1}{2}CV_x^2 \qquad E = \frac{1}{4}CV_x^2$



⇒



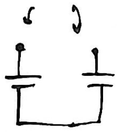
$CV_x = 2CV_1$

$V_1 = \frac{V_x}{2}$

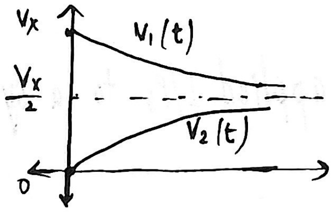
$V_1(0) = V_x \qquad V_2(0) = 0$

$V_1(\infty) = V_2(\infty) = \frac{V_x}{2}$

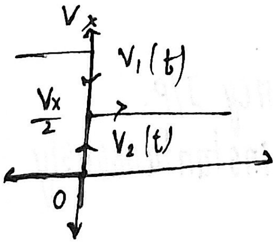
Assume finite resistance of switch.



$\tau = \frac{RC}{2}$



If $R \to 0 \quad \tau \to 0$



$V_2(t) = \frac{V_x}{2}\left[1 - e^{-t/\tau}\right]u(t)$

$V_1(t) = \left(\frac{V_x}{2} + \frac{V_x e^{-t/\tau}}{2}\right)u(t)$

$= \frac{V_x}{2}\left[1 + e^{-t/\tau}\right]u(t)$

$V_R(t) = V_1 - V_2$

$= V_x e^{-t/\tau} u(t)$

$\tau = \frac{RC}{2}.$

$i_R(t) = \frac{V_x}{R} e^{-t/\tau} u(t)$

$E_R = \frac{1}{4}CV_x^2$

$P_R(t) = \frac{V_x^2}{R} e^{-2t/\tau} u(t)$

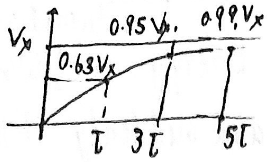
Even if the switch resistance is 0, power dissipated will become ∞.

$= \frac{V_x^2}{(4RC)}(4C) e^{-2t/\tau} u(t)$

$= \frac{1}{4}CV_x^2 \left[\frac{2}{\tau} e^{-2t/\tau} u(t)\right]$

Lecture 12a :- Time Domain Analysis of first order systems for various I/Ps

$$\tau \frac{dV_o}{dt} + V_o = V_i$$



$V_o(t) = V_x(1 - e^{-t/\tau})\, u(t)$

$t_s \rightarrow$ settling time

$t_s \approx \infty$ for many systems

95% $\quad$ 99% $\rightarrow t_s = 5\tau$

$\downarrow$

$V_o(t_s) = 0.95 V_x$

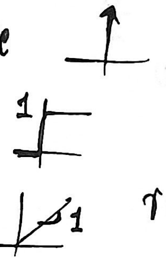
$0.95 V_x = V_x(1 - e^{-t/\tau})$

$e^{-t/\tau} = 0.05$

$t_s \approx 3\tau$

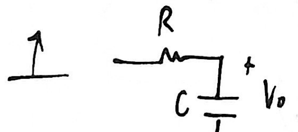
3 most commonly applied I/Ps to a system:-

1) Impulse (Area = 1.)
2) Step
3) Ramp $\quad r(t) = t\,u(t)$



1) Unit Impulse

$\delta(t) \rightarrow \boxed{FOS} \rightarrow$



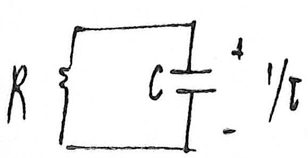
FOS will see the I/P as a very high frequency I/P.
So, the capacitor voltage won't change instantaneously.
$\therefore$ Voltage drop only across resistor.

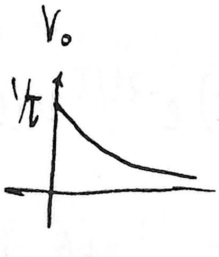
$i = \frac{1}{R}\delta(t) \rightarrow$ This current flows into the capacitor.

$$\therefore V_o(0^+) = \frac{1}{C}\int_0^t \frac{1}{R}\delta(t)\,dt$$

$$= \frac{1}{\tau} \rightarrow V_c \text{ suddenly } \uparrow\text{es to } \frac{1}{\tau}$$

The next moment $t = 0^+$, impulse drops down to 0.





$$\delta(t) \rightarrow \frac{1}{\tau} e^{-t/\tau} u(t)$$

2) Step Response $\quad u(t) = \int_{-\infty} \delta(t)\,dt$

$\rightarrow \boxed{\int} \rightarrow \boxed{FOS} \rightarrow y$

$\rightarrow \boxed{FOS} \rightarrow \boxed{\int} \rightarrow y$

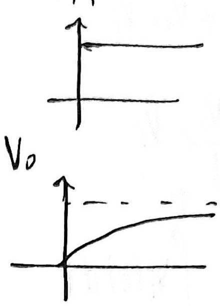
Impulse response $= \frac{1}{\tau} e^{-t/\tau} u(t)$

$\therefore y = \int_0^t \frac{1}{\tau} e^{-t/\tau}\,dt$

$\boxed{y = (1 - e^{-t/\tau})\,u(t)}$

Vin

Vo



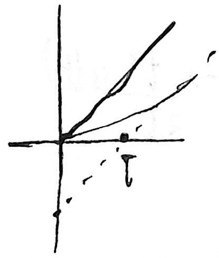
3) Ramp Response

$r(t) = \int_{-\infty}^{t} u(t)\,dt$

$\delta(t) \rightarrow \boxed{FOS} \rightarrow \boxed{\int} \rightarrow \boxed{\int} \rightarrow$ ramp response

$y = \int_{-\infty}^{t} (1 - e^{-t/\tau})\,dt$

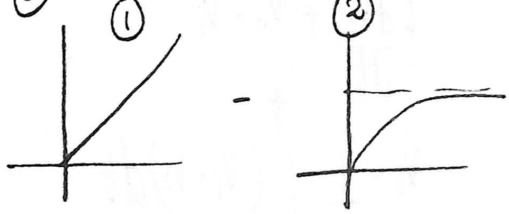
$\boxed{y = t\,u(t) - \tau(1 - e^{-t/\tau})\,u(t)}$



$t \rightarrow \infty$

③ $\boxed{y = u(t)(t - \tau)}$ # Delayed ramp fn.

③ = ① − ②



$V_{in} - V_o = \tau\,u(t) \rightarrow$ Steady State Error betn I/P & O/P.

Lecture 12b :- Initial and steady state response

$t = 0$

Know :- $V_o(0)$

Want :- $V_o(t)\big|_{t=0}$

$t = \infty$

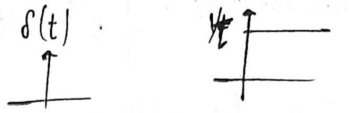
$V_o(\infty)$

$V_o(t)\big|_{t=\infty}$

1st order systems doesn't (change instantaneously) respond to sudden changes to the I/p

$V_i(t)$ $\quad$ $V_o(t)$

$\delta(t)$

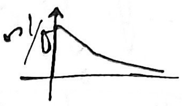


$$T\frac{dV_o}{dt} + V_o = V_i$$

$$T\frac{dV_o}{dt} = V_i$$

$$V_o = \frac{1}{T}\int_{-\infty}^{t} V_i(t)\,dt$$

Actual response $\sim 1/\delta$

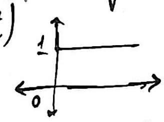


when there is a sudden change in the I/p

$\boxed{\text{At } t=0, \text{ FOS is behaving like a pure integrator}}$

For Step I/P :- $y = (1-e^{-t/T})u(t)$

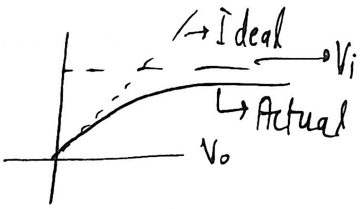
$e^{-x} \approx 1 - x.$



$$V_o(t) = \frac{1}{T}\int_{-\infty}^{t} V_i(t)\,dt$$

$$= \frac{t}{T}u(t)$$

$\therefore (1-e^{-t/T}) \approx t/T$ ← Matches with the actual o/P for small values of t.



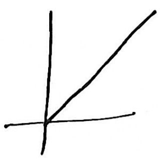
$$y = tu(t) - T(1-e^{-t/T})u(t)$$

$$e^{-x} \approx 1 - x + \frac{x^2}{2!} \qquad x = \frac{t}{T}$$

$$tu(t) - T\left(\frac{t}{T} - \frac{t^2}{2T^2}\right)$$

$$tu(t) - t + \frac{t^2}{2T}$$

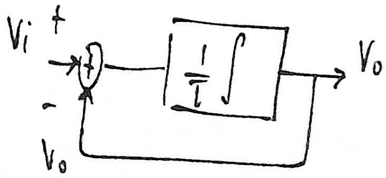
$$\rightarrow \frac{t^2}{2T}u(t)$$



$\rightarrow \frac{t^2}{2T}u(t)$

$$T\frac{dV_o}{dt} + V_o = V_i$$

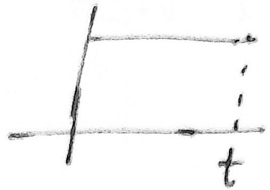
$$V_o = \frac{1}{T}\int_{-\infty}^{t}(V_i - V_o)\,dt$$

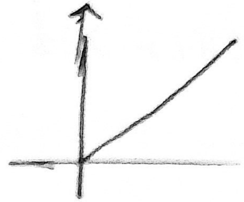


$$V_i - V_o = T\frac{dV_o}{dt}.$$

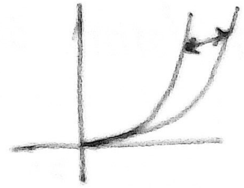
$$\approx T\frac{dV_i}{}$$

$V_i = V_o$ at $t = \infty$.





$v_i - v_o = \tau$



$e = v_i - v_o = \tau\, t\, u(t)$

$v_i = \frac{t^2}{2} u(t)$

$(v_i - v_o) \propto \left(\frac{dv_i}{dt}\right)\Big|_{\text{steady state}}$

Lecture 15- Laplace transforms and effects of zeroes and poles on 1st order systems.

$s = \sigma + j\omega$

$x(t) \rightarrow X(s)$

$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$

$\frac{dx(t)}{dt} \rightarrow sX(s)$

$\int x(t) dt \rightarrow \frac{X(s)}{s}$

$\delta(t) \rightarrow 1$.

$u(t) \rightarrow 1/s$.

$r(t) \rightarrow 1/s^2$

$e^{-at} u(t) \rightarrow \frac{1}{s+a}$

$e^{at} u(t) \rightarrow \frac{1}{s-a}$

$\delta(t) = \frac{du(t)}{dt} = s \cdot \frac{1}{s} = 1$

$\tau \frac{dV_o}{dt} + V_o = V_i$

$s\tau V_o(s) + V_o(s) = V_i(s)$

$\frac{V_o(s)}{V_i(s)} = \frac{1}{s\tau + 1} = H(s)$

$h(t) \rightarrow H(s)$

$\uparrow$

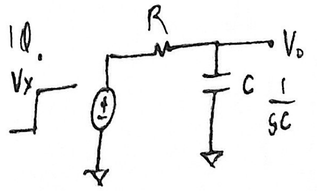
Impulse response

$b_n \frac{d^n V_o}{dt^n} + \ldots + b_0 = a_M \frac{d^m V_i}{dt^m} + \ldots + a_0$.

$H(s) = \frac{\sum_{i=0}^{M} a_i s^i}{\sum_{i=0}^{N} b_i s^i} = \frac{N(s)}{D(s)}$

$H(s)\big|_{s=s_z} \rightarrow 0 \qquad H(s)\big|_{s=s_p} \rightarrow \infty$

↳ max. power of s = order of the ckt.



$\frac{V_o}{V_i} = \frac{1}{1+sRC} = \frac{1}{1+s\tau} = \frac{1}{1+s/\omega_p}$

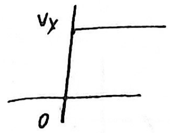
$\boxed{\omega_p = 1/\tau}$ → Pole is inverse of time constant in a 1st order system.

$H(s) = \frac{(1 + s/\omega_z)}{(1 + s/\omega_p)}$

$\omega_z = 0$

$s_p = -\omega_p$

$= -1/\tau$



$$V_o(s) = V_i(s)\, H(s)$$

$$V_o(s) = \frac{V_x}{s} \times \frac{1}{(1+s/\omega_p)}$$

$$= V_x \left[\frac{1}{s} - \frac{1/\omega_p}{1+s/\omega_p} \right]$$

$$\boxed{V_o(s) = V_x \left[\frac{1}{s} - \frac{1}{s+\omega_p} \right]}$$

$$V_o(t) = V_x \left[1 - e^{-\omega_p t} \right] u(t)$$

Ex:- $H(s) = \dfrac{(1+s/\omega_z)}{(1+s/\omega_p)}$

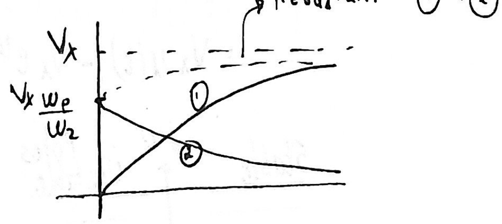
$$V_o(s) = \frac{V_x}{s} \frac{(1+s/\omega_z)}{(1+s/\omega_p)}$$

$$= \frac{V_x}{s} \times \frac{1}{(1+s/\omega_p)} + \left(\frac{s}{\omega_z}\right)\left[\frac{V_x}{s} \times \frac{1}{1+s/\omega_p}\right]$$

$$= V_x\left[1-e^{-\omega_p t}\right] u(t) + \frac{1}{\omega_z}\frac{d}{dt}\left[V_x\left(1-e^{-\omega_p t}\right) u(t)\right]$$

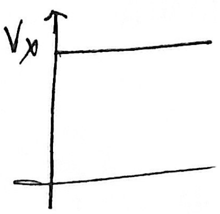
$$= V_x\left[1-e^{-\omega_p t}\right] u(t) + V_x \frac{\omega_p}{\omega_z} e^{-\omega_p t} u(t)$$

$\omega_p < \omega_z$.

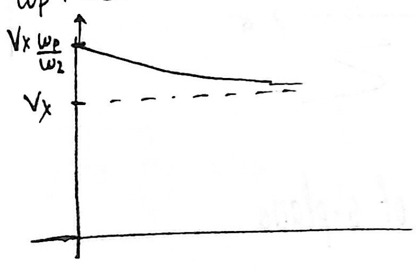


Effect of adding zero is equivalent to adding initial condition.

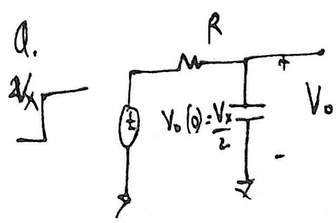
If $\omega_p = \omega_z$

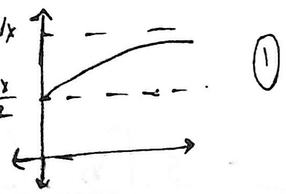


$\omega_p > \omega_z$.



Q.

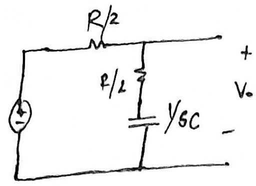




$H(s) = \dfrac{1}{1+sRC}$ $\omega_p = 1/RC$

$S_p = -1/RC$

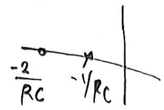
Q.



$$H(s) = \frac{R/2 + 1/sC}{R + 1/sC} = \frac{1 + \frac{sRC}{2}}{1 + sRC}$$

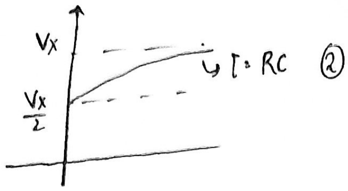
$\omega_z = \frac{2}{RC}$

$\omega_p = 1/RC$



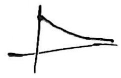
$\frac{\omega_p}{\omega_z} = \frac{1}{2}$

$V_x \frac{\omega_p}{\omega_z}$

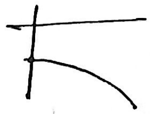


① & ② same → depicts effect of zeroes

$\frac{1}{1+s/\omega_p} = \frac{\omega_p}{s+\omega_p} \Rightarrow h(t) = \omega_p e^{-\omega_p t} u(t)$



$\frac{1}{1-s/\omega_p} = \frac{\omega_p}{s-\omega_p} \Rightarrow h(t) = -\omega_p e^{\omega_p t} u(t)$



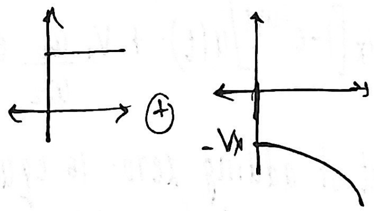
When the pole is in the right pole, the impulse response goes down to $-\infty$, which makes it unstable.

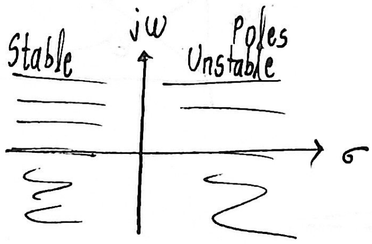
Ex:-

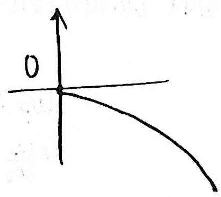
$$H(s) = \frac{V_x}{s} \times \frac{1}{(1-s/\omega_p)} = V_x\left[\frac{1}{s} + \frac{1/\omega_p}{1-s/\omega_p}\right]$$

$$= V_x\left[\frac{1}{s} - \frac{1}{s-\omega_p}\right]$$

$$= V_x u(t) - V_x e^{\omega_p t} u(t)$$







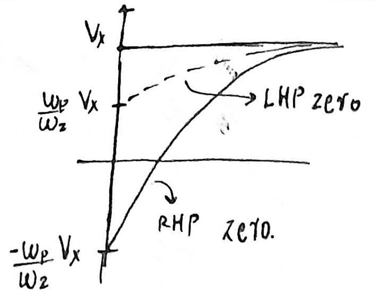
10
V_x
—

Ex:- A zero in the right of s-plane

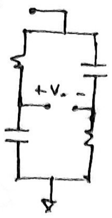
$$H(s) = \left(\frac{1 - s/\omega_z}{1 + s/\omega_p}\right)$$

$$V_o(s) = \frac{V_x}{s} \frac{(1 - s/\omega_z)}{(1 + s/\omega_p)} = \frac{V_x}{s} \times \frac{1}{1+s/\omega_p} - \frac{s}{\omega_z}\left[\frac{V_x}{s} \times \left(\frac{1}{1+s/\omega_p}\right)\right]$$

$$= V_x\left(1 - e^{-\omega_p t}\right)u(t) - \frac{\omega_p}{\omega_z} V_x e^{-\omega_p t} u(t)$$



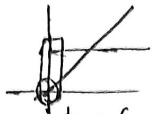
Ies settling time → Not desired.
↳ in case of RHP zero



① $\frac{1}{1+sRC} - \frac{R}{R+1/sC}$

$\frac{1-sRC}{1+sRC}$

$H(s) = \frac{\omega_I}{s}$.

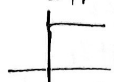


If V_x is given as step I/P
Output will be Ramp $\left(\frac{1}{s} \to \int\right)$

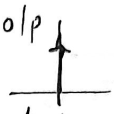
$t=0$ $(\omega=\infty)$
O/P $= 0$

$H(s) = \frac{s}{\omega_D}$.

I/P



O/P

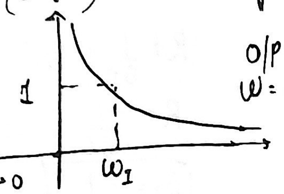


$t=\infty$
$\omega=0$.
O/P $= 0$.

$t=0$
$\omega=\infty$
O/P $= \infty$

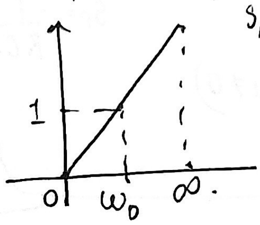
s/ω_D.

$H(s)\,(s=j\omega)\,\frac{\omega_I}{j\omega}$ Integrator

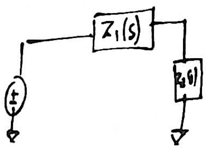


O/P $= 0$
$\omega = \infty$.

$\omega = 0$
O/P $= \infty$.



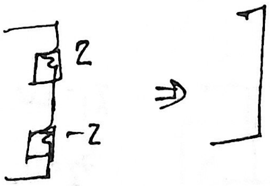
Lecture 16 - Simple & Intuitive tricks to find poles & zeroes in 1st order RC ckts.

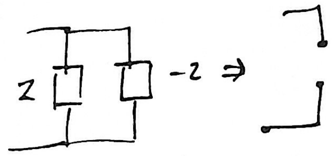


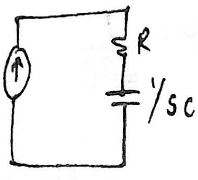
i) Poles :- $Z_1 + Z_2 = 0$. $\Rightarrow \boxed{Z_1(s) = -Z_2(s)}$
& $Z_1(s) = Z_2(s) \neq 0$.

$H(s) = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$

ii) Zeroes :- $Z_2(s) = 0$ $\quad (Z_1(s) \neq 0)$
$Z_1(s) = \infty$ $\quad (Z_2(s) \neq \infty)$

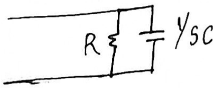






$R = -\frac{1}{sC}$ $\Rightarrow \boxed{s_z = -\frac{1}{RC}.}$

$\frac{1}{sC} = \infty$ $\Rightarrow \boxed{s_p = 0.}$



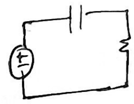
$\frac{1}{sC} = -R.$

$S_p = -\frac{1}{RC}$

For zero:-

$\frac{1}{sC} = 0 \Rightarrow \boxed{S_z = \infty.}$

Q2.



$Z_1 = \frac{1}{sC}$

$Z_2 = R$

$Z_1 = -Z_2$

$\boxed{S_p = -\frac{1}{RC.}}$

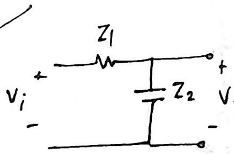
$\frac{Z_2}{Z_1 + Z_2}$

$Z_1 = \infty.$

$\frac{1}{sC} = \infty$

$\boxed{S_z = 0}$

~~1Q.~~



$\frac{Z_2}{Z_1 + Z_2.}$

$R + \frac{1}{sC} = 0$

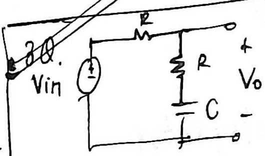
$S_p = -\frac{1}{RC.}$

$Z_2 = 0. \quad (Z_1 \neq 0)$

$\boxed{S_z = \infty}$

$H(s) = \frac{1}{1+sRC}$

~~2Q.~~



$Z_2 = R + 1/sC$

$Z_1 = R.$

$Z_1 = -Z_2$

$R + \frac{1}{sC} = -R$

$2R = -\frac{1}{sC}$

$\boxed{S_p = -\frac{1}{2RC}}$

$Z_2 = 0$

$Z_1 \neq 0.$

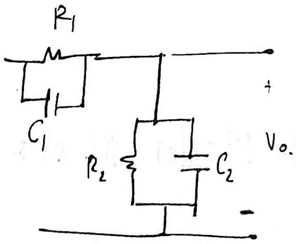
$R + \frac{1}{sC} = 0.$

$R = -\frac{1}{sC}$

$\boxed{S_z = -\frac{1}{RC}}$

$H(s) = \frac{R + 1/sC}{R + R + 1/sC} = \frac{RCs+1}{2RCs+1}$

Q.



$Z_2 = R_2 \parallel \frac{1}{sC_2}$

$Z_2 = \frac{\frac{R_2}{sC_2}}{R_2 + \frac{1}{sC_2}}$

$= \frac{R_2}{sR_2C_2 + 1.}$

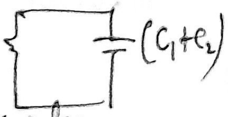
$H(s) = \frac{Z_2}{Z_1 + Z_2.}$

$Z_1 = \frac{R_1}{sR_1C_1 + 1}$

$= \frac{\frac{R_2}{sR_2C_2+1}}{\frac{R_1}{sR_1C_1+1} + \frac{R_2}{sR_2C_2+1}}$

$= \frac{R_2(sR_1C_1+1)}{R_1(sR_2C_2+1) + R_2(sR_1C_1+1)}$

$R_1 \parallel R_2$

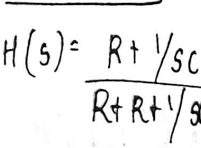


For pole:-

$Z_1 = -Z_2$

$S_z = -\frac{1}{R_1C_1}$

$S_p = \frac{-1}{(C_1+C_2)(R_1 \parallel R_2)}$

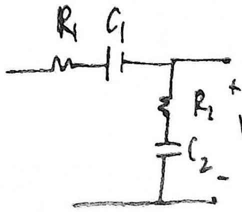


$Z_1 = \infty$

$S_z = 0.$

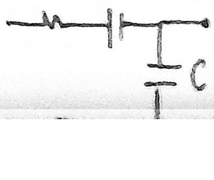
$\frac{1}{sC} + R = -R$

$S_p = -\frac{1}{2RC}$



$S_p = \frac{-1}{(C_1 \parallel C_2)(R_1 + R_2)}$

$S_z = \frac{-1}{R_1C_1}$



$\neq$ No zeroes in the c...

Lecture 17:- Freq...

T.T. tells about...

$H(\omega) = A(\omega)$...

$\frac{1}{1+s/\omega_p} \rightarrow$ ...

$\frac{s/\omega_p}{1+s/\omega_p}$ ...

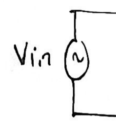
$H(0) \rightarrow$ DC g...

$\frac{\omega_p}{s} H(0)$

Integrator (D.C. g...

Can't be realize...

$\Rightarrow$ ~~Q4.~~



$H(s) = \frac{...}{...}$

Lecture 17:— Frequency response of 1^{st} order systems and bode plots.

F.T. tells about the distribut^n of energy of the signal across frequencies.

$H(\omega) = A(\omega)$

$\omega = \omega_p$

$20\log\left(\frac{1}{\sqrt{2}}\right) = -3\,dB$

$\frac{1}{1+s/\omega_p} \rightarrow lpf$

For high frequencies, the plot is same for a lpf as well as an integrator.

$\frac{s/\omega_p}{1+s/\omega_p} \rightarrow hpf$

$H(0) \rightarrow$ DC gain

$\frac{\omega_p}{s} H(0)$

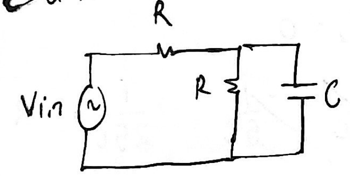
$\downarrow$

Integrator (D.C. gain is ∞)

$\Downarrow$

Can't be realized using RC ckts.

$\Rightarrow$ ~~Q4.~~



$\frac{Z_2}{Z_1 + Z_2}$ -

$$\boxed{Z_2 = \frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}} = \frac{RCs}{RCs+1}}$$

$$H(s) = \frac{\frac{RCs}{RCs+1}}{\frac{RCs}{RCs+1} + R} = \frac{RCs}{RCs + R^2Cs + R} = \frac{Cs}{Cs + RCs + 1} \Rightarrow$$

~~$Z_2 = 0 \; (Z_1 \neq 0)$~~

~~$sC = 0$~~

~~$\boxed{s_z = 0}$~~

$\boxed{s_z = \infty.}$

$Z_1 = -Z_2.$

$\frac{\cancel{R}}{RCs+1} = -\cancel{R}$

$1 = -RCs - 1$

~~$Cs+1 =$~~

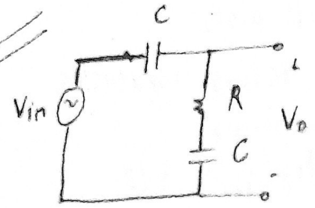
$1 + RCs = -1$

~~$s(RC+C) = -1$~~

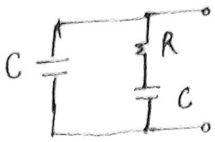
~~$s_p = -\frac{1}{C(R+1)}$~~

$\boxed{s_p = \frac{-2}{RC}}$

Q5.



For pole:-
Short the I/P.



$R + \frac{1}{sC} = -\frac{1}{sC}$

$-\frac{2}{sC} = R$

$\boxed{s_p = -\frac{2}{RC}}$

$Z_2 = 0$

$R + \frac{1}{sC} = 0$

$\boxed{s_z = \frac{-1}{RC}}$

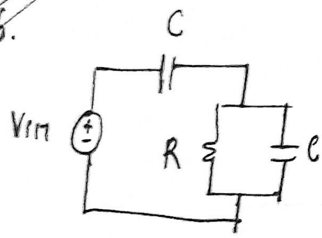
$Z_1 = \infty$

$\frac{1}{sC} = \infty$

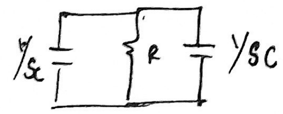
$\boxed{s_z = 0}$

$$H(s) = \frac{R + 1/sC}{R + \frac{2}{sC}} = \frac{RCs + 1}{RCs + 2}$$

Q6.



$\Rightarrow$



$Z_1 + Z_2 = 0$

$\cancel{R + \frac{1}{sC}} = \cancel{-\frac{1}{sC}}$ $\quad \frac{1}{2sC} = -R$

$Z_1 = \frac{1}{sC}$

$Z_2 = R + \frac{1}{sC}$

$R = \frac{-2}{sC}$

$\boxed{s_p = -\frac{1}{2RC}}$

$Z_2 = 0.$

$R + \frac{1}{sC} = 0$

$\boxed{s_z = \infty}$ $\Rightarrow Z_1 = 0.$

$$H(s) = \frac{\frac{R}{1+sRC}}{\frac{R}{1+sRC} + \frac{1}{sC}} = \frac{sRC}{1+s2CR}$$

$Z_1 \to \infty.$

$\frac{1}{sC} = \infty.$

$\boxed{s_z = 0}$

Lecture 17 :- Frequency Response of 1st order systems and Bode plots

$H(s) = \dfrac{1}{1+s/\omega_p}$

$s = j\omega$ $\qquad \phi(\omega) = -\tan^{-1}(\omega/\omega_p)$

$H(\omega) = \dfrac{1}{1+j\omega/\omega_p}$

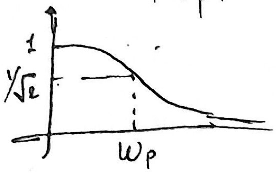
$|H(\omega)| = \dfrac{1}{\sqrt{1+(\omega/\omega_p)^2}}$ $\qquad$ $\omega = 0 \Rightarrow 1$

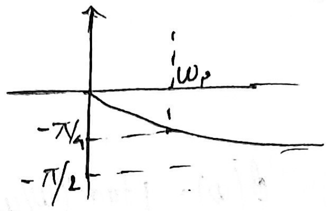
$\omega = \omega_p \Rightarrow 1/\sqrt{2}$

$\omega = \infty \Rightarrow 0$

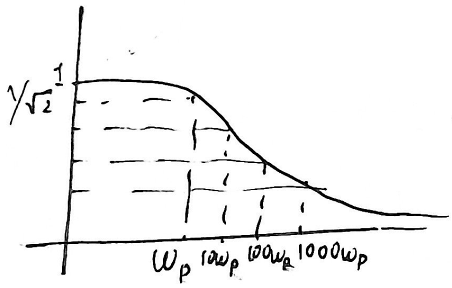
$\cos\omega_p t \Leftrightarrow \frac{1}{\sqrt{2}}\cos(\omega_p t - \pi/4)$

$|H(\omega)|$ # LPF.





Consider 3 more pts.



$\dfrac{1}{\sqrt{1+(\omega/\omega_p)^2}}$ For $\omega >> \omega_p$

$\Rightarrow \left(\dfrac{\omega_p}{\omega}\right) = 0.1$ for $10\omega_p$

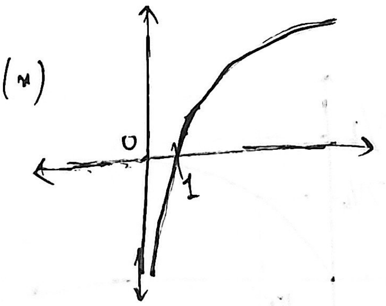
$= 0.01$ for $100\omega_p$

$= 0.001$ for $1000\omega_p$

$\therefore$ $0.1 - 0.01 \simeq 0.1$

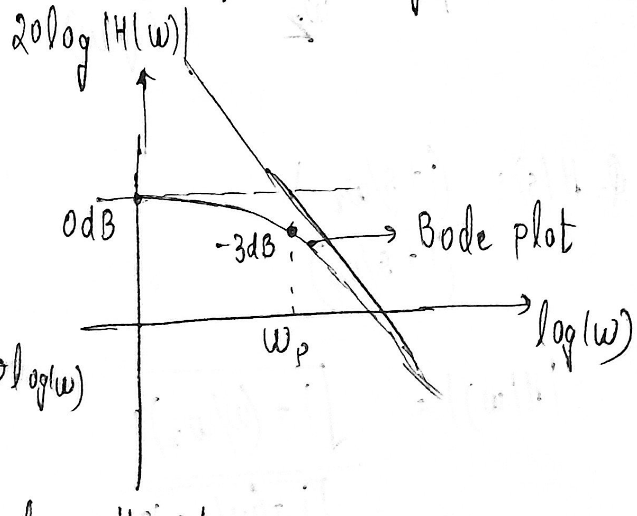
$0.01 - 0.001 \simeq 0.01$

$\log_{10}(x)$



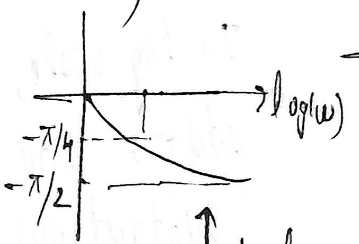
Bode plot touches ① asymptotically at $\omega = 0$ & $\omega = \infty$.

We always plot $20\log|H(\omega)|$



$H(\omega) = \dfrac{1}{\sqrt{1+(\omega/\omega_p)^2}}$ $\qquad \angle H(\omega)$

$20\log\left(\dfrac{1}{\sqrt{1+(\omega/\omega_p)^2}}\right)$



o/p lags the I/p

If $\omega << \omega_p \Rightarrow 20\log(1) = 0$.

$\omega >> \omega_p \Rightarrow 20\log\omega_p - 20\log(\omega) \rightarrow$ ①

$20\log\left(\frac{1}{\sqrt{2}}\right) = -3\text{dB}$.

$20\log|H(\omega)| = 20\log(\omega_p) - 20\log(\omega)$

At $\omega = 10\omega_p$.

$\Rightarrow 20\log\left(\frac{\omega_p}{10\omega_p}\right) = -20$

At $\omega = 100\omega_p$

$\Rightarrow 20\log\left(\frac{\omega_p}{100\omega_p}\right) = -40$

$\therefore$ Slope is -20dB/decade

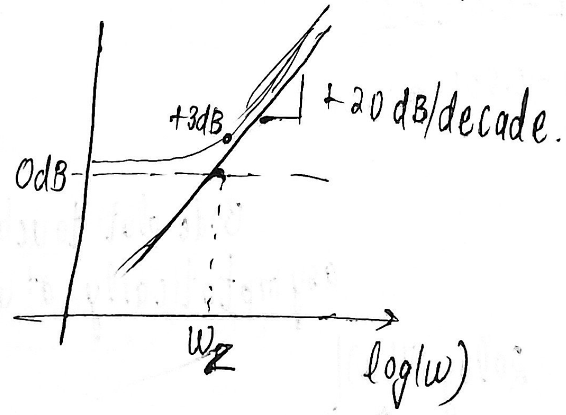
Ex:- $H(s) = 1 + \frac{s}{\omega_z}$

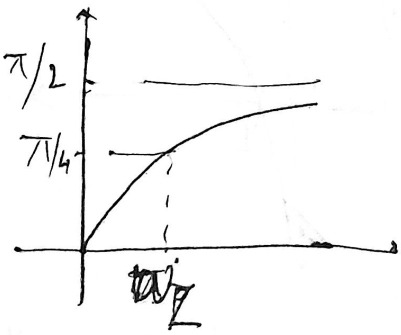
$H(j\omega) = 1 + \frac{\omega}{\omega_z}$ $\qquad \phi(\omega) = +\tan^{-1}(\omega/\omega_p)$

$|H(\omega)| = \sqrt{1+(\omega/\omega_z)^2}$

$20\log|H(\omega)| = 20\log\left(\sqrt{1+(\omega/\omega_z)^2}\right)$

For $\omega >> \omega_p \Rightarrow 20\log(\omega) - 20\log(\omega_z)$





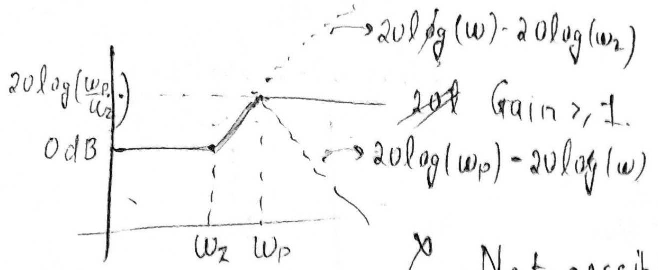
Q. $H(s) = \dfrac{(1+s/\omega_z)}{(1+s/\omega_p)}$

$|H(\omega)| = \dfrac{\sqrt{1+(\omega/\omega_z)^2}}{\sqrt{1+(\omega/\omega_p)^2}}$

In log scale, multiplications becomes additions and division becomes subtractions

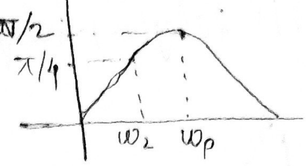
1) $\omega_p > \omega_z$.

Encounter zero before a pole.



$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega}{\omega_p}\right)$

Max^m: $\pi/2$ $-\pi/2$



$\times$ Not possible

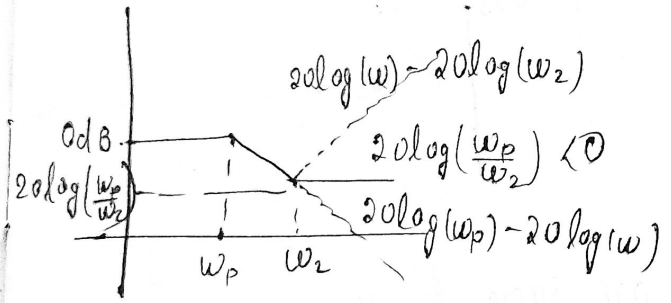
I/P $\cos\omega_0 t$ O/P $\frac{\omega_p}{\omega_z}\cos\omega_0 t$

For $\omega \gg \omega_p, \omega_z$

$$H(\omega) = \frac{(\omega/\omega_z)}{(\omega/\omega_p)} = \frac{\omega_p}{\omega_z}.$$

$$\boxed{H(\infty) = H(0)\,\frac{\omega_p}{\omega_z}.}$$

2) $\omega_p < \omega_z$.



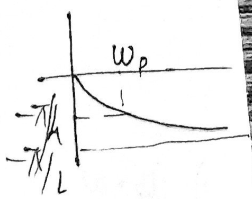
Ex:- $H(s) = \frac{s/\omega_p}{(1 + s/\omega_p)}$ # High pass filter

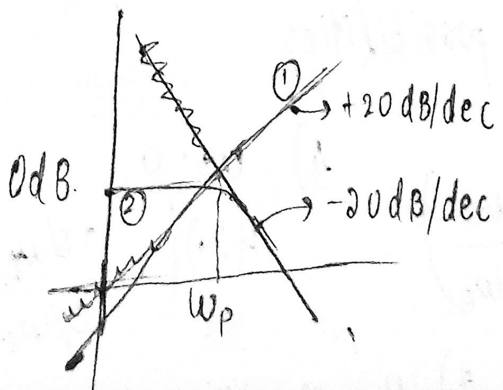
$$\boxed{H(\omega) = \frac{|\omega/\omega_p|}{\sqrt{1 + (\omega/\omega_p)^2}}}$$

$$\frac{j\omega/\omega_p}{1 + j\omega/\omega_p}$$

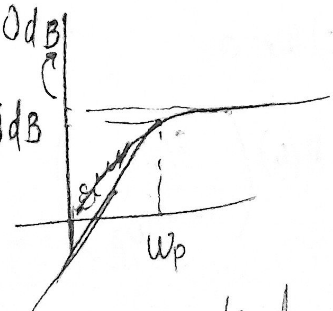
$\phi(\omega) = \frac{\pi}{2} - \tan^{-1}(\omega/\omega_p)$

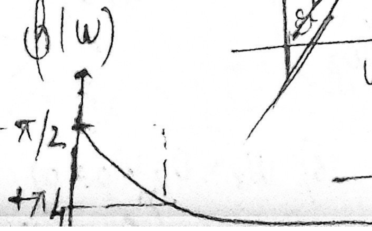
$\Downarrow$





1 + 2 $\Rightarrow$



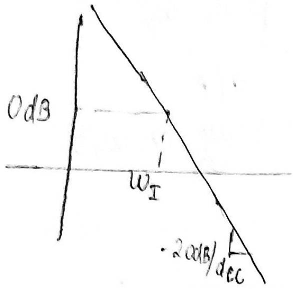


$\rightarrow$ O/P leads the I/P

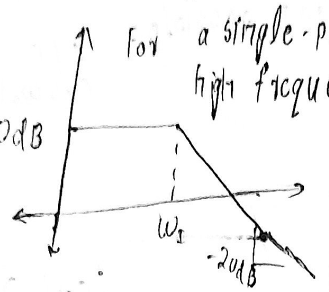
Ideal Integrator

$H(s) = \frac{\omega_I}{s}$ $\quad \phi = -\pi/2$

$20\log|H(\omega)| = 20\log(\omega_I) - 20\log(\omega)$



If $H(s) = \frac{1}{1+s/\omega_I}$



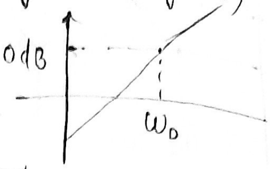
For a simple-pole system, high frequency phase is $-\pi/2$

For $\omega \gg \omega_I$; the frequency response of an integrator is same as that of a 1^{st} order low pass filter.

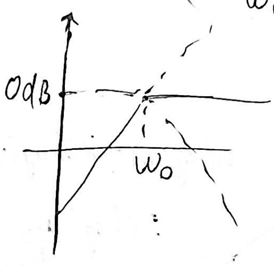
Ideal Differentiator

$H(s) = \frac{s}{\omega_0}$

$20\log(\omega) - 20\log(\omega_0)$



$H(s) = \frac{\frac{s}{\omega_p}}{\left(1+\frac{s}{\omega_0}\right)}$



1^{st} order hpf & a differentiator are same for $\omega \ll \omega_0$.

Lecture 18 :- Realizable 1^{st} order transfer fn using passive RC ckts.

General 1^{st} order transfer fn

$H(s) = H(0)\frac{(1+s/\omega_z)}{(1+s/\omega_p)}$

D.C gain

$H(s)|_{s=0} = H(0)$

s_z ω_z - $0, \omega_z, \infty$

s_p ω_p - $0, \omega_p, \infty$.

$(0,0)$, (∞,∞) → Not possible

Total 7 possibilities.

1) $\omega_z = \infty$

$\left(\frac{H(0)}{1+s/\omega_p}\right)$

2) $\omega_z = 0$

$H(0)\left(\frac{s/\omega_z}{1+s/\omega_p}\right)$

3) Finite

$H(0)\left(\frac{1+s/\omega_z}{1+s/\omega_p}\right)$

4) $\omega_p = 0$

$H(0)\left(\frac{1+s/\omega_z}{s/\omega_p}\right)$

5) $\omega_p = \infty$

$H(0)\left(1+s/\omega_z\right)$

6) $\omega_z = 0$; $\omega_p = \infty$

$H(0)\left(s/\omega_z\right)$

7) $\omega_z = \infty$; $\omega_p = 0$

$\frac{H(0)}{(s/\omega_p)} = \frac{H(0)\omega_p}{s}$

(5) & (6) are not 1st order systems because they don't have a pole.

Only (1), (2) & (3) are realizable

(3) → only under certain conditions.

For passive ckts; ~~H(0) ≤ 1~~ $\boxed{|H(\omega)|^2 \le 1}$

$$X(s) \rightarrow \boxed{H(s)} \rightarrow Y(s)$$

$$Y(s) = H(s)\,X(s)$$

$$|Y(\omega)| = |H(\omega)X(\omega)|$$

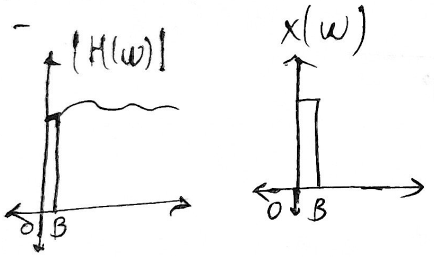
$$\frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(\omega)|^2 \, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 |X(\omega)|^2 \, d\omega$$

$$E_y = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$$

For a passive system; energy at the O/P is lesser than that at the I/P.

$$E_y \le E_x.$$

Ex:-



$$E_y = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(0)|^2 |X(\omega)|^2 \, d\omega$$

$$E_y = E_x \, |H(0)|^2 < E_x$$

For an integrator; DC gain is ∞ ($\omega_z = \infty$)

∴ It can't be realized using a passive RC ckt.

(4) It has a pole at D.C. ∴ D.C gain is ∞.

$$H(s) = \frac{s/\omega_z}{1 + s/\omega_p} \approx \frac{s/\omega_z}{s/\omega_p} = \frac{\omega_p}{\omega_z}.$$

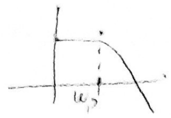
$$\omega_p < \omega_z \quad (H(0) < 1)$$

→ Integrator

① $H(s) = \frac{1}{1+s/\omega_p}$

② $H(s) = \frac{s/\omega_z}{1+s/\omega_p} \approx \frac{s/\omega_z}{s/\omega_p} = \frac{\omega_p}{\omega_z}$

$\therefore \omega_p < \omega_z$



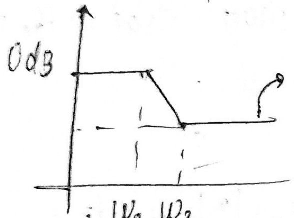
The slope with which the graph rises has to be smaller than ~~wt~~ the slope with which it is going to fall after ω_p.

If $\omega_p = \omega_z \Rightarrow 0dB$

$20\log\left(\frac{\omega_p}{\omega_z}\right)$

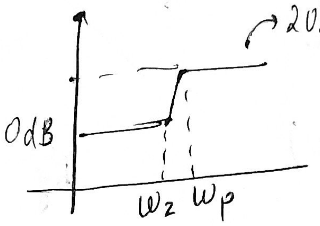
③ $H(s) = \frac{(1+s/\omega_p)}{(1+s/\omega_z)}$

$\omega_p < \omega_z$



$20\log\left(\frac{\omega_p}{\omega_z}\right)$

$\omega_z < \omega_p$

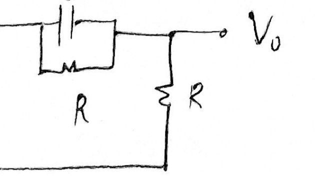


$20\log\left(\frac{\omega_p}{\omega_z}\right) \geq 0dB$

$\geq 1 \Rightarrow$ Not possible

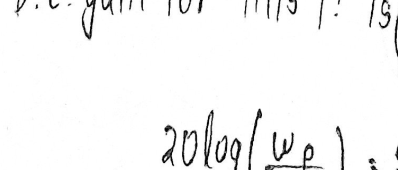
$\therefore$ ③ is realizable only if $\omega_p < \omega_z$.

Ex:-



$S_p = \frac{-1}{\frac{R}{2}C} = \frac{-2}{RC}$

Short I/P to find pole.



$S_p = \frac{-2}{RC}$

$\omega_p > \omega_z$.

D.C. gain for this fn is $\left(\frac{1}{2}\right)$

$S_z = \frac{-1}{RC}$

$20\log\left(\frac{\omega_p}{\omega_z}\right) = 20\log(2) = +6.$

$$H(s) = H(0)\left(\frac{1+s/\omega_z}{1+s/\omega_p}\right)$$

If $s_z = 0$

$$H(s) = H(0)\frac{(s/\omega_z)}{(1+s/\omega_p)}$$

For $s_z = 0$; D.C. gain $(H(0)) = 0$.

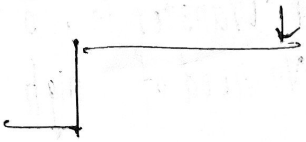
We need to know the slope i.e, the way fn behaves for low frequencies

$$H(s) = H(0)\frac{s}{\omega_z}$$ # for low frequencies.

Step / Impulse responses ⇒ Sudden I/Ps $(f = \infty)$

$$H(\infty) = H(0)\frac{\left(1+j\omega/\omega_z\right)}{\left(1+j\omega/\omega_p\right)}$$

$$\boxed{H(\infty) = H(0)\left(\frac{\omega_p}{\omega_z}\right)}$$



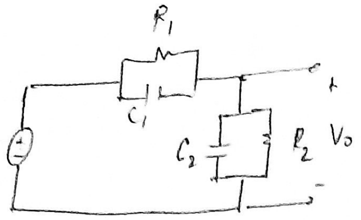
Once the I/P attains a steady state $(t = \infty)$ the value of the O/P at $t = \infty$.
(only true for steady state I/Ps)

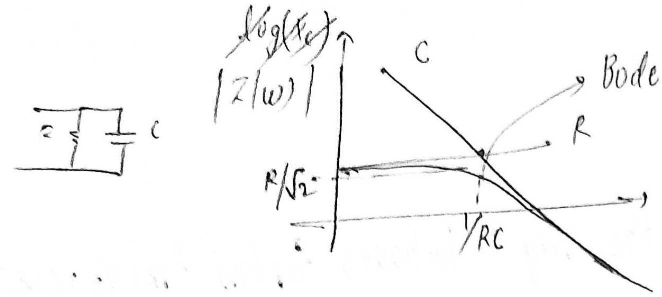
Steady state is like DC.

$$\boxed{y(\infty) = H(0)\,x(\infty)}$$

$$\boxed{y(0^+) = H(\infty)\,x(0)}$$

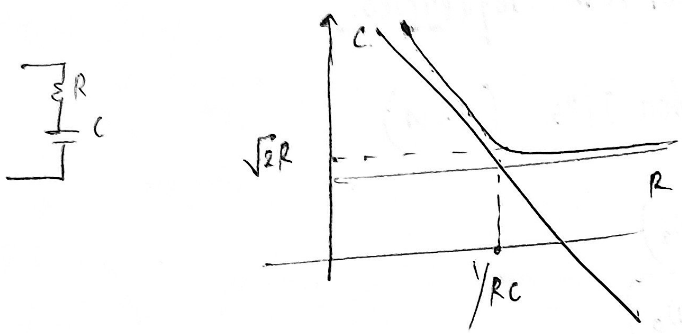
(i).





Bode plot for impedance

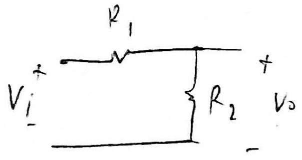
Lower impedance dominates in parallel.



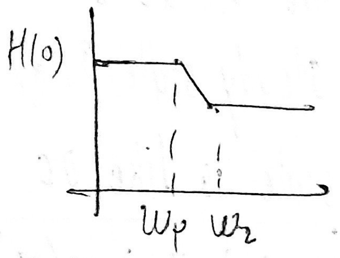
To find transfer fn, 3 things are needed:- DC gain, pole, zero

No need of high frequency gain.

For DC gain $s=0$.



$$\frac{V_o}{V_i} = \frac{R_2}{R_1+R_2}$$

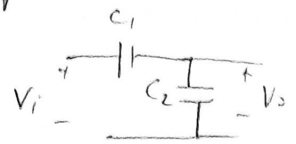


- $\omega_p = \dfrac{-1}{(R_1 \| R_2)(C_1+C_2)}$

- $\omega_z = \dfrac{-1}{R_1 C_1}$ (Z_1 should blow up)

$$\therefore H(s) = \left(\frac{R_2}{R_1+R_2}\right)\left(\frac{1+sR_1C_1}{1+s(R_1\|R_2)(C_1+C_2)}\right)$$

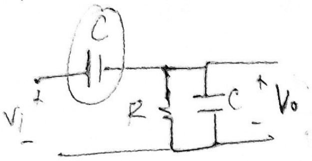
For very high frequencies; $X_C << R$



$$\frac{C_1}{C_1+C_2}$$

$$\left(\frac{R_2}{R_1+R_2}\right)\left(\frac{sR_1C_1}{s(R_1 \parallel R_2)(C_1+C_2)}\right) = \frac{C_1}{C_1+C_2}$$

Ex:- To find transfer f^n of system where it has zero at D.C.

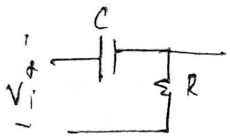


$s_z = 0.$

$s_p = \frac{-1}{2RC}$ → This ckt. behaves like a differentiator.

$$H(s) = \frac{sk}{1+s2RC} = \frac{sRC}{1+2sRC}$$

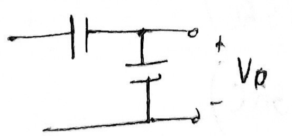
At low frequencies:-



$V_i(sC)R = V_o.$

$$\frac{V_o}{V_i} = sRC$$

At high freq:-

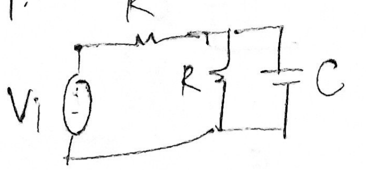


$$\frac{V_o}{V_i} = \frac{1}{2}$$

$$\frac{\cancel{s}k}{2\cancel{s}RC} = \frac{1}{2}$$

$$\boxed{k = RC.}$$

Lecture 19:-

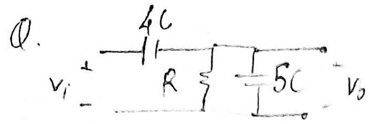


$$s_p = \frac{-2}{RC.}$$

$$H(0) = 1/2$$

$$= \frac{1}{2}\left(\frac{1}{1+\frac{sRC}{2}}\right)$$

Q.



At $s=0 \Rightarrow$ It is an open ckt

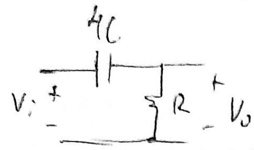
$S_z = 0.$

$S_p = \dfrac{-1}{9RC}$

$H(s) = \dfrac{s \cdot x}{(1 + s9RC)}$ $\quad x = 4RC$

$H(s) = \dfrac{4sRC}{1+s9RC}$

At low freq.

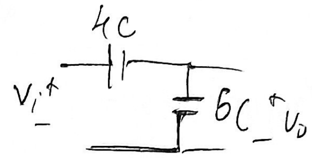


$V_i(4sC)R = V_o$

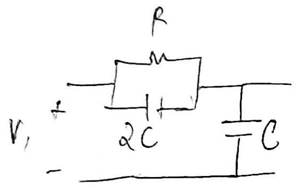
$\dfrac{V_o}{V_i} = 4sRC.$

At high freq

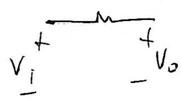
$H(s) = \dfrac{4}{9}$



Q.



At $s=0$



$S_p = \dfrac{-1}{3RC}$

$V_o = V_i$

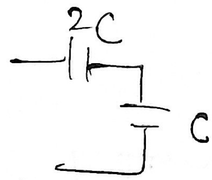
$H(0) = 1$

$S_z = \dfrac{-1}{2RC}$

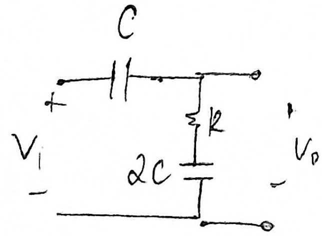
$H(s) = \dfrac{1(1+2sRC)}{(1+3sRC)}$

At high f

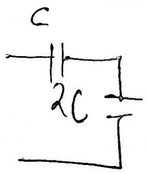
$H(s) = \dfrac{2}{3}$



Q.



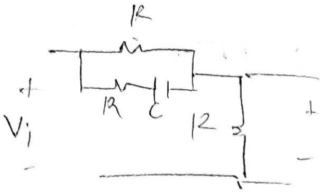
$S_z = \dfrac{-1}{2RC}$



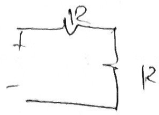
$1/3$

$S_p = \dfrac{-3}{2RC}$

$H(s) = \dfrac{1}{3}\dfrac{(1+s2RC)}{\left(1+\frac{2}{3}sRC\right)}$

Q. 

$s \to 0$



$H(0) = 1/2$

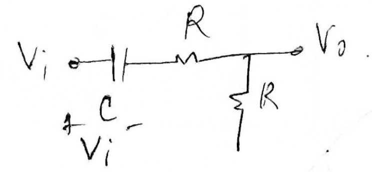
$R = -R - 1/sC$

$$\boxed{S_z = -\frac{1}{2RC}}$$

$$\frac{1}{sC} + R = -R/2$$

$$\boxed{S_p = -\frac{2}{3RC}}$$

$$H(s) = \frac{1}{2}\frac{(1+2sRC)}{\left(1+\frac{3}{2}sRC\right)} \approx \boxed{\frac{1}{3}}$$

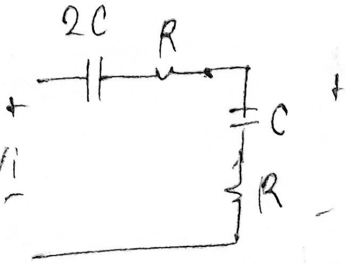
Q. 

$S_z = 0.$

$$S_p = \frac{-1}{2RC}.$$

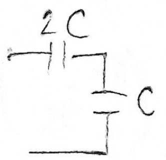
$$\frac{s(RC)}{1+s2RC} = 1/2 \; V_i(sC)$$

$\rightarrow$ High freq. gain

Q. 

$S_z = \frac{-1}{RC}$ $\quad Z_2 = 0$

$$S_p = -\frac{1}{\frac{4}{3}RC}.$$



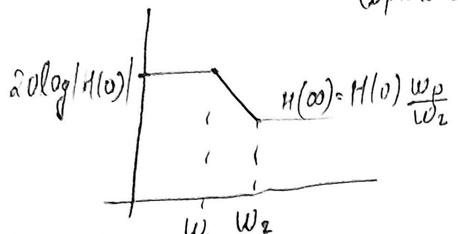
$2/3$

$$\frac{2}{3}\left(\frac{1+sRC}{1+\frac{4}{3}sRC}\right)$$

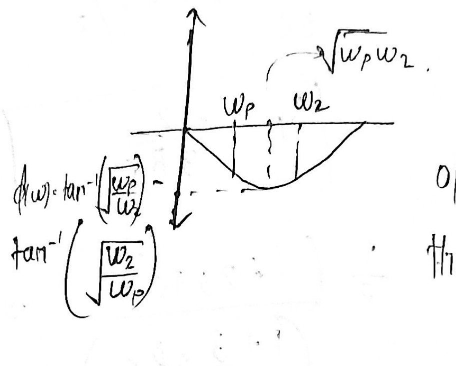
Lecture 20:- Relat: betn frequency & time domain response of first order system

$$H(s) = H(0)\,\frac{(1+s/\omega_z)}{(1+s/\omega_p)}$$

$\omega_p < \omega_z$

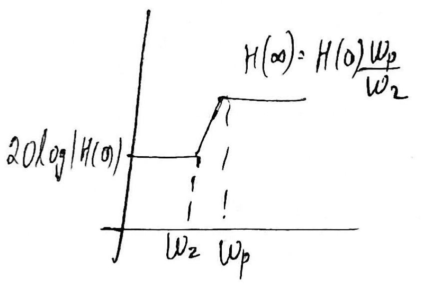


$\phi(\omega) = \tan^{-1}(\omega/\omega_z) - \tan^{-1}(\omega/\omega_p)$

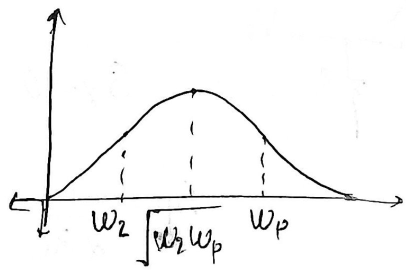


o/p lagging the I/P.

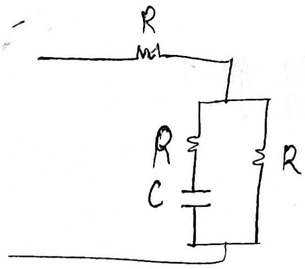
$\omega_p > \omega_z$.



$\phi(\omega) = \tan^{-1}(\omega/\omega_z) - \tan^{-1}(\omega/\omega_p)$



Q.



D.C. at gain (low freq.)

$H(0) = \frac{1}{2}$

For zero; $Z_2 = 0 \,/\, Z_1 = \infty$.

$$Z_2 = R \,\|\, \left(R + \frac{1}{sC}\right)$$

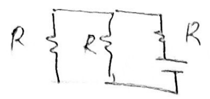
$$= \frac{R\left(R+\frac{1}{sC}\right)}{2R + \frac{1}{sC}} = \frac{\frac{R(RCs+1)}{Cs}}{\frac{2RCs+1}{sC}}$$

For $Z_2 = 0 \longrightarrow R = 0 \,/\, R + \frac{1}{sC} = 0$.

$R = -\frac{1}{sC}$

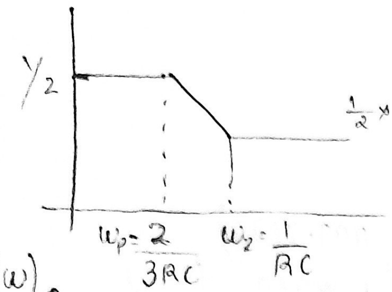
$s_z = -\frac{1}{RC}$

$g_p = \frac{-2}{3RC}$



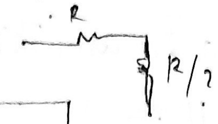
$\frac{R}{2} + R = \frac{3R}{2}$

$$H(s) = \frac{1}{2}\left(\frac{1+sRC}{1+\frac{3}{2}sRC}\right) \qquad \omega_p = \frac{2}{3RC} \; ; \; \omega_z = \frac{1}{RC}$$

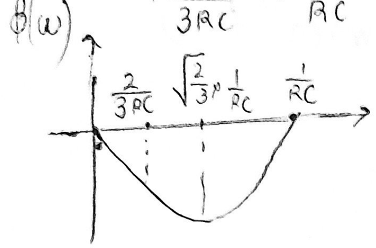


$\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$ → Other way

At high freq $X_C = 0$



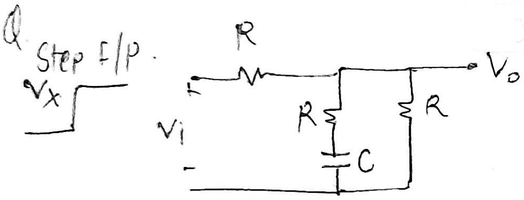
$A_v = \frac{1}{3}$.



$$\boxed{H(\infty) = H(0)\frac{\omega_p}{\omega_z}}$$

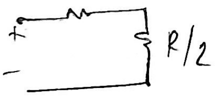
$$\boxed{H(\infty) = H(0)\frac{2}{3}}$$

Q



When step I/P is given; assume cap. is short

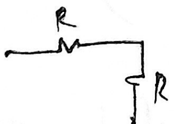
As $t \to 0$; $f \to \infty$ $\quad X_C = \frac{1}{2\pi fC} = 0$.



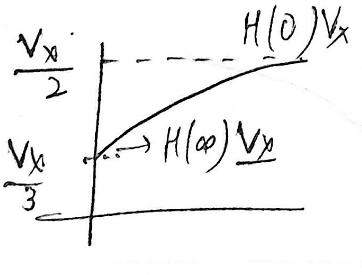
I/P - V_x

$\boxed{O/P - V_x/3.}$ → This is at $t=0$.

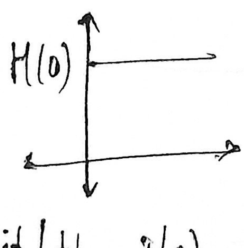
When $t \to \infty / s \to 0$, $X_C = \frac{1}{2\pi fC} = \infty$.



$H(0) = \frac{1}{2} \cdot \frac{V_x}{2}$.

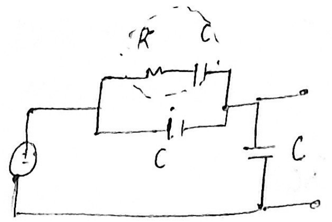


If there were no poles:-



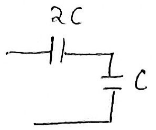
Because of presence of poles; it (the I/P) tries to catch up with the steady state value (H(0)).

Q.



DC gain $= H(0)$

$s = 0$



$H(0) = \frac{2}{3}$

$s_z =$ ~~0~~ Zero when $Z_1 = \infty$. (This should just be non-zero)

$\because Z_2 = \infty$ only when $f = 0$.

$Z_1 = \left(R \parallel \frac{1}{sC}\right) \parallel \frac{1}{sC}$

$-\frac{1}{sC} = \frac{1}{sC} + R$

$R = \frac{-2}{sC}$

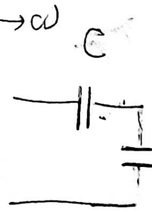
$\boxed{s_z = \frac{-2}{RC}}$

$s_p = \frac{-3}{2RC}$

$\boxed{s_p = \frac{-3}{2RC}}$

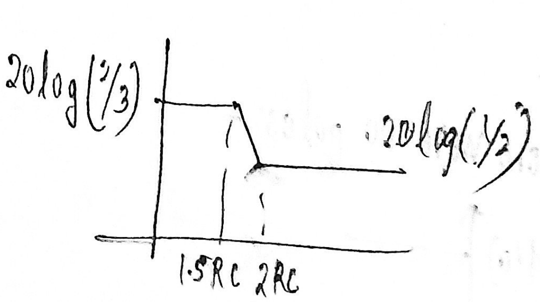
$\boxed{\omega_z = \frac{2}{RC}}$ $\boxed{\omega_p = \frac{3}{2RC}}$

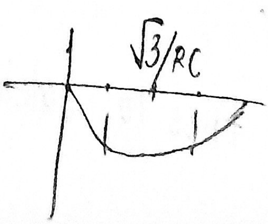
$s \to \infty$



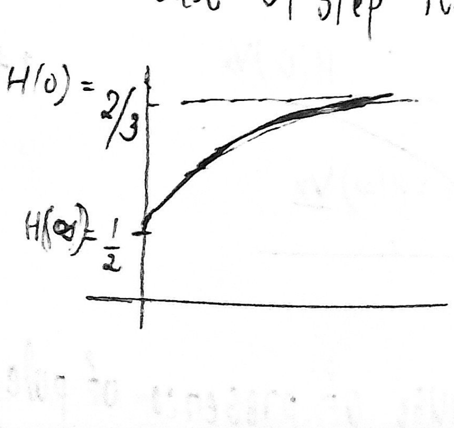
$H(\infty) = 1/2$

$$H(s) = \frac{2}{3} \frac{\left(1 + \frac{sRC}{2}\right)}{\left(1 + \frac{2}{3} sRC\right)}$$

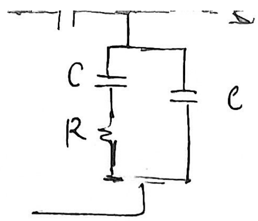




In case of step response:-



Q.



Z_2 as well

$\therefore s=0$ is not a zero.

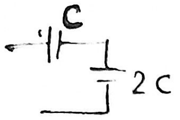
Check if $Z_1 = 0$

$R + \frac{1}{sC} = 0$

$\boxed{s_z = -\frac{1}{RC}}$

$\boxed{s_p = -\frac{3}{2RC}}$

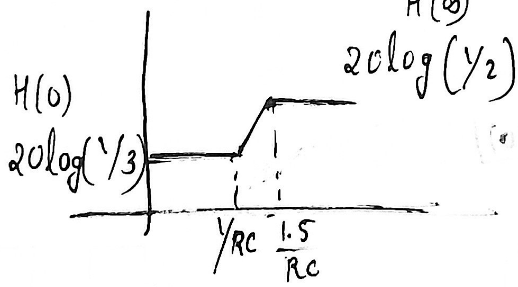
$\underset{s=0}{H(0)} = 1/3.$

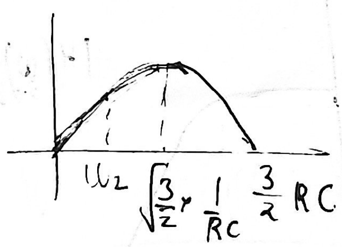


$= 1/3$

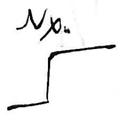
$H(\infty) = 1/2$

$H(s) = \frac{1}{3} \frac{(1+sRC)}{\left(1+\frac{2}{3}sRC\right)}$



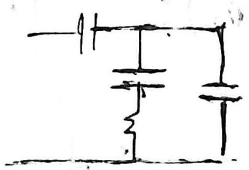


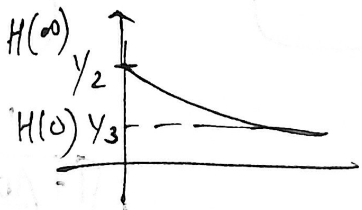
In case of a step I/P:-



$t = 0.$

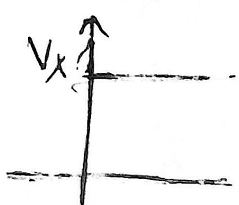
$\boxed{H(\infty) = \frac{1}{2}}$



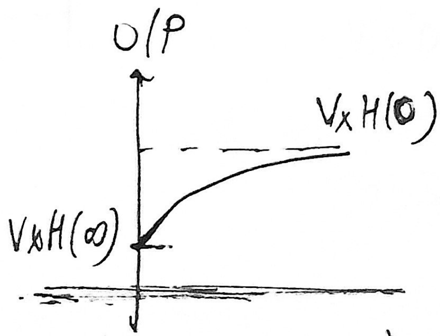


$$H(s) = H(0) \frac{(1+s/\omega_z)}{(1+s/\omega_p)}$$

I/P

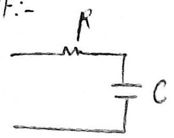


O/P



$$V_{out}(t) = V_x H(\infty) e^{-\omega_p t} + V_x H(0) (1 - e^{-\omega_p t})$$

LPF:-



$H(\infty) = 0$

$H(0) = 1.$

$V_o(t) = V_x\left(1 - e^{-\omega_p t}\right)$

HPF:-

$H(\infty) = 1$

$H(0) = 0$

$V_o(t) = V_x e^{-\omega_p t}$

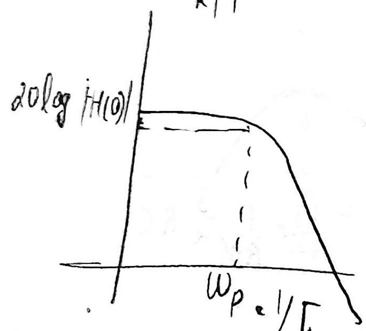
$H(\infty)$ depends on zero.

$$\boxed{H(\infty) = H(0)\frac{\omega_p}{\omega_z}}$$

$H(0) \rightarrow$ ckt dependent parameter.

$$V_o(t) = V_x H(0)\frac{\omega_p}{\omega_z} e^{-\omega_p t} + V_x H(0)\left(1 - e^{-\omega_p t}\right)$$

LPF



$$|H(\omega_{3\text{-}db})| = \frac{|H(0)|}{\sqrt{2}}.$$

$$\boxed{\omega_{3\text{-}db} = \omega_p.}$$

$$\frac{1}{\left(1 + s/\omega_p\right)^N}$$

$|H(\omega)| = 1/\sqrt{2}$

$|H(0)|^2 = 1/2.$

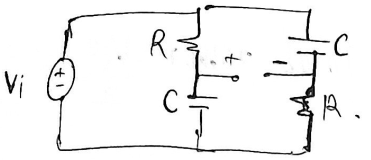
$$\frac{1}{\left[1 + \left(\frac{\omega}{\omega_p}\right)^2\right]^N} = \frac{1}{2}..$$

$$\boxed{\omega_{3\text{-}db} = \omega_p\sqrt{2^{1/N} - 1}}$$

@ $N = 1$

$\omega_{3\text{-}dB} = \omega$

$N \geq 2$



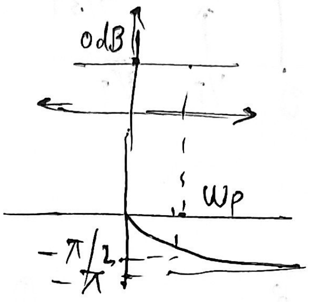
$$\frac{1}{1+sRC} - \frac{sRC}{1+sRC} = \frac{1-sRC}{1+sRC}$$

I/P- $V_p \cos(\omega_p t + \phi)$

$$\boxed{\omega_p = \frac{1}{RC}}$$

Mag. response

$$\frac{\sqrt{1+(\omega RC)^2}}{\sqrt{1+(\omega RC)^2}} = 1$$



$RC = 1/\omega_p$

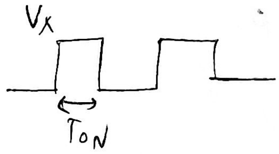
$\phi(\omega) = -2\tan^{-1}(\omega/\omega_p)$

O/P- $V_p \cos(\omega_p t + \phi - \pi/2)$

$= V_p \sin(\omega_p t + \phi)$

Lecture 21:- Pulse response of 1st order RC ckts.

$p(t)$



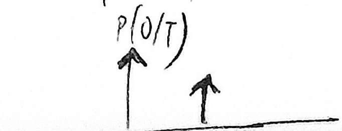
$T_{ON} + T_{OFF} = T$

$P(f) = V_x T_{ON} \operatorname{sinc}(\pi f T_{ON})$

$$x(t) = \sum_{k=-\infty}^{\infty} p(t-kT)$$

$$X(f) = \frac{1}{T} |P(f)| \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$$

$$= \frac{|P(k/T)|}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$$



$|P(f)| = V_x T_{ON} \operatorname{sinc}(\pi f T_{ON})$

$$\frac{|P(k/T)|}{T} = V_x \frac{T_{ON}}{T} \operatorname{sinc}(\pi f T_{ON})$$

$$= \frac{V_x T_{ON}}{T} \frac{\sin(\pi f T_{ON})}{\pi (k)\left(\frac{T_{ON}}{T}\right)}$$

$f = k/T$

$\boxed{D = 1/2}$

$$= D V_x \frac{\sin(\pi k D)}{\pi k D}$$

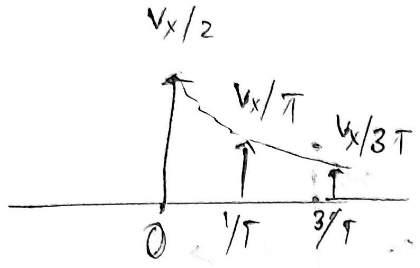
$$= \frac{V_x}{2} \frac{|\sin(\pi k/2)|}{(\pi k/2)}$$

$|\sin(\pi k/2)| \rightarrow$ 1 for odd k

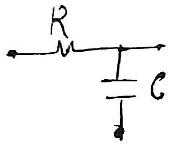
$\rightarrow$ 0 for even k

$$\frac{|P(k/T)|}{T} = \begin{cases} \frac{V_x}{\pi k} & \text{for odd } k \\ 0 & \text{for even } k \end{cases}$$

Total power $= \frac{V_x^2}{2}$



Q.



① $\tau << T$

⇓

The O/P will settle within 5 time constants (5τ).

$5\tau < T/2$

② $\tau >> T$

At this pt., the system will think of it as a step I/p.

At $t = T/2$ Cap. stores a charge of V_x

After the falling; capacitor discharges thro' the resistor.

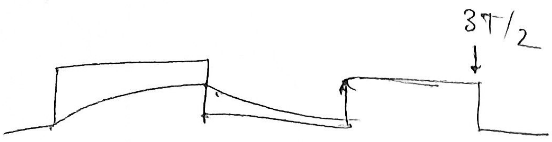
$V_x(1-e^{-t/\tau}) \to t = T/2 \Rightarrow V_x$

$\to V_x e^{-t/\tau} \to t = T/2 \Rightarrow 0$

There are no sudden transitions (S.T. ⇒ high frequency)

High frequency changes are all eliminated

② $\tau >> T$



$V_x(1-e^{-T/2\tau}) \cdot e^{-T/2\tau} > 0$

↓

At $t = T/2$

↓

At $t = T$

At $t = 3T/2$

$$\boxed{V_x(1-e^{-T/2\tau}) + V_x(1-e^{-T/2\tau})\, e^{-T/2\tau} \cdot e^{-T/2\tau}}$$

$e^{-T/2\tau} = x$

$V_x(1-x) + V_x(1-x)x^2$

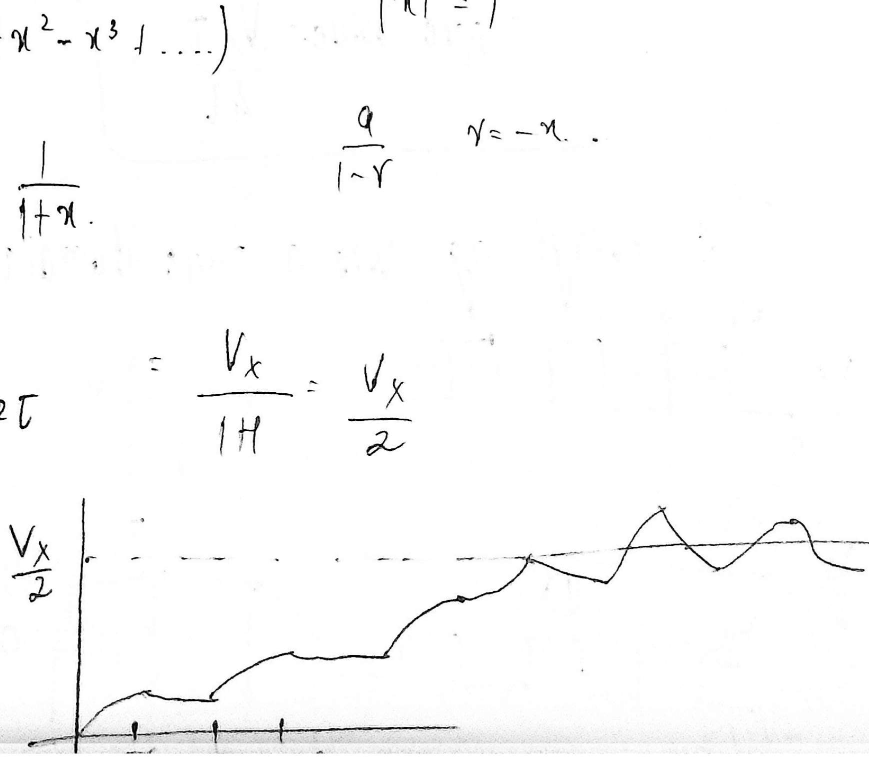
$V_x(1-x+x^2-x^3)$

At $t = nT/2$

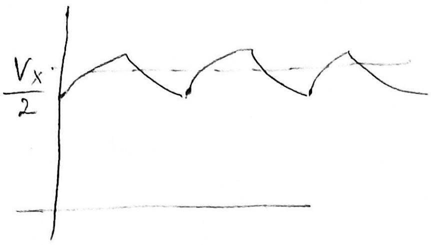
$V_x(1-x+x^2-x^3+\dots)$ $\quad |x| \le 1$

$V_x \cdot \frac{1}{1+x}$ $\quad \frac{a}{1-r}$ $\quad r = -x$

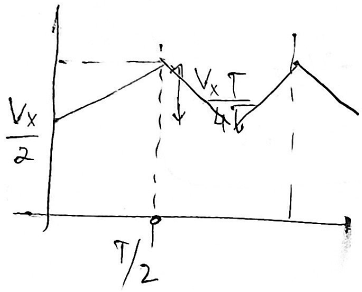
$$\frac{V_x}{1+e^{-T/2\tau}} = \frac{V_x}{1+1} = \frac{V_x}{2}$$



Applicatn :- To generate a constant D.C. voltage by applying pulse I/p.



Assume Initial Voltage on capacitor $\approx \frac{V_x}{2}$.



$$\frac{V_x}{2} + \frac{V_x}{2}\left(1-e^{-t/\tau}\right)$$

$$\frac{V_x}{2}\left(1-e^{-T/2\tau}\right)$$

$$\boxed{1-e^{-x} = x.}$$

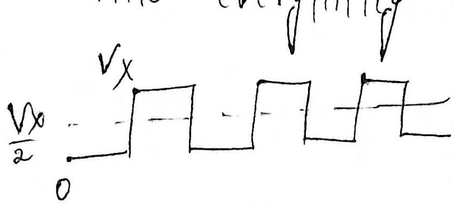
$$\frac{V_x}{2} \times \frac{T}{2\tau} = \frac{V_x T}{4\tau}$$

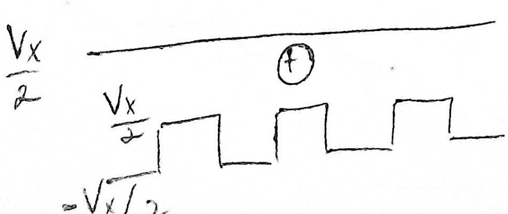
In (-)ve half-cycle:-

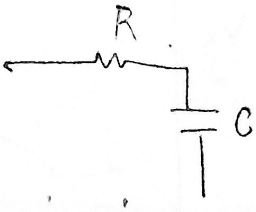
$$\frac{V_x}{2}\left(1-\frac{T}{2\tau}\right) = \frac{V_x}{2} - \frac{V_x T}{4\tau}$$

$$\therefore \boxed{\text{Ripple Value} = \frac{V_x T}{4\tau}}$$

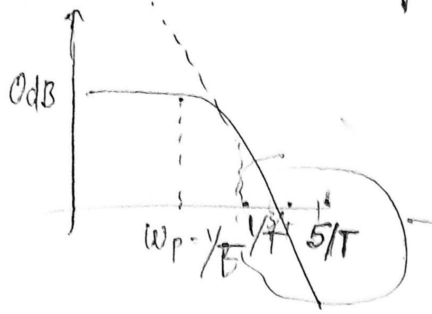
This everything was in time domain.







D.C gain of a 1st order system is 1.

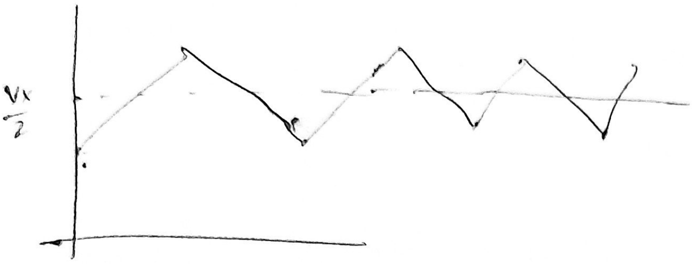


$\frac{1}{1+s/\omega_p}$

$\frac{\omega_p}{s}$

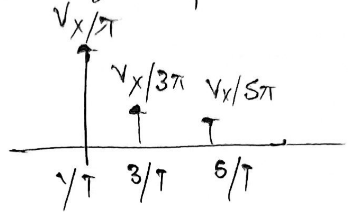
→ For this part, system behaves like an ideal integrator

Steady State Transient O/P :-

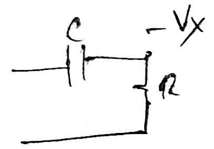


$\tau >> T$

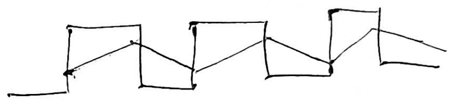
$\frac{1}{\tau} << \frac{1}{T}$

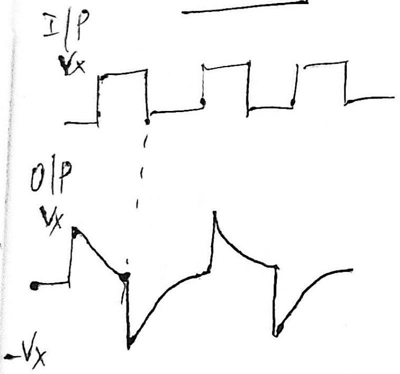


Hpf :-

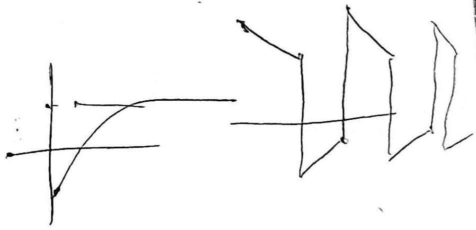


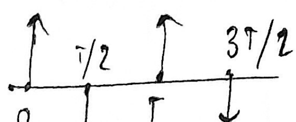
$\tau << T$.





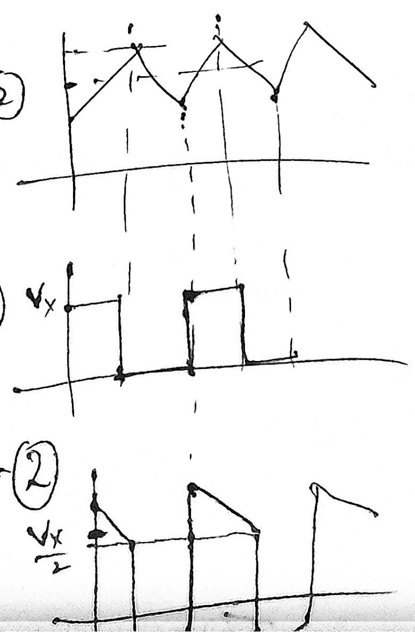
$\frac{s/\omega_p}{1+s/\omega_p}$.

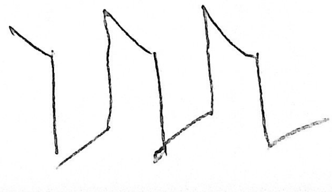




$\tau >> T$

$V_R(t) = V_i(t) - V_c(t)$

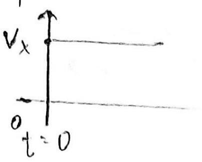




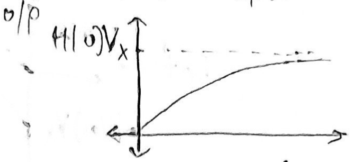
Lecture 22:- Step Response of first order RC ckts.

$$H(s) = \frac{H(0)}{(1+s/\omega_p)}$$

I/P



o/p

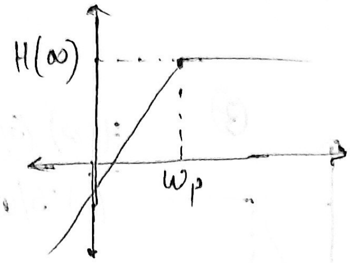


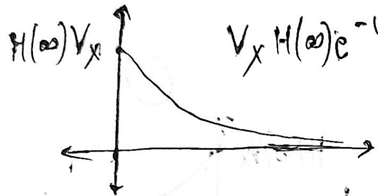
$\boxed{H(0) \le 1.}$

$H(0)\,V_x\left(1-e^{-\omega_p t}\right)u(t)$

$\omega_p = \frac{1}{\tau}$

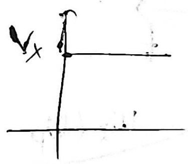
$$\frac{H(\infty)(s/\omega_p)}{(1+s/\omega_p)}$$





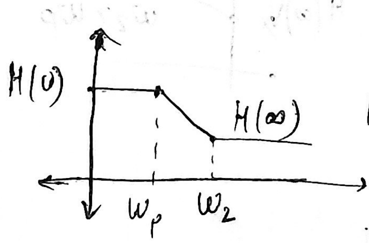
$V_x H(\infty) e^{-\omega_p t}\,u(t)$

$$H(0)\frac{(1+s/\omega_z)}{(1+s/\omega_p)}$$

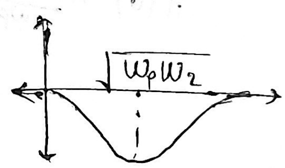


$\omega_p < \omega_z$

lagging phase

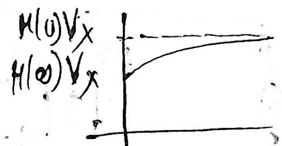


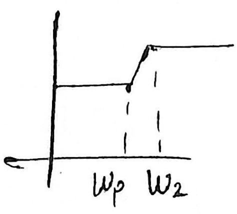
$H(\infty) < H(0)$



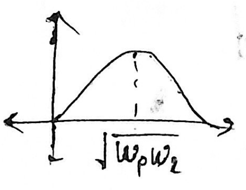
$\omega_p > \omega_z$.

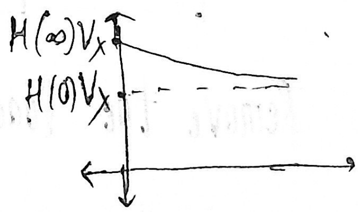
leading phase

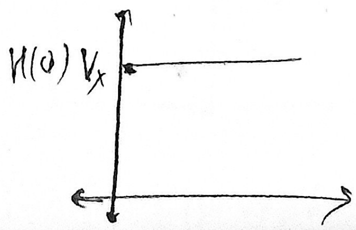




$H(\infty) > H(0)$

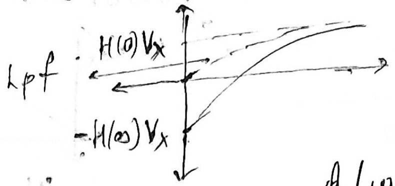




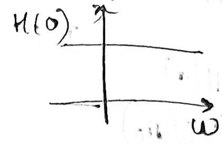


$$\frac{H(0)\,(1-s/\omega_z)}{(1+s/\omega_p)}$$

①-② gives:-

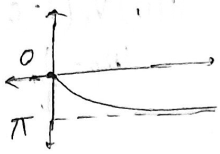


Magnitude Response



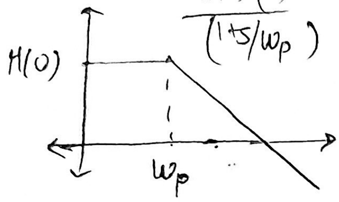
$$\phi(\omega) = -2\tan^{-1}(\omega/\omega_p)$$

Phase is always lagging.



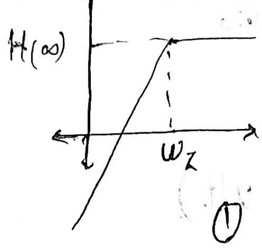
LPF ①

$$\frac{H(0)(1)}{(1+s/\omega_p)}$$



+

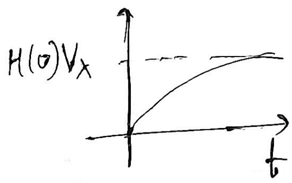
②



$$\frac{H(\infty)\,(s/\omega_p)}{(1+s/\omega_p)}$$

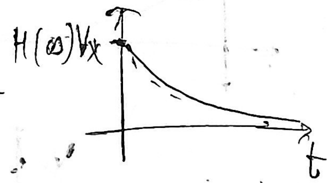
$$\frac{H(0)(1+s/\omega_z)}{(1+s/\omega_p)}$$

①



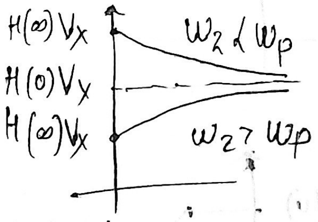
+

②

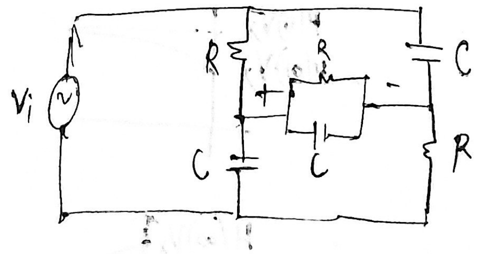


$H(\infty) = H(0)\,\frac{\omega_p}{\omega_z}$.

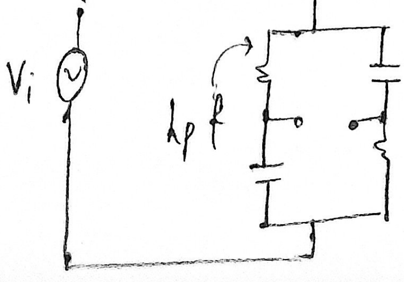
$H(0) = H(\infty)\frac{\omega_z}{\omega_p} \rightarrow \frac{s}{\omega_z}$



Q.



Remove the load.



$$V_{th} = \frac{(1-sRC)}{1+sRC}\,V_i$$

To find

$$\frac{V_i(1-sRC)}{1+sRC}$$

$\frac{1}{3}V_x$

$V_o(t) =$

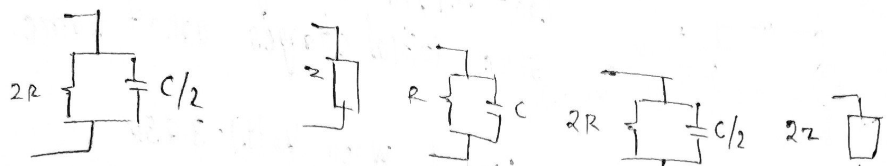
Q.

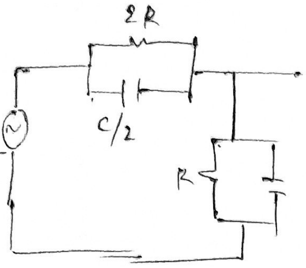
10

Magnitude Respons.

$^{-1}(\omega/\omega_p)$

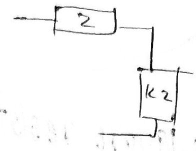
To find R_{th}, short the I/P





$V_i(1-sRC)$

$1+sRC$

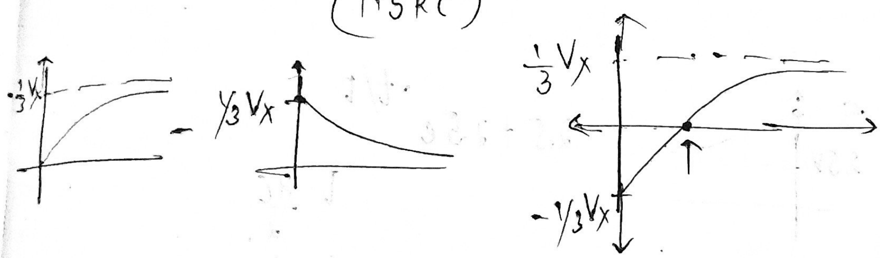


$\frac{kz}{(k+1)z} = \frac{k}{k+1}$

$k = 1/2$

$\frac{1/2}{1/2+1} = \frac{1}{3}$

$V_0 = \frac{1}{3}\left(\frac{1-sRC}{1+sRC}\right)V_i$

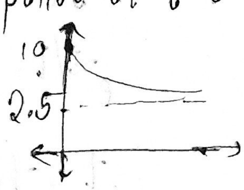


$\frac{1}{3}V_x\left(1-e^{-t/\tau}\right) \quad -\frac{1}{3}V_x e^{-t/\tau}$

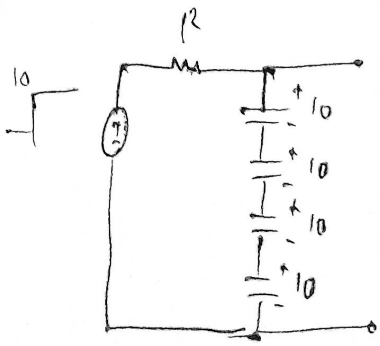
$$\boxed{V_0(t) = \frac{1}{3}V_x\left(1-2e^{-t/\tau}\right)}$$

To find response at $t=0$.

$2.5 + 7.5e^{-\omega_p t}$



Q.

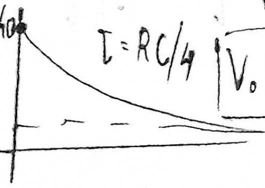


$V_0(t)$

Find t when $V_0 = 25V$.

$V_0(\infty) = 10V \quad V_0(0) = 40V$

$V_0(t) = 10 + (40-10)e^{-\omega_p t}$



$\tau = RC/4$

$$\boxed{V_0(t) = 10 + 30e^{-\omega_p t}}$$

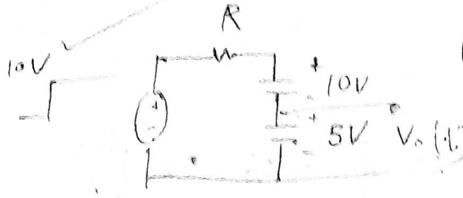
$40 + 4\Delta V = 10$

$4\Delta V = -30$

$\boxed{\Delta V = -7.5V}$

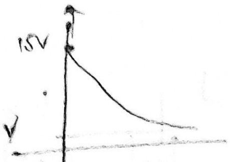
15

$\boxed{\therefore t = \tau \ln 2}$

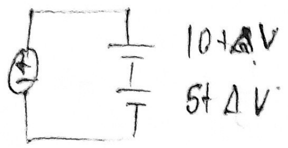


Can't reduce to a single capacitor since initial charges aren't same.

Find t when $V_o(t) = 3.25V$



In steady state, ignore resistor



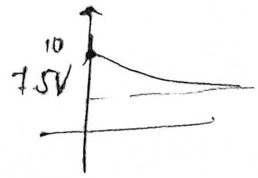
$15 + 2\Delta V = 10$

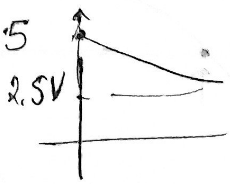
$2\Delta V = -5$

$\Delta V = -2.5$

$V(\infty) = 7.5\,V$

$V(\infty) = 2.5\,V$





$2.5 + 2.5e^{-t/\tau}$

$\tau = \frac{RC}{2}$

$\therefore 2.5 + 2.5e^{-t/\tau} = 3.25$

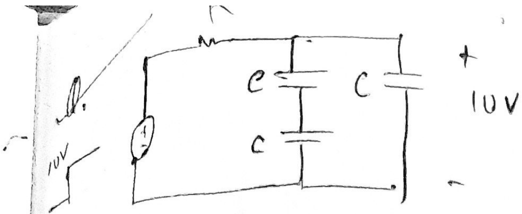
$2.5e^{-t/\tau} = 0.75$

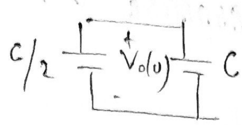
$e^{-t/\tau} = 0.3$

$\frac{t}{\tau} = -\ln(0.3)$

$\frac{-t}{\tau} = \ln(0.3)$

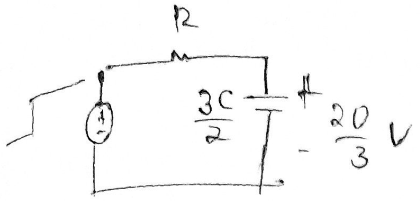
$t = 1.2\tau$

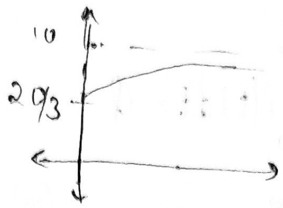




$V_0(0)\cdot\frac{3C}{2} = 10C$

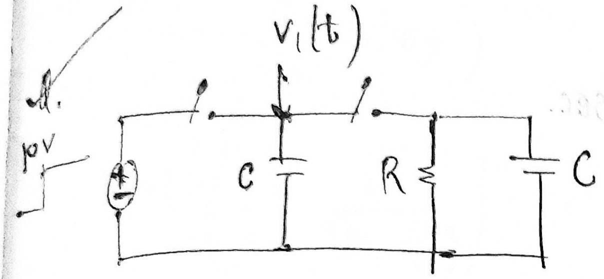
$\boxed{V_0(0) = 6.67V}$

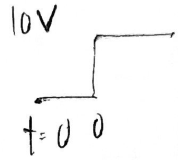


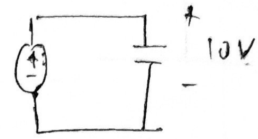


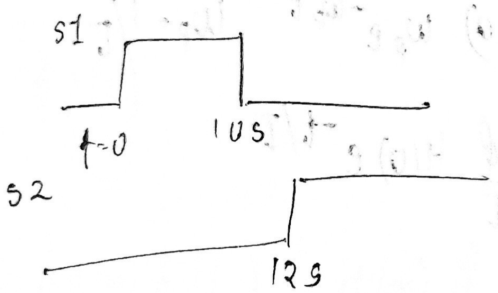
$\frac{20}{3} + \frac{10}{3}\left(1 - e^{-t/\tau}\right)$

$V(t) = \boxed{10 - \frac{10}{3} e^{-t/\tau}}$



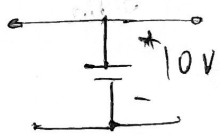




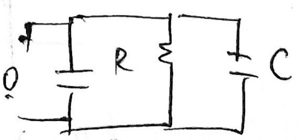


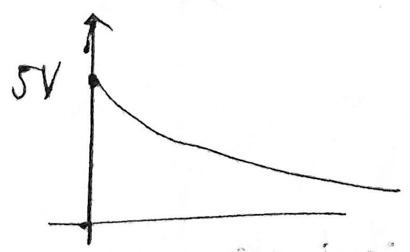
After 10s

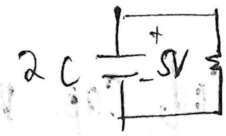
10-12s

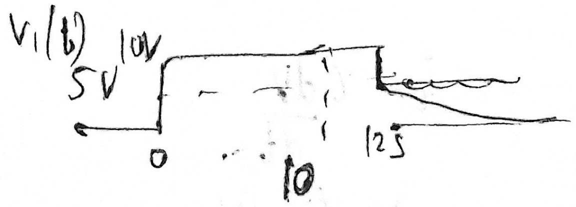


After 12s









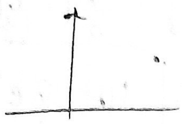
Lecture 23:- Impulse Response

$\delta(t)$

$t=0$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

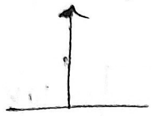
$i(t)$ $\quad Q\,\delta(t)$



$$\int_{-\infty}^{\infty} i(t)\,dt = Q$$

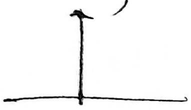
$$\int_{-\infty}^{\infty} v(t)\,dt = \phi$$

$v(t)$ $\quad \phi\,\delta(t)$



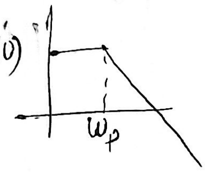
ϕ = Voltage sec.

$\phi\,\delta(t)$



$$\frac{H(0)}{1+s/\omega_p}$$

$H(0)$

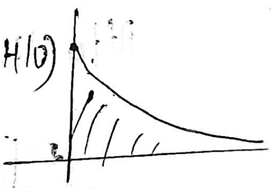


ω_p

$$\phi H(0)\,\omega_p e^{-\omega_p t} \qquad \omega_p = 1/\tau$$

$$\frac{\phi}{\tau} H(0)\, e^{-t/\tau}$$

$\frac{\phi}{\tau}H(0)$



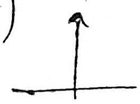
$$\int_{-\infty}^{\infty} x(t)\,dt = X(0)$$

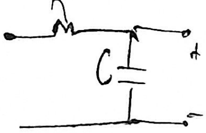
$$\int = \phi H(0)$$

When impulse is applied, Cap. will behave as short ckt.

LPF:-

$v(t)$ $\quad \phi\delta(t)$





$$i = \frac{\phi}{R}\delta(t)$$

$$i_c = C\frac{dV}{dt}$$

$$V = \int \frac{i_c}{C}\,dt$$

$$V_c = \frac{\phi}{RC}u(t) \quad \text{at } t=0$$

$\therefore$ Cap. is now charged to $V = \frac{\phi}{\tau}$

And then the I/P goes to 0.

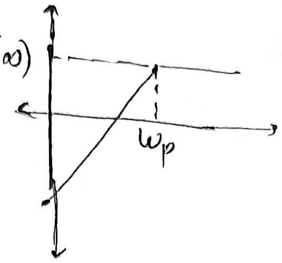
So, cap. will start discharging.

$\phi = V_x \tau$

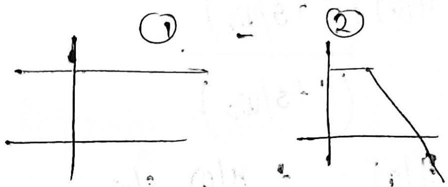
$V_x e^{-t/\tau} u(t)$

HPF:-

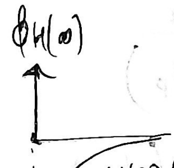
$$H(s) = \frac{H(\infty)(s/\omega_p)}{(1+s/\omega_p)}$$



$$H(\infty)\left(1 - \frac{1}{(1+s/\omega_p)}\right)$$



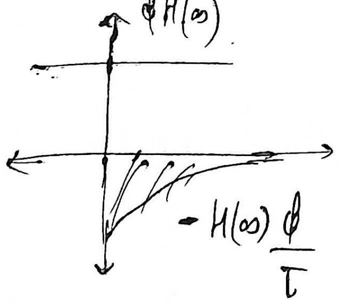
$\phi\delta(t)$



$\rightarrow \phi H(\infty)$ $\Big] = 0.$

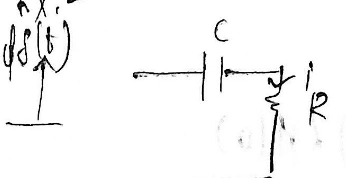
$-\frac{H(\infty)\phi}{\tau}$ $\int$ $-\phi H(\infty)$

$$\phi H(\infty)\delta(t) - H(\infty)\phi\,\omega_p e^{-\omega_p t} u(t)$$



$\phi H(\infty)$ $\Big] + -\phi H(\infty) = 0.$ D.C. gain is 0 for Hpf.

Ex:-



$H(\infty) = 1.$ $\phi\delta(t)$

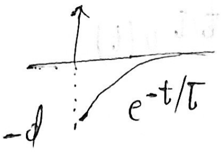
Cap. is short $t=0$.

Current flows thro resistor.

$i = \frac{\phi}{R}\delta(t)$

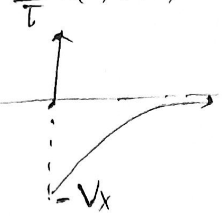
This i sets up voltage in the cap $\boxed{V_c = \frac{\phi}{\tau}}$

i/p is 0 o/p will go to $-\frac{\phi}{\tau}$.



$\phi = V_x \tau$

$\frac{V_x}{\tau} H(\infty) \delta(t)$



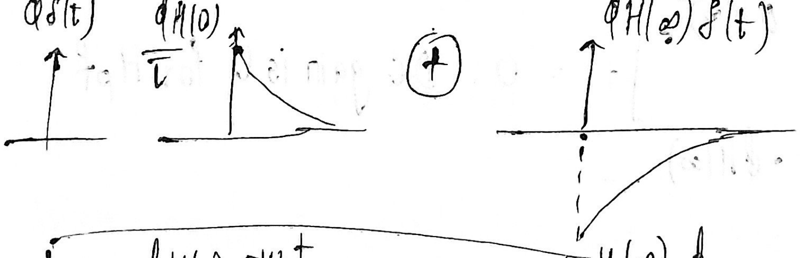
Ex:- General f^n

$$H(0)\frac{(1+s/\omega_z)}{(1+s/\omega_p)}$$

$$H(\infty) = H(0)\frac{\omega_p}{\omega_z}$$

$$\frac{H(0)}{1+s/\omega_p} + H(0)\frac{s/\omega_z}{(1+s/\omega_p)}$$

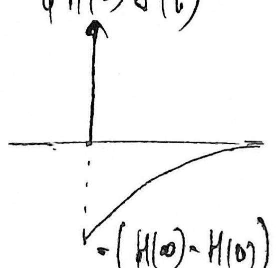
$$\frac{H(0)}{(1+s/\omega_p)} + \frac{H(\infty)(s/\omega_p)}{(1+s/\omega_p)}$$

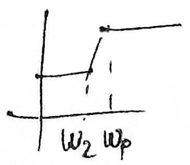


$$\boxed{\omega_p \phi H(0) e^{-\omega_p t} + H(\infty)\phi\left(\delta(t) - \omega_p e^{-\omega_p t}\right)}$$

$H(\infty) > H(0)$

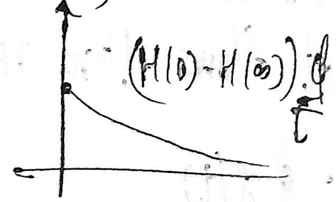
$\omega_z < \omega_p$



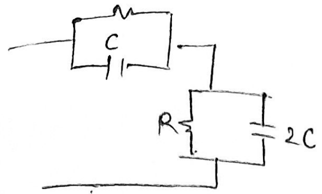


$H(\infty) < H(0)$

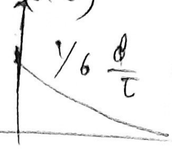
$\omega_p < \omega_z$



ℓ



$\frac{1}{3}V_x \tau \delta(t)$



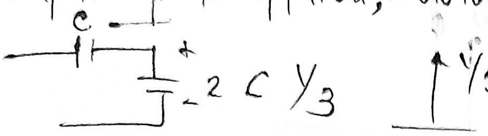
$\frac{1}{6}\frac{\phi}{\tau} = \frac{1}{6}V_x$

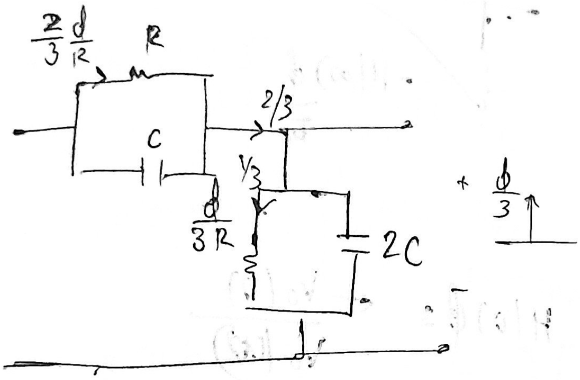
$H(\infty) = \frac{1}{3}$

$H(0) > H(\infty)$

$H(0) = \frac{1}{2}$.

When impulse $\frac{2}{3}\phi$ is applied, resistors can be ignored





Remaining current $= \frac{1}{3}\frac{\phi}{R}\delta(t)$
will split bet. 2 capacitors

C.i

$\therefore i_{2C} = \frac{2}{3} \times \frac{1}{3} \times \frac{\phi}{R}\delta(t)$

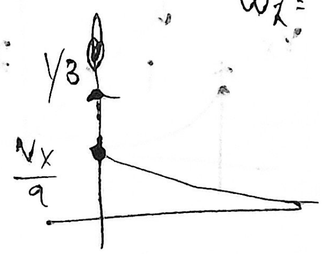
$V_{2C} = \int \frac{1}{2C} \times \left(\frac{2}{3} \times \frac{1}{3}\frac{\phi}{R}\delta(t)\right)$

$= \frac{1}{9}\frac{\phi}{\tau}u(t)$

$\boxed{V_{2C} = \frac{\phi}{9\tau} = \frac{V_x}{9}}$

$\omega_p = \frac{2}{3RC}$

$\omega_z = \frac{1}{RC}$.

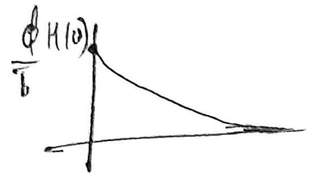


$\boxed{\omega_p < \omega_z}$

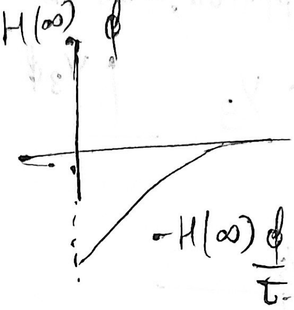
$$\frac{H(0)\left(1 - s/\omega_z\right)}{1+s/\omega_p}$$

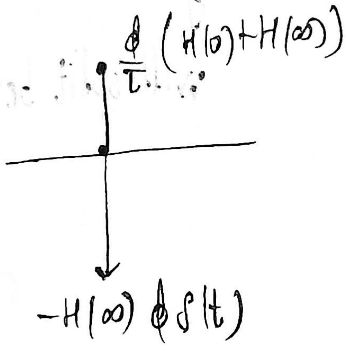
$$\frac{H(0)}{1+s/\omega_p} - \frac{H(\infty)\, s/\omega_p}{\left(1+s/\omega_p\right)}$$

$$\phi\, H(0)\,\omega_p e^{-\omega_p t} u(t) - H(\infty)\,\phi\left(\delta(t) - \omega_p e^{-\omega_p t} u(t)\right)$$



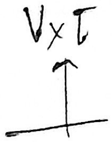
(—)

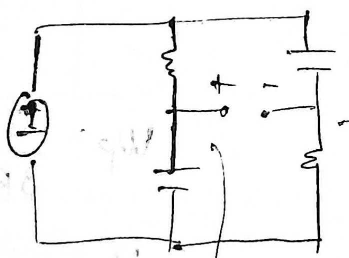


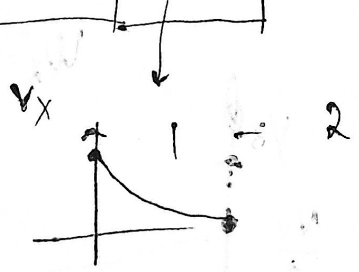


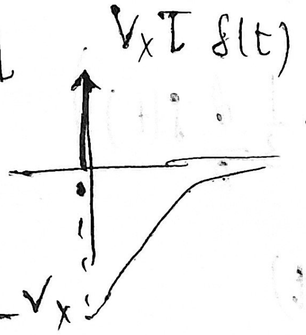
$$H(0)\,\phi = \leftarrow \frac{V_o(0)}{V_o(\infty)}$$

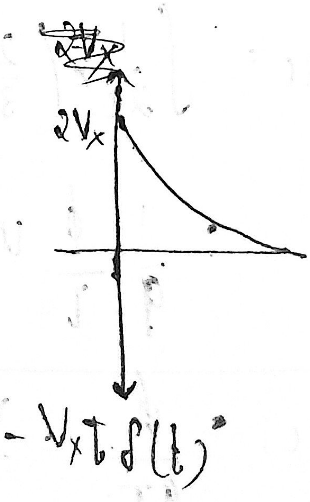
$\phi = V_x \tau$







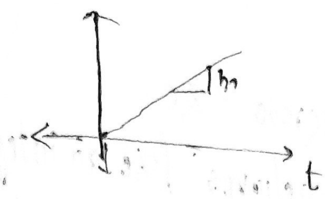


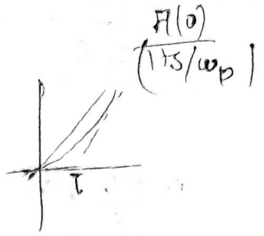


Lecture 24 :- Ramp response of 1st order ckts.

$v(t) = mt\, u(t)$

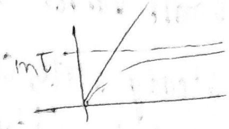
$L.T.\, [r(t)] = \frac{m}{s^2}$

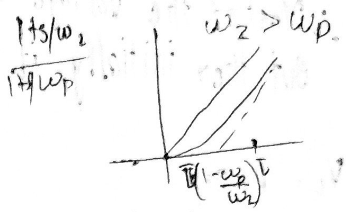




LPF:-

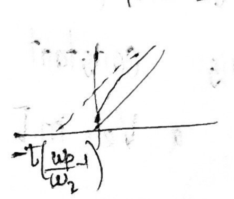
$H(s) = \frac{H(0)}{(1+s/\omega_p)}$





$V_i H(s) = \frac{m\, H(0)}{s^2} \frac{1}{(1+s/\omega_p)}$

$= H(0) m \frac{1}{s}\left[\frac{1}{s} \frac{1}{(1+s/\omega_p)}\right]$



$= m\, H(0) \frac{1}{s}\left[\frac{1}{s} - \frac{1/\omega_p}{1+s/\omega_p}\right]$

$= m\, H(0) \left[\frac{1}{s^2} - \left(\frac{1}{\omega_p}\right)\left[\frac{1}{s} - \frac{1/\omega_p}{1+s/\omega_p}\right]\right]$

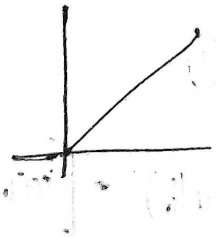
↓

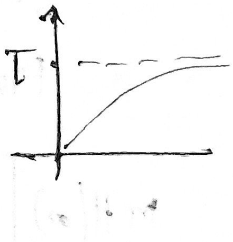
ILT

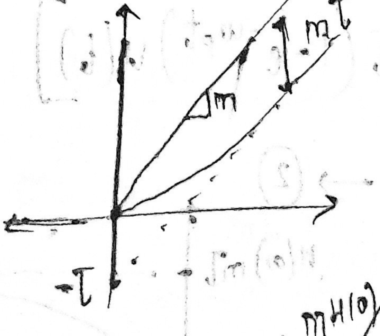
$m H(0) \left[t\, u(t) - \tau \left(1 - e^{-\omega_p t}\right) u(t)\right] \longrightarrow 1$

If $H(0) = 1$ & $t >> \tau$

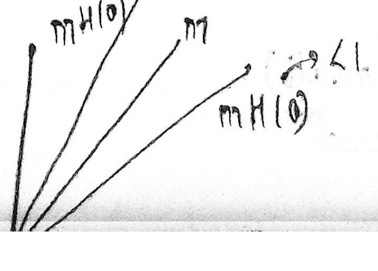
$m H(0) \left[(t-\tau)\, u(t)\right]$

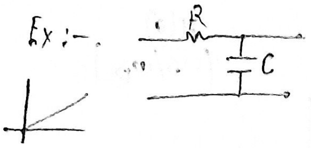






$H(0) \neq 1$



Ex:-  $H(0) = 1$ The moment ramp is applied Cap acts as a short ckt

$i = \frac{mt}{R} u(t)$ flows across R & C.

For small values of t, f^n behaves like an integrator

At steady state, Cap. Z imp becomes very large

$\therefore$ Most of the voltage would appear across the capacitor.

But then initially, it was taking time to build that voltage.

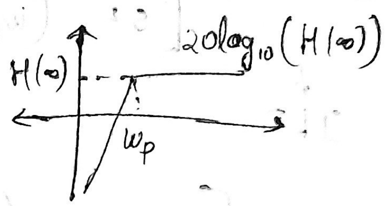
V_c = ramp

i_c = step (constant)

$\therefore$ This constant current i_c flows through R generating a $V_R = m\tau$

* High Pass Filter:-

$$H(s) = \frac{H(\infty)\,(s/\omega_p)}{(1+s/\omega_p)}$$



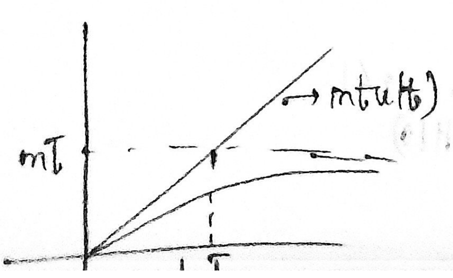
$$H(\infty)\left[1 - \frac{1}{(1+s/\omega_p)}\right]$$

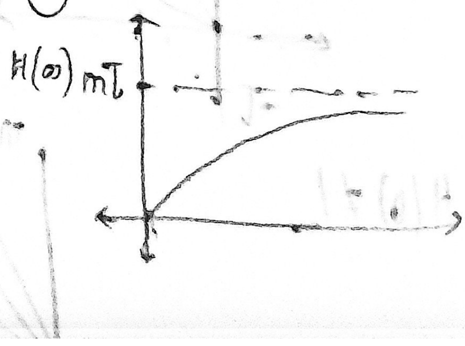
$\frac{m}{s^2}$

From (1)

$$m\,H(\infty)\left[mt\,u(t) - \left[mt\,u(t) - \tau\left(1-e^{-\omega_p t}\right)u(t)\right]\right]$$

$$H(\infty)\left[m\tau\left(1-e^{-\omega_p t}\right)u(t)\right] \rightarrow (2)$$





$H(\infty) = 1$ (Assume)

$$\boxed{\frac{s/\omega_p}{1+s/\omega_p} \approx 1.}$$ At $t=0$.

$$\boxed{\therefore V_o = V_i}$$

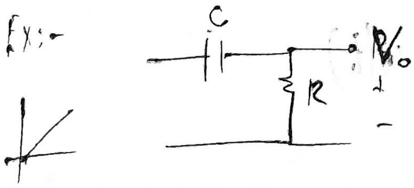
At $t >> T$ (steady state)

$$\frac{s/\omega_p}{1+s/\omega_p} \approx s/\omega_p = sT \text{ (Ideal differentiator)}$$

$$\frac{d}{dt}(mt\,u(t)) = T\,\frac{du}{dt} = mT$$

D.C. gain is 0 for a hpf.

Ex:-



$$H(s) = \frac{sRC}{1+sRC}$$

At $t=0$; when ramp is applied; C is short and voltage appears across R

$V_o = V_i$ at $t=0$

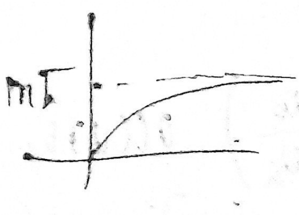
At time Tes; V_c = ramp and i_R = constant

$v = mt\,u(t)$

$i = mC\,u(t)$

$$\boxed{V_R = mRC\,u(t)}$$

$$\boxed{V_R = mT}$$



General Transfer fn:-

i) Finite zero & a pole

$$\frac{H(0)\,(1+s/\omega_z)}{(1+s/\omega_p)}$$

$$H(0)\frac{\omega_p}{\omega_z} \times \frac{s}{\omega_p} = H(\infty)$$

$$= \frac{H(0)}{1+s/\omega_p} + \frac{H(0)\, s/\omega_z}{1+s/\omega_p}$$

$$= \underbrace{\frac{H(0)}{1+s/\omega_p}}_{Lpf} + \underbrace{\frac{H(\infty)\, s/\omega_p}{(1+s/\omega_p)}}_{Hpf}$$

From (1) & (2):-

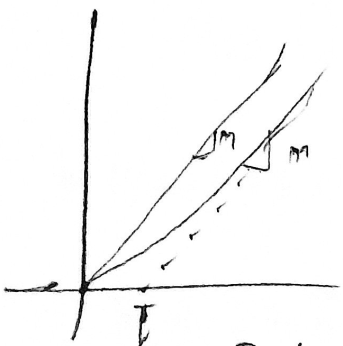
$$= H(0)\, m\left[t\,u(t) - \tau\left(1-e^{-\omega_p t}\right)u(t)\right] + H(\infty)\, m\left[\tau\left(1-e^{-\omega_p t}\right)u(t)\right]$$

$$= H(0)\, m\, t\, u(t) - m\tau\left[H(0) - H(\infty)\right]\left(1-e^{-\omega_p t}\right)u(t)$$

$$= H(0)\, m\left[t\,u(t) - \left(1-\frac{\omega_p}{\omega_z}\right)\tau\left(1-e^{-\omega_p t}\right)u(t)\right]$$

For a 1st order system (lpf) $\omega_z \to \infty$.

Here; ω_z is at a finite frequency

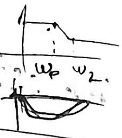


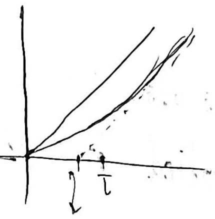
$m(t-\tau)u(t)$

Factor of $\left(1-\frac{\omega_p}{\omega_z}\right)$ occurs.

a) $\omega_p < \omega_z$. $\Rightarrow \left(1 - \frac{\omega_p}{\omega_z}\right) < 1$

Lagging phase $(t - \alpha T)$ $\alpha < 1$





$T\left(1 - \frac{\omega_p}{\omega_z}\right)$

Delay is being reduced.

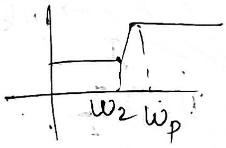
$\omega_p = \omega_z \Rightarrow$ Overlap.

$$\boxed{\text{Steady state error} = mT\left(1 - \frac{\omega_p}{\omega_z}\right)}$$

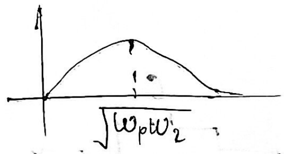
b) $\omega_p > \omega_z$.

$\left(1 - \frac{\omega_p}{\omega_z}\right)$ Assume $H(0) = 1$. $\therefore H(\infty) > 1$

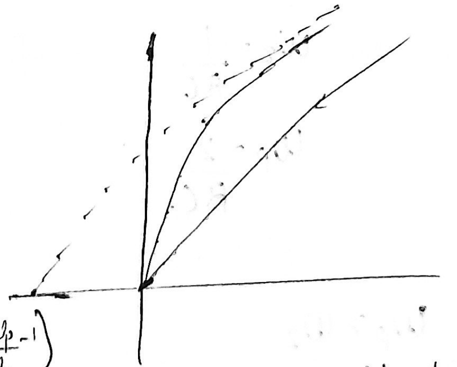
$$m\left(t\,u(t) - \left(1 - \frac{\omega_p}{\omega_z}\right)T\left(1 - e^{-\omega_p t}\right)u(t)\right)$$



$$m\left(t\,u(t) + \left(\frac{\omega_p}{\omega_z} - 1\right)T\left(1 - e^{-\omega_p t}\right)u(t)\right)$$



$m\left(t + T\left(\frac{\omega_p}{\omega_z} - 1\right)\right)$



At $t = 0 \Rightarrow$ The O/P rises faster.

Steady state error $= mT\left(\frac{\omega_p}{\omega_z} - 1\right)$

$z = -\frac{S}{RC}$

$S = -2RC$

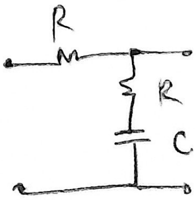
$H(0) = 1/2$

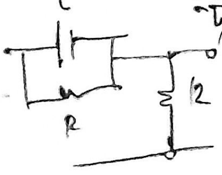
$\cancel{H(\infty)} =$

$\frac{1}{1 + \frac{1}{RCS+1}}$

$\frac{RCS+1}{RCS+2}$

Q.





$\frac{1}{CS} \| R$

$\frac{R \cdot \frac{1}{SC}}{R + \frac{1}{SC}}$

$\frac{R}{RCS+1}$

$\frac{R}{R + \frac{R}{RCS+1}}$

$R(RCS+1)$

$H(0) = 1$. $H(\infty) = \frac{1}{2}$

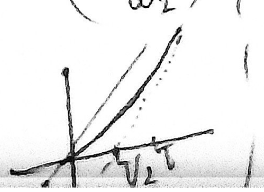
$\boxed{\omega_z > \omega_p}$.

$\omega_p < \omega_z$ $\frac{1 + SRC}{1 + 2SRC}$

$H(\infty) = H(0)\frac{\omega_p}{\omega_z}$

$\boxed{\frac{\omega_p}{\omega_z} = \frac{1}{2}}$

$\left(1 - \frac{\omega_p}{\omega_z}\right)$



$\omega_z = -\frac{1}{RC}$ $\omega_p > \omega_z$

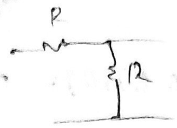
$\omega_p = -\frac{2}{RC}$

D.C. gain is 1

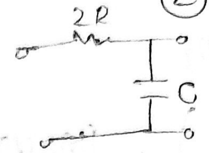
$\therefore$ Slope of the o/p is also 1.

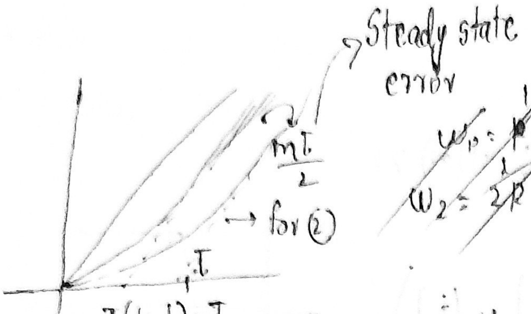
When the I/P is applied at $t=0$ ckt will become

o/p of slope $= \frac{m}{2}$



②





$T\left(1-\frac{1}{2}\right) = \frac{T}{2}$

$\omega_p = \frac{1}{RC}$ ~~$\omega_p = \frac{1}{RC}$~~

~~$\omega_2 = \frac{1}{2RC}$~~

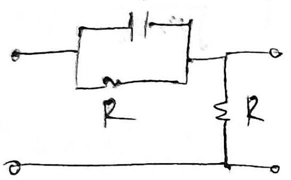
$\omega_p = \frac{1}{2RC}$

$\omega_2 = \frac{1}{RC}$

$H(\infty) = 0$

$T = 2RC$

Q.

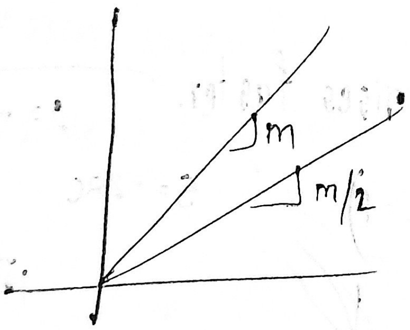


$$\frac{1}{2}\frac{(1+sRC)}{\left(1+\frac{sRC}{2}\right)}$$

$\omega_2 = \frac{1}{RC}$

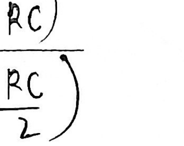
$\omega_p = \frac{2}{RC}$.

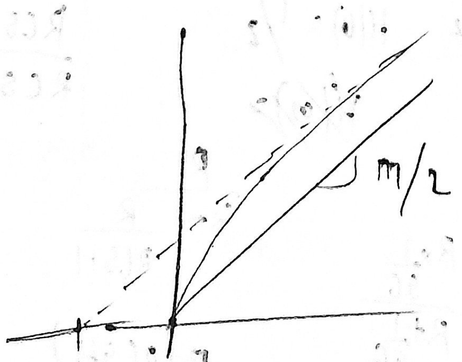
D.C. gain $= H(0) = \frac{1}{2}$.



$\omega_p > \omega_2$

$T = \frac{RC}{2}$

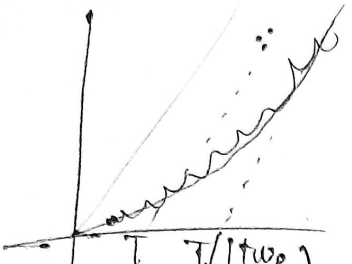




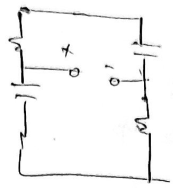
$T = -\frac{RC}{2}$

$H(0)\dfrac{(1-s/\omega_z)}{(1+s/\omega_p)}$

$$mH(0)\left[t\,u(t) - \left(1+\frac{\omega_p}{\omega_z}\right)\tau\left(1-e^{-\omega_p t}\right)u(t)\right]$$



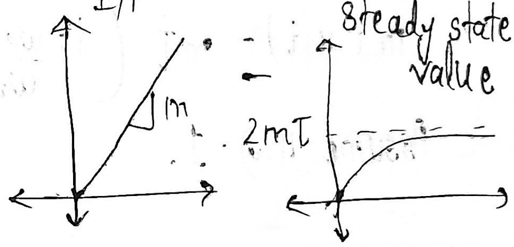
Extra Delay occurs.



Here, slope of O/P doesn't rise parabolically, instead it rises linearly or same as the I/P

$$\frac{1}{1+s(2\tau)}$$

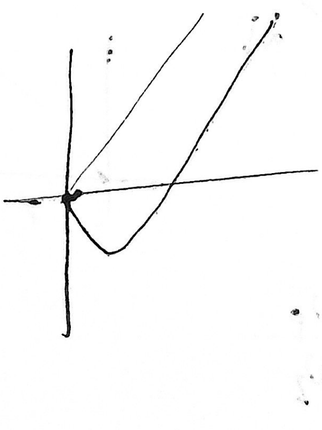
For a 1st order system; $H(\infty) = 0$.



$$t\,u(t) - 2\tau\left(1-e^{-\omega_p t}\right)u(t)$$

$$t\,u(t) - 2\tau\left(t/\tau\right)u(t)$$

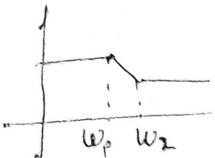
$$-t\,u(t)$$

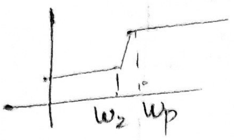


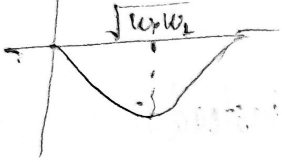
Lecture 24a:- Ramp response of 1st order systems with right half plane zero.

$$H(s) = H(0)\,\frac{(1 - s/\omega_z)}{(1 + s/\omega_p)} \qquad \boxed{H(\infty) = H(0)\,\frac{\omega_p}{\omega_z}}$$

$\omega_p < \omega_z$







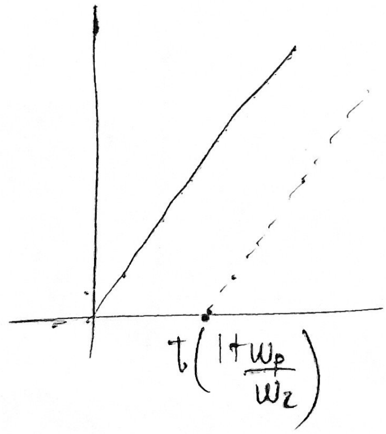
Phase will be lagging in both cases as it is a RHP zero.

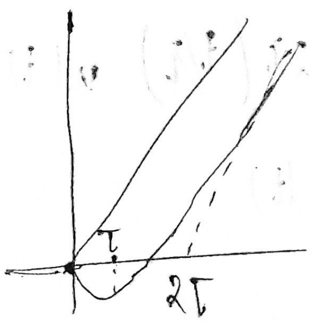
$$m\,t\,u(t) - m\tau\left(1 + \frac{\omega_p}{\omega_z}\right)\left(1 - e^{-\omega_p t}\right)u(t)$$

Assume $H(0) = 1$.

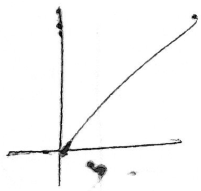
$$m\left[t - \tau\left(1 + \frac{\omega_p}{\omega_z}\right)\right]$$

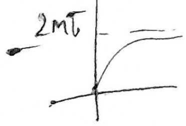
$$m(t - 2\tau)$$





$$m\left(t - 2\tau\left(1 - e^{-\omega_p t}\right)u(t)\right)$$





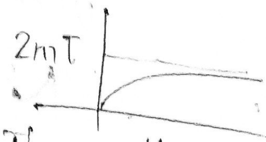
$$\frac{1 - s/\omega_z}{1 + s/\omega_p} \approx \frac{-(s/\omega_z)}{s/\omega_p} = -\frac{\omega_p}{\omega_z}$$

I/P

$$m\,t\,u(t) \rightarrow -m\,t\,\frac{\omega_p}{\;}\,u(t)$$

At $t = 2\tau$

I/P is at $2m\tau$

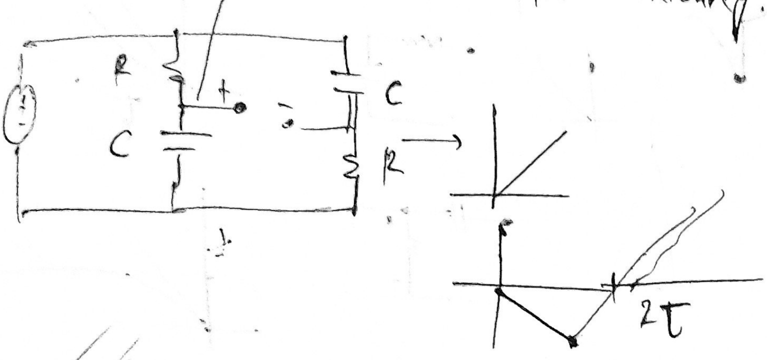


This will reach $2m\tau$ at $t = \infty$.

(x) at $t = 2\tau$ is a (+)ve value fn.

Ex:-

This O/P rises parabolically.



$$m(t\,u(t) - \tau(1 - e^{-\omega_p t})) - m(\tau(1 - e^{-\omega_p t}))$$

$$m(t - 2\tau(1 - e^{-\omega_p t}))$$

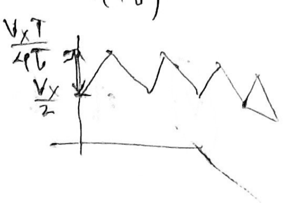
Pulse Response:-

8Q.

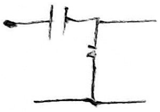
i) $T >> \tau$

ii) $\tau >> T$ $V_c(t)$

$V_c(t)$

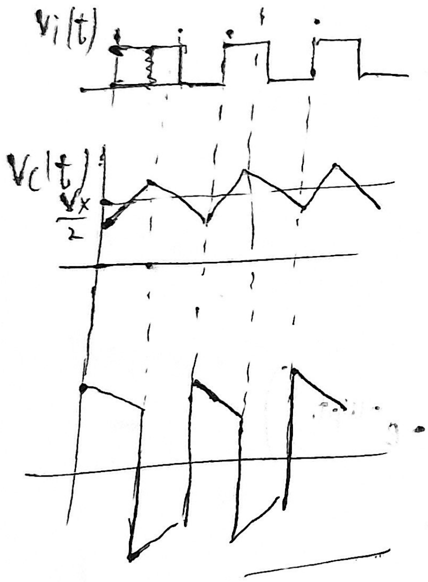


9Q.

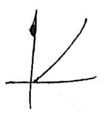


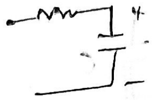
i) $T >> \tau$

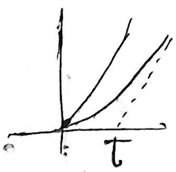
ii) $V_R(t) = V_i(t) - V_c(t)$

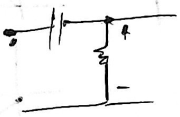


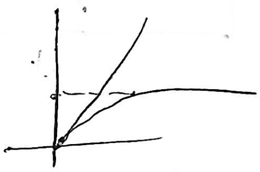
Ramp Response:-







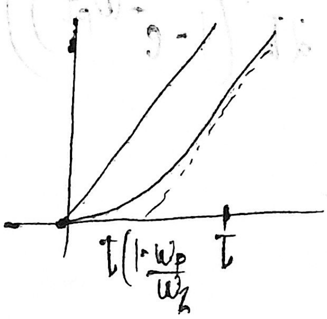




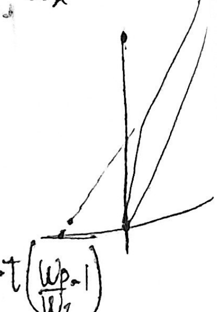
$$\frac{(1+s/\omega_z)}{(1+s/\omega_p)}$$

$$mH(0)\left[tu(t) - \tau\left(1-\frac{\omega_p}{\omega_z}\right)\left(1-e^{-\omega_p t}\right)u(t)\right]$$

$\frac{\omega_p}{\omega_z} < 1$



$\frac{\omega_p}{\omega_z} > 1$



$$\left(\frac{1-s/\omega_z}{1+s/\omega_p}\right)$$