MILNE'S PREDICTOR - CORRECTOR METHOD :-

To solve $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$, Milne's predicted formula is given as

$$\boxed{y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2f_{n-2} - f_{n-1} + 2f_n\right]}$$

Milne's corrected formula is ~~y_{n+1}/c~~

$$\boxed{y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[f_{n-1} + 4f_n + f_{n+1,p}\right]}$$

$f_1 = f(x_1, y_1)$, $f_2 = f(x_2, y_2)$, $f_3 = f(x_3, y_3)$,
$f_4 = f(x_4, y_4)$

Using Milne's method compute, $y(0.8)$, $y(1.0)$ given $y' = \frac{1}{x+y}$; $y(0) = 2$; $y(0.2) = 2.0933$

$y(0.4) = 2.1755$, $y(0.6) = 2.2493$.

$y_0 = 2$; $x_0 = 0$

$y_1 = 2.0933$; $x_1 = 0.2$

$y_2 = 2.1755$; $x_2 = 0.4$

$y_3 = 2.2493$; $x_3 = 0.6$.

$x_4 = 0.8$; $h = 0.2$. ; $b = \frac{1}{x+y}$

$x_5 = 1.0$

$n = 3$

$$\boxed{y_4 = y_0 + \frac{4h}{3}(2b_1 - b_2 + 2b_3)}$$

$b_1 = \frac{1}{x_1+y_1} = \frac{1}{(0.2)+(2.0933)} = 0.4361$.

$b_2 = 0.3883$ $\qquad b_0 = 0.5$

$b_3 = 0.3510$.

$$y_4 = 2 + \frac{4\times0.2}{3}(2(0.4361) - 0.3883 + 2(0.3510))$$

$= 2 + 0.2667(1.1859)$

$= 2 + 0.3163$

$y_{4p} \Rightarrow y(0.8) = \underline{2.3163}$.

$$\boxed{y_{4c} = y_2 + \frac{h}{3}(b_2 + 4b_3 + b_{4p})}$$

$b_{4p} = 0.3209$.

$$y_{4c} = 2.1755 + \frac{0.2}{3}\left[0.3883 + 1.4040 + 0.3209\right]$$

$$= 2.1755 + \frac{0.2}{3}(2.1132).$$

$$y_{4c} = \underline{\underline{2.3164}} = y(0.8)$$

$n = 4$

$$y(1.0) = y_{5p} = y_1 + \frac{4h}{3}(2b_2 - b_3 + 2b_4)$$

$$= 2.0933 + \frac{4(0.2)}{3}\left(2(0.3883) - 0.351 + 2(0.3209)\right)$$

$$= 2.0933 + 0.2667(1.0674)$$

$$y_{5p} = \underline{\underline{2.3780}}.$$

$$b_5 = \underline{\underline{0.2960}}.$$

$$y_{5c} = y_3 + \frac{h}{3}(b_3 + 4b_4 + b_{5p})$$

$$= 2.2493 + \frac{0.2}{3}\left(0.3510 + 4(0.3209) + 0.2960\right)$$

$$= 2.2493 + \frac{0.2}{3}(1.9306)$$

$$= \underline{\underline{2.3780}} = y(1.0).$$

2] Solve the initial value problem $\frac{dy}{dx} = x^2(1+y)$, $y(1) = 1$ using Taylor Series method taking the terms upto 4^{th} degree for $x = 1.1, 1.2, 1.3$ further compute $y(1.4)$ using Milne's Predictor Corrector method:-

$$y' = x^2(1+y) \xrightarrow{} x^2 + x^2 y, \quad x_0 = 1, \quad y_0 = 1$$

$$y'(1) = 2.$$

$$y'' = 2x + x^2 y' + 2xy.$$

$$y''(1) = 2 + 2 + 2$$

$$= \underline{\underline{6}}.$$

$$y''' = 2 + x^2 y'' + 4xy' + \cancel{2xy'} + 2y.$$

$$y'''(1) = 2 + 6 + 4 + 4 + 2$$

$$= \underline{\underline{18}}.$$

$$y^{iv} = x^2 y''' + 2xy'' + 4xy'' + 4y' + 2y'$$

$$y^{iv}(1) = 18 + 12 + 24 + 8 + 4.$$

$$= \underline{\underline{66}}.$$

$$y(x) = 1 + (x-1)2 + \frac{(x-1)^2}{2}(6) + \frac{(x-1)^3}{6}(18) + \frac{(x-1)^4}{24} \times 66.$$

$$y(x) = 1 + (x-1)2 + 3(x-1)^2 + 3(x-1)^3 + \frac{11}{4}(x-1)^4.$$

$$y(1) = 1 + 0 + 3(0)$$

$$y(1) = \underline{\underline{1}}.$$

$$y(1.2) = 1 + (0.2)2 + 3(0.2)^2 + 3(0.2)^3 + \frac{11}{4}(0.2)^4$$

$$= 1 + 0.4 + 0.12 + 0.0240 + 0.0044.$$

$$= \underline{\underline{1.5484}}$$

$$y(1.1) = 1 + (0.1)2 + (0.1)^2(3) + (0.1)^3(3) + \frac{11}{4}(0.1)^4$$

$$= 1 + 0.2 + 0.03 + 0.003 + 0.0003$$

$$= \underline{1.2333}$$

$$y(1.3) = 1 + (0.3)2 + (0.3)^2(3) + (0.3)^3(3) + (0.3)^4 \times \frac{11}{4}$$

$$= 1 + 0.6 + 0.27 + 0.081 + 0.0223$$

$$= \underline{\underline{1.9733}}$$

		$f_0 = 2$
$x_0 = 1$	$y_0 = 1$	$f_1 = 2.7023$
$x_1 = 1.1$	$y_1 = 1.2332$	
$x_2 = 1.2$	$y_2 = 1.5484$	$f_2 = 3.6697$
$x_3 = 1.3$	$y_3 = 1.9733$	$f_3 = 5.0249$
$x_4 = 1.4$		

$$y_{4_p}(1.4) = y_0 + \frac{4h}{3}(2f_1 - f_2 + 2f_3)$$

$$= 1 + \frac{4 \times 0.1}{3}(2(2.7023) - 3.6697 + 2(5.024$$

$$= 1 + \frac{0.4}{3}(11.7831)$$

$$y_{4(p)}(1.4) = \underline{\underline{2.5711}}$$

$$y_{4_c}(1.4) = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$$

$$f_4 = x^2 + x^2y = 1.96 + 5.0394 \Rightarrow 6.9994.$$

$$y_{4_c}(1.4) = 1.5484 + \frac{0.1}{3}[3.6697 + 4(5.0249) + 6.999$$

$$= \underline{\underline{2.5740}}$$