



**Department of Mathematics**

<b>Course:</b> Vector Calculus, Laplace Transform and Numerical Methods	<b>Improvement Quiz &amp; Test</b>	<b>Maximum Marks:</b> 10+50
<b>Course Code:</b> MA221TA	<b>Second Semester 2023-2024</b> <b>Branch:</b> EC, EE, EI, ET	<b>Time:</b> 02:00 PM to 04:00 PM <b>Date:</b> 01/07/2024

*Instructions to students: Answer all questions.*

*Quiz should be answered in the first two pages of answer booklet.*

Q.No.	Quiz	M	BTL	CO
1	The real root of the equation $x = e^{-x}$ lies in the interval _____.	1	1	1
2	The Runge-Kutta method for solution of $y' = \frac{y^2 - x^2}{y^2 + x^2}$ , $y(0) = 1$ at $x = 0.2$ , $h = 0.2$ yields $k_1 = 0.2$ , $k_2 = 0.19672$ , then values of $k_3 =$ _____ and $k_4 =$ _____.	2	2	1
3	The unit normal to the surface $\phi(x, y, z) = xy^2 + 3x^2 - z^3$ at the point $(2, -1, 4)$ is _____.	2	1	2
4	If the vector $\vec{F} = (2x^2y^2 + z^2)\hat{i} + (3xy^3 - x^2z)\hat{j} + (\lambda xy^2z + xy)\hat{k}$ is solenoidal, then the value of the constant ' $\lambda$ ' is _____.	2	2	3
5	The line integral $\frac{1}{2} \int_C (Pdx + Qdy)$ represents the area of region bounded by $C$ if $P =$ _____ and $Q =$ _____.	1	1	2
6	The work done by the force $\vec{F} = 5xy\hat{i} + 2y\hat{j}$ , in displacing a particle from $x = 1$ to $x = 2$ along $y = x^3$ is _____.	2	3	4

Q.No.	Test	M	BTL	CO
1a	Find the directional derivative of $\phi = x^2y^2z^2$ at the point $(1, 1, -1)$ in the direction of the tangent to the curve $x = e^t$ , $y = 1 + 2 \sin t$ , $z = t - \cos t$ , $-1 \leq t \leq 1$ .	6	2	3
1b	If $\vec{f} = (x^2y^3 - z^4)\hat{i} + 4x^5y^2z\hat{j} - y^4z^6\hat{k}$ , find $\text{div}(\text{curl } \vec{f})$ .	4	1	1
2a	If $r = \sqrt{x^2 + y^2 + z^2}$ , show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ , hence deduce that $\frac{1}{r}$ is harmonic function.	6	2	3
2b	Show that the vector $\vec{f} = \frac{\cos\theta}{r^3} \left[ \left( \frac{1}{\sin\theta} \right) \hat{e}_r + \frac{1}{\cos\theta} \hat{e}_\theta + r^4 \hat{e}_\phi \right]$ in spherical polar coordinate system is solenoidal.	4	3	1

3	Verify Green's theorem for $\int_C (xy + y^2)dx + x^2dy$ , where $C$ is the closed curve bounded by the line $y = x$ and the parabola $y = x^2$ .	10	2	4
4a	Find a positive real root of the equation $x \log_{10}(x) = 1.2$ in $[2.6, 3]$ by the method of false position up to four places of decimals. Perform four iterations.	6	1	2
4b	By using Newton-Raphson method find a real root of the equation $3x = \cos x + 1$ correct to four places of decimals by taking initial approximation to root as 0.5	4	2	2
5a	Apply Milne's to find the solution of initial value problem $y' + y^2 = x$ at $x = 0.8, 1.0$ given $y(0) = 0, y(0.2) = 0.020, y(0.4) = 0.0795$ and $y(0.6) = 0.1762$ .	10	2	2

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Marks Distribution Test	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Max Marks	08	20	12	10	10	36	4	--	--	--
Quiz		03	03	02	02	04	04	02	--	--	--

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks