UNIT-II

DIFFERENTIAL CALCULUS

TUTORIAL SHEET - 1

1. If (-1, -√3) are Cartesian coordinates of a point in plane, the corresponding polar coordinates are ___________
   Ans: (2, 4π/3)

2. If (√2, 5π/4) are the polar coordinates of a point in plane, the corresponding Cartesian Coordinates are __________
   Ans: (-1, -1)

3. The circle $x^2 + y^2 - 2ax = 0$ in polar form is ____________
   Ans: ($r = 2a\cos(\theta)$)

4. The polar equation $\theta - k = 0$, geometrically represents _________
   Ans: (straight lines)

5. If two polar curves C_1 and C_2 are orthogonal, then value of $\cot(\varphi_1)\cot(\varphi_2) =$ _____ Ans: -1

6. Find the angle of intersection between the polar curves $r = \frac{k\theta}{1+\theta}$ and $r = \frac{k}{1+\theta^2}$ Ans: $\tan^{-1}(3)$

7. Show that the angle made by the tangent and the normal at any point $P(r,\theta)$ on the curve Lemniscate $r^2 = a^2\cos(2\theta)$ with the initial line is '3θ'.

8. Show that the tangents to the cardioid $r = a(1 + cos\theta)$ at θ = π/3 and θ = 2π/3 are respectively parallel and perpendicular to the initial line.

9. Show that the circle $r = b$ intersects the curve $r^2 = a^2 \cos(2\theta) + b^2$, at an angle given by $tan^{-1}\left(\frac{a^2}{b^2}\right)$

10. Find the angle of intersection between the curves $r = a(1 + sin\theta)$ and $r = a(1 - sin\theta)$: Ans: π/2

TUTORIAL SHEET - 2

1. The curvature of a circle $s = a\psi$ at any point is ________
   Ans:(κ = 1/a)

2. The radius of curvature for straight line $y = mx + c$ is ______
   Ans: ($\rho = \infty$, not defined)

3. The curvature of the curve $y = e^x$ at the point where it crosses the y-axis is ______ Ans: ($\kappa = \frac{1}{2^{3/2}}$)

4. The Taylor series expansion of $\log(x)$ about $x = 1$ up to second degree term is ______
   Ans: $\log(x) = (x-1) - \frac{(x-1)^2}{2} + - \cdots \infty$

5. The Maclaurin series expansion of $\cos(x)$ is __________
   Ans: $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \infty$

6. Show that the radius of curvature of the Folium $x^3 + y^3 = 3axy$ at the point (3a/2, 3a/2) is given by $-\frac{3a}{8\sqrt{2}}$.

7. Find the radius of curvature of the curve $y^2 = \frac{4a^2(2a-x)}{x}$ where the curve meets the x-axis.

8. For the curve $y = \frac{ax}{a+x}$, show that $\left(\frac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$

9. Find the radius of curvature of the $x = a\log(\sec t + \tan t)$, $y = a\sec t$. Ans: $\rho = a\, sec^2 t$

10. Show that the curvature of the tractrix $x = a[\cos t + log tan(\frac{t}{2})]$, $y = a\, sin\, t$ at any point is given by $\kappa = \frac{tan\, t}{a}$

11. Find the coordinates of the centre of curvature at (at^2, 2at) on the parabola $y^2 = 4ax$.
Ans: $((\bar{x}, \bar{y}) = ((2+3t)at^2,\ -4\sqrt{2}at^{3/2})$

12. Find the circle of curvature at the point (a/4, a/4) for the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$.
Ans: $\left(x - \frac{3a}{4}\right) + \left(y + \frac{3a}{4}\right) = \frac{a^2}{2}$

13. Find the radius of curvature of the curve $r^n = a^n \cos(n\theta)$
Ans: $\frac{a^n r^{1-n}}{n+1}$

14. Show that the radius of curvature at any point (r, θ) on the Cardiod $r = a(1 - \cos\theta)$ varies as $\sqrt{r}$

15. Find the radius of curvature for the parabola $\frac{2a}{r} = 1 - \cos\theta$ at any point (r, θ)
Ans: $2\sqrt{\frac{r^3}{a}}$

TUTORIAL SHEET -3

1. Match the following:

i)	The angle between radius vector and tangent for the polar curve at any point $P(r,\theta)$ is	a)	$\rho \propto y^2$
ii)	The angle between radius vector and tangent for the Cartesian curve at any point $P(x,y)$ is	b)	$\rho \propto \frac{1}{y^2}$
iii)	The radius of curvature at any point $P(x,y)$ on the catenary $y = c.cosh\left(\frac{x}{c}\right)$ is	c)	$\cot(\phi) = \frac{1}{r}.\frac{dr}{d\theta}$
		d)	$\tan(\phi) = r.\frac{dr}{d\theta}$
		e)	$\tan(\phi) = \frac{xy' - y}{x + yy'}$
		h)	$\tan(\phi) = \frac{xy' + y}{x - yy'}$

Ans: (i) - (c) (ii) – (e) (iii) – (a)

2. Find the Taylor series expansion of the function $y = \log(cosx)$ about the point $x = \pi/3$.
   Ans: $\log(cosx) = -log2 - \sqrt{3}\left(x - \frac{\pi}{3}\right) - 2\left(x - \frac{\pi}{3}\right)^2 - \frac{4}{\sqrt{3}}\left(x - \frac{\pi}{3}\right)^2 - \frac{10}{\sqrt{3}}\left(x - \frac{\pi}{3}\right)^3 - \cdots$

3. Obtain the expansion of the function $e^{\sin(x)}$ in ascending powers of ‘x’ up to terms containing ‘x^4’
   Ans: $e^{\sin(x)} = 1 + x + \frac{x^2}{2} - \frac{x^4}{8} \ldots$

4. Obtain the Maclaurin series expansion for the function $f(x) = tan^{-1}(x)$ and hence deduce that $\pi = 4\left[1 - \frac{1}{3} + \frac{1}{5} - \; + \cdots\right]$

   Ans: $tan^{-1}(x) = \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots\right]$

5. Using Maclaurin's series, prove that
   $\sqrt{1+\sin(2x)} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \cdots$

6. Show that $\left(\frac{x}{sinx}\right) = 1 + \frac{x^2}{6} + \frac{7x^4}{360} + \cdots$