RV COLLEGE OF ENGINEERING

(An autonomous institution affiliated to VTU, Belgaum)

DEPARTMENT OF MATHEMATICS

FUNDAMENTS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS (MAT211AT)

Multiple Integrals

TUTORIAL SHEET-1

1. $\int_1^4 \int_0^{\sqrt{4-x}} xy\, dy\, dx =$ ______________
2. $\int_0^2 \int_0^x (x+y) dy\, dx =$ ______________
3. $\int_0^1 \int_0^1 \frac{\mathrm{dx\ dy}}{\sqrt{1-x^2}\sqrt{1-y^2}} =$ ______________
4. Find the area bounded between the parabola $y^2 = 4ax$ and $x^2 = 4ay$.
5. Show that the area of one loop of the lemniscates $r^2 = a^2 \cos 2\theta$ is $a^2/2$.
6. Find the area of one petal of the rose $r = a \sin 3\theta$.
7. Find the area of the circle $r = a \sin\theta$ outside the cardioid $r = a(1 - cos\theta)$.
8. Find the volume of the paraboloid of revolution $x^2 + y^2 = 4z$ cut off by the plane $z = 4$.
9. Find the volume of the region bounded by the paraboloid $az = x^2 + y^2$ and the cylinder $x^2 + y^2 = r^2$.
10. Find the volume of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ lying inside the cylinder $x^2 + y^2 = ax$.
11. Find the volume cut off the sphere $x^2 + y^2 + z^2 = a^2$by the cone $x^2 + y^2 = z^2$.
12. Change the order of the integration in the integrals:

a) $\int_0^a \int_0^x \frac{\cos y}{\sqrt{(a-x)\ (a-y)}}$ dy dx

b) $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}$ dy dx

c) $\int_0^a \int_y^a \frac{y}{x^2+y^2}$ dx dy

d) $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)$ dx dy

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Multiple Integrals

TUTORIAL SHEET-2

1. Given $\int_0^1 \int_0^1 dx\, dy$, the region of integration is __________________ and the integral value is ________________________.
2. The value of the integral $\int_0^{\pi} \int_0^{a\, sin\theta} r\, dr\, d\theta$ is______________
3. The value of the integral $\iint_R x^2 y^3 \text{dx dy}$ over the rectangle $0 \le x \le 1$ and $0 \le y \le 3$ is ________.
4. Area of the plane region R in the Cartesian coordinates using double integral is ________.
5. Prove that $\int_0^a \frac{dx}{\sqrt{ln\left(\frac{a}{x}\right)}} = a\sqrt{\pi}$ 8. Evaluate $\int_0^{\pi/2} \sqrt{tan\,\theta\, d\theta}$
6. Evaluate (i) $\int_0^3 \int_1^2 x(1 + x + y) dxdy$ (ii) $\int_0^{\pi/2} \int_0^a r^2\, sin\,\theta\, drd\theta$
7. Evaluate (i) $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dydx$ (ii) $\int_1^a \int_1^b \frac{1}{xy} dydx.$
8. Evaluate $\int_1^3 \int_{\frac{1}{x}}^1 \int_0^{\sqrt{xy}} xyz\, dz\, dy\, dx.$
9. Evaluate $\int_0^{\frac{\pi}{2a}} \int_0^{\cos\theta} \int_0^{\sqrt{a^2-r^2}} \text{rdz dr d}\theta$
10. Evaluate $\int_0^1 \int_0^{\sqrt{1\text{-x}^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}.$
11. Evaluate$\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) \text{dx dy dz}.$

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Multiple Integrals

TUTORIAL SHEET- 3

1. Change the order of integration $\int_0^a \int_0^x f(x,y)\ dx\, dy$
2. Change the variables in the polar coordinates in the integrals $\int_0^a \int_y^a \frac{x}{(x^2+y^2)}$ dx dy
3. Area of the plane region R in the Polar coordinates using double integral is _________
4. Volume of the region R in Cartesian coordinates in the form of triple integral is ...
5. The value of the integral $\int_0^1 \int_{y^2}^1 \int_0^{1-x} dz\, dx\, dy$ is______________
6. Evaluate$\iint_A xy\, dx\, dy$, Where A is the domain bounded by the x-axis , ordinate x = 2a
7. and the curve $x^2 = 4ay$.
8. $\iint (x+y)^2$dy dx over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

9. Change the order of integration and hence evaluate the following integral.
   a. $\int_0^1 \int_{\sqrt{y}}^{2-y} xydxdy$ b. $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ c. $\int_0^a \int_0^x \frac{\cos y}{\sqrt{(a-x)(a-y)}} dydx$
10. d. $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xydydx$
11. Change to polar coordinates and evaluate the following integral.
   a. $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}} dydx$ b. $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2} dydx$ c. $\int_0^1 \int_x^{\sqrt{2x-x^2}} x^2 +$
12. $y^2\ dydx$
12. Using the triple integrals, find the volume of the sphere x2+ $y^2 + z^2 = a^2$.
13. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
14. Change the order of the integration in the integrals:
   a) $\int_{-a}^a \int_0^{\sqrt{a^2-y^2}} f(x,y)\ dx\, dy$
   b) $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} f(x,y)\ dy\, dx$
   c) $\int_0^1 \int_{x0}^{\sqrt{y}} xy\, dx\, dy$
   d) $\int_0^a \int_0^x f(x,y)\ dx\, dy$