VECTOR CALCULUS, LAPLACE TRANSFORM & NUMERICAL METHODS (MA221TA)

UNIT-III- LAPLACE TRANSFORM

TUTORIAL SHEET-1

Find the following:

1. $L\left[\sqrt{t}-\frac{1}{\sqrt{t}}\right]=$____________. Ans: $\sqrt{\pi}\left(\frac{1}{2s^{\frac{3}{2}}}-\frac{1}{s^{\frac{1}{2}}}\right)$

2. $L[e^{-t}2^{t}]=$_______________. Ans: $\frac{1}{s+1-\log 2}$

3. $L\,[\sin 4t+\cos t\cos 2t]=$_______________. Ans: $\frac{4}{s^2+16}+\frac{1}{2}\left(\frac{s}{s^2+9}+\frac{s}{s^2+1}\right)$

4. $L\left[a^{t}\right]=$___________. Ans: $\frac{1}{s-\log a}$

5. Evaluate the Laplace transform of the following signals:

(i) $f(t)=e^{-3t}(t+1)^2$ Ans: $\frac{2}{(s+3)^3}+\frac{2}{(s+3)^2}+\frac{1}{s+3}$

(ii) $f(t)=e^{2t}\,sin\,3\,t$ Ans: $\frac{3}{(s-2)^2+9}$

(iii) $f(t)=e^{at}\cosh bt$ Ans: $\frac{s-2}{(s-2)^2-b^2}$

(iv) $f(t)=e^{\frac{3}{2}t}\,cos^3\,t$ Ans: $\frac{1}{4}\left[\frac{s-\frac{3}{2}}{\left(s-\frac{3}{2}\right)^2+9}+\frac{3\left(s-\frac{3}{2}\right)}{\left(s-\frac{3}{2}\right)^2+1}\right]$

(v) $f(t)=t(1-\sqrt{t}e^{t})^3$ Ans: $\frac{1}{s^2}-\frac{15\sqrt{\pi}}{8(s-3)^{\frac{7}{2}}}-\frac{9\sqrt{\pi}}{(s-1)^{\frac{5}{2}}}+\frac{6}{(s-2)^3}$

(vi) $f(t)=e^{\mathrm{at}}\cos(\mathrm{bt}+a)$ Ans: $\cos a\,\frac{s-a}{(s-a)^2+b^2}-\sin a\,\frac{b}{(s-a)^2+b^2}$

(vii) $f(t)=sint\,sin2t\,sin3t+e^{-3t}\cosh^2 2t$

Ans: $\frac{1}{4}\left[\frac{2}{s^2+4}+\frac{4}{s^2+16}-\frac{6}{s^2+36}+\frac{1}{s-1}+\frac{1}{s+7}+\frac{2}{s+3}\right]$

(viii) $f(t)=\begin{cases} t^2, & 0\le t<1 \\ 2-t, & 1\le t<2 \\ 7, & t\ge 2\end{cases}$. Also draw the graph of f(t).

Ans: $\frac{e^{-2s}}{s^2} - \frac{3e^{-s}}{s^2} + \frac{7e^{-2s}}{s} - \frac{2e^{-s}}{s^3} + \frac{2}{s^3}$

(ix) f(t) = $\begin{cases} e^t, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$ Ans: $\frac{1}{1-s}[1 + e^{1-s}]$

(x) f(t)=$\sin^5 t$ Ans: $\frac{5}{16(s^2+25)} - \frac{15}{16(s^2+9)} + \frac{5}{8(s^2+1)}$

(xi) f(t) = $\begin{cases} (t-a)^3, & t > a \\ 0, & t < a \end{cases}$ Ans: $\frac{6}{s^4} e^{-as}$

RV College of Engineering®
Mysore Road, RV Vidyaniketan Post,
Bengaluru - 560059, Karnataka, India

Department of Mathematics

VECTOR CALCULUS, LAPLACE TRANSFORM & NUMERICAL METHODS (MA221TA)

UNIT-III- LAPLACE TRANSFORM

TUTORIAL SHEET-2

1. If $L\{f(t) = F(s)$ then $L\{f(3t)\} =$ ____________. Ans: $\frac{1}{3}F\left(\frac{s}{3}\right)$

2. $s^3F(s) - s^2f(0) - sf'(0) - f''(0) =$ ____. Ans: $L\{f^3(t)\}$

3. Evaluate $\int_0^{\infty} e^{-t}t^2 \; cos \; 2t \, dt$. Ans: $-\frac{22}{125}$

4. Evaluate L $\left\{\int_0^t \frac{\sin 4t}{e^{-t}\,\mathrm{t}} dt\right\}$. Ans: $\frac{1}{s}\cot^{-1}\left(\frac{s-1}{4}\right)$

5. Find the Laplace transform of $\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^3$. Ans: $\frac{\sqrt{\pi}}{s}\left(\frac{3}{4s^2} - \frac{3}{2s} + 3 + 2s\right)$

6. Find $L\{t^n + sinat\}$, where n is a positive integer & $a > 0$. Ans: $\frac{n!}{s^{n+1}} + \frac{a}{s^2+a^2}$

7. Find $L\left\{cos^3 2t + e^{-3t}(2\,cos5t - 3\,sin5t)\right\}$.

   Ans: $\frac{1}{4}\left[\frac{s}{s^2+36} + \frac{3s}{s^2+4}\right] + \frac{2(s+3)}{(s+3)^2+25} - \frac{15}{(s+3)^2+25}$

8. Given $L\{2\sqrt{t/\pi}\} = \frac{1}{s^{3/2}}$, show that $L\left\{\frac{1}{\sqrt{\pi t}}\right\} = \frac{1}{\sqrt{s}}$.

9. Prove that $e^{2t}\int_0^{\infty}(te^{-2t} \sin 3t \, dt) = \frac{12}{169}$.

10. Find $L\left(\frac{cos\,5t - cos\,6t}{t}\right)$. Ans: $\log\left(\sqrt{\frac{s^2+36}{s^2+25}}\right)$

11. Obtain $L\left(\int_0^t \frac{sin^2\,t}{t} dt\right)$. Ans: $\frac{1}{2s}\log\left(\frac{\sqrt{s^2+4}}{s}\right)$

12. Find the Laplace transform of $t^2e^t \, sin\,4\,t$. Ans: $\frac{8(3s^2-6s-13)}{(s^2-2s+17)^{\,3}}$

13. Find the Laplace transform of $t^2(e^{-2t} - \cos 2t + 4)$ Ans: $\frac{2}{(s+2)^3} + \frac{8}{s^3} - \frac{2s(s^2-12)}{(s^2+4)^3}$

14. Prove that $\int_0^{\infty} \frac{e^{-t}-e^{-3t}}{t} dt = log3$

VECTOR CALCULUS, LAPLACE TRANSFORM & NUMERICAL METHODS (MA221TA)

UNIT-III- LAPLACE TRANSFORM

TUTORIAL SHEET-3

1. A rectangular wave $f(t)$ of period $2a,\ a>0$ is defined by

$$f(t)=\begin{cases}E, & 0\le t\le a\\ -E, & a<t\le 2a\end{cases}$$

Show that $L\{f(t)\}=\frac{E}{s}tanh\left(\frac{as}{2}\right)$. Also draw the graph of the wave function.

2. A periodic function of period $2\pi/_{\omega}$ is defined by

$$f(t)=\begin{cases}E\,sin\,\omega\,t, & 0\le t<\pi/\omega\\ 0, & \pi/\omega\le t\le 2\pi/\omega\end{cases},$$ where E & ω are positive constants.

Show that $L\{f(t)\}=\frac{E\omega}{(s^2+\omega^2)(1-e^{-\pi s/\omega})}$. Also draw the graph of the function.

3. Find the Laplace transform of function f(t)= $t\,u(t-4)-t^2\delta(t-2)$

Ans: $\frac{e^{-4s}1}{s^2}(1+4s)-4e^{-2s}$

4. Evaluate $\int_0^{\infty}t^m(\log t)^n\,\delta(t-3)dt$. Ans:$3^m\log 3^n$

5. Express $f(t)=\begin{cases}sin\,t\,, & 0<t\le\pi/2\\ cos\,t\,, & t>\pi/2\end{cases}$ in terms of the unit step function and hence find its Laplace Transform. Ans: $\frac{1}{s^2+1}\left\{1-e^{-\frac{\pi s}{2}}(s+1)\right\}$

6. Express $f(t)=\begin{cases}2, & 0<t\le\pi\\ 0, & \pi<t<2\pi\\ sin\,t\,, & t>2\pi\end{cases}$ in terms of the unit step function and hence find its Laplace Transform.

7. Express $f(t)=\begin{cases}t^2, & 0<t\le 2\\ 4t, & t>2\end{cases}$ in terms of the unit step function and hence find its Laplace Transform. Ans: $\frac{2}{s^3}+e^{-2s}\left(-\frac{2}{s^3}+\frac{4}{s}\right)$

8. Find Laplace of periodic function $f(t)$ of period $2a, a>0$ defined by

$f(t) = \begin{cases} t, & 0 \le t \le a \\ -t+2a, & a < t \le 2a \end{cases}$. Also draw the graph of the function. Ans: $\frac{1}{s^2}\tanh\frac{as}{2}$