

1(a) What is the probability, f , to find an electron when $E=E_F$ (fermi level) according to the Fermi-Dirac equation?

Ans: $f=0.5$

(b) For any given semiconductor at $E=E_F+0.5$ eV, what is the temperature required to have 1%, 5%, and 10% probability of finding an electron at that energy?

Hint: Substitute $E_F+0.5$ eV into E in $f(E)$ equation 1. Next for each value of f , evaluate temperature T .

Answer (i) $f=0.01$, $T=1260\text{K}$ or 990C ; $f=0.05$, Ans $T=1970\text{K}$ or 1700C ; $f=0.10$ Ans $T=2640\text{K}$ or 2370C

Ridiculously high numbers for temperature, but this gives you an idea of how the fermi-dirac equation relates temperature to probability of electron occupation at a given energy level.

2. Use of fermi function to obtain the value of $f(E)$ for $E-E_F = 0.01$ at 200 K .

Hint: The problem is as simple as it can get!!!!

3. Calculate the probability that an energy level (a) kT (b) $3kT$ (c) $10kT$ above the fermi-level is occupied by an electron?

Answer:

$$\text{Probability that an energy level } E \text{ is occupied is given by } f(E) = \frac{1}{e^{\frac{E - E_F}{kT}} + 1}$$

$$\text{For } (E-E_F) = kT, f(E) = \frac{1}{e^{\frac{kT}{kT}} + 1} = \frac{1}{e + 1} = 0.268$$

$$\text{For } (E-E_F) = 3kT, f(E) = \frac{1}{(e^3 + 1)} = 0.047$$

$$\text{For } (E-E_F) = 10kT, f(E) = \frac{1}{(e^{10} + 1)} = 4.5 \times 10^{-5}$$

4. The fermi-level in a semiconductor is 0.35 eV above the valence band. What is the probability of non-occupation of an energy state *at the top* of the valence band, at (i) 300 K (ii) 400 K ?

The probability that an energy state in the valence band is not occupied is

(i) $T=300\text{K}$

$$1-f(E) = 1 - \frac{1}{(e^{\frac{E_V - E_F}{kT}} + 1)} = 1 - \frac{1}{(e^{\frac{-0.35}{0.0259}} + 1)} = 1.353 \times 10^{-6}$$

$$(ii) T=400K \quad 1-f(E) \cong \frac{e^{\frac{E_V-E_F}{kT}}}{e^{\frac{E_V-E_F}{kT}} + 1} = 3.9 \times 10^{-5}$$

5. The fermi-level in a semiconductor is 0.35 eV above the valence band. What is the probability of non-occupation of an energy state at a level kT below the top of the valence band, at (i) 300 K (ii) 400 K?

The probability that an energy state in the valence band is not occupied is

$$(i) T=300K \quad 1-f(E) = 1 - \frac{1}{e^{\frac{E-E_F}{kT}} + 1} \cong e^{\frac{E-E_F}{kT}} \quad \text{for } E_F - E > kT$$

$$e^{\frac{E-E_F}{kT}} = e^{\frac{-(0.35+0.0259)}{0.0259}} = 4.97 \times 10^{-7}$$

(ii) T=400K

$$1-f(E) \cong e^{\frac{E-E_F}{kT}} = e^{\frac{-(0.35+0.0345)}{0.0345}} = 1.448 \times 10^{-5} \quad \text{Note } (E-E_F) \text{ is -ve}$$

6. For copper at 1000K find the energy at which the probability P(E) that a conduction electron state will be occupied is 90%. The Fermi energy is 7.06eV.

$$\text{The fermi factor } f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} = 0.90$$

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$$e^{\frac{E-E_F}{kT}} = \left[\frac{1}{0.90} - 1 \right] = 0.11$$

$$E = E_F + kT (\ln 0.11) = 7.06 - 0.19 = 6.87 \text{ eV}$$

7. The Fermi energy for potassium is 2.1 eV. Calculate the velocity of the electrons at the Fermi level.

$$E_F = \frac{1}{2}mv^2 = 0.74 \times 10^{12} \text{ m}^2\text{s}^{-2}$$

8. At what temperature we can expect a 10% probability that electrons in silver have an energy which is 1% above the Fermi energy? The Fermi energy of silver is 5.5 eV.

$$\text{The fermi factor } f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} = 290 \text{ K}$$