Unit - 3 (a)

Transformers.

Introduction

* Most of the loads are located far away from the generating stations. Depending on the type of load, the voltage at which it operates also differs. For example, an industry might need a 3ϕ, 4-wire system at 400V, while our homes need 230V, 1ϕ supply. However, both these loads operate at the same frequency of 50Hz.

* The electrical generator usually generates power at a voltage of 11kV or 33kV at the generating station (power plant). Therefore, it will be necessary to reduce the voltage to a value compatible to the load, at the same frequency (50Hz).

* In a similar way, two circuits might work at the same frequency but at different voltage levels. In such scenarios also, voltage needs to be changed.

* This can be achieved using a transformer.

* A transformer is a static device which transfers power from one circuit to another, at the same or different voltage levels, without any change in frequency and power.

* There exists no simple device that can accomplish such changes in DC voltages. Thus, the transformer has provided a feature to ac power system that lacks in dc power systems.

* The transformation of power usually involves a change in voltage level. In other words, the basic use of a transformer is to increase or decrease ac voltage.

* A transformer that increases or raises the voltage (i.e, the output voltage (V_o) will be more than the input voltage (V_s)) is called a step-up transformer.

* A transformer that decreases the voltage, making the output voltage more than the input voltage, is called a step-down transformer.

* If the power transfer happens at the same voltage, (i.e., the output & input voltages are equal), such a transformer is called one-to-one transformer.

* A transformer is a static device. This implies that a transformer has no moving parts. Hence, the efficiency of a transformer is very high.

* Transformer works on the principle of electromagnetic induction. There are two coils linked magnetically but separated electrically, and are used to transfer power.

* In brief, a transformer is a device that
  i) transfers electric power from one circuit to another
  ii) it does so without a change of frequency
  iii) it accomplishes this by electromagnetic induction, and
  iv) where the two circuits are magnetically coupled but electrically isolated (separated).

Electromagnetic Induction

* Any material that has both attractive & repulsive properties is called a magnet.

* In nature, magnets are found in the form of an iron ore, which is an oxide of iron Fe_3O_4 (ferric oxide) known as magnetite. Artificial or temporary magnets can be created by winding certain materials with a coil and passing current through the coil. Such a magnet is called an electromagnet. As long as a current flows, an electromagnet behaves like a magnet. If the current is stopped, the lose the magnetic properties. Iron, cobalt, Nickel & their alloys such as silicon steel can behave like magnets when current is passed as discussed. Therefore these materials behave as electromagnets.

* The strength (of attractiveness or repulsiveness) depends on,
  i) the number of turns of the coil
  ii) the magnitude of current in the coil.

  Hence, by suitably increasing the number of turns of a coil and the current passing through it, an electromagnet of any strength can be built.

* The region or space around a magnet where the magnetic effects are felt is known as a magnetic field. The magnetic field is represented by magnetic lines of force, which start from the north pole & go to the south pole, completing their path in the surrounding medium and the material of the magnet.

* The total number of magnetic lines of force around a magnet (in a magnetic field) is called the magnetic flux. It is represented by the Greek letter phi (ϕ). The S.I unit of magnetic flux is weber (Wb).

$$\boxed{1\ \text{Wb} = 10^8 \text{ magnetic lines of force}}$$

* The magnetic flux density at any point in a field is given by the flux (number of magnetic lines of force) passing through unit area at that point, through a plane that is at right angles to the flux. It is represented by 'B' and is given by

$$\text{flux density} = \frac{\text{total magnetic flux}}{\text{area}}$$

$$\therefore \boxed{B = \frac{\phi}{A}}$$

where ϕ is the total magnetic flux (in Wb) passing through an area of A (in m^2). Hence, the unit of flux density is webers per metre-square (Wb/m^2) or tesla (T).

* Just like how current flows in an electric circuit, flux flows in a magnetic circuit. All materials do not allow flux to pass through them easily. The ability of a material or a medium to allow the flow of magnetic flux through it is called permeability. In other words, permeability is the property of a material by the virtue of which the magnetic flux can easily be created in it.

* On the other hand, the property of a material or medium by the virtue of which it opposes the creation of magnetic flux in it is called its <u>reluctance</u>, represented by $\mathcal{R}$. Its unit is ampere-turns per weber (AT/Wb)
* The magnetic force, which creates magnetic flux in a magnetic material is called the <u>magnetomotive force</u> or <u>MMF</u>. Its unit is ampere-turns (AT)

$$\therefore \boxed{MMF = NI} \text{ AT}$$

where

N = number of turns of the coil

I = current through the coil.

Another equation for MMF is

$$MMF = \text{flux} * \text{Reluctance}$$

$$\therefore \boxed{MMF = \phi * \mathcal{R}} \text{ AT}$$

* Imagine a conducting wire which makes a closed loop. The wire can be of any shape and can have any number of turns. The flux going through the closed surface created by the boundary of wire is called the <u>flux linkage</u> of the wire. In other words, flux linkage deals with the number of lines of force that 'links' the given loop.

* If flux density of a magnetic field is 'B' and a circular coil of 'N' turns is placed perpendicular to this field, then

$$\boxed{\text{flux linkages} = B * A * N}$$

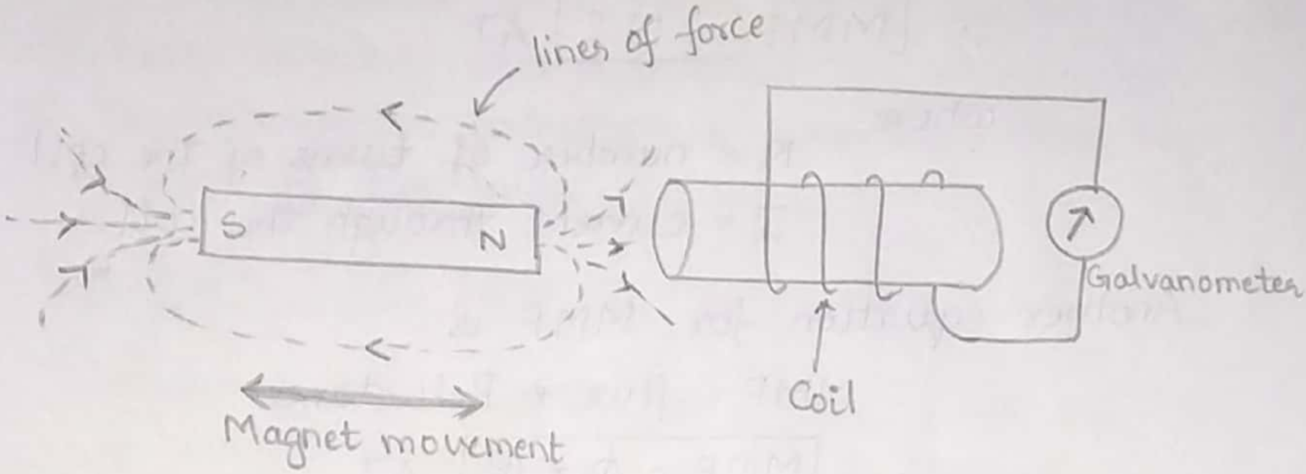
where 'A' is the area of the circle.

Since $\phi = BA$,

$$\boxed{\text{flux linkages} = \phi N}$$

* When an electric current passes through a conductor, a magnetic field is set up which surrounds or links the conductor. Thus, magnetism can be created by means of electric current.
* The converse of this is also true. i.e., electricity can be created by magnetism by changing the magnetic flux linking the conductor.

* This process is called <u>electromagnetic induction</u>.



* Consider an insulated coil be wound on a base material (core) for a large number of turns be connected to a galvanometer, as shown above.

* If the permanent magnet NS is moved towards the coil, the galvanometer pointer deflects in one direction. If the magnet is moved away (moved in the opposite direction), the pointer deflects in the opposite direction. If the movement of the magnet is stopped, the galvanometer shows zero deflection.

* The galvanometer deflection indicates that an emf is induced in the coil when the flux linking the coil changes.

* The direction of pointer deflection depends on the direction of magnet movement, and indicates the direction of induced emf.

* If the magnet is moved faster, the pointer also deflects faster, while a slow movement of magnet gives a slower deflection.

* These observations were made by a British Scientist Michael Faraday who discovered the phenomenon of electromagnetic induction & enunciated them in the form of two laws.

* <u>Faraday's First Law</u>

  "Whenever the magnetic flux linking a coil or a circuit changes, an e.m.f is induced in it."

→ This was based on the observation of deflection of galvanometer pointer when the magnet was moved towards the coil.

* <u>Faraday's Second Law</u>

  "The magnitude of the induced e.m.f is directly proportional to the rate of change of flux linkages."

→ This was based on the observation of the deflection when the magnet was moved faster & slower.

* Consider a coil of 'N' turns. Let the flux linking the coil change from an initial value of ϕ_1 to a new value ϕ_2 in 't' seconds.

* Thus,

Initial flux linkages = $N\phi_1$

Final flux linkages = $N\phi_2$.

$\therefore$ Change of flux linkages = $N\phi_2 - N\phi_1$.

and

Rate of change of flux linkages = $\dfrac{N\phi_2 - N\phi_1}{t}$

* As per Faraday's second law, the magnitude of the induced emf (e) is,

$$|e| \propto \text{rate of change of flux linkages.}$$

$$\therefore |e| \propto \frac{N\phi_2 - N\phi_1}{t}$$

$$\Rightarrow |e| = K \cdot N\frac{(\phi_2 - \phi_1)}{t}$$

taking 'K' as unity & putting the equation in differential form, we get the magnitude of induced emf to be

$$\boxed{|e| = N\frac{d\phi}{dt}} \text{ volts.}$$

* There is a definite relationship between the direction of induced emf, direction of flux and the direction of motion of the conductor. Two methods are used to relate these. They are :-

a) Lenz's Law

b) Flemming's right hand rule

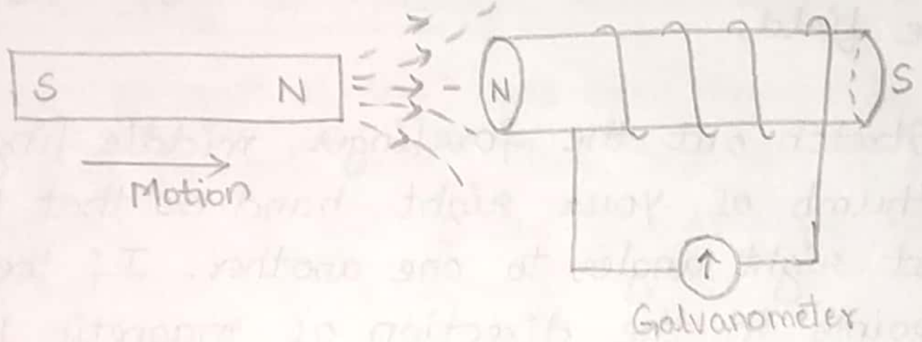
* Lenz's Law

"The direction of the induced emf and hence current is such that it opposes the very cause producing it."

→ Lenz's law is reflected mathematically with a minus sign in the RHS of Faraday's second Law. i.e.,

$$\boxed{e = -N\frac{d\phi}{dt}} \text{ volts}$$

The negative sign simply reminds us that the induced current opposes the changing magnetic field that caused the induced current. The negative sign has no other meaning.



→ Consider the case when the N-pole of the magnet is moving towards a coil of 'N' turns. As the magnet moves towards the coil, the flux linking the coil changes (here it increases). Therefore, an e.m.f and hence a current is induced in the coil.

→ According to Lenz's law, the direction of induced emf and current will be such that it opposes the cause producing it.

→ Here, the cause of the induced current is the increasing magnetic field linking the coil. Therefore, the induced current will set up a flux that opposes the increase in flux through the coil.

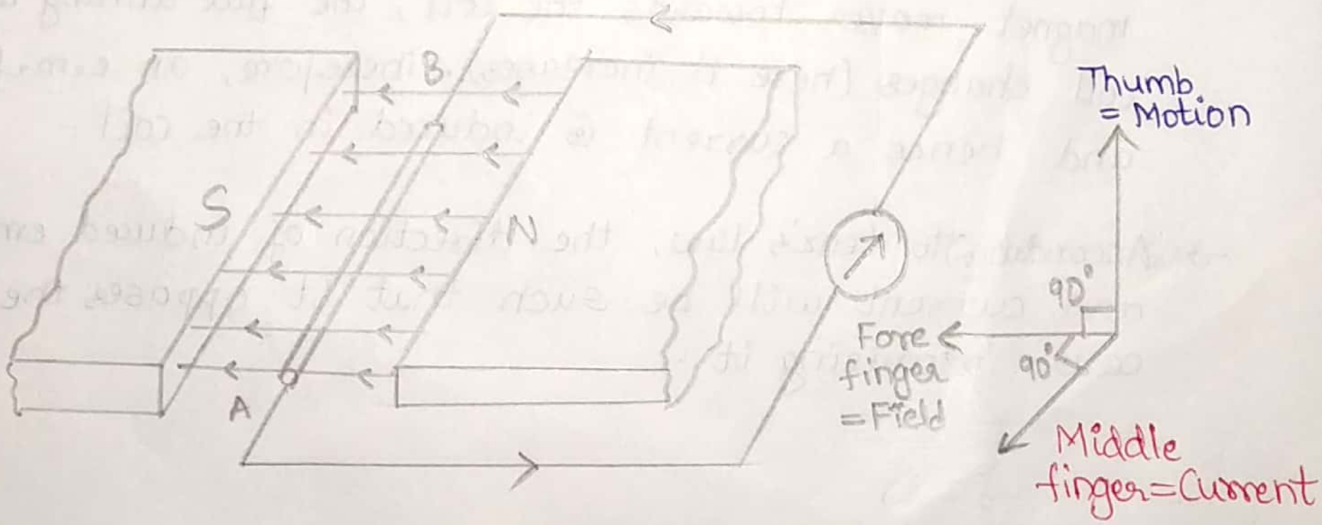
→ This is possible only if the left hand face of the coil becomes N - pole.

→ Once we know the magnetic polarity of the coil face, the direction of induced current can be determined by applying Flemming's right hand rule.

* Flemming's Right hand rule (Generator Rule).

→ This rule is particularly suitable to find the direction of the induced emf and hence current when the conductor moves at right angles to the field.

→ " Stretch out the forefinger, middle finger and thumb of your right hand so that they are at right angles to one another. If the forefinger points in the direction of magnetic field, thumb in the direction of motion of the conductor, then the middle finger will point in the direction of induced current."



THUMB ⟶ MOTION
FORE FINGER ⟶ FIELD
MIDDLE FINGER ⟶ CURRENT (I)

→ Consider a conductor AB moving upwards at right angles to a uniform magnetic field as shown. Applying Flemming's right-hand rule, it is clear that the direction of induced current is from B to A.

→ If the conductor is moved downwards, keeping the direction of magnetic field unchanged, then the current will flow from A to B.

Note :-

1. Why is 'k' unity in Faraday's second law?

* One volt (SI unit of emf) has been defined so that the value of 'k' becomes unity. Thus, one volt is said to be induced in a coil if the flux linkages change by 1 Wb-turn in 1 second.

* Here,

$$N\phi_2 - N\phi_1 = 1 \text{ Wb-turn}$$

$$t = 1 s$$

$$e = 1 V.$$

$$\therefore e = k \frac{N\phi_2 - N\phi_1}{t}$$

$$\Rightarrow 1 = k\left(\frac{1}{1}\right)$$

$$\Rightarrow \boxed{k = 1}$$

2. Lenz's law obeys Law of conservation of energy.

* In the setup shown to explain Lenz's law, when the N-pole of the magnet is approaching the coil, the induced current will flow in the coil in such a direction that the left-hand face of the coil becomes N-pole. The result is that the motion of the magnet is opposed. The mechanical energy spent in overcoming this opposition (repulsion) is converted into electrical energy which appears in the coil. Thus, Lenz's Law is consistent with the law of conservation of energy.

* When the N-pole of the magnet is moved away from the coil, the left-hand face of the coil becomes the S-pole. Therefore, again the motion of magnet is opposed and the mechanical energy spent in overcoming this opposition is converted into electrical energy.

* When there is no motion of the magnet, induced emf and hence current in the coil is zero i.e., no electrical energy is available. This is consistent with law of conservation of energy since no mechanical energy is spent.

* Changing of flux

→ We have established that an emf is induced in a conductor when the rate of change of flux linking the conductor changes.

→ There are two ways to achieve this.

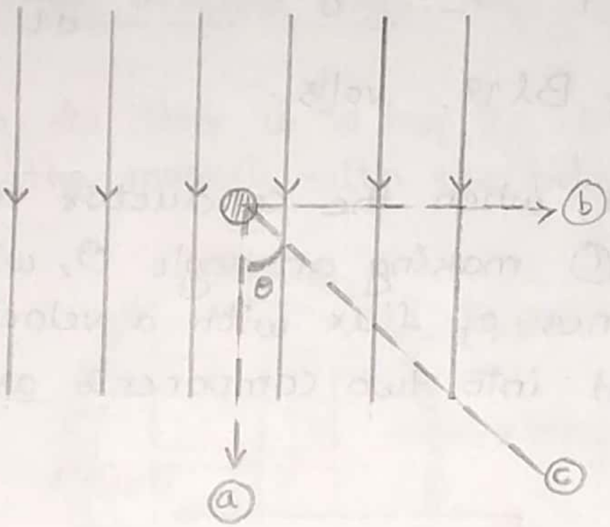
a) By keeping the field stationary & moving the conductor

b) by moving the magnetic field & keeping conductor stationary.

* Dynamically Induced EMF

→ When the magnetic field is stationary and the conductor is in motion, the rate of change of flux linking the conductor changes and an emf is induced. This type of emf induced is known as dynamically induced emf.

→ Consider a magnetic field of constant flux density 'B' (in Wb/m^2), which is represented by the magnetic lines of flux as shown below.



→ Let a conductor of uniform length 'l' and area of cross-section 'a' be placed perpendicular to the lines of flux.

→ When the conductor moves with a velocity 'v' in the direction ⓐ, it moves parallel to the lines of flux. Hence, there is no rate of change of flux linkages, and therefore, the emf induced is zero.

→ When the conductor moves with a velocity 'v' in direction ⓑ, it moves perpendicular to the flux lines & there is maximum rate of change of flux linking the coil. Hence, maximum emf is induced in it. Let the conductor move a small distance dx in time dt seconds. Then the flux cut by the conductor is given by

$$d\phi = B \times l \times dx.$$

The rate at which the flux links will be,

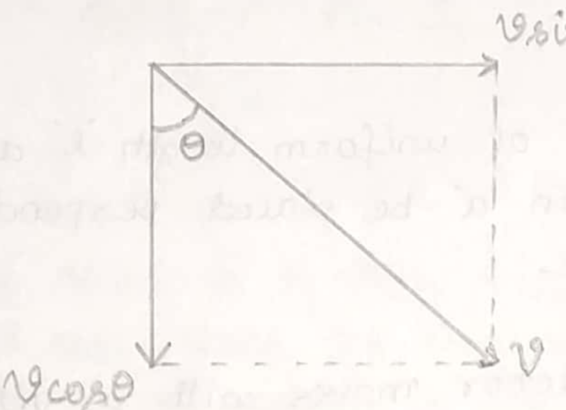
$$\frac{d\phi}{dt} = Bl\frac{dx}{dt}.$$

since $\frac{dx}{dt} = v$, the velocity

and as per faraday's laws, $\frac{d\phi}{dt}$ = emf induced.

$$\therefore \quad E_m = Blv. \quad \text{volts}.$$

→ In general, when the conductor moves in the direction © making an angle θ, with the direction of the lines of flux with a velocity v, it can be divided into two components as shown below.



→ The component $v\cos\theta$, which is in the direction of flux, does not contribute anything for the emf induced as it is parallel to the flux lines.

→ The component $v\sin\theta$ is perpendicular to the direction of flux and leads to emf induction, given by

$$\boxed{e = Blv\sin\theta} \text{ volts}$$

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since $B l v = E_m$,

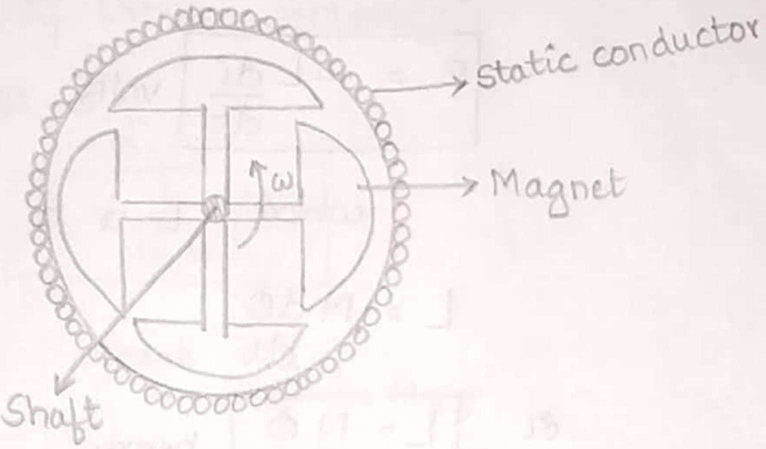
$$\boxed{e = E_m \sin\theta} \text{ volts}$$

The direction of this emf is given by Flemming's right hand rule.

* Statically Induced EMF

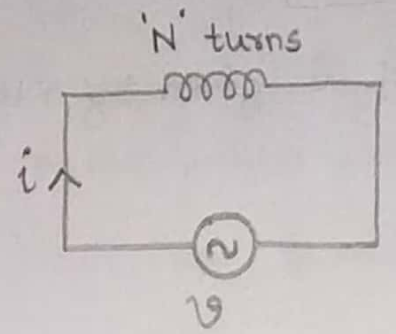
→ When a conductor is stationary and the magnetic field is moving or changing, the EMF induced is called statically induced emf.

→ One way to do this is to keep the conductor stationary and rotate the magnets with the help of a common shaft



→ However, there is another way of obtaining statically induced emf. When the electric circuit is in the form of a coil, which is stationary and when an alternating current is passed through it, an alternating flux is produced, which links the coil. Since the conductor is stationary & the (alternating) flux is changing, this is also a type of method to obtain statically induced emf.

→ Consider a coil of 'N' turns which is connected to an alternating voltage 'v' due to which, an alternating current 'i' flows through it.



→ This alternating current, produces an alternating flux (ϕ), which links with the coil. Then an emf is induced in the coil. The induced emf is given by,

$$e = -N\frac{d\phi}{dt}$$

$$= -N\frac{d\phi}{di} * \frac{di}{dt}$$

$$\boxed{e = -L\frac{di}{dt}} \text{ volts}$$

where L is a constant.

$$L = N\frac{d\phi}{dt}$$

or $$\boxed{L = \frac{N\phi}{I}} \text{ henry}$$

This is known as the <u>self-inductance of the coil</u>.

<u>Note</u> :- $\frac{d\phi}{di} = \frac{\phi}{i}$ because $\phi \propto i$

→ Since the emf induced here is due to the rate of change of flux linkages with the same coil, this type of statically induced emf is also known as <u>self-induced emf</u>.

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→ The equation for self-inductance is,

$$L = \frac{N\phi}{I}.$$

But the magnetomotive force is,

$$MMF = NI = \phi \mathcal{R}.$$

$$\Rightarrow \phi = \frac{NI}{\mathcal{R}}.$$

$$\therefore L = \frac{N}{I}\left(\frac{IN}{\mathcal{R}}\right)$$

$$\boxed{L = \frac{N^2}{\mathcal{R}}} \text{ henry.}$$

For a magnetic material, the reluctance is directly proportional to its length (l) & inversely proportional to the area of cross-section. (A)

$$\therefore \mathcal{R} \propto \frac{l}{a}$$

$$\text{or } \boxed{\mathcal{R} = \frac{1}{\mu_0 \mu_r}\left(\frac{l}{A}\right)}$$

where

$\mu = \mu_0 \mu_r$

$\mu \to$ absolute permeability of the material

$\mu_0 \to$ permeability of free space
$\mu_0 = 4\pi * 10^{-7}$ H/m

$\mu_r \to$ relative permeability of the material.

Substituting, we get

$$L = \frac{N^2}{l/\mu_0 \mu_r A} \Rightarrow \boxed{L = \frac{\mu_0 \mu_r A N^2}{l}} \text{ henry}$$

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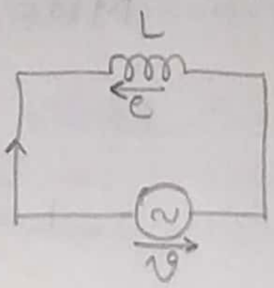
* Consider a coil of self-inductance 'L' henrys, through which, an alternating current (i) is flowing. This produces an alternating flux, which links the coil, and an emf is induced in it. This emf is given by

$$|e| = L\frac{di}{dt}$$



However,

$$|e| = v$$

$$\therefore \quad v = L\frac{di}{dt}$$

* A pure inductor does not consume any power. The energy supplied to the coil is stored in the form of an electromagnetic field.

* The induced emf opposes any change in the value of supply. Hence in order to establish a current of 'I' amperes in 't' seconds, work has to be done to overcome this opposition. The word done in dt seconds is given by

$$dW = v\,i\,dt$$

$$= L\frac{di}{dt}\,i\,dt$$

$$dW = L\,i\,di$$

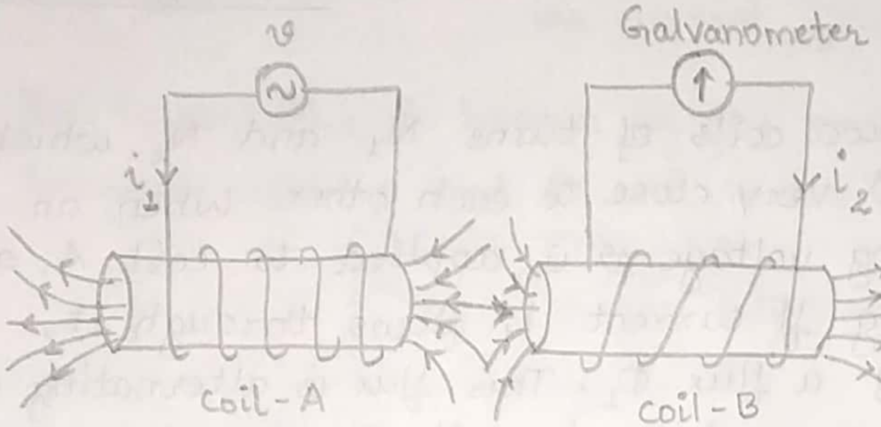
The work done in 't' seconds will be,

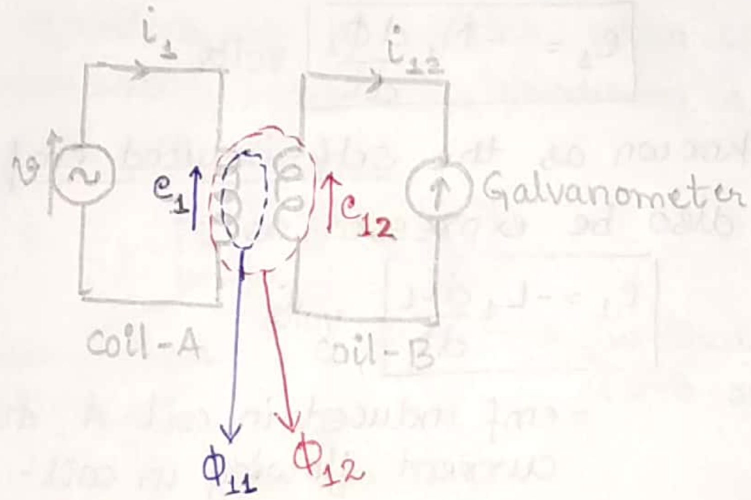
$$W = \int_0^t dW = \int_0^I L\,i\,di = L\int_0^I i\,di$$

$$\Rightarrow \boxed{W = \frac{1}{2}LI^2} \quad \ldots \text{energy stored in inductor.}$$

→ Until now, we have discussed the production of emf in one coil due to change of flux linking the same coil. Another interesting case is to be studied, where two coils are placed very close to each other and there is interaction of flux between them.

→ Consider two coils A and B placed close to each other so that the flux created by one coil due to the AC supply, completely links with the other coil.





* Two coils, which are placed close to each other are said to be <u>mutually coupled</u>, when the full or a part of the flux produced in one coil links with the other coil.

→ Since the supply is alternating, the flux produced is also of alternating type, and hence emf is induced in both coils.

→ The emf induced in coil-A where the flux is produced, is called the self-induced emf as it is the emf induced in the coil due to change of flux linking the same coil.

→ The emf induced in the second coil is due to the alternating flux of coil-A that links with coil-B. Such an emf which is induced due to the change of flux in another coil is called mutually induced emf.

→ Consider two coils of turns N_1 and N_2, which are placed very close to each other. When an alternating voltage v is applied to coil-A, an alternating ~~ft~~ current i_1 flows through it, producing a flux ϕ_1. This flux is alternating & induces an emf e_1 in coil-A, given by

$$\boxed{e_1 = -N_1 \frac{d\phi_1}{dt}} \text{ volts}$$

This is known as the self-induced emf of coil-A.

It can also be expressed as,

$$\boxed{e_1 = -L_1 \frac{di_1}{dt}} \text{ volts}$$

= emf induced in coil-A, due to the current flowing in coil-A.

→ Out of the flux ϕ_1 produced in coil-A, some of it links coil-B. This flux ϕ_{12} which links both coil-A and coil-B is called the mutual flux between them. A flux ϕ_{11} (out of ϕ_1) links only with coil-A. Thus,

$$\boxed{\phi_1 = \phi_{11} + \phi_{12}} \text{ webers}$$

→ The mutual flux Φ_{12} linking coil - B, induces an emf e_{12} in coil - B. This emf is known as mutually induced emf and is given by

$$\boxed{e_{12} = -N_2 \frac{d\Phi_{12}}{dt}} \text{ volts}$$

It can also be expressed as,

$$\boxed{e_{12} = -M_{12} \frac{di_1}{dt}} \text{ volts}$$

= emf induced in coil - B due to the current flowing in coil - A.

The term M_{12} is known as the mutual - inductance between coils - A and B and is expressed as

$$\boxed{M_{12} = N_2 \frac{d\Phi_{12}}{di_1}} \text{ henry}$$

→ Similar equations can be written, when coil - B is energized by an alternating current i_2, producing a total flux of Φ_2 in it. Therefore,

$$\boxed{\Phi_2 = \Phi_{22} + \Phi_{21}}$$

total flux produced in coil - B

flux that links both coil - B and coil - A.

flux that links only coil - B

Self - induced emf in coil - B is

$$e_2 = -L_2 \frac{di_2}{dt} = -N_2 \frac{d\Phi_2}{dt}$$

Due to the mutual flux, the mutually induced emf will be

$$e_{21} = -N_1 \frac{d\phi_{21}}{dt} = -M_{21} \frac{di_2}{dt}$$

→ As the coupling between the two coils is bilateral (i.e., the coupled circuit has the same characteristic in both directions),

$$\boxed{M_{12} = M_{21} = M}$$

Hence, the mutual inductance between any two coils, which are placed close to each other, may be defined as the ability of one coil to induce an emf in the other coil, when an alternating current flows through one of the coil.

$$\boxed{M = N_2 \frac{d\phi_{12}}{dt} = N_1 \frac{d\phi_{12}}{dt}}$$

→ The ratio of mutual flux to total flux is known as coefficient of coupling (k)

$$k_{12} = \frac{\phi_{12}}{\phi_1}, \qquad k_{21} = \frac{\phi_{21}}{\phi_2}$$

Since coupling is bilateral, $k_{12} = k_{21} = k$.

$$\therefore \phi_{12} = k\phi_1, \qquad \phi_{21} = k\phi_2.$$

Since $M_{12} = M_{21} = M$,

$$M^2 = M_{12} * M_{21} = N_2 \frac{d\phi_{12}}{dt} * N_1 \frac{d\phi_{21}}{dt} = N_1 N_2 \frac{d(k\phi_1)}{dt} \cdot \frac{d(k\phi_2)}{dt}$$

$$= N_1 N_2 k^2 \frac{d\phi_1}{dt} \frac{d\phi_2}{dt} = k^2 N_1 \frac{d\phi_1}{dt} \cdot N_2 \frac{d\phi_2}{dt}$$

$$\Rightarrow M^2 = k^2 L_1 L_2.$$

$$\therefore \boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$ ..... coefficient of coupling.

→ The following points may be noted :

a) The mutually induced emf in coil-B persists so long as the current in coil-A is changing (alternating). If the supply to coil-A is removed, then the current in coil-A, the flux in coil-A and the flux linking the two coils all become zero. Hence the self & mutually induced emf are all zero.

b) The magnitude of mutually induced emf depends on the number of turns of coil-B (when coil-A is excited with an alternating voltage) and the rate of change of flux linking coil B $(d\phi_{12}/dt)$

c) The direction of induced emf (both self & mutual) is such that it opposes the very cause producing it.

→ Consider two closely placed coils with turns N_1 and N_2. Let current i_1 flow in coil-A to produce a flux ϕ_1 through it. Out of this, let a flux ϕ_{12} link with coil-B. Then the flux linkages in coil-B due to current in coil-A is,

$$\text{flux linkages} = \frac{N_2 \phi_{12}}{I_1}.$$

As per definition, this represents the mutually inductance.

$$\therefore \boxed{M = \frac{N_2 \phi_{12}}{I_1}} \quad \text{... when coil-A is excited.}$$

→ The flux in coil-A will be given by,

$$\phi_1 = \frac{N_1 I_1}{R_1} \qquad (\because \text{MMF} = NI = \phi R)$$

since $R = \dfrac{l}{\mu_0 \mu_r A}$, we get

$$\phi_1 = \frac{N_1 I_1}{(l/\mu_0 \mu_r A)}$$

Flux per ampere is,

$$\frac{\phi_1}{I_1} = \frac{N_1}{(l/\mu_0 \mu_r A)}$$

~~If the entire~~

Suppose the entire flux links with coil-B, which has N_2 turns (i.e., $\phi_{12} = \phi_1$), then

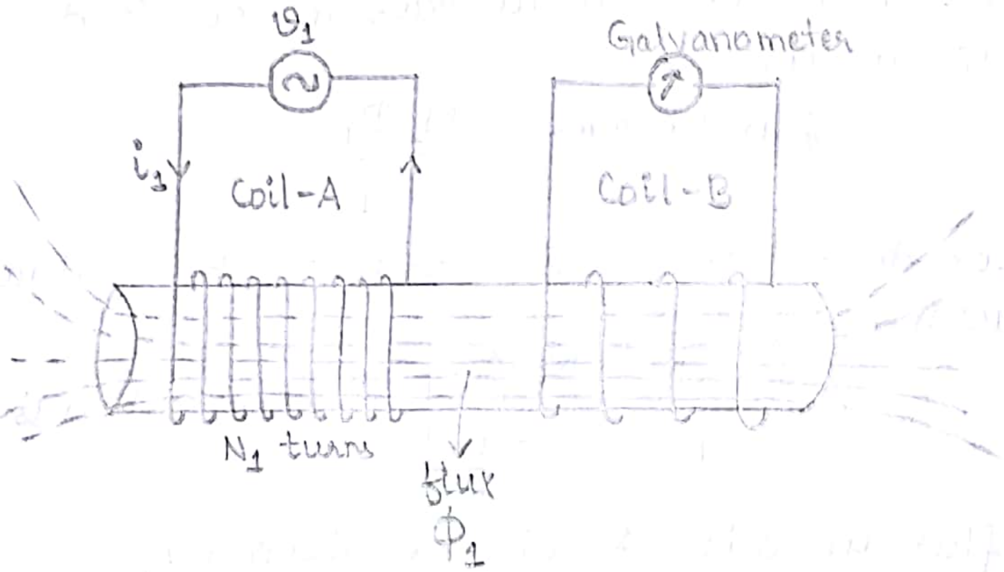
$$M = \frac{N_2 \phi_1}{I_1}$$

Substituting for ϕ_1 / I_1, we get

$$M = \frac{N_2 N_1}{l/\mu_0 \mu_r A}$$

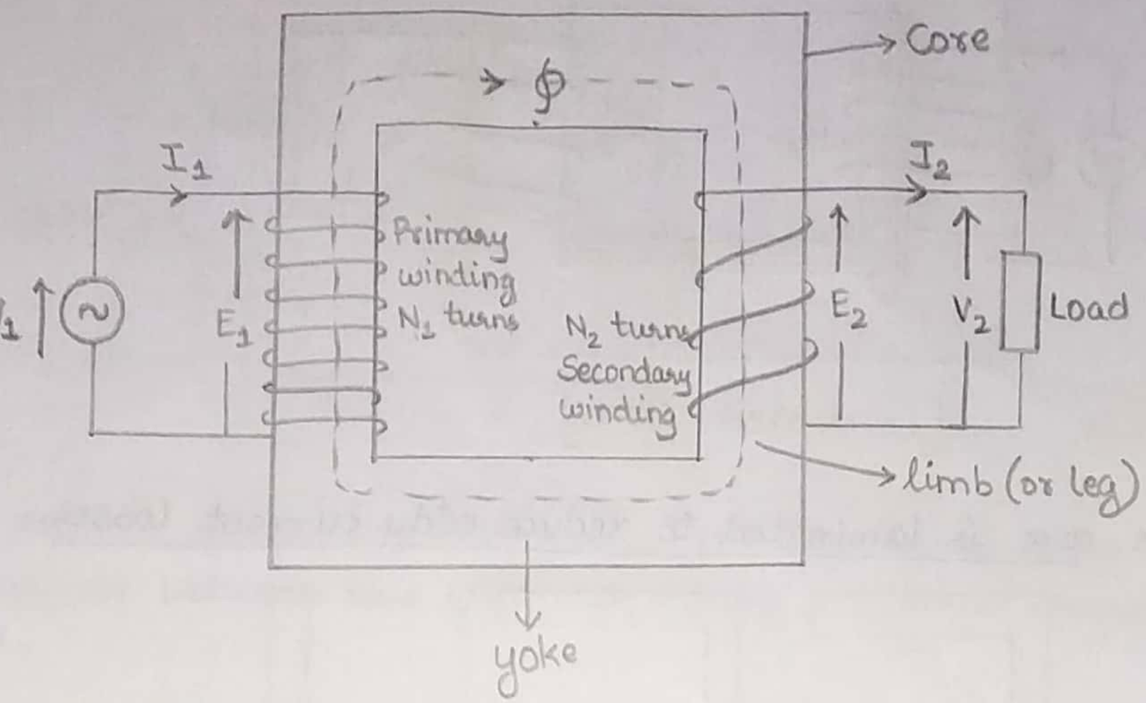
$$\boxed{M = \frac{\mu_0 \mu_r A N_1 N_2}{l}} \text{ henry}$$

This is also the mutual inductance between two coils wound on the same magnetic material.



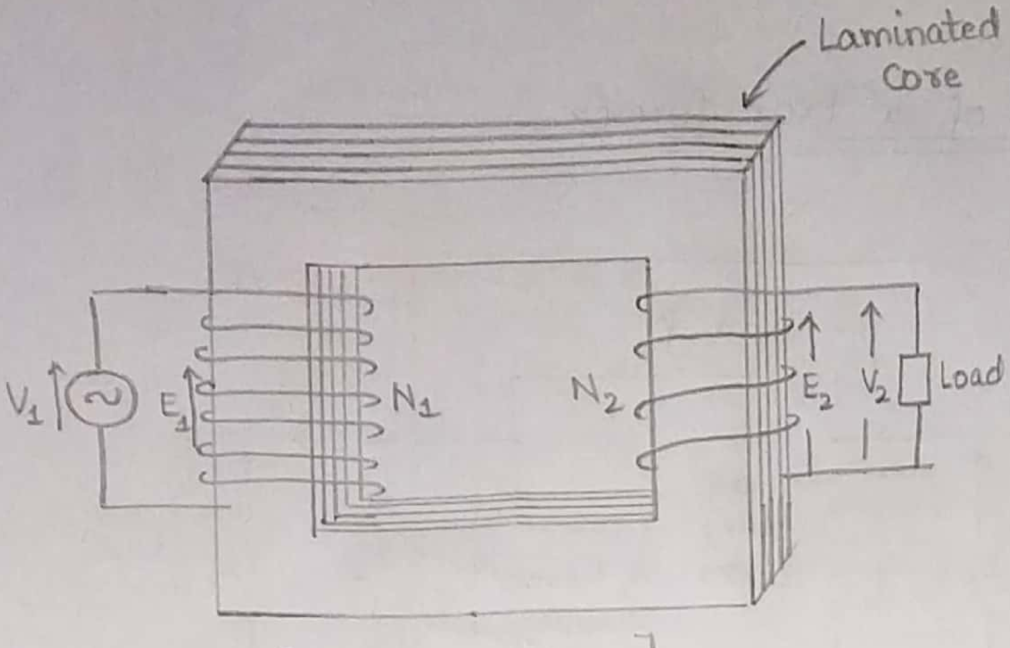
→ This forms the basis of a transformer where two windings of different number of turns are wound on the same magnetic material.

Construction of a transformer



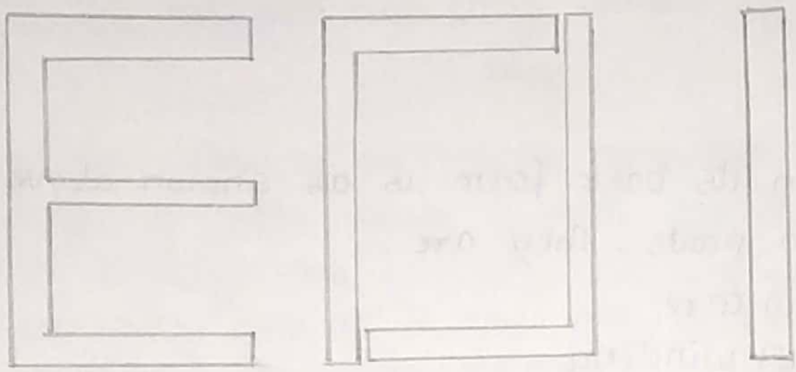
* A transformer in its basic form is as shown above. It consists of two parts. They are
  i) core
  ii) winding

* The core is the magnetic material on which the windings are wound. Since it has to carry the flux, it must have low reluctance (or high permeability). Hence it is usually made of silicon steel or CRGO steel (cold rolled grain oriented steel). The vertical parts of a core are called limbs while the horizontal parts are called yoke. The core is usually not a single solid block of steel, but is made of several thin sheets of material, each insulated from the other by varnish or paper insulator. For small transformers, each lamination is a single piece, whereas for large transformers each lamination is made of two or more sections like E, T, L, or I, so that, when they are joined together, they form a complete lamination. The laminations are staggered, while forming the core, so that, the reluctance offered by these joints is minimum.



[Laminated Core]

* The core is laminated to reduce eddy current losses.

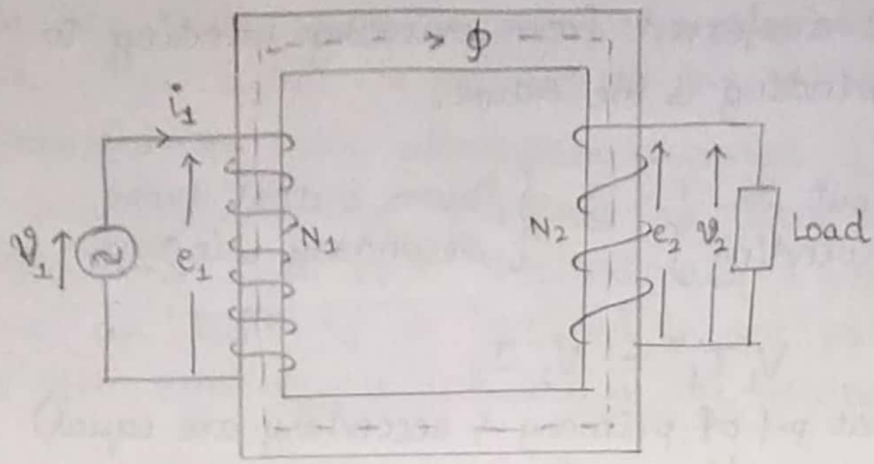


(E, L, I sections of laminations)

* The transformer uses two windings, both wound on the common magnetic core, on its limbs. In its most basic form, one winding is wound on the left side limb while the other on the right side limb. The windings must have very low resistance & hence are made of copper. Cylindrical or rectangular conductors may be used. The winding which is connected to the supply is called the primary winding and the winding connected to the load is called secondary winding.

* The windings are always insulated from each other and from the core to avoid any short circuit. The transformer is placed in an enclosure (tank) and suitable bushings are used to bring out the winding terminals.

Working principle of a transformer



* The physical basis of working of a transformer is mutual induction between two coils linked by a common magnetic flux.

* When the primary winding is connected to an alternating voltage of RMS value V_1 volts, an alternating current flows through the primary winding and sets up an alternating flux ϕ in the core material. This alternating flux links with coil-1 (primary winding) and an emf e_1 is induced in the primary winding. The alternating flux also links with the secondary winding (coil-2) and an emf e_2 is induced in the secondary winding.

* If the primary and secondary windings have N_1 & N_2 turns respectively, then the self-induced emf (e_1) and mutually induced emf (e_2) are given by

$$e_1 = -N_1 \frac{d\phi}{dt} \qquad e_2 = -N_2 \frac{d\phi}{dt}$$

$$\therefore \quad \frac{e_1}{e_2} = \frac{-N_1 \frac{d\phi}{dt}}{-N_2 \frac{d\phi}{dt}} = \frac{N_1}{N_2}$$

$$\therefore \quad \boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{1}{K}}$$

$K \rightarrow$ transformation ratio

* If a load is connected to the secondary winding, a current I_2 flows through the load. V_2 is the terminal voltage across the load.

* As power transferred from primary winding to secondary winding is the same,

$$\left.\begin{matrix}\text{Power input to}\\ \text{primary winding}\end{matrix}\right\} = \left\{\begin{matrix}\text{Power output from}\\ \text{Secondary winding.}\end{matrix}\right.$$

$$V_1 I_1 = V_2 I_2.$$

(assuming that p.f of primary & secondary are equal)

$$\therefore \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

* In an <u>ideal transformer,</u>

$$V_1 = E_1$$
$$V_2 = E_2.$$

* Therefore, we can write

$$\boxed{\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K} \text{ .... Ideal transformer}$$

* Thus, the primary & secondary currents are inversely proportional to their respective turns.

* For a step-up transformer,

$V_2 > V_1$	$k > 1$
$N_2 > N_1$	

* For a step-down transformer,

$V_1 > V_2$	$k < 1$
$N_1 > N_2$	

EMF Equation

Method-1

* When an alternating voltage $v_1 = V_m \sin \omega t$ with RMS value $V_1 = V_m/\sqrt{2}$ is applied to the primary winding of a transformer, the alternating current flowing through the primary winding produces an alternating flux ϕ, which links with both the primary & secondary winding. Hence an EMF e_1 is induced in the primary winding and an EMF e_2 is induced in the secondary winding.

* The equation for e_1 is

$$e_1 = -N_1 \frac{d\phi}{dt} \longrightarrow (1)$$

* As the applied voltage to the primary winding is sinusoidal, the current drawn and the resulting flux are also sinusoidal in nature.

$$\therefore \quad \phi = \phi_m \sin \omega t \longrightarrow (2)$$

$$\Rightarrow \frac{d\phi}{dt} = \phi_m \omega \cos \omega t.$$

* Thus, the EMF equation would be

$$e_1 = -N_1 \phi_m \omega \cos \omega t$$

$$e_1 = \omega N_1 \phi_m \sin(\omega t - 90^\circ) \longrightarrow (3)$$

Since $\quad \omega = 2\pi f,$

$$e_1 = 2\pi f N_1 \phi_m \sin(\omega t - 90^\circ) \longrightarrow (4)$$

* This emf will be maximum when $\sin(\omega t - 90^\circ) = 1$. The magnitude of the ~~is~~ maximum value of the induced emf will be

$$E_{m1} = 2\pi f N_1 \phi_m \longrightarrow (5).$$

* Therefore, the RMS value of the induced voltage in the primary winding will be,

$$E_1 = \frac{E_{m1}}{\sqrt{2}}$$

$$= \frac{2\pi f N_1 \phi_m}{\sqrt{2}}$$

$$\boxed{E_1 = 4.44 f \phi_m N_1} \text{ volts} \longrightarrow (6)$$

* Similarly, the RMS value of the emf induced in the secondary winding can be written as,

$$\boxed{E_2 = 4.44 f \phi_m N_2} \text{ volts} \longrightarrow (7)$$

* Dividing (7) by (6), we get

$$\frac{E_2}{E_1} = \frac{4.44 f \phi_m N_2}{4.44 f \phi_m N_1}.$$

$$\therefore \boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = K}$$

* Comparing equations (2) and (4), we can say that the induced emf lags flux by 90°

* Since

$$\phi_m = B_m * A,$$

$$\boxed{E_1 = 4.44 f B_m A N_1}$$

$$\boxed{E_2 = 4.44 f B_m A N_2}$$

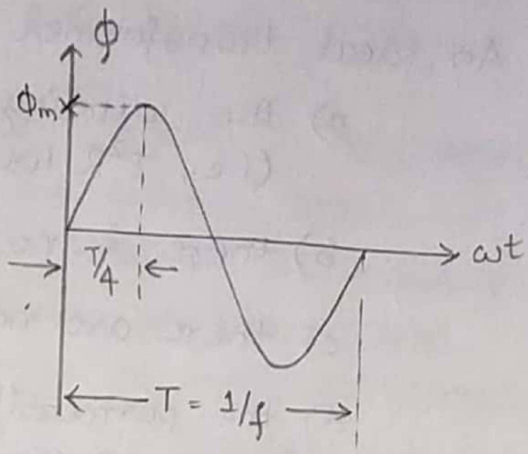
* The EMF per turn will be,

$$\frac{E_1}{N_1} = 4.44 f B_m A \qquad \frac{E_2}{N_2} = 4.44 f B_m A.$$

Therefore, emf per turn is same for primary & secondary.

Method-2

* Let N_1 = No. of turns of primary
  N_2 = No. of turns in secondary
  Φ_m = max. flux in the core (Wb)
  $\Phi_m = B_m * A$
  f = frequency of ac supply.



* When the supply voltage is sinusoidal, the current drawn & the flux set up are also sinusoidal. The flux increases from zero to Φ_m in time $T/4$ where $T = 1/f$, the periodic time.

$$\therefore \text{Average rate of change of flux} = \frac{\Phi_m - 0}{T/4 - 0} = \frac{\Phi_m}{T/4} = 4f\Phi_m$$

* As per Faraday's Law, rate of change of flux per turn is equal to the induced emf per turn (with k = 1).

$$\therefore \text{Average emf/per turn} = 4f\Phi_m \text{ volts.}$$

* The RMS value of this emf can be found using form factor as,

$$\text{RMS value of emf/turn} = \text{Average value} * \text{form factor}$$
$$= 4f\Phi_m * 1.11$$
$$= 4.44 f\Phi_m \text{ volts}$$

* RMS value of EMF induced in the entire primary winding will be

$$\text{RMS value of emf} = \text{emf per turn} * N_1$$
$$\therefore \boxed{E_1 = 4.44 f \Phi_m N_1 \text{ volts}}$$

* Similarly for the secondary,

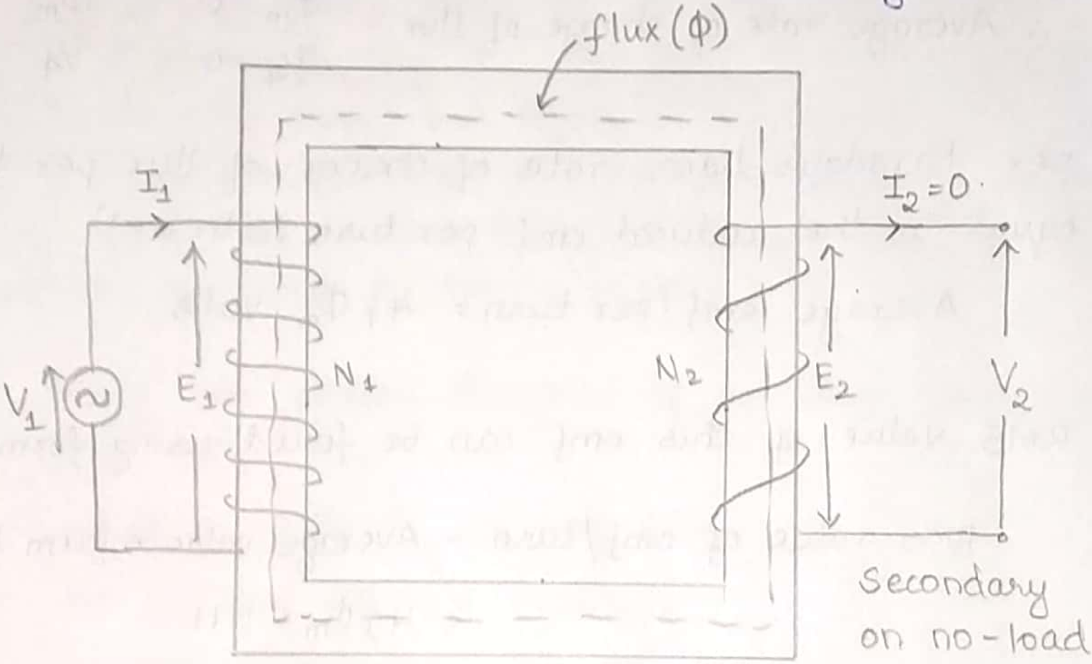
$$\boxed{E_2 = 4.44 f \Phi_m N_2 \text{ volts}}$$

Ideal Transformer

* An ideal transformer is a transformer in which
  - a) the winding resistances are neglected (i.e., I^2R loss or copper loss is zero).
  - b) there is no leakage flux
  - c) there are no iron losses in the core.
  - d) the permeability (μ_r) of the core is very large.

Ideal transformer on no-load.

* Consider an ideal transformer on no-load as shown below. Let the primary winding be supplied by an ac voltage with RMS value V_1. Since the transformer is on no-load, the current in the secondary is zero.



* The primary winding draws a current I_1 whose value is just sufficient to magnetize the core and set up flux in the core. This current is called magnetizing current (I_μ)

* Since the transformer is ideal, resistance of primary winding is zero and it is a purely inductive coil. Hence, primary current (I_1) lags supply (V_1) by 90°.

* Since this current is responsible ($I_1 = I_\mu$) for production of flux, we say that the flux and I_μ are in phase.

* The no-load current (magnetizing current) is very small and is usually only 3-4% of the current drawn by the transformer when it is fully loaded. For example for a 1.1kVA transformer operating at 50Hz, the no-load current is around 3.04% of full-load current. The value is different for different transformers.

* The flux (ϕ_m) links with both primary & secondary windings, inducing self- and mutually-induced EMFs, E_1 and E_2 respectively, in the primary & secondary windings. According to Lenz's law, the induced EMF opposes the very cause producing it. Therefore, E_1 is in antiphase with V_1 (180° away) but equal in magnitude. The induced EMF E_2 also opposes V_1, but its magnitude depends on N_2. Thus, E_2 is also in antiphase with V_1. In other words, E_1 and E_2 are in phase.

* To draw the phasor diagram of an ideal transformer on no-load,

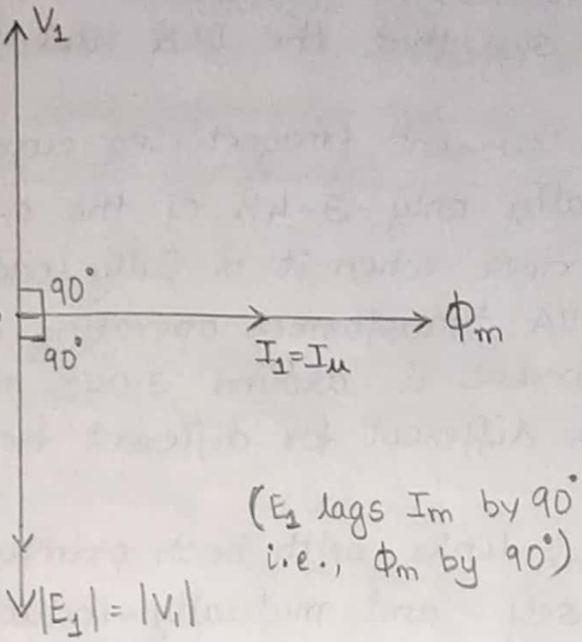
    → take flux ϕ_m as reference

    → I_μ and ϕ_m are in phase

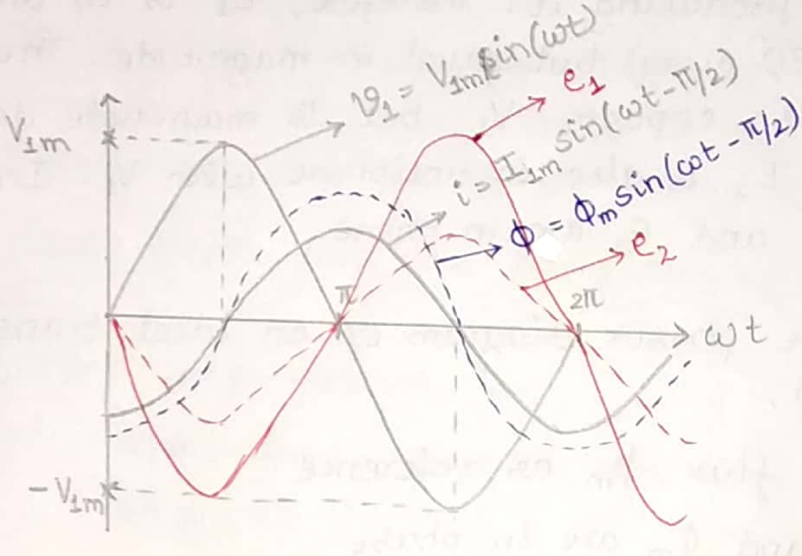
    → This current lags V_1 by 90°. Hence draw V_1 at 90° lead w.r.t I_μ.

    → Draw E_1 in antiphase with V_1 such that their magnitudes are equal.

    → Draw E_2 also in antiphase with V_1.



(phasor diagram of ideal transformer on no-load)



* The instantaneous values of applied voltage (v_1), induced EMFs (e_1 and e_2), the flux, and the magnetizing current are shown in the above waveforms.

* It is not possible to realize an ideal transformer in practice. But, the concept of ideal transformer is very helpful in understanding the working of an actual transformer.

Practical Transformer

* A practical transformer is one in which the windings have resistances, there are losses in the core, and the effect of leakage fluxes are considered.

* <u>Winding resistance</u>

→ The primary and secondary windings are made of copper and therefore have small values of resistances.

→ The primary winding resistance, R_1, is represented in series with the primary winding, while the resistance of the secondary winding, R_2, is connected in series with the secondary winding.

* <u>Iron losses</u> (core losses)

→ As the core is subjected to alternating fluxes, two kinds of losses are seen. These losses are known collectively as core losses or iron losses.

→ The core losses are the eddy current & hysteresis loss.

→ The iron loss depends on,

a) maximum value of core flux density (B_m)

b) supply frequency (f)

c) volume of the core (V)

Since all these are constants, the core losses are considered to be constant.

* <u>Leakage fluxes</u>

→ In an ideal transformer, it is assumed that the flux ϕ produced due to the primary winding current links completely with both primary and secondary windings. But in practice, this does not happen.

→ In a practical transformer, ϕ is the flux, which is linking both primary & secondary winding, and is called the mutual flux.

→ But all the flux produced due to the current in primary winding does not link with the secondary winding. A flux ϕ_{L1} completes its magnetic path through the surrounding medium and not through the core. This is called the primary leakage flux. This links only the primary winding. The primary leakage flux is proportional to the primary ampere turns $(N_1 I_1)$ alone. Due to this, an EMF e_{L1} is induced in the primary winding. This induced emf opposes the applied voltage. Hence, it is considered as an equivalent drop across a fictitious inductive reactance X_1 such that

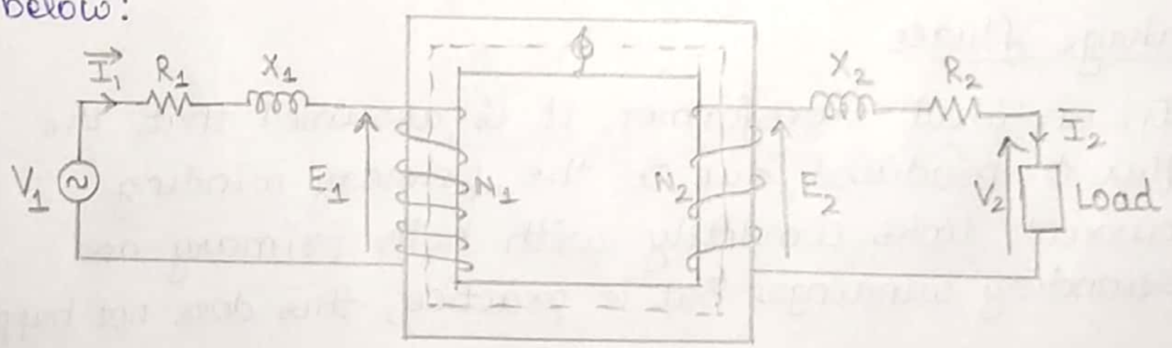
$$e_{L1} = I_1 X_1 .$$

→ Similarly when I_2 flows in the secondary winding, it creates a secondary leakage flux ϕ_{L2} which links only the secondary winding, and is due to the secondary ampere turns $(N_2 I_2)$. This induces an emf e_{L2} in the secondary winding such that e_{L2} opposes the supply voltage. This is considered as an equivalent drop across a fictitious inductive reactance X_2 such that

$$e_{L2} = I_2 X_2 .$$

→ The X_1 and X_2 are connected in series with the primary & secondary windings respectively.

* Overall, a practical transformer is represented as shown below:



Practical transformer considering iron loss but no leakage or resistance.

* Since we are considering a transformer made of a magnetic material like Silicon steel or CRGO steel, they do not have infinite permeability. In other words, the material has a finite reluctance & a finite MMF is required to set up flux in the core.

* Since the core has finite reluctance, there are losses associated with it. These losses are called iron losses or core losses.

No - Load Condition.

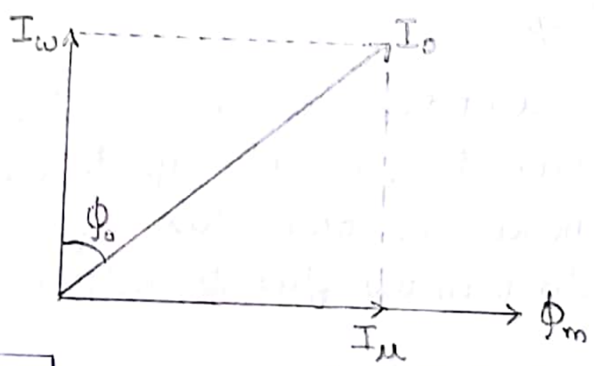
* Therefore when such a transformer is on no-load, the current in the primary should not only magnetize the core, but it should also supply the core losses. Thus, the no-load current will have two components:

  i) the magnetizing component (I_μ) that is required to set up the flux in the core, and hence I_μ is in phase with ϕ_m.

  ii) the core-loss component or the active component or the working component (I_ω)

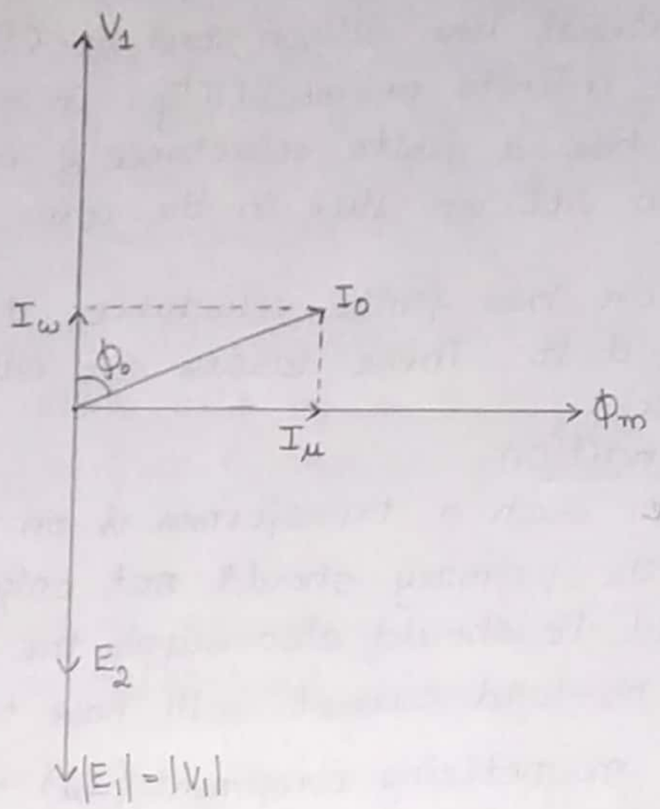
* These two components, when added vectorially, give the no-load current (I_o). Therefore,

$$\boxed{I_o = \sqrt{I_\omega^2 + I_\mu^2}}$$



$\therefore \boxed{I_\mu = I_o \sin\phi_o}$ and $\boxed{I_\omega = I_o \cos\phi_o}$

* Thus, such a transformer which is partially ideal, has a phasor diagram as shown below.



(Partly ideal transformer on no-load)

* Therefore when the ideal transformer is on no-load, the power drawn from the input is practically equal to the no-load (core) losses (neglecting the small primary copper loss).

Transformer on load

* When the secondary is loaded, secondary current I_2 is set up. The magnitude of I_2 is determined by the load characteristics.

* Due to the secondary current I_2 flowing in the primary turns N_2, it sets up its own secondary MMF $N_2 I_2$ and hence its own flux ϕ_2 which is in opposition to the main primary flux ϕ, which is due to I_0.

Scanned by CamScanner

* The opposing secondary flux ϕ_2 weakens the primary flux momentarily and the primary induced EMF E_1 tends to reduce. For a moment V_1 gains the upper hand over E_1. Thus, the primary current increases. The current drawn is due to the secondary flux ϕ_2. This current is known as the load component of primary current and is represented as I_2'.

* This current is in phase opposition with I_2. The additional current sets up a primary MMF $N_2 I_2'$ with sets up a flux ϕ_2' which is equal & opposite to ϕ_2.

* Thus, the effect of secondary current gets neutralized immediately by additional primary current I_2'.

* Hence, whatever may be the conditions of the load, the net-flux passing through the core is approximately the same as at no-load.

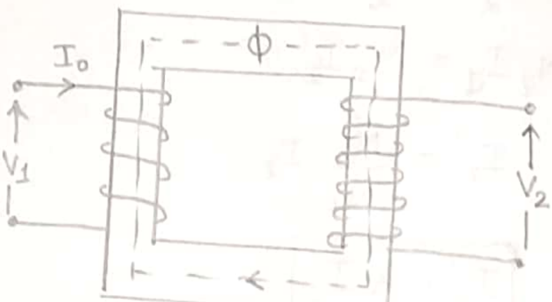


fig-(a)

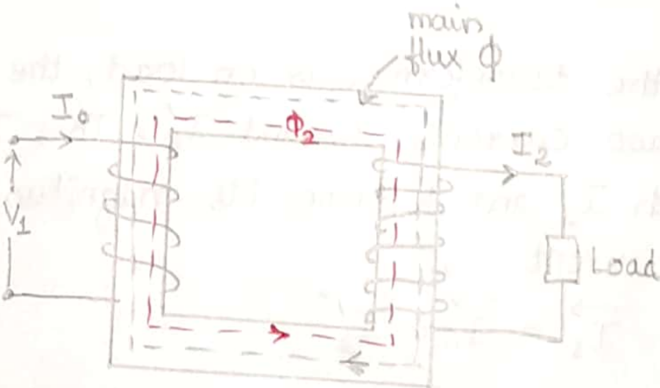


fig-(b)

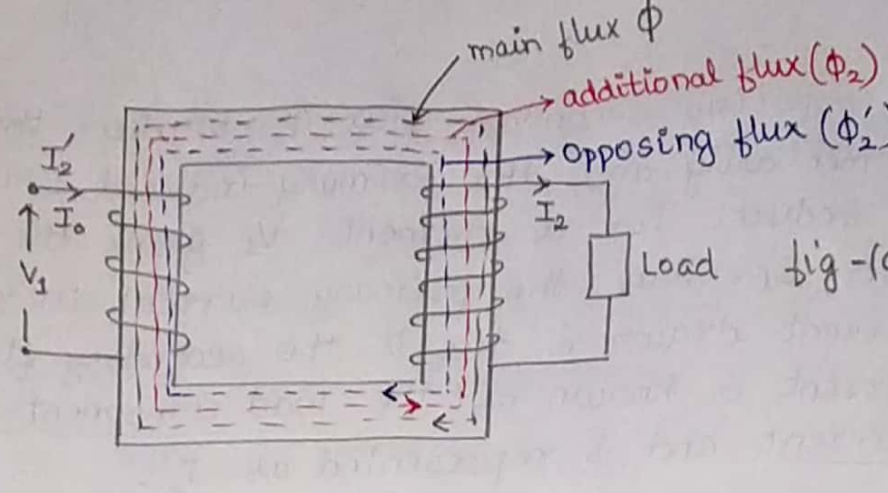


fig -(c)

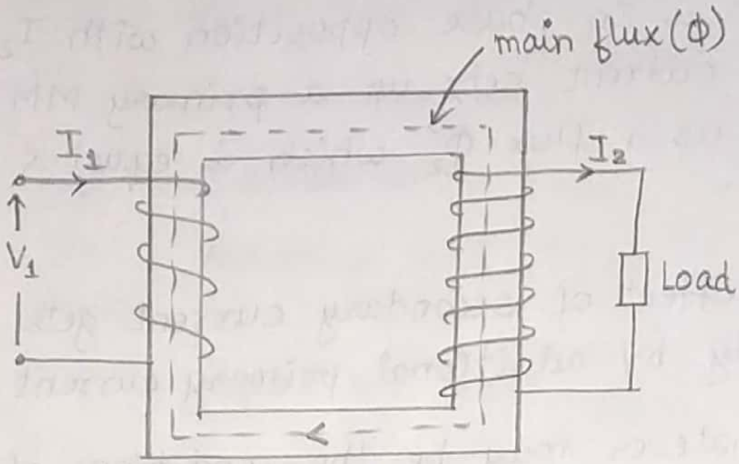


fig - (d)

* Figures (a) to (d) describe the above process.

$$\phi_2 = \phi_2'$$

$$N_2 I_2 = N_1 I_2'$$

$$I_2' = \frac{N_2}{N_1} I_2$$

$$\boxed{I_2' = k I_2}$$

* Hence, when the transformer is on load, the primary winding has two currents I_0 and I_2'. This I_2' is in antiphase with I_2 and k times its magnitude. Thus, the primary current is,

$$\vec{I_1} = \vec{I_0} + \vec{I_2'}$$

Scanned by CamScanner

* When partly ideal transformer is on <u>purely resistive load</u>,

→ take flux as reference

→ I_μ is in phase with ϕ_m

→ I_w is at $90°$ to I_μ (and ϕ_m)

→ The resultant of I_μ and I_w is I_0

→ V_1 is at $90°$ lead w.r.t I_μ (and ϕ_m)

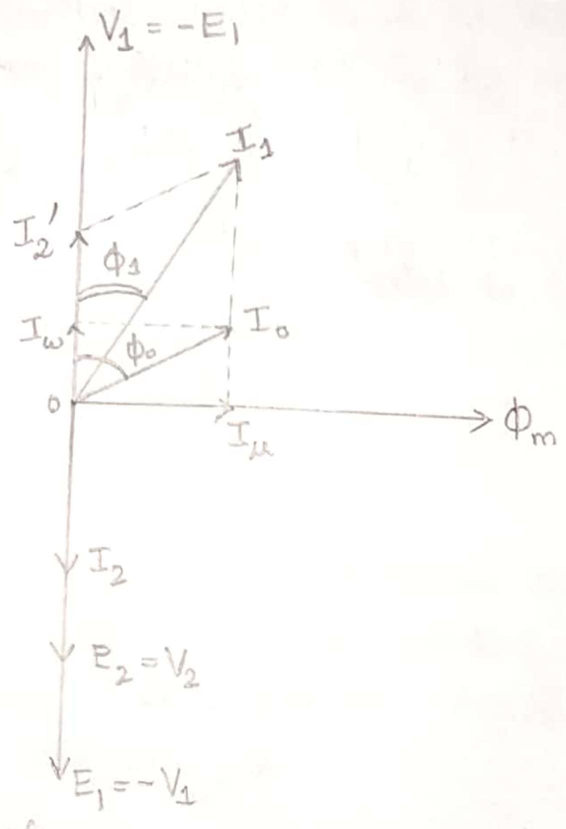
→ E_1 is in antiphase with V_1, and $|E_1| = |V_1|$.

→ E_2 is in phase with E_1.

→ For a resistive load, I_2 and E_2 are in phase.

→ I_2' is equal and opposite to I_2.

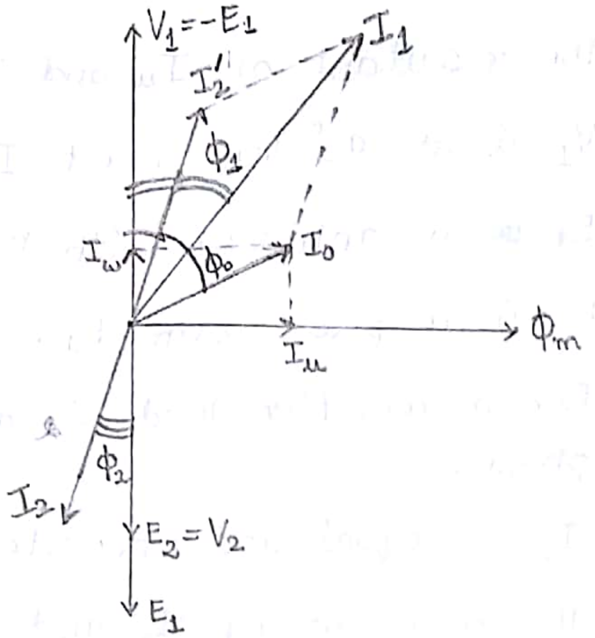
→ The resultant of I_2' and I_0 is I_1.



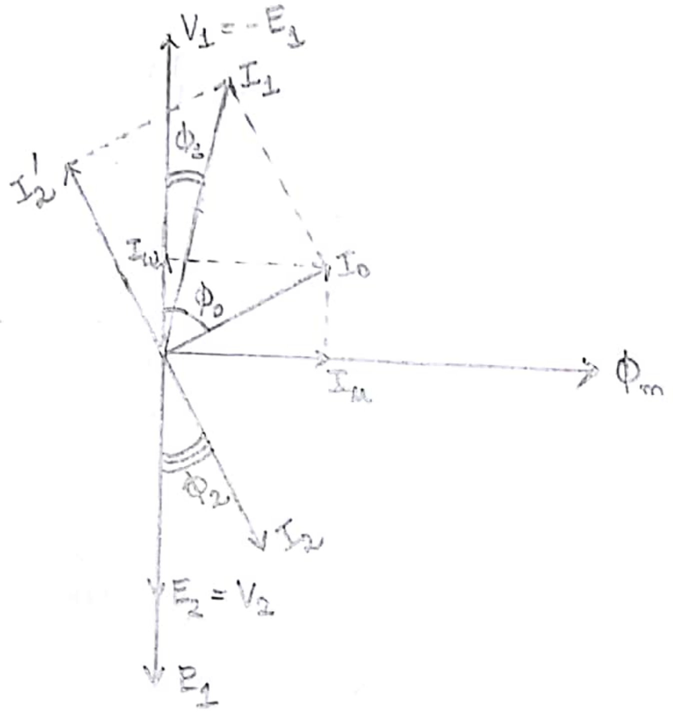
(partly ideal transformer on resistive load)

* When on <u>inductive load</u>, current I_2 lags E_2 by ϕ_2. Then I_2' will be in anti-phase with I_2.

* When on <u>capacitive load</u>, current I_2 leads E_2 by ϕ_2 and I_2' will be in anti-phase with I_2.



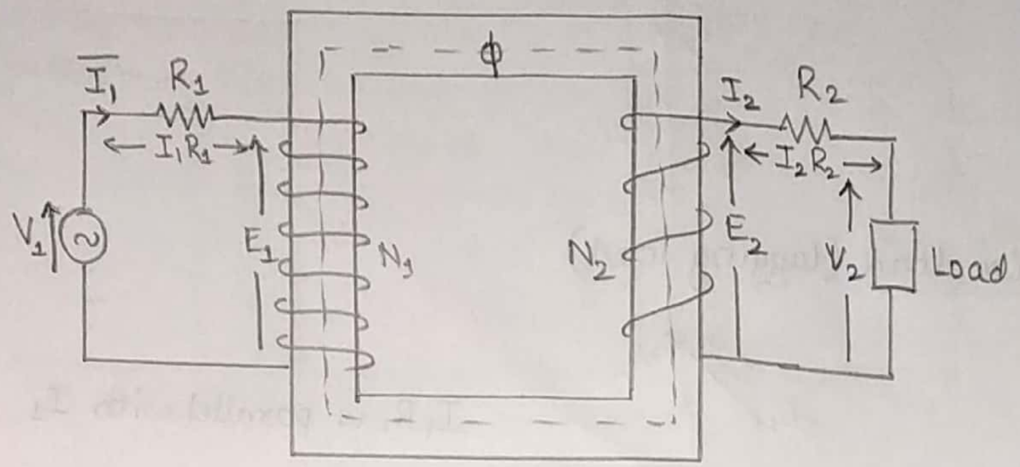
(Partly ideal transformer on inductive load)



(Partly ideal transformer on capacitive load)

Practical transformer considering iron loss & resistance but no magnetic leakage.

* The representation of a transformer considering only resistance but no leakage fluxes is shown below.



* Due to this resistance, there is some voltage drop in the windings. The result is that :

  a) the secondary terminal voltage V_2 is vectorially less than the secondary induced EMF E_2 by an amount I_2R_2. Therefore,

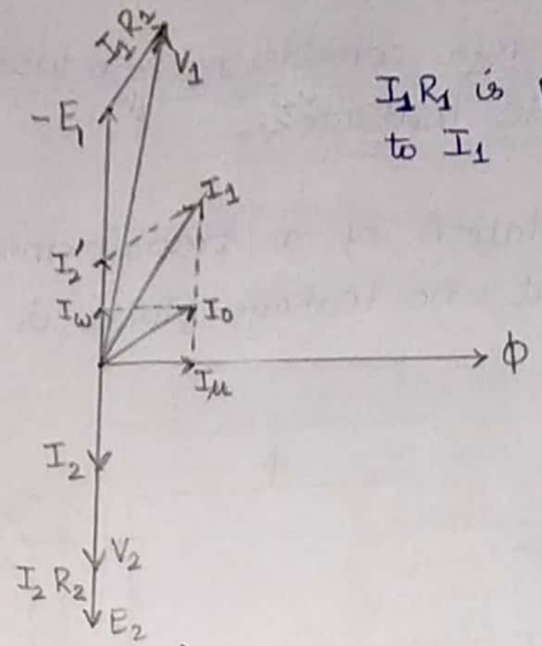
$$\boxed{\vec{V_2} = \vec{E_2} - \vec{I_2}R_2}$$

  b) the primary induced emf E_1 is equal to the phasor difference of V_1 and I_1R_1. i.e.,

$$\boxed{\vec{E_1} = \vec{V_1} - \vec{I_1}R_1}$$

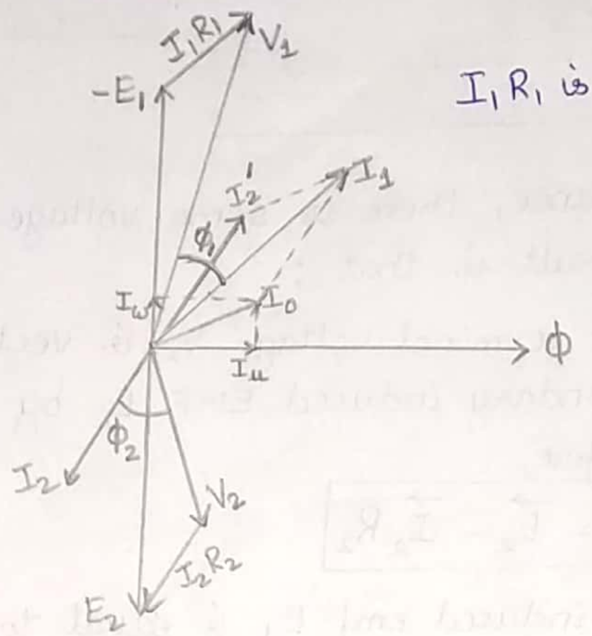
* Thus, the vector diagram for transformer on load while considering the iron losses & winding resistance & neglecting leakage fluxes should incorporate the above phasor relationships.

* Resistive load



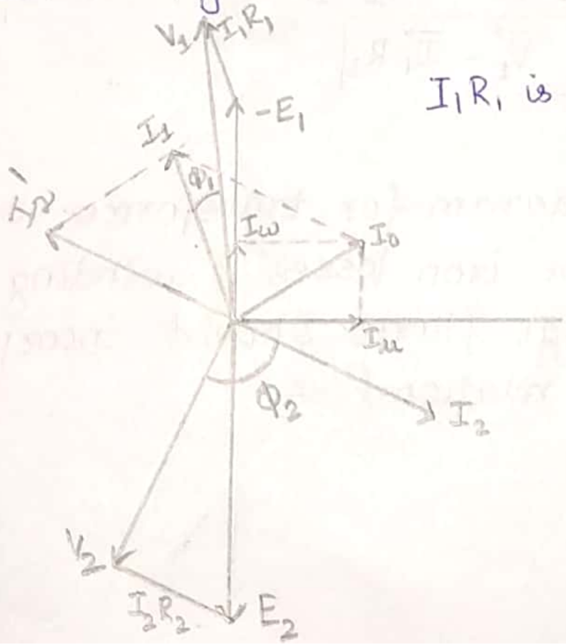
I_1R_1 is parallel to I_1

* Inductive load (lagging load)



I_1R_1 is parallel with I_1

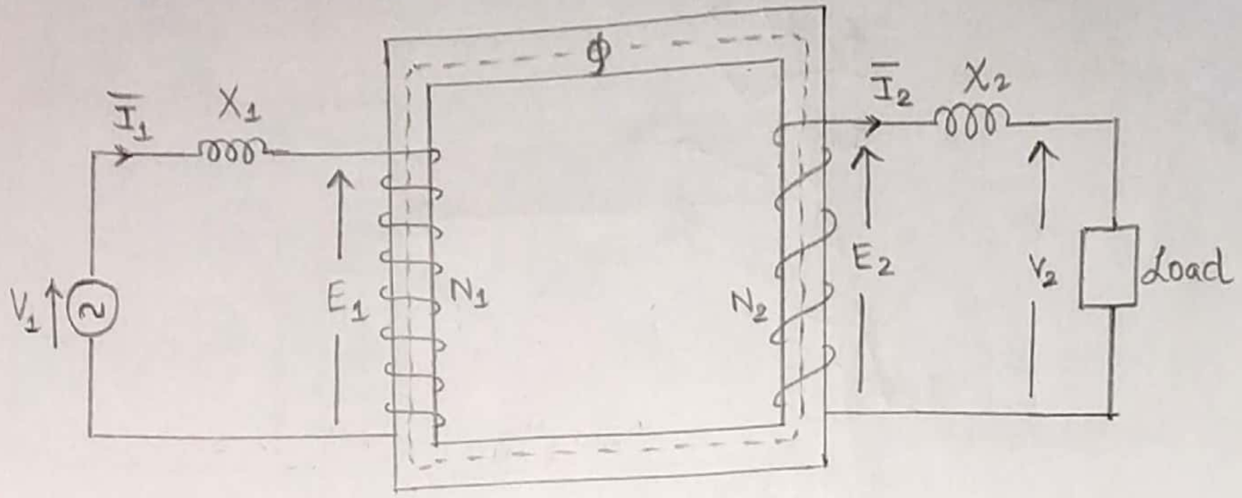
* Capacitive load (leading load)



I_1R_1 is parallel with I_1.

Practical transformer considering iron loss and magnetic leakage, but no resistance.

* The representation of a transformer considering magnetic leakage, but no resistance is shown below.

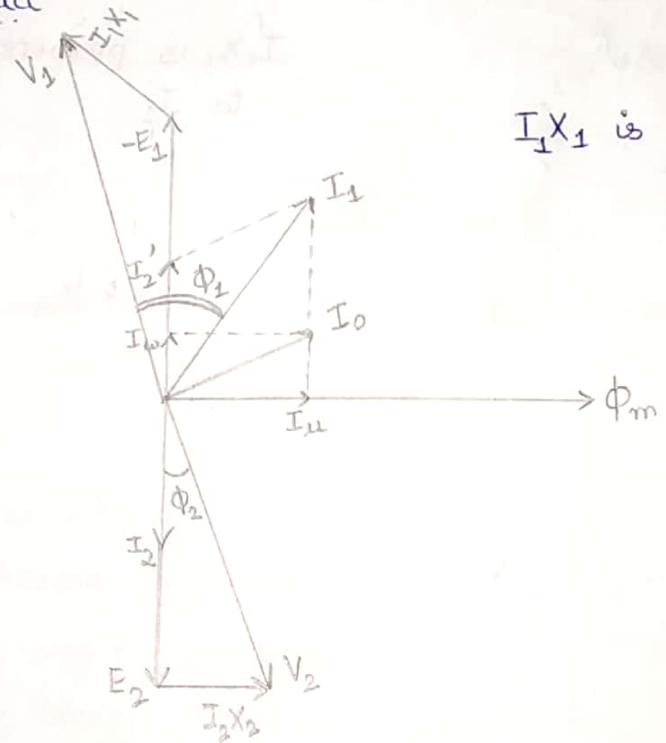


* The vector relationships are:

$$\bar{V}_2 = \bar{E}_2 - \bar{I}_2 X_2$$

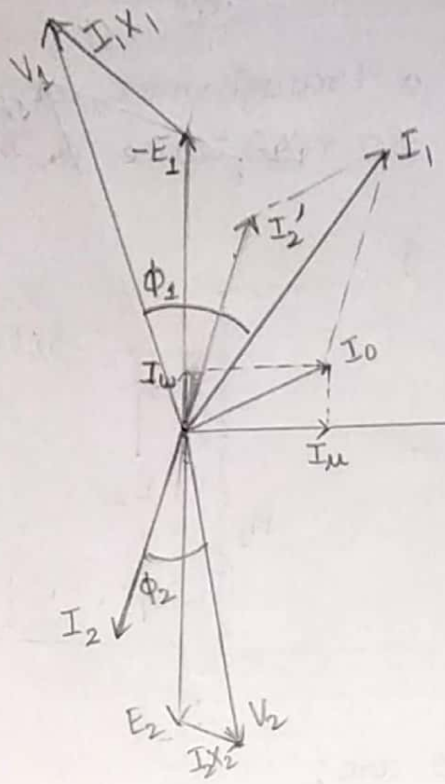
$$\bar{E}_1 = \bar{V}_1 - \bar{I}_1 X_1$$

* Resistive load



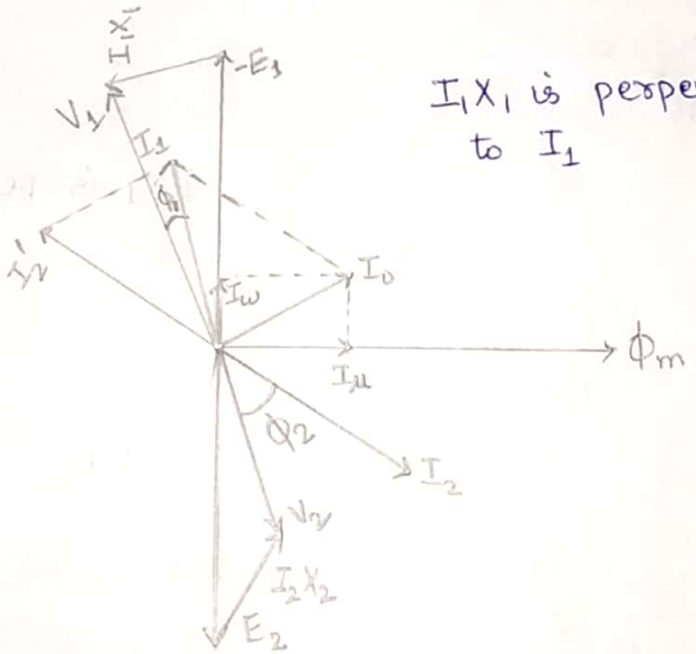
I_1X_1 is perpendicular to I_1

* <u>Inductive load</u>



I_1X_1 is perpendicular to I_1

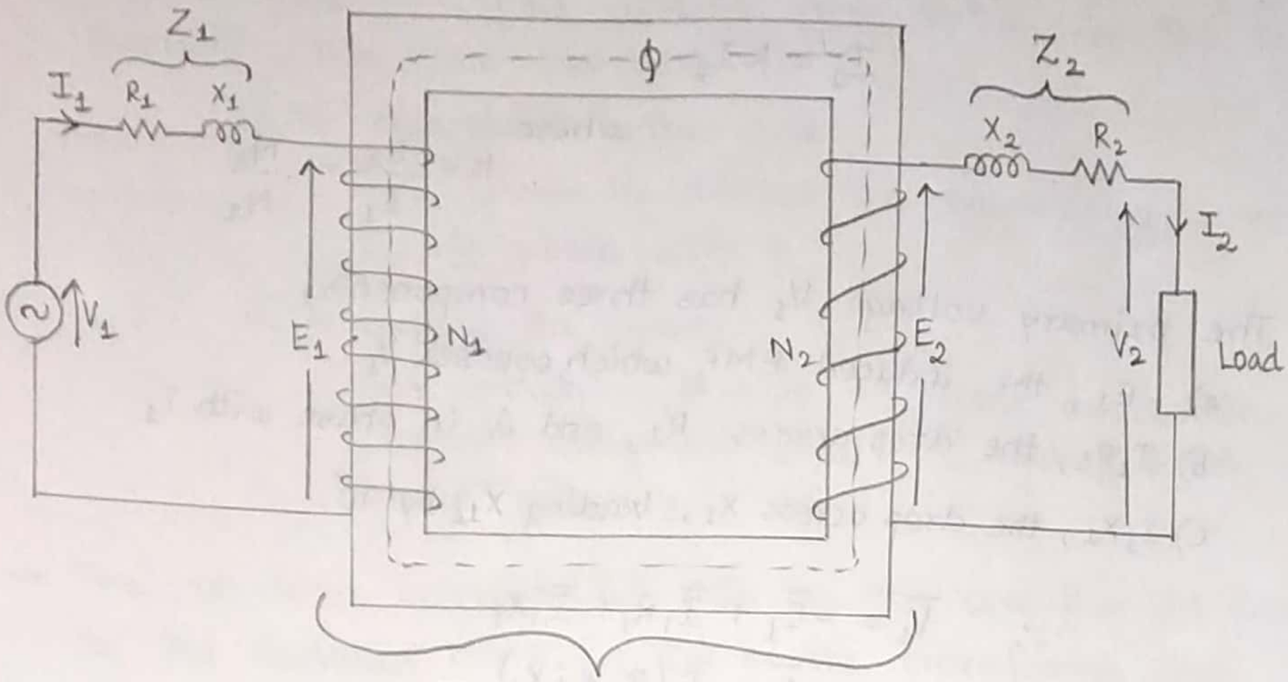
* <u>Capacitive load</u>



I_1X_1 is perpendicular to I_1

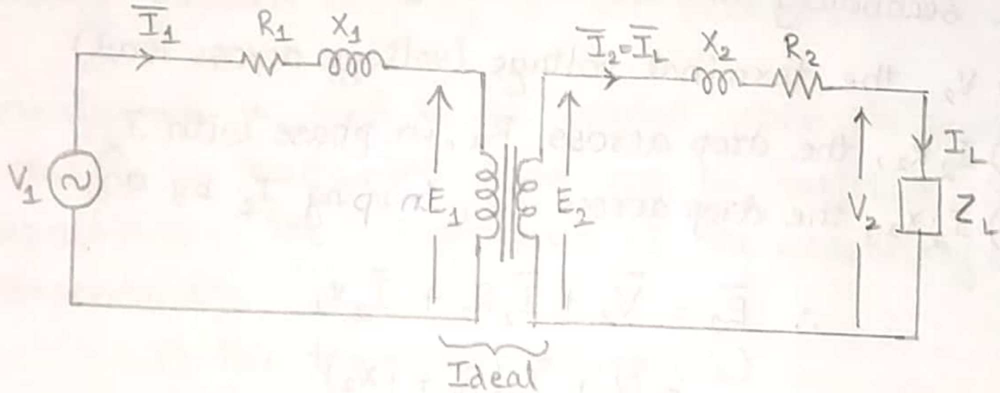
Practical transformer with leakage flux, resistances & losses.

* A practical transformer considering all the non-idealities are shown below.



Ideal transformer.

* The equivalent circuit is as shown below.



* Let,

R_1 = Primary winding resistance

X_1 = Primary leakage reactance

R_2 = Secondary winding resistance

X_2 = Secondary leakage reactance

Z_L = Load impedance

$\overline{I}_1$ = Primary current

$\overline{I}_2$ = Secondary current = $\overline{I}_L$ = Load current

* Now,

$$\bar{I}_1 = \bar{I}_2' + \bar{I}_0$$

where

I_0 = no-load current

I_2' = load component of primary current

$$I_2' = k I_2$$

where

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

* The primary voltage V_1 has three components,
  a) $-E_1$, the induced EMF which opposes V_1
  b) I_1R_1, the drop across R_1, and is in phase with I_1
  c) I_1X_1, the drop across X_1, leading I_1 by 90°.

$$\therefore \quad \bar{V}_1 = -\bar{E}_1 + \bar{I}_1R_1 + \bar{I}_1X_1$$

$$= -\bar{E}_1 + \bar{I}_1(R_1 + jX_1)$$

$$\boxed{\bar{V}_1 = -\bar{E}_1 + Z_1\bar{I}_1}$$

* The secondary induced EMF, E_2, also has three components,
  a) V_2, the terminal voltage (voltage across load)
  b) I_2R_2, the drop across R_2, in phase with I_2
  c) I_2X_2, the drop across X_2, leading I_2 by 90°.

$$\therefore \quad \bar{E}_2 = \bar{V}_2 + \bar{I}_2R_2 + \bar{I}_2X_2$$

$$= \bar{V}_2 + \bar{I}_2(R_2 + jX_2)$$

$$\boxed{\bar{E}_2 = \bar{V}_2 + \bar{I}_2Z_2}$$

* The phasor diagram of a practical transformer on load depends on the load type.

* Practical transformer operation

→ When the secondary of the transformer is kept open (i.e., the transformer is on no-load), the primary winding draws the no-load current from the supply. This no-load current has two functions:

i) to magnetize the core
for which it draws the magnetizing current I_μ, in phase with ϕ

ii) to supply the losses
for which it draws an active or working current component (I_ω), 90° leading ϕ. (or, in phase with V_1).

→ The no-load current (I_0) sets up the core flux (ϕ) due to the induced MMF ($N_1 I_0$) in the transformer core.

→ This flux links both primary & secondary windings and induces a self-induced EMF E_1 in the primary and a mutually induced emf E_2 in the secondary.

→ A transformer is said to be loaded when its secondary is connected to a load. The load can be resistive, inductive or capacitive. The magnitude of the secondary current (I_2) depends on

i) the terminal voltage (V_2)

ii) the load impedance (Z_L)

The phase angle between the secondary current and the voltage depends on the type of the load. If the load is resistive ($Z_L = R$) then I_2 and V_2 are in phase. If the load is inductive, then I_2 lags V_2 by ϕ_2. If the load is capacitive, then I_2 leads V_2 by ϕ_2.

→ When current I_2 flows through the secondary winding, it sets up its own MMF $N_2 I_2$, which produces a flux ϕ_2, which opposes the main flux ϕ, tending to decrease the core flux momentarily. Hence $N_2 I_2$ are called '<u>demagnetizing ampere-turns</u>'.

→ Momentarily, due to the opposing flux ϕ_2, the flux in the core gets reduced. Hence, the EMF E_1 induced in the primary winding also gets reduced. This increases the difference between V_1 and E_1, and the primary draws an extra current I_2', which is known as the <u>load component of primary current</u>. The current I_2' is in antiphase with I_2.

→ Now the primary winding sets up extra ampere turns $N_1 I_2'$ which sets up its own flux ϕ_2', which is equal in magnitude to ϕ_2 but in opposite direction. ϕ_2 and ϕ_2' cancel each other and hence, the flux in the core remains constant at ϕ.

→ Under load conditions, the primary current is the phasor sum of I_0 and I_2'

$$\therefore \bar{I_1} = \bar{I_0} + \bar{I_2'}$$

→ The primary and secondary winding impedances are,

$$Z_1 = R_1 + jX_1$$

$$Z_2 = R_2 + jX_2.$$

→ Therefore, for primary winding,

$$V_1 = -E_1 + I_1 Z_1 = -E_1 + I_1 R_1 + j I_1 X_1$$

and for secondary winding,

$$E_2 = V_2 + I_2 Z_2 = V_2 + I_2 R_2 + j I_2 X_2.$$

* Phasor diagram for resistive load

→ Consider flux ϕ as reference

→ Draw I_μ in phase with ϕ.

→ E_1 lags ϕ by 90°. Reverse E_1 to get $-E_1$. This is equal to V_1.

→ E_1 and E_2 are in phase.

→ Draw I_ω in phase with V_1. The resultant of I_μ and I_ω is I_o.

→ Assume V_2 in a particular direction

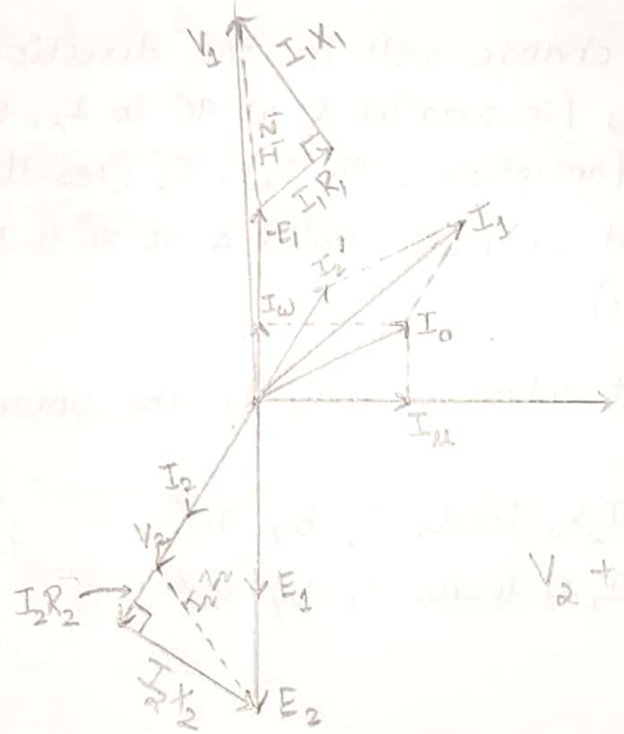
→ I_2 is in-phase with V_2 ($\phi_2 = 0$ for resistive load)

→ Starting from V_2, draw I_2R_2 in parallel to I_2. Then draw I_2X_2 perpendicular to I_2, starting from I_2R_2. The resultant of V_2, I_2R_2 and I_2X_2 is E_2.

→ Reverse I_2 to get I_2'

→ The resultant of I_2' and I_o is I_1.

→ Draw I_1R_1 (from $-E_1$) in parallel to I_1, and I_1X_1 at 90° to I_1. The resultant of $-E_1$, I_1R_1 and I_1X_1 is V_1.

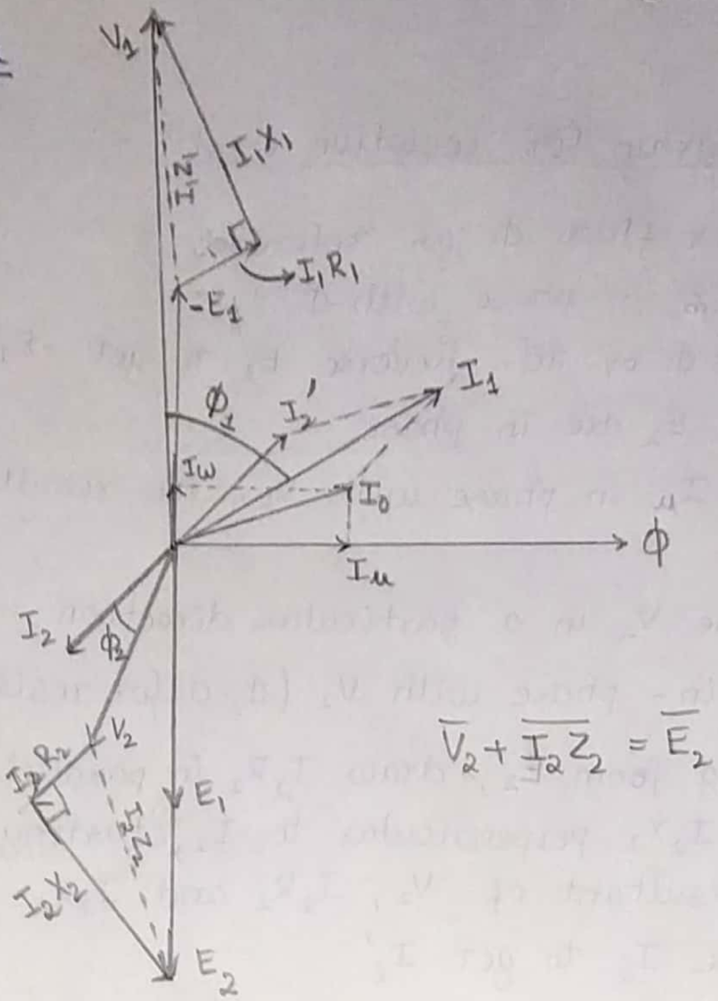


$I_2' = -I_2$

$-E_1 + I_1R_1 + I_1X_1 = V_1$

$V_2 + I_2R_2 + I_2X_2 = E_2$

* Inductive load
(lagging load)



$$\overline{V_2} + \overline{I_2 Z_2} = \overline{E_2}$$

→ For an inductive load, power factor $\cos\phi_2$ is lagging. The current I_2 lags V_2 by ϕ_2.

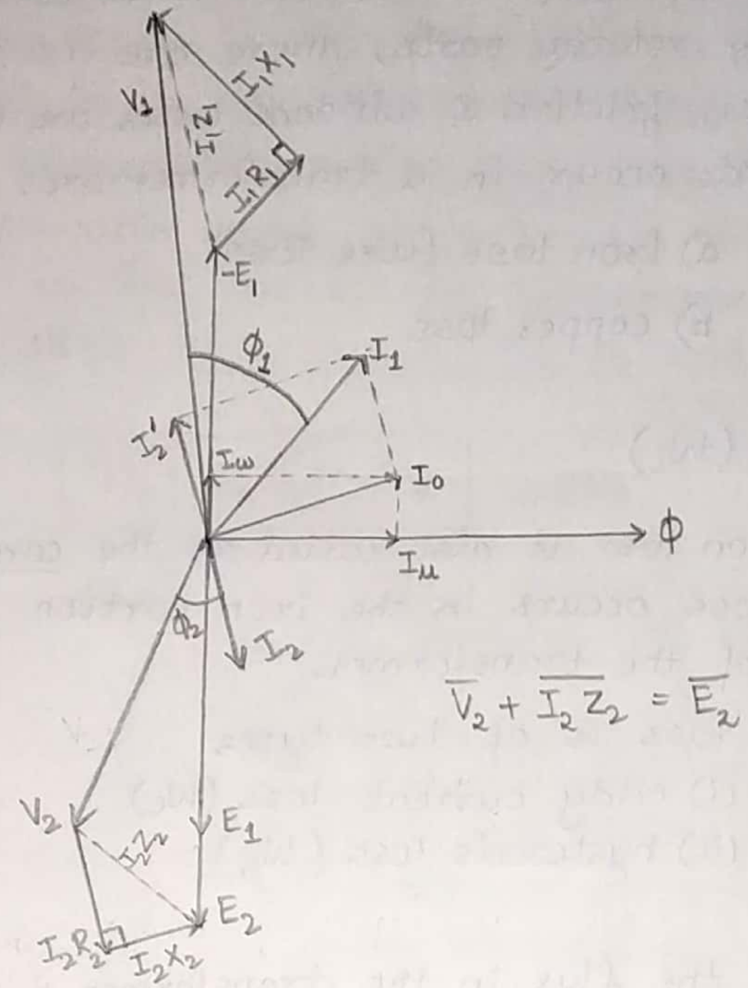
→ So the first change in phasor diagram is to draw I_2 lagging V_2 by ϕ_2 in step-7 discussed earlier.

→ The next change will be the directions of I_2R_2, I_2X_2 (in parallel & at 90° to I_2, starting from V_2), I_2' (antiphase with I_2), I_1 (resultant of I_0 & I_2'), I_1R_1 and I_1X_1 (in parallel & at 90° to I_1, starting from $-E_1$).

→ Note that whatever may be the power factor of the load,

- I_2X_2 leads I_2 by 90°
- I_1X_1 leads I_1 by 90°

* Capacitive load (leading load)



→ For a capacitive load, I_2 leads V_2. In addition to this, the direction of I_2R_2, I_2X_2, I_2', I_1, I_1R_1 and I_1X_1 all change.

→ Following steps similar to the steps discussed earlier, the phasor diagram is drawn.

Note:-

* Regardless of the load type,

$\phi_1 \rightarrow$ Angle between V_1 and I_1

$\phi_2 \rightarrow$ Angle between V_2 and I_2

Scanned by CamScanner

Losses in a transformer

* As the transformer is a static device and does not contain any rotating parts, there are no mechanical losses i.e., friction & windage losses are absent. The losses that occur in a transformer are:

a) iron loss (core loss)

b) copper loss

* Iron loss (W_i)

→ The iron loss is also called as the core loss and this loss occurs in the iron portion i.e., the core of the transformer.

→ Iron loss is of two types

(i) eddy current loss (W_e)

(ii) hysteresis loss (W_h)

→ Since the flux in the transformer is alternating, some power is required for the continuous reversal of the core molecules which form the electromagnet. Since this power is not sent to the load but is dissipated as heat, it is a loss component & is called hysteresis loss. It depends on the core flux density, the frequency of flux reversal and on the core volume, & is given by

$$\boxed{W_h = K_h B_m^{1.6} f V} \text{ watts}$$

where

K_h = a constant whose value depends on core material

B_m = maximum value of core flux density (Wb/m^2)

f = supply frequency (Hz)

V = core volume (m^3).

→ The flux setup in the core not only links with the primary and secondary windings, but also in the metallic core itself. Therefore an emf will be induced in it and a current flows in the core. These currents are called eddy currents. These eddy currents cause power loss in the core and heats up the core of the transformer. These losses are called eddy current losses and are given by,

$$\boxed{W_e = K_e B_m^2 f^2 t^2 V} \text{ watts}$$

where

K_e = a constant that depends on core material

B_m = maximum value of core flux density (Wb/m^2)

f = supply frequency (Hz)

t = thickness of core (in m) (or thickness of core laminations)

V = core volume (m^3).

→ Therefore,

Iron loss = $W_e + W_h$

$$\therefore \boxed{W_i = K_e B_m^2 f^2 t^2 V + K_h B_m^{1.6} f V}$$

From this, we see that iron loss in the transformer depends on the maximum flux density (B_m) & supply frequency (f), as other parameters like, the thickness and volume of the core are constants. As long as the supply voltage remains constant, B_m and f remain constants. Therefore, iron loss in the transformer is considered to be a constant from no-load to full-load. The core losses are therefore considered to be a constant for all practical purposes.

* ## Copper loss (W_{cu})

→ The copper losses occur due to the ohmic resistances in both the primary & secondary windings.

$$\text{Cu loss in primary} = I_1^2 R_1$$
$$\text{Cu loss in secondary} = I_2^2 R_2$$

∴ Total copper loss is,

$$\boxed{W_{cu} = I_1^2 R_1 + I_2^2 R_2}$$

→ The copper loss depends on I_1 and I_2, which vary with load. Hence, copper loss in the transformer is a variable loss.

* The total loss in a transformer is,

$$\boxed{\text{Transformer loss} = \text{Iron loss} + \text{Copper loss}}$$
$$= (W_e + W_h) + W_{cu}$$

$$\boxed{\text{Total transformer loss} = k_e B_m^2 f^2 t^2 V + K_h B_m^{1.6} f V + I_1^2 R_1 + I_2^2 R_2}$$

Reduction of iron losses

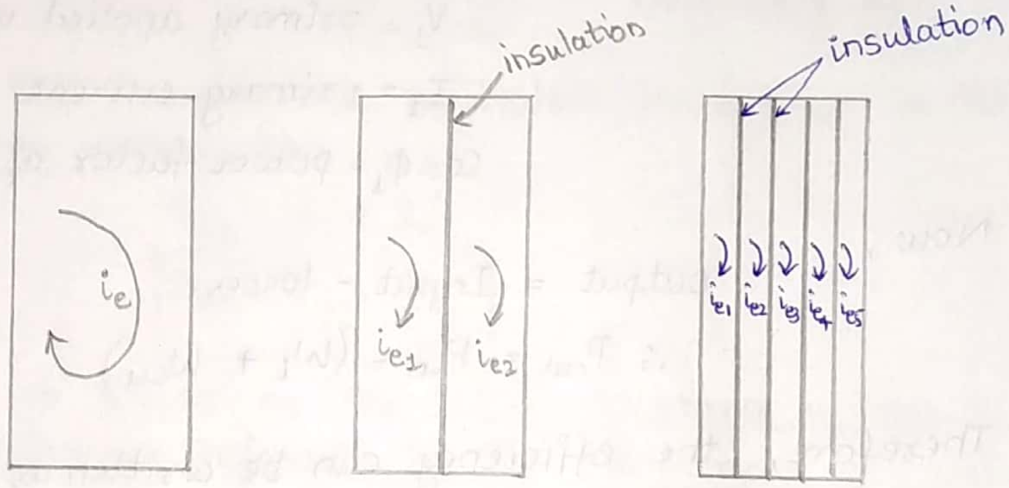
* Hysteresis loss can be reduced by selecting a core material with low hysteresis coefficient (k_h) and high permeability.

* Eddy current loss can be reduced by laminating the core using thin sheets of core material which are insulated from each other.

* The core of a transformer is constructed by stacking thin pieces known as laminations. The laminations are insulated from each other by thin layers of insulating materials like varnish, paper or mica.

* If the transformer core were to be constructed using a single block, the effective area available for the flow of eddy currents would be high. Since $R \propto 1/a$, the core would pose less resistance, leading to high eddy currents & higher eddy current losses.

* When laminations are used, they restrict the paths of eddy currents to respective laminations only (since the laminations are insulated). So the area through which eddy currents can flow, decreases. This increases the resistance, and in turn considerably reduces the magnitude of eddy currents. This reduces the eddy current losses.



(Solid core) (thick laminations) $(i_{e1} + i_{e2}) < i_e$ (thin laminations) $(i_{e1} + i_{e2} + \dots + i_{e5}) < i_e$

* Use of laminations also help reduce the width of hysteresis loop & thereby reduce hysteresis losses.

* Hence, core loss can be reduced by laminating the core using laminations of high permeability and low hysteresis coefficient.

Transformer Efficiency

* The efficiency of a transformer at a particular load & power factor is defined as the ratio of the output power to the input power.

$$\therefore \text{Efficiency} = \frac{\text{Output power in watts}}{\text{Input power in watts}}$$

* It is represented as η.

$$\therefore \boxed{\eta = \frac{P_{out}}{P_{in}}}$$

* The input power of the transformer is

$$P_{in} = V_1 I_1 \cos\phi_1$$

where

V_1 = primary applied voltage

I_1 = primary current

$\cos\phi_1$ = power factor of primary.

* Now,

$$\text{output} = \text{Input} - \text{losses}.$$

$$\therefore P_{out} = P_{in} - (W_i + W_{cu})$$

* Therefore, the efficiency can be written as

$$\eta = \frac{\text{Input} - \text{losses}}{\text{Input}}$$

$$\boxed{\eta = \frac{P_{in} - W_i - W_{cu}}{P_{in}}}$$

Equivalent resistance & reactances

* While calculating the losses in a transformer we have to deal with two windings. If we transfer all the resistances to one side (primary or secondary), the calculations would be simpler.

* The copper loss in secondary is $I_2^2 R_2$. This loss is supplied by the primary which takes a current of I_1. Thus, R_2' is the equivalent resistance in the primary which would have caused the same loss as R_2 in the secondary.

$$\therefore I_1^2 R_2' = I_2^2 R_2$$

$$R_2' = \frac{I_2^2}{I_1^2} R_2$$

Since $\frac{I_1}{I_2} = K$, we get

$$R_2' = \frac{R_2}{K^2} \quad \text{(neglecting } I_0\text{)}$$

* Then the transformer equivalent impedance in the primary would be,

$$R_{01} = R_1 + R_2'$$

$$\boxed{R_{01} = R_1 + \frac{R_2}{k^2}}$$

This is called as the total resistance of the transformer referred to the primary.

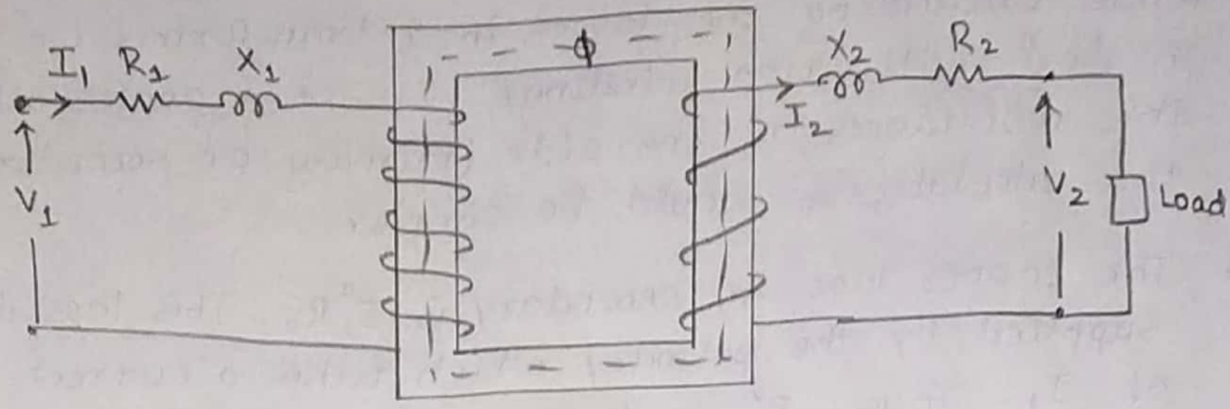
* In a similar manner, the total resistance of the transformer referred to the secondary is,

$$\boxed{R_{02} = R_2 + k^2 R_1}$$

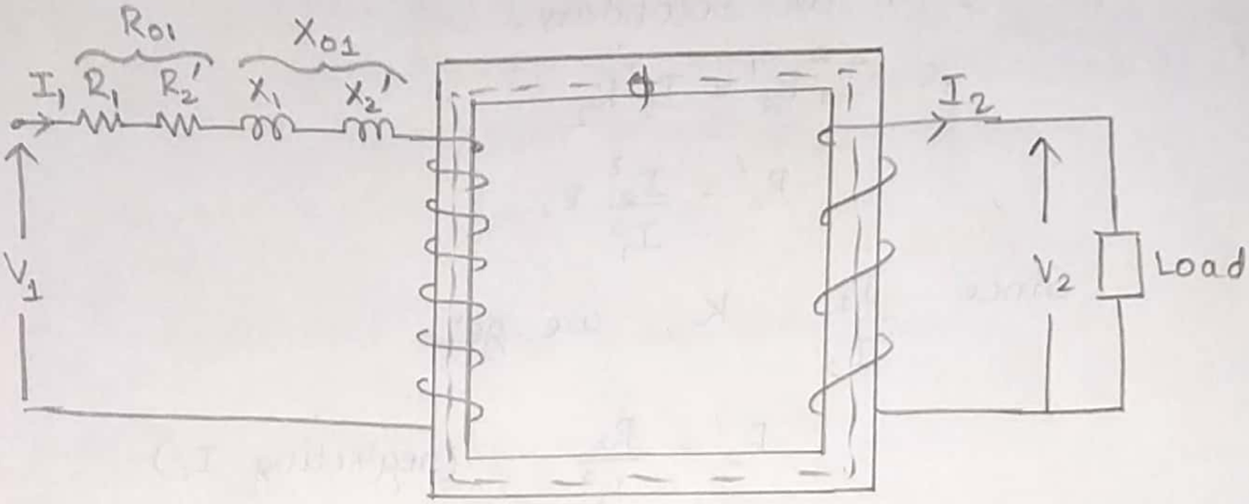
* For reactances we can write

$$\boxed{X_{01} = X_1 + X_2/k^2} \text{ ---- referred to primary}$$

$$\boxed{X_{02} = X_2 + k^2 X_1} \text{ ---- referred to secondary.}$$



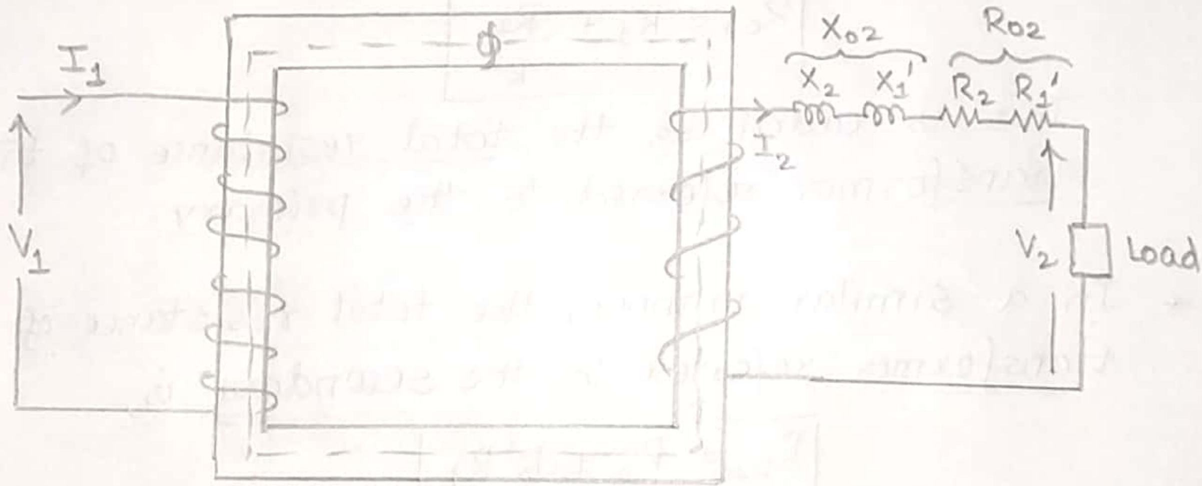
(Actual transformer)



(Referred to primary)

Total Cu loss $= I_1^2 (R_1 + R_2') = I_1^2 R_{01}$

and $Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$



(Referred to secondary)

Total cu loss $= I_2^2 (R_2 + R_1') = I_2^2 R_{02}$

and $Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$

Condition for maximum efficiency

* Efficiency of a transformer is,

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{P_{out}}{P_{out} + \text{losses}}$$

* The output power is,

$$P_{out} = V_2 I_2 \cos\phi_2 .$$

* Let W_i be the iron loss and W_{cu} be the copper loss at any given load. If R_{02} is the total resistance of the transformer referred to the secondary, then

$$W_{cu} = I_2^2 R_{02} .$$

* Therefore the efficiency can be written as,

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}}$$

dividing both numerator & denominator by I_2,

$$\Rightarrow \eta = \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + (W_i/I_2) + I_2 R_{02}} \longrightarrow (1).$$

* The efficiency will be maximum when the denominator of equation (1) above is minimum, for which we must have

$$\frac{d}{dI_2}\left[V_2 \cos\phi_2 + (W_i/I_2) + I_2 R_{02}\right] = 0$$

$$\Rightarrow \frac{d}{dI_2}(V_2 \cos\phi_2) + \frac{d}{dI_2}\left(\frac{W_i}{I_2}\right) + \frac{d}{dI_2}(I_2 R_{02}) = 0$$

→ $V_2 \cos\phi_2$ does not depend on I_2. Hence its differential is zero.

→ W_i does not depend on I_2 & is a constant w.r.t I_2.

$$\therefore\ 0 - \frac{W_i}{I_2^2} + R_{02} = 0$$

$$\Rightarrow \frac{W_i}{I_2^2} = R_{02}$$

$$\Rightarrow \boxed{W_i = I_2^2 R_{02}}$$

$$\Rightarrow \boxed{\text{Iron loss} = \text{Copper loss}}$$

* Thus, the efficiency at a given terminal voltage and load power factor is maximum for such a load current I_2 which makes the variable loss (copper loss) equal to the constant losses (iron losses).

* This load current at which efficiency is maximum can be calculated as follows:

$$I_2^2 R_{02} = W_i$$

$$\Rightarrow I_2^2 = \frac{W_i}{R_{02}}$$

$$\Rightarrow \boxed{I_2 = \sqrt{\frac{W_i}{R_{02}}}}$$

Since $R_{02} = R_2 + R_1' = R_2 + (R_1/k^2)$

$$\Rightarrow \boxed{I_2 = \sqrt{\frac{W_i}{R_2 + (R_1/k^2)}}} \text{ ---- gives } \eta_{max}.$$

This is the load current corresponding to η_{max}

Load corresponding to maximum efficiency

* Once the losses in the transformer are obtained using transformer tests, we can determine the load (in kVA) at which the transformer operates at maximum efficiency.

* Let

x = kVA output at which efficiency is maximum.

W_i = Iron loss (which is constant)

W_{cu} = full- load copper loss

* The efficiency is maximum when the iron loss W_i is equal to full-load copper loss W_{cu}.

* Since 'x' is the kVA output at which efficiency is maximum,

$$W_{cu} \propto (\text{full load kVA})^2$$

As W_i is equal to the copper loss at 'x' kVA,

$$W_i \propto x^2$$

Thus,

$$\frac{x^2}{(\text{full-load kVA})^2} = \frac{W_i}{W_{cu}}$$

$$\therefore \boxed{x = (\text{full load kVA}) * \sqrt{\frac{W_i}{W_{cu}}}}$$

$$\therefore \boxed{x = (\text{full load kVA}) * \sqrt{\left(\frac{\text{Iron loss}}{\text{Full load copper loss}}\right)}}$$

* Thus, for maximum efficiency,

$$W_{cu} = W_i$$

When this happens, the load will have a particular kVA, given by

$$\therefore \boxed{x = \left(\sqrt{\frac{\text{Iron loss}}{\text{Full load copper loss}}}\right) * (\text{full load kVA})}$$

Efficiency at any load & power factor

* The efficiency at any load and power factor is given by

$$\boxed{\eta_x = \frac{(x * kVA * 1000 * pf)}{(x * kVA * 1000 * pf) + W_i + (x^2 W_{cu})}}$$

where

x = load expressed as a fraction of full-load

e.g. → $x = 1$ for full load

$x = \frac{1}{2}$ for half full load

$x = \frac{1}{4}$ for quarter full load, etc.

Note:- The value of kVA at which transformer efficiency is maximum is independent of the load power factor.

Transformer Ratings

* The following are the principal ratings of a single-phase transformer

	Item	Specification
1.	Continuous rated capacity	5kVA
2.	System voltage (max.)	12kV (line-to-line)
3.	Rated voltage HV	11kV
4.	Rated voltage LV	250V
5.	Line current HV	0.454 A
6.	Line current LV	20 A
7.	Frequency	50Hz ± 5%
8.	No. of phases	01
9.	Type of cooling	ONAN
10.	Vector group	–
11.	Tap changing arrangement	Not applicable
12.	Permissible temperature rise over ambient	
	i) of top oil measured by thermometer	30°C
	ii) of winding measured by resistance	35°C

$$\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

$$I_{1(\text{full load})} = (\text{kVA rating} * 1000)/V_1$$

$$I_{2(\text{full load})} = (\text{kVA rating} * 1000)/V_2$$

Why are transformers rated in kVA and not in kW?

* There are two major kinds of losses in a transformer. They are :-

  a) copper loss

$$W_{cu} = (I_1^2 R_1 + I_2^2 R_2)$$

  This loss depends on the magnitude of the currents in the windings (the RMS value).

  b) iron loss

$$W_i = W_h + W_e.$$

  Here,

$$W_e \propto B_m^2 \quad \text{and} \quad W_h \propto B_m^{1.6}$$

  The flux density depends on the applied voltage (or rated voltage) of the primary.

Hence, total transformer loss depends on volt ampere (VA) and not on the phase angle between voltage & current. i.e., the total losses are independent of the load power factor. This is why transformers are rated in kVA and not in kW.

* The transformer can be used to supply any kind of load (including reactive loads). In such a case, not only should the transformer supply active power to the load, but it should also help in transferring reactive power. Even this reactive component has to be accounted for while calculating the total power (and energy). Hence, transformers are rated using apparent power i.e., kVA. If the transformer capacity was expressed in kW, we would not have considered the type of load (i.e., the reactive power). If a load that consumes reactive power is connected then the transformer would be overloaded. Therefore the total power required in kVA is expressed.

Can D.C. Supply be used for transformers?

* The d.c. supply cannot be used for transformers.

* Transformer works on the principle of mutual induction, for which there has to be continuous change of flux linkages, caused by continuous (and uniform) change of current.

* If d.c. supply is given, the current will not change, thereby no emf will be induced and the transformer will not work.

* Practically, transformer winding resistance is very small. For d.c., the inductive reactance X_L is zero as d.c. has no frequency. So the total impedance offered by a transformer to a d.c. supply is very low. Therefore, the windings will draw a very high d.c. current in this case. This may cause burning of transformer winding due to the extra heat (I^2Rt) generated and may cause permanent, irreversible damage to the transformer.

* In addition, the core of the transformer gets saturated (constant, very high magnetization with no reversals at 'f' cycles per second) due to which the windings draw a very large current from the d.c. supply. This will only increase the damage.

* Therefore, a transformer should never be operated on d.c.

Types of transformer constructions

* Depending on how the windings are wound on the core, the transformers are classified into two types:

  a) Core - type

  b) shell - type

* Core - type transformer

  → It has a single magnetic circuit with two limbs on the core. The winding is wound around the limbs i.e., the winding encircles the core.

  → Generally, cylindrical coils are used. For the purposes of representation, we show the primary & secondary windings as being wound on separate limbs.

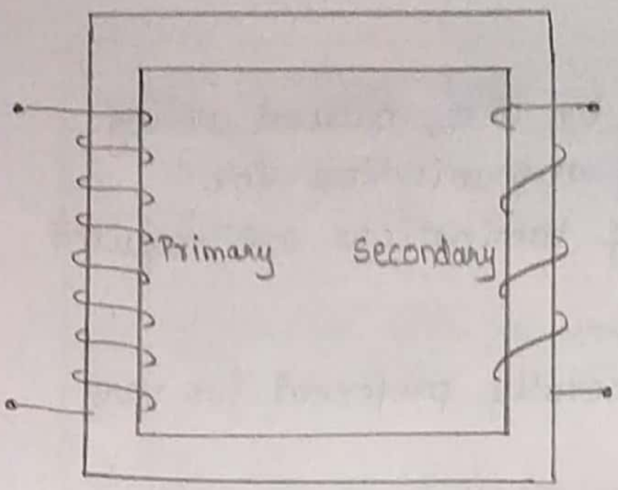
  → However in an actual transformer, the two windings are interleaved to reduce leakage flux. i.e., half the primary winding and half the secondary winding are placed concentrically on one limb, and the other halves on the other limb. In other words, both coils are placed on both limbs.

  → Generally, the low voltage winding is placed nearer to the core while the high voltage coil surrounds the low voltage coil.

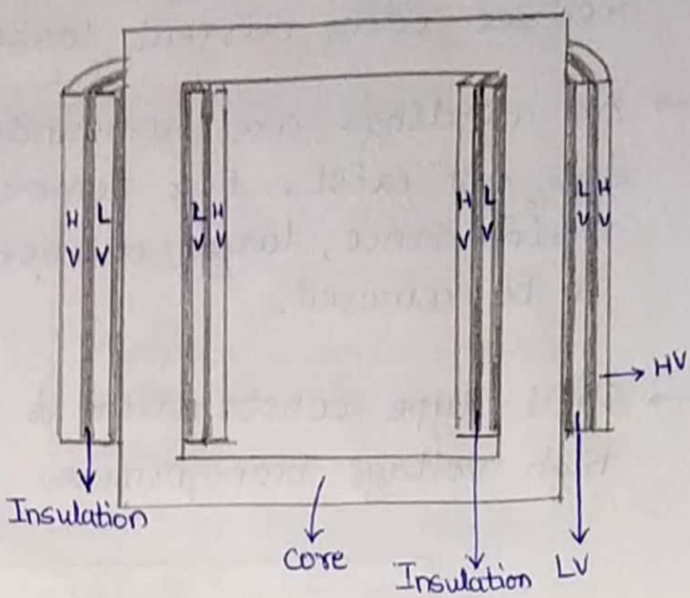
  → As the windings are uniformly distributed over the two limbs, the natural cooling is more effective.

  → By removing the top yoke laminations, the windings can be removed easily for replacement or maintenance.

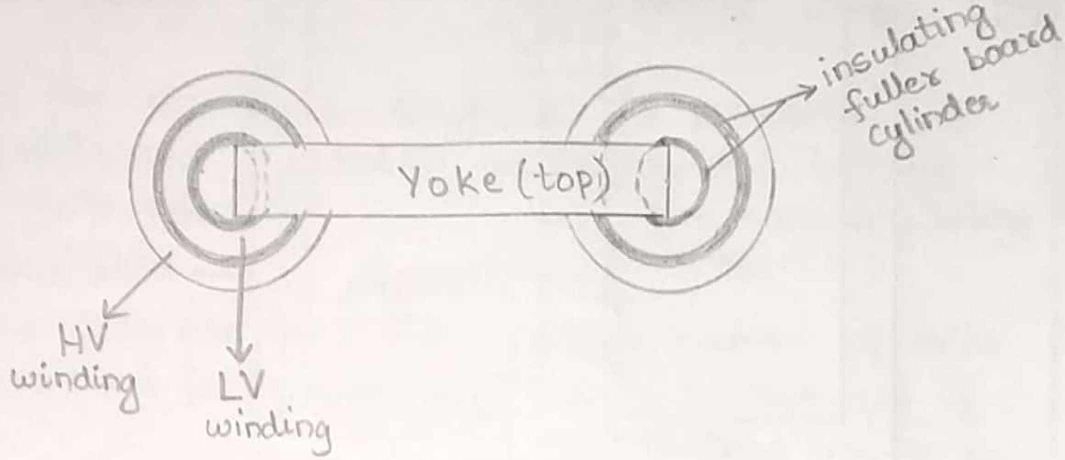
  → The winding layers are insulated from each other & from the core using fuller board insulators.



(Representation)



[cut-away section front view]



[top-view of core-type transformer]

* <u>Shell-type</u>.

→ It has a double magnetic circuit. The core has three limbs. Both primary and secondary windings are placed on the central limb.

→ Therefore, the core encircles most part of the windings.

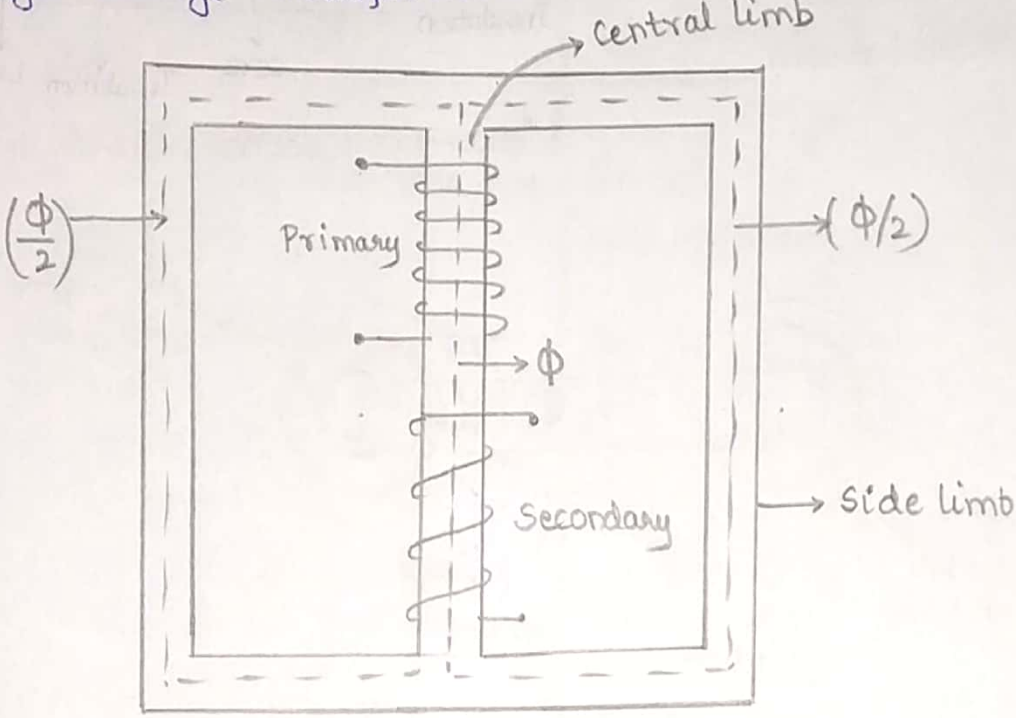
→ Generally, multilayer disc type windings are used.

→ Each high voltage coil is in between two low voltage coils and the low voltage coil is nearest to the top and bottom yokes.

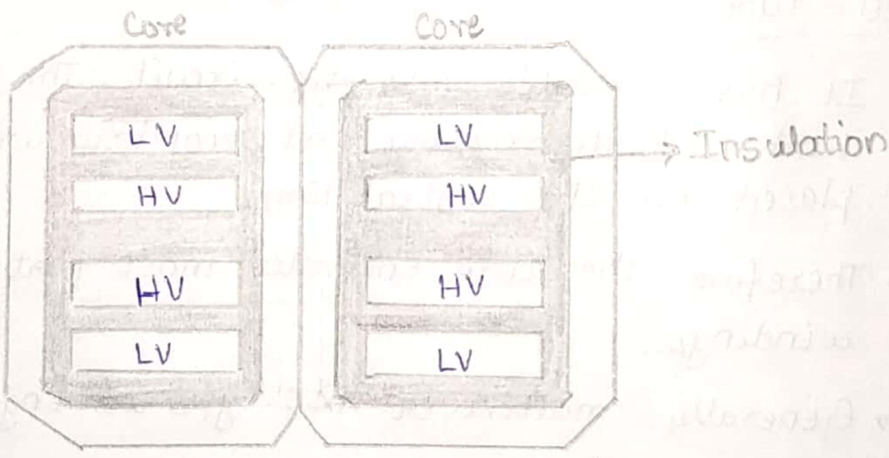
→ The core is rectangular in shape & is laminated to reduce eddy current losses.

→ As windings are surrounded by core, natural cooling does not exist. For removing any winding for maintenance, large number of laminations are required to be removed.

→ Shell-type construction is generally preferred for very high voltage transformers.



(Representation)



[Cut-away section top view]

Sl. No.	Core Type	Shell type
1.	The winding encircles the core	The core encircles the windings.
2.	Cylindrical coils are used	Multilayer disc type or sandwich coils are used.
3.	It has a single magnetic circuit	It has double magnetic circuit.
4.	In 1ϕ type, the core has two limbs	In 1ϕ type, the core has three limbs.
5.	As the windings are distributed, natural cooling is more effective	As the windings are surrounded by the core, the natural cooling does not exist.
6.	The coils can be easily removed for maintenance	Large number of coils are to be removed if any winding has to be removed for maintenance
7.	Generally preferred for low voltage transformers	Usually preferred for high voltage transformers
8.	Less mechanical protection to the coils	Better mechanical protection to the coil due to the surrounding core.