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Department of Mathematics

VECTOR CALCULUS, LAPLACE TRANSFORM AND NUMERICAL METHODS (MA221TA)

UNIT-1V: INVERSE LAPLACE TRANSFORM

TUTORIAL SHEET-1

I. Objective type questions:

1) Laplace transform of the signal is found to be $\frac{1}{(S+3)^3}$, then the corresponding signal in time domain is ______ **Ans:** $\frac{e^{-3t}t^2}{2}$

2) The frequency response of a system is found to be $\frac{4}{4s^2-3}$, find the corresponding time response. **Ans:** $\frac{2}{\sqrt{3}}\sinh\frac{\sqrt{3}}{2}t$

3) $L^{-1}\left(\frac{1}{s^{3/2}}\right)$=___________. **Ans:** $2\sqrt{\frac{t}{\pi}}$

4) $L^{-1}\left(\frac{1+e^{-3s}}{s^2}\right)$ =___________. **Ans:** $t+(t-3)u(t-3)$

5) $L^{-1}\left(\frac{1}{s^2}-\frac{48}{s^5}\right)$ =_____________. **Ans:** $t-2t^4$

6) $L^{-1}\left\{\frac{(s+2)^2}{s^3}\right\}$ = ____________. **Ans:** $1+2t^2+4t$

II. Find the inverse Laplace transform of the following signals:

i) $\frac{s+2}{s^2+36}+\frac{4s-1}{s^2+25}+\frac{(s+3)^3}{s^6}$

Ans: $\cos 6t+\frac{1}{3}\sin 6t+4\cos 5t-\frac{1}{5}\sin 5t+\frac{t^2}{2}+\frac{t^3}{2}+\frac{9t^4}{8}+\frac{9t^5}{40}$

ii) $\frac{2s+1}{s^2+3s+1}$ **Ans:** $e^{-t/2}\left\{\cos\left(\frac{\sqrt{3}}{2}t\right)+1/\sqrt{3}\sin\left(\frac{\sqrt{3}}{2}t\right)\right\}$

iii) $\frac{7s+4}{4s^2+4s+9}$ **Ans:** $\frac{e^{-t/2}}{4}\{7\cos(\sqrt{2}\,t)+1/2\sqrt{2}\sin(\sqrt{2}\,t)\}$

iv) $\frac{2(s^2+2a^2)e^{-2s}}{s^4+4a^4}$ **Ans:** $\frac{2}{a}\{\sin \text{a}(t-2)\cosh a(t-2)\}u(t-2)$

v) $\frac{5s+3}{(s-1)(s^2+2s+5)}$ **Ans:** $e^t+e^{-t}\left[\frac{3}{2}\sin 2t-\cos 2t\right]$

vi) $\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$ **Ans:** $\frac{e^{-t}}{3}[\sin t+\sin 2t]$

vii) $\frac{1}{s}\left(\cos\frac{1}{s}\right)$ **Ans:** $1-\frac{t^2}{(2!)^2}+\frac{t^4}{(4!)^2}-\cdots$

viii) $\frac{e^{4-3s}}{(s+4)^{\frac{5}{2}}}$ **Ans:** $\frac{4}{3\sqrt{\pi}}e^{-4(t-4)}(t-3)^{\frac{3}{2}}u(t-3)$

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TUTORIAL SHEET-II

Find the inverse Laplace transform of the following signals.

1. $\log\left[\frac{s^2+1}{s(s-1)}\right]$ **Ans:** $\frac{\mathbf{1+e^{-t}-2\cos t}}{\mathbf{t}}$
2. $s\log\left(\frac{s+1}{s-1}\right)+2$ **Ans:** $\frac{2}{t^2}(\sinh t - t\cosh t)$
3. $tan^{-1}\left(\frac{2}{s^2}\right)$ **Ans:** $\frac{2\sin t\sinh t}{t}$
4. $\log\left(1+\frac{a^2}{s^2}\right)$ **Ans:** $2\left(\frac{1-\cos at}{t}\right)$
5. $\frac{e^{-3s}}{s}-\frac{e^{-s}}{s^2}$ **Ans:** $f(t)=\begin{cases} 0 & t<1 \\ 1-t & 1<t<3 \\ 4-t & t\geq 3\end{cases}$
6. $\frac{e^{-3s}}{(s+1)^3}$ **Ans:** $u(t-3)\left(\frac{1}{2}e^{-(t-3)}(t-3)^2\right)$
7. $\frac{s+1}{(s^2+2s+2)^2}$ **Ans:** $\frac{t}{2}e^{-t}\sin t$
8. $\frac{1}{s(s^2+2s+2)}$ **Ans:** $\frac{1}{2}[1-e^{-t}(\sin t+\cos t)]$

Applying convolution theorem, find the inverse transform of the following functions.

1) $\frac{\mathbf{1}}{(\mathbf{s^2+6})^2}$ **Ans:** $\frac{1}{12}\left(\frac{sin\sqrt{6}t}{\sqrt{6}}+tcos\,\sqrt{6}t\right)$
2) $\frac{\mathbf{s}}{(\mathbf{s^2+6})^2}$ **Ans:** $\frac{t}{2\sqrt{6}}sin\sqrt{6}t$
3) $\frac{\mathbf{1}}{\mathbf{s^2(s^2+6)}}$ **Ans:** $\frac{1}{6}\left(t-\frac{sin\sqrt{6}t}{\sqrt{6}}\right)$
4) $\frac{3s+1}{(s-2)(s^2+1)}$ **Ans:** $\frac{1}{5}(7e^{2t}-7\cos t+\sin t)$
5) $\frac{1}{(s^2+4)(s+1)^2}$ **Ans:** $\frac{\mathbf{e^{-t}}}{\mathbf{50}}[10e^{-t}-3\sin 2t-4\cos 2t]$

Verify convolution theorem for the following functions.

1) $f(t)=sin\,t\,,\quad g(t)=e^{-t}$
2) $f(t)=t,\quad g(t)=te^{-t}$
3) $f(t)=\sin at\,,\quad g(t)=\cos at$
4) $f(t)=t,\quad g(t)=\cos t$

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TUTORIAL SHEET-III

Solve the following differential equations using Laplace transform method.

1. $y'' - 3y' + 2y = 4t + e^{3t}, y(0) = 1, y'(0) = 1.$
Ans: $y = 3 + 2t - \frac{5}{2}e^{t} + \frac{1}{2}e^{3t}$

2. $\frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} - 4\frac{dy}{dt} - 4y = t + 1, y(0) = y'(0) = 0$

Ans: $y = \frac{1}{8}(\sinh 2t - 2t)$

3. $y'' + y = f(t), y(0) = 1, y'(0) = 0$, where $f(t) = \begin{cases} 3, & 0 \le t \le 4 \\ 2t - 5, & t > 4 \end{cases}$

Ans: $y = 3 - 2\cos t + 2[t - 4) - \sin(t - 4)]u(t - 4)$

4. $\frac{d^2x}{dt^2} + 9x = \cos 2t \;\; x(0) = 1, x\left(\frac{\pi}{2}\right) = -1$
Ans: $x = \frac{4}{5}\cos 3t + \frac{4}{5}\sin 3t + \frac{1}{5}\cos 2t$

5. The current i flowing in an electric circuit is governed by the differential equation $\frac{di}{dt} + i = E(t), e$ is a positive constant in $0 < t < 1$ and $E(t) = 0$ for $t > 1$. The circuit carries no current at time $t = 0$. Find the current at any time $t > 0$.
Ans: $i = \begin{cases} E(1 - e^{-t}), & 0 < t \le 1 \\ E(e - 1)e^{-t}, & t > 1. \end{cases}$

6. $y' + y - 2\int_0^t y \; dt = \frac{t^2}{2}, y(0) = 1, y'(0) = -2$
Ans: $y(t) = \frac{1}{3}e^{t} + \frac{11}{12}e^{-2t} - \frac{t}{2} - \frac{1}{4}$

7. $(D^2 + 1)y = \sin t \sin 2t, y(0) = 1, y'(0) = 0$
Ans: $y(t) = \frac{15}{16}\cos t + \frac{t}{4}\sin t + \frac{1}{16}\cos 3t$