R V COLLEGE OF ENGINEERING

(An autonomous institution affiliated to VTU, Belgaum)

DEPARTMENT OF MATHEMATICS

VECTOR CALCULUS, LAPLACE TRANSFORM AND NUMERICAL METHODS (22MA21A)

UNIT 1: VECTOR DIFFERENTIATION

TUTORIAL SHEET-1

1. If $\phi(x,y,z) = xy^2z^3 - x^3y^2z$, then $|\nabla\phi|$ at $(1,-1,1)$ is _____________.
Ans: $\mathbf{2\sqrt{2}}$

2. The maximum directional derivative of $\phi(x,y,z) = x^2y + yz^2 - xz^3$ at $(-1,2,1)$ is _____.
Ans: $\mathbf{\sqrt{78}}$

3. If $\phi(x,y,z) = x^2 + \sin y + z$ then $|\nabla\phi|$ at $\left(0,\frac{\pi}{2},1\right)$ is _____________.
Ans: $\boldsymbol{\hat{k}}$

4. Find the unit normal vector to the surface $\phi(x,y,z) = x^2y + y^2z + xz^2 - 5$ at the point $(1,-1,2)$.
Ans: $\frac{\mathbf{1}}{\sqrt{\mathbf{38}}}(\mathbf{2\hat{\imath} - 3\hat{\jmath} + 5\hat{k}})$

5. Find the constants a and b so that the surface $3x^2 - 2y^2 - 3z^2 + 8 = 0$ is orthogonal to the surface $ax^2 + y^2 = bz$ at the point $(-1,2,1)$.
Ans: $\boldsymbol{a} = \frac{\mathbf{4}}{\mathbf{9}},\ \boldsymbol{b} = \frac{\mathbf{40}}{\mathbf{9}}$

7. Find the directional derivative of $\phi(x,y,z) = xyz - xy^2z^3$ at $(1,2,-1)$ in the direction of $\hat{\imath} - \hat{\jmath} - 3\hat{k}$.
Ans: $\frac{\mathbf{29}}{\sqrt{\mathbf{11}}}$

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TUTORIAL SHEET-2

1. If $\vec{f} = 3x^2\hat{\imath} + 5xy^2\hat{\jmath} + xyz^3\hat{k}$ then find $div\,\vec{f}$ at $(1, 2, 3)$ is _____.
Ans: 80

2. If $\vec{f} = (y^2 + z^2 - x^2)\hat{\imath} + (z^2 + x^2 - y^2)\hat{\jmath} + (x^2 + y^2 - z^2)\hat{k}$ then find $div\,\vec{f}$ and $curl\,\vec{f}$
Ans: $\boldsymbol{div\,\vec{f} = -2(x + y + z)}$ $\boldsymbol{curl\,\vec{f} = 2[(y - z)\hat{\imath} + (z - x)\hat{\jmath} + (x - y)\hat{k}]}$

3. Show that the vector field $\vec{f} = (x + 3y)\hat{\imath} + (y - 3z)\hat{\jmath} + (x - 2z)\hat{k}$ is solenoidal.

4. Determine the constant a such that the vector field $\vec{f} = 3x\hat{\imath} + (x + y)\hat{\jmath} - az\hat{k}$ is solenoidal.
Ans: 4

5. If $\vec{f} = x^2\hat{\imath} + y^2\hat{\jmath} + z^2\hat{k}$ and $\vec{g} = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$ then show that $\vec{f} \times \vec{g}$ is solenoidal.

6. If $\vec{f} = (2x + 3y + az)\hat{\imath} + (bx + 2y + 3z)\hat{\jmath} + (2x + cy + 3z)\hat{k}$ is irrotational vector field, then find the constants a, b, c.
Ans: $\boldsymbol{a = 2, b = 3, c = 3}$

7. If $\phi = x^2y + 2xy + z^2$ then show that $|\nabla\phi|$ is irrotational.

8. If $\phi = x^2 - y^2$ then show that ϕ satisfies the Laplacian equation.

9. If $\phi = 2x^2yz^3$ then find $\nabla^2\phi$ at $(1, 1, 1)$.
Ans: 1

10. If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$ then show that $r^n\vec{r}$ is irrotational for all values of n and solenoidal for $n = -3$.

11. Show that $\vec{f} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational. Find the function ϕ such that $\vec{f} = \nabla\phi$.
Ans: $\boldsymbol{\phi = 3x^2y + xz^3 - yz}$.

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UNIT 1: VECTOR DIFFERENTIATION

TUTORIAL SHEET-3

1. Compute the gradient of the scalar potentials
   a) $f = rz\cos\theta$ in the cylindrical coordinate system (r, θ, z).
   b) $f = r^2 \sin 2\theta \sin\phi$ in the spherical coordinates (r, θ, ϕ).
   c) $f = r^2 + 2r\cos\theta - e^z \sin\theta$ in the cylindrical coordinate system (r, θ, z).
   d) $f = 3r^2 \sin\theta + e^r \cos\phi - r$ in the spherical coordinates (r, θ, ϕ).
   e) $f = r\sin\theta\cos\phi$ in the spherical coordinates (r, θ, ϕ).
   f) $g = \frac{zr^3}{\cos\theta}$ in the cylindrical coordinate system (r, θ, z).
2. If $\vec{F} = r^2\cos\theta\,\hat{e}_r + \frac{1}{r}\hat{e}_\theta + \frac{1}{r\sin\theta}\hat{e}_\phi$ in the spherical coordinates (r, θ, ϕ), determine $div\,\vec{F}$ and $curl\,\vec{F}$.
3. Prove that $\vec{F} = rz\sin 2\theta\left(\hat{e}_r + \cot 2\theta\,\hat{e}_\theta + \frac{r}{2z}\hat{e}_z\right)$ is irrotational in the cylindrical coordinate system (r, θ, z).
4. Prove that $\vec{F} = \frac{\cos\theta}{r^3}\left(\frac{1}{\sin\theta}\hat{e}_r - \frac{1}{\cos\theta}\frac{1}{r}\hat{e}_\theta + r^4\hat{e}_\phi\right)$ is solenoidal in spherical coordinates (r, θ, ϕ).
5. Show that $\nabla^2 f = 2r^2\cos\theta$, if $f = r^2 z^2 \cos 2\theta$.
6. Show that $\vec{F} = \frac{2\cos\theta}{r^3}\hat{e}_r + \frac{\sin\theta}{r^3}\hat{e}_\theta$ is solenoidal in spherical coordinates (r, θ, ϕ).
7.