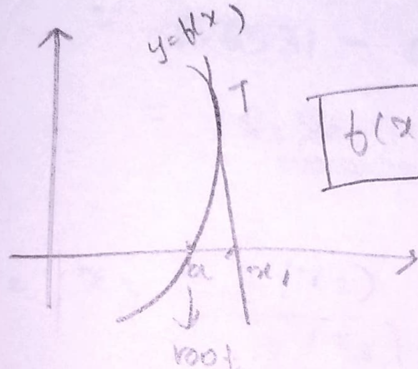
# NEWTON RAPHSON METHOD:-

In Newton Raphson method, the curve $y = f(x)$ is approximated by the tangent and the point A where the tangent meets x-axis is taken as initial approximation.

By Newton Raphson method,

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$




$\boxed{f(x) = 0.}$

<u>Note</u>:- The order of convergence of Newton Raphson method is Quadratic or two.

1) Find the real root of the equation $f(x) = 3x - \cos x + 1$ in $[0, 1]$:-

$\Rightarrow$ $f(x) = 3x - \cos x + 1$

$f'(x) = 3 + \sin x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(3x_n - \cos x_n + 1)}{3 + \sin x_n}.$$

~~f(0) = 1/0.~~

$f(0.5) = 1.6224$

$f(1) = 3.4597$

2] Find the root of the equation $\cos x - xe^x = 0$ at $x_0 = 0$

$\rightarrow$ $f(x) = \cos x - xe^x$

$f'(x) = -\sin x - xe^x - e^x$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow x_1 = 0 - \frac{1}{-1}$$

$$x_1 = 1$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1 - \frac{(0.5403 - 2.7183)}{(-0.8415 - 2.7183 - 2.7183)}$$

$$x_2 = 1 - \left(\frac{\cancel{-}2.1780}{\cancel{-}6.2781}\right)$$

$$= 1 - 0.3469$$

$$x_2 = \underline{\underline{0.6531}}.$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.6531 - \left[\frac{-0.4607}{-3.7841}\right]$$

$$= 0.6531 - 0.1217$$

$$= \underline{\underline{0.5314}}$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 0.5314 - \left(\frac{-0.0420}{-3.1121}\right)$$

$$= 0.5314 - 0.0135$$

$$= \underline{\underline{0.5179}}.$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$

$$= 0.5179 - \left[\frac{-0.0004}{-3.0429}\right]$$

$$= 0.5179 - 0.0001$$

$$= \underline{\underline{0.5178}}.$$

$$x_6 = x_5 - \frac{f(x_5)}{f'(x_5)}$$

$$= 0.5178 - \left(\frac{-0.0001}{-3.0423}\right)$$

$$= 0.5178 - 0.0000$$

$$= \underline{\underline{0.5178}}$$

Real, root $\Rightarrow x = 0.5178$.

3) Find the real root of equation $x \sin x + \cos x = 0$ near $x_0 = \pi$.

$\Longrightarrow$ $f(x) = x \sin x + \cos x$

$f'(x) = x \cos x + \sin x - \sin x$

$= x \cos x$.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

$x_1 = 2.8233$

$x_2 = 2.7986$

$x_3 = 2.7984$

$x_4 = \underline{\underline{2.7984}}$

Real root, $x = \underline{\underline{2.7984}}$.

4] Calculate root of $\sqrt{10}$ ; $x_0 = 3$.

$\rightarrow$ $x = \sqrt{10}$.

$f(x) = x - \sqrt{10}$

$f'(x) = 1$

$x_1 = 3.1623$.

$x_2 = 3.1623$.

Real root, $x = 3.1623$

** 5] Evaluate $\sqrt[4]{22}$

$\Rightarrow$ $x^4 = 22$. ; $x_0 = 1.9$.

$\therefore f(x) = x^4 - 22$

$f'(x) = 4x^3$

$x_1 = 2.2269$

$x_2 = 2.1682$

$x_3 = 2.1657$

$x_4 = 2.1657$.

Real root, $x = 2.1657$.

6] Evaluate $x \log_{10} x = 1.2$. , $x_0 = 2$.

$\Rightarrow$ $f(x) = x \log_{10} x - 1.2$

$f'(x) = \frac{x}{x} + \log_{10} x$.

$= 1 + \log_{10} x$.

$x_1 = 2.4596$ ; $x_3 = 2.6977$ ; $x_5 = 2.7340$

$x_2 = 2.6312$ ; $x_4 = 2.7238$ ; $x_6 = 2.7380$

$x_7 = 2.7396$

$x_8 = 2.7402$

$x_9 = 2.7405$

$x_{10} = 2.7406$

$x_{11} = 2.7406$.

$\therefore$ Real root, $x = 2.7406$.

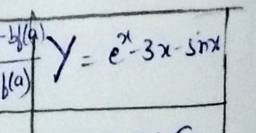
REVISION :-

Solve the following equations by regula falsi method & NR method :-

1) $e^x = 3x + \sin x$

Regula-Falsi method :-

$f(x) = e^x - 3x - \sin x$. $[0, 1]$




$f(0) = 1$.

$f(1) = 2.7183 - 3 - 0.8415$

$= -1.1232$.

| $a = A$ | $b = B$ | $f(a) = C$ | $f(b) = D$ | $X = \frac{af(b) - bf(a)}{f(b) - f(a)}$ | $Y = e^x - 3x - \sin x$ |
|---|---|---|---|---|---|
| 0 | 1 | 1 | -1.1232 | 0.4710 | -0.2652 |
| 0 | 0.4710 | 1 | -0.2652 | 0.3723 | -0.0295 |
| 0 | 0.3723 | 1 | -0.0295 | 0.3616 | -0.0030 |
| 0 | 0.3616 | 1 | -0.0030 | 0.3605 | -0.0002 |
| 0 | 0.3605 | 1 | -0.0002 | 0.3604 | 0.0000 |

Real root, $x = \underline{\underline{0.3604}}$.

By NF method :-

$f(x) = e^x - 3x - \sin x$

$f'(x) = e^x - 3 - \cos x$.

$x_0 = 0$

$x_1 = 0.3333$

$x_2 = 0.3602$

$x_3 = 0.3604$

$x_4 = \underline{\underline{0.3604}}$.

$f'(x) =$

# NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS.

• Taylor Series method:

To solve the differential equations of the form $\dfrac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ → Initial value problems.

$\Rightarrow$ $y' = f(x, y)$ ; $y_0' = f(x_0, y_0)$

$$\boxed{y(x) = y_0 + (x - x_0) y_0' + \frac{(x - x_0)^2}{2!} y_0'' + \cdots}$$

1] Using Taylor series method, solve:

$$\frac{dy}{dx} = 2y + 3e^x, \quad y(0) = 0.$$

Find $y$ at $x = 0.2$ & $x = 0.4$

$\Rightarrow$ $y' = 2y + 3e^x$, $x_0 = 0$, $y_0 = 0$

$y'(0) = 3$

$y'' = 2y' + 3e^x$ $\qquad y''(0) = 9$

$y''' = 2y'' + 3e^x$ $\qquad y'''(0) = 21$

$$\therefore y(x) = 0 + (x - 0)\,3 + \frac{(x - 0)^2}{2!}(9) + \frac{(x - 0)^3}{3!}(21) + \cdots$$

$$y(x) = 3x + \frac{9x^2}{2} + \frac{7}{2}x^3 + \cdots$$

$\rightarrow x = 0.2$

$$y(0.2) = 3(0.2) + \frac{9(0.2)^2}{2} + \frac{7(0.2)^3}{2} + \cdots$$

$$= 0.6 + 0.18 + 0.028 + \cdots$$

$$y(0.2) = \underline{\underline{0.808}}.$$

$x = 0.4$

$$y(0.4) = 3(0.4) + \frac{9(0.4)^2}{2} + \frac{7}{2}(0.4)^3$$

$$= 1.2 + 0.72 + 0.224$$

$$y(0.4) = \underline{\underline{2.144}}.$$

Compare with exact soln :-

2] Apply Taylor series method to find $y(4.1)$, $y(4.2)$ given that $(x^2+1)\frac{dy}{dx} = 1$, $y(4) = 4$.

$\Rightarrow$ $\frac{dy}{dx} = \frac{1}{x^2+1} = y'$ ; $x_0 = 4$ ; $y_0 = 4$

$y'(4) = 1/17$

$y'' = \frac{-2x}{(x^2+1)^2}$ $\quad y''(4) = \frac{-8}{17 \times 17} = \frac{-8}{289}$

$$y''' = -\left[\frac{(x^2+1)^2(2) - 2x(2(x^2+1)(2x))}{(x^2+1)^4}\right]$$

$$= -\left[\frac{2(x^2+1)^2 - 2x(4x^3+4x)}{(x^2+1)^4}\right]$$

$$y'''(4) = -\left[\frac{2(289) - 8(256+16)}{(17)^4}\right]$$

$$= -\left[\frac{578 - 2176}{83521}\right]$$

$$y'''(4) = \frac{\cancel{1598}\ 94}{\cancel{83521}\ 4913}$$

$$y'''(4) = \frac{94}{4913}$$

$$y(x) = 4 + (x-4)\frac{1}{17} + \frac{(x-4)^2}{2!} \times \frac{-8}{289} +$$

$$\frac{(x-4)^3}{8} \times \frac{94}{4913}.$$

At $x = 4.1$.

$$y(4.1) = 4 + \frac{0.1}{17} - \frac{0.01}{2} \times \frac{8}{289} + \frac{0.001}{6} \times \frac{94}{4913}$$

$$= 4 + 0.0059 - 0.0001 + 0$$

$$y(4.1) = 4.0058$$

At $x = 4.2$

$$y(4.2) = 4 + \frac{0.2}{17} + \frac{0.04}{2} \times \frac{-8}{289} + \frac{0.008}{8} \times \frac{94}{4913}$$

$$= 4 + 0.0118 - 0.0006 + 0$$

$$= 4.0112$$

3] Find $y(1.1)$, $y(1.2)$, $y(1.3)$, $y(1.4)$, $\frac{dy}{dx} = x^2(1+y)$

$y(1) = 1$.

$\Rightarrow$ $y' = x^2 + x^2 y$, $x_0 = 1$, $y_0 = 1$

$y'(1) = 2$.

$y'' = 2x + x^2 y' + 2xy$ ; $y''(1) = 2 + 2 + 2$

$= 6$.

$y''' = 2 + x^2 y'' + 2xy' + 2xy' + 2y$

$y'''(1) = 2 + 6 + 4 + 4 + 2$.

$= 18$.

$$y(x) = 1 + (x-1)2 + \frac{(x-1)^2}{2!}(6) + \frac{(x-1)^3}{6}(18)$$

$$= 1 + 2(x-1) + 3(x-1)^2 + \frac{9}{4}(x-1)^3$$

At $x = 1.1$

$y(1.1) = 1 + 2(0.1) + 3(0.1)^2 + 3(0.1)^3$

$= 1 + 0.2 + 0.03 + 0.003$

$= 1.233$

$y(1.2) = 1 + 2(0.2) + 3(0.2)^2 + 3(0.2)^3$

$= 1 + 0.4 + 0.12 + 0.024$

$= \cancel{1.5330}\ 1.544$

$y(1.3) = 1 + 2(0.3) + 3(0.3)^2 + 3(0.3)^3$

$= 1 + 0.6 + 0.27 + \cancel{0.0608}\ 0.0810$

$= 1.9510$

$y(1.4) = 1 + 2(0.4) + 3(0.4)^2 + 3(0.4)^3$

$= 1 + 0.8 + 0.48 + 0.1920$

$= 2.4720$

** 4) Find the value of $y$ at $x = 0.1$, $x = 0.2$ given that $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$.

$\Rightarrow$ $y' = x^2y - 1$ ; $y_0 = 1$, $x_0 = 0$.

$y'(0) = -1$.

$y'' = x^2y' + 2xy$

$y''(0) = 0$.

$y''' = 2xy' + x^2y'' + 2xy' + 2y$

$y'''(0) = \underline{\underline{2}}.$

$y^{iv} = 2xy'' + 2y' + x^2y''' + 2xy'' + 2xy'' + 2y' + 2y'$

$y^{iv}(0) = -2 - 2 - 2.$

$= \underline{\underline{-6}}.$

$y(x) = 1 + (x-0)(-1) + \frac{(x-0)^2}{2!}(0) + \frac{(x-0)^3}{3!}(2) + \frac{(x-0)^4}{4!}(-6)$

$= \underline{\underline{1 - x + \frac{x^3}{3} - \frac{x^4}{4}}}.$

At $x = 0.1$

$y(0.1) = 1 - 0.1 + 0.0003 - 0.0002$

$= \underline{\underline{0.9003}}.$

At $x = 0.2$.

$y(0.2) = 1 - 0.2 + 0.0027 - 0.0084$

$= \underline{\underline{0.8023}}.$