Model papers

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model question ...

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RV COLLEGE OF ENGINEERING®
(An Autonomous Institution Affiliated to VTU)
I Semester B. E. Examinations May-2023
(Common to EC, EE, EI, ET)
FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS

Time: 03 Hours | Maximum Marks: 100

Instructions to candidates:
1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of scientific calculator is permitted.
4. Use of Handbook of Mathematics is permitted.

PART-A

1	1.1	Given $A=\begin{bmatrix}0&2&4\\3&5&13\\4&4&12\end{bmatrix}$, then the rank of A is 2	02
	1.2	Sum and product of the eigenvalues of the matrix $A=\begin{bmatrix}-2&2&-3\\2&1&-6\\-1&-2&0\end{bmatrix}$ is -1 and 39	02
	1.3	The angle between the radius vector and tangent to the curve $r=e^{\theta}$ is $\pi/2$	02
	1.4	The Maclaurin series expansion for $\sinh x$ is ______	02
	1.5	If $u=\frac{2x}{y}$ and $v=\frac{2y}{x}$, then $\frac{\partial(u,v)}{\partial(x,y)}=$ ______	02
	1.6	Given $f_{xx}=6x$, $f_{xy}=0$ and $f_{yy}=6y$, then the nature of stationary point at $(-1,2)$ is ______	02
	1.7	Evaluate $\int_0^2\int_1^3 y\,dy\,dx$	02
	1.8	Sketch the domain of the integral $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xy\,dy\,dx$	02
	1.9	Construct the table of differences for the data below. (x: 0, 10, 20, 30, 40; y: 1, 1.5, 2.2, 3.1, 4.6) Evaluate $\Delta^2 f(20)$ and $\Delta^3 f(10)$.	02
	1.10	If $y_0=0$, $y_1=2$, $y_2=2.5$, $y_3=2.3$, $y_4=2$, $y_5=1.7$, $y_6=1.5$, then compute the area using Weddle's rule.	02

x	0	10	20	30	40
y	1	1.5	2.2	3.1	4.6

PART-B

2	a	In control theory, the rank of a matrix is used to determine whether a linear system is controllable or observable. To analyze one such system, reduce the given matrix into the row echelon form and hence obtain rank of a matrix $A=\begin{bmatrix}1&2&3\\3&2&1\\2&4&3\\6&8&7\end{bmatrix}$	06
	b	Eigenvalue analysis is used to design the car stereo system. Apply Rayleigh's power method to compute the largest eigenvalue and the corresponding eigenvector for the matrix representing the position of car stereo given by $A=\begin{bmatrix}25&1&2\\1&3&0\\2&0&-4\end{bmatrix}$ taking initial vector as $\begin{bmatrix}1\\0\\0\end{bmatrix}$. Carry out 6 iterations.	10
3	a	Determine the angle between the radius vector and tangent for the curve $r=a(1+\sin\theta)$ at any point θ. Hence find the slope at $\theta=\frac{\pi}{2}$.	08
	b	Find the centre of curvature and circle of curvature on the curve $y=e^{3x}$ at a point where the curve crosses the y-axis.	08
		OR	
4	a	Obtain the radius of curvature at any point $P(x,y)$ on the curve $y=a\cosh\left(\frac{x}{a}\right)$, $a>0$.	08
	b	Expand $y=\log_e(\sec x)$ as a series in powers of 'x'. Hence obtain the series expansion for $\tan x$.	08
5	a	If $u=\log_e(x^2+y^2)+\tan^{-1}\left(\frac{y}{x}\right)$, show that u satisfies the Laplace equation $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$.	08
	b	Evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ if $u=\frac{yz}{x}$, $v=\frac{zx}{y}$ and $w=\frac{xy}{z}$.	08
		OR	
6	a	If $u=f(x-y,y-z,z-x)$, then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.	08
	b	Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16}=1$.	08
7	a	Evaluate $\iint_R dx\,dy$ where R is the region bounded by the lines $y=x$, $x+y=4$, $y=1$ and $y=0$.	08
	b	Evaluate $\int_0^{\pi}\int_0^{a\sin\theta}\int_0^{\frac{a^2-r^2}{a}} r\,dz\,dr\,d\theta$.	08
		OR	
8	a	Find the area bounded by the curves $xy=2$, $4y=x^2$, $y=4$ and the co-ordinate axes using double integrals.	08
	b	Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}dx\,dy$ by changing to polar co-ordinates.	08
9	a	A survey conducted in a locality reveals the following information as classified below. Estimate the probable number of persons in the income group 20k – 25k.	08
	b	Applying Lagrange's formula in appropriate form fit an interpolating polynomial of the form $x=f(y)$ for the given data and hence find $x(5)$ and $y(5)$.	08
		OR	
10	a	Evaluate $\int_0^6 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}$th rule. Hence deduce the value of ln 2.	08
	b	Find the radius of curvature of a Cartesian curve $y=f(x)$ when $x=23$ given the following data using numerical differentiation.	08

Income (per month) Rs.	Under 10k	(10-20)k	(20-30)k	(30-40)k	(40-50)k
Number of persons	20	45	115	210	115

x	2	10	17
y	1	3	4

x	19	20	21	22	23
y	91	100.25	110	120.25	131

1.1) $\begin{matrix}3&5&13\\0&2&4\\4&4&12\end{matrix}$

$\det(A)=0$ & $\begin{vmatrix}0&2\\3&5\end{vmatrix}=-6\neq0$

$\rightarrow \rho(A)=2$

1.2) sum $=-2+1+0=-1$

product = det $=-2(-12)-2(-6)-3(-2+1)$
$=24+12+3=39$

1.3) $r=e^{\theta}$

$\frac{dr}{d\theta}=e^{\theta}$

$\tan\phi=r\frac{d\theta}{dr}=e^{\theta}\times\frac{1}{e^{\theta}}=1$

$\phi=\pi/4$

1.4) $f(x)=f(0)+x\cdot f'(0)+\frac{x^2}{2!}f''(0)$

$f(x)=\sinh x$ $\quad f(0)=0$
$f'(x)=\cosh x$ $\quad f'(0)=1$
$f''(x)=\sinh x$ $\quad f''(0)=0$
$f'''(x)=\cosh x$ $\quad f'''(0)=1$
$f^{iv}(x)=\sinh x$ $\quad f^{iv}(0)=0$
$f^{v}(x)=\cosh x$ $\quad f^{v}(0)=1$

$\sinh x=x+\frac{x^3}{6}+\frac{x^5}{120}\ldots$

1.5) $u=\frac{2x}{y}$ $\quad v=\frac{2y}{x}$

$\frac{\partial(u,v)}{\partial(x,y)}=\begin{vmatrix}\partial u/\partial x&\partial u/\partial y\\\partial v/\partial x&\partial v/\partial y\end{vmatrix}$

$J=\begin{vmatrix}2/y&-2x/y^2\\-2y/x^2&2/x\end{vmatrix}$

$J=\frac{4}{xy}-\frac{4xy}{(xy)^2}=0$

1.6) $r=6x=-6$
$s=0=0$
$t=6y=12$
$rt-s^2=-72-0<0$
$\rightarrow$ saddle point

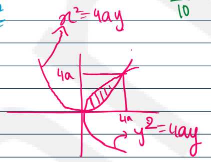
1.7) $\int_0^2\int_1^3 y\,dy\,dx$
$=\int_0^2\left[\frac{y^2}{2}\right]_1^3 dx$
$=\left(\frac{9}{2}-\frac{1}{2}\right)\times|x|_0^2$
$=4\times2=8$

1.8) $y=\frac{x^2}{4a}$

$y=2\sqrt{ax}$

$\frac{x^2}{4a}=2\sqrt{ax}$

$x^3\cancel{x^4}=4^3a^3\cancel{x}$ $\quad x=4a$



1.9)

x	y	Δy	Δ^2	Δ^3	Δ^4
0	1				
		0.5			
10	1.5		0.2		
		0.7		0	
20	2.2		0.2		0.4
		0.9		0.4	
30	3.1		0.6		
		1.5			
40	4.6				

$\Delta^2 f(20)=0.6$

$\Delta^3 f(10)=0.4$

1.10) Weddles $=\frac{3h}{10}(y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6)$

$=\frac{3}{10}(38.3)$
$=11.49$

2a) $A=\begin{bmatrix}1&2&3\\3&2&1\\2&4&3\\6&8&7\end{bmatrix}$

$R_2\rightarrow R_2-3R_1$
$R_3\rightarrow R_3-2R_1$
$R_4\rightarrow R_4-6R_1$

$=\begin{bmatrix}1&2&3\\0&-4&-8\\0&0&-3\\0&-4&-14\end{bmatrix}$

$R_4\rightarrow R_4-R_2$

$=\begin{bmatrix}1&2&3\\0&-4&-8\\0&0&-3\\0&0&-6\end{bmatrix}$

$R_4\rightarrow R_4-2R_3$

$=\begin{bmatrix}1&2&3\\0&-4&-8\\0&0&-3\\0&0&0\end{bmatrix}$

$\rho(A)=3$

2b) $A=\begin{bmatrix}25&1&2\\1&3&0\\2&0&-4\end{bmatrix}$ $\quad X^{(0)}=\begin{bmatrix}1\\0\\0\end{bmatrix}$

$AX^{(0)}=\begin{bmatrix}25\\1\\2\end{bmatrix}=25\begin{bmatrix}1\\0.04\\0.08\end{bmatrix}$

$\lambda^{(6)}=25.186$ $\quad X^{(6)}=\begin{bmatrix}1\\0.045\\0.0685\end{bmatrix}$

3a) $r=a(1+\sin\theta)$

$\frac{dr}{d\theta}=a\cos\theta$

$\tan\phi=r\frac{d\theta}{dr}=\frac{\cancel{a}(1+\sin\theta)}{\cancel{a}\cos\theta}$

$=\frac{(\cos\theta/2+\sin\theta/2)^2}{\cos^2\theta/2-\sin^2\theta/2}$

$=\frac{\cos\theta/2+\sin\theta/2}{\cos\theta/2-\sin\theta/2}$

$=\frac{1+\tan\theta/2}{1-\tan\theta/2}$

$\tan\phi=\tan(\pi/4+\theta/2)$

$\phi=\frac{\theta}{2}+\frac{\pi}{4}$

$\psi=\theta+\phi=\frac{3\theta}{2}+\frac{\pi}{4}$

$\tan\psi|_{\theta=\frac{\pi}{2}}=\tan\left(\frac{3\pi}{4}+\frac{\pi}{4}\right)$
$=\tan\pi$

slope $=0$

3b) COC$(\bar{x},\bar{y})$

$y=e^{3x}$ at $x=0, y=1$

$\rightarrow y_1=3e^{3x}=3$

$y_2=9e^{3x}=9$

$\rightarrow \bar{x}=x-\frac{y_1}{y_2}(1+y_1^2)$

$=0-\frac{3}{9}(1+3^2)$

$\bar{x}=-\frac{10}{3}$

$\bar{y}=y+\frac{1}{y_2}(1+y_1^2)$; $\rho=\frac{(1+y_1^2)^{3/2}}{y_2}$

$=1+\frac{1}{9}(1+3^2)$ $\quad \rho^2=\frac{(1+3^2)^3}{9^2}$

$\bar{y}=\frac{19}{9}$ $\quad =\frac{10^3}{9^2}=\frac{10k}{81}$

CoC $\Rightarrow$

$\left(x+\frac{10}{3}\right)^2+\left(y-\frac{19}{9}\right)^2=\frac{10k}{81}$

4a) $y=a\cosh(x/a)$ $\quad \rho=?$

$\rightarrow y_1=\frac{\cancel{a}}{\cancel{a}}\sinh\left(\frac{x}{a}\right)$

$y_2=\frac{1}{a}\cosh\left(\frac{x}{a}\right)$

$\rightarrow \rho=\frac{(1+y_1^2)^{3/2}}{y_2}$

$=\frac{(1+\sinh^2(x/a))^{3/2}}{\frac{1}{a}\cosh(x/a)}$

$=\frac{a\cosh^3(x/a)}{\cosh(x/a)}$

$\rho=a\cosh^2(x/a)$

4b) $y=\log_e(\sec x)$ $\quad y(0)=0$

$y'=\frac{1}{\sec x}\times\sec x\tan x=\tan x$ $\quad y'(0)=0$

$y''=\sec^2 x$ $\quad y''(0)=1$

$y'''=2\sec^2 x\tan x$ $\quad y'''(0)=0$

$y''''=2(y'')(y')$ $\quad y^{iv}(0)=2$

$y^{v}=2(y''y''+y'''y')$ $\quad y^{v}(0)=0$

$y^{v}=2(y'''y''+y''y''+y''y'''+y^{iv}y')$

$y^{vi}=2(y''y^{v}+y^{v}y''+y^{iv}y''+y^{iv}y'')$

$y^{vi}(0)=2(1\times2+2\times1+2\times1+2\times1)$
$=16$

$y=\log(\sec(x))=\frac{x^2}{2!}+\frac{2x^4}{4!}+\frac{16x^6}{6!}$

$\tan x=x+\frac{\cancel{4}x^3}{\cancel{4}\times3\times\cancel{x}}+\frac{\cancel{16}x\,\cancel{6}x^5}{\cancel{6}\times5\times4\times3\times2}$

$\tan x=x+\frac{x^3}{3}+\frac{2x^5}{15}\ldots$

5a) $u=\log_e(x^2+y^2)+\tan^{-1}(y/x)$

$\frac{\partial u}{\partial x}=\frac{2x}{x^2+y^2}+\frac{1}{1+\frac{y^2}{x^2}}\times\frac{-y}{x^2}$

$\frac{\partial u}{\partial x}=\frac{2x-y}{x^2+y^2}$

$\frac{\partial^2u}{\partial x^2}=\frac{2(x^2+y^2)-(2x-y)(2x)}{x^2+y^2}$

$=\frac{2x^2+2y^2-4x^2+2xy}{x^2+y^2}$

$\frac{\partial u}{\partial y}=\frac{2y}{x^2+y^2}+\frac{1}{1+\frac{y^2}{x^2}}\times\frac{1}{x}$

$=\frac{2y+x}{x^2+y^2}$

$\frac{\partial^2u}{\partial y^2}=\frac{2(x^2+y^2)-(2y+x)(2y)}{x^2+y^2}$

$=\frac{2x^2+2y^2-4y^2-2xy}{x^2+y^2}$

$(1)+(2)\rightarrow \frac{-2x^2+2y^2+2xy+2x^2-2y^2-2xy}{x^2+y^2}$

5b) $u=\frac{yz}{x}$, $v=\frac{zx}{y}$, $w=\frac{xy}{z}$

$\frac{\partial(u,v,w)}{\partial(x,y,z)}=\begin{vmatrix}\partial u/\partial x&\partial u/\partial y&\partial u/\partial z\\\partial v/\partial x&\partial v/\partial y&\partial v/\partial z\\\partial w/\partial x&\partial w/\partial y&\partial w/\partial z\end{vmatrix}$

$=\begin{vmatrix}-yz/x^2&z/x&y/x\\z/y&-zx/y^2&x/y\\y/z&x/z&-xy/z^2\end{vmatrix}$

$=\frac{1}{xyz}\begin{vmatrix}-yz/x&z&y\\z&-zx/y&x\\y&x&-xy/z\end{vmatrix}$

$=\frac{1}{xyz}\left[-\frac{yz}{x}\left(\frac{-zx}{y}\cdot\frac{-xy}{z}-x^2\right)\right.$

6a) $u=f(x-y,y-z,z-x)$ $\quad p=x-y, q=y-z, r=z-x$

$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial p}\cdot\frac{\partial p}{\partial x}+\frac{\partial u}{\partial q}\cdot\frac{\partial q}{\partial x}+\frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x}$

$=\frac{\partial u}{\partial p}(1)+\frac{\partial u}{\partial q}(0)+\frac{\partial u}{\partial r}(-1)\rightarrow(1)$

$\frac{\partial u}{\partial y}=\frac{\partial u}{\partial p}\cdot\frac{\partial p}{\partial y}+\frac{\partial u}{\partial q}\cdot\frac{\partial q}{\partial y}+\frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial y}$

$=\frac{\partial u}{\partial p}(-1)+\frac{\partial u}{\partial q}(1)+\frac{\partial u}{\partial r}(0)$

$= \frac{\partial u}{\partial p}(1) + \frac{\partial u}{\partial q}(0) + \frac{\partial u}{\partial r}(-1) \rightarrow (1)$

$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial p}\cdot\frac{\partial p}{\partial y} + \frac{\partial u}{\partial q}\cdot\frac{\partial q}{\partial y} + \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial y}$

$= \frac{\partial u}{\partial p}(-1) + \frac{\partial u}{\partial q}(1) + \frac{\partial u}{\partial r}(0) \rightarrow (2)$

$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial p}\cdot\frac{\partial p}{\partial z} + \frac{\partial u}{\partial q}\cdot\frac{\partial q}{\partial z} + \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial z}$

$= \frac{\partial u}{\partial p}(0) + \frac{\partial u}{\partial q}(-1) + \frac{\partial u}{\partial r}(1) \rightarrow (3)$

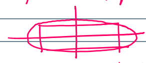
$(1)+(2)+(3) = 0$

$= \frac{2x+2y-2y-2xy}{x^2+y^2}$

$(1)+(2) \rightarrow \frac{-2x^2+2y^2+2xy+2x^2-2y^2-2xy}{x^2+y^2}$

$= 0$

6b) Volume of largest rec. parallelopiped in ellipsoid



$f(x,y,z) = 2x\cdot 2y\cdot 2z$ $\quad \pm x, \pm y, \pm z$

$V = 8xyz$

$F(x,y,z) = f(x,y,z) + \lambda\left(\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16}-1\right)$

$F = 8xyz + \lambda\left(\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16}-1\right)$

$\frac{\partial f}{\partial x} = 8yz + \lambda\cdot\frac{2x}{4} = 0 \Rightarrow \lambda = -8yz\cdot\frac{4}{2x} = -\frac{16yz}{x}$

$\frac{\partial F}{\partial y} = 8xz + \lambda\cdot\frac{2y}{9} = 0 \Rightarrow \lambda = -8xz\cdot\frac{9}{2y} = -\frac{36xz}{y}$

$\frac{\partial f}{\partial z} = 8xy + \lambda\cdot\frac{2z}{16} = 0 \Rightarrow \lambda = -8xy\cdot\frac{16}{2z} = -\frac{64xy}{z}$

$\rightarrow \lambda = -\frac{16yz}{x} = -\frac{36xz}{y} = -\frac{64xy}{z}$

$\frac{x^2}{y^2} = \frac{16}{36} \Rightarrow y^2 = \frac{9}{4}x^2 \qquad \frac{x^2}{z^2} = \frac{16}{64} \Rightarrow z^2 = 4x^2$

$\rightarrow \frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16} = 1 \qquad y = \frac{3}{2}\cdot\frac{2}{\sqrt3} = \sqrt3 \qquad z = 2x = 2\cdot\frac{2}{\sqrt3} = \frac{4}{\sqrt3}$

$\frac{x^2}{4} + \frac{9x^2}{4\cdot 9} + \frac{4x^2}{16} = 1$

$\frac{3x^2}{4} = 1 \Rightarrow x = \frac{2}{\sqrt3}$

$\rightarrow V = 8xyz = 8\cdot\frac{2}{\sqrt3}\cdot\sqrt3\cdot\frac{4}{\sqrt3} = \frac{64}{\sqrt3} = \frac{64\sqrt3}{3}$

$= \frac{1}{xyz}\left[-\frac{yz}{x}\left(-\frac{2x}{y}\cdot -\frac{xy}{z} - x^2\right) - z\left(-\frac{xy}{z}\cdot x - xy\right) + y\left(xz + \frac{xz}{y}\cdot y\right)\right]$

$= \frac{1}{xyz}\left[2xyz + xyz + xyz\right]$

$= 4$

7b) $\int_0^{\pi/2}\int_0^{a\sin\theta}\int_0^{\frac{a^2-r^2}{a}} r\,dz\,dr\,d\theta$

$\theta \rightarrow r \rightarrow z$

$= \int_0\int_0 z\Big|_0^{\frac{a^2-r^2}{a}} r\,dr\,d\theta$

$= \iint \frac{a^2-r^2}{a}\cdot r\,dr\,d\theta$

$= \int_0 \frac{1}{a}\int_0^{a\sin\theta} a^2r - r^3\,dr\,d\theta$

$= \int_0 \frac{1}{a}\left[a^2\frac{r^2}{2} - \frac{r^4}{4}\right]_0^{a\sin\theta} d\theta$

$\int_0 \frac{1}{a}\left(\frac{a^2\cdot a^2\sin^2\theta}{2} - \frac{a^4\sin^4\theta}{4}\right)$

$\int_0 \frac{1}{a}\, a^4\sin^2\theta\left(\frac{2}{4} - \frac{\sin^2\theta}{4}\right)$

$\frac{a^3}{4}\int_0 \sin^2\theta(2-\sin^2\theta)$

$\frac{a^3}{4}\int_0 2\sin^2\theta - \int(\sin^2\theta)^2$

$\frac{a^3}{4}\left[\int_0 2\frac{(1-\cos2\theta)}{2} - \int\frac{(1-\cos2\theta)^2}{2^2}\right]$

$\frac{a^3}{4}\left[\int_0 1-\cos2\theta - \frac{1}{4}\int 1+\cos^2 2\theta - 2\cos2\theta\right]$

$\frac{a^3}{4}\left[\theta - \frac{\sin2\theta}{2}\right]_0^{\pi/2} - \frac{1}{4}\int 1 + \left(\frac{1+\cos4\theta}{2}\right) - 2\cos2\theta$

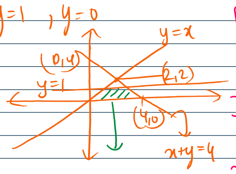
$\frac{a^3}{4}\left[\left(\frac{\pi}{2}-0\right) - \frac{1}{4}\left(\frac{3\theta}{2} + \frac{1}{2}\frac{\sin4\theta}{4} - \frac{2\sin2\theta}{2}\right)_0^{\pi/2}\right]$

$\frac{a^3}{4}\left[\frac{\pi}{2} - \frac{1}{4}\times\frac{3\pi}{4}\right]$

$\frac{a^3}{4}\left(\frac{\pi}{2} - \frac{3\pi}{16}\right) = \frac{a^3}{4}\cdot\frac{5\pi}{16} = \frac{5\pi a^3}{64}$

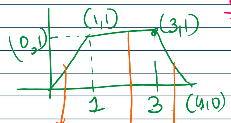
7a) $y = x,\ x+y=4,\ y=1,\ y=0$

x	0	4	2
y	4	0	2



$= \int_0^1\int_0^x dy\,dx + \int_1^3\int_0^1 dy\,dx + \int_3^4\int_0^{4-x} dy\,dx$

$x\ y \qquad x\ y \qquad x\ y$



$= \int_0^1 y\Big|_0^x dx + \int_1^3 y\Big|_0^1 dx + \int_3^4 y\Big|_0^{4-x} dx$

$= \int_0^1 x\,dx + \int_1^3 dx + \int_3^4 (4-x)\,dx$

$= \frac{x^2}{2}\Big|_0^1 + x\Big|_1^3 + 4x - \frac{x^2}{2}\Big|_3^4$

$= \frac{1}{2} + 3 + 16 - 8 - \left(12 - \frac{9}{2}\right)$

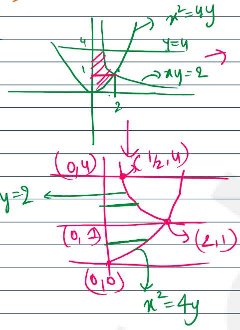
$= \frac{1}{2} + 11 - 12 + \frac{9}{2} = 5-1 = 4$

8a) $xy = 2,\ 4y = x^2,\ y = 4$

x	2
y	2	1

$4\cdot\frac{2}{x} = x^2$, $x^3 = 8$, $x = 2, y = 1$

$x\cdot 4 = 2$, $x = \frac{1}{2}$



$A = \int_0^1\int_0^{2\sqrt y} dx\,dy + \int_1^4\int_0^{2/y} dx\,dy$

$= \int_0^1 x\Big|_0^{2\sqrt y} dy + \int_1^4 x\Big|_0^{2/y} dy$

$= \int_0^1 2\sqrt y\,dy + \int_1^4 \frac{2}{y}\,dy$

$= 2\frac{y^{3/2}}{3/2}\Big|_0^1 + 2\log y\Big|_1^4$

$= \frac{4}{3}\times 1 + 2(\log 4 - \log 1)$

$= \frac{4}{3} + 2(2\log 2 - 0)$

$= \frac{4}{3} + 4\log 2 = 4\left(\frac{1}{3} + \log 2\right)$

8b) $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ $\qquad x = r\cos\theta,\ y = r\sin\theta,\ r^2 = x^2+y^2$

$= \int_0^{\pi/2}\int_0^\infty e^{-r^2}\, r\,dr\,d\theta$ $\qquad y=0, y=\infty$

$= \int_0\int_0^\infty e^{-t}\frac{dt}{2}\,d\theta$ $\qquad$ let $r^2 = t$, $2r\,dr = dt$, $r=0, t=0$; $r=\infty, t=\infty$

$= \int_0 \frac{1}{2}\frac{e^{-t}}{-1}\Big|_0^\infty d\theta$

$= \int_0^{\pi/2} -\frac{1}{2}(e^{-\infty} - e^0)\,d\theta$

$= \int_0 -\frac{1}{2}(0-1)\,d\theta$

$= \frac{1}{2}\,\theta\Big|_0^{\pi/2} = \pi/4$

9a)

x	y	Δy	Δ^2	Δ^3	Δ^4
10k	20	45	70	25	-215
20k	65	115	95	-190	
30k	180	210	-95		
40k	390	115			
50k	505				

$p = \frac{x - x_0}{h} = \frac{25k - 10k}{10k} = \frac{15k}{10k} = \frac{3}{2}$

$\rightarrow f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$

$f(25k) = 20 + \frac{3}{2}(45) + \frac{3}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(70)$

$+ \frac{3}{2}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{6}\right)(25) + \frac{3}{2}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{1}{24}\right)(-215)$

$f(25k) = 107.1484$

$20k - 25k = 107.1484 - 65 \approx 42$

9b)

x	y
2	1
10	3
17	4

$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \times y_0$

$y = \frac{(x-10)(x-17)}{(2-10)(2-17)} \times 1 + \frac{(x-2)(x-17)}{(10-2)(10-17)} \times 3 + \frac{(x-2)(x-10)}{(17-2)(17-10)} \times 4$

$= \frac{x^2 - 27x + 170}{(-8)(-15)} + \frac{x^2 - 19x + 34}{(8)(7)}\times(3) + \frac{x^2 - 12x + 20}{(15)(7)}\times(4)$

$=$

10b) $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$

x	y	Δy	Δ^2	Δ^3
19	91	9.25	0.5	0
20	100.25	9.75	0.5	0
21	110	10.25	0.5	
22	120.25	10.75		
23	131			

$y_1 = \frac{1}{h}\left(\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n \cdots\right)$

$= \frac{1}{1}\left(10.75 + \frac{1}{2}(0.5) + \frac{1}{3}(0)\right)$

$y_1 = 11$

$y_2 = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n \cdots\right)$

$= \frac{1}{1^2}(0.5 + 0)$

$y_2 = 0.5$

10a) $\int_0^1 \frac{dx}{1+x} = \frac{3h}{8}\left(y_0 + y_6 + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right)$

x	0	1/6	2/6	3/6	4/6	5/6	1
y	1	6/7	3/4	2/3	3/5	6/11	1/2
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$\ln(1+x)\Big|_0^1 = \frac{3(1/6)}{8}\left(1 + 1/2 + 3(6/7 + 3/4 + 3/5 + 6/11) + 2(2/3)\right)$

$= \frac{1}{16}\left(3/2 + 8.2578 + 4/3\right)$

$\ln(2) = \frac{11.0911}{16} = 0.6932$

$$= \frac{1}{16}\left(3/2 + 8.2578 + 4/3\right)$$

$$\ln(2) = \frac{11.0911}{16} = 0.6932$$

$$= \frac{1}{1^2}(0.5 + 0)$$

$$y_2 = 0.5$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{(1+11^2)^{3/2}}{0.5} = 2(122)^{3/2}$$

$$\rho = 2695.0680$$

model question ...

MA211TA

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RV COLLEGE OF ENGINEERING
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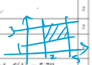
I Semester B.E. February -2024 Examinations
DEPARTMENT OF MATHEMATICS
FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS
(2022 SCHEME)
(Non-Integrated Course)

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, and 9 and 10.

PART-A (Objective type for one or two marks)
(True & false and match the following questions are not permitted)

1	1.1	Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 5 \\ 0 & 0 & 3 \end{bmatrix}$, then the eigen values of the matrix A^{-2} are ______. $(1)^{-2}$ $(-1)^{-2}$ $(3)^{-2}$	1
	1.2	The reduced system of set of linear equations is $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$. Then the solution of the system is -4 3 0	1
	1.3	Angle between radius vector and tangent to the curve $r = e^{-\theta}$ is ______.	2
	1.4	If rank of the matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ k & 13 & 10 \end{bmatrix}$ is 2, then the value of k = 2 $\lvert A \rvert = 0$	2
	1.5	The Maclaurin series expansion for $\cosh x$ is $1 + \frac{x^2}{2!} + \frac{x^4}{4!}$	2
	1.6	If $x = r\cos\theta$, $y = r\sin\theta$, then $J\left(\frac{x,y}{r,\theta}\right)$ = r	2
	1.7	If $w(x,y) = x^2y$ then $\frac{\partial^2 w}{\partial x \partial y}$ at the point (-1, 1)= -2	2
	1.8	Evaluate $\int_0^1 \int_2^3 xy\,dy\,dx$. 5/4	2
	1.9	Sketch the domain of integral $\int_2^5 \int_1^3 x(x^2+y^2)dydx$. 	2
	1.10	If $f(0) = 0$, $f\left(\frac{1}{4}\right) = 2.45$, $f\left(\frac{2}{4}\right) = 3.97$, $f\left(\frac{3}{4}\right) = 5.58$ and $f(1) = 5.78$, then $\int_0^1 f(x)dx$ = 3.82	2
	1.11	Given $y(2) = -2$, $y(4) = 4$, $y(6) = 10$, $y(8) = 12$, $y(10) = 14$, then $\nabla^3 y(8)$= -4	2

PART-B

UNIT-I

2 a. Investigate for what values of λ and μ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions. 5

$\lambda = 5$ $\mu \neq 9$ $\quad \lambda = 5,\ \mu = 9 \quad \lambda \neq 5,\ \mu \in R$

b. The currents I_1, I_2, I_3 in an electrical network follow the linear equations $8I_1 - 3I_2 + 2I_3 = 20$, $4I_1 + 11I_2 - I_3 = 33$, $2I_1 + I_2 + 4I_3 = 12$. Compute I_1, I_2, I_3 using Gauss-Seidel iteration method. Perform four iterations. 5

3.01
1.99
0.99

c. Eigen values and eigen vectors are used to calculate the theoretical limit of how much information can be carried via a communication channel. Identify the dominant eigen value and corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

by taking the initial approximation $\begin{bmatrix} 1 \\ 0.8 \\ 0.8 \end{bmatrix}$. Perform 5 iterations.

$\lambda^{(5)} = 4.7620$ $\begin{bmatrix} 1 \\ 0.9960 \\ 0.0279 \end{bmatrix}$

UNIT-II

3 a. Find the slope of the tangent to the parabola $\frac{2a}{r} = 1 - \cos\theta$ at $\theta = \frac{2\pi}{3}$. $\sqrt{3}$

b. Obtain the circle for curvature of the curve $\sqrt{x} + \sqrt{y} = 4$ at the point where it cuts the line passing through the origin making an angle 45^0 with x-axis.

$(x-12)^2 + (y-12)^2 = 128$

OR

4 a. Show that the radius of curvature at any point (r, θ) on the Cardiod $r = a(1 - \cos\theta)$ varies as $\sqrt{r}$.

b. Expand $f(x) = \tan^{-1} x$ in ascending powers of x upto the term containing x^5, hence obtain the expansion of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$. $= 2 - \frac{2x^3}{3} + \frac{2x^5}{5}$

$= 2\tan^{-1} x$

UNIT-III

5 a. Prove that $v = e^{a\theta}\cos(a \log r)$ is the solution of the Laplace equation

$$v_{rr} + \frac{1}{r}v_r + \frac{1}{r^2}v_{\theta\theta} = 0.$$

b. If $u = x^2 + y^2 + z^2$, $v = x + y + z$ and $w = xy + yz + zx$, prove that $J(u, v, w)$ vanishes identically. Also find the relation between the given functions. $v^2 = u - 2w$

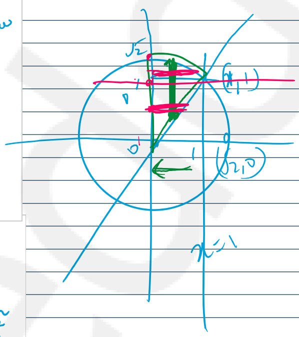
OR

6 a. (i) If $z = f(y - 3x) + g(y + 2x) + \sin x - y\cos x$, show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x.$$

(ii) Find $\frac{du}{dt}$ given $u = \sin\left(\frac{x}{y}\right)$ where $x = e^t$, $y = t^2$. $\cos\frac{e^t}{t^2}\left(\frac{e^t(t-2)}{t^3}\right)$

b. The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the unit sphere using Lagrange multiplier. $\frac{200\sqrt{3}}{3}$



UNIT-IV

7 a. Evaluate $\iint_R xy\,dx\,dy$ where R is the triangular bounded by the axes of coordinates and the line $\frac{x}{2} + \frac{y}{3} = 1$. 3/2

b. Show that $\int_0^{\frac{\pi}{2}} \int_0^{a\sin\theta} \int_0^{\frac{a^2-r^2}{a}} r\,dz\,dr\,d\theta = \frac{5\pi a^3}{64}$.

OR

8 a. Using double integral find the area enclosed by the curve $r = a(1 + \cos\theta)$ and lying above the initial line. $\frac{3\pi a^2}{4}$

b. Change the order of integration and evaluate $\int_1^{?} \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$. $2-\sqrt{2}$

$\int_0^1 \int_0^y + \int_1^{\sqrt{2}} \int_0^{\sqrt{2-y^2}}$

UNIT-V

9 a. In an experiment, the values of the output y were recorded for input x from 1.0 to 3.5 at intervals of 0.5. Estimate by using appropriate interpolation formulae, the values of y for input (i) $x = 1.2$, (ii) $x = 3.4$ (iii) $x = 3.8$.

x	1.0	1.5	2.0	2.5	3.0	3.5
y	177	166	146	130	115	102

b. Fit a cubic polynomial for the following data and hence find $f(5)$ and $f(6)$.

x	1	2	3	7
$f(x)$	2	4	8	128

OR

10 a. Numerical integration is used to simulate devices such as semiconductors and antennas. Estimate the value of the integral $\int_0^2 e^{-x^2}dx$ using Simpson's 1/3, Simpson's 3/8 and Weddle's rules, by dividing the interval [0,2] into six equal sub intervals.

b. The following data defines the sea-level concentration of dissolved oxygen for fresh water as a function of temperature:

T (°C)	0	8	15	25	32
O_2 (mg/L)	14.621	11.843	9.870	8.418	7.305

Calculate the amount of oxygen when temperature 10°C and 35°C.

Signature of Scrutinizer: Signature of Chairman

Name: Name:

model question ...

MA211TA

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(2022 SCHEME)
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Time: 03 Hours **Maximum Marks: 100**

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1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, and 9 and 10.

PART-A (Objective type for one or two marks)
(True & false and match the following questions are not permitted)

1	1.1	Product of the eigen values of the matrix $A=\begin{bmatrix}1&2&3\\-1&-1&-1\\3&6&4\end{bmatrix}$ is -5	1
	1.2	Rank of the matrix $A=\begin{bmatrix}1&4&3\\0&2&1\\0&6&3\end{bmatrix}$ is 2.	1
	1.3	Curvature of a straight line $y=3x+2$ is 0.	1
	1.4	Given $y(1)=2.5, y(3)=5.6, y(5)=7.2, y(7)=8.5$, then $\Delta y(3)=$ 1.6	1
	1.5	Maclaurin series expansion of $y=e^{-\frac{x}{2}}$ is $1-\frac{x}{2}+\frac{x^2}{8}-\frac{x^3}{48}\ldots$	1
	1.6	If two characteristic roots of the matrix $\begin{bmatrix}1&1&3\\1&5&1\\3&1&1\end{bmatrix}$ are 3 and 6, then the third characteristic root is -2.	1
	1.7	The angle between the radius vector and tangent for the curve $r=ae^{\theta\cot\alpha}$ is α.	2
	1.8	If $z=(\cos x)^y$, then $\frac{\partial z}{\partial y}=$ $(\cos x)^y\ln(\cos x)$	2
	1.9	Given that $z=2xy-3x^2y$ and x increases at the rate of 2cm/sec. Find the rate at which y changes at the instant when x =3cm and y=1cm in order that the z shall be neither increasing nor decreasing. $-32/21$	2
	1.10	Evaluate $\int_0^2\int_1^3 x^2y\,dy\,dx$. $32/3$	2
	1.11	Sketch the region of integration $\int_0^{\frac{\pi}{2}}\int_0^{2a\cos\theta} r^2\sin\theta\ dr\ d\theta$. $(a,0)$	2
	1.12	The value of $\Delta^4[(x-2)(2x-3)(3x-4)]$ with $h=5$ is 0	2
	1.13	If $f(0)=1,\ f(0.25)=1.03,\ f(0.5)=1.39,\ f(0.75)=1.97$ and $f(1)=2.56$, then $\int_0^1 f(x)dx=$ 1.5283	2

PART-B

		UNIT-I	
2	a	Compute the rank of the matrix $A=\begin{bmatrix}1&2&3&2\\2&3&5&1\\1&3&4&5\end{bmatrix}$. 2	4
	b	The currents i_1, i_2, i_3 in the paths of an electrical network follow the linear equations $i_1-i_2+i_3=0,\ 3i_1+2i_3=7,\ 2i_2+4i_3=8$. Determine i_1,i_2,i_3 using Gauss-Jordan elimination method.	8
	c	Determine the largest eigenvalue and the corresponding eigenvector of the matrix $A=\begin{bmatrix}-2&0&-1\\1&-1&1\\2&2&0\end{bmatrix}$. Choose the initial vector as $\begin{bmatrix}1\\1\\1\end{bmatrix}$. Perform 5 iterations.	4

17/13
20/13
3/13

		UNIT-II	
3	a	Expand $y=\log_e\sec x$ in ascending powers of x up to and including the term in x^4 and hence deduce the expansion of $\tan x$.	8
	b	For the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that ρ at any point is equal to $\frac{(CD)^3}{ab}$ where C is the centre of the ellipse and D is an extremity of the diameter conjugate to CP.	8
		OR	
4	a	Find the radius of curvature of Folium $x^3+y^3=3axy$ at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$. $-3a/8\sqrt{2}$	8
	b	Show that the angle of intersection of the Lemniscate $r^2=a^2\cos 2\theta$ and Cardioid $r=a(1+\cos\theta)$ intersect at angle $3\sin^{-1}\left(\frac{3}{4}\right)^{\frac{1}{4}}$. Yay!)	8

		UNIT-III	
5	a	If $u=f\left(\frac{y-x}{xy},\frac{z-x}{xz}\right)$, find the value of $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}+z^2\frac{\partial u}{\partial z}$.	8
	b	Show that by Lagrange's method of undetermined multipliers the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.	8
		OR	
6	a	In robotics, the functions representing robotic arm from cartesian to any system (x,y) are given by $x=e^u\cos v$ and $y=e^u\sin v$. As the Jacobian represents transformation factor between different systems, verify $\frac{\partial(x,y)}{\partial(u,v)}\times\frac{\partial(u,v)}{\partial(x,y)}=1$.	8
	b	If $z=x^2\tan^{-1}\left(\frac{y}{x}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right)$, verify that $\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}$.	8

		UNIT-IV	
7	a	By changing the order of integration and hence evaluate $\int_0^3\int_1^{\sqrt{4-y}}(x+y)\,dx\,dy$. $241/60$	8
	b	Find the volume of the tetrahedron $x\geq 0, y\geq 0, z\geq 0, \frac{x}{a}+\frac{y}{b}+\frac{z}{c}\leq 1$.	8
		OR	

$abc/6$

8	a	Transform to polar coordinates and hence evaluate $\int_0^2\int_1^{\sqrt{4-y^2}} y\sqrt{x^2+y^2}\,dx\,dy$.	8
	b	Determine the centre of gravity of the triangular lamina bounded by the coordinate axis and the line $x+y=1$. $\left(\frac{1}{3},\frac{1}{3}\right)$	8

47/12

		UNIT-V	
9	a	Using Lagrange's interpolation, find the polynomial of lowest degree which agrees with the point (x,y) given in the following table. Hence find $y(2.5)$.	8
	b	The following table gives the result of an observation. The temperature T in degree centigrade of a vessel of cooling water is given in different time t (in minutes). (i) Estimate the temperature at $t=1.5$. (ii) Estimate the approximate rate of cooling at $t=3$.	8

x	3	2	1	0	-1
$y(x)$	8	26	32	14	-40

t	1	3	5	7	9
T	85.3	74.5	67.0	60.5	54.3

OR

10	a	The table gives the distance in nautical miles of the visible horizon for the given heights in feet, above the earth's surface. Find the distances when the heights are $160\ ft$ and $410\ ft$.	8
	b	Estimate the value of the integral $\int_1^4\sqrt{x}e^{-x}dx$ using Simpson's 1/3, Simpson's 3/8 and Weddle's rules, by diving the interval [1,4] into six equal sub intervals.	8

Height	100	150	200	250	300	350	400
Distance	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Signature of Scrutinizer: Signature of Chairman

Name: Name:

model question ...

Course: Fundamentals Of Linear algebra, Calculus and Numerical Methods

Course Code: MA211TA

PART A

1.1	A is a 4 × 4 matrix. The third row is a combination of the first and second rows. The fourth row is combination of second and third rows, then rank of A = ______ 2	1
1.2	The Eigen values of $A = \begin{bmatrix} 0 & 5 \\ 3 & 0 \end{bmatrix}$ are ______ $\pm\sqrt{15}$	1
1.3	If Eigen values of A are 1, 3, 5 then $trace(A^2)$ = ______ 35	1
1.4	The system $kx + 3y = 6$; $3x + ky = -6$ is inconsistent for k = ______ ± 3	1
1.5	If $u = xy^2z^3$, then the total differential du = ______ $y^2z^3\,dx + 2xyz^3\,dy + 3xy^2z^2\,dz$	1
1.6	If $u = x(1-y)$ and $v = x^2y$, then $\frac{\partial(u,v)}{\partial(x,y)}$ = ______ $x^2(1+y)$	2
1.7	The Maclaurin series expansion for $\log(1+x)$ is ______ $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$	2
1.8	Two sound waves are represented as polar curves intersecting orthogonally, in the usual notation at the point of intersection $\tan(\theta_1) = a$ and $\tan(\theta_2) = -\frac{4}{a}$. The value of a is ______ ± 2	2
1.9	Evaluate: $\int_0^1\int_0^{1-x} dy\,dx$ ______ $1/2$	2
1.10	Express the integral $\int_0^2\int_0^{\sqrt{4-x^2}} dydx$ as equivalent integral by changing to polar coordinates. $\int_0^{\pi/2}\int_0^{2} r\,dr\,d\theta$	2
1.11	Construct a forward difference table for the following values of x, y.	2
1.12	The velocity of a train which starts from rest is given by the table. The approximate distance travelled in 8 minutes by using Simpson's 1/3rd rule is ______ 136.67	2

x	0	5	10	15	20
y	6	10	13	17	23

Minutes	0	2	4	6	8
Miles/hour	0	10	18	25	29

PART – B

2a	Examine the consistency of the following system of equations and solve, if consistent. $2x - y - z = 2$, $x + 2y + z = 2$, $4x - 7y - 5z = 2$	6
2b	Solve the following system of equations by Gauss Siedel Method. $x + 2y + z = 0$, $3x + y - z = 0$, $x - y + 4z = 3$. Carry out 3 iterations by taking the initial approximation as (1,1,1).	5
2c	Compute the largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by applying Rayleigh's power method taking $X^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ as the initial vector. Carry out 4 iteration.	5
3a	Show that the two curves $r^2 = a^2\cos 2\theta$ and $r = a(1+\cos\theta)$ intersect at an angle $3\sin^{-1}\left(\frac{3}{4}\right)^{\frac{1}{4}}$.	8
3b	Expand $f(x) = (x+2)^4 + 5(x+2)^3 + 6(x+2)^2 + 7(x+2) + 8$ in ascending powers of $(x-1)$. taylors $a=1$	8
4a	Obtain the radius of curvature at any point $P(x,y)$ on the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$; $a > 0$. $3a\cos\theta\sin\theta$	8
4b	Find the centre of curvature and circle of curvature of the curve $y = x^3 - 6x^2 + 3x + 1$ at a point (1, -1). $\bar{x}, \bar{y} = -36, -43/6$; $\rho^2 = \frac{37^3}{36}$	8
5a	The two-dimensional Laplace equation in polar coordinates is given by $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. Any function $u(r,\theta)$ satisfying this equation called a harmonic function. Show that $u = e^{a\theta}\cos(a\log r)$ is a harmonic function.	8
5b	Evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. Given $x = u$, $y = u\tan v$, $z = w$. $x^2 + y^2$	8
	OR	
6a	If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.	8
6b	Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum. Solve by Lagrange method of undetermined multipliers. $x, y, z = 4, 8, 12$	8
7a	Compute the area enclosed by the parabola $y^2 = 4ax$ and the lines $x + y = 3a$, $y = 0$ in the first quadrant. $\frac{10a^2}{3}$	8
7b	By changing the order of integration, evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xy\,dy\,dx$. $\frac{64}{3}$	8
	OR	
8a	Evaluate: $\iiint x^2yz\,dx\,dy\,dz$ over the region bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$. $\frac{1}{2520}$	8
8b	Evaluate $\iint_R r\sqrt{a^2 - r^2}\,dr\,d\theta$, over the upper half of the circle $r = a\cos\theta$.	8
9a	Find the number of persons getting wages less than Rs. 15 from the following data: 24	8
9b	A body moving with velocity 'v' at any time 't' satisfies the data. Obtain the distance travelled in 4 seconds.	8
	OR	
10a	A rod is rotating in a plane. The following table gives the angle A (in radians) through which the rod has turned for various values of time T (seconds). Calculate the angular velocity and angular acceleration of the rod at T = 0.6 second. $3.8167 = V$, $6.75 = a$	8
10b	Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's 1/3 rule, 3/8 rule and Weddle's rule. 1.3662, 1.3571, 1.3735	8

Wages in Rs.	0-10	10-20	20-30	30-40
No. of persons	9	30	35	42

t	0	1	3	4
v	21	15	12	10

T	0	0.2	0.4	0.6	0.8	1
A	0	0.12	0.49	1.12	2.02	3.20

$(\frac{6+c}{5}, \frac{2-3c}{5}, c)$

$\theta(=)$

$|A - \lambda I| = 0$

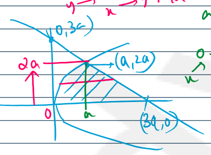
$= \begin{matrix} 0 & 5 \\ 3 & 0 \end{matrix} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{vmatrix} -\lambda & 5 \\ 3 & -\lambda \end{vmatrix} = 0$

$\lambda^2 - 15 = 0$

$\lambda = \pm\sqrt{15}$

7a) $\int_{y \to 0}^{2a}\int_{x \to y^2/4a}^{3a-y}$



$\int_{x \to 0}^{a}\int_0^{2\sqrt{ax}} + \int_a^{3a}\int_0^{3a-x}$

$y^2 = 4ax$ $\quad x + y = 3a$

$x = 3a - y$

$y^2 = 4a(3a - y)$

$y^2 + 4ay - 12a^2 = 0$

6 -2

$y^2 + 6ay - 2ay - 12a^2 = 0$

$y(y + 6a) - 2a(y + 6a) = 0$

$y = 2a, x = a$.

8a) $\int_{x \to 0}^{1}\int_{y \to 0}^{1-x}\int_{z \to 0}^{1-x-y} x^2yz\,dx\,dy\,dz$

$= \int_x\int_y x^2y\,\frac{z^2}{2}\Big|_0^{1-x-y}\,dx\,dy$

$\int_x\int_y \frac{x^2y}{2}(1-x-y)^2\,dx\,dy$ $\quad ((1-x) - (y))$

$\int_x \frac{x^2}{2}\int_y y((1-x) - y)^2\,dy\,dx$

$\int_x \frac{x^2}{2}\int_y y((1-x)^2 + y^2 - 2y(1-x))\,dy\,dx$

$\int_x \frac{x^2}{2}\int_y (1-x)^2y + y^3 - 2(1-x)y^2\,dy\,dx$

$\int_x \frac{x^2}{2}\,(1-x)^2\frac{y^2}{2} + \frac{y^4}{4} - 2(1-x)\frac{y^3}{3}\Big|_0^{1-x}\,dx$

$\int_x \frac{x^2}{2}\left[\frac{(1-x)^4}{2} + \frac{(1-x)^4}{4} - \frac{2(1-x)^4}{3}\right]dx$

$\int_x \frac{x^2}{2}\cdot\frac{(1-x)^4}{12}\,dx$ $\quad \left\{\frac{t}{2} + \frac{t}{4} - \frac{2t}{3} = \frac{t}{12}\right\}$

$\frac{1}{24}\int_x x^2(1-x)^4\,dx$

$x^2(1-x)^2(1-x)^2$

$(x(1-x))^2(1-x)^2$

$(x - x^2)^2(1-x)^2$

$(x^2 + x^4 - 2x^3)(1 + x^2 - 2x)$

$x^2 + \cancel{x^4} - \cancel{2x^3} + \cancel{x^4} + \cancel{x^6} - \cancel{2x^5} - \cancel{2x^3} - \cancel{2x^5} + \cancel{4x^4}$

$\frac{1}{24}\int_x x^6 - 4x^5 + 6x^4 - 4x^3 + x^2$

$\frac{1}{24}\left[\frac{x^7}{7} - \frac{4x^6}{6} + \frac{6x^5}{5} - \frac{4x^4}{4} + \frac{x^3}{3}\right]_0^1$

8b) $\iint_R r\sqrt{a^2 - r^2}\,dr\,d\theta$

$r = a\cos\theta$

$r^2 = ar\cos\theta$

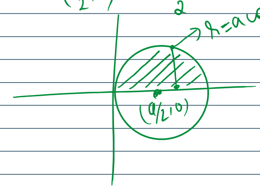
$x^2 + y^2 = ax$

$x^2 - \frac{2ax}{2} + y^2 = 0$

$x^2 - 2(x)\left(\frac{a}{2}\right) + \left(\frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2 + y^2 = 0$

$\left(x - \frac{a}{2}\right)^2 + y^2 = \left(\frac{a}{2}\right)^2$

$\left(\frac{a}{2}, 0\right)$ $\quad r = \frac{a}{2}$



$\int_0^{\pi/2}\int_0^{a\cos\theta} r\sqrt{a^2 - r^2}\,dr\,d\theta$

$\int_\theta -\frac{1}{2}\int_{a^2}^{a^2\sin^2\theta}\sqrt{t}\,dt\,d\theta$ $\quad a^2 - r^2 = t$; $-2r\,dr = dt$; $r\,dr = \frac{dt}{-2}$

$\int_\theta -\frac{1}{\cancel{2}}\,\frac{t^{3/2}}{3/\cancel{2}}\Big|_{a^2}^{a^2\sin^2\theta}$ $\quad r = 0, t = a^2$; $r = a\cos\theta$, $t = a^2 - a^2\cos^2\theta = a^2\sin^2\theta$

$\int_\theta -\frac{1}{3}(a^3\sin^3\theta - a^3)\,d\theta$

$-\frac{a^3}{3}\left[\int\sin^3\theta\,d\theta - \int d\theta\right]$

$-\frac{a^3}{3}\left[\int\sin\theta(1 - \cos^2\theta)\,d\theta - \theta\Big|_0^{\pi/2}\right]$

$\cos\theta = t$; $-\sin\theta\,d\theta = dt$; $\theta = 0, t = 1$; $\theta = \pi/2, t = 0$

$-\frac{a^3}{3}\left[-\int 1 - t^2\,dt - \frac{\pi}{2}\right]$ $\quad -\pi$

$\frac{a^3}{3}\left[t - \frac{t^3}{3}\Big|_1^0 + \frac{\pi}{2}\right]$ $\quad +\pi$

$\frac{a^3}{3}\left[0 - \left(1 - \frac{1}{3}\right) + \frac{\pi}{2}\right]$

$\frac{1}{24}\pi\int x^6 - 4x^5 + 6x^4 - 4x^3 + x^2 \quad \leftarrow \quad \frac{1}{24}\pi\int x^2(1-x)^4\,dx$

$\frac{1}{24}\left[\frac{x^7}{7} - \frac{4x^6}{6} + \frac{6x^5}{5} - \frac{4x^4}{4} + \frac{x^3}{3}\right]_0^1$

$\frac{1}{24}\left(\frac{1}{7} - \frac{2}{3} + \frac{6}{5} - 1 + \frac{1}{3}\right) = \frac{1}{2520}$

$\frac{a^3}{3}\left[0 - \left(1 - \frac{1}{3}\right) + \frac{\pi}{2}\right]$

$\frac{a^3}{3}\left[-\frac{2}{3} + \frac{\pi}{2}\right]$

$\frac{a^3}{3} \times \frac{3\pi - 4}{6}$

$\frac{(3\pi-4)a^3}{18}$

$\left(1+\frac{1}{3}\right) - \left(1 - \frac{1}{3}\right)$

$-1 + \frac{1}{3} - 1 + \frac{1}{3}$

$\frac{2}{3} - 2 \quad \frac{2-6}{3} = -\frac{4}{3} + \pi$

$-\frac{a^3}{3}\left(\frac{4}{3} - \pi\right)$

model question ...

MA211TA/ 22MA11A

USN

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

I Semester B. E. Supplementary Examinations Aug-2024

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS

(Common to EC, EE, EI, ET)

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of mathematics Handbook is permitted. Do not write anything on handbook.

PART-A

			M	BT	CO
1	1.1	Rank of singular matrix of order 4 is ______. <4	01	2	1
	1.2	In solving $n \times n$ non-homogeneous system of equations $AX = B$, using Gauss-Jordan method, the coefficient matrix A is reduced to ______. identity matrix	01	1	1
	1.3	The radius of curvature for straight line $y = mx + c$ is ______. ∞	01	1	1
	1.4	Transform the circle $x^2 + y^2 - 2x = 0$ in polar form. $r = 2\cos\theta$	01	2	2
	1.5	Simpson's three-eight rule is used when number of subintervals is multiple of ______. 3	01	1	1
	1.6	The value of $\Delta^4[(x-2)(2x-3)(3x-4)]$ is ______. 0	01	2	2
	1.7	Eigenvalues of the matrix $A = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}$ are ______. $\pm 2\sqrt{2}i$	02	1	2
	1.8	Find the angle between the radius vector and tangent to the curve $r = ae^{\theta\cot\alpha}$. α	02	1	1
	1.9	If $w(x,y) = y^2\sin(x)$, then at the point $(\pi, 1)$, $\frac{\partial^2 w}{\partial x \partial y}$ = ______. -2	02	1	1
	1.10	Given $z = xy^2 + x^3y$ where x and y are functions of t with $x(1) = 1$, $y(1) = 2$, $x'(1) = 3$ and $y'(1) = 4$. The value of $\frac{dz}{dt}$ at $t = 1$ is ______. 50	02	2	2
	1.11	Evaluate $\int_1^2\int_1^2 xdx\,dy$. 3	02	3	2
	1.12	Transform the integral to polar form $\int_0^2\int_0^{\sqrt{4-x^2}} dydx$. $\int_0^{\pi/2}\int_0^2 r\,dr\,d\theta$	02	3	3
	1.13	Construct the difference table for the following data points: (table below)	02	1	1

x	2	4	6	8
y	3	8	12	7

x y Δ Δ² Δ³

2 3 → 5 → −1

4 8 → 4 → −9 → −8

6 12 → −5

8 7

PART-B

			M	BT	CO
2	a	Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & 1 \\ 3 & 1 & 0 \\ 1 & 1 & -2 \end{bmatrix}$. 2	05	2	2
	b	Estimate the values of p and q for which the system of linear equations: $x + y + z = 1$, $2x + y + 4z = 2$, $4x + y + pz = q$ has i) a unique solution $p \neq 10$, $q \in R$ ii) no solution $p = 10$, $q \neq 4$ iii) an infinite number of solutions. $p = 10$, $q = 4$	05	3	2
	c	The eigenvalues given the displacement of an atom or a molecule from its equilibrium position and the direction of displacement is given by eigenvectors. Use the Rayleigh's power method to identify the dominant eigenvalue and corresponding eigenvector of the matrix. $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ with initial vector $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Carry out four iterations. 5.4 $\begin{bmatrix} -0.3169 \\ 1 \\ -0.3498 \end{bmatrix}$	06	3	3
3	a	Determine the angle of intersection of the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$. $\pi/4$	08	2	2
	b	Find the centre of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$. $3a/4, 3a/4$	08	2	2
		OR			
4	a	Obtain the radius of curvature at any point of the cycloid $x = a(t + \sin t)$, $y = a(1 - \cos t)$. $4a\cos t/2$	08	2	2
	b	Using Maclaurin series expand $e^x\cos x$ in powers of x up to fourth degree terms. $1 + x - \frac{x^3}{3} - \frac{x^4}{6} \ldots$	08	3	3
5	a	The two-dimensional Laplace equation in polar coordinates is given by $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. Any function $u(r,\theta)$ satisfying this equation called a harmonic function. Show that $u = e^{a\theta}\cos(a\log r)$ is a harmonic function.	08	3	2
	b	Compute $J\left(\frac{u,v,w}{x,y,z}\right)$ when $u = 3x + 2y - z$, $v = x - 2y + z$ and $w = x(x + 2y - z)$. Interpret the result. 0 functionally dependent	08	2	3
		OR			
6	a	If z is a function of x and y and if $x = e^u\sin v$, $y = e^u\cos v$, prove that i) $\frac{\partial z}{\partial u} = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ ii) $\frac{\partial z}{\partial x} = e^{-u}\left(\sin v\frac{\partial z}{\partial u} + \cos v\frac{\partial z}{\partial v}\right)$.	08	3	3
	b	A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point (x,y,z) on the probe's surface is $T(x,y,z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's surface, using Lagrange's method of multipliers.	08	4	4
7	a	Evaluate $\iint_R xy\,dxdy$ where R is the triangular region bounded by the x-axis, y-axis and the line $x + y = 2$. $2/3$	08	3	2
	b	Determine the centroid of the rectangular lamina bounded by $x = 0$, $x = 4$, $y = 0$, $y = 3$ when the density is xy at the point (x,y). $\bar{x}, \bar{y}$ $\left(\frac{8}{3}, 2\right)$	08	4	4
		OR			
8	a	Change the order of integration and hence evaluate $\int_0^2\int_{x/2}^{\sqrt{x/2}}(x^2 + y^2)\,dydx$. $27/70$	08	3	3
	b	Determine the volume of the tetrahedron $x \geq 0, y \geq 0, z \geq 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} \leq 1$. $\frac{abc}{6}$	08	4	4

			M	BT	CO
9	a	The following data defines the sea-level concentration of dissolved oxygen for fresh water as a function of temperature: (table 9a below). Use appropriate Newton's interpolation formula to calculate the amount of oxygen when temperature is 10 ℃ and 35 ℃.	08	3	3
	b	For the given data: (table 9b below). Compute $\int_2^5 y\,dx$ using: i) Simpson's one-third rule ii) Simpson's three-eighth rule and iii) Weddle's rule.	08	2	2
		OR			
10	a	The following table gives the viscosity of oil as a function of temperature. Use Lagrange's interpolation formula to find viscosity of oil at a temperature of 120° and 140°. (table 10a below) 9.3578 7.0333	08	2	2
	b	A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned for various values of the time t (in seconds). (table 10b below) Calculate the angular velocity and the angular acceleration at t = 0.4 and 0.8 using numerical differentiation.	08	3	3

Table 9a:

T (℃)	0	8	16	24	32
O_2 (mg/L)	14.621	11.843	9.870	8.418	7.305

Table 9b:

x	2	2.5	3	3.5	4	4.5	5
y	1.3863	1.4351	1.4816	1.5260	1.5686	1.6094	1.6486

Table 10a:

Temperature	110	130	160	190
Viscosity	10.8	8.1	5.5	4.8

Table 10b:

t	0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.12	0.49	1.12	2.02	3.2	4.67

	v	a
0.4	2.4917	5.1917
0.8	6.5	7