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RV COLLEGE OF ENGINEERING
Autonomous Institution affiliated to VTU
I Semester B.E. February -2024 Examinations
DEPARTMENT OF MATHEMATICS
FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS AND NUMERICAL METHODS
(2022 SCHEME)
(Non-Integrated Course)

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:
1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, and 9 and 10.

PART-A (Objective type for one or two marks)
(True & false and match the following questions are not permitted)

1	1.1	Product of the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 3 & 6 & 4 \end{bmatrix}$ is _____.	1
	1.2	Rank of the matrix $A = \begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 1 \\ 0 & 6 & 3 \end{bmatrix}$ is _____.	1
	1.3	Curvature of a straight line $y = 3x + 2$ is _____.	1
	1.4	Given $y(1) = 2.5, y(3) = 5.6, y(5) = 7.2, y(7) = 8.5$, then $\Delta y(3)$=_____.	1
	1.5	Maclaurin series expansion of $y = e^{-\frac{x}{2}}$ is _____.	1
	1.5	If two characteristic roots of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ are 3 and 6, then the third characteristic root is _____.	1
	1.6	The angle between the radius vector and tangent for the curve $r = ae^{\theta \cot\propto}$ is_____.	2
	1.7	If $z = (\cos x)^y$, then $\frac{\partial z}{\partial y}$ =_____.	2
	1.8	Given that $z = 2xy - 3x^2y$ and x increases at the rate of 2cm/sec. Find the rate at which y changes at the instant when x =3cm and y=1cm in order that the z shall be neither increasing nor decreasing.	2
	1.9	Evaluate $\int_0^2 \int_1^3 x^2 y dy dx$.	2
	1.10	Sketch the region of integration $\int_{\frac{\pi}{2}}^{\pi} \int_0^{2\cos\theta} r^2 \sin\theta \ \ dr\, d\theta$.	2
	1.11	The value of $\nabla^4[(x-2)(2x-3)(3x-4)]$ with $h = 5$ is _____.	2
	1.13	If $f(0) = 1$, $f(0.25) = 1.03$, $f(0.5) = 1.39$, $f(0.75) = 1.97$ and $f(1) = 2.56$, then $\int_0^1 f(x)dx$ =_____.	2

PART-B

		UNIT-I	
2	a	Compute the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$.	4
	b	The currents i_1, i_2, i_3 in the paths of an electrical network follow the linear equations $i_1 - i_2 + i_3 = 0, 3i_1 + 2i_2 = 7, 2i_2 + 4i_3 = 8$. Determine i_1, i_2, i_3 using Gauss-Jordan elimination method.	6
	c	Determine the largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}$. Choose the initial vector as $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Perform 5 iterations.	6

		UNIT-II	
3	a	Expand $y = \log_e \sec x$ in ascending powers of x up to and including the term in $x^6$and hence deduce the expansion of $\tan x$.	8
	b	For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that ρ at any point is equal to $\frac{(CD)^3}{ab}$ where C is the centre of the ellipse and D is an extremity of the diameter conjugate to CP.	8
		OR	
4	a	Find the radius of curvature of Folium $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.	8
	b	Show that the angle of intersection of the Lemniscate $r^2 = a^2 \cos 2\theta$ and Cardioid $r = a(1 + \cos\theta)$ intersect at angle $3 \sin^{-1}\left(\frac{3}{4}\right)^{\frac{1}{4}}$.	8

		UNIT-III	
5	a	If $u = f(\frac{y-x}{xy}, \frac{z-x}{xz})$, find the value of $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z}$.	8
	b	Show that by Lagrange's method of undetermined multipliers the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.	8
		OR	
6	a	In robotics, the functions representing robotic arm from cartesian to any system (x, y) are given by $x = e^u \cos v$ and $y = e^u \sin v$. As the Jacobian represents transformation factor between different systems, verify $\frac{\partial(x,y)}{\partial(u,v)} \times \frac{\partial(u,v)}{\partial(x,y)} = 1$.	8
	b	If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, verify that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.	8

		UNIT-IV	
7	a	By changing the order of integration and hence evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x + y)\, dx\, dy$.	8
	b	Find the volume of the tetrahedron $x \geq 0, y \geq 0,\ z \geq 0,\ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} \leq 1$.	8
		OR	

8	a	Transform to polar coordinates and hence evaluate $\int_0^2 \int_1^{\sqrt{4-y^2}} y\sqrt{x^2+y^2}\, dx\, dy$.	8
	b	Determine the centre of gravity of the triangular lamina bounded by the coordinate axis and the line $x + y = 1$.	8

UNIT-V

9 a Using Lagrange's interpolation, find the polynomial of lowest degree which agrees with the point (x, y) given in the following table. Hence find $y(2.5)$. — 8

x	3	2	1	0	-1
$y(x)$	8	26	32	14	-40

b The following table gives the result of an observation. The temperature T in degree centigrade of a vessel of cooling water is given in different time t (in minutes). — 8

t	1	3	5	7	9
T	85.3	74.5	67.0	60.5	54.3

(i) Estimate the temperature at $t = 1.5$.

(ii) Estimate the approximate rate of cooling at $t = 3$.

OR

10 a The table gives the distance in nautical miles of the visible horizon for the given heights in feet, above the earth's surface. — 8

Height	100	150	200	250	300	350	400
Distance	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the distances when the heights are $160\ ft$ and $410\ ft$.

b Estimate the value of the integral $\int_1^4 \sqrt{x}e^{-x}dx$ using Simpson's 1/3, Simpson's 3/8 and Weddle's rules, by diving the interval [1,4] into six equal sub intervals. — 8

Signature of Scrutinizer: Signature of Chairman

Name: Name: