Problems on Single - Phase A.C. Circuits

1. An inductance of 1H is in series with a capacitance of $1\mu F$. Calculate the circuit impedance when,

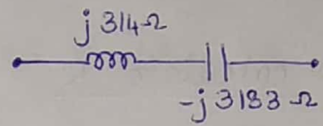
   a) the frequency is 50 Hz

   b) the frequency is 1 kHz.

Sol^n

$L = 1H$, $C = 1\mu F$.

Case-1 :- When $f = 50Hz$.

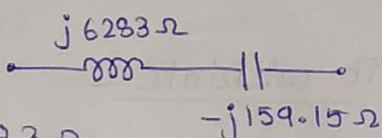


$$X_L = 2\pi f L = 2\pi * 50 * 1 = 314\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi * 50 * 1 * 10^{-6}} = 3183\,\Omega$$

$$\therefore jX_{eq} = jX_L - jX_C = j314 - j3183 = -j2869\,\Omega$$

Case-2 :- When $f = 1kHz$



$$jX_L = j\,2\pi f L = j\,2\pi * 1000 * 1 = j\,6283\,\Omega$$

$$-jX_C = \frac{-j}{2\pi f C} = \frac{-j * 1}{2\pi * 1000 * 1 * 10^{-6}} = -j\,159.15\,\Omega$$

$$jX_{eq} = jX_L - jX_C = j6283 - j159.15 = j\,6123.84\,\Omega$$

Note:-

* In case-1, $jX_{eq} = -j\,2869\,\Omega$. The $-j$ factor indicates that the overall effect is capacitive.
* In case-2, $jX_{eq} = +j\,6123\,\Omega$. The $+j$ factor indicates that the overall effect is inductive.
* As frequency increases, inductive reactance increases ($X_L \propto f$), while the capacitive reactance reduces ($X_C \propto 1/f$).

2. A coil when connected to a 200V, 50Hz supply, takes a current of 10A and dissipates 1200W. Calculate the resistance and inductance of the coil.

Sol^n^

Given,

RMS value of voltage, $V = 200V$

RMS value of current, $I = 10A$.

Since it is an inductive circuit, the current lags the applied voltage.

To calculate R

The power is dissipated only in the resistance.

$$\therefore \quad P = I^2 R$$

$$1200 = 10^2 * R$$

$$\Rightarrow \boxed{R = 12\,\Omega}$$

To calculate L

The circuit impedance is,

$$Z = \frac{V}{I} = \frac{200}{10} = 20\,\Omega$$

Also, $Z = \sqrt{R^2 + X_L^2} \Rightarrow X_L = \sqrt{Z^2 - R^2}$

$$= \sqrt{20^2 - 12^2}$$

$$\Rightarrow X_L = 16\,\Omega.$$

But $X_L = 2\pi f L$

$$\Rightarrow L = \frac{X_L}{2\pi f}$$

$$= \frac{16}{2\pi * 50}$$

$$\boxed{L = 0.0509\,H}$$

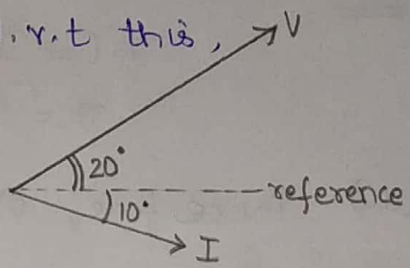
3. A current $i = \sin(31t - 10^\circ)$ produces a potential drop $v = 200\sin(31t + 20^\circ)$ in a circuit. Calculate the value of the circuit parameters assuming a series combination.

Sol^n

Consider 0° as the reference. W.r.t this,

→ Current lags by 10°

→ Voltage leads by 20°.



Therefore, phase difference between V and I is

$$\phi = 20 - (-10) = 30^\circ$$

This means the current lags voltage by 30°.

Also, $I_m = 1A$ $\quad \omega = 31$ rad/sec

$V_m = 200V$ $\quad \Rightarrow f = \frac{\omega}{2\pi} = 4.93\,Hz.$

The circuit impedance is,

$$Z = \frac{V}{I} = \frac{V_m/\sqrt{2}}{I_m/\sqrt{2}} = \frac{V_m}{I_m} = \frac{200}{1} = 200\,\Omega$$

Given that the circuit parameters are in series. Since current lags voltage, the circuit is inductive.

$$\phi = 30^\circ$$

$$\Rightarrow \cos\phi = \cos 30^\circ = 0.866$$

$$\sin\phi = \sin 30^\circ = 0.5$$

$\therefore R = Z\cos\phi$

$= 200 * 0.866$

$\boxed{R = 173.2\,\Omega}$

$X_L = Z\sin\phi$

$= 200 * 0.5$

$X_L = 100\,\Omega$

$\Rightarrow 2\pi f L = 100$

$\Rightarrow \boxed{L = 3.23\,H}$

4. An EMF given by $100 \sin\left(314t - \frac{\pi}{4}\right)$ is applied to a circuit ~~so~~ and the current is $20 \sin(314t - 1.5708)$ A. Find

(i) frequency

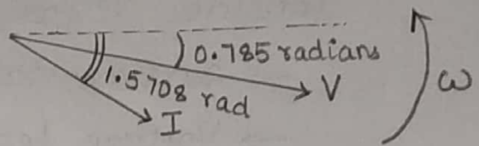
(ii) circuit elements

(iii) phase difference between V and I.

Sol^n

$$i = I_m \sin(\omega t - \phi_1)$$

$$v = V_m \sin(\omega t - \phi_2)$$



(iii) There fore, phase angle between Voltage & current is,

$$\phi = 1.5708 - 0.785$$

$$\Rightarrow \boxed{\phi = 0.785 \text{ radians}}$$

$$= 0.785 * \frac{180}{\pi}$$

$$\boxed{\phi = 45^\circ} \text{ lagging.}$$

Current lags voltage by 45°.

(i) $\omega = 314$ rad/sec

$$\Rightarrow 2\pi f = 314 \Rightarrow f = \frac{314}{2\pi} \Rightarrow \boxed{f = 50\text{Hz}}$$

(ii) Since the current lags voltage, it is an R-L circuit. Given $V_m = 100V$, $I_m = 20A$. Thus the circuit impedance is,

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{(V_m/\sqrt{2})}{(I_m/\sqrt{2})} = \frac{V_m}{I_m} = \frac{100}{20} = 5\,\Omega$$

Since $\phi = 45^\circ$, $\cos\phi = \sin\phi = 0.707$.

$$\therefore R = Z\cos\phi = 5 * 0.707 \Rightarrow \boxed{R = 3.535\,\Omega}$$

$$X_L = Z\sin\phi = 5 * 0.707 \Rightarrow X_L = 3.535\,\Omega$$

$$\Rightarrow 2\pi f L = 3.535\,\Omega$$

$$\therefore L = \frac{3.535}{2\pi f} = \frac{3.535}{2\pi * 50}$$

$$\Rightarrow \boxed{L = 0.01125\text{H}}$$

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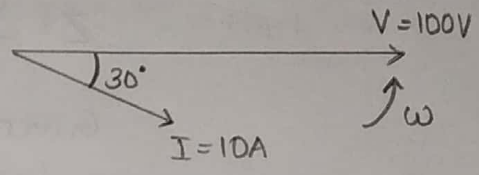
5. An inductive circuit takes a current of 10A, lagging the voltage by 30° when connected to a 100V, 50Hz supply. Calculate the circuit components. Draw the phasor diagram.

Solⁿ

Given that current lags voltage by 30°.
Since $\phi = 30°$ ($\phi < 90°$), the circuit is an R-L circuit.

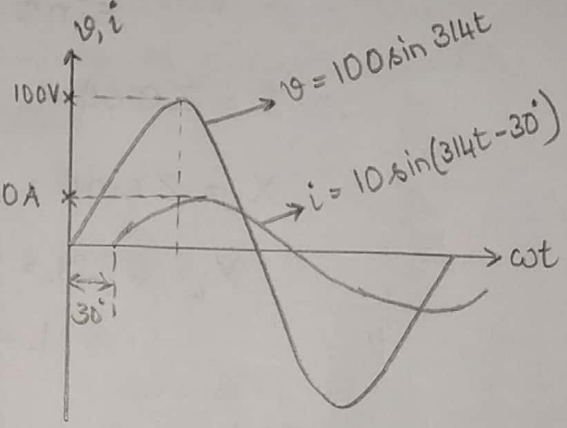
$$Z = \frac{V}{I} = \frac{100}{10} = 10\,\Omega.$$



$\phi = 30°$

$\Rightarrow \cos\phi = \cos 30° = 0.866$

$\sin\phi = \sin 30° = 0.5.$



$\therefore \quad R = Z\cos\phi$

$= 10 * 0.866$

$\boxed{R = 8.66\,\Omega}$

$X_L = Z\sin\phi$

$= 10 * 0.5$

$X_L = 5\,\Omega$

$\Rightarrow 2\pi f L = 5$

$\Rightarrow L = \frac{5}{2\pi f}$

$= \frac{5}{2\pi * 50}$

$\boxed{L = 0.01591\ H}$

6. A choke coil takes a current of 2A, lagging 60° behind the applied voltage of 200V at 50Hz. Calculate the inductance, resistance and impedance of the circuit. Also calculate the power consumed when the circuit is connected to a 100V, 25Hz supply.

Solⁿ:

Case-1 :- When $V = 200V$, $f = 50Hz$

$V = 200V$, $I = 2A$. The circuit impedance is

$$Z = \frac{V}{I} = \frac{200}{2} = 100\Omega$$

Given $\phi = 60^\circ$ lag

$$\therefore \cos\phi = \cos 60^\circ = 0.5$$

$$\sin\phi = \sin 60^\circ = 0.866.$$

$$\therefore R = Z\cos\phi = 100 * 0.5 \Rightarrow \boxed{R = 50\Omega}$$

$$X_L = Z\sin\phi = 100 * 0.866 \Rightarrow X_L = 86.6\Omega.$$

$$X_L = 2\pi f L = 86.66$$

$$\Rightarrow L = \frac{86.66}{2\pi f} = \frac{86.66}{2\pi * 50}$$

$$\Rightarrow \boxed{L = 0.275H}$$

Case-2 :- When $V' = 100V$, $f' = 25Hz$

Since frequency is changed, X_L, Z and I will change. However, R and L remain the same.

$$\therefore X_L' = 2\pi f' L = 2\pi * 25 * 0.275 = 43.2\Omega.$$

$$R' = R = 50\Omega$$

$$\therefore Z' = \sqrt{R'^2 + X_L'^2} = \sqrt{50^2 + 43.2^2} = 66.1\Omega.$$

$\therefore$ New value of current is

$$I' = \frac{V'}{Z'} = \frac{100}{66.1} = 1.512A.$$

Power is consumed only in the resistance.

$$\therefore \quad P' = (I')^2 R$$

$$= (1.5128)^2 * 50$$

$$\boxed{P' = 114.45\ W}$$

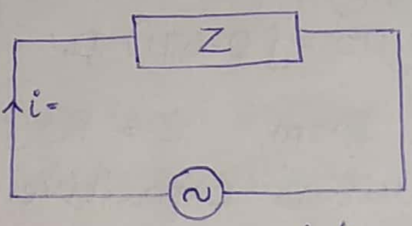
(OR).

$$\cos\phi' = \frac{R'}{Z'} = \frac{50}{66.1} = 0.75$$

$$P' = V' I' \cos\phi' = 100 * 1.5128 * 0.75 \approx \underline{114.45\ W}.$$

7. A black box contains a two-element series circuit. A voltage of $(40 - j30)$ volts drives a current of $(40 - j3)$ amps, in the circuit. Calculate the values of circuit elements when the supply frequency is 50Hz.

$\underline{Sol^n}$



$$\omega = 2\pi f$$

$$= 2 * \pi * 50$$

$\vartheta = 70.71 \sin(314t - 36.86^\circ)$ $\quad \omega \approx 314$ rad/sec

$$V = (40 - j30)\ V = \sqrt{40^2 + (-30)^2}\ \angle \tan^{-1}(-30/40) = 50 \angle -36.86^\circ \text{ Volts}$$

Therefore,

$$\vartheta = V_m \sin(\omega t + \phi)$$

$$\vartheta = 50\sqrt{2} \sin(314t - 36.86^\circ)$$

$$\Rightarrow \boxed{\vartheta = 70.71 \sin(314t - 36.86^\circ)} \text{ volts}$$

$$I = (40 - j3)\ A = \sqrt{40^2 + (-3)^2}\ \angle \tan^{-1}(-3/40) = 40.11 \angle -4.28^\circ\ A$$

$$\therefore \quad i = I_m \sin(\omega t + \phi)$$

$$= 40.11\sqrt{2} \sin(314t - 4.28^\circ)$$

$$\Rightarrow \boxed{i = 56.72 \sin(314t - 4.28^\circ)} \text{ amperes.}$$

The circuit impedance is

$$Z = \frac{V}{I}$$

$$= \frac{50\angle -36.86^\circ}{40.11\angle -4.28^\circ}$$

$$= \frac{50}{40.11}\angle -36.86-(-4.28)$$

$$Z = 1.246\angle -32.58^\circ$$

$\therefore\ |Z| = 1.246\,\Omega$

$\phi = -32.58^\circ$

$\therefore\ R = |Z|\cos\phi = 1.246\cos(-32.58^\circ) = 1.05\,\Omega$

$X = |Z|\sin\phi = 1.246\sin(-32.58^\circ) = -0.671\,\Omega.$

$\therefore$ The impedance is,

$$Z = R + jX$$

$$= 1.05 + j(-0.671)$$

$$Z = (1.05 - j0.671)\,\Omega$$

This is of the form $Z = R - jX$

Hence, the circuit is capacitive.

$\therefore$ The circuit resistance is,

$$\boxed{R = 1.05\,\Omega}$$

$X_c = 0.671\,\Omega$

$$\frac{1}{2\pi fC} = 0.671\,\Omega$$

$$\Rightarrow C = \frac{1}{2\pi f * 0.671} = \frac{1}{2\pi * 50 * 0.671}$$

$$\Rightarrow C = 4.7438 * 10^{-3}\,F$$

$$\Rightarrow \boxed{C = 4743.8\,\mu F}$$

8. A voltage of 100V at 50 Hz is applied to a series R-L circuit. The current in the circuit is 5A lagging behind the voltage by 35°. Write the expression for current and voltage. Draw the phasor diagram.

Sol^n:

$V = 100V \qquad f = 50Hz$

$I = 5A \qquad \phi = 35°.$

$\therefore \quad \omega = 2\pi f = 2\pi \times 50 \simeq 314$ rad/sec.

$\phi = 35° = \frac{35\pi}{180}$ radians $= 0.611$ radians

$V_m = V\sqrt{2} = 100\sqrt{2} = 141.4V$

$I_m = \sqrt{2}I = 5\sqrt{2} = 7.071A.$

Thus, the equation for voltage will be

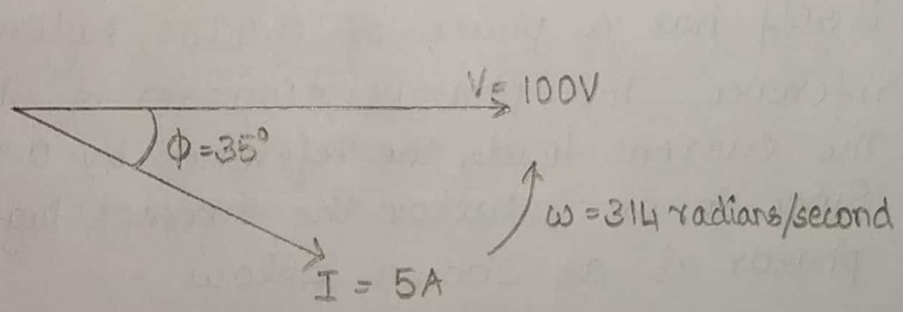
$v = V_m \sin \omega t$

$\boxed{v = 141.4 \sin(314t)}$ volts

Since current lags voltage by 0.611 radians, the equation of current will be

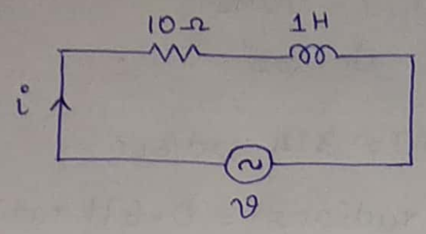
$i = I_m \sin(\omega t - \phi)$

$\boxed{i = 7.071 \sin(314t - 0.611)}$ amperes.



9. A current of $i = 2.8 \sin(94.3t + \frac{\pi}{6})$ is flowing through a resistance of $10\,\Omega$ connected in series with an inductance of 1H. Obtain the expression for the voltage across the R-L combination. Draw the phasor diagram.

Sol^n:



$L = 1H$ $\qquad\qquad \omega = 94.3$

$R = 10\,\Omega$ $\qquad\qquad \Rightarrow f = \frac{94.3}{2\pi} = 15\,Hz.$

$\therefore X_L = 2\pi f L = 2\pi * 15 * 1 = 94.25\,\Omega.$

$\therefore \tan\phi = \frac{X_L}{R}$

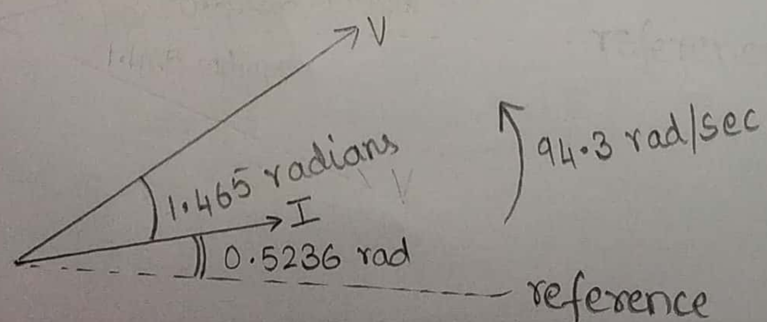
$\Rightarrow \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{94.25}{10}\right)$

$\Rightarrow \phi = 1.465$ radians.

The current expression has

$\phi_1 = \frac{\pi}{6} = 0.5236$ radians.

This means the voltage and current have a phase difference of 1.465 radians, while the current itself has a phase of 0.5236 radians w.r.t a reference. The phasor diagram is shown below. The current leads the reference by 0.5236 radians. Since in an inductor the current lags voltage, the phasor is as shown below.



From the phasor diagram, the voltage should lead the current by 1.465 radians, and it should lead the reference by

$$\phi_2 = 1.465 + 0.5236 = 1.9886 \text{ radians}$$

Now,

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 94.25^2} = 94.77\,\Omega$$

$$\therefore \quad V_m = I_m Z$$

$$= 2.8 * 94.77$$

$$V_m = 265.36\ V$$

Therefore, the equation for voltage is

$$\boxed{v = 265.36 \sin(94.3t + 1.988\ \text{rad})} \text{ volts.}$$

10) A series R-L circuit takes 384 watts of power at a power factor of 0.8 from a 120V, 60Hz supply. Find the value of the circuit components.

Sol^n:

$P = 384W$ .... dissipated only in 'R'.

$\cos\phi = 0.8$

$V = 120V$

$f = 60Hz.$

$$P = VI\cos\phi \Rightarrow I = \frac{P}{V\cos\phi} = \frac{384}{120*0.8} = 4A.$$

Also, $P = I^2R \Rightarrow R = \frac{P}{I^2}$

$$= \frac{384}{4^2}$$

$$\Rightarrow \boxed{R = 24\,\Omega}$$

Now, $Z = \frac{V}{I} = \frac{120}{4} = 30\Omega$

$$Z = \sqrt{R^2 + X_L^2} \Rightarrow X_L = \sqrt{Z^2 - R^2}$$

$$= \sqrt{30^2 - 24^2}$$

$$X_L = 18\Omega$$

$$X_L = 2\pi f L = 18$$

$$\Rightarrow L = \frac{18}{2\pi f}$$

$$= \frac{18}{2\pi * 60}$$

$$\boxed{L = 0.0477H}$$

1. An alternating voltage $(80+j60)$ V is applied across a circuit in which the current drawn is $(-4+j10)$ A. Calculate,

a) circuit element values
b) circuit impedance
c) power factor
d) power consumed

Also draw the phasor diagram. Take $f = 50Hz$

Solⁿ:

$$V = 80 + j60V = \sqrt{80^2 + 60^2} \angle \tan^{-1}(60/80) = 100 \angle 0.643^{rad} V$$

$$I = -4 + j10\ A = \sqrt{(-4)^2 + 10^2} \angle \tan^{-1}(-4/10) = 10.77 \angle 1.95^{rad} A$$

∴ Circuit impedance is

$$Z = \frac{V}{I} = \frac{100 \angle 0.643}{10.77 \angle 1.95} = \frac{100}{10.77} \angle 0.643 - 1.95 = 9.28 \angle -1.307^{r} \Omega$$

∴ $\phi = -1.307$ radians.

∴ $\cos\phi = 0.2607$

$\sin\phi = -0.965$

$\therefore \quad R = |Z| \cos\phi$

$= 9.28 * 0.2607$

$\boxed{R = 2.42\,\Omega}$

$X = |Z| \sin\phi$

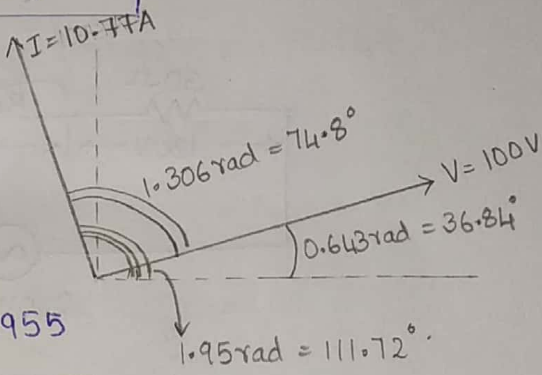
$= 9.28 * (-0.965) = -8.9952\,\Omega.$

$\boxed{X = -8.9552\,\Omega}$ --- negative indicates capacitive reactance.

$\therefore$ The impedance is,

$Z = R + jX$

$\boxed{Z = (2.42 - j\,8.955)\,\Omega}$



Thus, $X_C = 8.955\,\Omega$

$\dfrac{1}{2\pi f C} = 8.955$

$\Rightarrow \quad C = \dfrac{1}{2\pi * 50 * 8.955}$

$\Rightarrow \boxed{C = 355.45\,\mu F}$

Power factor is,

$Pf = \cos\phi \Rightarrow \boxed{Pf = 0.2607}$

Power consumed is,

$P = I^2 R$

$= (10.77)^2 (2.42)$

$\boxed{P = 280.702\,W}$

Phase angle between V and I is

$\phi = \tan^{-1}\left(\dfrac{X_C}{R}\right) = \tan^{-1}\left(\dfrac{8.955}{2.42}\right) = 1.306$ radians $= 74.8°$

Current leads voltage by $74.8°$.

12) A current of 5A flows through an inductive coil which is in series with a pure resistance of 30 Ω when connected across the 240V, 50Hz supply. The voltage across the coil is 180V and across the resistance is 130V. Calculate,

(i) resistance & inductance of the coil

(ii) power dissipated in the coil & the circuit

Also draw the phasor diagram.

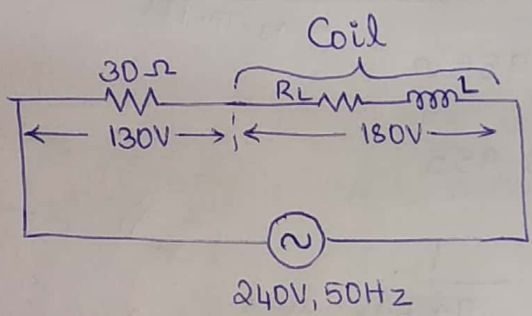
<u>Sol</u>

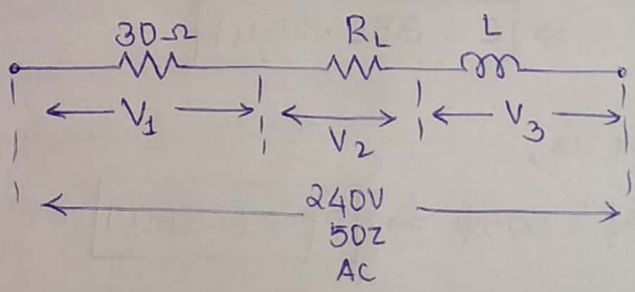
Sol^{n}:

Let L = inductance of the coil

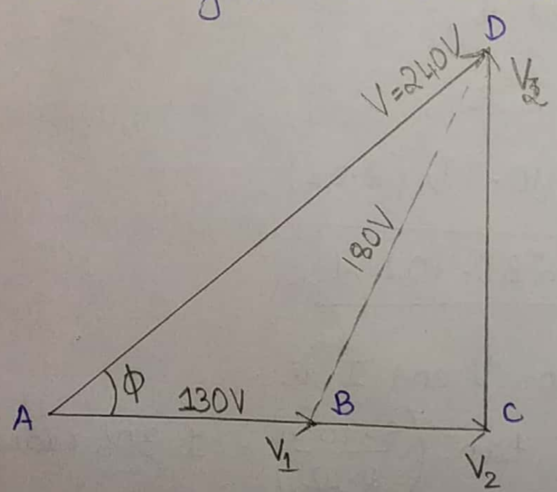
R_L = resistance of the inductive coil

R = 30 Ω resistance





The phasor diagram is as shown below.



From the phasor diagram,

$$BC^2 + CD^2 = 180^2 \longrightarrow (1)$$

and $$(130 + BC)^2 + CD^2 = 240^2 \longrightarrow (2)$$

Subtracting (1) from (2),

$$(130 + BC)^2 - BC^2 + CD^2 - CD^2 = 240^2 - 180^2$$

$$130^2 + BC^2 + 260BC - BC^2 = 25200$$

$$16900 + 260BC = 25200$$

$$\Rightarrow BC = 31.92V.$$

$$\therefore CD = \sqrt{180^2 - BC^2} = \sqrt{180^2 - 31.92^2} = 177.14V.$$

Given $I = 5A$,

$V = 240V$.

$$\therefore \text{coil impedance, } Z_c = \frac{V_c}{I} = \frac{180}{5} = 36\Omega$$

$$V_2 = BC = IR_L$$

$$\Rightarrow R_L = \frac{BC}{I} = \frac{31.92}{5}$$

$$\Rightarrow \boxed{R_L = 6.384\Omega}$$

Inductive reactance of the coil,

$$X_L = \sqrt{Z_c^2 - R_L^2} = \sqrt{36^2 - 6.384^2} = 35.43\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{35.43}{2\pi * 50} \Rightarrow \boxed{L = 0.11277H}$$

Power dissipated in the coil, $P_c = I^2R_L = 5^2(6.384)$

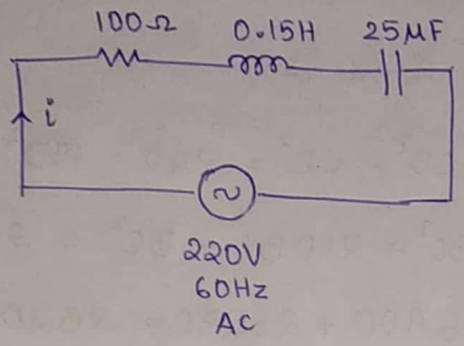
$$\Rightarrow \boxed{P_c = 159.6W}$$

Power dissipated in the circuit $= I^2(R+R_L) = 5^2(30+6.384)$

$$\Rightarrow \boxed{P = 909.6W}$$

13). A series circuit with $R = 100\Omega$, $C = 25\mu F$ and $L = 0.15H$ is connected across 220V, 60Hz supply. Calculate
(i) current (ii) power
(iii) power factor in the circuit

Sol^n:



The circuit impedance magnitude is,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$R = 100\Omega$

$X_L = 2\pi f L = 2\pi * 60 * 0.15 = 56.55\Omega$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi * 60 * 25 * 10^{-6}} = 106.103\Omega$.

$$\therefore \quad Z = \sqrt{100^2 + (56.55 - 106.103)^2}$$

$$\Rightarrow Z = 111.60\Omega$$

Current in the circuit is

$$I = \frac{V}{Z} = \frac{220}{111.60}$$

$$\Rightarrow \boxed{I = 1.97A}$$

Power factor, $pf = \dfrac{R}{Z} = \dfrac{100}{111.60} \Rightarrow \boxed{pf = 0.896}$

Since $X_C > X_L$, the pf is a leading one.

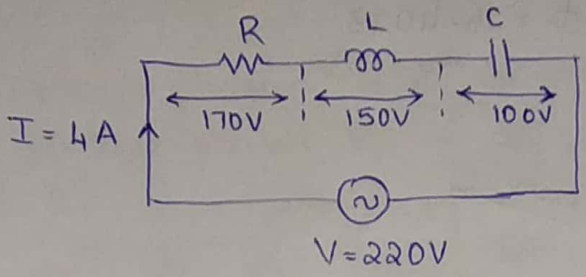
Power in the circuit is,

$$P = VI\cos\phi = 220 * 1.97 * 0.896$$

$$\Rightarrow \boxed{P = 388.32\,W}$$

14. A voltage of 220V is applied to a series circuit consisting of a resistor, an inductor & a capacitor. The respective voltages across these components are 170V, 150V and 100V and the current is 4A. Find the power factor of the circuit. Draw the phasors.

Solⁿ:



Given the voltage across resistor is 170V.

$$\therefore V_R = IR$$

$$\Rightarrow R = \frac{V_R}{I} = \frac{170}{4} = 42.5\,\Omega.$$

Voltage across inductor is

$$V_{X_L} = 150 = IX_L$$

$$\Rightarrow X_L = \frac{150}{I} = \frac{150}{4} = 37.5\,\Omega.$$

Voltage across capacitor is

$$V_{X_C} = 100 = IX_C$$

$$\Rightarrow X_C = \frac{100}{I} = \frac{100}{4} = 25\,\Omega$$

The circuit impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{42.5^2 + (37.5-25)^2} = 44.3\,\Omega$$

The power factor is

$$pf = \frac{R}{Z} = \frac{42.5}{44.3}$$

$$\Rightarrow \boxed{pf = 0.9593}$$

To draw the phasor diagram we need the voltages.

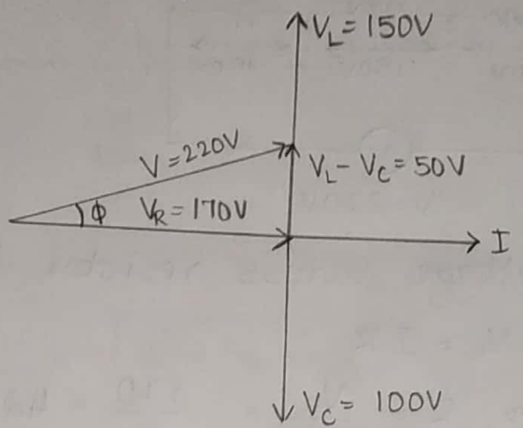
$V_R = IR = 170V$ $\quad\quad I = 4A.$

$V_L = IX_L = 150V$

$V_C = IX_C = 100V$

$$pf = \cos\phi = 0.9593$$

$$\Rightarrow \phi = 16.4028^\circ$$



15). A metal filament lamp, rated at 750W, 100V, is to be used on a 230V, 50Hz supply, by connecting a capacitor of suitable value in series. Determine

a) the capacitance required

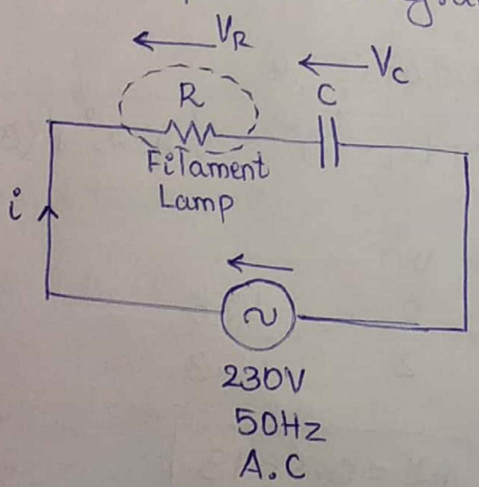
b) the phase angle

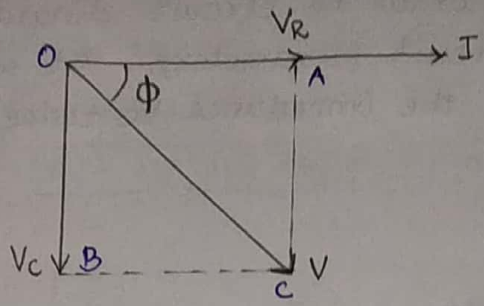
c) the power factor

d) the apparent, active & reactive powers

Also draw the phasor diagram.

Sol^n





The voltage phasor V_R is in phase with 'I' while the voltage phasor V_c is 90° out of phase (lagging) with 'I'.

In Δ^{le} OAC,

$$V_c^2 = \sqrt{V^2 - V_R^2}$$

$$I = \frac{P}{V_R} = \frac{750}{100} = 7.5A$$

$V = 230V$

$V_R = 100V$

$$\therefore \; V_c^2 = \sqrt{230^2 - 100^2} = 207V.$$

$$X_c = \frac{V_c}{I} = \frac{207}{7.5} = 27.6\Omega$$

$$\Rightarrow \frac{1}{2\pi fC} = 27.6\Omega$$

$$\Rightarrow C = \frac{1}{2\pi * 50 * 27.6}$$

$$\Rightarrow \boxed{C = 115.33\mu F}$$

Phase angle, $\phi = \cos^{-1}\left(\frac{V_R}{V}\right) = \cos^{-1}\left(\frac{100}{230}\right) = 64.22^\circ$ (lead)

power factor, $pf = \cos\phi = \cos(64.22^\circ) = 0.4347$ leading

Apparent power, $S = VI = 230 * 7.5 = 1725VA$

Active power, $P = VI\cos\phi = 1725 * 0.4347 = 749.85W$

Reactive power, $Q = \sqrt{S^2 - P^2} = \sqrt{1725^2 - 749.85^2} = 1553.49 VAr$

16> In a particular circuit, a voltage of 10V at 25Hz produces 100mA, while the same voltage at 75Hz produces 60mA. Draw the circuit diagram and find out the circuit parameters. At what frequency will the value of the impedance be twice that at 25Hz ?

Solⁿ.

Let

R = resistance

X_1 = reactance at 25Hz

X_2 = reactance at 75Hz

If $f_1 = 25Hz$, then $f_2 = 75Hz = 3f_1$.

Since $\omega \propto f$, $\Rightarrow \omega_2 = 3\omega_1$

Since $X \propto \omega$, $\Rightarrow X_2 = 3X_1$.

At 25Hz, the circuit impedance is

$$Z_1 = \frac{V_1}{I_1} = \frac{10}{100*10^{-3}} = 100\Omega$$

At 75Hz, the circuit impedance is,

$$Z_2 = \frac{V_2}{I_2} = \frac{V_1}{I_2} = \frac{10}{60*10^{-3}} = 166.67\Omega$$

$$\therefore \quad R^2 + X_1^2 = 100^2$$

$$R^2 + (3X_1)^2 = 166.67^2$$

Subtracting, we get

$$9X_1^2 - X_1^2 + R^2 - R^2 = 166.67^2 - 100^2$$

$$8X_1^2 = 17778.8889$$

$$\Rightarrow \boxed{X_1 = 47.14\Omega} \text{ --- at 25Hz}$$

$$\therefore \quad 2\pi f_1 L_1 = 47.14 \Rightarrow L_1 = \frac{47.14}{2\pi * 25}$$

$$\Rightarrow \boxed{L_1 = 0.3 H}$$

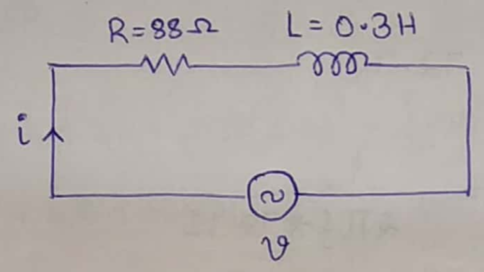
The reactance at 75Hz is

$$X_2 = 3X_1$$
$$= 3 * 47.14$$
$$\boxed{X_2 = 141.42\,\Omega} \quad \cdots \text{ at } 75\text{Hz}.$$

Now,

$$R = \sqrt{Z_1^2 - X_1^2} = \sqrt{Z_2^2 - X_2^2}$$
$$= \sqrt{100^2 - 47.14^2}$$
$$\Rightarrow \boxed{R = 88.2\,\Omega}$$

The circuit is as shown below



Impedance at 25Hz = 100 Ω.

Let Z_3 be the impedance which is twice that of Z_1. This occurs at a frequency f_3. The resistance & inductance are frequency independent while the inductive reactance is frequency-dependent.

$$Z_3 = 2Z_1 = 200\,\Omega$$
$$\Rightarrow \sqrt{R^2 + X_3^2} = 200$$
$$R^2 + X_3^2 = 200^2$$
$$\Rightarrow X_3^2 = 200^2 - R^2 = 200^2 - 88.2^2$$
$$\Rightarrow X_3 = 179.5\,\Omega$$
$$\Rightarrow 2\pi f_3 L = 179.5$$
$$f_3 = \frac{179.5}{2\pi L} = \frac{179.5}{2\pi * 0.3}$$
$$\Rightarrow \boxed{f_3 = 95.22\text{Hz}} \cdots \text{gives } Z_3 = 2Z_1.$$

17. A resistor 'R' in series with a capacitance 'C' is connected to a 50Hz, 240V supply. Calculate the value of 'C' so that 'R' absorbs 300W at 100V.

Sol^n:

$$I = \frac{P_R}{V_R} = \frac{300}{100} = 3A.$$

Let V_c be the voltage across the capacitor.

$$\therefore V_c = \sqrt{240^2 - 100^2} = 218.17V.$$

The capacitive reactance is,

$$X_C = \frac{V_c}{I} = \frac{218.17}{3} = 72.72\Omega.$$

$$\Rightarrow \frac{1}{2\pi f C} = 72.72$$

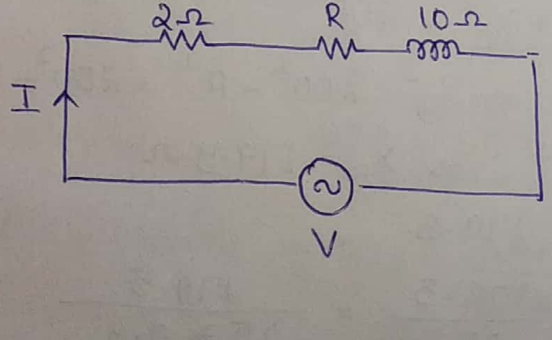
$$\Rightarrow C = \frac{1}{2\pi f * 72.72}$$

$$= \frac{1}{2\pi * 50 * 72.72}$$

$$\boxed{C = 43.77\mu F}$$

18. A circuit has a fixed resistance of 2Ω and a reactance of 10Ω in series with a resistor R across a 100V constant frequency mains. For what value of R is the power consumed in it a maximum?

Sol^n:



The RMS value of current is,

$$I = \frac{100}{\sqrt{(R+2)^2 + 10^2}}$$

The power is,

$$P = I^2 R$$

$$P = \frac{100^2 * R}{(R+2)^2 + 100}$$

The power is maximum when $\frac{dP}{dR} = 0$.

$$\therefore \frac{dP}{dR} = \frac{[((R+2)^2 + 100)(100^2)] - [100^2 R\,(2(R+2))]}{[(R+2)^2 + 100]^2} = 0$$

$$\Rightarrow (R^2 + 4 + 4R + 100)(100)^2 - [100^2 R(2R+4)] = 0$$

$$(R^2 + 4R + 104)\cancel{(100)^2} = \cancel{100^2} R(2R+4)$$

$$R^2 + \cancel{4R} + 104 - 2R^2 - \cancel{4R} = 0$$

$$-R^2 + 104 = 0$$

$$R^2 = 104$$

$$\Rightarrow \boxed{R = 10.2\,\Omega}$$

19) A non-inductive load takes 10A at 100V. Calculate the inductance of a reactor connected in series in order that the same current is supplied from 220V, 50Hz supply. Also calculate the phase angle between the supply voltage & current. Neglect the resistance of the reactor.

Sol^n: Without the reactor,

$$R = \frac{100V}{10A} = 10\,\Omega.$$

To have the same current with a reactor,

$$Z = \frac{220}{10} = 22\Omega.$$

Let 'X' be the reactance of the reactor.

Then

$$X^2 = Z^2 - R^2$$

$$\Rightarrow X = (22^2 - 10^2)^{1/2}$$

$$X = 19.59\Omega$$

$$\Rightarrow 2\pi f L = 19.59$$

$$\Rightarrow L = \frac{19.59}{2\pi f} = \frac{19.59}{2\pi * 50}$$

$$\Rightarrow \boxed{L = 0.0623H}$$

Phase angle between voltage & current is,

$$\phi = \tan^{-1}\left(\frac{X}{R}\right)$$

$$= \tan^{-1}\left(\frac{19.59}{10}\right)$$

$$\boxed{\phi = 62.95^\circ}$$

20) When a voltage of 100V at 50Hz is applied to a coil-A, the current taken is 8A, and the power is 120W. When applied to coil-B the current is 10A and the power 500W. What current and power will be taken when the voltage is applied to the two coils A and B connected in series?

Sol^n.

Since both coils have associated power, there is a dissipation, implying that both coils have series resistance. Let their impedances be $Z_A = R_A + jX_A$ and $Z_B = R_B + jX_B$.

For coil-A,

$$Z_A = \frac{V}{I_A} = \frac{100}{8} = 12.5\,\Omega.$$

$$P_A = 120W = I_A^2 R_A \Rightarrow R_A = \frac{120}{I_A^2} = \frac{120}{8^2} = 1.875\,\Omega$$

$$\therefore X_A = \sqrt{12.5^2 - 1.875^2} = 12.36\,\Omega.$$

$$\therefore Z_A = (1.875 + j12.36)\,\Omega$$

For coil-B,

$$Z_B = \frac{100}{10} = 10\,\Omega$$

$$P_B = 500 = I_B^2 R_B \Rightarrow R_B = \frac{500}{10^2} = 5\,\Omega$$

$$X_B = \sqrt{Z_B^2 - R_B^2} = \sqrt{100-25} = 8.66\,\Omega.$$

$$\therefore Z_B = (5 + j8.66)\,\Omega$$

When connected in series,

$$Z = Z_A + Z_B = (6.875 + 21.02)\,\Omega.$$

$$\Rightarrow |Z| = \sqrt{6.875^2 + 21.02^2} = 22.11\,\Omega.$$

$\therefore$ The current will be

$$I = \frac{V}{Z} = \frac{100}{22.11} \Rightarrow \boxed{I = 4.52A}$$

The power will be

$$P = I^2 R = (4.52)^2 * 6.875$$

$$\Rightarrow \boxed{P = 140.459\,W}$$

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21. Calculate the admittance, conductance and susceptance of a circuit consisting of a resistor of 10Ω in series with an inductor of 0.1H, when the frequency is 50Hz.

Solⁿ

$R = 10\,\Omega$

$L = 0.1\,H$

$\therefore X_L = 2\pi f L = 2\pi * 50 * 0.1 = 31.416\,\Omega$

$\therefore$ Coil impedance is

$Z = R + jX_L$

$Z = 10 + j\,31.416\,\Omega$

The admittance (Y) is

$$Y = \frac{1}{Z} = \frac{1}{10 + j31.416}$$

$$= \frac{10 - j\,31.416}{(10+j31.416)(10-j31.416)}$$

$$= \frac{10 - j\,31.416}{10^2 - (j^2 * 31.416^2)}$$

$$= \frac{10 - j\,31.416}{100 - (-986.96)} \qquad (\because j^2 = -1)$$

$$= \frac{10 - j\,31.416}{1086.96}$$

$$= \frac{10}{1086.96} - j\,\frac{31.416}{1086.96}$$

$$\boxed{Y = (0.00917 - j\,0.0288)\,℧}$$

This is of the form $Y = G + jB$ where 'G' is the conductance & B is the susceptance.

$\therefore$ $\boxed{G = 0.00917\,℧}$ and $\boxed{B = -0.0288\,℧}$

22) . A coil having a resistance R ohms and inductance L henry is connected across a variable frequency A.C supply of 110V. An ammeter in the circuit showed 15.6A when the frequency of 80Hz and 19.7A when the frequency was 40Hz. Find the values of R and L of the coil.

Sol^n:

Let X_1 = reactance at 40Hz.

Then $X_2 = 2X_1$ = reactance at 80Hz

At 40 Hz,

$$R^2 + X_1^2 = Z_1^2$$

$$Z_1 = \frac{V_1}{I_1} = \frac{110}{19.7} = 5.583$$

$$\therefore R^2 + X_1^2 = 5.583^2 \longrightarrow (1)$$

At 80 Hz,

$$R^2 + X_2^2 = Z_2^2$$

$$Z_2 = \frac{V_2}{I_2} = \frac{V_1}{I_2} = \frac{110}{15.6} = 7.051$$

$$\therefore R^2 + X_2^2 = 7.051^2$$

$$\Rightarrow R^2 + (2X_1)^2 = 7.051^2$$

$$R^2 + 4X_1^2 = 7.051^2 \longrightarrow (2)$$

(2) − (1) ⇒

$$3X_1^2 = 18.546$$

$$\Rightarrow X_1 = 2.486\Omega$$

$$\therefore L = \frac{X_1}{2\pi f_1} = \frac{2.486}{2\pi * 40} \Rightarrow L = 0.00989\,H$$

or $\boxed{L = 0.01\,H}$

$$Z_1 = \sqrt{R_1^2 + X_1^2} \Rightarrow R_1^2 = \sqrt{Z_1^2 - X_1^2} = \sqrt{24.956}$$

$$\Rightarrow \boxed{R_1 = 5\Omega}$$

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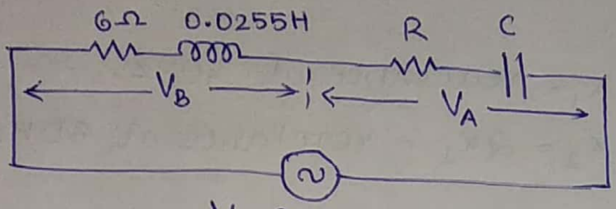
23> Find the values of R and C in the circuit shown below so that $V_B = 3V_A$, given that V_A and V_B are in quadrature. Also find the phase relationship between

a) V and V_B

b) V_A and I



V = 240V, 50Hz

Solⁿ.

$$V_B = 3V_A$$

$$\Rightarrow I Z_B = 3 I Z_A$$

$$\Rightarrow Z_B = 3Z_A.$$

$$X_L = 2\pi f L = 2\pi * 50 * 0.0255 = 8\Omega$$

$$\therefore Z_B = 6 + j8\Omega = 10\angle 53^\circ$$

Since $Z_B = 3Z_A$

$$\Rightarrow Z_A = \frac{Z_B}{3}$$

Since V_A and V_B are at right angles,

$$Z_A = \frac{10}{3}\angle 53^\circ - 90^\circ = 3.33\angle -37^\circ$$

$$Z_A = (2.688 - j1.965)\Omega \simeq (2.7 - j2)\Omega$$

$$\therefore \boxed{R = 2.7\Omega}$$

$$X_C = 2\Omega \Rightarrow \frac{1}{2\pi f C} = 2\Omega \Rightarrow C = \frac{1}{2\pi f * 2}$$

$$= \frac{1}{4\pi * 50}$$

$$\Rightarrow \boxed{C = 1.591 mF}$$

To obtain the phase relationship, Let us take current as the reference.

$$\bar{I} = I\angle 0^\circ.$$

$$\bar{V}_A = I\bar{Z}_A = 3.33I\angle -37^\circ.$$

In other words, V_A lags I by 37°.

$$\therefore \boxed{\phi_{(V_A - I)} = 37^\circ \text{ lag}}$$

$$\bar{V}_B = I\bar{Z}_B = 10I\angle 53^\circ. \longrightarrow (1).$$

This means V_B leads I by 53°.

$$Z = Z_A + Z_B$$

$$= (2.659 - j2) + (6 + j8)$$

$$= (8.659 + j6)\,\Omega$$

$$Z = 10.5\angle 34.7$$

$$\bar{V} = IZ = 10.5I\angle 34.7^\circ \longrightarrow (2)$$

i.e. the supply voltage leads current by 34.7°.

From (1) and (2), V_B leads V by

$$\phi_{(V_B - V)} = 53 - 34.7^\circ$$

$$\boxed{\phi_{(V_B - V)} = 18.3^\circ \text{ lead}}$$

24. Two impedances having the same numerical value are connected in series. If one impedance has a power factor of 0.866 lagging and the other has pf of 0.6 leading, calculate the power factor of the series combination.

Sol^{n}:

For the first impedance,

$$\cos\phi_1 = 0.866 \text{ lag}$$

$$\Rightarrow \sin\phi_1 = 0.5.$$

For the second impedance,

$$\cos\phi_2 = 0.6 \text{ lead}$$

$$\Rightarrow \sin\phi_2 = 0.8.$$

Given

$$|Z_1| = |Z_2|$$

Let

$$|Z_1| = |Z_2| = Z$$

Then,

$$Z_1 = R_1 + jX_1 = Z\cos\phi_1 + jZ\sin\phi_1$$

$$\therefore Z_1 = (0.866 + j0.5)Z \longrightarrow (1).$$

$$Z_2 = R_2 + jX_2 = Z\cos\phi_2 - jZ\sin\phi_2$$

$$\therefore Z_2 = (0.6 - j0.8)Z. \longrightarrow (2).$$

When connected in series,

$$Z_{eq} = Z_1 + Z_2$$

$$= Z[0.866 + 0.6 + j(0.5 - 0.8)]$$

$$Z_{eq} = Z(1.466 - j0.3)$$

$$Z_{eq} = 1.466Z - j0.3Z.$$

The power factor will be

$$pf = \cos\phi = \cos\left[\tan^{-1}\left(\frac{X}{R}\right)\right]$$

$$= \cos\left[\tan^{-1}\left(\frac{0.3Z}{1.466Z}\right)\right]$$

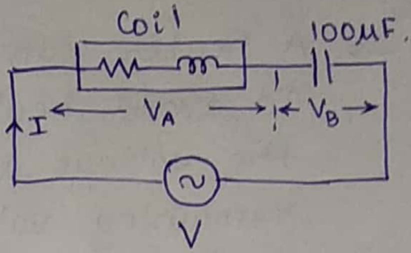
$$= 0.9796.$$

Since Z_{eq} is of the form $R - jX$,

$$\boxed{\text{power factor} = \cos\phi = 0.9796 \text{ lead}}$$

25. A coil of power factor 0.6 is in series with a 100 μF capacitor. When connected to a 50Hz supply, the potential difference across the coil is equal to the potential difference across the capacitor. Calculate the resistance & inductance of the coil.

Sol^{n}



For the coil,

$\cos\phi = 0.8$

Given $|V_A| = |V_B|$

$\omega = 2\pi f = 2\pi * 50 = 314$ rad/sec.

$|V_A| = |V_B|$

$\Rightarrow |I||Z_A| = |I||Z_B|$

$\Rightarrow |Z_A| = |Z_B|$.

$$|Z_B| = X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi * 50 * 100 * 10^{-6}} = 31.83\,\Omega.$$

$\Rightarrow |Z_A| = 31.83\,\Omega$.

Then for the coil, $\cos\phi = 0.6$

$\Rightarrow \sin\phi = 0.8$.

$\therefore R_{coil} = |Z_A| \cos\phi$

$= 31.83 * 0.6$

$\boxed{R_{coil} = 19.1\,\Omega}$

$X_{coil} = |Z_A| \sin\phi = 31.83 * 0.8 = 25.464$

$\Rightarrow 2\pi f L = 25.464$

$$\Rightarrow L = \frac{25.464}{2\pi * 50}$$

$\Rightarrow \boxed{L = 0.081\ H}$

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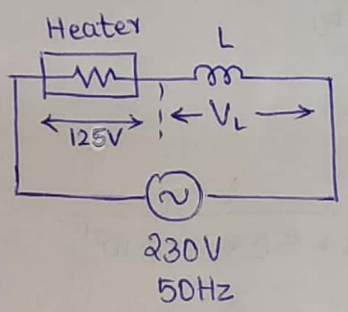
26. A room heater of 2kW, 125V rating is to be operated on 230V, 50Hz A.C supply. Calculate the value of inductance to be connected in series with the heater so that the heater will not get damaged ~~to~~ due to overvoltage.

Sol^n:

→ A heater can be modeled as a purely resistive load. To avoid heater overvoltage, we must ensure that the voltage across the heater is 125V. Thus, the remaining voltage should appear across the inductor.

→ Note that the voltage across the inductor is not merely an algebraic difference between the supply & heater voltages, but is the phasor difference.



$$V_L \neq 230 - 120.$$

The supply voltage is

$$V = 230V$$

$$V = \sqrt{120^2 + V_L^2} = 230$$

$$\Rightarrow V_L = \sqrt{230^2 - 125^2}$$

$$V_L = 193.067V$$

But $V_L = IX_L$

Power dissipated by the heater is,

$$P = \frac{V_{heater}^2}{R_{heater}} = 2kW$$

$$\Rightarrow R_{heater} = \frac{125^2}{2000} = 7.8125\Omega.$$

$$\therefore I = \frac{V_{heater}}{R_{heater}} = \frac{125}{7.8125} = 16A$$

Thus,

$$V_L = IX_L$$

$$\Rightarrow X_L = \frac{V_L}{I} = \frac{193.067}{16} = 12.066\,\Omega.$$

$$\Rightarrow 2\pi f L = 12.066\,\Omega$$

$$L = \frac{12.066}{2\pi \times 50}$$

$$\boxed{L = 0.0384\,H}$$

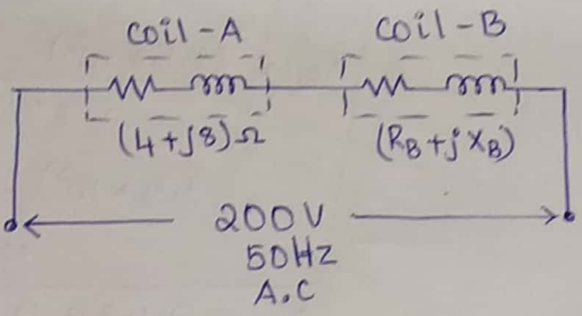
27. Two coils A and B are connected in series across 200V, 50Hz A.C supply. The power input to the circuit is 2.2kW and 1.5kVAr. If the resistance & reactance of coil A are 4 Ω and 8 Ω respectively, calculate,

a) resistance & reactance of coil B

b) active power consumed by the two coils

c) total impedance of the circuit.

Sol^n:



$R_A = 4\,\Omega$

$X_A = 8\,\Omega$

$$\Rightarrow Z_A = (4 + j8)\,\Omega.$$

Total power,

$$P = 2.2\,kW$$

$$Q = 1.5\,kVAr.$$

$\therefore$ Apparent power is

$$S = \sqrt{P^2 + Q^2} = \sqrt{(2.2 \times 10^3)^2 + (1.5 \times 10^3)^2}$$

$$S = 2.66\,kVA.$$

$$S = VI$$

$$\Rightarrow 2.66 * 10^3 = 200 * I$$

$$\Rightarrow I = 13.3A.$$

Total power = Power in R_A + Power in R_B.

$$2.2kW = I^2 R_A + I^2 R_B$$

$$\Rightarrow I^2 R_B = 2.2 * 10^3 - I^2 R_A$$

$$\Rightarrow R_B = \frac{(2.2 * 10^3) - (I^2 R_A)}{I^2}$$

$$= \frac{(2.2 * 10^3) - (13.3^2 * 4)}{13.3^2}$$

$$\boxed{R_B = 8.437 \Omega}$$

~~∴ R_B = 2.2kW →~~

Total reactive power = $I^2 X_A + I^2 X_B$

$$1.5kVAr = I^2 X_A + I^2 X_B$$

$$\Rightarrow X_B = \frac{(1.5 * 10^3) - I^2 X_A}{I^2}$$

$$= \frac{(1.5 * 10^3) - (13.3^2 * 8)}{13.3^2}$$

$$\boxed{X_B = 0.479 \Omega}$$

Active power consumed by the two coils are,

$$P_A = I^2 R_A = 13.3^2 * 4 \Rightarrow \boxed{P_A = 707.56 W}$$

$$P_B = I^2 R_B = 13.3^2 * 8.437 \Rightarrow \boxed{P_B = 1492.42 W}$$

Total impedance of the circuit,

$$Z = Z_A + Z_B = (4 + j8) + (8.437 + j0.479)$$

$$\Rightarrow \boxed{Z = (12.437 + j8.479) \Omega}$$