

Title: Generative model

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In statistical classification, two main approaches are called the generative approach and the discriminative approach. These compute classifiers by different approaches, differing in the degree of statistical modelling. Terminology is inconsistent, but three major types can be distinguished:

A generative model is a statistical model of the joint probability distribution $P(X, Y)$ on a given observable variable X and target variable Y ; A generative model can be used to "generate" random instances (outcomes) of an observation x .

A discriminative model is a model of the conditional probability $P(Y \mid X = x)$ of the target Y , given an observation x . It can be used to "discriminate" the value of the target variable Y , given an observation x .

Classifiers computed without using a probability model are also referred to loosely as "discriminative".

The distinction between these last two classes is not consistently made; Jebara (2004) refers to these three classes as generative learning, conditional learning, and discriminative learning, but Ng & Jordan (2002) only distinguish two classes, calling them generative classifiers (joint distribution) and discriminative classifiers (conditional distribution or no distribution), not distinguishing between the latter two classes. Analogously, a classifier based on a generative model is a generative classifier, while a classifier based on a discriminative model is a discriminative classifier, though this term also refers to classifiers that are not based on a model.

Standard examples of each, all of which are linear classifiers, are:

generative classifiers: naive Bayes classifier and linear discriminant analysis

naive Bayes classifier and

linear discriminant analysis

discriminative model: logistic regression

logistic regression

In application to classification, one wishes to go from an observation x to a label y (or probability distribution on labels). One can compute this directly, without using a probability distribution (distribution-free classifier); one can estimate the probability of a label given an observation, $P(Y \mid X = x)$ (discriminative model), and base classification on that; or one can estimate the joint distribution $P(X, Y)$ (generative model), from that compute the conditional probability $P(Y \mid X = x)$, and then base classification on that. These are increasingly indirect, but increasingly probabilistic, allowing more domain knowledge and probability theory to be applied. In practice different approaches are used, depending on the particular problem, and hybrids can combine strengths of multiple approaches.

Definition

An alternative division defines these symmetrically as:

a generative model is a model of the conditional probability of the observable X , given a target y , symbolically, $P(X \mid Y = y)$

a discriminative model is a model of the conditional probability of the target Y , given an observation x , symbolically, $P(Y \mid X = x)$

Regardless of precise definition, the terminology is constitutional because a generative model can be used to "generate" random instances (outcomes), either of an observation and target (x, y) , or of an observation x given a target value y , while a discriminative model or discriminative classifier (without a model) can be used to "discriminate" the value of the target variable Y , given an observation x . The difference between "discriminate" (distinguish) and "classify" is subtle, and these are not consistently distinguished. (The term "discriminative classifier" becomes a pleonasm when "discrimination" is equivalent to "classification".)

The term "generative model" is also used to describe models that generate instances of output variables in a way that has no clear relationship to probability distributions over potential samples of input variables. Generative adversarial networks are examples of this class of generative models, and are judged primarily by the similarity of particular outputs to potential inputs. Such models are not classifiers.

Relationships between models

In application to classification, the observable X is frequently a continuous variable, the target Y is generally a discrete variable consisting of a finite set of labels, and the conditional probability $P(Y \mid X)$ can also be interpreted as a (non-deterministic) target function $f: X \rightarrow Y$, considering X as inputs and Y as outputs.

Given a finite set of labels, the two definitions of "generative model" are closely related. A model of the conditional distribution $P(X \mid Y = y)$ is a model of the distribution of each label, and a model of the joint distribution is equivalent to a model of the distribution of label values $P(Y)$, together with the distribution of observations given a label, $P(X \mid Y)$; symbolically, $P(X, Y) = P(X \mid Y) P(Y)$. Thus, while a model of the joint probability distribution is more informative than a model of the distribution of label (but without their relative frequencies), it is a relatively small step, hence these are not always distinguished.

Given a model of the joint distribution, $P(X, Y)$, the distribution of the individual variables can be computed as the marginal distributions $P(X) = \sum_y P(X, Y = y)$ and $P(Y) = \int_x P(Y, X = x)$ (considering X as continuous, hence integrating over it, and Y as discrete, hence summing over it), and either conditional distribution can be computed from the definition of conditional probability: $P(X \mid Y) = P(X, Y) / P(Y)$ and $P(Y \mid X) = P(X, Y) / P(X)$.

Given a model of one conditional probability, and estimated probability distributions for the variables X and Y , denoted $P(X)$ and $P(Y)$, one can estimate the opposite conditional probability using Bayes' rule:

For example, given a generative model for $P(X \mid Y)$, one can estimate:

and given a discriminative model for $P(Y \mid X)$, one can estimate:

Note that Bayes' rule (computing one conditional probability in terms of the other) and the definition of conditional probability (computing conditional probability in terms of the joint distribution) are frequently conflated as well.

Contrast with discriminative classifiers

A generative algorithm models how the data was generated in order to categorize a signal. It asks the question: based on my generation assumptions, which category is most likely to generate this signal? A discriminative algorithm does not care about how the data was generated, it simply categorizes a given signal. So, discriminative algorithms try to learn $p(y \mid x)$ directly from the data and then try to classify data. On the other hand, generative algorithms try to learn $p(x, y)$ which can be transformed into $p(y \mid x)$ later to classify the data. One of the advantages of generative algorithms is that you can use $p(x, y)$ to generate new data similar to existing data. On the other hand, it has been proved that some discriminative algorithms give better performance than some generative algorithms in classification tasks.

Despite the fact that discriminative models do not need to model the distribution of the observed variables, they cannot generally express complex relationships between the observed and target variables. But in general, they don't necessarily perform better than generative models at classification and regression tasks. The two classes are seen as complementary or as different views of the same procedure.

Deep generative models

With the rise of deep learning, a new family of methods, called deep generative models (DGMs), is formed through the combination of generative models and deep neural networks. An increase in the scale of the neural networks is typically accompanied by an increase in the scale of the training data, both of which are required for good performance.

Popular DGMs include variational autoencoders (VAEs), generative adversarial networks (GANs), and auto-regressive models. Recently, there has been a trend to build very large deep generative models. For example, GPT-3, and its precursor GPT-2, are auto-regressive neural language models that contain billions of parameters, BigGAN and VQ-VAE which are used for image generation that can have hundreds of millions of parameters, and Jukebox is a very large generative model for musical audio that contains billions of parameters.

Types

Generative models

Types of generative models are:

Gaussian mixture model (and other types of mixture model)

Hidden Markov model

Probabilistic context-free grammar

Bayesian network (e.g. Naive bayes, Autoregressive model)

Averaged one-dependence estimators

Latent Dirichlet allocation

Boltzmann machine (e.g. Restricted Boltzmann machine, Deep belief network)

Variational autoencoder

Generative adversarial network

Flow-based generative model

Energy based model

Diffusion model

If the observed data are truly sampled from the generative model, then fitting the parameters of the generative model to maximize the data likelihood is a common method. However, since most statistical models are only approximations to the true distribution, if the model's application is to infer about a subset of variables conditional on known values of others, then it can be argued that the approximation makes more assumptions than are necessary to solve the problem at hand. In such cases, it can be more accurate to model the conditional density functions directly using a discriminative model (see below), although application-specific details will ultimately dictate which approach is most suitable in any particular case.

Discriminative models

k-nearest neighbors algorithm

Logistic regression

Support Vector Machines

Decision Tree Learning

Random Forest

Maximum-entropy Markov models

Conditional random fields

Examples

Simple example

Suppose the input data is $x \in \{1, 2\}$, the set of labels for x is $y \in \{0, 1\}$, and there are the following 4 data points: $(x, y) = \{(1, 0), (1, 1), (2, 0), (2, 1)\}$

For the above data, estimating the joint probability distribution $p(x, y)$ from the empirical measure will be the following:

while $p(y | x)$ will be following:

Text generation

Shannon (1948) gives an example in which a table of frequencies of English word pairs is used to generate a sentence beginning with "representing and speedily is an good"; which is not proper English but which will increasingly approximate it as the table is moved from word pairs to word triplets etc.

See also

Mathematics portal

Discriminative model

Graphical model

Notes

References

External links

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v

t

e

Outline

Index

Mean Arithmetic Arithmetic-Geometric Contraharmonic Cubic Generalized/power Geometric Harmonic Heronian Heinz Lehmer

Arithmetic

Arithmetic-Geometric

Contraharmonic
Cubic
Generalized/power
Geometric
Harmonic
Heronian
Heinz
Lehmer
Median
Mode
Average absolute deviation
Coefficient of variation
Interquartile range
Percentile
Range
Standard deviation
Variance
Central limit theorem
Moments Kurtosis L-moments Skewness
Kurtosis
L-moments
Skewness
Index of dispersion
Contingency table
Frequency distribution
Grouped data
Partial correlation
Pearson product-moment correlation
Rank correlation Kendall's τ Spearman's ρ
Kendall's τ
Spearman's ρ
Scatter plot
Bar chart
Biplot
Box plot
Control chart
Correlogram
Fan chart

Forest plot
Histogram
Pie chart
Q–Q plot
Radar chart
Run chart
Scatter plot
Stem-and-leaf display
Violin plot
Effect size
Missing data
Optimal design
Population
Replication
Sample size determination
Statistic
Statistical power
Sampling Cluster Stratified
Cluster
Stratified
Opinion poll
Questionnaire
Standard error
Blocking
Factorial experiment
Interaction
Random assignment
Randomized controlled trial
Randomized experiment
Scientific control
Adaptive clinical trial
Stochastic approximation
Up-and-down designs
Cohort study
Cross-sectional study
Natural experiment
Quasi-experiment
Population

Statistic

Probability distribution

Sampling distribution Order statistic

Order statistic

Empirical distribution Density estimation

Density estimation

Statistical model Model specification L_p space

Model specification

L_p space

Parameter location scale shape

location

scale

shape

Parametric family Likelihood (monotone) Location–scale family Exponential family

Likelihood (monotone)

Location–scale family

Exponential family

Completeness

Sufficiency

Statistical functional Bootstrap U V

Bootstrap

U

V

Optimal decision loss function

loss function

Efficiency

Statistical distance divergence

divergence

Asymptotics

Robustness

Estimating equations Maximum likelihood Method of moments M-estimator Minimum distance

Maximum likelihood

Method of moments

M-estimator

Minimum distance

Unbiased estimators Mean-unbiased minimum-variance Rao–Blackwellization Lehmann–Scheffé theorem Median unbiased

Mean-unbiased minimum-variance Rao–Blackwellization Lehmann–Scheffé theorem

Rao–Blackwellization
Lehmann–Scheffé theorem
Median unbiased
Plug-in
Confidence interval
Pivot
Likelihood interval
Prediction interval
Tolerance interval
Resampling Bootstrap Jackknife
Bootstrap
Jackknife
1- & 2-tails
Power Uniformly most powerful test
Uniformly most powerful test
Permutation test Randomization test
Randomization test
Multiple comparisons
Likelihood-ratio
Score/Lagrange multiplier
Wald
Z -test (normal)
Student's t -test
F -test
Chi-squared
G -test
Kolmogorov–Smirnov
Anderson–Darling
Lilliefors
Jarque–Bera
Normality (Shapiro–Wilk)
Likelihood-ratio test
Model selection Cross validation AIC BIC
Cross validation
AIC
BIC
Sign Sample median
Sample median

Signed rank (Wilcoxon) Hodges–Lehmann estimator
Hodges–Lehmann estimator
Rank sum (Mann–Whitney)
Nonparametric anova 1-way (Kruskal–Wallis) 2-way (Friedman) Ordered alternative (Jonckheere–Terpstra)
1-way (Kruskal–Wallis)
2-way (Friedman)
Ordered alternative (Jonckheere–Terpstra)
Van der Waerden test
Bayesian probability prior posterior
prior
posterior
Credible interval
Bayes factor
Bayesian estimator Maximum posterior estimator
Maximum posterior estimator
Correlation
Regression analysis
Pearson product-moment
Partial correlation
Confounding variable
Coefficient of determination
Errors and residuals
Regression validation
Mixed effects models
Simultaneous equations models
Multivariate adaptive regression splines (MARS)
Simple linear regression
Ordinary least squares
General linear model
Bayesian regression
Nonlinear regression
Nonparametric
Semiparametric
Isotonic
Robust
Homoscedasticity and Heteroscedasticity
Exponential families

Logistic (Bernoulli) / Binomial / Poisson regressions

Analysis of variance (ANOVA, anova)

Analysis of covariance

Multivariate ANOVA

Degrees of freedom

Cohen's kappa

Contingency table

Graphical model

Log-linear model

McNemar's test

Cochran–Mantel–Haenszel statistics

Regression

Manova

Principal components

Canonical correlation

Discriminant analysis

Cluster analysis

Classification

Structural equation model Factor analysis

Factor analysis

Multivariate distributions Elliptical distributions Normal

Elliptical distributions Normal

Normal

Decomposition

Trend

Stationarity

Seasonal adjustment

Exponential smoothing

Cointegration

Structural break

Granger causality

Dickey–Fuller

Johansen

Q-statistic (Ljung–Box)

Durbin–Watson

Breusch–Godfrey

Autocorrelation (ACF) partial (PACF)

partial (PACF)

Cross-correlation (XCF)
ARMA model
ARIMA model (Box–Jenkins)
Autoregressive conditional heteroskedasticity (ARCH)
Vector autoregression (VAR) (Autoregressive model (AR))
Spectral density estimation
Fourier analysis
Least-squares spectral analysis
Wavelet
Whittle likelihood
Kaplan–Meier estimator (product limit)
Proportional hazards models
Accelerated failure time (AFT) model
First hitting time
Nelson–Aalen estimator
Log-rank test
Bioinformatics
Clinical trials / studies
Epidemiology
Medical statistics
Chemometrics
Methods engineering
Probabilistic design
Process / quality control
Reliability
System identification
Actuarial science
Census
Crime statistics
Demography
Econometrics
Jurimetrics
National accounts
Official statistics
Population statistics
Psychometrics
Cartography
Environmental statistics

[Geographic information system](#)

[Geostatistics](#)

[Kriging](#)

[Category](#)

[Mathematics portal](#)

[Commons](#)

[WikiProject](#)