Instrumental Variables and Systems of Equations

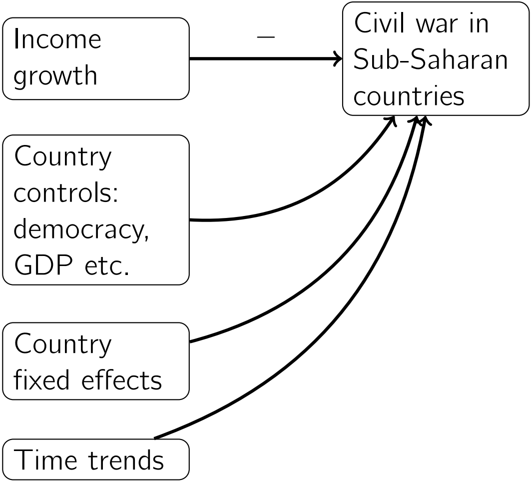
Philip Leifeld

GV903: Advanced Research Methods, Week 10

University of Essex

1. Conceptual Introduction

Miguel et al (2004): Economic Shocks and Civil Conflict



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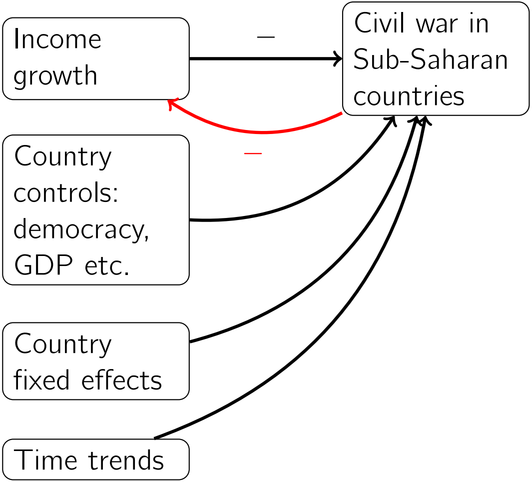

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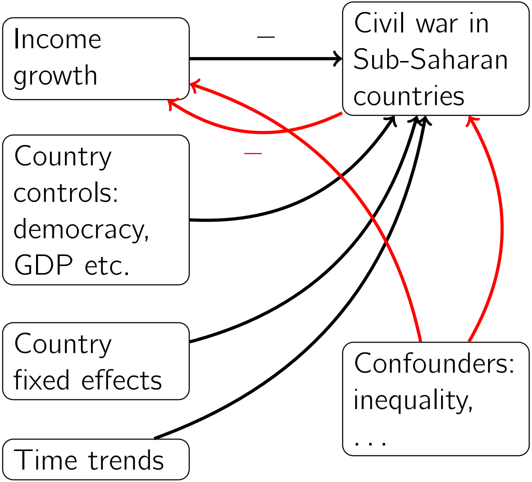

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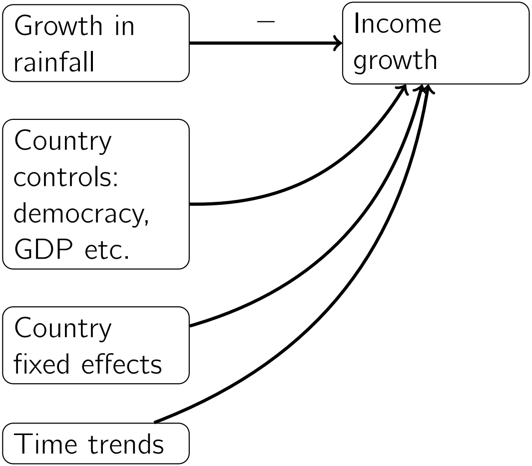
Instrumental Variable: Growth in Rainfall



$$\text{growth}_{it} = a_i + \mathbf{X}_{it}^{\top}\mathbf{b} + c_1 \Delta R_{it} + c_2 \Delta R_{i,t-1} + d_i \text{year}_t + e_{it}$$

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More generally:

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As a consequence, y_2 is correlated with the error term.

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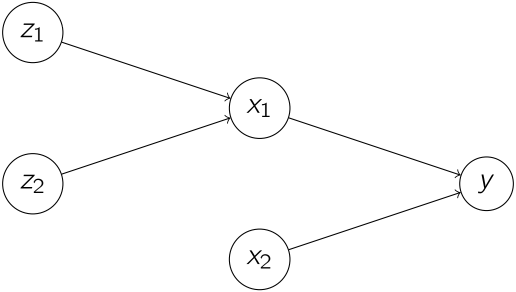

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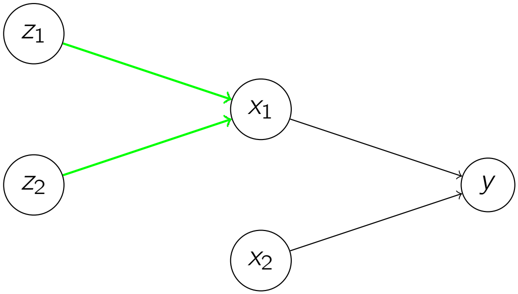

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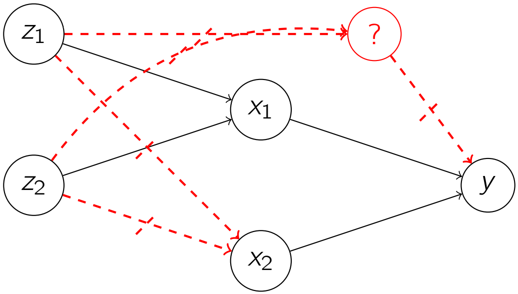

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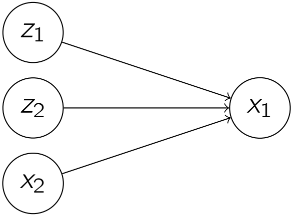

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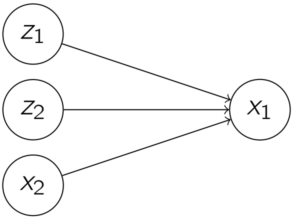

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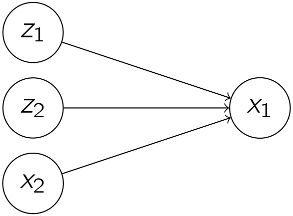

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We do this to produce a predicted version of x_1 that is "purged" of any confounding (i. e., correlations with the error term u).

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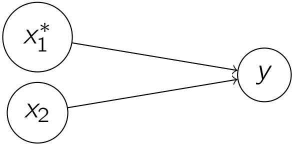

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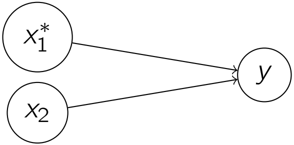

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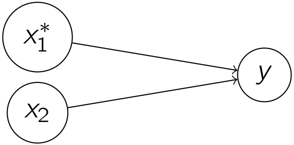

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This is like the original OLS estimation, but with the predicted version of x_1 that is not subject to endogeneity (i. e., "purged").

Let's Reconsider the Rainfall and Conflict Example...

The endogenous equation (endogenous term highlighted):

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Explanatory Variable	Dependent Variable: Civil Conflict ≥25 Deaths						Dependent Variable: Civil Conflict ≥1,000 Deaths
	Probit (1)	OLS (2)	OLS (3)	OLS (4)	IV-2SLS (5)	IV-2SLS (6)	IV-2SLS (7)
Economic growth rate, t	−.37	−.33	−.21	−.21	−.41	−1.13	−1.48*
	(.26)	(.26)	(.20)	(.16)	(1.48)	(1.40)	(.82)
Economic growth rate, $t-1$	−.14	−.08	.01	.07	−2.25**	−2.55**	−.77
	(.23)	(.24)	(.20)	(.16)	(1.07)	(1.10)	(.70)
Log(GDP per capita), 1979	−.067	−.041	.085		.053		
	(.061)	(.050)	(.084)		(.098)		
Democracy (Polity IV), $t-1$	.001	.001	.003		.004		
	(.005)	(.005)	(.006)		(.006)		
Ethnolinguistic fractionalization	.24	.23	.51		.51		
	(.26)	(.27)	(.40)		(.39)		
Religious fractionalization	−.29	−.24	.10		.22		
	(.26)	(.24)	(.42)		(.44)		
Oil-exporting country	.02	.05	−.16		−.10		
	(.21)	(.21)	(.20)		(.22)		
Log(mountainous)	.077**	.076*	.057		.060		
	(.041)	(.039)	(.060)		(.058)		
Log(national population), $t-1$	.080	.068	.182*		.159*		
	(.051)	(.051)	(.086)		(.093)		
Country fixed effects	no	no	no	yes	no	yes	yes
Country-specific time trends	no	no	yes	yes	yes	yes	yes
R^2	…	.13	.53	.71	…	…	…
Root mean square error	…	.42	.31	.25	.36	.32	.24
Observations	743	743	743	743	743	743	743

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- Only theoretical thinking can help here. Which variables z cause y *only* through x?
- Make sure there are at least as many instruments as there are endogenous variables. Only then the model is *identified.*

Exercise

1. In the growth and conflict example, is the *relevance* condition met? Why? How can you test this empirically?
2. Is the *exogeneity* assumption met? Are there possible alternative causal pathways between rainfall and conflict? How can you assess whether they could be problematic?

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- Application of OLS to binary data is called the linear probability model.
- It is not ideal (predictions outside [0, 1] etc), but can be regarded as an acceptable fix because having biased estimates due to endogeneity would be worse.

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- No natural interpretation of R^2, thus no F test.
- In practice, it is *hard* to come up with relevant and exogenous instruments.

2. Estimation in `R`

Instrumental Variables Estimation in R

Let's load some panel data on cigarette consumption for the 48 continental US States from 1985 to 1995.

```r
library("AER")
data("CigarettesSW")
```

price	Average price during year, including tax.
cpi	Consumer price index.
income	State personal income.
population	State population.
tax	Average tax per year, federal, state, and local.
taxs	Average tax per year, state level.

Some Data Preparation. . .

We need to create some additional variables for the analysis. . .

```
# real prices
CigarettesSW$rprice <- with(CigarettesSW, price / cpi)

# real income per capita
CigarettesSW$rincome <- with(CigarettesSW, income / population / cpi)

# real state tax relative to everywhere
CigarettesSW$tdiff <- with(CigarettesSW, (taxs - tax) / cpi)
```

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where $\log \mathit{rprice}$ is endogenous and instrumented as follows:

$$\log \mathit{rprice}^* = \hat{\pi}_0 + \hat{\pi}_1 \log \mathit{rincome} + \hat{\pi}_2 \mathit{tdiff} + \hat{\pi}_3 \frac{\mathit{tax}}{\mathit{cpi}} + e$$

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In R with the `ivreg` function:

```
m <- ivreg(log(packs) ~ log(rprice) + log(rincome) | log(rincome) +
  tdiff + I(tax / cpi), data = CigarettesSW, subset = year == "1995")
```

Cigarette Demand Results

```
summary(m)
##
## Call:
## ivreg(formula = log(packs) ~ log(rprice) + log(rincome) | log(rincome) +
##     tdiff + I(tax/cpi), data = CigarettesSW, subset = year ==
##     "1995")
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.6006931 -0.0862222 -0.0009999  0.1164699  0.3734227
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    9.8950     1.0586   9.348 4.12e-12 ***
## log(rprice)   -1.2774     0.2632  -4.853 1.50e-05 ***
## log(rincome)   0.2804     0.2386   1.175    0.246
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1879 on 45 degrees of freedom
## Multiple R-Squared: 0.4294,Adjusted R-squared: 0.4041
## Wald test: 13.28 on 2 and 45 DF,  p-value: 2.931e-05
```

```r
# heteroskedasticity-consistent SEs + diagnostics; Inf: z- or chi^2 test
summary(m, vcov = sandwich, df = Inf, diagnostics = TRUE)
##
## Call:
## ivreg(formula = log(packs) ~ log(rprice) + log(rincome) | log(rincome) +
##     tdiff + I(tax/cpi), data = CigarettesSW, subset = year ==
##     "1995")
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.6006931 -0.0862222 -0.0009999  0.1164699  0.3734227
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    9.8950     0.9288  10.654  < 2e-16 ***
## log(rprice)   -1.2774     0.2417  -5.286 1.25e-07 ***
## log(rincome)   0.2804     0.2458   1.141    0.254
##
## Diagnostic tests:
##                  df1 df2 statistic p-value
## Weak instruments   2  44   228.738  <2e-16 ***
## Wu-Hausman         1  44     3.823  0.0569 .
## Sargan             1  NA     0.333  0.5641
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
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```

F Test for Restricted IV Specification

Let's compute a restricted model with only one instrument:

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anova(m, m2)
## Analysis of Variance Table
##
## Model 1: log(packs) ~ log(rprice) + log(rincome) | log(rincome) + tdiff +
##     I(tax/cpi)
## Model 2: log(packs) ~ log(rprice) | tdiff
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1     45 1.5880
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```

Here, we conclude that the simpler model is sufficient.

Manual Steps for 2SLS in R

We can replicate the `ivreg` results using `lm`:

```
cig <- subset(CigarettesSW, subset = year == "1995")
s1 <- lm(log(rprice) ~ log(rincome) + tdiff + I(tax / cpi), data = cig)
cig$pred1 <- predict(s1)
s2 <- lm(log(packs) ~ pred1 + log(rincome), data = cig)
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For multiple endogenous variables: repeat Stage 1 multiple times and replace each endogenous variable with its predicted values in Stage 2.

Here, $log(rincome)$ is assumed to be exogenous, so not necessary.

Let's Compare the Manual Results to `ivreg`...

```
library("texreg")
screenreg(list(m, s2), single.row = TRUE)
##
## =================================================
##                Model 1           Model 2
## -------------------------------------------------
## (Intercept)    9.89 (1.06) ***   9.89 (1.14) ***
## log(rprice)   -1.28 (0.26) ***
## log(rincome)   0.28 (0.24)       0.28 (0.26)
## pred1                           -1.28 (0.28) ***
## -------------------------------------------------
## R^2            0.43              0.34
## Adj. R^2       0.40              0.31
## Num. obs.     48                48
## =================================================
## *** p < 0.001; ** p < 0.01; * p < 0.05
```

Same coefficients. SEs differ and would need adjustment. R^2 lower (can be negative in principle; no natural interpretation).

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The syntax is like in `ivreg`:

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formula = y ~ x1 + x2 + x3 | x3 + z1 + z2
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```

Several instrument variable transformation methods are available. See `?plm`.

Exercise

- Can you come up with a new research example in political science where instrumental variables would make sense?
- Write down the equations for both stages.
- What is the dependent variable?
- What is the endogenous variable that is problematic?
- What is the nature of the endogeneity? Why is the variable not exogenous?
- What control variables would you include?
- What are possible instruments? Discuss their relevance and exogeneity.

3. Manual Computation Using Matrix Algebra

Manual Computation of IV using Matrix Algebra

See also: https://stats.stackexchange.com/questions/265780/
See also slides on Heteroskedasticity earlier this semester.

First, we create a projection matrix for the space spanned by $\mathbf{Z}$, where $\mathbf{Z}$ is the instrument design matrix including exogenous variables and actual instruments):

$$\mathbf{P}_Z = \mathbf{Z}\left(\mathbf{Z}^\top \mathbf{Z}\right)^{-1}\mathbf{Z}^\top$$

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This would now in principle permit us to generate predicted values if we wanted to (Stage 1):

$$\hat{\mathbf{X}}_i = \mathbf{P}_Z \mathbf{X}_i$$

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$$\hat{\boldsymbol{\beta}} = \left(\mathbf{X}^\top \mathbf{P}_Z \mathbf{X}\right)^{-1}\mathbf{X}^\top \mathbf{P}_Z \mathbf{y}$$

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You will recognise this as WLS where the weight matrix $\mathbf{W} := \mathbf{P}_Z$.

Calculation of Variances and SEs

See also: https://stats.stackexchange.com/questions/265780/
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Calculate the VCOV matrix:

$$\mathrm{Cov}(\hat{\beta}) = \left(\mathbf{X}^\top \mathbf{P}_Z \mathbf{X}\right)^{-1} \mathbf{X}^\top \mathbf{P}_Z \mathbf{\Omega} \mathbf{P}_Z \mathbf{X} \left(\mathbf{X}^\top \mathbf{P}_Z \mathbf{X}\right)^{-1}$$

where $\mathbf{\Omega} = \mathrm{Cov}(\mathbf{y}) = \sigma_2 \mathbf{I}$ (or plug in residuals to do FGLS. . .).

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We can then extract the standard errors as the square root of $\widehat{\mathrm{Var}}(\hat{\boldsymbol{\beta}})$ or of the diagonal elements of $\mathrm{Cov}(\hat{\boldsymbol{\beta}})$.

4. Simultaneous Equation Models

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- This is true because each equation is essentially used as a Stage 1 model in 2SLS for the respective other equation.
- Systems with more than two equations are possible.
- Look at Reuveny and Li on the reading list for an interesting example. They have three equations to model joint democracy and conflicts between countries simultaneously.

Simultaneous Equation Models (SEM)

- In addition to 2SLS, there are more complicated estimation procedures for SEMs.
- SEMs have been implemented in several `R` packages:
  - `lavaan`
  - `systemfit`
  - `sem`
- If you do not carefully consider the issue of identification by exogenous variables, they will throw error messages.

Exercise

- Can you come up with a political science example of simultaneous equations that would be identified?
- What are the endogenous variables and exogenous instruments, respectively? Write down the equations.
- Are your exogenous variables really exogenous? Are they relevant? Discuss.
- The example can be based on your own research, the previous IV exercise, or something completely new.