Panel and Multilevel Data

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GV903: Advanced Research Methods, Week 9

University of Essex

1. Concepts and Data Structures

Multilevel Data

Consider this data structure:

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1	A	12.24	0.23	-33.23
2	A	11.09	0.19	-43.01
3	A	13.94	0.56	-28.75
4	B	16.41	0.68	-9.33
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Examples: students in classes; voters in districts; candidates in parties; municipalities in states; respondents in scenarios.

Panel Data

Now consider this data structure:

Obs.	Respondent	Time	DV	IV_1	IV_2
1	1	1	12.24	0.23	-33.23
2	1	2	11.09	0.19	-43.01
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Examples: voters' party preferences; countries' democracy scores.

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- But: TSCS makes more use of time series methods and spatial regression.

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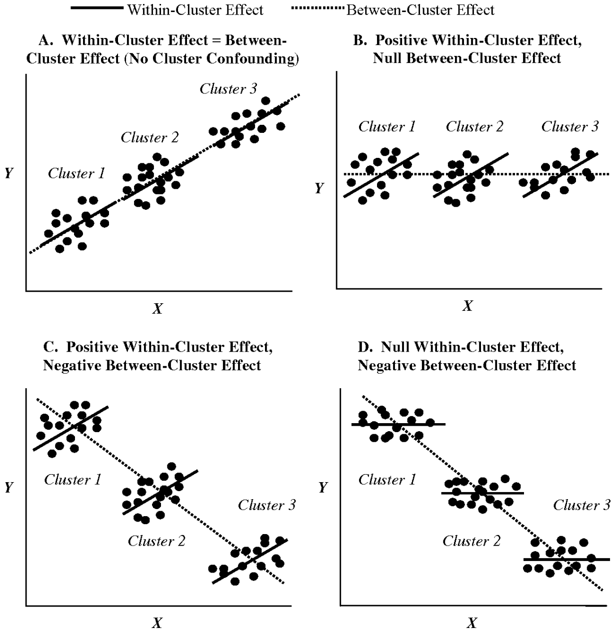
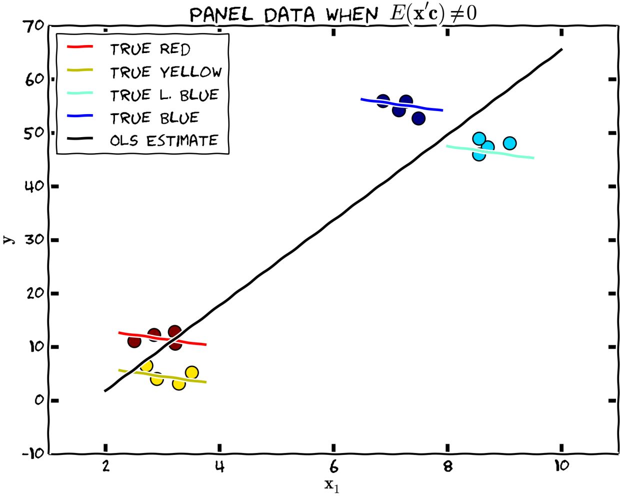
- Example: When measuring democracy in countries, there may be more variation across countries than within each time series (due to serial autocorrelation).
- The error clustering may also be correlated with the IVs.
- This violates the OLS assumption of unsystematic errors.

The Consequence: Cluster Confounding (Bartels 2011)



Case 1: Slopes (= IVs) are Affected

Source: `https://rlhick.people.wm.edu/econ407/notes/intro_panel_data.html`



In this case, we can use a Fixed Effects Model.

Case 2: Only Intercepts are Affected

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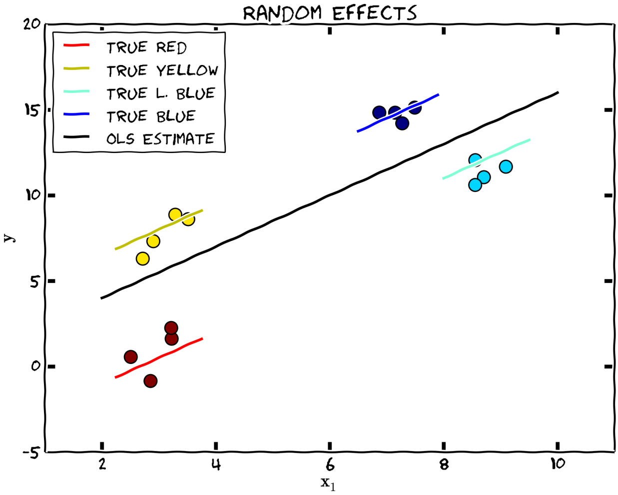

In this case, we can use a Random Effects Model.

Exercise

1. Can you come up with a political science example of a panel dataset, TSCS dataset, and multilevel dataset each, along with a research question (DV and IV)?
2. In each of these examples, what are the individual-level units and what are the higher-level units/groups?
3. Do you expect any cluster confounding in these cases?

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- These and random effects models are both sub-types of mixed-effects models.
- Let's start with fixed effects models!

2. Fixed Effects

The Fixed Effects Model

You can control for any between-group variance directly by using dummy variables. I. e., one binary indicator D_j per group j:

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There is a fix for this: group demeaning!

The Fixed Effects Model: Group Demeaning

We can decompose the error term into a group-level component (a_j) and the remaining, "idiosyncratic" error (u_{ij}):

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Note how a_j drops out of the equation. We got rid of any group-level variation and do not have to include dummies.

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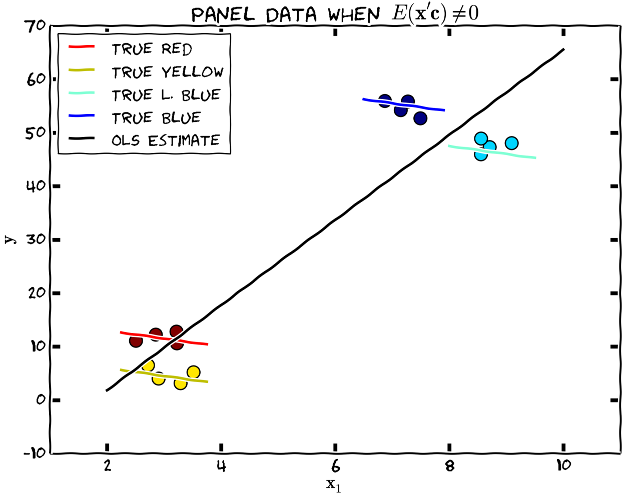
- In a panel context, we do time demeaning instead: for each observation, we subtract the individual's mean over time.
- This puts the different individuals' time series on a comparable scale.
- Then we can compare the remaining variation over different time points because we have fixed all individual variation.

Fixed Effects



Think of the demeaning as moving all these groups or time series onto a common scale (on both axes)! Then estimate pooled OLS.

Let's Do Fixed Effects in R!

Source: https://www.princeton.edu/~otorres/Panel101R.pdf

First, load some panel data.

```
library("foreign")
Panel <- read.dta(
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Now with country demeaning. `plm` is a package for panel data.

```r
library("plm")
fixed <- plm(y ~ x1, data = Panel,
             index = c("country", "year"),
             model = "within")
```

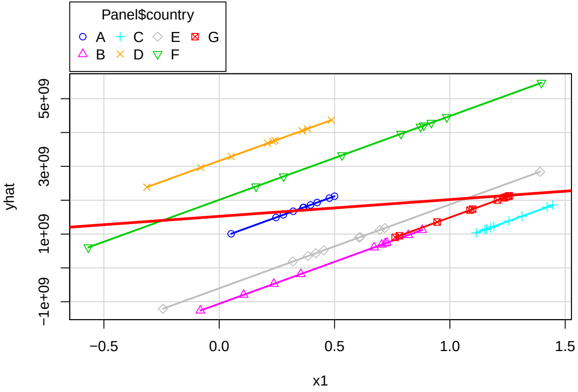
Results of the Two Models

	Model 1	Model 2
x1	2475617827.10*	2475617827.10*
	(1106675593.60)	(1106675593.60)
factor(country)A	880542403.99	
	(961807052.24)	
factor(country)B	−1057858363.16	
	(1051067684.19)	
factor(country)C	−1722810754.55	
	(1631513751.40)	
factor(country)D	3162826897.32***	
	(909459149.66)	
factor(country)E	−602622000.33	
	(1064291684.41)	
factor(country)F	2010731793.24	
	(1122809097.35)	
factor(country)G	−984717493.45	
	(1492723118.24)	
R^2	0.44	0.07
Adj. R^2	0.37	−0.03
Num. obs.	70	70

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

We can Inspect the Dummy Variable Groups

```
yhat <- predict(fixed.dum)
library("car")
scatterplot(yhat ~ Panel$x1|Panel$country, boxplots = FALSE,
            xlab = "x1", ylab = "yhat", smooth = FALSE)
abline(lm(Panel$y ~ Panel$x1), lwd = 3, col = "red")
```



Within- and Between-Estimation

On the previous slides, we did within-estimation. That means we want to look at the variation within each time series and fix the cross-series variance (or generally, the cross-group variance).

We could also do between-estimation, thereby fixing the group-internal or time-series-internal variance (e. g., time demeaning) and focusing on the variance between groups or time series.

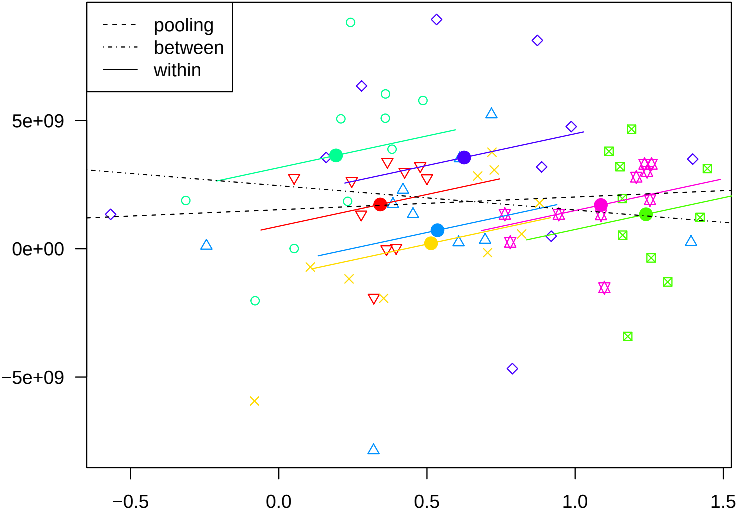
This would be equivalent to including time-point dummies in the panel case.

```
fixed2 <- plm(y ~ x1, data = Panel,
              index = c("country", "year"),
              model = "between")
```

Plotting Within- and Between-Estimation

This time the observed values, not fitted values.

```
plot(fixed, within = TRUE, between = TRUE)
```



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ranef <- plm(y ~ x1, data = Panel,
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phtest(fixed, ranef)
##
##  Hausman Test
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Close call. . . probably random effects are sufficient.

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We can include further controls. The interaction tells us if the intervention works.

Exercise

1. Can you come up with an example where fixed-effects estimation makes sense?
2. Describe the DV and IV, the nesting structure of the data, and the research question.
3. Would you need a within- or between-estimator to answer this question?
4. Describe a natural or field experiment in which you could use a difference-in-difference estimator. How would you do this from an analysis perspective? That is, try to write down the regression equation to identify the causal effect of the treatment.

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- Need to decide among within- and between-estimation.
- Some people (e. g., Bell and Jones 2014, PSRM) argue that fixed effects are never needed because mixed-effects models can do all of this, but better.

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- For mixed/random effects, packages `nlme` and `lme4` are more general and more powerful.
- Let's look at random effects with `lme4`...

3. Random Effects and Mixed Effects

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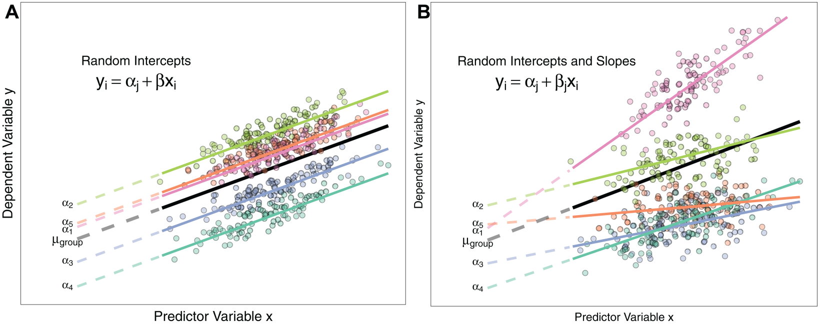
model <- lmer(dv ~ (1 + x1|x2), data = d)
```

`(1 + x1|x2)` introduces random intercepts (`1`) *and* `x1` slopes for the `x2` grouping variable.

Random Intercepts and Random Slopes

Source: Harrison et al. (2018), *PeerJ*. `https://peerj.com/articles/4794/`



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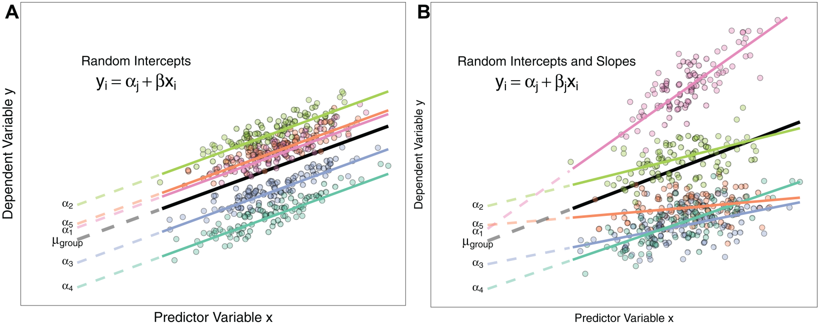

Example: Pupils' grades are a function of class attendance. Pupils are nested in schools with all sorts of characteristics, which we can't control for but which affect grades. Hence random intercepts for the schools.

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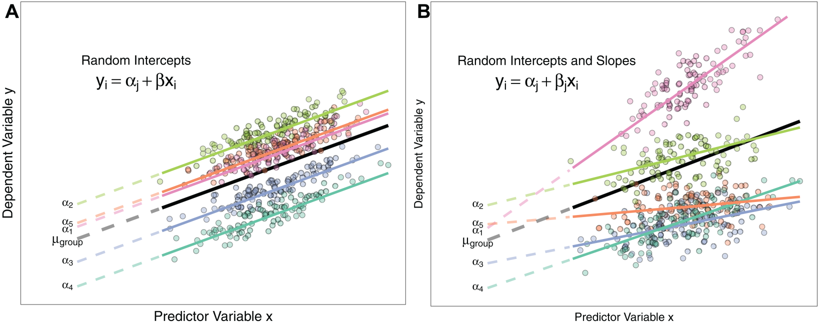

If we think the link between attendance and grades differs by school, perhaps because teacher quality varies, we need random intercepts *and* slopes.

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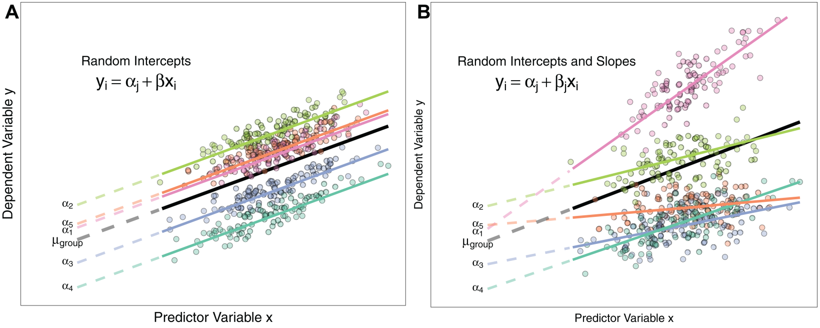

Another example: Countries' foreign direct investment (FDI) over time. If we explain FDI by lagged democracy, we need to account for baseline differences in FDI across countries using random intercepts.

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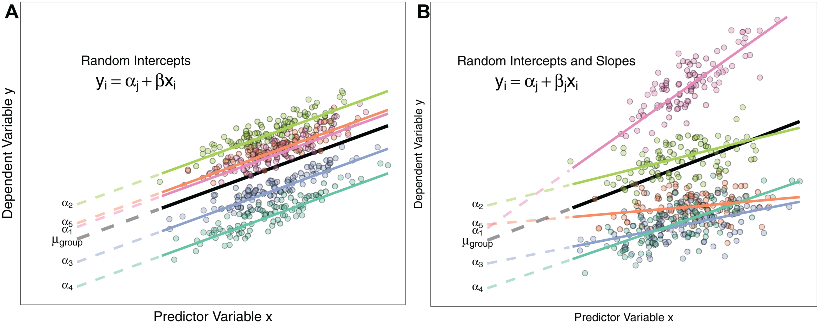

If we think the link between democracy and FDI differs by country, we need random intercepts *and* slopes.

Random Intercepts and Random Slopes

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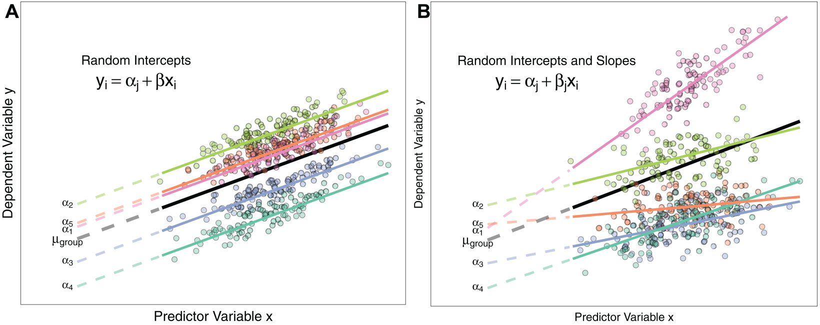

μ_{group} is the mean of the intercept of all groups. The model output reports the variance across groups around this random effect. If we fit random slopes, we also get a variance for those.

Random Intercepts and Random Slopes

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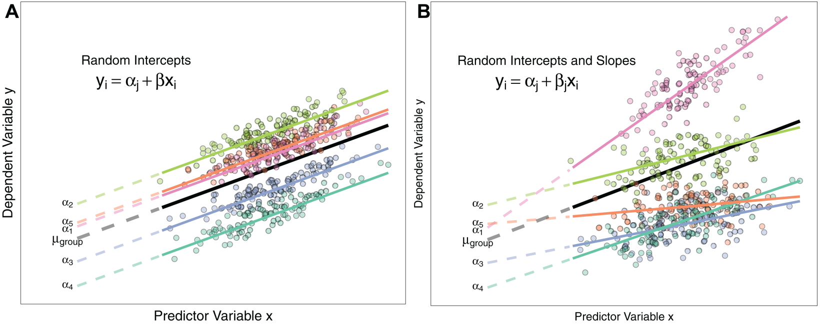

Note how β is replaced by β_j in the random slopes equation.
Note also how the random intercepts are denoted by a_j.
Estimation is complicated; `lme4` uses Bayesian estimation.

Interviewer Ratings of Respondent Political Knowledge

Data from the 2000 American National Election Study

```
library("pscl")
data("politicalInformation")
head(politicalInformation)
##              y collegeDegree female age homeOwn govt length id
## 1 Fairly High           Yes     No  49     Yes   No  58.40  1
## 2     Average            No    Yes  35     Yes   No  46.15  2
## 3   Very High            No    Yes  57     Yes   No  89.52  3
## 4     Average            No     No  63     Yes   No  92.63  4
## 5 Fairly High           Yes    Yes  40     Yes   No  58.85  4
## 6     Average            No     No  77     Yes   No  53.82  4

politicalInformation$y2 <- as.numeric(politicalInformation$y)
```

Interviewers may have different baseline rates of judging respondents' knowledge, perhaps conditioned on their own expertise, age, gender etc. Better use a random intercept for interviewers.

Interviewer Ratings of Respondent Political Knowledge

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```
library("lme4")
model1 <- lm(y2 ~ collegeDegree + female + age + homeOwn + govt +
  length, data = politicalInformation)
model2 <- lmer(y2 ~ collegeDegree + female + age + homeOwn +
  govt + length + (1|id), data = politicalInformation)
```

	model1	model2
(Intercept)	$2.28\ (0.10)^{***}$	$2.29\ (0.10)^{***}$
collegeDegreeYes	$0.82\ (0.05)^{***}$	$0.83\ (0.05)^{***}$
femaleYes	$-0.36\ (0.05)^{***}$	$-0.39\ (0.05)^{***}$
age	$0.00\ (0.00)^{**}$	$0.00\ (0.00)^{*}$
homeOwnYes	$0.27\ (0.05)^{***}$	$0.27\ (0.05)^{***}$
govtYes	$0.10\ (0.08)$	$0.10\ (0.07)$
length	$0.01\ (0.00)^{***}$	$0.01\ (0.00)^{***}$

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

There is not a big difference here. But there could have been...

Do We Need a Random Effect Here?

Some additional output from `summary(model2)`:

```
Random effects:
 Groups   Name        Variance Std.Dev.
 id       (Intercept) 0.1349   0.3673
 Residual             0.8907   0.9438
Number of obs: 1790, groups:  id, 115

Fixed effects:
                 Estimate Std. Error t value
(Intercept)      2.292365   0.103488  22.151
collegeDegreeYes 0.825025   0.048383  17.052
femaleYes       -0.385396   0.046078  -8.364
age              0.003748   0.001473   2.545
homeOwnYes       0.266954   0.051635   5.170
govtYes          0.095319   0.072731   1.311
length           0.008518   0.001114   7.645
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The variance of 0.13 indicates it is better than not to include a random effect. We also have theoretical reasons to do it.

When Should You Use Random versus Fixed Effects?

Dieleman and Templin (2014), *PLoS ONE*.

Both RE and FE estimation rely on the assumptions of OLS. The estimated models [. . .] must be correctly specified, each variable of x must be strictly exogenous and linearly independent, and the residual must be independently and identically distributed. When these conditions are met, theory states that FE estimation is unbiased and consistent. RE estimation requires an additional assumption—the group-level effect and the included explanatory variables must be independent in order to avoid [omitted variable bias]. When this assumption is met, RE estimation is unbiased, consistent, and, because it utilized both the within- and between-group variation, efficient. Under this assumption, FE estimation is not efficient because it only utilizes the within-group variation [. . .]. Thus, the correlation between the explanatory variable(s) and group-level effects distinguishes which of these two estimators to utilize.

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However, simulations have shown that RE is robust to even extreme correlations. So people in practice mostly use RE because more flexible. (See Clark/Linzer and Bell/Jones in PSRM.)

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- Mixed effects are more flexible, but fixed effects make more sense when you want to completely eliminate any between- or within-variation.
- Kreft and de Leeuw (1998): "Once you know that hierarchies exist, you see them everywhere."

Exercise

1. Can you find a political science example where you need a random intercept?
2. Can you find a political science example where you might need random slopes?
3. Write down the `lme4` formula for estimating the (imaginary) model.