

AN ECONOMIC MODEL OF REPRESENTATIVE DEMOCRACY*

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This paper develops an approach to the study of democratic policy-making where politicians are selected by the people from those citizens who present themselves as candidates for public office. The approach has a number of attractive features. First, it is a conceptualization of a pure form of representative democracy in which government is *by*, as well as *of*, the people. Second, the model is analytically tractable, being able to handle multidimensional issue and policy spaces very naturally. Third, it provides a vehicle for answering normative questions about the performance of representative democracy.

“In the real world, individuals, as such, do not seem to make fiscal choices. They seem limited to choosing ‘leaders,’ who will, in turn, make fiscal decisions” [Buchanan 1967, p. v].

I. INTRODUCTION

The principal role of political economy is to yield insights into the formation of policy. To this end, the model put forward by Downs [1957] has played a central role in studies of democratic settings. This paper develops an alternative theory of policy choice in representative democracies. The primitives of the approach are the citizens of a polity, their policy alternatives, and a constitution that specifies the rules of the political process. The theory builds from these to provide an account of citizens’ decisions to participate as candidates for public office, their voting decisions, and the policy choices of elected representatives. No preexisting political actors are assumed, and no restrictions are made on the number or type of policy issues to be decided. Political outcomes are thus derived directly from the underlying tastes and policy technology.

The paper tackles the standard case where a community elects a single representative to choose policy for one period.¹ Citizens care about policy outcomes, and may also have intrinsic

*We thank an anonymous referee, Abhijit Banerjee, Gene Grossman, Robert Inman, John Lott, Jr., Gillian Paull, Stephen Morris, Martin Osborne, Andrei Shleifer, Alex Tabarrok, Sharon Tennyson, a number of seminar participants, and, especially, Howard Rosenthal for comments and encouragement. The authors are grateful to the Institute for Policy Reform and the University of Pennsylvania Research Foundation for their respective support.

1. Osborne [1995] surveys the large literature that adopts this perspective on representative democracy.

preferences about the identity of the representative. Citizens can also differ in their policy-making abilities. The political process is modeled as a three-stage game. Stage 1 sees each citizen deciding whether or not to become a candidate for public office. Each citizen is allowed to run, although doing so is costly. At the second stage, voting takes place over the declared candidates, with all citizens having the right to vote. At stage 3 the candidate with the most votes chooses policy.

This game-theoretic structure implies that candidates who win implement their preferred policies; they cannot credibly commit to do otherwise. Understanding this, citizens will vote for candidates on the basis of their policy preferences and policy-making abilities. A voting equilibrium is then a set of voting decisions such that each citizen's vote is a (weakly undominated) best response to others. Citizens contemplating standing for office must anticipate who else will enter the race and the resulting voting equilibrium. An equilibrium at the entry stage is therefore a set of entry decisions such that each citizen's decision is optimal given the decisions of others and the anticipated voting behavior.

We investigate the positive and normative implications of this theory. The key positive issues concern the number and policy preferences of candidates who choose to run. In addition, we study the possibility of "spoiler" candidates who run simply to prevent others from winning. The principal normative concern is with efficiency. The social choice problem faced by the polity has two components: selecting a policy-maker and a policy alternative. Representative democracy provides a particular method of generating such selections, and we ask whether these selections are Pareto efficient.

The same basic model of democratic policy-making to be studied here was suggested, independently, by Osborne and Slivinski [1996], who coined the term "citizen-candidates" to describe the approach. There are, however, some important differences between our setup and theirs. First, they focus exclusively on a one-dimensional model with Euclidean preferences, and second, they work with a continuum of citizens who are assumed to vote sincerely. The sincerity assumption produces very different implications from the model, which we discuss below. In terms of scope, the analyses are complementary. We develop a more general version of the model and explore the normative issues discussed above. They use their one-dimensional version to derive

some interesting implications of different electoral systems (plurality rule and majority rule with runoffs) for the number and type of candidates.

The remainder of the paper is organized as follows. Section II lays out the model and shows that an equilibrium exists, in either pure or mixed strategies. Section III provides a fairly complete characterization of pure strategy equilibria. Section IV develops the implications of our theory for the standard one-dimensional policy model with Euclidean preferences and compares the findings with those of Osborne and Slivinski [1996]. Section V develops the normative analysis, and Section VI concludes.

II. THE MODEL

A community made up of N citizens, labeled $i \in \mathcal{N} = \{1, \dots, N\}$, must choose a representative to select and implement a policy alternative, denoted by x . In many applications, these are conventional policy instruments, such as taxes and public expenditures. The set of policy alternatives available if individual i is the policy-maker is denoted by \mathcal{A}^i . This set takes account of both technological and constitutional constraints on policy choices. Differences in \mathcal{A}^i across citizens reflect varying levels of policy-making competence. Let $\mathcal{A} = \cup_{i=1}^N \mathcal{A}^i$ be the set of all possible policy alternatives.

Each citizen's utility depends upon the policy outcome and the identity of the representative. The latter captures the possibility of idiosyncratic utility from holding office oneself ("ego rent") or from having another making policy (for example, liking a "good-looking" representative). We denote the utility of individual i when the policy choice is $x \in \mathcal{A}$ and the representative is $j \in \mathcal{N} \cup \{0\}$ by $V^i(x, j)$. The notation $j = 0$ refers to the case in which the community has no representative.

The polity selects its representative in an election. All citizens can run for office, but face a (possibly small) utility cost δ , if they do so. The constitution governing elections specifies that all citizens have one vote that, if used, must be cast for one of the self-declared candidates. The candidate who receives the most votes is elected, and in the event of ties, the winning candidate is chosen with equal probability from among the tying candidates. If only one candidate runs, then he is automatically selected to

choose policy, and if no one runs, a default policy x_0 ($\in \cap_i \mathcal{A}^i$) is implemented.

The political process has three stages. At stage 1 candidates declare themselves. At stage 2 citizens choose for whom to vote from among the declared candidates. At the final stage the elected candidate makes a policy choice. These stages are analyzed in reverse order.

Policy Choice. The citizen who wins the election implements his preferred policy—promising anything else is not credible.² Citizen i 's preferred policy is given by

$$(1) \quad x_i^* = \arg \max_x \{V^i(x, i) \mid x \in \mathcal{A}^i\}.$$

We assume that the solution to (1) is unique. Associated with each citizen's election, therefore, is a utility imputation (v_{1i}, \dots, v_{Ni}) , where $v_{ji} = V^j(x_i^*, i)$ is individual j 's utility if i is elected. If no citizen stands for office, the default policy x_0 is selected, with the utility imputation in this case being (v_{10}, \dots, v_{N0}) , where $v_{j0} = V^j(x_0, 0)$.

Voting. Given a candidate set $\mathcal{C} \subset \mathcal{N}$, each citizen may decide to vote for any candidate in \mathcal{C} or abstain. Let $\alpha_j \in \mathcal{C} \cup \{0\}$ denote citizen j 's decision. If $\alpha_j = i$, then j casts his vote for candidate i ; while if $\alpha_j = 0$, he abstains. A vector of voting decisions is denoted by $\alpha = (\alpha_1, \dots, \alpha_N)$.

The set of winning candidates (i.e., those who receive the most votes) when voting decisions are α is denoted by $W(\mathcal{C}, \alpha)$. Since if only one candidate runs he is automatically elected, we adopt the convention that $W(\mathcal{C}, \alpha) = \mathcal{C}$ (for all α) when $\#\mathcal{C} = 1$. Given our assumptions, the probability that candidate i wins, denoted $P^i(\mathcal{C}, \alpha)$, is then $1/\#W(\mathcal{C}, \alpha)$ if i is in the winning set and 0 otherwise.

Citizens correctly anticipate the policies that would be chosen by each candidate and vote strategically. A *voting equilibrium* is thus a vector of voting decisions α^* such that for each citizen $j \in \mathcal{N}$ (i) α_j^* is a best response to α_{-j}^* , i.e.,

2. Standard models assume that candidates can credibly commit to implement any policy promise. While legitimate in models where candidates have no policy preferences, one has otherwise to explain why winning candidates keep their promises [Alesina 1988].

$$(2) \quad \alpha_j^* \in \arg \max \left\{ \sum_{i \in \mathcal{C}} P^i(\mathcal{C}, (\alpha_j, \alpha_{-j}^*)) v_{ji} \mid \alpha_j \in \mathcal{C} \cup \{0\} \right\},$$

and (ii) α_j^* is not a weakly dominated voting strategy.³ Ruling out the use of weakly dominated voting strategies implies sincere voting in two-candidate elections. It is straightforward to show that a voting equilibrium exists for any nonempty candidate set. Indeed, in elections with three or more candidates, there will typically be multiple voting equilibria.

Entry. Each citizen must decide whether or not to run for office. The potential benefit from running is either directly from winning or indirectly by affecting who else is victorious. Since an individual's benefit from running depends on the entire candidate set, the entry decision is strategic.

Citizen i 's pure strategy is $s^i \in \{0, 1\}$, where $s^i = 1$ denotes entry, and a pure strategy profile is $s = (s^1, \dots, s^N)$. Given s , the set of candidates is $\mathcal{C}(s) = \{i \mid s^i = 1\}$. Each citizen's expected payoff from this strategy profile depends on voting behavior. Let $\alpha(\mathcal{C})$ denote the commonly anticipated voting decisions when the candidate set is \mathcal{C} .

Given $\alpha(\cdot)$, the expected payoff to a citizen i from the pure strategy profile s is

$$(3) \quad U^i(s; \alpha(\cdot)) = \sum_{j \in \mathcal{C}(s)} P^j(\mathcal{C}(s), \alpha(\mathcal{C}(s))) v_{ij} + P^0(\mathcal{C}(s)) v_{i0} - \delta s^i.$$

The notation $P^0(\mathcal{C})$ denotes the probability that the default outcome is selected. Thus, $P^0(\mathcal{C}(s))$ equals 1 if $\mathcal{C}(s) = \emptyset$ and 0 otherwise. Citizen i 's payoff represents the probability that each candidate j wins multiplied by i 's payoff from j 's preferred policy, less the entry cost if i is a candidate.

To ensure the existence of an equilibrium at the entry stage, we need to allow for mixed strategies. Let γ^i be a mixed strategy for citizen i , giving the probability that i runs for office. The set of mixed strategies for each citizen is then the unit interval $[0, 1]$.

3. A voting decision α_j is weakly dominated for citizen j if there exists $\hat{\alpha}_j \in \mathcal{C} \cup \{0\}$ such that

$$\sum_{i \in \mathcal{C}} P^i(\mathcal{C}, (\hat{\alpha}_j, \alpha_{-j})) v_{ji} \geq \sum_{i \in \mathcal{C}} P^i(\mathcal{C}, (\alpha_j, \alpha_{-j})) v_{ji}$$

for all α_{-j} with the inequality holding strictly for some α_{-j} .

A mixed strategy profile is denoted by $\gamma = (\gamma^1, \dots, \gamma^N)$, and citizen i 's expected payoff from γ is denoted by $u^i(\gamma; \alpha(\cdot))$.⁴ An *equilibrium of the entry game* given $\alpha(\cdot)$ is a mixed strategy profile γ such that for each citizen i , γ^i is a best response to γ_{-i} given $\alpha(\cdot)$. The entry game is *finite* since each citizen has only two alternatives: enter or not enter. We may therefore apply the standard existence result due to Nash [1950] to conclude that an equilibrium of the entry game exists.

Combining the analysis of the three stages, we define a *political equilibrium* to be a vector of entry decisions γ and a function describing voting behavior $\alpha(\cdot)$ such that (i) γ is an equilibrium of the entry game given $\alpha(\cdot)$ and (ii) for all nonempty candidate sets \mathcal{C} , $\alpha(\mathcal{C})$ is a voting equilibrium. Given that a voting equilibrium exists for any nonempty candidate set and that an equilibrium of the entry game exists for any specification of voting behavior, we have

PROPOSITION 1. A political equilibrium exists.

A political equilibrium $\{\gamma, \alpha(\cdot)\}$ is a *pure strategy* equilibrium if citizens employ pure strategies at the entry stage (i.e., $\gamma = s$ for some $s \in \{0, 1\}^N$) and a *mixed strategy* equilibrium otherwise. Since pure strategy equilibria exist quite broadly, they are the main focus of our attention.⁵

III. CHARACTERIZATION OF PURE STRATEGY POLITICAL EQUILIBRIA

This section characterizes pure strategy political equilibria with one, two, and three or more candidates. Our characteriza-

4. This is given by

$$u^i(\gamma; \alpha(\cdot)) = \prod_{j=1}^N \gamma^j U^i(1, \dots, 1; \alpha(\cdot)) + \prod_{j=2}^N \gamma^j (1 - \gamma^1) U^i(0, 1, \dots, 1; \alpha(\cdot)) \\ + \dots \prod_{j=1}^N (1 - \gamma^j) U^i(0, \dots, 0; \alpha(\cdot)).$$

5. Nonetheless, there are reasonable environments where pure strategy political equilibria do not exist. Following Harsanyi [1973], mixed strategy equilibria can be interpreted as the limit of pure strategy equilibria of a perturbed game of incomplete information, where each citizen i has a slightly different entry cost given by $\delta_i = \delta + \varepsilon \cdot \theta_i$, with $\varepsilon \in (0, 1)$ and θ_i is the realization of a random variable with range $(-\delta, \delta)$ and distribution function $G(\theta)$. In this game, θ_i , and hence citizen i 's entry cost, is private information. A pure strategy for citizen i is then a mapping $\sigma^i: (-\delta, \delta) \rightarrow \{0, 1\}$, where $\sigma^i(\theta_i)$ denotes citizen i 's entry decision when his "type" is θ_i . The relevant limit for our mixed strategy equilibria is as ε goes to zero.

tion exploits the fact that s is a pure strategy equilibrium of the entry game given the voting function $\alpha(\cdot)$ if and only if the following two conditions are satisfied. First, for all $i \in \mathcal{C}(s)$,

$$(4) \quad \sum_{j \in \mathcal{C}(s)} P^j(\mathcal{C}(s), \alpha(\mathcal{C}(s))) v_{ij} - \delta \geq \sum_{j \in \mathcal{C}(s)/\{i\}} P^j(\mathcal{C}(s)/\{i\}, \alpha(\mathcal{C}(s)/\{i\})) v_{ij} + P^0(\mathcal{C}(s)/\{i\}) v_{i0},$$

where $\mathcal{C}/\{i\}$ is the candidate set with individual i removed. This says that each candidate must be willing to run given who else is in the race. Second, for all $i \notin \mathcal{C}(s)$,

$$(5) \quad \sum_{j \in \mathcal{C}(s)} P^j(\mathcal{C}(s), \alpha(\mathcal{C}(s))) v_{ij} + P^0(\mathcal{C}(s)) v_{i0} \geq \sum_{j \in \mathcal{C}(s) \cup \{i\}} P^j(\mathcal{C}(s) \cup \{i\}, \alpha(\mathcal{C}(s) \cup \{i\})) v_{ij} - \delta.$$

This says that the equilibrium is *entry proof*; i.e., there is no individual not in the race who would like to enter. The analytical work largely involves a more detailed appreciation of what conditions (4) and (5) imply.

The results employ the notion of a *sincere partition*. Given a candidate set \mathcal{C} , a partition⁶ of the electorate $(N_i)_{i \in \mathcal{C} \cup \{0\}}$ is said to be *sincere* if and only if (i) $l \in N_i$ implies that $v_{li} \geq v_{lj}$ for all $j \in \mathcal{C}$ and (ii) $l \in N_0$ implies that $v_{li} = v_{lj}$ for all $i, j \in \mathcal{C}$. Intuitively, a sincere partition divides the electorate among the candidates so that every citizen is associated with his/her preferred candidate. There are many such partitions if some voters are indifferent between candidates.

One-Candidate Equilibria. In some situations there is an equilibrium in which a single citizen runs and is elected unopposed. The following proposition develops the necessary and sufficient conditions for this to arise.⁷

PROPOSITION 2. A political equilibrium in which citizen i runs unopposed exists if and only if

- (i) $v_{ii} - v_{i0} \geq \delta$, and
- (ii) for all $k \in \mathcal{N}/\{i\}$ such that $\#N_k \geq \#N_i$ for all sincere partitions (N_i, N_k, N_0) , then $\frac{1}{2}(v_{kk} - v_{ki}) \leq \delta$ if there exists a sincere partition such that $\#N_i = \#N_k$ and $v_{kk} - v_{ki} \leq \delta$ otherwise.

6. A *partition* is a collection of disjoint, nonempty subsets of \mathcal{N} , $(N_j)_{j \in \mathcal{J}}$, such that $\bigcup_{j \in \mathcal{J}} N_j = \mathcal{N}$.

7. The proof of this and all subsequent results can be found in the Appendix.

Condition (i) guarantees that the hypothesized candidate's gain from running is sufficient to compensate him for the entry cost. Condition (ii) guarantees that no other citizen has an incentive to enter the race. Since citizens vote sincerely in two-candidate races, any entrant who is preferred by a majority could win and hence must have no incentive to enter.

Finding an individual for whom condition (i) is satisfied is not a problem if the default option is poor enough and the costs of running are small. Condition (ii) is much more difficult to satisfy. It requires that citizen i 's policy alternative be preferred by a majority to the policy alternative of any other citizen with significantly different policy preferences. If entry costs are small, this condition is satisfied if and only if citizen i 's policy choice is a Condorcet winner in the set of preferred policy alternatives of the N citizens.⁸ Formally, we have

COROLLARY 1. Suppose that for all $j \in N$, $\mathcal{A}^j = \mathcal{A}$ and $V^j(x, h) = V^j(x)$ for all $h \in N$ and $x \in \mathcal{A}$. Then

- (i) if for sufficiently small δ a political equilibrium exists in which citizen i runs unopposed, then x_i^* must be a Condorcet winner in the set of alternatives $\{x_j^* : j \in N\}$, and
- (ii) if x_i^* is a strict Condorcet winner in the set of alternatives $\{x_j^* : j \in N\}$ and if $x_i^* \neq x_0$, then a political equilibrium exists in which citizen i runs unopposed for sufficiently small δ .

The conditions for the existence of a Condorcet winner are well-known to be extremely restrictive, making it unlikely that one-candidate pure strategy equilibria exist in most environments. Nonetheless, since the standard model of political competition, introduced in Downs [1957], only produces a prediction in such cases, such equilibria will exist in most cases where that model is used (see Section IV for an example).⁹

8. Suppose that for all $j \in N$, $V^j(x, h) = V^j(x)$ for all $h \in N$. Then an alternative $x \in \mathcal{S} \subset \mathcal{A}$ is a *Condorcet winner* in \mathcal{S} if for all $z \in \mathcal{S} \setminus \{x\}$,

$$\#\{j \mid V^j(x) \geq V^j(z)\} \geq \#\{j \mid V^j(x) < V^j(z)\}.$$

It is a *strict Condorcet winner* if the inequality is strict.

9. In Downs's model, two candidates, who care only about winning, compete by offering the electorate different platforms. There is an equilibrium in pure strategies only if a Condorcet winner exists in the set of feasible policies. One-candidate pure strategy equilibria are more likely in our setup, since we only require a Condorcet winner to exist in the set of policies that would be chosen by *some* citizen, rather than in the set of *all* feasible policies.

Two-Candidate Equilibria. The majority of formal models in political science begin with the assumption of two competing political actors.¹⁰ This makes two-candidate pure strategy equilibria of our model especially interesting. As the following result demonstrates, they exist in our model under fairly weak conditions.

PROPOSITION 3. Suppose that a political equilibrium exists in which citizens i and j run against each other. Then

(i) there exists a sincere partition (N_i, N_j, N_0) such that $\#N_i = \#N_j$, and

(ii) $\frac{1}{2}(v_{ii} - v_{ij}) \geq \delta$ and $\frac{1}{2}(v_{jj} - v_{ji}) \geq \delta$.

Furthermore, if $N_0 = \{l \in N \mid v_{li} = v_{lj}\}$ and $\#N_0 + 1 < \#N_i = \#N_j$, then these conditions are sufficient for a political equilibrium to exist in which i and j run against each other.

To find two candidates who are willing to run against each other, both must believe that they stand some chance of winning. Since citizens vote sincerely in two-candidate races, this implies that condition (i) must be satisfied. In addition, the expected utility gain from being elected and implementing one's preferred policy must be sufficient to compensate both candidates for incurring the entry costs. This is the content of condition (ii). It requires either that the two candidates' preferred policies be sufficiently different or that there is an intrinsic benefit from holding office.

The remainder of the proposition states that if N_i and N_j consist solely of citizens who have a strict preference for one candidate over the other and if strictly less than one-third of the electorate is indifferent between the two candidates, conditions (i) and (ii) are sufficient for i and j running against each other to be an equilibrium. The voting behavior which justifies this is that supporters of i and j continue to vote for their candidates even if a third candidate enters, so that an entrant can pick up at most

10. Notable exceptions are Palfrey [1984] and Feddersen, Sened, and Wright [1990]. Palfrey analyzes a one-dimensional model with three vote-maximizing parties. The two "dominant" parties are assumed to announce their platforms before the "new" third party. The main result is that the two dominant parties offer divergent platforms and the entrant party loses. Feddersen et al. consider a one-dimensional model in which a fixed number of parties must decide whether to enter the race and what platform to adopt. Parties are assumed to care only about winning, and entry is costly. In contrast to Palfrey, voters are assumed to vote strategically. Their main result is that all entering parties adopt the median position, with the number of entering parties depending upon the entry costs and the benefits of holding office.

all of the voters who are currently abstaining. Intuitively, this captures the idea that, even though they may prefer the entrant, supporters of i (j) will be reluctant to switch their votes for fear that they will cause j (i) to win. Entry is therefore deterred.

The Proposition provides a fairly weak condition for the existence of a pure strategy equilibrium with two candidates if δ is small. Basically, *any* pair of candidates who split the voters evenly can be an equilibrium of this form, provided that they are not “too close” together. In many environments, even those with multiple policy dimensions, it will be possible to find such pairs of candidates.

Equilibria with Three or More Candidates. Equilibria with three or more candidates are perfectly possible in our framework. Our first result develops some conditions that must be satisfied by the set of winning candidates in any multicandidate equilibrium.

PROPOSITION 4. Let $\{s, \alpha(\cdot)\}$ be a political equilibrium with $\#\mathcal{C}(s) \geq 3$, and let $\hat{W}(s) = W(\mathcal{C}(s), \alpha(\mathcal{C}(s)))$ denote the set of winning candidates. If $\#\hat{W}(s) \geq 2$, there must exist a sincere partition $(N_i)_{i \in \hat{W}(s) \cup \{0\}}$ for the candidate set $\hat{W}(s)$ such that

- (i) $\#N_i = \#N_j$ for all $i, j \in \hat{W}(s)$, and
- (ii) for all $i \in \hat{W}(s)$

$$\sum_{j \in \hat{W}(s)} \left(\frac{1}{\#\hat{W}(s)} \right) v_{\ell_j} \geq \max \{v_{\ell_j} \mid j \in \hat{W}(s) \setminus \{i\}\} \text{ for all } \ell \in N_i.$$

To understand this result, observe that, in a multicandidate election where two or more candidates are tying, each voter is decisive. This implies that each citizen is either voting for his most preferred candidate among the set of winners or is indifferent between all the winning candidates. If this were not the case, the citizen could switch his vote to his most preferred candidate in the set of winners and cause his election (see also Lemma 1 of Feddersen, Sened, and Wright [1990]). Thus, there must exist a sincere partition for the set of winning candidates where (i) is satisfied. The inequality condition in (ii) should also hold; each citizen must prefer the lottery over all the winning candidates to the certain victory of his next most preferred winning candidate.

In many applications, Proposition 4 can be used to rule out multicandidate equilibria with three or more (nonidentical) tying candidates. In a large community with continuous variation in

citizens' preferences, then for any set of three or more candidates, there will be some set of citizens nearly indifferent between two candidates.¹¹ The inequality in condition (ii) then fails. In the next section we use Proposition 4 to rule out equilibria with three or more winning candidates in the one-dimensional model.¹²

Proposition 4 provides us with conditions that the set of winning candidates must satisfy. The next Proposition deals with the losing candidates.

PROPOSITION 5. Let $\{s, \alpha(\cdot)\}$ be a political equilibrium with $\#\mathcal{C}(s) \geq 3$, and let $\hat{W}(s) = W(\mathcal{C}(s), \alpha(\mathcal{C}(s)))$ denote the set of winning candidates. Then, for each losing candidate $j \in \mathcal{C}(s)/\hat{W}(s)$,
 (i) $W(\mathcal{C}(s)/\{j\}, \alpha(\mathcal{C}(s)/\{j\})) \neq \hat{W}(s)$, and
 (ii) there exists $k \in \mathcal{C}(s)$ such that

$$\sum_{i \in \hat{W}(s)} \left(\frac{1}{\#\hat{W}(s)} \right) v_{ji} - \delta > v_{jk}.$$

These conditions follow directly from considering the incentives for losing candidates to run. If a losing candidate is in the race, he must affect the outcome, which implies condition (i). In addition, he must prefer the lottery over the current winners' policies to what would happen if he dropped out, which implies condition (ii).

Proposition 5 provides some useful necessary conditions for political equilibria with losing candidates. However, it does not tell us about their plausibility. The following example studies a model, due to Stiglitz [1974], where the policy-maker can choose to publicly provide a private good at different quality levels and citizens can choose whether to opt for market or public sector consumption of the good. We show that it can support a three-candidate equilibrium where only one candidate has a chance of winning.

Example: Public Provision of a Private Good with Opt-Out.

The community chooses the level of a publicly provided private

11. Feddersen [1992] exploits this fact in a related model. In his setup, voters may cast their votes for one of an infinite number of policy alternatives. The alternative that gets the most votes is implemented. Voting is costly, and voters vote strategically. His main result, which exploits an inequality similar to that in Proposition 4, is that only two alternatives receive support in equilibrium.

12. While multicandidate equilibria in which three or more candidates are in the winning set may be unusual, they are not entirely ruled out by our framework. An earlier version of the paper developed a set of sufficient conditions for an equilibrium with three or more candidates in which all candidates tie.

good, such as education or health care. Citizens consume at most one unit of the good, but may do so at different quality levels. Each citizen may buy the good in the market, opting out of the public sector in this instance. The quality level provided in the public sector can be “low,” q_L , or “high,” q_H . The set of policy alternatives is therefore $\{0, q_L, q_H\}$ with 0 denoting no provision. Higher quality public provision leads to larger tax bills for the citizens.¹³

Citizens are assumed to be indifferent to the identity of their representative (for all citizens i , $V^i(x, j) = V^i(x)$ for all $j \in \mathcal{N}$ and all $x \in \mathcal{A}$). We suppose that there are five groups of policy preferences, indexed by $\tau \in \{a, b, c, d, e\}$. Type a citizens do not consume the good in question and therefore dislike any public expenditures on it. They have preference ordering $V^a(0) > V^a(q_L) > V^a(q_H)$. Type b citizens prefer to use the private sector, but will use the public sector if quality is high. Thus, since they get no benefit from low-quality public provision, their preferences are $V^b(0) > V^b(q_H) > V^b(q_L)$. Type c citizens prefer to consume in the public sector if quality is high, with preference ordering $V^c(q_H) > V^c(0) > V^c(q_L)$. Type d citizens always choose the public sector, but prefer high to low quality so that $V^d(q_H) > V^d(q_L) > V^d(0)$. Finally, type e citizens always choose the public sector but, since they have low incomes, prefer low quality to high quality so that $V^e(q_L) > V^e(q_H) > V^e(0)$.

Let T^τ be the number of citizens of type τ . Assume that (i) $T^a + T^b + T^c > T^d + T^e$; (ii) $T^b + T^c + T^d > T^a + T^e$, and (iii) $T^e > \max\{T^a + T^b, T^c + T^d\} + 1$. Part (i) says that a majority of the population prefers no public provision to low quality public provision, and part (ii) says that a majority prefers high quality public provision to low quality provision. Part (iii) says that, in a three-way race, low quality public provision would receive a plurality. Under these assumptions there is a three-candidate equilibrium in which a citizen from groups a , d , and e contest the election. In this equilibrium citizens from group e vote for the type e candidate; citizens from groups a and b vote for the type a candidate; and the remaining citizens vote for the type d candidate. Thus, by (iii) the type e candidate wins, and the policy choice is low quality provision. The type a citizen stays in the race because he knows that if he exited, then by (ii) the type d candidate would

13. To save space, we work with citizens’ “reduced-form” preferences over $\{0, q_L, q_H\}$ with the taxes used to finance public provision being implicit.

defeat the type e candidate resulting in high quality public provision. Similarly, the type d citizen stays in the race because he knows that if he exited, then by (i) the type a candidate would win resulting in no public provision. Voting behavior is such that new entrants receive no votes. Thus, additional citizens have no incentive to enter.

In this example preferences are not single-peaked, and each spoiler candidate stays in the race to prevent the other from winning. There are many interesting environments where this logic can be applied. Constructing political equilibria with four or more candidates is even more straightforward—it is even possible in a one-dimensional policy model with single peaked preferences. This takes advantage of multiple voting equilibria that permit flexibility in constructing voting outcomes to support losing candidates' fears about what would happen if they withdrew from the race.

The results of this section provide a fairly complete account of pure strategy equilibria. Since one-candidate equilibria parallel the existence of a Condorcet winner, we expect them to be rare in practice. Thus, our model reinforces the idea that building theories of political equilibrium resting on the existence of a Condorcet winner is unlikely to be fruitful. This mirrors the fact that we so rarely find uncontested elections.

Two-candidate equilibria are more promising as far as existence goes, with any pair of sufficiently antagonistic candidates who split the space being an equilibrium. The theory suggests that two-candidate competition can become a self-fulfilling prophecy, with citizens' beliefs in the inevitability of two-candidate competition guaranteeing that the system survives by deterring costly political entry. In many environments, including that studied in the next section, there will be many two-candidate equilibria, and some will involve candidates who are "far apart." Hence, our model does not yield any central tendency for political outcomes. On the other hand, extremism does require a counterweight; if a very right-wing individual is running, then a very left-wing one must be opposing him.

While two-candidate competition is considered the norm under plurality rule, our model does not rule out equilibria with more than two candidates. It is true that races in which the outcome is a close run between three or more candidates are unlikely to exist in most environments. However, multicandidate races

with one or two winning candidates and one or more losers are a possibility. These equilibria make sense of the commonly held notion that candidates sometimes run as spoilers, preventing another candidate from winning.

For those who would like a clean empirical prediction, our multiple equilibria will raise a sense of dissatisfaction. However, this finding squares with the more familiar problem of game-theoretic models: that rationality alone does not typically pin down equilibrium play with complete precision (a message that echoes Myerson and Weber's [1993] discussion of voting behavior). This suggests the need to understand better the role of political institutions as coordinating devices, giving some greater determinacy to equilibrium outcomes.

IV. A ONE-DIMENSIONAL MODEL WITH EUCLIDEAN PREFERENCES

The standard one-dimensional issue space model from formal political science is ideal to illustrate the model at work. It also highlights some differences between our approach and that of Osborne and Slivinski [1996]. The set of policy alternatives is the unit interval $[0,1]$. Each citizen i has Euclidean preferences over these alternatives with distinct ideal point ω_i and cares only about policy outcomes, not the identity of their representative. Thus, for all $i \in \mathcal{N}$, $V^i(x,j) = -|\omega_i - x|$.¹⁴ The default policy alternative is $x_0 = 0$. For simplicity, we assume that the number of citizens in the community is odd, with m denoting the median ideal point.

Using Proposition 2, we obtain the following result.

PROPOSITION 6. A political equilibrium exists in which citizen i runs unopposed if and only if

- (i) $\omega_i \geq \delta$, and
- (ii) there is no citizen k such that $2m - \omega_i < \omega_k < \omega_i - \delta$ or $\omega_i + \delta < \omega_k < 2m - \omega_i$.

The first condition guarantees that citizen i wishes to run against the default outcome. The second condition guarantees that citizen i 's ideal point is not too far away from the median. Corollary 1 may be verified by noting that (given that there exists a citizen k such that $\omega_k = m$) condition (ii) is satisfied for sufficiently small δ if and only if $\omega_i = m$. Thus, for sufficiently small entry costs,

14. Osborne and Slivinski [1996] assume a continuum of citizens who receive some independent benefit from holding office— $V^i(x,i) = b - |\omega_i - x|$.

the policy choice in a one-candidate equilibrium is the ideal point of the median voter—the same as that emerging from the Downsian model.

Turning to two-candidate equilibria, we apply Proposition 3 to obtain

PROPOSITION 7. There exists a political equilibrium in which citizens i and j run against each other if and only if

- (i) $(\omega_i + \omega_j)/2 = m$, and
- (ii) $|\omega_j - \omega_i| \geq 2\delta$.

The first condition says that the ideal points of the two candidates must be on opposite sides and equidistant from the median, ensuring that the two candidates split the electorate and the race is tied. The second condition says that the candidates must be far enough apart so that each finds it worthwhile to compete against the other. This prevents policy convergence. These two-candidate equilibria are at variance with the predictions of the standard Downsian model. Our model predicts a seesaw across the political spectrum by candidates whose ideologies counterbalance each other. Osborne and Slivinski [1996] show that the two candidates cannot be too far apart if citizens vote sincerely. With sufficient distance between them, a third candidate could enter in the middle and attract sufficient support to win the race. However, if citizens vote strategically, such “consensus” candidates are not guaranteed support.

Finally, we turn to races with more than two candidates. We first show how Proposition 4 rules out equilibria where three or more candidates tie provided that citizens’ preferences are not clumped together. Our “nonclumping” assumption is extremely mild:

ASSUMPTION 1. Let I be any interval of the policy space $[0,1]$. Then, if there exists an interval $I' \subset [0,1]$ of smaller length that contains the ideal points of at least one-third of the citizens, the interval I must contain the ideal point of at least one citizen.

We can then establish:

PROPOSITION 8. Suppose that Assumption 1 is satisfied. Then, there are no pure strategy political equilibria in which three or more candidates tie.

The proof of this result draws on Proposition 4. By considering the implications of condition (ii) of that proposition for those

citizens who are running, we first establish that there can be only three winning candidates in such an equilibrium. We then show that, if condition (ii) is satisfied for all citizens in the polity, Assumption 1 must be violated.

It remains to examine the possibility of multicandidate equilibria in which one or two candidates win. Our next result shows that there are no three-candidate equilibria of this form provided that voting behavior satisfies a mild restriction. The restriction, which we call *Abstinence of Indifferent Voters* (AIV), is that citizens will abstain whenever they are indifferent between *all* candidates.¹⁵

PROPOSITION 9. Suppose that Assumption 1 is satisfied. Then, there are no pure strategy political equilibria involving three candidates in which voting behavior satisfies AIV.

If there was an equilibrium with three candidates, only one of whom was winning, then the winner would be the candidate whose ideal point is in-between those of the other two. The logic of the example developed in the previous section suggests that each losing “extremist” must then anticipate that the centrist candidate would lose to the other candidate in a two-way race. However, this is inconsistent with voting equilibrium. In an equilibrium with three candidates involving two candidates winning, the median citizen must be indifferent between the two winners and be voting for the losing candidate. If voting behavior satisfies AIV, the median citizen would abstain if the losing candidate dropped out, and thus his presence can have no effect on the outcome, violating condition (ii) of Proposition 5.

Proposition 9 contrasts with Osborne and Slivinski [1996] whose model yields two kinds of three-candidate equilibria. In the first there are three tying candidates, while the second has two tying candidates and a losing spoiler candidate. Both of these rest on sincere voting and independent benefits to holding office. Without such benefits, at least one candidate would be better off withdrawing and transferring his supporters to a contiguous candidate.¹⁶ As noted earlier, Proposition 9 notwithstanding, pure

15. Formally, voting behavior satisfies AIV if for all citizens $k \in \mathcal{N}$ and candidate sets \mathcal{C} , if $v_{ki} = v_{kj}$ for all $i, j \in \mathcal{C}$ then $\alpha_k(\mathcal{C}) = 0$.

16. Introducing independent benefits from office into our model would not, however, restore the possibility of three-candidate equilibria in which all candidates tie. If Assumption 1 is satisfied, there will exist at least one voter for whom the inequality in Proposition 4 fails.

strategy equilibria of the entry game involving four or more candidates in which only one or two candidates winning are possible. We leave to the interested reader the task of constructing examples.¹⁷

V. NORMATIVE ANALYSIS OF REPRESENTATIVE DEMOCRACY

A long-standing concern in political economy is whether outcomes in political equilibrium are efficient. Writers in the Chicago tradition, such as Stigler [1982] and Becker [1985], have argued that political competition should ensure efficient policy choices. However, the legitimacy of this view remains unresolved. We now study this issue in the current model.

Representative democracy produces a *selection* $(x, i) \in \mathcal{A} \times \mathcal{N} \cup \{0\}$ consisting of a policy-maker i and a policy alternative x . A selection (x, i) with $i \in \mathcal{N}$ is *feasible* if the policy selected can be implemented by citizen i ($x \in \mathcal{A}^i$). (The case of $i = 0$ requires that the policy is the default outcome, $x = x_0$.) A selection (x, i) is *efficient* if it is feasible and there exists no alternative feasible selection (x', j) such that $V^h(x', j) > V^h(x, i)$ for all $h \in \mathcal{N}$. Thus, it must not be possible to find a citizen to govern and a policy choice that makes everyone better off.¹⁸

Any political equilibrium *generates* a set of possible selections for the community. If $\{s, \alpha(\cdot)\}$ is a pure strategy political equilibrium, it generates the set of selections $\{(x_i^*, i): i \in W(\mathcal{C}(s), \alpha(\mathcal{C}(s)))\}$ if $s \neq 0$ and $\{(x_0, 0)\}$ if $s = 0$. If $\{\gamma, \alpha(\cdot)\}$ is a mixed strategy political equilibrium, the set of selections it generates is simply those associated with all the vectors of entry decisions that may arise with positive probability in equilibrium. We now investigate whether the selections generated by representative democracy are efficient.¹⁹

Identical Policy-Making Abilities. We begin with the case in which all citizens have identical policy-making abilities; i.e., for

17. One such example is available from the authors.

18. We use this more permissive notion of efficiency to avoid some odd special cases that arise in the heterogeneous policy-making abilities case.

19. We neglect two other possible costs of democratic selection. First, the randomness in the selection if the winning set contains more than one candidate or individuals use mixed strategies may reduce citizens' ex ante expected utilities. Second, resources are used up in the process of generating the selection; a candidate set \mathcal{C} results in aggregate utility costs $\#\mathcal{C} \cdot \delta$. Even if representative democracy produces an efficient selection, there may be a method of selecting policy that is both ex post efficient and uses fewer "campaign" resources.

all $i \in \mathcal{N}$, $\mathcal{A}^i = \mathcal{A}$. In this case, given that holding office is desirable, it is clearly not possible to give citizen i any higher level of utility than $V^i(x_i^*, i)$. But if (x, i) is a selection generated by a political equilibrium and $(x, i) \neq (x_0, 0)$, then $x = x_i^*$. Thus, it is clearly not possible to make citizen i better off. (Indeed, since each citizen has a unique optimal policy, any change must make him worse off.) This yields

PROPOSITION 10. Suppose that citizens have identical policy-making abilities and that for all $i \in \mathcal{N}$ and $x \in \mathcal{A}$, $V^i(x, i) \geq V^i(x, j)$ for all $j \in \mathcal{N}$. Let $\{\gamma, \alpha(\cdot)\}$ be a political equilibrium in which $\gamma^i = 1$, for some $i \in \mathcal{N}$. Then, the selections generated by $\{\gamma, \alpha(\cdot)\}$ are efficient.

This is a powerful (if obvious) result.²⁰ Consistent with the Chicago view, it implies that policy choices made in representative democracy will be efficient when citizens have identical policy-making abilities. The result holds because representative democracy vests policy authority in a particular citizen who makes an optimal policy choice.²¹

A common reaction is to suggest that the preferences of policy-makers should not count. This is understandable given the tradition of modeling policy choices by planners or political parties whose political action is not rooted in citizens' preferences. However, policies *are* chosen and implemented by citizens, and Pareto efficiency properly demands that the policy-maker's preferences be counted. To do otherwise would be to make an implicit distributional judgment about the social value of different individuals' utilities.

This efficiency result does require that at least one citizen enter with probability one. If $\gamma^i < 1$ for all $i \in \mathcal{N}$, the selection $(x_0, 0)$ is in the set of those generated by γ , and there is no guaran-

20. Our model of representative democracy relates to the study of implementation in Nash equilibrium by Hurwicz and Schmeidler [1978]. They investigate the existence of a nondictatorial mechanism for selecting a social outcome such that (i) for every preference profile there exists a Nash equilibrium and (ii) such equilibria are efficient. They prove by construction that there exists such a mechanism which they call the *kingmaker outcome function*. This involves one individual, or a group of individuals, selecting another to make social decisions. Our model of representative democracy can be thought of as a particular kingmaker outcome function. Propositions 1 and 10 confirm its desirable properties. Bergson [1976] discusses the social choice properties of "representative democracy," interpreted as selecting a citizen to decide. He observes that it satisfies all of the axioms of Arrow [1963], except Independence of Irrelevant Alternatives.

21. Besley and Coate [1996] considers the conflict between economic efficiency and payoff maximization by the incumbent that arises in a dynamic model.

tee that this is efficient. Equilibria in which no citizens enter the race with probability one may arise when the preferences of the electorate are similar or the entry cost is high. In such cases, citizens might decide to subsidize others' entry costs, establish public funding of candidates, or set an attractive salary for the community's representative, or some combination of the three.

Heterogeneous Policy-Making Abilities. The idea that candidates differ in their policy-making abilities appears to be a presumption of political campaigns and has figured in previous theoretical literature (for example, Rogoff [1990]). In this model such differences can be captured by supposing that feasible policy sets \mathcal{A}^i differ. The following example demonstrates that in such circumstances representative democracy can yield inefficient selections.

Example: Public Goods Provision with Differing Competence Levels. There are two kinds of citizens, labeled α and β , with the latter in the majority. There are two goods: a private good and a public good g . Each citizen is endowed with y units of the private good. The task of the representative is to choose a level of the public good for the community that must be financed with a head tax T . The default outcome is that no public good is provided.

Citizens of type $\gamma \in \{\alpha, \beta\}$ have Cobb-Douglas preferences $g^\gamma(y - T)^{1-\gamma}$. Is it assumed that $\alpha < \beta$, so that type β citizens have a stronger taste for public goods than type α citizens. When in office, citizens of type γ are assumed able to provide g units of the public good at cost $\theta_\gamma g$. The feasible set of policy alternatives for a type γ citizen is therefore

$$\mathcal{A}^\gamma = \{(T, g) \in [0, y] \times \mathfrak{R}_+ : \theta_\gamma g \leq NT\}.$$

We assume that type α citizens are more competent policy-makers than type β citizens, so that $\theta_\alpha < \theta_\beta$. This implies that $\mathcal{A}^\beta \subset \mathcal{A}^\alpha$.

If a type γ citizen is selected to govern, he will choose the policy alternative $(T_\gamma^*, g_\gamma^*) = (\gamma y, \gamma y / \theta_\gamma)$. It is easy to show that if $[(1 - \alpha)/(1 - \beta)]^{(1 - \beta)/\beta} \alpha/\beta < \theta_\alpha/\theta_\beta$, type β citizens prefer not to have a type α citizen in power. Since the latter are a majority, Proposition 2 implies the existence (for sufficiently small δ) of a political equilibrium in which a type β citizen runs unopposed. However, all citizens would be better off with a type α citizen as policy-maker, selecting the alternative $(\beta y, \beta y / \theta_\alpha)$.

Here, citizens who are better at policy-making (type α) cannot be trusted to serve the interests of the majority (type β). Hence to actually generate a Pareto improvement would require some way of forcing a type α citizen to act faithfully on behalf of the majority. If there were some citizens who shared type β citizens' preferences but had the policy-making abilities of type α citizens, then this problem ought not to arise. This is like saying that the space of types is sufficiently rich to encompass a broad array of tastes and policy-making abilities. An assumption along these lines is

ASSUMPTION 2. For every citizen $i \in \mathcal{N}$, if there exists some citizen j and policy choice $x \in A^j$ such that $V^h(x, j) > v_{hi}$ for all $h \in \mathcal{N}$, then there exists a citizen k such that $v_{hk} > v_{hi}$ for all $h \in \mathcal{N}$.

This says that, if there is a citizen who could in principle Pareto dominate another by virtue of his superior policy-making abilities, then there must be a citizen who would actually deliver a Pareto superior policy choice if elected. This failed in the example because there was no citizen who shared the type β citizens' preferences and who could produce public goods at low cost. The assumption permits some positive results.

PROPOSITION 11. Suppose that Assumption 2 holds, and let $\{s, \alpha(\cdot)\}$ be a political equilibrium in which a single citizen runs unopposed. Then, if δ is sufficiently small, the selection generated by $\{s, \alpha(\cdot)\}$ is efficient.

An appealing logic underlies this result. Suppose that the single candidate running is inefficient in the sense that (x_i^*, i) is an inefficient selection. Then, under Assumption 2 there would exist another citizen who, if elected, would produce a Pareto superior outcome. Since voting sincerely is the only weakly undominated strategy in two-candidate races, if this citizen entered, he would win. Thus, he will enter if the entry cost is small enough. Political competition therefore ensures the selection of citizens with superior policy-making abilities.

Unfortunately, this logic does not generalize to political equilibria in which two candidates run against each other. Suppose that one of the candidates is inefficient. If a Pareto superior candidate entered, there is no guarantee that the supporters of the inefficient candidate would switch their votes. They may fear that switching their votes would result in the opposing candidate win-

ning.²² As a consequence, the more efficient citizen is deterred from entering.

An efficiency result can be obtained by further restricting voting behavior. One could, for example, assume that Pareto-dominated candidates will attract no votes, which we call *Irrelevance of Inefficient Candidates* (IIC). Thus, whenever there are two candidates i and j such that (x_i^*, i) Pareto dominates (x_j^*, j) , then $\alpha_k \neq j$ for all citizens $k \in \mathcal{N}$. Under this assumption, we obtain

PROPOSITION 12. Suppose that Assumption 2 holds, and let $\{s, \alpha(\cdot)\}$ be a political equilibrium in which two candidates run against each other and voting behavior satisfies IIC. Then, if δ is sufficiently small, the selections generated by $\{s, \alpha(\cdot)\}$ are efficient.

However, even the assumption of IIC is not sufficient to guarantee that political equilibria involving three or more candidates produce efficient selections. Consider, for example, a three-candidate race in which all candidates are in the winning set, but one would produce an inefficient selection. There is no guarantee that a Pareto-dominant candidate would be in the winning set if he entered, even if the inefficient candidate received no votes. If the entrant is preferred by all the supporters of the inefficient candidate (say, candidate 1) together with a small number of another candidate's (say, candidate 2), the remaining supporters of candidate 2 may switch their votes to candidate 3 causing the entrant to lose! Thus, there seems to be little hope of obtaining a general efficiency result for multicandidate elections.

To summarize, our analysis identifies three reasons why representative democracy may not produce efficient selections when citizens differ in their policy-making abilities. First, if policy-making talent is concentrated among groups with certain policy preferences, then individuals may opt for a less able citizen who better represents their views. Second, even if the space of types is rich in the sense of Assumption 2, a problem can arise in elections with two (or more) winning candidates if voters are reluctant to switch their votes from an inefficient to an efficient

22. A similar problem arises in Myerson's [1993] study of the effectiveness of different electoral systems in reducing government corruption. Under plurality rule, voters may be unwilling to switch their votes to less corrupt parties who represent their policy preferences, for fear that this will simply result in the victory of parties with opposing policy preferences.

candidate because they fear that transferring their support will simply result in another less preferred candidate winning. Finally, in races with three or more candidates, entry by Pareto superior candidates might simply produce a higher probability of winning for a candidate whom they like less than the inferior candidate whom they displace.

VI. CONCLUDING REMARKS

This paper has developed a rudimentary understanding of an alternative model of representative democracy. The theory introduces the indisputable fact that representative democracy is about the participation of citizens in the political process. In addition, it has the merit that all decisions by citizens as voters, candidates, and policy-makers are derived from optimizing behavior. The model facilitates a rigorous normative analysis of political outcomes, which suggests an interesting agenda for future work linking normative public economics and political economy.

Nonetheless, the theoretical framework studied here is stark. A single elected official makes policy choices in an atemporal world without political parties or interest groups. Moreover, voters have complete information about the policy preferences and policy-making abilities of candidates. It is clear, therefore, that much remains to be done to develop the approach. Extensions that incorporate the election of representatives to a legislature and repeated elections are of interest. It will also be important to bring in uncertainty about candidates' preferences and abilities and to understand how campaigns convey information. With respect to political parties the model will hopefully facilitate the modeling of the formation of parties endogenously, rather than assuming them *deus ex machina*.

APPENDIX: PROOFS OF RESULTS

Proof of Proposition 2: (Sufficiency). Let \hat{s} be the vector of entry decisions such that $\hat{s}^i = 1$ and $\hat{s}^j = 0$ for all citizens $j \neq i$. We will show that if (i) and (ii) are satisfied, there exists a voting function $\hat{\alpha}(\cdot)$ such that $\{\hat{s}, \hat{\alpha}(\cdot)\}$ is a political equilibrium. We construct the voting function $\hat{\alpha}(\cdot)$ as follows. For all candidate sets $\{i, k\}$ with $k \neq i$, let $(\hat{N}_i, \hat{N}_k, \hat{N}_0)$ denote the sincere partition in which $\#N_i - \#N_k$ is maximized. Then, let $\hat{\alpha}(\{i, k\})$ be the vector of voting decisions generated by $(\hat{N}_i, \hat{N}_k, \hat{N}_0)$; that is, $\hat{\alpha}_j(\{i, k\}) = i$ if j

$\in \hat{N}_i$, $\hat{\alpha}_j(\{i, k\}) = k$ if $j \in \hat{N}_k$, and $\hat{\alpha}_j(\{i, k\}) = 0$ if $j \in \hat{N}_0$. Clearly, $\hat{\alpha}(\{i, k\})$ is a voting equilibrium. For all other candidate sets \mathcal{C} , let $\hat{\alpha}(\mathcal{C})$ be any voting equilibrium.

We now claim that \hat{s} is an equilibrium of the entry game given $\hat{\alpha}(\cdot)$. Condition (i) guarantees that citizen i 's entry decision is optimal. With anticipated voting behavior $\hat{\alpha}(\{i, k\})$, no citizen $k \neq i$ for whom there is a sincere partition (N_i, N_k, N_0) with $\#N_i > \#N_k$ will enter, since he will anticipate losing. No citizen $k \neq i$ for whom $(\hat{N}_i, \hat{N}_k, \hat{N}_0)$ is such that $\#\hat{N}_i = \#\hat{N}_k$ will enter since he will anticipate tying with citizen i , and the first part of condition (ii) says that, under these circumstances, entry will not be worthwhile. The second part of condition (ii) implies that the remaining citizens $k \neq i$ have no incentive to enter.

(*Necessity*). Suppose now that either (i) or (ii) is not satisfied. We must show that there exists no voting function $\alpha(\cdot)$ such that $\{\hat{s}, \alpha(\cdot)\}$ is a political equilibrium. When (i) fails, citizen i is unwilling to run against the default option and hence will not be willing to enter if nobody else is running. Suppose that (ii) fails for some citizen k . Since voting sincerely is the only weakly undominated strategy in two-candidate races, we know that if α is a voting equilibrium when the candidate set is $\{i, k\}$ there must exist a sincere partition (N_i, N_k, N_0) which generates α . It follows that any voting equilibrium $\alpha(\{i, k\})$ has individual k winning if $\#N_i < \#N_k$ for all sincere partitions and at least tying if $\#N_i = \#N_k$ for some sincere partition. Thus, whatever voting equilibrium $\alpha(\{i, k\})$ is anticipated, k will enter if citizen i is running unopposed. \square

Proof of Corollary 1. This is a straightforward consequence of Proposition 2, which we leave to the reader. \square

Proof of Proposition 3: (Necessity). If i and j wish to run against each other, then it must be the case that $W(\{i, j\}, \alpha(\{i, j\})) = \{i, j\}$. Since any voting equilibrium with two candidates involves sincere voting, it follows that there must exist a sincere partition (N_i, N_j, N_0) such that $\#N_i = \#N_j$ which gives condition (i). Furthermore, since each candidate wins with probability $1/2$, condition (ii) must hold if both candidates are willing to run against each other.

(*Sufficiency*). The proof is completed by showing that conditions (i) and (ii) are sufficient for the existence of a political equilibrium in which i and j run against each other if $N_0 = \{l \in N \mid v_{li} = v_{lj}\}$ and $\#N_0 + 1 < \#N_i = \#N_j$. Let \hat{s} be the vector of entry decisions such that $\hat{s}^i = \hat{s}^j = 1$ and $\hat{s}^k = 0$ for all citizens $k \notin \{i, j\}$.

Now construct the voting function $\hat{\alpha}(\cdot)$ as follows: first, let $\hat{\alpha}(\{i,j\})$ be the voting decisions generated by the sincere partition (N_i, N_j, N_0) ; that is, $\hat{\alpha}_l(\{i,j\}) = i$ if $l \in N_i$, $\hat{\alpha}_l(\{i,j\}) = j$ if $l \in N_j$, and $\hat{\alpha}_l(\{i,j\}) = 0$ if $l \in N_0$. Clearly, $\hat{\alpha}(\{i,j\})$ is a voting equilibrium. Second, for all citizens $k \in N/\{i,j\}$, let $\bar{N}_k = \{l \in N \mid v_{lk} > v_{li} = v_{lj}\}$ and $\underline{N}_k = \{l \in N \mid v_{lk} < v_{li} = v_{lj}\}$. For any citizen in \bar{N}_k , voting for any candidate other than k is weakly dominated. Similarly, for any citizen in \underline{N}_k voting for candidate k is weakly dominated. Notice that both \bar{N}_k and \underline{N}_k are subsets of N_0 , under our assumptions. Now if $v_{ki} \geq v_{kj}$, let $\hat{\alpha}(\{i,j,k\})$ be the vector of voting decisions generated by the partition $(N_i, N_j \cup N_k, \bar{N}_k, N_0/(\bar{N}_k \cup \underline{N}_k))$. On the other hand, if $v_{ki} < v_{kj}$, let $\hat{\alpha}(\{i,j,k\})$ be the vector of voting decisions generated by the partition $(N_i \cup \underline{N}_k, N_j, \bar{N}_k, N_0/(\bar{N}_k \cup \underline{N}_k))$. Since $\#\bar{N}_k < \#N_0 + 1 < \#N_i = \#N_j$, then it is clear that $\hat{\alpha}(\{i,j,k\})$ is a voting equilibrium for all citizens $k \in N/\{i,j\}$ and that candidate k must lose. Finally, for all remaining candidate sets \mathcal{C} , let $\hat{\alpha}(\mathcal{C})$ be any voting equilibrium.

We now claim that \hat{s} is an equilibrium of the entry game given $\hat{\alpha}(\cdot)$. Under the assumed voting behavior, if citizens i and j run against each other, they will both win with probability $1/2$ and hence condition (ii) implies that their voting decisions are optimal. All other citizens have no incentive to enter, since, given the assumed voting behavior, they will either not change the outcome (if $\underline{N}_k = \emptyset$) or will cause their preferred candidate of i and j to lose (if $\underline{N}_k \neq \emptyset$). \square

Proof of Proposition 4. For all $i \in \hat{W}(s)$ let $N_i = \{l \in N \mid \alpha_l(\mathcal{C}(s)) = i\}$, and let $N_0 = \{l \in N \mid \alpha_l(\mathcal{C}(s)) \notin \hat{W}(s)\}$. Then we know that $\#N_i = \#N_j$ for all $i, j \in \hat{W}(s)$, since all the candidates in $\hat{W}(s)$ are receiving an equal number of votes. It is also clear that $(N_i)_{i \in \hat{W}(s) \cup \{0\}}$ is a sincere partition for the candidate set $\hat{W}(s)$. If some citizen $l \in N_i$ did not prefer candidate i to another candidate $j \in \hat{W}(s)$, then by switching his vote to j , he could cause j to win, thereby improving his utility. Similarly, if some citizen $l \in N_0$ was not indifferent between all candidates in $\hat{W}(s)$, he would switch his vote to his preferred candidate in $\hat{W}(s)$ causing him to win. The inequality in condition (ii) of the proposition follows immediately from the observation that by simply switching his vote to any other candidate in $\hat{W}(s)$, citizen $l \in N_i$ could cause that candidate to win. \square

Proof of Proposition 5. This follows immediately from considering citizen j 's incentive to enter the race. By hypothesis, candi-

date j has no chance of winning. Thus, the only reason he has for being in the race is to prevent some other candidate from winning. This means that the winning set must be affected by his exit (condition (i)) and that there must exist a candidate $k \in \mathcal{C}(s)$ such that $\sum_{i \in W(s)} (1/\#W(s)) v_{ji} - v_{jk} \geq \delta$ (condition (ii)). \square

Proof of Proposition 6. It is clear that condition (i) of this proposition is equivalent to condition (i) of Proposition 2, and thus to prove the result, we need to show that condition (ii) of this proposition is equivalent to condition (ii) of Proposition 2. This is a straightforward exercise that we leave to the reader. \square

Proof of Proposition 7: (Necessity). Suppose that there exists a political equilibrium in which citizens i and j run against each other. Then conditions (i) and (ii) of Proposition 3 must be satisfied. Since

$$v_{ii} - v_{ij} = v_{jj} - v_{ji} = |\omega_i - \omega_j|,$$

condition (ii) of Proposition 3 immediately implies condition (ii) of the proposition. Condition (i) of Proposition 3, together with the fact that $\omega_i \neq \omega_j$ implies that $(\omega_i + \omega_j)/2 = m$, which is condition (i) of the proposition. To see this, note that if $(\omega_i + \omega_j)/2 < m$, then assuming $\omega_i < \omega_j$, the median citizen must prefer candidate j to candidate i . This means that all those citizens with ideal points greater than or equal to m prefer candidate j to candidate i . Thus, every sincere partition would involve $\#N_i < \#N_j$. Similarly, if $(\omega_i + \omega_j)/2 > m$, every sincere partition would involve $\#N_i > \#N_j$.

(*Sufficiency*). Now suppose that conditions (i) and (ii) of the proposition are satisfied. Then it is immediate that condition (ii) of Proposition 3 is satisfied. In addition, since there is a single citizen with ideal point m , there exists a sincere partition (N_i, N_j, N_0) , such that $\#N_i = \#N_j$, $N_0 = \{l \in \mathcal{N} \mid v_{li} = v_{lj}\}$, and $\#N_0 = 1$. Proposition 3 then implies that there exists a political equilibrium in which citizens i and j run against each other. \square

Proof of Proposition 8. Let $\{\hat{s}, \alpha(\cdot)\}$ be a pure strategy equilibrium of the entry game, and let $\hat{W}(s) = W(\mathcal{C}(s), \alpha(\mathcal{C}(s)))$ be the set of winning candidates. Suppose that $r = \#\hat{W}(s) \geq 3$, and label the ideal points of the r winning candidates as $\{\beta_1, \dots, \beta_r\}$. Relabeling as necessary, we may assume that $\beta_1 < \dots < \beta_r$. We will prove the proposition by showing that the necessary conditions stated in Proposition 4 cannot be satisfied.

Proposition 4 tells us that there must exist a sincere parti-

tion $(N_i)_{i \in \hat{W}(s) \cup \{0\}}$ for the candidate set $\hat{W}(s)$ such that $\#N_i = \#N_j$ for all $i, j \in \hat{W}(s)$, and for all $i \in \hat{W}(s)$, condition (ii) holds for all $l \in N_i$. To be sincere, the partition must satisfy

$$\left\{ \ell: \omega_\ell \in \left[0, \frac{\beta_1 + \beta_2}{2} \right) \right\} \subset N_1 \subset \left\{ \ell: \omega_\ell \in \left[0, \frac{\beta_1 + \beta_2}{2} \right] \right\},$$

$$\left\{ \ell: \omega_\ell \in \left(\frac{\beta_{i-1} + \beta_i}{2}, \frac{\beta_i + \beta_{i+1}}{2} \right] \right\} \subset N_i,$$

and

$$N_i \subset \left\{ \ell: \omega_\ell \in \left[\frac{\beta_{i-1} + \beta_i}{2}, \frac{\beta_i + \beta_{i+1}}{2} \right] \right\} \text{ for all } i \in \{2, \dots, r-1\},$$

$$\left\{ \ell: \omega_\ell \in \left(\frac{\beta_{r-1} + \beta_r}{2}, 1 \right] \right\} \subset N_r \subset \left\{ \ell: \omega_\ell \in \left[\frac{\beta_{r-1} + \beta_r}{2}, 1 \right] \right\},$$

and

$$N_0 = \emptyset.$$

It is clear that candidate 1 is in N_1 and candidate r is in N_r . Condition (ii) of Proposition 4 therefore implies that

$$(6) \quad \frac{1}{r} [\|\beta_2 - \beta_1\| + \dots + \|\beta_r - \beta_1\|] \leq \|\beta_2 - \beta_1\|$$

and

$$(7) \quad \frac{1}{r} [\|\beta_r - \beta_1\| + \dots + \|\beta_r - \beta_{r-1}\|] \leq \|\beta_r - \beta_{r-1}\|.$$

Noting that for all $j = 2, \dots, r$, $\|\beta_j - \beta_1\| = \|\beta_j - \beta_{j-1}\| + \dots + \|\beta_2 - \beta_1\|$ and for all $j = 1, \dots, r-1$, $\|\beta_r - \beta_j\| = \|\beta_r - \beta_{r-1}\| + \dots + \|\beta_{j+1} - \beta_j\|$, we see that (6) and (7) can be written as

$$(8) \quad \|\beta_2 - \beta_1\| \geq (r-2) \|\beta_3 - \beta_2\| + \dots + \|\beta_r - \beta_{r-1}\|$$

and

$$(9) \quad \|\beta_r - \beta_{r-1}\| \geq \|\beta_2 - \beta_1\| + \dots + (r-2) \|\beta_{r-1} - \beta_{r-2}\|.$$

For both (8) and (9) to hold, it is necessary that $r = 3$ and that

$$(10) \quad \beta_3 - \beta_2 = \beta_2 - \beta_1.$$

Assume, therefore, that $r = 3$ and that (10) holds. It is clear that condition (ii) of Proposition 4 cannot be satisfied if there exists any citizen l such that $\omega_l = (\beta_1 + \beta_2)/2$ or $\omega_l = (\beta_2 + \beta_3)/2$.

Thus,

$$N_1 = \{\ell: \omega_\ell \in [0, (\beta_1 + \beta_2)/2)\},$$

$$N_2 = \{\ell: \omega_\ell \in ((\beta_1 + \beta_2)/2, (\beta_2 + \beta_3)/2)\},$$

and

$$N_3 = \{\ell: \omega_\ell \in ((\beta_2 + \beta_3)/2, 1]\}.$$

Moreover, each of these sets must contain exactly one-third of the citizens.

It is straightforward to show that condition (ii) of Proposition 4 does not hold for all those citizens in N_1 for whom $\omega_\ell \in (\beta_1, (\beta_1 + \beta_2)/2)$; all those citizens in N_2 for whom $\omega_\ell \in ((\beta_1 + \beta_2)/2, (3\beta_2 + \beta_1)/4)$ or $\omega_\ell \in ((3\beta_2 + \beta_3)/4, (\beta_2 + \beta_3)/2)$; and all those citizens in N_3 for whom $\omega_\ell \in ((\beta_2 + \beta_3)/2, \beta_3]$. It follows that these intervals cannot contain the ideal point of any citizen. Consequently, the interval $((3\beta_2 + \beta_1)/4, (3\beta_2 + \beta_3)/4) \subset N_2$ must contain the ideal points of exactly one-third of the citizens, while the interval $(\beta_1, (3\beta_2 + \beta_1)/4)$ contains the ideal points of none of the citizens. But this violates Assumption 1 because the latter interval is longer than the former. \square

Proof of Proposition 9. Proposition 8 tells us that there exist no three-candidate equilibria in which all candidates win. It remains to rule out the possibility of a three-candidate equilibrium in which one or two candidates are winning.

We begin with the one-candidate winning scenario. Let $\{s, \alpha(\cdot)\}$ be a pure strategy equilibrium such that $\#\mathcal{C}(s) = 3$, and suppose that $\#W(\mathcal{C}(s), \alpha(\mathcal{C}(s))) = 1$. Label the ideal points of the three candidates as $\{\beta_1, \beta_2, \beta_3\}$ and do so in such a way that $\beta_1 < \beta_2 < \beta_3$. Condition (ii) of Proposition 5 implies that candidate 2 must be the winning candidate. Thus, for candidate 1 to wish to remain in the race, equilibrium voting behavior must be such that $3 \in W(\{2, 3\}, \alpha(\{2, 3\}))$, while for candidate 3 to remain in the race, voting behavior must be such that $1 \in W(\{1, 2\}, \alpha(\{1, 2\}))$. Since citizens vote sincerely in two-candidate races, if $3 \in W(\{2, 3\}, \alpha(\{2, 3\}))$, then it must be the case that

$$(\beta_2 + \beta_3)/2 \leq m,$$

while if $1 \in W(\{1, 2\}, \alpha(\{1, 2\}))$, it must be the case that

$$(\beta_1 + \beta_2)/2 \geq m.$$

But the former inequality implies that $\beta_2 < m$, and the latter inequality implies that $\beta_2 > m$ —a contradiction.

We now turn to the scenario in which two candidates are winning. Again, let $\{s, \alpha(\cdot)\}$ be a pure strategy equilibrium such that $\#\mathcal{C}(s) = 3$, and suppose that $\#W(\mathcal{C}(s), \alpha(\mathcal{C}(s))) = 2$. Label the ideal points of the three candidates as $\{\beta_1, \beta_2, \beta_3\}$, and do so in such a way that $\beta_1 < \beta_2 < \beta_3$.

We show first $W(\mathcal{C}(s), \alpha(\mathcal{C}(s))) \neq \{1, 2\}$. Suppose, to the contrary, that the winning set did consist of candidates 1 and 2. Then Proposition 4 implies that there must exist a sincere partition (N_1, N_2, N_0) such that $\#N_1 = \#N_2$. This, in turn, implies that $(\beta_1 + \beta_2)/2 = m$. It follows that all those citizens with ideal points smaller than m will be voting for candidate 1, while all those with ideal points larger than m will be voting for candidate 2. But since $m < \beta_2 < \beta_3$, the citizen with the median ideal point prefers candidates 1 and 2 to candidate 3. Weak dominance therefore implies that the median citizen will vote for either candidate 1 or candidate 2. It follows that candidates 1 and 2 cannot have the same number of votes—a contradiction.

In a similar manner, it can be shown that $W(\mathcal{C}(s), \alpha(\mathcal{C}(s))) \neq \{2, 3\}$. The remaining possibility is that $W(\mathcal{C}(s), \alpha(\mathcal{C}(s))) = \{1, 3\}$. In this case, Proposition 4 implies that $(\beta_1 + \beta_3)/2 = m$, which means that all those citizens with ideal points smaller than m will be voting for candidate 1 and all those with ideal points larger than m will be voting for candidate 3. Since β_2 is closer to m than β_1 or β_3 , the citizen with the median ideal point prefers candidate 2 to candidates 1 and 3. Weak dominance therefore implies that the median citizen will vote for candidate 2. Candidate 2 thus receives one vote, and the remaining voters are divided equally between candidates 1 and 3. Now suppose that candidate 2 were to drop out of the race. Voters vote sincerely in two-candidate races, so that those citizens supporting candidates 1 and 3 would continue to do so. Since voting behavior satisfies AIV, the median citizen will abstain. Thus, $W(\mathcal{C}(s)/\{2\}, \alpha(\mathcal{C}(s)/\{2\})) = \{1, 3\}$ which violates condition (i) of Proposition 5.

We have now ruled out the possibility of a three-candidate equilibrium in which two candidates win. \square

Proof of Proposition 11. Let i be the citizen who is running (i.e., $s^i = 1$). Then the selection generated by $\{s, \alpha(\cdot)\}$ is (x_i^*, i) . If (x_i^*, i) were inefficient, there would exist an alternative selection (x, j) such that $V^h(x, j) > v_{hi}$ for all $h \in \mathcal{N}$. By Assumption 2, there-

fore, there would exist some citizen k such that $v_{hk} > v_{hi}$ for all $h \in \mathcal{N}$. It follows that $\#N_k > \#N_i$ for all sincere partitions (N_k, N_i, N_0) . For sufficiently small δ , therefore, condition (ii) of Proposition 2 would be violated. Thus, for sufficiently small δ , (x_i^*, i) must be efficient. \square

Proof of Proposition 12. Let i and j be the citizens who are running (i.e., $s^i = s^j = 1$). Then the selections generated by $\{s, \alpha(\cdot)\}$ are (x_i^*, i) and (x_j^*, j) . Suppose that, say, (x_i^*, i) were inefficient. Then there would exist an alternative selection (x, j) such that $V^h(x, j) > v_{hi}$ for all $h \in \mathcal{N}$. By Assumption 2, therefore, there would exist some citizen k such that $v_{hk} > v_{hi}$ for all $h \in \mathcal{N}$. Suppose that citizen k were to enter the race. Proposition 3 implies the existence of a sincere partition (N_i, N_j, N_0) such that $\#N_i = \#N_j$. We know that if $h \in N_i \cup N_0$ it must be the case that $v_{hk} > v_{hi} \geq v_{hj}$, which implies that voting for j and abstaining are weakly dominated voting strategies. Moreover, since voting behavior satisfies IIC, no citizen in $N_i \cup N_0$ would vote for candidate i . It follows that all citizens in $N_i \cup N_0$ would vote for citizen k . Since this group constitutes at least half the population, citizen k must win with a probability of at least one-half. For sufficiently small δ , therefore, citizen k would prefer to enter the race, contradicting the fact that i and j running against each other is an equilibrium. \square

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