Last Step to the Throne: The Conflict between Rulers and Their Successors

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Abstract

I model the dynamic between ruler and successor. The ruler wants to cultivate a successor for a smooth power transition but fears being ousted by him, while the successor fears being removed by the ruler; these mutual fears may induce ruler–successor conflict. Each party follows a non-monotonic equilibrium strategy. The successor accumulates power while not threatening the ruler, and he prolongs their relationship by maintaining a low profile. The ruler gradually becomes more intolerant of the successor's growing power but, as his life nears its end, has less incentive to replace the successor. Thus conflict is most probable in the middle of their relationship. Although an institutionalized procedure may render conflict less likely, a predetermined succession order could increase its likelihood by restricting the ruler's optimal time to select a successor. If there are two candidates then a ruler prefers the weaker one only if their capabilities are similar.

Key words: Conflict, Succession, Authoritarian Regimes, Dynamic Game

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1 Introduction

One of the most important issues in an autocracy is how to arrange for the succession of leadership. Svolik (2012) reports that, following World War II, most leadership changes in authoritarian regimes were non-constitutional. Hence selecting a successor and arranging a power transition, if properly handled, enable the ruler to maintain a particular regime.

Yet rulers face a trade-off in the succession procedure. On the one hand, allowing the heir apparent to accumulate power facilitates the transition when a ruler dies or steps down. On the other hand, a successor who is delegated too much power may seek to claim the throne earlier than the ruler desires. Herz (1952) refers to this phenomenon as the "crown prince problem".[1]

Although this problem has been recognized in Tullock (1987), several questions remain unanswered. Selecting and cultivating successors may take years. When should a ruler choose the successor? How can the "crown prince problem" affect the long-term relationship between rulers and their successors? This study attempts to fill these gaps by presenting a dynamic model for analyzing the long-term relationship between a ruler and a successor.

The crown prince problem underscores the successor's incentive to maximize his power; however, there is still another source of conflict in the succession procedure. In particular, a dilemma for both the ruler and the successor is that the fear of being prematurely replaced motivates the ruler to monitor the successor's behavior and, perhaps, to remove him from that position; hence the successor fears being stripped of his title. It follows that, in assuming his position is not secure, the successor may challenge the ruler—that is, even before accruing enough power to do so with any reasonable expectation of success.

This paper argues that the conflict between ruler and successor may be caused by mutual fear and not by the successor's greed. This fear shapes the two parties' interaction, leading each to adjust the strategic behaviors over time toward the end of moderating the successor's

[1]When King Saud took the throne in Saudi Arabia in 1952, he appointed his brother Faisal as the crown prince and also as prime minister. After serving as crown prince for 11 years, Faisal overthrew Saud and became the new king in 1963.

power accumulation and thereby maintaining mutual trust.

In the model, a successor can choose different levels of effort to increase his power in each period—during which he can also decide whether to challenge the ruler. In turn, a ruler who is not challenged decides—depending on how much power the successor has already accrued—whether to strip the successor of that position. The ruler's health is expected to worsen with time, which results in non-monotonic equilibrium strategies being followed by both parties. The ruler monitors the successor's behavior during each period. In equilibrium, this monitoring becomes stricter over time and, if the successor's power exceeds a certain threshold, the ruler will revoke the successor's title. Yet a ruler who is near the end of his life has no incentive to replace the successor. Thus the successor's equilibrium strategy in each period is based on two thresholds in his level of power that trigger different responses from the ruler. When the successor's power is low, the ruler's monitoring is also low and so the successor exerts considerable effort to obtain more power. When the successor's power is greater but not yet sufficient to challenge the ruler, keeping a low profile is his optimal choice. Once the successor's power is great enough that he constitutes a potential threat to the ruler, the successor should focus on increasing his power—that is, in preparation for the conflict to arise from the ruler's anticipated attempt to replace him.

According to the baseline model, if the ruler can head the regime for a long period then the ruler–successor relationship comprises three phases. During the *honeymoon* phase, at the beginning of the successor's tenure, the probability of conflict is zero owing to the low threat he poses. As the ruler's life nears its end, there is a *power transition* phase in which the probability of conflict is low simply because the ruler is dying and therefore has no incentive to replace the successor. Between these phases is a middle, *mutual suspicion* phase in which the likelihood of conflict cannot be ignored—especially if the successor has accumulated considerable power and the ruler remains healthy.

Allowing the successor to accrue power is a power-sharing procedure (Francois et al. 2015). One prominent emphasis in the study of rulerships is that rulers face the threat of betrayal by high-level officials (Tullock 1987; Wintrobe 2000; Gregory 2009; Svolik 2012; McMahon and Slantchev 2015; Debs 2016; Bueno de Mesquita and Smith 2017). Even so, a ruler needs the support of skilled lieutenants to defend9– the regime from outside threats (Myerson 2008; Svolik 2009; Egorov and Sonin 2011; McMahon and Slantchev 2015)—support that requires credible institutions to guarantee payoffs for political stability (Gandhi and Przeworski 2007; Myerson 2008; Gandhi 2013). Egorov and Sonin (2011) and Zakharov (2016) focus on this loyalty–competence trade-off involving the ruler's chosen agents. Konrad and Mui (2017) explicitly question whether a power-sharing model implies that rulers truly need successors.

This paper differs from the previous research on power-sharing models in three respects. First, its assumption that the ruler's health worsens with time means that the two parties' interests are initially unaligned but eventually converge. In this setup, the changing equilibrium strategies result from changes in the two parties' respective preferences. In the game's early stage, the ruler is more concerned about maintaining his reign than about ensuring a smooth power transition; however, he tends to care more about that transition when his health worsens. This change in preference plays a key role in the parties' middle-stage relationship difficulties.

Second, the equilibrium results presented here indicate that conflict between these two parties may arise from a mutual fear of being ousted by the other. The model also explicitly shows that the successor (i) may maintain the ruler's trust by adopting a non-monotonic strategy and (ii) may—to protect his own position—end up challenging the ruler when he hasn't accumulate enough power.

Third, the model focuses on incentives created by the need to guarantee succession and therefore the model setup abstracts from the possibility of a ruler needing support from the successor to maintain the regime before he dies. This setup reflects not only the value of designating successors to help guarantee a smooth power transition but also the observation that rulers need time to cultivate their successors. In Syria, for example, Hafez al-Assad spent six years building popular support for Bashar al-Assad. Both Kim Il-sung (in North Korea) and Chiang Kai-shek (in Taiwan) made a series of power arrangements to ensure that their sons inherited power.

The baseline model bears several implications, especially for institutional design. On the one hand, if some form of institutional protection is in place for the succession process—for example, a tradition of primogeniture—then conflict is less likely. When institutions serve to protect succession, the successor is less motivated to challenge the ruler and so the ruler's fear of being ousted declines; thus the probability of conflict is reduced.

On the other hand, a predetermined order of succession may actually increase the probability of conflict. The ruler's optimal time to designate a successor is when the successor poses no threat to the throne. Yet the ruler lacks this timing flexibility when the succession order is predetermined, which means that conflict may occur immediately if the successor has accrued enough power. This result extends studies that address the institutionalization of authoritarian regimes (Geddes 2003; Malesky and Schuler 2010; Boix and Svolik 2013; Pepinsky 2014). In an empirical paper, Frantz and Stein (2017) find that institutionalized rules of succession can discourage coups against rulers. However, the results derived here suggest that such institutionalization is a double-edged sword—as shown (in Section 2.4) by modern examples reflecting Saudi Arabia's succession rules.

Finally, the baseline model can be extended to the case of multiple potential successors. Apart from the ruler's motivation to protect his power, he may replace a successor deemed insufficiently competent to assume the throne. This dynamic renders the weaker candidate's position precarious because he will not be chosen unless his competence approaches that of the stronger candidate. Yet even though a strong candidate is thus advantaged in the succession game, a ruler who has "backups" for the current successor will be less tolerant of that heir apparent accruing power—which in turn will reduce interparty trust and so may well lead to conflict.

This paper's model is also related to succession's *coordination problem*, which underlies the ruler's need to appoint a successor (Bueno de Mesquita et al. 2003; Gandhi and Przeworski 2007; Magaloni 2008; Svolik 2012). By appointing a clear successor, the ruler gives the elite supporters a long-term incentive to remain loyal; in other words, that loyalty is rewarded even after the ruler dies (Kurrild-Klitgaard 2000; Brownlee 2007). Kokkonen and

Sundell (2014) use historical data from ancient Europe to undertake an empirical test of this coordination problem. Eisner (2011) finds that autocrats have been murdered less frequently over time; in the case of Europe, Blaydes and Chaney (2013) report that the length of rulers' reigns increased with the spread of feudalism. Kurrild-Klitgaard (2000, 2004) analyze the effect—on rulers being deposed—of changed succession laws and practices in Denmark and Sweden. Iqbal and Zorn (2008) examine the consequences of political assassinations.

The paper proceeds as follows. Section 2.1 explains the model's setup. The equilibrium strategy is solved in Section 2.2, after which Section 2.3 presents the strategic implications and comparative statics. I analyze conflict under a predetermined order of succession in Section 2.4 and discuss the multi-candidate case in Section 3. Section 4 concludes with a summary of findings.

2 Baseline Model

2.1 Model Setup

I consider an infinite-horizon environment with complete information and two strategic players: a ruler and a candidate for the successor position. The ruler takes the throne at the beginning of period 1; the candidate is endowed with initial power $\widetilde{S} \in [0, 1)$, which represents his power relative to that of the ruler.[2]

Each period t consists of four stages. At Stage 1 of any period t, if the successor position is vacant then the ruler must decide whether (or not) to formally designate the candidate as the successor. The candidate can begin to accrue power beyond $\widetilde{S}$ only if he is designated as the successor. The details will be explained when the game reaches the second stage. If the successor position is not vacant at stage 1, then the ruler needs make no decision. Thereafter, nature will randomly determine whether the ruler dies in period t with probability p_t, which is interpreted as the ruler's health condition. I assume that $p_{t-1} < p_t$, which captures the deterioration of a ruler's health with age, and also that $\lim_{t\to\infty} p_t = 1$, which means death

[2]If $\widetilde{S} = 1$, then the game is trivial, I briefly discuss this case in Section 2.2.

is unavoidable if the game lasts a long enough time. $\{p_t\}$ is known to both players at the beginning of the game. This assumption indicates that the ruler's life expectancy is common knowledge among the players. I do not consider the successor's health condition explicitly.[3] It is because the changes of p_t can also represent the deteriorating rate of the ruler's health relative to that of the successor. That is, a large $p_t - p_{t-1}$ implies that the ruler's health deteriorates much more quickly than that of the successor.

If a ruler dies naturally after stage 1 yet the successor position is vacant, then the game ends with the regime's collapse.[4] However, if the successor position is *not* vacant and the ruler dies, then the successor assumes the throne with probability $\min\{S_{t-1} + w, 1\}$. Here S_{t-1} represents the successor's power inherited from the preceding period and w $(0 < w < 1)$ is a lump-sum power increase, interpreted as the strength of institutional protection or of political will. If the ruler is still alive after stage 1 but has not designated a successor, then no strategic move (by either party) will be initiated until the next period $(t + 1)$. If the end of stage 1 finds that the ruler is still alive and a successor has been designated, then the game proceeds to the next stage.

At stage 2, the designated successor chooses either high (h) or low (l) effort to increase the power. A successor who adopts the high effort strategy h increases his power by H unit with probability $p_h \in (0, 1)$ but increases it by only L unit with probability $1 - p_h$, where $0 < L < H < 1$. If the successor's strategic choice is l, then his power can increase only by L.[5] Thus the successor's total power in period t is realized as $S_t = \min\{S_{t-1} + Outcome_t, 1\}$. That is: the successor's power is now equal to that inherited from the preceding period (S_{t-1})

[3]In general, the chosen successor, such as the son or younger brother of the incumbent ruler, is younger and healthier than the ruler.

[4]This assumption is a simple description of the chaos that ensues when the ruler's death leads to a power vacuum. Although the regime may not actually collapse, a power struggle and/or general chaos is more likely to occur absent a smooth succession to the highest power.

[5]Adding uncertainty to the outcome when the successor's effort is l would not change the results—provided the likelihood of realizing the high outcome after exerting effort l is *less* than that likelihood after choosing h.

plus the outcome, H or L, of his effort to accrue power in this period; and his relative power to the ruler is restricted less than or equal to 1.

At stage 3, both players observe S_t and the successor decides whether to challenge the ruler—for example, via a palace coup or an open rebellion.[6] If the successor does mount a challenge, then the game ends with him either seizing the throne with probability S_t or losing his position with probability $1 - S_t$. If the successor chooses to remain loyal, then the game proceeds to stage 4.

At stage 4, the ruler must decide whether or not to strip the successor of his title. If the ruler does, then the game ends and the successor loses that position. If instead the ruler lets him remain as successor, then the game continues to the next period.[7]

The timing of events within period t can be summarized as follows:

Stage 1 (Designation Stage). If the position of successor is vacant, then the ruler decides whether to designate one. Nature determines whether the ruler remains alive throughout this period. If the ruler is still alive and a successor has been designated, then . . .

Stage 2 (Power Acquisition Stage). The successor may choose to increase his power.

Stage 3 (Challenge Stage). Both players observe the successor's current power (S_t), after which the successor decides whether to challenge the ruler. If the successor remains loyal, then . . .

Stage 4 (Ruler's Decision Stage). The ruler decides whether to strip the successor of his title.

[6]An example of a palace coup is the one orchestrated by Qatar's Sheikh Hamad bin Khalifa Al Thani: this heir apparent deposed his father, Emir Khalifa, in a bloodless coup in 1995. An example of an open rebellion is the Holy Roman Emperor Henry IV, who was challenged by his two successors—Conrad II and Henry V—in turn (Robinson 2003).

[7]This sequential-move setup reflects that the successor, when opposing the rule, adopts an "underdog" position and so will look to seize the first-mover advantage. The results are qualitatively unaffected, however, if the setup is changed to a simultaneous-move game.

Table 1: Payoffs

	Cash Flow in each period	Lump-sum payment when the game ends			
Successor	0	Take the throne		Fail to take the throne	
		R		$-b$	
Ruler	r	Die naturally	Be overthrown	Die naturally	No successor
		ηR	$-b$	0	0

Payoffs: The game's payoff structure is summarized in Table 1. For any candidate who is never designated as a successor, the payoff is normalized to zero. The payoff to a candidate who is appointed successor depends on whether he assumes the throne. This successor receives a lump-sum payment R, the expected value of the regime, if he becomes the new ruler; but if he loses the title of successor or fails to seize power after the ruler's death, he receives $-b$ (where $b > 0$). Given that the designation of a successor serves to prepare for the regime's future, becoming the new ruler is presumably the successor's ultimate goal. For this reason, the game abstracts from any cash flows or other wealth that the successor may gain upon assuming the crown.

The ruler in power receives cash flow r in each period. If there is no conflict between the two parties and if the successor assumes the throne upon the ruler's death—but under no other circumstances—then the ruler receives an additional lump-sum payment of ηR ($0 < \eta < 1$); this payment is a measure of how much the ruler cares about the regime's future or his legacy. In any period, a ruler who is overthrown by the successor incurs a lump-sum loss equal to $-b$. If no successor is designated or if the successor loses his title in any period t, then the ruler continues to receive cash flow r (until he dies) but will not receive additional payments because he has no successor. All payments are discounted by the factor $\delta \in (0, 1)$.

Equilibrium Concept: The ruler and successor take action sequentially in each period, so the strategies and states can be considered separately in each stage. The state variables are the crown prince's power at the beginning of each stage (S) and the monarch's health

(p_t).[8] Hence, the set of state variables is $\mathbb{S} = \mathcal{S} \times \mathcal{P}$. The monarch's set of actions is {designate, not designate} at stage 1 and {strip, not strip} at stage 4. The crown prince's set of actions at stage 2 is $\{h, l\}$ and at stage 3 is {challenge, not challenge}.

The model's equilibrium concept follows the Markov perfect equilibrium (MPE) (cf. Maskin and Tirole 2001) *except* that the payoffs are not discounted at the various stages within each period (A more precise statement of this MPE is defined formally in the Online Appendix).[9] From this point forward, the focus will be on the players' Markov strategies and on the entire game's Markov perfect equilibrium, in which Markov strategies ignore all historical details except for the history's length and the current state (S, p_t).[10]

Model Interpretation and Discussion: Before solving the equilibrium strategies for the players, it is necessary to further interpret the ruler-successor relationship and the model setup.

In our baseline model, I assume that the ruler cares about whether the successor can seize power after he dies. This assumption reflects the ruler's concern about the regime's future. First, when rulers and successors are related by blood, securing a bright future for one's descendants is part of human nature. For instance, Kim Il-sung in North Korea and Chiang Kai-shek in Taiwan purged powerful subordinates to guarantee that power can be transferred to their sons. Second, rulers also care about whether their legacies can be continued by their successors. In 1974, Mao Zedong was persuaded to reinstate Deng Xiaoping to a leadership position, and Deng was treated as Mao' successor. However, when Mao suspected that Deng would destroy the positive reputation of the Cultural Revolution (which Mao considered one

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[8]A more precise state variable would be the monarch's condition in current *and* future periods—that is, $\hat{p}_t = \{p_t, p_{t+1}, \ldots\}$. But since the monarch's health is a fixed variable in each period, it follows that knowing the current time t or p_t is enough to know the rest.

[9]The definition of equilibrium employed here is of the form described by Mailath and Samuelson (2006, p. 191).

[10]Since the state variables include the monarch's health condition, which is related to time, it follows that the equilibrium describe in this study is *not* a stationary Markov equilibrium (in which the length of time can safely be ignored).

of his greatest policy initiatives), Mao dismissed Deng from his position in 1976. Third, when a ruler steps down voluntarily, he requires a powerful successor to provide protection. For instance, on his first day in office, acting Russian President Putin pardoned his predecessor Yeltsin for any possible prosecutions.

As the successor position is prepared for the future of the regime after the incumbent's reign, a key difference exits between the ruler-successor relationship and the general principal-agent counterpart. The ruler does not directly claim any residuals from the successor's effort of accruing power, but the designated successor is a "legalized" backup of the ruler. When Faisal overthrew Saud and became the new king of Saudi Arabia in 1963, the legitimacy of Faisal's reign was not challenged by others. In Qatar, Sheikh Hamad deposed his father Emir Khalifa in a palace coup in 1995, and he reigned the country until he died. These examples suggest that the designated successor could be a more dangerous threat to the throne than other subordinates. Such observation arises because the successor may accrue considerable power, and his position as successor provides legitimacy to assume the throne even through non-peaceful means. Given that the successor has the incentive to take the throne earlier than planned, a commitment problem may arise if an agency model setup was adopted. Hence, a general dynamic setup without commitment is used in our baseline setup.

The baseline model ignores the selection problem in the succession procedure. Such circumstance focuses on the long-term relationship between a ruler and a chosen successor. This setup can aptly explain the succession problem under an authoritarian regime in which the selection issue has been partly solved by a predetermined arrangement. For instance, acts of settlement in several European countries have already chosen successors for rulers. Similarly, the primogeniture tradition has selected the ruler's oldest son as the successor directly. In modern society, Kim Il-sung chose his oldest son Kim Jong-il, Chiang Kai-shek trained his only son Chiang Ching-kuo.[11] The stability of different selection rules has been discussed by Tullock (1987), Gandhi and Przeworski (2007) and Svolik (2012). I will further discuss the selection problem under the current model setup in Section 3.

[11]Chiang Kai-shek has another adopted son Chiang Wei-kuo

For simplicity, I assume that the potential successor cannot accumulate any power except the initial endowment $\widetilde{S}$ before he becomes the successor. Real-world rulers are always alert to increases in the power of their subordinates, so only the designated successor has the privilege of increasing his power with the ruler's permission. This procedure represents the ruler's "cultivation" of a successor.[12]

I also assume whether the successor takes the throne after the ruler's death depends on his accumulated power plus the institutional protection $(S + w)$. This setup captures the difficulty of arranging a smooth power transition in autocracies. When a ruler dies, his subordinates do not always transfer their loyalty to the new autocrat. Especially when the designated successor exhibits weakness in assuming power, the new crown may be challenged immediately by forces within and outside the regime. Thus only a powerful successor can firmly grasp the throne. Institutional protection (w) reflects the efforts of autocracies to protect power transitions by establishing a constitutional procedure, such as by adopting primogeniture or passing an act of settlement.[13]

When the successor seeks a high level of increased power, the potential high outcome H is the upper bound of that power increase; it occurs when the successor takes full advantage of his position to build his coalition and to raise his reputation among citizens and the military. The term H also represents the possibility that the successor's ability will affect the game. For example, a large H implies that he is capable of accruing more power in each period. The low outcome L is nonzero, which captures the idea that elites—in light of the succession

[12] I ignore the situation that the powerful potential candidate challenges the ruler before he is chosen as the successor, because this scenario has been studied in the existing literature, such as Myerson (2008), McMahon and Slantchev (2015), and Debs (2016).

[13] In East Asia, primogeniture is a form of institutional protection. Although it is more tradition than law, respect for this tradition continues to figure prominently in the process of choosing successors. Huang (1982) cites the example of an entire civil official group vehemently opposing the Wanli Emperor's attempt to choose his favorite younger son as the successor rather than Zhu Changluo, his eldest son.

order—will tend to gather around the designated successor even if he does not actively build such support. That process implies a nontrivial increase in the successor's power over time.

To simplify the model setup, no additional restrictions were placed on the scale of certain parameters, such as the levels of power increase H and L, and the probability of high outcome p_h. The technical assumptions needed to avoid the trivial solutions will be explicitly discussed in the next section.

2.2 Equilibrium Strategies

This section characterizes the equilibrium strategies for each player. Before calculating those strategies, I assume that $p_h(H-L)+L<w$. Under this assumption, the successor's power increase that is due to institutional protection w is *greater* than the highest expected power increase from a single period $p_h(H-L)+L$. The implication is that a nontrivial effort is required for autocrats to institutionalize the transition of power. We shall also need a second assumption: during any period t, the total cash flow that a ruler can collect is *less* than his expected utility when a successor who is initially powerless (i.e., for whom $S_{t-1}=0$) achieves a large increase in power and then challenges the ruler. More precisely, in any period t, we assume $(1-p_t)k_t < p_t w\eta R + (1-p_t)(-(b+k_t)H+k_t)$, where $k_t \equiv r + r\sum_{i=t+1}^{\infty}\delta^{i-t}\prod_{j=t+1}^{i}(1-p_j)$. This assumption implies that, if a successor has no initial power, then his maximum single-period increase in power is not sufficient to threaten the ruler. The motive for assuming this is the need to avoid a situation in which a low-power successor quickly becomes powerful enough to challenge the ruler and thus leaves no time for the ruler to react.

To begin the analysis, we will first calculate the equilibrium strategies when the candidate has been appointed as successor. This calculation is then followed by a discussion of the optimal time for the ruler to designate a successor.

At the ruler's decision stage (stage 4) of any period t, the *expected utility* of a ruler who strips his successor's title can be written as

$$\underbrace{r}_{\text{payoff in } t} + \underbrace{r \sum_{i=t+1}^{\infty} \delta^{i-t} \prod_{j=t+1}^{i} (1 - p_j)}_{\text{expected payoffs since } t+1}, \tag{1}$$

which also represents the total cash that a ruler can collect from period t until his death. Yet if the successor is not removed from that position, then the ruler's expected payoff is

$$\underbrace{r}_{\text{payoff in } t} + \delta \underbrace{(p_t \overbrace{\min(S_t + w, 1)\eta R}^{\text{ruler dies}} + (1 - p_t) \overbrace{V^m_{t+1,2}(S_t)}^{\text{ruler lives}})}_{\text{expected payoffs since } t+1}; \tag{2}$$

here $V^m_{t+1,2}(S_t)$ is the ruler's value function at stage 2 of period $t + 1$.[14] As the ruler's health worsens over time (i.e., as p_t to 1), his payoff from stripping the successor converges to r while the payoff from retaining the successor converges to $r+\delta \min(S_t+w, 1)\eta R$; note that this sum is strictly greater than r. In other words, if no successor is available then it becomes more likely that the regime will collapse after the ruler's death. So as the life approaches its end, a ruler should be more concerned about maintaining the regime than about betrayal by the successor—from which it follows that the former will not strip the latter of his designation. This notion is expressed formally as follows.

Lemma 1. *There exists a period $\bar{t}^m$ after which the ruler will not strip the successor of that title.*

Lemma 1 stipulates the existence of a period $\bar{t}^m$ that separates the game into two parts. Let us first consider the subgame after period $\bar{t}^m$; for this purpose, we need only consider the successor's strategy. After that, we can address—within a finite-horizon environment—each party's equilibrium strategies in the periods *preceding* $\bar{t}^m$.

Given that being stripped of his position no longer concerns the successor after period $\bar{t}^m$, he will choose an effort level h to increase his power at the power acquisition stage (stage 2) of any such period. Therefore, we need only consider the conditions under which a successor

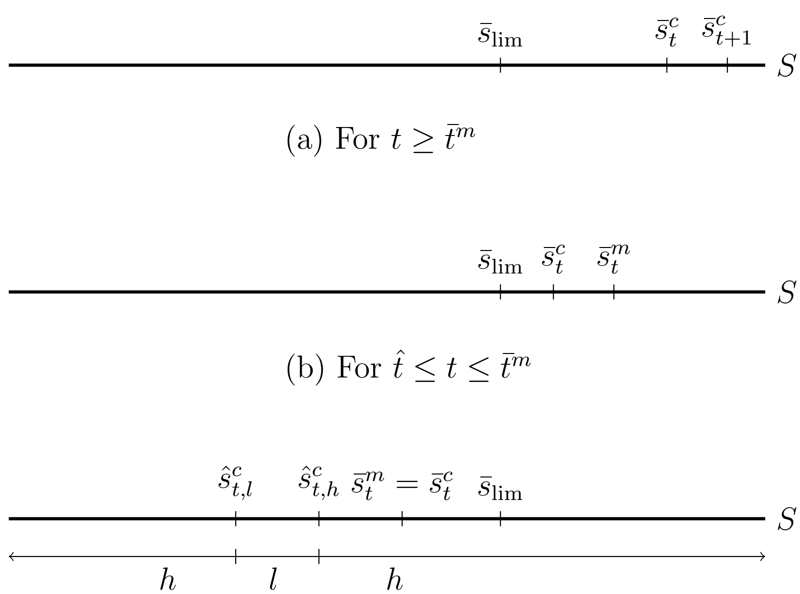
[14]Another state variable p_t is dropped from function V without confusion, because p_t is time related and the subscript t is sufficient to indicate the ruler's health condition.

will challenge the ruler at the challenge stage (stage 3). A successor who challenges the leader receives a payoff of $\min(S_t, 1)(R + b) - b$; for a loyal successor, the payoff is

$$\delta \underbrace{\left(p_t \overbrace{(\min(S_t + w, 1)(R + b) - b)}^{\text{ruler dies}} + (1 - p_t) \overbrace{V^c_{t,2}(S_t)}^{\text{ruler lives}}\right)}_{\text{expected payoffs since t+1}}, \tag{3}$$

The successor compares these two payoffs and thereby follows a cut-off rule: when he is powerful enough to challenge the ruler, the successor prefers to seize power as soon as possible. However, if he has not accrued sufficient power then the successor's optimal decision is to remain loyal. This result is summarized in the following proposition.

Figure 1: Successor's Equilibrium Cut-off Strategies for Challenging the Ruler



(a) For $t \geq \bar{t}^m$

(b) For $\hat{t} \leq t \leq \bar{t}^m$

(c) For $t < \hat{t}$

Notes: The terms $\bar{s}^c_t$ and $\bar{s}^m_t$ represent (respectively) the successor's challenge threshold and the ruler's tolerance threshold; $\bar{s}_{\text{lim}}$ is the lower bound of each challenge threshold when $t \geq \bar{t}^m$. In parts (a) and (b) of this figure, the successor always chooses a high effort level (h) at the power acquisition stage. In part (c), $\hat{s}^c_{t,h}$ and $\hat{s}^c_{t,l}$ are the successor's thresholds that determine his choice between h and l at the power acquisition stage.

Proposition 1. *A unique MPE exists for any subgame starting in period $\bar{t}^m$. In any period $t \geq \bar{t}^m$, if there is a designated successor then the following statements hold.*

1. *The successor chooses the high effort level (h) at the power acquisition stage.*

2. *At the challenge stage, a threshold $\bar{s}_t^c$ exists such that the successor will challenge the ruler if $S_t > \bar{s}_t^c$ but will otherwise remain loyal.*

3. *The "challenge threshold" $\bar{s}_t$ adopted by the successor increases with time, and it is bounded from below by $\bar{s}_{\text{lim}}$. That is $\bar{s}_{\text{lim}} \leq \bar{s}_t^c \leq \bar{s}_{t+1}^c$, where $\bar{s}_{\text{lim}} = b/(R+b) + \delta(p_h(H-L)+L+w)/(1-\delta)$.*

Part 3 of this proposition indicates that the successor has incentive to seize the throne early because at that point, when he has lower power, the reward is substantially larger. Yet remaining loyal is a better and safer option, regardless of the timing, if the successor's power is weaker than a specific threshold, $\bar{s}_{\text{lim}}$. The relationship of these bounded thresholds described in the third part of this proposition is shown in Figure 1 (a).

We can now discuss the equilibrium strategies adopted *before* period $\bar{t}^m$. During these periods, the ruler cares more about maintaining his own reign than about enabling a s-mooth future power transition. So if the successor becomes strong enough to threaten the throne, the ruler will no longer tolerate the successor accruing additional power. To stave off such a threat, the ruler must set a "tolerance threshold" $\bar{s}_t^m$ in each period and then, if that threshold is exceeded, strip the successor of his position. The ruler may set difference tolerance thresholds in different periods because the goal is simply to ensure that, in each period, the successor's increase in power remains within an acceptable range—namely, such that $\bar{s}_{t-1}^m \leq \bar{s}_t^m$. Also, the more powerful is the successor, the less *additional* power the ruler wants the successor to accrue. Hence the ruler's control tightens as the successor's power increases over time: $\bar{s}_t^m - \bar{s}_{t-1}^m \geq \bar{s}_{t+1}^m - \bar{s}_t^m$.

These circumstances differ—except for the successor's continued incentive to seize the throne earlier than expected by the ruler—than those prevailing in periods that follow $\bar{t}^m$ a successor who risks being stripped of his title may challenge the ruler to protect that position. It follows that the threshold at which the successor challenges the ruler cannot exceed the latter's tolerance threshold; that is, $\bar{s}_t^c \leq \bar{s}_t^m$.

In the relatively late periods, a successor may be able to accumulate sufficient power that his challenge thresholds become strictly lower than the ruler's tolerance thresholds ($\bar{s}_t^c < \bar{s}_t^m$); see Figure 1(b). The reason is that then the successor's incentive to take the throne early dominates the incentive to protect his position. In this scenario, the successor will definitely choose h, and not l to increase his power at the power acquisition stage.

However, the successor's "greed" incentive is bounded from below by $\bar{s}_{\text{lim}}$, which implies that he will not challenge the ruler unless fully prepared to do so. This situation is most likely to occur during the early periods of a ruler–successor relation. As a consequence, the successor will establish a challenge threshold that is *equal* to the tolerance threshold: $\bar{s}_t^c = \bar{s}_t^m$. In this situation, A successor's challenging the ruler can be viewed as a proactive strategy adopted to protect the former's position. Thus the successor's strategy at the power acquisition stage exhibits the non-monotonically increasing property illustrated in Figure 1(c). More precisely: the successor's decision to increase power depends on the difference between his power in the previous period, S_{t-1}, and his challenge threshold $\bar{s}_t^c$. If the gap is large (i.e., if S_{t-1} is much less than the threshold $\hat{s}_{t,h}^c$), then a weak successor poses no threat to the ruler; it is therefore unlikely that the ruler will strip such a successor of his position. In that case, the successor can take advantage of the opportunity to increase the power as much as possible (and so chooses h). Yet if a successor becomes powerful enough to warrant increased attention from the ruler—that is, if $\hat{s}_{t,h}^c < S_{t-1} \leq \hat{s}_{t,l}^c$)—then the former must maintain a low profile (and so chooses l) to avoid conflict with the latter because the successor is not yet strong enough to win that battle. Finally, if the gap is small ($S_{t-1} > \hat{s}_{t,l}^c$) then the successor will definitely be incentivized to initiate conflict at the challenge stage. Hence the successor must increase his power as much as possible (and so chooses h) to prepare for conflict with the ruler. Proposition 2 stipulates each party's equilibrium strategies.

Proposition 2. *In any period $t < \bar{t}^m$, if there is a designated successor after stage 1 then the following statements hold.*

1. *The ruler's strategy has a unique threshold* $\bar{s}_t^m$ *such that the ruler strips the successor of his title if* $S_t > \bar{s}_t^m$*; and keep the successor's position otherwise. Furthermore,* $\bar{s}_t^m$ *weakly increases and becomes stricter over time i.e.* $\bar{s}_{t-1}^m \leq \bar{s}_t^m$ *and* $\bar{s}_t^m - \bar{s}_{t-1}^m \geq \bar{s}_{t+1}^m - \bar{s}_t^m$*.*

2. *For the successor, there exists a time* $\hat{t} \leq \bar{t}^m$ *such that in a relative late period* $\hat{t} \leq t \leq \bar{t}^m$*, the successor always chooses high effort* (h) *and sets his challenge threshold equal to the ruler's tolerance threshold; that is, he sets* $\bar{s}_t^c = \bar{s}_t^m$*.*

3. *In a relatively early period* $(t < \hat{t})$*, the successor's strategy depends on two thresholds:* $\hat{s}_{t,h}^c \leq \hat{s}_{t,l}^c$*. He chooses a high effort level* (h) *if* $S_{t-1} < \hat{s}_{t,h}^c$ *but only a lower effort level* (l) *if* $\hat{s}_{t,h}^c \leq S_{t-1} < \hat{s}_{t,l}^c$*. In the latter case, however, he will switch back to exerting effort* h *if* $\hat{s}_{t,l}^c \leq S_{t-1}$*. The successor's challenge threshold is strictly lower than the ruler's tolerance threshold—that is,* $\bar{s}_t^c < \bar{s}_t^m$*.*

It is worthy to note that Proposition 2 includes some degenerated situations. For example, if $\tilde{S}$ is large, then $\hat{t}$ may be equal to 1, which implies that if a powerful successor is designated as the successor at an early period of the game, he will always set the challenge threshold equal to the ruler's tolerance threshold. Also, if the ruler's initial health is bad (large p_0), then $\bar{t}^m$ could be 1, which implies the ruler will not strip any successor since the first period. I will further discuss the possible equilibrium outcomes from the macroscopic perspective in the next section.

Having characterized the parties' equilibrium behaviors following designation of a successor, we can now address the question of *when* the ruler should appoint a successor. The ruler faces a trade-off: a successor who is designated too early will be better able to accumulate sufficient power to mount a challenge, but choosing a successor too late may lead to chaos when the ruler dies.

According to Propositions 1 and 2, the threat from a successor depends on his accumulation of power. Nonetheless, it is unwise for the ruler to leave the position of successor unfilled. This generalization holds because, for any initial level of the successor's power $\widetilde{S}$, the value function of a ruler who designates his successor in period t is $p_t \min\{\widetilde{S} + w, 1\}\eta R +$ $(1 - p_t)V_{t,2}^m(\widetilde{S})$. Here $V_{t,2}^m$ is the ruler's value function at the power acquisition stage of period t, a payoff that is strictly greater than zero when t is sufficiently large. The payoff to a ruler who decides *never* to appoint a successor is $(1-p_t)k_t$; here k_t is the total cash flow that he can collect in the future, and this payoff eventually converges to zero. Thus there is a period during which it makes more sense for the ruler to appoint a successor than to leave the position vacant. For the extreme case where $\widetilde{S}=1$, the successor could challenge the ruler at any time after being so designated; in that case, no heir apparent will be appointed until the ruler's life is nearly over. Hence we may conclude that, for any candidate with initial power $\widetilde{S}$, the ruler's optimal time to designate a successor is bounded from both above and below. This outcome is summarized as follows.

Proposition 3. *Given an initial power $\widetilde{S}$, there exist two periods—t' and t'', with $1 \le t'(\widetilde{S}) \le t''(\widetilde{S})$—such that the ruler will not designate a successor sooner than t' or later than t''. Both t' and t'' are weakly increasing functions of $\widetilde{S}$.*

This proposition states that the optimal time for the ruler to choose a crown prince lies in the range $[t', t'']$, referred to hereafter as the *designation interval*. The more powerful is the candidate, the later the ruler should designate him as successor.

From the candidate's perspective, once appointed as successor his power will increase with time. If the trend in this increase is uninterrupted, then the successor will become powerful enough that challenging the ruler becomes unavoidable. In particular: if the successor's power increases at the lowest rate of L units in each period, then his power will exceed 1 after $1/L$ periods. The implication is that, in this situation, the ruler–successor relationship cannot be stabilized even though the game proceeds under an infinite time horizon. This outcome is formalized in the following corollary.[15]

[15]The result presented in Corollary 1 is not restricted to the case in which the successor's power increases linearly with time; in other words, it is here assumed only that the successor receives a *nontrivial* power increase in each period. As the successor's power approaches 1, his weakly dominant strategy is to challenge the ruler (i.e., given the former's incentives to move earlier rather than later). Hence this corollary holds provided that the successor's power (a) is an increasing function of his effort level and (b) approaches 1.

Corollary 1. *If a candidate is designated as successor in period t^a, then there exists a time $\bar{t}^c(t^a, \tilde{S})$ such that the successor will challenge the ruler no later than $\bar{t}^c$.*

Corollary 1 indicates that, once a successor is appointed, the game will last at most some finite number of periods. This situation results in conflict between ruler and successor or in the ruler's (natural) death.

2.3 Conflict Analysis and Comparative Statics

The previous section described the equilibrium behavior of the ruler and his successor. The aims of this section are to assess the likelihood of conflict between them and to identify the leading cause of such conflict. The resulting analysis establishes the dynamic, macroscopic change in the relationship between these two parties.

Suppose a successor is designated no later than period t. Then: (i) conflict will arise if the ruler remains alive in the current period, $\prod_{i=1}^{t}(1-p_i)$; (ii) no conflict will arise before the current period, $S_i < \bar{s}_i^c$ for all $i < t$; and (iii) the successor's power exceeds his challenge threshold $\bar{s}_t^c$ after stage 3, $S_t \geq \bar{s}_t^c$. Hence we can formally express the *conflict probability* (CP) in period t as follows:

$$\mathrm{CP}_t \equiv \Pr(S_t \geq \bar{s}_t^c \cap (S_i < \bar{s}_i^c \ \forall i < t)) \prod_{i=1}^{t}(1-p_i).$$

Proposition 3 implies that a ruler who is still in good health will not designate a successor who could pose an immediate threat to the throne. So when the successor is finally appointed, the gap between his initial power and his challenge threshold is large enough to preclude immediate conflict. On the equilibrium path, then, $\mathrm{CP}_t = 0$ during the period soon after the successor is designated. Because such a newly designated successor is no threat to the throne, the two parties enjoy a "honeymoon" phase during this period.

When the ruler's health begins to deteriorate (i.e., as $1 - p_t \to 0$), the probability of conflict likewise converges to 0 when t is sufficiently large. It is intuitive that a ruler who is likely to die soon has no motivation to strip the successor of his position. These results are formalized in the next proposition.

Proposition 4. *Suppose that, on the equilibrium path, a successor is designated in period t^a. Then the following statements hold.*

(1) *If the successor's endowed initial power $\widetilde{S}$ is less than a constant $\hat{S}$, then there is a period $t^h(t^a, \widetilde{S})$ of no conflict* $(\mathrm{CP}_t = 0)$ *before t^h.*

(2) *When t is sufficiently large, the probability of conflict converges to zero:* $\lim_{t\to\infty} \mathrm{CP}_t = 0$.

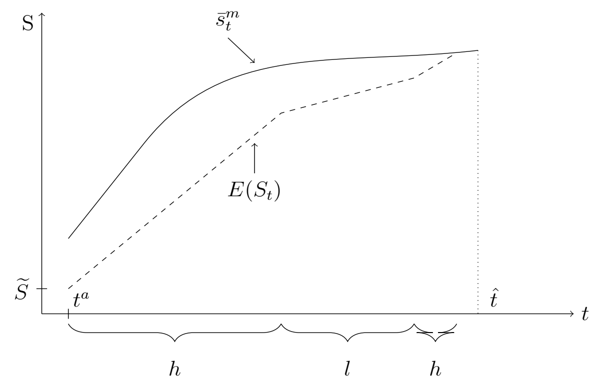
Part 1 of the proposition corresponds to the ruler–successor honeymoon phase during the early periods (i.e., before t^h) following the heir apparent's selection. This phase occurs provided that the successor's initial power is not too high ($\widetilde{S} < \hat{S}$). The second part of Proposition 4 characterizes the "power transition" phase. During this phase, the successor is still incentivized to undertake an early challenge to the ruler; however, the upper bound on the probability of such conflict is reduced by the ruler's declining health.

However, there need not be a direct transition from the honeymoon phase to the power transition phase. Under certain conditions, the probability of conflict cannot be ignored indefinitely. Thus we have the following proposition.

Proposition 5. *If the successor's initial power is low* (*sufficiently small* $\widetilde{S}$) *and the ruler begins his reign in a healthy condition and remains healthy for a long period* (*sufficiently small p_1 and $p_t - p_{t-1}$*), *then the conflict probability is bounded away from zero in some periods after t^h:* $\mathrm{CP}_t \geq \bar{c} > 0$.

The implication here is that, if a candidate with low initial power is designated successor early in the ruler's reign, then the former must maintain a long-term relationship with the latter—who is presumed to be in good physical condition at this point. The situation described in Proposition 5 is illustrated by Figure 2, where the solid curve represents the evolution in the ruler's tolerance thresholds and the dashed curve marks the expected evolution in the successor's power. The resulting equilibrium strategies imply that a successor may take advantage of the honeymoon phase to increase his power substantially. After such an immediate power increase achieved via the effort level h, the ruler can no longer ignore his

Figure 2



Notes: This figure depicts the scenario under which a ruler remains healthy for a relatively long time. Solid curve: change over time in the ruler's tolerance thresholds $\bar{s}_t^m$; dashed line: change over time in the level of power that the successor is expected to accrue.

successor's accumulated power; hence their relationship enters a "mutual suspicion" phase in which the successor must (at least temporarily) try to maintain a low profile and avoid possible conflict. However, keeping a low profile will not prevent conflict if the ruler remains healthy for a sufficiently long period. When a ruler's tolerance threshold tightens faster than the speed at which the successor's power increases, the latter must attempt to increase his power in preparation for conflict. Under these circumstances, the conflict probability rises and so cannot be ignored.

In short, Propositions 4 and 5 imply that, if the ruler reigns for a long time, then his relationship with the anointed successor will likely experience three phases: the honeymoon, mutual suspicion, and power transition phases. So any successor who seeks to maintain a long relationship with the ruler is in a race against time. After the honeymoon phase, the successor's gaining priority in that relationship depends on whether he can maintain a low profile until the ruler's health deteriorates.[16].

[16]It is worthy to note that the conflict between the two parties is unlikely to occur under certain degenerated situation, such as the ruler's health rapidly deteriorates or worsens at a moderate speed. These situations imply the ruler-successor relationship will transit from

The aforementioned theoretical results describe a dynamic relationship between a ruler and his successor. The history of succession in Imperial China offers a good illustration of the dynamics at play because, for nearly every regime in ancient China, designating a crown prince required a formal edict and ceremony. There is, in fact, precise information available on both timing and identity for some ruler-successor relationships.[17]

The Kangxi Emperor of the Qing dynasty ruled China for 61 years and was the longest-reigning emperor in Chinese history. When the emperor was 21 years old, he designated his one-year old son Yinreng as the crown prince. The emperor personally oversaw the upbringing of the crown prince, thereby grooming Yinreng to be the perfect successor. However, when the crown prince grew up, Yinreng and his supporters formed a Crown Prince Party that aimed to help Yinreng claim the throne as soon as possible. After being a crown prince for 38 years, Yinreng was stripped of the title by the emperor after a series of political struggles, and Yinreng was placed in perpetual confinement.[18] The emperor reigned the country for another 10 years and chose his second successor near his death. Emperor Wu of Han reigned China for 54 years, making him the third longest reigning emperor in Chinese history. He chose his seven-year-old son Ju as the crown prince in 122 BCE. After being a crown prince for 31 years, Ju could not maintain the trust of the emperor, and he was killed in a palace revolt.

Furthermore, in Imperial China there were 22 emperors who ruled for more than 30 years. Eleven of them experienced successor-related conflicts, which occurred 15.2 years (on

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the honey moon phase to the power transition phase quickly (the details are shown in Figure A.1 and A.2 in the Online Appendix)

[17]The literature (e.g., Kokkonen and Sundell 2014) has, to date, lacked the type of information—especially as regards the *timing* of designations and conflicts—needed to examine this relationship closely.

[18]Yinreng was stripped of the crown prince title in 1708 and was restored as crown prince in 1709. In 1712, Yinreng was stripped of the title again and was placed in perpetual confinement.

average) after their successors' designations.[19] After those conflicts, the emperors reigned for an average of another 18 years.[20] These findings confirm the model's posited mutual suspicion phase in long-term relationships between monarchs and crown princes.

Although arranging a smooth power transition is difficult under autocracy, history is replete with efforts to provide weak institutional protection (e.g., primogeniture) to apparent heirs. Therefore, I next analyze the effects of such institutional protection. If the power transition procedure is completely institutionalized, then there is no point in the ruler designating a successor. In authoritarian regimes, however, no institution can guarantee sufficient protection for a successor because the ruler's death creates a power vacuum. So if institutional protection is increased, then the ruler is less concerned about being betrayed and hence is less motivated to strip the successor of his position. With his designation thereby further secured, the successor has little incentive to challenge the ruler. The results are a reduction in the parties' mutual fear and perhaps a considerably earlier designation. These considerations lead to following summary proposition.

Proposition 6. i *For a given initial power $\widetilde{S}$, there exists a $\widetilde{w}$ such that both t' and t''—the lower and upper bounds (respectively) of the designation interval—are weakly decreasing in the institutional protection w when w is less a constant $\widetilde{w}$.*

(ii) *If the optimal designation period t^a does not change with w, then the length of the honeymoon phase $(t^h - t^a)$ is weakly increasing in w. If $\widetilde{S}$ and p_t satisfy the conditions stipulated in Proposition 5, then the conflict probability's lower bound $\bar{c}$ is weakly decreasing in w.*

The Chinese succession procedure also illustrates the effect of the weak institutional protection of succession under autocracy. An adjusted primogeniture was the prevailing custom in ancient China.[21] Under the traditional polygamy, the relative ranks of imperial

[19]The standard deviation is 11.1 years.

[20]The standard deviation is 11.4 years.

[21]The *Gongyang Zhuan*, a commentary on the Spring and Autumn Annals that was written in China's "warring states" period (475–221 BCE), stated explicitly that primogeniture was

consorts play a key role in imperial succession, which ranks heirs according not only to the birth order but also to the mother's status. The position of empress is always unique, and the son of an empress has priority over other princes in the succession order.[22] If the empress does not have a son or there is no empress, then the emperor's oldest son has the priority to be the successor. Still, this succession tradition provided at least some institutional protection for a successor following primogeniture. One example to illustrate this protection happened in the Ming dynasty. The Wanli Emperor wanted to choose his third son who was born by his favorite concubine, however, many of his powerful ministers were opposed, and this led to a clash between sovereign and ministers that lasted more than 15 years. In 1601, the Wanli Emperor finally gave in and designated his oldest son crown prince. Among 147 crown princes whose mothers' information can be found in the Chinese history, after a monarch assumed the throne or the existing successor died (or was stripped of his title), it took an average of 1.56 years for the ruler to appoint a new crown prince if the prince's mother was an empress (the high w case);[23] otherwise, the appointment process consumed an average of 3.56 years (the low w case).[24] Following designation of a successor, moreover, the conflict rate was 8.7% (4 out of 45) for the son of an empress but more than twice that (20.1%) for other designated successors. These examples accord with the model in showing that increased institutional protection may result in both an earlier designation and a lower rate of conflict.

2.4 Predetermined Succession Order

The analysis so far has been restricted to the case where the ruler can designate a successor in any period. This arrangement gives a ruler the flexibility to handle possible conflict with his

followed.

[22]There are a few exceptions to the uniqueness of an empress; for instance, Emperor Xuan of Northern Zhou had four empresses simultaneously. However, the data set used for this research includes none of these cases.

[23]with a standard deviation of 2.12 years

[24]with a standard deviation of 5.38 years

successor. In reality, however, the ruler is not always afforded that flexibility. One example of legislation providing for a clear succession order is the Act of Settlement established by England in 1701. The motive for such approaches is that stipulating a predetermined order of succession prevents the chaos caused by more ambiguous succession customs and the resulting destructive power struggles.

Yet a predetermined order of succession is, as mentioned previously, a double-edged s-word. On the one hand, it may provide increased protection for the successor's position and thereby reduce the likelihood of conflict (Proposition 6). On the other hand, a pre-determined procedure leaves the ruler with limited flexibility, which could lead to conflict because designating a successor at the game's outset might not be the ruler's optimal choice (Proposition 3).

The negative effect of a predetermined succession order could be significant when the successor has considerable power. By Proposition 3, the ruler is inclined to appoint a powerful successor only in the later periods of his reign. Unless his health has already begun to deteriorate, the ruler prefers a situation where the gap between the successor's initial power and his own tolerance threshold is large enough that immediate conflict can be avoided. In contrast, if a ruler is in good health and if his health is slow to deteriorate, then the existence of a predetermined successor could eliminate the two parties' honeymoon phase and thus result in immediate conflict.

Corollary 2. *Suppose that a candidate with initial power $\widetilde{S}$ is designated as successor in period 1 and that p_1 is not too large. Then there will be conflict in that period if the ruler's health deteriorates slowly (i.e., if $p_t - p_{t-1}$ is bounded by a constant).*

In today's world, absolute monarchy still exists in the Gulf countries. In Saudi Arabia, ruler–successor conflicts embody the negative effect of a predetermined succession order. A "deputy crown prince" position has been used to identify the heir of the current heir. Thus a successor who assumes the throne has a (predetermined) successor that was chosen by the former ruler. The succession order in Saudi Arabia follows *agnatic seniority* in that the throne is handed down among the sons of a single individual (here, Ibn Saud). This type of succession implies that the age gap between rulers and their successors is small. Moreover, successors have already amassed considerable power when they are appointed, which can create conflicts with the ruler.

After the death of two successive heirs in one year, King Abdullah designated Prince Salman bin Abdulaziz as his successor in 2012 and appointed Prince Muqrin as the deputy crown prince in 2014. When Salman ascended to the throne in 2015 and so Muqrin became crown prince, Prince Nayef was designated as deputy crown prince. Yet once in power, King Salman immediately stripped that successor position from the 71-year-old Muqrin. And within two years of the 51-year-old Nayef being appointed crown prince, he was deposed by King Salman and replaced by his 31-year-old son: Mohammad bin Salman.

Although the House of Saud attempted to enhance institutional protection by establishing the deputy successor position to predetermine the order of succession, it seems likely that power struggles between rulers and their heirs will intensify and lead the kingdom to forgo such protections.

3 Multi-candidate Selection

In this section, the model is extended to a context in which more than one candidate can be chosen as the successor. For simplicity, we shall assume that only two candidates (candidate 1 and candidate 2) are eligible for the successor title at the beginning of the game. Each candidate has an initial power endowment $\widetilde{S}^i$, where $i \in \{1, 2\}$. The ruler can decide when to designate his heir apparent.

When a candidate is designated as successor in period t, the game proceeds as in the baseline model described in Section 2.1. If a selected heir apparent successfully challenges the ruler in any period, he becomes the new ruler and the game ends. Any successor who loses his title can never again be appointed successor. In that case, the ruler decides whether and when to select the next successor.

In this game, those in the candidate pool do not have an active move. This setup precludes an "ordinary" candidate—that is, one who has not been cultivated by the ruler to become a future leader and so does not face an appointed successor's dilemma—initiating a coup or open rebellion. Insurgencies led by an ordinary prince are discussed in the literature on coups and rebellions.[25] Also, I ignore the possibility that other candidates will conspire to sabotage the relationship between the ruler and any chosen successor. The success of this type of palace intrigue relies on the ruler being suspicious of the successor. Such suspicions are already captured by the baseline model.

We first consider the scenario with a fixed succession order, such as one determined by birth order. In such cases, the ruler must designate candidate 1 *before* designating candidate 2. If the first chosen successor is stripped, then the game between the ruler and the remaining candidate is exactly the same as in the baseline model with the unique candidate. The subsequent analysis here therefore focuses on the interaction between a ruler and the first chosen successor. To simplify the presentation, in this section I highlight the difference between the one- and two-candidate cases; details regarding their equilibrium strategies are given in the appendix.

When there are two candidates, the ruler has a backup to fill the position if the first chosen successor is stripped of his title. So besides the motivation to protect his own power, the ruler has another incentive to replace the existing successor—namely, if the current successor is not competent enough to assume the throne and carry on the ruler's legacy. Therefore, the first successor always faces a risk of losing the position due to competition, even if the ruler's health deteriorates (the formal result is in the appendix). In this situation, a weak successor will be pressured because his title will be stripped if he cannot prove his ability to seize the throne. Whether a weak candidate still has the opportunity to succeed the throne depends on the initial power gap between the candidates. When candidate 1 has less initial power than candidate 2 and this power gap is sufficiently large, then the optimal strategy for the ruler is to strip the first successor of his title to make space for candidate 2.

[25] Most of these studies employ a coordination game to model regime change; see, for example, Angeletos et al. (2007) and Bueno de Mesquita (2010).

Moreover, conflict will occur right after the designation, because the ruler will not want to risk the first successor dethroning him. When the initial power gap is small, then designating candidate 1 could be a safer choice for the ruler as long as the sacrifice of the competency is not too large. These results are summarized as follows.

Proposition 7. *If candidate 1 has lower initial power than candidate 2* ($\widetilde{S}^1 < \widetilde{S}^2$), *then there exists a* $d_2 \geq 0$ *such that the ruler and his first successor will not immediately conflict only if* $\widetilde{S}^1 \geq \widetilde{S}^2 - d_2$.

This proposition implies there will an immediately conflict if candidate 1's initial power is too low ($\widetilde{S}^1 < \widetilde{S}^2 - d_2$).

When candidate 1's initial power is greater than that of candidate 2, the former experiences no direct competitive pressure from the latter. Yet because the first successor is replaceable, the ruler will be more concerned about maintaining his grip on throne than about the particulars of a power transition. Hence he will prefer to choose any successor in a later period, thereby avoiding (or at least forestalling) potential challenges. The ruler's tolerance threshold for the existing successor will tighten in this case, which could increase the probability of conflict. Thus we have our next proposition.

Proposition 8. (i) *If* $\widetilde{S}^1 > \widetilde{S}^2$, *then both the lower and upper bounds (i.e.,* t_1' *and* t_1''*) of the first successor's designation interval are weakly greater than their counterparts in the single-candidate case; that is* $t_1' \geq t'$ *and* $t_1'' \geq t''$.

(ii) *If the optimal designation time for the first successor does not differ from that in the single-candidate case, then the duration of this successor–ruler's honeymoon phase will be weakly lower than that in the single-candidate case.*

In ancient China, it took a ruler an average of 2.56 years to appoint a successor when no other son was available,[26] but 3.66 years to appoint a successor when more than one candi-

[26]with a s.d. of 3.57 years

date was available.[27][28] This descriptive phase is consistent with the first part of Proposition 8. The second part of this proposition indicates that the probability of conflict will increase with the number of candidates for the successor position. An intuitive example is the relationship between Alexander the Great and his father Philip. Alexander was his father's sole successor, until Philip married again and had another son. The relationship between Philip and Alexander then became very intense until Philip was assassinated. In ancient China, 13.1% of crown princes who had no more than 5 brothers, had conflicts with emperors; this conflict rate increased to 17.6% for 6–10 brothers and to 30% when the successor had more than 10 brothers.[29]

Now the succession order restriction can be relaxed by allowing the monarch to freely choose the heir apparent. The analysis of Proposition 7 and Proposition 8 can be easily extended to this scenario. The weak candidate does not have the chance to be selected as the first successor if the initial power gap between him and the strong candidate is large. When the power gap is small, the weak candidate may obtain a chance to become the successor first. However, the power gap restriction is stronger in this situation because the monarch does not have to designate the weak candidate first. Thus, to gain the opportunity to be named heir apparent, the weak candidate must provide the monarch a higher expected payoff (i.e. be more capable).

Corollary 3. *If there are two candidates for whom* $\widetilde{S}^1 > \widetilde{S}^2$ *and if the ruler is free to designate his successor, then the weaker candidate will be chosen as the first successor only if his initial power is close to that of the strong candidate—that is, only if* $\widetilde{S}^1 - \widetilde{S}^2 < \tilde{d}_2$ *and* $0 \leq \tilde{d}_2 \leq d_2$.

Proposition 7 and Corollary 3 together imply that, as one might expect, a strong candidate may have an advantage in the race to succession. These results offer another explanation

[27] with a s.d. of 5.53 years

[28] In this descriptive phase, we focus on 147 crown princes whose mothers were not empresses. It is because the empress's son has the priority be the crown prince.

[29] The average number of such sons is 7, with a standard deviation of 6.

for why primogeniture prevails under authoritarian regimes. In such regimes, it is typical for the oldest son to have more time (than do his younger brothers) for establishing coalitions and garnering support. These results also imply that when there are multiple candidates, choosing and keeping a weak candidate as a successor first, then replace this successor with a more competent successor until the ruler's health deteriorate may not be an optimal solution for the rule. It is because this strategy will increase the risk for the ruler to loose power in the conflict with the first successor.

4 Conclusion

This paper proposes a dynamic game to endogenize the relationship between rulers and their successors. The model captures the essential ruler–successor conflict that arises from a mutual fear of being ousted by the other party. Until his life nears its end, the ruler constantly monitors the successor's increasing power. At the same time, a successor who is not strong enough to threaten the ruler will devote his efforts to accruing additional power. When his power has increased enough to make the ruler suspicious, the successor should maintain a low profile to avoid conflict. Yet when he is sufficiently powerful to arouse the ruler's suspicion, the successor should strive to accrue still more power in preparation for contesting the throne.

There are several noteworthy implications of the model developed here. First, conflict between a ruler and a successor is more likely to occur during the middle stage of their relationship. Second, although institutional protection for power transitions can reduce the likelihood of conflict, a predetermined succession order may increase that likelihood. Finally, a ruler who can choose from among (say) two candidates prefers the weaker one only if this candidate's capabilities differ little from those of the stronger candidate; if the stronger candidate is chosen, this tends to occur later and requires the ruler to monitor the candidate strictly.

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Appendices

A Math Proofs

We first formally define the equilibrium concept used in the model. The set of state variables is $\mathbb{S} = \mathcal{S} \times \mathcal{P}$. The ruler's set of actions is $A_1^m = \{\text{designate, not designate}\}$ at stage 1 and $A_4^m = \{\text{strip, not strip}\}$ at stage 4. The successor's set of actions at stage 2 is $A_2^c = \{h, l\}$ and at stage 3 is $A_3^c = \{\text{challenge, not challenge}\}$.

At stage j of period t, the set of ex *ante* histories $\mathcal{H}_t$ is $(\mathbb{S} \times A)_{t,j}$, which identifies both the state and the action profile at each stage of each period. The set of ex *post* histories $\widetilde{\mathcal{H}}_{t,j}$ at stage j of period t is $(\mathbb{S} \times A)_{t,j} \times \mathbb{S}$. Let $\mathbb{H} = \bigcup_t \bigcup_j \widetilde{\mathcal{H}}_{t,j}$. Hence the pure strategy for player i at stage j is a mapping $\sigma_j^i \colon \mathbb{H} \to A_j^i$ for $i \in \{m, c\}$.

The strategy profile σ is a Markov strategy if, for any two ex post histories $h_{t,j}$ and $h'_{t,j}$ of the same length and terminating in the same state, the equality $\sigma(h_{t,j}) = \sigma(h'_{t,j})$ holds. The strategy profile σ is a *Markov perfect equilibrium* (MPE) if σ is both a Markov strategy profile and a subgame-perfect equilibrium. It is worth noting that this equilibrium concept is like the Markov perfect equilibrium (Maskin and Tirole (2001)) except that the payoffs are not discounted at the stages within each period. So we keep using Markov perfect equilibrium in the paper.[30] From this point forward, the focus will be on the players' Markov strategies and on the entire game's (period) Markov perfect equilibrium, in which (period) Markov strategies ignore all historical details except for the history's length and the current state.[31]

We restate the assumptions in the paper here:

Assumption 1. $p_h(H - L) + L < w$

[30]The definition of equilibrium employed here is of the form described by Mailath and Samuelson (2006, p. 191).

[31]Since the state variables include the ruler's health condition, which is strongly related to time, it follows that the equilibrium described in this study is *not* a stationary Markov equilibrium (in which the length of time can safely be ignored).

Assumption 2. *In any period t,*

$$p_t w\eta R + (1-p_t)(-(b+k_t)H + k_t) > (1-p_t)k_t, \tag{4}$$

where $k_t \equiv r + r\sum_{i=t+1}^{\infty} \delta^{i-t} \prod_{j=t+1}^{i}(1-p_j)$.

Next, let's introduce some notations first: $V_{t,j}^{g}(S)$ denotes the value function of player g at stage j in period t with status variable S. Furthermore, to simplify the notation for the case that the power of the successor is no greater than 1, let $V_{t,j}^{g}(S) = V_{t,j}^{g}(1)$ if $S > 1$. When $j = 0$, we sometime drop j to represent the value function of that period, i.e. $V_t^g \equiv V_{t,1}^g$.

For the ruler, in period t, his value function at stage 1 of this period is

$$V_{t,1}^{m}(S_{t-1}) = p_t \min(S_{t-1} + w, 1)\eta R + (1-p_t)V_{t,2}^{m}(S_{t-1}). \tag{5}$$

The value function at the beginning of the stage 2 is

$$V_{t,2}^{m}(S_{t-1}) = \begin{cases} p_h V_{t,3}^{m}(S_{t-1} + H) + (1-p_h)V_{t,3}^{m}(S_{t-1} + L) & \text{if } e = h, \\ V_{t,3}^{m}(S_{t-1} + L) & \text{if } e = l. \end{cases} \tag{6}$$

The value function at the beginning of the stage 3 is

$$V_{t,3}^{m}(S_t) = \begin{cases} -(b+k_t)\min(S_t, 1) + k_t & \text{if being challenged,} \\ V_{t,4}^{m}(S_t) & \text{if not,} \end{cases} \tag{7}$$

where $k_t \equiv r + r\sum_{i=t+1}^{\infty} \delta^{i-t} \prod_{j=t+1}^{i}(1-p_j)$.

The value function at the beginning of the stage 4 is

$$V_{t,4}^{m}(S_t) = \begin{cases} r + \delta V_{t+1,0}^{m}(S_t) & \text{if keep the successor,} \\ k_t & \text{if strip the successor} \end{cases} \tag{8}$$

Proof of Lemma 1. This lemma is equivalent that maintaining the successor's position is a dominant strategy for the ruler in any subgame-perfect equilibrium of the subgame starting in period $t \geq \bar{t}^m$.

In this proof, we discuss the players' strategies that are not restricted in period Markov strategies. At stage 3 of any t, if the ruler keeps the successor, the value function has the following property:

$$V^m_{t,4}(S_t) = r + \delta[p_{t+1}(\min(S_t + w, 1))\eta R + (1 - p_{t+1})V^m_{t+1,1}(S_t)], \tag{9}$$
$$\geq r + \delta[p_{t+1}(\min(\widetilde{S} + w, 1)\eta R + b) - b]. \tag{10}$$

The inequality comes from that $V^m_{t+1,1}(S_t) > -b$ and $S_t \geq \widetilde{S}$. If the ruler strips the successor, his expected payoff is k_t.

We denote the right hand side of (10) as f_t. It is an increasing function of t, and $\lim_{t\to\infty} f_t = r + \delta \min(\widetilde{S} + w, 1)\eta R$. We also know k_t is decreasing with t and $\lim_{t\to\infty} k_t = r$. Therefore, there exists t' such that $f_t \geq k_t$ when $t > t'$. Furthermore, there exists $\bar{t}^m$ with $\bar{t}^m \leq t'$ such that $V^m_{t,4}(S) \geq k_t$ for any $S \in [0, 1]$ when $t > \bar{t}^m$. So the ruler will not strip the successor after period $\bar{t}^m$. □

For the successor, in period t, his value function at stage 1 of this period is

$$V^c_{t,1}(S_{t-1}) = p_t(\min(S_{t-1} + w, 1)(R + b) - b) + (1 - p_t)V^c_{t,2}(S_{t-1}). \tag{11}$$

The value function at the beginning of the stage 2 is

$$V^c_{t,2}(S_{t-1}) = \begin{cases} p_h V^c_{t,3}(S_{t-1} + H) + (1 - p_h)V^c_{t,3}(S_{t-1} + L) & \text{if } e = h, \\ V^c_{t,3}(S_{t-1} + L) & \text{if } e = l. \end{cases} \tag{12}$$

The value function at the beginning of the stage 3 is

$$V^c_{t,3}(S_t) = \begin{cases} \min(S_t, 1)(R + b) - b & \text{if challenge,} \\ V^c_{t,4}(S_t) & \text{if not.} \end{cases} \tag{13}$$

The value function at the beginning of the stage 4 is

$$V^c_{t,4}(S_t) = \begin{cases} \delta V^c_{t+1,0}(S_t) & \text{if not being stripped,} \\ -b & \text{if being stripped} \end{cases} \tag{14}$$

Proof of Proposition 1. In period t with $t \geq \bar{t}^m$, at stage 2, since the successor will not be stripped by the ruler, he chooses high effort level always.

At stage 3, if he challenges the ruler his payoff is

$$(R + b)S_t - b. \tag{15}$$

If he remains loyal, his payoff is

$$\delta[p_{t+1}(\min(S_t+w,1)(R+b)-b)+(1-p_{t+1})(p_h V^c_{t+1,2}(S_t+H)+(1-p_h)V^c_{t+1,2}(S_t+L))]. \quad (16)$$

Since $lim_{t\to\infty}p_t = 1$, for a given extreme small $\epsilon > 0$, there exists a t^b such that equation (16) is less than $\delta(\min(S_t + w, 1)(R + b) - b) + \epsilon$.

From now on we only consider the strategies of the successor when $t \leq t^b$, the successor's strategies after t^b all follow the same equilibrium strategies when $t = t^b$.

When $t = t^b$, without loss of generality, equation (16) can be rewrite as $\delta(\min(S_t + w, 1)(R + b) - b)$ by ignoring the error term.

Consider equation

$$(R + b)S - b = \delta((S + w)(R + b) - b), \quad (17)$$

the root is $S_2 = \frac{b}{R+b} + \frac{\delta w}{1-\delta}$.

For any $t' > \bar{t}^m$ and $t' < t^b$ (t' always exists because we find let t^b sufficiently large such that $t^b > \bar{t}^m$):

C1, $V^c_{t',3}$ is a continuous piecewise linear function when $S \in [0, 1]$, i.e. $\{a^c_j S+b^c_j | q^c_j \leq S < q^c_{j+1}\}$ with $q^c_1 = 0 < q^c_2 < \cdots < q^c_n < q^c_{n+1} = 1$;

C2, $a^c_j < a^c_{j'}$ with $j < j'$, except when $a^c_{j'} = 0$,

C3, the last segment of $V^c_{t',3}$ is $S(R + b) - b$.

Let $\bar{\delta} \equiv 1 - w - wb/R$, then we consider two cases.

When $\delta > \bar{\delta}$, if the successor remains loyal, his expected payoff is $\delta(\min(S+w, 1)(R+b)-b)$, which intersects $(R+b)S-b$ at $S_3 \equiv (\delta R+b)/(R+b) < S_2$. Therefore, the successor will challenge the ruler if $S > S_3$, and remain loyal otherwise. Therefore $V^c_{t,3} = \delta(S(R + b) - b)$ if $S < 1 - w$, $V^c_{t,3} = \delta R$ if $1 - w \leq S < S_3$, and $V^c_{t,3} = S(R + b) - b$ if $S_3 \leq S$. So it satisfies C1-C3. Now we also assume:

C4, the first segment of $V^c_{t,3}$ has the linear form $a \min(S, 1) + b$ and ends at the intersect with δR which is less than S_2, when $\delta > \bar{\delta}$.

Then, for any t', if the successor remains loyal, his expected payoff at stage 3 is $\min(\delta[p_{t'}((S_{t'-1}+w)(R + b) - b) + (1 - p_{t'})(aS_{t'-1} + p_h(H - L) + L + b)], \delta R)$. Then $\delta[p_{t'}(S_{t'-1} + w(R + b) - b) + (1 - p_{t'})(aS_{t'-1} + p_h(H - L) + L + b)$ intersects $(R + b)S - b$ at $S'_2 > S_2$ and reach δR before S_3. Therefore in period t', the payoff of remaining loyal is $\delta[p_{t'}(\min(S_{t'-1} + w, 1)(R + b) - b) + (1 - p_{t'})(a \min(S_{t'-1} + p_h(H - L) + L, 1) + b)]$.

So $V^c_{t',2}$ satisfies C1-C4, and the cutoff threshold to challenge the ruler is still S_3. This situation represents the case that the successor is patient, he will not challenge the successor unless his power is sufficient large which is unrelated to the time.

When $\delta < \bar{\delta}$, we have $S_2 < S_3$. Then in period t', let $\bar{s}^c_{t'} = S_2$, and the successor should challenge if $S_{t'} > \bar{s}^c_{t'}$ and remain loyal otherwise. Therefore $V^c_{t',3} = \delta(S_{t'}(R+b)-b)$ if $S_{t'} < S_2$, and $V^c_{t',3} = S(R+b) - b$ if $S_2 \leq S_{t'}$. So it satisfies C1-C3.

Assume in t', $V^c_{t',2}$ satisfies C1-C3. At $t'-1$, if the successor remains loyal, then his expected payoff is $\delta(p_{t'}((S_{t'}+w)(R+b)-b))-(1-p_{t'})(p_h V^c_{t',3}(S_{t'}+H)+(1-p_h)(V^c_{t',3}(S_{t'}+L))))$. This function is still piecewise linear. Also, the slope of each segment of this function satisfies C2, and the last segment $\delta(R+b)(S_{t'-1}+p_{t'}w+(1-p_{t'})(p_h(H-L)+L))-b$ has the largest slope, and it intersects $S_{t'-1}(R+b)-b$ at $\bar{s}^c_{t'-1} \equiv b/(R+b)+(p_{t'-1}w+(1-p_{t'-1})(p_h(H-L)+L))\delta/(1-\delta)$. By assumption 1, we also have $\bar{s}^c_{t'-1} < \bar{s}^c_{t'}$ and less than S_2 and S_3. Therefore the successor will challenge the ruler if $S_{t'-1} > \bar{s}^c_{t'-1}$, and not otherwise. Furthermore, $\bar{s}^c_{t'-1}$ approaches $\bar{s}_{lim} = b/(R+b) + \delta(p_h(H-L)+L+w)/(1-\delta)$ when p_t goes to 0. Therefore, no $\bar{s}^c_{t'}$ is less than $\bar{s}_{lim}$.

Moreover $V^c_{t'-1,3}(S) = \delta(p_{t'}((S+w)(R+b)-b))-(1-p_{t'})(p_h V^c_{t',3}(S+H)+(1-p_h)(V^c_{t'-1,3}(S+L))))$ if $S_{t'-1} \leq \bar{s}^c_{t'-1}$, and $V^c_{t'-1,3} = S(R+b)-b$ if $S > \bar{s}^c_{t'-1}$, which satisfies C1-C3. □

Proof of Proposition 2. We begin to consider the ruler's strategy first. In period t, the ruler's expected payoff at stage 4 is

$$r + \delta[p_{t+1}(S_t + w)\eta R + (1-p_{t+1})V^m_{t+1,1}(S_t)] \tag{18}$$

if the successor is not stripped. Let's start in a sufficient large period $t^b > \bar{t}^m$ such that equation (18) can be rewritten as $r+\delta(S_t+w)\eta R+\epsilon$ and simply denoted as $r+\delta(S_t+w)\eta R$ by dropping the error term. Since the successor always chooses the high effort level after period $\bar{t}^m$, and challenge the ruler at stage 3 if $S_t > \bar{s}^c_t$, then the value function of ruler at stage 2 of period t^b is

$$V^m_{t^b,2}(S_{t^b-1}) = \begin{cases} r+\delta(S_{t^b-1}+p_h(H-L)+L+w)\eta R & \text{if } S_{t^b-1} < \bar{s}^c_t - H, \\ p_h(r+\delta(S_{t^b-1}+w)\eta R)+(1-p_h)(-(b+k_t)S_{t^b-1}+k_t) & \text{if } \bar{s}^c_t - H \leq S_{t^b-1} < \bar{s}^c_t - L, \\ -(b+k_t)(S_{t^b-1}+p_h(H-L)+L)+k_t & \text{if } \bar{s}^c_t - L \leq S_{t^b-1}. \end{cases} \tag{19}$$

Then at stage 1, $V^m_{t^b,1}(S_{t^b-1}) = p_t(S_{t^b-1}+w)\eta R + (1-p_t)V^m_{t^b,2}(S_{t^b-1})$.

Now we assume at any $t^b > t > \bar{t}^m$, $V^m_{t,1}(S)$ has the following properties.

M1, $V^m_{t,1}$ is a piecewise linear function when $S \in [0,1]$, i.e. $\{a^m_j S + b^m_j | q^m_j \leq S < q^m_{j+1}\}$ with

$q_1 = 0 < q_2 < \cdots < q_n < q_{n+1} = 1$;
M2, $a_j^m > a_{j+1}^m$;
M3, $a_j^m q_{j+1}^m + b_j^m > a_{j+1}^m q_{j+1}^m + b_{j+1}^m$;
M4, $b_j^m > b_{j+1}^m$.

M2 indicates the slopes of the linear segments is decreasing; M3 indicates the end point of each segment is greater than the starting point of the next segment; M4 indicates the intercepts of the linear segment is decreasing.

At stage 4 of period $t-1$, $V_{t-1,4}^m(S) = r + \delta k_t$ if the ruler strips the successor and $r + \delta V_{t,1}^m(S)$ otherwise. At stage 3, since the successor challenges the ruler if $S > \bar{s}_{t-1}^c$, then $V_{t-1,3}^m = r + \delta V_{t,1}^m(S)$ if $S \leq \bar{s}_{t-1}^c$, and $-(b + r + \delta k_t)S + r + \delta k_t$ if $S > \bar{s}_{t-1}^c$.

$V_{t-1,2}^m = p_h V_{t-1,3}^m(S+H) + (1-p_h)V_{t-1,3}^m(S+L)$. It is easy to find $V_{t-1,3}^m$ is a piecewise linear function. Any segment O_v of $V_{t-1,2}^m$ can be written as $p_h(a_j(S+H)+b_j)+(1-p_h)(a_{j'}(S+L)+b_{j'})$. The next segment of O_v is either $O_{v+1} = p_h(a_{j''}(S+H)+b_{j''})+(1-p_h)(a_{j'}(S+L)+b_{j'})$ with $j \geq j''$ (case 1) or $O_{v+1} = p_h(a_j(S+H)+b_j)+(1-p_h)(a_{j'''}(S+L)+b_{j'''})$ with $j''' \geq j'$ (case 2).

In either case, the slope of O_v is greater than the slope of O_{v+1}. Then end point of O_v must be either $q_{j+1} - H$ or $q_{j'+1} - L$. When it is the former, $O_v(q_{j+1}-H) > O_{v+1}(q_{j+1}-H)$ (case 1). When it is the latter, we have $O_v(q_{j'+1}-L) > O_{v+1}(q_{j'+1}-L)$ (case 2). Therefore M1, M2 and M3 are satisfied. The intercept of O_v is $p_h(a_j H + b_j) + (1-p_h)(a_{j'}L + b_{j'})$, and the intercept of O_{v+1} is either $p_h(a_{j''}H + b_{j''}) + (1-p_h)(a_{j'}L + b_{j'})$ or $p_h(a_j H + b_j) + (1-p_h)(a_{j'''}L + b_{j'''})$, then $V_{t-1,2}^m(S)$ also satisfied M4. Since $V_{t-1,1}^m(S) = p_{t-1}(S+w)\eta R + (1-p_{t-1})V_{t-1,2}^m$, it preserves all four properties. Furthermore, the last segment of $V_{t-1,1}^m(S)$ is

$$p_{t-1}(S+w)\eta R + (1-p_{t-1})[-(b+k_{t-1})(S+p_h(H-L)+L)+k_{t-1}], \tag{20}$$

and the segment before the last is

$$p_{t-1}(S+w)\eta R + (1-p_{t-1})[p_h(-(b+k_{t-1})(S+H)+k_{t-1}) + (1-p_h)V_{t-1,3}^m(S+L)]. \tag{21}$$

We know $V_{t-1,3}^m(S+L) \geq -(b+k_{t-1})(S+H)+k_{t-1}$, it is because $V_{t-1,3}^m(S+L)$ is either $-(b+k_{t-1})(S+L)+k_{t-1}$, which is greater than $-(b+k_{t-1})(S+H)+k_{t-1}$, or $V_{t-1,4}^m(S+L)$ which is greater or equal to k_{t-1} due to the ruler can strip the successor at stage 4. Therefore equation (21) is greater or equal to $p_{t-1}(S+w)\eta R + (1-p_{t-1})[(-(b+k_{t-1})(S+H)+k_{t-1})]$. By Assumption 2, equation (21) is greater than $(1-p_{t-1})k_{t-1}$. Then all other segments' intercept is greater than $(1-p_{t-1})k_{t-1}$, except the last one.

In each period t, the last segment's slope is increasing with t, it is because p_t and $-(b+k_t)$ is increasing with t. Similarly, for any segment of $V^m_{t,1}$ between $\bar{s}^c_t - H$ and $\bar{s}^c_t - L$ we have the form $p_t(S+w)\eta R + (1-p_t)[p_h(-(b+k_t)(S+H)+k_t) + (1-p_h)(\delta V^m_{t+1,1}(S+L)+r)]$. Since $p_{t+1} > p_t$, the segment between $\bar{s}^c_t - H$ and $\bar{s}^c_t - L$ has a smaller slope in period t than the slope of the corresponding part in $t+1$. Therefore, when t decreases, we can find the ruler should strip the successor if $S_t > \bar{s}^m_{\bar{t}^m-1}$ in period $\bar{t}^m - 1$.

Now we begin to consider the period before $\bar{t}^m$. Since the successor will not set up the challenging threshold greater than the tolerance threshold, i.e $\bar{s}^m_t \geq \bar{s}^c_t$ when $t < \bar{t}^m$, if $\bar{s}^m_t > \bar{s}^c_t$, the successor still choose high effort at t. Since the slope of the segment of $V^m_{t,1}$ becomes smaller when t decreases, so $\bar{s}^m_t - \bar{s}^m_{t-1} \leq \bar{s}^m_{t-1} - \bar{s}^m_{t-2}$. When the successor does not considering the risk of being stripped, then his challenging threshold is determined by the intersect of $(R+b)S_{t-1} - b$ and $\delta[p_t((S_t+w)(R+b)-b) + (1-p_t)(p_h V^c_{t,2}(S_{t-1}+H) + (1-p_h)V^c_{t,2}(S_{t-1}+L))$. This intersection is bounded by $\bar{s}_{\lim}$ from below. Moreover, since $p_{t-1} < p_t$, the gap between the successive these intersections decreases as t decreases. On the contrary, since the gap between the successive tolerance thresholds increases as t deceases, therefore, a $\hat{t}$ exists such that $\bar{s}^m_{\hat{t}} = \bar{s}^c_{\hat{t}}$, otherwise let $\hat{t} = 0$.

Next we focus on the equilibrium strategies when $t \leq \hat{t}$, and assume $\delta < \bar{\delta}$. In period $\hat{t}$, stage 1, the value function $V^c_{\hat{t},3}(S_{\hat{t}})$ is a continuous piecewise linear function with property C1-C3.

At stage 4 of period $\hat{t}-1$, if the successor is stripped, then his payoff is $-b$. Therefore, at stage 3, he cannot remain loyal if his power $S_{\hat{t}-1} > \bar{s}^m_{\hat{t}-1}$, instead he should challenge. So the challenge cutoff for the successor is $\bar{s}^c_{\hat{t}-1} = \bar{s}^m_{\hat{t}-1}$, the value function can be written as

$$V^c_{\hat{t}-1,3}(S_{\hat{t}-1}) = \begin{cases} \delta V^c_{\hat{t},1}(S_{\hat{t}-1}) & \text{if} \quad S_{\hat{t}-1} \leq \bar{s}^c_{\hat{t}-1}, \\ S_{\hat{t}-1}(R+b) - b & \text{if} \quad S_{\hat{t}-1} > \bar{s}^c_{\hat{t}-1} \end{cases} \tag{22}$$

This value function is not continuous at $\bar{s}^c_{\hat{t}-1}$, because $\delta V^c_{\hat{t},1}(\bar{s}^c_{\hat{t}-1}) > \bar{s}^c_{\hat{t}-1}(R+b) - b$. Define $\Delta = \delta V^c_{\hat{t},1}(\bar{s}^c_{\hat{t}-1}) - \bar{s}^c_{\hat{t}-1}(R+b) + b$.

At stage 2 of period $\hat{t}-1$, if the successor chooses low effort, then his power becomes $S_{\hat{t}-1} = S_{\hat{t}-2} + L$. So his value function with low effort level is $V^c_{\hat{t}-1,3}(S_{\hat{t}-2}+L)$. When he chooses the high effort level, his value function is $p_h V^c_{\hat{t}-1,3}(S_{\hat{t}-2}+H) + (1-p_h)V^c_{\hat{t}-1,3}(S_{\hat{t}-2}+L)$. When $S_{\hat{t}-2} < \bar{s}^c_{\hat{t}-1} - H$, $V^c_{\hat{t}-1,3}(S_{\hat{t}-2}+H) > V^c_{\hat{t}-1,3}(S_{\hat{t}-2}+L)$. Then there exists a $\hat{s}^c_{\hat{t}-1,l}$ such that $V^c_{\hat{t}-1,3}(S_{\hat{t}-2}+H) < V^c_{\hat{t}-1,3}(S_{\hat{t}-2}+L)$ when $\bar{s}^c_{\hat{t}-1} - H < S_{\hat{t}-2} \leq \hat{s}^c_{\hat{t}-1,l}$, and $V^c_{\hat{t}-1,3}(S_{\hat{t}-2}+H) >$

$V^c_{\hat{t}-1,3}(S_{\hat{t}-2}+L)$ when $S_{\hat{t}-2} > \hat{s}^c_{\hat{t}-1,l}$. Intuitively, the existence of $\hat{s}^c_{\hat{t}-1,2}$ comes from that 1, at point $\bar{s}^c_{\hat{t}-1}-H$, $\delta V^c_{\hat{t}-1,3}(S+H) > \delta V^c_{\hat{t}-1,3}(S+L) > (S+H)(R+b)-b$, it guarantees the payoff from the low effort is large than the high effort level due to the gap at point $\bar{s}^c_{\hat{t}-1}-H$; 2, because the slope of $(S+H)(R+b)-b$ is greater than the slope of any segment of $\delta V^c_{\hat{t}-1,3}(S+L)$ when $S \in [\bar{s}^c_{\hat{t}-1}-H, \hat{s}^c_{\hat{t}-1,l}]$, we have the payoff from the high effort level is increasing faster than the payoff from the low effort level; 3, when $S > \hat{s}^c_{\hat{t}-1,l}$, $(S+H)(R+b)-b > (S+L)(R+b)-b$, which can make sure that when $S > \hat{s}^c_{\hat{t}-1,l}$, the payoff from the high effort is greater than that from the low effort. Now we define $\hat{s}^c_{\hat{t}-1,h} \equiv \bar{s}^c_{\hat{t}-1}-H$.

The value function $V^m_{\hat{t},1}$ preserves the properties M1-M4, because the successor always chooses high effort in each period since $\hat{t}$. Consider the ruler's strategy at period $\hat{t}-2$, stage 3. If he strips the successor, the expected payoff is $r+\delta k_{\hat{t}-2}$. If he keeps the successor, the expected payoff is $r+\delta V^m_{\hat{t}-1,1}$. So we focus on $V^m_{\hat{t}-1,1}$. At period $\hat{t}-1$, stage 2, the successor challenges the ruler if $S_{\hat{t}-1} > \bar{s}^c_{\hat{t}-1} = \bar{s}^m_{\hat{t}-1}$. Since, in stage 1, the successor chooses high effort if $S_{\hat{t}-1} \geq \bar{s}^c_{\hat{t}-1}-H$, therefore the segment of $V^m_{\hat{t}-1,1}$ before $\bar{s}^c_{\hat{t}-1}-H$ is $r+\delta(S+w+p_h(H-L)+L)\eta R$. Since the successor chooses low effort if $\bar{s}^c_{\hat{t}-1}-H \leq S_{\hat{t}-1} < \hat{s}^c_{\hat{t}-1,h}$, then the segment of $V^m_{\hat{t}-1,1}$ is $r+\delta(S+w+L)\eta R$ between $\bar{s}^c_{\hat{t}-1}-H$ and $\hat{s}^c_{\hat{t}-1,h}$. Due to M2-M4, any segments of $r+\delta V^m_{\hat{t},1}$ moved paralleled into the interval $[\bar{s}^c_{\hat{t}-1}-H, \bar{s}^c_{\hat{t}-1}-L]$ has smaller slope than $r+\delta(S+w+L)\eta R$ and lower value. Therefore the segments of $V^m_{\hat{t}-1,2}$ after $\hat{s}^c_{\hat{t}-1,h}$ are $(r+\delta(p_h\delta V^m_{\hat{t},3}(S+H))+(1-p_h)V^m_{\hat{t},3}(S+L))$. Now $V^m_{\hat{t}-1,2}$ satisfies M1-M4. Finally $V^m_{\hat{t}-1,1} = p_t((S_{\hat{t}-1}+w)\eta R)+(1-p_t)V^m_{\hat{t}-1,2}(S_{\hat{t}-1})$ satisfies M1-M4.

Now we restate the assumption for the successor as follows for any $t < \hat{t}$:

C1', $V^c_{t',2}$ is a piecewise linear function when $S \in [0,1]$, i.e. $\{a^c_j S + b^c_j | q^c_j \leq S < q^c_{j+1}\}$ with $q^c_1 = 0 < q^c_2 < \cdots < q^c_n < q^c_{n+1} = 1$;

C2', the last segment of $V^c_{t',2}$ is $S(R+b)-b$.

C3', At stage 2, there exists a $\bar{s}^c_t$ such that the successor challenges the ruler if $S_t > \bar{s}^c_t$, and not otherwise. At stage 1, there exist $\hat{s}^c_{t,h}$ and $\hat{s}^c_{t,l}$, with $\hat{s}^c_{t,h} = \bar{s}^c_t - H \leq \hat{s}^c_{t,l} \leq \bar{s}^c_t - L$, such that the successor puts high effort when $S_{t-1} \leq \hat{s}^c_{t,1}$, low effort when $\hat{s}^c_{t,l} < S_{t-1} \leq \hat{s}^c_{t,h}$, and high effort when $\hat{s}^c_{t,h} < S_{t-1}$.

In period $t-1$, stage 3, if the successor remain loyal, his payoff if $\delta V^c_{t,1}(S)$ if the ruler does not strip him and $-b$ if he is stripped. He needs to compare this payoff with the payoff if he challenges the ruler now $(S(R+b)-b)$. Since $\delta V^c_{t,1}(S)$'s last segment is $\delta(p_t((S+w)(R+b)-b)+(1-p_t)((S+p_h(H-L)+L)(R+b)-b))$ which intersects with $S(R+b)-b$ after

$\bar{s}^m_{t-1}$, then $\bar{s}^c_{t-1} = \bar{s}^m_{t-1} \in [\bar{s}^c_t - H, \bar{s}^c_t - L]$.

At stage 3, once the successor observes S_{t-1}, his value function at that moment can be written as

$$V^c_{t-1,3}(S) = \begin{cases} \delta(p_t((S+w)(R+b)-b) + (1-p_t)V^c_{t,1}(S)) & \text{if} \quad S \le \bar{s}^c_{t-1}, \\ S(R+b)-b & \text{if} \quad S > \bar{s}^c_{t-1} \end{cases} \tag{23}$$

To calculate the strategy, we need to compare $V^c_{t-1,3}(S+H)$ and $V^c_{t-1,3}(S+L)$. First we have $V^c_{t-1,3}(S+H) > V^c_{t-1,3}(S+L)$ when $S < \bar{s}^c_t - H$. Then let $\hat{s}^c_{t-1,h} \equiv \bar{s}^c_t - H$, and we know the successor should choose high effort, if $S_t < \hat{s}^c_{t-1,h}$. When $S \ge \bar{s}^c_t - H$, we have $V^c_{t-1,2}(S+H) \ge (S+H)(R+b)-b$. Also we have $\delta V^c_{t,1}(\bar{s}^c_t - H + H) > \delta V^c_{t,1}(\bar{s}^c_t - H + L) > (\bar{s}^c_t - H + H)(R+b)-b$. Moreover, we have $R+b > p_h(a^c_i) + (1-p_h)(a^c_j)$ for any i and j when $S < \bar{s}^c_{t-1} - L$. Furthermore, when $S > \bar{s}^c_{t-1} - L$, we have $(S+H)(R+b)-b > (S+L)(R+b)-b$. Then there exists a $\hat{s}_{t-1,l}$, such that $V^c_{t-1,2}(S_{t-2}+H) < V^c_{t-1,2}(S_{t-2}+L)$ when $\hat{s}_{t-1,h} < S_{t-2} < \hat{s}_{t-1,l}$, and $V^c_{t-1,2}(S_{t-2}+H) > V^c_{t-1,2}(S_{t-2}+L)$ when $S_{t-2} > \hat{s}_{t-1,l}$. In other word, the successor should chooses high effort when $S_{t-2} < \hat{s}_{t-1,h}$, low effort when $\hat{s}_{t-1,h} < S_{t-2} < \hat{s}_{t-1,l}$, and switch to high effort again when $S_{t-2} > \hat{s}_{t-1,l}$.

It is easy to find that $V^c_{t-1,2}(S_{t-2})$ is still a function consist of a set of linear segments, with positive slope for each segment. Also, the last segment is $(S_{t-2} + p_h(H-L) + L)(R+b) - b$ when $S_{t-2} > \bar{s}^c_{t-1} - L$, and $V^c_{t-1,2}(S_{t-2}) > -b$. In summary $V^c_{t-1,2}$ satisfies C1'-C3'.

For the case when $\delta \ge \bar{\delta}$, for any $t \ge \bar{t}^m$, the ruler does not strip the successor, so $\bar{s}^c_t = S_3$ always (S_3 is defined in the proof of proposition 1). For any $t < \bar{t}^m$, we follow the same procedure as the case when $\delta > \bar{\delta}$, and can get the same results. □

Proof of Proposition 3. For a given $\widetilde{S}$, if $\widetilde{S} \ge \bar{s}^c_1$, then let $t' = 1$. Otherwise, since $\bar{s}^c_t$ is an increasing function of t, we can always find $t' > 1$ such that $\widetilde{S} \ge \bar{s}^c_{t'}$. Then $\widetilde{S} \ge \bar{s}^c_t$ for $t < t'$. Since the appointed successor always challenge the ruler right after the appointment, the ruler's value function is increasing function of t when $t < t'$, therefore the optimal choosing time must greater or equal to t'. From the proof of proposition 1, we the value function of the ruler is pairwise linear function, it implies the slope of the last segment of the value function is negative when t is small, and this slope is an increasing function of time. When t is large, the slope becomes positive. So for a given $\widetilde{S}$, we can find a t' such that the intersection of the last segment of ruler's value function and $(1 - p_{t'}k_{t'})$ is less than $\widetilde{S}$, but that intersection is greater than $\widetilde{S}$ in period $t'+1$. So t' increases with $\widetilde{S}$.

From the proof of proposition 2, we know in a sufficient large period t^b, the ruler's value function at stage 1 can be written as $(S_t + w)\eta R$ for $S_t \in [0,1]$. Furthermore, the value function $V^m_{t^b-1,1}(S)$ in period $t^b - 1$ is a piecewise linear function and its first segment has the form $a_1 S + b_1$ with $a_1 > 0$ for any $S \in [1, \bar{s}^c_{t^b-1} - H]$. Therefore, for any given initial power $\widetilde{S}$, there exists a period $t'' \leq t^b$ such that $\widetilde{S}$ belongs to the first segment of $V^m_{t'',1}(S)$ and this segment has the positive slope, moreover $\widetilde{S}$ belongs to the first segment of $V^m_{t^{cc},1}(S)$ and the segment has the positive slope for any $t^b \geq t^{cc} > t''$.

If the successor is appointed in $t''+1$, and the first segment of $V^m_{t'',1}(S)$ is denoted as $aS+b$, then the value function in period t'' is $(1-p_{t''})(r+\delta(aS+b))$. If the successor is appointed in t'', the ruler's value function is $p_{t''-1}(S+w)\eta R + (1-p_{t''-1})(r+\delta(S+p_h(H-L)+L)+b)$. Therefore, appointing the successor in period t'' is better than $t''+1$ for the ruler. Also, it is easy to show that postponing the appointment will make the ruler worse off for any $t > t''$. Therefore, the optimal choosing time should be less than or equal to t''. When $\widetilde{S}$ increase, using the backward induction to calculate the value function of $V^m_{t,1}$ from t^b, then $t''(\widetilde{S})$ is increasing with $\widetilde{S}$. $\square$

Proof of Corollary 1. There exists $\widetilde{t}$ such that $\widetilde{t}L \geq 1$, then $S_{\widetilde{t}} \geq 1$. When the successor can win the fight with the ruler, challenge the ruler is a dominant strategy. Therefore $\bar{t}^c$ exists and $\bar{t}^c \leq \widetilde{t}$. $\square$

Proof of Proposition 4. We prove the second part of the proposition first. Since $1 - p_t$ converge to zeros when t goes to infinite, therefore for a given ϵ, there exist t^ϵ such that $1 - p_t < \epsilon$. Then we have $CP_t < \epsilon$ by the definition.

When the initial power $\widetilde{S} = 1$, then the chosen successor will challenge the ruler immediately, therefore the ruler will not designate the heir apparent until his life close to the end. Therefore we can find CP_{t^a} must be less than a given ϵ, with $t^a \geq t'$ (proposition 3). Furthermore, there exists $\widetilde{S}_1 \in [0,1]$ such that the successor always challenge the successor right after his designation. Also,

When $\widetilde{S} < \widetilde{S}_1$, from the proof of proposition 3, we know the ruler also will not designate the successor in any period in which the successor will challenge him when the successor receives a high outcome in this period. It means that the optimal choosing period must satisfy the condition that $\widetilde{S} < \bar{s}^c_{ta} - H$. Also we can find another $\widetilde{S}_2$ such that when $\widetilde{S} < \widetilde{S}_2$, there exists a period t if the successor is designated in this period, then $\widetilde{S} < \bar{s}^c_t - H$. Now let $\hat{S} = \min\{\widetilde{S}_1, \widetilde{S}_2\}$. Then we know in the optimal designation time $\widetilde{S} < \bar{s}^c_{ta} - H$. Therefore we know t^h exists with $t^h \geq t^a$. This result implies that t^h is the minimal period such that $\widetilde{S}+(t-t^a+1)H < \widetilde{s}_t^c$, in other words, the last period that the upper bound of the successor's power is less than the challenging threshold. $\square$

Proof of Proposition 5.

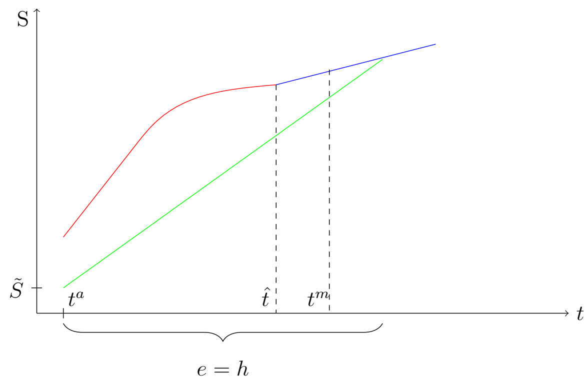
$$CP_t \equiv Pr(S_t \geq \bar{s}_t^c \cap (\cap_{t'=1}^{t-1} S_{t'} < \bar{s}_{t'}^c)) \prod_{i=1}^{t}(1-p_i).$$

Since the tolerance threshold $\bar{s}_t^m$ will vanish after period $\bar{t}^m$, while $\bar{t}^m$ is determined by the ruler's health. Both $\bar{t}^m$ and $\bar{s}_t^m$ are independent with $\widetilde{S}$. Also $t''(\widetilde{S})$ increases with $\widetilde{S}$. Therefore we can find constants p' and δ' and $\widetilde{S}^v$ such that when $p_1 < p'$, $p_t - p_{t-1} < \delta'$, and $\widetilde{S} \leq \widetilde{S}^v$, $\bar{t}^m - t''$ is greater than $\bar{t}^c - t^a$. This set up is not trivial because when $\widetilde{S}^v = 0$, we can find p' and δ' such that $\bar{t}^m$ is sufficiently large. Furthermore, from the proof of proposition 2, when p_t increases in a sufficient small scale in each period, there exists a t^u such that $\bar{s}_t^m = \bar{s}_t^c$ when $t \leq t^u$ and $\bar{s}_t^m = \bar{s}_{t+1}^m - H$. Therefore, we can choose a even smaller p' and δ' such that $t'' \leq t^u$.

The chosen $\widetilde{S}^v$, p' and δ' is to guarantee that the successor is designated in a early period, and there exist sufficiently long periods after the honeymoon phase and before the ruler's health gets worse.

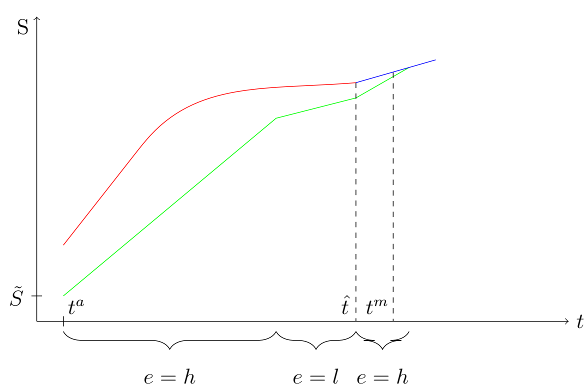
At the end of the honeymoon phase ($t = t^h$), the possible outcome of the successor's power is between $[\widetilde{S} + t^h L, \widetilde{S} + t^h H]$, since the increase rate of tolerance thresholds $\bar{s}_t^m$ converges to zeros before $\bar{t}^m$ and the increase rate of the successor's power is at least L, therefore there exists a period such that the increase rate of $\bar{s}_{t-1}^m$ is less than L after this period. Furthermore, it implies that a period t^β exists such that the smallest possible $S_{t^\beta - 1}$ is greater than $\bar{s}_{t^\beta, l}^c$, and $S_{t^\beta - 2}$ is less than $\bar{s}_{t^\beta - 1, l}^c$. It means, the successor will choose high effort in period t^β, and once the outcome is high, then he will challenge the ruler. Then conditional on no conflict in period $t^\beta - 1$, the conflict probability in t^β is $p_h(1 - p_{t^\beta})$. From the perspective of period t^a, the conflict probability in period t^β is $CP_{t^\beta} = p_h(1-p_{t^\beta})(1-p_h)^{t^\beta - t^a} \prod_{i=1}^{t^\beta - 1}(1-p_i)$. The conflict probability of the next period, $CP_{t^\beta+1}$, is either $p_h(1-p_{t^\beta+1})(1-CP_{t^\beta})$ or $(1-p_{t^\beta+1})(1-CP_{t^\beta})$. Then choose a small enough constant $0 < \phi < CP_{t^\beta}$ and let t^γ be the maximal period such that $CP_t > \phi$ for all consecutive $t > t^\alpha$, and $\bar{c}$ is the minimal CP_t with $t \in [t^\beta, t^\gamma]$. $\square$

Figure A.1



Notes: This figure represents the scenario that the monarch's health deteriorates quickly such that the honeymoon phase is connected with the power transition phase directly. The red curve represents the change of the monitoring thresholds $\bar{s}_t^m$ with time. The blue curve represents the change of the challenge thresholds $\bar{s}_t^c$ with time. When $t \leq \hat{t}$, $\bar{s}_t^m = \bar{s}_t^c$; when $\hat{t} < \bar{t}^m$, $\bar{s}_t^c < \bar{s}_t^m$; when $\bar{t}^m \leq t$, $\bar{s}_t^m$ does not exists. The green line represents the change of the expected power accrued by the crown prince with time. Since the crown prince always chooses high effort, the average power increase rate is $p_h(H - L) + L$.

Figure A.2



Notes: This figure represents the scenario that after the honeymoon phase the crown prince has to keep a low profile to avoid the conflict and wait for the deterioration of the monarch's health. The red curve represents the change of the monitoring thresholds $\bar{s}_t^m$ with time. The blue curve represents the change of the challenge thresholds $\bar{s}_t^c$ with time. When $t \leq \hat{t}$, $\bar{s}_t^m = \bar{s}_t^c$; when $\hat{t} < \bar{t}^m$, $\bar{s}_t^c < \bar{s}_t^m$; when $\bar{t}^m \leq t$, $\bar{s}_t^m$ does not exists. The green line represents the change of the expected power accrued by the crown prince with time. When the crown prince chooses high effort, the average power increase rate is $p_h(H - L) + L$. When the crown prince chooses low effort, the power increase rate is L.

Proof of Proposition 6. For a given initial power $\widetilde{S}$, let $\widetilde{w} = 1 - \widetilde{S}$. When $w < \widetilde{w}$, the ruler's value function can be written as $\min(S + w, 1)\eta R$ as proof of Proposition 2 when he designate a successor at stage 1 of a sufficient large period t^b. Therefore, when $S > 1 - w$, the value function is ηR, which is the last segment of $V^m_{t^a,1}$. Furthermore, the last segment of any $V^m_{t,1}$ can be written as $p_t(\eta R) + (1 - p_t)(-(b + r + k_{t+1})S + r + k_{t+1})$ when $\bar{s}^c_t > 1 - w$. It implies $\bar{s}^m_t$ is not affected by w when $\bar{s}^c_t > 1 - w$.

When $\widetilde{S} < 1 - w$, from the proof of proposition 3, we know t'' is the first period such that $\widetilde{S} > \bar{s}^c_t$. Therefore, when w increases, both S_1 and S_2 in Proposition 2 will decrease. It indicates the successor is less likely to challenge the ruler when the ruler does not have incentive to strip him. Following the same induction procedure, we can find the thresholds $\bar{s}^c_t$, $\bar{s}^m_t$ in each period will increase with w. Then following the proof in proposition 3 and the increase of $\bar{s}^c_t$, we have both $t'(\widetilde{S})$ and $t''(\widetilde{S})$ will decrease.

Given w and an optimal designation period t^a, when w increases, the optimal designation period will not increase. It comes from the proof that t' decreases with w such that the optimal designation time cannot increase when w increases. If the optimal designation time does not change, since $\bar{s}^c_t$ weakly increases with w, it indicates the gap between $\widetilde{S}$ and $\bar{s}^c_{t^a}$ increases. Therefore, the length of the honeymoon phase will weakly increase.

From the proof of part 1, we know when w increase, t^h weakly increase with w, which indicates the gap between $\bar{s}^c_t$ and S_t in the last period of the honeymoon phase before will increase with w. It also indicate that t^β weakly increases with w, i.e. the successor need more time to reach period t^β. Therefore, from the proof of proposition 5, we know CP_{t^β} will decreases with w. Since the conflict probability in each period decreases in the interval $[t^\beta, t^\gamma]$, $\bar{c}$ will decreases with w. □

Proof of Corollary 2. At stage 4 of period 1, if the successor has received a power such that he will receive the power reach 1 in the next period, then he will challenge the ruler in period two. In this situation, if the ruler keeps the successor, his expected payoff is $r + \delta(p_2\eta R - (1 - p_2)b)$, and if he strips the successor, his expected payoff is k_1. Then if we have $k_1 > r + \delta(p_2\eta R - (1 - p_2)b)$, i.e. $((k_1 - r)/\delta + b)/(\eta R + b) > p_2$, when we know $\bar{s}^m_1$ exists because the ruler has an incentive to strip the successor. Let $d_1 = p_2 - p_1$, then when $p_1 < ((k_1 - r)/\delta + b)/(\eta R + b) - d_1$, $\bar{s}^m_1$ exists. This result indicates when p_1 is not sufficiently large, then $t_1 < \bar{t}^m$. In other words, the ruler's health is not worse enough such that he gives up the chance to strip the successor.

For given $p_1, p_2, ...$ with $p_1 < ((k_1 - r)/\delta + b)/(\eta R + b) - d_1$, if $bars_1^c \leq \widetilde{S}$, then our proposition holds immediately. If not, then let $d_t = p_t - p_{t-1}$ and $d_m = sup_t(d_t)$. In the first case that $\bar{s}_1^m = \bar{s}_1^c$, then we fix p_1 and let d_m decreases, it implies the except p_1, all other p_t decreases. From the proof of proposition 2, we know the value function of ruler at stage 1 $V_{t,1}^m(S)$ is a pairwise linear function and its the last segment is $p_t(S_{t-1} + w) + (1 - p_t)(-(b + k_t)(S + p_h(H - L) + L) + k_t)$ when $S \leq \bar{s}_t^c$, and it is decreasing with p_t. Therefore, repeat the induction in the proof of proposition 2, we have $\bar{s}_t^m$ decreases with p_t. Therefore, for given $\widetilde{S}$, when there exist a $\widetilde{d}$ such that when $d_m < \widetilde{d}$, then $\bar{s}_1^m \leq \widetilde{S}$.

In the second case that $\bar{s}_1^m > \bar{s}_1^c$, when d_m decreases, though the proof of the existence of $\hat{t}$ in proposition 2. Then $\bar{t}$ is increasing with d_m, it implies that a d' exists such that when $d_m < d'$, $\bar{s}_1^c = \bar{s}_1^m$. Then further decreases d_m may lead $\bar{s}_1^m \leq \widetilde{S}$. This procedure indicates the existence of $\widetilde{d}$ in the second case such that the conflict will occur in the first period, when $p_t - p_{t-1} < \widetilde{d}$.

□

Lemma A.1. *For any period t, suppose that candidate 1 has been designated as successor. Then a tolerance threshold $\bar{s}_{t,1}^m$ exists such that the ruler will strip this successor's title if that threshold is exceeded by the successor's power—that is, if $S_t^1 > \bar{s}_{t,1}^m$.*

Proof of Lemma A.1. The proof of this lemma can be found in the proof of part 1 of Proposition A.1. □

In the next proposition, I calculate the equilibrium strategies in any subgame when the successor has been chosen.

Proposition A.1. *Given $\widetilde{S}_2$, in any period t when Candidate 1 has been designated as the successor,*

1. At stage 4, a threshold $\bar{s}_{t,1}^m$ exists such that the ruler will strip the successor when $S_t^1 \leq \bar{s}_{t,1}^m$. There exists a time period $\bar{t}^m$, when $t < \bar{t}^m$, a threshold $\bar{s}_{t,2}^m$ exists such that the ruler will strip the the successor when $S_t^1 > \bar{s}_{t,2}^m$, and $\bar{s}_{t,2}^m$ weakly increases with time.

2. At stage 3, two thresholds $\bar{s}_{t,1}^c$ and $\bar{s}_{t,2}^c$ exist such that the successor challenges the ruler if $S_t \leq \bar{s}_{t,1}^c$ or $S_t > \bar{s}_{t,2}^c$; otherwise, he remains loyal. Moreover, a $\hat{t}$ exists with $\hat{t} \leq \bar{t}^m$ such that $\bar{s}_{t,2}^c < \bar{s}_{t,2}^m$ when $\hat{t} \leq t \leq \bar{t}^m$, and $\bar{s}_{t,2}^c = \bar{s}_{t,2}^m$ when $t < \hat{t}$.

3. At stage 2, then $\bar{s}^c_t = \bar{s}^m_t$, two thresholds, $\hat{s}^c_{t,h}$ and $\hat{s}^c_{t,l}$, exist such that the successor will choose a low effort level if $\hat{s}^c_{t,h} \leq S_{t-1} < \hat{s}^c_{t,l}$; he will then switch back to a high effort level if $\hat{s}^c_{t,l} \leq S_{t-1}$.

Proof of Proposition A.1. The proof of Proposition A.1 is similar as the proof of Proposition 2. The difference between these two propositions will be highlighted. Assume Candidate 1 has been designated as the successor. I use $V^g_{i,j}(S, \widetilde{S}^2)$ to denote player g's expected payoff when the first successor's power is S and Candidate 2's initial power is $\widetilde{S}^2$ at the beginning of stage j of period i, meanwhile, $V^g_{i,j}(S)$ is denoted player g's expected payoff when the second successor's power is S, which is equivelent to the case when there is only one candidate.

Start in a very large t, similar as the proof of Proposition 2, such that the ruler's expected payoff when he keeps Candidate 1 as the successor at stage 4 can be rewritten as $r + \delta(S^1_t + w)\eta R$. If the ruler strip the successor, then he should appoint Candidate 2 as the successor at stage 1 of period $t + 1$. The ruler's payoff is denoted as $V^m_{t,4}(\widetilde{S}^2)$ which can be written as $r + \delta(\widetilde{S}^2 + w)\eta R$. Therefore, Candidate 1 should be stripped if $S^1_t < \widetilde{S}^2$. So we have $\bar{s}^m_{t,1} = \widetilde{S}^2$.

At stage 3, When the successor challenge the ruler, his expected payoff is $(R + b)S^1_t - b$ (hereafter we drop the super- and sub-script for the existing successor), and when he remains loyal, his expect payoff is $\delta((S + w)(R + b) - b)$ when $\delta > \bar{\delta} \equiv R/((1 + w)(R + b) - b)$, or $\delta((S + w)(R + b) - b)$ if $S < (\delta R + b)/(R + b)$ and δR if $S \geq (\delta R + b)/(R + b)$ when $\delta > \bar{\delta} \equiv R/((1 + w)(R + b) - b)$. Therefore, he challenges the ruler if $S < \bar{s}^c_{t,1} \equiv \bar{s}^m_{t,1}$ or $S \geq \bar{s}^c_{t,2}$, where $\bar{s}^c_{t,2} = (\delta R + b)/(R + b)$.

At stage 2, in the case that $\delta < \bar{\delta}$, we have $\delta((S+w)(R+b)-b)$ intersects $\delta((S+w)(R+b)-b)$ less than $S = 1$. It implies $V^m_{t,3}(S + H, \widetilde{S}^2) > V^m_{t,3}(S + L, \widetilde{S}^2)$ for all $S \leq 1$. Therefore, the successor should choose high effort at this stage, and $V^m_{t,2}(S, \widetilde{S}^2) = p_h V^m_{t,3}(S + H, \widetilde{S}^2) + (1-p_h)V^m_{t,3}(S+L, \widetilde{S}^2)$. In the case that $\delta \geq \bar{\delta}$, we will have the same result that the successor will choose the high effort level. In the special case that $\widetilde{S}^2 > \bar{s}^c_{t,1}$, we define $\bar{s}^c_{t,1} = \bar{s}^c_{t,2}$. Now the expected payoff function of the successor can be denoted as a piecewise linear function. Moreover, the last segment is $(R + b)S - b$ with starting point $\bar{s}^c_{t,2}$. Hereafter we focus on the cast that $\delta < \bar{\delta}$. The case that $\delta < \bar{\delta}$ is similar as the analysis in Proposition 2.

At stage 1, the ruler's expected payoff is denoted as $V^m_{t,1}(S, \widetilde{S}^2) = p_t(S + w)\eta R + (1 - p_t)V^m_{t,2}(S, \widetilde{S}^2)$. We make the following claim

Claim 1. *When $S < \widetilde{S}^2$, $V^m_{t,1}(S, \widetilde{S}^2) < V^m_{t,1}(\widetilde{S}^2, \widetilde{S}^2) = p_t(\widetilde{S}^2 + w)\eta R + (1 - p_t)V^m_{t,2}(\widetilde{S}^2, \widetilde{S}^2)$.*

Proof of Claim 1. For $V_{t,2}^m(S, \widetilde{S}^2)$, if $\widetilde{S}^2 - L < \bar{s}_{t,2}^c - H$, then $V_{t,1}^m(\widetilde{S}^2, \widetilde{S}^2) = p_t(\widetilde{S}^2 + w)\eta R + (1-p_t)(r + \delta(\widetilde{S}^2 + p_h(H-L) + L)\eta R)$. If $\widetilde{S}^2 - L < S \geq \widetilde{S}^2 - L$, then we have $V_{t,1}^m(\widetilde{S}^2, \widetilde{S}^2) > p_t(S+w)\eta R + (1-p_t)(r + \delta(S + p_h(H-L) + L)\eta R)$. If $S \geq \widetilde{S}^2 - L$, then $V_{t,2}^m(S, \widetilde{S}^2)$ is either $p_t(S+w)\eta R + (1-p_t)(r + \delta(p_h(-(b + V_t^m(\widetilde{S}^2))S + b) + (1-p_h)(S+L)\eta R))$ or $p_t(S+w)\eta R + (1-p_t)(-(b + V_t^m(\widetilde{S}^2))(S + p_h(H-L) + L) + b)$. In either case, we have $V_{t,1}^m(S, \widetilde{S}^2) < p_t(\widetilde{S}^2 + w)\eta R + (1-p_t)V_{t,2}^m(\widetilde{S}^2, \widetilde{S}^2)$.

If $\widetilde{S}^2 - L \geq \bar{s}_{t,2}^c - H$, then $V_{t,1}^m(\widetilde{S}^2, \widetilde{S}^2) = p_t(S+w)\eta R + (1-p_t)(r + \delta(p_h(-(b+V_t^m(\widetilde{S}^2))\widetilde{S}^2 + b) + (1-p_h)(\widetilde{S}^2 + L)\eta R))$. For any $S < \widetilde{S}^2$, $V_{t,2}^m(S, \widetilde{S}^2)$ is either $p_t(S+w)\eta R + (1-p_t)(r + \delta(p_h(-(b+V_t^m(\widetilde{S}^2))S + b) + (1-p_h)(S+L)\eta R))$ or $p_t(S+w)\eta R + (1-p_t)(-(b+V_t^m(\widetilde{S}^2))(S + p_h(H-L) + L) + b)$. In either case, we have $V_{t,1}^m(S, \widetilde{S}^2) < p_t(\widetilde{S}^2 + w)\eta R + (1-p_t)V_{t,2}^m(\widetilde{S}^2, \widetilde{S}^2)$. □

We simply assume that $\widetilde{S}^2 - L < \bar{s}_{t,2}^c - H$, and discuss the case that $\widetilde{S}^2 - L \geq \bar{s}_{t,2}^c - H$ later. After calculate the expected payoffs in period t, we know $\widetilde{S}^2$ belongs to the segment of $V_{t,1}^m(S, \widetilde{S}^2)$ with the largest slope and the ruler will not be challenged by the successor when the successor's power S_{t-1}^1 at stage 1 of period t belongs to this segment because of the assumption. We name this segment the "untouched segment" $O_{t,u}$. This segment's starting and end points are denoted as $S_{t,s}$ and $S_{t,e}$ respectively.

Claim 2. *At stage 1 of period $t-1$, if $S < S_{t,e} - \bar{s}_{t,2}^c - 2H$, then for any S that belongs to the untouched segment $O_{t,u}$, we have $V_{t-1,1}^m(S) > (1-p_{t-1})(r + \delta V_{t,1}^m(S))$.*

Proof of Claim 2. Since S belongs to the untouched segment in period t, then the ruler also does not face any challenge when the successor's power is S at stage 1 of period $t-1$ if $S < S_{t,e} - \bar{s}_{t,2}^c - 2H$. Therefore, we have $V_{t-1,1}^m(S) = p_{t-1}(S+w)\eta R + (1-p_{t-1})(r + \delta V_{t,1}^m(S + p_h(H-L) + L))$, which is greater than $(1-p_{t-1})(r + \delta V_{t,1}^m(S))$. □

Now we can extend the definite of untouched segment to any period: For any given period t', the untouched segment at stage one of period t', $O_{u_{t'}}$, consists of the S such that the ruler has zero probability of being challenged in the period. It is easy to show if this segment is not empty, then this is continuous inside with starting and end points, $S_{t',s}$ and $S_{t',e}$ respectively. It is worth to note that the untouched segment may be an empty set in some periods.

The ruler's value function, $V_{t,1}^m(S, \widetilde{S}^2)$, at stage 1 of period t is a piecewise linear function, $\{O_{i_t} | i_t = 1, \cdots, n_t\}$. For the untouched segment of period t, we have O_{u_t} with $u_t \in \{1, \cdots, n_t\}$ if it is not an empty set.

Let's consider the situation in period $t-1$. Assume $\widetilde{S}^2 < \bar{s}^c_{t,2} - 2H$, at stage 4 of period $t-1$, the ruler's payoff is $r + \delta V^m_{t,1}(\widetilde{S}^2)$ and $\widetilde{S}^2$ is on the untouched segment of period t. It implies that the current $V^m_{t,1}(\widetilde{S}^2)$ provides the ruler the optimal payoff in period t if Candidate 1 has been stripped (or equivelent to the case that Candidate 2 is the unique candidate). If the ruler keeps Candidate 1 at stage 4 of period $t-1$, then from Claim 1, we know any S less than $\widetilde{S}^2$ cannot give the ruler higher payoff than $r + \delta V^m_{t,1}(\widetilde{S}^2)$. Therefore the ruler strips Candidate 1 if the successor's power at this moment is less than $\widetilde{S}^2$, and we have $\bar{s}^m_{t-1,1} = \widetilde{S}^2$.

The ruler's expected payoff when he strips the current successor is $r + \delta V^m_{t,1}(\widetilde{S}^2)$, and this payoff does not satisfies Assumption 2. So for any segment O_{i_t} with $i_t > u_t$, it is not guaranteed that O_{i_t} does not intersect with $V^m_{t,1}(\widetilde{S}^2)$ if the slope of O_{i_t} is positive, but we do know the last segment has the form $O_{n_t} = p_t(S+w)\eta R + (1-p_t)(-(b + V^m_{t,1}(\widetilde{S}^2))(S + p_h(H-L) + L) + V^m_{t,1}(\widetilde{S}^2))$. Then we put all the intersect points of $V^m_{t,1}(S, \widetilde{S}^2)$ and $V^m_{t,1}(\widetilde{S}^2)$ together with all the start points of each segment except 0, then delete the repeated points. This set is denoted as $\{\bar{s}^{m,t-1}_j | j = 1, \cdots, k\}$ with $\bar{s}^{m,t-1}_j < \bar{s}^{m,t-1}_{j'}$ when $j < j'$. Then the ruler's strategy is described as follow: Given a sufficient small number ϵ, for each $\bar{s}^{m,t-1}_j$, consider a point $\bar{s}^{m,t-1}_j + \epsilon$. If the slope of the segment of $V^m_{t,1}(S, \widetilde{S}^2)$ at this point is positive, then the ruler keeps the successor when $\bar{s}^{m,t-1}_j < S \leq \bar{s}^{m,t-1}_{j+1}$. If this slope is negative, then the ruler strips the successor when $\bar{s}^{m,t-1}_j < S \leq \bar{s}^{m,t-1}_{j+1}$. Consider a point $\bar{s}^{m,t-1}_j - \epsilon$, if the slope of the segment of $V^m_{t,1}(S, \widetilde{S}^2)$ at this point is positive, then the ruler keeps the successor when $\bar{s}^{m,t-1}_{j-1} < S \leq \bar{s}^{m,t-1}_j$. If this slope is negative, then the ruler strips the successor when $\bar{s}^{m,t-1}_{j-1} < S \leq \bar{s}^{m,t-1}_j$. Now define a new set SW^m_{t-1} that contains the points in $\bar{s}^{m,t-1}_j + \epsilon$ and the ruler switches the strategy across these points. So SW^m_{t-1} is set of all thresholds. Moreover, if the last segment, O_{n_t}'s slope is negative and it intersects with $V^m_{t,1}(S, \widetilde{S}^2)$, then let $\bar{s}^m_{t,2}$ be this intersect and it must be $\bar{s}^{m,t-1}_k$. If O_{n_t}'s starting point is less than $V^m_{t,1}(\widetilde{S}^2)$, find the large S such that $V^m_{t,1}(S', \widetilde{S}^2) \geq V^m_{t,1}(\widetilde{S}^2)$ with $S' \geq S$, then let $\bar{s}^m_{t-1,2}$ be this S, and also $\bar{s}^m_{t-1,2} \in SW^m_{t-1}$. Otherwise $\bar{s}^m_{t-1,2} = \emptyset$.

At stage 3 of period $t-1$, the last segment of $V^c_{t,1}(S, \widetilde{S}^2)$ equals $p_t((S+w)(R+b)-b) + (1-p_t)((R+b)(S+p_h(H-L)+L)-b)$, which is denoted as $O^c_{t,l}$. If $\delta(O^c_{t,l})$ intersects $(R+b)S-b$ before $\bar{s}^m_{t-1,2}$ or $\bar{s}^m_{t-1,2} = \emptyset$, then let $\bar{s}^c_{t-1,2}$ be the intersection. If $\delta(O^c_{t,l})$ intersects $(R+b)S-b$ after $\bar{s}^m_{t-1,2}$, then $\bar{s}^c_{t-1,2} = \bar{s}^m_{t-1,2}$. Let $SW^c_{t-1} = SW^m_{t-1} \cup \{\bar{s}^m_{t-1,2}\}$, then delete any points in SW^m_{t-1} but greater than $\bar{s}^m_{t-1,2}$ from SW^c_{t-1}. Then SW^c_{t-1} is denoted as $\{\bar{s}^{c,t-1}_j | j = 1, \cdots, k'\}$. The successor's equilibrium strategy is described as follow: If $k' > 2$, for any threshold,

$\bar{s}_j^{c,t-1}$ with $1 < j \leq k'$, if ruler strips the successor when $S \in (\bar{s}_{j-1}^{c,t-1}, \bar{s}_j^{c,t-1}]$, then the successor should challenge the ruler is this interval. If ruler keeps the successor in this interval, then the successor should remain loyal. When $j = k'$, the successor should challenge the ruler when $S > \bar{s}_{k'}^{c,t-1} = \bar{s}_{t-1,2}^c$. If $k' = 2$, then the successor challenges the ruler when $S > \bar{s}_{k'}^{c,t-1}$ or $S \leq \bar{s}_{k'-1}^{c,t-1}$, and remain loyal otherwise.

At stage 2 of period $t-1$, since we know at each S, the successor makes the decision by comparing $V_{t-1,3}^c(S+H, \widetilde{S}^2)$ and $V_{t-1,3}^c(S+L, \widetilde{S}^2)$; and both these value functions are piecewise linear functions. Therefore there is a unique optimal action for the successor at each S. When $S < \bar{s}_{t-1,1}^c$, all segments of $V_{t-1,3}^c(S, \widetilde{S}^2)$ have the property that the maximal value of each segment is less than the minimal value of its next segment and they all have positive slope. Therefore, for any given S, we have $V_{t-1,3}^c(S+H, \widetilde{S}^2) > V_{t-1,3}^c(S+L, \widetilde{S}^2)$. So the successor always choose high effort. For the segments that contain S that larger than $\widetilde{S}^2$, we know the last segment, always has the form $(R+b)S-b$. If the last segment and the one before the last segment are continuous at threshold $\bar{s}_{t,2}^c$, then if $S > S_s - L$, the successor will high effort, where S_s is the start point of the last segment. If the last segment and the one before the last segment are not continuous at threshold $\bar{s}_{t,2}^c$, then we must have the value of the start point in the last segment is less than the value of the end point in the segment before the last. Similar as Proposition 2, there exist an interval such that the successor chooses the low effort in this interval, and high effort when S is larger than the upper bound of this interval. Also there must exists a adjunct interval such that the successor chooses the high effort also. Then the lower bound and upper bound of this interval are denoted as $\hat{s}_{t,h}^c$ and $\hat{s}_{t,l}^c$ respectively.

Now we can repeat the induction with same procedure in the proof of Proposition 2 to get the conclusion.

It is worth to mention that the existence of $\bar{t}^m$ is guaranteed because the last segment of $V_{t,1}^m$ always has the form $p_t(S_{t-1}+w)+(1-p_t)(-(b+\hat{V}_{t,1}^m(\bar{S}^2))(S+p_h(H-L)+L)+\hat{V}_{t,1}^m(\bar{S}^2))$, where $\hat{V}_{t,1}^m(\bar{S}^2))$ is the ruler's maximal expected payoff at stage 1 of period t when Candidate 2 is the unique candidate.

The existence of $\hat{t}$ is also guaranteed. From the proof of Proposition 2, we know when t is large, $\bar{s}_{t,2}^c$ is either fixed when $\delta \geq \bar{\delta}$ or decreases but bounded from below when $\delta \geq \bar{\delta}$. Also $\bar{s}_{t,2}^c$ cannot greater than $\bar{s}_{t,2}^m$. When $\bar{s}_{t,2}^m$ decreases when t becomes small, there is not lower bound for $\bar{s}_{t,2}^m$. Hence there exists a $\hat{t}$ such that $\bar{s}_{t,2}^c = \bar{s}_{t,2}^m$ when $t < \hat{t}$. It is worth to mention, it is possible that $\bar{s}^c_{t,2} = \bar{s}^m_{t,2}$ for all t when $\bar{t}^m$ exists. In this case, $\hat{t} = \bar{t}^m$.

It is also worth to point out, when $\widetilde{S}^2$ is in the untouched segment of $V^m_{t,1}(S, \widetilde{S}^2)$, $\hat{V}^m_{t,1}(\bar{S}^2))$ is always $V^m_{t,1}(\bar{S}^2)$ because of Claim 2. Once $t < t^*(\widetilde{S}^2)$, where $t^*(\widetilde{S}^2)$ is the optimal time to choose Candidate 2 as the successor in the signal candidate case. Then $\hat{V}^m_{t,1}(\widetilde{S}^2) = r(1 - \delta^{t^*-t})/(1-\delta) + \delta^{t^*-t} V^m_{t^*,1}(\widetilde{S}^2)$. Also if $t^*(\widetilde{S}^2)$ is not unique, we can use any optimal time without changing $\hat{V}^m_{t,1}(\bar{S}^2))$. Furthermore, when t decrease, there must exist a time $\hat{t}'$ such that when $t < \hat{t}'$, the untouched segment is empty. In the previous proof when we assume that $\widetilde{S}^2 - L < \bar{s}^c_{t,2} - H$. If $\widetilde{S}^2 - L \geq \bar{s}^c_{t,2} - H$, then this is a special case that the untouched segment becomes empty in period $t-1$. Also if $\widetilde{S}^2 \geq \bar{s}^c_{t,2} - 2H$, this is another special case that the untouched segment becomes empty in period $t-1$. Both these two situations do not affect any previous proof. □

Proof of Proposition 7. Part 1: In any period t, when $\widetilde{S}^2$ is in the untouched segment, we know if the successor's power $S_{t-1} < \widetilde{S}^2$, then the ruler strips the successor at stage 4 of period $t-1$ (Claim 1 and Claim 2). It implies the ruler will not appoint Candidate 1 as the successor in any these period. Since as long as $\widetilde{S}^2$ is in the untouched segment of period t, $V^m_{t,1}(\bar{S}^2)$ increases with decreasing t. From the proof of Proposition A.1, there exists period $\hat{t}_2 \geq \hat{t}$ such that $\widetilde{S}^2$ is not on the untouched segment when $t < \hat{t}_2$, where $\hat{t}$ is the time that the untouched segment becomes an empty set when $t < \hat{t}$.

If $\hat{t}_2$ is the optimal time to appoint Candidate 2 as the successor in the single candidate case, then let $d' = L - p_h(H-L)$. It implies if the successor's power, $S_{\hat{t}_2-1}$, is greater $\widetilde{S}^2 - d'$, then the ruler's expected payoff is greater than stripping the successor and appoint Candidate 2 as the successor in period $\hat{t}_2$. However, for any successor whose power cannot reach $\widetilde{S}^2 - d'$ in period $S_{\hat{t}_2-1}$, he cannot provide better expected payoff to the ruler than choosing Candidate 2. It means for any Candidate 1, if his initial power is less than $\widetilde{S}^2 - (p_h(H-L)+L)\hat{t}_2\hat{t}_2$, then his expected power increase cannot exceeds $\widetilde{S}^2$ in period $\hat{t}_2$. Therefore, to make the space to Candidate 2, an appointed the successor should be removed immediately, otherwise, from the ruler's perspective, the probability of losing the conflict will increase. Then $d_2 = \widetilde{S}^2 - (p_h(H-L)+L)\hat{t}_2$.

In the case that $\hat{t}_2$ is not the optimal time to appoint Candidate 2 as the successor in the single candidate, it implies that the optimal time $t^* < \hat{t}_2$. Let O_v be the segment contains $\widetilde{S}^2$ in period t^*, this segment cannot be the untouched segment. Then there exist at least one segment before O_v, since these segments' slopes must be positive, then the set of S_{t^*-1}

such that $V^m_{t^*,1}(S_{t^*-1}, \bar{S}^2) > V^m_{t^*,1}(\bar{S}^2)$ must non-empty. Then let the distance between the lower bound of this set and $\bar{S}^2$ be d'. After that we can repeat the procedure in the previous paragraph to find d_2. □

Proof of Proposition 8. Part 1: In the single candidate case with only Candidate 1, the proof of Proposition 3 indicates $t'' - 1$ is the first period that the untouched segment does not includes $\widetilde{S}^1$. In the two candidate case, whenever there is a conflict between the ruler and the first successor, the ruler's expected payoff is $-(b + \hat{V}^m_{t,1}(\widetilde{S}^2))S^1_t + \hat{V}^m_{t,1}(\widetilde{S}^2)$, where $\hat{V}^m_{t,1}(\widetilde{S}^2)$ is the ruler's maximal expected payoff in period t when there is one candidate, Candidate 2. We have $\hat{V}^m_{t,1}(\widetilde{S}^2) \leq k_t$. Therefore from the proof of Proposition A.1, we know $\widetilde{S}^1$ will be excluded from the untouched segment with weakly large t. Therefore we have $t''_1 \leq t''$.

Similarly t' is the time that $\widetilde{S}^1$ is included into the segment with from $-(b+\hat{V}^m_{t,1}(\widetilde{S}^2))S + \hat{V}^m_{t,1}(\widetilde{S}^2)$. By the proof of Proposition 3, t'_1 exists, since $-(b + \hat{V}^m_{t,1}(\widetilde{S}^2))S^1_t + \hat{V}^m_{t,1}(\widetilde{S}^2)$ has a more negative slope than $-(b + k_t)S^1_t + k_t$, then we have $t'_1 \leq t'$.

Part 2: Assume t^a is the optimal time to designate Candidate 1 successor in the single candidate case, then if now add Candidate 2 into the game dose not change the optimal designation time t^a. Then we can just follow the proof of Proposition 4, the difference is the lowest challenging threshold $\bar{s}^c_t$ in each period is weakly less than the challenging threshold in the single candidate case. □

Proof of Corollary 3. This corollary can be derived from the proof of Proposition 7. To choose Candidate 2 as the first as successor, first Candidate 2's initial power cannot less than d_2. The proof of Proposition 7 indicates that if Candidate 2 is chosen first, his expected power increase needs to exceed Candidate 1's initial power at the optimal time t^* of choosing Candidate 1 in the single candidate case. If choosing Candidate 1 as the first successor gives the ruler less expected utility at time t^*, then choosing Candidate 2 is still better than choosing 1. If choosing Candidate 1 as the first successor gives the ruler higher expected utility at time t^*, then it implies Candidate 2 has to obtain additional power to making choosing Candidate 2 first the optimal choice. So we can obtain another $\tilde{d}_2$ with the same procedure in the proof of Proposition 8, which is weakly less than d_2. When $\widetilde{S}^1 - \widetilde{S}^2 > \tilde{d}_2$, Candidate 2 has no change to acquire enough power before t^*. □

B Historical Data

The information of monarch-crown prince relationship are available on request from the author