MATHEMATICS - IA

BLUE PRINT

S.NO	NAME OF THE CHAPTER	WEIGHTAGE MARKS
ALGEBRA		
1.	**Functions**	**11 (7+2+2)**
2.	**Mathematical induction**	**07 (7)**
3.	**Matrices**	**22 (7+7+4+2+2)**
VECTOR ALGEBRA		
4.	**Addition of vectors**	**08 (4+2+2)**
5.	**Product of Vectors**	**13 (7+4+2)**
TRIGONOMETRY		
6.	**Trigonometry upto transformations**	**15 (7+4+2+2)**
7.	**Trigonometric equations**	**04 (4)**
8.	**Inverse trigonometric Functions**	**04 (4)**
9.	**Hyperbolic functions**	**02 (2)**
10.	**Properties of triangles**	**11 (7+4)**
	Total marks	**97**

QUESTION BANK ANALYSIS

S.NO	TOPIC NAME	LAQ			SAQ			VSAQ	TOTAL
		***	**	*	***	**	*		
ALGEBRA									
1.	FUNCTIONS	7	3	-	-	-	-	23	33
2.	MATHEMATICAL INDUCTION	11	1	1	-	-	-	-	13
3.	MATRICES	11	3	-	11	3	2	20	50
VECTOR ALGEBRA									
4.	ADDITION OF VECTORS	-	-	-	6	6	-	15	27
5.	PRODUCT OF VECTORS	6	2	2	15	10	5	25	65
TRIGONOMETRY									
6.	TRIGNOMETRY UPTO TRANSFORMATIONS	13	2	5	11	3	4	28	66
7.	TRIGNOMETRIC EQUATIONS	-	-	-	10	7	0	-	17
8.	INVERSE TRIGNOMETRIC FUNCTIONS	-	-	-	11	9	2	-	22
9.	HYPERBOLIC FUNCTIONS	-	-	-	-	-	-	13	13
10.	PROPERTIES OF TRIANGLES	11	8	3	10	6	5	26	69
SUBTOTAL		59	19	11	74	44	18	150	375
TOTAL		89			136				

LONG ANSWER QUESTIONS (7 Marks)

FUNCTIONS

***1. Let $f: A \to B, g: B \to C$ be bijections. Then show that $gof: A \to C$ is a bijection

(March - 2009, May-2006,2008,2010, 2012)

***2. Let $f: A \to B, g: B \to C$ be bijections. Then show that $(gof)^{-1} = f^{-1}og^{-1}$

(March-06,10,11, 2014, May-09,11)

***3. Let $f: A \to B$ be a bijection. Then show that $fof^{-1} = I_B$ and $f^{-1}of = I_A$

(March-07,12, May-2005,2007)

***4. Let $f: A \to B$, I_A and I_B be Identity functions on A and B respectively. Then show that $foI_A = f = I_Bof$ **(March - 2013, May-2005,2008)**

***5. Let $f: A \to B$ be a bijection.Then show that f is a bijection if and only if there exists a function $g: B \to A$ such that $fog = I_B$ and $gof = I_A$ and in this case, $g = f^{-1}$

***6. I) If $f: R \to R, g: R \to R$ are defined by $f(x) = 4x - 1$ and $g(x) = x^2 + 2$ then find

(i) $(gof)(x)$ ii) $(gof)\left(\frac{a+1}{4}\right)$ iii) $fof(x)$ iv) $go(fof)(0)$

II) Let $A = \{1,2,3\}, B = \{a,b,c\}, C = \{p,q,r\}$. If $f: A \to B, g: B \to C$ are defined by $f = \{(1,a),(2,c),(3,b)\}$, $g = \{(a,q),(b,r),(c,p)\}$ then show that $f^{-1}og^{-1} = (gof)^{-1}$

***7. If $f: Q \to Q$ is defined by $f(x) = 5x + 4 \ \forall x \in Q$ then show that f is a bijection and find f^{-1}.

(Mar-2010)

**8. Let $f: A \to B$, $g: B \to C$ and $h: C \to D$. Then show that $ho(gof) = (hog)of$.

**9. If the function f is defined by $f(x) = \begin{cases} x+2, & x > 1 \\ 2, & -1 \le x \le 1 \\ x-1, & -3 < x < -1 \end{cases}$, then find the values of

(a) $f(3)$ (b) $f(0)$ (c) $f(-1.5)$ (d) $f(2) + f(-2)$ (e) $f(-5)$

**10. If the function f is defined by $f(x) = \begin{cases} 3x-2, & x > 3 \\ x^2 - 2, & -2 \le x \le 2 \\ 2x+1, & x < -3 \end{cases}$

Then find the values of

$f(4)$ (**Mar-14**), $f(2.5)$ (**Mar-14**), $f(-2)$, $f(-4)$, $f(0)$, $f(-7)$

MATHEMATICAL INDUCTION

***11. Show that $1^2+\left(1^2+2^2\right)+\left(1^2+2^2+3^2\right)+....$ upto n terms = $\dfrac{n\left(n+1\right)^2\left(n+2\right)}{12}$, $\forall n\in N$

(March-09,12, May-2009)

***12. Show that $\dfrac{1^3}{1}+\dfrac{1^3+2^3}{1+3}+\dfrac{1^3+2^3+3^3}{1+3+5}+....$ upto n terms = $\dfrac{n}{24}\left[2n^2+9n+13\right]$

(March-05,07, 2014)

***13. Show that $\forall n\in N, \dfrac{1}{1.4}+\dfrac{1}{4.7}+\dfrac{1}{7.10}+....$ upto n terms = $\dfrac{n}{3n+1}$

(Mar-06,11, May-2011)

***14. Show that $2.3+3.4+4.5+....$ upto n terms = $\dfrac{n\left(n^2+6n+11\right)}{3}$ $\forall n\in N$

(March - 2013, May-2006)

***15. Show that $\dfrac{1}{1.3}+\dfrac{1}{3.5}+\dfrac{1}{5.7}+...+\dfrac{1}{\left(2n-1\right)\left(2n+1\right)}=\dfrac{n}{2n+1}$, $\forall n\in N$

***16. Show that $1.2.3+2.3.4+3.4.5+....$ upto n terms = $\dfrac{n\left(n+1\right)\left(n+2\right)\left(n+3\right)}{4}$, $\forall n\in N$

***17. Prove by Mathematical induction

$a+(a+d)+(a+2d)+.....$ upto n terms $=\dfrac{n}{2}[2a+(n-1)d]$ **(Mar-2010)**

***18 $a+ar+ar^2+......upto\ \ n\ \ terms=\dfrac{a(r^n-1)}{(r-1)},\ r\neq 1$ **(Mar-11)**

***19. Show that $49^n+16n-1$ divisible by 64 for all positive intergers n. **(May-2005)**

***20. Show that $3.5^{2n+1}+2^{3n+1}$ is divisible by 17, $\forall n\in N$. **(May-08,10,12)**

***21. Use mathematical induction $2.4^{2n+1}+3^{3n+1}$ is divisible by 11

**22. Use mathematical induction to prove the statement

$2+3.2+4.2^2+.....$ up to n terms $=n.2^n$, $\forall n\in N$. (**May-07)**

*23. i) Using mathematical induction, show that x^m+y^m is divisible by $x+y$. If 'm' is an odd natural number and x, y are natural numbers.

ii) If x & y are natural numbers and $x\neq y$ .using mathematical induction. Show that x^n-y^n is divisible by $x-y, \forall n\in N$.

MATRICES

***24. $If \quad A=\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is a non-singular matrix, then show that A is invertible and $A^{-1}=\dfrac{adjA}{\det A}$

(Mar-07, June-10)

***25. Show that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ **(Oct-96)**

***26. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \neq 0$ then show that abc = -1 **(Mar-04, 2014)**

***27. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \left(a^3+b^3+c^3-3abc\right)^2$

(Mar-01,12,May-09)

***28. Show that $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$ **(Mar-09)**

***29. Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$ **(Mar-11,May-11)**

***30. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$ **(Mar-10, June-10)**

***31. Show that $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix} = a^3+b^3+c^3-3abc$ **(Mar-08, May-07)**

***32. Show that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = \left(a-1\right)^3$ **(March-07,13)**

***33. Solve the following simultaneous linear equations by using Cramer's rule **[Mar-12,May-09]** Matrix inversion **[March-2013, May-11,12]** and Gauss -Jordan method **[Mar-09,10,2014 May-2010]**

(i) $3x+4y+5z=18,\ 2x-y+8z=13,\ 5x-2y+7z=20$ **(Mar-08,12,13, May-09)**

(ii) $x+y+z=9,\ 2x+5y+7z=52,\ 2x+y-z=0$ **(Mar-07,09, May-10,11)**

(iii) $2x-y+3z=9,\ x+y+z=6,\ x-y+z=2$ **(Mar-10, 2014)**

***34. Examine whether the following system of equations is consistent or inconsistent. If consistent find the complete solutions.

i) $x+y+z=4,\ 2x+5y-2z=3,\ x+7y-7z=5$

ii) $x+y+z=3,\ 2x+2y-z=3,\ x+y-z=1$ **(June 02)**

iii) $x+y+z=6,\ x-y+z=2,\ 2x-y+3z=9$ **(Mar-05,11)**

35. If $A=\begin{bmatrix}1 & -2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}$ then find A^3-3A^2-A-3I **(Mar-11)

**36. Show that $\begin{vmatrix}-2a & a+b & c+a\\ a+b & -2b & b+c\\ c+a & c+b & -2c\end{vmatrix}=4(a+b)(b+c)(c+a)$

**37. By using Gaus-Jordan method, show that the following system has no solution $2x+4y-z=0, x+2y+2z=5, 3x+6y-7z=2$

PRODUCT OF VECTORS

***38. i)Find the shortest distance between the skew lines **r** = (6**i** + 2**j** + 2 **k**) + t (**i** - 2**j** + 2 **k**) and **r** = (-4 **i** - **k**) + s (3**i** - 2**j** - 2**k**) where s, t are scalars **(March-08,09)**

. ii)If $A=(1,-2,-1), B=(4,0,-3), C=(1,2,-1)$ and $D=(2,-4,-5)$, find the distance between AB and CD. **(March-07,12, 2014)**

***39. Let **a, b, c** be three vectors. Then show that

i) $(\bar{a}\times\bar{b})\times\bar{c}=(\bar{a}.\bar{c})\bar{b}-(\bar{b}.\bar{c})\bar{a}$ ii) $\bar{a}\times(\bar{b}\times\bar{c})=(\bar{a}.\bar{c})\bar{b}-(\bar{a}.\bar{b})\bar{c}$ **(May-2006,09)**

***40. Find the equation of the plane passing to the points A=(2,3,-1), B=(4,5,2) and C=(3,6,5). **(March-10,11)**

***41. A line makes angles $\theta_1, \theta_2, \theta_3$ and θ_4 with the diagonals of a cube. Show that

$$\cos^2\theta_1+\cos^2\theta_2+\cos^2\theta_3+\cos^2\theta_4=\frac{4}{3}$$

***42. Show that in any triangle the altitudes are concurrent. **(March-2013)**

***43. Find the vector equation of the plane passing through the intersection of the planes $\bar{r}.(\bar{i}+\bar{j}+\bar{k})=6$ and $\bar{r}.(2\bar{i}+3\bar{j}+4\bar{k})=-5$ and the point $(1,1,1)$

44. **a, b, c are non-zero vectors and **a** is perpendicular to both **b** and **c**.

If |**a**| = 2, |**b**| = 3, |**c**| = 4 and (**b, c**) = $\frac{2\pi}{3}$, then find |[**a b c**]| **(May-2008)**

45. If [b c d**] + [**c a d**] + [**a b d**] = [**a b c**], then show that the points with position vectors **a, b, c** and **d** are coplanar.

*46. For any four vectors $\bar{a},\bar{b},\bar{c}$ *and* $\bar{d}$, prove that

$$\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)=\left[\bar{a}\,\bar{c}\,\bar{d}\right]\bar{b}-\left[\bar{b}\,\bar{c}\,\bar{d}\right]\bar{a}\ \ and\ \ \left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)=\left[\bar{a}\,\bar{b}\,\bar{d}\right]\bar{c}-\left[\bar{a}\,\bar{b}\,\bar{c}\right]\bar{d}$$

*47. If $\bar{a}=\bar{i}-2\bar{j}+\bar{k},\bar{b}=2\bar{i}+\bar{j}+\bar{k},\bar{c}=\bar{i}+2\bar{j}-\bar{k}$, find $\bar{a}\times\left(\bar{b}\times\bar{c}\right)$ and $\left|\left(\bar{a}\times\bar{b}\right)\times\bar{c}\right|$

TRIGNOMETRY UPTO TRANSFORMATIONS

***48. In triangle ABC, prove that $\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}=4\cos\frac{\pi-A}{4}\cos\frac{\pi-B}{4}\cos\frac{\pi-C}{4}$

(March-07,10, May-07)

***49. If A,B,C are angles of a triangle, then prove that

$$\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$ **(May-06,11)**

***50. If $A+B+C=\pi$, then prove that

$$\cos^2\frac{A}{2}+\cos^2\frac{B}{2}-\cos^2\frac{C}{2}=2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$ **(May-2010)**

***51. If A,B,C are angles in a triangle, then prove that

$$\cos A+\cos B-\cos C=-1+4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$ **(May-2006)**

***52. If A,B,C are angles in a triangle, then prove that

$$\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}=1+4\sin\frac{\pi-A}{4}.\sin\frac{\pi-B}{4}.\sin\frac{\pi-C}{4}$$ **(Mar-11, 2014)**

***53. If $A+B+C=180^0$, then prove that

$$\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=2\left(1+\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$$ **(Mar-12)**

***54. If A,B,C are angles in a triangle, then prove that

$$\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$ **(May-2009)**

***55. In triangle ABC, prove that

$$\cos\frac{A}{2}+\cos\frac{B}{2}-\cos\frac{C}{2}=4\cos\frac{\pi+A}{4}\cos\frac{\pi+B}{4}\cos\frac{\pi-C}{4}$$ **(March-2005)**

***56. If $A+B+C=2S$, then prove that

$$\cos(S-A)+\cos(S-B)+\cos C=-1+4\cos\frac{S-A}{2}\cos\frac{S-B}{2}\cos\frac{C}{2}$$

***57. If $A+B+C=2S$, then prove that

$$\cos(S-A)+\cos(S-B)+\cos(S-C)+\cos S=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

***58. Suppose $(\alpha-\beta)$ is not an odd multiple of $\frac{\pi}{2}$, m is a non zero real number such that $m\neq-1$ and $\frac{\sin(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{1-m}{1+m}$. Then prove that $\tan\left(\frac{\pi}{4}-\alpha\right)=m.\tan\left(\frac{\pi}{4}+\beta\right)$

***59. If $A+B+C=\frac{3\pi}{2}$, prove that $Cos2A+\cos 2B+\cos 2C=1-4\sin A\sin B\sin C$

(Mar 2013)

***60. If none of A,B,A+B is an integral multiple of π, then prove that

$$\frac{1-\cos A+\cos B-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)}=\tan\frac{A}{2}\cot\frac{B}{2}$$

**61. In triangle ABC, prove that

$$\sin\frac{A}{2}+\sin\frac{B}{2}-\sin\frac{C}{2}=-1+4\cos\frac{\pi-A}{4}\cos\frac{\pi-B}{4}\sin\frac{\pi-C}{4}$$

**62. If A,B,C are angles in a triangle, then prove that

$\sin 2A-\sin 2B+\sin 2C=4\cos A\sin B\cos C$

*63. If $A+B+C=90^0$ then show that

(i) $\sin^2 A+\sin^2 B+\sin^2 C=1-2\sin A\sin B\sin C$

(ii) $\sin 2A+\sin 2B+\sin 2C=4\cos A\cos B\cos C$

*64. If $A+B+C=0^0$, then prove that $\cos^2 A+\cos^2 B+\cos^2 C=1+2\cos A\cos B\cos C$

*65. If A,B,C are angles in a triangle, then prove that

$$\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

*66. If $A+B+C=270^0$, then prove that $\cos^2 A+\cos^2 B-\cos^2 C=-2\cos A\cos B\sin C$

*67. If $A+B+C+D=360^0$, prove that $\cos 2A+\cos 2B+\cos 2C+\cos 2D=4\cos(A+B)\cos(A+C)\cos(A+D)$

PROPERTIES OF TRIANGLES

***68. If a = 13, b = 14, c = 15, show that $R=\frac{65}{8}, r=4, r_1=\frac{21}{2}, r_2=12$ and $r_3=14$ **(March-2014),(May-10), (Jun-11)**

***69. i) If $r_1=2, r_2=3, r_3=6$ and $r=1$, Prove that $a=3, b=4$ and $c=5$ **(Mar-2009)**

ii) In ΔABC, if $r_1=8, r_2=12, r_3=24$, find a, b, c

***70. Show that $\frac{r_1}{bc}+\frac{r_2}{ca}+\frac{r_3}{ab}=\frac{1}{r}-\frac{1}{2R}$ **(May-09)**

***71. (i) Show that $r+r_1+r_2-r_3=4R\cos C$ **(Mar-12)**

(ii) Show that $r+r_3+r_1-r_2=4R\cos B$ **(Mar-13)**

***72. In ΔABC, prove that $r_1+r_2+r_3-r=4R$ **(Mar-2006)**

***73. If P_1, P_2, P_3 are the altitudes drawn from vertices A, B, C to the opposite sides of a triangle respectively, then show that

i) $\frac{1}{P_1}+\frac{1}{P_2}+\frac{1}{P_3}=\frac{1}{r}$ ii) $P_1P_2P_3=\frac{(abc)^2}{8R^3}=\frac{8\Delta^3}{abc}$ **(March-2010)**

***74. Show that $\frac{ab-r_1r_2}{r_3}=\frac{bc-r_2r_3}{r_1}=\frac{ca-r_3r_1}{r_2}$ **(March-2008, May-2008)**

***75. Show that $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=2+\frac{r}{2R}$ **(March-2005)**

***76. If $r:R:r_1=2:5:12$, then prove that the triangle is right angled at A. **(May-2007,2009)**

***77. Prove that $a^3\cos(B-C)+b^3\cos(C-A)+c^3\cos(A-B)=3abc$

***78. Show that $a\cos^2\frac{A}{2}+b\cos^2\frac{B}{2}+c\cos^2\frac{C}{2}=s+\frac{\Delta}{R}$ **(Mar-09)**

**79. Show that i) $a=(r_2+r_3)\sqrt{\frac{rr_1}{r_2r_3}}$ ii) $\Delta=r_1r_2\sqrt{\frac{4R-r_1-r_2}{r_1+r_2}}$

**80. In ΔABC show that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$ where R is the circumradius

**81. If $\cos A+\cos B+\cos C=3/2$, then show that the triangle is equilateral

**82. Prove that $\dfrac{\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}}{\cot A+\cot B+\cot C}=\dfrac{(a+b+c)^2}{a^2+b^2+c^2}$

**83. Prove that $\dfrac{r_1(r_2+r_3)}{\sqrt{r_1r_2+r_2r_3+r_3r_1}}=a$

**84. If $a^2+b^2+c^2=8R^2$, then prove that the triangle is right angled.

**85. In ΔABC, show that i) $b^2=c^2+a^2-2ca\cos B$ ii) $c^2=a^2+b^2-2ab\cos C$

iii) $a^2=b^2+c^2-2bc\cos A$

**86. The angle of elevation of the top point P of the vertical tower PQ of height h from a point A is 45^0 and from a point B is 60^0, where B is a point at a distance 30 meters from the point A measured along the line AB which makes an angle 30^0 with AQ. Find the height of the tower.

*87. A lamp post is situated at the middle point M of the side AC of a triangular plot ABC with BC= 7m, CA= 8m and AB= 9m. Lamp post subtends an angle 15^0 at the point B. Find the height of the lamp post.

*88. The upper $\dfrac{3}{4}$ th portion of a vertical pole subtends an angle $\tan^{-1}\dfrac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40m from the foot. Given that the vertical pole is at a height less than 100m from the ground, find its height.

*89. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60^0. He moves away from the pole along the line BC to a point D such that CD= 7m. From D, the angle of elevation of the point A is 45^0. Find the height of the pole.

SHORT ANSWER QUESTIONS (4 Marks)

MATRICES

***01. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ then show that for all positive integers 'n', $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$ **(Nov-98)**

***02. If $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$ then for any integer $n\geq 1$ show that $A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}$

***03. If $\theta-\phi=\frac{\pi}{2}$, then show that $\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}\begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}=0$ **(Mar-04, May-09,12)**

***04. If $3A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ -2 & 2 & -1\end{bmatrix}$ then show that $A^{-1}=A^T$ **(Mar-09, 2014)**

***05. If $A=\begin{bmatrix}2 & -1 & 2\\ 1 & 3 & -4\end{bmatrix}$, $B=\begin{bmatrix}1 & -2\\ -3 & 0\\ 5 & 4\end{bmatrix}$ then verify that $(AB)^{\prime}=B^{\prime}A^{\prime}$ **[March-2013]**

***06. Show that $\begin{vmatrix}y+z & x & x\\ y & z+x & y\\ z & z & x+y\end{vmatrix}=4xyz$

***07. Find the value of x, if $\begin{vmatrix}x-2 & 2x-3 & 3x-4\\ x-4 & 2x-9 & 3x-16\\ x-8 & 2x-27 & 3x-64\end{vmatrix}=0$ **(Mar-06)**

***08. Show that $\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}=(a-b)(b-c)(c-a)$ **(Mar-2005)**

***09. If $A=\begin{bmatrix}2 & 1 & 2\\ 1 & 0 & 1\\ 2 & 2 & 1\end{bmatrix}$ Find the adjoint and inverse of A. **(Mar-05,08)**

***10. If $A=\begin{bmatrix}1 & 2 & 1\\ 3 & 2 & 3\\ 1 & 1 & 2\end{bmatrix}$ then find A^{-1} **(Mar-12)**

***11. If $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $E=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ then show that $(aI+bE)^3 = a^3I + 3a^2bE$ **(Mar-10,J-05)**

**12. If $A=\begin{bmatrix}1 & 4 & 7\\ 2 & 5 & 8\end{bmatrix}$, $B=\begin{bmatrix}-3 & 4 & 0\\ 4 & -2 & -1\end{bmatrix}$ then prove that $(A+B)^T = A^T + B^T$

13. If $A=\begin{bmatrix}2 & -4\\ -5 & 3\end{bmatrix}$ Find $A+A'$, $A.A'$. **(May-07)

14. If A and B are invertible then show that AB is also invertible and $(AB)^{-1} = B^{-1}A^{-1}$ **(Jun-03)

*15. For any nxn matrix. A prove that A can be uniquely expressed as a sum of a symmetric matrix and a skew symmetric matrix.

*16. Show that the determinant of skew-symmetric matrix of order 3 is always zero.

ADDITION OF VECTORS

***17. Let A B C D E F be a regular hexagon with centre 'O'. Show that
AB + **AC** + **AD** + **AE** + **AF** = 3**AD** = 6**AO**. **(May-09,11)**

***18. In ΔABC, if 'O' is the circumcentre and H is the orthocentre, then show that
i) **OA** + **OB** + **OC** = **OH** ii) **HA** + **HB** + **HC** = 2 **HO**

***19. If the points whose position vectors are 3 **i** - 2 **j** - **k**, 2 **i** + 3 **j** - 4 **k**, - **i** + **j** + 2 **k** and 4 **i** + 5 **j** + λ **k** are coplanar, then show that $\lambda = \frac{-146}{17}$

***20 **a, b, c** are non-coplanar vectors. Prove that the following four points are coplanar
i) -**a** + 4**b** - 3**c**, 3**a** + 2**b** - 5**c**, -3**a** + 8**b** - 5**c**, -3**a** + 2**b** + **c** (**May-10)**
ii) 6**a** + 2**b** - **c**, 2**a** - **b** + 3**c**, -**a** + 2**b** - 4**c**, -12**a** - **b** -3**c**

***21. If $\bar{i}, \bar{j}, \bar{k}$ are unit vectors along the positive directions of the coordinate axes, then show that the four points $4\bar{i}+5\bar{j}+\bar{k},\ -\bar{j}-\bar{k}, 3\bar{i}+9\bar{j}+4\bar{k}$ and $-4\bar{i}+4\bar{j}+4\bar{k}$ are coplanar. **(Mar-2014)**

***22. In the two dimensional plane, prove by using vector method, the equation of the line whose intercepts on the axes are 'a' and 'b' is $\frac{x}{a}+\frac{y}{b}=1$ **(May-2005)**

23. Show that the line joining the pair of points 6a** - 4**b** + 4**c**, -4**c** and the line joining the pair of points -**a** - 2**b** - 3**c**, **a** + 2**b** - 5**c** intersect at the point -4**c** when **a, b, c** are non-coplanar vectors

24. Find the vector equation of the plane passing through points 4i** - 3**j** - **k**, 3**i** + 7**j** - 10**k** and 2**i** + 5**j** - 7**k** and show that the point **i** + 2**j** - 3**k** lies in the plane. **[March-2013]**

**25. Find the vector equation of the line parallel to the vector $2\bar{i}-\bar{j}+2\bar{k}$ and passing through the point A whose position vector is $3\bar{i}+\bar{j}-\bar{k}$. If P is a point on this line such that AP=15 then find the position vector of P.

**26. Let $\bar{a},\bar{b}$ be non-collinear vectors. If $\bar{\alpha}=(x+4y)\bar{a}+(2x+y+1)\bar{b}$ and $\bar{\beta}=(y-2x+2)\bar{a}+(2x-3y-1)\bar{b}$ are such that $3\bar{\alpha}=2\bar{\beta}$ then find x *and* y.

**27. If $\bar{a}+\bar{b}+\bar{c}=\alpha\bar{d}$, $\bar{b}+\bar{c}+\bar{d}=\beta\bar{a}$ and $\bar{a},\bar{b},\bar{c}$ are non-coplanar vectors, then show that $\bar{a}+\bar{b}+\bar{c}+\bar{d}=0$

**28. If $\bar{a},\bar{b},\bar{c}$ are non-coplanar vectors, then test for the collinerarity of the following points whose position vectors are given by

i) $\bar{a}-2\bar{b}+3\bar{c},\quad 2\bar{a}+3\bar{b}-4\bar{c},\quad -7\bar{b}+10\bar{c}$

ii) $3\bar{a}-4\bar{b}+3\bar{c},\quad -4\bar{a}+5\bar{b}-6\bar{c},\quad 4\bar{a}-7\bar{b}+6\bar{c}$

iii) $2\bar{a}+5\bar{b}-4\bar{c},\bar{a}+4\bar{b}-3\bar{c},4\bar{a}+7\bar{b}-6\bar{c}$

PRODUCT OF VECTORS

***29. Prove that the smaller angle θ between any two diagonals of a cube is given by $\cos\theta=\frac{1}{3}$

(Mar-10,May-10, Jun-11)

***30. Find the unit vector perpendicular to the plane passing through the points $(1,2,3)$, $(2,-1,1)$ and $(1,2,-4)$. **(May-2010)**

***31. Find the area of the triangle whose vertices are $A(1,2,3)$, $B(2,3,1)$ and $C(3,1,2)$

(March-08, 2014)

***32. Find a unit vector perpendicular to the plane determined by the points $P(1,-1,2)$, $Q(2,0,-1)$ and $R(0,2,1)$

***33. If **a** = 2**i** + 3**j** + 4**k**, **b** = **i** + **j** - **k** and **c** = **i** - **j** + **k**, then compute $\bar{a}\times(\bar{b}\times\bar{c})$ and verify that it is perpendicular to $\bar{a}$.

***34. Find the volume of the tetrahedron whose vertices are (1, 2, 1), (3, 2, 5), (2, -1, 0) and (-1, 0, 1). **(May-2007)**

35. Find the volume of the parallelopiped whose coterminus edges are represented by the vectors ***2i-3j+k, i-j+2k and 2i+j-k.

***36. Determine λ, for which the volume of the parallelopiped having coterminus edges **i** + **j**, 3**i** - **j** and 3**j** + λ **k** is 16 cubic units **(May-2005)**

***37. Find the volume of the tetrahedron having the edges **i** + **j** + **k**, **i** - **j** and **i** + 2 **j** + **k** **(May-2009)**

***38. If **a**=**i**-2**j**-3**k**,**b**=2**i**+**j**-**k** and **c** =**i**+3**j**-2**k**,verify that $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})\neq(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$. **(Mar-08, May-11)**

***39. **a** = 3**i** - **j** + 2**k**, **b** = -**i** + 3**j** + 2**k**, **c** = 4**i** + 5**j** - 2**k** and **d** = **i** + 3**j** + 5**k**, then compute the following i) $\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)$ and ii) $\left(\bar{a}\times\bar{b}\right).\bar{c}-\left(\bar{a}\times\bar{d}\right).\bar{b}$

***40. If $\bar{a}=2\bar{i}+\bar{j}-\bar{k},\ \bar{b}=-\bar{i}+2\bar{j}-4\bar{k},\ \bar{c}=\bar{i}+\bar{j}+\bar{k}$ then find $\left(\bar{a}\times\bar{b}\right).\left(\bar{b}\times\bar{c}\right)$ **(Mar-09)**

***41. Show that angle in a semi circle is a right angle **(May-2008)**

***42. If **a** + **b** + **c** = **0,** |**a**| = 3, |**b**| = 5 and |**c**| = 7, then find the angle between **a** and **b.**

***43. Let **a**=4**i** + 5**j**- **k**, **b**=**i** - 4**j** + 5**k** and **c**=3**i** + **j** - **k.** Find the vector which is perpendicular to both **a** and **b** whose magnitude is twenty one times the magnitude of **c**.

**44. Show that for any two vectors $\bar{a}$ and $\bar{b}$, $\left|\bar{a}\times\bar{b}\right|^2=\left(\bar{a}.\bar{a}\right)\left(\bar{b}.\bar{b}\right)-\left(\bar{a}.\bar{b}\right)^2=\bar{a}^{\,2}\bar{b}^{\,2}-\left(\bar{a}.\bar{b}\right)^2$

45. Show that the points (5, -1, 1), (7, -4, 7), (1, -6, 10) and (-1, -3, 4) are the vertices of a rhombus by vectors **(Mar-2013)

46. Let $\bar{a}$ and $\bar{b}$ be vectors, satisfying $\left|\bar{a}\right|=\left|\bar{b}\right|=5$ and $\left(\bar{a},\bar{b}\right)=45^0$. Find the area of the triangle having $\bar{a}-2\bar{b}$ and $3\bar{a}+2\bar{b}$ as two of its sides **(March-2008)

**47. Find the vector having magnitude $\sqrt{6}$ units and perpendicular to both $2\bar{i}-\bar{k}$ and $3\bar{j}-\bar{i}-\bar{k}$

48. For any three vectors **a, b, c prove that **[b** $\times$ **c c** $\times$ **a a** $\times$ **b] =** $\left[\bar{a}\ \bar{b}\ \bar{c}\right]^2$

49. Let **a, b and **c** be unit vectors such that **b** is not parallel to c and $\bar{a}\times\left(\bar{b}\times\bar{c}\right)=\frac{1}{2}\bar{b}$. Find the angles made by **a** with each of **b** and **c.**

**50. $\bar{A}=\left(1,a,a^2\right),\bar{B}=\left(1,b,b^2\right)$ and $\bar{c}=\left(1,c,c^2\right)$ are non-coplanar vectors and

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$$

, then show that a b c + 1 = 0

**51. $\bar{a},\bar{b}$ and $\bar{c}$ are non-zero and non-collinear vectors and $\theta\neq\{0,\pi\}$ is the angle between $\bar{b}$ and $\bar{c}$. If $\left(\bar{a}\times\bar{b}\right)\times\bar{c}=\frac{1}{3}\left|\bar{b}\right|\left|\bar{c}\right|\bar{a}$, then find $\sin\theta$

**52. If $\bar{a}=2\bar{i}+\bar{j}-3\bar{k},\bar{b}=\bar{i}-2\bar{j}+\bar{k},\bar{c}=-\bar{i}+\bar{j}-4\bar{k}$ and $\bar{d}=\bar{i}+\bar{j}+\bar{k}$ then compute $\left|\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)\right|$

**53. For any two vectors $\bar{a}$ and $\bar{b}$. Then show that

$$\left(1+|\bar{a}|^2\right)\left(1+|\bar{b}|^2\right)=|1-\bar{a}.\bar{b}|^2+|\bar{a}+\bar{b}+\bar{a}\times\bar{b}|^2$$

*54. Show that the points $2\bar{i}-\bar{j}+\bar{k}, \bar{i}-3\bar{j}-5\bar{k}$ and $3\bar{i}-4\bar{j}-4\bar{k}$ are the vertices of a right angled triangle. Also find the other angles.

*55. Show that for any four vectors **a, b, c** and **d** $\left(\bar{a}\times\bar{b}\right).\left(\bar{c}\times\bar{d}\right)=\begin{vmatrix}\bar{a}.\bar{c} & \bar{a}.\bar{d}\\ \bar{b}.\bar{c} & \bar{b}.\bar{d}\end{vmatrix}$

and in particular $\left(\bar{a}\times\bar{b}\right)^2=\bar{a}^2\ \bar{b}^2-\left(\bar{a}.\bar{b}\right)^2$

*56. Show that in any triangle, the perpendicular bisectors of the sides are concurrent.

*57. If **a, b, c** are unit vectors such that **a** is perpendicular to the plane of **b, c** and the angle between **b** and **c** is $\frac{\pi}{3}$, then find $|\bar{a}+\bar{b}+\bar{c}|$

*58. If **a** = (1, -1, -6), **b** = (1, -3, 4) and **c** = (2, -5, 3), then compute the following
i) **a**. (**b** $\times$ **c**) ii) **a** $\times$ (**b** $\times$ **c**) iii) (**a** $\times$ **b**) $\times$ **c**

TRIGNOMETRY UPTO TRANSFORMATIONS

***59. If $A+B=45^0$, then prove that

i) $(1+\tan A)(1+\tan B)=2$ **(May-11)**

ii) $(\cot A-1)(\cot B-1)=2$ **(March-07, May-09)**

iii) If $A-B=\frac{3\pi}{4}$, then show that $(1-TanA)(1+TanB)=2$

***60. Prove that $\frac{Tan\theta+\sec\theta-1}{Tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$ **(March-2014)**

***61. Prove that $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)=\frac{1}{16}$

***62. If A is not an integral multiple of π, prove that $\cos A.\cos 2A.\cos 4A.\cos 8A=\frac{\sin 16A}{16\sin A}$ and hence deduce that $\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}=\frac{1}{16}$ **(March-09,12)**

***63. Let ABC be a triangle such that $\cot A+\cot B+\cot C=\sqrt{3}$. then prove that ABC is an equilateral triangle.

***64. Prove that $\tan 70^0-\tan 20^0=2\tan 50^0$

***65. For $A \in R$, prove that i) $\sin A.\sin(60+A)\sin(60-A)=\frac{1}{4}\sin 3A$

ii) $\cos A.\cos(60+A)\cos(60-A)=\frac{1}{4}\cos 3A$ and hence deduce that

iii) $\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0 = \frac{3}{16}$ iv) $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9}=\frac{1}{16}$

***66. If 3A is not an odd multiple of $\frac{\pi}{2}$, prove that $\tan A.\tan(60+A).\tan(60-A)=\tan 3A$ and hence find the value of $\tan 6^0 \tan 42^0 \tan 66^0 \tan 78^0$

***67. i) Prove that $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}=\frac{3}{2}$

ii) Show that $\cos^2\left(\frac{\pi}{10}\right)+\cos^2\left(\frac{2\pi}{5}\right)+\cos^2\left(\frac{3\pi}{5}\right)+\cos^2\left(\frac{9\pi}{10}\right)=2$

***68. Prove the following

i) $\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}.\cos\frac{8\pi}{7}=\frac{1}{8}$ ii) $\cos\frac{\pi}{11}.\cos\frac{2\pi}{11}.\cos\frac{3\pi}{11}.\cos\frac{4\pi}{11}.\cos\frac{5\pi}{11}=\frac{1}{32}$

***69. Prove that $\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5}=\frac{5}{16}$ **[March-2013]**

**70. If $\sec(\theta+\alpha)+\sec(\theta-\alpha)=2\sec\theta$ and $\cos\alpha \neq 1$, then show that $\cos\theta=\pm\sqrt{2}\cos\frac{\alpha}{2}$

**71. If $\cos x+\cos y=\frac{4}{5}$ and $\cos x-\cos y=\frac{2}{7}$ find the value of $14\tan\frac{x-y}{2}+5\cot\frac{x+y}{2}$

**72. Prove that $\cos^2 76^0+\cos^2 16^0-\cos 76^0\cos 16^0=\frac{3}{4}$

*73. If $0<A<B<\frac{\pi}{4}, \sin(A+B)=\frac{24}{25}, \cos(A-B)=\frac{4}{5}$, find the value of $\tan 2A$

*74. Prove that i) $\sin 18^0=\frac{\sqrt{5}-1}{4}$ (**May-10)** ii) $\cos 36^0=\frac{\sqrt{5}+1}{4}$

*75. Prove that $\sin^2(\alpha-45)^0+\sin^2(\alpha+15)^0-\sin^2(\alpha-15)^0=\frac{1}{2}$

*76. If $\cos n\alpha \neq 0$ and $\cos\frac{\alpha}{2}\neq 0$, then show that

$$\frac{\sin(n+1)\alpha-\sin(n-1)\alpha}{\cos(n+1)\alpha+2\cos n\alpha+\cos(n-1)\alpha}=\tan\frac{\alpha}{2}$$

TRIGONOMETRIC EQUATIONS

***77. Solve the following and write the general solution

i) $2\cos^2\theta - \sqrt{3}\sin\theta + 1 = 0$ **(May-2009)**

ii) $\sqrt{2}\left(\sin x + \cos x\right) = \sqrt{3}$ **(May-12)**

iii) $\tan\theta + 3\cot\theta = 5\sec\theta$

***78. If $\tan\left(\pi\cos\theta\right) = \cot\left(\pi\sin\theta\right)$, then prove that $\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$

***79. If $\tan p\theta = \cot q\theta$, and $p \neq -q$ then show that the solutions are in A.P. with common difference $\frac{\pi}{p+q}$.

***80. If θ_1, θ_2 are solutions of the equation $a\cos 2\theta + b\sin 2\theta = c$, $\tan\theta_1 \neq \tan\theta_2$ and $a + c \neq 0$, then find the values of

i) $\tan\theta_1 + \tan\theta_2$, ii) $\tan\theta_1 . \tan\theta_2$ (iii) $\tan\left(\theta_1 + \theta_2\right)$ **(May-10)**

***81. If α, β are solutions of the Equation

$aCos\theta + bSin\theta = c \quad a,b,c \in R \ and \ a^2 + b^2 > 0, Cos\alpha \neq Cos\beta, S\text{in}\,\alpha \neq Sin\beta$, then show that

$i) Sin\alpha + Sin\beta = \frac{2bc}{a^2 + b^2}$ $ii) Sin\alpha . Sin\beta = \frac{c^2 - a^2}{a^2 + b^2}$

$iii) Cos\alpha + Cos\beta = \frac{2ac}{a^2 + b^2}$ $iv) Cos\alpha . Cos\beta = \frac{c^2 - b^2}{a^2 + b^2}$

***82. Solve i) $\sin 2x - \cos 2x = \sin x - \cos x$

ii) $\sin x + \sqrt{3}\cos x = \sqrt{2}$ **(Mar-10)**

iii) $1 + \sin^2\theta = 3\sin\theta\cos\theta$ **(Mar-11)**

***83. If $0 < \theta < \pi$, solve $\cos\theta . \cos 2\theta \cos 3\theta = \frac{1}{4}$

***84. Solve the equation $\cot^2 x - \left(\sqrt{3} + 1\right)\cot x + \sqrt{3} = 0;\ \left(0 < x < \frac{\pi}{2}\right)$ **(Mar-12, 2014)**

***85. Find all values of x in $\left(-\pi, \pi\right)$ satisfying the equation $8^{1+\cos x+\cos^2 x+.....} = 4^3$ **(Mar-2009)**

***86. Solve $4\,sin\,x\ \ sin\,2x\ \ sin\,4x = sin\,3x$ **(Mar 2013)**

**87. If x is acute and $\sin\left(x + 10^0\right) = \cos\left(3x - 68^0\right)$ find x.

**88. Find the general solution of the equations $\cos ec\theta = -2, \cot\theta = -\sqrt{3}$

**89. Solve $Tan\theta + Sec\theta = \sqrt{3},\quad 0 \leq \theta \leq 2\pi$.

**90. Solve $Cos3x + Cos2x = Sin\frac{3x}{2} + Sin\frac{x}{2} \qquad 0 \leq x \leq 2\pi$

**91. Solve and write the general solution of the equation $4\cos^2\theta+\sqrt{3}=2\left(\sqrt{3}+1\right)\cos\theta$

**92. If $x+y=\frac{2\pi}{3}$ and $\sin x+\sin y=\frac{3}{2}$ then find x and y

**93. Solve $\sin 3\alpha=4\sin\alpha\ \sin\left(x+\alpha\right)\sin\left(x-\alpha\right)$ where $\alpha\neq n\pi, n\in z$

INVERSE TRIGONOMETRIC FUNCTIONS

***94. Prove that $Tan^{-1}\frac{1}{2}+Tan^{-1}\frac{1}{5}+Tan^{-1}\frac{1}{8}=\frac{\pi}{4}$ **(Mar-11) (May-06,10,11)**

***95. Prove that i) $Sin^{-1}\frac{4}{5}+Sin^{-1}\frac{5}{13}+Sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$

ii) $Sin^{-1}\frac{4}{5}+2Tan^{-1}\frac{1}{3}=\frac{\pi}{2}$ **(Mar-10)**

iii) $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{36}{85}$ **(May-09)**

iv) $2Sin^{-1}\left(\frac{3}{5}\right)-Cos^{-1}\frac{5}{13}=Cos^{-1}\left(\frac{323}{325}\right)$ **(March-2014)**

***96. Find the value of $\tan\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]$ **(Mar-12)**

***97. Prove that $Sin^{-1}\left(\frac{4}{5}\right)+Sin^{-1}\left(\frac{7}{25}\right)=Sin^{-1}\left(\frac{117}{125}\right)$ **(Mar-2013)**

***98. If $Sin^{-1}x+Sin^{-1}y+Sin^{-1}z=\pi$, then prove that $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$
(March-2006, May-2005)

***99. If $Cos^{-1}p+Cos^{-1}q+Cos^{-1}r=\pi$, then prove that $p^2+q^2+r^2+2pqr=1$

***100. i) If $Tan^{-1}x+Tan^{-1}y+Tan^{-1}z=\pi$, then prove that $x+y+z=xyz$

ii) If $Tan^{-1}x+Tan^{-1}y+Tan^{-1}z=\frac{\pi}{2}$, then prove that $xy+yz+zx=1$

***101. If $Cos^{-1}\frac{p}{a}+Cos^{-1}\frac{q}{b}=\alpha$, then prove that $\frac{p^2}{a^2}-\frac{2pq}{ab}.\cos\alpha+\frac{q^2}{b^2}=\sin^2\alpha$

***102. Solve the following equations for x.

i) $3Sin^{-1}\frac{2x}{1+x^2}-4Cos^{-1}\frac{1-x^2}{1+x^2}+2Tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$ **(Mar-09)**

ii) $Tan^{-1}\frac{x-1}{x-2}+Tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

***103. Prove that $\cos\left[Tan^{-1}\left\{\sin\left(Cot^{-1}x\right)\right\}\right]=\sqrt{\frac{x^2+1}{x^2+2}}$

***104. Show that $\sec^2\left(Tan^{-1}2\right)+\cos ec^2\left(Cot^{-1}2\right)=10$

**105. Find the value of $\tan\left(2Tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right)$

**106. Prove that $\tan\left(\frac{\pi}{4}+\frac{1}{2}Cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}Cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

**107. Prove that $\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$

**108. Prove that $Cot^{-1}9+Co\sec^{-1}\frac{\sqrt{41}}{4}=\frac{\pi}{4}$

**109. Prove that $\cos\left[2Tan^{-1}\frac{1}{7}\right]=\sin\left[4Tan^{-1}\frac{1}{3}\right]$.

**110. Prove that $Tan^{-1}\frac{3}{4}+Tan^{-1}\frac{3}{5}-Tan^{-1}\frac{8}{19}=\frac{\pi}{4}$.

111. Prove that $\sin\left[\cot^{-1}\frac{2x}{1-x^2}+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]=1$ **(March-2004)

**112. Solve $\tan^{-1}\left[\frac{1}{2x+1}\right]+\tan^{-1}\left[\frac{1}{4x+1}\right]=\tan^{-1}\left[\frac{2}{x^2}\right]$

113. Prove that $\cos^{-1}\frac{4}{5}+\sin^{-1}\frac{3}{\sqrt{34}}=\tan^{-1}\left(\frac{27}{11}\right)$ **(May-2013)

*114. If $sin^{-1}x+sin^{-1}y+sin^{-1}z=\pi$ then prove that

$x^4+y^4+z^4+4x^2y^2z^2=2\left(x^2y^2+y^2z^2+z^2x^2\right)$

*115 (i) Solve arc $\sin\left(\frac{5}{x}\right)+arc\sin\frac{12}{x}=\frac{\pi}{2}\ \left(x>0\right)$

(ii) Solve $\sin^{-1}x+\sin^{-1}2x=\frac{\pi}{3}$

PROPERTIES OF TRIANGLES

***116. If i) $a=\left(b-c\right)\sec\theta$, prove that $\tan\theta=\frac{2\sqrt{bc}}{b-c}\sin\frac{A}{2}$ **(Mar-10,11)**

ii) $a=\left(b+c\right)\cos\theta$, prove that $\sin\theta=\frac{2\sqrt{bc}}{b+c}\cos\frac{A}{2}$ **(May-11)**

iii) $\sin\theta=\frac{a}{b+c}$, prove that $\cos\theta=\frac{2\sqrt{bc}}{b+c}\cos\frac{A}{2}$ **(March-2011,12, May -2011)**

***117. $\cot A+\cot B+\cot C=\frac{a^2+b^2+c^2}{4\Delta}$ **(March-2010)(May-2012)**

***118. Show that $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a^2+b^2+c^2}{2abc}$ **(May-2010)**

***119. Show that $\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\frac{a^2+b^2+c^2}{\Delta^2}$

***120. Show that $\cos A+\cos B+\cos C=1+\frac{r}{R}$

***121. In ΔABC, if $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$, show that $C=60^0$

***122. If $C=60^0$, then show that i) $\frac{a}{b+c}+\frac{b}{c+a}=1$ ii) $\frac{b}{c^2-a^2}+\frac{a}{c^2-b^2}=0$

***123. Show that in ΔABC, $a=b\cos C+c\cos B$ **(May-2009)**

***124. Show that in ΔABC, $\tan\left(\frac{B-C}{2}\right)=\frac{b-c}{b+c}\cot\frac{A}{2}$ **(May-2008)**

***125. Show that $(b-c)^2\cos^2\frac{A}{2}+(b+c)^2\sin^2\frac{A}{2}=a^2$ **(May-2008)**

126. Show that $a^2\cot A+b^2\cot B+c^2\cot C=\frac{abc}{R}$ **(March-2014)

**127. If p_1, p_2, p_3 are the altitudes of the vertices A, B, C of a triangle respectively, show that

$\frac{1}{p_1^2}+\frac{1}{p_2^2}+\frac{1}{p_3^2}=\frac{\cot A+\cot B+\cot C}{\Delta}$ **[Mar-2013]**

**128. If $a:b:c=7:8:9$, find $\cos A:\cos B:\cos C$

**129. If $\cot\frac{A}{2},\cot\frac{B}{2},\cot\frac{C}{2}$ are in A.P., then prove that a, b, c are in A.P.

**130. If $(r_2-r_1)(r_3-r_1)=2r_2r_3$. Show that $A=90^0$

**131. If $\frac{a^2+b^2}{a^2-b^2}=\frac{\sin C}{\sin(A-B)}$, prove that ΔABC is a right angled.

*132. Show that $b^2\sin 2C+c^2\sin 2B=2bc\sin A$.

*133. Show that $(a+b+c)\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)=2c\cot\frac{C}{2}$.

*134. Prove that $\tan\frac{A}{2}+\tan\frac{B}{2}+\tan\frac{C}{2}=\frac{bc+ca+ab-s^2}{\Delta}$

*135. If $\cot\frac{A}{2}:\cot\frac{B}{2}:\cot\frac{C}{2}=3:5:7$, show that $a:b:c=6:5:4$

*136. If $\sin^2\frac{A}{2},\sin^2\frac{B}{2},\sin^2\frac{C}{2}$ are in H.P., then show that a, b, c are in H.P.

VERY SHORT ANSWER QUESTIONS (2 Marks)

FUNCTIONS

1. Find the domain of the following real valued functions

i) $f(x)=\dfrac{1}{6x-x^2-5}$

ii) $f(x)=\sqrt{x^2-1}+\dfrac{1}{\sqrt{x^2-3x+2}}$

iii) $f(x)=\dfrac{1}{\sqrt{|x|-x}}$

iv) $f(x)=\sqrt{x+2}+\dfrac{1}{\log_{10}(1-x)}$

v) $f(x)=\dfrac{\sqrt{3+x}+\sqrt{3-x}}{x}$ **(Mar-2007)**

vi) $f(x)=\sqrt{4x-x^2}$ **(May-10)**

vii) $f(x)=\log\left(x^2-4x+3\right)$ **(Mar-08, 10, May-07)**

viii) $f(x)=\sqrt{x^2-25}$ **(Mar-12)**

ix) $f(x)=\log\left(x-[x]\right)$

x) $f(x)=\dfrac{1}{\left(x^2-1\right)\left(x+3\right)}$ **(March-2014)**

2. If $f=\{(1,2),\ (2,-3),\ (3,-1)\}$ then find

i) 2f ii) $2+f$ iii) f^2 iv) $\sqrt{f}$ **(Mar-08,12)**

3. If $f=\{(4,5),(5,6),(6,-4)\}$ and $g=\{(4,-4),(6,5),(8,5)\}$ then find

i) $f+g$ ii) $f-g$ iii) $2f+4g$ iv) $f+4$ v) fg

vi) f/g vii) $|f|$ viii) $\sqrt{f}$ ix) f^2 x) f^3

4. If f *and* g are real valued functions defined by $f(x)=2x-1$ and $g(x)=x^2$ then find

i) $(3f-2g)(x)$ ii) $(fg)(x)$ iii) $\left(\dfrac{\sqrt{f}}{g}\right)(x)$ iv) $(f+g+2)(x)$ **(Mar-09)**

5. If $f:R\to R,\ \ g:R\to R$ are defined by $f(x)=3x-1,\ \ g(x)=x^2+1$ then find

i) $fof\left(x^2+1\right)$ ii) $fog(2)$ iii) $gof(2a-3)$

6. Find the range of the following real valued functions

i) $\log\left|4-x^2\right|$ ii) $\dfrac{x^2-4}{x-2}$

7. If $f(x)=2, g(x)=x^2, h(x)=2x$ for all $x\in R$, then find $\left(fo(goh)(x)\right)$

8. Find the inverse of the following functions

i) If $a,b\in R,\ f:R\to R$ defined by $f(x)=ax+b\left(a\neq 0\right)$ **(Mar - 2013)**

ii) $f:R\to(0,\infty)$ defined by $f(x)=5^x$ **(Mar-06,11)**

iii) $f:(0,\infty)\to R$ defined by $f(x)=\log_2 x$

9. If $A=\left\{0,\dfrac{\pi}{6},\dfrac{\pi}{4},\dfrac{\pi}{3},\dfrac{\pi}{2}\right\}$ and $f:A\to B$ is a surjection defined by $f(x)=\cos x$ then find B.

(Mar-11,Jun-11)

10. If $A=\{-2,-1,0,1,2\}$ and $f:A\to B$ is a surjection defined by $f(x)=x^2+x+1$, then find B.

(May-2010)

11. If $f(x)=\dfrac{x+1}{x-1}(x\neq 1)$ then find i) $(fofof)(x)$ ii) $(fofofof)(x)$

12. Find the domain and range of the following real valued functions

i) $f(x)=\dfrac{x}{1+x^2}$ ii) $f(x)=\sqrt{9-x^2}$ iii) $f(x)=|x|+|1+x|$

13. If the function $f:R\to R$ defined by $f(x)=\dfrac{3^x+3^{-x}}{2}$, then show that

$f(x+y)+f(x-y)=2f(x)f(y)$

14. If $f:R\to R, g:R\to R$ defined by $f(x)=3x-2, g(x)=x^2+1$, then find

i) $(gof^{-1})(2)$ ii) $(gof)(x-1)$ iii) $(fog)(2)$ **(March - 2013)**

15. Define the following functions and write an example for each
i) One-One (Injection) ii) Onto (Surjection) iii) Even and Odd iv) Bijection

16. If $f:N\to N$ is defined as $f(x)=2x+3$, Is 'f' onto? Explain with reason. **(May-08)**

17. (i)If $f:R\to R$ is defined by $f(x)=\dfrac{1-x^2}{1+x^2}$, then show that $f(\tan\theta)=\cos 2\theta$

(ii) If $f:R-\{\pm 1\}\to R$ is defined by $f(x)=\log\left|\dfrac{1+x}{1-x}\right|$ then show that $f\left(\dfrac{2x}{1+x^2}\right)=2f(x)$

18. If $f(x)=\cos(\log x)$, then show that $f\left(\dfrac{1}{x}\right)f\left(\dfrac{1}{y}\right)-\dfrac{1}{2}\left(f\left(\dfrac{x}{y}\right)+f(xy)\right)=0$

19. If $f(x)=\dfrac{1}{x},\ g(x)=\sqrt{x}$ for all $x\in(0,\infty)$ then find $(gof)(x)$

20. If $f:R-\{0\}\to R$ is defined by $f(x)=x^3-\dfrac{1}{x^3}$, then show that $f(x)+f(1/x)=0$

21. Prove that the real valued function $f(x)=\dfrac{x}{e^x-1}+\dfrac{x}{2}+1$ is an even funciton on $R-\{0\}$.

22. If $A=\{1,2,3,4\}$ and $f:A\to R$ is a function defined by $f(x)=\dfrac{x^2-x+1}{x+1}$, then find the range of 'f'.

23. If $f(x)=\dfrac{\cos^2 x+\sin^4 x}{\sin^2 x+\cos^4 x}$ $\forall x\in R$ then show that $f(2012)=1$.

MATRICES

24. If $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}$ $B=\begin{bmatrix}3&8\\7&2\end{bmatrix}$ and 2X + A = B then find X. **(March-95,11,13)**

25. If $A=\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}$ and $B=\begin{pmatrix}3&2&1\\1&2&3\end{pmatrix}$ find 3B - 2A **(Mar-12)**

26. If $\begin{bmatrix}x-3&2y-8\\z+2&6\end{bmatrix}=\begin{bmatrix}5&2\\-2&a-4\end{bmatrix}$, find x, y, z and a

27. Define trace of a matrix and find the trace of A, if $A=\begin{bmatrix} 1 & 2 & -\frac{1}{2} \\ 0 & -1 & 2 \\ -\frac{1}{2} & 2 & 1 \end{bmatrix}$ **(June-10)**

28. Define symmetric matrix and skew-symmetric matrix **(Mar-05, June 05, May 07)**

29 If $A=\begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$ is a symmetric matrix, find x **(Mar 05)**

30. If $A=\begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ is a skew-symmetric matrix, find the value of x **(May-11)**

31. If $A=\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ and det A = 45, then find x **(Mar-03,07, May-09)**

32. Find the determinant of $\begin{pmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{pmatrix}$ **(Mar-10)**

33. If ω is a complex (non - real) cube root of unity, then show that $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}=0$ **(Mar-11, 2014)**

34. If $A=\begin{pmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{pmatrix}$ and $B=\begin{pmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{pmatrix}$, then find $2A+B^T$ and $3B^T-A$. **(Mar-10)**

35. If $A=\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{pmatrix}$ and $B=\begin{pmatrix} -3 & 4 & 0 \\ 4 & -2 & -1 \end{pmatrix}$ then show that $(A+B)^T=A^T+B^T$ **(May-09)**

36. If $A=\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then show that $AA^1=A^1A=I$ **(Mar-07)**

37. Find the adjoint and the inverse of the matrix $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ **(Mar-09,13)**

38. If $A=\begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2=0$ find the value of k. **(Mar-05, 2014 May-11)**

39. If $A=\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, find A^2 **(Mar 08)**

40. Find the rank of each of the following matrices

i) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **(Mar 08, June-10)** ii) $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ iii) $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$ **(Mar-12)**

iv) $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ v) $\begin{bmatrix} 1 & 0 & -4 \\ 2 & -1 & 3 \end{bmatrix}$

41. Write the definitions of singular and non-singular matrices and give examples.

42. A certain book shop has 10 dozen Chemistry books, 8 dozen Physics books, 10 dozen Economics books. Their selling prices are Rs. 80, Rs.60 and Rs. 40 each respectively. Using matrix algebra, find the total value of the books in the shop.

43. Construct a 3×2 matrix whose elements are defined by $a_{ij} = \frac{1}{2}|i-3j|$

ADDITION OF VECTORS

44. (i) Find the unit vector in the direction of vector $\bar{a} = 2i + 3j + k$. **(March-2014)**
(ii) Let **a** = 2**i** + 4**j** - 5**k**, **b** = **i** + **j** + **k** and **c** = **j** + 2**k**. Find the unit vector in the opposite direction of **a** + **b** + **c** **(Mar-09,10,12)**

45. Show that the points whose position vectors are -2**a** + 3**b** + 5**c**, **a** + 2**b** + 3**c**, 7**a** - **c** are collinear when **a, b, c** are non-coplanar vectors.

46. If the position vectors of the points A, B and C are -2 **i** + **j** - **k**, -4**i** + 2**j** + 2**k** and 6**i** - 3**j** - 13**k** respectively and **AB** = λ **AC**, then find the value of λ **(Mar-11)**

47. If the vectors $-3\vec{i} + 4\vec{j} + \lambda\vec{k}$ and $\mu\vec{i} + 8\vec{j} + 6\vec{k}$ are collinear vectors, then find λ *and* μ. **(March-2014, May-2010)**

48. If $\bar{a} = 2\bar{i} + 5\bar{j} + \bar{k}$ and $\bar{b} = 4\bar{i} + m\bar{j} + n\bar{k}$ are collinear vectors then find the values of m and n **(Jun-11)**

49. If **OA** = **i + j + k, AB =** 3**i** - 2**j** + **k**, **BC** = **i** + 2**j**-2**k** and **CD** = 2**i** + **j** + 3**k**, then find the vector **OD** **(March - 2013)**

50. OABC is a parallelogram. If **OA** = **a** and **OC** = **c**, then find the vector equation of the side BC. **(March-2009)**

51. Find the equation of the plane which passes through the points $2\bar{i} + 4\bar{j} + 2\bar{k}$, $2\bar{i} + 3\bar{j} + 5\bar{k}$ and parallel to the vector $3\bar{i} - 2\bar{j} + \bar{k}$ **(Mar-12)**

52. Find the vector equation of the line joining the points 2**i** + **j** + 3**k** and -4**i** + 3**j** - **k**. **(Mar-11)**

53. Find the vector equation of the line passing through the point $2\bar{i} + 3\bar{j} + \bar{k}$ and parallel to the vector $4\bar{i} - 2\bar{j} + 3\bar{k}$. **(June-10)**

54. Find the vector equation of the plane passing through the points.
$\bar{i} - 2\bar{j} + 5\bar{k},\ -5\bar{j} - \bar{k}$ and $-3\bar{i} + 5\bar{j}$

55. If $\bar{a},\bar{b},\bar{c}$ are the position vectors of the vertices A,B and C respectively of ΔABC then find the vector equations of the median through the vertex A. **(Mar-04,13,May-08)**

56. Is the triangle formed by the vectors 3**i** + 5**j** + 2**k**, 2**i** - 3**j** - 5**k** and -5**i** - 2**j** + 3**k** equilateral?

57. Find the vector equation of the plane passing through the points (0, 0, 0), (0, 5, 0), and (2, 0, 1).

58. ABCDE is a pentagon. If the sum of the vecotrs $\overline{AB}, \overline{AE}, \overline{BC}, \overline{DC}, \overline{ED}$ *and* $\overline{AC}$ is $\lambda\,\overline{AC}$ then find the value of λ.

PRODUCT OF VECTORS

59. If **a** = **i** + 2**j** - 3**k** and **b** = 3 **i** - **j** + 2**k**, then show that **a** + **b** and **a** - **b** are perpendicular to each other. **(May-11)**

60. If the vectors $\lambda\bar{i}-3\bar{j}+5\bar{k}$ and $2\lambda\bar{i}-\lambda\bar{j}-\bar{k}$ are pependicular to each other, find λ

61. If $4\bar{i}+\frac{2p}{3}\bar{j}+p\bar{k}$ is parallel to the vector $\bar{i}+2\bar{j}+3\bar{k}$, find p **(Mar-11)**

62. Find the angle between the vectors $i+2j+3k$ and $3i-j+2k$. **(March-10, 2014)**

63. Find the cartesian equation of the plane through the point $A(2,-1,-4)$ and parallel to the plane $4x-12y-3z-7=0$

64. Find the angle between the planes **r**. (2**i** - **j** + 2 **k**) = 3 and **r**. (3**i** +6 **j** + **k**) = 4.

65. Find the area of the parallelogram having $2\bar{i}-3\bar{j}$ and $3\bar{i}-\bar{k}$ as adjacent sides. **(May 12)**

66. Let **a** = **i** + **j** + **k** and **b** = 2 **i** + 3 **j** + **k** find
i) The projection vector of **b** and **a** and its magnitude
ii) The vector components of **b** in the direction of **a** and perpendicular to **a**

67. If **a** =2**i**+2**j**-3**k** , **b** = 3**i** - **j** + 2**k**, then find the angle between 2**a** + **b** and **a** + 2**b**

68. If |**a**| = 2, |**b**| = 3 and |**c**| = 4 and each of **a, b, c** is perpendicular to the sum of the other two vectors, then find the magnitude of **a** + **b** + **c**

69. Find the unit vector perpendicular to the plane determined by the vectors
$\bar{a}=4\bar{i}+3\bar{j}-\bar{k},\ \bar{b}=2\bar{i}-6\bar{j}-3\bar{k}$ **(May-09)**

70. If $\bar{a}=2i-j+k$ and $\bar{b}=i-3j-5k$, then find $\left|\bar{a}\times\bar{b}\right|$ **(Mar-2013)**

71. If **a** = 2**i** - 3**j** + 5**k** , **b** = -**i** + 4**j** + 2**k** then find $\bar{a}\times\bar{b}$ and unit vector perpendicular to both **a** and **b.**

72. Let **a** =2**i**-**j**+**k** and **b** = 3**i**+4 **j**-**k**. If θ is the angle between **a** and **b**, then find $\sin\theta$

73. For any vector **a**, show that $\left|\bar{a}\times\bar{i}\right|^2+\left|\bar{a}\times\bar{j}\right|^2+\left|\bar{a}\times\bar{k}\right|^2=2\left|\bar{a}\right|^2$

74. If $\left|\bar{p}\right|=2,\left|\bar{q}\right|=3$ and $\left(\bar{p},\bar{q}\right)=\frac{\pi}{6}$, then find $\left|\bar{p}\times\bar{q}\right|^2$

75. Compute $\bar{a}\times\left(\bar{b}+\bar{c}\right)+\bar{b}\times\left(\bar{c}+\bar{a}\right)+\bar{c}\times\left(\bar{a}+\bar{b}\right)$

76. Find the area of the parallelogram having $\bar{a}=2\bar{j}-\bar{k}$ and $\bar{b}=-\bar{i}+\bar{k}$ as adjacent sides

77. Find the area of the parallelogram whose diagonals are $3\bar{i}+\bar{j}-2\bar{k}$ and $\bar{i}-3\bar{j}+4\bar{k}$

78. If the vectors **a** = 2**i** - **j** + **k**, **b** = **i** + 2**j** - 3**k** and **c** = 3**i** + p **j** + 5**k** are coplanar, then find **p**

79. Show that $\bar{i}\times\left(\bar{a}\times\bar{i}\right)+\bar{j}\times\left(\bar{a}\times\bar{j}\right)+\bar{k}\times\left(\bar{a}\times\bar{k}\right)=2\bar{a}$ for any vector $\bar{a}$

80. Prove that for any three vectors **a, b, c, [b + c c + a a + b] = 2 [a b c]**

81. Compute $\left[\bar{i}-\bar{j}\quad \bar{j}-\bar{k}\quad \bar{k}-\bar{i}\right]$

82. Let **b**=2**i**+**j**-**k**,**c**=**i**+3**k**.If **a** is a unit vector then find the maximum value of [**a b c**]

83. If $\frac{1}{2}\left|\bar{e}_1-\bar{e}_2\right|=\sin\lambda\theta$ where $\bar{e}_1$ and $\bar{e}_2$ are unit vectors including an angle θ, show that $\lambda=\frac{1}{2}$.

TRIGNOMETRY UPTO TRANSFORMATIONS

84. If $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$, prove that $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$ **(Mar-09,Jun-11)**

85. If $3\sin\theta+4\cos\theta=5$, then find the value of $4\sin\theta-3\cos\theta$ **(Mar-12)**

86. Prove that $\cot\frac{\pi}{20}.\cot\frac{3\pi}{20}.\cot\frac{5\pi}{20}.\cot\frac{7\pi}{20}.\cot\frac{9\pi}{20}=1$ **(March-2005)**

87. Find the period of the following functions

 i) $f\left(x\right)=\tan 5x$ ii) $f\left(x\right)=\cos\left(\frac{4x+9}{5}\right)$ **(March-2005, 2014, May-10)**

 iii) $f\left(x\right)=\left|\sin x\right|$

 iv) $f\left(x\right)=\cos^4 x$

 v) $f\left(x\right)=2\sin\frac{\pi x}{4}+3\cos\frac{\pi x}{3}$

 vi) $f(x)=\tan\left(x+4x+9x+...+n^2x\right)$ (n is any positive integer).

 vii) $f(x)=\cos(3x+5)+7$

88. Prove that $\cos 12^0+\cos 84^0+\cos 132^0+\cos 156^0=-\frac{1}{2}$

89. Prove that $\cos 100^0\cos 40^0+\sin 100^0\sin 40^0=\frac{1}{2}$

90. Find the value of $\cos 42^0+\cos 78^0+\cos 162^0$ **(May-11)**

91. Find the maximum and minimum values of the following functions over R

i) $f(x)=7\cos x-24\sin x+5$

ii) $f(x)=\sin 2x-\cos 2x$

iii) $\cos\left(x+\frac{\pi}{3}\right)+2\sqrt{2}\sin\left(x+\frac{\pi}{3}\right)-3$ **(March-2009)**

iv) $f(x)=13\cos x+3\sqrt{3}\sin x-4$

v) $f(x)=3\sin x-4\cos x$ **(March-2014)**

92. Find the value of

i) $\sin^2 82\frac{1}{2}^0-\sin^2 22\frac{1}{2}^0$ ii) $\cos^2 112\frac{1}{2}^0-\sin^2 52\frac{1}{2}^0$

iii) $\sin^2 52\frac{1}{2}^0-\sin^2 22\frac{1}{2}^0$

93. Prove that $\frac{1}{\sin 10^0}-\frac{\sqrt{3}}{\cos 10^0}=4$

94. If $\sec\theta+\tan\theta=\frac{2}{3}$, find the value of $\sin\theta$ and determine the quadrant in which θ lies.

95. Show that $\cos^4\alpha+2\cos^2\alpha\left(1-\frac{1}{\sec^2\alpha}\right)=1-\sin^4\alpha$

96. Prove that $(\tan\theta+\cot\theta)^2=\sec^2\theta+\cos ec^2\theta=\sec^2\theta.\cos ec^2\theta$

97. If $\frac{2\sin\theta}{1+\cos\theta+\sin\theta}=x,$ find the value of $\frac{1-\cos\theta+\sin\theta}{1+\sin\theta}$

98. i) If $\tan 20^0=p$, then prove that $\frac{\tan 610^0+\tan 700^0}{\tan 560^0-\tan 470^0}=\frac{1-p^2}{1+p^2}$

ii) If $Tan20^0=\lambda$, then show that $\frac{Tan160^0-Tan110^0}{1+Tan160^0 Tan110^0}=\frac{1-\lambda^2}{2\lambda}$

99. i) Draw the graph of $y=\tan x$ in between $\left[0,\frac{\pi}{4}\right]$

ii) Draw the graph of $y=\cos^2 x$ in $[0,\pi]$

iii) Draw the graph of $y=\sin 2x$ in $(0,\pi)$

100. If θ is not an integral multiple of $\frac{\pi}{2}$, prove that $\tan\theta+2\tan 2\theta+4\tan 4\theta+8\cot 8\theta=\cot\theta$

101. Prove that $4\left(\cos 66^0+\sin 84^0\right)=\sqrt{3}+\sqrt{15}$

102. Prove that $\cos 20^0\cos 40^0-\sin 5^0\sin 25^0=\frac{\sqrt{3}+1}{4}$

103. If A, B, C are angles of a triangle and if none of them is equal to $\frac{\pi}{2}$, then prove that $\tan A + \tan B + \tan c = \tan A \tan B \tan C$

104. If $\sin\theta = -\frac{1}{3}$ and θ does not lie in the third quadrant. Find the value of $\cos\theta$. **(Mar-13)**

105. Find the cosine function whose period is 7 **(Mar-13)**

106. Find a sine function whose period is $\frac{2}{3}$

107. Prove that $\frac{\cos 9^0 + \sin 9^0}{\cos 9^0 - \sin 9^0} = \cot 36^0$ **(Mar-11)**

108. If $\cos\theta = -\frac{5}{13}$ and $\frac{\pi}{2} < \theta < \pi$ then find $\sin 2\theta$.

109. For what values of x in the first quadrant $\frac{2\tan x}{1-\tan^2 x}$ is positive?

110. If $0 < \theta < \frac{\pi}{8}$, show that $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4\theta}}} = 2\cos\frac{\theta}{2}$

111. Prove that $\tan\alpha = \frac{\sin 2\alpha}{1+\cos 2\alpha}$ and hence deduce the values of $\tan 15^0$ and $\tan 22\frac{1}{2}^0$

HYPERBOLIC FUNCTIONS

112. If $\cosh x = \frac{5}{2}$, find the values of (i) $\cosh(2x)$ and (ii) $\sinh(2x)$ **(Mar-10,11)(May-06,11)**

113. $\sinh x = \frac{3}{4}$, find $\cosh(2x)$ and $\sinh(2x)$ **(Mar-12, 2014, May-09)**

114. If $\cosh x = \sec\theta$ then prove that $\tanh^2 x/2 = \tan^2\theta/2$ **(Mar -2013)**

115. For $x, y \in R$ i) $\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$
ii) $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

116. Prove that
i) $(\cosh x - \sinh x)^n = \cosh(nx) - \sinh(nx)$, for any $n \in R$ **(March-06,07)**
ii) $(\cosh x + \sinh x)^n = \cosh(nx) + \sinh(nx)$, for any $n \in R$

117. For any $x \in R$, Prove that $\cosh^4 x - \sinh^4 x = \cosh(2x)$

118. *If* $\theta \in \left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$ *and* $x = \log\left[\cot\left(\frac{\pi}{4}+\theta\right)\right]$, prove that $\sinh x = -\tan 2\theta$ and $\cosh x = \sec 2\theta$

119. If $u = \log_e\left(\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right)$ and if $\cos\theta > 0$, then prove that $\cosh u = \sec\theta$.

120. Prove that $\tanh(x-y) = \frac{\tanh x - \tanh y}{1 - \tanh x.\tanh y}$

121. Prove that $\dfrac{\cosh x}{1-\tanh x}+\dfrac{\sinh x}{1-\coth x}=\sinh x+\cosh x$, for $x\neq 0$.

122. Theorem : for $x\in(-1,1), \tanh^{-1}(x)=\dfrac{1}{2}\log_e\left(\dfrac{1+x}{1-x}\right)$

123. Show that $\tanh^{-1}\left(\dfrac{1}{2}\right)=\dfrac{1}{2}\log_e 3$ **(Mar-2005, 2007, May-2005,2007)**

124. If $\sinh x=5$, show that $x=\log_e\left(5+\sqrt{26}\right)$

PROPERTIES OF TRIANGLES

(NOT GIVEN IN EXAMS BUT THE QUESTIONS USEFUL TO SOLVE LAQ'S & SAQ'S)

125. In an equilateral triangle, find the value of r/R.

126. If the lengths of the sides of a triangle are 3, 4, 5, find the circumradius of the triangle

127. In ΔABC, show that $\sum(b+c)\cos A=2s$

128. If the sides of a triangle are 13, 14, 15, then find the circum diameter

129. In ΔABC, if $(a+b+c)(b+c-a)=3bc$, find A **(March-2008)**

130. In ΔABC, find $b\cos^2\dfrac{C}{2}+c\cos^2\dfrac{B}{2}$ **(Mar-10,12)**

131. If $\tan\dfrac{A}{2}=\dfrac{5}{6}$ and $\tan\dfrac{C}{2}=\dfrac{2}{5}$, determine the relation between a, b, c **(May-2005)**

132. If $\cot\dfrac{A}{2}=\dfrac{b+c}{a}$, find angle B

133. In ΔABC, express $\sum r_1\cot\left(\dfrac{A}{2}\right)$ in terms of s. **(May-06,11)**

134. Show that $\dfrac{c-b\cos A}{b-c\cos A}=\dfrac{\cos B}{\cos C}$

135. If $a=\sqrt{3}+1 cms., \underline{|B}=30^0, \underline{|C}=45^0$, then find c

136. If a = 26 cms., b = 30 cms. and $\cos C=\dfrac{63}{65}$, then find c. **(Mar-11)**

137. If $a=6,\ b=5,\ c=9$ then find angle A. **(May-10)**

138. If $a=4,\ b=5,\ c=7$ then find $\cos\dfrac{B}{2}$

139. If the angles are in the ratio 1 : 5 : 6, then find the ratio of its sides **(May-2007)**

140. Prove that $\dfrac{a^2+b^2-c^2}{c^2+a^2-b^2}=\dfrac{\tan B}{\tan C}$

141. Prove that $(b-a\cos C)\sin A=a\cos A\sin C$ **(March-2006)**

142. If $\dfrac{a}{\cos A}=\dfrac{b}{\cos B}=\dfrac{c}{\cos C}$, then show that ΔABC is equilateral **(March-2009)**

143. In ΔABC, prove that $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{1}{r}$

144. Show that $r\,r_1\,r_2\,r_3=\Delta^2$

145. In $\Delta ABC, \Delta = 6$ sq.cm and s = 1.5 cm., find r.

146. If $rr_2 = r_1r_3$, then find B

147. If $A=90^0$, show that $2(r+R)=b+c$

148. Show that $\sum\frac{r_1}{(s-b)(s-c)}=\frac{3}{r}$

149. Show that $a^2\sin 2C + c^2\sin 2A = 4\Delta$

150. If $a\cos A = b\cos B$, prove that the triangle is either isosceles or right angled.

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