JUNIOR IPE IMPORTANT QUESTION BANK
MATHEMATICS – IB

BLUE PRINT		
S.NO	NAME OF THE CHAPTER	WEIGHTAGE MARKS
CO-ORDINATE GEOMETRY		
1.	Locus	04 (4)
2.	Transformation of axes	04 (4)
3.	Straight line	15 (7+4+2+2)
4.	Pair of straight lines	14 (7+7)
3D GEOMETRY		
5.	3D – coordinates	02 (2)
6.	Direction Cosines & Direction Ratios	07 (7)
7.	The Plane	02 (2)
CALCULUS		
8.	Limits & Continuity	08 (4+2+2)
9.	Differentiation	15 (7+4+2+2)
10.	Errors – Approximations	02 (2)
11.	Tangent & Normal	11 (7+4)
12.	Rate measure	04 (4)
13.	Rolle's & Lagrange's Theorems	02 (2)
14.	Maxima & Minima	07 (7)
	Total marks	**97**

QUESTION BANK ANALYSIS									
S.NO	TOPIC NAME	LAQ			SAQ			VSAQ	TOTAL
		***	**	*	***	**	*		
CO-ORDINATE GEOMETRY									
1	LOCUS				8	3	2		13
2	TRANSFORMATIONS OF AXES				6	1	1		8
3	STRAIGHT LINES	12	-	2	13	2	10	30	69
4	PAIR OF STRAIGHT LINES	12	5	5					22
3D GEOMETRY									
5	3D CO-ORDINATES							14	14
6	DIRECTION COSINES & DIRECTION RATIOS	4	2	2					8
7	PLANES							13	13
CALCULUS									
8	LIMITS							35	35
9	CONTINUITY				4	1	3		8
10	DIFFERENTIATIONS	7	7	2	7	7	7	39	76
11	ERRORS & APPROXIMATIONS							14	14
12	TANGENT & NORMALS	9	6	3	9	2	4		33
13	RATE MEASURE				8	2	3		13
14	ROLLE'S & LAGRANGE'S THEOREMS							13	13
15	MAXIMA & MINIMA	4	3	3					10
	SUBTOTAL	48	23	17	55	18	30	158	349
	TOTAL	88			103				

LONG ANSWER QUESTIONS (7 Marks)

STRAIGHT LINES

1***. Find the circumcentre of the triangle with the vertices(-2, 3), (2, -1) and (4, 0). **(Mar-11)**

2***. Find the circumcentre of the triangle whose vertices are (1, 3), (0, -2) and (-3, 1).

3***. Find the orthocentre of the triangle with the vertices(-2,-1), (6,-1) and (2,5). **(Mar-04,07)**

4***. Find the orthocentre of the triangle with the vertices(-5,-7), (13, 2) and (-5, 6). **(Mar-12)**

5***. Find the circumcentre of the triangle whose sides are $3x-y-5=0, x+2y-4=0$ and $5x+3y+1=0$.**(June 2005, Mar-2006)**

6***. Find the circumcentre of the triangle whose sides are $x+y+2=0$, $5x-y-2=0$ and $x-2y+5=0$. **(March-2014)**

7***. Find the orthocentre of the triangle formed by the lines $x+2y=0$, $4x+3y-5=0$ and $3x+y=0$. **(Mar-10)**

8***. If the equations of the sides of a triangle are $7x+y-10=0$, $x-2y+5=0$ and $x+y+2=0$. Find the orthocentre of the triangle. **(May 2009)**

9***. If Q(h,k) is the image of the point P (x_1, y_1) w.r.t. the straight line ax + by + c = 0. Then $(h-x_1):a=(k-y_1):b=-2(ax_1+by_1+c):a^2+b^2$ **(or)** $\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}$ and find the image of (1, -2) w.r.t. The straight line 2x-3y+5=0. **(Mar-2013, May-2004)**

10***. If Q(h, k) is the foot of the perpendicular from $P(x_1, y_1)$ on the line ax + by + c = 0, then prove that $(h-x_1):a=(k-y_1):b=-(ax_1+by_1+c):a^2+b^2$ (or) $\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}$. Also find the foot of the perpendicular from (-1,3) on the line 5x − y − 18=0 . **(May 2007)**

11***. If p and q are the lengths of the perpendiculars from the origin to the straight lines xsec α +ycosec α =a and xcos α −ysin α =acos2 α ,prove that $4p^2+q^2=a^2$ **(Mar-08)**

12***. Find the equations of the straight lines passing through the point of intersection of the lines 3x+2y+4=0,2x+5y=1 and whose distance from (2,-1) is 2. **(Mar- 09,May-09)**

13*. Show that the origin is with in the triangle whose angular points are (2, 1) (3, -2) and (-4, -1)

14* Find the equation of the straight lines passing through $(1,2)$ and making an angle of 60^0 with the line $\sqrt{3}x+y+2=0$. **(Board Paper)**

PAIR OF STRAIGHT LINES

15.*** Find the values of k, if the lines joining the origin to the points of intersection of the curve $2x^2-2xy+3y^2+2x-y-1=0$ and the line x + 2y = k are mutually perpendicular.
(Mar-12, Mar-11, May-10, Mar 2005, June 2005, Mar 2006, May 2007,)

16.*** Find the angle between the lines joining the origin to the points of intersection of the curve $x^2+2xy+y^2+2x+2y-5=0$ and the line 3x−y+1 = 0. **(Mar-13,May-11,June-04,Mar07,09)**

17.*** Show that the lines joining the origin to the points of intersection of the curve $x^2-xy+y^2+3x+3y-2=0$ and the straight line $x-y-\sqrt{2}=0$ are mutually perpendicular.
(Mar-08,12)

18.*** Find the condition for the lines joining the origin to the points of intersection of the circle $x^2+y^2=a^2$ and the line lx + my = 1 to coincide.

19.*** Find the condition for the chord lx+my=1 of the circle $x^2+y^2=a^2$ (whose center is the origin) to subtend a right angle at the origin.**(Mar-2014)**

20.*** Let the equation $ax^2+2hxy+by^2=0$ represents a pair of straight lines. Then the angle θ between the lines is given by $\cos\theta=\dfrac{a+b}{\sqrt{(a-b)^2+4h^2}}$. **(Mar-11)**

21.*** Show that the product of the perpendicular distances from a point (α,β) to the pair of straight lines $ax^2+2hxy+by^2=0$ is $\dfrac{|a\alpha^2+2h\alpha\beta+b\beta^2|}{\sqrt{(a-b)^2+4h^2}}$
(May-11,Mar-04, 07, May-07, 2008)

22.*** If the equation $ax^2+2hxy+by^2=0$ represents a pair of distinct (i.e., intersecting) lines, then the combined equation of the pair of bisectors of the angles between these lines is $h(x^2-y^2)=(a-b)xy$ **(Mar 2009, May 2009)**

23.*** Show that the area of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my+n=0$ is $\dfrac{n^2\sqrt{h^2-ab}}{|am^2-2hlm+bl^2|}$ sq.units. **(Mar-13, May-10)**

24.*** If the equation $S\equiv ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of parallel straight lines, then show that $(i)\, h^2=ab$ $(ii)\, af^2=bg^2$ and (iii) the distance between the parallel lines is $2\sqrt{\dfrac{g^2-ac}{a(a+b)}}=2\sqrt{\dfrac{f^2-bc}{b(a+b)}}$. **(March-10,12, May 2006)**

25.*** If the second degree equation $S\equiv ax^2+2hxy+by^2+2gx+2fy+c=0$ in two variables x and y represents a pair of straight lines, then
i) $abc+2fgh-af^2-bg^2-ch^2=0$ and ii) $h^2\ge ab, g^2\ge ac\ and\ f^2\ge bc$ **[Mar-12,14]**

26.*** Find the centroid and area of the triangle formed by the lines $12x^2 - 20xy + 7y^2 = 0$ and $2x - 3y + 4 = 0$. **(Mar 2005)**

27.** Show that the straight lines represented by $(x+2a)^2 - 3y^2 = 0$ and $x = a$ from an equilateral triangle.

28.** Show that the pairs of straight lines $6x^2 - 5xy - 6y^2 = 0$ and $6x^2 - 5xy - 6y^2 + x + 5y - 1 = 0$ forms a square.

29**. If the equation $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines then find λ **(May-09)**

30.** Show that the product of the perpendicular distances from the origin to the pair of straight lines represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\frac{|c|}{\sqrt{(a-b)^2 + 4h^2}}$

31.** Show that the equation $2x^2 - 13xy - 7y^2 + x + 23y - 6 = 0$ represents a pair of straight lines. Also find the angle between them and the coordinates of the point of intersection of the lines. **[Mar-04]**

32.* Show that the straight lines represented by $3x^2 + 48xy + 23y^2 = 0$ and $3x - 2y + 13 = 0$ form an equilateral triangle of area $13/\sqrt{3}$ sq.units.

33.* Show that the lines represented by $(lx+my)^2 - 3(mx-ly)^2 = 0$ *and* $lx + my + n = 0$ forms an equilateral triangle with area $\frac{n^2}{\sqrt{3}(l^2+m^2)}$ square units.

34.* If (α, β) is the centroid of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$, prove that $\frac{\alpha}{bl - hm} = \frac{\beta}{am - hl} = \frac{2}{3(bl^2 - 2hlm + am^2)}$.

35.* Find the value of k, if the equation $2x^2 + kxy - 6y^2 + 3x + y + 1 = 0$ represents a pair of straight lines. Find the point of intersection of the lines and the angle between the straight lines for this value of 'k'.

36*. Find the centroid and the area of the triangle formed by the following lines

i) $2y^2 - xy - 6x^2 = 0$; $x + y + 4 = 0$

ii) $3x^2 - 4xy + y^2 = 0$; $2x - y = 6$

DIRECTION COSINES AND DIRECTION RATIOS

37.*** If a ray makes the angles α, β, γ and δ with four diagonals of a cube then find $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$. **(Mar 2005, June 2005, Mar 2008, May 2008)**

38.*** Find the angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$, $l^2 + m^2 - n^2 = 0$ **(Mar-07,11,13) (June 2004, May 2007)**

39.*** Find the angle between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$ **(May-06,10, Mar-06,09,10)**

40.*** Show that direction cosines of two lines which are connected by the relations $l + m + n = 0$ and $2mn + 3nl - 5lm = 0$ are perpendicular to each other. **(Mar-12)**

41.** Find the angle between two diagonals of a cube.

42.** Find the direction cosines of two lines which are connected by the relations $l+m+n=0$ and $mn-2nl-2lm=0$

43.* Find the direction cosines of two lines which are connected by the relations $l-5m+3n=0$ and $7l^2+5m^2-3n^2=0$ **(May 2007)**

44.* The vertices of triangle are A(1,4,2),B(-2,1,2).C(2,3-4)).Find $\underline{|A}, \underline{|B}, \underline{|C}$. **(March-2014)**

DIFFERENTIATION

45.*** If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ then show that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$. **(Mar 2005, 2008,2011)**

46.*** If $y=Tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$ for $0<|x|<1$, find $\frac{dy}{dx}$ **(Mar-04,09,10,12)**

47.*** If $y=x^{\tan x}+(\sin x)^{\cos x}$, find $\frac{dy}{dx}$. **(May 2006, Mar-07,11,2014)**

48.*** Find the derivative of $(\sin x)^{\log x}+x^{\sin x}$ with respective x. **(Mar-2013)**

49.*** If $x^y+y^x=a^b$ then show that $\frac{dy}{dx}=-\left[\frac{yx^{y-1}+y^x\log y}{x^y\log x+xy^{x-1}}\right]$. **(Mar-11)**

50.*** If $y=x\sqrt{a^2+x^2}+a^2\log\left(x+\sqrt{a^2+x^2}\right)$ then prove that $\frac{dy}{dx}=2\sqrt{a^2+x^2}$ **(M.08,M.09)**

51.*** If $a>0, b>0$ and $0<x<\pi$ and

$f(x)=(a^2-b^2)^{-1/2}\cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$ then $f^1(x)=(a+b\cos x)^{-1}$ **(May-09)**

52.** If $x=\frac{3at}{1+t^3}, y=\frac{3at^2}{1+t^3}$. Find $\frac{dy}{dx}$ **(Board paper)**

53**. If $f(x)=S\text{in}^{-1}\sqrt{\frac{x-\beta}{\alpha-\beta}}$ and $g(x)=Tan^{-1}\sqrt{\frac{x-\beta}{\alpha-x}}$ then show that

$f'(x)=g'(x)(\beta<x<\alpha)$. **(Mar 2006)**

54.** If $y=Tan^{-1}\left(\frac{2x}{1-x^2}\right)+Tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)-Tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)$ then show that $\frac{dy}{dx}=\frac{1}{1+x^2}$ **.(May07)**

55.** Find the derivative of $f(x)=Tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r .to $g(x)=Sin^{-1}\left(\frac{2x}{1+x^2}\right)$ **(Mar 2004)**

56.** If $x^y=y^x$ then show that $\frac{dy}{dx}=\frac{y(x\log y-y)}{x(y\log x-x)}$.

57.** If $x^{\log y}=\log x$ then show that $\frac{dy}{dx}=\frac{y}{x}\frac{[1-\log x\log y]}{\log_x^2}$

58.** If $ax^2+2hxy+by^2=1$ then prove that $\frac{d^2y}{dx^2}=\frac{h^2-ab}{(hx+by)^3}$

59.* Find $\frac{dy}{dx}$ of the function $y = \frac{(1-2x)^{\frac{2}{3}}(1+3x)^{\frac{-3}{4}}}{(1-6x)^{\frac{5}{6}}(1+7x)^{\frac{-6}{7}}}$ **(May-10)**

60.* Find derivatives of the $\sin^{-1}\left(\frac{b+a\sin x}{a+b\sin x}\right)(a>0,\, b>0)$

TANGENTS AND NORMALS

61.*** If the tangent at any point P on the curve $x^m y^n = a^{m+n}\ (mn \neq 0)$ meets the coordinate axes in A and B then show that AP : BP is a constant. **(March-10, May 2006, 2008)**

62.*** If the tangent at any point on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ intersects the coordinate axes in A and B, then show that the length AB is a constant. **(May-10, Mar 2006, 2007, 2008, 2013,2014)**

63.*** Show that curves $y^2 = 4(x+1)$ and $y^2 = 36(9-x)$ intersect orthogonally.**(Mar-06,09,11)**

64.*** Find the angle between the curves $y^2 = 4x$ and $x^2 + y^2 = 5$ **(May-07,Mar-12)**

65.*** Find the angle between the curves $2y^2 - 9x = 0,\ 3x^2 + 4y = 0$ (in the 4th quadrant).**(May-09)**

66.*** At any point 't' on the curve x = a (t + sin t), y = a (1 – cos t), find the lengths of tangent, normal, subtangent and subnormal. **(Board paper)**

67.*** At a point (x_1, y_1) on the curve $x^3 + y^3 = 3axy$ show that the tangent is
$(x_1^2 - ay_1)x + (y_1^2 - ax_1)y = ax_1y_1$

68.*** Find the value of K so that the length of the sub-normal at any point on the curve $xy^k = a^{k+1}$ is a constant.

69.*** (i) Define the angle between two curves
(ii) Find the angle between the curves $xy = 2$, and $x^2 + 4y = 0$ **(May 2011)**

70.** Find the angle between the curves $y^2 = 8x,\ \ 4x^2 + y^2 = 32$ **(May 2012)**

71.** Show that the curves $6x^2 - 5x + 2y = 0$ and $4x^2 + 8y^2 = 3$ touch each other at $\left(\frac{1}{2}, \frac{1}{2}\right)$ **(Mar-10)**

72.** Show that the equation of tangent at the point (x_1, y_1) on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is
$xx_1^{-1/2} + yy_1^{-1/2} = a^{1/2}$ **(June 2004)**

73.** Find the lengths of sub tangent, sub normal at a point t on the curve
$x = a(\cos t + t\sin t),\ y = a(\sin t - t\cos t)$

74.** Find the angle between the curves $x^2 y = 4,\ y(x^2+4) = 8$.

75.** Show that the condition for the orthogonality of the curves $ax^2+by^2=1$ and $a_1x^2+b_1y^2=1$ is $\frac{1}{a}-\frac{1}{b}=\frac{1}{a_1}-\frac{1}{b_1}$.

76.* If the slope of the tangent to the curve $y = x\log x$ at a point on it is $\frac{3}{2}$, then find the equations of tangent and normal at that point.

77.* Show that the square of the length of subtangent at any point on the curve $by^2=(x+a)^3 (b\neq 0)$ varies with the length of the subnormal at that point.

78.* Find the equations of the tangents to the curve $y=3x^2-x^3$, where it meets the X-axis.

MAXIMA AND MINIMA

79.*** If the curved surface of right circular cylinder inscribed in a sphere of radius 'r' is maximum, show the height of the cylinder is $\sqrt{2}r$. **(May-10, Mar 2004, June 2004, Mar 2008)**

80.*** From a rectangular sheet of dimensions 30cm $\times$ 80cm. four equal squares of side x cm. are removed at the corners, and the sides are then turned up so as to form an open rectangular box. Find the value of x, so that the volume of the box is the greatest. **(Mar-09,2014)**

81.*** A window is in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 20ft., find the maximum area. **(May 2009)**

82.*** A wire of length 'l' is cut into two parts which are bent respectively in the form of a square and a circle. What are the lengths of the pieces of the wire respectively so that the sum of the areas is the least. **(Mar-11)**

83**. Find two positive numbers whose sum is 15 so that the sum of their squares is minimum.

84**. A manufacturer can sell x items at a price of rupees (5 - x/100) each. The cost price of x items is Rs. (x/5 + 500). Find the number of items that the manufacturer should sell to earn maximum profits.

85**. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

86*. Find the absolute maximum and absolute minimum of $f(x)=8x^3+81x^2-42x-8$ on [-8, 2].

87.* Find the absolute maximum value and the absolute minimum value of the function $f(x)=x+\sin 2x$ in $[0,\pi]$ **(May 2007)**

88.* Find the maximum area of the rectangle that can be formed with fixed perimeter 20.

SHORT ANSWER QUESTIONS (4 Marks)

LOCUS

1.*** A(1,2), B(2,-3)andC(-2,3) are three points. A point 'P' moves such that $PA^2+PB^2=2PC^2$. Show that the equation to the locus of P is 7x–7y+4=0. **(Mar-11) (May 2007, 2009)**

2.*** Find the equation of locus of P, if the ratio of the distance from P to (5, -4) and (7, 6) is 2 : 3. **(March-2014, May 2008, July 2001)**

3.*** A (5, 3) and B (3, -2) are two fixed points. Find the equation of locus of P, so that the area of triangle PAB is 9 sq.units. **(Mar 2006, 2009)**

4.*** A (2,3); B(-3,4) are two given points. Find the equation of locus of P so that the area of ΔPAB is 8.5 sq.units. **(March -2011)**

5.*** Find the equation of locus of a point, the difference of whose distances from (-5, 0) and (5, 0) is 8. **(Mar 2004, May 2006)**

6.*** Find the equation of locus of P, if A = (2, 3), B = (2, -3) and PA + PB = 8. **(March 2003, 08)**

7.*** Find the equation of locus of P, if the line segment joining (2, 3) and (-1, 5) subtends a right angle at P. **(March-2013, May 2012, March-2005)**

8.*** The ends of the hypotenuse of a right angled triangle are (0, 6) and (6, 0). Find the equation of locus of its third vertex. **(May-10, March-2008)**

9.** Find the equation of the locus of a point P such that the distance of P from the origin is twice the distance of P from A(1,2). **(Mar-12, Jun 2005)**

10.** Find the equation of locus of P, if A=(4,0), B=(-4,0) and $|PA-PB|=4$. **(May-13, Mar-07)**

11.** Find the equation of locus of a point, the sum of whose distances from (0, 2) and (0, -2) is 6 units. **(Mar-10)**

12.* Find the equation of locus of a point P, if the distance of P from A (3, 0) is twice the distance of P from B (-3, 0).

13.* Find the equation of the locus of a point P such that $PA^2+PB^2=2c^2$ where $A=(a,0)$, $B=(-a,0)$ and $0<|a|<|c|$

TRANSFORMATION OF AXES

14.*** When the origin is shifted to the point (2,3), the transformed equation of a curve is $x^2+3xy-2y^2+17x-7y-11=0$. Find the original equation of the curve. **(Mar- 09,10,11 May 09)**

15.*** When the axes are rotated through an angle $\frac{\pi}{4}$, find the transformed equation of $3x^2+10xy+3y^2=9$. **(June 2005, May 08, 12)**

16.*** When the axes are rotated through an angle $\frac{\pi}{6}$, find the transformed equation of $x^2+2\sqrt{3}xy-y^2=2a^2$. **(May 2006, Mar 07,12, May 2013)**

17.*** When the axes are rotated through an angle 45°, the transformed equation of a curve is $17x^2-16xy+17y^2=225$. Find the original equation of the curve. **(May-10, Mar 2008)**

18.*** Show that the axes are to be rotated through an angle of $\frac{1}{2}Tan^{-1}\left(\frac{2h}{a-b}\right)$ so as to remove the xy term from the equation $ax^2+2hxy+by^2=0$, if $a \neq b$ and through the angle $\pi/4$, if a = b. **(Mar-2006, 2013)**

19.*** When the axes are rotated through an angle α, find the transformed equation of $x\cos\alpha + y\sin\alpha = p$. **(May 2007, Mar-14)**

20.** When the origin is shifted to (-1,2) by the translation of axes, find the transformed equation of $x^2+y^2+2x-4y+1=0$.

21.* When the origin is shifted to the point (-1,2), the transformed equation of a curve is $x^2+2y^2+16=0$. Find the original equation of the curve.

STRAIGHT LINES

22.*** Transform the equation $\sqrt{3}x+y=4$ into (a) slope-intercept form (b) intercept form and (c) normal form. **(Mar 2004, 2008)**

23.*** Transform the equation $\frac{x}{a}+\frac{y}{b}=1$ into the normal form when a>0 and b>0.If the perpendicular distance of straight line from the origin is p, deduce that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$.**(Jun 04, Mar 07, May 08)**

24.*** A straight line through $Q\left(\sqrt{3},2\right)$ makes an angle $\pi/6$ with the positive direction of the X-axis. If the straight line intersects the line $\sqrt{3}x-4y+8=0$ at P, find the distance PQ.**(Mar04)**

25.*** A straight line through $Q(2,3)$ makes an angle $\frac{3\pi}{4}$ with the negative direction of the X-axis. If the straight line intersects the line $x+y-7=0$ at P, find the distance PQ.

26.*** Find the points on the line $3x-4y-1=0$ which are at a distance of 5 units from the point (3,2)

27.*** Find the value of k, if the angle between the straight lines $4x-y+7=0$ and $kx-5y-9=0$ is 45^0. **(Mar-12)**

28.*** Find the values of k, if the angle between the straight lines $kx+y+9=0$ and $3x-y+4=0$ is $\frac{\pi}{4}$.

29.*** Find the equations of the straight lines passing through the point (-3, 2) and making an angle of 45° with the straight line $3x-y+4=0$. **(Mar 2009)**

30.*** Find the equation of straight line making non-zero equal intercepts on the coordinate axes passing through the point of intersection of lines $2x-5y+1=0$ and $x-3y-4=0$.**(Mar 2006)**

31.*** Find the point on the line $3x+y+4=0$ which is equidistant from the points (-5,6) and (3,2) **(March-2013)**

32.*** A triangle of area 24 sq. units is formed by a straight line and the co-ordinate axes in the first quadrant. Find the equation of that straight line if it passes through (3,4) **(May 2007)**

33.*** Find the value of k, if the lines $2x-3y+k=0, 3x-4y-13=0$ and $8x-11y-33=0$ are concurrent. **(Mar 2005)**

34.*** If the straight lines $ax+by+c=0, bx+cy+a=0$ and $cx+ay+b=0$ are concurrent, then prove that $a^3+b^3+c^3=3abc$.

35.** If $3a+2b+4c=0$, then show that the equation $ax+by+c=0$ represents a family of concurrent straight lines and find the point of concurrency. **(May-10)**

36.** Find the equation of the line perpendicular to the line $3x+4y+6=0$ and making an intercept -4 on the X-axis. **(March-10)**

37.* Find the equation of the straight line parallel to the line 3x + 4y = 7 and passing through the point of intersection of the lines x - 2y – 3 = 0 and x + 3y - 6 = 0.

38.* Find the equation of the straight line perpendicular to the line 2x + 3y = 0 and passing through the point of intersection of the lines x + 3y – 1 = 0 and x – 2y + 4 = 0.

39*. Find the value of 'a' if the distances of the points (2, 3) and (-4, a) fromthe straight line $3x+4y-8=0$ are equal.

40*. Find the equations of the straight lines passing through (1, 3) and (i) parallel to (ii) perpendicular to the line passing through the points (3, -5) and (-6, 1)

41.* Find the angles of the triangle whose sides are $x+y-4=0$, $2x+y-6=0$ and $5x+3y-15=0$ **(May 2007)**

42.* Line 'L' has intercepts a and b on the axes of cooridnates. When the axes are rotated through a given angle, keeping the origin fixed, the same line 'L' has intercepts p and q on the transformed axes prove that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

43.* $A(10,4),\ B(-4,9)\ and\ C(-2,-1)$ are the vertices of a triangle. Find the equations of

i) $\overleftrightarrow{AB}$ ii) The median through A

iii) the altitude through B iv) the perpendicular bisector of the side $\overleftrightarrow{AB}$

44.* A variable stright line drawn through the point of intersection of stright lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ meets the co-ordinate axes at A and B. Show that the locus of the mid point of $\overline{AB}$ is $2(a+b)xy=ab(x+y)$. **(May2005)**

45.* The length of the perpendicular from the point $P(x_0, y_0)$ to the straight line $ax+by+c=0$ is $\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$.

46.* A(-1, 1), B(5,3) are opposite vertices of a square in the XY-plane. Find the equation of the other diagonal (not passing through A, B) of the square.

CONTINUITY

47.*** Show that $f(x)=\begin{cases}\dfrac{\cos ax-\cos bx}{x^2} & if\ x\neq 0\\ \dfrac{1}{2}\left(b^2-a^2\right) & if\ x=0\end{cases}$ where a and b are real constants, is continuous at x = 0. **(May 2006, Mar 2009)**

48.*** Is 'f' defined by $f(x)=\begin{cases}\dfrac{\sin 2x}{x} & ,if\ x\neq 0\\ 1 & ,if\ x=0\end{cases}$ continuous at x = 0 ? **(May-10, Mar-06, 09)**

49.*** Check the continuity of 'f' given by

$$f(x)=\begin{cases}\left(x^2-9\right)/\left(x^2-2x-3\right) & if\ 0<x<5\ and\ x\neq 3\\ 1.5 & if\ x=3\end{cases}$$ at the point x = 3.

(Mar-11,13,14)

50.*** Check the continuity of the following function at 2.

$$f(x)=\begin{cases}\dfrac{1}{2}\left(x^2-4\right) & if\ 0<x<2\\ 0 & if\ x=2\\ 2-8x^{-3} & if\ x>2\end{cases}$$

51.** If 'f' given by $f(x)=\begin{cases}k^2x-k & if\ x\geq 1\\ 2 & if\ x<1\end{cases}$, is a continuous function on R, then find the values of k.

52.* Find real constants a, b so that the funciton f given by

$$f(x)=\begin{cases}\sin x & if\ x\leq 0\\ x^2+a & if\ 0<x<1\\ bx+3 & if\ 1\leq x\leq 3\\ -3 & if\ x>3\end{cases}$$ is continuous on **R.**

53.* Is the function f , defined by $f(x)=\begin{cases}x^2 & if\quad x\leq 1\\ x & if\quad x>1\end{cases}$ continuous on R?

54.* Show that f, given by $f(x)=\dfrac{x-|x|}{x}$ $(x\neq 0)$ is continuous on $R-\{0\}$

DIFFERENTIATION

55.*** Find the derivative of the following functions from the first principles w.r.to x.

i) $\cos^2 x$ **(JUNE-2004, MAY 2008)** ii) $\tan 2x$ **(MARCH 2005, May 2011)**

iii) $\sqrt{x+1}$ **(JUNE 2005, May-2012)** iv) $\sec 3x$ **(Mar-08,12)**

v) $\cos(ax)$ **(Mar-09,11,13)** vi) $\sin 2x$ **(May-10)**

vii) $x\sin x$ **(Mar-10,May-09)** viii) $\log x$

ix) ax^2+bx+c x) a^x

56.*** If $x^y = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$. **(Mar 2007, 2008)**

57.*** If $\sin y = x\sin(a+y)$, then show that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ (a *is not a multiple of* π) **(Mar-11)**

58.*** If $f(x) = Tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $g(x) = Tan^{-1}x$ then differentiate $f(x)$ w.r.to $g(x)$ **(May09)**

59.*** If $y = x^y$ then show that $\frac{dy}{dx} = \frac{y^2}{x(1-\log y)} = \frac{y^2}{x(1-y\log x)}$. **(Mar 2004)**

60.*** Find $\frac{dy}{dx}$ for the functions, $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$. **(May 2008)**

61*** If $x^{2/3} + y^{2/3} = a^{2/3}$ then $\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$

62.** If $y = a\cos x + (b+2x)\sin x$, then show that $y'' + y = 4\cos x$ **(May 2007)**

63.** If $x = 3\cos t - 2\cos^3 t$, $y = 3\sin t - 2\sin^3 t$, then find $\frac{dy}{dx}$. **(Mar-10)**

64.** If $x = a(t-\sin t), y = a(1+\cos t)$ then find $\frac{d^2y}{dx^2}$.

65.** Find derivative of $\tan^{-1}\left[\frac{3a^2x - x^3}{a(a^2-3x^2)}\right]$

66.** If $y = \sin^{-1}\left[\frac{2^{x+1}}{1+4^x}\right]$ then find $\frac{dy}{dx}$

67.** If $y^x = x^{\sin y}$ then find $\frac{dy}{dx}$

68.** If $y = \log(4x^2-9)$ then find y^{11}

69.* If $x = a\left[\cos t + \log\left(\tan\left(\frac{t}{2}\right)\right)\right]$, $y = a\sin t$ then find $\frac{dy}{dx}$

70.* Find $\frac{dy}{dx}$ of the functions $x = a\left(\frac{1-t^2}{1+t^2}\right)$, $y = \frac{2bt}{1+t^2}$

71.* If $y = Tan^{-1}\sqrt{\frac{1-x}{1+x}}$ $(|x|<1)$, then find $\frac{dy}{dx}$.

72.* If $f(x)=\sqrt{\frac{1+x^2}{1-x^2}}\,(|x|<1)$ then find $f^1(x)$

73.* If $ay^4=(x+b)^5$ then ST $5yy^{11}=(y^1)^2$

74.* Show that $f(x)=|x|$ is differentiable at any $x\neq 0$ and is not differentiable at 0.

75.* Check the differentiability of function $f(x)=\begin{cases}3+x & x>0\\ 3-x & x<0\end{cases}$ $at\ 0$

TANGENTS & NORMALS

76.*** Show that the length of the subnormal at any point on the curve $y^2=4ax$ is a constant. **(Mar-05,11, May 2009)**

77.*** Show that the length of the subtangent at any point on the curve $y=a^x\,(a>0)$ is a constant. **(Mar-12)**

78.*** Find the lengths of normal and sub normal at a point on the curve $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{\frac{-x}{a}}\right)$ **(Mar13)**

79.*** Find the equations of the tangent and normal to the curve $y=x^2-4x+2$ at (4, 2) **(Mar-09)**

80.*** Show that at any point (x, y) on the curve $y=be^{x/a}$, the length of the sub-tangent is a constant and the length of the sub normal is $\frac{y^2}{a}$ **(Mar-10)**

81.*** Find the equations of tangent and normal to the curve xy = 10 at (2, 5). **(Mar-11)**

82.*** Find the equations of the tangent and normal to the curve $y=5x^4$ at the point (1, 5). **(May-10)**

83.*** Show that the tangent at any point θ on the curve $x=C\sec\theta, y=C\tan\theta$ is $y\sin\theta=x-C\cos\theta$

84.*** Find the angle between the curves $x+y+2=0$, $x^2+y^2-10y=0$ **(March-2014)**

85.** Find the equations of tangent and normal to the curve $y=x^3+4x^2$ at (-1, 3).

86.** Find the equations of tangent and normal to the curve $y^4=ax^3$ at (a,a)

87.* Find the slope of the normal to the curve $x=a\cos^3\theta, y=a\sin^3\theta\ at\ \theta=\frac{\pi}{4}$.

88.* Find the slope of the tangent to the curve $y=x^3-3x+2$ at the point whose x-coordinate is 3.

89.* Find the points at which the tangent to the curve $y=x^3-3x^2-9x+7$ is parallel to the x-axis.

90.* Show that the length of the subnormal at any point on the curve $xy=a^2$ varies as the cube of the ordinate of the point

RATE MEASURE

91***. A particle is moving in a straight line so that after t seconds its distance is s (in cms) from a fixed point on the line is given by $s=f(t)=8t+t^3$. Find
(i) the velocity at time t = 2sec (ii) the initial velocity (iii) acceleration at t = 2sec.

92***. The distance - time formula for the motion of a particle along a straight line $S=t^3-9t^2+24t-18$ then find when and where the velocity is zero. **(Mar-12)**

93.*** A point P is moving on the curve $y=2x^2$. The x co-ordinate of P is increasing at the rate of 4 units per second. Find the rate at which the y co-ordinate is increasing when the point is at (2, 8). **(May 2008)**

94***. A container in the shape of an inverted cone has height 12cm and radius 6cm at the top. If it is filled with water at the rate of 12cm^3/sec., what is the rate of change in the height of water level when the tank is filled 8cm?

95***. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of the edge is 10 centimetres? **[March-2013]**

96*** The volume of a cube is increasing at the rate of $8\,cm^3/\sec$. How fast is the surface area increasing when the length of an edge is $12cm$. **(March-2014)**

97***. A stone is dropped into a quiet lake and ripples move in circles at the speed of 5cm/sec. At the instant when the radius of circular ripple is 8cm., how fast is the enclosed area increases?

98***. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900cubic centimeters of gas per second. Find the rate at which the radius of balloon increases when the radius is 15cm.

99**. Suppose we have a rectangular acuarium with dimensions of length 8m, width 4m and height 3m. Suppose we are filling the tank with water at the rate of 0.4m^3/sec. How fast is the height of water changing when the water level is 2.5m?

100**. The total cost *C(x)* in rupees associated with productions of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 500$. Find the marginal cost when 3 units are produced.

101*. Find the average rate of change of $s = f(t) = 2t^2 + 3$ between $t = 2$ and $t = 4$.

102*. The radius of a circle is increasing at the rate of 0.7cm/sec. What is the rate of increase of its circumference.

103*. The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/sec. At what rate is the volume of the bubble increasing when the radius is 1 cm?

VERY SHORT ANSWER QUESTIONS (2 Marks)

STRAIGHT LINES

1. Prove that the points (1,11), (2,15) and (-3, -5) are collinear and find the equation of the stright line containing them.

2. Find the condition for the points $(a,0)$, (h,k) and $(0,b)$ where $ab \neq 0$ to be collinear. **(Mar-10)**

3. Transform the equations into normal form.
   i) $x + y + 1 = 0$ **(May-10)** ii) $x + y - 2 = 0$ **(Mar-12)**

4. If the area of the triangle formed by the straight lines x=0, y= 0 and $3x + 4y = a$ $(a > 0)$ is 6. Find the value of 'a'. **(May 2007, Mar 2009)**

5. If the product of the intercepts made by the straight line $x\tan\alpha + y\sec\alpha = 1$ $\left(0 \leq \alpha < \frac{\pi}{2}\right)$ on the co-ordinate axes is equal to $\sin\alpha$, find α.

6. Find the area of the triangle formed by the straight line $x - 4y + 2 = 0$ with the coordinate axes.

7. Find the equation of the straight line passing through (-4, 5) and cutting off equal nonzero intercepts on the coordinate axes. **(May-10, Mar 2007, May 2008)**

8. Find the equation of the straight line passing through the point (3, -4) and making X and Y-intercepts which are in the ratio 2 : 3. **(Mar 2008)**

9. Find the equation of the straight line passing through the points $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_2\right)$ **(Mar 2014)**

10. Find the length of the perpendicular drawn from the point (-2, -3) to the straight line 5x–2y+4=0.

11. Find the distance between the parallel straight lines 5x–3y–4=0, 10x–6y–9=0 **(Mar 09)**

12. Find the equation of straight line passing through the point (5, 4) and parallel to the line 2x + 3y + 7 = 0. **(March-13)**

13. Find the value of y, if the line joining (3,y) and (2,7) is parallel to the line joining the points (-1,4) and (0,6). **(March-2014 SAQ) (Mar-08)**

14. Find the value of k, if the straight lines 6x – 10y + 3 = 0 and kx – 5y + 8 = 0 are parallel.

15. Find the value of p, if the straight lines 3x + 7y – 1 = 0 and 7x – py + 3 = 0are mutually perpendicular.

16. Find the equation of the straight line passing through (2, 3) and making non-zero intercepts on the co-ordinate axes whose sum is zero. **(Mar-12)**

17. Find the equation of the straight line passing through (-2, 4) and making non-zero intercepts on the co-ordinate axes whose sum is zero. **(May-09)**

18. Find the equations of the straight lines passing through the origin and making equal angles with the co-ordinate axes. **(May 2005)**

19. Find the value of P, if the straight lines x + p = 0, y + 2 = 0, 3x + 2y + 5 = 0 are concurrent. **(March-13)**

20. If $2x-3y-5=0$ is the perpendicular bisector of the line segment joining (3, -4) and (α,β), find $\alpha+\beta$. **(Mar-11)**

21. Find the value of k, if the straight lines $y-3kx+4=0$ and $(2k-1)x-(8k-1)y-6=0$ are perpendicular. **(Mar-10)**

22. If θ is the angle between the lines $\frac{x}{a}+\frac{y}{b}=1$, $\frac{x}{b}+\frac{y}{a}=1$ then find the value of $\sin\theta$ (a>b) **(May-09)**

23. Transform the equation $(2+5k)x-3(1+2k)y+(2-k)=0$ into the form $L_1+\lambda L_2=0$ and find the point of concurrency of the family of straight lines.

24. Find the ratio in which the straight line $2x+3y-20=0$ divides the join of the points (2, 3) and (2, 10).

25. If a portion of a straight line intercepted between the axes of coordinates is bisected at $(2p,2q)$ write the equation of the straight line.

26. Find the foot of the perpendicular drawn from (4, 1) upon the straight line $3x-4y+12=0$.

27. Find the orthocenter of the triangle whose sides are given by $4x-7y+10=0$, $x+y=5$ and $7x+4y=15$

28. Find the incentre of the triangle whose sides are $x=1$, $y=1$ and $x+y=1$.

29. If a, b, c are in arithmetic progression, then show that the equation $ax+by+c=0$ represents a family of concurrent lines and find the point of concurrency.

30. Find the ratio in which the straight line $2x+3y=5$ divides the join of the points (0, 0) and (-2, 1). **(Mar-2014)**

3D – GEOMETRY

31. Find the centroid of the triangle whose vertices are (5,4,6),(1,-1,3) and (4,3,2). **(Mar 04)**

32. Find the coordinates of the vertex 'C' of ΔABC if its centroid is the origin and the vertices A, B are (1, 1,1) and (-2,4,1) respectively.

33. Find the centroid of the tetrahedron whose vertices are $(2,3,-4),(-3,3,-2),(-1,4,2),(3,5,1)$

34. If (3, 2, -1), (4, 1, 1) and (6, 2, 5) are three vertices and (4, 2, 2) is the centroid of a tetrahedron, find the fourth vertex. **(Mar-09, 2014)**

35. Show that the points $A(3,-2,4), B(1,1,1), C(-1,4,-2)$ are collinear.

36. Show that the points A(1, 2, 3) B(7, 0, 1) C (–2, 3, 4) are collinear **(Mar –2013)**

37. Find the fourth vertex of the parallelogram whose consecutive vertices are (2,4,-1), (3,6,-1) and (4,5,1). **(June 2003, Mar-11)**

38. Find the ratio in which YZ – plane divides the line joining A (2, 4, 5) and B(3, 5, -4). Also find the point of intersection. **(May-10)**

39. Find x if the distance between (5, -1,7) and (x, 5, 1) is 9 units. **(May-11)**

40. Show that the points (1,2,3) , (2,3,1) and (3,1,2) form an equilateral triangle.

41. If H, G, S and I respectively denotes orthocentre, centroid, circumcentre and in-centre of a triangle formed by the points (1, 2, 3) (2,3,1) and (3,1,2) then find H,G,S,I

42. Show that the points A(-4,9,6) B(-1,6,6) and C(0,7,10) form a rightangled isosceles triangle.

43. If the point (1,2,3) is changes to the point (2,3,1) through translation of axes. find the new origin

44. Find the ratio in which the point P(5,4,-6) divides the line segment joining the points A(3,2,-4) and B(9,8,-10) . Also, find the harmonic conjugate of P.

THE PLANE

45. Find the angle between the planes $x+2y+2z-5=0$ and $3x+3y+2z-8=0$. **(M 09)**

46. Find the angle between the planes $2x - y + z = 6$ and $x + y + 2z = 7$. **(Mar-11)**

47. Find the equation of the plane whose intercepts on X, Y, Z - axes are 1, 2,4 respectively. **(March-10)**

48. Transform the equation $4x - 4y + 2z + 5=0$ into intercept form. **(Mar-12)**

49. Find the intercepts of the plane $4x + 3y - 2z + 2 = 0$ on the coordinate axes.

50. Find the direction cosines of the normal to the plane x+2y+2z-4=0. **(March-2013)**

51. Reduce the equation x + 2y – 3z – 6 = 0 of the plane in to the normal form. **(March-2014)**

52. Find the equation of the plane passing through the point (1,1,1) and parallel to the plane x + 2y + 3z – 7 =0.

53. Find the equation of the plane passing through the point (-2,1,3) and having (3,-5,4)as direction ratios of its normal. **(Mar-11)**

54. Find the equation to the plane parallel to the ZX-plane and passing through (0, 4,4).

55. Find the midpoint of the line joining the points (1,2,3) and (-2, 4, 2) **(May-12)**

56. Find the equation of the plane passing through the points (2,0,1) and (3,-3,4) and perpendicular to x-2y+z=6 **(May-09,10,11)**

57. Find the equation of the plane passing through (2,3,4) and perpendicular to X-axis.

LIMITS

58. Find $\lim_{x\to 0}\left(\frac{\sqrt{1+x}-1}{x}\right)$. **(Mar 04, 07,2014, May-10)**

59. Find $\lim_{x\to 0}\left(\frac{3^x-1}{\sqrt{1+x}-1}\right)$. **(Mar 2005)**

60. Compute $\lim_{x\to 0}\left(\frac{e^x-1}{\sqrt{1+x}-1}\right)$. **(Mar 2009)**

61. Compute $\lim_{x\to 0}\frac{a^x-1}{b^x-1}$ $(a>0,\ b>0,\ b\neq 1)$ **(Mar 02,08,13, June 2002)**

62. Find $\lim_{x\to 0}\frac{\sin(a+bx)-\sin(a-bx)}{x}$ **(Mar 2005, May 2009,Mar-12)**

63. Find $\lim_{x\to 0}\left(\frac{\cos ax-\cos bx}{x^2}\right)$. **(Mar 2004, 2007)**

64. Find $\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$. **(Mar-08,09,10,11)**

65. Find $\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\left(x-\frac{\pi}{2}\right)}$ **(June 2005, Mar 2008)**

66. Compute $\mathop{Lt}_{x\to 0}\frac{1-\cos 2mx}{\sin^2 nx}$ $(m,\ n\in Z)$ **(Mar-10)**

67. Find $\lim_{x\to\infty}\frac{11x^3-3x+4}{13x^3-5x^2-7}$ **(March-2014, May 2007)**

68. Evaluate $\mathop{Lt}_{x\to 1}\frac{\sin(x-1)}{x^2-1}$ **(May 2006)**

69. Compute $\lim_{x \to 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1-x}}{x}$. **(May 2006)**

70. Find $\lim_{x \to a} \frac{\sin(x-a)\tan^2(x-a)}{\left(x^2-a^2\right)^2}$. **(Mar 2006)**

71. Campute $\underset{x\to a}{Lim} \left(\frac{x \sin a - a \sin x}{x-a}\right)$ **(Mar-11)**

72. Show that $\lim_{x \to 2^-} \frac{|x-2|}{x-2} = -1$. **(June 2004)**

73. show that $\lim_{x \to 0^+} \left(\frac{2|x|}{x} + x + 1\right) = 3$. **(May 2008)**

74. Find $\lim_{x \to \infty} \frac{8|x|+3x}{3|x|-2x}$. **(May-09,10, Mar-12)**

75. Compute $\lim_{x\to 2} \frac{\left(2x^2-7x-4\right)}{(2x-1)\left(\sqrt{x}-2\right)}$. **(May 2007)**

76. Compute $\underset{x\to 0}{Lt} \frac{e^{3+x}-e^3}{x}$

77. Compute $\underset{x\to 0}{Lt} \frac{\log_e(1+5x)}{x}$

78. Compute $\underset{x\to 0}{Lt} \frac{e^{3x}-1}{x}$

79. Compute $\lim_{x \to 2+} ([x]+x)$ and $\lim_{x \to 2-} ([x]+x)$.

80. Find $\lim_{x \to -\infty} \left(\frac{2x+3}{\sqrt{x^2-1}}\right)$

81. Compute $\lim_{x\to 2} \left(\frac{x-2}{x^3-8}\right)$

82. Compute $\lim_{x\to 2} \left(\frac{1}{x-2} - \frac{4}{x^2-4}\right)$.

83. Compute $\lim_{x\to\infty} \left(\sqrt{x+1} - \sqrt{x}\right)$.

84. Compute $\lim_{x\to 0} \frac{1-\cos mx}{1-\cos nx}$, $n \neq 0$.

85. Show that $\underset{x\to 3}{Lim} \frac{x-3}{\sqrt{|x^2-9|}} = 0$

86. Compute $\underset{x\to a}{Lim} \frac{\tan(x-a)}{x^2-a^2}$

87. Compute $\underset{x\to\infty}{Lim} \frac{x^2-\sin x}{x^2-2}$

88. Show that $\lim\limits_{x\to 0} \dfrac{|x|}{x}$ does not exist.

89. Compute $\lim\limits_{x\to 0} \dfrac{\sin ax}{x\cos x}$

90. Compute $\lim\limits_{x\to 0}\dfrac{\sin ax}{\sin bx}$, $b \neq 0,\ a \neq b$

91. Evaluate $\lim\limits_{x\to 1}\dfrac{\log_e x}{x-1}$

92. Compute $\underset{x\to 0}{Lt}\ \dfrac{e^x - \sin x - 1}{x}$ **(Mar-2013)**

DIFFERENTIATION

93. If $y = ax^{n+1} + bx^{-n}$ then $P.T\ x^2 y'' = n(n+1)y$. **(Mar-06,May-10)**

94. If $y = \sec\left(\sqrt{\tan x}\right)$, then find $\dfrac{dy}{dx}$ **(May 2007)**

95. Find the derivative of the function $f(x) = a^x.e^{x^2}$ **(May 2008)**

96. If $f(x) = 7^{x^3+3x}(x>0)$, then find $f^1(x)$. **(May 2005)**

97. If $x = \tan\left(e^{-y}\right)$, then show that $\dfrac{dy}{dx} = \dfrac{-e^y}{1+x^2}$. **(Mar 2005)**

98. If $f(x) = \log(\sec x + \tan x)$, then find $f^1(x)$. **(March-2014, May-2011)**

99. If $y = \left(Cot^{-1}x^3\right)^2$, then find $\dfrac{dy}{dx}$. **(May-09)**

100. If $y = \log(\sin^{-1}(e^x))$ then find $\dfrac{dy}{dx}$. **(Mar-10)**

101. If $f(x) = x^2\ 2^x \log x (x>0)$, then find $f'(x)$. **(May-10)**

102. If $y = \cos\left[\log\left(\cot x\right)\right]$ then find $\dfrac{dy}{dx}$. **(Mar-09)**

103. If $y = \log\left(\cosh 2x\right)$ then find $\dfrac{dy}{dx}$ **(Mar-12)**

104. Find $\dfrac{dy}{dx}$ if y= $\sin^{-1}(\sqrt{x})$ **(Mar–13)**

105. Find the derivative of $f(x) = \dfrac{ax+b}{cx+d}$ **(May-12)**

106. If y= $\sec^{-1}\left(\dfrac{1}{2x^2-1}\right)$ find $\dfrac{dy}{dx}$ **(Mar –13)**

107. If $y = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ then find $\dfrac{dy}{dx}$ **(Mar-12)**

108. If $x = a\cos^3 t,\ y = a\ \sin^3 t,$ then find $\dfrac{dy}{dx}$ **(May-12, May-11)**

109. If $y = \log\left(\sin\left(\log x\right)\right)$, find $\dfrac{dy}{dx}$.

110. If $y = x^x$ then find $\frac{dy}{dx}$ **(Mar-2011)**

111. If $x^3 + y^3 - 3axy = 0$, find $\frac{dy}{dx}$. **(Apr 2000)**

112. Find the derivative of the following functions w.r.to x.

i) $Cos^{-1}\left(4x^3 - 3x\right)$ **(March-2014)** ii) $Tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ **(June 2002)**

113. Differentiate $f(x)$ with respect to $g(x)$ if $f(x) = e^x$, $g(x) = \sqrt{x}$ **(Mar 2003)**

114. Find the derivative of the following functions w.r.to x.

i) $Tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ **(June 2003)** ii) $\tan^{-1}\left(\frac{1+x}{1-x}\right)$ **(May-12)**

115. If $f(x) = x\,e^x \sin x$, *then find* $f^1(x)$

116. If $y = ae^{nx} + be^{-nx}$ then prove that $y'' = n^2 y$.

117. If $y = \sin(\log x)$, *then find* $\frac{dy}{dx}$

118. If $f(x) = 1 + x + x^2 + ... + x^{100}$ then find $f^1(1)$.

119. If $y = e^{a\sin^{-1}x}$ then prove that $\frac{dy}{dx} = \frac{ay}{\sqrt{1-x^2}}$

120. Find the derivative of $20^{\log(\tan x)}$

121. Find the derivative of $f(x) = e^x(x^2+1)$ w.r.t x

122. If $f(x) = \frac{a-x}{a+x}$ then find $f^1(x)$

123. If $y = \left(x^3 + 6x^2 + 12x - 13\right)^{100}$ then find $\frac{dy}{dx}$

124. If $f(x) = \log_7(\log x)$ then find $f^1(x)$

125. If $y = \frac{1}{ax^2 + bx + c}$ then find $\frac{dy}{dx}$

126. If $y = \cos ec^{-1}(e^{2x+1})$, find $\frac{dy}{dx}$

127. If $y = \frac{1-\cos 2x}{1+\cos 2x}$ then find $\frac{dy}{dx}$

128. If $f(x) = \sinh^{-1}\left(\frac{3x}{4}\right)$ then find $f^1(x)$

129. If $y = \sin^{-1}\left(3x - 4x^3\right)$ then find $\frac{dy}{dx}$

130. If $y = \frac{\cos x}{\sin x + \cos x}$ then find $\frac{dy}{dx}$

131. If $x=at^2$, $y= 2at$ find $\frac{dy}{dx}$

ERRORS AND APPROXIMATIONS

132. Find Δy and dy if $y = x^2 + 3x + 6$. When x = 10, $\Delta x = 0.01$. **(MAR-2005,2011,2014)**

133. Find Δy and dy if $y = x^2 + x$, at x = 10, $\Delta x = 0.1$

134. Find Δy and dy if $y = \frac{1}{x+2}$ when x = 8, $\Delta x = 0.02$

135. Find Δy and dy for $y = e^x + x$, when x = 5, $\Delta x = 0.02$

136. Find Δy and dy if $y = \cos x, x = 60^0$ and $\Delta x = 1^0$

137. Find the approximate value of $\sqrt{82}$ **(March-2013, May-2009)**

138. Find the approximate value of $\cos\left(60^o 5^{|}\right)$ ($\therefore 1^0$=0.0174 radians)

139. Find the approximate value of $\sqrt[3]{65}$.

140. Find the approximate value of $\sqrt[3]{7.8}$

141. (i) If the increase in the side of a square is 4%. Then find the approximate percentage of increase in the area of square.
(ii) If the increase in the side of a square is 2%. Then find the approximate percentage of increase in the area of square.

142. If the radius of a sphere is increased from 7cm to 7.02cm then find the approximate increase in the volume of the sphere

143. if $y = f(x) = kx^n$ show that the approximate relative error in y is n times the relative error in x where n and k are constants

144. The diameter of sphere is measured to be 40cm. If an error of 0.02cm is made in it. Then Find approximate errors in volume and surface area of the sphere.

145. The time t, of a complete oscillation of a simple pendulum of length 'l' is given by the equation $t = 2\pi\sqrt{\frac{l}{g}}$ where 'g' is gravational constant. Find approximate percentage error in 't' when the perecentage of error in 'l' is 1%.

ROLLE'S AND LAGRANGE'S THEOREMS

146. State Rolle's theorem

147. State Lagrange's theorem

148. Let $f(x) = (x-1)(x-2)(x-3)$. Prove that there is more than one c in $(1,3)$ such that $f'(c) = 0$ **(Mar-2013)**

149. Find the value of c in Rolle's theorem for the function $y=f(x)=x^2+4$ on [-3, 3].

150. Find the value of c from Rolle's theorem for the function $f(x) = x^2 - 1$ *on* $[-1,1]$ **(Mar-14)**

151. It is given that Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax$ on [1, 3] with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b.

152. Verify Rolle's theorem for the function $f(x) = \sin x - \sin 2x$ on $[0,\pi]$

153. Verify the Rolle's theorem for the function $\left(x^2-1\right)\left(x-2\right)$ on [-1, 2]. Find the point in the interval where the derivate vanishes.

154. Verify Rolle's theroem for the function $f\left(x\right)=x\left(x+3\right)e^{-x/2}$ on $\left[-3,0\right]$

155. Show that there is no real number k for which the equation $x^2-3x+k=0$ has two distinct roots in [0, 1]

156. Find c, so that $f^{'}\left(c\right)=\dfrac{f\left(b\right)-f\left(a\right)}{b-a}$ in the following cases:

(i) $f\left(x\right)=x^2-3x-1;\ a=\dfrac{-11}{7},b=\dfrac{13}{7}$ (ii) $f\left(x\right)=e^x;a=0,b=1$

157. Verify the conditions of the Lagrange's mean value theorem for the following functions. In each case find a point 'c' in the interval as stated by the theorem.

(i) x^2-1 on [2,3] (ii) $\sin x-\sin 2x$ on $\left[0,\pi\right]$ (iii) $\log x$ on [1,2]

158. Find a point on the graph of the curve y = x^3 where the tangent is parallel to chord joining the points (1, 1) and (3, 27).

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