Fairness-Aware Performance Evaluation for Multi-Party Multi-Objective Optimization

Zifan Zhao, Peilan Xu, *Member, IEEE*, Wenjian Luo, *Senior Member, IEEE*

Abstract—In multi-party multi-objective optimization problems (MPMOPs), the evaluation of solution sets is commonly conducted using classical performance metrics, such as IGD and HV, aggregated across decision makers (DMs). However, such mean-based evaluations may yield unfair evaluation by implicitly favoring certain parties, as they assume that identical geometric approximation quality with respect to each party's Pareto front (PF) carries comparable evaluative significance. Moreover, prevailing notions of MPMOP optimal solutions are largely restricted to strictly common Pareto-optimal solutions, representing a narrow and sometimes impractical form of co-operation in multi-party decision-making scenarios. These limitations obscure whether a solution set reflects balanced relative gains or meaningful consensus among heterogeneous DMs. To address these issues, this paper develops a fairness-aware performance evaluation framework grounded in a generalized notion of consensus solutions. From a cooperative game-theoretic perspective, we formalize four axioms that a fairness-aware evaluation function for MPMOPs should satisfy. By introducing a concession rate vector to quantify acceptable compromises by individual DMs, we generalize the classical definition of MPMOP optimal solutions and embed classical performance metrics into a Nash-product-based evaluation framework, which is theoretically shown to satisfy all four axioms. To support empirical validation, we further construct benchmark problems that extend existing MPMOP suites by incorporating consensus-deficient negotiation structures. Experimental results demonstrate that the proposed evaluation framework is able to distinguish algorithmic performance in a manner consistent with consensus-aware fairness considerations. Specifically, algorithms converging toward strictly common solutions are assigned higher evaluation scores when such solutions exist, whereas in the absence of strictly common solutions, algorithms that effectively cover the commonly acceptable region are more favorably evaluated.

Index Terms—Multi-party Multi-objective Optimization, Fairness-aware Evaluation, Nash Bargaining, Performance metrics.

I. INTRODUCTION

Multi-party multi-objective optimization problems (MP-MOPs) arise in decision-making scenarios where multiple decision makers (DMs) simultaneously pursue their own sets of objectives, which may conflict both within and across parties [1]. Typical applications include UAV path planning, where regulatory authorities emphasize safety and compliance, while commercial operators prioritize cost efficiency and operational flexibility [2]. In such scenarios, no centralized authority exists to aggregate heterogeneous objectives into a single decision criterion, and feasible outcomes must emerge from compromises among self-interested and heterogeneous DMs.

Within evolutionary computation, population-based meta-heuristics have demonstrated strong effectiveness in approximating PFs, owing to their ability to maintain diversity and to explore trade-offs simultaneously [3]–[5]. Extensions to multi-party scenarios exploit this population-based nature to incorporate the perspectives of multiple DMs. Representative approaches include multi-party non-dominated sorting [6], [7], which extends NSGA-II [8] by intersecting party-specific Pareto layers to identify mutually acceptable solutions, multi-objective immune algorithms [9], [10] incorporating multi-party dominance relations, and party-by-party evolutionary update strategies [11], [12] based on MOEA/D [13], [14]. Indicator-based algorithms further integrate classical indicator-driven selection with multi-party dominance relations to guide the search toward compromise-oriented solutions [15]. In addition, optimization methods tailored for discrete multi-party problems [16], theoretical analyses of runtime and convergence behavior [17], privacy-preserving mechanisms [18], and benchmark problems specifically designed for MPMOPs [19], [20] have also been actively investigated.

Despite these advances, the evaluation of solution sets for MPMOPs still largely relies on classical performance metrics such as IGD [21] and HV [13], typically aggregated across parties to form metrics such as meanIGD and meanHV. Such aggregation implicitly assumes that identical geometric approximation quality with respect to each party's PF carries comparable evaluative significance across all DMs [22]–[24]. However, in multi-party decision-making contexts, this assumption may not align with the notion of consensus. That is, a solution set may achieve excellent approximation for one party while providing only marginal benefits to others, yet still obtain a favorable averaged score. Consequently, mean-based metrics may fail to distinguish between outcomes exhibiting balanced relative gains and those biased toward a subset of parties.

This limitation is further amplified by the solution structure of MPMOPs. Under strict Pareto rationality, solutions acceptable to all parties must lie in the intersection of their Pareto sets

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(PSs), which is often sparse or even empty. Classical metrics such as IGD and HV evaluate geometric approximation quality over the entire PF and may penalize solution sets concentrated in small common regions, even when such regions correspond to the only mutually acceptable outcomes. As a result, geometric balance or uniform coverage of the objective space, implicitly rewarded by these metrics, does not necessarily reflect consensus-oriented fairness in MPMOPs.

Given that cooperation in MPMOPs cannot be adequately characterized by strictly common Pareto-optimal solutions alone, a more general notion of consensus is required. In this work, consensus refers to solution sets that yield mutually acceptable relative gains across DMs, rather than strictly dominating outcomes under classical Pareto rationality. From a cooperative bargaining perspective, each DM is associated with a bargaining region representing the set of solutions it considers acceptable. Rather than treating the existence of strictly common solutions as a prerequisite, we allow parties to tolerate controlled concessions, thereby inducing a unified notion of consensus solutions that subsumes strictly common Pareto-optimal solutions as a special case. Such compromises are quantified through a concession rate vector, which characterizes the acceptable degree of deviation for each DM.

Building on this, we integrate classical performance metrics into a Nash-product-based evaluation framework so that solution sets yielding balanced relative gains across parties are consistently favored. To avoid superficially balanced yet non-consensual outcomes, we further introduce a penalty mechanism for solutions lying outside the jointly acceptable consensus region. The proposed framework is formulated as a performance evaluation methodology and is independent of specific optimization algorithms. The main contributions of this paper are summarized as follows:

- *Fairness limitations of existing performance metrics and evaluation axioms.* We identify fundamental limitations of existing mean-based performance metrics for MP-MOPs, showing that they may yield biased or incomplete evaluation by failing to capture consensus and balanced relative gains among DMs. From a cooperative game-theoretic perspective, we formalize four axioms that a fairness-aware performance evaluation function for MP-MOPs should satisfy: (A1) Pareto Monotonicity, (A2) Symmetry, (A3) Balance Preference, and (A4) Acceptability Monotonicity.
- *Consensus optimal solutions and Nash-product-based evaluation framework.* By introducing a concession rate vector to quantify acceptable compromises for each DM, we generalize the classical notion of MPMOP optimal solutions and define a unified concept of consensus solutions. Building on this formulation, we propose a Nash-product-based evaluation framework that integrates classical performance metrics and theoretically prove that it satisfies all four axioms. The proposed framework provides a principled mechanism for jointly evaluating efficiency and fairness across heterogeneous DMs.
- *Benchmark extensions for consensus-deficient negotiation structures.* To enable more rigorous and representative testing of multi-party multi-objective optimization algo-

rithms, we extend existing MPMOP benchmark suites by constructing problem instances with highly conflicting negotiation structures among DMs. These benchmarks exhibit empty intersections of party-specific PSs and therefore pose significant challenges for algorithmic convergence toward cooperative outcomes. They also provide an experimental basis for examining how different performance evaluation criteria evaluate algorithmic behavior under consensus-oriented settings.

The rest of this paper is organized as follows. Section II introduces the preliminaries. Section III analyzes the limitations of prevailing performance metrics for MPMOPs. Section IV presents the proposed fairness-aware performance evaluation framework based on the Nash product. Section V reports experimental results comparing the proposed metric with meanIGD across multiple representative algorithms on two MPMOP benchmarks. Section VI discusses the validity of the proposed evaluation function and its potential extension to problems without a known PF. Finally, Section VII concludes the paper and outlines directions for future work.

II. PRELIMINARIES

This section introduces the basic concepts required for the subsequent analysis, including MPMOPs, Pareto dominance relations in multi-party settings, the Nash bargaining framework, and commonly used performance metrics for evaluating multi-party multi-objective evolutionary algorithms.

A. Multi-party Multi-objective Optimization Problem

MPMOPs extend classical multi-objective optimization problems by considering multiple DMs, each associated with an individual set of potentially conflicting objectives. A minimization MPMOP can be formally defined as follows.

Definition 1 (MPMOP [1]). *An MPMOP involves M DMs. For each DM $m \in \{1, \ldots, M\}$, let*

$$F_m(\mathbf{x}) = \big(f_{m,1}(\mathbf{x}), \ldots, f_{m,K_m}(\mathbf{x})\big)$$

denote its K_m-dimensional objective vector. The overall problem is defined as

$$\min_{\mathbf{x} \in \mathcal{X}} F(\mathbf{x}) = \big(F_1(\mathbf{x}), \ldots, F_M(\mathbf{x})\big),$$

where $\mathbf{x} \in \mathcal{X} \subseteq \mathbb{R}^n$ is the decision vector and $\mathcal{X}$ denotes the feasible decision space.

To evaluate solution quality from the perspective of each individual DM, Pareto dominance is recalled for party-wise objective vectors.

Definition 2 (Pareto Dominance). *For a given party m, let $F_m(\mathbf{x}) = (f_{m,1}(\mathbf{x}), \ldots, f_{m,K_m}(\mathbf{x}))$. For two solutions $\mathbf{x}, \mathbf{y} \in \mathcal{X}$:*

1) *$\mathbf{x}$ weakly dominates $\mathbf{y}$ with respect to party m, denoted $\mathbf{x} \preceq_m \mathbf{y}$, if $f_{m,k}(\mathbf{x}) \leq f_{m,k}(\mathbf{y})$ for all $k \in \{1, \ldots, K_m\}$;*

2) *$\mathbf{x}$ dominates $\mathbf{y}$ with respect to party m, denoted $\mathbf{x} \prec_m \mathbf{y}$, if $\mathbf{x} \preceq_m \mathbf{y}$ and there exists at least one $k \in \{1, \ldots, K_m\}$ such that $f_{m,k}(\mathbf{x}) < f_{m,k}(\mathbf{y})$.*

Based on party-wise dominance relations, a joint dominance notion across all DMs can be defined.

Definition 3 (Multi-party Pareto Dominance). *For two solutions* $\mathbf{x}, \mathbf{y} \in \mathcal{X}$ *in an MPMOP:*

1) $\mathbf{x}$ *weakly dominates* $\mathbf{y}$ *in the multi-party sense, denoted* $\mathbf{x} \preceq^{\mathrm{MP}} \mathbf{y}$*, if* $\mathbf{x} \preceq_m \mathbf{y}$ *holds for all* $m \in \{1, \ldots, M\}$*;*
2) $\mathbf{x}$ *dominates* $\mathbf{y}$ *in the multi-party sense, denoted* $\mathbf{x} \prec^{\mathrm{MP}} \mathbf{y}$*, if* $\mathbf{x} \preceq_m \mathbf{y}$ *holds for all* $m \in \{1, \ldots, M\}$ *and there exists at least one party* m *such that* $\mathbf{x} \prec_m \mathbf{y}$*.*

Accordingly, multi-party Pareto-optimal solutions correspond to the intersection of party-wise Pareto-optimal sets.

B. Nash Bargaining Framework

The Nash bargaining problem [25] is a canonical cooperative game-theoretic model for analyzing negotiated outcomes among multiple participants. Let $\mathcal{S} \subseteq \mathbb{R}^M$ denote the feasible utility set, where each vector $\mathbf{u} = (u_1, \ldots, u_M)$ represents the utilities simultaneously attainable by all M parties through cooperation. Let $\mathbf{d} = (d_1, \ldots, d_M)$ denote the disagreement (or fallback) utility vector, specifying the utility received by each party if no agreement is reached. The Nash bargaining problem is thus defined as the pair $(\mathcal{S}, \mathbf{d})$, subject to the feasibility condition

$$\mathcal{S} \cap \{\mathbf{u} : \mathbf{u} \geq \mathbf{d}\} \neq \emptyset.$$

To ensure the existence and uniqueness of the Nash bargaining solution, $\mathcal{S}$ is typically assumed to be closed and convex. In this work, the Nash bargaining framework is used as an analytical tool for performance evaluation, without serving as a solution concept or optimization mechanism for MPMOPs.

Within multi-objective game environments, cooperative and bargaining-based solution concepts have been extensively studied to characterize rational and fair outcomes [26], [27]. Existing approaches can be broadly categorized as follows: (a) extensions of Nash equilibrium to vector-valued payoffs, which replace scalar utility comparisons with Pareto dominance or aggregated utility mechanisms [28]–[31]; (b) multi-objective bargaining solutions, which employ constructs such as the Nash product to formalize equitable compromise in vector payoff spaces [32]–[35]; and (c) preference-aware optimization frameworks, which explicitly incorporate heterogeneous preferences or individual utility regions into the decision-making process [36]–[38].

While these studies establish important theoretical foundations for fairness and cooperation, fairness is predominantly embedded in solution construction or equilibrium definitions. In contrast, existing performance metrics for MPMOPs are typically inherited from single DM multi-objective optimization and lack algorithm-independent mechanisms for evaluating balanced cooperation and consensus quality across heterogeneous DMs.

C. Performance metrics

Performance metrics play a central role in evaluating multi-objective optimization algorithms by quantifying the quality of obtained solution sets. Among the most widely adopted

metrics are the IGD and HV, which are commonly used to evaluate convergence and diversity in classical multi-objective optimization. In MPMOPs, however, each DM is associated with an individual PF, which complicates direct application of classical metrics. As a result, existing studies have proposed multi-party extensions of IGD and HV by aggregating party-wise performance evaluation.

1) Inverted Generational Distance (IGD): The IGD evaluates how well an approximate solution set P represents a reference PF by measuring the average distance from each point in PF to its nearest point in P:

$$\mathrm{IGD}(\mathrm{PF}, P) = \frac{1}{|\mathrm{PF}|} \sum_{\mathbf{v} \in \mathrm{PF}} d(\mathbf{v}, P), \tag{1}$$

where $d(\mathbf{v}, P)$ denotes the minimum Euclidean distance between $\mathbf{v}$ and any point in P. Smaller IGD values indicate closer approximation of the reference PF.

For MPMOPs, since each DM m is associated with an individual PF_m, Liu et al. [1] proposed a multi-party extension of IGD that aggregates distances across all parties. Specifically, the distance between a reference point $\mathbf{v}$ and a candidate solution set P is defined as

$$d(\mathbf{v}, P) = \min_{\mathbf{p} \in P} \sum_{m=1}^{M} \sqrt{\sum_{k=1}^{K_m} (v_{m,k} - p_{m,k})^2}, \tag{2}$$

where K_m denotes the number of objectives of party m, and $v_{m,k}$ and $p_{m,k}$ represent the k-th objective value of party m for $\mathbf{v}$ and $\mathbf{p}$, respectively. The remaining computation follows the classical IGD formulation. This multi-party IGD measures the overall geometric proximity of P to all party-wise PFs.

2) Hypervolume (HV): The HV metric measures the volume of the objective space dominated by a solution set P with respect to a predefined reference point $\mathbf{r}$. For a minimization problem associated with party m, each solution $\mathbf{p} \in P$ is represented in the corresponding objective space as

$$\mathbf{p} = (p_{m,1}, \ldots, p_{m,K_m}).$$

The HV of P with respect to party m is defined as

$$\mathrm{HV}_m(P) = \lambda \left(\bigcup_{\mathbf{p} \in P} [p_{m,1}, r_{m,1}] \times \cdots \times [p_{m,K_m}, r_{m,K_m}] \right), \tag{3}$$

where $\lambda(\cdot)$ denotes the Lebesgue measure.

In MPMOPs, a unified joint hypervolume across all parties is generally unavailable due to heterogeneous objective spaces. A common practice is therefore to evaluate the hypervolume separately for each party and then aggregate the results. For example, Chen et al. [9] proposed computing the mean hypervolume as

$$\mathrm{meanHV}(P) = \frac{1}{M} \sum_{m=1}^{M} \mathrm{HV}_m(P), \tag{4}$$

which treats the party-wise HV values symmetrically.

III. FAIRNESS LIMITATIONS OF CLASSICAL METRICS

In MPMOPs, classical performance metrics such as IGD and HV are typically evaluated with respect to each DM's PF and then aggregated across parties using arithmetic means, yielding metrics such as $\mathrm{meanIGD}$ and meanHV. While computationally convenient, this practice implicitly treats the problem as if it involved a single representative DM, thereby overlooking fairness considerations among heterogeneous DMs.

As a consequence, the aggregated metric value may improve even when one or more DMs experience substantial utility degradation. Such evaluations are inconsistent with classical cooperative bargaining principles, in which balanced concessions and symmetry among participants are central. From a negotiation perspective, each DM evaluates the same solution set through a distinct utility mapping [1], [39], [40]. Simple averaging of per-party IGD or HV values can therefore obscure asymmetric sacrifices, assigning similar scores to fundamentally different cooperative outcomes.

To make this limitation explicit, we conduct visualization-based comparative analyses on multi-party distance minimization problems (MPDMPs) [7]. These problems exhibit a transparent geometric structure that allows simultaneous inspection of solution behavior in both decision and objective spaces, making them particularly suitable for illustrating how solution sets relate to different PFs and whether their trade-offs align with intuitive notions of cooperative fairness. MPDMPs model scenarios in which multiple DMs share a common decision space but evaluate solutions using distinct distance-based objectives reflecting heterogeneous preferences. We consider two DMs, denoted as DM A and DM B, sharing a common decision variable $\mathbf{x} \in \mathbb{R}^2$. Each DM seeks to minimize Euclidean distances from $\mathbf{x}$ to two distinct target points. Two representative cases are constructed.
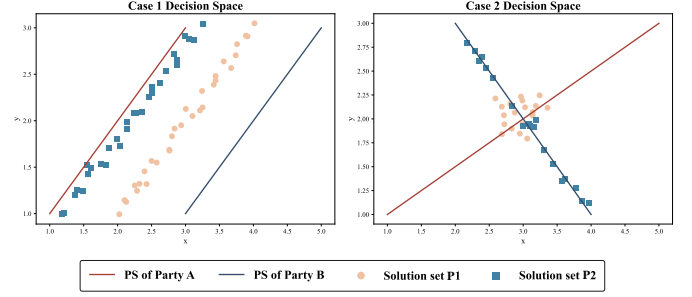
Case 1 (Disjoint Pareto-optimal regions): DM A minimizes distances to $(1, 1)$ and $(3, 3)$, while DM B minimizes distances to $(3, 1)$ and $(5, 3)$. The resulting PSs in the decision space are parallel and disjoint, implying that no solution simultaneously lies on both DMs' PSs.

Case 2 (Intersecting Pareto-optimal regions): DM A minimizes distances to $(1, 1)$ and $(5, 3)$, while DM B minimizes distances to $(2, 3)$ and $(4, 1)$. In this configuration, the two PSs intersect at $\mathbf{x}^* = (3, 2)$, yielding a common Pareto-optimal region that naturally represents a balanced compromise.
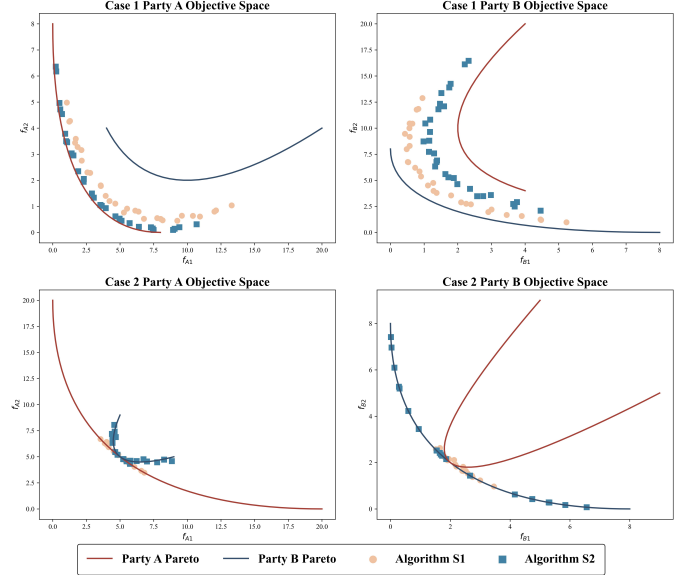
Fig. 1 visualizes both cases in the decision and objective spaces, together with two representative solution sets, denoted as P_1 and P_2.

TABLE I: IGD and HV comparison for representative solution sets in Case 1 and Case 2.

	$\mathrm{IGD_A}$	$\mathrm{IGD_B}$	$\mathrm{meanIGD}$	$\mathrm{HV_A}$	$\mathrm{HV_B}$	meanHV
Case 1						
P_1	0.73	0.73	0.73 $\approx$	3.90	3.90	3.90 $\approx$
P_2	0.14	1.36	0.75 $\approx$	5.42	2.06	3.74 $\approx$
Case 2						
P_1	5.18	1.78	3.48 $\downarrow$	9.65	5.19	7.42 $\approx$
P_2	5.42	0.40	2.91 $\uparrow$	8.19	6.68	7.43 $\approx$

(a) Decision spaces. Left: the PSs of the two DMs are parallel and disjoint, indicating the absence of a common Pareto-optimal region. In this case, solution set P_2 is strongly biased toward DM A, whereas P_1 exhibits a more symmetric compromise. Right: the PSs of the two DMs intersect, forming a common Pareto-optimal region. Here, P_1 concentrates around this intersection, while P_2 is biased toward DM B.

(b) Objective spaces. The first row corresponds to Case 1, and the second row corresponds to Case 2. In Case 1, P_2 closely approximates the PF of DM A but remains far from that of DM B, whereas P_1 maintains a more balanced approximation for both DMs. In Case 2, P_2 provides relatively uniform coverage of DM B's PF, while P_1 concentrates near the intersection of the two PFs, reflecting a more consensus-oriented outcome.

Fig. 1: Visualization of fairness limitations of mean-based performance metrics in representative MPMOP cases. Decision and objective spaces for two representative MPDMP cases are shown, illustrating scenarios with and without a common Pareto-optimal region.

Table I reports the IGD and HV values for both cases. In Case 1, solution set P_2 strongly favors DM A, achieving a very small $\mathrm{IGD_A}$ at the expense of a large $\mathrm{IGD_B}$. However, $\mathrm{meanIGD}$ assigns nearly identical scores to P_1 and P_2, effectively masking this imbalance. A similar effect is observed for HV, where aggregation masks the underlying imbalance. From a bargaining perspective, P_1 corresponds to a more symmetric compromise, whereas P_2 does not satisfy the symmetry principle central to cooperative bargaining.

In Case 2, where a common Pareto-optimal region exists, P_1 concentrates near the intersection of the two PFs and represents a balanced cooperative outcome. Nevertheless, $\mathrm{meanIGD}$

slightly favors P_2, which is clearly biased toward DM B, due to its uneven distance profile. meanHV again fails to distinguish the two solution sets. This behavior is inconsistent with the bargaining intuition that solution sets close to the common region should be preferred as fair and effective cooperative outcomes.

These examples demonstrate that mean-based performance metrics not only mask severe imbalance when no common Pareto-optimal region exists, but also fail to appropriately favor solution sets that concentrate near the common Pareto-optimal region when such solutions are available.

IV. A FAIRNESS-AWARE NASH-PRODUCT-BASED EVALUATION FRAMEWORK FOR MPMOPS

This section develops a fairness-aware evaluation method for MPMOPs. Motivated by the limitations of mean-based performance aggregation, we propose an evaluation function that explicitly accounts for balanced compromise, impartiality across DMs, and sensitivity to mutual acceptability. Our construction proceeds in three steps. First, we formalize a set of axioms that capture rationality, symmetry, equity, and acceptability requirements for evaluating solution sets in MPMOPs. Second, we show that these axioms are satisfied by a *Nash-product-based evaluation function*, which aggregates per-DM utilities multiplicatively. Third, we extend this evaluation function to explicitly penalize violations of inter-party acceptability through concession-based penalty mechanism that augments individual loss terms. These steps yield a unified and axiomatically grounded evaluation framework, whose theoretical properties are formally established in the remainder of this section.

A. Axioms for Fairness-Aware Evaluation

We consider an MPMOP involving M DMs, indexed by $\mathcal{M} = \{1, \ldots, M\}$. Each DM evaluates a candidate solution set P using a common performance metric, interpreted from its own objective perspective. Let $u_m(P) \in \mathbb{R}_+$ denote the resulting utility for DM m, where larger values indicate better performance. An overall evaluation function aggregates individual utilities as

$$\Psi(P) = \Psi\big(u_1(P), \ldots, u_M(P)\big).$$

The following axioms characterize structural properties that such an aggregation should satisfy in order to reflect fairness considerations inherent in MPMOPs.

Axiom 1 (Pareto Monotonicity). *For any two solution sets P_1 and P_2, if*

$$u_i(P_1) \geq u_i(P_2) \; \forall i \in \mathcal{M},$$

and there exists $j \in \mathcal{M}$ such that $u_j(P_1) > u_j(P_2)$, then

$$\Psi(P_1) > \Psi(P_2).$$

Axiom 2 (Symmetry). *For any permutation π of $\mathcal{M}$, assuming that all DM-specific parameters are permuted consistently with their corresponding utilities,*

$$\Psi\big(u_1(P), \ldots, u_M(P)\big) = \Psi\big(u_{\pi(1)}(P), \ldots, u_{\pi(M)}(P)\big).$$

Axiom 3 (Balance Preference). *Let $\mathbf{u}(P_1)$ and $\mathbf{u}(P_2)$ denote the utility vectors of two solution sets such that*

$$\sum_{m=1}^{M} u_m(P_1) = \sum_{m=1}^{M} u_m(P_2).$$

If $\mathbf{u}(P_1)$ is obtained from $\mathbf{u}(P_2)$ by a mean-preserving contraction, then

$$\Psi(P_1) > \Psi(P_2).$$

Axiom 4 (Acceptability Monotonicity). *Let $\mathcal{A} \subseteq \mathcal{X}$ denote the common acceptable region. For any two solution sets $P_1, P_2 \subseteq \mathcal{X}$ such that*

$$P_2 = P_1 \cup \{\mathbf{x}'\} \setminus \{\mathbf{x}\}, \quad \mathbf{x} \in P_1 \cap \mathcal{A}, \quad \mathbf{x}' \notin \mathcal{A},$$

then

$$\Psi(P_1) > \Psi(P_2).$$

Collectively, these axioms encode rationality (Pareto Monotonicity), impartiality across DMs (Symmetry), preference for equitable compromise under fixed total utility (Balance Preference), and strict sensitivity to violations of mutual acceptability (Acceptability Monotonicity).

B. Nash-Product-Based Evaluation Function

The axioms impose strong structural constraints on $\Psi(\cdot)$. In particular, Pareto Monotonicity and Symmetry rule out purely ordinal or identity-dependent aggregations, while Balance Preference is incompatible with purely additive or mean-based aggregation schemes, which, as shown in Section III, may assign similar scores to highly imbalanced outcomes. These requirements collectively make a multiplicative form of aggregation particularly suitable, due to its sensitivity to imbalance and proportional losses.

In MPMOP, cooperation typically does not generate surplus gains; instead, each DM incurs unavoidable performance losses relative to its individual Pareto-optimal set. Fairness therefore corresponds to distributing these losses in a balanced manner. To formalize this principle, we adopt the Nash product as a representative realization.

Let $L_m(P) \geq 0$ denote a loss-based performance metric for DM m, where smaller values indicate better performance and $L_m(P) = 0$ corresponds to achieving the ideal PF. We define the utility of DM m as

$$u_m(P) = C - L_m(P), \quad (5)$$

Here, C is introduced solely to ensure positivity of utilities and does not affect the relative comparison between solution sets.

Following the Nash bargaining, we aggregate individual utilities multiplicatively and define the evaluation function as

$$\Psi_{\mathrm{NP}}(P) = \prod_{m=1}^{M} u_m(P) = \prod_{m=1}^{M} \left(C - L_m(P)\right). \quad (6)$$

This function penalizes disproportionate losses multiplicatively and strictly favors balanced compromises when total utility is fixed. Importantly, Ψ_{NP} is introduced as an *evaluation metric*, rather than a solution or decision rule.

C. Consensus Solutions via Concession-Based Penalty

If a strictly common Pareto-optimal solution exists, it represents the most stringent form of cooperation in a MPMOP, and an effective algorithm is expected to approximate it as closely as possible. However, such solutions may not always exist, and even when they do, fairness alone does not guarantee joint acceptability. In particular, balanced performance improvements may still impose unacceptable sacrifices on individual parties. This makes it necessary to explicitly characterize and quantify consensus in multi-party settings.

In many practical MPMOPs, the Pareto-optimal sets of different DMs exhibit little or no overlap. In such cases, cooperation can only be achieved if each party concedes part of its individually optimal outcome, allowing a mutually acceptable region to emerge. Moreover, cooperative outcomes typically involve asymmetric and explicitly quantifiable sacrifices across DMs, reflecting differences in how far each party deviates from its own Pareto-optimal set. Consequently, cooperative outcomes generally involve unequal and explicitly quantifiable levels of sacrifice across parties.

To formally characterize such sacrifices, we introduce the notion of a *concession rate*, which measures the relative performance loss tolerated by each DM with respect to its own Pareto-optimal set.

Definition 4 (Concession Rate). *Let $\mathcal{X}$ denote the feasible decision space, and let $\mathcal{P}_m^*$ be the Pareto-optimal set of DM m. For any candidate solution $\mathbf{x} \in \mathcal{X}$ and any reference solution $\mathbf{y} \in \mathcal{P}_m^*$, define the normalized component-wise deviation as*

$$\delta_m(\mathbf{x}, \mathbf{y}) = \max_{k \in K_m} \frac{f_{m,k}(\mathbf{x}) - f_{m,k}(\mathbf{y})}{f_{m,k}^{\max} - f_{m,k}^{\min}},$$

where K_m denotes the set of objective indices of DM m, and $f_{m,k}^{\max}$ and $f_{m,k}^{\min}$ are the bounds of the k-th objective over $\mathcal{X}$.

The deviation of $\mathbf{x}$ from DM m's Pareto-optimal set is defined as

$$D_m(\mathbf{x}) = \min_{\mathbf{y} \in \mathcal{P}_m^*} \delta_m(\mathbf{x}, \mathbf{y}).$$

Let $\mathcal{P}_{-m}^* = \bigcup_{j \neq m} \mathcal{P}_j^*$ denote the union of Pareto-optimal sets of all other DMs, and define

$$\Delta_m = \max_{\mathbf{q} \in \mathcal{P}_{-m}^*} D_m(\mathbf{q}).$$

The concession rate of $\mathbf{x}$ with respect to DM m is then given by

$$\gamma_m(\mathbf{x}) = \frac{D_m(\mathbf{x})}{\Delta_m} \in [0, 1].$$

The concession rate quantifies the relative sacrifice tolerated by each DM in achieving cooperative outcomes. Unlike classical ε-based metrics in multi-objective optimization, which measure dominance relaxation or approximation quality [41], γ_m explicitly captures inter-party compromise across heterogeneous Pareto-optimal sets.

Using the concession rate, we extend the classical notion of multi-party Pareto optimal solutions to explicitly incorporate acceptability constraints.
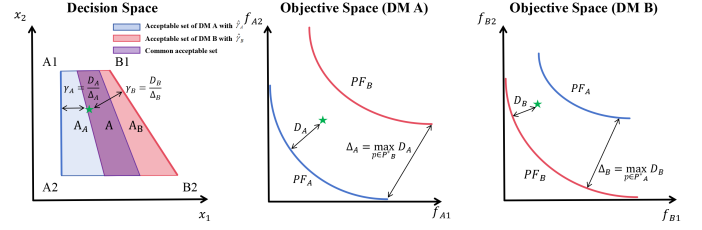

Fig. 2: Illustration of concession rates and the mutually acceptable region. γ_m quantifies the normalized deviation from it. The intersection $\mathcal{A}$ indicates solutions meeting all DMs' concession thresholds.

Definition 5 (Generalized Multi-party Pareto-optimal Solutions). *Let $\mathcal{P}^* \subset \mathcal{X}$ denote the set of solutions that are non-dominated with respect to the joint objective vector*

$$F(\mathbf{x}) = (f_{1,1}(\mathbf{x}), \ldots, f_{1,K_1}(\mathbf{x}), \ldots, f_{M,1}(\mathbf{x}), \ldots, f_{M,K_M}(\mathbf{x})).$$

Given concession thresholds $\hat{\gamma}_m \in [0, 1]$ for each DM m, define the extended acceptable set

$$\mathcal{A}_m(\hat{\gamma}_m) = \{\mathbf{x} \in \mathcal{P}^* \mid \gamma_m(\mathbf{x}) \leq \hat{\gamma}_m\}.$$

The mutually acceptable region is defined as

$$\mathcal{A} = \bigcap_{m=1}^{M} \mathcal{A}_m(\hat{\gamma}_m).$$

If $\hat{\gamma}_m = 0$ for all m, the mutually acceptable region reduces to the classical strictly common Pareto-optimal set. If $\mathcal{A} = \emptyset$, no solution satisfies all DMs' concession thresholds simultaneously. Otherwise, each $\mathbf{x} \in \mathcal{A}$ represents a consensus solution that balances efficiency, fairness, and acceptability.

Having characterized the mutually acceptable region induced by concession constraints, we now incorporate acceptability into the Nash-product-based evaluation function Ψ_{NP}, which introduced earlier satisfies Pareto monotonicity, symmetry, and balance preference, but does not explicitly penalize violations of acceptability.

Definition 6 (Non-consensus Penalty). *For a solution $\mathbf{x}$ and DM m, define the acceptability violation as*

$$\phi_m(\mathbf{x}) = \max\{0, \gamma_m(\mathbf{x}) - \hat{\gamma}_m\}.$$

For a solution set $P \subset \mathcal{X}$, the total non-consensus penalty incurred by DM m is

$$L_m^{\text{pen}}(P) = \sum_{\mathbf{x} \in P} \phi_m(\mathbf{x}).$$

Accordingly, the utility of DM m is given by

$$u_m(P) = C - (L_m(P) + \lambda_m L_m^{\text{pen}}(P)),$$

where $\lambda_m > 0$ controls the strictness of concession enforcement. The Nash-product evaluation function is redefined as

$$\Psi_{\text{NP}}(P) = \prod_{m=1}^{M} \left(C - (L_m(P) + \lambda_m L_m^{\text{pen}}(P)) \right).$$

This formulation preserves the structural properties of the original Nash-product-based criterion while explicitly discouraging non-consensus solutions. As a result, Ψ_{NP} provides

a unified performance evaluation that jointly accounts for efficiency, fairness, and mutual acceptability in MPMOPs.

D. Axiomatic Analysis of the Proposed Evaluation Criterion

In this subsection, we provide that the Nash-product-based formulation satisfies the axioms. The four axioms are verified separately through Lemma 1–4.

Lemma 1 (Pareto Monotonicity). *The Nash-product-based evaluation function $\Psi_{\mathrm{NP}}(P)$ satisfies Axiom 1.*

Proof. Consider two solution sets P_1 and P_2 such that $u_m(P_1) \geq u_m(P_2)$ for all m, and there exists at least one DM j with $u_j(P_1) > u_j(P_2)$. Because the product of positive numbers is strictly increasing in each component, we have

$$\prod_{m=1}^{M} u_m(P_1) > \prod_{m=1}^{M} u_m(P_2).$$

Hence, $\Psi_{\mathrm{NP}}(P_1) > \Psi_{\mathrm{NP}}(P_2)$, establishing Pareto monotonicity. $\square$

Lemma 2 (Symmetry). *The Nash-product-based evaluation function $\Psi_{\mathrm{NP}}(P)$ satisfies Axiom 2.*

Proof. Let π be any permutation of DM indices. Permuting both the utilities and all associated DM-specific parameters yields

$$\prod_{m=1}^{M} u_{\pi(m)}(P) = \prod_{m=1}^{M} u_m(P),$$

since multiplication is commutative. Therefore, the evaluation is invariant under any permutation of DMs, and symmetry is satisfied. $\square$

Lemma 3 (Balance Preference). *The Nash-product-based evaluation function $\Psi_{\mathrm{NP}}(P)$ satisfies Axiom 3.*

Proof. Consider two solution sets P_1 and P_2 with equal total utility and such that $\mathbf{u}(P_1)$ is obtained from $\mathbf{u}(P_2)$ by a mean-preserving contraction. The logarithm of the Nash product is strictly concave:

$$\log \Psi_{\mathrm{NP}}(P_1) = \sum_{m=1}^{M} \log u_m(P_1).$$

By the definition of mean-preserving contraction and Jensen's inequality, we have

$$\sum_{m=1}^{M} \log u_m(P_1) > \sum_{m=1}^{M} \log u_m(P_2),$$

and exponentiating both sides yields

$$\Psi_{\mathrm{NP}}(P_1) > \Psi_{\mathrm{NP}}(P_2),$$

as required. $\square$

Lemma 4 (Acceptability Monotonicity). *The Nash-product-based evaluation function $\Psi_{\mathrm{NP}}(P)$ satisfies Axiom 4 for sufficiently large penalty coefficients $\{\lambda_m\}_{m=1}^{M}$.*

Proof. Consider two solution sets

$$P_2 = P_1 \cup \{\mathbf{x}'\} \setminus \{\mathbf{x}\}, \quad \mathbf{x} \in P_1 \cap \mathcal{A}, \quad \mathbf{x}' \notin \mathcal{A}.$$

By definition of $\mathcal{A}$, there exists at least one DM $m_0 \in \{1, \ldots, M\}$ such that

$$\phi_{m_0}(\mathbf{x}') = \max\{0, \gamma_{m_0}(\mathbf{x}') - \hat{\gamma}_{m_0}\} > 0.$$

The utility of DM m is

$$u_m(P) = C - \big(L_m(P) + \lambda_m L_m^{\mathrm{pen}}(P)\big).$$

Since P_1 and P_2 differ in exactly one solution, the losses change as

$$L_{m_0}(P_2) = L_{m_0}(P_1) + \mu_{m_0}(\mathbf{x}'),$$
$$L_{m_0}^{\mathrm{pen}}(P_2) = L_{m_0}^{\mathrm{pen}}(P_1) + \phi_{m_0}(\mathbf{x}').$$

Hence,

$$u_{m_0}(P_2) = u_{m_0}(P_1) - \big(\mu_{m_0}(\mathbf{x}') + \lambda_{m_0}\phi_{m_0}(\mathbf{x}')\big). \quad (7)$$

For all $m \neq m_0$, let $\mu_m(\mathbf{x}') \geq 0$ denote the maximal possible utility increase due to replacing $\mathbf{x}$ by $\mathbf{x}'$. Let $\underline{u}_m$ denote a positive lower bound of $u_m(P_1)$ ensured by the choice of C. Then

$$u_m(P_2) \leq u_m(P_1) + \mu_m(\mathbf{x}') \leq \underline{u}_m + \mu_m(\mathbf{x}').$$

By bounding each factor with the lower bound $\underline{u}_m$, we obtain

$$\prod_{m \neq m_0} \left(1 + \frac{\mu_m(\mathbf{x}')}{u_m(P_1)}\right) \leq \prod_{m \neq m_0} \left(1 + \frac{\mu_m(\mathbf{x}')}{\underline{u}_m}\right) = \alpha^{M-1}.$$

Then a sufficient condition for strict decrease of Nash-product ratio is

$$\frac{\Psi_{\mathrm{NP}}(P_2)}{\Psi_{\mathrm{NP}}(P_1)} = \left(1 - \frac{\mu_{m_0}(\mathbf{x}') + \lambda_{m_0}\phi_{m_0}(\mathbf{x}')}{u_{m_0}(P_1)}\right) \alpha^{M-1} < 1.$$

which yields

$$\lambda_{m_0} > \frac{u_{m_0}(P_1)\big(1 - \alpha^{1-M}\big) - \mu_{m_0}(\mathbf{x}')}{\phi_{m_0}(\mathbf{x}')}.$$

Therefore, choosing λ_{m_0} sufficiently large ensures $\Psi_{\mathrm{NP}}(P_2) < \Psi_{\mathrm{NP}}(P_1)$. $\square$

Remark 1. *Lemma 4 establishes a sufficient condition on the penalty coefficients λ_m under which acceptability violations dominate any potential utility gains of other DMs. The bound is derived from a worst-case analysis and is not intended to be tight. In practice, moderate penalty values are sufficient to enforce acceptability, while the existence result provides theoretical consistency of the proposed evaluation function.*

The above result formally shows that the Nash-product-based evaluation function simultaneously captures efficiency improvement (Pareto monotonicity), fair treatment of DMs (symmetry), balanced compromise across parties (balance preference), and concession rationality for DMs (acceptability monotonicity), which are essential for evaluating solution sets in MPMOPs. Thus, we conclude that the proposed Nash-product-based evaluation metric framework identifies the utility-balanced solution set among all equally efficient outcomes. This provides a theoretically grounded and consistent measure of fairness in multi-party multi-objective optimization.

Theorem 1 (Axiomatic Soundness of the Proposed Evaluation Framework)**.** *The fairness-aware Nash-product-based evaluation function*

$$\Psi_{\text{NP}}(P) = \prod_{m=1}^{M} \left(C - \left(L_m(P) + \lambda_m L_m^{\text{pen}}(P) \right) \right).$$

satisfies all four axioms introduced in Section IV-A, namely Pareto Monotonicity (A1), Symmetry (A2), Balance Preference (A3), and Acceptability Monotonicity (A4).

Proof. Lemmas 1–3 establish that the Nash-product aggregation satisfies Axioms (A1)–(A3). Lemma 4 further shows that the introduction of non-consensus penalties enforces Acceptability Monotonicity (A4). Therefore, the proposed evaluation function satisfies all axioms simultaneously. ☐

V. EXPERIMENTS

This section presents comprehensive experiments to demonstrate the effectiveness of the proposed fairness-aware evaluation framework. Two representative classes of multi-party optimization problems are considered. These experiments complement the theoretical analysis and evaluate the practical behavior of the proposed framework under different consensus conditions.

A. Experimental Design and Setup

1) Test Problems: Two categories of benchmark problems are considered. The first category consists of continuous multi-party multi-objective optimization problems (MPMOPs), including MPMOP1–MPMOP11 from [1] and six newly constructed benchmarks, MPMOP12–MPMOP17, designed to cover non-consensus scenarios. The second category focuses on multi-party distance minimization problems (MPDMPs), including MPDMP1–MPDMP12 from [7] and six additional instances, MPDMP13–MPDMP18. Detailed definitions of the extended benchmark problems are provided in the supplement document. For all benchmarks, each algorithm is independently run 30 times, and the mean and variance of the results are reported.

2) Compared Algorithms: Several representative algorithms are selected for comparison. For the MPMOP experiments, OptAll, which aggregates all DMs' objectives into a single multi-objective optimization problem, and OptMPDNS [1], one of the earliest algorithms tailored for multi-party multi-objective optimization, are considered. For the MPDMP experiments, OptAll and OptMPDNS are further compared with two improved variants, OptMPDNS2 [6] and OptMPDNS3 [7]. OptMPDNS2 extends OptMPDNS by incorporating nondominated sorting information from all parties, while OptMPDNS3 further enhances the search process using a PFt-based initialization strategy and an adaptive differential evolution operator. Among these methods, OptMPDNS3 has been reported to achieve strong performance on MPDMPs with existing consensus solutions.

3) Performance metrics: All obtained solution sets are evaluated under two classes of metrics.
- Classical metrics: meanIGD, computed as the arithmetic mean across all DMs.
- Fairness-aware metric: the proposed Nash-product-based evaluation function Ψ_{NP}, where larger values indicate better-balanced inter-party compromise.

4) Parameter Settings: To ensure a fair comparison, all algorithms are implemented under identical experimental settings. The penalty factor λ and the constant C are specified in the supplement document. To ensure consistency, the concession rates of all DMs are set to zero in the original benchmarks. All remaining parameters follow the configurations reported in the corresponding original studies, unless otherwise stated.

Furthermore, to evaluate algorithmic robustness in scenarios without common solutions, six additional benchmark problems are constructed. For each problem, six test cases with different concession rate configurations are designed. The specific parameter configurations are detailed in the supplement document.

B. Experimental Results

This subsection reports and analyzes the experimental results on both MPMOPs and MPDMPs, focusing on the comparison between the proposed Nash-product-based evaluation function (Ψ_{NP}) and the conventional meanIGD metric.

TABLE II: Comparison of OptMPNDS and OptAll under Ψ_{NP} and meanIGD for MPMOP benchmarks ($D = 2$).

Problem	Metric	OptMPNDS	OptAll
MPMOP1	Ψ_{NP}	**4.21E+07 (5.94E+06)**	7.79E+05 (1.37E+06)
	meanIGD	1.44E+00 (2.96E-01)	**9.52E-01 (2.56E-02)**
MPMOP2	Ψ_{NP}	**8.11E+02 (2.49E+01)**	5.26E+01 (2.13E+01)
	meanIGD	9.36E-02 (1.72E-02)	**7.53E-02 (9.64E-03)**
MPMOP3	Ψ_{NP}	**6.12E+05 (3.26E+03)**	9.02E+04 (2.65E+04)
	meanIGD	**1.08E-01 (3.16E-04)**	1.15E-01 (1.85E-03)
MPMOP4	Ψ_{NP}	1.18E+01 (5.58E+00)	**1.24E+01 (8.35E+00)**
	meanIGD	**6.59E-02 (2.77E-03)**	6.63E-02 (2.87E-03)
MPMOP5	Ψ_{NP}	8.86E+01 (1.85E+01)	**8.87E+01 (1.38E+01)**
	meanIGD	**1.05E-01 (5.40E-03)**	1.06E-01 (5.11E-03)
MPMOP6	Ψ_{NP}	**7.10E+00 (2.08E+00)**	6.32E+00 (2.16E+00)
	meanIGD	**1.30E-01 (5.97E-03)**	1.32E-01 (6.97E-03)
MPMOP7	Ψ_{NP}	**2.87E+12 (2.25E+11)**	1.51E+09 (6.23E+09)
	meanIGD	**1.27E+00 (4.44E-01)**	1.28E+00 (2.06E-01)
MPMOP8	Ψ_{NP}	**4.91E+05 (6.40E+03)**	3.68E+03 (3.06E+03)
	meanIGD	**1.97E-01 (2.74E-02)**	2.50E-01 (3.68E-02)
MPMOP9	Ψ_{NP}	4.20E+13 (7.97E+13)	**4.78E+13 (8.21E+13)**
	meanIGD	**1.95E-01 (3.96E-03)**	1.95E-01 (4.22E-03)
MPMOP10	Ψ_{NP}	**3.20E+03 (5.91E+02)**	3.16E+03 (5.58E+02)
	meanIGD	1.42E-01 (8.52E-03)	**1.40E-01 (8.33E-03)**
MPMOP11	Ψ_{NP}	**8.10E+05 (6.96E+03)**	1.09E+05 (4.90E+04)
	meanIGD	**1.97E-01 (8.66E-03)**	2.91E-01 (2.08E-02)

1) Results on MPMOP Benchmarks: Table II shows the evaluation results of OptMPNDS and OptAll on MPMOP1–MPMOP11 under both Ψ_{NP} and meanIGD. For most prob-
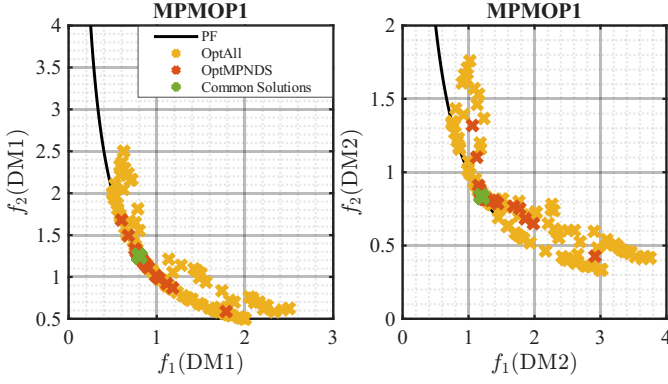
Fig. 3: Representative solution sets of OptMPNDS and OptAll on objective space of MPMOP1.

lems, the solutions achieving the best Ψ_{NP} and the best meanIGD do not coincide. Only in a few cases (MP-MOP3, MPMOP6, MPMOP7, MPMOP8 and MPMOP11) does OptMPNDS achieve superior performance under both metrics simultaneously. This discrepancy indicates meanIGD emphasizes proximity and uniformity relative to individual DMs' PFs, while Ψ_{NP} additionally accounts for fairness and penalizes solutions falling outside commonly acceptable regions.

To illustrate these differences, Fig. 3 shows representative solutions on MPMOP1. The solution set produced by OptAll (yellow points) is more evenly distributed along the PFs of individual DMs, even though it does not fully lie within the mutually acceptable region. In contrast, OptMPNDS (red points) generates solutions that are more concentrated around the common acceptable region (green points). Consequently, OptAll attains a better meanIGD score due to its emphasis on coverage of individual PFs, whereas OptMPNDS achieves a higher Ψ_{NP} by avoiding non-acceptable solutions and focusing on fairness-sensitive compromises.

The evaluation is further extended to non-consensus scenarios with MPMOP12–MPMOP17, as shown in Fig. 4, where six cases of concession thresholds are considered to progressively enlarge the acceptable region. In most cases, OptAll achieves higher scores under Ψ_{NP}. In the absence of common solutions, however, OptMPNDS tends to converge to the PS of one DM, resulting in asymmetric concessions, one DM incurs heavy penalties while the other receives minor penalties, leading to a lower Nash-product-based score. By contrast, OptAll, which simultaneously considers all four objectives, distributes more evenly between DMs. As a result, the product of penalties is relatively smaller, yielding higher Ψ_{NP} values. As the concession threshold increases, scores of both algorithms rise. The performance gap gradually narrows, especially for MPMOP13, MPMOP14, and MPMOP17. This trend confirms that the unfairness of OptMPNDS in non-consensus problems is alleviated as the acceptable region expands, allowing previously heavily penalized solutions to contribute positively to the overall score. Fig. 5 presents the PSs and solution distributions for MPMOP12 and MPMOP13. OptAll produces solutions that are relatively uniform across the objective space,

while OptMPNDS tends to generate solutions closer to the PS of DM2. This observation aligns with the trends reported in Fig. 4.
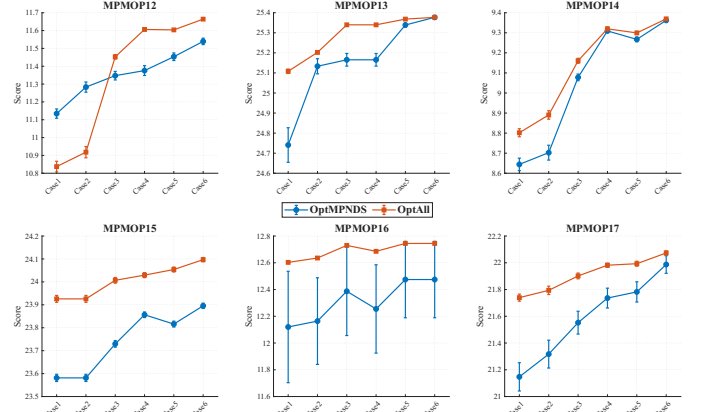

Fig. 4: Evaluation scores Ψ_{NP} of OptMPNDS and OptAll under varying concession thresholds in no common solution scenarios of MPMOPs.
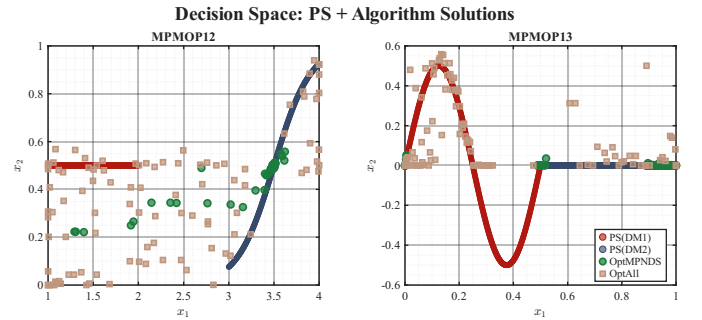

Fig. 5: Representative solution distributions of OptMPNDS and OptAll on decision space of MPMOP12 and MPMOP13.

2) Results on MPDMP Benchmarks: Table III summarizes the performance of different algorithms on the original MPDMP benchmarks using both meanIGD and Ψ_{NP}. Since all algorithms are able to achieve relatively good convergence on these problems, the resulting meanIGD values are generally small. Consequently, the corresponding values of Ψ_{NP} become very large, and thus the logarithmic scale is adopted for numerical stability and clearer comparison.

From Table III, it can be observed that, similar to the phenomena reported on the MPMOP benchmarks, meanIGD and Ψ_{NP} do not always yield consistent rankings across algorithms on MPDMPs. Specifically, the two metrics agree on the best-performing algorithm for MPDMP1, MPDMP5, MPDMP7, MPDMP8, and MPDMP11, whereas for the remaining problems the optimal results are achieved by different algorithms under the two metrics. This discrepancy again confirms that meanIGD and Ψ_{NP} emphasize fundamentally different aspects of solution quality: the former focuses on geometric proximity to individual PSs, while the latter explicitly evaluates mutual acceptability and fairness among DMs. Nevertheless, under both metrics, OptMPNDS3 achieves the highest Ψ_{NP} values across most MPDMP instances, whereas OptAll attains the

lowest, indicating that both metrics implicitly favor algorithms that actively seek common solutions.
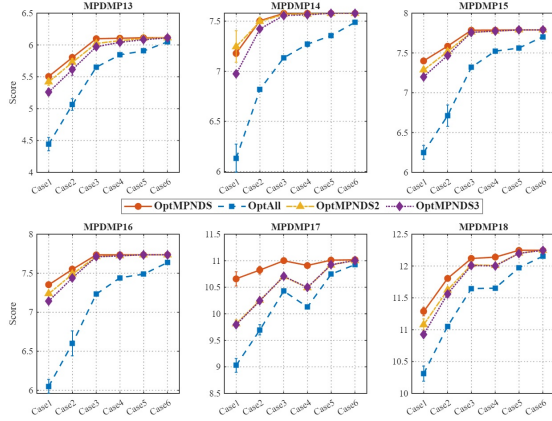

Fig. 6: Evaluation scores $\log \Psi_{\text{NP}}$ under varying concession thresholds in non common solution scenarios of MPDMPs.

Fig. 8: Contour plots of Ψ_{NP} under symmetric concession thresholds and population dispersion.
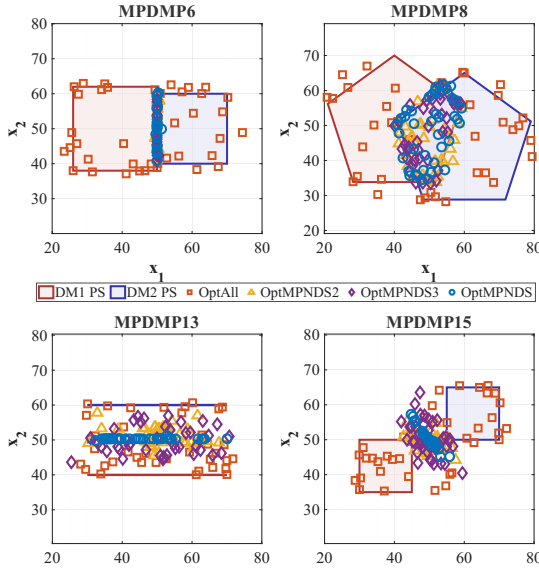

Fig. 7: Representative solution distributions of algorithms on decision space of MPDMP6, 8, 13, and 15.

Fig. 6 further shows the evaluation results on the constructed MPDMP instances with varying sizes of the common acceptable region $\mathcal{A}$. OptMPNDS achieves the highest evaluation scores across all cases, while OptAll remains the worst-performing algorithm. Moreover, for all algorithms, the evaluation score Ψ_{NP} increases monotonically as the acceptable region expands, reflecting the reduced difficulty in satisfying mutual acceptability. The performance gap among algorithms is most pronounced in Case 1, where the acceptable region is empty, whereas in Case 6 the scores of different algorithms become much closer, indicating diminishing fairness discrimination when concessions are sufficiently large.

More detailed insights can be obtained from Fig. 7, which visualizes representative decision-space distributions for problems with and without common acceptable solutions. For
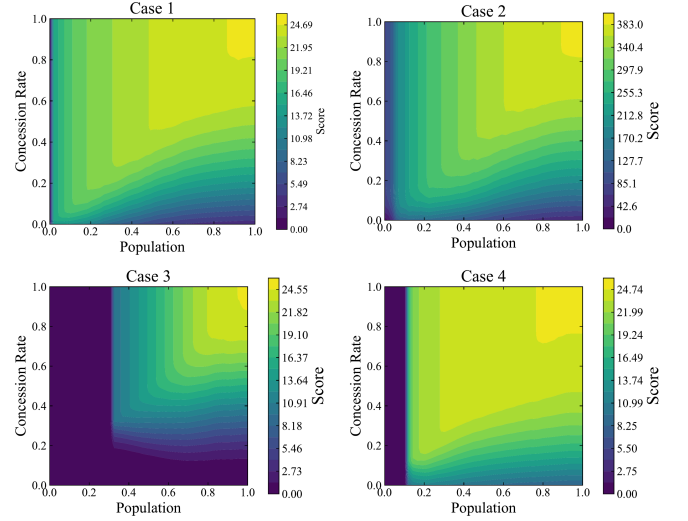
MPDMP6 and MPDMP8, where common acceptable regions exist, all three variants of OptMPNDS converge tightly toward the shared region, leading to very similar Ψ_{NP} values. In contrast, OptAll produces solutions that are more dispersed and significantly farther from the common region.

For MPDMP13 and MPDMP15, where no common solution exists, OptMPNDS still exhibits a distinctive behavior. Its solutions concentrate along a central compromise manifold between the Pareto sets of the two DMs, indicating a tendency toward balanced trade-offs. In contrast, the remaining algorithms generate solutions scattered around this region, with OptAll showing the widest dispersion. As a result, OptMPNDS attains the highest evaluation scores, while OptAll is subject to substantially stronger penalties under the concession-aware evaluation.

Overall, two observations can be made from these results. First, the proposed evaluation framework is capable of distinguishing algorithms not only by convergence quality but also by their ability to achieve fairness and mutual acceptability. Second, These results suggest that algorithm design for multi-party decision-making should not be limited to pursuing consensus solutions alone. The degree of concession between parties, which effectively acts as a controllable parameter in practical negotiations, must also be explicitly incorporated into both algorithmic mechanisms and performance evaluation.

VI. DISCUSSION

A. Sensitivity of Ψ_{NP} to Concession Levels

This subsection examines how the proposed Nash-product-based evaluation function Ψ_{NP} responds to variations in concession, with particular attention to the interaction between the intrinsic concession level of a solution population and the concession threshold prescribed in the evaluation metric. The experiments aim to demonstrate whether the observed score landscapes align with the design principles of Ψ_{NP}.
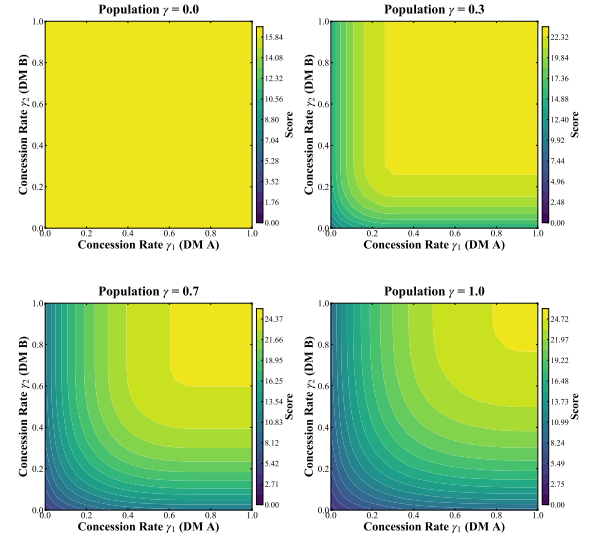
TABLE III: Comparison of OptMPNDS variants and OptAll under $\log \Psi_{\mathrm{NP}}$ and meanIGD for MPDMP benchmarks.

Problem	Metric	OptAll	OptMPNDS	OptMPNDS2	OptMPNDS3
MPDMP1	$\log \Psi_{\mathrm{NP}}$	6.6979 (0.1748)	8.6562 (0.0028)	8.6556 (0.0017)	**8.6565 (0.0024)**
	meanIGD	4.04E+00 (2.46E+00)	5.45E-02 (2.95E-02)	4.76E-02 (2.77E-02)	**4.47E-02 (2.85E-02)**
MPDMP2	$\log \Psi_{\mathrm{NP}}$	8.3224 (0.1470)	9.2360 (0.0051)	**9.2362 (0.0044)**	9.2354 (0.0037)
	meanIGD	3.94E+00 (2.06E+00)	6.49E-02 (3.44E-02)	6.17E-02 (3.25E-02)	**5.77E-02 (3.57E-02)**
MPDMP3	$\log \Psi_{\mathrm{NP}}$	4.6996 (0.1737)	**6.4921 (0.0082)**	6.4905 (0.0076)	6.4855 (0.0087)
	meanIGD	2.73E+00 (6.63E-01)	2.11E-01 (4.80E-02)	2.15E-01 (4.19E-02)	**1.13E-01 (3.53E-02)**
MPDMP4	$\log \Psi_{\mathrm{NP}}$	9.3076 (0.1440)	10.1946 (0.0028)	**10.1949 (0.0033)**	10.1826 (0.0039)
	meanIGD	1.78E-02 (1.38E-02)	**9.58E-04 (2.11E-03)**	1.05E-03 (1.66E-03)	1.33E-02 (6.03E-03)
MPDMP5	$\log \Psi_{\mathrm{NP}}$	9.5794 (0.1495)	10.4618 (0.0048)	10.4620 (0.0041)	**10.4666 (0.0027)**
	meanIGD	6.90E+00 (6.07E-01)	2.97E+00 (9.16E-02)	2.94E+00 (1.23E-01)	**2.66E+00 (9.88E-02)**
MPDMP6	$\log \Psi_{\mathrm{NP}}$	8.9463 (0.1413)	**9.9652 (0.0013)**	9.9645 (0.0011)	9.9630 (0.0003)
	meanIGD	3.23E+00 (7.96E-01)	1.32E-01 (8.91E-03)	1.30E-01 (9.17E-03)	**8.38E-02 (1.37E-02)**
MPDMP7	$\log \Psi_{\mathrm{NP}}$	9.5011 (0.1493)	10.5529 (0.0032)	10.5530 (0.0028)	**10.5549 (0.0020)**
	meanIGD	8.93E+00 (1.25E+00)	2.45E+00 (1.33E-01)	2.45E+00 (1.31E-01)	**2.26E+00 (1.10E-01)**
MPDMP8	$\log \Psi_{\mathrm{NP}}$	9.7392 (0.1291)	11.0936 (0.0043)	11.0935 (0.0046)	**11.0970 (0.0026)**
	meanIGD	6.23E+00 (8.02E-01)	2.46E+00 (6.52E-02)	2.46E+00 (8.63E-02)	**2.25E+00 (5.57E-02)**
MPDMP9	$\log \Psi_{\mathrm{NP}}$	14.9919 (0.1150)	**16.1469 (0.0010)**	16.1464 (0.0022)	16.1363 (0.0040)
	meanIGD	6.71E+00 (3.17E+00)	3.10E-02 (2.25E-02)	**2.15E-02 (1.85E-02)**	2.03E-01 (8.75E-01)
MPDMP10	$\log \Psi_{\mathrm{NP}}$	14.3334 (0.1151)	15.4952 (0.0021)	**15.4957 (0.0017)**	15.4953 (0.0017)
	meanIGD	6.55E+00 (3.20E+00)	9.64E-02 (4.75E-02)	8.45E-02 (4.78E-02)	**6.99E-02 (3.51E-02)**
MPDMP11	$\log \Psi_{\mathrm{NP}}$	17.1521 (0.1428)	18.8487 (0.0062)	18.8495 (0.0053)	**18.8566 (0.0029)**
	meanIGD	1.79E+00 (1.79E+00)	1.39E+00 (9.76E-02)	1.38E+00 (7.83E-02)	**1.19E+00 (4.58E-02)**
MPDMP12	$\log \Psi_{\mathrm{NP}}$	17.2519 (0.1562)	18.5885 (0.0032)	**18.5891 (0.0032)**	18.5889 (0.0023)
	meanIGD	1.02E+01 (1.62E+00)	**2.74E+00 (1.49E-01)**	2.78E+00 (1.42E-01)	2.79E+00 (3.17E-01)

(1) Symmetric concession thresholds. The first experiment considers four multi-party distance minimization problems. The first two problems admit inherent common solutions, whereas the latter two do not. For each problem, a solution population is generated within the acceptable region $\mathcal{A}$ with a prescribed intrinsic concession level γ, while the evaluation metric adopts an identical concession threshold $\hat{\gamma}$ for all DMs. By varying $(\gamma, \hat{\gamma})$, contour plots of Ψ_{NP} are obtained, as shown in Fig. 8.

The contour plots can be interpreted from two complementary perspectives. First, fixing a population generated in $\mathcal{A}$ with concession level γ, if the evaluation parameter satisfies $\hat{\gamma} \geq \gamma$, the resulting Ψ_{NP} value remains unchanged. In this case, the population fully satisfies the prescribed concession requirement and no penalty is incurred. In contrast, when $\hat{\gamma} < \gamma$, the evaluation score decreases monotonically as $\hat{\gamma}$ decreases, since part of the population is regarded as insufficiently acceptable and is therefore penalized.

Second, fixing the concession threshold $\hat{\gamma}$ in the evaluation metric and varying the intrinsic concession level γ of the solution population, two trends are observed. When $\hat{\gamma}$ is relatively small, the evaluation score initially increases with γ and then decreases. At small γ, the population is highly concentrated and its coverage of $\mathcal{A}$ is limited, resulting in a low score. As γ increases and approaches $\hat{\gamma}$, the penalty vanishes and the score reaches its maximum. Further increasing γ enlarges the population range, but since part of the population exceeds the prescribed threshold, penalties are reintroduced and the score decreases. In contrast, when $\hat{\gamma}$ is sufficiently large, the score increases monotonically with γ, because the

Fig. 9: Contour plots of Ψ_{NP} under asymmetric concession thresholds.

population expansion improves coverage without triggering penalties.

For the two problems without inherent common solutions, the same qualitative trends are observed. However, an extended zero-score region appears at small γ, reflecting the absence of any common acceptable region $\mathcal{A}$. Only after sufficient concession enlarges $\mathcal{A}$ does Ψ_{NP} become positive. This behavior is consistent with the design of the metric, which assigns no value when mutual acceptability is not achieved.

(2) Asymmetric concession thresholds. The second experiment investigates asymmetric concession thresholds in a multi-party distance minimization problem with inherent common solutions. Solution populations are generated under four intrinsic concession levels ($\gamma = 0, 0.3, 0.7$, and 1.0), while DMs adopt different concession thresholds. The resulting contour plots are shown in Fig. 9.

For a fixed solution population, relaxing the concession threshold of one DM leads to a monotonic increase in Ψ_{NP}, indicating reduced penalties. Meanwhile, populations with larger intrinsic concession levels require correspondingly larger thresholds to avoid score degradation. The resulting score contours form a convex frontier, reflecting the trade-off induced by asymmetric concession requirements among DMs.

Overall, the two experiments illustrate that Ψ_{NP} produces structured and interpretable score landscapes in response to both population-level concession and evaluation-level concession thresholds. The observed trends are consistent across problems with and without inherent common solutions, and align with the intended penalty and acceptability mechanisms embedded in the metric. While these experiments do not constitute a definitive validation, they provide empirical evidence that the proposed evaluation function behaves in a manner consistent with its conceptual design.

B. PF-Free Evaluation: A UAV Path Planning Case Study

One practical concern of the proposed Nash-product-based evaluation framework is its applicability when true PFs are unavailable. In such scenarios, conventional PF-dependent penalties cannot be defined, and the evaluation necessarily focuses on fairness and relative balance rather than absolute . To examine whether the proposed framework remains usable under this constraint, we consider the multi-party UAV path planning problems [2], where explicit PFs cannot be obtained.

Table IV reports the evaluation results using three PF-free metrics: the sum of hypervolumes across DMs (sumHV), the product of absolute hypervolume values (Absolute), and a comparative Nash-product-based metric derived from normalized utilities (Comparative). This comparison is not intended to claim the superiority of any particular metric, but rather to illustrate how the proposed Nash-based formulation can still be instantiated in PF-free settings.

Let $G(P)$ denote a common strictly monotone gain-type performance metric used to evaluate solution sets when PFs are unknown. For each DM m, a comparative utility is defined as

$$u_m(P) = \frac{G_m(P)}{\max_{P' \in \mathcal{P}} G_m(P')} \in (0, 1],$$

where $\mathcal{P}$ denotes the collection of solution sets generated by all compared algorithms. This normalization preserves ordinal preferences within each DM while mapping heterogeneous performance scales to a common unit interval. Based on these comparative utilities, a PF-free Nash-type score is defined as

$$\Psi_{\mathrm{NP}} = \prod_{m=1}^{M} u_m(P).$$

which evaluates the relative balance of performance across DMs without relying on explicit PF information or concession-based penalties.

Overall, the numerical differences among these metrics are relatively small across all MPUAV instances. More importantly, the resulting algorithm rankings are largely consistent: algorithms that perform well under sumHV also tend to obtain high scores under the Absolute and Comparative metrics. The numerical differences are small, and algorithm rankings are largely consistent across metrics. From an empirical perspective, these results suggest that the proposed PF-free Nash-type metric behaves similarly to the commonly used meanHV or sumHV criteria on this class of problems. In this sense, the comparative Nash formulation does not introduce unexpected or contradictory evaluation, and its evaluation outcomes remain reasonable and interpretable when PF information is unavailable.

Fig. 10 provides a qualitative illustration of the solution distributions on MPUAV5. In this problem, MPAIMA achieves a wider spread of solutions in the objective space, which corresponds to a larger sumHV value. By contrast, OptMPNDS2 produces solutions that are more evenly balanced between the two DMs. While this difference does not lead to a pronounced discrepancy in the PF-free quantitative metrics reported in Table IV, it highlights a qualitative distinction between efficiency-oriented coverage and fairness-oriented compromise in solution distributions.
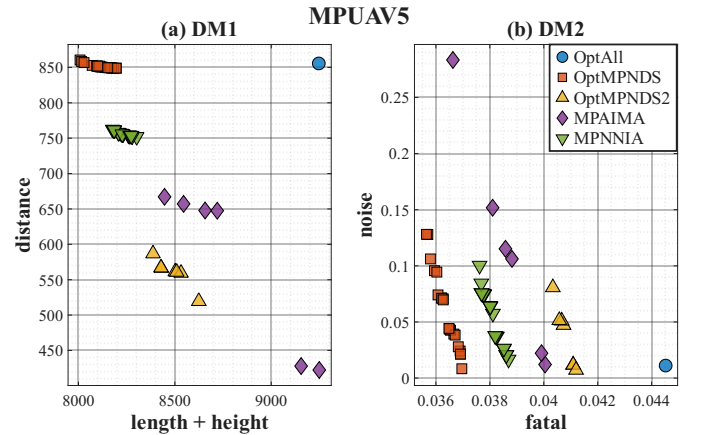

Fig. 10: Solution distributions of comparative algorithms in the objective spaces of MPUAV5.

Overall, the UAV path planning example demonstrates that, in the absence of explicit PFs, the proposed Nash-product-based framework can still be instantiated in a PF-free manner and yields evaluation results generally aligned with conventional hypervolume-based metrics. This subsection demonstrates that the proposed Nash-product-based framework remains applicable and produces reasonable rankings in PF-free settings. How to systematically incorporate fairness penalties or concession mechanisms when PFs are unknown is an important open issue and is left for future investigation.

TABLE IV: Comparison of multi-party multi-objective optimization algorithms under Ψ_{NP} and sumHV for MPUAV path planning benchmarks.

Problem	metric	OptAll	OptMPNDS	OptMPNDS2	MPNNIA	MPAIMA
MPUAV1	Comparative	3.50E-01 (1.10E-01)	5.70E-01 (9.47E-02)	6.20E-01 (1.07E-01)	6.11E-01 (1.06E-01)	**7.41E-01 (1.23E-01)**
	Absolute	1.22E+04 (3.84E+03)	1.98E+04 (3.29E+03)	2.16E+04 (3.73E+03)	2.13E+04 (3.68E+03)	**2.58E+04 (4.28E+03)**
	sumHV	9.51E+05 (2.16E+05)	1.23E+06 (1.69E+05)	1.25E+06 (1.80E+05)	1.23E+06 (1.77E+05)	**1.42E+06 (1.63E+05)**
MPUAV2	Comparative	3.51E-01 (1.05E-01)	6.19E-01 (1.09E-01)	6.44E-01 (1.37E-01)	6.51E-01 (1.51E-01)	**7.54E-01 (1.27E-01)**
	Absolute	2.65E+04 (7.90E+03)	4.66E+04 (8.25E+03)	4.86E+04 (1.03E+04)	4.91E+04 (1.14E+04)	**5.69E+04 (9.58E+03)**
	sumHV	1.96E+06 (4.23E+05)	2.94E+06 (3.43E+05)	2.89E+06 (4.63E+05)	2.86E+06 (5.27E+05)	**3.10E+06 (3.61E+05)**
MPUAV3	Comparative	3.69E-01 (1.26E-01)	6.25E-01 (1.27E-01)	6.17E-01 (1.31E-01)	6.84E-01 (1.03E-01)	**7.51E-01 (1.08E-01)**
	Absolute	4.46E+04 (1.52E+04)	7.56E+04 (1.53E+04)	7.46E+04 (1.59E+04)	8.27E+04 (1.24E+04)	**9.08E+04 (1.31E+04)**
	sumHV	2.86E+06 (7.40E+05)	4.04E+06 (5.99E+05)	3.84E+06 (6.75E+05)	4.10E+06 (5.34E+05)	**4.35E+06 (4.87E+05)**
MPUAV4	Comparative	3.98E-01 (1.37E-01)	6.35E-01 (1.00E-01)	6.00E-01 (1.27E-01)	5.68E-01 (1.26E-01)	**6.83E-01 (1.19E-01)**
	Absolute	1.50E+04 (5.15E+03)	2.39E+04 (3.77E+03)	2.26E+04 (4.79E+03)	2.14E+04 (4.73E+03)	**2.57E+04 (4.49E+03)**
	sumHV	1.06E+06 (2.35E+05)	1.20E+06 (1.57E+05)	1.12E+06 (2.32E+05)	1.03E+06 (2.42E+05)	**1.31E+06 (1.98E+05)**
MPUAV5	Comparative	3.47E-01 (1.16E-01)	6.23E-01 (1.19E-01)	**6.53E-01 (1.48E-01)**	5.84E-01 (1.28E-01)	6.13E-01 (1.27E-01)
	Absolute	2.75E+04 (9.18E+03)	4.93E+04 (9.40E+03)	**5.17E+04 (1.17E+04)**	4.63E+04 (1.02E+04)	4.86E+04 (1.01E+04)
	sumHV	1.91E+06 (4.49E+05)	2.51E+06 (4.64E+05)	2.55E+06 (5.29E+05)	2.25E+06 (4.92E+05)	**2.60E+06 (3.38E+05)**
MPUAV6	Comparative	2.84E-01 (1.04E-01)	6.01E-01 (1.44E-01)	6.00E-01 (1.35E-01)	5.76E-01 (1.11E-01)	**6.10E-01 (1.48E-01)**
	Absolute	2.82E+04 (1.04E+04)	5.96E+04 (1.43E+04)	5.95E+04 (1.34E+04)	5.72E+04 (1.10E+04)	**6.06E+04 (1.47E+04)**
	sumHV	2.60E+06 (7.37E+05)	3.68E+06 (7.23E+05)	3.48E+06 (6.70E+05)	3.31E+06 (6.13E+05)	**3.83E+06 (7.33E+05)**

VII. Conclusion

This paper proposes a Nash-product-based performance evaluation framework for multi-party multi-objective optimization, with explicit emphasis on fairness across heterogeneous decision makers. By transforming heterogeneous performance metrics into strictly monotone utility representations and aggregating them multiplicatively, the proposed framework provides an evaluation criterion that favors balanced compromises rather than mean-dominated outcomes. To address scenarios in which strictly common Pareto-optimal solutions do not exist, we introduce the notion of concession rates, which leads to a generalized definition of multi-party optimal solutions and a corresponding penalty mechanism that discourages solution sets deviating excessively from mutually acceptable regions. Experiments on both MPMOP and MPDMP benchmarks show that the proposed metric Ψ_{NP} often yields rankings inconsistent with meanIGD, confirming that the two metrics capture fundamentally different aspects of solution quality. In particular, Ψ_{NP} consistently reflects inter-party balance and mutual acceptability under both consensus and non-consensus settings, as well as under symmetric and asymmetric concession configurations. Additional experiments indicate that the framework remains well defined in PF-free scenarios and produces evaluation outcomes consistent with classical indicators, although explicit fairness enforcement in such settings remains an open challenge.

Future work will focus on the development of fairness-aware evaluation criteria for PF-free settings, where acceptability and fairness must be inferred without explicit Pareto references. In addition, we plan to investigate fairness-aware optimization algorithms that incorporate fairness considerations directly into the search process, enabling principled trade-offs between efficiency and fairness in multi-party optimization.

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Supplementary document of Fairness-Aware Performance Evaluation for Multi-Party Multi-Objective Optimization

S-I. EXPERIMENTAL PARAMETER SETTINGS

A. Setting of the Constant C

To determine an appropriate value of the constant C, a preliminary empirical analysis is conducted using NSGA-II. Specifically, NSGA-II is independently executed for $R = 10$ runs. For each decision maker (DM) m, the maximum loss observed across all runs is recorded as

$$\widehat{L}_m = \max_{r=1,\ldots,R} \ \max_{\mathbf{x} \in \mathcal{P}_{\mathrm{NSGA\text{-}II}}^{(r)}} L_m(\mathbf{x}),$$

where $\mathcal{P}_{\mathrm{NSGA\text{-}II}}^{(r)}$ denotes the final population obtained in the r-th run of NSGA-II.

The constant C is then defined as

$$C = \max_{m=1,\ldots,M} \widehat{L}_m + \xi,$$

where $\xi > 0$ is a small safety margin introduced to account for potential unseen loss values.

This setting ensures that C serves as an empirical upper bound for all loss values encountered during the search process. NSGA-II is employed in this preliminary step due to its robustness in approximating diverse Pareto-optimal solutions, making it suitable for estimating conservative loss bounds.

B. Setting of the Penalty Factor λ_m

For each DM m, the Pareto-optimal solutions of all other DMs are projected onto the objective subspace of DM m, forming the set $\mathcal{P}_{-m}$. Let PF_m denote the Pareto front of DM m.

We first define

$$\delta_m = \max_{v \in \mathcal{P}_{-m}} \mathrm{IGD}(v, PF_m),$$

which represents the worst-case approximation error induced by solutions optimized for other DMs when evaluated against PF_m.

The penalty factor λ_m is then determined as

$$\lambda_m = \max_{v \in \mathcal{P}_{-m}} \frac{\delta_m - \mathrm{IGD}(v, PF_m)}{\gamma_m(v) - \hat{\gamma}},$$

where $\gamma_m(v)$ denotes the concession rate of solution v with respect to DM m, and $\hat{\gamma}$ is the concession-rate threshold specified in the evaluation function.

C. Setting of the Concession Rate $\hat{\gamma}$

To evaluate algorithmic behavior under varying degrees of compromise among DMs, multiple experimental scenarios are further defined by specifying different concession-rate thresholds. The same set of threshold configurations is consistently applied to all benchmark categories, enabling a unified and fair comparison across problem types. The considered concession-rate settings are reported in Table S-I.

TABLE S-I: Concession Rate Threshold Configurations

Case	Description	2 DMs		3 DMs		
		γ_1	γ_2	γ_1	γ_2	γ_3
1	No concession	0.0	0.0	0.0	0.0	0.0
2	Partial concession	0.3	0.0	0.3	0.0	0.0
3	Equal partial	0.3	0.3	0.3	0.3	0.3
4	Asymmetric	0.3	0.7	0.3	0.1	0.6
5	Equal moderate	0.5	0.5	0.5	0.5	0.5
6	Equal high	0.7	0.7	0.7	0.7	0.7

S-II. MPMOP12–MPMOP17 BENCHMARK PROBLEMS

This section provides detailed descriptions of the MPMOP12–MPMOP17 benchmark problems. These problems are specifically constructed such that the Pareto sets (PSs) of different DMs are mutually disjoint, i.e., no solution is Pareto-optimal for all DMs simultaneously. This property distinguishes MPMOP12–MPMOP17 from MPMOP1–MPMOP11, where common Pareto-optimal solutions across DMs may exist.

The detailed configurations of the benchmark problems are summarized in Table S-III.

S-III. MPDMP13–MPDMP18 BENCHMARK PROBLEMS

The MPDMP13–MPDMP18 benchmark suite extends the MPDMP framework by incorporating more complex geometric configurations with multiple DMs and heterogeneous

numbers of objectives. These problems are constructed based on diverse Pareto set (PS) geometries, including line segments, triangles, squares, and pentagons, which induce varied trade-off structures among DMs. As a result, the suite provides controlled yet nontrivial test scenarios for evaluating algorithm performance in multi-DM optimization settings.

For all problems, the decision vector is defined over a two-dimensional continuous domain, i.e., $\mathbf{x} \in [0, 100]^2$. For each DM $m \in \{1, \ldots, M\}$, let

$$F_m(\mathbf{x}) = \big(f_{m,1}(\mathbf{x}), \ldots, f_{m,K_m}(\mathbf{x})\big)$$

denote its objective vector. Each objective function is defined as the Euclidean distance from the decision vector $\mathbf{x}$ to a reference point associated with the underlying geometric structure:

$$f_{m,i}(\mathbf{x}) = \sqrt{(x_1 - p_{m,i,1})^2 + (x_2 - p_{m,i,2})^2}, \quad i = 1, \ldots, K_m, \tag{1}$$

where $\mathbf{p}_{m,i} = [p_{m,i,1}, p_{m,i,2}]^{\mathsf{T}}$ denotes the i-th reference point defining the PS geometry for DM m, and K_m is the number of objectives associated with DM m. The reference points for all benchmark problems are summarized in Table S-II.
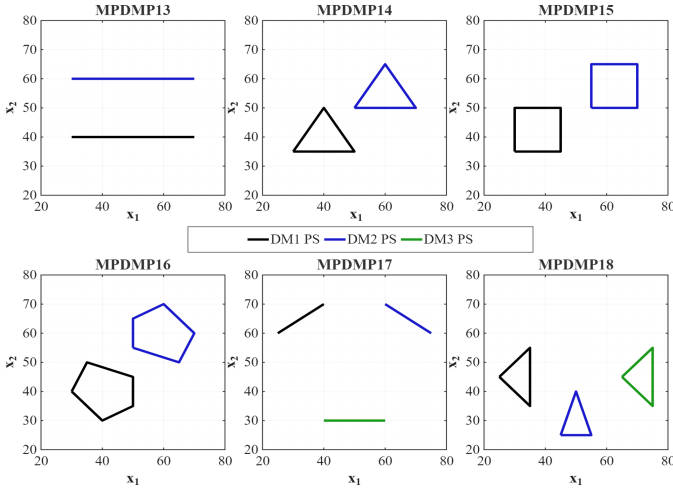

Fig. 1: MPDMPs with no common Pareto optimal solutions.

TABLE S-II: Reference points for MPDMP13–MPDMP18

Problem	DM	Objective number	PS	$p_{i,1}$	$p_{i,2}$
MPDMP13	1	2	Line segment	30 70	40 40
	2	2	Line segment	30 70	60 60
MPDMP14	1	3	Triangle	30 50 40	35 35 50
	2	3	Triangle	50 70 60	50 50 65
MPDMP15	1	4	Square	30 45 45 30	35 35 50 50
	2	4	Square	55 70 70 55	50 50 65 65
MPDMP16	1	5	Pentagon	30 40 50 50 35	40 30 35 45 50
	2	5	Pentagon	50 65 70 60 50	55 50 60 70 65
MPDMP17	1	2	Line segment	25 40	60 70
	2	2	Line segment	60 75	70 60
	3	2	Line segment	40 60	30 30
MPDMP18	1	3	Triangle	25 35 35	45 35 55
	2	3	Triangle	45 55 50	25 25 40
	3	3	Triangle	65 75 75	45 35 55

Problem	DM	Objective Functions	Variable Bounds	Pareto Set (PS)
MPMOP12	DM$_1$	$F_1 = (f_{11}(\mathbf{x}), f_{12}(\mathbf{x}))$, $g_1(\mathbf{x}) = 1 + \sum_{i=2}^{D}(x_i - 0.5)^2$, $f_{11}(\mathbf{x}) = g_1(\mathbf{x}) \cdot \dfrac{2}{x_1}, \quad f_{12}(\mathbf{x}) = g_1(\mathbf{x}) \cdot \dfrac{x_1}{2}$	$x_1 \in [1, 2]$, $x_{i \geq 2} \in [0, 1]$	$\{\mathbf{x} \mid x_1 \in [1, 2], \ x_i = 0.5 \ (i \geq 2)\}$
	DM$_2$	$F_2 = (f_{21}(\mathbf{x}), f_{22}(\mathbf{x}))$, $g_2(\mathbf{x}) = 1 + \sum_{i=2}^{D}\left(x_i - \dfrac{1}{1 + e^{-5(x_1 - 3.5)}}\right)^2$, $f_{21}(\mathbf{x}) = g_2(\mathbf{x}) \cdot \dfrac{3}{x_1}, \quad f_{22}(\mathbf{x}) = g_2(\mathbf{x}) \cdot \dfrac{x_1}{3}$	$x_1 \in [3, 4]$, $x_{i \geq 2} \in [0, 1]$	$\{\mathbf{x} \mid x_1 \in [3, 4], \ x_i = \frac{1}{1 + e^{-5(x_1 - 3.5)}} \ (i \geq 2)\}$
MPMOP13	DM$_1$	$F_1 = (f_{11}(\mathbf{x}), f_{12}(\mathbf{x}))$, $g_1(\mathbf{x}) = 1 + \sum_{i=2}^{D}\left(x_i - \dfrac{\sin(4\pi x_1)}{2}\right)^2 + 100 \max(0, x_1 - 0.5)^2$, $f_{11}(\mathbf{x}) = g_1(\mathbf{x})\big(x_1 + 0.1\sin(3\pi x_1)\big)$, $f_{12}(\mathbf{x}) = g_1(\mathbf{x})\big(1 - x_1 + 0.1\sin(3\pi x_1)\big)^{4.25}$	$x_i \in [0, 1]$	$\{\mathbf{x} \mid x_1 \in [0, 0.5), \ x_i = \frac{\sin(4\pi x_1)}{2} \ (i \geq 2)\}$
	DM$_2$	$F_2 = (f_{21}(\mathbf{x}), f_{22}(\mathbf{x}))$, $g_2(\mathbf{x}) = 1 + \sum_{i=2}^{D} x_i^2 + 100 \max(0, 0.5 - x_1)^2$, $f_{21}(\mathbf{x}) = g_2(\mathbf{x})\big(x_1 + 0.1\sin(3\pi x_1)\big)$, $f_{22}(\mathbf{x}) = g_2(\mathbf{x})\big(1 - x_1 + 0.1\sin(3\pi x_1)\big)^{4.25}$	$x_i \in [0, 1]$	$\{\mathbf{x} \mid x_1 \in [0.5, 1], \ x_i = 0 \ (i \geq 2)\}$
MPMOP14	DM$_1$	$F_1 = (f_{11}(\mathbf{x}), f_{12}(\mathbf{x}), f_{13}(\mathbf{x}))$, $g_1(\mathbf{x}) = 1 + \sum_{i=3}^{D}\left(x_i - \dfrac{\sin(2\pi(x_1 + x_2) + \pi/4)}{2}\right)^2$, $f_{11}(\mathbf{x}) = g_1(\mathbf{x})\sin^2\left(\dfrac{\pi}{2}x_1\right)$, $f_{12}(\mathbf{x}) = g_1(\mathbf{x})\sin^2\left(\dfrac{\pi}{2}x_2\right)\cos^2\left(\dfrac{\pi}{2}x_1\right)$, $f_{13}(\mathbf{x}) = g_1(\mathbf{x})\cos^2\left(\dfrac{\pi}{2}x_2\right)\cos^2\left(\dfrac{\pi}{2}x_1\right)$	$x_i \in [0, 1]$	$\{\mathbf{x} \mid x_1 \in [0, 0.5], \ x_2 \in [0, 1], \ x_i = \frac{\sin(2\pi(x_1 + x_2) + \pi/4)}{2} \ (i \geq 3)\}$
	DM$_2$	$F_2 = (f_{21}(\mathbf{x}), f_{22}(\mathbf{x}), f_{23}(\mathbf{x}))$, $g_2(\mathbf{x}) = 1 + \sum_{i=3}^{D}\left(x_i - \sin\left(2\pi(x_1 + x_2) + \dfrac{\pi}{2}\right)\right)^2$, $f_{21}(\mathbf{x}) = g_2(\mathbf{x})\sin^2\left(\dfrac{\pi}{2}x_1\right)$, $f_{22}(\mathbf{x}) = g_2(\mathbf{x})\sin^2\left(\dfrac{\pi}{2}x_2\right)\cos^2\left(\dfrac{\pi}{2}x_1\right)$, $f_{23}(\mathbf{x}) = g_2(\mathbf{x})\cos^2\left(\dfrac{\pi}{2}x_2\right)\cos^2\left(\dfrac{\pi}{2}x_1\right)$	$x_i \in [0, 1]$	$\{\mathbf{x} \mid x_1 \in (0.5, 1], \ x_2 \in [0, 1], \ x_i = \sin\left(2\pi(x_1 + x_2) + \frac{\pi}{2}\right) \ (i \geq 3)\}$

Prob.	DM	Objective Functions	Variable Bounds	Pareto Set (PS)
MPMOP15	DM_1	$F_1 = (f_{11}(\mathbf{x}), f_{12}(\mathbf{x})),$ $g_1(\mathbf{x}) = 1 + \sum_{i=2}^{D}(x_i - 0.5)^2,$ $f_{11}(\mathbf{x}) = g_1(\mathbf{x})\dfrac{2}{x_1}, \quad f_{12}(\mathbf{x}) = g_1(\mathbf{x})\dfrac{x_1}{2}$	$x_1 \in [1, 2],$ $x_{i\geq 2} \in [0, 1]$	$\{\mathbf{x} \mid x_1 \in [1, 2], \ x_i = 0.5 \ (i \geq 2)\}$
	DM_2	$F_2 = (f_{21}(\mathbf{x}), f_{22}(\mathbf{x})),$ $g_2(\mathbf{x}) = 1 + \sum_{i=2}^{D}\left(x_i - \dfrac{1}{1 + e^{-5(x_1 - 2.5)}}\right)^2,$ $f_{21}(\mathbf{x}) = g_2(\mathbf{x})\dfrac{3}{x_1}, \quad f_{22}(\mathbf{x}) = g_2(\mathbf{x})\dfrac{x_1}{3}$	$x_1 \in [2, 3],$ $x_{i\geq 2} \in [0, 1]$	$\{\mathbf{x} \mid x_1 \in [2, 3], \ x_i = \frac{1}{1 + e^{-5(x_1 - 2.5)}} \ (i \geq 2)\}$
	DM_3	$F_3 = (f_{31}(\mathbf{x}), f_{32}(\mathbf{x})),$ $g_3(\mathbf{x}) = 1 + \sum_{i=2}^{D}(x_i - 0.5)^2,$ $f_{31}(\mathbf{x}) = g_3(\mathbf{x})\dfrac{4}{x_1}, \quad f_{32}(\mathbf{x}) = g_3(\mathbf{x})\dfrac{x_1}{4}$	$x_1 \in [3, 4],$ $x_{i\geq 2} \in [0, 1]$	$\{\mathbf{x} \mid x_1 \in [3, 4], \ x_i = 0.5 \ (i \geq 2)\}$
MPMOP16	DM_1	$F_1 = (f_{11}(\mathbf{x}), f_{12}(\mathbf{x})),$ $g_1(\mathbf{x}) = 1 + \sum_{i=2}^{D}\left(x_i - \dfrac{\sin(4\pi x_1)}{2}\right)^2 + 100\max(0, x_1 - 1/3)^2,$ $f_{11}(\mathbf{x}) = g_1(\mathbf{x})(x_1 + 0.1\sin(3\pi x_1)),$ $f_{12}(\mathbf{x}) = g_1(\mathbf{x})(1 - x_1 + 0.1\sin(3\pi x_1))^{2.5}$	$x_i \in [0, 1]$	$\{\mathbf{x} \mid x_1 \in [0, 1/3], \ x_i = \frac{\sin(4\pi x_1)}{2} \ (i \geq 2)\}$
	DM_2	$F_2 = (f_{21}(\mathbf{x}), f_{22}(\mathbf{x})),$ $g_2(\mathbf{x}) = 1 + \sum_{i=2}^{D} x_i^2 + 100\left(\max(0, 1/3 - x_1)^2 + \max(0, x_1 - 2/3)^2\right),$ $f_{21}(\mathbf{x}) = g_2(\mathbf{x})(x_1 + 0.1\sin(3\pi x_1)),$ $f_{22}(\mathbf{x}) = g_2(\mathbf{x})(1 - x_1 + 0.1\sin(3\pi x_1))^{3.0}$	$x_i \in [0, 1]$	$\{\mathbf{x} \mid x_1 \in [1/3, 2/3], \ x_i = 0 \ (i \geq 2)\}$
	DM_3	$F_3 = (f_{31}(\mathbf{x}), f_{32}(\mathbf{x})),$ $g_3(\mathbf{x}) = 1 + \sum_{i=2}^{D}\left(x_i + \dfrac{\sin(4\pi x_1)}{2}\right)^2 + 100\max(0, 2/3 - x_1)^2,$ $f_{31}(\mathbf{x}) = g_3(\mathbf{x})(x_1 + 0.1\sin(3\pi x_1)),$ $f_{32}(\mathbf{x}) = g_3(\mathbf{x})(1 - x_1 + 0.1\sin(3\pi x_1))^{3.5}$	$x_i \in [0, 1]$	$\{\mathbf{x} \mid x_1 \in [2/3, 1], \ x_i = -\frac{\sin(4\pi x_1)}{2} \ (i \geq 2)\}$

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Prob.	DM	Objective Functions	Variable Bounds	Pareto Set (PS)
MPMOP17	DM$_1$	$F_1 = (f_{11}(\mathbf{x}), f_{12}(\mathbf{x}), f_{13}(\mathbf{x}))$, $g_1(\mathbf{x}) = 1 + \sum_{i=3}^{D}(x_i - 0.5x_1)^2$, $y_1 = \frac{\pi}{6} + \frac{\pi}{3}x_1, \quad y_2 = \frac{\pi}{2}x_2$, $f_{11}(\mathbf{x}) = g_1(\mathbf{x})\sin(y_1)$, $f_{12}(\mathbf{x}) = g_1(\mathbf{x})\sin(y_2)\cos(y_1)$, $f_{13}(\mathbf{x}) = g_1(\mathbf{x})\cos(y_2)\cos(y_1)$	$x_i \in [0,1]$	$\{\mathbf{x} \mid x_1 < 0.33,\ x_2 \in [0,1],\ x_i = 0.5x_1\ (i \geq 3)\}$
	DM$_2$	$F_2 = (f_{21}(\mathbf{x}), f_{22}(\mathbf{x}), f_{23}(\mathbf{x}))$, $g_2(\mathbf{x}) = 1 + \sum_{i=3}^{D}x_i^2$, $y_1 = \frac{\pi}{2}x_1, \quad y_2 = \frac{\pi}{2}x_2$, $f_{21}(\mathbf{x}) = g_2(\mathbf{x})\sin(y_1)$, $f_{22}(\mathbf{x}) = g_2(\mathbf{x})\sin(y_2)\cos(y_1)$, $f_{23}(\mathbf{x}) = g_2(\mathbf{x})\cos(y_2)\cos(y_1)$	$x_i \in [0,1]$	$\{\mathbf{x} \mid 0.33 \leq x_1 < 0.66,\ x_2 \in [0,1],\ x_i = 0\ (i \geq 3)\}$
	DM$_3$	$F_3 = (f_{31}(\mathbf{x}), f_{32}(\mathbf{x}), f_{33}(\mathbf{x}))$, $g_3(\mathbf{x}) = 1 + \sum_{i=3}^{D}(x_i - 0.5x_1)^2$, $y_1 = \frac{\pi}{6} + \frac{\pi}{3}x_1, \quad y_2 = \frac{\pi}{2}x_2$, $f_{31}(\mathbf{x}) = g_3(\mathbf{x})\sin(y_1)$, $f_{32}(\mathbf{x}) = g_3(\mathbf{x})\sin(y_2)\cos(y_1)$, $f_{33}(\mathbf{x}) = g_3(\mathbf{x})\cos(y_2)\cos(y_1)$	$x_i \in [0,1]$	$\{\mathbf{x} \mid x_1 \geq 0.66,\ x_2 \in [0,1],\ x_i = 0.5x_1\ (i \geq 3)\}$