

Using External Archive for Improved Performance in Multi-Objective Optimization

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Abstract

It is shown that the use of an external archive, purely for storage purposes, can bring substantial benefits in multi-objective optimization. A new scheme for archive management for the above purpose is described. The new scheme is combined with the NSGA-II algorithm for solving two multi-objective optimization problems, and it is demonstrated that this combination gives significantly improved sets of Pareto-optimal solutions. The additional computational effort because of the external archive is found to be insignificant when the objective functions are expensive to evaluate.

1 Introduction

An external archive for storing non-dominated solutions plays an important role in a variety of multi-objective evolutionary algorithms (MOEAs). In algorithms based on the genetic algorithm (GA), an external archive is generally used to store the non-dominated solutions and propagate them to the next generation through a suitable selection mechanism [1]-[15]. In algorithms based on particle swarm optimization (PSO), an external archive is used in the selection of the globally best particle [16]-[21]. In some of the evolutionary algorithms (e.g., [4], [10], [17]), the external archive also serves as the final output, i.e., the Pareto-optimal solution set.

It is the purpose of this paper to show that an external archive, even when used purely for storage, can lead to a significant improvement in the output, viz., the set of Pareto-optimal solutions, of an MOEA. In particular, we take NSGA-II [22], one of the industry-standard MOEAs, as an example and present results for some multi-objective optimization problems, with and without an external archive, to demonstrate the advantage of using an external archive.

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The paper is organized as follows. In Section 2, we present a brief review of the existing schemes for external archive management. In Section 3, we discuss a new scheme for archive management which is efficient in terms of memory requirement. In Section 4, we describe a new algorithm which is a simple combination of the NSGA-II algorithm and the archive management scheme of Section 3. Finally, results obtained with the new algorithm are compared with NSGA-II in Section 5.

2 Archive Management: Review

The various archiving strategies reported in the literature for multi-objective optimization may be broadly categorized as follows.

(a) Unconstrained archive: In this case, the archive is not limited, i.e., it can hold any number of solutions (see [1], [8], [18], for example). This scheme has the obvious advantage that all desirable (non-dominated) solutions are preserved, and the efficacy of the search process is not compromised on account of limited archive size. However, the memory requirement for this approach is clearly larger as compared to a limited archive. Equally important, the archive operations are more time-consuming due to a larger number of archive members. Special data structures can be employed [8] to speed up computations related to the archive.

In a practical situation, an unlimited archive is generally not required from the utility perspective as long as a sufficiently large number of well-spread Pareto-optimal solutions are produced by the concerned MOEA. It is therefore more common to employ a limited archive.

(b) Limited archive: In this case, an upper limit (N_A^{\max}) is placed on the total number of solutions to be accommodated in the external archive. To decide whether to admit a new candidate (C) into the archive, it is compared with the existing solutions (A_i) in the archive. If C is dominated by each A_i , the archive remains unchanged. If C dominates any of the archive solutions, they are removed from the archive and C is admitted. In the situation where the archive is full (i.e., it has N_A^{\max} solutions already)

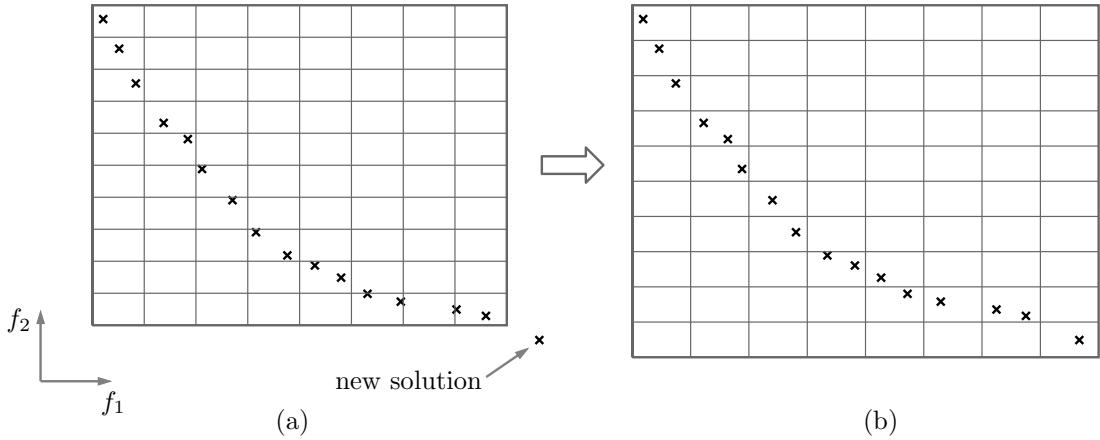


Figure 1: Schematic diagram showing the hypergrid (a) before and (b) after a new candidate lying outside the current hypergrid is admitted in [17].

and C is non-dominated with respect to each A_i , one solution needs to be removed before admitting C . In other words, the archive needs to be “pruned” or “truncated.”

Depending on how the non-dominated solutions are organized in the archive, we can have the following sub-divisions of limited archive schemes.

- (i) Archive with hypergrid: In this scheme, the archive is divided into an M -dimensional [3], [5], [17], [21] or L -dimensional [4] “hypergrid” where M is the number of objective functions and L is the number of decision variables. The smallest unit or cell of the hypergrid is called “hypercube” or “hyperbox.” For pruning of the archive, some measure of the density of solutions in the hypercubes is used, and a solution from a hypercube with a higher density is preferred for removal. In Section 3, we will look at the hypergrid scheme of [17] in more detail.
- (ii) Archive without hypergrid: In this case, the non-dominated solutions are stored as N_A individual solutions and are not organized into hypercubes. Various approaches have been used for pruning, such as clustering [2], [16], distance to the closest neighbour [6], and crowding distance [9], [10], [12]-[15], [20], the intention always being to remove a solution from a dense region of the archive.

Hypergrid-based archive management schemes are attractive from the computational perspective since they involve only *local* calculations for pruning, and in the simplest case, plain *counting* of solutions in a given hypercube [17].

3 Archive Management: New Scheme

In this section, we present a new scheme for archive management. Before describing the new scheme, however, it is instructive to look at the scheme used in the MOPSO algorithm [17]. Fig. 1 (a) shows a hypergrid example for minimisation of two objective functions f_1 and f_2 , with $N_{f1} = 8$ and $N_{f2} = 10$, where N_{fk} is the number of divisions for the k^{th} objective function. The crosses in the figure represent the non-dominated solutions in the archive.

If a new candidate being considered for entry into the archive falls inside the current hypergrid – the outer rectangle in Fig. 1 (a) – no changes are required in the hypergrid boundaries. If it falls outside (see the solution marked as “new candidate” in the figure), the hypergrid boundaries need to be recalculated, and the existing solutions in the hypergrid need to be relocated, as some of them may now fall in a different hypercube (see Fig. 1(b)).

In practice, this grid recalculation would be required more frequently in the initial stages of the MOEA; as the algorithm converges, the hypergrid boundaries would tend to become constant. Nevertheless, a hypergrid scheme which does not require recalculation of the boundaries is desirable.

Another important aspect of hypergrid management is memory requirement. Let N_{sols}^{\max} be the maximum number of solutions allowed in a given hypercube. In that case, we need to allocate memory for $N_{\text{sols}}^{\max} N_{f1} N_{f2}$ solutions for $M = 2$ and for $N_{\text{sols}}^{\max} \times \prod_{k=1}^M N_{fk}$ solutions in the general case. Of these hypercubes, only a small fraction (typically 10 to 20 %) would be occupied, and in that sense, improvement in memory utilization is desirable.

With these two improvements in mind, viz., avoiding grid boundary recalculation and more efficient use of memory, we propose the following hypergrid management scheme.

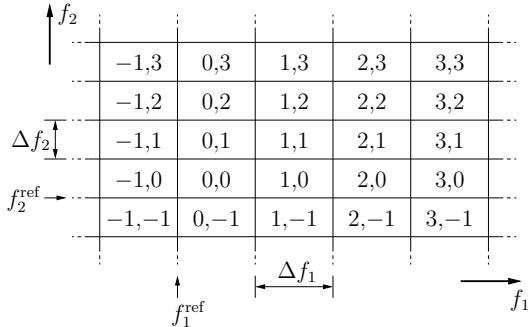


Figure 2: Proposed hypergrid management scheme for $M=2$ (two objective functions).

In the new scheme, shown in Fig. 2, the hypergrid does not have boundaries. Each hypercube is characterized by M indices governed by the position of the hypercube and two parameters for each of the objective functions: a reference value f_k^{ref} and a spacing Δf_k . A maximum of N_{cells}^{\max} hypercubes (cells) are allowed, and each hypercube can hold up to N_{sols}^{\max} solutions. Memory allocation for $N_{\text{cells}}^{\max} \times N_{\text{sols}}^{\max}$ solutions is required in this scheme, which we will refer to as the “fixed hypergrid” (FH) scheme.

Note that the parameter N_{cells}^{\max} needs to be only as large as the maximum number of occupied cells during the evolution of the population. The advantage of the FH scheme can be seen from the results shown in Fig. 3 for the CTP1 two-objective problem [22]. At the end of the first iteration, we see that 9 hypercubes (cells) have non-dominated solutions (indicated by squares in the figure). As the evolutionary algorithm progresses, some of the previously occupied cells can become empty and some additional cells can become occupied because of new non-dominated solutions appearing in the archive. After the second iteration, four of the previously occupied cells have become empty, two new cells have become occupied, and the total number of cells is 11. At the end of the 40th iteration, the number of occupied cells is 15, and the number of empty cells (which were occupied at some point) is 9. We would therefore require storage for 15 cells if empty cells are removed during the evolution and 24 if they are not. In contrast, with the adaptive grid scheme described earlier (Fig. 1), we would have started with a grid of, say 10×8 (assuming roughly the same resolution as in Fig. 3, and kept on changing it as new solutions joined the archive).

The FH scheme requires some parameters to be specified by the user, viz., a reference (f_k^{ref}) and resolution (Δf_k) for each objective function, maximum number of cells (N_{cells}^{\max}), and maximum number of solutions for each cell (N_{sols}^{\max}). In practice, the user may have sufficient knowledge about the optimization problem, enabling a judicious choice for the parameters. If not, the user can use a relatively coarse grid (large values of Δf_k), run the MOEA, look at the results, and then fine-tune the parameters.

It is possible during the initial stages of the MOEA that

a large number of cells, which were once occupied, become empty later because of new non-dominated solutions entering the archive. As new cells get occupied, the total number of cells (both occupied and empty) may exceed N_{cells}^{\max} . When that happens, we can “pack” the hypergrid by removing the empty cells, as shown in Fig. 4. After the packing operation, the total number of cells would become equal to the number of occupied cells.

Fig. 5 illustrates how N_{cells}^{\max} affects the total number of cells for the CTP1 example. For $N_{\text{cells}}^{\max} = 25$, packing does not take place because the total number of cells is always less than N_{cells}^{\max} . For $N_{\text{cells}}^{\max} = 20$, the total number of cells exceeds N_{cells}^{\max} at the 6th iteration, and therefore a packing step is carried out, i.e., the vacant cells are removed.

4 NSGA-II with External Archive

The FH scheme can be easily combined, *purely* as a storage mechanism, with an existing MOEA. We choose NSGA-II (real-coded), one of the industry-standard MOEAs, for this purpose. The resulting algorithm (see Algorithm 1) will be referred to as NSGA-II-FH, i.e., NSGA-II with fixed hypergrid. The only new step in this

Algorithm 1 NSGA-II-FH

- 1: Initialize parent population.
 - 2: Initialize archive.
 - 3: Evaluate parent population.
 - 4: Assign rank and crowding distance to each individual.

 - 5: **for** $i_{\text{gen}} = 1$ to N_{gen} **do**
 - 6: Perform selection and crossover.
 - 7: Perform mutation.
 - 8: Evaluate child population.
 - 9: Merge child and parent populations and obtain
 - 10: mixed population.
 - 11: Perform non-dominated sorting on mixed population
 - 12: and obtain the next parent population.
 - 13: Update archive.
 - 14: **end for**
-

algorithm, as compared to the NSGA-II algorithm, is step 13, that of updating the archive. Note that the archive does not participate in the evolution of the population; it only stores non-dominated individuals from the population in a cumulative manner, as and when they become available.

Algorithm 2 describes the archive update procedure. The operations involved in archive update are comparisons (to check dominance) and copying of decision variable and objective function values when a new solution is admitted into the archive. The FH scheme has the advantage that it does not require recalculation of hypergrid boundaries. However, locating j_{cell} corresponding to a population member i_{pop} (step 14 in Algorithm 2)

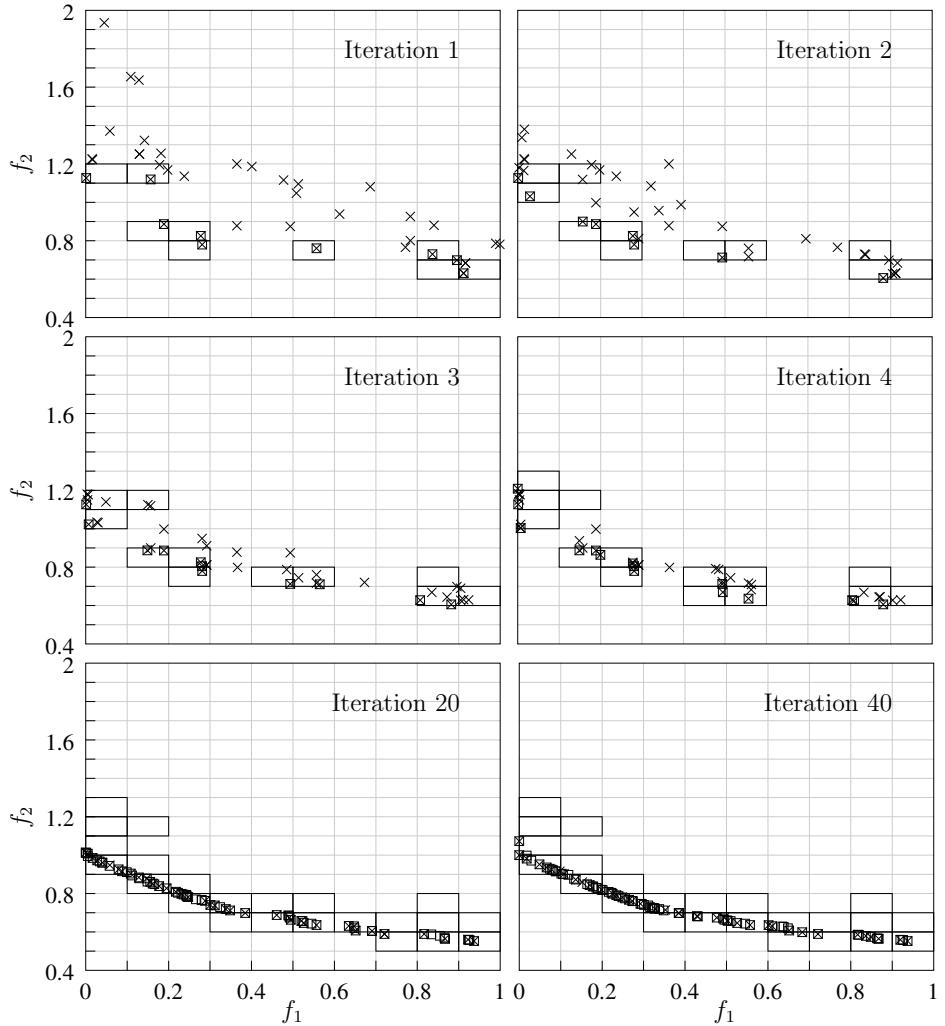


Figure 3: Illustration of the fixed grid scheme using the CTP1 two-objective problem [22]. The real-coded NSGA-II algorithm with a population size of $N=40$ was used. Crosses indicate the current positions of individuals in the population, and squares indicate all of the non-dominated solutions obtained up to the given iteration.

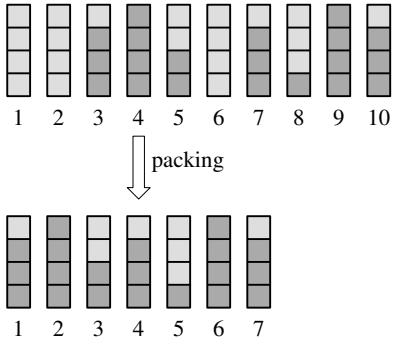


Figure 4: Illustration of packing of the hypergrid, with $N_{\text{cells}}^{\max} = 10$, $N_{\text{sols}}^{\max} = 4$. Occupied and empty solution slots are shown in dark grey and light grey, respectively. After packing, three additional cells can be added to the archive in this example.

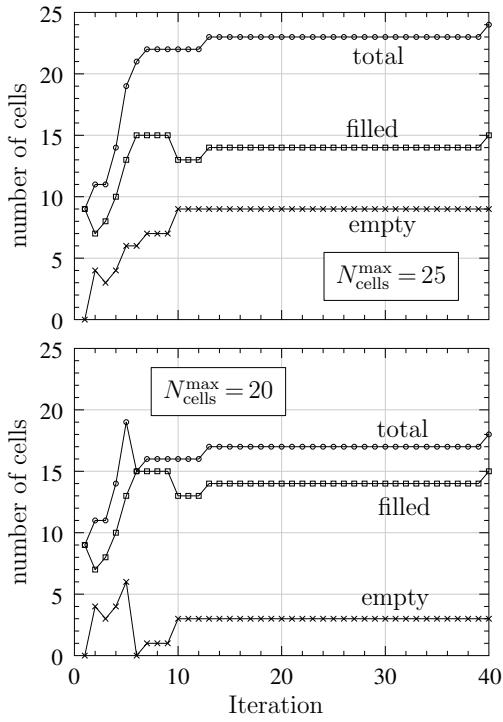


Figure 5: Filled, empty, and total number of cells versus iteration for the CTP1 example for two values of N_{cells}^{\max} .

is more expensive in this scheme – compared to the hypergrid approach of [17] – because it involves comparing the position of i_{pop} with each of the occupied hypercubes (until a match is found). In the next section, through specific examples, we will discuss how the additional work involved in updating the archive affects the overall performance of the NSGA-II-FH algorithm with respect to the standard NSGA-II algorithm.

Algorithm 2 Update archive

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1: for  $i_{\text{pop}} = 1$  to  $N$  do
2:   for  $i_{\text{cell}} = 1$  to  $N_{\text{cells}}$  do
3:     for each occupied solution  $i_{\text{sol}}$  in  $i_{\text{cell}}$  do
4:       Compare  $i_{\text{pop}}$  and  $i_{\text{sol}}$  for dominance.
5:       if  $i_{\text{pop}}$  and  $i_{\text{sol}}$  are non-dominating then
6:         if  $i_{\text{pop}}$  and  $i_{\text{sol}}$  are identical then
7:           Next  $i_{\text{pop}}$ 
8:         end if
9:       else if  $i_{\text{sol}}$  dominates then
10:        Next  $i_{\text{pop}}$ 
11:      end if
12:      Find all solutions in archive dominated by  $i_{\text{pop}}$ 
13:      and remove them.
14:      Find  $j_{\text{cell}}$  corresponding to  $i_{\text{pop}}$ .
15:      if  $j_{\text{cell}}$  exists then
16:        if  $j_{\text{cell}}$  is full then
17:          Randomly remove one solution from  $j_{\text{cell}}$ .
18:        end if
19:        Add  $i_{\text{pop}}$  to  $j_{\text{cell}}$ .
20:      else
21:        if  $N_{\text{cells}} = N_{\text{cells}}^{\max}$  then
22:          if there are vacant cells then
23:            Pack archive (Remove vacant cells).
24:            Create a new cell and add  $i_{\text{pop}}$  to it.
25:          else
26:            Declare “archive full” and stop.
27:          end if
28:        end if
29:      end if
30:    end for
31:  end for
32: end for

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5 Results and Discussion

Two multi-objective optimization problems are considered in this section. The two algorithms, NSGA-II and NSGA-II-FH, are used for each of the problems, keeping the algorithmic parameters the same. In particular, the following parameter values are used: crossover probability $p_c = 0.8$, mutation probability $p_m = 1/L$ (where L is the number of decision variables), distribution index for crossover $\eta_c = 10$, and distribution index for mutation $\eta_m = 10$.

As our first example, we consider the VNT problem (see

[22]) which has $L=2$, $M=3$ (i.e., two decision variables and three objective functions). All three functions (f_1 , f_2 , f_3) are to be minimized, with the decision variables in the range $-3 \leq (x_1, x_2) \leq 3$. We keep the population size constant ($N=60$) and run NSGA-II and NSGA-II-FH for different values of N_{gen} (number of generations). The hypergrid parameters used for the NSGA-II-FH algorithm are $N_{\text{cells}}^{\max} = 1000$, $N_{\text{sols}}^{\max} = 10$, $f_1^{\text{ref}} = f_2^{\text{ref}} = f_3^{\text{ref}} = 0$, $\Delta f_1 = 0.1$, $\Delta f_2 = 0.01$, $\Delta f_3 = 0.1$.

The results are shown in Figs. 6 and 7. The true Pareto front is also shown for comparison. It can be observed that the NSGA-II-FH method produces a significantly larger number of Pareto-optimal solutions for each value of N_{gen} . The advantage of using an external archive to store a large number of solutions visited by the population cumulatively is clear from the figures. This was in fact one of the motivations behind using an external archive in the AMGA algorithm [10], for example. The NSGA-II-FH results demonstrate that an external archive is beneficial even when it does not participate in the underlying algorithm.

Our second example is an engineering problem, the inverting amplifier shown in Fig. 8(a). It is well known that a high gain and a high bandwidth for an amplifier are conflicting objectives. If a simple op-amp model is used, the gain versus bandwidth relationship can be obtained analytically. However, if a realistic op-amp model is used, circuit simulation is required.

The optimization problem considered here can be stated as follows. There are two design variables, the resistance R_1 and R_2 , with $1\text{k}\Omega < R_1 < 10\text{k}\Omega$, $5\text{k}\Omega < R_2 < 50\text{k}\Omega$. There are three objective functions: gain, bandwidth (i.e., the high cut-off frequency f_H), and input resistance of the amplifier. The gain and bandwidth are to be maximised, and the input resistance minimised. Two constraints are imposed: a minimum gain of 5 and a minimum bandwidth of 1 kHz.

Computation of the objective functions involves setting the parameter values (R_1 and R_2) in a circuit file, simulating the circuit using the circuit simulator NGSPICE [23], computing the mid-band gain, cut-off frequency, and input resistance from the simulator output. Note that, compared to our first example, function evaluations are much more expensive in this case.

Each algorithm was run once with $N=20$ and $N_{\text{gen}}=100$. The hypergrid parameters for the NSGA-II-FH were $N_{\text{cells}}^{\max} = 1000$, $N_{\text{sols}}^{\max} = 3$, $f_1^{\text{ref}} = f_2^{\text{ref}} = f_3^{\text{ref}} = 0$, $\Delta f_1 = 1$, $\Delta f_2 = 1\text{kHz}$, $\Delta f_3 = 0.5\text{k}\Omega$.

Figs. 8(b) and 8(c) show the results, and it is observed once again that NSGA-II-FH produces a substantially larger number of Pareto-optimal solutions. NSGA-II, because of the absence of an external archive, can only give up to N Pareto-optimal solutions whereas NSGA-II-FH is not limited by the population size. In other words, NSGA-II gives only a sample – a snapshot – of all the Pareto-optimal solutions visited by the population while NSGA-II-FH gives a more complete picture.

The benefit of the NSGA-II-FH algorithm comes with an additional computational cost. To gauge the impact of this additional cost, we present in Table 1 the CPU time required for the two algorithms on a desktop computer (Linux) with 3.3 GHz clock and 4 GB RAM without any parallelization.

From the table, we observe that NSGA-II-FH takes substantially longer than NSGA-II for the VNT problem. On the other hand, for the amplifier problem, there is virtually no difference between the two because the time taken by function evaluations dominates in this case, and the overhead due to archive manipulation is negligibly small.

In conclusion, a new scheme for external archive management has been presented. It is demonstrated that the new scheme, combined with the NSGA-II algorithm, produces a substantially larger number of Pareto-optimal solutions as compared to the original NSGA-II algorithm. The new scheme is particularly attractive for optimization problems in which the computing time is dominated by objective function evaluations.

References

- [1] G. T. Parks and I. Miller, “Selective breeding in a multiobjective genetic algorithm,” in *Parallel Problem Solving from Nature*, A. E. Bäck et al, Ed. Berlin, Germany: Springer, 1998, pp. 250–259.
- [2] E. Zitzler and L. Thiele, “Multiobjective evolutionary algorithms: a comparative case study and the strength pareto approach,” *IEEE Trans. Evol. Comput.*, vol. 3, no. 4, pp. 257–271, 1999.
- [3] J. D. Knowles and D. W. Corne, “Approximating the nondominated front using the pareto archived evolution strategy,” *IEEE Trans. on Evol. Comput.*, vol. 8, pp. 149–172, 2000.
- [4] D. W. Corne, J. D. Knowles, and M. J. Oates, “The pareto envelope-based selection algorithm for multi-objective optimisation,” in *Parallel Problem Solving from Nature*, M. S. et al, Ed. Berlin, Germany: Springer, 2000, pp. 839–848.
- [5] D. W. Corne, J. D. Knowles, M. J. Oates, and J. Martin, “PESA-II: region-based selection in evolutionary multiobjective optimization,” in *Proceedings of the 3rd Annual Conference on Genetic and Evolutionary Computation*. Morgan Kaufmann Publishers Inc., 2001, pp. 283–290.
- [6] E. Zitzler, M. Laumanns, and L. Thiele, “SPEA2: Improving the strength pareto evolutionary algorithm,” *TIK-report*, vol. 103, 2001.
- [7] M. Laumanns, L. Thiele, K. Deb, and E. Zitzler, “Combining convergence and diversity in evolutionary multiobjective optimization,” *Evolutionary computation*, vol. 10, no. 3, pp. 263–282, 2002.

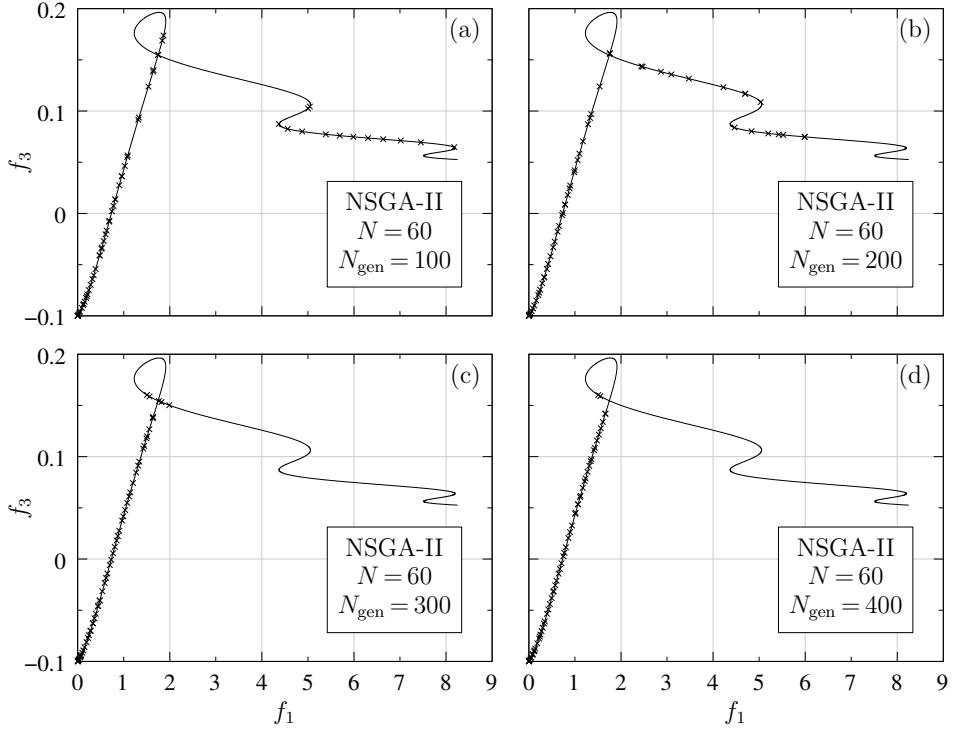


Figure 6: Pareto-optimal solutions for the VNT problem obtained with the NSGA-II algorithm for a population size $N = 60$ and different values of N_{gen} . (a) $N_{\text{gen}} = 100$, (b) $N_{\text{gen}} = 200$, (c) $N_{\text{gen}} = 300$, (d) $N_{\text{gen}} = 400$.

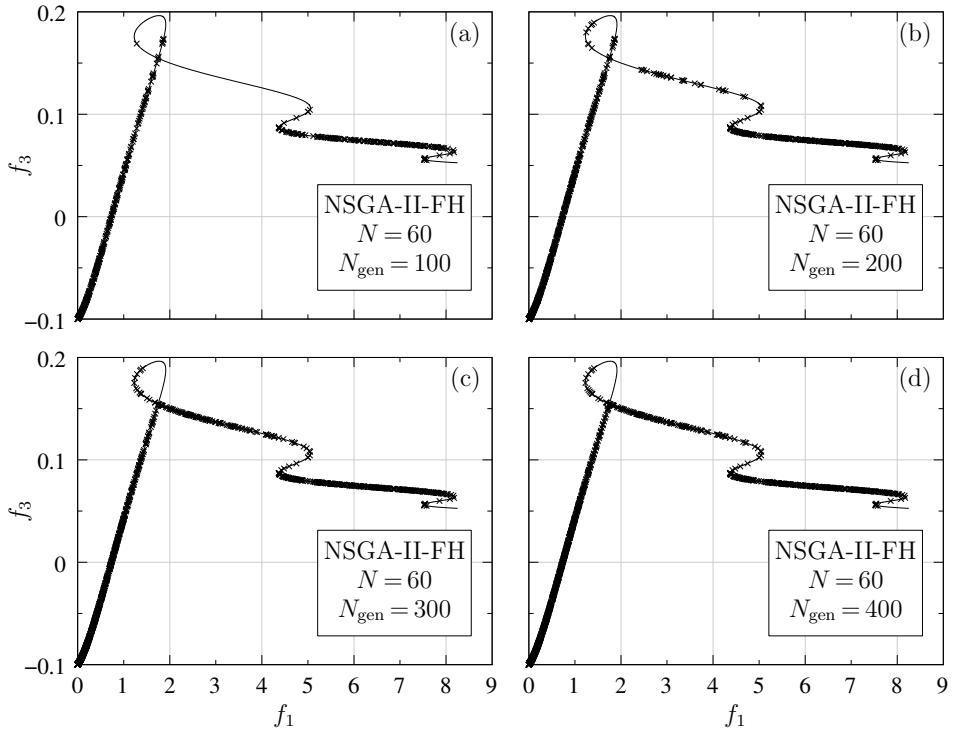


Figure 7: Pareto-optimal solutions for the VNT problem obtained with the NSGA-II-FH algorithm for a population size $N = 60$ and different values of N_{gen} . (a) $N_{\text{gen}} = 100$, (b) $N_{\text{gen}} = 200$, (c) $N_{\text{gen}} = 300$, (d) $N_{\text{gen}} = 400$.

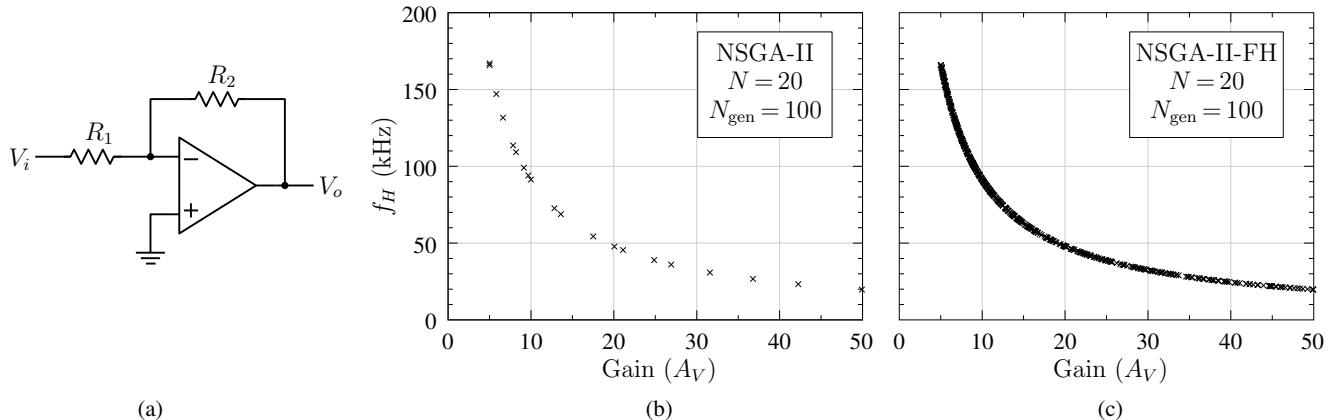


Figure 8: (a) Op-amp based amplifier circuit, (b) NSGA-II result, (c) NSGA-II-FH result obtained for a population size $N = 20$ and $N_{\text{gen}} = 100$.

Table 1: CPU time taken by NSGA-II and NSGA-II-FH algorithms.

N_{gen}	VNT ($N = 60$)		Amplifier ($N = 20$)	
	NSGA-II	NSGA-II-FH	NSGA-II	NSGA-II-FH
100	13 msec	31 msec	33 sec	34 sec
200	21 msec	77 msec	67 sec	67 sec
300	33 msec	128 msec	101 sec	101 sec
400	41 msec	177 msec	134 sec	135 sec

- [8] J. E. Fieldsend, R. M. Everson, and S. Singh, “Using unconstrained elite archives for multi-objective optimisation,” *IEEE Trans. Evol. Comput.*, vol. 7, no. 3, pp. 305–323, 2003.
- [9] V. L. Huang, P. N. Suganthan, A. K. Qin, and S. Baskar, “Multiobjective differential evolution with external archive and harmonic distance-based diversity measure,” *School of Electrical and Electronic Engineering Nanyang, Technological University Technical Report*, 2005.
- [10] S. Tiwari, P. Koch, G. Fadel, and K. Deb, “AMGA: an archive-based micro genetic algorithm for multi-objective optimization,” in *Proceedings of the 10th annual conference on Genetic and evolutionary computation*. ACM, 2008, pp. 729–736.
- [11] S. Z. Martínez and C. A. C. Coello, “An archiving strategy based on the convex hull of individual minima for MOEAs,” in *Evolutionary Computation (CEC), 2010 IEEE Congress on*. IEEE, 2010, pp. 1–8.
- [12] D. Sharma and P. Collet, “An archived-based stochastic ranking evolutionary algorithm (ASREA) for multi-objective optimization,” in *Proceedings of the 12th annual conference on Genetic and evolutionary computation*. ACM, 2010, pp. 479–486.
- [13] S. Tiwari, G. Fadel, and K. Deb, “AMGA2: improving the performance of the archive-based micro-genetic algorithm for multi-objective optimization,” *Engineering Optimization*, vol. 43, no. 4, pp. 377–401, 2011.
- [14] X. Cai, Y. Li, Z. Fan, and Q. Zhang, “An external archive guided multiobjective evolutionary algorithm based on decomposition for combinatorial optimization,” *IEEE Trans. Evol. Comput.*, vol. 19, no. 4, pp. 508–523, 2015.
- [15] M. Song and D. Chen, “An improved knowledge-informed NSGA-II for multi-objective land allocation (MOLA),” *Geo-spatial Information Science*, pp. 1–15, 2018.
- [16] S. Mostaghim and J. Teich, “Strategies for finding good local guides in multi-objective particle swarm optimization (MOPSO),” in *Swarm Intelligence Symposium, 2003. SIS’03. Proceedings of the 2003 IEEE*. IEEE, 2003, pp. 26–33.
- [17] C. A. C. Coello, G. T. Pulido, and M. S. Lechuga, “Handling multiple objectives with particle swarm optimization,” *IEEE Trans. Evol. Comput.*, vol. 8, no. 3, pp. 256–279, 2004.
- [18] J. E. Alvarez-Benitez, R. M. Everson, and J. E. Fieldsend, “A MOPSO algorithm based exclusively

- on pareto dominance concepts,” in *International Conference on Evolutionary Multi-Criterion Optimization*. Springer, 2005, pp. 459–473.
- [19] P. Tripathi, S. Bandyopadhyay, and S. K. Pal, “Adaptive multi-objective particle swarm optimization algorithm,” in *Evolutionary Computation, 2007. CEC 2007. IEEE Congress on*. IEEE, 2007, pp. 2281–2288.
 - [20] Y. Zhang, D.-W. Gong, and Z. Ding, “A bare-bones multi-objective particle swarm optimization algorithm for environmental/economic dispatch,” *Information sciences*, vol. 192, pp. 213–227, 2012.
 - [21] H. Han, W. Lu, and J. Qiao, “An adaptive multiobjective particle swarm optimization based on multiple adaptive methods,” *IEEE Trans. Cybern.*, vol. 47, no. 9, pp. 2754–2767, 2017.
 - [22] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, “A fast and elitist multiobjective genetic algorithm: NSGA-II,” *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, 2002.
 - [23] P. Nenzi, “Ngspice circuit simulator release 26, 2014.”