Robust Control of a Hard Disk Drive

Control Systems Identification

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Outline

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2. General concepts of H_∞ and μ Theory
3. Description of an Hard Disk Driver
4. Designed Controllers
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   - Frequency response of the closed-loop systems
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Project goal

Design of a robust servo system of *hard disk drive* (HDD).

Figure: An Hard Disk Driver.

General concepts of H_∞ Controller

General H_∞ Control

Given the control system in Figure and the partition of $P(s)$

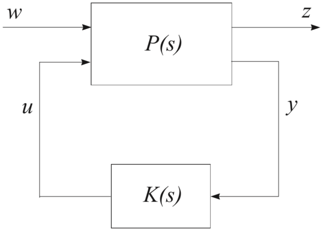

Figure: P-K structure.

$$P(s) = \begin{bmatrix} P_{zw} & P_{zu} \\ P_{yw} & P_{yu} \end{bmatrix}$$

General H_∞ Control

The closed-loop system is given by the transfer matrix

$$F[P(s), K(s)] = T_{zw}(s) = P_{zw} + P_{zu}K(I - P_{yu}K)^{-1}P_{yw}$$

Then, the H_∞ optimal control problem consists in finding a causal controller $K(s)$ which stabilizes $P(s)$ and which minimizes the cost function

$$J_\infty(K) = \|F[P(s), K(s)]\|_\infty = \|T_{zw}(s)\|_\infty = \sup_{\omega \in \mathbf{R}} \sigma_{max}[T_{zw}(j\omega)]$$

General H_∞ Control

The minimization of $J_\infty(K)$ is a very hard problem. Therefore conditions to ensure the existence of a stabilizing controller have been found, such that the H_∞-norm bound holds for a given $\gamma > \gamma_{min} > 0$:

$$J_\infty(K) < \gamma, \quad where \quad \| T_{zw}(s) \|_\infty := \sup_{\omega \in \mathbf{R}} \sigma_{max}[T_{zw}(j\omega)] = \gamma_{min}$$

The procedure to find $\gamma > \gamma_{min} > 0$ is called $\gamma - iteration$ and lets to find γ with any degree of accuracy.

General concepts of μ Controller

μ Analysis and Synthesis

The quest is to develop a general procedure for the synthesis of a robust controller, with respect to structured and unstructured uncertainties, that can verity and guarantee stability and performance.

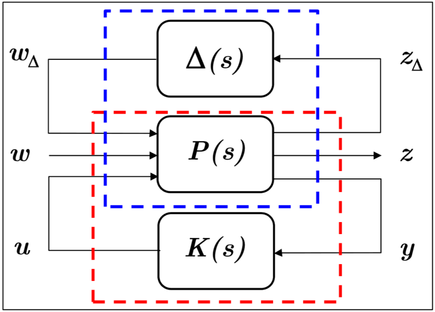

Figure: Block diagram of the closed-loop system for μ analysis and synthesis.

μ Analysis and Synthesis

Robustness Synthesis is a 2 - Block representation, which includes the uncertainty structure in $G(s)$:

$$\begin{bmatrix} z \\ y \end{bmatrix} = G(s) \begin{bmatrix} w \\ u \end{bmatrix}$$

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

$$G_{zw}(s) = F_L\{G(P, \Delta), K\}$$

It means to find $K(s)$ to obtain robustness.

μ Analysis and Synthesis

Robust Analysis is a 2 - Block representation, which includes the closed-loop controller in $N(s)$:

$$\begin{bmatrix} z_\Delta \\ yz \end{bmatrix} = N(s) \begin{bmatrix} w_\Delta \\ w \end{bmatrix}$$

$$N(s) = \begin{bmatrix} N_{11}(s) & N_{12}(s) \\ N_{21}(s) & N_{22}(s) \end{bmatrix}$$

$$N_{zw}(s) = F_U\{N(P,K), \Delta\}$$

It means, given $K(s)$, verify where robustness is obtained.

μ Analysis and Synthesis

The controller $K(s)$, within the feedback $N(s)$ stabilises the normal system (i.e. when $\Delta = 0$) as well as the closed-loop uncertain 2 - Block structure, for all the uncertainties assumed in Δ.
The critical relationship for robust stability is that, given

$$z = N_{22} + N_{21}\Delta(I - M\Delta)^{-1}N_{12} \qquad N_{11}(s) = M(s)$$

$I - M\Delta$ has to be stable and invertible.
In order to verify robustness, a parameter r is introduced, which shrinks or expands the set Δ. Then we must determine the biggest $r*$ such that

$$det[I - M\Delta] \neq 0 \quad \forall \Delta \in r\Delta$$

μ Analysis and Synthesis

The **structured singular value** of the complex matrix $M(s)$, with respect to Δ, is defined as:

$$\mu_{\Delta} = \frac{1}{sup\{r|det(I - M\Delta) \neq 0, \forall \Delta \in r\Delta\}}$$

It can be shown that $I - M\Delta$ has a proper and stable inverse if $\mu_{\Delta} \leq 1$. Therefore, the lower the value of μ_{Δ}, the biggest the set Δ for which robustness is guaranteed.

μ Analysis and Synthesis

$\mu[M(s)]$ is frequency dependent and so it should be calculated for each frequency over a certain range and its computation is a hard problem.

Basing on some properties of μ, it can be shown that

$$\rho(M) \leq \mu_{\Delta}(M) \leq \sigma_{max}(M)$$

μ Analysis and Synthesis

The numerical procedure to compute the upper bound is called **D-K iteration**.

Given scaling matrices on $M(s)$ and a controller $K(s)$, the iteration reduces the value of μ until it cannot be decreased any more. The procedure make use of a cost function in order to minimise the controller $K(s)$.

System

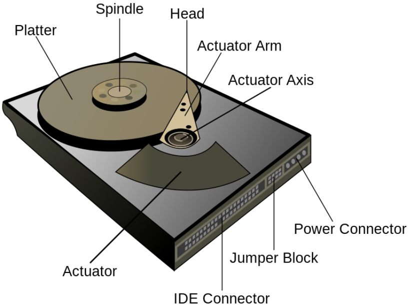

Figure: Schematic diagram of a hard disk drive.

System

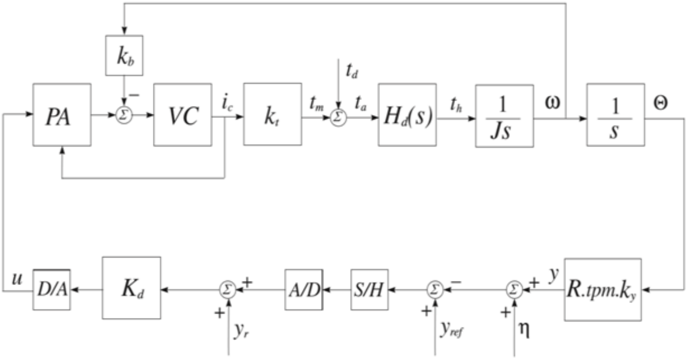

Figure: Block diagram of the hard disk drive servo system.

Uncertainties

System structured uncertainties:

- Mechanical resonant modes:
  - Frequency ω
  - Damping coefficient ξ
- Rigid body parameters:
  - VCM torque constant k_t
  - Arm moment of inertia J
  - Position measurement gain k_y

Uncertainty representation

Each uncertainty can be represented as:

- $\omega = \bar{\omega}(1 + p_\omega \delta_\omega)$
- $\xi = \bar{\xi}(1 + p_\xi \delta_\xi)$
- $k_t = \bar{k_t}(1 + p_{k_t} \delta_{k_t})$
- $J = \bar{J}(1 + p_J \delta_J)$
- $k_y = \bar{k_y}(1 + p_{k_y} \delta_{k_y})$

Uncertainty model

The uncertainties can be pulled out to generate a *Linear Fractional Transformation* (LFT) model.

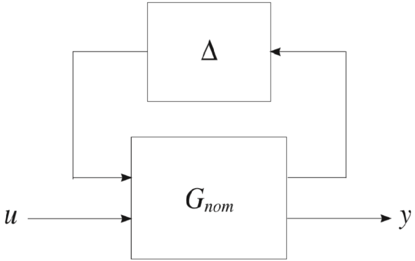

Figure: Plant model in the form of an upper LFT.

Closed-loop System Design Specifications

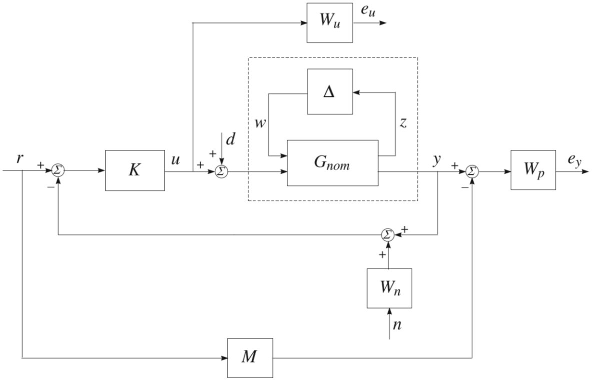

Figure: Block diagram of the closed-loop system with performance specifications.

Closed-loop System Design Specifications

Transfer function matrix from r, d and n to e_y and e_u.

$$\begin{bmatrix} e_y \\ e_u \end{bmatrix} = \begin{bmatrix} W_p(S_oGK - M) & W_pS_oG & -W_pS_oGKW_n \\ W_uS_iK & -W_uKS_oG & -W_uKS_oW_n \end{bmatrix} \begin{bmatrix} r \\ d \\ n \end{bmatrix}$$

The performance objective can be recast as that the $\|\cdot\|_\infty$, of this transfer function matrix, is less than 1.

Closed-loop System Design Specifications

Function	**Description**
$W_p(S_o GK - M)$	Weighted difference between the ideal and actual closed-loop systems
$W_p S_o G$	Weighted disturbance sensitivity
$W_p S_o GKW_n$	Weighted noise sensitivity
$W_u S_i K$	Weighted control effort due to reference
$W_u KS_o G$	Weighted control effort due to disturbance
$W_u KS_o W_n$	Weighted control effort due to noise

Table: H_∞ functions to be minimized.

Closed-loop System Design Specifications

The controller synthesis problem of the Hard Disk Drive Servo System is to find a linear, output feedback controller $K(s)$ which has to ensure the following properties of the closed-loop system.

Closed-loop System Design Specifications

Nominal Performance

The closed-loop system achieves nominal performance if the performance objective is satisfied for the nominal plant model.

$$\left\| \begin{bmatrix} W_p(S_o G_{nom} K - M) & W_p S_o G_{nom} & -W_p S_o G_{nom} K W_n \\ W_u S_i K & -W_u K S_o G_{nom} & -W_u K S_o W_n \end{bmatrix} \right\|_\infty < 1$$

Closed-loop System Design Specifications

Robust Stability

The closed-loop system achieves robust stability if the closed-loop system is internally stable for each possible plant dynamics $G = F_U(G_{nom}, \Delta)$.

Closed-loop System Design Specifications

Robust Performance
The closed-loop system must remain internally stable for each $G = F_U(Gnom, \Delta)$ and in addition the performance criterion should be satisfied for each $G = F_U(Gnom, \Delta)$.

$$\left\| \begin{bmatrix} W_p(S_o G K - M) & W_p S_o G & -W_p S_o G K W_n \\ W_u S_i K & -W_u K S_o G & -W_u K S_o W_n \end{bmatrix} \right\|_\infty < 1$$

System Specifications

- Peak closed-loop gain <4 dB
- Open-loop gain >20 dB at 100 Hz
- Steady state error <0.1 μm
- Settling time <1.5 ms
- Closed-loop bandwidth >1000 Hz
- Gain Margin >5 dB
- Phase margin >40 deg
- Control action <1.2 V
- Good disturbance rejection and noise attenuation

Model Transfer Function

The model transfer function is chosen so that the time response to the reference signal would have an overshoot less than 20 % and a settling time less than 1 *ms*.

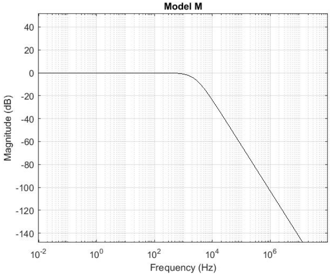

Figure: Frequency response of the ideal model M.

Weighting Functions

The noise weighting function W_n is determined on the basis of the spectral density of the position noise signal, whose spectral content is usually above 500 Hz.

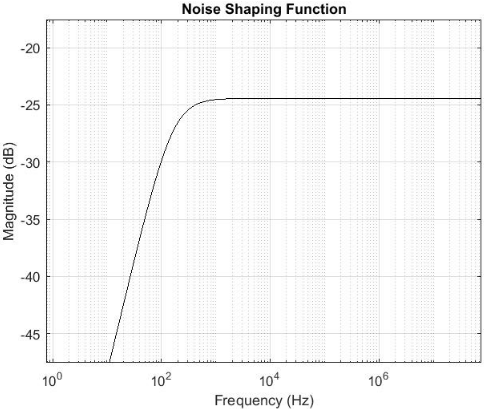

Figure: Frequency response of the noise shaping function.

Weighting Functions

The closed-loop system performance specifications are reflected by the weighting performance function $W_p(s)$.

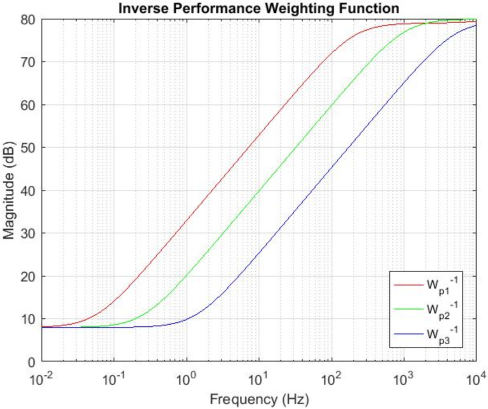

Figure: Frequency response of the inverse of W_p.

Weighting Functions

The control weighting function W_u is usually chosen as high pass filter in order to ensure that the control action does not exceed the maximum admissible value. Three weighting functions are considered in the design.

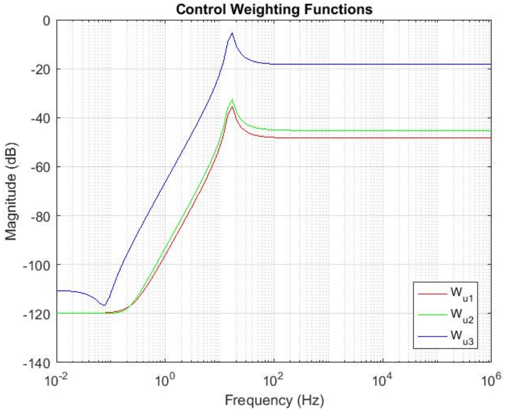

Figure: Frequency response of the three control weighting functions.

μ Controller

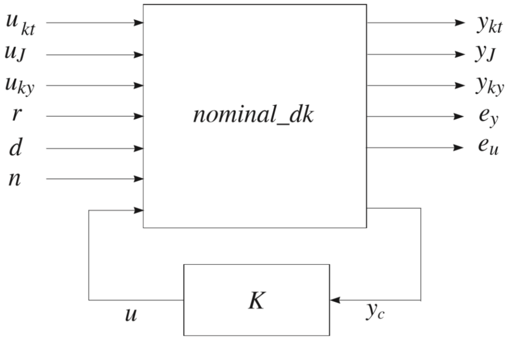

Figure: Block diagram of the closed-loop system with μ controller.

μ Controller

Denote by $P(s)$ the transfer matrix of the seven-input, six-output, open-loop system **nominal_dk** and let the block structure of the uncertainty Δ_P be defined as

$$\Delta_P := \left\{ \begin{bmatrix} \Delta_r & 0 \\ 0 & \Delta_F \end{bmatrix} : \Delta_r \in \mathbf{R}^{3\times 3}, \Delta_F \in \mathbf{C}^{3\times 2} \right\}$$

Where the block Δ_r contains the uncertainties of the rigid body model, while the second block Δ_F is a fictitious uncertainty block which is used for the performance requirements

μ Controller

To guarantee robust performance, it needs to find a stabilising controller $K(s)$ such that, for each frequency $\omega \in [0, \infty]$, the following holds:

$$\mu_{\Delta_P}[F_L(P, K)(j\omega)] < 1$$

$$\left\| \begin{bmatrix} W_p(S_o G K - M) & W_p S_o G & -W_p S_o G K W_n \\ W_u S_i K & -W_u K S_o G & -W_u K S_o W_n \end{bmatrix} \right\|_\infty < 1$$

μ Controller

The μ controller is designed three times, each time with the corresponding weighting functions pair (i.e. W_p and W_u). In each case six DK iteration are used.

Controller	**Order**	**RS** μ_{max}	**RP** μ_{max}
1	38	0.37296	0.43625
2	38	0.41163	0.54945
3	26	0.34509	1.9333

Table: Robust stability and robust performance for the three K_μ.

Controller	**Gain margin**dB	**Phase margin** deg
1	10.9009	59.0071
2	9.838	50.9925
3	-10.4504	36.3051

Table: Gain and phase margins for the three K_μ.

μ Controller

The best μ controller is the first one, therefore the graphics of robust stability and robust performance relative to it are reported.

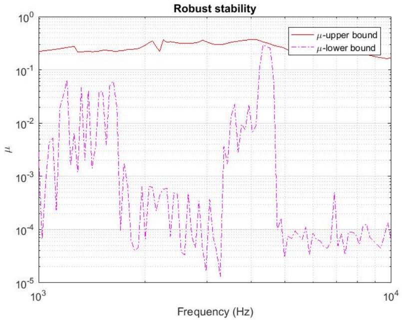

Figure: Robust Stability.

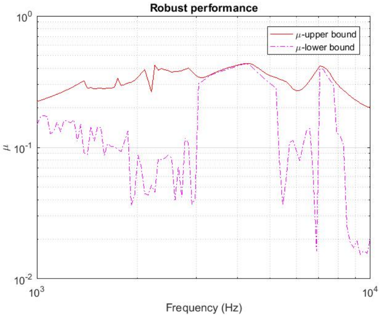

Figure: Robust Perfomance.

μ Controller

Specification	Constraint	Value
Peak closed-loop gain	$<4\ dB$	$2.04\ dB$
Open-loop gain	$>20\ dB$ at $100\ Hz$	$29.39\ dB$
Steady state error	$<0.1\ \mu m$	$0\ \mu m$
Settling time	$<1.5\ ms$	$1.6\ ms$
Closed-loop bandwidth	$>1000\ Hz$	$1060\ Hz$
Gain margin	$>5\ dB$	$10.9009\ dB$
Phase margin	$>40\ deg$	$59.0071\ deg$
Max control action	$<1.2\ V$	$1.193\ V$
Overshoot	$<20\ \%$	$20.356\ \%$

Table: Specifications due to the first μ controller.

H_∞ Controller

The aim of the design in this case is to find an H_∞(*sub*)optimal, output controller for the interconnection shown in Figure, in which the inputs and outputs of the uncertainty block are excluded.

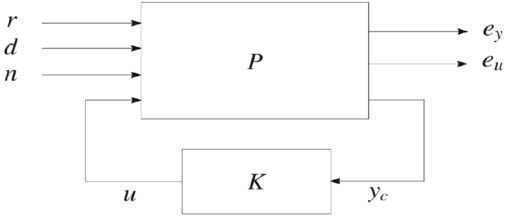

Figure: Closed-loop system with H_∞ controller.

The optimal control minimises the $\infty - norm$ of $F_L(P, K)$ in respect to the transfer function K of the controller.

H_∞ Controller

The controller obtained is of 18th order and guarantees robust stability and robust performance.

Order	Robust Stability μ_{max}	Robust Performance μ_{max}
18	0.43293	0.48774

Table: Robust stability and robust performance for the H_∞ controller.

H_∞ Controller

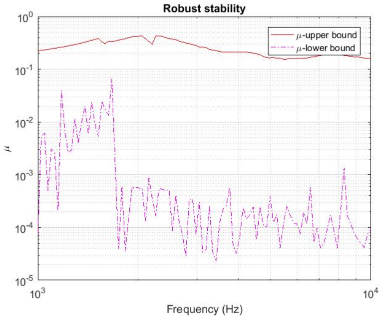

Figure: Robust Stability.

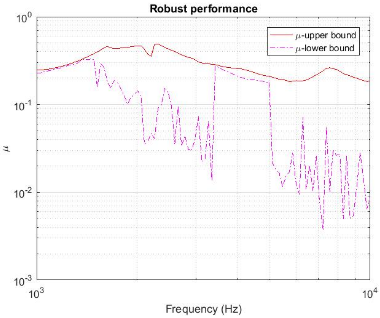

Figure: Robust Perfomance.

H_∞ Controller

Specification	Constraint	Value
Peak closed-loop gain	$<4\ dB$	$4.895\ dB$
Open-loop gain	$>20\ dB$ at $100\ Hz$	$35.88\ dB$
Steady state error	$<0.1\ \mu m$	$0\ \mu m$
Settling time	$<1.5\ ms$	$1.87\ ms$
Closed-loop bandwidth	$>1000\ Hz$	$1500\ Hz$
Gain margin	$>5\ dB$	$12.4245\ dB$
Phase margin	$>40\ deg$	$36.187\ deg$
Max control action	$<1.2\ V$	$0.789\ V$
Overshoot	<20 %	48.8677 %

Table: Specifications due to H_∞ controller.

LQG Controller

The *Linear* − *Quadratic* − *Gaussian* (LQG) regulator is the combination of a *Kalman filter*, i.e. a *Linear* − *Quadratic* − *Estimator* (LQE), with a *Linear* − *Quadratic* − *Regulator* (LQR).

LQG optimality does not automatically ensure good robustness properties. The robust stability of the closed loop system must be checked separately after the LQG controller has been designed.

LQG Controller

Loop Transfer Recovery

The *Loop Transfer Recovery* method (LTR) is a modification of the LQG controller, where design parameters are changed in order to achieve robustness similar to the LQR (at input) or to the KBF (at output).

This method can be applied only to *minimum phase* and *square* systems. In this case it cannot be used since the transfer function $P_{yu}(s)$ has a zero in the right part of the s plane. Therefore, a standard LQG is designed.

LQG Controller

The LQG compensator is designed respect to the nominal transfer function $P_{yu}(s)$.

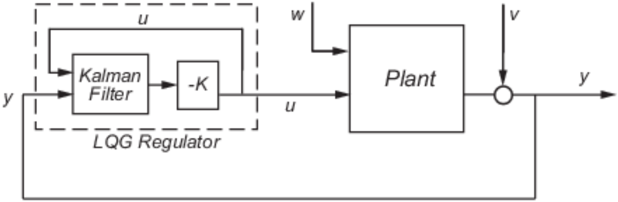

Figure: Closed-loop system with *LQG* controller.

Then the structure singular value analysis is applied to verify robustness.

LQG Controller

The LQG is very sensible to parameter variations as shown in the following Table.

Robust Stability μ_{max}	**Robust Performance** μ_{max}
74.9181	437.985

Table: Robust stability and robust performance for the *LQG* controller.

LQG Controller

Specification	Constraint	Value
Peak closed-loop gain	<4 *dB*	0 *dB*
Open-loop gain	>20 *dB* at 100 *Hz*	19.3 *dB*
Steady state error	<0.1 μm	0 μm
Settling time	<1.5 *ms*	1.93 *ms*
Closed-loop bandwidth	>1000 *Hz*	500 *Hz*
Gain margin	>5 *dB*	13.0669 *dB*
Phase margin	>40 *deg*	64.6897 *deg*
Max control action	<1.2 *V*	0.7131 *V*
Overshoot	<20 %	6.6058 %

Table: Specifications due to *LQG* controller.

PID Controller

Proportional − *Integral* − *Derivative*(PID) controllers are very common in industrial sectors due to its simplicity.

$$C(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{T_f s + 1}$$

where K_p,K_i and K_d denote the *proportional*, *integrative* and *derivative* term, respectively.

PID Controller

As well as before, the *PID* controller is designed respect to the nominal plant $P_{yu}(s)$.

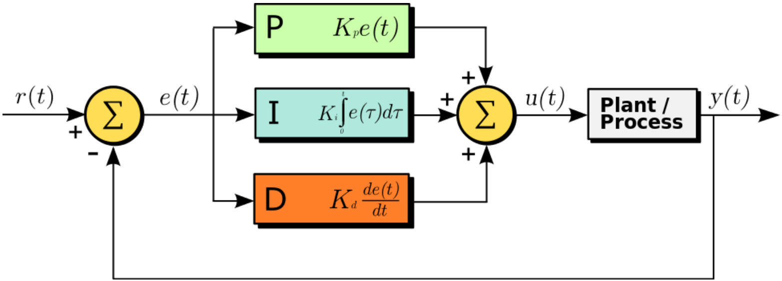

Figure: Closed-loop system with *PID* controller.

PID Controller

The controller parameters are showed in the following Table.

K_p	K_i	K_d	T_f
0.0132	0.734	8.69e-06	1.45e-05

Table: *PID* controller parameters.

PID Controller

Then the structured singular value analysis is applied.

Robust Stability μ_{max}	**Robust Performance** μ_{max}
0.45838	0.5429

Table: Robust stability and robust performance for the *PID* controller.

PID Controller

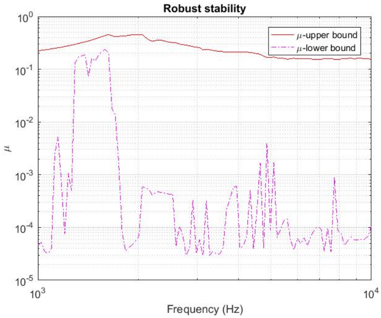

Figure: Robust Stability.

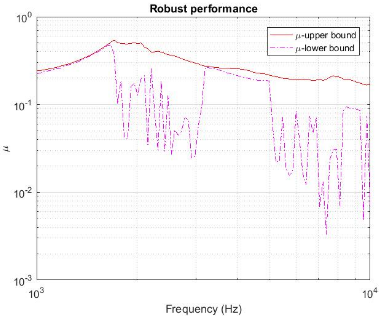

Figure: Robust Perfomance.

PID Controller

Specification	**Constraint**	**Value**
Peak closed-loop gain	<4 *dB*	3.945 *dB*
Open-loop gain	>20 *dB* at 100 *Hz*	29 *dB*
Steady state error	<0.1 μm	0 μm
Settling time	<1.5 *ms*	1.151 *ms*
Closed-loop bandwidth	>1000 *Hz*	1600 *Hz*
Gain margin	>5 *dB*	8.4856 *dB*
Phase margin	>40 *deg*	37.0338 *deg*
Max control action	<1.2 *V*	0.735 *V*
Overshoot	<20 %	41.367 %

Table: Specifications due to *PID* controller.

Comparison between controllers

- Frequency response of the controllers
- Frequency response of the closed-loop systems
- Transient response of the closed-loop systems

Frequency response of the controllers

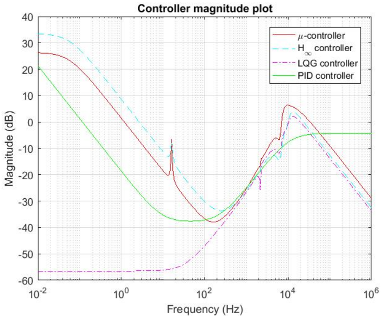

Figure: Controller magnitude.

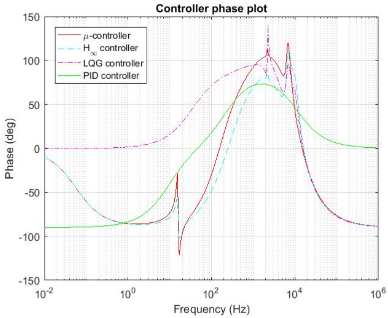

Figure: Controller phase.

Frequency response of the controllers

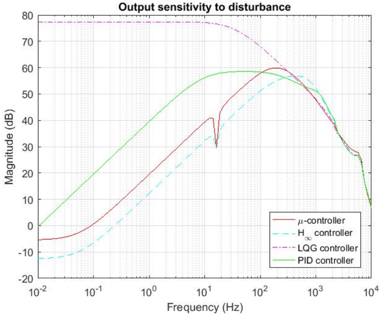

Figure: Output sensitivity to disturbance.

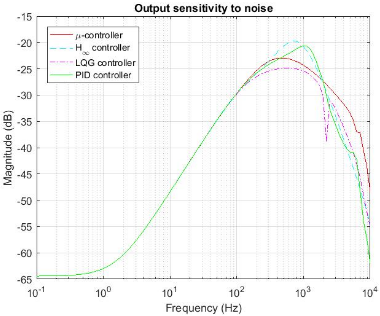

Figure: Output sensitivity to noise.

Transient responses of the closed loop systems

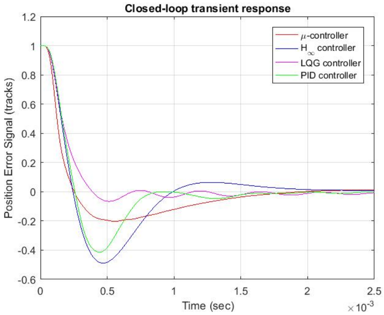

Figure: Closed-loop transient response.

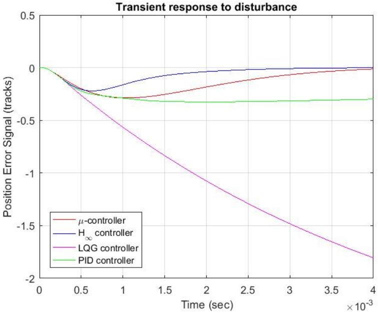

Figure: Transient response to disturbance.

Transient responses of the closed loop systems

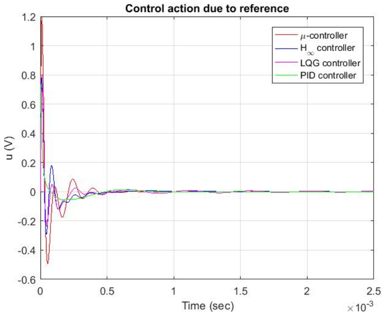

Figure: Control action due to reference.

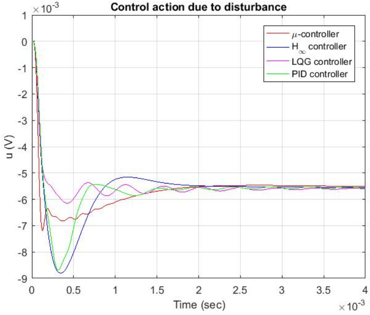

Figure: Control action due to disturbance.

Nominal Performance

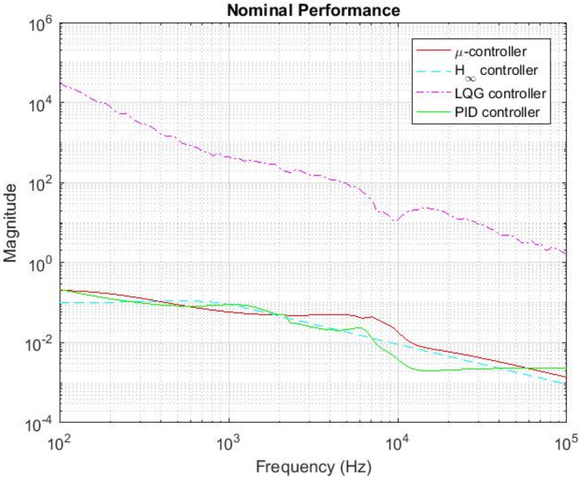

Figure: Nominal performance of the closed-loop systems.

Robust Stability

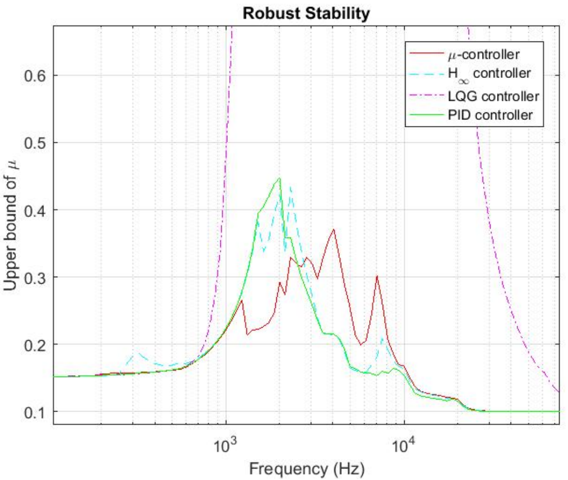

Figure: Robust stability of the closed-loop systems.

Robust Performance

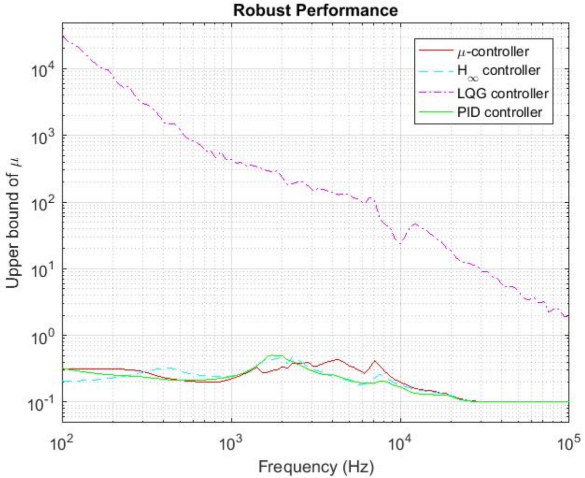

Figure: Robust performance of the closed-loop systems.