PHY 300 HW 8, Carl Liu

2.8

For the vector $w = (1, \sqrt{3})$ we have a linear polarization of 60^o from the x axis. An orthogonal vector v would have the property that $w \cdot v = 0$. Meaning $v_1 + \sqrt{3}v_2 = 0$. Thus $v_1 = -\sqrt{3}v_2$ meaning $v = (-\sqrt{3}, 1)$ is orthogonal to w

For the vector $w = (i, -1)$ we have $w = -i(1, -i) = e^{i3\pi/2}(1, -i)$ and thus is a right circular polarization that is phase shifted $3\pi/2$. An orthogonal vector v would have the property of $v \cdot w = 0$ which means $-v_1 i + v_2 = 0$. Thus $v_2 = v_1 i$ resulting in an orthogonal vector of $(1, i)$

For the vector $w = (1-i, 1+i)$ we have $w = (1, 1)+(-i, i) = (1, 1)+i(-1, 1)$. This is a superposition of a linear polarization, 45^o from the x axis, and a linear polarization -45^o from the x axis with a phase shift of $\pi/2$ meaning a left polarization. An orthogonal vector would have the property $v \cdot w = 0$ meaning $v_1(1 + i) + v_2(1 - i) = 0$. Thus $v_1 = -v_2(1 - i)/(1 + i)$. So we have an orthogonal vector of $(-(1 - i)/(1 + i), 1)$ which means $(-1 + i, 1 + i)$ is also orthogonal.

2.10

Consider an arbitrary polarized light $w = (A, B)$. Suppose the light is first sent through a quarter-wave plate then a linear polarizer of 45^o. The Jones calculus would result in a vector

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 & -i \\ 1 & -i \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A - iB \\ A - iB \end{bmatrix} = (A - iB) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

which is a 45^o linear polarization with a phase shift or/and scaling of $A-iB$. whereas when the 45^o polarizer is first applied then the circular polarizer, we have

$$\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -i & -i \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A + B \\ -i(A + B) \end{bmatrix} = (A + B) \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

which is a right polarization with a phase shift or/and scaling of $A + B$

2.12

This is represented by the Jones calculus

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

which is vertically polarized as required.

2.16

We have for the TE case $R_s = |r_s|^2$ and for the TM case $R_p = |r_p|^2$. So for the reflectance of water with an incidence of 45^o we have

$$R_s = \left(\frac{\cos(45^o) - \sqrt{1.33^2 - \sin^2(45^o)}}{\cos(45^o) + \sqrt{1.33^2 - \sin^2(45^o)}} \right)^2 = \left(\frac{\frac{1}{\sqrt{2}} - \sqrt{1.33^2 - \frac{1}{2}}}{\frac{1}{\sqrt{2}} + \sqrt{1.33^2 - \frac{1}{2}}} \right)^2 = 0.0523$$

and

$$R_p = \left(\frac{-1.33^2 \cos(45^o) + \sqrt{1.33^2 - \sin^2(45^o)}}{1.33^2 \cos(45^o) + \sqrt{1.33^2 - \sin^2(45^o)}} \right)^2 =$$

$$\left(\frac{\frac{-1.33^2}{\sqrt{2}} + \sqrt{1.33^2 - \frac{1}{2}}}{\frac{1.33^2}{\sqrt{2}} + \sqrt{1.33^2 - \frac{1}{2}}} \right)^2 = 0.0027$$

For diamond we have

$$R_s = \left(\frac{\cos(45^o) - \sqrt{2.42^2 - \sin^2(45^o)}}{\cos(45^o) + \sqrt{2.42^2 - \sin^2(45^o)}} \right)^2 = \left(\frac{\frac{1}{\sqrt{2}} - \sqrt{2.42^2 - \frac{1}{2}}}{\frac{1}{\sqrt{2}} + \sqrt{2.42^2 - \frac{1}{2}}} \right)^2 = 0.282$$

$$R_p = \left(\frac{-2.42^2 \cos(45^o) + \sqrt{2.42^2 - \sin^2(45^o)}}{2.42^2 \cos(45^o) + \sqrt{2.42^2 - \sin^2(45^o)}} \right)^2 =$$

$$\left(\frac{-\frac{2.42^2}{\sqrt{2}} + \sqrt{2.42^2 - \frac{1}{2}}}{\frac{2.42^2}{\sqrt{2}} + \sqrt{2.42^2 - \frac{1}{2}}} \right)^2 = 0.08$$