PHY 300 HW 6, Carl Liu

7-5

A) We have as the velocity of the wave to be $\sqrt{T/\mu}$ for a string. This velocity is independent of the driving frequency. Thus we have $v = \sqrt{T/\mu} = \sqrt{50/0.1} = 22.36m/s$

B) The wavelength is then just $v * T$ where v is the velocity, and T, the period of the oscillations. Thus we have $\lambda = v * T = 22.36 * 0.1 = 2.236m$

C) The form of our progressive wave is

$$y(x,t) = A \sin\left(\frac{2\pi}{\lambda}(x - vt) + \phi\right)$$

where ϕ is some phase shift. Since at $x = 0$, $t = 0$, we have $y = 0.01m$, we must have $A\sin(\phi) = 0.01$ but $A = 0.02$. So $\sin(\phi) = 1/2$. That means $\phi = \pi/6$ or $\phi = 5\pi/6$. We must also have $dy/dt < 0$ meaning

$$-\frac{Av2\pi}{\lambda}\cos\left(\frac{2\pi}{\lambda}(x - vt) + \phi\right) < 0$$

Taking this at $x = 0, t = 0$, we have

$$-\frac{Av2\pi}{\lambda}\cos(\phi) < 0$$

Since A and v have been established as positive, we must have $-\cos(\phi) < 0$ and so $\cos(\phi) > 0$. Thus $\phi = \pi/6$ and so the final form of our equation is

$$y(x,t) = 0.02\sin\left(\frac{2\pi}{2.236}(x - 22.36t) + \frac{\pi}{6}\right) = 0.02\sin(2.81x - 62.83t + 0.524)$$

7-6

A) We have $v = \sqrt{T/\mu}$. We have $T = 100 * g * \mu * L$. So $v = \sqrt{100 * g * L}$ and since it took $0.1s$ for the wave pulse to travel the length, we must have $L = 0.1 * v = 0.1 * \sqrt{100 * g * L}$. Thus $L^2 = 0.1^2 * 100 * g * L$ and so $L = 0.1^2 * 100 * g = 9.8m$

B) The equation for the normal mode of the string is of the form

$$y(x,t) = A_n \sin\left(\frac{n\pi x}{L}\right)\cos(\omega_n t)$$

where

$$\omega_n = \frac{2\pi v}{\lambda} = \frac{n\pi}{L} v$$

Thus we have $\omega_n = n * 31.416$ and $n\pi/L = n * 0.321$ resulting in

$$y(x,t) = A_n \sin(n * 0.321x) \cos(n * 31.416t)$$

where the third normal mode is

$$y(x,t) = A_3 \sin(0.962x) \cos(94.25t)$$

7-8

A) The frequency is that of the frequency of the transverse motion and so we have $\omega = 3\pi$ so $f = \omega/2\pi = 1.5Hz$.

B) Since the form of a traveling wave is

$$y(x,t) = A \sin\left(\frac{2\pi}{\lambda}(x \pm vt)\right)$$

and we have

$$y(0,t) = A \sin\left(\frac{2\pi}{\lambda}(\pm vt)\right) = 0.2 \sin(3\pi t)$$

and

$$y(1,t) = A \sin\left(\frac{2\pi}{\lambda}(1 \pm vt)\right) = 0.2 \sin(3\pi t + \pi/8)$$

That means

$$\frac{2\pi}{\lambda}(\pm vt) + n * 2\pi = 3\pi t \quad \frac{2\pi}{\lambda}(1 \pm vt) + n * 2\pi = 3\pi t + \frac{\pi}{8} + m * 2\pi$$

for integers $m \geq 0$. This results in

$$\frac{2\pi}{\lambda} = \frac{\pi}{8} + m * 2\pi$$

So

$$\lambda_m = \frac{2\pi}{\frac{\pi}{8} + m * 2\pi} = \frac{2}{\frac{1}{8} + 2 * m} = \frac{16}{1 + 16m}$$

When $m < 0$, we have

$$\frac{16}{1+16m}$$

is negative so

$$\lambda_n = \frac{16}{16n-1}$$

for $n > 0$ where now the wave is traveling in the opposite direction as that of the m index

C) Since $v = \lambda_m * f$, we have for $m \geq 0$ that

$$v = \frac{24}{1+16m}$$

for one direction and

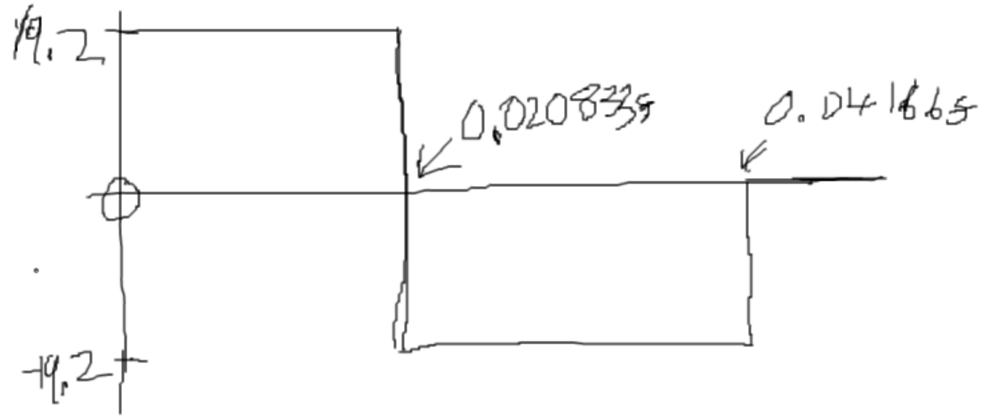
$$v = \frac{24}{16m-1}$$

for a wave traveling in the opposite direction.

D) The information given cannot be used to establish whether the wave is moving left or right. There are many waves that fit such a description and as shown earlier these waves can be traveling in opposite directions.

7-9

In the case of a triangle we have $v_y = H/\Delta t$ where $\Delta t = 0.5/v$ where v is velocity when looking at $x = 1m$ for the first half of the triangle moving through. Then for the last half we have $v_y = -H/\Delta t$ where $\Delta t = 0.5/v$ and so we have $v_y = 0.4 * 24/0.5 = 19.2m/s$ for $0s \leq t < 0.5/v = 0.020833s$ and $v_y = -19.2m/s$ for $0.020833s < v \leq 1/24 = 0.04166s$. Thus we have



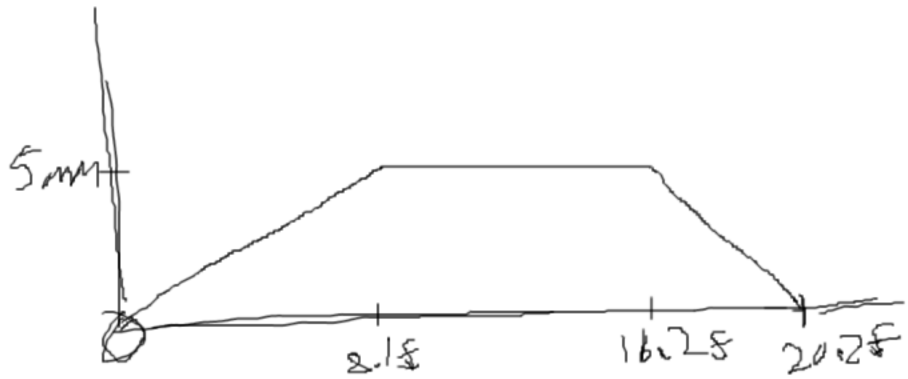
7-16

A) Since we have

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10 * 9.8}{0.005/12.5}} = 495m/s$$

and from the graph we see that $d = 5.6 - 5.2 = 0.4m$ is the length when the string is at it's highest amplitude, we can conclude that the time the switch was fully closed is $d/v = 0.4m/495m/s = 0.00081s = 8.1 * 10^{-4}s$

B) The graph of displacement vs time would start by having a constant positive slope for a time $(6.0 - 5.6)/495 = 8.1 * 10^{-4}s$ until it reaches $5mm$. Then it is fully closed for $8.1*10^{-4}s$ meaning it has a displacement of $5mm$ for that whole time. Then the switch moves away back to 0 displacement with a negative constant slope in the time $(5.2 - 5.0)/495 = 4 * 10^{-4}s$. Thus we



have

C) the maximum speed of the contact happened when the switch was re-open. During the reopening there was a slope of $5/4 * 10^{-4} = 12500mm/s = 12.5m/s$ whereas we only had $5/8.1 * 10^{-4} = 6172mm/s = 6.172m/s$ during closing. When it was closed completely velocity was 0.

D) Since the start of the wave has traveled to 6m and we know

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10 * 10}{0.005/12.5}} = 500$$

we can conclude that the picture was taken at $t = 6/500 = 0.012s = 12ms$