MAT 341 HW12, Carl Liu

1.

We have

$$\nabla^2 p = \frac{\partial^2 p}{\partial x^2}(x,y) + \frac{\partial^2 p}{\partial y^2}(x,y) = -H$$

since

$$\frac{\partial^2 p}{\partial x^2} = 2D \quad \frac{\partial^2 p}{\partial y^2} = 2F$$

we have

$$\nabla^2 p = 2D + 2F = -H$$

meaning $D = -(H+2F)/2$ is the required conditions on the coefficients, all other coefficients can be arbitrary.

2.

Since

$$v(0,\theta) = a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)d\theta$$

1) for $f(\theta) = |\theta|$

$$v(0,\theta) = a_0 = \frac{2}{2\pi}\int_{0}^{\pi} \theta d\theta = 1$$

2) for $f(\theta) = \theta$

$$v(0,\theta) = a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \theta d\theta = 0$$

3) for $f(\theta) = \cos(\theta), -\pi/2 < \theta < \pi/2$ and $f(\theta) = 0$ otherwise, we have

$$v(0,\theta) = a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)d\theta = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \cos(\theta)d\theta = \frac{1}{\pi}$$

4) for $f(\theta) = -1, -\pi < \theta < 0$ and $f(\theta) = 1$, $0 < \theta < \pi$, we have

$$v(0,\theta) = a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)d\theta = \frac{1}{2\pi}\int_{0}^{\pi} d\theta + \frac{1}{2\pi}\int_{-\pi}^{0} -d\theta = 0$$