PHY 300 HW 10, Carl Liu

5.4

Since we have $(\sin(\beta)/\beta)^2 = I/I_0$, we have maximums when $\beta = (2n+1)\pi/2$ where $n \geq 1$ is an integer. We can assume $\sin(\beta) = 1$ at all times in order to obtain an envelope to the equation. This results in $(1/\beta)^2 = I/I_0$. We want $I/I_0 = 0.005$ (corresponds to 0.5%), resulting in $1/\sqrt{0.005} = \beta$ and so $\beta = 14.142$. Which means $14.142 = (2n+1)\pi/2$ and so $((14.142 * 2/\pi) - 1)/2 = 4.001 = n$. Thus we have the 4th maxima being what is required, this is considering the central maxima as the 0th.

5.9

We have $I/I_0 = (\sin(\beta)/\beta)^2 \cos^2(\gamma)$ where

$$\beta = \frac{1}{2}kb\sin(\theta) \quad \gamma = \frac{1}{2}kh\sin(\theta)$$

But because $h = b$, we have $\gamma = \beta$, resulting in

$$I/I_0 = \frac{\sin^2(\beta)}{\beta^2}\cos^2(\beta) = \frac{4\sin^2(\beta)}{4\beta^2}\cos^2(\beta) = \left(\frac{2\sin(\beta)\cos(\beta)}{2\beta}\right)^2 = \left(\frac{\sin(2\beta)}{2\beta}\right)^2$$

but the last equation is equivalent to the single slit equation with the length of the slit being $2b$ as required.

5.12

We have $D = I/I_0 = (\sin(\beta)/\beta)^2 \cos^2(\gamma)$ where

$$\beta = \frac{1}{2}kb\sin(\theta) \quad \gamma = \frac{1}{2}kh\sin(\theta)$$

The envelope we can consider is $(\sin(\beta)/\beta)^2$. The central maxima is between $\beta = \pm\pi$. This means $\frac{1}{2}kb\sin(\theta) = \pm\pi$ and so

$$\sin(\theta) = \frac{\pm 2\pi}{kb}$$

resulting in

$$\theta = \sin^{-1}\left(\frac{\pm 2\pi}{kb}\right)$$

But that means

$$\gamma = \frac{1}{2}kh\sin\left(\sin^{-1}\left(\frac{\pm 2\pi}{kb}\right)\right) = \frac{1}{2}kh\frac{\pm 2\pi}{kb} = \pm\frac{h\pi}{b}$$

is where γ ranges. Since we have $\cos^2(\gamma)$ as a factor in the diffraction pattern, we must have maximas at $\gamma = n\pi$ for $n \geq 0$. Since γ ranges $2h\pi/b$, we would have $2h/b$ maximas. But there are also maximas near $\pm h\pi/b$ due to the $\sin(\beta)$ factor going to 0 this creates a maxima that isn't counted above since it is not due to the $\cos(\gamma)$ factor being at a maximum. Thus we must add on two more maximas for a total of $2h/b + 2$ maximas.

5.14

We have

$$\Delta\theta = \frac{\lambda}{Nh\cos(\theta)} \quad \Delta\theta = \frac{n\Delta\lambda}{h\cos(\theta)}$$

by the Rayleigh criterion. Our goal is to find $\Delta\lambda$. Since we have

$$\frac{\lambda}{\Delta\lambda} = Nn$$

where n is the order and N is the number of lines, we have from a grating of 1200 lines/mm with width $5cm$ and first order resulting in

$$\frac{\lambda}{\Delta\lambda} = 1200 * 50 = 60000$$

This results in $\Delta\lambda = \lambda/60000$ as the minimum resolvable wavelength.