PHY 300 HW 9, Carl Liu

3.6

We have $n = c/u$ where c is the speed of light in vacuum, and u, the speed of light in the medium. In an evacuated tube of length l. The light would have to go back and forth for a total length of $2l$ at a speed of c. This means it would take $2l/c = t_e$ time for the light to travel the full length. Now in a full cell, we would have $2l/u = t_f$ from the same logic. The time difference, $t_f - t_e$ multiplied by c would then give us the extra length that the evacuated tube light would have effectively traveled as compared to the non evacuated tube. Thus the effective path difference is

$$(t_f - t_e)c = \left(\frac{2l}{u} - \frac{2l}{c}\right) c = \left(\frac{2lc}{u} - 2l\right) = (2ln - 2l) = 2l\,(n-1)$$

Thus we have $4l(n-1)/\lambda$ fringes that pass. For $n = 1.0003$, $l = 0.1m$ $\lambda = 590 * 10^{-9}m$, we have

$$4 * 0.1 \left(\frac{1.0003 - 1}{590 * 10^{-9}}\right) = 203.4\ fringes$$

4.1

The maximum transmittance occurs when $\Delta = 0$. So

$$\mathcal{F}_{max} = \frac{I_{max}}{I_0} = \frac{T^2}{(1-R)^2} = \frac{0.05^2}{(1-0.9)^2} = \frac{1}{4}$$

The minimum reflectance occurs when $\Delta = \pi$. Meaning

$$\mathcal{F}_{min} = \frac{I_{max}}{I_0} = \frac{T^2}{(1-R)^2}\frac{1}{1+F}$$

Since $F = \frac{4R}{(1-R)^2}$, we have

$$\mathcal{F}_{min} = \frac{T^2}{(1-R)^2}\frac{1}{1+F} = \frac{T^2}{(1-R)^2}\frac{1}{1+\frac{4R}{(1-R)^2}} = \frac{T^2}{(1-R)^2+4R} =$$

$$\frac{T^2}{1-2R+R^2+4R} = \frac{T^2}{(1+R)^2} = \frac{0.05^2}{(1+0.9)^2} = \frac{1}{1444}$$

The coefficient of finesse F is

$$F = \frac{4R}{(1-R)^2} = \frac{4 * 0.9}{0.1^2} = 360$$

and the reflecting finesse is

$$\mathcal{F} = \frac{\pi}{2}\sqrt{F} = \frac{\pi}{2}\sqrt{360} = \frac{\pi}{2}6\sqrt{10} = 29.8$$

4.5

Since we are considering a single layer, we have

$$t = \frac{2n_0}{\cos(kd)n_0 - \frac{i}{n}\sin(kd)n_T n_0 - in\sin(kd) + \cos(kd)n_T}$$

Because $n_T = n_0 = 1$, we can then conclude

$$t = \frac{2}{2\cos(kd) - i\left(\frac{1}{n}\sin(kd) + n\sin(kd)\right)}$$

This means

$$T = |t|^2 = tt^* =$$

$$\frac{4}{4\cos^2(kd) + \sin^2(kd)\left(\frac{1}{n} + n\right)^2} = \frac{4}{4(1-\sin^2(kd)) + \sin^2(kd)\left(\frac{1}{n^2} + 2 + n^2\right)} =$$

$$\frac{4}{4 + \sin^2(kd)\left(\frac{1}{n^2} - 2 + n^2\right)} = \frac{4}{4 + \sin^2(kd)\left(\frac{n^2-1}{n}\right)} =$$

$$\frac{1}{1 + \sin^2(kd)\left(\frac{n^2-1}{2n}\right)^2}$$

where we have $k = 2\pi/\lambda = 2\pi n/\lambda_0$. Thus

$$T = \left(1 + \sin^2(2\pi nd/\lambda_0)\left(\frac{n^2-1}{2n}\right)^2\right)^{-1}$$

So we have a maximum when $2\pi nd/\lambda_0 = N\pi$, where N is an integer. Meaning

$$\lambda_N = \frac{2\pi nd}{N\pi} = \frac{2nd}{N}$$