MAT 312 HW 5 Carl Liu

4.1:4

i) Starting with 1 and using the switch strategy we obtain $\pi(1) = 8, \pi(8) = 4, \pi(4) = 6, \pi(6) = 2, \pi(2) = 3, \pi(3) = 1$ and so a cycle $(1, 8, 4, 6, 2, 3)$. The smallest number that is not in this cycle is 5 and $\pi(5) = 5$ which is fixed. The next smallest number is 7 and $\pi(7) = 7$ is also fixed. Thus we can represent this product as $(1, 8, 4, 6, 2, 3)$

ii) Starting with 1 and using the switching strategy we obtain $\pi(1) = 2, \pi(2) = 7, \pi(7) = 5, \pi(5) = 4, \pi(4) = 9, \pi(9) = 3, \pi(3) = 12, \pi(12) = 10, \pi(10) = 1$ and so we have a cycle $(1, 2, 7, 5, 4, 9, 3, 12, 10)$. The smallest number not in this cycle is 6 and so we have $\pi(6) = 6$ making it fixed. We also have $\pi(8) = 8$ and $\pi(11) = 11$. Thus we can represent the product as $(1, 2, 7, 5, 4, 9, 3, 12, 10)$

iii) Starting with 1 and using the switching strategy we obtain $\pi(1) = 5, \pi(5) = 9, \pi(9) = 4, \pi(4) = 8, \pi(8) = 3, \pi(3) = 7, \pi(7) = 2, \pi(2) = 6, \pi(6) = 1$. Thus we have a cycle $(1, 5, 9, 4, 8, 3, 7, 2, 6)$. The smallest number not in this cycle is 10 and we have $\pi(10) = 11, \pi(11) = 10$. Thus we have a cycle $(10, 11)$. This means we can represent the product as $(1, 5, 9, 4, 8, 3, 7, 2, 6)(10, 11)$ which are two disjoint sets

4.1:5

	id	(1234)	(13)(24)	(1432)	(13)	(24)	(12)(34)	(14)(23)
id	id	(1234)	(13)(24)	(1432)	(13)	(24)	(12)(34)	(14)(23)
(1234)	(1234)	(13)(24)	(1432)	id	(14)(23)	(12)(34)	(13)	(24)
(13)(24)	(13)(24)	(1432)	id	(1234)	(24)	(13)	(14)(23)	(12)(34)
(1432)	(1432)	id	(1234)	(13)(24)	(12)(34)	(14)(23)	(24)	(13)
(13)	(13)	(12)(34)	(24)	(14)(23)	id	(13)(24)	(1234)	(1432)
(24)	(24)	(14)(23)	(13)	(12)(34)	(24)(13)	id	(1432)	(1234)
(12)(34)	(12)(34)	(24)	(14)(23)	(13)	(1432)	(1234)	id	(13)(24)
(14)(23)	(14)(23)	(13)	(12)(34)	(24)	(1234)	(1432)	(13)(24)	id

4.2:1

i) We see that the permutation is composed of disjoint cycles and thus the order is $lcm(5, 3, 2) = 30$. Since we have $sgn(\omega\tau\psi) = sgn(\omega)sgn(\tau)sgn(\psi)$ and the permutation is a composition of 3 cycles, we thus have $(-1)^{5-1}(-1)^{3-1}(-1)^{2-1} = -1$ being the sign of the permutation.

ii) Following the same process as above we have the order $lcm(6, 5) = 30$ and the sign being $(-1)^{6-1}(-1)^{5-1} = -1$

iii) the order is $lcm(2,2,4,2) = 4$ and the sign is $(-1)^{2-1}(-1)^{2-1}(-1)^{4-1}(-1)^{2-1} = 1$

iv) the permutation above is equivalent to id and thus means it has an order of 1 as well as a sign of 1

4.2:6

The order of the first one is 5, the second is $lcm(2,3) = 6$, the third is $lcm(2,2) = 2$

4.2:9

$A(4)$ has $4!/2 = 12$ elements. We can consider cycles that are only of odd length as well as cycles that have two transpositions The permutations of $A(4)$ and their order are

$Permutation$	$Order$
id	1
(123)	3
(124)	3
(132)	3
(134)	3
(142)	3
(143)	3
(234)	3
(243)	3
(12)(34)	2
(13)(24)	2
(14)(23)	2

4.2:13

All permutations of the puzzle is in $S(16)$. Since all possible moves are due to transpositions with 16 the puzzle must be of the permutation $\pi = (16, x_1)...(16, x_k)$ where $1 \leq x_j < 16$ and $\pi(16) = 16$. But for this to happen, we must have $k = 2n$ since every transposition up will require one down and every transposition left would require one right. That would mean $sgn(\pi) = (-1)^{2n} = 1$. Since the end permutation is a composition of disjoint transpositions $\pi = (1+0, 15-0)...(1+j, 15-j)...(1+14, 15-14)$. Since this is 15 transpositions, we thus have $sgn(\pi) = -1$ a contradiction. Therefore such a permutation is not possible.