PHY 300 Coupled Oscillators: Lab 1

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Introduction

In this lab we study how two masses coupled together using springs oscillate. The system was driven at a range of frequencies near the frequency of the normal modes and the amplitude was then recorded for the differing frequencies. The forces equations governing this motion is

$$F_1 = -kx_1 - k(x_1 - x_2)$$
$$F_2 = -kx_2 - k(x_2 - x_1)$$

Thus the differential equation is

$$m\frac{d^2x_1}{dt^2} = -2kx_1 - kx_2$$

$$m\frac{d^2x_2}{dt^2} = -2kx_2 - kx_1$$

From here we can obtain the frequencies for the two normal modes of

$$\omega_1 = \sqrt{\frac{k}{m}}$$

$$\omega_2 = \sqrt{\frac{3k}{m}}$$

ω_1 corresponds to the frequency of when the two masses are in phase and have the same amplitude. ω_2 corresponds to the frequency of when the two masses have the same magnitude but opposite amplitudes and are in phase.

By scanning through the frequencies near resonance, we can determine γ by taking the width of the frequencies at half the max amplitude. This is because the amplitude is linearly proportional to power, so half the amplitude is the same as half the power. Using γ we have $Q = \omega_1/\gamma_1 = \omega_2/\gamma_2$ where γ_1 and γ_2 are the damping terms for their respective normal modes.

Methods/Measurements

In this lab we had two masses connected vertically by springs. All springs were the same and we had a spring from the bottom mass to a driver, one between the two masses, and another from the top mass to a fixed point.

The first step was to measure ω_1 aka the frequency of the first normal mode. This was done by displacing the masses by the same amplitude in the same direction and graphing it's movement. This resulted in $\omega_1 = 10.23 rad/s$ and $f_1 = 2\pi/10.23 = 1.643 Hz$. For the frequency of the second normal mode ω_2, we displaced the two masses the same amount but in opposite directions and graphed it's motion. This resulted in a frequency of $\omega_2 = 17.944 rad/s$ and $2.856 Hz$. Then we applied a driving force at frequencies close to resonance for the two normal modes, the frequencies being, $[f_1 \pm 0.2Hz, f_1 \pm 0.15Hz, f_1 \pm 0.1Hz, f_1 \pm 0.08Hz, f_1 \pm 0.06Hz, f_1 \pm 0.05Hz, f_1 \pm 0.04Hz, f_1 \pm 0.03Hz, f_1 \pm 0.02Hz, f_1 \pm 0.01Hz, f_1]$ for the first normal mode. For the second normal mode we have $[f_2 \pm 0.2Hz, f_2 \pm 0.15Hz, f_2 \pm 0.1Hz, f_2 \pm 0.08Hz, f_2 \pm 0.06Hz, f_2 \pm 0.05Hz, f_2 \pm 0.04Hz, f_2 \pm 0.03Hz, f_2 \pm 0.02Hz, f_2 \pm 0.01Hz, f_2]$. Along with $(f_1 + f_2/2)$ We drive the system at these frequencies for around 30 seconds each before taking measurements in order for the transient to die out and get a more accurate amplitude for the steady state. We measured the amplitude at $(f_1 + f_2/2) = 2.25Hz$ of $0.0016m$

Amplitude around f_1

$Frequency\ (Hz)$	$Uncertainty(Hz)$	$Amplitude(m)$	$Uncertainty(m)$
1.44	±0.01	0.0059	±0.00001
1.49	±0.01	0.0019	±0.00001
1.54	±0.01	0.0053	±0.00001
1.56	±0.01	0.0073	±0.00001
1.58	±0.01	0.0062	±0.00001
1.59	±0.01	0.0091	±0.00001
1.6	±0.01	0.0091	±0.00001
1.61	±0.01	0.0081	±0.00001
1.62	±0.01	0.0078	±0.00001
1.63	±0.01	0.0293	±0.00001
1.64	±0.01	0.0325	±0.00001
1.65	±0.01	0.0517	±0.00001
1.66	±0.01	0.052	±0.00001
1.67	±0.01	0.0456	±0.00001
1.68	±0.01	0.0349	±0.00001
1.69	±0.01	0.0176	±0.00001
1.7	±0.01	0.012	±0.00001
1.72	±0.01	0.0108	±0.00001
1.74	±0.01	0.0081	±0.00001
1.79	±0.01	0.0064	±0.00001
1.84	±0.01	0.0051	±0.00001

Amplitude around f_2

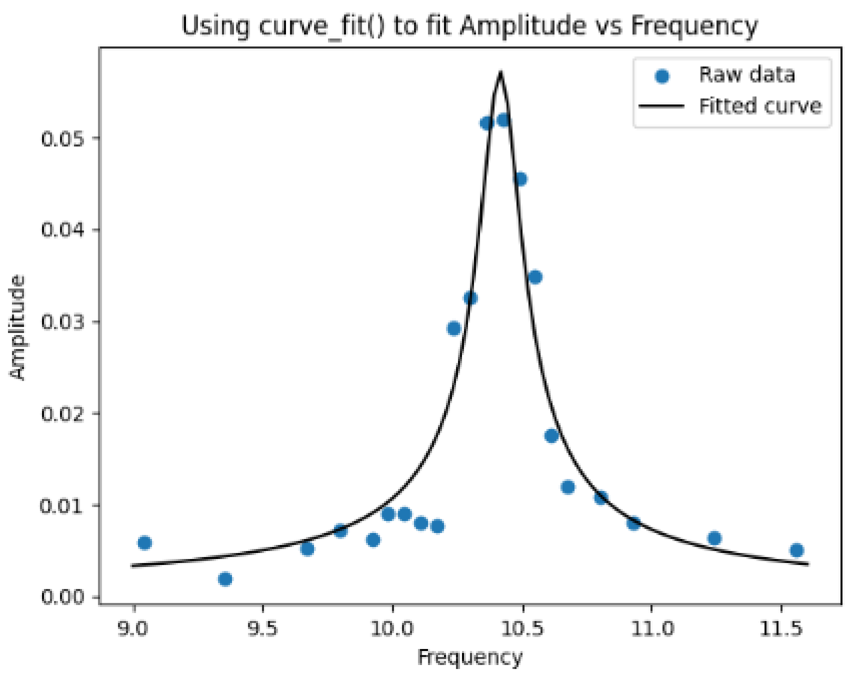
$Frequency\ (Hz)$	$Uncertainty(Hz)$	$Amplitude(m)$	$Uncertainty(m)$
2.65	±0.01	0.002	±0.00001
2.7	±0.01	0.0015	±0.00001
2.75	±0.01	0.0043	±0.00001
2.77	±0.01	0.005	±0.00001
2.79	±0.01	0.0066	±0.00001
2.8	±0.01	0.0082	±0.00001
2.81	±0.01	0.0084	±0.00001
2.82	±0.01	0.0098	±0.00001
2.83	±0.01	0.011	±0.00001
2.84	±0.01	0.04	±0.00001
2.85	±0.01	0.0499	±0.00001
2.86	±0.01	0.039	±0.00001
2.87	±0.01	0.0164	±0.00001
2.88	±0.01	0.0123	±0.00001
2.89	±0.01	0.0112	±0.00001
2.9	±0.01	0.01	±0.00001
2.91	±0.01	0.009	±0.00001
2.93	±0.01	0.0078	±0.00001
2.95	±0.01	0.0073	±0.00001
3	±0.01	0.0061	±0.00001
3.05	±0.01	0.0053	±0.00001

Analysis/Discussion

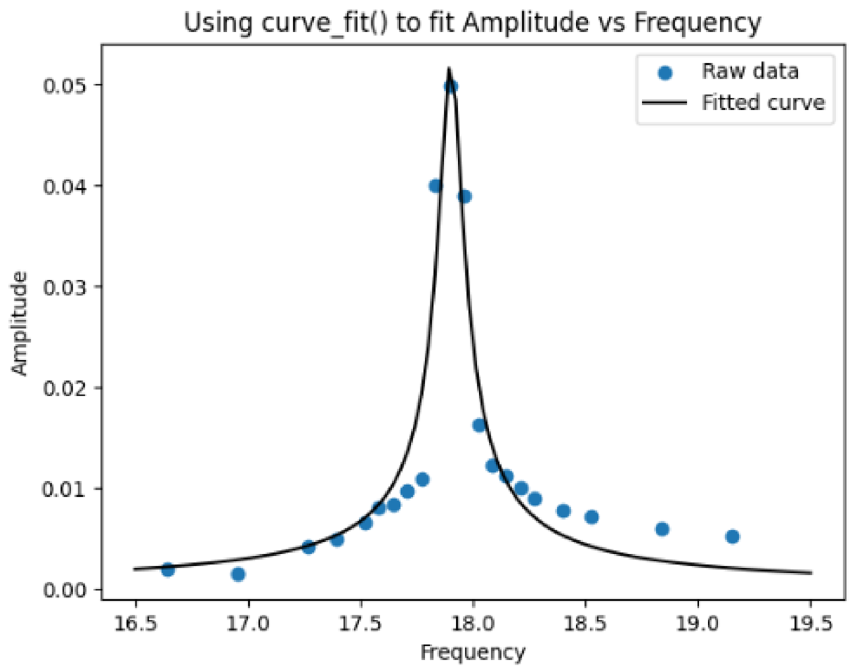
The frequencies that we got for the two normal modes from the peaks of the below graph should be in a ratio of $\omega_2 = \sqrt{3}\omega_1$. So since we obtained $\omega_1 = 10.4 rad/s$, we should expect $\omega_1 = \sqrt{3} * 10.4 rad/s = 18.01 rad/s$ our experimental value was $17.9 rad/s$. So we have a percent error of 0.6%

We fit a curve to the data that we got for the amplitudes around each of the normal modes.

For the first normal mode the curve we have is



For the second normal mode the curve we have is



From these graphs we can observe that at half the amplitude we have a width of 0.2 for ω_1 and 0.3 for ω_2. These values are the corresponding γ values for the two normal modes. Thus we have

$$Q = \frac{\omega_1}{\gamma_1} = 51.15$$

and

$$Q = \frac{\omega_2}{\gamma_2} = 59.8$$

The two resonance frequency we got from the peak of the graphs were $10.4 rad/s$ and $17.9 rad/s$. These values have only a 1.6% error and 0.2% error respectively compared to their natural frequencies at the normal mode. This is to be expected as the natural frequency is where the masses want to oscillate.

Some successes of the lab were that we were able to get all the data necessary. But we did not have a lot of time to wait for the transient to die down as much as we would like. This may have affected the final measurement. There was also some horizontal movements in the masses which may have affected the measuring of the distance that was traveled by the spring.

Conclusion

The measurements that were performed agreed with what we expected. The frequencies of the two normal modes were indeed in a ratio of $\sqrt{3}$ as expected. The error in the expected and measured was only that of 0.6%. The γ values for the respective normal modes results in a $Q = 51.15$ and $Q = 59.8$ which is very similar as is to be expected for the value of Q. The resonant frequencies were also similar to their respective natural frequencies as is also to be expected