Inverse Kinematics of the 6DOF C12XL Robotic Arm

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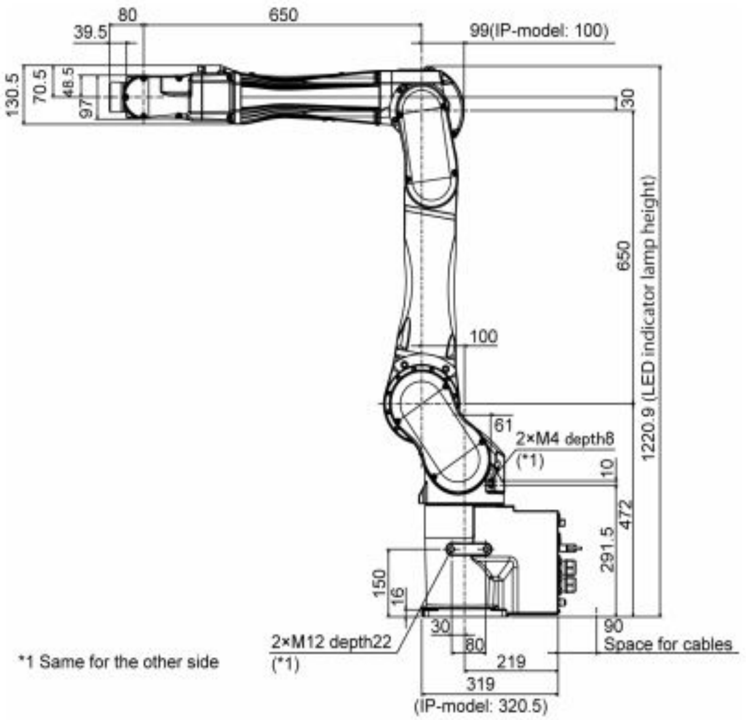
Introduction/Background Information

Robotic manipulators have been widely used in the industry performing repetitive tasks. Our work investigated the inverse engineering of the C12XL robot. The performed analyses and derivations help for the design of other robotic manipulators of similar kind.

Usage of DH convention to assign frames and create a DH table
(Scaled down by a factor of 100)

$Frames_i$	(Joint Variable) θ_i	α_i	a_i	d_i
1	q1	pi/2	1	2.672
2	q2	0	6.5	0
3	q3	-pi/2	0.3	0
4	q4	pi/2	0	-6.50
5	q5	-pi/2	0	0
6	q6	pi	0	-0.8

DH variables allow for joint axis to be related to one another, giving a simplified description of a robotic arm.



Dimensions of the C12XL Robotic Arm (Unit: mm)

The inverse kinematics of a robotic arm is used to determine the joint variables that control the movement of each joint in a robotic arm. This makes it possible to command the robot's end effector to reach a desired position and orientation in space.

Materials and Methods

Materials:

- Matlab R2021a
- Peter Corke’s Matlab Robotic Toolbox
- Epson C12XL specification sheet

Desired orientation and position of the end effector

$$EOP = \begin{bmatrix} nx & ox & ax & px \\ ny & oy & ay & py \\ nz & oz & az & pz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$EO = \begin{bmatrix} nx & ox & ax \\ ny & oy & ay \\ nz & oz & az \end{bmatrix} \quad EP = \begin{bmatrix} px \\ py \\ pz \end{bmatrix}$$

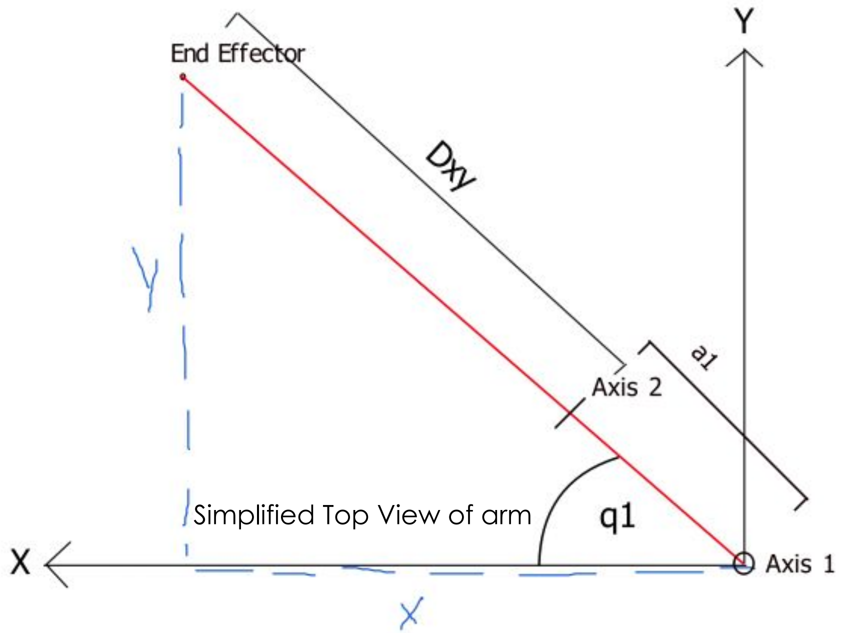
$$EO*[0\ 0\ 0.8]^T=[d_{x6}\ d_{y6}\ d_{z6}]^T$$
$$[p_x\ p_y\ p_z]^T - [d_{x6}\ d_{y6}\ d_{z6}]^T = [x\ y\ z]^T$$

To obtain the first three joint variables, the desired orientation of the end effector was multiplied with the final link’s vector then the XYZ components of the modified link vector was subtracted from the desired position.

Materials and Methods

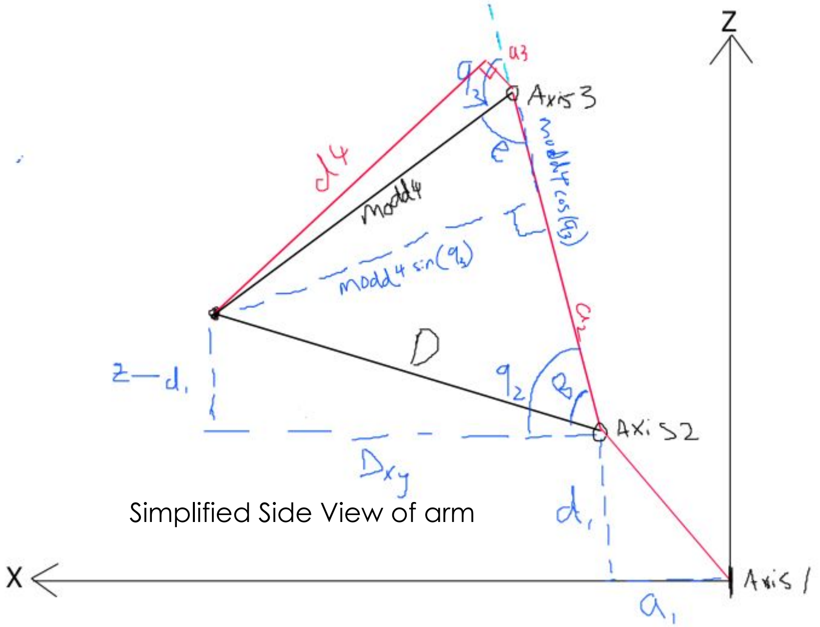
The models shown below have the last link subtracted from its position and solves for the position resulted from the subtraction. The wrist is also removed to solve for the position of the end effector, which is based on the first three joints of the robotic arm.

Red - Simplified links of the robot



Simplified Top View of arm

Due to the first joint rotating on the Z-axis. Second, third joints rotating on axis parallel to one another and perpendicular to the Z-axis the first joint variable is based on the angle created at q1 when looking at the robot from top down.



Simplified Side View of arm

The third axis would determine the length of D. The length of D is based on the desired position and is found by subtracting the d_1 and a_1 component of the link between Axis 2 and 1 from the desired position. Axis 2 is then used to alter the Z value of the end effector.

Materials and Methods

Solving for Positioning

modd4 is the hypotenuse of a right triangle with sides a3 and d4. This is used instead of d4 because of the a3 offset

modd4 = sqrt($a_3{}^2$+$d_4{}^2$)

D = sqrt($D_{xy}{}^2$+(z-d1)^2)

Law of Cosine: $c^2 = a^2+b^2-2ab\cos(C)$

cos(C) = c^2-a^2-b^2/-2ab

-cos(C) = c^2-a^2-b^2/2ab

-cos(e) = D^2-modd4^2-$a_2{}^2$/(2$a_2$modd4)

sin(e) = 1-(-cos(e))^2

-cos(e)=cos(q3)

sin(e) = sin(q3)

q3 = pi/2* + tan^{-1}(sin(e)/-cos(e)) - sin(a3/modd4)*

B = tan^{-1}((modd4*sin(q3))/(a_2+modd4*cos(q3)))

q2=tan^{-1}(z-d1/D_{xy}) - tan^{-1}((modd4*sin(q3))/(a_2+modd4*cos(q3)))

q2 = tan^{-1}(z-d1/D_{xy})-B

Second Set of Solutions for Positioning of End Effector

q1 = tan^{-1}(y/x)

Law of Cosine: $c^2 = a^2+b^2-2ab\cos(C)$

cos(C) = a^2+b^2-c^2 /2ab

cos(e) =modd4^2+$a_2{}^2$-D^2/(2$a_2$modd4)

sin(e) = 1-cos(e)^2

q3 = pi - tan^{-1}(sin(e)/cos(e)) - sin(a3/modd4)+pi/2*

q2 = tan^{-1}(z-d1/D_{xy})-B

To solve for positioning, the law of cosine was used to find joint variables 2 and 3. Joint variable 1 was solved by using the pythagorean theorem

Finding End Effector Orientation

RA = -(q3+q2) -pi
mq1 = -q1+pi
Zrot = q4
Yrot = -q5

$$Ry(RA) = \begin{bmatrix} cosRA & 0 & sinRA \\ 1 & 1 & 1 \\ -sinRA & 0 & cosRA \end{bmatrix}$$

$$Rz(mq1) = \begin{bmatrix} \cos(mq1) & -\sin(mq1) & 0 \\ \sin(mq1) & \cos(mq1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(RA) * \cos(mq1) & \cos(RA) * -\sin(mq1) & \sin(RA) \\ \sin(mq1) & \cos(mq1) & 0 \\ -\sin(RA) * \cos(mq1) & -\sin(RA) * -\sin(mq1) & \cos(RA) \end{bmatrix} = {}^0_3R$$

$$\begin{bmatrix} \cos(RA) * \cos(mq1) & \cos(RA) * -\sin(mq1) & \sin(RA) \\ \sin(mq1) & \cos(mq1) & 0 \\ -\sin(RA) * \cos(mq1) & -\sin(RA) * -\sin(mq1) & \cos(RA) \end{bmatrix}^T = InverseR$$

$$\begin{bmatrix} \cos(Yrot) * \cos(Zrot) & \cos(Yrot) * -\sin(Zrot) & \sin(Yrot) \\ \sin(Zrot) & \cos(Zrot) & 0 \\ -\sin(Yrot) * \cos(Zrot) & -\sin(Yrot) * -\sin(Zrot) & \cos(Yrot) \end{bmatrix} = Ry(Yrot)\,Rz(Zrot)$$

$${}^0_5R = Ry(Yrot)\,Rz(Zrot)\,{}^0_3R$$

EO_{F5} = ${}^0_5$R*EO

q6 = -tan^{-1}(EO_{F521}/EO_{F511})

To find the orientation of $[d_{x6}\ d_{y6}\ d_{z6}]^T$ (last link) relative to the 3rd frame.

$[d_{x6F3}\ d_{y6F3}\ d_{z6F3}]^T={}^0_3R*[d_{x6}\ d_{y6}\ d_{z6}]^T$

d_{xyF3}= sqrt($d_{x6F3}{}^2$ +$d_{y6F3}{}^2$)

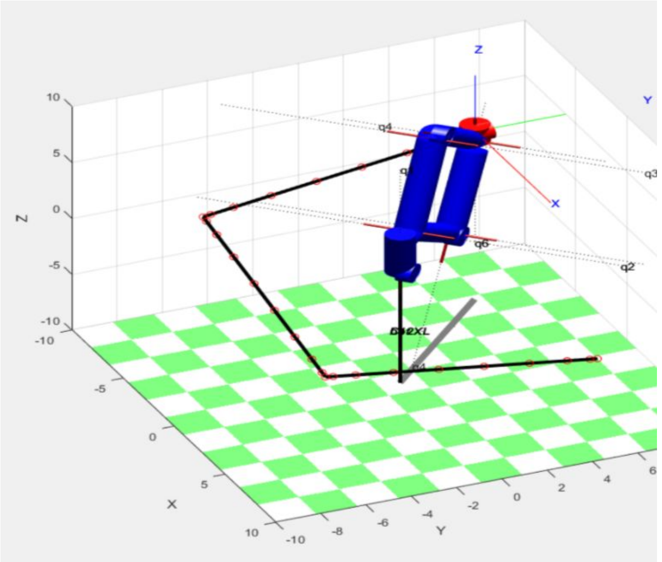
Because the z axis of the final frame is aligned with vector $[d_{x6}\ d_{y6}\ d_{z6}]^T$,

q5=tan^{-1}(d_{xyF3}/d_{z6F3})

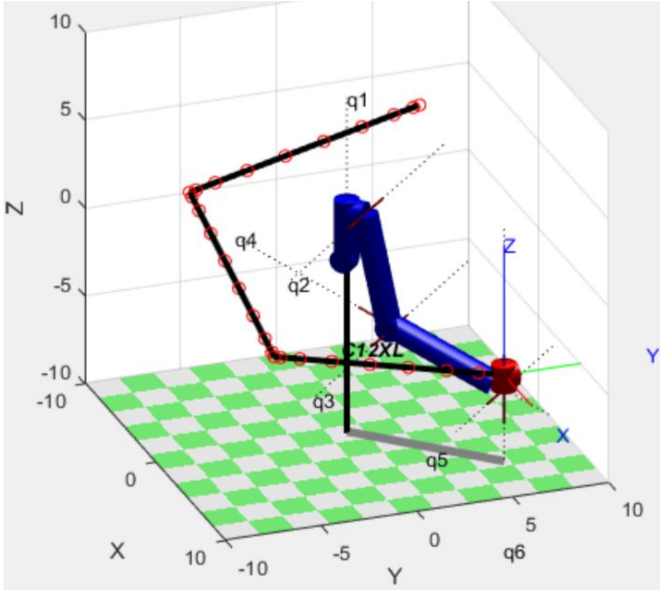
q4=-tan^{-1}(d_y/d_x)

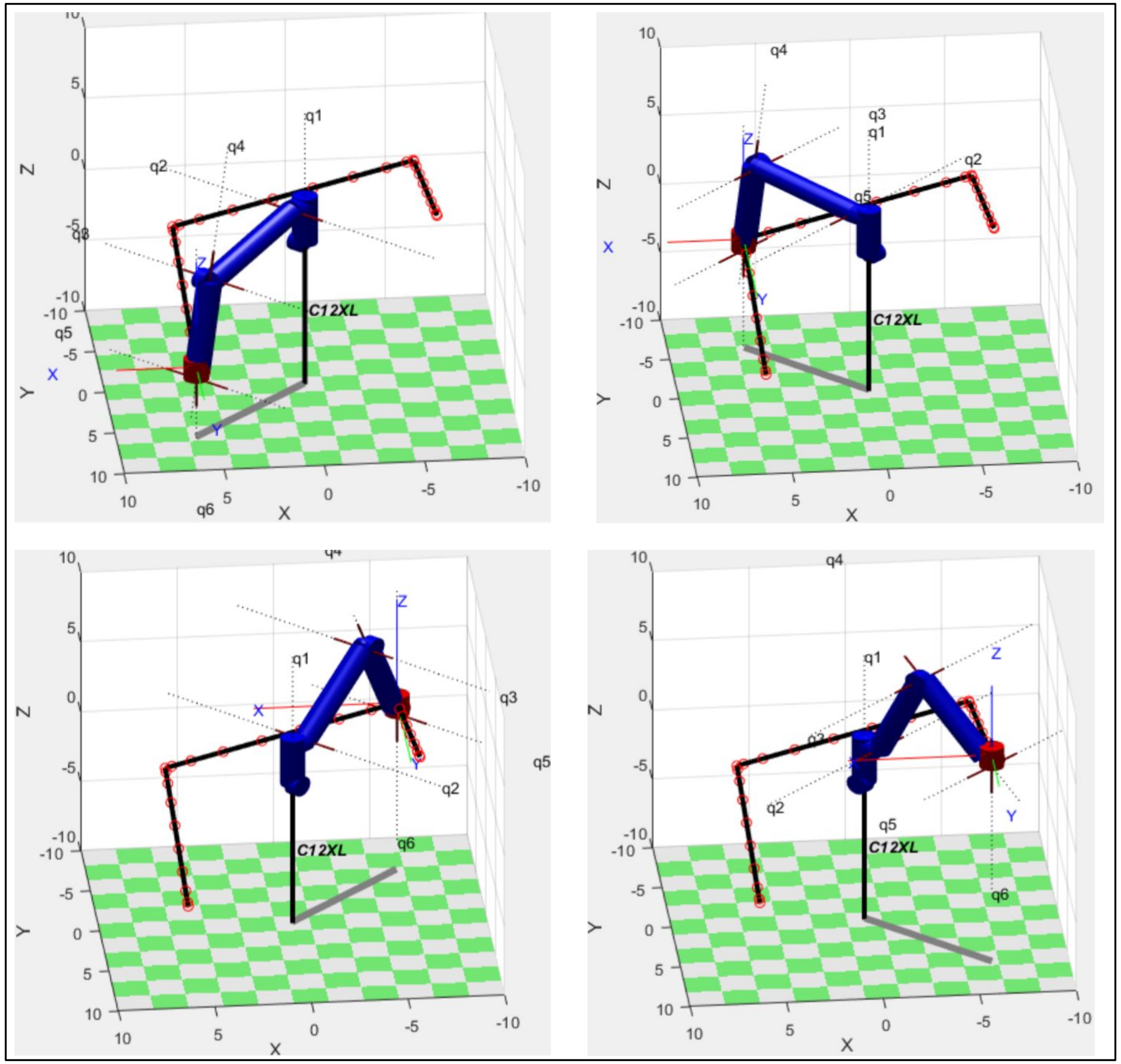
Simulation Results

Two sets of solutions of joint variables for the same positioning



Same end effector position and orientation as the images to the left but using a different set of solutions. In this case the links are likely bumping into one another or the actual robot can't rotate this far down which is why many sets of solutions are necessary for positioning and orienting the end effector





Given the desired position and orientation of the end effector, (in this case the desired orientation is a 3x3 identity matrix) the inverse kinematics can find the joint variables that allow the arm to position and orient the end effector to what is desired.

Conclusion

The Inverse kinematic of the C12XL Robotic arm is found by using a combination of trigonometry and rotation matrices. There are also two sets of solutions found for the robotic arm throughout this research. There is likely still many more sets of solutions for the C12XL robotic arm.

Acknowledgement

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Citations

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