

12.1

The Ideal Gas Law

Section Preview/ Specific Expectations

In this section, you will

- **state** Avogadro's hypothesis, and **explain** how it contributes to our understanding of the reactions of gases
- **describe** the quantitative relationships that exist among pressure, volume, temperature, and amount of substance, based on the ideal gas law
- **solve** quantitative problems involving the ideal gas law
- **use** and **convert** appropriate units to express pressure and temperature
- **communicate** your understanding of the following terms: *law of combining volumes, law of multiple proportions, Avogadro's hypothesis, molar volume, ideal gas law*

Near the beginning of the nineteenth century, Joseph Gay-Lussac experimented with the volumes of gases. He found that adding two volumes of gas to one volume of gas produced only two volumes of gas. Puzzled, Gay-Lussac tried adding three volumes of gas to one volume. The result was still two! When he tried adding one volume of gas to a second volume of gas, again the result was two. What was going on?

In England, around the same time, John Dalton studied the masses of compounds as they reacted to produce products. After Dalton read about the similar work of other scientists, such as Lavoisier and the British scientist Joseph Priestley, he contacted Gay-Lussac. He described his results and hypotheses to Gay-Lussac. In 1808, both men published their theories. After examining the theories of Dalton and Gay-Lussac, an Italian scientist named Amedeo Avogadro formulated a hypothesis that combined their theories.



Figure The information that was shared by these three scientists led to the gas laws that we use today.

The Molar Volume of Gases

Gay-Lussac measured the *volumes* of gases before and after a reaction. His research led him to devise the **law of combining volumes**: When gases react, the volumes of the reactants and the products, measured at equal temperatures and pressures, are always in whole number ratios. For example, 2 volumes of hydrogen gas react with 1 volume of oxygen gas to produce 2 volumes of water vapour.

John Dalton examined the *masses* of compounds before and after a reaction. Dalton's research led him to propose the **law of multiple proportions**: The masses of the elements that combine can be expressed in small whole number ratios.

By combining these ideas, Avogadro related the *volume* of a gas to the *amount* that is present (calculated from the mass). Avogadro divided Dalton's mass ratios by the molar masses of the elements to obtain the mole ratios. He realized that these mole ratios were the same as the volume ratios that Gay-Lussac had obtained. For example, 1 L of hydrogen gas reacts with 1 L of chlorine gas. Avogadro decided that there must be the same number of molecules in each litre of gas. Thus, **Avogadro's hypothesis** was formulated: Equal volumes of all ideal gases at the same temperature and pressure contain the same number of molecules.

Figures 12.2, 12.3, and 12.4 show the three reactions that produced Gay-Lussac's confusing observations. You can see that the mole ratios are the same as the volume ratios. Today our knowledge of atoms and molecules helps us understand Gay-Lussac's results. We know that gases are made of molecules that may contain more than one atom.

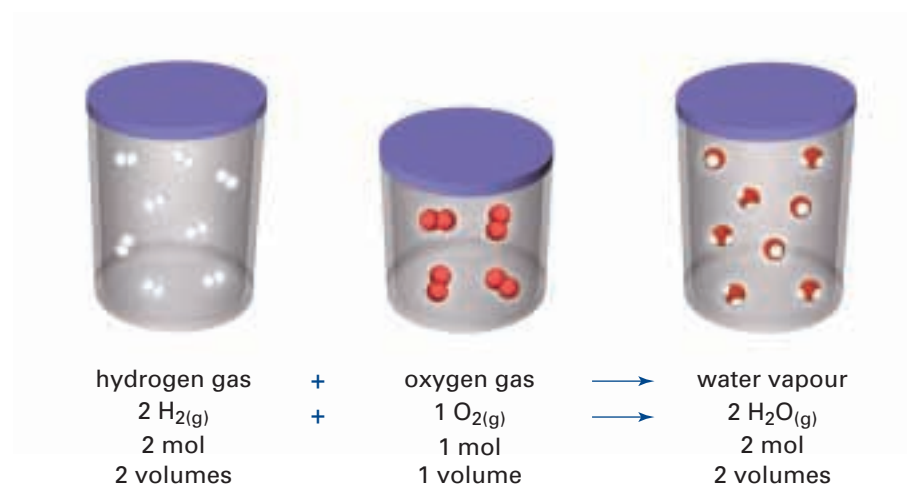


Figure 12.2 Hydrogen and oxygen gases combine to form water vapour.

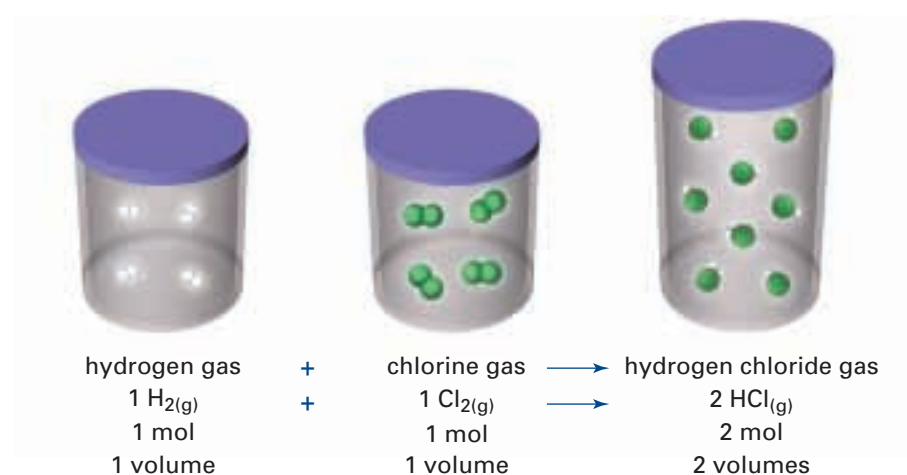


Figure 12.3 Hydrogen and chlorine gases combine to form hydrogen chloride gas.

Language

LINK

The word *symposium* comes from the ancient Greeks. A symposium was a gathering of intellectuals to drink, feast, and talk. All sorts of new ideas arose from these meetings. The participants took away the new ideas to work on them further. Then they brought their findings back to the next symposium.

Many scientific laws are given the name of one person. It is important to remember, however, that most laws are the culmination of many scientists' work over a long time. Science would never move forward without ideas being shared.

Look up the word "symposium" in a dictionary. What meaning does it have today? Do a quick Internet search of the word "symposium." What modern symposiums can you find? What subjects do they cover?

mind STRETCH

As a class, hold a symposium about gas balloons. Research hot air balloons and helium balloons. Also research any other information about balloons, the history of balloons, and balloon travel that interests you. Prepare papers and posters to share your information.

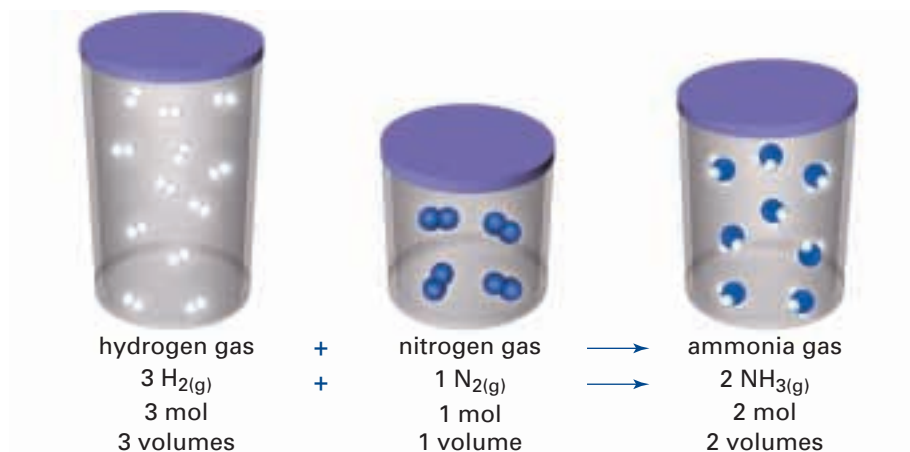


Figure 12.4 Hydrogen and nitrogen combine to form ammonia gas.

Avogadro's hypothesis can be written as a mathematical law, shown here.

Avogadro's Law

Avogadro's law gives us a mathematical relationship between the volume of a gas (V) and the number of moles of gas present (n).

$$n \propto V \quad \text{or} \quad n = kV \quad \text{or} \quad \frac{n_1}{V_1} = \frac{n_2}{V_2}$$

where n = number of moles

V = volume

k = a constant

CHECKPOINT

The next Sample Problem involves the combined gas law. Review this law in section 11.4 before continuing.

Based on Avogadro's law, one mole of a gas occupies the same volume as one mole of another gas at the same temperature and pressure. The molar volume of a gas is the space that is occupied by one mole of the gas. **Molar volume** is measured in units of L/mol. You can find the molar volume of a gas by dividing its volume by the number of moles that are present ($\frac{V}{n}$). Look at the Sample Problem below to find out how to calculate molar volume. Then complete the following Thought Lab to find the molar volumes of carbon dioxide gas, oxygen gas, and methane gas at STP.

Sample Problem

The Molar Volume of Nitrogen

Problem

A resealable 1.30 L container has a mass of 4.73 g. Nitrogen gas, $\text{N}_{2(\text{g})}$, is added to the container until the pressure is 98.0 kPa at 22.0°C. Together, the container and the gas have a mass of 6.18 g. Calculate the molar volume of nitrogen gas at STP.

What Is Required?

You need to find the volume of one mole of nitrogen (the molar volume) at STP.

Continued ...

What Is Given?

Set out all the data in a table like the one shown below.

Situation 1: in the container	Situation 2: at STP
$P_i = 98.0 \text{ kPa}$	$P_f = 101.3 \text{ kPa}$
$V_i = 1.30 \text{ L}$	$V_f = ?$
$T_i = 22.0^\circ\text{C}$, or 295 K	$T_f = 0^\circ\text{C}$, or 273 K
$m_i = 6.18 \text{ g} - 4.73 \text{ g} = 1.45 \text{ g}$ (Subtract the mass of the container.)	$m_f = 1.45 \text{ g}$ (The mass remains the same.)
$n_i = ?$	$n_f = n_i = ?$

Plan Your Strategy

Algebraic method

Step 1 Calculate the number of moles of nitrogen gas (n_i) by dividing the mass of the nitrogen in the container by the molar mass of nitrogen gas (28.02 g/mol).

$$n = \frac{m}{M}$$

Step 2 Use the combined gas law from Chapter 11 to find the volume of nitrogen at STP (V_f).

Step 3 Use the volume (V_f) and the number of moles ($n_f = n_i$) to find the molar volume ($\frac{V}{n}$). The molar volume is the volume of one mole of gas.

$$\text{Molar volumes} = \frac{V}{n}$$

Ratio method

Step 1 Calculate the number of moles of nitrogen gas (n_i) by dividing the mass of the nitrogen in the container by the molar mass of nitrogen gas (28.02 g/mol).

$$n = \frac{m}{M}$$

Step 2 Since the pressure increases from 98.0 kPa to 101.3 kPa , the volume will decrease. Multiply the initial volume by a pressure ratio that is less than 1. Since the temperature decreases from 295 K to 273 K , the volume will decrease further. Multiply the initial volume by a temperature ratio that is less than 1.

Step 3 Since there is less than 1 mol of nitrogen gas present, the volume of 1 mol of nitrogen gas (the molar volume) will be greater than the volume you calculated in step 2. To find the molar volume, multiply by a mole ratio that is greater than 1.

PROBLEM TIP

In the Sample Problem, you will see two different methods of solving the problem: the algebraic method and the ratio method. Choose the method you prefer to solve this type of problem.

Act on Your Strategy

Algebraic method

Step 1 Calculate the number of moles.

$$\begin{aligned} n &= \frac{m}{M} \\ &= \frac{1.45 \text{ g}}{28.02 \text{ g/mol}} \\ &= 0.0517 \text{ mol} \end{aligned}$$

Step 2 Find the volume of nitrogen at STP.

$$\begin{aligned} \frac{P_i V_i}{T_i} &= \frac{P_f V_f}{T_f} \\ \therefore V_f &= \frac{P_i V_i T_f}{T_i P_f} \\ &= \frac{98.0 \text{ kPa} \times 1.30 \text{ L} \times 273 \text{ K}}{295 \text{ K} \times 101.3 \text{ kPa}} \\ &= 1.16 \text{ L} \end{aligned}$$

Step 3 Find the molar volume.

$$\begin{aligned} \text{Molar volume} &= V/n \\ &= \frac{1.16 \text{ L}}{0.0517 \text{ mol}} \\ &= 22.4 \text{ L/mol} \end{aligned}$$

Therefore, the molar volume of nitrogen at STP is 22.4 L/mol.

Ratio method

Step 1 Calculate the number of moles.

$$\begin{aligned} n &= \frac{m}{M} \\ &= \frac{1.45 \text{ g}}{28.02 \text{ g/mol}} \\ &= 0.0517 \text{ mol} \end{aligned}$$

Step 2 V_f is the final volume of the nitrogen gas.

$$\begin{aligned} V_f &= 1.30 \text{ L} \times \frac{98.0 \text{ kPa}}{101.3 \text{ kPa}} \times \frac{273 \text{ K}}{295 \text{ K}} \\ &= 1.16 \text{ L} \end{aligned}$$

Step 3 V_m is the volume of 1 mol of the nitrogen gas.

$$\begin{aligned} V_m &= 1.16 \text{ L} \times \frac{1.00 \text{ mol}}{0.0517 \text{ mol}} \\ &= 22.4 \text{ L} \end{aligned}$$

The molar volume is equal to the volume V_m divided by 1.00 mol. Thus the molar volume is 22.4 L/mol.

Check Your Solution

The answer is expressed in the correct units. It agrees with the accepted value. There are three significant digits in the answer. This is consistent with the least number of significant digits in the question.

Practice Problems

- At 19°C and 100 kPa, 0.021 mol of oxygen gas, $\text{O}_{2(g)}$, occupy a volume of 0.50 L. What is the molar volume of oxygen gas at this temperature and pressure?
- What is the molar volume of hydrogen gas, $\text{H}_{2(g)}$, at 255°C and 102 kPa, if a 1.09 L volume of the gas has a mass of 0.0513 g?
- A sample of helium gas, $\text{He}_{(g)}$, has a mass of 11.28 g. At STP, the sample has a volume of 63.2 L. What is the molar volume of this gas at 32.2°C and 98.1 kPa?
- In the Sample Problem, you discovered that the molar volume of nitrogen gas is 22.4 L at STP.
 - How many moles of nitrogen are present in 10.0 L at STP?
 - What is the mass of this gas sample?

ThoughtLab



Molar Volume of Gases

Two students decided to calculate the molar volumes of carbon dioxide, oxygen, and methane gas. First they measured the mass of an empty 150 mL syringe under vacuum conditions. This ensured that the syringe did not contain any air. Next they filled the syringe with 150 mL of carbon dioxide gas. They measured and recorded the mass of the syringe plus the gas. The students repeated their procedure for oxygen gas and for methane gas.

Finally, the students found the temperature of the room to be 23.0°C (296 K). They found the pressure to be 98.7 kPa. They took these values to be the temperature and pressure of the three gases. The students' results are given in the table.

Three Gases at 296 K and 98.7 kPa

Gas	carbon dioxide	oxygen	methane
Volume of gas (V)	150 mL	150 mL	150 mL
Mass of empty syringe	25.08 g	25.08 g	25.08 g
Mass of gas + syringe	25.34 g	25.27 g	25.18 g
Mass of gas (m)			
Molar mass (M)			
Number of moles of gas ($n = m/M$)			
Volume of gas at STP (273 K and 101.3 kPa)			
Molar volume at STP ($MV = V/n$)			

Procedure

- Copy the table into your notebook.
- Calculate the molar volume of carbon dioxide gas at the given temperature and pressure, and at STP. Write your calculations and answers in the table.
- Do the same calculations for oxygen and methane gas. Write your calculations and answers in the table.

Analysis

- Compare the three molar volumes at STP. What do you observe?
- The accepted molar volume of a gas at STP is 22.4 L/mol. Use this value to calculate the percent error in your experimental data for each gas.

In the Thought Lab, you found that the molar volume of one gas is roughly the same as the molar volume of another gas at the same temperature and pressure. In fact, the molar volume of an *ideal* gas at STP is 22.4 L/mol. Figure 12.5 shows a balloon with a volume of 22.4 L compared to some common objects. This is a fairly large volume of gas. For example, a basketball has a volume of only 7.5 L.



Figure 12.5 One mole of any gas at STP occupies 22.4 L (22.4 dm³). How large is 22.4 L? The other objects are shown for comparison.

CHECKPOINT

What are the temperature and pressure at STP? Go back to section 11.4 to remind yourself.

In Chapter 11, you learned that temperature, pressure, and volume are related. Based on Avogadro's law, the number of moles is related to the temperature, pressure, and volume of a gas. Therefore, Avogadro's law can be applied to solve gas problems involving moles and volume, when the temperature and pressure remain constant. Figure 12.6 explains the relationship among temperature, pressure, volume, and number of moles of a gas.

The following Sample Problems show you how to do gas calculations using Avogadro's law.

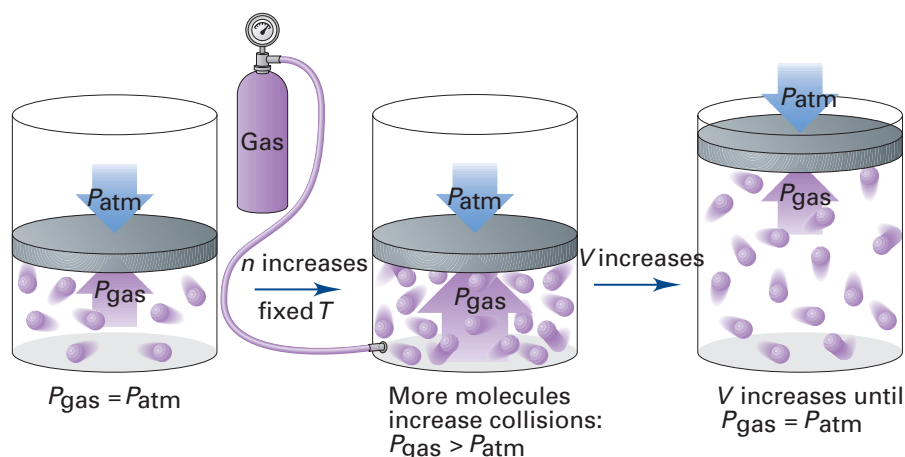


Figure 12.6 At a temperature (T), a given amount of a gas (n) produces a pressure (P). When more gas is added, the number of moles (n) increases. This results in more molecular collisions with the container walls, thus increasing the pressure of the gas. The volume (V) of the gas increases until the pressure is again equal to the pressure of the surroundings.

Sample Problem

Volumes of Gases

Problem

What is the volume of 3.0 mol of nitrous oxide, $\text{NO}_{2(g)}$, at STP?

What Is Required?

You need to find the volume of nitrous oxide at STP (V_f).

What Is Given?

You know that 1.00 mol of a gas occupies 22.4 L at STP.

$$\therefore n_i = 1.0 \text{ mol}$$

$$V_i = 22.4 \text{ L}$$

$$n_f = 3.0 \text{ mol of NO}_2$$

Plan Your Strategy

Algebraic method

Use Avogadro's law: $\frac{n_i}{V_i} = \frac{n_f}{V_f}$

Cross multiply to solve for V_f , the unknown volume of NO_2 .

Ratio method

There are 3 mol of nitrous oxide. Thus, the volume of nitrous oxide at STP must be larger than the volume of 1 mol of gas at STP. Multiply by a mole ratio that is greater than 1.

Act on Your Strategy

Algebraic method

$$\begin{aligned} V_f &= \frac{n_f V_i}{n_i} \\ &= \frac{3.0 \text{ mol} \times 22.4 \text{ L}}{1.00 \text{ mol}} \\ &= 67 \text{ L} \end{aligned}$$

Ratio method

$$\begin{aligned} V_f &= 22.4 \text{ L} \times \frac{3.0 \text{ mol}}{1.0 \text{ mol}} \\ &= 67 \text{ L} \end{aligned}$$

Therefore, there are 67 L of nitrous oxide.

Check Your Solution

The significant digits and the units are all correct.

The volume of nitrous oxide is three times the volume of 1 mol of gas at STP. This makes sense, since there are 3 mol of nitrous oxide.

PROBLEM TIP

In these Sample Problems, you will see two different methods of solving the problem: the algebraic method and the ratio method. Choose the method you prefer to solve this type of problem.

Sample Problem

Moles of Gas

Problem

Suppose that you have 44.8 L of methane gas at STP.

- (a) How many moles are present?
- (b) What is the mass (in g) of the gas?
- (c) How many molecules of gas are present?

What Is Required?

- (a) You need to calculate the number of moles.
- (b) You need to calculate the mass of the gas.
- (c) You need to calculate the number of molecules.

What Is Given?

The gas is at STP. Thus 1.00 mol of gas has a volume of 22.4 L. You know that one mole contains 6.02×10^{23} molecules. There are 44.8 L of gas.

Plan Your Strategy

Algebraic method

- (a) Use Avogadro's law. Solve for the number of moles by cross multiplying.
- (b) Multiply the number of moles (n) by the molar mass (M) to find the mass of the gas (m).

$$m = n \times M$$

- (c) Multiply the number of moles (n) by the Avogadro constant (6.02×10^{23}) to find the number of molecules.

$$\# \text{ of molecules} = n \times 6.02 \times 10^{23} \text{ molecules/mol}$$

Ratio method

- (a) The volume of the unknown gas is 44.8 L. Since the volume is greater than 22.4 L, there is more than 1 mol of gas. To find the unknown number of moles (n), multiply by a volume ratio that is greater than 1.
- (b) Multiply the number of moles (n) by the molar mass (M) to find the mass of the gas (m).

$$m = n \times M$$

- (c) Multiply the number of moles (n) by the Avogadro constant (6.02×10^{23}) to find the number of molecules.

$$\# \text{ of molecules} = n \times 6.02 \times 10^{23} \text{ molecules/mol}$$

Continued ...

Act on Your Strategy**Algebraic method**

$$(a) \frac{n_i}{V_i} = \frac{n_f}{V_f}$$

$$\begin{aligned} n_f &= \frac{n_i V_f}{V_i} \\ &= \frac{1.00 \text{ mol} \times 44.8 \cancel{\text{L}}}{22.4 \cancel{\text{L}}} \\ &= 2.00 \text{ mol} \end{aligned}$$

(b) Find the molar mass of methane, CH₄.

$$\begin{aligned} 1\text{C} &= 1 \times 12.01 \text{ g/mol} \\ 4\text{H} &= 4 \times 1.01 \text{ g/mol} \\ \hline M_{\text{CH}_4} &= 16.05 \text{ g/mol} \\ m &= n \times M \\ &= 2.00 \cancel{\text{mol}} \times 16.05 \text{ g}/\cancel{\text{mol}} \\ &= 32.1 \text{ g} \end{aligned}$$

$$\begin{aligned} (c) \text{ \# molecules} &= 2.00 \cancel{\text{mol}} \times 6.02 \times 10^{23} \frac{\text{molecules}}{\cancel{\text{mol}}} \\ &= 1.20 \times 10^{24} \text{ molecules} \end{aligned}$$

Ratio method

$$\begin{aligned} (a) \ n &= 1.00 \text{ mol} \times \frac{44.8 \cancel{\text{L}}}{22.4 \cancel{\text{L}}} \\ &= 2.00 \text{ mol} \end{aligned}$$

(b) Find the molar mass of methane, CH₄.

$$\begin{aligned} 1\text{C} &= 1 \times 12.01 \text{ g/mol} \\ 4\text{H} &= 4 \times 1.01 \text{ g/mol} \\ \hline M_{\text{CH}_4} &= 16.05 \text{ g/mol} \\ m &= n \times M \\ &= 2.00 \text{ mol} \times 16.05 \text{ g/mol} \\ &= 32.1 \text{ g} \end{aligned}$$

$$\begin{aligned} (c) \text{ \# molecules} &= 2.00 \cancel{\text{mol}} \times 6.02 \times 10^{23} \frac{\text{molecules}}{\cancel{\text{mol}}} \\ &= 1.20 \times 10^{24} \text{ molecules} \end{aligned}$$

Therefore, 2.00 mol of methane are present. The mass of the gas is 32.1 g. 1.20×10^{24} molecules are present.

Check Your Solution

The significant digits are correct.

The volume of methane is double the volume of 1 mol of gas.

It makes sense that 2 mol of methane are present. It also makes sense that the number of molecules present is double the Avogadro constant.

PROBLEM TIP

To solve many of these problems, try setting up a proportion and solving by cross multiplication.

Table 12.1 Molar Volume of Several Real Gases at STP

Gas	Molar volume (L/mol)
helium, He	22.398
neon, Ne	22.401
argon, Ar	22.410
hydrogen, H ₂	22.430
nitrogen, N ₂	22.413
oxygen, O ₂	22.414
carbon dioxide, CO ₂	22.414
ammonia, NH ₃	22.350

Practice Problems

- A balloon contains 2.0 L of helium gas at STP. How many moles of helium are present?
- How many moles of gas are present in 11.2 L at STP? How many molecules?
- What is the volume, at STP, of 3.45 mol of argon gas?
- A certain set of conditions allows 4.0 mol of gas to be held in a 70 L container. What volume do 6.0 mol of gas need under the same conditions of temperature and pressure?
- At STP, a container holds 14.01 g of nitrogen gas, 16.00 g of oxygen gas, 66.00 g of carbon dioxide gas, and 17.04 g of ammonia gas. What is the volume of the container?
- (a) What volume do 2.50 mol of oxygen occupy at STP?
(b) How many molecules are present in this volume of oxygen?
(c) How many oxygen atoms are present in this volume of oxygen?
- What volume do 2.00×10^{24} atoms of neon occupy at STP?

Volumes of Real Gases

You know that ideal gases have a volume of 22.4 L at STP. Do *real* gases have the same volume? The volumes of several real gases at STP are given in Table 12.1. All the volumes are very close to 22.4 L/mol, the molar volume of an ideal gas. Scientists have decided that 22.4 L/mol is an acceptable approximation for *any* gas at STP when using gas laws. Although the volumes are the same, one mole of a gas will have a different mass and density than one mole of another gas. (See Figure 12.7.)

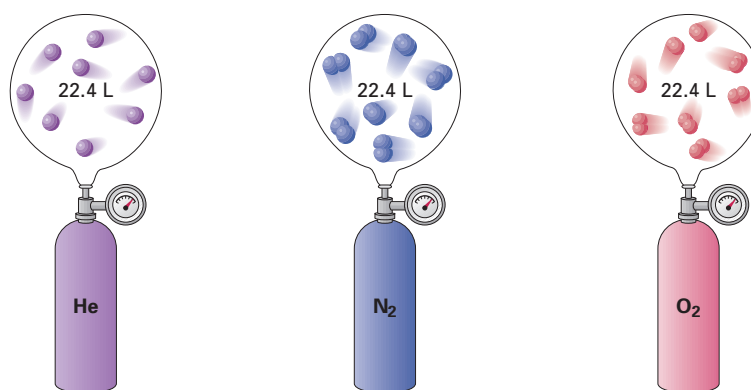


Figure 12.7 At STP, gases such as helium, nitrogen, and oxygen behave as ideal gases. They have a molar volume of 22.4 L.

$n = 1 \text{ mol}$	$n = 1 \text{ mol}$	$n = 1 \text{ mol}$
$P = 1 \text{ atm (760 torr)}$	$P = 1 \text{ atm (760 torr)}$	$P = 1 \text{ atm (760 torr)}$
$T = 0^\circ\text{C (273 K)}$	$T = 0^\circ\text{C (273 K)}$	$T = 0^\circ\text{C (273 K)}$
$V = 22.4 \text{ L}$	$V = 22.4 \text{ L}$	$V = 22.4 \text{ L}$
Number of gas particles $= 6.022 \times 10^{23}$	Number of gas particles $= 6.022 \times 10^{23}$	Number of gas particles $= 6.022 \times 10^{23}$
Mass = 4.003 g	Mass = 28.02 g	Mass = 32.00 g
$d = 0.179 \text{ g/L}$	$d = 1.25 \text{ g/L}$	$d = 1.43 \text{ g/L}$

A Deeper Look: Real Gas Deviations

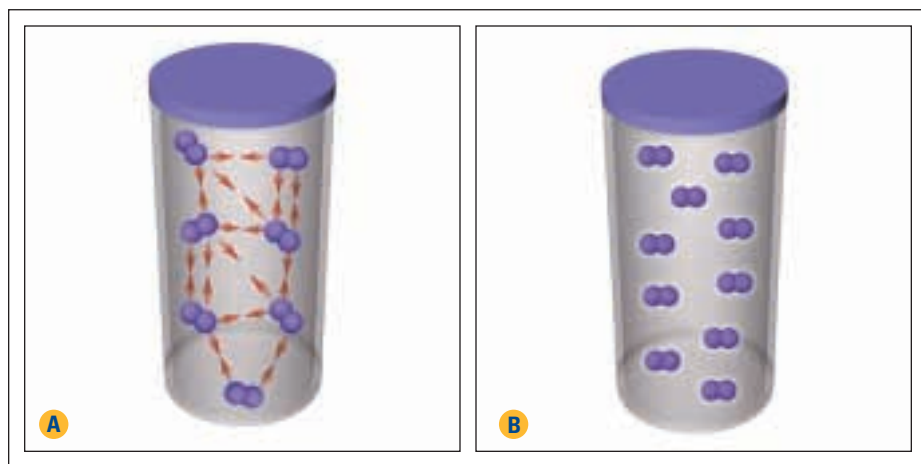
Do real gases *always* behave like ideal gases? At STP, you have seen that most real gases behave like ideal gases. At high pressures and/or low temperatures, however, gases no longer behave ideally. To understand why, recall the characteristics of an ideal gas, according to the kinetic molecular theory. (You learned this in Chapter 11.)

- Gas molecules have insignificant volume of their own.
- Molecules have no attractive forces between each other or with their container.
- Molecules move in perfectly straight lines.
- Collisions are completely elastic and, thus, do not use up energy.

These characteristics led to the ideal gas law. They are accurate enough for most applications. They are not perfect, however. In fact, most gases fall short of these characteristics in many ways:

- The particles of a real gas have a significant volume of their own.
- Molecules do attract each other.
- Molecules do not necessarily move in straight lines.
- Collisions are not completely elastic.

At high pressures and/or low temperatures, gases have smaller volumes. This means that the gas molecules are closer together. Since they are closer together, the molecules interact more than when they are far apart. It is no longer true to say that the molecules have no attractive forces between each other or with their container. Part A of Figure 12.8 illustrates how this affects the pressure of a gas. Also, since the total volume is smaller, the amount of space taken up by the gas molecules is more important. You can no longer ignore the volume of the gas molecules. Part B of Figure 12.8 illustrates that gas molecules do occupy part of the volume of the container.



CHECKPOINT

Look back at Table 12.1. It gives more accurate molar volumes for several gases. Using what you have learned about the way real gases behave, explain why these molar volumes are slightly different from the molar volume of an ideal gas.

Figure 12.8 (A) Because particles are attracted to each other, the pressure is reduced. (B) Since particles take up space, the total volume of empty space is smaller than the volume of the container.

Scientists who want more accuracy in their experiments have adapted the ideal gas law to reflect the behaviour of real gases. In later chemistry courses, you will learn more about this corrected version of the ideal gas law, called the *Van der Waals equation*.

To summarize, at low pressures and high temperatures, most gases behave as ideal gases. Under any conditions that allow attractive forces between molecules to occur, gases no longer behave as an ideal gas. They behave as real gases.

History

LINK

The French chemist, Antoine Lavoisier (1743–1794), was the first person to notice the gas-volume relationship. He observed water decomposing to give two volumes of hydrogen and one volume of oxygen. He mentioned this relationship in his 1789 textbook *Éléments de la Chimie*.

Arriving at the Ideal Gas Law

In the Sample Problem earlier, you used three steps to find the molar volume of nitrogen gas. After calculating the number of moles, you calculated the final volume using the combined gas law. Then you used Avogadro's law to find the volume of one mole of nitrogen. There is an easier way to do problems like this—by combining the two gas laws. After Avogadro's work, it did not take scientists long to connect $V \propto n$ (Avogadro's law) with $V \propto \frac{T}{P}$ (the combined gas law).

$$\begin{aligned} V &\propto n \\ V &\propto \frac{T}{P} \\ \therefore V &\propto \frac{nT}{P} \\ \therefore \frac{PV}{nT} &= R, \text{ where } R \text{ is a constant} \end{aligned}$$

As you know, all gases behave in a similar way. The *universal gas constant* (R), which applies to all gases, was derived for the final equation given above. Examine the calculation below to see how R was derived.

For one mole of gas at STP,

$$\begin{aligned} P &= 101.3 \text{ kPa} \\ T &= 273 \text{ K} \\ V &= 22.4 \text{ L} \\ n &= 1.00 \text{ mol} \end{aligned}$$

$$\begin{aligned} R &= \frac{PV}{nT} \\ &= \frac{101.3 \text{ kPa} \times 22.4 \text{ L}}{1.00 \text{ mol} \times 273.15 \text{ K}} \\ &= 8.31 \frac{\text{kPa} \cdot \text{L}}{\text{mol} \cdot \text{K}} \end{aligned}$$

When it is measured more accurately, the universal gas constant (R) has a value of $8.314 \frac{\text{kPa} \cdot \text{L}}{\text{mol} \cdot \text{K}}$. This was the final piece of the equation. The resulting equation is an efficient tool for solving many problems that involve gases.

The **ideal gas law** states that the pressure multiplied by the volume is equal to the number of moles multiplied by the universal gas constant and the temperature.

$$PV = nRT$$

Guidelines for Using the Ideal Gas Law

- Always convert the temperature to kelvins (K).
- Always convert the masses to moles (mol).
- Always convert the volumes to litres (L).
- Using the ideal gas law will be easier if you always convert the pressures to kilopascals (kPa). Then you can memorize the value of R ($8.314 \frac{\text{kPa} \cdot \text{L}}{\text{mol} \cdot \text{K}}$) and use it for every calculation. If you happen to forget the value of R , you can calculate it by finding R for 1 mol of gas at STP.



Electronic Learning Partner

Go to the Chemistry 11 Electronic Learning Partner for an interactive simulation on the ideal gas law.

Converting the Units of the Universal Gas Constant

Some people prefer to use a converted value of R , rather than converting all the pressures to kilopascals, as suggested in the guidelines above.

What are the units for R if the pressure is given in atmospheres?

$$\frac{8.314 \text{ kPa}\cdot\text{L}}{\text{mol}\cdot\text{K}} \times \frac{1 \text{ atm}}{101.3 \text{ kPa}} = 0.08206 \frac{\text{atm}\cdot\text{L}}{\text{mol}\cdot\text{K}}$$

What are the units for R if the pressure is given in torr (mm Hg)?

$$8.314 \frac{\text{kPa}\cdot\text{L}}{\text{mol}\cdot\text{K}} \times \frac{760 \text{ mmHg}}{101.3 \text{ kPa}} = 62.37 \frac{\text{mmHg}\cdot\text{L}}{\text{mol}\cdot\text{K}}$$

Since R is an exact constant, always express it with *four* significant digits. Do not worry about changing the significant digits of R in your calculations. Always keep four digits, and then fix the number of significant digits in your answer.

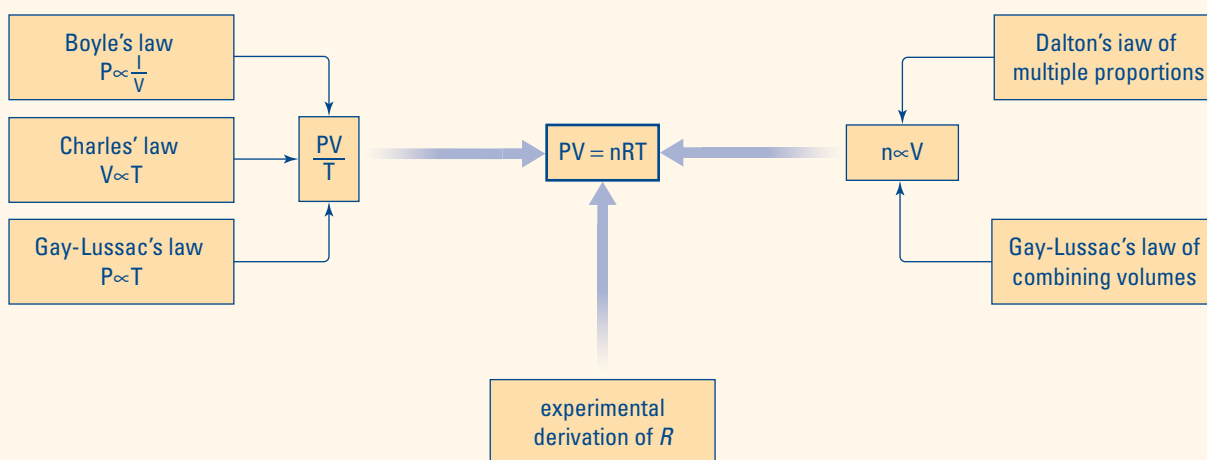
The diagram below illustrates the evolution of the ideal gas law. It is a brief summary of what you have learned so far in this section. The following Sample Problems show you how to do calculations using the ideal gas law.

CHECKPOINT

Go back to section 11.2 to review the units for pressure.

Concept Organizer

The Evolution of the Ideal Gas Law



Sample Problem

Calculating Molar Volume Using $PV = nRT$

Problem

Use the ideal gas law to calculate the molar volume of a gas at standard ambient temperature and pressure (SATP). The conditions for SATP are 298 K and 100 kPa.

What Is Required?

You need to find the molar volume, $MV = \frac{V}{n}$, at SATP. The units for your answer will be in mol/L.

Continued ...

PROBLEM TIP

The molar volume is in L/1.00 mol. If you let $n = 1.00$ mol, you only need to find V .

What Is Given?

$$P = 100 \text{ kPa}$$

$$n = 1.00 \text{ mol}$$

$$R = 8.314 \text{ kPa}\cdot\text{L/mol}\cdot\text{K}$$

$$T = 298 \text{ K}$$

Plan Your Strategy

Let $PV = nRT$, and solve for V . Then substitute the known values into the equation.

Act on Your Strategy

$$PV = nRT$$

$$\begin{aligned}\therefore V &= \frac{nRT}{P} \\ &= \frac{1.00 \text{ mol} \times 8.314 \text{ kPa}\cdot\text{L/mol}\cdot\text{K} \times 298 \text{ K}}{100 \text{ kPa}} \\ &= 24.8 \text{ L}\end{aligned}$$

The molar volume of a gas at SATP is 24.8 L/mol.

Check Your Solution

The molar volume is in units of L/mol. 24.8 L/mol is slightly larger than the 22.4 L/mol molar volume at STP. This is the expected result, since the pressure has decreased and the temperature has increased.

Sample Problem**A Laughing Matter****Problem**

Dentists sometimes use laughing gas (dinitrogen oxide, N_2O) to keep patients relaxed during dental procedures. A cylinder of laughing gas has a diameter of 23.0 cm and a height of 140 cm. The pressure is 108 kPa, and the temperature is 294 K. How many grams of laughing gas are in the cylinder?

What Is Required?

You need to find the mass (in g) of laughing gas in a cylinder at 108 kPa and 294 K.

What Is Given?

$$P = 108 \text{ kPa}$$

$$V = \pi r^2 h, \text{ where } r = 11.5 \text{ cm and } h = 140 \text{ cm}$$

$$R = 8.314 \text{ kPa}\cdot\text{L/mol}\cdot\text{K}$$

$$T = 294 \text{ K}$$

Plan Your Strategy

- Step 1** Use the formula for the volume of a cylinder ($\pi r^2 h$) to approximate the volume of the gas inside.
- Step 2** Solve the ideal gas equation for n . Put the known values into the equation to find the number of moles present (n).
- Step 3** Calculate the molar mass (M) of dinitrogen oxide. Use it to find the mass (m) of the gas present.

Act on Your Strategy

- Step 1** Find the volume of the cylinder.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 11.5^2 \text{ cm}^2 \times 140 \text{ cm} \\ &= 5.73 \times 10^4 \text{ cm}^3 \\ &= 57.3 \text{ L} \end{aligned}$$

- Step 2** Use the gas law to find the number of moles.

$$\begin{aligned} PV &= nRT \\ \therefore n &= \frac{PV}{RT} \\ &= \frac{108 \text{ kPa} \times 57.3 \text{ L}}{8.314 \text{ kPa}\cdot\text{L/mol}\cdot\text{K} \times 294 \text{ K}} \\ &= 2.53 \text{ mol} \end{aligned}$$

- Step 3** Calculate the mass of the gas.

For N_2O , the molar mass is

$$\begin{array}{r} 2\text{N} = 2 \times 14.01 \text{ g/mol} \\ 1\text{O} = 1 \times 16.00 \text{ g/mol} \\ \hline M_{\text{N}_2\text{O}} = 44.02 \text{ g/mol} \end{array}$$

For 2.53 mol, the mass (m) is

$$\begin{aligned} m &= n \times M \\ &= 2.53 \text{ mol} \times 44.02 \text{ g/mol} \\ &= 111 \text{ g} \end{aligned}$$

Therefore, 111 g of laughing gas is in the cylinder.

Check Your Solution

The answer is in grams, which is the unit you were asked for. There are three significant digits. This is consistent with the least number of significant digits in the question.

Practice Problems

12. 4.00 L of ammonia gas in a container holds 2.17 mol at 206 kPa. What is the temperature inside the container?

13. How many kilograms of chlorine gas are contained in 87.6 m^3 at 290 K and 2.40 atm ? **Hint:** $1 \text{ m}^3 = 1000 \text{ L}$
14. Calculate the volume of 3.03 g of hydrogen gas at a pressure of 560 torr and a temperature of 139 K .
15. A $6.0 \times 10^2 \text{ L}$ reaction tank contains 5.0 mol of oxygen gas and 28 mol of nitrogen gas. If the temperature is 83°C , what is the pressure of the oxygen, in kPa ?

Section Wrap-up

In this section, you learned that the ideal gas law was derived from Avogadro's law and the combined gas law. You can use the ideal gas law to calculate the pressure, temperature, volume, or number of moles of a gas. In the next section, you will learn how to apply the ideal gas law to identify unknown gases on the basis of their densities and/or molar masses.

Remember that you can manipulate the ideal gas law to solve for any variable.

$$P = \frac{nRT}{V} \quad V = \frac{nRT}{P} \quad n = \frac{PV}{RT} \quad T = \frac{PV}{nR}$$

The universal gas constant (R) always has the same value: $8.314 \text{ kPa}\cdot\text{L/mol}\cdot\text{K}$. (You need to use an alternative value of R if pressure is given in atmospheres or in torr.)

Section Review



PROBEWARE

If you have access to probe-ware, do the Gas Laws lab, or a similar lab available from a probeware company.

- 1 **I** How many grams of sulfur dioxide are in 36.2 L at STP?
- 2 **I** At certain conditions, 20 mol of a gas occupy 498 L . What volume do 35 mol occupy?
- 3 **I** What is the pressure of 0.76 mol of gas at 48°C in 8.0 L ?
- 4 **I** How many grams of bromine gas, Br_2 , are in 3.12 L at a pressure of 2.4 atm and a temperature of -20°C ? How many molecules of bromine gas are there?
- 5 **I** What is the temperature of 6.02 mol of hydrogen sulfide in a 132 L tank at 0.95 atm ?
- 6 **K/U** What was Avogadro's hypothesis? Explain how Avogadro contributed to our understanding of gases and the relationships among the properties of gases.
- 7 **K/U** 1.00 m^3 of regular air at 1.0 atm is compressed to one eighth of the volume at a constant temperature. What is the pressure contribution of the nitrogen? **Hint:** See the Table 11.2 in Chapter 11, section 11.4.