Math and Chemistry

In chemistry, quantities and values are often very, very large or very, very small. One mole of contains a huge number (602 214 199 000 000 000 000 000) of molecules. The mass of each molecule is minuscule (0.000 000 000 000 000 000 000 029 9 g). To calculate the mass of one mole of using the above data, you multiply the number of molecules in a mole by the mass of an individual molecule. Imagine doing this by hand:

There is another problem with writing all these zeros: It implies that they are all significant digits. Yet the non-final zeros to the right of the decimal point in the mass of an molecule are used as placeholders. They are not significant digits. There are only three significant digits in

0.000 000 000 000 000 000 000 0**29 9** g

Scientific Notation

To simplify reporting large numbers and doing calculations, you can use scientific notation. As you learned in an earlier mathematics course, scientific notation works by using *powers of 10* as multipliers.

The number of particles in one mole of a substance (the *Avogadro constant*) is often rounded down to 602 000 000 000 000 000 000 000. When performing calculations, however, this number is still too unwieldy. To simplify it, you can express it as a number between 1 and 10, multiplied by factors of 10. To do this, move the decimal point behind the left-most non-zero digit, counting the number of places that the decimal point moves. The number of places moved is the exponent for the base 10.

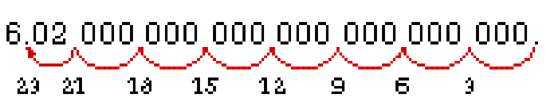

Figure E.1 **The decimal point moves to the left.**

Numbers less than 1 are converted into scientific notation by moving the decimal point to the right of the left-most non-zero digit. The exponent is expressed as a negative power of 10.

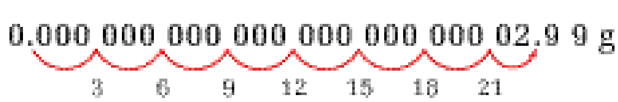

Figure E.2 **The decimal point moves to the right.**

Once you have numbers in scientific notation, calculations become much simpler. To find the mass of one mole of , you can multiply (/mol) × (g). To do this without a calculator, you first multiply.

 (rounded up to three significant digits)

When multiplying, exponents are added to find the new exponent: .

This gives g/mol. In scientific notation, however, there must be only one digit before the decimal place. Here the decimal place must be moved one place to the left: g/mol.

Figure E.3 shows how to do this calculation with a scientific calculator. When you enter an exponent on a scientific calculator, you do not have to enter (× 10) at any time.

Keystrokes	Display
6 . 0 2 EXP 2 3	6.02 23
× 2 . 9 9 EXP 2 3 ±	2.99 −23
=	17.998

Round to three significant digits and express in scientific notation:

Figure E.3 **On some scientific calculators, the EXP key is labelled EE. Key in negative exponents by entering the exponent, then striking the ± key.**

Rules for Scientific Notation

- When multiplying, exponents are added algebraically.
- When dividing, exponents are subtracted algebraically.
- When adding or subtracting numbers in scientific notation, the numbers must be converted to the same power of 10 as the measurement with the greatest power of 10. Once the numbers are all expressed to the same power of 10, the power of 10 is neither added nor subtracted in the calculation.

$$(9.73 \times 10^1\ dm^3) - (8.11 \times 10^{-1}\ dm^3)$$
$$= (9.73 \times 10^1\ dm^3) - (.0811 \times 10^1\ dm^3)$$
$$= 9.6489 \times 10^1\ dm^3 \rightarrow 9.65 \times 10^1\ dm^3$$

Practice Problems

Scientific Notation

1. Convert each number into scientific notation.

(a) 0.000 945
(b) 39 230 000 000 000
(c) 0.000 000 000 000 003 497
(d) 879×10^5
(e) 0.00142×10^3
(f) 31271×10^{-6}

2. Add, subtract, multiply, or divide. Round your answer, and express it in scientific notation to the correct number of significant digits.

(a) $(3.00 \times 10^3) + (8.50 \times 10^1)$
(b) $(6.99 \times 10^3) + (8.13 \times 10^2)$
(c) $(1.01 \times 10^1) - (9.34 \times 10^{-2})$
(d) $(12.01 \times 10^6) \times (8.32 \times 10^7)$
(e) $(4.75 \times 10^{-3}) \div (3.21 \times 10^{-2})$

Logarithms

Calculators have made multiplication and division as easy as adding and subtracting. Before calculators were invented, however, what did people do? In 1614, John Napier (1550–1617) invented *logarithms* to help people multiply and divide large numbers.

Why do logarithms make multiplication and division easier? Recall, from the power laws, how you multiply and divide powers. To multiply powers with the same base, you add the exponents. To divide powers with the same base, you subtract the exponents. Logarithms are exponents. Therefore, to multiply large numbers using logarithms, you simply add the logarithms. To divide large numbers using logarithms, you subtract the logarithms.

Find the log (for "logarithm") key on your calculator. Enter 1000 or 10^3 into your calculator, then press the log key. The number 3 will be in the display. Now enter 10 000 or any other power of 10. Press the log key. What is displayed for the log of the number? (The logarithm of any power of 10 is just the exponent.)

How do you multiply one million by one billion, or $10^6 \times 10^9$, using logarithms? You add the exponents, or logarithms, of these two numbers: $6 + 9 = 15$. The answer is 10^{15}, a power of 10. This power is called the *antilogarithm* of the logarithm 15.

Logarithms may not seem like a major discovery, but John Napier also evaluated logarithms for numbers such as 2, 3, 5, and 764. (Table E.1 has more examples of logarithms.) Enter 2 into your calculator, and press the log key. The display will show 0.3010, to four significant digits. This means that $10^{0.3010} = 2$. Repeat to find the log of 3. The display will show 0.4771. This means that $10^{0.4771} = 3$.

You know that $2 \times 3 = 6$. If you add 0.3010 to 0.4771, you get 0.7781. What is $10^{0.7781}$? Enter 0.7781 into your calculator. Find the 10^x key. (Usually it is above the log key, as a second function.) Press the 10^x key. If you round the answer in the display, you will get 6.

Find the log of 8 and the log of 4. You know that $8 \div 4 = 2$. Subtract the log of 4 from the log of 8: $0.9031 - 0.6021 = 0.3010$. This answer is the log of 2: $10^{0.9031} \div 10^{0.6021} = 10^{0.3010} = 2$. Evaluate $10^{0.3010}$ on your calculator by typing in 0.3010 and using the 10^x key.

Technology LINK

Logarithms were used for 350 years to do many complex calculations. Even today, calculators are programmed to do mathematical manipulations with logarithms. So, indirectly, we are still using John Napier's invention.

Table E.1 Some Numbers and Their Logarithms

Number	Scientific notation	As a power of 10	Logarithm
1 000 000	1×10^6	10^6	6
7 895 900	7.8590×10^5	$10^{5.8954}$	5.8954
1	1×10^0	10^0	0
0.000 001	1×10^{-6}	10^{-6}	−6
0.004 276	4.276×10^{-3}	$10^{-2.3690}$	−2.3690

Notice that numbers above 1 correspond to logarithms whose value is positive; numbers below 1 correspond to logarithms whose value is negative. Over the years, scientists have found logarithms convenient for more than calculations. Logarithms can be used to express values that span a range of powers of 10, such as pH. In Chapter 10, section 10.2, concentrations of acids are discussed. The pH of an acid solution is defined as $-\log[H_3O^+]$. (The square brackets mean "concentration.") For example, suppose that the hydronium ion concentration in a solution is 0.0001 mol/L (10^{-4} mol/L). The pH is −log(0.0001). To calculate this, enter 0.0001 into your calculator. Then press the log key. Since the logarithm is negative, press the − key. The answer in the display is 4. Therefore the pH of the solution is 4.

As stated above, there are logarithms for all numbers, not just whole multiples of 10. What is the pH of a solution if $[H_3O^+] = 0.00476$ mol/L? Enter 0.00476. Press the log key and then the − key. The answer is 2.322. Remember that the concentration was expressed to three significant digits. The pH to three significant digits is 2.32.

What if you want to find the $[H_3O^+]$ from the pH? You would need to find 10^{-pH}. For example, what is the $[H_3O^+]$ if the pH is 5.78? Enter 5.78, and press the − key. Then use the 10^x function. The answer is $10^{-5.78}$. Therefore, the $[H_3O^+]$ is 1.66×10^{-6} mol/L.

Remember that the pH scale is a negative log scale. Thus, a decrease in pH from pH 7 to pH 4 is an increase of 10^3, or 1000, in the acidity of a solution. An increase from pH 3 to pH 6 is a decrease of 10^3, or 1000, in acidity.

CHEM FACT

You may be familiar with two other logarithmic scales: the decibel scale for sound, and the Richter scale for earthquake strength.

Practice Problems

Logarithms

1. Calculate the logarithm of each number. Note the trend in your answers.

(a) 1 **(b)** 5 **(c)** 10 **(d)** 50 **(e)** 100 **(f)** 500 **(g)** 1000 **(h)** 5000 **(i)** 10 000 **(j)** 50 000 **(k)** 100 000 **(l)** 500 000 **(m)** 1 000 000

2. Calculate the antilogarithm for each number.

(a) 0 **(b)** 1 **(c)** –1 **(d)** 2 **(e)** –2 **(f)** 3 **(g)** –3

3. (a) How are your answers for question 2, parts (b) and (c) related?

(b) How are your answers for question 2, parts (d) and (e) related?

(c) How are your answers for question 2, parts (f) and (g) related?

(d) Calculate the antilogarithm of 3.5.

(e) Calculate the antilogarithm of –3.5.

(f) Take the reciprocal of your answer for part (d).

(g) How are your answers for parts (e) and (f) related?

4. (a) Calculate log 76 and log 55.

(b) Add your answers for part (a).

(c) Find the antilogarithm of your answer for part (b).

(d) Multiply 76 and 55.

(e) How are your answers in parts (c) and (d) related?

Linear Graphs

Suppose that you conduct a Boyle's law experiment. (See Chapter 11, section 11.2.) You obtain data that are similar to the data in Table E.2.

Table E.2 Boyle's Law Data

1/Pressure (kPa^{-1})	Volume (mL)
1.00×10^{-2}	50.0
8.93×10^{-3}	44.6
7.87×10^{-3}	39.4
7.04×10^{-3}	35.2
6.13×10^{-3}	30.7

Looking at the data does not tell you much about the relationship between the two variables: the volume (V) and the inverse pressure ($\frac{1}{P}$). What if you graph the data? Put $\frac{1}{P}$ (the independent variable) on the x-axis. Put V (the dependent variable) on the y-axis. The scale for $\frac{1}{P}$ should go from 0 to at least 0.01. The scale for V should go from 0 to at least 50.0. To plot each point, go along the x-axis until you reach the value of $\frac{1}{P}$ (such as 0.01). Then go up from this point to the corresponding value of V (50.0). After you have plotted all the points, draw the line of best fit through the points. (See Figures E.4A and E.4B).

Finding a Point on a Graph

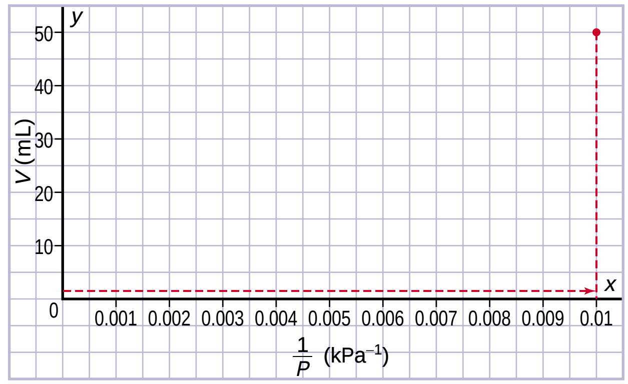

Figure E.4A

Drawing a Line of Best Fit

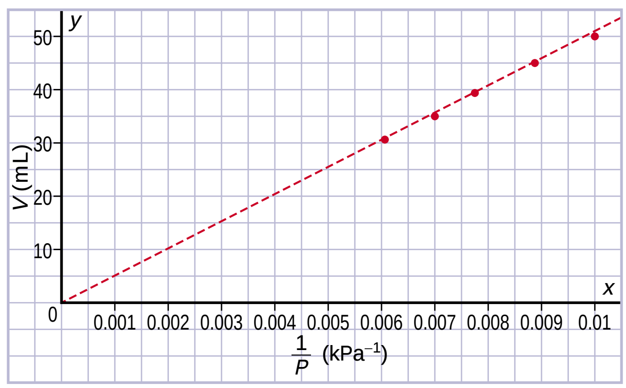

Figure E.4B

Examine the graph in Figure E.4B. You can see that the points form a straight line that goes through the origin, (0,0). You can also see that there is a linear relationship between the volume and the inverse pressure.

Mathematically, this means that the volume is directly proportional to the inverse pressure, or $V \propto \frac{1}{P}$. To remove the proportionality sign and replace it with an equal sign, V must be multiplied by a proportionality constant, k.

$$\therefore V = k\left(\frac{1}{P}\right)$$
$$V = \frac{k}{P}$$
$$PV = k$$

How can you determine the proportionality constant? Recall, from mathematics courses, that the equation of a straight line that goes through the origin is given by $y = mx$, where m is the slope of the line. Also recall the equation for the slope of a straight line.

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

This equation is illustrated in Figure E.4C.

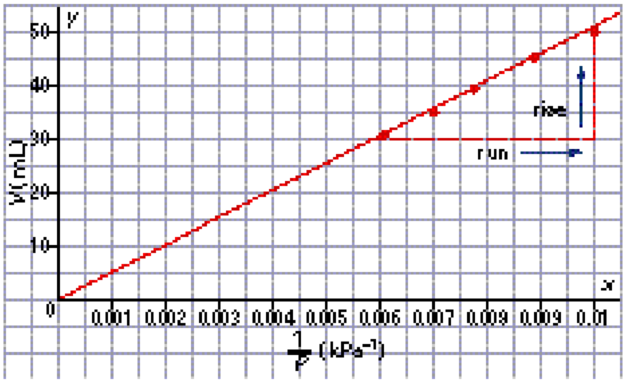

Figure E.4C

For this graph, the change in the y values corresponds to a change in V, from 50.0 mL to 30.7 mL. The change in the x values corresponds to a change in , from
 to .

The equation of the line is therefore .

What happens if the data do not give a line that goes through the origin? The data in Table E.3 came from an experiment to determine the density of an unknown liquid. The student forgot to measure the mass of the container that was used to hold the various volumes of the liquid.

Table E.3 Mass and Volume of a Liquid

Volume of liquid (mL)	Mass of liquid and container (g)
22	58.4
41	71.7
65	87.2
83	99.2
100	110.6

Graph of Mass and Volume of a Liquid

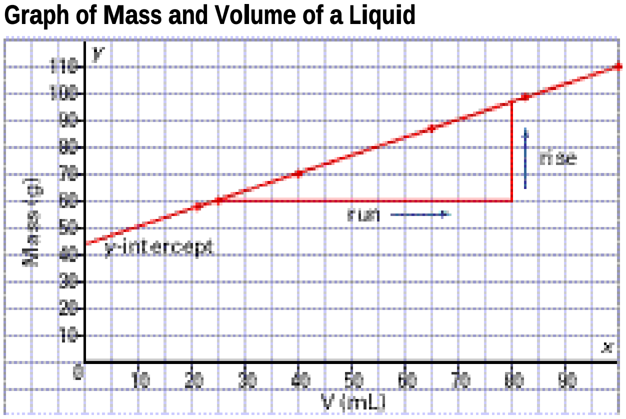

Figure E.5

These data are plotted, as shown in Figure E.5, with the volume on the x-axis and the mass on the y-axis. The equation of this line is in the form , where m is the slope of the line and b is the y-intercept. (The y-intercept is the value of y where the line crosses the y-axis. This value occurs when .) Thus, the relationship here is , where the k is the slope of the line. As before,

Therefore, the equation of the line is .
If there is no liquid, both the mass and the volume of the liquid are zero. Notice, however, that there is still a value for mass on the graph. This is the value of b, the y-intercept, which is 43.8 g. It represents the mass of the container. Therefore, the final equation is .

Consider one more example. Two different sets of data for Charles' law experiments are given in Table E.4.

Table E.4 Charles' Law Data

Set 1

Temperature (°C)	Volume (mL)
20	150
34	157
42	161
55	168
68	175

Set 2

Temperature (°C)	Volume (mL)
15	240
26	249
38	259
51	270
80	294

The linear graphs of both sets of data are given in Figure E.6, on the same pair of axes. Since two different samples of gas were used, there are two different lines in the form . Notice that the lines intersect at −273°C, or 0 K, the theoretical lowest temperature possible. (The linear graphs for the temperature and volume of all gases go through this point.) If the y-axis was moved to this point and a new origin was established here, then the equations for the lines would be in the form . This is what happens when you convert temperatures to the Kelvin scale.

Charles' Law Graph

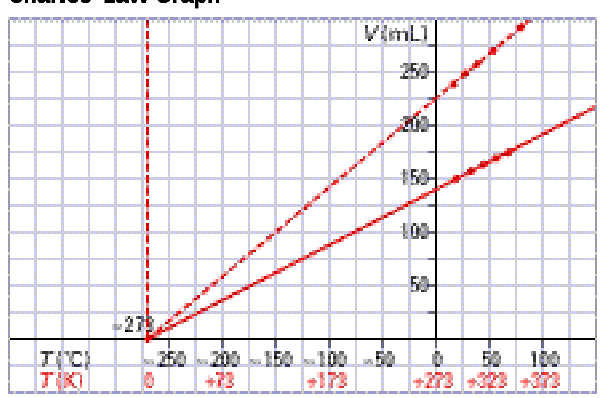

Figure E.6

Practice Problems

Linear Graphs

1. A spring, 20 cm long, has hooks on each end. One end is attached to the ceiling. The other has small, metal blocks, with hooks hanging from it. The following data are generated.

Mass attached (g)	Spring length (cm)
50	20.1
200	20.4
300	20.7
500	21.0
600	21.3

(a) Which variable is the independent variable?
(b) Which variable is the dependent variable?
(c) Graph these data. Is the graph linear?
(d) Determine the slope and the *y*-intercept for your graph. Do not forget to use appropriate units.
(e) Write an equation to represent your graph.

2. An object is dropped off the roof of a building that is 100 m high. Its position at various times is given below.

Time (s)	Distance fallen (m)
0	0
0.5	1.3
1	4.9
1.5	11
2	20

(a) Which variable is the independent variable?
(b) Which variable is the dependent variable?
(c) Graph these data. Is the graph linear?
(d) Try graphing *distance* versus some function of *time* (such as 1 or) until a linear graph appears to be generated.
(e) Write an equation from your linear graph in part (d).
(f) How long will it take for the object to reach the ground?

The Unit Analysis Method of Problem Solving

The unit analysis method of problem solving is extremely versatile. You can use it to convert between units or to solve simple formula problems. If you forget a formula during a test, you may still be able to solve the problem using unit analysis.

The unit analysis method involves analyzing the units and setting up conversion factors. You match and arrange the units so that they divide out to give the desired unit in the answer. Then you multiply and divide the numbers that correspond with the units.

Steps for Solving Problems Using Unit Analysis

Step 1 Determine which data you have and which conversion factors you need to use. (A conversion factor is usually a ratio of two numbers with units, such as 1000 g/1 kg. You multiply the given data by the conversion factor to get the desired units in the answer.) It is often convenient to use the following three categories to set up your solution: Have, Need, and Conversion Factors.

Step 2 Arrange the data and conversion factors so that you can cross out the undesired units. Decide whether you need any additional conversion factors to get the desired units in the answer.

Step 3 Multiply all the numbers on the top of the ratio. Then multiply all the numbers on the bottom of the ratio. Divide the top result by the bottom result.

Step 4 Check that the units have cancelled correctly. Also check that the answer seems reasonable, and that the significant digits are correct.

Sample Problem

Active ASA

Problem

In the past, pharmacists measured the active ingredients in many medications in a unit called grains (gr). A grain is equal to 64.8 mg. If one headache tablet contains 5.0 gr of active acetylsalicylic acid (ASA), how many grams of ASA are in two tablets?

What Is Required?

You need to find the mass in grams of ASA in two tablets.

What Is Given?

There are 5.0 gr of ASA in one tablet. A conversion factor for grains to milligrams is given.

Plan Your Strategy

Multiply the given quantity by conversion factors until all the unwanted units cancel out and only the desired units remain.

Have	Need	Conversion factors
5.0 gr	? g	64.8 mg/1 gr and 1 g/1000 mg

Act on Your Strategy

Continued...

Continued...

There are 0.65 g of active ASA in two headache tablets.

Check Your Solution

There are two significant digits in the answer. This is the least number of significant digits in the given data.

Notice how conversion factors are multiplied until all the unwanted units are cancelled out, leaving only the desired unit in the answer.

The next Sample Problem will show you how to solve a simple stoichiometric problem.

Sample Problem
Stoichiometry and Unit Analysis

Problem

What mass of oxygen, , can be obtained by the decomposition of 5.0 g of potassium chlorate, ? The balanced equation is given below.

What Is Required?

You need to calculate the amount of oxygen, in grams, that is produced by the decomposition of 5.0 g of potassium chlorate.

What Is Given?

You know the mass of potassium chlorate that decomposes.

Mass = 5.0 g

From the balanced equation, you can obtain the molar ratio of the reactant and the product.

Plan Your Strategy

Calculate the molar masses of potassium chlorate and oxygen. Use the molar mass of potassium chlorate to find the number of moles in the sample.

Use the molar ratio to find the number of moles of oxygen produced. Use the molar mass of oxygen to convert this value to grams.

Act on Your Strategy

The molar mass of potassium chlorate is

The molar mass of oxygen is

g/mol

Continued...

Find the number of moles of potassium chlorate.

Find the number of moles of oxygen produced.

Convert this value to grams.

Therefore, 2.0 g of oxygen are produced by the decomposition of 5.0 g of potassium chlorate.
As you become more familiar with this type of question, you will be able to complete more than one step at once. Below, you can see how the conversion factors we used in each step above can be combined. Set these conversion ratios so that the units cancel out correctly.

Check Your Solution

The oxygen makes up only part of the potassium chlorate. Thus, we would expect less than 5.0 g of oxygen, as was calculated.

The smallest number of significant digits in the question is two. Thus, the answer must also have two significant digits.

Practice Problems

Unit Analysis

Use the unit analysis method to solve each problem.

1. The molecular mass of nitric acid is 63.02 g/mol. What is the mass, in g, of 7.00 mol of nitric acid?
2. To make a salt solution, 0.50 mol of NaCl are dissolved in 750 mL of water. What is the concentration, in g/L, of the salt solution?
3. The density of solid sulfur is 2.07 g/. What is the mass, in kg, of a 345 sample?
4. How many grams of dissolved potassium iodide are in 550 mL of a 1.10 mol/L solution?