5.1 Isotopes and Average Atomic Mass

Section Preview/ Specific Expectations

In this section, you will

- **describe** the relationship between isotopic abundance and average atomic mass
- **solve** problems involving percentage abundance of isotopes and relative atomic mass
- **explain** the significance of a weighted average
- **communicate** your understanding of the following terms: *isotopic abundance, average atomic mass, mass spectrometer, weighted average*

How does the mass of a substance relate to the number of atoms in the substance? To answer this question, you need to understand how the relative masses of individual atoms relate to the masses of substances that you can measure on a balance.

The head of a pin, like the one shown in Figure 5.1, is made primarily of iron. It has a mass of about 8×10^{-3} g, yet it contains about 8×10^{19} atoms. Even if you could measure the mass of a single atom on a balance, the mass would be so tiny (about 1×10^{-22} g for an iron atom) that it would be impractical to use in everyday situations. Therefore, you need to consider atoms in bulk, not individually.

How do you relate the mass of individual atoms to the mass of a large, easily measurable number of atoms? In the next two sections, you will find out.

Figure 5.1 The head of a typical pin contains about 80 quintillion atoms.

Relating Atomic Masses to Macroscopic Masses

In Chapter 2, you learned that the mass of an atom is expressed in atomic mass units. Atomic mass units are a relative measure, defined by the mass of carbon-12. According to this definition, one atom of carbon-12 is assigned a mass of 12 u. Stated another way, 1 u = $\frac{1}{12}$ of the mass of one atom of carbon-12.

The masses of all other atoms are defined by their relationship to carbon-12. For example, oxygen-16 has a mass that is 133% of the mass of carbon-12. Hence the mass of an atom of oxygen-16 is $\frac{133}{100} \times 12.000 \text{ u} = 16.0 \text{ u}$.

Usually, not all the atoms in an element have the same mass. As you learned in Chapter 2, atoms of the same element that contain different numbers of neutrons are called isotopes. Most elements are made up of two or more isotopes. Chemists need to account for the presence of isotopes when finding the relationship between the mass of a large number of atoms and the mass of a single atom. To understand why this is important, consider the following analogy.

Imagine that you have the task of finding the total mass of 10 000 spoons. If you know the mass of a dessertspoon, can you assume that its mass represents the average mass of all the spoons? What if the 10 000 spoons include soupspoons, dessertspoons, and tablespoons? If you use the mass of a dessertspoon to calculate the total mass of all the spoons, you may obtain a reasonable estimate. Your answer will not be accurate, however, because each type of spoon has a different mass. You cannot calculate an accurate average mass for all the spoons based on knowing the mass of only one type. How can you improve the accuracy of your answer without determining the mass of all the spoons?

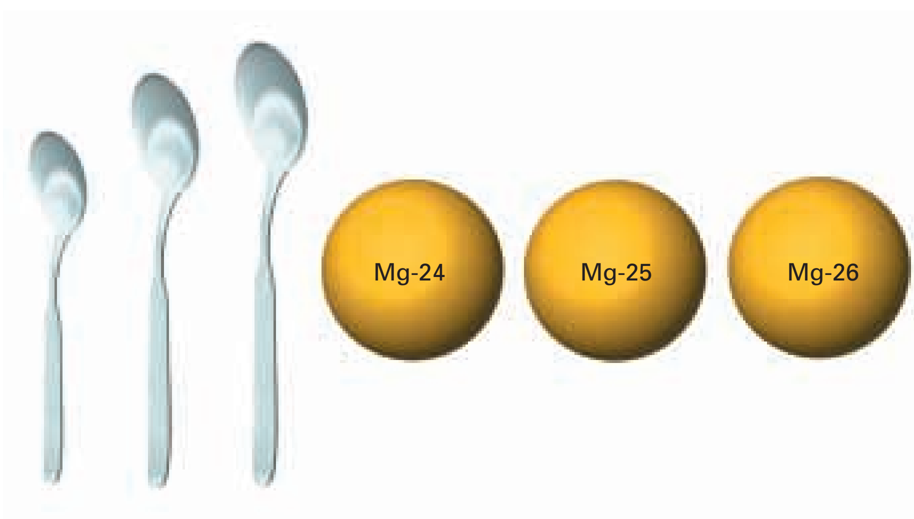


Figure 5.2 **Think about finding the average mass of a group of objects that have different masses. How is this similar to finding the average mass of an element that is composed of different isotopes?**

Isotopic Abundance

Chemists face a situation similar to the one described above. Because all the atoms in a given element do not have the same number of neutrons, they do not all have the same mass. For example, magnesium has three naturally occurring isotopes. It is made up of 79% magnesium-24, 10% magnesium-25, and 11% magnesium-26. Whether the magnesium is found in a supplement tablet (like the ones on the right) or in seawater as $Mg(OH)_2$, it is always made up of these three isotopes in the same proportion. The relative amount in which each isotope is present in an element is called the **isotopic abundance**. It can be expressed as a percent or as a decimal fraction. When chemists consider the mass of a sample containing billions of atoms, they must take the isotopic abundance into account.

CHEM FACT

Magnesium plays a variety of roles in the body. It is involved in energy production, nerve function, and muscle relaxation, to name just a few. The magnesium in these tablets, like all naturally occurring magnesium, is made up of three isotopes.

Average Atomic Mass and the Periodic Table

The **average atomic mass** of an element is the average of the masses of all the element's isotopes. It takes into account the abundance of each isotope within the element. The average atomic mass is the mass that is given for each element in the periodic table.

It is important to interpret averages carefully. For example, in 1996, the average size of a Canadian family was 3.1. Of course, no one family actually has 3.1 people. In the same way, while the average atomic mass of carbon is 12.01 u, no one atom of carbon has a mass of 12.01 u.

Examine Figure 5.3. Since the atomic mass unit is based on carbon-12, why does the periodic table show a value of 12.01 u, instead of exactly 12 u? Carbon is made up of several isotopes, not just carbon-12. Naturally occurring carbon contains carbon-12, carbon-13, and carbon-14. If all these isotopes were present in equal amounts, you could simply find the average of the masses of the isotopes. This average mass would be about 13 u, since the masses of carbon-13 and carbon-14 are about 13 u and 14 u respectively.

The isotopes, though, are not present in equal amounts. Carbon-12 comprises 98.9% of all carbon, while carbon-13 accounts for 1.1%. Carbon-14 is present in a very small amount—about 1×10^{-10}%. It makes sense that the average mass of all the isotopes of carbon is 12.01 u—very close to 12—since carbon-12 is by far the predominant isotope.

CHEM FACT

The only elements with only one naturally occurring isotope are beryllium, sodium, aluminum, and phosphorus.

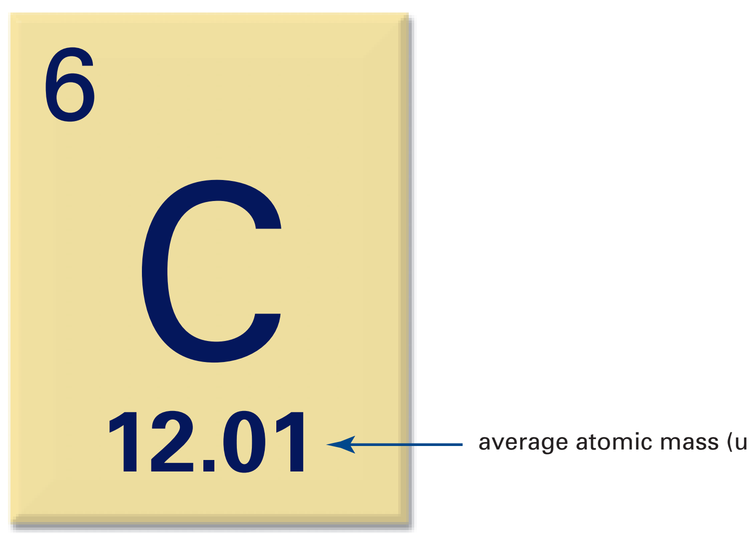


Figure 5.3 The atomic mass that is given in the periodic table represents the average mass of all the naturally occurring isotopes of the element. It takes into account their isotopic abundances.

Thus chemists need to know an element's isotopic abundance and the mass of each isotope to calculate the average atomic mass. How do chemists determine the isotopic abundance associated with each element? How do they find the mass of each isotope? They use a **mass spectrometer**, a powerful instrument that generates a magnetic field to obtain data about the mass and abundance of atoms and molecules. You will learn more about the mass spectrometer in Tools & Techniques on page 166. You can use the data obtained with a mass spectrometer to calculate the average atomic mass given in the periodic table.

Working with Weighted Averages

If you obtain the isotopic abundance of an element from mass spectrometer data or a table, you can calculate the average atomic mass of the element. You do this by calculating the **weighted average** of each isotope's mass. A weighted average takes into account not only the values associated with a set of data, but also the abundance or importance of each value.

Normally, when you calculate the average of a set of data, you find the equally weighted average. You add the given values and divide the total by the number of values in the set. Each value in the average is given equal weight. For example, imagine that you have three objects: A, B, and C. A has a mass of 1.0 kg, B has a mass of 2.0 kg, and C has a mass of 3.0 kg. Their average mass is

$$\frac{\text{Mass of (A + B + C)}}{\text{Number of items}} = \frac{1.0\text{ kg} + 2.0\text{ kg} + 3.0\text{ kg}}{3}$$
$$= 2.0\text{ kg}$$

What if you have a set containing two of A, one of B, and three of C? Their average mass becomes

$$\frac{2(1.0\text{ kg}) + 2.0\text{ kg} + 3(3.0\text{ kg})}{6} = 2.2\text{ kg}$$

This is a weighted average.

Another way to calculate the same weighted average is to consider the relative abundance of each object. There are six objects in total. A is present as $\frac{2}{6}$ (33%) of the total, B is present as $\frac{1}{6}$ (17%) of the total, and C is present as $\frac{3}{6}$ (50%) of the total. Thus their average mass can be calculated in the following way:

$$(0.33)(1.0\text{ kg}) + (0.17)(2.0\text{ kg}) + (0.50)(3.0\text{ kg}) = 2.2\text{ kg}$$

mind STRETCH

We use weighted averages all the time! For example, course marks are often based on weighted averages. Suppose that the final mark in a chemistry course is determined as follows: laboratory 25%, tests 30%, homework and quizzes 5%, project 10%, and final exam 30%. A student obtains the following marks: laboratory $\frac{114}{130}$, tests $\frac{261}{300}$, homework and quizzes $\frac{90}{95}$, project $\frac{21}{25}$, and final exam $\frac{70}{80}$. What is the student's final mark in chemistry?

Calculating Average Atomic Mass

You can use a similar method to calculate average atomic mass. If you know the atomic mass of each isotope that makes up an element, as well as the isotopic abundance of each isotope, you can calculate the average atomic mass of the element.

For example, lithium exists as two isotopes: lithium-7 and lithium-6. As you can see in Figure 5.4, lithium-7 has a mass of 7.015 u and makes up 92.58% of lithium. Lithium-6 has a mass of 6.015 u and makes up the remaining 7.42%. To calculate the average atomic mass of lithium, multiply the mass of each isotope by its abundance.

$$\left(\frac{92.58}{100}\right)(7.015\text{ u}) + \left(\frac{7.42}{100}\right)(6.015\text{ u}) = 6.94\text{ u}$$

Looking at the periodic table confirms that the average atomic mass of lithium is 6.94 u. The upcoming Sample Problem gives another example of how to calculate average atomic mass.

CHECKPOINT

How is the atomic mass of an atom different from the mass number of the atom? How are the atomic mass and mass number similar?

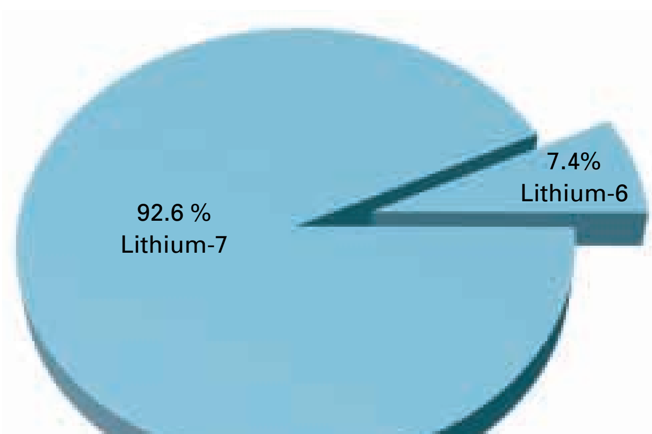


Figure 5.4 Naturally occurring lithium consists of two isotopes, $^{7}_{3}Li$ and $^{6}_{3}Li$.

The Mass Spectrometer

Many chemists depend on instruments known as mass spectrometers. Mass spectrometers can detect trace pollutants in the atmosphere, provide information about the composition of large molecules, and help to determine the age of Earth's oldest rocks.

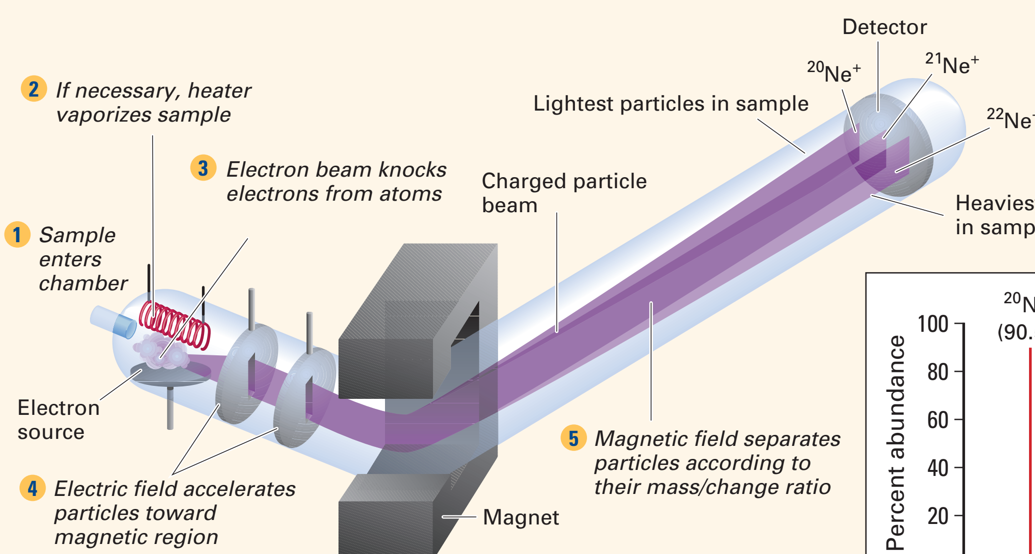
As well, mass spectrometers can find the relative abundance of each isotope in an element. In 1912, J.J. Thompson first detected neon-20 and neon-22 in a sample of neon gas by using a magnetic field to separate the isotopes.

Today's mass spectrometers also use a magnetic field to separate the isotopes of an element. Since a magnetic field can only affect the path of a charged particle, the atoms must first be charged, or "ionized." The magnetic field then deflects ions with the same charge, but different masses, onto separate paths. Imagine rolling a tennis ball at a right angle to the air current from a fan. The air current deflects the path of the ball. The same air current deflects a Ping-Pong™ ball more than a tennis ball. Similarly, a magnetic field deflects isotopes of smaller mass more than isotopes of larger mass.

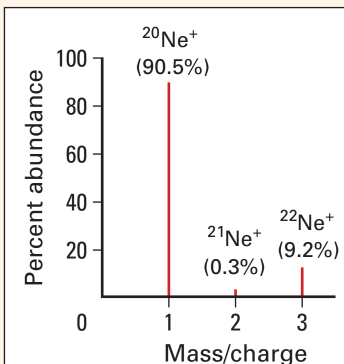
In a mass spectrometer, elements that are not gases are vaporized by heating. Next, the gas atoms are ionized. In electron impact ionization, the gas atoms are bombarded with a stream of electrons from a heated filament. These electrons collide with the gas atoms, causing each atom to lose an electron and become a positive ion.

The ions are focussed and accelerated by electric fields, toward a magnetic field. The magnetic field deflects them and forces them to take a curved path. Lighter ions curve more than heavier ions. Therefore ions from different isotopes arrive at different destinations. Since ions have a charge, a detector can be used to register the current at each destination. The current is proportional to the number of ions that arrive at the destination. An isotope that has a larger relative abundance generates a larger current. From the currents at the different destinations, chemists can deduce the proportion of each isotope in the element.

Mass spectral data for neon, below, show the relative abundance of the isotopes of neon. Using mass spectrometers, chemists now detect a signal from neon-21, in addition to the signals from neon-20 and neon-22.



Mass spectroscopy of neon



Mass spectral data for isotopes of neon

Sample Problem

Average Atomic Mass

Problem

Naturally occurring silver exists as two isotopes. From the mass of each isotope and the isotopic abundance listed below, calculate the average atomic mass of silver.

Isotope	Atomic mass (u)	Relative abundance (%)
$^{107}_{47}Ag$	106.9	51.8
$^{109}_{47}Ag$	108.9	48.2

What Is Required?

You need to find the average atomic mass of silver.

What Is Given?

You are given the relative abundance and the atomic mass of each isotope.

Plan Your Strategy

Multiply the atomic mass of each isotope by its relative abundance, expressed as a decimal. That is, 51.8% expressed as a decimal is 0.518 and 48.2% is 0.482.

Act on Your Strategy

$$\begin{aligned}\text{Average atomic mass of Ag} &= 106.9\text{ u }(0.518) + 108.9\text{ u }(0.482)\\ &= 107.9\text{ u}\end{aligned}$$

Check Your Solution

In this case, the abundance of each isotope is close to 50%. An average atomic mass of about 108 u seems right, because it is between 106.9 u and 108.9 u. Checking the periodic table reveals that the average atomic mass of silver is indeed 107.9 u.

Practice Problems

1. The two stable isotopes of boron exist in the following proportions: 19.78% $^{10}_{5}B$ (10.01 u) and 80.22% $^{11}_{5}B$ (11.01 u). Calculate the average atomic mass of boron.
2. In nature, silicon is composed of three isotopes. These isotopes (with their isotopic abundances and atomic masses) are $^{28}_{14}Si$ (92.23%, 27.98 u), $^{29}_{14}Si$ (4.67%, 28.97 u), and $^{30}_{14}Si$ (3.10%, 29.97 u). Calculate the average atomic mass of silicon.

Continued ...

CHEM FACT

Why are the atomic masses of individual isotopes not exact whole numbers? After all, ^{12}C has a mass of exactly 12 u. Since carbon has 6 neutrons and 6 protons, you might assume that protons and neutrons have masses of exactly 1 u each. In fact, protons and neutrons have masses that are close to, but slightly different from, 1 u. As well, the mass of electrons, while much smaller than the masses of protons and neutrons, must still be taken into account.

CHECKPOINT

Why is carbon-12 the only isotope with an atomic mass that is a whole number?

CHEM FACT

In some periodic tables, the average atomic mass is referred to as the atomic weight of an element. This terminology, while technically incorrect, is still in use and is generally accepted.

Continued ...
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3. Copper is a corrosion-resistant metal that is used extensively in plumbing and wiring. Copper exists as two naturally occurring isotopes: $^{63}_{29}Cu$ (62.93 u) and $^{65}_{29}Cu$ (64.93 u). These isotopes have isotopic abundances of 69.1% and 30.9% respectively. Calculate the average atomic mass of copper.

4. Lead occurs naturally as four isotopes. These isotopes (with their isotopic abundances and atomic masses) are $^{204}_{82}Pb$ (1.37%, 204.0 u), $^{206}_{82}Pb$ (26.26%, 206.0 u), $^{207}_{82}Pb$ (20.82%, 207.0 u), and $^{208}_{82}Pb$ (51.55%, 208.0 u). Calculate the average atomic mass of lead.

ExpressLab A Penny for your Isotopes

The mass of a Canadian penny has decreased several times over the years. Therefore you can use pennies to represent different "isotopes" of a fictitious element, *centium*. That is, each "atom" of *centium* reacts the same way—it is still worth 1¢—but the various isotopes have different characteristic masses.

Safety Precautions

Procedure

1. Obtain a bag of pennies from your teacher. Since the mass of a penny decreased in 1982 and 1997, your bag will contain pennies dated anywhere from 1982 to the present date.
2. Sort your pennies into groups of pre-1997 "isotopes" and post-1997 "isotopes" of *centium*.
3. Count the number of pennies in each group.
4. Find the mass of ten pennies from each group. Divide the total mass by 10 to get the mass of each *centium* "isotope."
5. Use the data you have just gathered to calculate the mass of the pennies, using a weighted average. This represents the "average atomic mass" of *centium*.

Analysis

1. In step 4, you used the average mass of ten pennies to represent the mass of one "isotope" of *centium*.
   (a) Why did you need to do this? Why did you not just find the mass of one penny from each group?
   (b) If you were able to find the mass of real isotopes for this experiment, would you need to do step 4? Explain.
2. Compare your "average atomic mass" for *centium* with the "average atomic mass" obtained by other groups.
   (a) Are all the masses the same? Explain any differences.
   (b) What if you were able to use real isotopes of an element, such as copper, for this experiment? Would you expect results to be consistent throughout the class? Explain.

Calculating Isotopic Abundance

Chemists use a mass spectrometer to determine accurate values for the isotopic abundance associated with each element. Knowing the average atomic mass of an element, you can use the masses of its isotopes to calculate the isotopic abundances.

Sample Problem

Isotopic Abundance

Problem

Boron exists as two naturally occurring isotopes: $^{10}_{5}B$ (10.01 u) and $^{11}_{5}B$ (11.01 u). Calculate the relative abundance of each isotope of boron.

What Is Required?

You need to find the isotopic abundance of boron.

What Is Given?

Atomic mass of $^{10}_{5}B$ = 10.01 u

Atomic mass of $^{11}_{5}B$ = 11.01 u

From the periodic table, the average atomic mass of boron is B = 10.81 u.

Plan Your Strategy

Express the abundance of each isotope as a decimal rather than a percent. The total abundance of both isotopes is therefore 1. Let the abundance of boron-10 be x. Let the abundance of boron-11 be $1 - x$. Set up an equation, and solve for x.

Act on Your Strategy

$$\begin{aligned}
\text{Average atomic mass} &= x(\text{atomic mass B-10}) + (1 - x)(\text{atomic mass B-11}) \\
10.81 &= x(10.01) + (1 - x)(11.01) \\
10.81 &= 10.01x + 11.01 - 11.01x \\
11.01x - 10.01x &= 11.01 - 10.81 \\
x &= 0.2000
\end{aligned}$$

The abundance of boron-10 is 0.2000.

The abundance of boron-11 is $1 - x$, or $1 - 0.2000 = 0.8000$.

The abundance of $^{10}_{5}B$ is therefore 20.00%. The abundance of $^{11}_{5}B$ is 80.00%.

Check Your Solution

The fact that boron-11 comprises 80% of naturally occurring boron makes sense, because the average atomic mass of boron is 10.81 u. This is closer to 11.01 u than to 10.01 u.

Continued ...

CHEM FACT

If you wear contact lenses, you may use boron every day. Boron is part of boric acid, H_3BO_3, which is contained in many cleaning solutions for contact lenses.

Continued ...
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Practice Problems

5. Hydrogen is found primarily as two isotopes in nature: $^{1}_{1}H$ (1.0078 u) and $^{2}_{1}H$ (2.0140 u). Calculate the percentage abundance of each isotope based on hydrogen's average atomic mass.

6. Lanthanum is composed of two isotopes: $^{138}_{57}La$ (137.91 u) and $^{139}_{57}La$ (138.91 u). Look at the periodic table. What can you say about the abundance of $^{138}_{57}La$?

7. Rubidium ignites spontaneously when exposed to oxygen to form rubidium oxide, Rb_2O. Rubidium exists as two isotopes: $^{85}_{37}Rb$ (84.91 u) and $^{87}_{37}Rb$ (86.91 u). If the average atomic mass of rubidium is 85.47 u, determine the percentage abundance of $^{85}_{37}Rb$.

8. Oxygen is composed of three isotopes: $^{16}_{8}O$ (15.995 u), $^{17}_{8}O$ (16.999 u), and $^{18}_{8}O$ (17.999 u). One of these isotopes, $^{17}_{8}O$, comprises 0.037% of oxygen. Calculate the percentage abundance of the other two isotopes, using the average atomic mass of 15.9994 u.

Section Wrap-up

In this section, you learned how isotopic abundance relates to average atomic mass. Since you know the average mass of an atom in any given element, you can now begin to relate the mass of a single atom to the mass of a large number of atoms. First you need to establish how many atoms are in easily measurable samples. In section 5.2, you will learn how chemists group atoms into convenient amounts.

Section Review

1. K/U The average atomic mass of potassium is 39.1 u. Explain why no single atom of potassium has a mass of 39.1 u.

2. I Naturally occurring magnesium exists as a mixture of three isotopes. These isotopes (with their isotopic abundances and atomic masses) are Mg-24 (78.70%, 23.985 u), Mg-25 (10.13%, 24.985 u), and Mg-26 (11.17%, 25.983 u). Calculate the average atomic mass of magnesium.

3. C Assume that an unknown element, X, exists naturally as three different isotopes. The average atomic mass of element X is known, along with the atomic mass of each isotope. Is it possible to calculate the percentage abundance of each isotope? Why or why not?

4. C You know that silver exists as two isotopes: silver-107 and silver-109. However, radioisotopes of silver, such as silver-105, silver-106, silver-108, and silver-110 to silver-117 are known. Why do you not use the abundance and mass of these isotopes when you calculate the average atomic mass of silver? Suggest two reasons.