Schrödinger Wave Function of Electrons in 3D space

Multi-electron atoms require approximative methods. The family of solutions are:[32]

$$\psi_{n\ell m}(r,\theta,\phi) = \sqrt{\left(\frac{2}{na_0}\right)^3 \frac{(n-\ell-1)!}{2n[(n+\ell)!]^3}} e^{-r/na_0} \left(\frac{2r}{na_0}\right)^{\ell} L_{n-\ell-1}^{2\ell+1}\left(\frac{2r}{na_0}\right) \cdot Y_{\ell}^{m}(\theta,\phi)$$

where:

- $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}$ is the Bohr radius,
- $L_{n-\ell-1}^{2\ell+1}(\cdots)$ are the generalized Laguerre polynomials of degree $n-\ell-1$.
- n, ℓ, m are the principal, azimuthal, and magnetic quantum numbers respectively: which take the values:

$$\begin{aligned} n &= 1,2,3\cdots \\ \ell &= 0,1,2\cdots n-1 \\ m &= -\ell\cdots\ell \end{aligned}$$