GRASP

Science Skills Toolkit 11

Sample Problem: Mirror Equations and Concave Surfaces

Problem

A concave mirror has a focal length of 12 cm. An object with a height of 2.5 cm is placed 40.0 cm in front of the mirror.

a. Calculate the image distance.

b. Calculate the image height.

Hint: Starting with the equation $\frac{1}{d_i} = \frac{1}{12\text{ cm}} - \frac{1}{40.0\text{ cm}}$, use the 1/x button to evaluate the second two terms. Put 12 in your calculator, and press the 1/x button. You will get 0.8333. Now put 40 in your calculator, and press the 1/x button. You will get 0.025. Your equation has become

$\frac{1}{d_i} = 0.083\,33 - 0.025$

$= 0.058\,333$

Press the 1/x button again, and the result is $d_i = 17.14$ cm, which rounds to 17 cm.

Solution

a. Use the mirror equation to find the image distance.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$= \frac{1}{12\text{ cm}} - \frac{1}{40.0\text{ cm}}$$

$$= \frac{10}{120\text{ cm}} - \frac{3}{120\text{ cm}}$$

$$= \frac{7}{120\text{ cm}}$$

$$d_i = \frac{120\text{ cm}}{7}$$

$$= 17.14\text{ cm}$$

The image is 17 cm (after rounding) from the mirror. The sign is positive, so the image is in front of the mirror.

b. Use the magnification equation to find h_i.

$$\frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

$$\frac{h_i}{2.5\text{ cm}} = \frac{-17.14}{40.0}$$

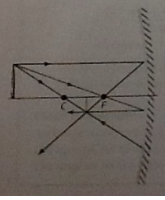
$$h_i = 2.5\text{ cm}\left(\frac{-17.14}{40.0}\right)$$

$$h_i = -1.07\text{ cm}$$

The height of the image is 1.1 cm (after rounding). The image height is negative, so the image is inverted.

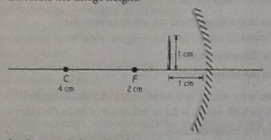
Check Your Solution

The value of C is twice the value of F, so C is 2 × 12 cm = 24 cm. The object is at 40 cm, so it is beyond C. Therefore, the image should be closer to the mirror than the object, smaller than the object, and inverted. All of these characteristics agree with the answers. The ray diagram on the left verifies the solution.

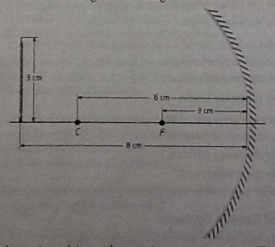


Practice Problems

1. A concave mirror has a focal length of 6.0 cm. An object with a height of 0.60 cm is placed 10.0 cm in front of the mirror.
   a. Calculate the image distance.
   b. Calculate the image height.
2. In the diagram below, the object is between the mirror and F. Use the data in the diagram to answer the questions below.
   a. Calculate the image distance.
   b. Calculate the image height.



3. In the diagram below, the object is beyond C. Use the data in the diagram to answer the questions below.
   a. Calculate the image distance.
   b. Calculate the height of the image.

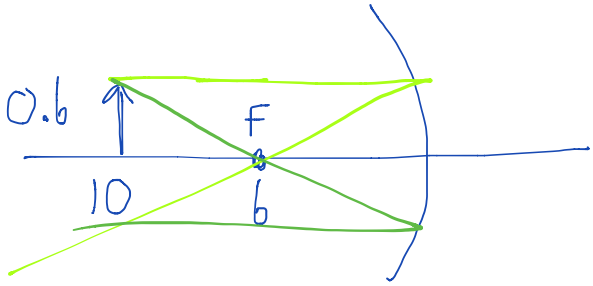


4. A dancer is applying make-up using a concave mirror. The dancer's face is 35 cm in front of the mirror, and the image is 72 cm behind the mirror. Use the mirror equation to calculate the focal length of the mirror.
5. A concave mirror magnifies an object placed 30.0 cm from the mirror by a factor of +3.0. Calculate the radius of curvature of the mirror.

Practice Problems

1. A concave mirror has a focal length of 6.0 cm. An object with a height of 0.60 cm is placed 10.0 cm in front of the mirror.
   a. Calculate the image distance.
   b. Calculate the image height.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$



$$\frac{1}{d_i} = \frac{1}{6} - \frac{1}{10}$$

$$\frac{h_i}{h_o} = \frac{-d_i}{d_o} \qquad h_i = \frac{(-15)(0.6)}{10}$$

$$\frac{1}{d_i} = \frac{10-6}{60}$$

$$\frac{h_i}{0.6} = \frac{15}{10} \qquad = -0.9\text{ cm}$$

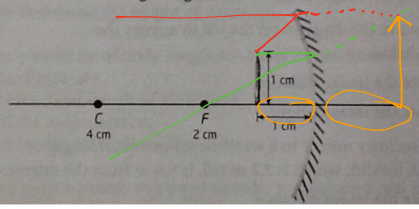
$$d_i = \frac{60}{4} = +15\text{ cm}$$

$$\frac{10\,h_i}{10} = \frac{(0.6)(15)}{10}$$

2. In the diagram below, the object is between the mirror and *F*. Use the data in the diagram to answer the questions below.

a. Calculate the image distance.

b. Calculate the image height.



$R = d_i$

$G = f = 2\,cm$

$d_o = 1\,cm$

$A = \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$

$\frac{h_i}{h_o} = \frac{-d_i}{d_o}$

$\frac{h_i}{1} = \frac{-(-2)}{1}$

$h_i = 2\,cm$

$\frac{1}{d_i} = \frac{1}{2} - \frac{1}{1}$ $\quad \frac{1-2}{2}$

$d_i = -2\,cm$ $\quad \frac{-1}{2} = \frac{1}{d_i}$

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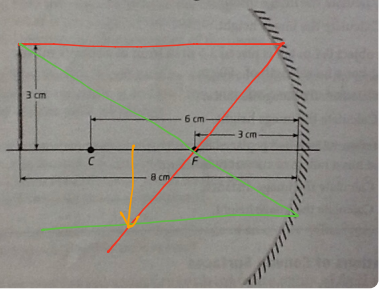
3. In the diagram below, the object is beyond *C*. Use the data in the diagram to answer the questions below.

a. Calculate the image distance.

b. Calculate the height of the image.



$R = d_i$ $\quad G = d_o = 8\,cm$ $\quad f = 3\,cm$ $\quad h_o = 3\,cm$

$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$

$\frac{1}{d_i} = \frac{1}{3} - \frac{1}{8}$

$\frac{1}{d_i} = \frac{8-3}{24}$

$\frac{1}{d_i} = \frac{5}{24}$

$d_i = \frac{24}{5}$

$d_i = 4.8\,cm$

$R = h_i$ $\quad G = h_o, d_i, d_o$

$\frac{h_i}{h_o} = \frac{-d_i}{d_o}$ $\quad \frac{h_i}{3\,cm} = \frac{-4.8\,cm}{8\,cm}$ $\quad h_i = \frac{3 \times -4.8}{8} = -1.8\,cm$

4. A dancer is applying make-up using a concave mirror. The dancer's face is 35 cm in front of the mirror, and the image is 72 cm behind the mirror. Use the mirror equation to calculate the focal length of the mirror.

$R = f$

$G = d_o = 35\text{cm}$

$d_i = -72\text{cm}$

$A = \frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$

$$\begin{cases} \frac{1}{f} = \frac{1}{35} - \frac{1}{72} \\ \frac{1}{f} = \frac{72-35}{2520} \\ \frac{1}{f} = \frac{37}{2520} \\ f = \frac{2520}{37} \end{cases}$$

$= 68.1\text{ cm}$

5. A concave mirror magnifies an object placed 30.0 cm from the mirror by a factor of +3.0. Calculate the radius of curvature of the mirror.

Chapte

$C = 2F$

$d_o = 30$

$M = 3$

$M = \frac{-d_i}{d_o}$

$M \times d_o = -d_i$

$30 \times 3 = -d_i$

$d_i = -90\text{ cm}$

$R = f$

$G =$ $\quad d_i = -90\ cm$

$\quad d_o = 30\ cm$

A - $\quad \frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$

$$\frac{1}{f} = \frac{1}{-90} + \frac{1}{30}$$

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{90}$$

$$= \frac{3-1}{90}$$

$$\frac{1}{f} = \frac{2}{90}$$

$$f = 45\ cm$$

$C = 2 \times 45\,cm$

$C = 90\,cm$