

Applications of the Ideal Gas Law

12.2

Studying gases and their properties becomes more interesting when you realize that your own body is a container for gases. Your lungs hold air, an important solution of gases that you need to live. As well, there is the embarrassing type of gas, called methane, that results from the digestive process. How many moles of oxygen do your lungs hold? Using the laws and properties you have learned, you can now calculate this fact about your own body. (See Figure 12.9.)

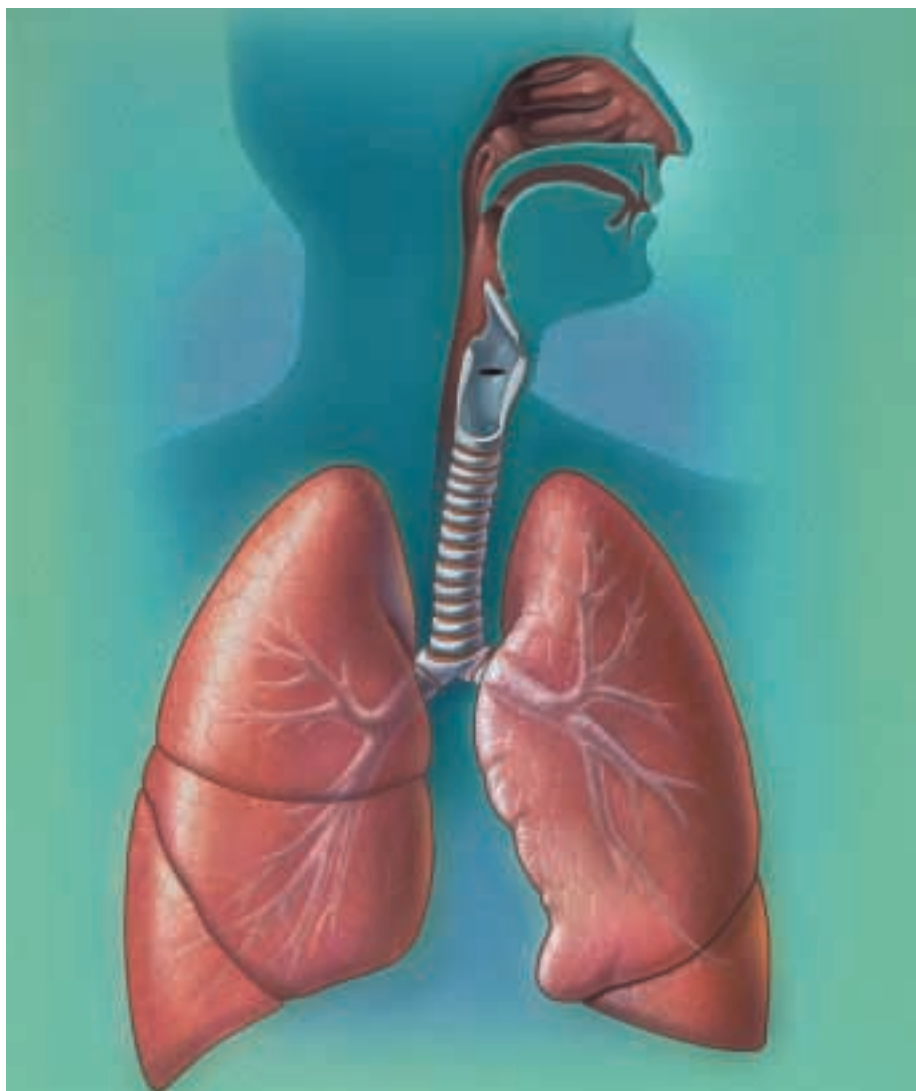


Figure 12.9 Your lungs hold about 4 L of air. When you breathe out, only 500 mL of air is expelled. The same amount of air is taken in when you inhale one breath. If air contains 20% oxygen gas, how many moles of oxygen gas do your lungs contain at 37°C and 100 kPa?

In this section, you will learn more about two properties of a gas that are closely related to molar volume: density and molar mass. You have already encountered these properties, but now you will use them to help you with your gas calculations.

Section Preview/ Specific Expectations

In this section, you will

- **solve** more quantitative problems using the ideal gas law
- **determine** the molar mass of an unknown gas through experimentation

Biology LINK

As you breathe in, your diaphragm moves down and your rib cage moves out to increase the volume of your lungs. As the volume increases, the air pressure in your lungs decreases. Air from the outside rushes in to fill the expanded volume. This is an application of Boyle's law. In the same way, your lungs decrease in volume when you exhale. The resulting increase in pressure pushes air out of your lungs.

CHECKPOINT

In this book, we use a capital *M* to symbolize molar mass. When working with solutions, you may see *M* used to express the molar concentration of a solution (for example, 6 M HCl_(aq)). Make sure that you do not become confused! Always check to make sure that you understand what *M* is referring to.

Density and Molar Mass

As you learned in the last section, the *molar volume* of a gas is defined as the space that is occupied by one mole of the gas. It is always given in units of L/mol.

The *density* of a gas is similar to the density of a solid or a liquid. Density is found by dividing mass by volume. The density of a gas is usually reported in units of g/L.

The *molar mass* of a gas refers to the mass (in g) of one mole of the gas. You can calculate molar mass by adding the masses of atoms in the periodic table. You can also calculate molar mass by dividing the mass of a sample by the number of moles that are present. Molar mass is always expressed in the units g/mol. Table 12.2 summarizes molar volume, density, and molar mass.

Table 12.2 Molar Volume, Density, and Molar Mass

	Molar volume	Density	Molar mass
Unit	L/mol	g/L	g/mol
Meaning	volume/amount	mass/volume	mass/amount
Calculations	$\text{Molar volume} = \frac{\text{Volume}}{\text{Number of moles}}$ $MV = V/n$	$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$ $D = m/V$	<p>Molar mass = Sum of molar masses of the atoms in the compound</p> <p>or</p> $\text{Molar mass} = \frac{\text{Mass}}{\text{Number of moles}}$ $M = m/n$

mind STRETCH

Molar volume (L/mol), density (g/L), and molar mass (g/mol) are closely related. You can calculate each property using the other two. Analyze the units to discover the exact relationship.

Dense Gases Can Be Deadly

You have learned that the volume of a gas at a certain temperature and pressure is the same as the volume of any other gas at the same temperature and pressure. For example, all gases have the approximate volume of 22.4 L at STP. The *molecular masses* of different gases, however, are all different. This means that each gas has a different density, or mass per unit of volume. (Look back at Figure 12.7 to see three examples.)

Understanding the densities of gases can be useful in the everyday world. For example, miners who drill deep into the ground must know which gases are present, and which have the highest densities. (See

Figure 12.10.) They must take appropriate safety precautions to avoid explosions, poisoning, or suffocation.

On December 3, 1984, an industrial accident in Bhopal, India, released a large quantity of methylisocyanate, CH₃NCO, a dense gas, into the air. This highly irritating and toxic gas caused the death of more than 3000 people living nearby. The Chemistry Bulletin on the next page gives an example of a natural disaster involving a dense gas. Figure 12.11, on page 492, illustrates a popular use for gases that are less dense than air.



Figure 12.10 Dense gases sit at the bottoms of pits, such as mines and wells. Miners must understand the behaviour of these gases to avoid accidents. For example, dense carbon dioxide gas can cause suffocation.

The Killing Lakes of Cameroon

On August 15, 1984, a cloud of deadly gas burst from Lake Monoun in Cameroon, a country in western Africa. Thirty-seven people died from suffocation. Two years later, on August 21, 1986, Lake Nyos, a larger and deeper lake, ejected a full cubic kilometre of the same gas. The gas travelled silently into neighbouring villages, killing 1700 people and thousands of livestock. What was this toxic gas?



Lake Nyos and Lake Monoun both sit in volcanic craters. The lakes are hazardous because of their volcanic origin, even though both volcanoes are dormant. Volcanoes are vents through Earth's crust. They carry *magma*, a mixture of molten rock and dissolved gases, to the surface of Earth. When magma rises to Earth's surface in volcanoes, the pressure is decreased. The gases come out of solution and expand.

Magma contains a large quantity of dissolved carbon dioxide and minerals. These are released from the magma into the ground water under a volcano's crater. Lake Nyos and Lake Monoun are both fed at the bottom by volcanic springs of mineral-rich carbonated ground water. In some crater lakes, there is enough water circulation for the carbon dioxide to bubble up from the bottom and be released at the surface. The surface waters of Lake Nyos and Lake Monoun, however, do not mix with deeper waters. The carbon dioxide remains at the bottom. Volcanic springs continue to supply

the lakes with carbon dioxide, which remains trapped at the bottom. Fresher surface water sits on this dense lower layer.

Scientists believe that the tragedies at Lake Nyos and Lake Monoun were caused by a disruption of the water layers, perhaps triggered by a landslide, an earthquake, or even a strong wind. Lower carbonated water was suddenly released into the upper water. As it moved up to the surface, the pressure on the gas decreased while the temperature increased. As a result, the carbon dioxide gas rapidly bubbled out of solution. At Lake Nyos, the sudden release of the lower lake water into the upper layers caused a plume of water and gas to rise high into the air.

How did this massive release of carbon dioxide cause so many deaths? Carbon dioxide gas is one-and-a-half times as dense as air. It sinks to the ground and displaces the oxygenated air we need to breathe. The invisible, odourless carbon dioxide that was released from Lake Nyos and Lake Monoun settled on the ground. It travelled rapidly down the slopes into populated regions. People quickly became unconscious and died of suffocation.

Carbon dioxide continues to accumulate at the bottoms of both lakes. An international team of scientists is developing a plan to release this carbon dioxide by controlled degassing. They plan to insert long pipes deep into both lakes and suck up some of the dense bottom water. This will create a pressure difference and cause a fountain of gas-rich water to jet from the pipes.

Making Connections

1. Calculate and compare the density of one mole of $\text{CO}_{2(g)}$ with the density of one mole of $\text{N}_{2(g)}$ at STP. (Nitrogen gas is the main constituent of air.) Explain the significance of your calculations.
2. How is carbonated water produced artificially, and what is it used for? Do research to find out.

www.school.mcgrawhill.ca/resources/

Many accidents have occurred because of dense gases in mines. Which gases are dangerous? What safety precautions should be taken? To learn more, go to the web site above. Go to **Science Resources**, then to **Chemistry 11** to find out where to go next.



Figure 12.11 Helium balloons and hot air balloons also depend on the density of gases. These objects are filled with gases that are less dense than air. Therefore, they are able to float.

In the following Sample Problem, you will use the ideal gas law to find the density of nitrogen gas.

Sample Problem

Finding the Density of Nitrogen Gas

Problem

Nitrogen gas makes up almost 80% of our atmosphere. What is the density of pure nitrogen gas, in g/L, at 12.50°C and 126.63 kPa?

What Is Required?

Find the density of nitrogen gas, in g/L, at 12.50°C and 126.63 kPa pressure.

What Is Given?

$$P = 126.63 \text{ kPa}$$

$$T = 12.50^{\circ}\text{C}$$

Plan Your Strategy

- Step 1** Change the temperature to kelvins. The temperature in this question is given to two decimal places. Therefore, use a conversion factor with two decimal places: + 273.15.
- Step 2** Calculate the molar mass (M) of nitrogen gas, N_2 , using the molar mass in the periodic table.
- Step 3** Since the volume is not given, set it as 1.00 L. Since the pressure is given in kilopascals, use $R = 8.314 \text{ kPa}\cdot\text{L}/\text{mol}\cdot\text{K}$. Substitute the numbers and units for P , T , R , and V into the ideal gas law equation. Solve for n .

Continued ...

In World War I, a highly irritating gas called phosgene, COCl_2 , was used against the Allied troops. Phosgene is about 3.4 times more dense than air. In concentrations above 50 ppm, it causes the lungs to fill up with fluid. This results in diminished lung capacity, and subsequent collapse of the heart. Phosgene can bring about death within hours.

Step 4 Convert n to the mass of nitrogen (m) by multiplying the number of moles by the molar mass (M) of N_2 .

Step 5 Find the density by dividing the mass by the volume (1.00 L).

Act on Your Strategy

Step 1 $T = (12.50^\circ\text{C} + 273.15)$
 $= 285.65 \text{ K}$

Step 2 $M_{N_2} = 2 \times 14.01 \text{ g/mol}$
 $= 28.02 \text{ g/mol}$

Step 3 $PV = nRT$
 $\therefore n = \frac{PV}{RT}$
 $= \frac{(126.63 \text{ kPa})(1.00 \text{ L})}{(8.314 \text{ kPa}\cdot\text{L/mol}\cdot\text{K})(285.65 \text{ K})}$
 $= 5.3320 \times 10^{-2} \text{ mol}$

Step 4 The mass of N_2 is

$$m = n \times M$$

$$= (5.3348 \times 10^{-2} \text{ mol})(28.02 \text{ g/mol})$$

$$= 1.4940 \text{ g}$$

Step 5 The mass of 1.00 L of nitrogen gas is 1.494 g. By dividing this by the volume, 1.00 L, we obtain a density of 1.494 g/L.

Check Your Solution

When the units cancel out in the ideal gas equation, mol remains.

When units cancel in the density equation, g/L remain.

The least number of digits in the question is four. Therefore, the answer should have four significant digits, which it does.

Practice Problems

- Oxygen makes up about 20% of our atmosphere. Find the density of pure oxygen gas, in g/L, for the conditions in the Sample Problem: 12.50°C and 126.63 kPa .
- Find the density of butane gas, C_4H_{10} , (in g/L) at SATP conditions: 298 K and 100 kPa .
- The atmosphere of the imaginary planet Xylo is made up entirely of poisonous chlorine gas, Cl_2 . The atmospheric pressure of this inhospitable planet is 155.0 kPa , and the temperature is 89°C . What is the density of the atmosphere?
- The atmosphere of planet Yaza, from the same star system as Xylo, is made of fluorine gas, F_2 . The density of the atmosphere on Yaza is twice the density of the atmosphere on Xylo. The temperature of both planets is the same. What is the atmospheric pressure of Yaza?

COURSE CHALLENGE

How can you use what you learn in this Sample Problem to help you identify the gas given off by water plants? You will apply your learning later on, in the Chemistry Course Challenge.

Molar Mass of a Gas

You can find the molar mass of a gaseous element or compound in the same way that you find the molar mass of any other element or compound: by adding up the masses of the atoms. You can also find the molar mass by dividing the mass by the number of moles. (See Figure 12.12.)



Figure 12.12 How could you determine the molar mass of vaporized iodine gas on paper? How could you determine it in a laboratory?

In the laboratory, calculating the molar mass of an unknown gas can help you identify it. The next Sample Problem will demonstrate this.

Sample Problem

Using Molar Mass to Identify an Unknown Gas

Problem

A scientist isolates 2.366 g of a gas. The sample occupies a volume of 800 mL at 78.0°C and 103 kPa. Use these data to calculate the molar mass of the gas. Is the gas most likely to be bromine, krypton, neon, or fluorine?

What Is Given?

$$P = 103 \text{ kPa}$$

$$V = 800 \text{ mL}$$

$$m = 2.366 \text{ g}$$

$$R = 8.314 \text{ kPa}\cdot\text{L/mol}\cdot\text{K}$$

$$T = 78.0^\circ\text{C}$$

What Is Required?

You need to find the number of moles. Then you need to use the mass and the number of moles to find the molar mass of the gas.

Continued ...

Plan Your Strategy

Convert the temperature to kelvins. Solve $PV = nRT$ for n . Then substitute in the known values to find the number of moles. Finally, set up a proportion to find the number of grams that would be in one mole, using the equation $M = \frac{m}{n}$, where M is the molar mass, m is the mass, and n is the number of moles.

Act on Your Strategy

$$\begin{aligned} T(\text{K}) &= T(^{\circ}\text{C}) + 273 \\ &= 78.0^{\circ}\text{C} + 273 \\ &= 351 \text{ K} \end{aligned}$$

Using the ideal gas law,

$$PV = nRT$$

$$\begin{aligned} \therefore n &= \frac{PV}{RT} \\ &= \frac{103 \text{ kPa} \times 0.800 \text{ L}}{8.314 \text{ kPa}\cdot\text{L/mol}\cdot\text{K} \times 351 \text{ K}} \\ &= 0.0282 \text{ mol} \end{aligned}$$

$$\begin{aligned} M &= \frac{m}{n} \\ &= \frac{2.366 \text{ g}}{0.0282 \text{ mol}} \\ &= 83.9 \text{ g/mol} \end{aligned}$$

The molar mass of the gas is 83.9 g/mol.

To identify the gas, compare the molar masses of the four gases mentioned.

Bromine, Br_2 , has a molar mass of $2 \times 79.9 \text{ g/mol} = 141.8 \text{ g/mol}$.

Krypton, Kr, has a molar mass of 83.8 g/mol.

Neon, Ne, has a molar mass of 20.2 g/mol.

Fluorine, F_2 , has a molar mass of $2 \times 18.9 \text{ g/mol} = 38.0 \text{ g/mol}$.

Therefore, the gas must be krypton.

Check Your Solution

The units of the answer are g/mol, the correct units for molar mass. The answer has three significant digits, equal to the least number of digits in the question.

The answer is probably correct, since it is so close to the molar mass of one of the given gases.

In the following investigation, you will find the molar mass of an unknown gas. You will use this mass to identify the gas.

The ideal gas law gives great flexibility for solving many different types of problems. After the investigation, you will find another Sample Problem. It illustrates how you can use the ideal gas law with methods you have previously learned, to identify an unknown gas. Practice problems are located at the end of this Sample Problem.

Calculating the Molar Mass of an Unknown Gas: Teacher Demonstration

Cigarette lighters contain a gaseous fuel that burns quickly. It produces a large amount of heat using only a small amount of gas. Your teacher will measure the volume and mass of a sample of this gas. Then you will use these data to calculate the molar mass of the gas.

Materials

4 L beaker or plastic pail
500 mL graduated cylinder
disposable cigarette lighter
needle nose pliers
plastic wrap
balance or scale
tap water
thermometer
barometer
hair dryer

Your teacher will take the following Safety Precautions and perform the following steps.

Safety Precautions

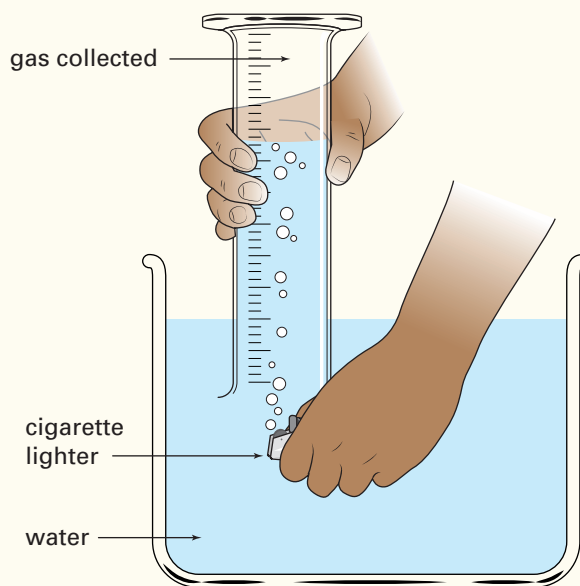


- Remember that this gas is flammable. Do not try to produce flames with the lighters. Before beginning, check that there are no flames (such as lit Bunsen burners) in the laboratory.
- If water is spilled on the floor, wipe it up immediately so that no one steps in it.
- Release all the gas collected into an operating fume hood after the investigation. Expel the remaining gas in the lighter outside before disposing of the lighter.

Procedure

- Use pliers to remove the striker, flint, and spring from the disposable lighter.

- Fill the 4 L beaker (or pail) about two-thirds full of tap water. Determine the temperature of the tap water. Use this measurement to approximate the temperature of the gas.
- Briefly immerse the lighter in the water, then shake it dry. Use the hair dryer on a low or cool setting to dry the lighter as much as possible. This is to set a standard for drying the empty lighter later. **CAUTION** Do not overheat the lighter.
- Determine the mass of the lighter.
- Fill the graduated cylinder with water. Cover the cylinder tightly with a piece of plastic wrap. With your hand over the plastic wrap, place the cylinder upside down into the beaker. Make sure that no air bubbles are trapped in the cylinder. Slide the plastic wrap away. A water-filled measuring tube to collect gas that has been created. The gas will displace the water as it rises, giving an accurate measurement.



6. As shown in the diagram, hold the lighter underwater, below the graduated cylinder in the beaker. Carefully depress the button on the lighter to release gas into the cylinder. The entire lighter does not need to be emptied. Just gather enough gas for an accurate measurement.
7. Add tap water to the beaker (or pail), or lift the cylinder, so that *the water inside the cylinder is at exactly the same level as the water in the beaker*. This equalizes the pressure in the cylinder with the pressure of the atmosphere. Record the volume of the gas collected when the water levels are equal inside and outside the cylinder.
8. Dry the lighter with the hair dryer. Measure its mass.
9. Record the air pressure in the room.
10. Wash your hands.

Analysis

1. (a) Subtract the final mass of the lighter from its initial mass. This will give you the mass of the gas used.
(b) Use the volume, the mass, the temperature of the water, and the air pressure to calculate the number of moles of gas.
2. Use the mass of the gas and the number of moles to calculate the molar mass.

Conclusions

3. The gas in the lighter has the formula C_nH_{2n+2} .
(a) Use the periodic table to calculate the molar masses of the compounds with $n = 1$ to $n = 5$.
(b) Identify the gas in the lighter. You will learn how to name this gas in Chapter 13.
4. How do your results compare with the theoretical molar mass calculated from the periodic table?
5. Calculate the percent error for your results.
6. What were the sources of error?



Figure 12.13 Methane gas, CH_4 , is used as a fuel for Bunsen burners. Suppose that you are given a container of an unknown gas. What methods could you use to find out if it is methane?

Sample Problem

Identifying a Compound Using Percent Composition and the Ideal Gas Law

Problem

As geologists study the area where an ancient marsh was located, they discover an unknown gas seeping from the ground. They collect a sample of the gas, and take it to a lab for analysis. Lab technicians find that the gas is made up of 80.0% carbon and 20.0% hydrogen. They also find that a 4.60 g sample occupies a volume of 2.50 L at 1.50 atm and 25.0°C. What is the molecular formula of the gas?

What Is Required?

You need to find the molecular formula (and thus the identity) of an unknown gas.

What Is Given?

$$P = 1.50 \text{ atm}$$

$$V = 2.50 \text{ L}$$

$$m = 4.60 \text{ g}$$

$$R = 8.314 \text{ kPa}\cdot\text{L/mol}\cdot\text{K}$$

$$T = 25.0^\circ\text{C}$$

The percentage composition of the gas is 80.0% carbon and 20.0% hydrogen.

Plan Your Strategy

- Step 1** Find the empirical formula of the gas, using the molar masses of carbon and hydrogen and the percent compositions.
- Step 2** Solve the ideal gas law for the number of moles. First change the temperature to kelvins. Then change the pressure to kilopascals and use $R = 8.314 \text{ kPa}\cdot\text{L/mol}\cdot\text{K}$. (You could also use $R = 0.08206 \text{ atm}\cdot\text{L/mol}\cdot\text{K}$.)
- Step 3** Find the molar mass (M) of the compound by dividing the mass of the sample (m) by the number of moles (n) in the sample.
- Step 4** Compare the molar mass of the unknown gas with the molar mass of the empirical formula. To find the molecular formula, multiply the empirical formula by the ratio of the two molar masses.

Act on Your Strategy

- Step 1** Find the empirical formula. Assume that the total mass of the sample is 100.0 g. Thus the mass of the carbon in the sample is $80.0\% \times 100.0 \text{ g} = 80.0 \text{ g}$. The mass of the hydrogen is $20.0\% \times 100.0 \text{ g} = 20.0 \text{ g}$.

Continued ...

Now find the number of moles of carbon and hydrogen using the formula $n = \frac{m}{M}$

For carbon,

$$n = \frac{80.0 \text{ g}}{12.01 \text{ g/mol}} \\ = 6.67 \text{ mol}$$

For hydrogen,

$$n = \frac{20.0 \text{ g}}{1.01 \text{ g/mol}} \\ = 20.0 \text{ mol}$$

Finally, find the simplest mole ratio of the two elements in the compound. This will be the empirical, or simplest, formula of the gas.

The ratio of the elements in the compound is

$$\frac{6.67}{6.67} \text{ mol of C to } \frac{20.0}{6.67} \text{ mol of H}$$

or 1.0 mol of C to 3.0 mol of H

The empirical formula of the unknown gas is CH_3 .

Step 2 Use the ideal gas law.

$$T = (25.0^\circ\text{C} + 273) \\ = 298 \text{ K}$$

$$PV = nRT$$

$$\therefore n = \frac{PV}{RT}$$

If you convert the units of R ...	If you convert the pressure to kPa ...
$R = 0.08206 \text{ atm}\cdot\text{L/mol}\cdot\text{K}$	$P = 1.50 \text{ atm} \times 101.3 \text{ kPa/atm}$ $= 152 \text{ kPa}$
$n = \frac{(1.50 \text{ atm})(2.50 \text{ L})}{(0.08206 \text{ atm}\cdot\text{L/mol}\cdot\text{K})(298 \text{ K})}$ $= 0.153 \text{ mol}$	$n = \frac{(1.52 \text{ kPa})(2.50 \text{ L})}{(8.314 \text{ kPa}\cdot\text{L/mol}\cdot\text{K})(298 \text{ K})}$ $= 0.153 \text{ mol}$

Step 3 Find the molar mass.

$$M = \frac{m}{n} \\ = \frac{4.60 \text{ g}}{0.153 \text{ mol}} \\ = 30.1 \text{ g/mol}$$

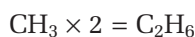
Step 4 Find the molecular formula.

The molar mass of the compound is 30.1 g/mol.

The molar mass of the empirical formula, CH_3 , is 15.04 g/mol.

$$\text{Ratio of molar masses} = \frac{30.07 \text{ g/mol}}{15.04 \text{ g/mol}} \\ = 2.00$$

Thus, the molecular formula of the unknown gas is twice the empirical formula.



Check Your Solution

In step 2, the units in the ideal gas equation cancel out to give moles. C_2H_6 is a reasonable answer for the molecular formula of the gas, since it is a simple integer ratio of the two molar masses.

Practice Problems

20. A 1.56 L gas sample has a mass of 3.22 g at 100 kPa and 281 K. What is the molar mass of the gas?
21. 2.0 L of haloethane has a mass of 14.1 g at 344 K and 1.01 atm. What is the molar mass of haloethane?
22. A vapour has a mass of 0.548 g in 237 mL, at 373 K and 755 torr. What is the molar mass of the vapour?
23. The mass of a 5.00 L evacuated container is 125.00 g. When the container is filled with argon gas at 298 K and 105.0 kPa, it has a mass of 133.47 g.
 - (a) Calculate the density of argon under these conditions.
 - (b) What is the density of argon at STP?
24. A gaseous compound contains 92.31% carbon and 7.69% hydrogen by mass. 4.35 g of the gas occupies 4.16 L at 22.0°C and 738 torr. Determine the molecular formula of the gas.

Section Wrap-up

In this section, you learned how density and molar mass are related to the ideal gas law. You also learned how to identify an unknown substance by calculating its molar mass, both theoretically and in the laboratory. Before you continue, take the time to complete the following Section Review questions. They will help you remember what you have learned.

Section Review

Unit Issue Prep

You will be debating a question related to gas pollution in the Unit Issue. How does the density of a gas determine whether the gas pollutes Earth's surface or the atmosphere?

- 1 **I** What is the density of methane, $CH_4(g)$, if 4.5 mol are in 100 L?
- 2 **I** 8.1 g of a gas occupy 12.3 L of space at 27°C and 8 atm.
 - (a) What is the molar mass of the gas?
 - (b) What might the gas be?
- 3 **I** A gas that consists of only nitrogen and oxygen atoms is found to contain 30% nitrogen. A 9.23 g sample of the gas occupies 2.2 L at STP. What is the gas?
- 4 **I** You are given a sample of an unknown gas. Describe how you can identify the gas in the laboratory. What measurements will you take? What apparatus might you need?
- 5 **K/U** How can the densities of two gases at STP be different, even though their volumes are the same?