

None of the problems on this assignment are graded. Instead, after completing it, please fill out the associated Google form. That form is the only graded component.

## Cryptography

The cryptography section is placed before the background here because it asks you to explicitly work together.

**Exercise 1.** *With your newfound abilities in modular arithmetic and the Caesar cipher, encrypt a message of your choice (be civil). Give your message, along with your shift key to a classmate who has not yet been given a message.*

**Exercise 2.** *Once you have received a message and key decrypt it.*

## Math Background

**Exercise 3.** *Let  $A = \{0, 1, 2, 3\}$ ,  $B = \{5, 7, 9, 11\}$ , and suppose  $f : A \rightarrow B$ . Write at least one such a function  $f$ . Make sure that every element of  $A$  is mapped to an element of  $B$ , and try to write it with fundamental operations like addition, subtraction, multiplication, division, etc ...*

**Exercise 4.** *Let  $A = \{0, 1, 2, 3\}$ ,  $B = \{5, 7, 9, 11\}$ , and suppose  $g : B \rightarrow A$ . Write at least one such a function  $g$ . Make sure that every element of  $B$  is mapped to an element of  $A$ , and try to write it with fundamental operations like addition, subtraction, multiplication, division, etc ...*

**Exercise 5.** *Consider the following congruence:  $x+3 \equiv 5 \pmod{26}$ . Try and find one solution (value for  $x$ ). If you can, try and find a formula for all solutions.*

## Bonus

**Exercise 6.** *Both the natural numbers ( $\mathbb{N}$ ) and the integers ( $\mathbb{Z}$ ) are sets of infinite size, but the natural numbers are a subset of the integers. Are the sets the same size? Try to justify why.*

*hint: In order to say whether two infinite sets are the same size, you need a suitable definition of size. If you aren't sure, start by looking up what makes two sets the same size.*