

This is not graded on correctness, merely the attempt. Please upload your solutions through the google form.

## Number Theory Proofs

**Exercise 1.** *Prove the following theorem: Let  $a$ ,  $b$ , and  $c$  be integers. If  $a|b$ , then  $a|bc$ .*

**Exercise 2.** *Prove the following theorem: Let  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $n$  be integers with  $n \neq 0$ . If  $a \equiv b \pmod{n}$  and  $c \equiv d \pmod{n}$ , then  $a + b \equiv c + d \pmod{n}$ .*

*Hint: It isn't sufficient to say that you can add the equations together, think about what it means for values to be congruent in this manner.*

**Exercise 3.** *Prove the following theorem: Let  $a$ ,  $b$ ,  $c$ , and  $n$  be integers with  $n > 0$ . If  $a \equiv b \pmod{n}$  and  $b \equiv c \pmod{n}$ , then  $a \equiv c \pmod{n}$ .*

## Bonus

**Exercise 4.** *Prove the following theorem: A natural number  $n$  is prime if and only if for all primes  $p \leq \sqrt{n}$ ,  $p$  does not divide  $n$ .*