

None of the problems on this assignment are graded. Instead, after completing it, please fill out the associated Google form. That form is the only graded component.

## Math Background

**Exercise 1.** Evaluate the following matrix products:

$$\bullet \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\bullet \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$$

**Exercise 2.** Invert the following matrices

$$\bullet \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\bullet \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$$

## Cryptography

**Exercise 3.** Consider the following matrix key with 3 known entries.

$$\bullet \begin{bmatrix} 1 & x \\ 1 & 2 \end{bmatrix}$$

How many values of  $x$  produce unique keys. Remember,  $x$  can take on unique values from 0 through 25. The determinant cannot be 0, divisible by 2, or divisible by 13.

**Exercise 4.** Write a message in plaintext, select a  $2 \times 2$  matrix as a key (make sure it is invertible modulo 26) and encrypt your message. Pass the message and the key to a friend and have them decrypt it (as always be civil!).

*Hint: There is a google colab notebook linked on the google classroom. This is not necessary to complete, but may make encryption and decryption easier if you complete it.*

## Bonus

Matrices are from a field of math called linear algebra. It is a super important and useful field of math. If you are curious about this field of math, feel free to read [this](#) or watch the corresponding videos. 3 blue 1 brown is an excellent resource for mathematics explanation and this is a good series.