Massachusetts Institute of Technology
6.046J/18.410J: Introduction to Algorithms
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Handout 9
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Master Theorem Worksheet

This is a worksheet to help you master solving recurrence relations using the Master Theorem. For each recurrence, either give the asympotic solution using the Master Theorem (state which case), or else state that the Master Theorem doesn't apply. You should be able to go through these **25** recurrences in **10** minutes.

Problem 1-1. $T(n) = 3T(n/2) + n^2$

case 3:
$f(n) = n^2$ dominates
$T(n) = \Theta(n^2)$

Problem 1-2. $T(n) = 7T(n/2) + n^2$

case 1: $f(n) = n^2$ dominated by the recursive term
$T(n) = \Theta(n^{\log_2 7})$

Problem 1-3. $T(n) = 4T(n/2) + n^2$

case 2: $f(n) = n^2$ matches the growth rate of the recursion
$T(n) = \Theta(n^2 \log n)$

Problem 1-4. $T(n) = 3T(n/4) + n \lg n$

case 3: $n \log n$ dominates
$T(n) = n \log n$

Problem 1-5. $T(n) = 4T(n/2) + \lg n$

case 1: $f(n) = \log n$ is significantly smaller
$T(n) = \Theta(n^2)$

Problem 1-6. $T(n) = T(n-1) + n$

$T(n-1)$ form is
not applicable
to the master theorem

Problem 1-7. $T(n) = 4T(n/2) + n^2 \lg n$

extended case 2:

$n^2 \log n$ dominates

$T(n) = \Theta(n^2 \log n)$

Problem 1-8. $T(n) = 5T(n/2) + n^2 \lg n$

case 1: $n^2 \log n$ is dominated by the recursive term

$T(n) = \Theta(n^{\log 5})$

Problem 1-9. $T(n) = 3T(n/3) + n/\lg n$

$\frac{n}{\log n}$ is not applicable to the master theorem

Problem 1-10. $T(n) = 2T(n/4) + c$

constant works outside of recursion

$T(n) = \Theta(n^{\log_4 2}) = \Theta(n^{0.5})$

Problem 1-11. $T(n) = T(n/4) + \lg n$

extended case 2:

$T(n) = \log^2 n$

Problem 1-12. $T(n) = T(n/2) + T(n/4) + n^2$

master theorem does not apply

Problem 1-13. $T(n) = 2T(n/4) + \lg n$

case 1: recursive term dominates $f(n) = \log n$

$T(n) = \Theta(n^{0.5})$

Problem 1-14. $T(n) = 3T(n/3) + n \lg n$

extended case 2:

$T(n) = \Theta(n \log^2 n)$

Problem 1-15. $T(n) = 8T((n - \sqrt{n})/4) + n^2$

$T((n-\sqrt{n})/4)$ does not apply to the master theorem

Problem 1-16. $T(n) = 2T(n/4) + \sqrt{n}$

case 2:

$T(n) = \Theta(n^{0.5} \log n)$

Problem 1-17. $T(n) = 2T(n/4) + n^{0.51}$

case 3: $f(n) = n^{0.51}$ dominates

$T(n) = \Theta(n^{0.51})$

Problem 1-18. $T(n) = 16T(n/4) + n!$

case 3: $f(n) = n!$ dominates

$T(n) = \Theta(n!)$

Problem 1-19. $T(n) = 3T(n/2) + n$

case 1:

$T(n) = \Theta(n^{\log 3})$

Problem 1-20. $T(n) = 4T(n/2) + cn$

case 1:

$T(n) = \Theta(n^2)$

Problem 1-21. $T(n) = 3T(n/3) + n/2$

case 2:

$T(n) = \Theta(n \log n)$

Problem 1-22. $T(n) = 4T(n/2) + n/\lg n$

Case 1:

$T(n) = \Theta(n^2)$

Problem 1-23. $T(n) = 7T(n/3) + n^2$

case 3: n^2 dominates

$T(n) = \Theta(n^2)$

Problem 1-24. $T(n) = 8T(n/3) + 2^n$

case 3: 2^n dominates

$T(n) = \Theta(2^n)$

Problem 1-25. $T(n) = 16T(n/4) + n$

case 1: $T(n) = \Theta(n^2)$