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Dynamic epistemic logic of belief change in legal judgments

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Abstract This study realizes belief/reliability change of a judge in a legal judgment by dynamic epistemic logic (DEL). A key feature of DEL is that possibilities in an agent's belief can be represented by a Kripke model. This study addresses two difficulties in applying DEL to a legal case. First, since there are several methods for constructing a Kripke model, our question is how we can construct the model from a legal case. Second, since this study employs several dynamic operators, our question is how we can decide which operators are to be applied for belief/reliability change of a judge. In order to solve these difficulties, we have implemented a computer system which provides two functions. First, the system can generate a Kripke model from a legal case. Second, the system provides an inconsistency solving algorithm which can automatically perform several operations in order to reduce the effort needed to decide which operators are to be applied. By our implementation, the above questions can be adequately solved. With our analysis method, six legal cases are analyzed to demonstrate our implementation.

Keywords Belief revision · Reliability change · Legal case · Dynamic epistemic logic

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1 Introduction

In legal proceedings, a judge has to reconsider his/her belief in light of new evidence in order to reach his/her own decision. That is, when a judge receives a piece of information, he/she has to decide if he/she will believe the received information or not. This is the same concept as *belief revision* which is a study of rationally revising beliefs according to new information. Let us briefly introduce an example taken from a legal case (the second legal case in "Appendix 1").

> In the inquiry stage, the witnesses gave the statements to the police, but after then all of them changed their statements in the Youth Court.

In order to analyze the legal case, belief revision is applied to illustrate belief change of a judge in a judgment process. Nowadays, belief revision is one of the areas that has been studied widely in the context of artificial intelligence (AI) such as in Dragoni and Giorgini (2001), Perrussel and Thévenin (2004) and Liu et al. (2014). In addition, *dynamic epistemic logic* (DEL) (Baltag et al. 2008), which is a branch of modal logic for studying about knowledge change, has been applied to formalize belief revision such as in van Ditmarsch et al. (2007), Kiel (2013) and Velázquez-Quesada (2014). These frameworks have been studied by several works. Roorda et al. (2002) presented a framework for belief revision based on modal logic, but their framework captured only belief expansion. van Benthem and Pacuit (2011) proposed more fine-grained models for supporting evidence dynamics. Their models provided three operations including addition, removal and modification for evidence management, though they omitted the notions of evidence sources and their reliability. This problem has been addressed by Liau (2003), Lorini et al. (2011) and Cholvy (2005). Their works focused on formalizing the information source and its reliability. However, any dynamics of reliability cannot be handled.

Since logic can be used to clarify the meaning and soundness of reasoning, several studies (Bench-Capon and Prakken 2008; Grossi and Rotolo 2011; Prakken and Sartor 2001) have presented logical approaches in the legal systems. An application of DEL to law was initially presented by Jirakunkanok et al. (2013) and then has been studied by Jirakunkanok et al. (2014, 2015a, b). In Jirakunkanok et al. (2013), two logical operators including *commitment* and *permission*[1] were presented in order to formalize belief re-revision which is not only a sequence of multiple belief revisions but also a restoration of former beliefs. An agent can remove some beliefs by applying the commitment operator, while the permission operator is used for restoring the former beliefs. Nevertheless, their work did not take the *reliability* of information sources into consideration. In Jirakunkanok et al. (2014), an influence of the reliability of information sources on belief/reliability change of an agent was stated. In order to formalize reliability change, two logical operators consisting of upgrade and downgrade operators were introduced for changing the grade of agents' reliability. Based on Jirakunkanok et al. (2013, 2014), a combination of logical operators was proposed for analyzing belief re-revision

[1] *Permission* in this context does not refer to an approval in a legal procedure, but an admission of a possibility in beliefs.

based on reliability change in Jirakunkanok et al. (2015a). In their work, a new logical operator called a joint downgrade operator was introduced in order to downgrade such agents in a specific group less reliable than the agents in the other groups and then make them equally reliable. According to Jirakunkanok et al. (2013, 2014), a new kind of permission called a private permission operator was introduced for capturing an action of an agent's privately permitting the possibility to his/her belief in Jirakunkanok et al. (2015b).

From the previous works in Jirakunkanok et al. (2013, 2014, 2015a, b), there are two difficulties in applying DEL to a legal case: First, since DEL provides a Kripke model which can be used to demonstrate possibilities, it is required to construct such a model. There are several ways for generating all possibilities and representing them by a Kripke model. In the previous works, they have different ways to construct the model for analyzing belief/reliability change of an agent. Thus, our question is how such models are constructed. Second, the previous works proposed many logical operators in terms of DEL for formalizing belief/reliability change of an agent and then applied them in different ways, that is, different sequences or combinations of such operators. Thus, our question is how such operators are to be applied for changing an agent's belief and/or reliability. For this reason, our motivation for this study is to solve these difficulties by addressing the following questions:

(Q1) How can we construct a model from a legal case?
(Q2) How can we decide which operators are to be applied for analyzing belief/reliability change of an agent?

In order to answer the above questions, we formalize belief/reliability change of a judge in a legal judgment and develop a computer system to realize this process. For our logical formalization, six operators based on Jirakunkanok et al. (2014, 2015a, b) are employed in order to analyze belief/reliability change of a judge. Three logical operators including *private announcement*, *private permission* and *careful policy* are used for formalizing belief re-revision. Reliability change can be formalized by three logical operators including *downgrade*, *upgrade* and *joint downgrade*. Our implementation provides two main functions:

(A1) The system can generate a model for analyzing belief/reliability change of a judge from a legal case according to our proposed method in Sect. 6.1.1.
(A2) The system can perform the inconsistency solving algorithm automatically. That is, when the system can detect that there is an agent giving inconsistent statements, the system will downgrade such agent by the joint downgrade and apply the private permission for restoring the former beliefs. After that, if there is the received information which is not inconsistent with the existing belief of an agent and is signed by the most reliable agent, the system will apply the private announcement for admitting such information. With this function, the second question (Q2) can be partially solved.

In order to answer the second question (Q2), we also propose a concrete algorithm to decide which operator should be applied in Sect. 6.1.1. In accordance with our implementation, we analyze six target legal cases.

This paper is organized as follows: Sect. 2 describes former formalisms including defeasible logic and belief revision. The related works are compared with our formalization in Sect. 3. Section 4 presents our dynamic logical formalization including the static logic of agents' beliefs equipped with the notion of signed information and dynamic operators for capturing belief re-revision and reliability change of an agent. An implementation of our logical formalization is proposed in Sect. 5. Section 6 introduces a method for analyzing the legal case. With this method, six target legal cases are analyzed by our implementation. Finally, Sect. 7 concludes this work and states our further directions.

2 Former formalisms

Non-monotonic reasoning is a viable tool for AI and can be described as a theory of reasoning. Human reasoning is *non-monotonic* because we sometimes need to accept a new knowledge which contradicts our former beliefs. Thus far, default logic, predicate completion, circumscription, autoepistemic logic, and so on have been proposed as non-monotonic logic in Obeid and Turner (1991), Cadoli and Schaerf (1993) and Prakken (1997). Furthermore, argumentation theory or argumentation (Dung 1995) has been used to provide non-monotonic reasoning in law. In order to deal with the dynamics of such human reasoning, two prominent formalisms based on non-monotonic reasoning including defeasible logic and belief revision are discussed in this section.

2.1 Defeasible logic

Defeasible logic (DL) (Nute 1994) is a simple and flexible non-monotonic formalism, based on deductive reasoning. The goal of this logic is to derive plausible conclusions from the knowledge base. There are five different kinds of features: facts, strict rules, defeasible rules, defeaters, and a superiority relation among rules. Strict rules cannot be defeated, while defeasible rules can be defeated by the contrary evidence. Defeaters are rules that cannot be used to draw any conclusions but to prevent some conclusions. Essentially, the superiority relation provides the priority orderings between rules where one rule may override the conclusion of another rule.

2.2 Belief revision

Belief revision (Gardenfors 1992) is a study of how an agent should revise his/her belief when he/she receives new information without generating inconsistencies. Belief revision composes of five basic operations, i.e., revision, contraction, expansion, consolidation, and merging. The difference between revision and merging operations can be described as follows: For revision, the new belief is considered to be more reliable than the old ones. When there is an inconsistency, some old beliefs are removed. For merging, the priority among the beliefs is considered to be the same. However, revision can be performed by first incorporating the new belief and then restoring the consistency by a consolidation operation. This can be considered as merging rather than revision because the new belief is not always treated as more reliable than the old ones.

2.3 Limitations

According to the above former formalisms, they cannot adequately handle belief re-revision and reliability change of an agent because of the following limitations:

- Both of them do not provide a process of restoring the former beliefs. DL provides only a process of retracting beliefs. Although belief revision provides a process of revising belief including removing and adding beliefs, it cannot restore the former beliefs. Therefore, both of them cannot deal with belief re-revision.
- Both of them do not consider reliability change of an agent. Although DL and belief revision consider the reliability by providing a concept of the priority, they do not consider it in terms of dynamics. That is, the priority ordering in DL or the reliability of beliefs in belief revision is static, i.e, it cannot be changed. Although both of them can be used to realize reliability change, DEL has a clearer formal semantics than DL and belief revision.

3 Related work

This section demonstrates that our work is different from recent developments in several aspects.

The first aspect is the issue of re-revision. Governatori and Rotolo (2010) investigated the notions of legal modifications in DL using techniques from revision based on belief sets and from base revision. A revision in their work was used to transform a normative system into another one by changing the rules in it. This is different from belief re-revision in our work which provides operations of removing and restoring accessible links. Aucher et al. (2009) presented a dynamic logic for capturing two operations including contraction and expansion of context. Their work provided a process of re-revision intended as restoration of former beliefs. However, belief re-revision in our work is different from that in Aucher et al. (2009) as follows: First, all operations in their framework including re-revision focus on the context of a theory, while our belief re-revision focuses on accessible links in Kripke semantics.

Second, let us focus on belief dynamics in a legal setting. Grossi and Velázquez-Quesada (2009) proposed a framework for formalizing three notions including awareness of, implicit and explicit information by a dynamic logic approach. In addition, their framework provided three operations including awareness, inference and announcement for demonstrating how agents change their beliefs. The first operation was used to extend an agent's awareness, while the second one was used to extend an agent's explicit information. The last one, announcement operation, represented an agent's communication. Although the announcement operation in Grossi and Velázquez-Quesada (2009) and the private announcement in our work have the same goal which aims to restrict the accessibility relation to some specific set, they give different results as follows: First, the announcement in Grossi and Velázquez-Quesada (2009) will remove both links and states, while our private announcement will remove only links. Second, our announcement will affect only one agent, while their announcement will affect all the agents.

Next, the sources of information or evidences are taken into account in belief change. Liu and Lorini (2016) introduced a new logic called Dynamic Epistemic Logic of Evidence Sources (DEL-ES) for reasoning about building and changing of an agent's belief by a consideration of evidence source. This logic allowed an agent to believe information φ if the weight of evidence supporting φ is considered to be sufficient. In addition, they demonstrated the proposed framework by a legal case. In their work, they did not study the relation between evidences and their sources. This point can be handled by *signature* operators in our work. Although their work formalized the reliability by the weight of evidence in a qualitative sense, they did not take agents' preferences into account. Thus, this framework does not suffice for our purpose of this study which aims to formalize an agent's changing of reliability orderings. Baltag et al. (2016) presented an extension of the evidence model framework in van Benthem and Pacuit (2011) for investigating several types of evidential dynamics based on DEL. In their work, a topological semantics for evidence, evidence-based justifications, belief and knowledge were introduced. Nevertheless, their framework did not cover the reliability issue that is one main target of our work.

Finally, let us consider software systems for DEL. van Benthem et al. (2015) proposed knowledge structures as a symbolic model checker for representing DEL models. Their framework was shown to work well on several benchmark problems such as the Muddy Children (van Ditmarsch et al. 2008) and Russian cards (van Ditmarsch 2003). However, their model checker cannot cover a representation of action models and a formalization of preference change that are our requirements. Therefore, their framework does not satisfy for our work. van Ditmarsch et al. (2012) presented an agent-based architecture in the setting of DEL, where agents' knowledge was represented in relational structures called Kripke models. They implemented their system in a DEL model checker called DEMO. With this system, virtual coding and decoding agents were proposed for simulating encryption and decryption. Nevertheless, their system had a problem for representing a large epistemic model by DEMO. Since our formalization provides both signature and belief operators, our Kripke model consists of several possible states representing belief and signed statements of agents. Therefore, the current DEMO may not suitable for our work.

4 Syntax and semantics of dynamic logical formalism

This section describes the syntax and semantics of our dynamic logical formalism. Firstly, we will explain the static logic of agents' belief for signed information. Then, we will present the syntax and semantics of our dynamic operators for formalizing belief re-revision and reliability change of an agent.

4.1 Static logic of agents' beliefs for signed information

To formalize belief re-revision and reliability change of an agent from a logical point of view, we introduce a modal language, based on previous works (Jirakunkanok et al. 2013, 2014), which enables us to formalize each agent's belief, the reliability of information sources, and signed information.

Let G be a fixed *finite* set of agents. Our syntax $\mathcal{L}_{BSR}$ consists of the following vocabulary: (i) a countably infinite set $\mathsf{Prop} = \{p, q, r, \ldots\}$ of propositional letters, (ii) Boolean connectives: $\neg$, $\wedge$, (iii) the belief operators $\mathsf{Bel}(a, \cdot)$ $(a \in G)$, (iv) the signature operators $\mathsf{Sign}(a, \cdot)$ $(a \in G)$, (v) the constants for reliability ordering $b \leqslant_a c$ $(a, b, c \in G)$, and (vi) the universal modality A. A set of formulas of $\mathcal{L}_{BSR}$ is inductively defined as follows:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathsf{Bel}(a, \varphi) \mid \mathsf{Sign}(a, \varphi) \mid b \leqslant_a c \mid \mathsf{A}\varphi,$$

where $p \in \mathsf{Prop}$ and $a, b, c \in G$. The intuitive readings of formulas are shown in Table 1. Note that $\mathsf{A}\varphi$ is read as "φ is true in all states." Signature operators are used to formalize information sources by a notion of signed information. For example, $\mathsf{Sign}(a, \varphi)$ represents information φ is given by agent a. This allows us to keep track of information sources. From Table 1, $\mathsf{Bel}\big(a, \mathsf{Sign}(b, \varphi)\big)$ can be interpreted by the context of information gathering in the judicial process as follows: When witness b gives information φ to judge a, judge a cannot believe that such witness tells the truth. That is, judge a cannot believe information φ, but judge a can believe that witness b signs or provides information φ. For the reliability ordering, we can define by the following notions: $b <_a c$ stands for b is strictly more reliable than c, i.e., $(b \leqslant_a c) \wedge \neg(c \leqslant_a b)$, and $b \approx_a c$ which stands for b and c are equally

Table 1 Examples of static logical formalization

$\mathsf{Bel}(a, \varphi)$	:	Agent a believes that φ.
$\mathsf{Sign}(a, \varphi)$	:	Agent a signs statement φ.
$\mathsf{Sign}(a, \mathsf{Sign}(b, \varphi))$	:	Agent a signs statement that agent b signs statement φ.
$b \leqslant_a c$	:	From agent a's perspective, agent b is at least as reliable as agent c.
$\mathsf{Bel}\big(a, \mathsf{Sign}(b, \varphi)\big)$	:	Agent a believes that agent b signs statement φ.
$\mathsf{Bel}\big(a, b \leqslant_a c\big)$	:	Agent a believes that from agent a's perspective, agent b is at least as reliable as agent c.

reliable can be defined as $(b \leqslant_a c) \wedge (c \leqslant_a b)$. We define $\vee$, $\rightarrow$, $\leftrightarrow$ as ordinary abbreviations.

Let us provide Kripke semantics for our syntax. A *model* $\mathfrak{M}$ is a tuple $\mathfrak{M} = (W, (R_a)_{a \in G}, (S_a)_{a \in G}, (\preccurlyeq_a)_{a \in G}, V)$, where W is a non-empty set of states, called the *domain*, $R_a \subseteq W \times W$ is an accessibility relation representing beliefs, $S_a \subseteq W \times W$ is an accessibility relation representing signatures, $\preccurlyeq_a$ is a function which maps from W to $\mathcal{P}(G \times G)$ representing the reliability ordering between agents by agent a, and $V : \mathsf{Prop} \rightarrow \mathcal{P}(W)$ is a valuation. In what follows, we simply write $b \preccurlyeq_a^w c$ for $(b, c) \in \preccurlyeq_a(w)$. Following Lorini et al. (2011) and Jirakunkanok et al. (2014), $\preccurlyeq_a^w$ is always required to be a *total preordering* between agents, i.e., $\preccurlyeq_a^w$ is reflexive ($b \preccurlyeq_a^w b$ for all b), transitive ($b \preccurlyeq_a^w c$ and $c \preccurlyeq_a^w d$ jointly imply $b \preccurlyeq_a^w d$ for all b, c, d), and comparable (for any b and c, $b \preccurlyeq_a^w c$ or $c \preccurlyeq_a^w b$). For any binary relation X on W and any state $w \in W$, we write $X(w)$ to mean $\{ v \in W \mid (w, v) \in X \}$.

Given any model $\mathfrak{M}$, any state $w \in W$ and any formula φ, we define the *satisfaction relation* $\mathfrak{M}, w \models \varphi$ inductively as follows:

$$
\begin{array}{lll}
\mathfrak{M}, w \models p & \text{iff} & w \in V(p) \\
\mathfrak{M}, w \models \neg\varphi & \text{iff} & \mathfrak{M}, w \not\models \varphi \\
\mathfrak{M}, w \models \varphi \wedge \psi & \text{iff} & \mathfrak{M}, w \models \varphi \text{ and } \mathfrak{M}, \mathrm{w} \models \psi \\
\mathfrak{M}, w \models b \leqslant_a c & \text{iff} & b \preccurlyeq_a^w c \\
\mathfrak{M}, w \models \mathsf{Sign}(a, \varphi) & \text{iff} & \mathfrak{M}, v \models \varphi \mathrm{forallvsuchthatwS_av} \\
\mathfrak{M}, w \models \mathsf{Bel}(a, \varphi) & \text{iff} & \mathfrak{M}, v \models \varphi \mathrm{forallvsuchthatwR_av} \\
\mathfrak{M}, w \models \mathsf{A}\varphi & \text{iff} & \mathfrak{M}, v \models \varphi \mathrm{forallv} \in \mathrm{W}
\end{array}
$$

A formula φ is *valid* in a model $\mathfrak{M}$ if $\mathfrak{M}, w \models \varphi$ for all states w of $\mathfrak{M}$.

For the semantics of signature operators, we can describe by an example in the left-hand side of Fig. 1. This example shows that agents b and c sign statement p or $\neg p$ that can be represented by S_b and S_c, respectively. Following S_b from state w_1, statement p holds at state w_5. Thus, $\mathsf{Sign}(b, p)$ is true at state w_1. In addition, the right-hand side of Fig. 1 can be used to illustrate the semantics of $\mathsf{Bel}(a, \mathsf{Sign}(b, p))$. In order to check if $\mathsf{Bel}(a, \mathsf{Sign}(b, p))$ is true or not, we need to check that $\mathsf{Sign}(b, p)$ is true at all states that have R_a for agent a from state w_0. Following R_a from state w_0, $\mathsf{Sign}(b, p)$ is true at states w_1 and w_3. Therefore, we obtain that $\mathsf{Bel}(a, \mathsf{Sign}(b, p))$ is true at state w_0.

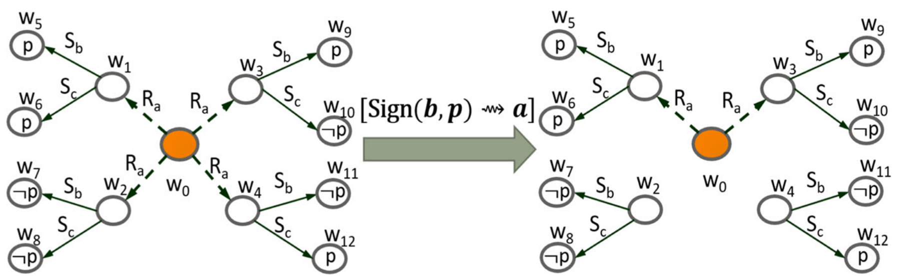

Fig. 1 Update operation of a private announcement of $\mathsf{Sign}(b, p)$ to agent a ($[\mathsf{Sign}(b, p) \rightsquigarrow a]$)

Table 2 presents the Hilbert-style axiomatization **HBSR** for $\mathcal{L}_{BSR}$. Note that K, T, B, 4 and *Incl* are called distribution, reflexivity, symmetry, transitivity and inclusion axioms. In addition, *MP* refers to modus ponens which is a rule of inference, and *Nec* refers to a necessitation rule. For reliability orderings $\preccurlyeq_a^w$, we regard that it is a total pre-ordering between agents, i.e., $\preccurlyeq_a^w$ is reflexive (see $(R_{\leqslant})$), transitive (see $(Tr_{\leqslant})$), and comparable (see $(To_{\leqslant})$). From Lorini et al. (2011) and Jirakunkanok et al. (2014), S_a has three properties of relations including serial, transitive, and Euclidean. That is, we ensures that an agent never signs a contradiction (due to the property of seriality of S_a) and has both positive and negative introspection of his/her signed information (due to the properties of transitivity and Euclideanness of S_a). However, in this study, there is no need to assume these properties of S_a for an agent in a legal case. For example, a witness first gave statement p in the inquiry stage, but after then he/she gave statement $\neg p$ in the court. Thus, the judge came to notice that the witness gave both p and $\neg p$. This example shows that the witness in a legal case can sign a contradiction. For belief operators $\mathsf{Bel}(a, \cdot)$, we suppose that R_a has no properties of relations because of the private announcement and the private permission (described in Sect. 4.2.1).

Theorem 1 *The set of all valid formulas of* $\mathcal{L}_{BSR}$ *is sound and complete with respect to the Hilbert-style axiomatization* **HBSR** *in Table* 2.

The proof of Theorem 1 is presented in "Appendix 2.1"

4.2 Dynamic operators for belief re-revision and reliability change of an agent

This section introduces dynamic logical operators for formalizing belief re-revision and reliability change. First, we describe three operators including private

Table 2 Hilbert-style axiomatization **HBSR** for $\mathcal{L}_{BSR}$

All instances of propositional tautologies	
(K_{B})	$\mathsf{Bel}(a, \varphi \rightarrow \psi) \rightarrow (\mathsf{Bel}(a, \varphi) \rightarrow \mathsf{Bel}(a, \psi))$
(K_{S})	$\mathsf{Sign}(a, \varphi \rightarrow \psi) \rightarrow (\mathsf{Sign}(a, \varphi) \rightarrow \mathsf{Sign}(a, \psi))$
(K_{A})	$\mathsf{A}(\varphi \rightarrow \psi) \rightarrow (\mathsf{A}\varphi \rightarrow \mathsf{A}\psi)$
(T_{A})	$\mathsf{A}\varphi \rightarrow \varphi$
(B_{A})	$\varphi \rightarrow \mathsf{AE}\varphi$
(4_{A})	$\mathsf{A}\varphi \rightarrow \mathsf{AA}\varphi$
$(Incl_{\mathsf{AB}})$	$\mathsf{A}\varphi \rightarrow \mathsf{Bel}(a, \varphi)$
$(Incl_{\mathsf{AS}})$	$\mathsf{A}\varphi \rightarrow \mathsf{Sign}(a, \varphi)$
$(R_{\leqslant})$	$b \leqslant_a b$
$(Tr_{\leqslant})$	$(b \leqslant_a c \wedge c \leqslant_a d) \rightarrow b \leqslant_a d$
$(To_{\leqslant})$	$b \leqslant_a c \vee c \leqslant_a b$
(MP)	From φ and $\varphi \rightarrow \psi$, infer ψ
(G)	From φ, infer $\mathsf{A}\varphi$
(Nec_{B})	From φ, infer $\mathsf{Bel}(a, \varphi)$
(Nec_{S})	From φ, infer $\mathsf{Sign}(a, \varphi)$

Table 3 Summary of six dynamic logical operators

Type	Operator name	Logical formula	Goal
Formalizing belief re-revision	Private announcement (Jirakunkanok et al. 2014)	$[\varphi \rightsquigarrow a]$	To restrict agent a's attention to the φ's states
	Careful policy (Lorini et al. 2011; Jirakunkanok et al. 2014)	$[\mathsf{Careful}(a, \varphi)]$	To aggregate information about φ
	Private permission (Jirakunkanok et al. 2015b)	$[\varphi \twoheadrightarrow a]$	To enlarge agent a's attention to cover all φ's states
Formalizing reliability change	Downgrade (Jirakunkanok et al. 2014)	$[H \Downarrow^a_\varphi]$	To make such agents who sign φ in H less reliable than all the other agents
	Upgrade (Jirakunkanok et al. 2014)	$[H \Uparrow^a_\varphi]$	To make such agents who sign φ in H more reliable than all the other agents
	Joint downgrade (Jirakunkanok et al. 2015a)	$[H \downdownarrows^a]$	To make such agents in H equally reliable and less reliable than the agents in the other groups

announcement, careful policy, and private permission which allow us to formalize belief re-revision. Then, three operators consisting of downgrade, upgrade, and joint downgrade for formalizing reliability change are presented. Our dynamic logical operators can be summarized in Table 3.

4.2.1 Private announcement and private permission

In this section, the private announcement and the private permission are captured in terms of *action model* (cf. van Ditmarsch et al. 2008; Baltag et al. 2008) in dynamic epistemic logic because the action model can be used for modeling a variety of events involving communication including public and private messages. The effect of the action model can be illustrated by Figs. 2 and 5. That is, when there is an announcement of an event, only an agent who knows about such event will change his/her belief. In addition, the careful policy is captured based on the private announcement.

Private announcement The private announcement $[\varphi \rightsquigarrow a]$ is introduced in Jirakunkanok et al. (2014). Our intended reading of $[\varphi \rightsquigarrow a]$ is "after a private announcement of φ to agent a." The main idea is that belief change of a will not be noticed by the other agents than a,[2] and only a recipient is defined as agent a. In this sense, this operator can be used for self-decision, i.e., the sender and the recipient

[2] Based on Jirakunkanok et al. (2014), $[\varphi \rightsquigarrow a]$ captures that the action of a's privately receiving message φ will not affect of the other agents' beliefs than a. Thus, this work considers only the case that the other agents than a do not know about such event.

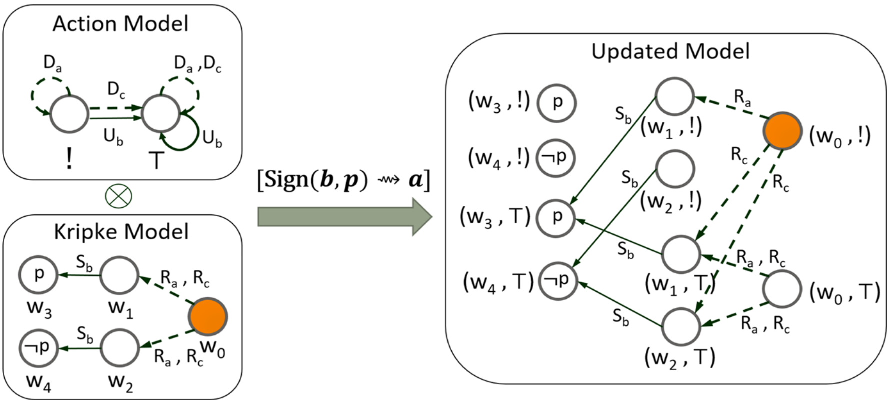

Fig. 2 Product update operation of a private announcement of $\mathsf{Sign}(b,p)$ to agent a ($[\mathsf{Sign}(b,p) \rightsquigarrow a]$). Note that an action ! represents the $\mathsf{Sign}(b,p)$-announcing action $!^a_{\mathsf{Sign}(b,p)}$ to agent a

are the same. For example, when a judge receives many signed information from several witnesses, he/she needs to decide which information he/she should believe by him/herself. This process can be captured by the careful policy which aims to derive an agent's belief from the received information (the more details of this policy will be described later). With this private announcement, an agent can remove some possibilities from his/her belief that can be described by Example 1.

Example 1 Figure 1 illustrates a process of a private announcement of $\mathsf{Sign}(b,p)$ to agent a. This private announcement can be interpreted by the tell-action in Jirakunkanok et al. (2014) as agent b privately tells information p to agent a. We can represent "agent b tells information p" by $\mathsf{Sign}(b,p)$. When agent a receives $\mathsf{Sign}(b,p)$, he/she commits him/herself to $\mathsf{Sign}(b,p)$ by $[\mathsf{Sign}(b,p) \rightsquigarrow a]$. Let us describe a process of this private announcement. Firstly, agent a does not believe that $\mathsf{Sign}(b,p)$, i.e., $\neg\mathsf{Bel}(a,\mathsf{Sign}(b,p))$ as in the left-hand side of Fig. 1. By the update of $[\mathsf{Sign}(b,p) \rightsquigarrow a]$, we delete all the links from state w_0 into the states where $\mathsf{Sign}(b,p)$ is false. That is, the links into states w_2 and w_4 will be eliminated. After this, agent a now believes that $\mathsf{Sign}(b,p)$, i.e., $\mathsf{Bel}(a,\mathsf{Sign}(b,p))$ as shown in the right-hand side of Fig. 1.

Definition 1 An action model $\mathbb{E}_{!^a_\varphi}$ for a private announcement of φ to agent a is defined as a tuple $(E,(D_c)_{c\in G},(U_a)_{a\in G},\mathrm{pre})$, where E is a set of two actions: φ-announcing action $!^a_\varphi$ to agent a and non-announcing action $\top$,[3] D_c is an accessibility relation representing beliefs such that $D_a = \{(!^a_\varphi, !^a_\varphi), (\top,\top)\}$ and $D_c = \{(!^a_\varphi, \top), (\top,\top)\}$ if $c \neq a$, U_c is an accessibility relation representing signatures such that $U_c = \{(!^a_\varphi, \top), (\top,\top)\}$ for all $c \in G$, and pre is a preconditions function that assigns a precondition to each action by $\mathrm{pre}(!^a_\varphi) = \varphi$ and $\mathrm{pre}(\top) = \top$.

[3] The φ-announcing action $!_\varphi$ is an action where there is an announcement of φ, while the non-announcing action is an action where nothing happens.

Definition 2 Given a Kripke model $\mathfrak{M} = (W, (R_c)_{c\in G}, (S_c)_{c\in G}, (\preccurlyeq_c)_{c\in G}, V)$, a semantic clause for $[\mathbb{E}_{!^a_\varphi}, e]$ on $\mathfrak{M}$ and $w \in W$ is defined as follows:

$$\mathfrak{M}, w \models [\mathbb{E}_{!^a_\varphi}, e]\psi \quad \text{iff} \quad \mathfrak{M}^{\otimes \mathbb{E}_{!^a_\varphi}}, (w, e) \models \psi,$$

where (w, e) is the updated state of $\mathfrak{M}^{\otimes \mathbb{E}_{!^a_\varphi}}$ (defined just below) by the action model of Definition 1, and $\mathfrak{M}^{\otimes \mathbb{E}_{!^a_\varphi}} = (W', (R'_c)_{c\in G}, (S'_c)_{c\in G}, (\preccurlyeq'_c)_{c\in G}, V')$ is the updated model by the action model of Definition 1. $\mathfrak{M}^{\otimes \mathbb{E}_{!^a_\varphi}}$ which is constructed with an operation called a *product update* (van Ditmarsch et al. 2008) is defined as follows:

- $W' := W \times E = W \times \{ !^a_\varphi, \top \}$.
- $(w, e)R'_c(v,f)$ iff $wR_c v$ and $(e,f) \in D_c$ and $\mathfrak{M}, v \models \mathrm{pre}(f)$ (for all $c \in G$).
- $(w, e)S'_c(v,f)$ iff $wS_c v$ and $(e,f) \in U_c$ (for all $c \in G$).
- $d \preccurlyeq'^{(w,e)}_c d'$ iff $d \preccurlyeq^w_c d'$.
- $(w, e) \in V'(p)$ iff $w \in V(p)$.

Finally, we define $[\varphi \rightsquigarrow a]\psi := [\mathbb{E}_{!^a_\varphi}, !^a_\varphi]\psi$, where we recall that $[\varphi \rightsquigarrow a]\psi$ is read as "after a private announcement of φ to agent a, ψ holds."

Proposition 1 (Recursive validities) *The following are valid on all models. Moreover, if ψ is valid on all models, then $[\mathbb{E}_{!^a_\varphi}, e]\psi$ is also valid on all models.*

$$\begin{array}{lcl}
[\mathbb{E}_{!^a_\varphi}, e]p & \leftrightarrow & p \\
[\mathbb{E}_{!^a_\varphi}, e]d \leqslant_c d' & \leftrightarrow & d \leqslant_c d' \\
[\mathbb{E}_{!^a_\varphi}, e]\neg\psi & \leftrightarrow & \neg[\mathbb{E}_{!^a_\varphi}, e]\psi \\
[\mathbb{E}_{!^a_\varphi}, e](\psi_1 \wedge \psi_2) & \leftrightarrow & [\mathbb{E}_{!^a_\varphi}, e]\psi_1 \wedge [\mathbb{E}_{!^a_\varphi}, e]\psi_2 \\
[\mathbb{E}_{!^a_\varphi}, e]\mathsf{Sign}(b, \psi) & \leftrightarrow & \bigwedge_{f\in U_b(e)} \mathsf{Sign}(b, [\mathbb{E}_{!^a_\varphi}, f]\psi) \\
[\mathbb{E}_{!^a_\varphi}, e]\mathsf{Bel}(a, \psi) & \leftrightarrow & \bigwedge_{f\in D_a(e)} \mathsf{Bel}\big(a, \mathrm{pre}(f) \rightarrow [\mathbb{E}_{!^a_\varphi}, f]\psi\big) \\
[\mathbb{E}_{!^a_\varphi}, e]\mathsf{A}\psi & \leftrightarrow & \bigwedge_{f\in E} \mathsf{A}[\mathbb{E}_{!^a_\varphi}, f]\psi
\end{array}$$

where $U_b(e)$, $D_a(e)$ and E are finite. Note that the axiom $[\mathbb{E}_{!^a_\varphi}, e]\mathsf{Bel}(a, \psi) \leftrightarrow \bigwedge_{f\in D_a(e)} \mathsf{Bel}\big(a, \mathrm{pre}(f) \rightarrow [\mathbb{E}_{!^a_\varphi}, f]\psi\big)$ captures the concept of the private announcement. For example, we suppose at first that agent a does not believe that p, i.e., $\neg\mathsf{Bel}(a, p)$, and agent c believes that $\neg p$, i.e., $\mathsf{Bel}(c, \neg p)$. When there is a private announcement of p to agent a, i.e., $[p \rightsquigarrow a]$, we obtain that $[\mathbb{E}_{!^a_p}, !^a_p]\mathsf{Bel}(a, p)$ and $[\mathbb{E}_{!^a_p}, !^a_p]\mathsf{Bel}(c, \neg p)$ are valid in all models. This means that after the private announcement $[p \rightsquigarrow a]$, agent a believes that p, i.e., $\mathsf{Bel}(a, p)$, and agent c believes that $\neg p$, i.e., $\mathsf{Bel}(c, \neg p)$. The proof of Proposition 1 is presented in "Appendix 2.2".

Example 2 Given a Kripke model $\mathfrak{M} = (W, (R_a)_{a\in G}, (S_a)_{a\in G}, (\preccurlyeq_a)_{a\in G}, V)$ (see a Kripke model in Fig. 2) and an action model $\mathbb{E}_{!^a_{\mathsf{Sign}(b,p)}}$ for a private announcement of $\mathsf{Sign}(b, p)$ to agent a (see an action model in Fig. 2). Let us describe how the Kripke model $\mathfrak{M}$ is updated with the action model $\mathbb{E}_{!^a_{\mathsf{Sign}(b,p)}}$ of Definition 1 by Fig. 2. First, agents a and c do not believe that $\mathsf{Sign}(b, p)$, i.e., $\neg\mathsf{Bel}(a, \mathsf{Sign}(b, p))$ and

$\neg\mathsf{Bel}(c, \mathsf{Sign}(b,p))$ (see a Kripke model in Fig. 2). By the product update operation of Definition 2, agent a believes that $\mathsf{Sign}(b,p)$, i.e., $\mathsf{Bel}(a, \mathsf{Sign}(b,p))$, but agent c still does not believe that $\mathsf{Sign}(b,p)$, i.e., $\neg\mathsf{Bel}(c, \mathsf{Sign}(b,p))$ as shown in the right-hand side of Fig. 2. We can explain the updated model in Fig. 2 as follows: When we focus on action '!' representing there is an announcement of $\mathsf{Sign}(b,p)$ to agent a, we obtain that only agent a changes his/her belief, and his/her belief change will not be noticed by agent c. On the other hand, when we focus on action $\top$, we obtain that both agents a and c do not change their belief because there is no announcement.

In addition, Jirakunkanok et al. (2014) proposed to capture the *careful policy* from Lorini et al. (2011) based on this private announcement as follows: Based on Lorini et al. (2011), a careful policy which is one of aggregation policies aims to derive an agent's beliefs from the received signed information. The policy is that only statements which are universally signed by a group of agents who are equally reliable are accepted as beliefs. Firstly, the careful policy is defined in terms of dynamic operators by $[\mathsf{Careful}(a,\varphi)]$, whose reading is "agent a aggregates information about φ". By using the total preordering $\preccurlyeq_a^w$, agents can be ranked by giving a partition $(\mathsf{C}_i^a)_{i \leq M}$ to G, where M is a natural number representing the maximum rank (such M always exists because G is finite). Note that $c \in \mathsf{C}_i^a$ can be read as "from agent a's perspective, the rank of agent c is i". As a result, agents who are equally reliable are categorized in the same group. Next, $\mathsf{Sign}(\mathsf{C}_i^a, \varphi)$ which stands for "all agents who are in the set C_i^a sign statement φ" is defined as follows:

$$\mathsf{Sign}(\mathsf{C}_i^a, \varphi) := \bigwedge\nolimits_{c \in \mathsf{C}_i^a} (\mathsf{Sign}(c, \varphi)).$$

After that, the *UniSign*, whose reading is "agent a believes that φ is universally signed by a group of agents who are equally reliable", is introduced as follows:

$$\mathsf{UniSign}(\varphi, a) := \bigvee\nolimits_{i \leq M} \begin{pmatrix} \mathsf{Bel}(a, \mathsf{Sign}(\mathsf{C}_i^a, \varphi)) \wedge \\ \mathsf{Bel}(a, \bigwedge_{1 \leq j \leq i-1} \neg\mathsf{Sign}(\mathsf{C}_j^a, \neg\varphi)) \end{pmatrix},$$

where M is the maximum natural number of $\{i \leq \#G \mid \mathsf{C}_i^a \neq \emptyset\}$. Finally, the careful policy is captured by the help of the private announcement as follows:

$$[\mathsf{Careful}(a,\varphi)]\psi := \mathsf{UniSign}(\varphi, a) \rightarrow [\varphi \rightsquigarrow a]\psi,$$

where $[\mathsf{Careful}(a,\varphi)]\psi$ can be read as "after agent a aggregates information about φ by the careful policy, ψ holds."

Example 3 Figure 3 illustrates how agent a aggregates information about $\neg p$. At state w_0, agent a believes that agent b is more reliable than agent c, i.e., $\mathsf{Bel}(a, b <_a c)$. In the initial situation, agent a believes that $\mathsf{Sign}(b, \neg p)$ but does not believe that $\mathsf{Sign}(c, \neg p)$, i.e., $\mathsf{Bel}(a, \mathsf{Sign}(b, \neg p))$ and $\neg\mathsf{Bel}(a, \mathsf{Sign}(c, \neg p))$ as shown in the left-hand side of Fig. 3. By $[\mathsf{Careful}(a, \neg p)]$, agent a aggregates information from agent b who is more reliable than agent c. Finally, agent a believes that $\neg p$, i.e., $\mathsf{Bel}(a, \neg p)$ as shown in the right-hand side of Fig. 3.

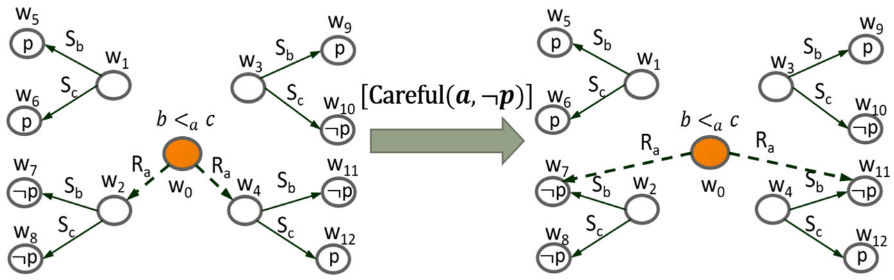

Fig. 3 Agent a aggregates information about $\neg p$ by $[\mathsf{Careful}(a, \neg p)]$

Private permission From Jirakunkanok et al. (2013), the permission is proposed to allow an agent to restore the former possibilities to his/her belief. While the previous study (Jirakunkanok et al. 2013) used the permission to revise an agent's belief locally in one specified world, this work assumes that an effect of an agent's permission is applied globally for all $w \in W$. Based on this idea and the private action, a new kind of permission called a *private permission* $[\varphi \twoheadrightarrow a]$ is introduced in Jirakunkanok et al. (2015b). Our intended reading of $[\varphi \twoheadrightarrow a]$ is "after a private permission of φ to agent a." The concept of this operator is to allow only agent a to notice his/her belief change after he/she permitted φ to be the case.[4]

Example 4 Figure 4 illustrates a process of a private permission of $\mathsf{Sign}(b, \neg p)$ to agent a. $[\mathsf{Sign}(b, \neg p) \twoheadrightarrow a]$ can be interpreted as agent a privately permits the possibility of $\mathsf{Sign}(b, \neg p)$ to his/her belief. Firstly, agent a believes that $\mathsf{Sign}(b, p)$, i.e., $\mathsf{Bel}(a, \mathsf{Sign}(b, p))$ as shown in the left-hand side of Fig. 4. Then, $[\mathsf{Sign}(b, \neg p) \twoheadrightarrow a]$ allows us to restore all the former links to the states where $\mathsf{Sign}(b, \neg p)$ is true. That is, the links into states w_2 and w_4 will be restored as shown in the right-hand side of Fig. 4. At this stage, agent a becomes undetermined on $\mathsf{Sign}(b, \neg p)$, i.e., $\neg\mathsf{Bel}(a, \mathsf{Sign}(b, p))$ and $\neg\mathsf{Bel}(a, \mathsf{Sign}(b, \neg p))$. Note that the right-hand side of Fig. 4 represents the result of $[\mathsf{Sign}(b, \neg p) \twoheadrightarrow a]$ when we focus on a current viewpoint of agent a representing by state w_0, that is, it shows only the links from state w_0.

From Figs. 1 and 4, let us describe a restoration process of the former links. First, agent a accepts $\mathsf{Sign}(b, p)$ by applying $[\mathsf{Sign}(b, p) \rightsquigarrow a]$. As a result, we delete all the links into the states where $\mathsf{Sign}(b, p)$ is false (see Fig. 1). Next, agent a reconsiders his/her decision and decides to reject $\mathsf{Sign}(b, p)$ but accept $\mathsf{Sign}(b, \neg p)$ instead. In order to overturn the decision, agent a first needs to permit the possibility of $\mathsf{Sign}(b, \neg p)$ by applying $[\mathsf{Sign}(b, \neg p) \twoheadrightarrow a]$. By the update of $[\mathsf{Sign}(b, \neg p) \twoheadrightarrow a]$, we add all the links into the states where $\mathsf{Sign}(b, \neg p)$ is true (see Fig. 4). Note that the states where $\mathsf{Sign}(b, p)$ is false are the same as the states where $\mathsf{Sign}(b, \neg p)$ is true. Therefore, we can regard that all the links into

[4] Based on the idea of the private action, $[\varphi \twoheadrightarrow a]$ captures that the action of a's privately permitting the possibility of φ will not affect of the other agents' beliefs than a. Thus, this work considers only the case that the other agents than a do not know about such event.

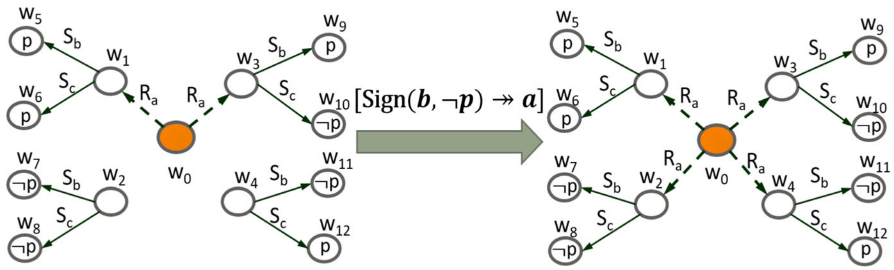

Fig. 4 Update operation of a private permission of $\mathsf{Sign}(b, \neg p)$ to agent a ($[\mathsf{Sign}(b, \neg p) \twoheadrightarrow a]$)

$\mathsf{Sign}(b, \neg p)$'s states (i.e., states where $\mathsf{Sign}(b, \neg p)$ is true), which are deleted by $[\mathsf{Sign}(b, \neg p) \rightsquigarrow a]$, can be restored by $[\mathsf{Sign}(b, \neg p) \twoheadrightarrow a]$.

Definition 3 An action model $\mathbb{E}_{¡^a_\varphi}$ for a private permission of φ to agent a is defined as a tuple $(E, (D_c)_{c\in G}, (U_a)_{a\in G}, \mathrm{pre})$, where E is a set of two actions: φ-announcing action $¡^a_\varphi$ to agent a and non-announcing action $\perp$,[5] D_c is an accessibility relation representing beliefs such that $D_a = \{ (¡^a_\varphi, ¡^a_\varphi), (\perp, \perp) \}$ and $D_c = \{ (¡^a_\varphi, \perp), (\perp, \perp) \}$ if $c \neq a$, U_c is an accessibility relation representing signatures such that $U_c = \{ (¡^a_\varphi, \perp), (\perp, \perp) \}$ for all $c \in G$, and pre is a preconditions function that assigns a precondition to each action by $\mathrm{pre}(¡^a_\varphi) = \varphi$ and $\mathrm{pre}(\perp) = \perp$.

Definition 4 Given a Kripke model $\mathfrak{M} = (W, (R_c)_{c\in G}, (S_c)_{c\in G}, (\preccurlyeq_c)_{c\in G}, V)$, a semantic clause for $[\mathbb{E}_{¡^a_\varphi}, e]$ on $\mathfrak{M}$ and $w \in W$ is defined as follows:

$$\mathfrak{M}, w \models [\mathbb{E}_{¡^a_\varphi}, e]\psi \quad \text{iff} \quad \mathfrak{M}^{\otimes \mathbb{E}_{¡^a_\varphi}}, (w, e) \models \psi,$$

where (w, e) is the updated state of $\mathfrak{M}^{\otimes \mathbb{E}_{¡^a_\varphi}}$ (defined just below) by the action model of Definition 3 and $\mathfrak{M}^{\otimes \mathbb{E}_{¡^a_\varphi}} = (W', (R'_c)_{c\in G}, (S'_c)_{c\in G}, (\preccurlyeq'_c)_{c\in G}, V')$ is the updated model by the action model of Definition 3. $\mathfrak{M}^{\otimes \mathbb{E}_{¡^a_\varphi}}$ which is constructed with an operation called a *product update* (van Ditmarsch et al. 2008) is defined as follows:

- $W' := W \times E = W \times \{ ¡^a_\varphi, \perp \}$.
- $(w, e)R'_c(v, f)$ iff $\big(wR_c v$ and $(e, f) \in D_c\big)$ or $\big(\mathfrak{M}, v \models \mathrm{pre}(f)$ and $(e, f) \in D_c\big)$ (for all $c \in G$).
- $(w, e)S'_c(v, f)$ iff $wS_c v$ and $(e, f) \in U_c$ (for all $c \in G$).
- $d \preccurlyeq_c'^{(w,e)} d'$ iff $d \preccurlyeq_c^w d'$.
- $(w, e) \in V'(p)$ iff $w \in V(p)$.

Finally, we define $[\varphi \twoheadrightarrow a]\psi := [\mathbb{E}_{¡^a_\varphi}, ¡^a_\varphi]\psi$, where we recall that $[\varphi \twoheadrightarrow a]\psi$ is read as "after a private permission of φ to agent a, ψ holds."

[5] The φ-announcing action $¡_\varphi$ is an action where there is an announcement of φ, while the non-announcing action is an action where nothing happens.

Proposition 2 (Recursive validities) *The following are valid on all models. Moreover, if* ψ *is valid on all models, then* $[\mathbb{E}_{¡^a_\varphi}, e]\psi$ *is also valid on all models.*

$$
\begin{array}{lll}
[\mathbb{E}_{¡^a_\varphi}, e]p & \leftrightarrow & p \\
[\mathbb{E}_{¡^a_\varphi}, e]d \leqslant_c d' & \leftrightarrow & d \leqslant_c d' \\
[\mathbb{E}_{¡^a_\varphi}, e]\neg\psi & \leftrightarrow & \neg[\mathbb{E}_{¡^a_\varphi}, e]\psi \\
[\mathbb{E}_{¡^a_\varphi}, e](\psi_1 \wedge \psi_2) & \leftrightarrow & [\mathbb{E}_{¡^a_\varphi}, e]\psi_1 \wedge [\mathbb{E}_{¡^a_\varphi}, e]\psi_2 \\
[\mathbb{E}_{¡^a_\varphi}, e]\mathsf{Sign}(b, \psi) & \leftrightarrow & \bigwedge_{f \in U_b(e)} \mathsf{Sign}(b, [\mathbb{E}_{¡^a_\varphi}, f]\psi) \\
[\mathbb{E}_{¡^a_\varphi}, e]\mathsf{Bel}(a, \psi) & \leftrightarrow & \bigwedge_{f \in D_a(e)} \big(\mathsf{Bel}(a, [\mathbb{E}_{¡^a_\varphi}, f]\psi) \wedge \mathsf{A}(\mathrm{pre}(f) \to [\mathbb{E}_{¡^a_\varphi}, f]\psi)\big) \\
[\mathbb{E}_{¡^a_\varphi}, e]\mathsf{A}\psi & \leftrightarrow & \bigwedge_{f \in E} \mathsf{A}[\mathbb{E}_{¡^a_\varphi}, f]\psi
\end{array}
$$

where $U_b(e)$, $D_a(e)$ and E are finite. Note that the axiom $[\mathbb{E}_{¡^a_\varphi}, e]\mathsf{Bel}(a, \psi) \leftrightarrow \bigwedge_{f \in D_a(e)} \big(\mathsf{Bel}(a, [\mathbb{E}_{¡^a_\varphi}, f]\psi) \wedge \mathsf{A}(\mathrm{pre}(f) \to [\mathbb{E}_{¡^a_\varphi}, f]\psi)\big)$ captures the concept of the private permission. For example, we suppose that agents a and c first believe that $\neg p$, i.e., $\mathsf{Bel}(a, \neg p)$ and $\mathsf{Bel}(c, \neg p)$. When there is a private permission of p to agent a, i.e., $[p \twoheadrightarrow a]$, we obtain that $[\mathbb{E}_{¡^a_p}, ¡^a_p]\neg\mathsf{Bel}(a, p \vee \neg p)$ and $[\mathbb{E}_{¡^a_p}, ¡^a_p]\mathsf{Bel}(c, \neg p)$ are valid in all models. This means that after the private permission $[p \twoheadrightarrow a]$, agent a becomes undetermined on p, i.e., $\neg\mathsf{Bel}(a, p)$ and $\neg\mathsf{Bel}(a, \neg p)$, and agent c believes that $\neg p$, i.e., $\mathsf{Bel}(c, \neg p)$. The proof of Proposition 2 is presented in "Appendix 2.3".

Example 5 Given a Kripke model $\mathfrak{M} = (W, (R_a)_{a \in G}, (S_a)_{a \in G}, (\preccurlyeq_a)_{a \in G}, V)$ (see a Kripke model in Fig. 5) and an action model $\mathbb{E}_{¡^a_{\mathsf{Sign}(b,\neg p)}}$ for a private permission of $\mathsf{Sign}(b, \neg p)$ to agent a (see an action model in Fig. 5). The product update of the Kripke model $\mathfrak{M}$ and the action model $\mathbb{E}_{¡^a_{\mathsf{Sign}(b,\neg p)}}$ of Definition 3 can be described by Fig. 5. First, agents a and c believe that $\mathsf{Sign}(b, p)$, i.e., $\mathsf{Bel}(a, \mathsf{Sign}(b, p))$ and $\mathsf{Bel}(c, \mathsf{Sign}(b, p))$ (see a Kripke model in Fig. 5). By the product update operation (defined in Definition 4), agent a does not believe that $\mathsf{Sign}(b, p)$ and $\mathsf{Sign}(b, \neg p)$, i.e., $\neg\mathsf{Bel}(a, \mathsf{Sign}(b, p))$ and $\neg\mathsf{Bel}(a, \mathsf{Sign}(b, \neg p))$, but agent c still believes that

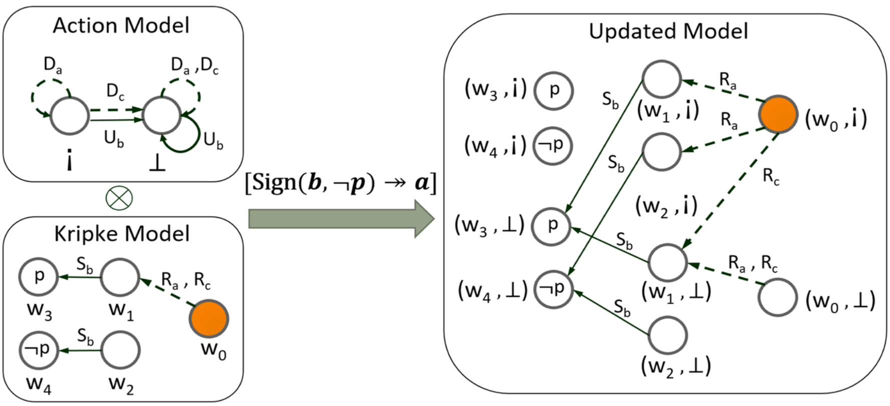

Fig. 5 Product update operation of a private permission of $\mathsf{Sign}(b, \neg p)$ to agent a ($[\mathsf{Sign}(b, \neg p) \twoheadrightarrow a]$). Note that an action ¡ represents the $\mathsf{Sign}(b, \neg p)$-announcing action $¡^a_{\mathsf{Sign}(b,\neg p)}$ to agent a

$\mathsf{Sign}(b,p)$, i.e., $\mathsf{Bel}(c,\mathsf{Sign}(b,p))$ as shown in the right-hand side of Fig. 5. The updated model in this figure can be described as follows: When we focus on action '¡', we obtain that only agent a changes his/her belief from $\mathsf{Bel}(a,\mathsf{Sign}(b,p))$ into $\neg\mathsf{Bel}(a,\mathsf{Sign}(b,p))$. This belief change of agent a can be noticed only by him/herself. On the other hand, when we focus on action $\perp$, we obtain that beliefs of agents a and b are the same as the initial situation before $[\mathsf{Sign}(b,\neg p)\twoheadrightarrow a]$. Note that the right-hand side of Fig. 5 represents the result of $[\mathsf{Sign}(b,\neg p)\twoheadrightarrow a]$ when we focus on a current viewpoint of agent a representing by state $(w_0,¡)$, that is, it shows only the links from state $(w_0,¡)$.

Frame properties and interaction between private announcement and private permission For R_c, we suppose that R_c has no properties of relations in order to enable us to apply both private announcement and private permission operators for capturing the non-monotonic change of an agent's belief. That is, we allow an agent to change his/her belief from $\mathsf{Bel}(a,\varphi)$ into $\neg\mathsf{Bel}(a,\varphi)$. For example, we assume that agent a first does not believe that φ, i.e., $\neg\mathsf{Bel}(a,\varphi)$. By the private announcement, agent a changes his/her belief from $\neg\mathsf{Bel}(a,\varphi)$ into $\mathsf{Bel}(a,\varphi)$ by removing the links into $\neg\varphi$'s states. If there is no link, we can regard that agent a believes that φ and $\neg\varphi$, i.e., $\mathsf{Bel}(a,\perp)$. In this sense, the property of seriality cannot be preserved. However, the private announcement can preserve the properties of transitivity and Euclideanness. Next, the private permission may be applied in order to change agent a's belief from $\mathsf{Bel}(a,\varphi)$ into $\neg\mathsf{Bel}(a,\varphi)$ by adding the links into $\neg\varphi$'s states. This process may break the properties of transitivity and Euclideanness. Nevertheless, the property of seriality can be preserved by the private permission. For this reason, we may regard that the repetitive application of the private announcement and the private permission could retrieve the properties of seriality, transitivity, and Euclideanness.

4.2.2 Downgrade, upgrade and joint downgrade

Downgrade and upgrade In Jirakunkanok et al. (2014), they introduced two dynamic operators consisting of downgrade $[H\Downarrow^a_\varphi]$ and upgrade $[H\Uparrow^a_\varphi]$, where $H\subseteq G$ is a set of agents, in order to change a reliability ordering between agents from a particular agent's perspective. Our intended reading of $[H\Downarrow^a_\varphi]\psi$ is "after agent a downgraded such agents who sign statement φ in H, ψ holds", and we can read $[H\Uparrow^a_\varphi]\psi$ as "after agent a upgraded such agents who sign statement φ in H, ψ holds." Semantically speaking, $[H\Downarrow^a_\varphi]$ makes such agents who sign φ in H less reliable than all the other agents, and $[H\Uparrow^a_\varphi]$ makes such agents who sign φ in H more reliable than all the other agents.

Before giving the detailed semantics, let us demonstrate the effects of $[H\Downarrow^a_\varphi]$ and $[H\Uparrow^a_\varphi]$ by figures. Firstly, we assume that a rectangle G of Fig. 6(i) represents a fixed finite set of agents. Secondly, we will select a specified set of agents in order to change their reliability ordering that can be represented by a rectangle H, and we assume that $c_1\approx_a c_2<_a b_1\approx_a b_2$ holds, i.e., agents c_1 and c_2 which are equally reliable are more reliable than agents b_1 and b_2 which are equally reliable from

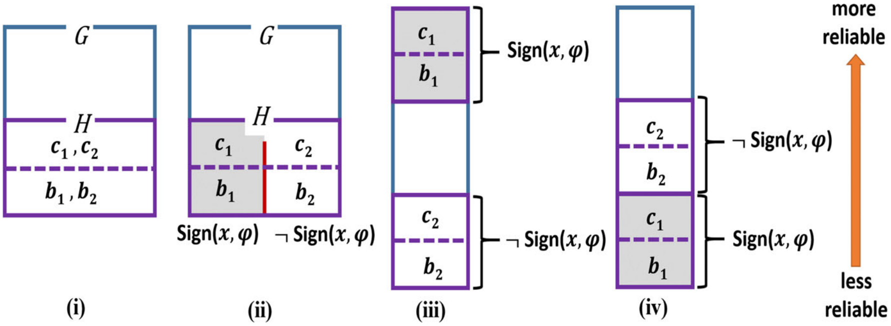

Fig. 6 Downgrading and Upgrading. (iii) is an effect of upgrading $[H \Uparrow^a_\varphi]$ to (ii), and (iv) is an effect of downgrading $[H \Downarrow^a_\varphi]$ to (ii)

agent a's perspective. In this sense, b_1, b_2, c_1 and c_2 are situated as in Fig. 6(i). Then, if we focus on agents who sign statement φ, H is divided into two equal vertical parts by $\mathsf{Sign}(x, \varphi)$ as in Fig. 6(ii), namely by the set $\{x \in H \mid \mathfrak{M}, w \models \mathsf{Sign}(x, \varphi)\}$ and the set $\{x \in H \mid \mathfrak{M}, w \models \neg\mathsf{Sign}(x, \varphi)\}$. Next, if agent a *upgrades* all the agents signing statement φ in H, we upgrade all of them more reliable than the other agents as in Fig. 6(iii). On the other hand, if agent a *downgrades* all the agents signing statement φ in H, we downgrade all of them less reliable than the other agents as in Fig. 6(iv).

Definition 5 Given a Kripke model $\mathfrak{M} = (W, (R_a)_{a \in G}, (S_a)_{a \in G}, (\preccurlyeq_d)_{d \in G}, V)$, a semantic clause for $[H \Downarrow^a_\varphi]$ on $\mathfrak{M}$ and $w \in W$ is defined by:

$$\mathfrak{M}, w \models [H \Downarrow^a_\varphi]\psi \quad \text{iff} \quad \mathfrak{M}^{H \Downarrow^a_\varphi}, w \models \psi,$$

where $\mathfrak{M}^{H \Downarrow^a_\varphi} = (W, (R_a)_{a \in G}, (S_a)_{a \in G}, (\preccurlyeq'_d)_{d \in G}, V)$ and $\preccurlyeq'_d$ is defined as: for all $u \in W$:

- if $d \neq a$, we put $\preccurlyeq'^u_d = \preccurlyeq^u_d$.
- otherwise (if $d = a$), we define $b \preccurlyeq'^u_a c$ iff[6]

$$\begin{aligned}
&\big(b, c \in H \,\mathrm{and}\, \mathfrak{M}, \mathrm{u} \models \mathsf{Sign}(\mathrm{b}, \varphi) \wedge \mathsf{Sign}(\mathrm{c}, \varphi) \,\mathrm{and}\, \mathrm{b} \preccurlyeq^{\mathrm{u}}_{\mathrm{a}} \mathrm{c}\big) \text{ or} \\
&\big(b, c \in (G \setminus H) \cup \{x \in H \mid \mathfrak{M}, u \models \neg\mathsf{Sign}(x, \varphi)\} \,\mathrm{and}\, \mathrm{b} \preccurlyeq^{\mathrm{u}}_{\mathrm{a}} \mathrm{c}\big) \text{ or} \\
&\big(b \in (G \setminus H) \cup \{x \in H \mid \mathfrak{M}, u \models \neg\mathsf{Sign}(x, \varphi)\} \,\mathrm{and}\, \mathrm{c} \in \mathrm{H} \,\mathrm{and}\, \mathfrak{M}, \mathrm{u} \models \mathsf{Sign}(\mathrm{c}, \varphi)\big).
\end{aligned}$$

Definition 6 Given a Kripke model $\mathfrak{M} = (W, (R_a)_{a \in G}, (S_a)_{a \in G}, (\preccurlyeq_d)_{d \in G}, V)$, a semantic clause for $[H \Uparrow^a_\varphi]$ on $\mathfrak{M}$ and $w \in W$ is defined by:

[6] In the last case, since there is no relation between agents b and c, $b \preccurlyeq^u_a c$ is omitted.

$$\mathfrak{M}, w \models [H \Uparrow^a_\varphi]\psi \quad \text{iff} \quad \mathfrak{M}^{H\Uparrow^a_\varphi}, w \models \psi,$$

where $\mathfrak{M}^{H\Uparrow^a_\varphi} = (W, (R_a)_{a\in G}, (S_a)_{a\in G}, (\preccurlyeq'_d)_{d\in G}, V)$ and $\preccurlyeq'_d$ is defined as: for all $u \in W$:

- if $d \neq a$, we put $\preccurlyeq'^u_d = \preccurlyeq^u_d$.
- otherwise (if $d = a$), we define $b \preccurlyeq'^u_a c$ iff[7]

$$\big(b, c \in H \,\text{and}\, \mathfrak{M}, \mathrm{u} \models \mathsf{Sign}(\mathrm{b}, \varphi) \wedge \mathsf{Sign}(\mathrm{c}, \varphi) \,\text{and}\, \mathrm{b} \preccurlyeq^{\mathrm{u}}_{\mathrm{a}} \mathrm{c}\big) \text{ or}$$
$$\big(b, c \in (G \setminus H) \cup \{x \in H \mid \mathfrak{M}, u \models \neg\mathsf{Sign}(x, \varphi)\} \,\text{and}\, \mathrm{b} \preccurlyeq^{\mathrm{u}}_{\mathrm{a}} \mathrm{c}\big) \text{ or}$$
$$\big(c \in (G \setminus H) \cup \{x \in H \mid \mathfrak{M}, u \models \neg\mathsf{Sign}(x, \varphi)\} \,\text{and}\, \mathrm{b} \in \mathrm{H} \,\text{and}\, \mathfrak{M}, \mathrm{u} \models \mathsf{Sign}(\mathrm{b}, \varphi)\big).$$

Note that downgrade and upgrade operators can preserve the property of total preordering of $(\preccurlyeq_d)_{d\in G}$.

Proposition 3 [Recursive validities (Jirakunkanok et al. 2014)] *The following are valid on all models. Moreover, if ψ is valid on all models, then $[H \Downarrow^a_\varphi]\psi$ is also valid on all models.*

$$\begin{array}{lcll}
[H\Downarrow^a_\varphi]p & \leftrightarrow & p & \\
[H\Downarrow^a_\varphi](b \leqslant_d c) & \leftrightarrow & b \leqslant_d c & (d \neq a) \\
[H\Downarrow^a_\varphi](b \leqslant_a c) & \leftrightarrow & b \leqslant_a c & (b, c \in G \setminus H) \\
[H\Downarrow^a_\varphi](b \leqslant_a c) & \leftrightarrow & \big(\mathsf{Sign}(b,\varphi) \wedge \mathsf{Sign}(c,\varphi) \wedge (b \leqslant_a c)\big) \vee & \\
 & & \big(\neg\mathsf{Sign}(b,\varphi) \wedge \neg\mathsf{Sign}(c,\varphi) \wedge (b \leqslant_a c)\big) \vee & \\
 & & \big(\neg\mathsf{Sign}(b,\varphi) \wedge \mathsf{Sign}(c,\varphi)\big) & (b, c \in H) \\
[H\Downarrow^a_\varphi](b \leqslant_a c) & \leftrightarrow & \mathsf{Sign}(c,\varphi) \vee \big(\neg\mathsf{Sign}(c,\varphi) \wedge (b \leqslant_a c)\big) & (c \in H, b \in G \setminus H) \\
[H\Downarrow^a_\varphi](b \leqslant_a c) & \leftrightarrow & \neg\mathsf{Sign}(b,\varphi) \wedge (b \leqslant_a c) & (b \in H, c \in G \setminus H) \\
[H\Downarrow^a_\varphi]\neg\psi & \leftrightarrow & \neg[H\Downarrow^a_\varphi]\psi & \\
[H\Downarrow^a_\varphi](\psi_1 \wedge \psi_2) & \leftrightarrow & [H\Downarrow^a_\varphi]\psi_1 \wedge [H\Downarrow^a_\varphi]\psi_2 & \\
[H\Downarrow^a_\varphi]\mathsf{Sign}(b,\psi) & \leftrightarrow & \mathsf{Sign}(b,[H\Downarrow^a_\varphi]\psi) & \\
[H\Downarrow^a_\varphi]\mathsf{Bel}(b,\psi) & \leftrightarrow & \mathsf{Bel}(b,[H\Downarrow^a_\varphi]\psi) & \\
[H\Downarrow^a_\varphi]\mathsf{A}\psi & \leftrightarrow & \mathsf{A}[H\Downarrow^a_\varphi]\psi &
\end{array}$$

Proposition 4 [Recursive validities (Jirakunkanok et al. 2014)] *The following are valid on all models. Moreover, if ψ is valid on all models, then $[H \Uparrow^a_\varphi]\psi$ is also valid on all models.*

[7] Also in the last case, since there is no relation between agents b and c, $b \preccurlyeq^u_a c$ is omitted.

$$
\begin{array}{lll}
[H\Uparrow^a_\varphi]p & \leftrightarrow & p \\
[H\Uparrow^a_\varphi](b\leqslant_d c) & \leftrightarrow & b\leqslant_d c \qquad\qquad (d\neq a) \\
[H\Uparrow^a_\varphi](b\leqslant_a c) & \leftrightarrow & b\leqslant_a c \qquad\qquad (b,c\in G\setminus H) \\
[H\Uparrow^a_\varphi](b\leqslant_a c) & \leftrightarrow & \big(\mathsf{Sign}(b,\varphi)\wedge\mathsf{Sign}(c,\varphi)\wedge(b\leqslant_a c)\big)\vee \\
 & & \big(\neg\mathsf{Sign}(b,\varphi)\wedge\neg\mathsf{Sign}(c,\varphi)\wedge(b\leqslant_a c)\big)\vee \\
 & & \big(\mathsf{Sign}(b,\varphi)\wedge\neg\mathsf{Sign}(c,\varphi)\big) \qquad\qquad (b,c\in H) \\
[H\Uparrow^a_\varphi](b\leqslant_a c) & \leftrightarrow & \neg\mathsf{Sign}(c,\varphi)\wedge(b\leqslant_a c) \qquad\qquad (c\in H, b\in G\setminus H) \\
[H\Uparrow^a_\varphi](b\leqslant_a c) & \leftrightarrow & \mathsf{Sign}(b,\varphi)\vee\big(\neg\mathsf{Sign}(b,\varphi)\wedge(b\leqslant_a c)\big) \qquad (b\in H, c\in G\setminus H) \\
[H\Uparrow^a_\varphi]\neg\psi & \leftrightarrow & \neg[H\Uparrow^a_\varphi]\psi \\
[H\Uparrow^a_\varphi](\psi_1\wedge\psi_2) & \leftrightarrow & [H\Uparrow^a_\varphi]\psi_1\wedge[H\Uparrow^a_\varphi]\psi_2 \\
[H\Uparrow^a_\varphi]\mathsf{Sign}(b,\psi) & \leftrightarrow & \mathsf{Sign}(b,[H\Uparrow^a_\varphi]\psi) \\
[H\Uparrow^a_\varphi]\mathsf{Bel}(b,\psi) & \leftrightarrow & \mathsf{Bel}(b,[H\Uparrow^a_\varphi]\psi) \\
[H\Uparrow^a_\varphi]\mathsf{A}\psi & \leftrightarrow & \mathsf{A}[H\Uparrow^a_\varphi]\psi
\end{array}
$$

Joint downgrade Based on Jirakunkanok et al. (2015a), a joint downgrade $[H\downdownarrows^a]$ is introduced in order to allow an agent to downgrade the agents in the specific group equally reliable and less reliable than the agents in the other groups. For $[H\downdownarrows^a]$, $H\subseteq G$ is a set of agents. The reading of $[H\downdownarrows^a]\psi$ is "after such agents in H are downgraded jointly by agent a, ψ holds". Semantically speaking, $[H\downdownarrows^a]$ makes such agents in H equally reliable and less reliable than the agents in the other groups. Note that the joint downgrade operator $[H\downdownarrows^a]$ is different from the downgrade operator $[H\Downarrow^a_\varphi]$ in two respects. First, $[H\downdownarrows^a]$ focuses only on the agents in H without consideration of information, while $[H\Downarrow^a_\varphi]$ considers both the agents in H and signed information of such agents, that is, $[H\Downarrow^a_\varphi]$ focuses on the agents who sign information φ in H. The second one is the result of downgrading, that is, $[H\downdownarrows^a]$ makes the reliability ordering between agents in H equal, while $[H\Downarrow^a_\varphi]$ keeps the same reliability ordering between agents in H.

Before giving the detailed semantics, let us demonstrate the effects of $[H\downdownarrows^a]$ by figures. Firstly, we assume that a rectangle G of Fig. 7(i) represents a fixed finite set

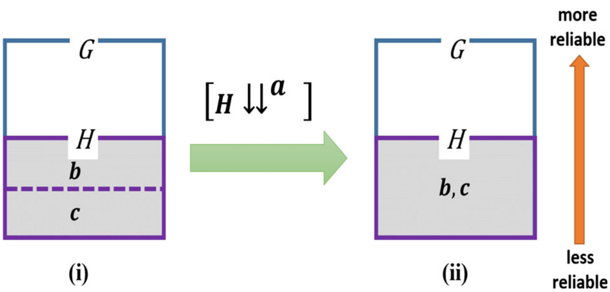

Fig. 7 A model for jointly downgrading by $[H\downdownarrows^a]$

of agents. Secondly, we will select a specified set of agents in order to jointly downgrade the reliability ordering that can be represented by a rectangle H, and we assume that $b <_a c$ holds, i.e., agent b is more reliable than agent c from agent a's perspective. In this sense, b and c are situated as in Fig. 7(i). Then, if the agents in H are *downgraded jointly* by agent a, all of them will be made to be equally reliable and less reliable than all the other agents as in Fig. 7(ii).

Definition 7 Given a Kripke model $\mathfrak{M} = (W, (R_a)_{a\in G}, (S_a)_{a\in G}, (\preccurlyeq_d)_{d\in G}, V)$, a semantic clause for $[H \downdownarrows^a]$ on $\mathfrak{M}$ and $w \in W$ is defined by:

$$\mathfrak{M}, w \models [H \downdownarrows^a]\psi \quad \text{iff} \quad \mathfrak{M}^{H\downdownarrows^a}, w \models \psi,$$

where $\mathfrak{M}^{H\downdownarrows^a} = (W, (R_a)_{a\in G}, (S_a)_{a\in G}, (\preccurlyeq'_d)_{d\in G}, V)$ and $\preccurlyeq'_d$ is defined as: for all $u \in W$:

- if $d \neq a$, we put $\preccurlyeq'^u_d = \preccurlyeq^u_d$.
- otherwise (if $d = a$), we define $b \preccurlyeq'^u_a c$ iff

$$\big(b, c \in H\big) \mathrm{or} \big(\mathrm{b, c} \in (\mathrm{G} \setminus \mathrm{H}) \,\mathrm{and}\, \mathrm{b} \preccurlyeq^{\mathrm{u}}_{\mathrm{a}} \mathrm{c}\big) \mathrm{or} \big(\mathrm{b} \in (\mathrm{G} \setminus \mathrm{H}) \,\mathrm{and}\, \mathrm{c} \in \mathrm{H}\big).$$

Note that this joint downgrade can preserve the property of total preordering of $(\preccurlyeq_d)_{d\in G}$.

Proposition 5 [Recursive validities (Jirakunkanok et al. 2015b)] *The following are valid on all models. Moreover, if ψ is valid on all models, then $[H \downdownarrows^a]\psi$ is also valid on all models.*

$$\begin{array}{lll}
[H \downdownarrows^a]p & \leftrightarrow & p \\
[H \downdownarrows^a](b \leqslant_d c) & \leftrightarrow & b \leqslant_d c \qquad (d \neq a) \\
[H \downdownarrows^a](b \leqslant_a c) & \leftrightarrow & b \leqslant_a c \qquad (b, c \in G \setminus H) \\
[H \downdownarrows^a](b \leqslant_a c) & \leftrightarrow & \top \qquad (b, c \in H) \\
[H \downdownarrows^a](b \leqslant_a c) & \leftrightarrow & \top \qquad (c \in H, b \in G \setminus H) \\
[H \downdownarrows^a](b \leqslant_a c) & \leftrightarrow & \bot \qquad (b \in H, c \in G \setminus H) \\
[H \downdownarrows^a]\neg\psi & \leftrightarrow & \neg[H \downdownarrows^a]\psi \\
[H \downdownarrows^a](\psi_1 \wedge \psi_2) & \leftrightarrow & [H \downdownarrows^a]\psi_1 \wedge [H \downdownarrows^a]\psi_2 \\
[H \downdownarrows^a]\mathsf{Sign}(b, \psi) & \leftrightarrow & \mathsf{Sign}(b, [H \downdownarrows^a]\psi) \\
[H \downdownarrows^a]\mathsf{Bel}(b, \psi) & \leftrightarrow & \mathsf{Bel}(b, [H \downdownarrows^a]\psi) \\
[H \downdownarrows^a]\mathsf{A}\psi & \leftrightarrow & \mathsf{A}[H \downdownarrows^a]\psi
\end{array}$$

Theorem 2 *The set of all valid formulas of the expanded syntax $\mathcal{L}^+_{BSR}$ of $\mathcal{L}_{BSR}$ with $[\mathbb{E}_{!^a_\varphi}, e]$, $[\mathbb{E}_{¡^a_\varphi}, e]$, $[H \Downarrow^a_\varphi]$, $[H \Uparrow^a_\varphi]$ and $[H \downdownarrows^a]$ is axiomatized by the Hilbert-style axiomatization* **HBSR** *in Table 2 as well as the axioms and the rules of Propositions 1, 2, 3, 4, and 5.*

The proof of Theorem 2 is presented in "Appendix 2.4".

5 Implementation

This section introduces an implementation of our logical formalization. First, the main algorithms for calculating our dynamic operators are presented in Sect. 5.1. Then, these algorithms together with a method for constructing an initial model and an inconsistency solving algorithm (described in Sect. 6.1.1) are implemented into our program that is described in Sect. 5.2.

5.1 Algorithms for changing belief and reliability

This section introduces seven algorithms. First, we describe how to calculate the truth value of the formulas $\mathsf{Bel}(a,\psi)$ and $\mathsf{Sign}(a,\psi)$ by Algorithms 1 and 2, respectively. Next, we present Algorithms 3 and 4 for calculating the updates by $[\mathbb{E}_{!^a_\varphi}, e]$ (for the private announcement) and $[\mathbb{E}_{¡^a_\varphi}, e]$ (for the private permission), respectively. After that, three algorithms for calculating the updates of reliability orderings by downgrade $[H \Downarrow^a_\varphi]$, upgrade $[H \Uparrow^a_\varphi]$ and joint downgrade $[H \downdownarrows^a]$ are stated in Algorithms 5, 6 and 7, respectively. For all algorithms, we assume that an input model $\mathfrak{M} = (W, (R_a)_{a\in G}, (S_a)_{a\in G}, (\preccurlyeq_a)_{a\in G}, V)$ (described in Sect. 4.1). The input models $\mathbb{E}_{!^a_\varphi}$ (in Algorithm 3) and $\mathbb{E}_{¡^a_\varphi}$ (in Algorithm 4) are defined as a tuple $(E, (D_c)_{c\in G}, (U_a)_{a\in G}, \mathrm{pre})$ according to Definitions 1 and 3 in Sect. 4.2.1, respectively.

First, let us explain Algorithm 1 for calculating the truth value of the formula $\mathsf{Bel}(a,\psi)$. The main operation is to investigate if ψ holds for all $v \in W$ such that wR_av or not. Algorithm 2 can be performed in a similar way to Algorithm 1, but we check the relation S_a instead of R_a.

Next, two algorithms for formalizing belief re-revision can be described as follows: Algorithm 3 concerns the private announcement operator $[\varphi \rightsquigarrow a]$ that can be calculated by $[\mathbb{E}_{!^a_\varphi}, e]$ according to Definition 2 (in Sect. 4.2.1). The main operation is to check if $\mathrm{pre}(f)$ holds for all v such that $(e,f) \in D_a$ and $(w,v) \in R_a$ or not. As a result, all the accessible worlds are restricted to those where $\mathrm{pre}(f)$ holds. Algorithm 4 concerns the private permission operator $[\varphi \twoheadrightarrow a]$ that can be calculated by $[\mathbb{E}_{¡^a_\varphi}, e]$ according to Definition 4 (in Sect. 4.2.1). The main operation is to enlarge all the accessible worlds to those where $\mathrm{pre}(f)$ holds for all $v \in W$ such that $(e,f) \in D_a$.

Finally, we will describe three algorithms for formalizing reliability change. Note that we fix the rank of reliability to be 0, 1, and 2 in our implementation. That is, we may regard that 0 represents unreliable, 1 represents neutral, and 2 represents reliable.[8] At the initial stage, our implementation will automatically define that all agents are equally reliable, i.e., their rank is 1. Then, if the agents are downgraded to be less reliable, we define their rank of reliability as 0. On the other hand, if the agents are upgraded to be more reliable, we define their rank of reliability as 2.

[8] In the real world, a judge cannot categorize the reliability of witnesses to be several groups as Lorini et al. (2011). Thus, this work proposes to simplify the rank of reliability in a real situation by fixing to be 0, 1, and 2.

Algorithm 5 concerns the downgrade operator $[H \Downarrow^a_\varphi]$. There are two main steps: (1) collecting all agents $c \in H$ where $\mathsf{Sign}(c, \varphi)$ holds at w into set X, and (2) setting $rank^u_a(c) := 0$, which stands for "from agent a's perspective, a rank of agent c at state u is 0", for all $c \in X$ and all $u \in W$. Algorithm 6 which concerns the upgrade operator $[H \Uparrow^a_\varphi]$ can be performed by two steps. The first step is the same as in Algorithm 5, while the second step defines the rank of an agent's reliability by $rank^u_a(c) := 2$ instead. The last algorithm, Algorithm 7, concerns the joint downgrade operator $[H \downdownarrows^a]$. Although $[H \Downarrow^a_\varphi]$ and $[H \downdownarrows^a]$ have the same goal which is to set $rank^u_a(c) := 0$, $[H \downdownarrows^a]$ does not consider signed statements. Thus, Algorithm 7 has only one step that is to set $rank^u_a(c) := 0$ for all $c \in H$ and all $u \in W$.

Algorithm 1 : Calculation of the truth of $\mathsf{Bel}(a, \psi)$

input $\mathfrak{M}$, $\mathsf{Bel}(a, \psi)$, $w \in W$
forall $((w, v) \in R_a,\ \mathfrak{M}, v \models \psi)$
  $\mathfrak{M}, w \models \mathsf{Bel}(a, \psi)$
else
  $\mathfrak{M}, w \not\models \mathsf{Bel}(a, \psi)$
end forall

Algorithm 2 : Calculation of the truth of $\mathsf{Sign}(a, \psi)$

input $\mathfrak{M}$, $\mathsf{Sign}(a, \psi)$, $w \in W$
forall $((w, v) \in S_a,\ \mathfrak{M}, v \models \psi)$
  $\mathfrak{M}, w \models \mathsf{Sign}(a, \psi)$
else
  $\mathfrak{M}, w \not\models \mathsf{Sign}(a, \psi)$
end forall

Algorithm 3 : Calculation of $[\mathbb{E}_{!^a_\varphi}, e]$

input $\mathfrak{M}$, $\mathbb{E}_{!^a_\varphi}$, $[\mathbb{E}_{!^a_\varphi}, e]$ $w \in W$, $e \in E$
for $(e, f) \in D_a$ **do**
  for $(w, v) \in R_a$ **do**
    add $((w, e), (v, f))$ **to** X
    if $\mathfrak{M}, v \models \mathrm{pre}(f)$ **then**
      add $((w, e), (v, f))$ **to** Y
    end if
  end for
end for
$R''_a := \{((w', e'), (v', f'))\,|(w', v') \in R_a \text{ and } (e', f') \in D_a\}$
$R'_a := (R''_a \setminus X) \cup Y$
$W' := \{(w', e')|w' \in W \text{ and } e' \in E\}$
$S'_a := \{((w', e'), (v', f'))\,|(w', v') \in S_a \text{ and } (e', f') \in U_a\}$
$\preccurlyeq'_a := \preccurlyeq_a$
$V'(p) := \{(w', e')|w' \in V(p) \text{ and } e' \in E\}$
output $\mathfrak{M}' = (W', (R'_a)_{a \in G}, (S'_a)_{a \in G}, (\preccurlyeq'_a)_{a \in G}, V')$

Algorithm 4 : Calculation of $[\mathbb{E}_{¡^a_\varphi}, e]$

input $\mathfrak{M}$, $\mathbb{E}_{¡^a_\varphi}$, $[\mathbb{E}_{¡^a_\varphi}, e]$, $w \in W$, $e \in E$
for $(e, f) \in D_a$ **do**
  findall $(v \in W,\ \mathfrak{M}, v \models \text{pre}(f))$
    add v **to** X
  end findall
end for
$Y := \{((w', e'), (v', f'))\,|w' \in W \text{ and } v' \in X \text{ and } (e', f') \in D_a\}$
$R''_a := \{((w', e'), (v', f'))\,|(w', v') \in R_a \text{ and } (e', f') \in D_a\}$
$R'_a := R''_a \cup Y$
$W' := \{(w', e')|w' \in W \text{ and } e' \in E\}$
$S'_a := \{((w', e'), (v', f'))\,|(w', v') \in S_a \text{ and } (e', f') \in U_a\}$
$\preccurlyeq'_a := \preccurlyeq_a$
$V'(p) := \{(w', e')|w' \in V(p) \text{ and } e' \in E\}$
output $\mathfrak{M}' = (W', (R'_a)_{a\in G}, (S'_a)_{a\in G}, (\preccurlyeq'_a)_{a\in G}, V')$

Algorithm 5 : Calculation of $[H \Downarrow^a_\varphi]$

input $\mathfrak{M}$, $[H \Downarrow^a_\varphi]$, $w \in W$, $H \subseteq G$, $(rank_a)_{a\in G}$
findall $(c \in H,\ \mathfrak{M}, w \models \mathsf{Sign}(c, \varphi))$
  add c **to** X
end findall
for $c \in X$ **do**
  for $u \in W$ **do**
    $rank^u_a(c) := 0$
  end for
end for
$rank'_a := rank_a$
output $(rank'_a)_{a\in G}$

Algorithm 6 : Calculation of $[H \Uparrow^a_\varphi]$

input $\mathfrak{M}$, $[H \Uparrow^a_\varphi]$, $w \in W$, $H \subseteq G$, $(rank_a)_{a\in G}$
findall $(c \in H,\ \mathfrak{M}, w \models \mathsf{Sign}(c, \varphi))$
  add c **to** X
end findall
for $c \in X$ **do**
  for $u \in W$ **do**
    $rank^u_a(c) := 2$
  end for
end for
$rank'_a := rank_a$
output $(rank'_a)_{a\in G}$

Algorithm 7 : Calculation of $[H \downdownarrows^a]$

input $\mathfrak{M}$, $[H \downdownarrows^a]$, $w \in W$, $H \subseteq G$, $(rank_a)_{a\in G}$
for $c \in H$ **do**
  for $u \in W$ **do**
    $rank^u_a(c) := 0$
  end for
end for
$rank'_a := rank_a$
output $(rank'_a)_{a\in G}$

5.2 Interfaces

We have implemented a Windows-Based Application with Visual C# ^{TM}. This program outputs the truth value of propositions, together with world accessibility relations in `dot` format. Thus, we can visualize the `dot` file by Graphviz^{TM}. The main features of our implementation are summarized as follows:

1. We can input and edit the definition of an initial model according to Sect. 4.1 including the following six items:

   - G : a finite set of agents
   - W : a finite non-empty set of states
   - $(R_a)_{a \in G}$: an accessibility relation representing beliefs of agent a
   - $(S_a)_{a \in G}$: an accessibility relation representing signatures of agent a
   - V : a valuation
   - $\preccurlyeq_a$: agent a's reliability ordering between agents

   In addition, we can input @ which is a current state representing an agent a's viewpoint. For the reliability ordering, the system will automatically define that all agents are equally reliable, i.e., their rank is 1 at the initial stage. Recall that this implementation fixes the rank of reliability to be 0, 1, and 2. That is, we may regard that 0 represents unreliable, 1 represents neutral, and 2 represents reliable. However, the system allows us to edit the reliability ordering of all agents manually. Furthermore, the system can export the initial model into the text file and import such initial model into the system. Figure 8 shows an example of inputting an initial model of the second legal case.

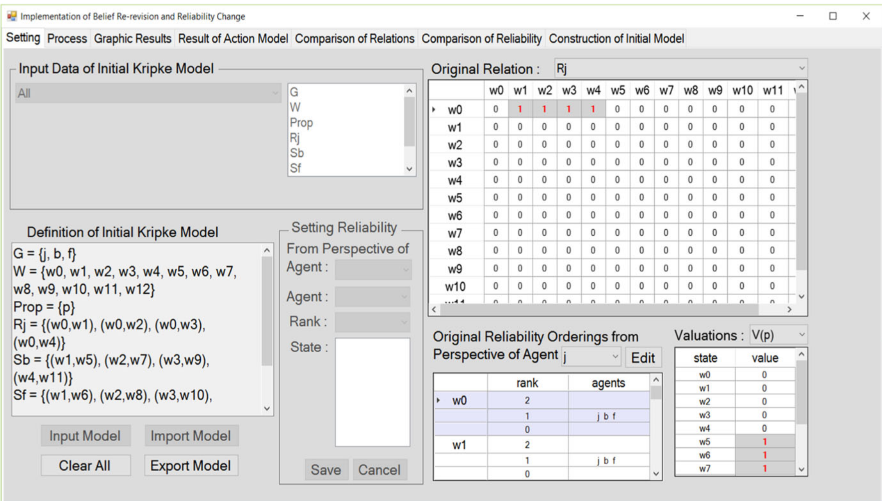

Fig. 8 Example of a result after inputting an initial model of the second legal case

2. The system can generate an initial model by the following steps:

   - The system first imports a set of agents (G) and a set of propositions (Prop). Then, the system allows us to input three key features of a legal case including a statement, a belief agent, and signed agents. Note that a statement represents a proposition that needs to be analyzed, a belief agent is an agent that needs to be analyzed his/her belief/reliability change, and signed agents are agents who give such statement in the legal case. After that, the system will generate all possibilities according to the input features.
   - The system allows us to edit such possibilities obtained from the previous step by removing some of them. Then, the system will generate an initial model such as in Fig. 9.
   - The system allows us to export such initial model into the text file.

   The more details of this process will be described in Sect. 6.1.1.

3. We can input any dynamic logical operators consisting of six operators as shown in Table 4. Then, the system automatically calculates such operator according to Sect. 4.2 and outputs the result on the screen such as in Fig. 10. In addition, the system can automatically visualize the resultant states such as Fig. 11 which represents a result after calculating $[\mathsf{Sign}(f, \neg p) \rightsquigarrow j]$ by a product update operation. Since the result of computing by an action model (in Fig. 11) is difficult to understand, the system will automatically convert such result into a simple one (which focuses only an announcing action '!') such as in the right-hand side of Fig. 15 in Sect. 6.1.2.
4. The system can verify if an input of dynamic logical operator is correct syntax or not by a logical formula parser. If the input has syntax errors, the parser can detect where an error occurred and may correct such error by a process of error recovery automatically. However, some errors cannot be automatically

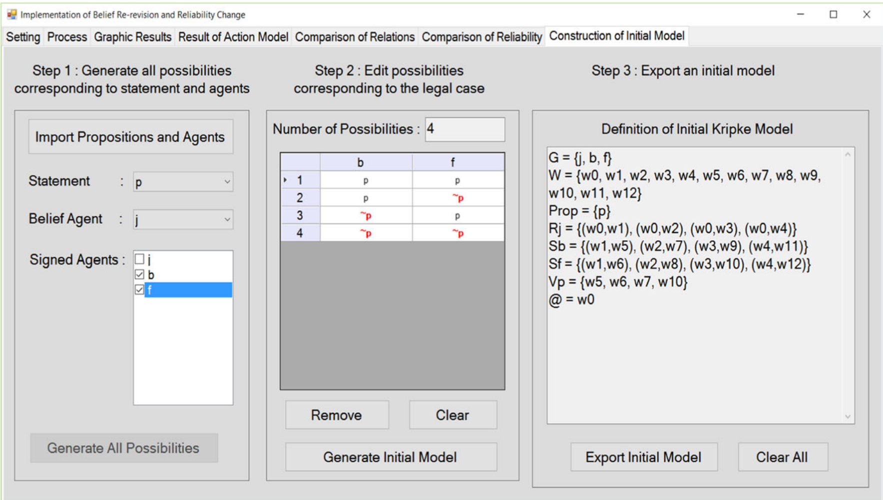

Fig. 9 Example of a result after generating an initial model of the second legal case

Table 4 Format of dynamic logical operators for inputting in the implementation

Operator name	Logical formula	Format in the system
Private announcement	$[p \rightsquigarrow a]$	`[p=>a]`
Private permission	$[p \twoheadrightarrow a]$	`[PriPer(a,p)]`
Careful policy	$[\mathsf{Careful}(a,p)]$	`[Agg(a,p)]`
Downgrade	$[H \Downarrow^a_p]$	`[downgrade(a,H,p)]`
Upgrade	$[H \Uparrow^a_p]$	`[upgrade(a,H,p)]`
Joint downgrade	$[H \downdownarrows^a]$	`[jdowngrade(a,H)]`

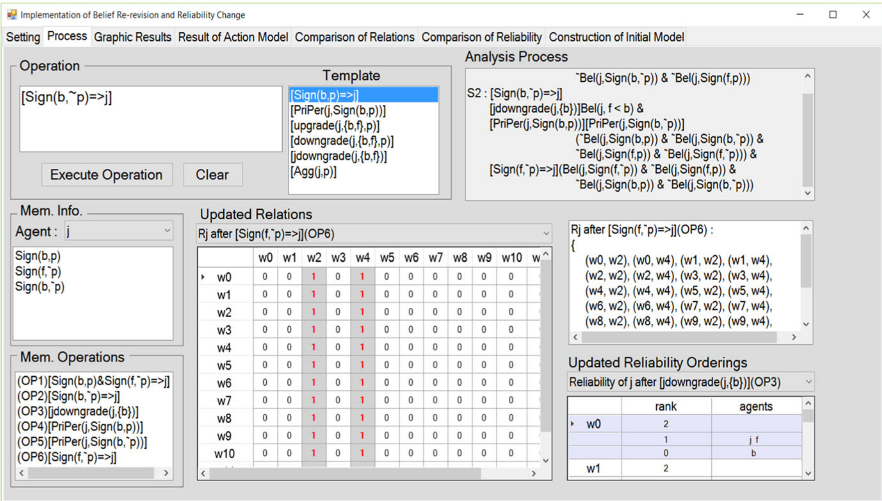

Fig. 10 Example of a result after inputting $[\mathsf{Sign}(b, \neg p) \rightsquigarrow j]$

corrected by the system. That is, it requires the user to correct them by him/herself. In this case, the system will indicate a position where the error occurred and generate an error message including information about such error and a suggestion for error recovery. A process of parsing in the logical formula parser is constructed based on the Earley algorithm (Jurafsky and Martin 2009) which is a well-known top-down parsing algorithm.

5. The system can perform an inconsistency solving algorithm as follows:

   5.1. The system will check if there is an inconsistency between the existing belief and new information or not.

   5.2. If there is an inconsistency, the system applies the joint downgrade and the private permission operators.

   5.3. The system will check if there is the received information which is not inconsistent with the existing belief and is signed by the most reliable agent or not. If there is such information, the system will apply the private announcement operator for admitting such information.

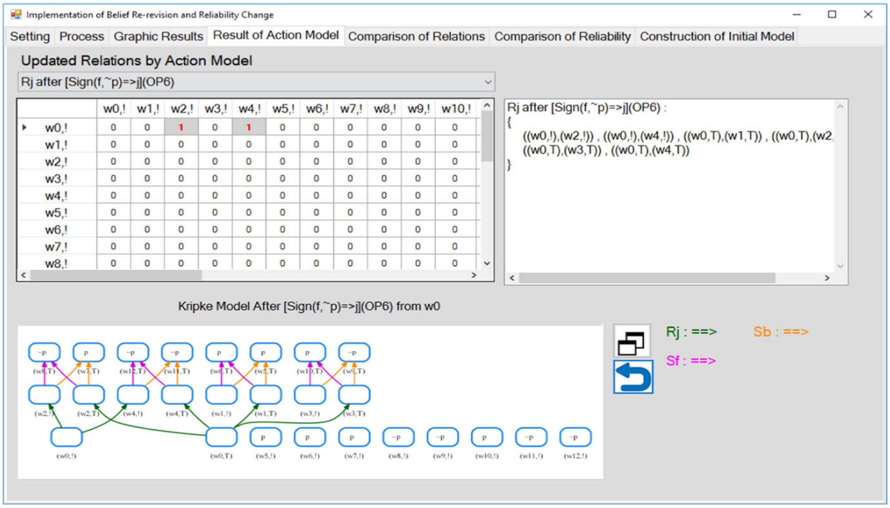

Fig. 11 Example of a result after calculating $[\mathsf{Sign}(f, \neg p) \rightsquigarrow j]$ by a product update operation

The above process can be illustrated by Fig. 10. When $[\mathsf{Sign}(b, \neg p) \rightsquigarrow j]$ is calculated and an inconsistency is detected, the system will perform four operations corresponding to the above process as follows: $[\{b\} \downdownarrows^{j}]$, $[\mathsf{Sign}(b, p) \twoheadrightarrow j]$, $[\mathsf{Sign}(b, \neg p) \twoheadrightarrow j]$, and $[\mathsf{Sign}(f, \neg p) \rightsquigarrow j]$. The more details of this process will be described in Sect. 6.1.2.

6. The system can keep track of all the changes including belief change and reliability change of an agent by showing a comparison of relations and a comparison of reliability orderings from a specific agent's perspective.

6 Dynamic logical analysis of six target legal cases

This section demonstrates our implementation by analyzing six target legal cases. Firstly, we will illustrate a method for analyzing the second legal case which was mentioned in Sect. 1. Then, we will show a summary of six target legal cases. After that, we will present the results of analyzing six target legal cases by our implementation.

6.1 Target legal case

In this section, our target legal case is the second legal case which is selected from our six target legal cases in Sect. 6.2. Before describing our analysis method, we will give a short description of our target legal case (cf. "Appendix 1") as follows:

> There was a fight between two groups of people, i.e., v's group (v and b) and d's group (d, f_1, f_2, and f_3). In the course of the fight, one of d's group pulled a knife and then stabbed v in the chest. Finally, v died.

6.1.1 Analysis method

In order to analyze this target legal case, we propose the following analysis method:

1. We will manually summarize the target legal case by extracting the facts and the decision (the more details of this step will be described in Sect. 6.2).[9] The result can be shown in Tables 6, 7, and 8.
2. We will construct an initial Kripke model from the target legal case. This model is used for analyzing belief/reliability change of a judge. A construction of the initial model can be done by our implementation, as mentioned in the second feature of our implementation in Sect. 5.2. This process can be summarized into the following steps:

   2.1. We will generate all possibilities which can be represented by possible belief states of an agent in a Kripke model. The number of all possible belief states can be calculated by 2^N where N is the number of signed agents or witnesses in the legal case. From Table 8, our target legal case consists of a statement p and two witnesses b and f. That is, we obtain that N is equal to two, and the number of all possible belief states is four states as follows:
   (w_1) Agent b gives statement p, and agent f gives statement p.
   (w_2) Agent b gives statement p, and agent f gives statement $\neg p$.
   (w_3) Agent b gives statement $\neg p$, and agent f gives statement p.
   (w_4) Agent b gives statement $\neg p$, and agent f gives statement $\neg p$.
   From the above possible belief states, we can regard that each belief state consists of two signed states representing signed statements of b and f. If there is a lot of possible belief states, we may remove some states which are considered to be not important for analyzing belief/reliability change of a judge. That is, we may regard that such possibilities cannot occur in the legal judgment. This is for simplifying the initial model to be easy for analyzing belief/reliability change of a judge.

   2.2 Firstly, j representing a judge is defined as a belief agent, b and f who are witnesses are defined as signed agents, and a current state representing j's viewpoint is defined as w_0. Then, we will add links from the current state (w_0) to all possible belief states (w_1, w_2, w_3, w_4) because we assume that the judge should open to all possibilities at the initial stage. Next, we also add links from each belief state to its signed states. Finally, the initial model of our target legal case can be constructed as in Fig. 12 which is the same as the left-hand side of Fig. 13 outputting from our implementation. Note that Fig. 12 presents one way for generating an initial model from the second legal case. However, an initial model of the second legal case can be constructed in

[9] In this study, we did not apply legal text processing in the area of natural language processing (NLP) for summarizing legal cases and generating an initial Kripke model from a legal case.

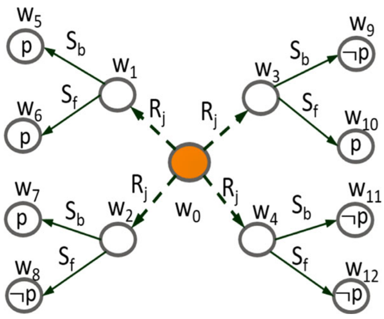

Fig. 12 An initial Kripke model from the second legal case. This model consists of 13 states including four belief states of j (w_1, w_2,w_3, w_4), eight signed states including four signed states of b (w_5, w_7, w_9, w_{11}) and four signed states of f (w_6, w_8, w_{10}, w_{12}), and one current state (w_0) representing j's viewpoint

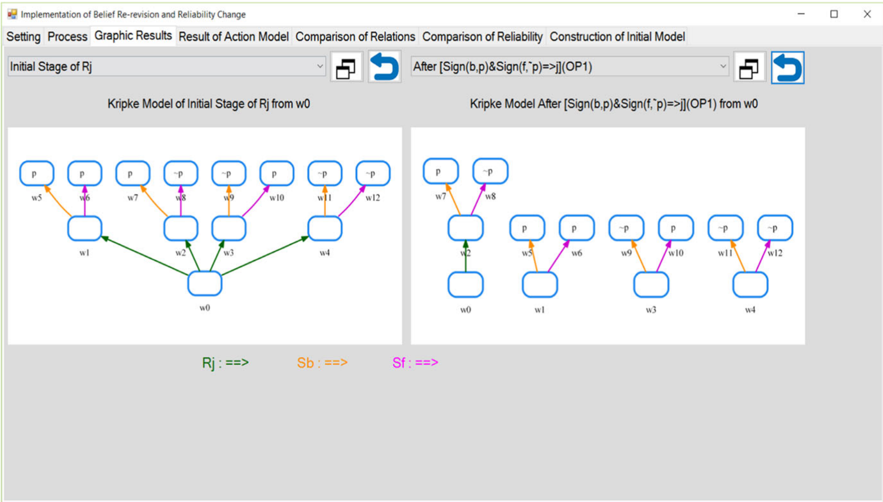

Fig. 13 The left-hand side is Kripke model of the initial stage, and the right-hand side is Kripke model after $[(\mathsf{Sign}(b,p) \wedge \mathsf{Sign}(f,\neg p)) \rightsquigarrow j]$

different ways, that is, identifying different key features of a legal case (i.e., a statement, a belief agent and signed agents) or removing some possibilities. For example, if we regard that giving statement p of agent f is not important, we can remove states w_1 and w_3. As a result, we obtain an initial Kripke model consisting of seven states (i.e., two belief states (w_2, w_4), two signed states of b (w_7, w_{11}), two signed states of f (w_8, w_{12}) and one current state (w_0)) that is different from Fig. 12. With different initial models, we can obtain the same analysis result if such models have the essential information which is sufficient for a judgment.

3. We can analyze the legal case by inputting any dynamic logical operators including private announcement, private permission, downgrade, upgrade, joint downgrade, and careful policy, as mentioned in Sect. 4.2. Recall that our

implementation aims at reducing the effort to decide which operators are to be applied for analyzing belief/reliability change of an agent. With this goal, we propose an application method of our dynamic operators as follows:

3.1. We assume that the agent needs to apply two basic operations including private announcement and careful policy. When the agent receives a piece of information, he/she will apply the private announcement for admitting such information. The careful policy is used for deriving beliefs from signed information. Based on this idea and the above goal, there are the following options (OP1) and (OP2):
(OP1) The agent needs to apply only two kinds of operators, i.e., private announcement and careful policy. This means that whenever the agent receives a piece of information, he/she will accept the received information by applying the private announcement. If there is an inconsistency, the system will handle such inconsistency instead of the agent. That is, the inconsistency solving algorithm will be automatically performed by applying joint downgrade, private permission, and/or private announcement operators. In this sense, the agent does not need to change his/her reliability or permit the possibility to his/her belief by him/herself.
(OP2) The agent needs to apply three kinds of operators, i.e., private announcement, downgrade/upgrade/joint downgrade, and careful policy. That is, the agent needs to apply the operation for changing his/her reliability, i.e., downgrade, upgrade, and joint downgrade. In this option, the agent needs to decide how to change his/her reliability ordering between other agents by him/herself.[10]

3.2. In order to analyze the legal case, we first use option (OP1) because it is an easy way that we do not need to take much effort to decide which operators are to be applied. If option (OP1) cannot work well, we will use option (OP2).

6.1.2 Analysis process

In order to analyze the target legal case from a judge's viewpoint, we will first focus on the court. At the initial stage, we assume that a judge j should open to all possibilities as in the left-hand side of Fig. 13. Next, when j receives information from witnesses b and f, the following steps are performed as in Table 5:

1. j admits the statements of witnesses b and f in the court, i.e., $\mathsf{Sign}(b,p)$ and $\mathsf{Sign}(f,\neg p)$ by $[(\mathsf{Sign}(b,p) \wedge \mathsf{Sign}(f,\neg p)) \rightsquigarrow j]$. As a result, j believes that $\mathsf{Sign}(b,p)$ and $\mathsf{Sign}(f,\neg p)$, i.e., $\mathsf{Bel}(j,\mathsf{Sign}(b,p))$ and $\mathsf{Bel}(j,\mathsf{Sign}(f,\neg p))$ as shown in the right-hand side of Fig. 13.

[10] In this study, we will not analyze how an agent decides to change his/her reliability ordering between the other agents because this is a psychological issue and is out of our scope.

Table 5 Summary of analysis process of the second legal case

Step	Operation	Meaning	Result
(1)	$[(\mathsf{Sign}(b,p) \wedge \mathsf{Sign}(f,\neg p)) \rightsquigarrow j]$	j admits $\mathsf{Sign}(b,p)$ and $\mathsf{Sign}(f,\neg p)$	$\mathsf{Bel}(j,\mathsf{Sign}(b,p)) \wedge \neg\mathsf{Bel}(j,\mathsf{Sign}(b,\neg p)) \wedge \neg\mathsf{Bel}(j,\mathsf{Sign}(f,p)) \wedge \mathsf{Bel}(j,\mathsf{Sign}(f,\neg p))$
(2)	$[\mathsf{Sign}(b,\neg p) \rightsquigarrow j]$	j admits $\mathsf{Sign}(b,\neg p)$	None
(2.1)	$[\{b\} \downdownarrows^{j}]$	j downgrades agent b	$\mathsf{Bel}(j, f <_j b)$
(2.2)	$[\mathsf{Sign}(b,p) \twoheadrightarrow j][\mathsf{Sign}(b,\neg p) \twoheadrightarrow j]$	j permits $\mathsf{Sign}(b,p)$ and $\mathsf{Sign}(b,\neg p)$	$\neg\mathsf{Bel}(j,\mathsf{Sign}(b,p)) \wedge \neg\mathsf{Bel}(j,\mathsf{Sign}(b,\neg p)) \wedge \neg\mathsf{Bel}(j,\mathsf{Sign}(f,p)) \wedge \neg\mathsf{Bel}(j,\mathsf{Sign}(f,\neg p))$
(2.3)	$[\mathsf{Sign}(f,\neg p) \rightsquigarrow j]$	j admits $\mathsf{Sign}(f,\neg p)$	$\neg\mathsf{Bel}(j,\mathsf{Sign}(b,p)) \wedge \neg\mathsf{Bel}(j,\mathsf{Sign}(b,\neg p)) \wedge \neg\mathsf{Bel}(j,\mathsf{Sign}(f,p)) \wedge \mathsf{Bel}(j,\mathsf{Sign}(f,\neg p))$
(3)	$[\mathsf{Sign}(f,p) \rightsquigarrow j]$	j admits $\mathsf{Sign}(f,p)$	None
(3.1)	$[\{f\} \downdownarrows^{j}]$	j downgrades agent f	$\mathsf{Bel}(j, b \approx_j f)$
(3.2)	$[\mathsf{Sign}(f,\neg p) \twoheadrightarrow j][\mathsf{Sign}(f,p) \twoheadrightarrow j]$	j permits $\mathsf{Sign}(f,\neg p)$ and $\mathsf{Sign}(f,p)$	$\neg\mathsf{Bel}(j,\mathsf{Sign}(b,p)) \wedge \neg\mathsf{Bel}(j,\mathsf{Sign}(b,\neg p)) \wedge \neg\mathsf{Bel}(j,\mathsf{Sign}(f,p)) \wedge \neg\mathsf{Bel}(j,\mathsf{Sign}(f,\neg p))$

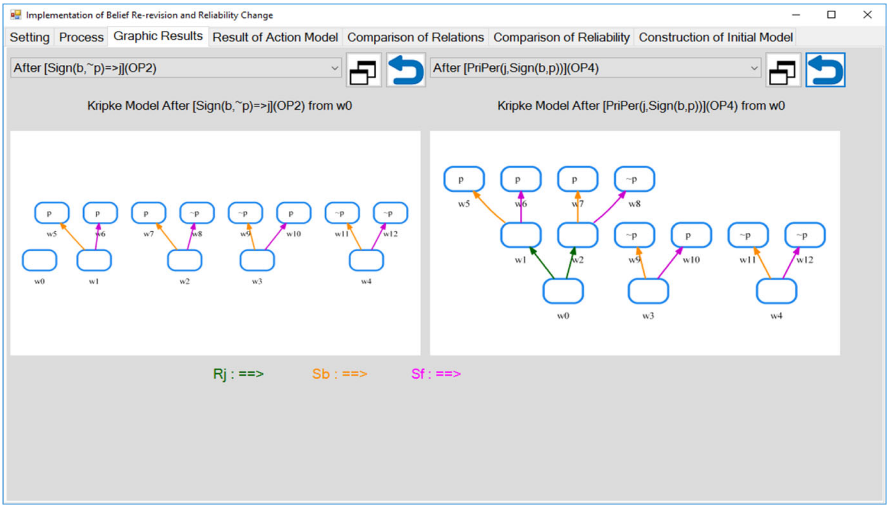

Fig. 14 The left-hand side is Kripke model after $[\mathsf{Sign}(b,\neg p)\rightsquigarrow j]$, and the right-hand side is Kripke model after $[\mathsf{Sign}(b,p)\twoheadrightarrow j]$

2. When j turns back to the inquiry stage, j commits him/herself to statement of b in the inquiry stage, i.e., $\mathsf{Sign}(b,\neg p)$ by $[\mathsf{Sign}(b,\neg p)\rightsquigarrow j]$, and the result can be shown in the left-hand side of Fig. 14. After that, the system can detect that the received statement $\mathsf{Sign}(b,\neg p)$ is inconsistent with j's belief. Thus, the system automatically performs the inconsistency solving algorithm including four operations as the following sequences:

   2.1. When the system finds that b gives inconsistent statements, we can regard that b is unreliable. Therefore, the system will apply $[\{b\}\downdownarrows^j]$ in order to downgrade b less reliable than the other agents. The result of this downgrading is $\mathsf{Bel}(j,f<_j b)$ which means j believes that b becomes less reliable than f from j's perspective.
   2.2. By the update of $[\mathsf{Sign}(b,\neg p)\rightsquigarrow j]$ in Step 2, there is no link from state w_0 (see the left-hand side of Fig. 14). That is, there is no possibility in j's belief. Thus, we can regard that j needs to permit the possibility of both $\mathsf{Sign}(b,p)$ and $\mathsf{Sign}(b,\neg p)$ by $[\mathsf{Sign}(b,p)\twoheadrightarrow j]$ and $[\mathsf{Sign}(b,\neg p)\twoheadrightarrow j]$, respectively. By these private permissions, j becomes undetermined on $\mathsf{Sign}(b,p)$ and $\mathsf{Sign}(b,\neg p)$, i.e., $\neg\mathsf{Bel}(j,\mathsf{Sign}(b,p))$ and $\neg\mathsf{Bel}(j,\mathsf{Sign}(b,\neg p))$. The results of $[\mathsf{Sign}(b,p)\twoheadrightarrow j]$ and $[\mathsf{Sign}(b,\neg p)\twoheadrightarrow j]$ are shown in the right-hand side of Fig. 14 and the left-hand side of Fig. 15, respectively.
   2.3. The system finds that there is the received statement of f, i.e., $\mathsf{Sign}(f,\neg p)$ which is not inconsistent with j's belief and is signed by f who is the most reliable agent. Thus, the system automatically employs $[\mathsf{Sign}(f,\neg p)\rightsquigarrow j]$

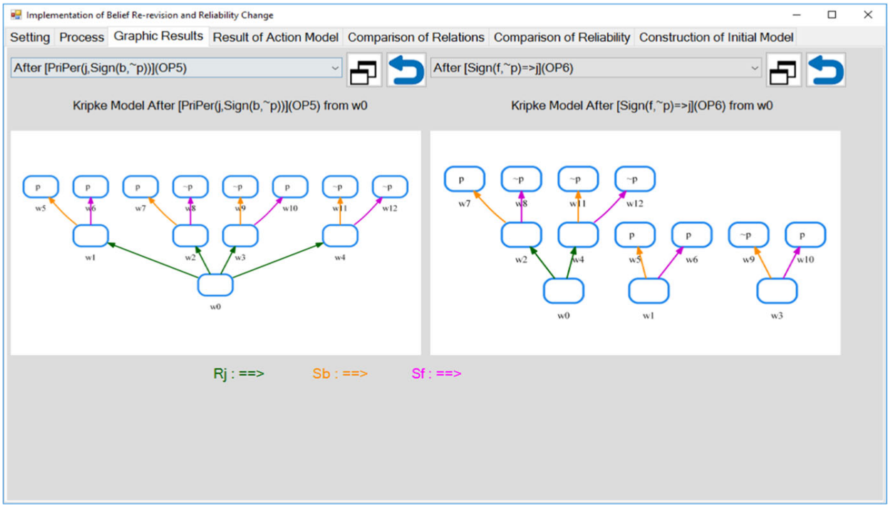

Fig. 15 The left-hand side is Kripke model after $[\mathsf{Sign}(b, \neg p) \twoheadrightarrow j]$, and the right-hand side is Kripke model after $[\mathsf{Sign}(f, \neg p) \rightsquigarrow j]$

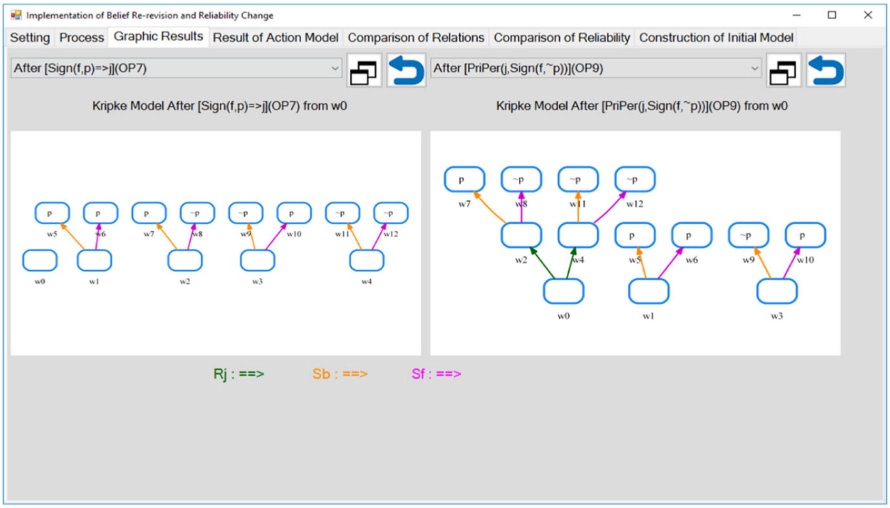

Fig. 16 The left-hand side is Kripke model after $[\mathsf{Sign}(f, p) \rightsquigarrow j]$, and the right-hand side is Kripke model after $[\mathsf{Sign}(f, \neg p) \twoheadrightarrow j]$

for admitting $\mathsf{Sign}(f, \neg p)$ to j. As a result, j believes that $\mathsf{Sign}(f, \neg p)$, i.e., $\mathsf{Bel}(j, \mathsf{Sign}(f, \neg p))$ as shown in the right-hand side of Fig. 15.

3. j also commits him/herself to statement of f in the inquiry stage, i.e., $\mathsf{Sign}(f, p)$ by $[\mathsf{Sign}(f, p) \rightsquigarrow j]$, and the result can be shown in the left-hand side of Fig. 16. After that, the system can detect that the received statement $\mathsf{Sign}(f, p)$ is

inconsistent with j's belief. Thus, the system automatically performs the inconsistency solving algorithm including three operations as the following sequences:

3.1. Since the system can detect that f gives inconsistent statements, f is regarded to be unreliable. Thus, $[\{f\} \downdownarrows^j]$ is employed in order to downgrade f. After this downgrading, j believes that b and f become equally reliable and less reliable than the other agents from j's perspective, i.e., $\mathsf{Bel}(j, b \approx_j f)$.

3.2. By the update of $[\mathsf{Sign}(f,p) \rightsquigarrow j]$ in Step 3, we can regard that there is no possibility in j's belief. For this reason, j needs to permit the possibility of both $\mathsf{Sign}(f,\neg p)$ and $\mathsf{Sign}(f,p)$ by $[\mathsf{Sign}(f,\neg p) \twoheadrightarrow j]$ and $[\mathsf{Sign}(f,p) \twoheadrightarrow j]$, respectively. These permissions will be performed by the system and the result is that j becomes undetermined on $\mathsf{Sign}(f,p)$ and $\mathsf{Sign}(f,\neg p)$, i.e., $\neg\mathsf{Bel}(j, \mathsf{Sign}(f,p))$ and $\neg\mathsf{Bel}(j, \mathsf{Sign}(f,\neg p)$ as shown in Fig. 17.

After the above process, if j employs the careful policy for information aggregation, we obtain that the careful policy cannot be done successfully because of the following reasons: By the careful policy, the system first finds a group of agents who are equally reliable. By Step 3.1, the system finds that agents b and f are equally reliable. Then, the system will find statements which are universally signed by b and f. In this step, the system cannot find such statements because there is an inconsistency between statements of b and f. Thus, the careful policy cannot be employed.

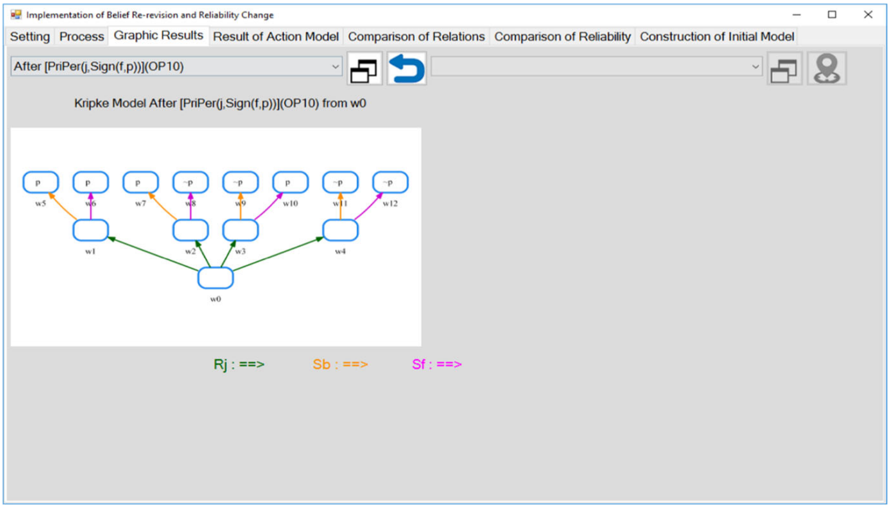

Fig. 17 Kripke model after $[\mathsf{Sign}(f,p) \twoheadrightarrow j]$

Table 6 Summary of the decisions of six target legal cases

Legal case	Defendants	Decisions: Inquiry	Decisions: Court
1	d	d was charged with attempted murder	j decided that d was guilty
2	d	d was charged with murder	j acquitted d
3	d_1 and d_2	d_1 and d_2 were charged with first degree murder	j acquitted d_1 and d_2 of first degree murder
4	d	d was charged with manslaughter	j decided that d was not guilty
5	d	d was charged with attempted murder	j decided that d was not guilty
6	d_1 and d_2	d_1 and d_2 were charged with manslaughter	j decided that d_1 and d_2 were not guilty

Nevertheless, we can interpret the result from our implementation corresponding to the actual decision as follows: From Fig. 17, we can regard that j becomes undetermined on the statements of all witnesses. That is, j cannot decide which information he/she should believe. This can be interpreted as there is an absence of sufficient evidence. Consequently, we can regard that j acquits the defendant as in Table 6.

6.2 Comparison of six legal cases

In this section, we present six target legal cases which have the following characteristics: First, these legal cases are published judgments of the Supreme Court that can be retrieved from an on-line database. Second, we suppose that the judges in these legal cases need to change their belief and/or reliability in order to derive their decision. This process can be demonstrated by our implementation. Third, these legal cases consist of three main components as follows:

- Facts provide the essential features of a legal case including all of the relevant people, actions, locations, evidences, and so on.
- Decision explains how a judge decides a legal case in a court starting from the original trial decision to the final one.
- Reasoning provides an explanation of how a judge justified application of the law including the legal rules or precedents.

Among the above components, we consider the facts and the decision to be essential for us to analyze our target legal cases by our implementation. Based on this idea, six target legal cases can be summarized in Tables 6 and 7 (the more details of all target legal cases are presented in "Appendix 1"). We can describe how to summarize our target legal cases as follows:

- Summarizing the decision of a legal case: this study focuses on only the inquiry stage and the court. Table 6 shows a summary of judgment in six target legal

Table 7 Summary of significant statements from witnesses in six target legal cases

Legal case	Witness	Statements		Note
		Inquiry	Court	
1	v	p	$\neg p$	$p : d$ was the offender
	f_1	p	$\neg p$	
	f_2	$\neg p$	None	
	mo	p	None	
	po	None	$\mathsf{Sign}(v, p)$, $\mathsf{Sign}(f_1, p)$, $\mathsf{Sign}(f_2, \neg p)$, $\mathsf{Sign}(mo, p)$	
2	b	$\neg p$	p	$p : d$ was the offender
	f_1	p	$\neg p$	
	f_2	p	$\neg p$	
	f_3	p	$\neg p$	
3	f_1	p	p	$p : d_2$ was the shooter
	f_2	$\neg p$	p	
4	f	$\neg p$	p	$p : d$ was the offender
5	v	p	$\neg p$	$p : d$ intended to kill v
	f_1	p	$\neg p$	
	f_2	p	p	
	b	p	p	
6	f_1	None	p, q, r	$p : d_2$ kicked v while v was on the ground, $q : d_1$ kicked v while v was on the ground, $r : d_2$ kicked v in the head
	f_2	None	$\neg p$, $\neg q$	
	f_3	$\neg p$, $\neg q$	$\neg p$, $\neg q$	
	f_4	p, $\neg r$	p, $\neg r$	
	f_5	$\neg r$	q, r	

cases. In Table 6, j represents a judge in a legal case. In this study, we regard the judges in each legal case as a single agent j.

- Summarizing the facts of a legal case: we will extract only statements and witnesses that are most important to the judge for deriving his/her decision as shown in Table 7. Note that, in the first legal case, po gives statements $\mathsf{Sign}(v, p)$, $\mathsf{Sign}(f_1, p)$, $\mathsf{Sign}(f_2, \neg p)$, and $\mathsf{Sign}(mo, p)$ that can be denoted by $\mathsf{Sign}(po, \mathsf{Sign}(v, p) \wedge \mathsf{Sign}(f_1, p) \wedge \mathsf{Sign}(f_2, \neg p) \wedge \mathsf{Sign}(mo, p))$. In this sense, we can regard that po first receives statements p, p, $\neg p$, and p from v, f_1, f_2, and mo, respectively, in the inquiry stage and then gives the received statements to the judge in the court.
- Simplifying a legal case: if there are some witnesses who give redundant statements, we can remove such witnesses. For example, in the second legal case, since three witnesses f_1, f_2, and f_3 give the same statements both in the inquiry stage and the court (see Table 7), we can merge them into one witness f as shown in Table 8.

Table 8 Simplified summary of significant statements from witnesses in six target legal cases

Legal case	Witness	Statements	
		Inquiry	Court
1	v (v, f_1)	p	$\neg p$
	f_2	$\neg p$	None
	mo	p	None
	po	None	$\mathsf{Sign}(v, p)$, $\mathsf{Sign}(f_2, \neg p)$, $\mathsf{Sign}(mo, p)$
2	b	$\neg p$	p
	f (f_1, f_2, f_3)	p	$\neg p$
3	f_1	p	p
	f_2	$\neg p$	p
4	f	$\neg p$	p
5	v (v, f_1)	p	$\neg p$
	b (b, f_2)	p	p
6	f_1	None	p, q, r
	f_3 (f_2, f_3)	$\neg p$, $\neg q$	$\neg p$, $\neg q$
	f_4	p, $\neg r$	p, $\neg r$
	f_5	$\neg r$	q, r

6.3 Analysis result

According to our analysis method in Sect. 6.1.1, six target legal cases are analyzed by our implementation, and the results can be shown in Tables 9 and 10.

Table 9 shows a process for analyzing each legal case by our implementation including statements from Table 8, operations for belief/reliability change of an agent, a goal of applying such operations, and the final result. In Table 9, the operations which an agent employed for his/her belief/reliability change are shown. Table 10 shows the results of analyzing six legal cases by our implementation including the following items:

- Number of statements are used to analyze the legal case (see Table 9).
- Number of steps are applied by an agent for his/her belief/reliability change (see Table 9).
- Number of all operations are employed by both an agent and our implementation for belief/reliability change of such agent.
- We will check if the inconsistency solving algorithm is applied or not (the more details of this algorithm are described in Sect. 5.2).
- We will check if the careful policy which is used to aggregate information can be applied or not.

From Table 9, we obtain the final result of all target legal cases, interpreted corresponding to the actual decision. From Table 10, we can interpret the results as follows:

Table 9 Summary of analysis process of six target legal cases. Note that 'Pri-Ann' stands for the private announcement and 'Agg' stands for the careful policy for information aggregation

Legal case	Statement	Operation	Goal	Final result
1	p	(1) Pri-Ann	To admit statements of v and po in the court	j believes p
		(2) Upgrade	To upgrade agent po who signs $\mathsf{Sign}(v,p)$	
		(3) Agg	To aggregate statements of po	
		(4) Upgrade	To upgrade agents v and mo who sign p	
		(5) Agg	To aggregate information about p	
2	p	(1) Pri-Ann	To admit statements of b and f in the court	j cannot determine on statements of both b and f
		(2) Pri-Ann	To admit statement of b in the inquiry stage	
		(3) Pri-Ann	To admit statement of f in the inquiry stage	
3	p	(1) Pri-Ann	To admit statements of f_1 and f_2 in the court	j believes p
		(2) Pri-Ann	To admit statement of f_2 in the inquiry stage	
		(3) Agg	To aggregate information about p	
4	p	(1) Pri-Ann	To admit statement of f in the court	j cannot determine on statements of f
		(2) Pri-Ann	To admit statement of f in the inquiry stage	
5	p	(1) Pri-Ann	To admit statements of v and b in the court	j believes p
		(2) Pri-Ann	To admit statement of v in the inquiry stage	
		(3) Agg	To aggregate information about p	
6	p	(1) Pri-Ann	To admit statements of f_1, f_3, and f_4 in the court	j believes $\neg p$
		(2) Downgrade	To downgrade agents f_1 and f_4 who sign p	
		(3) Agg	To aggregate information about $\neg p$	
	q	(1) Pri-Ann	To admit statements of f_1, f_3, and f_5 in the court	j believes q
		(2) Downgrade	To downgrade agent f_3 who signs $\neg q$	
		(3) Agg	To aggregate information about q	
	r	(1) Pri-Ann	To admit statements of f_1, f_4, and f_5 in the court	j believes $\neg r$
		(2) Downgrade	To downgrade agents f_1 and f_5 who sign r	
		(3) Agg	To aggregate information about $\neg r$	

Table 10 Result of analyzing six target legal cases by our implementation

Legal case	Number of statements	Number of steps	Number of operations	Triggering inconsistency solving algorithm	Capability for aggregation
1	1	5	5	No	Yes
2	1	3	10	Yes	No
3	1	3	7	Yes	Yes
4	1	2	5	Yes	No
5	1	3	7	Yes	Yes
6	3	9	9	No	Yes

- The number of statements can affect the number of steps and operations. Since this study assumes that an agent can consider only one information, our implementation will allow us to analyze only one statement at one time. However, we may need several statements for analyzing some legal case. For example, in the sixth legal case, we need three statements including p, q and r. In order to analyze such legal case, it is required to analyze each statement separately as in Table 9. From this table, three steps including three operations are performed for analyzing each statement. Thus, we need nine steps (i.e., nine operations) for analyzing this legal case.
- The number of steps depends on a way for applying our logical operations, as mentioned in the third step of Sect. 6.1.1. If we use option (OP1), the number of steps is less than the number of operations because some operations are performed by our implementation automatically. On the other hand, if we use option (OP2), the number of steps is equal to the number of operations. In this sense, our implementation cannot reduce the effort to apply logical operations.
- Triggering the inconsistency solving algorithm can reduce the number of operations which are employed by the agent. That is, our implementation can help the agent to reduce the effort to apply logical operations. For example, in the second legal case, when the inconsistency solving algorithm is applied, the system will automatically perform seven operations from the total of 10 (see Table 10). Thus, the agent needs to apply only three operations by him/herself.
- Although the careful policy cannot be applied for aggregating information in some legal cases, we can interpret the final result from our implementation corresponding to the actual decision. This can be illustrated by an example of analyzing the second legal case in Sect. 6.1.2.

7 Conclusion

This study proposed an implementation of our logical formalization for analyzing belief/reliability change of a judge in a legal judgment. Several dynamic logical operators consisting of private announcement, private permission, careful policy, upgrade, downgrade, and joint downgrade were presented in terms of dynamic epistemic logic. With a combination of these operators, six legal cases were analyzed and illustrated in our implementation with a graphic representation. This study contributes to two main issues. First, we can analyze belief/reliability change of an agent by a combination of logical operators. Second, our dynamic logical formalization was implemented in a computer system in order to demonstrate its adequacy. Our implementation consists of two main features. First, the system can construct an initial model for formalizing belief/reliability change of a judge from a legal case. Second, the system can perform an algorithm for handling inconsistency by applying a combination of dynamic operators. With the help of our implementation, we do not need to take much effort for analyzing belief/reliability change of a judge. That is, our implementation can automatically perform some operators instead. Therefore, this can be a helpful tool in analyzing the legal case. Furthermore, our implementation can aid an understanding of a judge's reasoning in a legal judgment.

Our future works consist of two goals:

(G1) To consider the reliability of statements

(G2) To consider more sophisticated ways to construct a restoration process of former beliefs by the private permission operator

For the first goal (G1), we can describe as follows: According to analysis results, our implementation cannot apply the aggregation policy in some legal cases because of the following reason: since the system cannot decide which possibilities should be restored, it will automatically restore all possibilities to the agent's beliefs. Nevertheless, this problem can be solved by the following steps: (1) formalizing the reliability of statements, (2) selecting which statement to be more reliable, and (3) performing a restoration process of the possibility of the statement from (2). In this study, the system cannot perform Step (2) automatically but can allow us to choose which statement an agent should permit. After that, the system can perform Step (3). For Step (1), this study only considered the reliability of agents but did not consider the reliability of statements. However, we may formalize the reliability of statements by employing a preference modality based on van Benthem and Liu (2007) or the framework by Ghosh and Velázquez-Quesada (2011).

The second goal (G2) can be explained as follows: Since this study supposed that an agent can consider only one information, we can analyze only one statement at one time. If we need to analyze several statements, it is required to analyze each statement separately. By this way, the private permission operator can work well. Nevertheless, if we consider multiple statements at the same time, it may cause a bad side-effect. For example, agent j first believes that p and q (i.e., $\mathsf{Bel}(j,p) \wedge \mathsf{Bel}(j,q)$). Then, if j needs to permit the possibility of $\neg p$, $[\neg p \twoheadrightarrow j]$ is employed. By the update of $[\neg p \twoheadrightarrow j]$, j will not believe that p (i.e., $\neg\mathsf{Bel}(j,p)$) and may not believe that q (i.e., $\neg\mathsf{Bel}(j,q)$). In other words, $[\neg p \twoheadrightarrow j]$ affects not only $\mathsf{Bel}(j,p)$ but also $\mathsf{Bel}(j,q)$. In fact, $[\neg p \twoheadrightarrow j]$ should not affect the other propositions than p. Therefore, our goal is to avoid this problem.

Appendix 1: Details of target legal cases

The story and the judgment of six target legal cases can be summarized as follows:

1. Legal case from Thailand [in Jirakunkanok et al. (2014)] occurred on January 26, 2003 in Trang province, Thailand.[11] The story can be summarized as follows:

   One day, Choochart (v) had a drink with his friends including Saichol (f_1), Ekachai (f_2), and Sommai (d) at f_2's house. After that, v was punched and stabbed with a hand scraper in the back by an offender, and as a result, v had bleeding in the lung. However, v was still alive.

   The details of judgment can be summarized as follows:

   In the inquiry stage, four witnesses v, f_1, f_2, and mo (mother of v) were interviewed by a police po, who is an official inquiry, as follows: v, f_1, and mo told that d was the offender, while f_2 told that d was not the offender. After the interview, d was charged with attempted murder. In the Civil Court, v and f_1 changed their statements, i.e., both of them told that d was not the offender. po was called to be a witness for testifying all statements in the inquiry stage. From these testimonies, the judge believed that the statements of v and f_1 in the Civil Court are less reliable than that in the inquiry stage. Thus, the judge believed that d was the offender and decided that d was guilty.
2. Legal case from Canada [in Jirakunkanok et al. (2015a)] occurred on April 24, 1988 in Ontario, Canada.[12] The story can be summarized as follows:

   One day, Joseph (v) and his brother, Steven (b), got off a bus at an intersection. At the same time, the respondent, K.G.B. (d), and three other men including P.L. (f_1), P.M. (f_2), and M.T. (f_3) were driving past the same intersection. An argument started among them and shortly thereafter a fight occurred. v and b were unarmed. In the course of the fight, one of the four men from the car pulled a knife and then stabbed v in the chest. Finally, v died.

   The details of judgment can be summarized as follows:

   In the inquiry stage, four witnesses b, f_1, f_2, and f_3 were interviewed as follows: b told that d was not the offender, while f_1, f_2, and f_3 told that d was the offender. After the interview, d was charged with murder. In the Youth Court, since all witnesses recanted their statements, the judge could not consider the prior statements of all witnesses as evidence. Thus, the judge acquitted d.

[11] This legal case can be referred from http://deka2007.supremecourt.or.th/deka/web/search.jsp (in Thai).

[12] This legal case can be referred from http://www.canlii.org/en/.

3. Legal case from British Columbia [in Jirakunkanok et al. (2015b)] occurred on August 31, 1996 in Surrey, British Columbia.[12] The story can be summarized as follows:

   One day, while Basant Singh (v) with new friends including Sher (f_1), Jarnail (f_2), and the others gathered for social purposes, a van consisting of two respondents Sukhminder (d_1), Ajmer (d_2), and the others slowly approached the group of v. A burst of gun fire swept the group of v, shooting on a low trajectory into the ground. As a result, v died and three others including f_2 were wounded.

   The details of judgment can be summarized as follows:

   In the inquiry stage, two witnesses f_1 and f_2 were interviewed as follows: f_1 told that d_1 was a driver of the van and d_2 was the shooter, while f_2 told that d_1 was a driver of the van and d_2 was not the shooter. After the interview, both d_1 and d_2 were charged with the first degree murder, the attempted murder of three other persons, and aggravated assault on the same three persons. In the Crown Court, f_2 changed his statement, i.e., he told that d_2 was the shooter. Since there is an inconsistency in the statements of f_2, the judge considered only f_1's statement for identifying the shooter to be truthful. In addition, the judge believed that both d_1 and d_2 did not intend to kill v. For this reason, the judge acquitted both d_1 and d_2 of first degree murder but convicted them of manslaughter. The judge also convicted them of aggravated assault instead of attempted murder of the other three victims.
4. Legal case from Nova Scotia occurred on December 31, 2009 in Halifax, Nova Scotia.[12] The story can be summarized as follows:

   One day, Welsh (v) went to a New Year's Eve Party with his girlfriend Gautreau (f). While f was drinking in the party, v went outside the party to have a cigarette. Later on, f went outside and found v was punched then fell backward and struck his head on the pavement. Finally, v died.

   The details of judgment can be summarized as follows:

   In the inquiry stage, only one witness f was interviewed as follows: f told that Leeds (d) was the offender. After the interview, d was charged with manslaughter. In the Crown Court, the judge found that f's recollection of the event was affected by her alcohol assumption, and there were many inconsistencies in f's evidence such as the identification of d as the offender. Thus, the judge believed that f was not a reliable witness. Accordingly, the judge decided that d was not guilty.
5. Legal case from Nova Scotia occurred on August 6, 2011 in Halifax, Nova Scotia.[12] The story can be summarized as follows:

   One day, Barry (v) and his friends including Fisher (f_1), Marsh (f_2), and Slaunwhite (f_3) were drinking alcohol and smoking marijuana at v's home. Then, v together with his friends f_1, f_2, and f_3 drove to the house of Neil

(d). While v was driving the vehicle at d, d was scared and fired the shot that injured v. However, v was still alive.

The details of judgment can be summarized as follows:

In the inquiry stage, four witnesses v, f_1, f_2, and Beaupre (b) who was d's neighbor were interviewed as follows: v and f_1 told that d intended to kill v, while f_2 and b told that d did not intend to kill v. After the interview, d was charged with the following offences: attempted murder, aggravated assault, using of a weapon in committing an assault, discharging a firearm with intent to endanger the life, intentionally discharging a firearm into a place, using of a firearm in a careless manner, and possessing a weapon for a purpose dangerous to the public peace. In the Crown Court, the judge considered v and f_1 to be unreliable because v could not recall the events because of a combination of his intoxication by both drugs and alcohol on the evening in the events, and f_1's evidence was inconsistent within itself. Thus, the judge only accepted the evidence from f_2 and b that d did not intend to kill v; in fact, d just defended himself against v's attack. That is, d's actions were justified to be self-defense. Therefore, the judge decided that d was not guilty of all counts in the indictment.

6. Legal case from Nova Scotia occurred on July 17, 2004 in Bedford, Nova Scotia.[12] The story can be summarized as follows:

One day, Bobby (v) was intoxicated at Busters Bar and having been denied further drinks from the bar. While Comer (d_1) and his friends including Warner (f_1), Maes (f_2), Southwell (f_3), and Morrison (f_4) were drinking, v approached d_1's table and asked for some beer, but his request was refused. Then, v attempted to take d_1's beer, but his attempt was prevented from f_1. After that, Smith (d_2) and his friend, Hodgson (f_5), arrived at the bar and joined the group at d_1's table. v left the bar first, then d_1, d_2, and f_5 left the bar. When f_1, f_2, f_3, and f_4 exited the bar, they came upon a verbal exchange between v, d_1, d_2, and f_5. Then, v kicked d_2 first, then all three including v, d_1, and d_2 were punching each other. The fight was of short duration. After v fell to the ground, d_1, d_2, and f_5 ran off.

The details of judgment can be summarized as follows:

In the inquiry stage, five witnesses f_1, f_2, f_3, f_4, and f_5 were interviewed as follows: f_1 and f_2 told that they could not see what happened when v was on the ground, but f_2 stated that he saw d_2 kicked v once above the belt. f_3 told that he did not see anyone kick v while v was on the ground. f_4 told that v kicked d_2 first, then d_2 kicked v while v was on the ground. However, f_4 was not sure if d_2's kick was to v's head or not. f_5 told that he could not say where d_1's kick landed on v. After the interview, both d_1 and d_2 were charged with manslaughter in the death of v. In the Crown Court, three witnesses f_1, f_2, and f_5 changed their statements as follows: f_1 testified that d_1 and d_2 kicked v while v was on the ground, and all the kicks he saw landed on v's upper body between the belt and the head. f_2

told that he could not say if anyone kicked v while v was on the ground because people were in front of him and blocking his view. f_5 told that v kicked d_2 first, then d_1 kicked v in the head while v was on the ground. The judge found that the reliability of evidence of all witnesses was questionable because of the following reasons: f_1 and f_5 gave inconsistent statements, f_2's view of the events was affected by the fact that he was not wearing his eyeglasses, and f_3 and f_4 turned away from the fight. Based on these reasons, the judge was not satisfied on the evidence that d_2 kicked v while v was on the ground. Thus, the judge believed that d_2's act was in self-defense and was not excessive. Accordingly, d_2 was found not guilty. On the other hand, the judge believed that the kicking of d_1 was not in self-defense and was excessive because of the evidence that d_1 kicked v while v was on the ground. However, the judge cannot conclude that the kicking of d_1 was the cause of v's death because there is no evidence to support a finding that d_1 kicked v in the head. Accordingly, d_2 was found not guilty.

Appendix 2: Omitted proofs

Appendix 2.1: Proof of Theorem 1

Proof Since the soundness is straightforward, we will focus on the completeness proof. Let us write our axiomatization **HBSR** in Table 2 by Λ. We show that any unprovable formula φ in Λ is falsified in some model. Let φ be an unprovable formula in Λ. We define the canonical model $\mathfrak{M}$ where φ is falsified at some point of $\mathfrak{M}$. We say that a set Γ of formulas is *Λ-consistent* (for short, *consistent*) if $\neg(\bigwedge \Gamma')$ is unprovable in Λ, for all finite subsets Γ' of Γ, and that Γ is *maximally consistent*, denoted Λ-MCS if Γ is consistent and $\varphi \in \Gamma$ or $\neg\varphi \in \Gamma$ for all formulas φ. Note that ψ is unprovable in Λ iff $\neg\psi$ is Λ-consistent, for any formula ψ. Let Σ be a Λ-MCS. We define the canonical model $\mathfrak{M}^{\Lambda}_{\Sigma} = (W^{\Lambda}, (R^{\Lambda}_a)_{a\in G}, (S^{\Lambda}_a)_{a\in G}, (\preccurlyeq^{\Lambda}_a)_{a\in G}, V^{\Lambda})$, for Λ by:

- $W^{\Lambda} := \{\Gamma | \Gamma$ is a Λ-MCS and $\{\varphi | \mathsf{A}\varphi \in \Sigma\} \subseteq \Gamma\}$;
- $\Gamma R^{\Lambda}_a \Delta$ iff $(\mathsf{Bel}(a, \psi) \in \Gamma$ implies $\psi \in \Delta)$ for all ψ;
- $\Gamma S^{\Lambda}_a \Delta$ iff $(\mathsf{Sign}(a, \psi) \in \Gamma$ implies $\psi \in \Delta)$ for all ψ;
- $b \preccurlyeq^{\Gamma}_a c$ iff $b \leqslant_a c \in \Gamma$;
- $\Gamma \in V^{\Lambda}(p)$ iff $p \in \Gamma$.

Then, we can show the following equivalence : $\mathfrak{M}^{\Lambda}_{\Sigma}, \Gamma \models \psi$ iff $\psi \in \Gamma$ for all formulas ψ and $\Gamma \in W$. Given any unprovable formula φ in Λ, we can find a maximal consistent set Σ such that $\neg\varphi \in \Sigma$. Then, by the equivalence above, φ is falsified at Σ of the canonical model $\mathfrak{M}^{\Lambda}_{\Sigma}$ for Λ, where we can assure that $\mathfrak{M}^{\Lambda}_{\Sigma}$ is our intended model. $\square$

Appendix 2.2: Proof of Proposition 1

Proof Our goal is to show that all axioms are valid with respect to the semantics of $[\mathbb{E}_{!^a_\varphi}, e]$ that is straightforward. We will show only the most important axiom, i.e.,

$$[\mathbb{E}_{!^a_\varphi}, e]\mathsf{Bel}(a,\psi) \leftrightarrow \bigwedge_{f \in D_a(e)} \mathsf{Bel}\big(a, \mathrm{pre}(f) \to [\mathbb{E}_{!^a_\varphi}, f]\psi\big).$$

Let us fix any model $\mathfrak{M}$ and any state w of $\mathfrak{M}$. We suffice to show:

$$\mathfrak{M}, w \models [\mathbb{E}_{!^a_\varphi}, e]\mathsf{Bel}(a,\psi) \text{ iff } \mathfrak{M}, w \models \bigwedge_{f \in D_a(e)} \mathsf{Bel}\big(a, \mathrm{pre}(f) \to [\mathbb{E}_{!^a_\varphi}, f]\psi\big).$$

The left-hand-side is equivalent to:

$$\begin{aligned}
&\mathfrak{M}^{\otimes \mathbb{E}_{!^a_\varphi}}, (w,e) \models \mathsf{Bel}(a,\psi)\\
&\text{iff } \forall (v,f)\Big((w,e)R'_a(v,f) \Rightarrow \mathfrak{M}^{\otimes \mathbb{E}_{!^a_\varphi}}, (v,f) \models \psi\Big)\\
&\text{iff } \forall (v,f)\Big((wR_a v \text{ and } (\mathrm{e},\mathrm{f}) \in \mathrm{D_a} \text{ and } \mathfrak{M}, \mathrm{v} \models \mathrm{pre(f)}) \Rightarrow \mathfrak{M}, \mathrm{v} \models [\mathbb{E}_{!^a_\varphi}, \mathrm{f}]\psi\Big)\\
&\text{iff } \forall f\Big((e,f) \in D_a \Rightarrow \forall v\Big((wR_a v \text{ and } \mathfrak{M}, \mathrm{v} \models \mathrm{pre(f)}) \Rightarrow \mathfrak{M}, \mathrm{v} \models [\mathbb{E}_{!^a_\varphi}, \mathrm{f}]\psi\Big)\Big)\\
&\text{iff } \forall f\Big((e,f) \in D_a \Rightarrow \forall v\Big(wR_a v \Rightarrow (\mathfrak{M}, v \models \mathrm{pre}(f) \Rightarrow \mathfrak{M}, v \models [\mathbb{E}_{!^a_\varphi}, f]\psi)\Big)\Big)\\
&\text{iff } \forall f\Big((e,f) \in D_a \Rightarrow \forall v\Big(wR_a v \Rightarrow (\mathfrak{M}, v \models \mathrm{pre}(f) \to [\mathbb{E}_{!^a_\varphi}, f]\psi)\Big)\Big)\\
&\text{iff } \forall f\Big((e,f) \in D_a \Rightarrow \mathfrak{M}, w \models \mathsf{Bel}(a, \mathrm{pre}(f) \to [\mathbb{E}_{!^a_\varphi}, f]\psi)\Big)\\
&\text{iff } \mathfrak{M}, w \models \bigwedge_{f \in D_a(e)} \mathsf{Bel}(a, \mathrm{pre}(f) \to [\mathbb{E}_{!^a_\varphi}, f]\psi).
\end{aligned}$$

Appendix 2.3: Proof of Proposition 2

Proof Our goal is to show that all axioms are valid with respect to the semantics of $[\mathbb{E}_{¡^a_\varphi}, e]$ that is straightforward. We will show only the most important axiom, i.e.,

$$[\mathbb{E}_{¡^a_\varphi}, e]\mathsf{Bel}(a,\psi) \leftrightarrow \bigwedge_{f \in D_a(e)} \big(\mathsf{Bel}(a, [\mathbb{E}_{¡^a_\varphi}, f]\psi) \wedge \mathsf{A}(\mathrm{pre}(f) \to [\mathbb{E}_{¡^a_\varphi}, f]\psi)\big).$$

Let us fix any model $\mathfrak{M}$ and any state w of $\mathfrak{M}$. We suffice to show:

$$\mathfrak{M}, w \models [\mathbb{E}_{¡^a_\varphi}, e]\mathsf{Bel}(a,\psi) \text{ iff } \mathfrak{M}, w \models \bigwedge_{f \in D_a(e)} \big(\mathsf{Bel}(a, [\mathbb{E}_{¡^a_\varphi}, f]\psi) \wedge \mathsf{A}(\mathrm{pre}(f) \to [\mathbb{E}_{¡^a_\varphi}, f]\psi)\big).$$

The left-hand-side is equivalent to:

$$
\begin{aligned}
&\mathfrak{M}^{\otimes \mathbb{E}_{\text{!`}\varphi}^{a}}, (w,e) \models \mathsf{Bel}(a,\psi) \\
&\text{iff } \forall (v,f)\Big((w,e)R'_a(v,f) \Rightarrow \mathfrak{M}^{\otimes \mathbb{E}_{\text{!`}\varphi}^{a}}, (v,f) \models \psi\Big) \\
&\text{iff } \forall (v,f)\Big(\big((wR_a v \text{ and } (\mathrm{e},\mathrm{f}) \in \mathrm{D_a}) \text{or} (\mathfrak{M},\mathrm{v} \models \mathrm{pre}(\mathrm{f}) \text{ and } (\mathrm{e},\mathrm{f}) \in \mathrm{D_a})\big) \Rightarrow \mathfrak{M},\mathrm{v} \models [\mathbb{E}_{\text{!`}\varphi}^{\mathrm{a}},\mathrm{f}]\psi\Big) \\
&\text{iff } \forall (v,f)\left(\begin{array}{c}\Big((wR_a v \text{ and } (\mathrm{e},\mathrm{f}) \in \mathrm{D_a}) \Rightarrow \mathfrak{M},\mathrm{v} \models [\mathbb{E}_{\text{!`}\varphi}^{\mathrm{a}},\mathrm{f}]\psi\Big) \text{ and} \\ \Big((\mathfrak{M},v \models \mathrm{pre}(f) \text{ and } (\mathrm{e},\mathrm{f}) \in \mathrm{D_a}) \Rightarrow \mathfrak{M},\mathrm{v} \models [\mathbb{E}_{\text{!`}\varphi}^{\mathrm{a}},\mathrm{f}]\psi\Big)\end{array}\right) \\
&\text{iff } \forall (v,f)\left(\begin{array}{c}\Big((e,f) \in D_a \Rightarrow (wR_a v \Rightarrow \mathfrak{M},v \models [\mathbb{E}_{\text{!`}\varphi}^{a},f]\psi)\Big) \text{ and} \\ \Big((e,f) \in D_a \Rightarrow (\mathfrak{M},v \models \mathrm{pre}(f) \Rightarrow \mathfrak{M},v \models [\mathbb{E}_{\text{!`}\varphi}^{a},f]\psi)\Big)\end{array}\right) \\
&\text{iff } \forall (v,f)\left(\begin{array}{c}\Big((e,f) \in D_a \Rightarrow (wR_a v \Rightarrow \mathfrak{M},v \models [\mathbb{E}_{\text{!`}\varphi}^{a},f]\psi)\Big) \text{ and} \\ \Big((e,f) \in D_a \Rightarrow (\mathfrak{M},v \models \mathrm{pre}(f) \to [\mathbb{E}_{\text{!`}\varphi}^{a},f]\psi)\Big)\end{array}\right) \\
&\text{iff } \forall (v,f)\Big((e,f) \in D_a \Rightarrow \Big((wR_a v \Rightarrow \mathfrak{M},v \models [\mathbb{E}_{\text{!`}\varphi}^{a},f]\psi) \text{ and } (\mathfrak{M},\mathrm{v} \models \mathrm{pre}(\mathrm{f}) \to [\mathbb{E}_{\text{!`}\varphi}^{\mathrm{a}},\mathrm{f}]\psi)\Big)\Big) \\
&\text{iff } \forall f\Big((e,f) \in D_a \Rightarrow \Big(\forall v(wR_a v \Rightarrow \mathfrak{M},v \models [\mathbb{E}_{\text{!`}\varphi}^{a},f]\psi) \text{ and } \forall \mathrm{v}(\mathfrak{M},\mathrm{v} \models \mathrm{pre}(\mathrm{f}) \to [\mathbb{E}_{\text{!`}\varphi}^{\mathrm{a}},\mathrm{f}]\psi)\Big)\Big) \\
&\text{iff } \forall f\Big((e,f) \in D_a \Rightarrow \Big(\mathfrak{M},w \models \mathsf{Bel}(a,[\mathbb{E}_{\text{!`}\varphi}^{a},f]\psi) \text{ and } \mathfrak{M},\mathrm{w} \models \mathsf{A}(\mathrm{pre}(\mathrm{f}) \to [\mathbb{E}_{\text{!`}\varphi}^{\mathrm{a}},\mathrm{f}]\psi)\Big)\Big) \\
&\text{iff } \forall f\Big((e,f) \in D_a \Rightarrow \Big(\mathfrak{M},w \models \mathsf{Bel}(a,[\mathbb{E}_{\text{!`}\varphi}^{a},f]\psi) \wedge \mathsf{A}(\mathrm{pre}(f) \to [\mathbb{E}_{\text{!`}\varphi}^{a},f]\psi)\Big)\Big) \\
&\text{iff } \mathfrak{M},w \models \bigwedge_{f \in D_a(e)} \Big(\mathsf{Bel}(a,[\mathbb{E}_{\text{!`}\varphi}^{a},f]\psi) \wedge \mathsf{A}(\mathrm{pre}(f) \to [\mathbb{E}_{\text{!`}\varphi}^{a},f]\psi)\Big).
\end{aligned}
$$

Appendix 2.4: Proof of Theorem 2

Proof By $\vdash \psi$ (or $\vdash^{+} \psi$), we mean that ψ is a theorem of the axiomatization for $\mathcal{L}_{BSR}$ (or, $\mathcal{L}_{BSR}^{+}$, respectively.) The soundness part is mainly due to Propositions 1, 2, 3, 4 and 5. One can also check that the necessitation rules for $[\mathbb{E}_{!\varphi}^{a}, e]$, $[\mathbb{E}_{\text{!`}\varphi}^{a}, e]$, $[H \Downarrow_{\varphi}^{a}]$, $[H \Uparrow_{\varphi}^{a}]$ and $[H \downdownarrows^{a}]$ preserve the validity on the class of all models. As for the completeness part, we can reduce the completeness of our dynamic extension to the static counterpart (i.e., Theorem 1) as follows. With the help of the reduction axioms of Propositions 1, 2, 3, 4 and 5, we can define a mapping t sending a formula ψ of $\mathcal{L}_{BSR}^{+}$ to a formula $t(\psi)$ of $\mathcal{L}_{BSR}$, where we start rewriting the *innermost occurrences* of $[\mathbb{E}_{!\varphi}^{a}, e]$, $[\mathbb{E}_{\text{!`}\varphi}^{a}, e]$, $[H \Downarrow_{\varphi}^{a}]$, $[H \Uparrow_{\varphi}^{a}]$ and $[H \downdownarrows^{a}]$. We can define this mapping t such that $\psi \leftrightarrow t(\psi)$ is valid on all models and $\vdash^{+} \psi \leftrightarrow t(\psi)$. Then, we can proceed as follows. Fix any formula ψ of $\mathcal{L}_{BSR}^{+}$ such that ψ is valid on all models. By the validity of $\psi \leftrightarrow t(\psi)$ on all models, we obtain $t(\psi)$ is valid on all models. By Theorem 1, $\vdash t(\psi)$, which implies $\vdash^{+} t(\psi)$. Finally, it follows from $\vdash^{+} \psi \leftrightarrow t(\psi)$ that $\vdash^{+} \psi$, as desired. $\square$

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