Assignment-2

$P(y|n) = b(y)\exp[n^T T(y) - a(n)]$

1) ~~$P(y|k) = \frac{y^k e^{-y}}{k!}$~~

$P(y|\lambda) = \frac{\lambda^y e^{-\lambda}}{y!}$

$\frac{1}{y!}\exp(y\log\lambda - \lambda)$

$b(y) = \frac{1}{y!}$ $\quad n^T = n = \log\lambda$ (only one y) $\quad T(y) = y$

$a(n) = \lambda = e^n$

2) Let ϕ_k is prob. of output y_k

We define $T(y) \in \mathbb{R}^{k-1}$

$T(1) = \begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix}$ $T(2) = \begin{bmatrix}0\\1\\0\\\vdots\\0\end{bmatrix}$ - - - - $T(k) = \begin{bmatrix}0\\0\\0\\\vdots\\0\end{bmatrix}$

$\phi_i = E[(T(y))_i] = P(y=i)$ where

$(T(y))_i = 1$ if $y = i$

otw 0

$$P(y|\phi) = \phi_1^{(T(y))_1} \phi_2^{(T(y))_2} \cdots \phi_k^{1-\sum (T(y))_i}$$

$$= \exp\left((T(y))_1 \log\left(\frac{\phi_1}{\phi_k}\right) \cdots\right)$$

$$= b(y) \exp(n_T T(y) - a(n))$$

$$n = \begin{bmatrix} \log\left(\frac{\phi_1}{\phi_k}\right) \\ \vdots \\ \log\left(\frac{\phi_{k-1}}{\phi_k}\right) \end{bmatrix}$$

$$a(n) = -\log(\phi_k)$$

$$b(y) = 1$$

$$n_i = \log \frac{\phi_i}{\phi_k}$$

$$e^{n_i} = \frac{\phi_i}{\phi_k} \qquad \phi_i = \phi_k e^{n_i}$$

$$\sum \phi_k e^{n_i} = \sum \phi_i = 1$$

$$\phi_k = \frac{1}{\sum e^{n_i}} \qquad \phi_i = \frac{e^{n_i}}{\sum_{j=1}^{k} e^{n_i}}$$