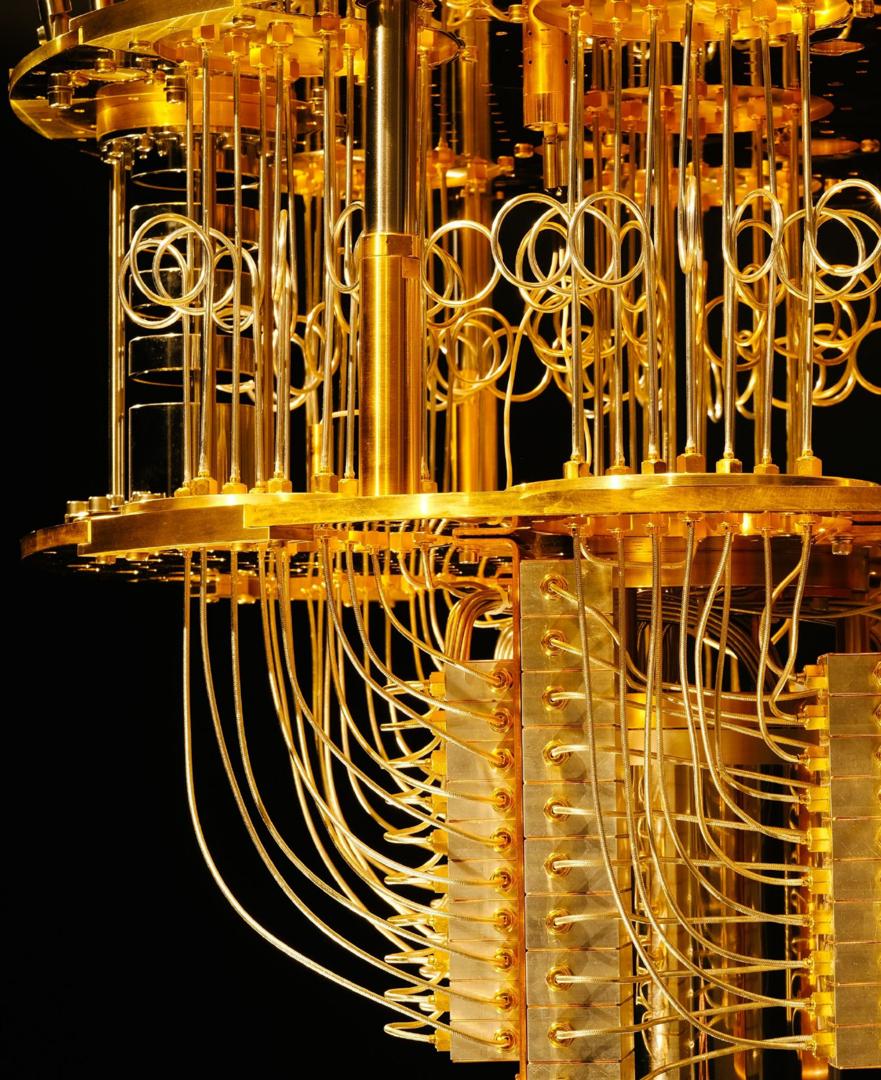
Quantum Computing – theoretical basics for the quantum coin game

—

Dr. Jan-R. Lahmann, IBM
Member of the IBM Academy of Technology

https://twitter.com/JanLahmann



Bit, Qubit, Measurement, State

The bit is the basic unit of information and has two possible states: 0 and 1.

The qubit is the basic *quantum* unit of information and also is 0 or 1 when you measure, or look at, it. This corresponds to the quantum states $|0\rangle$ or $|1\rangle$

A (general) quantum state can be written as

$$\alpha \cdot |0\rangle + \beta \cdot |1\rangle$$

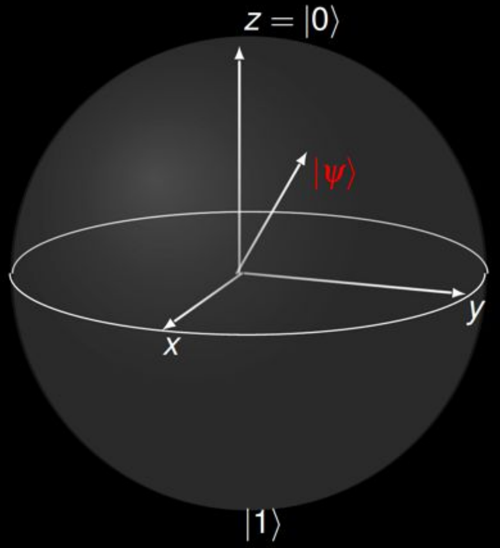
It is called a superposition of $|0\rangle$ and $|1\rangle$.

α, β are generalized probabilities (for measuring 0 or 1) with $\alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1$

It provides a probability distribution for each possible outcome of a measurement on the system.

Bloch Sphere

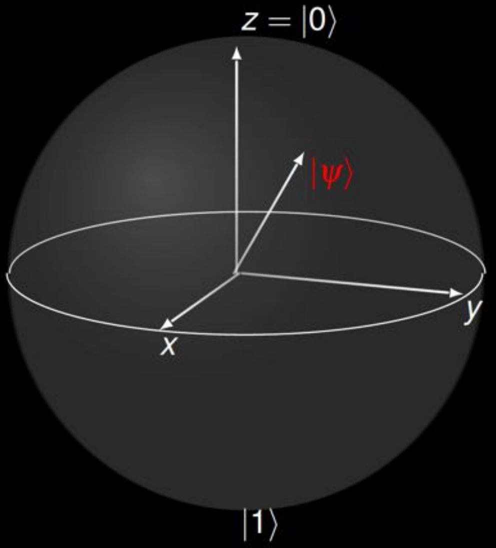
The Bloch sphere is a geometrical representation of the state of a qubit: $|0\rangle$ is at the north pole, $|1\rangle$ is at the south pole



Quantum Gates

A quantum gate is a basic ”operator”, acting on a small number of qubits. Gates are the building blocks of quantum circuits.

A quantum gate acting on a single qubit can be defined by its action on the basis vectors $|0\rangle$ and $|1\rangle$.



Examples:

X-Gate

It equates to a rotation around the X-axis of the Bloch sphere by 180°. It maps

$|0\rangle$ to $|1\rangle$ and $|1\rangle$ to $|0\rangle$.

It is called “bit-flip”.

Hadamard-Gate

The Hadamard gate acts on a single qubit.
It maps the basis state

$$|0\rangle \text{ to } \frac{|0\rangle+|1\rangle}{\sqrt{2}} \quad \text{and} \quad |1\rangle \text{ to } \frac{|0\rangle-|1\rangle}{\sqrt{2}},$$

which means that a measurement will have equal probabilities to become 0 or 1.
On the Bloch sphere, it is the combination of two rotations, 180° about the Z-axis followed by 90° about the Y-axis.

Systems with N Qbits:
Superposition and Entanglement

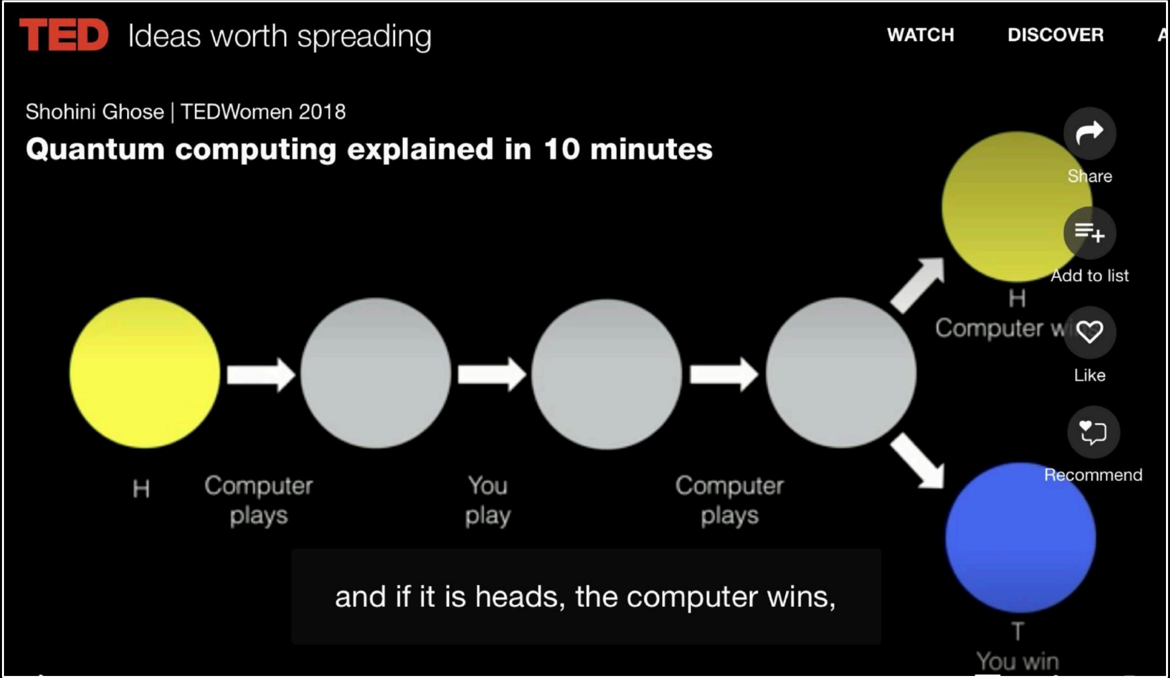
A single quantum bit can exist in a superposition of $|0\rangle$ and $|1\rangle$, and N qubits allow for a superposition of all possible 2^N combinations.

Example for N=2:

$$\alpha \cdot |00\rangle + \ \beta \cdot |01\rangle + \ \gamma \cdot |10\rangle + \ \delta \cdot |11\rangle$$

$\frac{1}{\sqrt{2}} \cdot (|00\rangle + \ |11\rangle)$ is an example for entanglement, i.e. a state of N qubits, which cannot be described independently of each other.

Heads or Tails: How Quantum Power Helps to Win a Coin Game
Play online: http://ibm.biz/QCoinGame



Inspired by the TED talk of Shohini Ghose
„Quantum computing explained in 10 minutes“
https://www.ted.com/talks/shohini_ghose_quantum_computing_explained_in_10_minutes