a) $T=(0,\infty)$, $A_t=\langle -t,t\rangle$. $\bigcup\limits_{t\in T} A_t$, $\bigcap\limits_{t\in T} A_t$ -?

① Najdeme $\bigcap\limits_{t\in T} A_t$.

1) $0\notin \bigcap\limits_{t\in T} A_t \iff \exists n\in T: 0\notin A_n \iff \exists n\in T: -n<n<0 \vee 0<-n<n \iff$

$\exists n\in T: (-n<n<0 \wedge \underbrace{0<n}_{=1}) \vee (0<-n<n \wedge \underbrace{-n<0}_{=1})$ — Rozpor, $0\in \bigcap\limits_{t\in T} A_t$

2) $x\in \bigcap\limits_{t\in T} A_t \overset{\wedge x\in R}{\wedge} x\neq 0 \iff \forall m\in T: x\in A_m \wedge x\neq 0 \iff \exists m=\frac{|x|}{2}\in T:$

$x\in A_m \wedge x\neq 0 \iff x\in\langle -\frac{|x|}{2},\frac{|x|}{2}\rangle \wedge x\neq 0 \iff (x>0 \wedge x\in\langle -\frac{x}{2},\frac{x}{2}\rangle) \vee$

$(x<0 \wedge x\in\langle \frac{x}{2},-\frac{x}{2}\rangle) \iff (x>0 \wedge -x\le 2x\le x) \vee (x<0 \wedge x\le 2x\le -x) \iff$

$(-1\le 2\le 1)\vee(1\ge 2\ge -1)$ — Rozpor

$$\boxed{\bigcap_{t\in T} A_t = \{0\}.}$$

② Najdeme $\bigcup\limits_{t\in T} A_t$.

1) $0\in \bigcap\limits_{t\in T} A_t \Rightarrow 0\in \bigcup\limits_{t\in T} A_t$

2) $y\in R: y\notin \bigcup\limits_{t\in T} A_t \overset{\wedge y\neq 0}{\iff} \forall k\in T: y\notin A_k \overset{\wedge y\neq 0}{\iff} \exists k=2|y|\in T: y\notin A_k \overset{\wedge y\neq 0}{\iff}$

$y\notin\langle -2|y|,2|y|\rangle \overset{\wedge y\neq 0}{\iff} |y|>2|y| \overset{\wedge y\neq 0}{\iff} 1>2$ — Rozpor, libovolné $(y\in R)\in \bigcup\limits_{t\in T} A_t$.

$$\boxed{\bigcup_{t\in T} A_t = R.}$$