Ťahák

Obraz množiny

$f(M) = \{b;\ \exists a \in M:\ f(a) = b\}$

Úplný vzor množiny

$f^{-1}(P) = \{a;\ \exists b \in P:\ f(a) = b\}$

$f^{-1}(\{0\})$ – uvm

$f^{-1}(0)$ – inverzná funkcia

Vlastnosti funkcií

- ohr. ↑ $\exists c \in \mathbb{R}\ \forall x \in D_f;\ f(x) \leq c$
  ↓ ohr. $\exists c, d \ldots$ $d \leq f(x) \leq c$
- monot
  rast. $\forall x_1, x_2 \in M;\ x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ $\leq\ >\ \geq$
  kles.
  neklés.
  nerast.
- parita
  párna (sudá) $\forall x \in D_f;\ f(-x) = f(x)$
  nep. (lichá) $f(-x) = -f(x)$
  ! def. obor musí být sym. $x \in D_f \Rightarrow -x \in D_f$
- periodicita
  $\exists p \in \mathbb{R} \setminus \{0\}\ \forall x \in D_f;\ f(x \pm p) = f(x)$

Okolí bodu

$\mathcal{U}(a, \varepsilon) = \{x \in \mathbb{R};\ |x-a| < \varepsilon\} = (a-\varepsilon, a+\varepsilon)$

a – stred, ε – polomer

Limita postupnosti

a: $\forall \varepsilon > 0\ \exists n_0;\ n > n_0 \Rightarrow |a_n - a| < \varepsilon$

Veta o dvoch policajtoch

Nech $\lim a_n = \lim b_n = a$, a nech ex. $n_1 \in \mathbb{N}$ tak, že pro $n > n_1$ je $a_n \leq c_n \leq b_n$. Potom ex. limita (c_n) a $\lim c_n = a$.

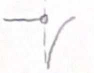
Limita funkcie

– č. A je l. f. $f(x)$ v b. c ak platí:

$\forall \varepsilon > 0\ \exists \delta > 0;\ 0 < |x-c| < \delta \Rightarrow |f(x) - A| < \varepsilon$

sprava $x - c$, zľava $c - x$

Nespojitosť

- 1. druhu (skok) 
- 2. druhu 

Asymptoty

- zvislá (bez smernice)
  $\lim_{x \to a^-} f(x) = \pm\infty$ alebo $\lim_{x \to a^+} f(x) = \pm\infty$
- šikmá (so smernicou)
  $\lim_{x \to \pm\infty} [f(x) - (ax+b)] = 0$

$$\boxed{\begin{aligned} a &= \lim_{x \to \pm\infty} \frac{f(x)}{x} \\ b &= \lim_{x \to \pm\infty} [f(x) - ax] \end{aligned}}$$

Derivácia v bode

$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$

Slovník pro derivacie

Funkce	Derivace	Funkce	Derivace
c (konst)	0	x	1
x^n	$n x^{n-1}$	x^α	$\alpha x^{\alpha-1}$
e^x	e^x	a^x	$a^x \ln a$
$\ln x$	$\frac{1}{x}$	$\log_a x$	$\frac{1}{x \ln a}$
$\sin x$	$\cos x$	$\cos x$	$-\sin x$
$\mathrm{tg}\, x$	$\frac{1}{\cos^2 x}$	$\mathrm{cotg}\, x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\mathrm{arctg}\, x$	$\frac{1}{1+x^2}$	$\mathrm{arccotg}\, x$	$-\frac{1}{1+x^2}$
$\sinh x$	$\cosh x$	$\cosh x$	$\sinh x$
$\mathrm{tgh}\, x$	$\frac{1}{\cosh^2 x}$	$\mathrm{cotgh}\, x$	$-\frac{1}{\sinh^2 x}$

Dotyčnica, normála:

$$\boxed{\begin{aligned} y - f(x_0) &= f'(x_0)(x - x_0) \\ y - f(x_0) &= -\frac{1}{f'(x_0)}(x - x_0) \end{aligned}}$$

diferenciál

Pravidla derivovania

- $(f(x) \pm g(x))' = f'(x) \pm g'(x)$
- $(c \cdot f(x))' = c \cdot f'(x)$
- $(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
- $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) g(x) - f(x) g'(x)}{g^2(x)}$
- $(f \circ g)'(x_0) = f'(g(x_0)) \cdot g'(x_0)$

Inflexní bod

konvexní ∪ ⇄ konkávna ∩

v něm ex. vlastní der-ce! (tečna)

Priebeh funkcie

1) D_f
2) Parita
3) As-ty
4) Priesečníky s osami
$f'(x) \to$ monot
$f''(x) \to$ konv/konk

Taylorov polyn.

$T_n(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \ldots + \frac{f^{(n)}(x_0)}{n!} \cdot (x - x_0)^n$

+C !

Num rieš. nelin. rovníc

- Grafický +
- m. bisekcie
- m. regula falsi $x_1 = b_1 - \frac{b_1 - a_1}{f(b_1) - f(a_1)} f(b_1)$
- Newtonova m. (dotyčnice)
  Podmínky konv:
  1) sp. $f'(x)$, $f''(x)$
  2) $f'(x) \neq 0$, $f'(x)$, $f''(x)$ nemení znam.!
  ⇓
  poč. $x_0 \in \langle a, b \rangle$
  $f(x_0) \cdot f''(x_0) > 0$
  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
- Prostej iterácie $f(x) = \cos x - x$, $\cos x = x$, $g(x) = \cos x$
  Podmínky:
  1) $g(x)$ dif. na ...
  $g(\langle a, b \rangle) \subseteq \langle a, b \rangle$
  2) Ak $\exists \alpha \in (0, 1)$: $|g'(x)| \leq \alpha$
  $\forall x \in \langle a, b \rangle$ $(|g'(x)| < 1)$
  ⇓
  $\exists$ pevný bod f-ce g na $\langle a, b \rangle$
  ∧ poč. ap. může být lib. $x_0 \in \langle a, b \rangle$
  a $x_1 = g(x_0)$
  ⋮
  $x_k = g(x_{k-1})$

Neurč. integrály

$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)|$

$\int f(ax+b) dx = \frac{1}{a} F(ax+b)$

$\int a f(x) + b g(x) dx = a \int f(x) dx + b \int g(x) dx$

$\int u(x) v'(x) dx = u(x) v(x) - \int u'(x) v(x) dx$

$\int f(x) dx = \int f[g(t)] g'(t) dt$ kde $x = g(t)$

$\int 0 dx$	c	$\int 1 dx$	x
$\int x^k dx$	$\frac{x^{k+1}}{k+1}$	$\int \frac{1}{x} dx$	$\ln|x|$
$\int \sin x\, dx$	$-\cos x$	$\int \cos dx$	$\sin x$
$\int \frac{1}{\sin^2 x} dx$	$-\mathrm{cotg}\, x$	$\int \frac{1}{\cos^2 x} dx$	$\mathrm{tg}\, x$
$\int e^x dx$	e^x	$\int a^x dx$	$\frac{a^x}{\ln a}$
$\int \frac{dx}{x^2+a^2}$	$\frac{1}{a} \mathrm{arctg} \frac{x}{a}$	$\int \frac{dx}{x^2-a^2}$	$\frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right|$
$\int \frac{dx}{\sqrt{x^2+b}}$	$\ln|x+\sqrt{x^2+b}|$	$\int \frac{dx}{\sqrt{a^2-x^2}}$	$\arcsin \frac{x}{a}$

$\int \sqrt{x^2+b}\, dx = \frac{x}{2}\sqrt{x^2+b} + \frac{b}{2} \ln|x + \sqrt{x^2+b}|$, $b \neq 0$

$\int_a^b f(x) dx = \frac{h}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4 \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))$

$\int \frac{P_m(x)}{Q_n(x)}$

$n > m$, $D < 0$

$Q_n(x) = (x-a)^2 \cdot (x-b)(x^2+cx+d)$

⇓

$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b} + \frac{Dx+E}{x^2+cx+d}$

Objem rot. telesa

$V = \pi \int_a^b (f(x))^2 dx$

Simpsonova metóda

$\int_a^b f(x) dx = \frac{b-a}{6} \cdot \left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right)$

L'Hospitalovo pravidlo

$\left(\frac{0}{0}\right)$ $\left(\frac{\infty}{\infty}\right)$

mení na derivace ↑↓

Lichobežník. met.

$\int_a^b f(x) dx = \int_a^b L(x) dx = \frac{b-a}{2}(f(b) + f(a))$

$\int_a^b f(x) dx = h\left(\frac{1}{2} f(x_0) + f(x_1) + \ldots + f(x_{n-1}) + \frac{1}{2} f(x_n)\right)$

Určitý integrál

$\int_a^b 0\, dx = 0$ $\int_a^b 1\, dx = b - a$

$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$

$$\boxed{\int_a^b f(x) = F(b) - F(a)}$$

$\int \mathrm{tg}(x) dx = -\int \frac{-\sin x}{\cos x} dx = -\ln|\cos x| + C$

$\int e^x \cos(e^x) dx = \left|\begin{matrix} e^x = t \\ e^x dx = dt \end{matrix}\right| = \int \cos t\, dt = \sin t + C = \sin(e^x) + C$

$\int_0^1 \mathrm{arctg}(x) dx = \left|\begin{matrix} u = \mathrm{arctg}(x) & u' = \frac{1}{1+x^2} \\ v' = 1 & v = x \end{matrix}\right| = [x\, \mathrm{arctg}(x)]_0^1 - \int_0^1 \frac{x}{1+x^2} dx = \frac{\pi}{4} - \frac{1}{2}[\ln|1+x^2|]_0^1 = \frac{\pi}{4} - \frac{1}{2} \ln 2$