

b)  $T = (0, 1)$  ,  $A_t = (t, t+1)$

① Najdeme  $\bigcap_{t \in T} A_t$

1)  $x = 1 \Rightarrow \forall t \in T: x > t \wedge x < t+1 \stackrel{\forall t \in T:}{\Rightarrow} t < x < t+1 \Rightarrow \forall t \in T: x \in A_t.$

2)  $x \in \mathbb{R} \wedge x \neq 1 \Rightarrow \forall y \in \mathbb{R}, y > 0: x = 1+y \Rightarrow \exists t = 0+\frac{y}{2} \in T: \frac{y}{2} = \frac{y}{2} \Rightarrow \exists t: A_t = (0+\frac{y}{2}, 1+\frac{y}{2}) \Rightarrow x \notin A_t$

$\forall y \in \mathbb{R}, y > 0: x = 1-y \Rightarrow \exists t = 1-\frac{y}{2} \in T: \frac{y}{2} = \frac{y}{2} \Rightarrow \exists t: A_t = (1-\frac{y}{2}, 2-\frac{y}{2}) \Rightarrow x \notin A_t$

$$\boxed{\bigcap_{t \in T} A_t = \{1\}}$$

② Najdeme  $\bigcup_{t \in T} A_t$

1)  $1 \in \bigcap_{t \in T} A_t \Rightarrow 1 \in \bigcup_{t \in T} A_t$

2)  $\forall x \in \mathbb{R}: x \in (0, 2) \wedge x \neq 1 \Rightarrow x = 0+\alpha \wedge 0 < \alpha < 1 \Rightarrow \exists t = \frac{\alpha}{2} \in T: 0 < t < x < 1 < t+1 \Rightarrow \exists t \in T: x \in (t, t+1) \Rightarrow \exists t: x \in A_t.$

$x = 2-\alpha \wedge 0 < \alpha < 1 \Rightarrow \exists t = 1-\frac{\alpha}{2} \in T: 0 < t < 1 < x < t+1 \Rightarrow \exists t \in T: x \in (t, t+1) \Rightarrow \exists t: x \in A_t.$

3)  $\forall x \in \mathbb{R}: x \leq 0 \Rightarrow \forall t \in T: t > x \Rightarrow \forall t \in T: x \notin (t, t+1) \Rightarrow \forall t \in T: x \notin A_t.$

4)  $\forall x \in \mathbb{R}: x \geq 2 \Rightarrow \forall t \in T: 1+t < x \Rightarrow \forall t \in T: x \notin (t, t+1) \Rightarrow \forall t \in T: x \notin A_t.$

$$\boxed{\bigcup_{t \in T} A_t = (0, 2)}$$

c)  $T = \mathbb{R}^+$ ,  $A_t = (1 - \frac{1}{t}, 2 + \frac{3}{t})$

1) Najdeme  $\bigcap_{t \in T} A_t$

$$\forall x \in \mathbb{R} \Rightarrow \exists t = -4 \in T; A_t = (\frac{5}{4}, \frac{5}{4}) \Rightarrow \exists t \in T: A_t = \emptyset$$

$$\boxed{\bigcap_{t \in T} A_t = \emptyset}$$

2) Najdeme  $\bigcup_{t \in T} A_t$

$$\forall x \in \mathbb{R} \Rightarrow \exists t \in T; t = \frac{1}{|x|+2} > 0 \Rightarrow \exists t \in T; A_t = (1 - |x| - 2, 2 + 3|x| + 6) \Rightarrow$$

$$-|x| - 1 < x < 3|x| + 8 \Rightarrow \exists t \in T: x \in A_t.$$

$$\boxed{\bigcup_{t \in T} A_t = \mathbb{R}}$$

d)  $T = (0, 2)$ ,  $A_t = (1 - \frac{1}{t}, 2 + \frac{3}{t})$

1) Najdeme  $\bigcap_{t \in T} A_t$

$$\forall x, y \in (0, 2), x, y \in \mathbb{R}; x > y \Rightarrow \frac{1}{x} < \frac{1}{y} \Rightarrow 1 - \frac{1}{y} < 1 - \frac{1}{x} < 2 + \frac{3}{x} < 2 + \frac{3}{y} \Rightarrow$$

$$A_x \subset A_y \Rightarrow \bigcap_{t \in T} A_t = A_2$$

$$\boxed{\bigcap_{t \in T} A_t = (0, 5; 3, 5)}$$

2) Najdeme  $\bigcup_{t \in T} A_t$

$$\forall x \in \mathbb{R} \Rightarrow \exists t \in T; t = \frac{1}{|x|+2} \wedge 0 < t \leq \frac{1}{2} \Rightarrow \exists t \in T; A_t = (1 - |x| - 2, 2 + 3|x| + 6) \Rightarrow$$

$$-|x| - 1 < x < 3|x| + 8 \Rightarrow \exists t \in T: x \in A_t.$$

$$\boxed{\bigcup_{t \in T} A_t = \mathbb{R}}$$



$$e) T = (0, 1), A_t = \left(1 - \frac{1}{1-t}, 2 + \frac{1}{t}\right)$$

① Najdeme  $\bigcap_{t \in T} A_t$

$$1) \forall x \in \mathbb{R}; x \in \langle 0, 3 \rangle \stackrel{1.}{\Rightarrow} \forall t \in T; 0 < 1-t < 1 \Rightarrow \forall t \in T; \frac{1}{1-t} > 1 \Rightarrow$$

$$\left( \forall t \in T; 1 - \frac{1}{1-t} < 0 \Rightarrow \forall t \in T; x > 1 - \frac{1}{1-t} \right.$$

$$\stackrel{2.}{\Rightarrow} \forall t \in T; \frac{1}{t} > 1 \Rightarrow \forall t \in T; 2 + \frac{1}{t} > 3 \Rightarrow \forall t \in T; x < 2 + \frac{1}{t}$$

$$\left. \begin{matrix} 1 \\ 2 \end{matrix} \right\} \Rightarrow \forall t \in T; x \in A_t.$$

$$2) \forall x, \alpha \in \mathbb{R}; x = 3 + \alpha \wedge \alpha > 0 \Rightarrow \exists t \in T; t = 1 - \frac{\alpha}{1+\alpha} \Rightarrow \exists t \in T; A_t = \left(1 - \frac{1}{1-t}, 2 + \frac{1}{t}\right) =$$

$$\Rightarrow \exists t \in T; A_t = \left(1 - \frac{1}{1 - 1 + \frac{\alpha}{1+\alpha}}, 2 + \frac{1}{1 - \frac{\alpha}{1+\alpha}}\right) \Rightarrow \exists t \in T; A_t = \left(1 - \frac{1+\alpha}{\alpha}, 2 + 1 + \alpha\right) \Rightarrow$$

$$\exists t \in T; A_t = \left(-\frac{1}{\alpha}, 3 + \alpha\right) \Rightarrow \exists t \in T; x \notin A_t.$$

$$3) \forall x, \alpha \in \mathbb{R}; x = 0 - \alpha \wedge \alpha > 0 \Rightarrow \exists t \in T; t = \frac{\alpha}{1+\alpha} \Rightarrow \exists t \in T;$$

$$A_t = \left(1 - \frac{1}{1 - \frac{\alpha}{1+\alpha}}, 2 + \frac{1}{\frac{\alpha}{1+\alpha}}\right) \Rightarrow \exists t \in T; A_t = \left(1 - 1 - \alpha, 2 + \frac{1+\alpha}{\alpha}\right) \Rightarrow$$

$$\exists t \in T; A_t = \left(-\alpha, 3 + \frac{1}{\alpha}\right) \Rightarrow \exists t \in T; x \notin A_t.$$

$$\boxed{\bigcap_{t \in T} A_t = \langle 0, 3 \rangle}$$

② Najdeme  $\bigcup_{t \in T} A_t$

$$1) \forall x \in \mathbb{R}; x > 0 \Rightarrow \exists t \in T; t = \frac{1}{x+1} \Rightarrow \exists t \in T; A_t = \left(1 - \frac{1}{1 - \frac{1}{x+1}}, 2 + \frac{1}{\frac{1}{x+1}}\right) \Rightarrow$$

$$\exists t \in T; A_t = \left(1 - \frac{x+1}{x}, 2 + x + 1\right) \Rightarrow \exists t \in T; A_t = \left(-\frac{1}{x}, 3 + x\right) \Rightarrow \exists t \in T; x \in A_t.$$

$$2) \forall x \in \mathbb{R}; x \leq 0 \Rightarrow \exists t \in T; t = 1 - \frac{1}{2-x} \Rightarrow \exists t \in T; A_t = \left(1 - \frac{1}{1 - (1 - \frac{1}{2-x})}, 2 + \frac{1}{1 - \frac{1}{2-x}}\right) \Rightarrow$$

$$\exists t \in T; A_t = \left(1 - 2 + x, 2 + \frac{2-x}{1-x}\right) \Rightarrow \exists t \in T; A_t = \left(x-1, 3 + \frac{1}{1-x}\right) \Rightarrow \exists t \in T; x \in A_t.$$

$$\boxed{\bigcup_{t \in T} A_t = \mathbb{R}}$$

f)  $T = (0, 1)$ ,  $A_t = \langle -\frac{1}{t}, \frac{1}{t} \rangle$

① Najdeme  $\bigcap_{t \in T} A_t$

1)  $\forall x \in \mathbb{R}; x \in \langle -1, 1 \rangle \Rightarrow \forall t \in T; \frac{1}{t} > 1 \wedge -\frac{1}{t} < -1 \Rightarrow \forall t \in T; -\frac{1}{t} < x \wedge \frac{1}{t} > x \Rightarrow \forall t \in T; x \in A_t.$

2)  $\forall x, \alpha \in \mathbb{R}; (x = 1 + \alpha \vee x = -1 - \alpha) \wedge \alpha > 0 \Rightarrow \exists t \in T; t = \frac{1}{1 + \frac{\alpha}{2}} \Rightarrow \exists t \in T; A_t = \langle -1 - \frac{\alpha}{2}, 1 + \frac{\alpha}{2} \rangle \Rightarrow \exists t \in T; x \notin A_t.$

$$\bigcap_{t \in T} A_t = \langle -1, 1 \rangle$$

② Najdeme  $\bigcup_{t \in T} A_t$

$\forall x \in \mathbb{R} \Rightarrow \exists t \in T; t = \frac{1}{|x| + 1} \Rightarrow \exists t \in T; A_t = \langle -|x| - 1, |x| + 1 \rangle \Rightarrow \exists t \in T; x \in A_t.$

$$\bigcup_{t \in T} A_t = \mathbb{R}$$

g)  $T = (0, 1)$ ,  $A_t = \langle 2 - \frac{1}{t}, 4 + \frac{2}{t} \rangle$

① Najdeme  $\bigcap_{t \in T} A_t$

1)  $\forall x \in \mathbb{R}; x \in \langle 1, 6 \rangle \Rightarrow \forall t \in T; -\frac{1}{t} < -1 \wedge \frac{1}{t} > 1 \Rightarrow \forall t \in T; 2 - \frac{1}{t} < 1 \wedge 4 + \frac{2}{t} > 6 \Rightarrow \forall t \in T; x \in A_t.$

2)  $\forall x, \alpha \in \mathbb{R}; (x = 1 - \alpha \vee x = 6 + \alpha) \wedge \alpha > 0 \Rightarrow \exists t \in T; t = \frac{1}{1 + \frac{\alpha}{4}} \Rightarrow \exists t \in T; A_t = \langle 2 - \frac{1}{1 + \frac{\alpha}{4}}, 4 + \frac{2}{1 + \frac{\alpha}{4}} \rangle \Rightarrow \exists t \in T; A_t = \langle 1 - \frac{\alpha}{4}, 6 + \frac{\alpha}{2} \rangle \Rightarrow \exists t \in T; x \notin A_t.$

$$\bigcap_{t \in T} A_t = \langle 1, 6 \rangle$$

② Najdeme  $\bigcup_{t \in T} A_t$

$\forall x \in \mathbb{R} \Rightarrow \exists t \in T; t = \frac{1}{|x| + 3} \Rightarrow \exists t \in T; A_t = \langle 2 - |x| - 3, 4 + 2|x| + 6 \rangle \Rightarrow \exists t \in T; A_t = \langle -1 - |x|, 10 + 2|x| \rangle \Rightarrow \exists t \in T; x \in A_t$

$$\bigcup_{t \in T} A_t = \mathbb{R}$$