Domací úkol №4 ISM

10) $z^3 = \frac{1}{8} + 0j$

$z^3 = |z^3| \cdot e^{\varphi j} = |z^3| \cdot e^{0 \cdot j}$

$z = \sqrt[3]{|z^3|} \cdot e^{\frac{0+2\pi k}{3} j}$, $k \in \mathbb{Z}$

$z_1 = \sqrt[3]{|z^3|} \cdot e^{0j} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

$z_2 = \frac{1}{2} \cdot e^{\frac{2\pi}{3} j} = \frac{1}{2}\left(\cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}\right) = -\frac{1}{4} + \frac{\sqrt{3}}{4} j$

$z_3 = \frac{1}{2} \cdot e^{\frac{4\pi}{3} j} = \frac{1}{2}\left(\cos\frac{4\pi}{3} + j\sin\frac{4\pi}{3}\right) = -\frac{1}{4} - \frac{\sqrt{3}}{4} j$

Výsledek: $\frac{1}{2}$; $-\frac{1}{4} \pm \frac{\sqrt{3}}{4} j$

8) $z_1 = \sqrt{3} + j$, $z_2 = 6\left(\cos\frac{\pi}{3} + j\sin\frac{\pi}{3}\right)$

$|z_1| = \sqrt{3+1} = 2$ $\quad \varphi_1 = \operatorname{arctg}\frac{\sqrt{3}}{3} = \frac{\pi}{6}$

$z_1 = |z_1| \cdot e^{\varphi j} = 2e^{\frac{\pi}{6} j}$

$z_2 = 6e^{\frac{\pi}{3} j}$

$z_1 \cdot z_2 = 2e^{\frac{\pi}{6} j} \cdot 6e^{\frac{\pi}{3} j} = 12e^{\frac{\pi}{2} j}$

$\frac{z_1}{z_2} = \frac{2e^{\frac{\pi}{6} j}}{6e^{\frac{\pi}{3} j}} = \frac{1}{3} e^{-\frac{\pi}{6} j}$

Výsledek: $12e^{\frac{\pi}{2} j}$, $\frac{1}{3} e^{-\frac{\pi}{6} j}$

6) $z = \sqrt{-5+12j}$ $\quad$ at' $z = a + bj$, $a, b \in \mathbb{R}$

$z^2 = -5 + 12j$

$(a+bj)^2 = -5 + 12j$

$a^2 + b^2 j^2 + 2abj = -5 + 12j$

$\begin{cases} a^2 - b^2 = -5 \\ 2abj = 12j \end{cases}$ $\begin{cases} a^2 - b^2 = -5 \\ ab = 6 \end{cases}$ $\begin{cases} a^2 - b^2 = -5 \\ a = \frac{6}{b} \end{cases}$

① $b = 0$:

$a \cdot 0 = 6 \quad , a \in \mathbb{R} \Rightarrow \emptyset$

② $b \neq 0$:

$\frac{36}{b^2} - b^2 = -5$

$\frac{36 + 5b^2 - b^4}{b^2} = 0$

$b^4 - 5b^2 - 36 = 0$

$b^2 = t, \ t \geq 0$

$t^2 - 5t - 36 = 0$

$D = 25 + 144 = 169$

$t_{1,2} = \frac{5 \pm 13}{2}$, ale $t \geq 0 \Rightarrow t = 9$

$b^2 = 9$

① $b = 3$ $\quad z_1 = 2 + 3j$

$a = \frac{6}{b} = 2$

② $b = -3$ $\quad z_2 = -2 - 3j$

$a = -2$

Výsledek: $z \in \{2 + 3j \ , -2 - 3j\}$