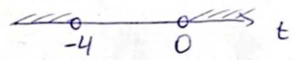
c) $T = \mathbb{R}^+$, $A_t = \left(1-\frac{1}{t}, 2+\frac{3}{t}\right)$

① $1-\frac{1}{t} < 2+\frac{3}{t}$

$\frac{4}{t}+1 > 0$

$\frac{t+4}{t} > 0$



$T' = (-\infty, -4) \cup (0, +\infty)$

② Najdeme $\bigcap_{t\in T'} A_t$

1) $x = \frac{5}{4} \Rightarrow 1-\frac{1}{t} < \frac{5}{4} \wedge \frac{5}{4} < 2+\frac{3}{t} \Rightarrow t \in (-\infty,-4)\cup(0,+\infty) \wedge$

$t \in (-\infty,-4)\cup(0,+\infty) \Rightarrow \forall t \in (-\infty,-4)\cup(0,+\infty) ;\ t \in T' \Rightarrow \forall t \in T' ;\ x \in A_t.$

$$\left\{\begin{array}{l} 1-\frac{1}{t} < \frac{5}{4} \\ \frac{5t+4-4t}{4t} > 0 \\ \frac{t+4}{t} > 0 \end{array}\right. \qquad \left\{\begin{array}{l} \frac{5}{4} < 2+\frac{3}{t} \\ \frac{8t+12-5t}{4t} > 0 \\ \frac{3t+12}{4t} > 0 \\ \frac{t+4}{t} > 0 \end{array}\right.$$

2) $x, \alpha \in \mathbb{R} ;\ x = \frac{5}{4}+\alpha \wedge \alpha \neq 0 \Rightarrow \exists t = -4-\frac{|\alpha|}{2} \in T' \Rightarrow$

$\exists t \in T' ;\ A_t = \left(1-\frac{1}{-4-\frac{|\alpha|}{2}}, 2+\frac{3}{-4-\frac{|\alpha|}{2}}\right) \Rightarrow \exists t \in T' ;\ A_t = \left(1+\frac{2}{8+|\alpha|}, 2-\frac{6}{8+|\alpha|}\right) \Rightarrow$

$\alpha > 0 \Rightarrow \exists t \in T' ;\ \frac{5}{4}+\alpha > 2-\frac{6}{8+\alpha} \Rightarrow \exists t \in T' ;\ x \notin A_t.$

$\alpha < 0 \Rightarrow \exists t \in T' ;\ \frac{5}{4}+\alpha < 1+\frac{2}{8-\alpha} \Rightarrow \exists t \in T' ;\ x \notin A_t.$

$$\left\{\begin{array}{l} \frac{5}{4}+\alpha \overset{\Delta}{>} 2-\frac{6}{8+\alpha} \\ \alpha \overset{\Delta}{>} \frac{3}{4}-\frac{6}{8+\alpha} \\ \alpha \overset{\Delta}{>} \frac{24+3\alpha-24}{4(8+\alpha)} \\ 1 \overset{\Delta}{>} \frac{3}{32+4\alpha} \end{array}\right. \qquad \left\{\begin{array}{l} \frac{5}{4}+\alpha \overset{\Delta}{<} 1+\frac{2}{8-\alpha} \\ \alpha \overset{\Delta}{<} -\frac{1}{4}+\frac{2}{8-\alpha} \\ \alpha \overset{\Delta}{<} \frac{-8+\alpha+8}{4(8-\alpha)} \\ 1 \overset{\Delta}{>} \frac{1}{\underset{>1}{4(8-\alpha)}} \end{array}\right.$$

$$\boxed{\bigcap_{t\in T'} A_t = \left\{\frac{5}{4}\right\}}$$

③ Najdeme $\bigcup_{t\in T'} A_t$

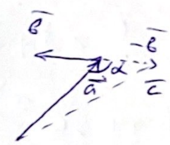
$\forall x \in \mathbb{R} \Rightarrow \exists t \in T' ;\ t = \frac{1}{|x|+2} > 0 \Rightarrow \exists t \in T' ;\ A_t = (1-|x|-2, 2+3|x|+6) \Rightarrow$

$-|x|-1 < x < 3|x|+8 \Rightarrow \exists t \in T' ;\ x \in A_t.$

$$\boxed{\bigcup_{t\in T'} A_t = \mathbb{R}}$$

14) $\left| \|\bar{a}\| - \|\bar{b}\| \right| \le \|\bar{a} - \bar{b}\|$?

at' $\bar{c} = \bar{a} + (-\bar{b})$. Podle Kosinové věty:



$$\|\bar{c}\|^2 = \|\bar{a}\|^2 + \|-\bar{b}\|^2 - 2\|\bar{a}\|\cdot\|-\bar{b}\|\cos\alpha$$

$$\underbrace{\left| \|\bar{a}\| - \|\bar{b}\| \right|}_{>0} \le \|\bar{a} - \bar{b}\| = \|\bar{c}\| = \underbrace{\sqrt{\|\bar{a}\|^2 + \|-\bar{b}\|^2 - 2\|\bar{a}\|\cdot\|-\bar{b}\|\cos\alpha}}_{>0} \qquad | \;{}^\wedge 2$$

$$\cancel{\|\bar{a}\|^2} + \cancel{\|\bar{b}\|^2} - 2\|\bar{a}\|\cdot\|\bar{b}\| \le \cancel{\|\bar{a}\|^2} + \cancel{\|-\bar{b}\|^2} - 2\|\bar{a}\|\cdot\|-\bar{b}\|\cos\alpha \qquad | \; \|\bar{b}\| = \|-\bar{b}\|$$

$$2\|\bar{a}\|\|\bar{b}\|(\cos\alpha - 1) \le 0$$

1) $\|\bar{a}\| = 0 \lor \|\bar{b}\| = 0$ $\quad 0 \le 0$ platí

2) $\|\bar{a}\| \ne 0 \land \|\bar{b}\| \ne 0$

$\cos\alpha - 1 \le 0$

$\cos\alpha \le 1$ to je pravda $\Rightarrow \left| \|\bar{a}\| - \|\bar{b}\| \right| \le \|\bar{a} - \bar{b}\|$.