Domací úloha 1

10) $x^5 + 5x^4 + 9x^3 + 13x^2 + 14x + 6$

	1	5	9	13	14	6
-1	1	4	5	8	6	0
-1	1	3	2	6	0	
-3	1	0	2	0		

$(x+1)^2 \cdot (x+3) \cdot (x^2+2)$

16) $x^5 + 5x^4 + 12x^3 + 24x^2 + 32x + 16$

	1	5	12	24	32	16
-2	1	3	6	12	8	0
-2	1	1	4	4	0	
-1	1	0	4	0		

$(x+2)^2 (x+1)(x^2+4)$

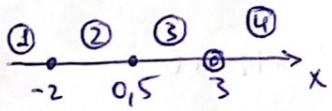
20) $x^5 - 5x^4 + 5x^3 - x^2 + 6x + 18$

	1	-5	5	-1	6	18
-1	1	-6	11	-12	18	0
3	1	-3	2	-6	0	
3	1	0	2	0		

$(x-3)^2 (x+1)(x^2+2)$

Domací úloha 2

19) $|x+2| + \frac{|2x-1|}{|x-3|} \leq 2$



$x \neq 3!$

① $x \leq -2$:

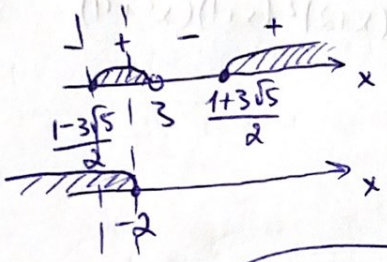
$$-x-2 + \frac{-2x+1}{-x+3} \leq 2$$

$$-x-4 + \frac{2x-1}{x-3} \leq 0$$

$$\frac{2x-1-(x+4)(x-3)}{x-3} \leq 0$$

$$\frac{2x-1-x^2-x+12}{x-3} \leq 0$$

$$\frac{x^2-x-11}{x-3} \geq 0$$



$$x^2-x-11=0$$
$$D = 1+44 = 45$$
$$x_{1,2} = \frac{1 \pm 3\sqrt{5}}{2}$$

$2 < \sqrt{5} < 3$ | $-3 < -\sqrt{5} < -2$
$6 < 3\sqrt{5} < 9$ | $-9 < -3\sqrt{5} < -6$
$7 < 1+3\sqrt{5} < 10$ | $-8 < 1-3\sqrt{5} < -5$
$3 < 3,5 < \frac{1+3\sqrt{5}}{2} < 5$ | $-4 < \frac{1-3\sqrt{5}}{2} < -2,5$ ✓

$$\left(-\infty ; \frac{1-3\sqrt{5}}{2} \right\rangle$$

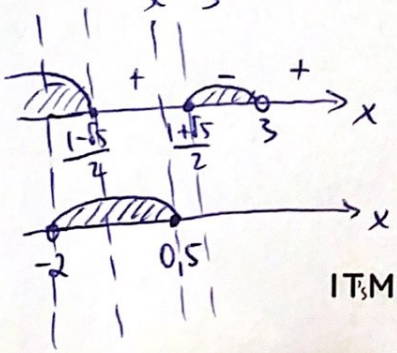
② $-2 < x \leq 0,5$:

$$x+2 + \frac{-2x+1}{-x+3} \leq 2$$

$$x + \frac{2x-1}{x-3} \leq 0$$

$$\frac{x^2-3x+2x-1}{x-3} \leq 0$$

$$\frac{x^2-x-1}{x-3} \leq 0$$



$$x^2-x-1=0$$
$$D = 1+4 = 5$$
$$x_{1,2} = \frac{1 \pm \sqrt{5}}{2}$$

$2 < \sqrt{5} < 3$ | $-3 < -\sqrt{5} < -2$
$3 < 1+\sqrt{5} < 4$ | $-2 < 1-\sqrt{5} < -1$
$0,5 < 1,5 < \frac{1+\sqrt{5}}{2} < 2 < 3$ | $-1 < \frac{1-\sqrt{5}}{2} < -0,5$

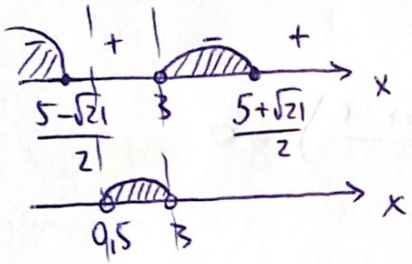
$$\left(-2 ; \frac{1-\sqrt{5}}{2} \right\rangle$$

③ $0{,}5 < x < 3$:

$$x+2+\frac{2x-1}{3-x} \leq 2$$

$$\frac{-x^2+5x-1}{3-x} \leq 0$$

$$\frac{x^2-5x+1}{x-3} \leq 0$$



$\emptyset$

$x^2-5x+1=0$

$D = 25-4 = 21$

$$x_{1,2} = \frac{5 \pm \sqrt{21}}{2}$$

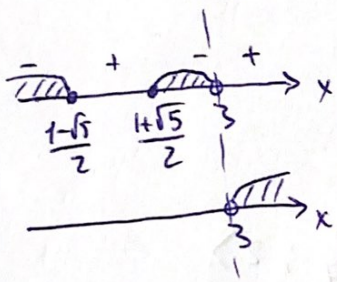
$-5 < -\sqrt{21} < -4$

$0 < 5-\sqrt{21} < 1$

$0 < \frac{5-\sqrt{21}}{2} < 0{,}5$

④ $x > 3$:

$$x+2+\frac{2x-1}{x-3} \leq 2$$ div. se ②



$\emptyset$

Výsledek: $\left\langle \frac{1-3\sqrt{5}}{2} ; \frac{1-\sqrt{5}}{2} \right\rangle$

20) $\frac{|x+2|}{|x+6|} \geq \frac{|x-1|}{|x-4|}$ $\quad x \neq 4 \wedge x \neq -6$!

$\left|\frac{x+2}{x+6}\right| \geq \left|\frac{x-1}{x-4}\right| \geq 0 \quad |\,()^2$

$\left(\frac{x+2}{x+6}\right)^2 \geq \left(\frac{x-1}{x-4}\right)^2$

$\left(\frac{x+2}{x+6} - \frac{x-1}{x-4}\right)\left(\frac{x+2}{x+6} + \frac{x-1}{x-4}\right) \geq 0$

$\left(\frac{x^2-2x-8-x^2-5x+6}{(x-4)(x+6)}\right)\left(\frac{x^2-2x-8+x^2+5x-6}{(x+6)(x-4)}\right) \geq 0$

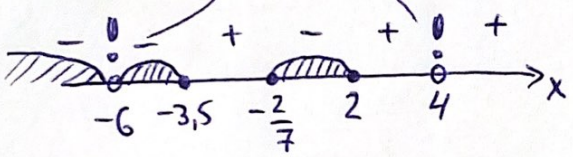
$\frac{(-7x-2)(2x^2+3x-14)}{(x-4)^2(x+6)^2} \geq 0$

$2x^2+3x-14=0$

$D = 9 + 112 = 121$

$x_{1,2} = \frac{-3 \pm 11}{4} = 2 \vee -3,5$

$\frac{(x+\frac{2}{7})(x-2)(x+3,5)\cdot 2}{(x-4)^2(x+6)^2} \leq 0$



Výsledek: $(-\infty; -6) \cup (-6; -\frac{7}{2}\rangle \cup \langle -\frac{2}{7}; 2\rangle$

24) atˡ odvěsny jsou x a y. $(x, y > 0)$

$\begin{cases} y - x = 1 \\ \sqrt{x^2+y^2} > 11 \end{cases}$ $\quad \begin{cases} y = 1 + x \\ \sqrt{x^2+(1+x)^2} > 11 \end{cases}$

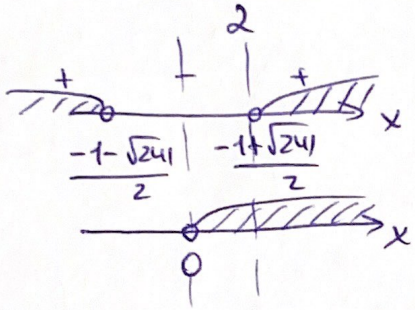
$x^2 + (1+x)^2 > 121$

$2x^2 + 2x + 1 - 121 > 0$

$x^2 + x - 60 > 0$

$D = 1 + 240 = 241$

$x_{1,2} = \frac{-1 \pm \sqrt{241}}{2}$



$x > \frac{-1+\sqrt{241}}{2}$; $y = 1 + x$

Výsledek: $\{[x, 1+x];\ x \in R,\ x > \frac{-1+\sqrt{241}}{2}\}$