9* s. 3.

$$J_n = \underbrace{\begin{vmatrix} 2 & 1 & 0 & \dots & 0 & 0 \\ 1 & 2 & 1 & 0 & \dots & 0 \\ 0 & 1 & 2 & & & 0 \\ \vdots & 0 & & \ddots & & 1 \\ 0 & \dots & 0 & 1 & & 2 \end{vmatrix}}_{n} \Bigg\} n$$

Najdeme determinant pomocí Laplaceova rozvoje 1. řádku:

$$J_n = 2 \cdot \underbrace{\begin{vmatrix} 2 & 1 & 0 & \dots & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & & \\ \vdots & 0 & & \ddots & 1 \\ 0 & 0 & & 1 & 2 \end{vmatrix}}_{n-1} \Bigg\} n-1 \cdot (-1)^{1+1} +$$

$$+ (-1)^{1+2} \cdot 1 \cdot \underbrace{\begin{vmatrix} 1 & 1 & 0 & \dots & 0 \\ 0 & 2 & 1 & & \vdots \\ 0 & 1 & 2 & & 0 \\ \vdots & & & \ddots & 1 \\ 0 & \dots & 0 & 1 & 2 \end{vmatrix}}_{n-1} \Bigg\} n-1 + 0 \cdot (n-2) \quad (1)$$

Najdeme doplňkový determinant $K_m = \underbrace{\begin{vmatrix} 1 & 1 & 0 & \dots & 0 \\ 0 & 2 & 1 & & \\ 0 & 1 & 2 & & 0 \\ \vdots & & & \ddots & 1 \\ 0 & \dots & 0 & 1 & 2 \end{vmatrix}}_{m} \Big\} m$ taky pomocí Laplaceova rozvoje, ale 1. sloupce:

$$K_m = (-1)^{1+1} \cdot 1 \cdot \underbrace{\begin{vmatrix} 2 & 1 & 0 & \dots & 0 \\ 1 & 2 & 1 & \dots & 0 \\ 0 & 1 & \ddots & & \\ 0 & & & & 2 \end{vmatrix}}_{m-1} \Bigg\} m-1 + 0 \cdot (m-1) \quad (2)$$

Přepíšeme (1) a (2):

$$\begin{cases} J_n = 2 \cdot J_{n-1} - K_{n-1} \\ K_m = J_{m-1} \end{cases}$$

Máme rekurentní vztah, ale potřebujeme ještě počateční podmínky:

$$J_2 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3$$

$$K_2 = \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = 2$$

Teď můžeme najít J_6:

$$J_6 = 2 \cdot J_5 - K_5 = 2 \cdot (2 \cdot J_4 - K_4) - J_4 = 3J_4 - 2K_4 = 3(2J_3 - K_3) -$$
$$-2J_3 = 4J_3 - 3K_3 = 4(2J_2 - K_2) - 3J_2 = 5J_2 - 4K_2 = 5 \cdot 3 - 4 \cdot 2 = 7$$

7* s. 4

$$\operatorname{adj}(A) = \begin{pmatrix} -1 & -2 & 1 \\ 3 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \operatorname{adj}(A)$$

$$A \cdot A^{-1} = A \cdot \frac{1}{|A|} \cdot \operatorname{adj}(A)$$

$$E = A \cdot \frac{1}{|A|} \cdot \operatorname{adj}(A)$$

$$\operatorname{adj} A^{-1} = A \cdot \frac{1}{|A|}$$

ať $\operatorname{adj}(A) = B$

$$B_{11} = (-1)^{1+1} \cdot (2) = 2$$
$$B_{12} = (-1)^{3} \cdot (3-1) = -2$$
$$B_{13} = (-1)^{4} \cdot (0+2) = 2$$
$$B_{21} = (-1)^{3} \cdot (-2) = 2$$
$$B_{22} = (-1)^{4} \cdot (-1+1) = 0$$
$$B_{23} = (-1)^{5} \cdot (0-2) = 2$$
$$B_{31} = (-1)^{4} \cdot (2-2) = 0$$
$$B_{32} = (-1)^{5} \cdot (1-3) = 2$$
$$B_{33} = (-1)^{6} \cdot (-2+6) = 4$$

$$|B| = -2 - 2 + 2 + 6 = 4$$

$$B^{-1} = \frac{1}{4} \begin{pmatrix} 2 & 2 & 0 \\ -2 & 0 & 2 \\ 2 & 2 & 4 \end{pmatrix} =$$

$$= \begin{pmatrix} 0{,}5 & 0{,}5 & 0 \\ -0{,}5 & 0 & 0{,}5 \\ 0{,}5 & 0{,}5 & 1 \end{pmatrix} = A \cdot \frac{1}{|A|}$$

ať $|A| = k$, tehdy

$$A = \begin{pmatrix} 0{,}5k & 0{,}5k & 0 \\ -0{,}5k & 0 & 0{,}5k \\ 0{,}5k & 0{,}5k & k \end{pmatrix}$$

$$|A| = 0 + 0{,}125k^3 + 0 - 0 - 0{,}125k^3 + 0{,}25k^3 = 0{,}25k^3 = k$$

$$k(0{,}25k^2 - 1) = 0$$

$$\begin{cases} k = 0 \\ k = \pm 2 \end{cases}$$

$$A = \pm \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

Uděláme zkoušku:

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

$A_{11} = (-1)^{1+1} \cdot (-1) = -1$ $\quad A_{21} = (-1)^{2+1} \cdot 2 = -2$ $\quad A_{31} = (-1)^{3+1} \cdot 1 = 1$

$A_{12} = (-1)^{1+2} \cdot (-3) = 3$ $\quad A_{22} = (-1)^{2+2} \cdot 2 = 2$ $\quad A_{32} = (-1)^{3+2} \cdot 1 = -1$

$A_{13} = (-1)^{1+3} \cdot (-1) = -1$ $\quad A_{23} = (-1)^{2+3} \cdot 0 = 0$ $\quad A_{33} = (-1)^{3+3} \cdot 1 = 1$

$$A\,adj = \begin{pmatrix} -1 & -2 & 1 \\ 3 & 2 & -1 \\ -1 & 0 & 1 \end{pmatrix} \quad \checkmark$$

Pro $A_2 = -A_1$ 1. činitel $(-1)^{i+j}$ se nezmění, pro 2. taky, protože $|A_{2k\,2k}| = |-A_{2k\,2k}|$, $k \in \mathbb{N}$ (pro každý řádek můžeme vytknout činitel -1 a sudý počet změn znaménka $|A|$ její nezmění.

Takže $(A_1)adj = (A_2)adj$.