***A Survey of Polar Codes***

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**ABSTRACT**

**In this paper, the specific polar codes adopted recently by the 5G (Fifth-generation) NR (New Radio) interface are investigated. Based on B-DMC (Binary Discrete Memoryless Channel), the purpose of each key component in these codes and the associated operations are explained. In this paper, we present a discussion about efficient method to construct polar codes coding and decoding. The simulation results are carried out by using a particular BEC (Binary Erasure Channel) channel and the SC (Successive Cancellation) decoding algorithm.**

**The obtained results are promising and will be a basis for the proposition of other high-performance algorithms which make it possible to overcome certain defects of the algorithms proposed in the literature.**

**Keywords**

Polar Code, successive cancelation, channel polarization, erasure channel,

# INTRODUCTION

The main idea of the FEC (Forward Error Correction) is to transmit enough redundant data with the useful data, to allow the receiver to correct, by itself, the errors generated by the transmission channel. In this case, no retransmission from the transmitter is required.

During the past decade, many FEC, like Turbo codes, RS (Reed Solomon)[1], BCH (Bose Chaudhuri Hocquenghem)[2], and LDPC (Low Density Parity Check) codes[3], have been proposed in the literature to increase Mobile devices of all shapes and sizes may communicate with each other, with fixed infrastructure and with satellites Wireless communications is susceptible to noise, interference, poor signal strength, jamming etc… Owing to these effects, the symbols in the received message may differ to the symbols in the transmitted message. To protect the communication from errors the systems use the FEC.

The 3G (Third Generation) named UMTS (Universal Mobile Telecommunication Service) and 4G (Fourth Generation) named LTE (Long Term Evolution) cellular systems, for example, use the Turbo code [1]. Compared to the 3G and 4G, in November 2016, 3GPP have been agreed to adopt a new FECs, for the future 5G standard: polar codes for control channels and LDPC codes for the corresponding Data. 5G uses a new radio interface which holds promise in fulfilling new communication requirements that enable ubiquitous, low-latency, high-speed, and high-reliability connections among mobile devices.

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While turbo and LDPC codes entered the consciousness of the communications community in 1993 [1] and 1996 [3] respectively, polar codes were not proposed until much more recently in 2008 [4]. Owing to this, turbo and LDPC codes have reached a much greater level of maturity than polar codes, as shown in [5]. In particular, turbo and LDPC codes can be found in many consumer devices, owing to their inclusion in 3G/4G and WiFi standards, respectively. By contrast, polar codes have not yet been adopted in any standards or consumer devices and so their maturity is limited to proof of concept demonstrators and academic publications [5].

the noisy channel coding theorem of shannon proove the possibility of achieving the capacity of channel but without explaining the method, the goal of polar coding was developed to achieve the goal of Shannon theorem for the class binary-input discrete memoryless channels.

Notably polar codes have modest encoding and decoding complexity O(N.log (N)) certainly,

These codes and their variants will find more deployment in many other applications and will be included in other new standards in the future. Nevertheless, the design of such codes for the next generation wireless communication systems is still in its infancy. There are a range of open issues waiting to be addressed. This why in this work, we investigate these codes and present a method to construct them.

This paper is organized as follows. Section 2 presents a brief overview of the coding and decoding algorithms for polar codes. Section 3 gives the simulation results in the case of the use of BEC channel. The conclusion of this this work is given in section 4.

# Polar codes: Overview

In this section we will try to give an overview of polar codes and explain a method of coding and decoding algorithm for polar codes,

## Polar codes

In information theory, a polar code is a linear block error correcting code, the code construction is based on multiple recursive concatenation of a short kernel code which transforms the physical channel into outer virtual channels, when the number of recursions becomes large, the virtual channels tend to either have high reliability or low reliability (in other words, they polarize), and the data bits are allocated to the most reliable channels,

The polar code is a new FEC invented by Arikan [4] based on a phenomenon called channel polarization, they are proved to achieve the symmetric capacity of any B-DMC using low complexity encoders and decoders, and their block error probability, is show to decrease exponentially in the square root of the block length , in fact two basic channel transformations lie at the heart of channel polarization, The recursive applications of these transformations result in the channel polarization which refers to the fact that the channels synthetized in these transformations become in the limit either almost reliable (perfect and free error) or unreliable (complexity noisy), In fact, channel polarization refers to these two extern situations, this is show by analyzing channel reliability parameters, such as the symmetric capacity and the Bhattacharya parameters. These two parameters standard respectively are measures of communication rate and reliability of the channel. On the other hand these two parameters are used jointly to prove that channel polarization occurs.

The operation of channel polarization consists of two phases: a channel combining and splitting phases. More details about this operation are investigated in the literature [4, 6].

In the following two sub-sections we will present the process and the structure of the encoder and decoder for polar codes.

## *Coding algorithm*

A polar codes (N, K) are a linear block code of length N=2n and rate K/N. They can be expressed mathematically by the multiplication between the vector information and matrix generator as.

(1)

Where u= (u1, u2… uN-1) denotes the input vector, and GN the matrix generator. The matrix generator are given by

 (2)

Where F denotes the Kronecker product, Where:

 denotes the Kronecker polarizing matrix shown below.

 (3)

 denotes the nth tensor power of F [4] and

 denotes the permutation matrix known as bit reversal.

The polarization effect brought by polar codes allows dividing the N-bit input vector u between reliable and unreliable bit-channels.

The K information bits are assigned to the most reliable bit-channels of u, while the remaining N-K bits, called frozen bits, are set to a predefined value (usually 0). They are assigned to the most unreliable bit-channel. Codeword X is transmitted through the channel, and the decoder receives the output sequence y=(y0,y1,…,yN-1) which is the noisy version of X=(x0,x1,…,xN-1)

The polar coding is characterized by transition probabilities between the source X and destination Y for which we write W: XY to denote a generic B-DMC.

X and Y are respectively the input and the output alphabets and W(x/y) is the transition probabilities where x ϵ X and y ϵY the x = {0, 1}.

We describe by WN the virtual channel corresponding to N independent uses of given B-DMC W, thus WN : XNYN.

 (4)

Where

 ,  and W(yi/xi) are the transition probability corresponding to each use of the given B-DMC. The special characteristic of polar coding is possibility to achieve the symmetric capacity I(W) of channel which defined through the formula illustrated by (5). I(W) is a measure of rate in a channel it is well-know that reliable communication is possible over a symmetric B-DMC at any rate less than I(W).

 (5)

The channel combining part is a recursive method to build a channel WN by N channels W.

For example a channel W2 illustrated in Fig.1 is result combining of two copies channel W it is defined by W2: X2Y2 with probability of transition defined by (6).

W2(y1y2|u1u2) = W(y1|u1⊕u2)W(y2|u2) (6)

Fig.2 presents a channel W4 which is the result combining of two copies channel W2.

The coding process is a function that shows how to construct xi from ui. The mapping can be described as shown by (7).

 (7)

Where is the input of channel WN and is the input of the virtual channel WN. The input of the channel WN can be written as show in (8)

 (8)

Where UA and UAc denote respectively the sets of the information and frozen bits. Consequently the input of the channel WN will be written expressed by (9).

 (9)

denotes the sub-matrix of GN formed by the rows with indices in A. A is an arbitrary subset of {1…N} of cardinal |A|=K, this set refers to the information bits positions. is the complement of A which refers to frozen bits.

If we fix the subset A and leave UAc a free variable, the mapping from source block UA to codeword blocks  will be very easy. For example, as illustrated in the Fig. 2, the mapping  from the input of W4 to the input of W4 can be described as shown in (10).

Fig. 1. Combination 2 channels

W

W

U1

U2

X1

X2

W2

Fig. 2. Combination 4 channels

W

W

U1

X1

X2

W

W

U3

U4

X3

X4

V1

V2

V3

V4

W4

s1

S2

S3

S4

 (10)

Where G4 is the generator matrix of size 4\*4 defined by (11).

 (11)

In this example we consider a polar code characterized by K=2, N=4, A={1, 3} and UAc={0, 0} the input of the virtual channel will be described by the (12).

 (12)

(13)

 (14)

For a source block (u1, u3) = (1, 0) the codeword will be given by (14).

 (15)

The choice of the information bits positions in the codeword influences very clearly the performance of polar codes. Hence their positions should be chosen carefully, depending on the Bhattacharya parameter. It can be see that smaller value of this parameter, the more reliable channel is.

The first step of the channel polarization is channel combining and the next and final step is the channel splitting. The channel splitting consists of split channel WN to construct N binary input coordinate channels.

Fig. 3. Encoding scheme of Polar code (4, 2)

X1

X1

X2

X3

X4

U1

U2

U3

U4

 (16)

defined by the following transition probabilities given by (17).

 (17)

Where  represents the output of  and  the input.

We use the channel splitting to construct polar codes that achieve channel capacity based on the idea that we only send data through those K channels  for which the Bhattacharya parameter of channel is near to 0. The parameter  is given by the (18).

 (18)

To construct a polar codes which achieve the symmetric capacity of a given B-DMC, for each (positions of information bits), the value of the corresponding  will be chose among the smallest K values from the set given by (19).

 (19)

The N-K remaining positions are chosen for frozen bits, the choice of these positions is unspecified and not important.

**Proposed coding algorithm:**

The proposed algorithm included two parts; the first one illustrated by the algorithm 1 computes the Bhattacharya parameter and the second one illustrated by the algorithm 2 computes the codeword.

**Algorithm 1: Cal-Bhattacharya (N, i, eps)**

**//**N is the code length

//i is bit position

// eps is the probability of erasure in BEC

**IF** (N=1 and i=1)

Exit

**ELSE**

IF (i odd)

N=N/2 and N=(N+1)/2

Cal-Bhattacharya(N,i,eps)

ELSE

N=N/2 and i=i/2

Cal-Bhattacharya (N,i, eps)

**END**

|  |
| --- |
| **Algorithm 2: Encoding** |

// N is the length of code and N is the vector information

**If** (N==1)

Result is u;// vector coding is U

**Else**

For e=1…N do

 s[2\*e-1] u[2\*e-1]^u[2\*e]

s[2\*e] u[2\*e]

va[e] s[2\*e-1]

vb[e]  s[2\*e]

encoding(Va, N/2)

encoding(Vb, N/2)

**For** e=1…N/2 do

x[e]  va[e]

x[e+N/2] vb[e]

**Result** is x // the vector coded is x

## Decoding algorithm

The SC (Successive Cancellation) decoding algorithm, which was introduced by E. Arikan [4], is the main and original algorithm used to decode polar codes. As its name shows, the SC decoding takes successive decisions on the information bits. More speciﬁcally, the receiver observes the channel output vector and estimates the elements of the input vector successively. The main task of this decoding algorithm is to generate an estimate vector corresponding to the input vector, given knowledge of,, and . All the input bits are transmitted by N channel Wi where i [1, N], the decoder observe (uAc), in reception we found N decision element, the decoder generate an estimation of the input vector. The decoder estimate exactly the value of frozen bits with 0% error =uAc, the error can achieve to generate of UA.

**Proposed SC decoding algorithm:**

As shown in the work proposed in [5], we proceed as shown in the algorithm 3 below to estimate the value of .

|  |
| --- |
| **Algorithm 3: SC Decoding** |

// out is the decoded vector

// U is the vector in the output

//N is the length of U

//cal\_lr: compute the likelihood ratio

For i=1...N

If U[i] is frozen bit

Out[i] U[i]

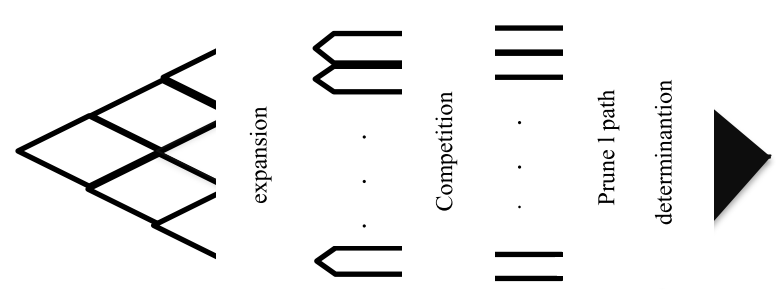
Else

If(cal\_lr(i)>=1)

 Out[i] 0;

Else

Out[i]  1;

 Fig 4. Procedure decoding of SCL

The SC decoding algorithm is optimal for infinite code lengths, but its error-correction performance degrades quickly at moderate and short code lengths [4] used, usually, in more applications. In its original formulation, it also suffers from long decoding latency.

The SC decoding algorithm have also another drawback, because the decoding process uses the old value of the estimated vector in order to make decision for the next estimated vectors, if we have error in one decision element, this error reverberate in the all estimation. To resolve this problem other proposed decoding algorithms in the literature are developed based on SC. Among these methods, we found the SCL (Successive Cancelation List) algorithm [6]. This algorithm was introduced by I. Tal et al. [6] to increasing the performance and reliability of polar codes despite the use of the most computing resources.

As seen in the Fig. 4, SCL decoding procedure generates two bits 0 and 1 in each iteration of decoding and keep L best sequences that are most probable.

In order to enhance the performance of polar codes, the authors of [7] have proposed an adaptive SCL decoder for with CRC (Cyclic Redundancy Check) algorithm. This concatenation was called CRC-aided SCL and uses the CRC to choosing the best codeword decoding by SCL.

If the CRC of the L (L fixed) most probable codewords is verified, the chosen codeword will be the one with the highest probability. In the other hand, if the CRC is not verified, the value of L are doubled by 2 (2\*L) and the CRC-aided SCL decoding are repeated.

**Proposed CRC-aided SCL algorithm:**

We propose here the algorithm of the original CRC-aided SCL proposed in [7] as shown in algorithm 4.

|  |
| --- |
| **Algorithm 4: CRC-aided SCL** |

// Lmax is fixed and l<Lmax

1 SCL with l

2 IF (CRC is verified)

3 The codewords is the most probable

4 Else

5 IF(2\*L<Lmax)

6 L=2\*L

7 Go to 1.

8 Else

9 The codewords is the most probable

Due to its simplicity, we are interested, in this first work, by the implementation of the SC algorithm. The simulation results will be presented in the following section.

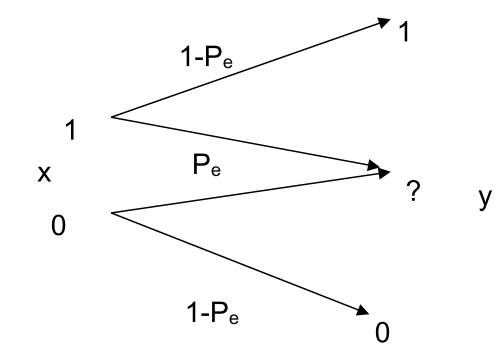


Fig. 5. Model of BEC Channel

# Simulation Results

In this simulation the W channel used is a BEC (Binary Erasure Channel) which is a common communications channel model used in coding and information theories. Its model is illustrated by the Fig 5. The transmitter sends a bit X (0 or 1) as input of the BEC channel, and the receiver takes the output of the BEC channel Y which can be a bit (0 or 1) or “?“ (that represents that the bit was erased), respectively with the probabilities and  (Probability of erasure).

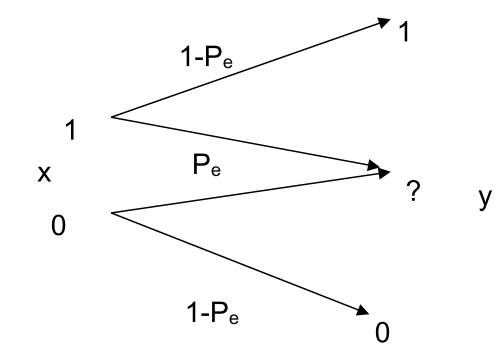


Fig. 5. Model of BEC Channel

Each channel created by channel polarisation is a BEC with erasure probability that can be computed by the recursion [8].

=2-

=, =

Z(W)= ;

We use here the polar code (8, 4) and the probability of erasure.=0.2.

The coding procedure of the polar code (8, 4) is composed by two blocks: the first one compute and choose the k most reliable positions in which the information bits are placed, by computing Bhattacharya parameter and the second one is the coding module. The k positions of information bits are corresponding to k small values of error-prone computed by the Bhattacharya parameter. According to the equation (17) shown by the work proposed in [8] to define the Bhattacharya parameter for the BEC, the computing of each will be as shown below.

Z()= 0.000003; Z()= 0.003197; Z()= 0.006147; Z()=0.150653 ; Z()=0.016796 ; Z()=0.242404 ; Z()=0.348572 and Z()=0.832228

In this case, the computed positions of information bits are {8, 7, 6, 4}. For example for source block information u={1,1,1,1}, uAc={0,0,0,0} the corresponding information vector will be (0,0,0,1,0,1,1,1). By using the algorithm 2 the vector X will be computed and its value is (0,1,1,0,0,1,1,1). This vector will be transmitted by virtual channels to carry out the vector Y. This transmission affected by the noise of the channel. The vector Y becomes the input to decoding module. We applied the algorithm 3 and the computed result will be (0, 0, 0, 1, 0, 1, 1, 1) which turn equal the vector information U**.**

**Fig 6 and Fig 7** present respectively the performance in term of BER (Bit Error Rate) and FER (Frame Error Rate) of polar coding for (N=8 ,K=4 ) and (N=512 ,K=256 ,) respectively by using a BEC

These figures show that BER and FER increased with erasure probability.

We show also in Tab 1 that BER are null for 3 Where N=8 also BER are null for N=2048 and that prouve the theorem of polar coding

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Fig. 6. Performance in terms of BER and FER of Polar code (8, 4)



Fig. 7. Performance in terms of BER and FER of Polar code (2048, 1024)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| BER( | 0.01 | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 | 0.25 | 0.29 | 0.31 |
| N=2048 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0,001 |
| N=8 | 0 | 0 | 0 | 0 | 0,003 | 0,007 | 0,017 | 0,0276 | 0,042 |

Tab BER for N=2048 and N=1024

# Conclusion

In this work, we have presented an overview about the polar codes and their characteristics to achieve the capacity of channel. The coding and decoding algorithms their implementations are investigated. The obtained and presented simulation results are promising. They will be a basis for the proposition of other high-performance algorithms which make it possible to overcome certain defects of the coding and decoding algorithms for polar codes.

As shown in the literature polar codes will find more deployment in many other applications and will be included in other new standards in the future. For this reason, in future work we will try to analyze the performance of these codes and implement them in hardware to meet the requirement of 5G.

Our study is specifically limited for binary channels, so polar codes for non-binary channel is interesting. The development of coding and decoding algorithms with more performance is another subject for future work.

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