

NØthing is Logical

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Slides: <https://www.marialoni.org/resources/Oxford25.pdf>

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NØthing is logical (Nihil)

- Goal of the project: a formal account of a class of natural language inferences which deviate from classical logic
- Common assumption: these deviations are not logical mistakes, but consequence of pragmatic enrichments (Grice)
- Strategy: develop *logics of conversation* which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency (a cognitive bias rather than a conversational principle) as crucial factor
- Main conclusion: deviations from classical logic consequence of enrichments albeit not (always) of the canonical Gricean kind



KONINKLIJKE NEDERLANDSE
AKADEMIE VAN WETENSCHAPPEN

Non-classical inferences

Free choice (FC)

- (1) FC: $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [von Wright 1968]
- (2) Deontic FC inference [Kamp 1973]
- You may go to the beach *or* to the cinema.
 - \rightsquigarrow You may go to the beach *and* you may go to the cinema.
- (3) Epistemic FC inference [Zimmermann 2000]
- Mr. X might be in Victoria *or* in Brixton.
 - \rightsquigarrow Mr. X might be in Victoria *and* he might be in Brixton.

Ignorance

- (4) The prize is either in the garden *or* in the attic \rightsquigarrow speaker doesn't know which [Grice 1989, p.45]
- (5) ? I have two *or* three children.

- In the standard approach, **ignorance** is a conversational implicature
- Less consensus on **FC** inference analysed as conversational implicature; grammatical (scalar) implicature; semantic entailment; ...

The challenge of **FC**: adding FC to classical modal logic implies the equivalence of any two possibility claims

$$\Diamond a \Rightarrow_{CML} \Diamond(a \vee b) \Rightarrow_{FC} \Diamond b$$

Novel hypothesis: neglect-zero

- FC and ignorance inferences are
 - Not the result of Gricean reasoning
 - Not the effect of applications of covert grammatical operators
- [\neq semantic entailments]
[\neq conversational implicatures]
[\neq grammatical (scalar) implicatures]
- They are rather a consequence of something else speakers do in conversation, namely,

NEGLECT-ZERO

when interpreting a sentence speakers construct models depicting reality (some verifying the sentence, some falsifying it) \mapsto common assumption¹ and in this process tend to neglect models that verify the sentence by virtue of an empty configuration (*zero-models*) \mapsto novel hypothesis

- Tendency to neglect zero-models follows from the cognitive difficulty of:
 - ① conceiving emptiness, the absence of things rather than their presence
 - ② evaluating truths with respect to empty witness sets

[Nieder 2016; Bott et al 2019]

¹Johnson-Laird (1983) *Mental Models*. Cambridge University Press.

Novel hypothesis: neglect-zero

Illustration

(6) Less than three squares are black.

a. Verifier: [■, □, ■]

b. Falsifier: [■, ■, ■]

c. Zero-models: [□, □, □]; [■, ■, ■]; [△, △, △]; [▲, ▲, ▲]; ...

Zero-models in (6-c) verify the sentence by virtue of an empty set of black squares

- Cognitive difficulty of zero-models confirmed by experimental findings and connected to / can be argued to explain:
 - the special status of 0 among the natural numbers [Nieder 2016]
 - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (*less* vs *more*) [Bott et al 2019]
- Core idea: neglect-zero also at the origin of many common departures from classical reasoning
 - FC and ignorance [MA 2022]
 - Existential Import: every A is B \Rightarrow some A is B
 - Aristotle's Thesis: if not A then A $\Rightarrow \perp$
 - Boethius' Thesis: if A then B & if A then not B $\Rightarrow \perp$
[Ziegler, Knudstorp & MA 2025]

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

(7) Maria ate an apple.

a. Verifier: [🍎]

b. Falsifiers: [🍌]; [🍏]; []

c. Zero-models: none

(8) Maria ate a banana.

a. Verifier: [🍌]

b. Falsifiers: [🍎]; [🍏]; []

c. Zero-models: none

(9) M ate an apple and a banana.

a. Verifier: [🍎 🍌]

b. Falsifiers: [🍏]; []

c. Zero-models: none

(10)

M ate an apple or a banana.

a. Verifier: ?

b. Falsifiers: [🍏]; []

c. Zero-models: ?

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

- | | |
|---------------------------------|---------------------------------|
| (11) Maria ate an apple. | (12) Maria ate a banana. |
| a. Verifier: [🍎] | a. Verifier: [🍌] |
| b. Falsifiers: [🍌]; [🥝]; [] | b. Falsifiers: [🍎]; [🥝]; [] |
| c. Zero-models: none | c. Zero-models: none |
-
- | | |
|--------------------------------------|-------------------------------------|
| (13) M ate an apple and a banana. | (14) M ate an apple or a banana. |
| a. Verifier: [🍎 🍌] | a. Verifier: ? |
| b. Falsifiers: [🥝]; [] | b. Falsifiers: [🥝]; [] |
| c. Zero-models: none | c. Zero-models: [🍎]; [🍌] |
-
- Two **zero-models** in (14-c): verify the sentence by virtue of an empty witness for one of the disjuncts

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

(15) Maria ate an apple.

a. Verifier: [apple]

b. Falsifiers: [banana]; [lime]; []

c. Zero-models: none

(16) Maria ate a banana.

a. Verifier: [banana]

b. Falsifiers: [apple]; [lime]; []

c. Zero-models: none

(17) M ate an apple and a banana. (18)

a. Verifier: [apple banana]

b. Falsifiers: [lime]; []

c. Zero-models: none

M ate an apple or a banana.

a. Verifier: [apple | banana] \Leftarrow 'split'

b. Falsifiers: [lime]; []

c. **Zero-models:** [apple]; [banana]

- Two **zero-models** in (18-c): verify the sentence by virtue of an empty witness for one of the disjuncts
- **Split state** in (18-a): simultaneously entertains different (possibly conflicting) alternatives
- **Neglect-zero hypothesis:** ignorance and FC arise because split states emerge as natural verifiers for disjunctions since zero-models, where only one of the disjuncts is depicted, are cognitively taxing and therefore kept out of consideration

A new conjecture: no-split

- (19) Maria ate an apple or a banana.

a. Verifier: [🍎 | 🍌]

[← split state]

b. Falsifiers: [🍏]; []

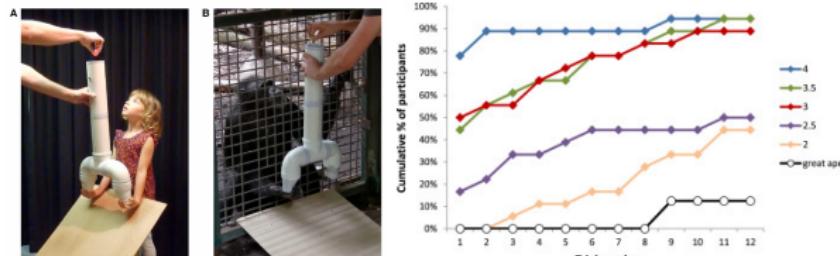
c. Zero-models: [🍎]; [🍌]

- **Split states:** multiple alternative possibilities processed in a parallel fashion ↫ also a cognitively taxing operation

NO-SPLIT CONJECTURE

[Klochowicz, Sbardolini & MA, SuB 2025]

the ability to split states (entertain multiple possibilities) is developed late



Children have trouble conceiving multiple possibilities [Redshaw & Suddendorf 2016]

- Combination of neglect-zero + no-split can explain non-classical inferences observed in pre-school children

A new conjecture: no-split

- Pre-school children sometimes (but systematically) interpret disjunctions conjunctively [Singh et al 2016; Cochard 2023; Bleotu et al 2024]

(20) M ate an apple or a banana = M ate an apple and a banana
 $(\alpha \vee \beta) \equiv (\alpha \wedge \beta)$

(21) M can eat an apple or a banana = M can eat an apple and a banana
 $\Diamond(\alpha \vee \beta) \equiv \Diamond(\alpha \wedge \beta)$

(22) M didn't eat an apple or/and a banana = M neither ate an apple nor a banana
 $\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \equiv \neg(\alpha \wedge \beta)$

- Proposal:** children have conjunctive readings as they (similarly to adults) neglect zero and, unlike adults, do not have the ability to split

① Deriving ignorance:

Apple OR banana \Rightarrow_{NZ} + \Rightarrow_{SPLIT} |

~ It might be an apple and it might be a banana

(adults)

② Deriving conjunctive reading:

Apple OR banana \Rightarrow_{NZ} + $\Rightarrow_{NO-SPLIT}$

~ Both an apple and a banana

(children)

③ In case of incompatible alternatives:

[Leahy & Carey 2020]

Left OR right \Rightarrow_{NZ} + $\Rightarrow_{NO-SPLIT}$ contradiction (\perp)

~ Random singular guess

(children)

Cognitive bias approach

Common assumption: Reasoning and understanding of natural language involve the creation of mental models [Johnson-Laird 1983]

- **Understanding** a sentence S means being able to mentally construct a model picturing the world which verifies S, and possibly also a model which falsifies it
- **Reasoning** depends on two main processes: first construct verifying models for the premises and then check the validity of the conclusion on these models

Novel hypothesis: biases can constrain the construction of these models and therefore impact both reasoning and interpretations:

- **Neglect-zero** prevents the constructions of zero-models;
- **No-split** expresses a dispreference for split-states.

Comparison with competing accounts

	Ignorance	FC inference	Quantifiers	Scalar impl.	Conjunctive or
Neo-Gricean Grammatical Nihil	reasoning debated neglect-zero	reasoning grammatical neglect-zero	reasoning grammatical neglect-zero	reasoning grammatical —	— grammatical negl-z + no-split

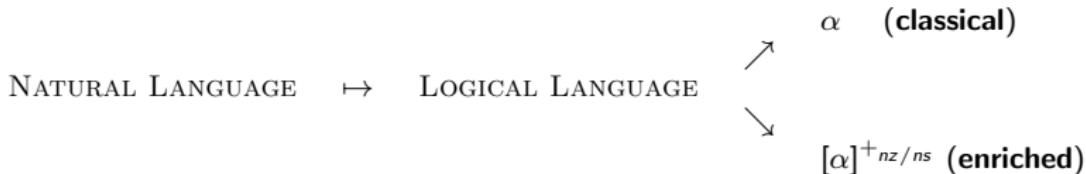
NEXT

- Logical modelling
- Experimental findings

Modelling biases in team semantics

General methodology

Natural language sentences translated into classical logic formulas interpreted in a **team semantics** which models both classical and enriched interpretations



Back to FC challenge

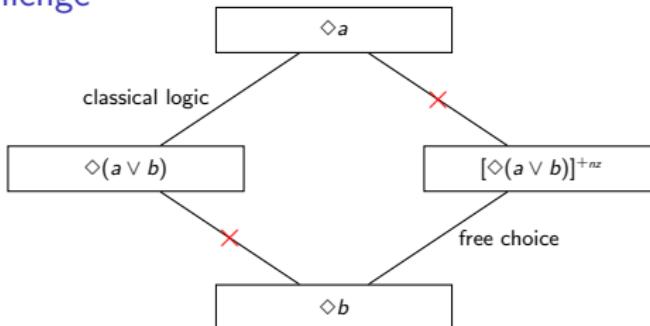


Figure: FC derived only for NZ enriched interpretations

Modelling biases in team semantics

Team semantics

- Formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Hodges 1997; Väänänen 2007; Yang & Väänänen 2017]

- Classical modal logic: $[M = \langle W, R, V \rangle]$

$$M, w \models \phi, \text{ where } w \in W$$

- Team-based modal logic:

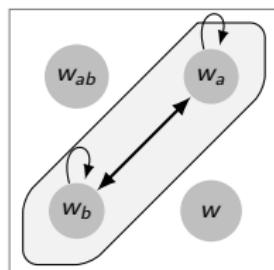
$$M, t \models \phi, \text{ where } t \subseteq W$$

- Two crucial features

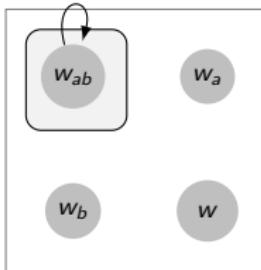
- The empty set is among the possible teams ($\emptyset \subseteq W$) \mapsto zero-models
- Multi-membered teams can model parallel processing of alternatives \mapsto split states

Illustrations

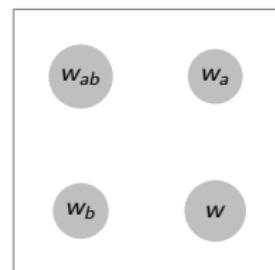
$$[W = \{w_{ab}, w_a, w_b, w\}]$$



(a) split



(b) no-split



(c) empty team

Modelling biases in team semantics

Team semantics

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- Classical modal logic: $[M = \langle W, R, V \rangle]$

$$M, w \models \phi, \text{ where } w \in W$$

- Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

- Two crucial features

- The empty set is among the possible teams ($\emptyset \subseteq W$) \mapsto zero-models
- Multi-membered teams can model parallel processing of alternatives \mapsto split states

Neglect-zero & no-split

- Neglect-zero modelled via **non-emptiness atom** NE which disallows empty teams as possible verifiers

$$M, t \models \text{NE} \text{ iff } t \neq \emptyset$$

- No-split modelled via **flattening operator** F which induces pointwise evaluations and therefore avoids simultaneous processing of alternatives

$$M, t \models F\phi \text{ iff for all } w \in t : M, \{w\} \models \phi$$

BSML: Classical Modal Logic + NE

Language

$$\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \text{NE}$$

Bilateral team semantics

Given a Kripke model $M = \langle W, R, V \rangle$ & states $s, t, t' \subseteq W$

$M, s \models p$ iff for all $w \in s : V(w, p) = 1$

$M, s \models p$ iff for all $w \in s : V(w, p) = 0$

$M, s \models \neg\phi$ iff $M, s \models \phi$

$M, s \models \neg\phi$ iff $M, s \models \phi$

$M, s \models \phi \vee \psi$ iff there are $t, t' : t \cup t' = s$ & $M, t \models \phi$ & $M, t' \models \psi$ ↖

$M, s \models \phi \vee \psi$ iff $M, s \models \phi$ & $M, s \models \psi$

$M, s \models \phi \wedge \psi$ iff $M, s \models \phi$ & $M, s \models \psi$

$M, s \models \phi \wedge \psi$ iff there are $t, t' : t \cup t' = s$ & $M, t \models \phi$ & $M, t' \models \psi$

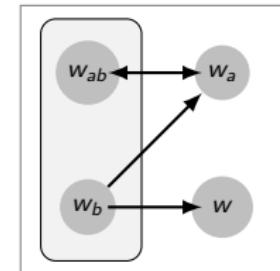
$M, s \models \Diamond\phi$ iff for all $w \in s : \exists t \subseteq R[w] : t \neq \emptyset$ & $M, t \models \phi$

$M, s \models \Diamond\phi$ iff for all $w \in s : M, R[w] \models \phi$ [where $R[w] = \{v \in W \mid wRv\}$]

$M, s \models \text{NE}$ iff $s \neq \emptyset$

$M, s \models \text{NE}$ iff $s = \emptyset$

Entailment: $\phi_1, \dots, \phi_n \models \psi$ iff for all M, s : $M, s \models \phi_1, \dots, M, s \models \phi_n \Rightarrow M, s \models \psi$



Proof Theory: MA, Anttila & Yang (2024); **Expressive completeness:** Anttila & Knudstorp (2025);

Comparisons via translation into Modal Information Logic: Knudstorp et al (2025)

Neglect-zero effects in BSML: split disjunction

- A state s supports a **disjunction** ($\alpha \vee \beta$) iff s is the union of two substates, each supporting one of the disjuncts

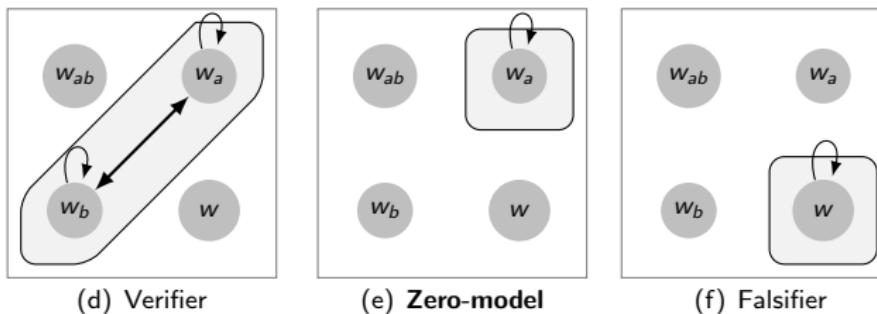


Figure: Models for $(a \vee b)$.

- $\{w_a\}$ verifies $(a \vee b)$ by virtue of an empty witness for the second disjunct,
 $\{w_a\} = \{w_a\} \cup \emptyset \& M, \emptyset \models b$ [\mapsto zero-model]
 - **Main idea:** define neglect-zero enrichments, $[]^+$, whose core effect is to rule out such zero-models
 - **Implementation:** $[]^+$ defined using NE ($s \models \text{NE}$ iff $s \neq \emptyset$), which models neglect-zero in the logic

BSML: neglect-zero enrichment

Neglect-zero enrichment

For NE-free α , $[\alpha]^+$ defined as follows:

$$\begin{aligned} [p]^+ &= p \wedge \text{NE} \\ [\neg\alpha]^+ &= \neg[\alpha]^+ \wedge \text{NE} \\ [\alpha \vee \beta]^+ &= ([\alpha]^+ \vee [\beta]^+) \wedge \text{NE} \\ [\alpha \wedge \beta]^+ &= ([\alpha]^+ \wedge [\beta]^+) \wedge \text{NE} \\ [\diamond\alpha]^+ &= \diamond[\alpha]^+ \wedge \text{NE} \end{aligned}$$

$[]^+$ enriches formulas with the requirement to satisfy NE distributed along each of their subformulas

Formal characterization of neglect-zero effects

$\alpha \rightsquigarrow_{nz} \beta$ (β is a neglect-zero effect of α) iff $\alpha \not\models \beta$ but $[\alpha]^+ \models \beta$

Formal characterization zero and no-zero models

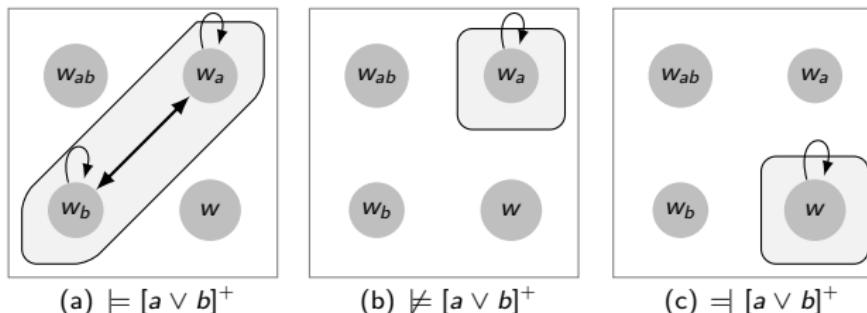
(M, s) is a **zero-model** for α iff $M, s \models \alpha$, but $M, s \not\models [\alpha]^+$

(M, s) is a **no-zero verifier** for α iff $M, s \models [\alpha]^+$

Neglect-zero effects in BSML: enriched disjunction

- s supports an **enriched disjunction** $[\alpha \vee \beta]^+$ iff s is the union of two **non-empty** substates, each supporting one of the disjuncts

$$[\alpha \vee \beta]^+ = (\alpha \wedge \text{NE}) \vee (\beta \wedge \text{NE}) \wedge \text{NE}$$



- An enriched disjunction requires both disjuncts to be live possibilities

[Zimmermann 2000]

(23) It is raining or snowing \rightsquigarrow_{nz} It might be raining and it might be snowing
 $[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$ (where R is state-based)

- Main result:** in BSML $[\]^+$ -enrichment has non-trivial effect only when applied to *positive* disjunctions

[MA 2022]

- we derive FC and related effects (for enriched formulas);
- $[\]^+$ -enrichment vacuous under single negation.

Neglect-zero effects in BSML: main results

After enrichment

- We derive both narrow and wide scope FC inferences:
 - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
 - Universal FC: $[\forall x \Diamond(\alpha \vee \beta)]^+ \models \forall x (\Diamond\alpha \wedge \Diamond\beta)$ (in qBSML)
 - Double negation FC: $[\neg\neg\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
 - Wide scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ (if R is indisputable)
- while no undesirable side effects obtain with other configurations:
 - Double prohibition: $[\neg\Diamond(\alpha \vee \beta)]^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Before enrichment

- The NE-free fragment of BSML is equivalent to classical modal logic:

$$\alpha \models_{BSML} \beta \text{ iff } \alpha \models_{CML} \beta \quad [\text{if } \alpha, \beta \text{ are NE-free}]$$

- But we can capture the infelicity of **epistemic contradictions** [Yalcin, 2007] by putting team-based constraints on the accessibility relation:
 - ① Epistemic contradiction: $\Diamond_e \alpha \wedge \neg\alpha \models \perp$ (if R is state-based)
 - ② Non-factivity: $\Diamond_e \alpha \not\models \alpha$
- Differences between modalities captured via constraints on R :
 - R **state-based** in (M, s) iff all and only worlds in s are accessible within s [\mapsto **epistemics** (always)]
 - R **indisputable** in (M, s) iff all worlds in s access exactly the same set of worlds [\mapsto **deontics** (sometimes)]

The data

(24) **Double Prohibition** [Alonso-Ovalle 2006, Marty *et al.* 2021]

- a. You are not allowed to eat the cake or the ice-cream \rightsquigarrow You are not allowed to eat either one
- b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$

(25) **Universal FC** [Chemla 2009]

- a. All of the boys may go to the beach or to the cinema.
 \rightsquigarrow All of the boys may go to the beach and all of the boys may go to the cinema.
- b. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$

(26) **Double Negation FC** [Gotzner *et al.* 2020]

- a. Exactly one girl cannot take Spanish or Calculus \rightsquigarrow One girl can take neither of the two and each of the others can choose between them.
- b. $\exists x(\neg\Diamond(\alpha(x) \vee \beta(x)) \wedge \forall y(y \neq x \rightarrow \neg\neg\Diamond(\alpha(y) \vee \beta(y)))) \rightsquigarrow$
 $\exists x(\neg\Diamond\alpha(x) \wedge \neg\Diamond\beta(x) \wedge \forall y(y \neq x \rightarrow (\Diamond\alpha(y) \wedge \Diamond\beta(y))))$

(27) **Wide Scope FC** [Zimmermann 2000, Cremers *et al.* 2017]

- a. Detectives may go by bus or they may go by boat \rightsquigarrow Detectives may go by bus and may go by boat
- b. Mr. X might be in Victoria or he might be in Brixton \rightsquigarrow Mr. X might be in Victoria and might be in Brixton
- c. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

(28) **FC cancellation** [sluice indicates wide scope disjunction]

- a. Detectives may go by bus or by boat, I don't know which $\not\rightsquigarrow$ Detectives may go by bus and may go by boat
- b. $\Diamond\alpha \vee \Diamond\beta \not\rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ (*R* not indisputable)

Experimental findings

Cognitive bias view

Many non-classical inferences prominently explained by neo-Gricean or grammatical mechanisms are instead consequence of a neglect-zero (+ no-split) tendency

Comparison with competing accounts²

	Ignorance	FC inference	Quantifiers	Scalar impl.	Conjunctive or
Neo-Gricean Grammatical Nihil	reasoning debated neglect-zero	reasoning grammatical neglect-zero	reasoning grammatical neglect-zero	reasoning grammatical —	— grammatical negl-z + no-split

Recent experiments

- Degano *et al*, NLS, 2025: on **ignorance inference** 
- Klochowicz *et al* (SuB24, CogSci25, SuB25): on **disjunction & quantifiers**
- Bleotu *et al* (TbiLLC 2025): on conjunctive *or*
- ...

²Neo-Gricean: Horn, Soames, Sauerland, ... Grammatical view: Chierchia, Fox, Singh *et al*, ...

Neglect-zero effects on plain disjunction

Enriched meanings for disjunction

- (29) Maria ate an apple or a banana \rightsquigarrow
- a. **Scalar implicature:** not both $(\alpha \vee \beta)$
 - b. **Conjunctive interpretation:** both $\neg(\alpha \wedge \beta)$
 - c. **Ignorance:** speaker doesn't know which $(\alpha \wedge \beta)$?

Two components of ignorance: possibility vs uncertainty

- (30) Maria ate an apple or a banana \rightsquigarrow speaker doesn't know which
[Degano et al 2025]³
- a. **Possibility:** $\Diamond_e \alpha \wedge \Diamond_e \beta$
It is possible that M ate an apple and it is possible that M ate a banana
 - b. **Uncertainty:** $\neg \Box_e \alpha \wedge \neg \Box_e \beta$
It is uncertain that M ate an apple and it is uncertain that M ate a banana

³Degano, Marty, Ramotowska, MA, Breheny, Romoli, Sudo. "The ups and downs of ignorance." *Nat Lang Sem*, 2025.

Neglect-zero effects on disjunction: predictions of BSML

Many no-zero verifiers for enriched disjunction

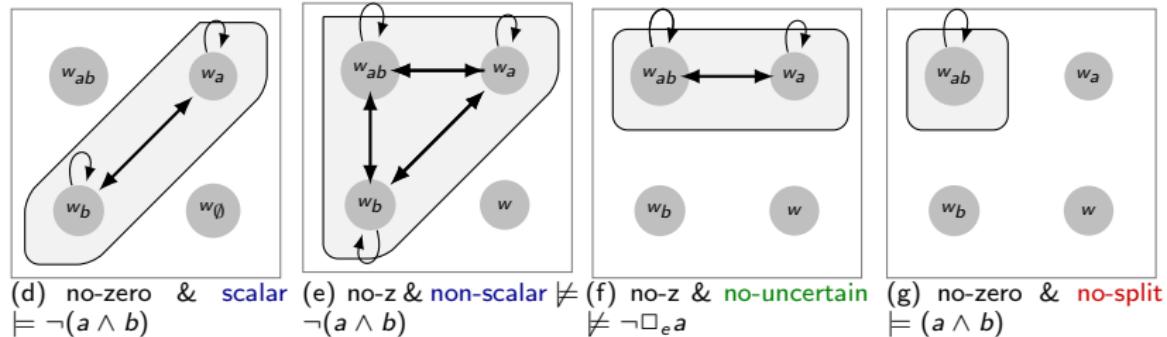


Figure: Models for enriched $[a \vee b]^+$.

- ① Neglect-zero enrichment does not derive **scalar implicatures**;
- ② Neglect-zero enrichment does not derive **uncertain inferences** \mapsto in contrast to standard neo-Gricean approach to ignorance ⇐
- ③ **No-split** verifiers compatible with neglect-zero enrichments
 - **No-split** conjecture: only **no-split** verifiers accessible to ‘conjunctive’ pre-school children. [Klochowicz, Sbardolini, MA, SuB, 2025]

Two derivations of full ignorance

① Standard neo-Gricean derivation

[Sauerland 2004]

(i) Uncertainty derived through **quantity** reasoning

$$(31) \quad \alpha \vee \beta \qquad \text{ASSERTION}$$

$$(32) \quad \neg \Box_e \alpha \wedge \neg \Box_e \beta \qquad \text{UNCERTAINTY (from QUANTITY)}$$

(ii) Possibility derived from uncertainty and **quality** about assertion

$$(33) \quad \Box_e(\alpha \vee \beta) \qquad \text{QUALITY ABOUT ASSERTION}$$

$$(34) \quad \Rightarrow \Diamond_e \alpha \wedge \Diamond_e \beta \qquad \text{POSSIBILITY}$$

② Neglect-zero derivation

(i) Possibility derived as **neglect-zero** effect

$$(35) \quad \alpha \vee \beta \qquad \text{ASSERTION}$$

$$(36) \quad \Diamond_e \alpha \wedge \Diamond_e \beta \qquad \text{POSSIBILITY (from NEGLECT-ZERO)}$$

(ii) Uncertainty derived from possibility and **scalar reasoning**

$$(37) \quad \neg(\alpha \wedge \beta) \qquad \text{SCALAR IMPLICATURE}$$

$$(38) \quad \Rightarrow \neg \Box_e \alpha \wedge \neg \Box_e \beta \qquad \text{UNCERTAINTY}$$

Neo-Gricean vs neglect-zero explanation

Contrasting predictions of competing accounts of ignorance

- Neo-Gricean: No possibility without uncertainty
- Neglect-zero: Possibility derived independently from uncertainty

Experimental findings

[Degano et al 2025]

- Using adapted mystery box paradigm, compared conditions in which
 - both uncertainty and possibility are false [zero-model]
 - uncertainty false but possibility true [no-zero, no-uncertain model]
- Less acceptance when possibility is false (95% vs 44%)
- ⇒ Evidence that possibility can arise without uncertainty
- A challenge for the traditional neo-Gricean approach

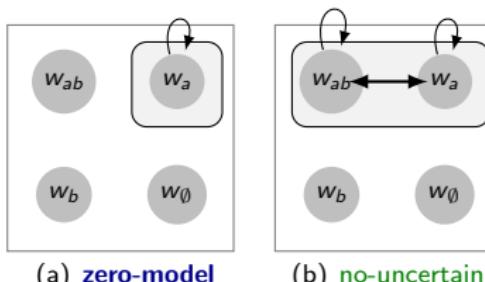


Figure: Models for $(a \vee b)$

Conclusions

- FC, possibility: a mismatch between logic and language
- Grice's insight:
 - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Nihil proposal:
some non-classical inferences due to cognitive bias rather than Gricean reasoning
 - FC, possibility and related inferences as neglect-zero effects
 $\text{Literal meanings (classical fragment)} + \text{cognitive factor (NE)} \Rightarrow \text{FC, possibility, etc}$
 - Conjunctive *or* as no-zero + no-split effect
 $\text{Literal meanings (classical fragment)} + \text{cognitive factors (NE, F)} \Rightarrow \text{conjunctive or}$
- Implementation in (extensions of) BSML, a team-based modal logic
- Recent experiments provide some first tentative evidence in agreement with the neglect-zero hypothesis
- Appendix:
 - Experimenting with disjunction and quantifiers
 - Comparison via translation into Modal Information Logic

Collaborators & related (future) research



Anttila



Degano



Klochowicz



Knudstorp



Ramotowska



Zhou

& many more ...

Logic

Proof theory ([Anttila, Yang](#)); expressive completeness ([Anttila, Knudstorp](#)); bimodal perspective ([Knudstorp, Baltag, van Benthem, Bezhanishvili](#)); qBSML ([van Ormondt](#)); BiUS & qBiUS ([MA](#)); typed BSML ([Muskens](#)); connexive logic ([Knudstorp, Ziegler & MA](#)); belief revision ([Klochowicz](#))

Language

FC cancellations ([Pinton, Hui](#)); modified numerals ([vOrmondt](#)); attitude verbs ([Yan](#)); conditionals ([Flachs, Ziegler](#)); questions ([Klochowicz](#)); quantifiers ([Klochowicz, Bott, Schlotterbeck](#)); indefinites ([Degano](#)); homogeneity ([Sbardolini](#)); acquisition ([Klochowicz, Sbardolini](#)); experiments ([Degano, Klochowicz, Ramotowska, Bott, Schlotterbeck, Marty, Breheny, Romoli, Sudo, Spychalska, Szymanik, Visser](#)); ...

THANK YOU!⁴

⁴This work is supported by NWO OC project *Nothing is Logical* (grant no 406.21.CTW.023).

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Neglect-zero effects on quantifiers: Empty Set (ES) inferences

Predictions of qBSML^{→5}

(39) Less than three squares are black $\mapsto \forall xyz((Sx \wedge Bx \wedge \dots) \rightarrow (x = y \vee \dots))$

- a. Verifier: [■, □, ■]
- b. Falsifier: [■, ■, ■]
- c. Zero-models: [□, □, □]; [▲, ▲, ▲]; ... \rightsquigarrow_{nz} there are black squares

(40) Every square is black. $\mapsto \forall x(Sx \rightarrow Bx)$

- a. Verifier: [■, ■, ■]
- b. Falsifier: [■, □, ■]
- c. Zero-models: [△, △, △]; [▲, ▲, ▲]; ... \rightsquigarrow_{nz} there are squares

(41) No squares are black. \mapsto (i) $\forall x(Sx \rightarrow \neg Bx)$; (ii) $\neg \exists x(Sx \wedge Bx)$

- a. Verifier: [□, □, □]
- b. Falsifier: [■, □, □]
- c. Zero-models for (i): [△, △, △]; [▲, ▲, ▲]; ... \rightsquigarrow_{nz} there are squares
- d. Zero-models for (ii): none no neglect-zero effect

(42) Every square is red or white. $\mapsto \forall x(Sx \rightarrow (Rx \vee Wx))$

- a. Verifier: [■, □, ■]
- b. Falsifier: [■, □, ■]
- c. Zero-models: [■, ■, ■]; [□, □, □]; ... \rightsquigarrow_{nz} there are white & red squares

These predictions tested in Bott, Klochowicz, Schlotterbeck et al (2024, 2025)

⁵MA & vOrmondt, Modified numerals and split disjunction. *J of Log Lang and Inf* (2023).

Experimenting with quantifiers and disjunction

Four non-classical interpretations

- (43)
- a. Some of the squares are black \Rightarrow not all of the squares are black [scalar UB]
 - b. Each square is red or white \Rightarrow there are white squares and red squares [DIST]
 - c. Less than 3 squares are black \Rightarrow there are some black squares [ES-scope]
 - d. Less than 3/every/no squares are black \Rightarrow there are some squares [ES-restrictor]

Three competing accounts

	UB	DIST	ES-scope	ES-restrictor
Alternative-based	implicature	implicature	implicature	implicature
Bott <i>et al.</i> , 2019	—	—	neglect-zero	presupposition
Nihil	—	neglect-zero	neglect-zero	neglect-zero

Two experiments

- Exp 1: Answering questions about the emptyset (O. Bott *et al.*, SuB 2024)
- Exp 2: Priming with zero-models (Klochowicz, Schlotterbeck *et al.*, CogSci 2025, SuB 2025)

Three main conclusions

- ① Evidence that ES-restrictor is a presupposition (Exp 1)
- ② Evidence that UB differs from both ES-scope and DIST (Exp1 and Exp2)
- ③ Some evidence that ES-scope and DIST involve the same cognitive process (Exp 2)

Experimenting with quantifiers and disjunction

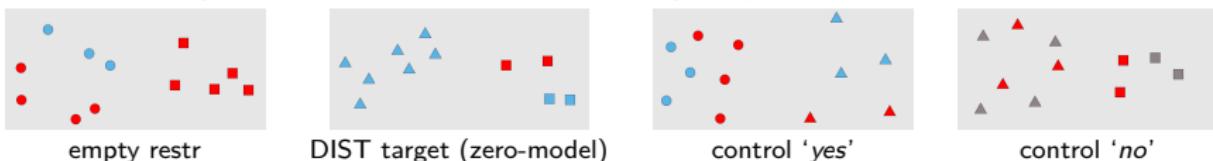
Non-classical interpretations

- (44)
- a. Some of the squares are black \Rightarrow not all of the squares are black [UB]
 - b. Each square is red or white \Rightarrow there are white squares and red squares [DIST]
 - c. Less than 3 squares are black \Rightarrow there are some black squares [ES-scope]
 - d. Less than 3/every/no squares are black \Rightarrow there are some squares [ES-restrictor]

Exp1: Bott et al, SuB 2024

- Question-answer task:

- (45) Ist jedes Dreieck entweder rot oder blau? Ja/Nein/Komische Frage
 (Is every triangle either red or blue?) Yes/No/Odd question



- Main results:

- ① Evidence that ES-restrictor is a presupposition: questions in empty restrictor models uniformly perceived as odd
- ② ES-scope (37%) and DIST (23%) unaffected by question environment; UB much less available (10%, while 40% when unembedded)
- ③ Inconclusive evidence on whether ES-scope and DIST had the same source

Experimenting with quantifiers and disjunction

Non-classical interpretations

- (46) a. Some of the squares are black \Rightarrow not all of the squares are black [UB/scalar]
 b. Each square is red or white \Rightarrow there are white and red squares [DIST]
 c. At most 2 squares are black \Rightarrow there are some black squares [ES-scope, sup]
 d. Less than 3 squares are black \Rightarrow there are some black squares [ES-scope, comp]

Two competing accounts

	UB	DIST	ES-scope
Alternative-based Nihil ($qBSML^{\rightarrow}$)	implicature —	implicature neglect-zero	implicature neglect-zero

Exp2: Klochowicz, Schlotterbeck *et al*, CogSci 2025, SuB 2025

- Tested whether frequency of enrichment in (46-d) changed after participants were primed to suspend other enrichments in (46-a-c):
 - UB \Rightarrow ES-scope[c]; DIST \Rightarrow ES-scope[c]; ES-scope[s] \Rightarrow ES-scope[c]
- Results:
 - ① Semantic priming between DIST and ES-scope (comp)
 - ② No priming between UB and ES-scope (comp)
 - ③ No trial-to-trial priming from ES-scope (sup) to ES-scope (comp) but spill-over and adaptation effects
- Tentative conclusion: ES-scope and DIST (but not UB) involve the same cognitive process, as predicted by neglect-zero hypothesis

BSML & related systems: information states vs possible worlds

- Failure of bivalence in BSML

$$M, s \not\models p \ \& \ M, s \not\models \neg p, \text{ for some info state } s$$

- Info states: less determinate than possible worlds
 - just like truthmakers, situations, possibilities, ...
- Technically:
 - Truthmakers/possibilities: points in a partially ordered set
 - Info states: sets of possible worlds, also elements of a partially ordered set, the Boolean lattice $\text{Pow}(W)$
- Thus systems using these structures are closely connected, although might diverge in motivation:
 - Truthmaker & possibility semantics: description of ontological structures in the world
 - BSML & inquisitive semantics: explaining patterns in inferential & communicative human activities
- NEXT:
 - Comparison via translations in Modal Information Logic [vBenthem19]

BSML & related systems: comparisons via translation

- **Modal Information Logic (MIL)** (van Benthem, 1989, 2019):⁶ common ground where related systems can be interpreted and their connections and differences can be explored
- **Goal:** translations into (extensions of) MIL of the following systems:
 - Truthmaker semantics (Fine)
 - Possibility semantics (Humberstone, Holliday)
 - BSML
 - Inquisitive semantics (Ciardelli, Groenendijk & Roelofsen)
- (cf. Gödel's (1933) translation of intuitionistic logic into modal logic)
- Here focus on propositional fragments
 - disjunction
 - negation
- (Based on work in progress with Søren B. Knudstorp, Nick Bezhanishvili, Johan van Benthem and Alexandru Baltag)

⁶Johan van Benthem (2019) Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic*.

Modal Information Logic (MIL)

Language

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \langle \text{sup} \rangle \phi \psi$$

where $p \in A$.

Models and interpretation

Formulas are interpreted on triples $M = (X, \leq, V)$ where \leq is a partial order

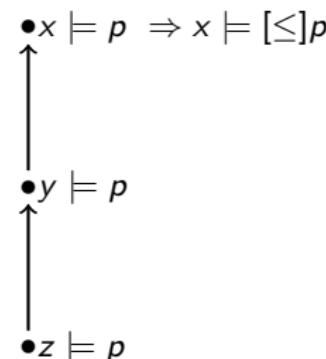
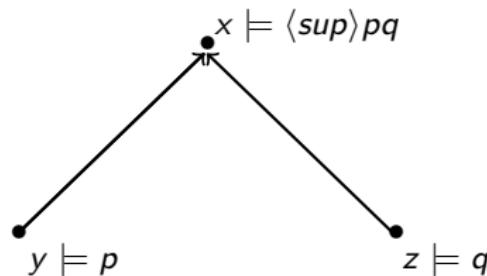
$$\begin{aligned}
 M, x \models p &\quad \text{iff} \quad x \in V(p) \\
 M, x \models \neg\phi &\quad \text{iff} \quad M, x \not\models \phi \\
 M, x \models \phi \wedge \psi &\quad \text{iff} \quad M, x \models \phi \text{ and } M, x \models \psi \\
 M, x \models \phi \vee \psi &\quad \text{iff} \quad M, x \models \phi \text{ or } M, x \models \psi \\
 M, x \models \langle \text{sup} \rangle \phi \psi &\quad \text{iff} \quad \text{there are } y, z : x = \text{sup}_{\leq}(y, z) \text{ & } M, y \models \phi \text{ & } M, z \models \psi
 \end{aligned}$$

$$[\leq]\phi = \neg\langle \text{sup} \rangle(\neg\varphi)\top$$

$$M, x \models [\leq]\phi \quad \text{iff} \quad \text{for all } y : y \leq x \Rightarrow M, y \models \phi$$

Modal Information Logic (MIL)

Examples



Translations into Modal Information Logic

- Possibility semantics (Humberstone, Holliday)⁷

$$\begin{array}{rcl} \vdots & & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & [\leq]\langle\leq\rangle(tr(\phi) \vee tr(\psi)) \\ \vdots & & \end{array}$$

- Inquisitive semantics (Groenendijk, Roelofsen and Ciardelli)

$$\begin{array}{rcl} \vdots & & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & tr(\phi) \vee tr(\psi) \\ \vdots & & \end{array}$$

⁷Johan van Benthem, Nick Bezhanishvili, Wesley H. Holliday, A bimodal perspective on possibility semantics, *Journal of Logic and Computation*, Volume 27, Issue 5, July 2017, Pages 1353–1389.

Translations into Modal Information Logic

- Truthmaker semantics (Fine)⁸

$$\begin{array}{lcl} \dots & & \\ (\neg\phi)^+ & = & (\phi)^- \\ (\neg\phi)^- & = & (\phi)^+ \\ (\phi \vee \psi)^+ & = & (\phi)^+ \vee (\psi)^+ \\ (\phi \vee \psi)^- & = & \langle \text{sup} \rangle (\phi)^- (\psi)^- \\ (\phi \wedge \psi)^+ & = & \langle \text{sup} \rangle (\phi)^+ (\psi)^+ \\ (\phi \wedge \psi)^- & = & (\phi)^- \vee (\psi)^- \end{array}$$

- BSML

$$\begin{array}{lcl} \dots & & \\ (\neg\phi)^+ & = & (\phi)^- \\ (\neg\phi)^- & = & (\phi)^+ \\ (\phi \vee \psi)^+ & = & \langle \text{sup} \rangle (\phi)^+ (\psi)^+ \\ (\phi \vee \psi)^- & = & (\phi)^- \wedge (\psi)^- \\ (\phi \wedge \psi)^+ & = & (\phi)^+ \wedge (\psi)^+ \\ (\phi \wedge \psi)^- & = & \langle \text{sup} \rangle (\phi)^- (\psi)^- \\ \dots & & \end{array}$$

⁸van Benthem, Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic* (2019).

Disjunction and Negation

- Three notions of disjunction expressible in MIL:
 - Boolean disjunction: $\phi \vee \psi$
[classical logic, intuitionistic logic, inquisitive logic]
 - Lifted/tensor/split disjunction: $\langle sup \rangle \phi \psi$
[BSML, dependence logic, team semantics, operational semantics for Positive R]
 - Cofinal disjunction: $[co](\phi \vee \psi)$ (where $[co]\phi =: [\leq] (\leq) \phi$)
[possibility semantics, dynamic semantics]
- Three notions of negation:
 - Boolean negation: $\neg \phi$
[classical logic, ...]
 - Bilateral negation: $(\neg \phi)^+ = (\phi)^-$ & $(\neg \phi)^- = (\phi)^+$
[truthmaker semantics, BSML, ...]
 - Intuitionistic-like negation: $[\leq] \neg \phi$
[possibility semantics, inquisitive semantics, intuitionistic logic]
- Some combinations:
 - Boolean disjunction + boolean negation \mapsto classical logic
 - Boolean notions in other combinations can generate non-classicality:
 - Boolean disjunction + intuitionistic negation \mapsto intuitionistic/inquisitive logic⁹
 - Classicality also generated by non-boolean combinations:
 - Split disjunction + bilateral negation (classical fragm. BSML)
 - Cofinal disjunction and intuitionistic negation (possibility semantics)

⁹Inquisitive & intuitionistic logic: same connectives but different translations for the atoms.