N∅thing is Logical

Maria Aloni
ILLC & Philosophy
University of Amsterdam
M.D.Aloni@uva.nl

Slides: https://www.marialoni.org/resources/Oxford25.pdf

Oxford Colloquium in Linguistics and Philosophy 2025
17 October 2025

N∅thing is logical (Nihil)

- Goal of the project: a formal account of a class of natural language inferences which deviate from classical logic
- Common assumption: these deviations are not logical mistakes, but consequence of pragmatic enrichments (Grice)
- Strategy: develop *logics of conversation* which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency (a cognitive bias rather than a conversational principle) as crucial factor
- Main conclusion: deviations from classical logic consequence of enrichments albeit not (always) of the canonical Gricean kind

abc amsterdam brain & cognition

KONINKLIJKE NEDERLANDSE AKADEMIE VAN WETENSCHAPPEN

PLEXUS
PHILOSOPHICAL, LOGICAL, AND EXPERIMENTAL ROUTES TO SUBSTRUCTURALITY

NWO

Non-classical inferences

Free choice (FC)

(1) FC: $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [von Wright 1968]

(2) Deontic FC inference [Kamp 1973]

a. You may go to the beach *or* to the cinema.
b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(3) Epistemic FC inference [Zimmermann 2000]

a. Mr. X might be in Victoria *or* in Brixton.
b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

Ignorance

(4) The prize is either in the garden *or* in the attic $\rightsquigarrow$ The prize might be in the garden *and* might be in the attic

[Grice 1989, p.45]

(5) ? I have two *or* three children.

- In the standard approach, ignorance is a conversational implicature
- Less consensus on FC inferences analysed as conversational implicatures; grammatical (scalar) implicatures; semantic entailments; ...

The challenge of FC: adding FC to classical modal logic implies the equivalence of any two possibility claims

$$\Diamond a \ \Rightarrow_{\mathrm{CML}} \ \Diamond(a \vee b) \ \Rightarrow_{\mathrm{FC}} \ \Diamond b$$

Novel hypothesis: neglect-zero

- FC and ignorance inferences are [$\neq$ semantic entailments]
  - Not the result of Gricean reasoning [$\neq$ conversational implicatures]
  - Not the effect of applications of covert grammatical operators [$\neq$ grammatical (scalar) implicatures]
- They are rather a consequence of something else speakers do in conversation, namely,

  NEGLECT-ZERO
  when interpreting a sentence speakers construct models depicting reality (some verifying the sentence, some falsifying it) $\mapsto$ common assumption
  and in this process tend to neglect models that verify the sentence by virtue of an empty configuration (*zero-models*) $\mapsto$ novel hypothesis

- Tendency to neglect zero-models follows from the cognitive difficulty of:
  1. conceiving emptiness, the absence of things rather than their presence
  2. evaluating truths with respect to empty witness sets

  [Nieder 2016; Bott *et al* 2019]

Novel hypothesis: neglect-zero

Illustration

(6) Less than three squares are black.
- a. Verifier: [■, □, ■]
- b. Falsifier: [■, ■, ■]
- c. Zero-models: [□, □, □]; [■, ■, ■]; [△, △, △]; [▲, ▲, ▲]; . . .

Zero-models in (6-c) verify the sentence by virtue of an empty set of black squares

- Cognitive difficulty of zero-models confirmed by experimental findings and connected to / can be argued to explain:
  - the special status of 0 among the natural numbers [Nieder 2016]
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (*less* vs *more*) [Bott *et al* 2019]
- NZ hypothesis: neglect-zero also at the origin of many common departures from classical reasoning
  - FC and ignorance [MA 2022]
  - Existential Import: every A is B $\Rightarrow$ some A is B
  - Aristotle's Thesis: if not A then A $\Rightarrow \bot$
  - Boethius' Thesis: if A then B & if A then not B $\Rightarrow \bot$

[Ziegler, Knudstorp & MA 2025]

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

(7) Maria ate an apple.
  a. Verifier: [🍎]
  b. Falsifiers: [🍌]; [🍐]; [ ]
  c. Zero-models: none

(8) Maria ate a banana.
  a. Verifier: [🍌]
  b. Falsifiers: [🍎]; [🍐]; [ ]
  c. Zero-models: none

(9) M ate an apple and a banana.
  a. Verifier: [🍎 🍌]
  b. Falsifiers: [🍐]; [ ]
  c. Zero-models: none

(10) M ate an apple or a banana.
  a. Verifier: ?
  b. Falsifiers: [🍐]; [ ]
  c. Zero-models: ?

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

(11) Maria ate an apple.

a. Verifier: [🍎]

b. Falsifiers: [🍌]; [🍐]; [ ]

c. Zero-models: none

(12) Maria ate a banana.

a. Verifier: [🍌]

b. Falsifiers: [🍎]; [🍐]; [ ]

c. Zero-models: none

(13) M ate an apple and a banana.

a. Verifier: [🍎 🍌]

b. Falsifiers: [🍐]; [ ]

c. Zero-models: none

(14) M ate an apple or a banana.

a. Verifier: ?

b. Falsifiers: [🍐]; [ ]

c. **Zero-models**: [🍎]; [🍌]

- Two **zero-models** in (14-c): verify the sentence by virtue of an empty witness for one of the disjuncts

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

(15) Maria ate an apple.
  a. Verifier: [🍎]
  b. Falsifiers: [🍌]; [🍐]; [ ]
  c. Zero-models: none

(16) Maria ate a banana.
  a. Verifier: [🍌]
  b. Falsifiers: [🍎]; [🍐]; [ ]
  c. Zero-models: none

(17) M ate an apple and a banana.
  a. Verifier: [🍎 🍌]
  b. Falsifiers: [🍐]; [ ]
  c. Zero-models: none

(18) M ate an apple or a banana.
  a. Verifier: [🍎 | 🍌] ⇐ 'split'
  b. Falsifiers: [🍐]; [ ]
  c. **Zero-models**: [🍎]; [🍌]

- Two **zero-models** in (18-c): verify the sentence by virtue of an empty witness for one of the disjuncts
- **Split state** in (18-a): simultaneously entertains different (possibly conflicting) alternatives
- Neglect-zero hypothesis: ignorance and FC arise because split states emerge as natural verifiers for disjunctions since zero-models, where only one of the disjuncts is depicted, are cognitively taxing and therefore kept out of consideration

A new conjecture: no-split

(19) Maria ate an apple or a banana.

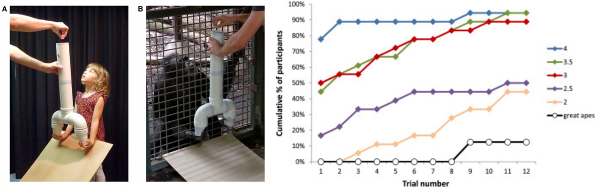
a. Verifier: [🍎 | 🍌] [⇐ **split state**]

b. Falsifiers: [🍐]; [ ]

c. **Zero-models**: [🍎]; [🍌]

- **Split states**: multiple alternative possibilities processed in a parallel fashion $\mapsto$ also a cognitively taxing operation

NO-SPLIT CONJECTURE [Klochowicz, Sbardolini & MA, SuB 2025]
the ability to split states (entertain multiple possibilities) is developed late



Children have trouble conceiving multiple possibilities [Redshaw & Suddendorf 2016]

- Combination of neglect-zero + no-split can explain non-classical inferences observed in pre-school children

A new conjecture: no-split

- Pre-school children sometimes (but systematically) interpret disjunctions conjunctively [Singh *et al* 2016; Cochard 2023; Bleotu *et al* 2024]

(20) M ate an apple or a banana = M ate an apple and a banana
$$(\alpha \vee \beta) \equiv (\alpha \wedge \beta)$$

(21) M can eat an apple or a banana = M can eat an apple and a banana
$$\Diamond(\alpha \vee \beta) \equiv \Diamond(\alpha \wedge \beta) \not\equiv \Diamond\alpha \wedge \Diamond\beta$$

(22) M didn't eat an apple or/and a banana = M neither ate an apple nor a banana
$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \equiv \neg(\alpha \wedge \beta)$$

- Proposal: children have conjunctive readings as they (similarly to adults) neglect zero and, unlike adults, do not have the ability to split

1. Deriving ignorance:

   Apple OR banana $\Rightarrow_{\text{NZ}}$ 🍎 + 🍌 $\Rightarrow_{\text{SPLIT}}$ [🍎 | 🍌]

   $\rightsquigarrow$ It might be an apple and it might be a banana (adults)

2. Deriving conjunctive reading:

   Apple OR banana $\Rightarrow_{\text{NZ}}$ 🍎 + 🍌 $\Rightarrow_{\text{NO-SPLIT}}$ [🍎 🍌]

   $\rightsquigarrow$ Both an apple and a banana (children)

3. In case of incompatible alternatives: [Leahy & Carey 2020]

   Left OR right $\Rightarrow_{\text{NZ}}$ ↙ + ↘ $\Rightarrow_{\text{NO-SPLIT}}$ **contradiction** ($\perp$)

   $\rightsquigarrow$ Random singular guess (children)

Cognitive bias approach

Common assumption: Reasoning and understanding of natural language involve the creation of mental models [e.g., Johnson-Laird 1983]

- **Understanding** a sentence S means being able to mentally construct a model picturing the world which verifies S, and possibly also a model which falsifies it
- **Reasoning** depends on two main processes: first construct verifying models for the premises and then check the validity of the conclusion on these models

Novel hypothesis: biases can constrain the construction of these models and therefore impact both reasoning and interpretation:

- **Neglect-zero** prevents the constructions of zero-models;
- **No-split** expresses a dispreference for split-states.

Comparison with competing accounts

	Ignorance	**FC & DIST**	**ES-Quant**	**Scalar impl.**	**Conjunctive *or***
Neo-Gricean	reasoning	reasoning	reasoning	reasoning	—
Grammatical	debated	grammatical	grammatical	grammatical	grammatical
Cognitive bias	neglect-zero	neglect-zero	neglect-zero	—	negl-z + no-split

Next

- Logical modelling of biases in team semantics
- Experimental findings
  - Degano *et al* (Nat Lang Sem, 2025): ignorance
  - Klochowicz *et al* (CogSci25, SuB25): on scalar, DIST & ES-Quant
  - Bleotu *et al* (TbiLLC 2025): on conjunctive *or*

Modelling biases in team semantics

General methodology

Natural language sentences translated into classical logic formulas interpreted in a team semantics which models both classical and enriched interpretations

NATURAL LANGUAGE $\mapsto$ LOGICAL LANGUAGE $\nearrow$ α (**classical**)

$\searrow$ $[\alpha]^{+\mathrm{NZ}/\mathrm{NS}}$ (**enriched**)

Back to FC challenge

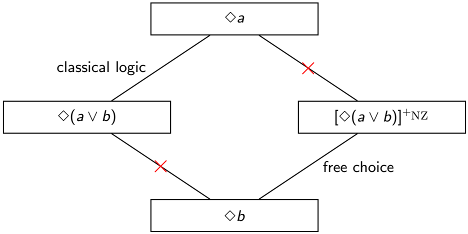


Figure: FC derived only for NZ enriched formulas

Modelling biases in team semantics

Team semantics

- Formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Hodges 1997; Väänänen 2007]
  - Classical modal logic: $[M = \langle W, R, V \rangle]$

$$M, w \models \phi, \text{ where } w \in W$$

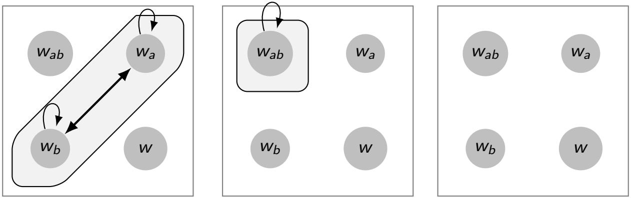
  - Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

- Two crucial features
  - The empty set is among the possible teams ($\emptyset \subseteq W$) $\mapsto$ zero-models
  - Multi-membered teams can model parallel processing of alternatives $\mapsto$ split states

Illustrations

$$[W = \{w_{ab}, w_a, w_b, w\}]$$



(a) split (b) no-split (c) empty team

Modelling biases in team semantics

Team semantics

- Formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Hodges 1997; Väänänen 2007]
- Two crucial features
  - The empty set is among the possible teams ($\emptyset \subseteq W$) $\mapsto$ zero-models
  - Multi-membered teams can model parallel processing of alternatives $\mapsto$ split states

Modelling neglect-zero & no-split

- Model-theoretically: by disallowing empty (neglect-zero) and multi-membered teams (no-split)
- Syntactically: via new logical atoms/operators
  - Neglect-zero modelled via **non-emptiness atom** NE which disallows empty teams as possible verifiers [Yang & Väänänen 2017]

$$M, t \models \mathrm{NE} \text{ iff } t \neq \emptyset$$

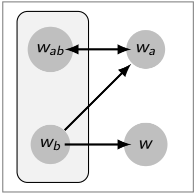
  - No-split modelled via **flattening operator** F which induces pointwise evaluations and therefore avoids simultaneous processing of alternatives

$$M, t \models \mathrm{F}\phi \text{ iff for all } w \in t : M, \{w\} \models \phi$$

BSML: Classical Modal Logic + NE

Language

$$\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \mathrm{NE}$$



Bilateral team semantics

Given Kripke model $M = \langle W, R, V\rangle$ & teams/states $s, t, t' \subseteq W$

$$
\begin{array}{rcll}
M, s \models p & \text{iff} & \text{for all } w \in s : V(w, p) = 1 & \\
M, s \dashv p & \text{iff} & \text{for all } w \in s : V(w, p) = 0 & \\
M, s \models \neg\phi & \text{iff} & M, s \dashv \phi & \\
M, s \dashv \neg\phi & \text{iff} & M, s \models \phi & \\
M, s \models \phi \vee \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \ \&\ M, t \models \phi \ \&\ M, t' \models \psi & \Leftarrow \\
M, s \dashv \phi \vee \psi & \text{iff} & M, s \dashv \phi \ \&\ M, s \dashv \psi & \\
M, s \models \phi \wedge \psi & \text{iff} & M, s \models \phi \ \&\ M, s \models \psi & \\
M, s \dashv \phi \wedge \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \ \&\ M, t \dashv \phi \ \&\ M, t' \dashv \psi & \\
M, s \models \Diamond\phi & \text{iff} & \text{for all } w \in s : \exists t \subseteq R[w] : t \neq \emptyset \ \&\ M, t \models \phi & \\
M, s \dashv \Diamond\phi & \text{iff} & \text{for all } w \in s : M, R[w] \dashv \phi \qquad [\text{where } R[w] = \{v \in W \mid wRv\}] & \\
M, s \models \mathrm{NE} & \text{iff} & s \neq \emptyset & \\
M, s \dashv \mathrm{NE} & \text{iff} & s = \emptyset &
\end{array}
$$

Entailment: $\phi_1, \ldots, \phi_n \models \psi$ iff for all M, s: $M, s \models \phi_1, \ldots, M, s \models \phi_n \Rightarrow M, s \models \psi$

Proof Theory: MA, Anttila & Yang (2024); Expressive completeness: Anttila & Knudstorp (2025); Comparisons via translation into Modal Information Logic: Knudstorp *et al* (2025)

Neglect-zero effects in BSML

BSML models both classical and enriched interpretations

- α (NE-free) $\Rightarrow$ empty team allowed $\mapsto$ **classical**
- $[\alpha]^+$ $\Rightarrow$ empty team not allowed $\mapsto$ **enriched**

Neglect-zero enrichment

For NE-free α, $[\alpha]^+$ defined as follows:

$$\begin{array}{rcl}
[p]^+ & = & p \wedge \mathrm{NE} \\
[\neg\alpha]^+ & = & \neg[\alpha]^+ \wedge \mathrm{NE} \\
[\alpha \vee \beta]^+ & = & ([\alpha]^+ \vee [\beta]^+) \wedge \mathrm{NE} \\
[\alpha \wedge \beta]^+ & = & ([\alpha]^+ \wedge [\beta]^+) \wedge \mathrm{NE} \\
[\Diamond\alpha]^+ & = & \Diamond[\alpha]^+ \wedge \mathrm{NE}
\end{array}$$

$[\ ]^+$ enriches formulas with the requirement to satisfy NE (non-emptiness) distributed along each of their subformulas

Formal characterization of neglect-zero effects

$\alpha \rightsquigarrow_{nz} \beta$ (β is a neglect-zero effect of α) iff $\alpha \not\models \beta$ but $[\alpha]^+ \models \beta$

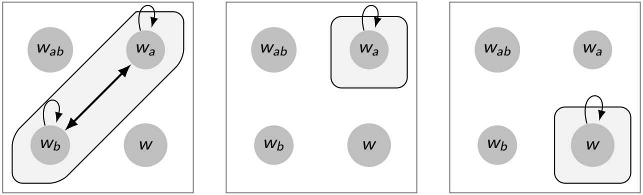
Formal characterization of zero and no-zero models

(M, s) is a zero-model for α iff $M, s \models \alpha$, but $M, s \not\models [\alpha]^+$
(M, s) is a no-zero verifier for α iff $M, s \models [\alpha]^+$

Neglect-zero effects in BSML: split disjunction

- A state s supports a **disjunction** iff s is the union of two substates, each supporting one of the disjuncts

$$M, s \models \phi \vee \psi \text{ iff } \exists t, t' : t \cup t' = s \;\&\; M, t \models \phi \;\&\; M, t' \models \psi$$



Verifier	**Zero-model**	Falsifier
$\models a \vee b$; $\models [a \vee b]^+$	$\models a \vee b$; $\not\models [a \vee b]^+$	$=\!\mid a \vee b$; $=\!\mid [a \vee b]^+$

Figure: Models for $a \vee b$

Why is $\{w_a\}$ a zero-model?

- Empty team allowed $\mapsto$ substates can be empty **(classical)**

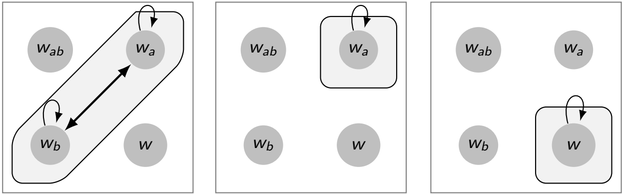
  $\{w_a\} \models a \vee b$ by virtue of an empty witness for b, $M, \emptyset \models b$
- Empty team not allowed $\mapsto$ substates cannot be empty **(enriched)**

  $\{w_a\} \not\models [a \vee b]^+$ because there is no non-empty subset supporting b

Neglect-zero effects in BSML: enriched disjunction

- *s* supports an **enriched disjunction** $[\alpha \vee \beta]^+$ iff *s* is the union of two non-empty substates, each supporting one of the disjuncts

$$[\alpha \vee \beta]^+ = (\alpha \wedge \mathrm{NE}) \vee (\beta \wedge \mathrm{NE}) \wedge \mathrm{NE}$$



(a) $\models [a \vee b]^+$ (b) $\not\models [a \vee b]^+$ (c) $\dashv [a \vee b]^+$

- An enriched disjunction requires both disjuncts to be live possibilities [Zimmermann 2000]

(23) It is raining or snowing $\rightsquigarrow_{nz}$ It might be raining and it might be snowing

$[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$ (where *R* is state-based)

- Main result: in BSML $[\ ]^+$-enrichment has non-trivial effect only when applied to *positive* disjunctions [MA 2022]
  - $\mapsto$ we derive ignorance, FC and related effects (for enriched formulas);
  - $\mapsto$ $[\ ]^+$-enrichment vacuous under single negation.

Neglect-zero effects in BSML: main results

After enrichment

- We derive ignorance and FC:
  - Ignorance: $[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$ (if R is state-based)
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond \alpha \wedge \Diamond \beta$
  - Double negation FC: $[\neg\neg\Diamond(\alpha \vee \beta)]^+ \models \Diamond \alpha \wedge \Diamond \beta$
  - Wide scope FC: $[\Diamond \alpha \vee \Diamond \beta]^+ \models \Diamond \alpha \wedge \Diamond \beta$ (if R is indisputable)
- while no undesirable side effects obtain with other configurations:
  - Double prohibition: $[\neg\Diamond(\alpha \vee \beta)]^+ \models \neg\Diamond \alpha \wedge \neg\Diamond \beta$

Before enrichment

- The NE-free fragment of BSML is equivalent to classical modal logic:

$$\alpha \models_{BSML} \beta \text{ iff } \alpha \models_{CML} \beta \qquad [\text{if } \alpha, \beta \text{ are NE-free}]$$

- But we can capture the infelicity of epistemic contradictions [Yalcin, 2007]
  - Epistemic contradiction: $\Diamond_e \alpha \wedge \neg \alpha \models \bot$ (if R is state-based)
  - Non-factivity: $\Diamond_e \alpha \not\models \alpha$

Team-based constraints on accessibility relation

- R state-based in (M, s) iff all and only worlds in s are accessible within s [$\mapsto$ **epistemics** (always)]
- R indisputable in (M, s) iff all worlds in s access exactly the same set of worlds [$\mapsto$ **deontics** (sometimes)]

The data

(24) Double Prohibition [Alonso-Ovalle 2006, Marty *et al.* 2021]

a. You are not allowed to eat the cake or the ice-cream $\rightsquigarrow$ You are not allowed to eat either one

b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$

(25) Double Negation FC [Gotzner *et al.* 2020]

a. Exactly one girl cannot take Spanish or Calculus $\rightsquigarrow$ One girl can take neither of the two and each of the others can choose between them.

b. $\exists x(\neg\Diamond(\alpha(x) \vee \beta(x)) \wedge \forall y(y \neq x \rightarrow \neg\neg\Diamond(\alpha(y) \vee \beta(y)))) \rightsquigarrow$
$\exists x(\neg\Diamond\alpha(x) \wedge \neg\Diamond\beta(x) \wedge \forall y(y \neq x \rightarrow (\Diamond\alpha(y) \wedge \Diamond\beta(y))))$

(26) Wide Scope FC [Zimmermann 2000, Cremers *et al* 2017]

a. Detectives may go by bus or they may go by boat $\rightsquigarrow$ Detectives may go by bus and may go by boat

b. Mr. X might be in Victoria or he might be in Brixton $\rightsquigarrow$ Mr. X might be in Victoria and might be in Brixton

c. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ (if R indisputable)

(27) FC cancellation [sluice indicates wide scope disjunction]

a. Detectives may go by bus or by boat, I don't know which $\not\rightsquigarrow$ Detectives may go by bus and may go by boat

b. $\Diamond\alpha \vee \Diamond\beta \not\rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ (if R not indisputable)

Experimental findings

Cognitive bias view

Non-classical inferences prominently explained by neo-Gricean or grammatical mechanisms are instead consequence of a neglect-zero (+ no-split) tendency

Comparison with competing accounts[1]

	Ignorance	**FC & DIST**	**ES-Quant**	**Scalar impl.**	**Conjunctive *or***
Neo-Gricean	reasoning	reasoning	reasoning	reasoning	—
Grammatical	debated	grammatical	grammatical	grammatical	grammatical
Cognitive bias	neglect-zero	neglect-zero	neglect-zero	—	negl-z + no-split

Recent experiments

1. Degano *et al* (Nat Lang Sem, 2025): ignorance $\Leftarrow$
2. Klochowicz *et al* (CogSci25, SuB25): on DIST, ES-Quant & scalar

   (28) a. Each square is red or white $\Rightarrow$ there are white and red squares [DIST]
   b. Less than 3 squares are black $\Rightarrow$ there are some black squares [ES-Quant]
   c. Some of the squares are black $\Rightarrow$ not all of the squares are black [scalar]

   Main result:
   - Semantic priming between DIST and ES-Quant;
   - No priming between scalar and ES-Quant.
3. Bleotu *et al* (TbiLLC 2025): on conjunctive *or*

[1] Neo-Gricean: Horn, Soames, Sauerland, ... Grammatical view: Chierchia, Fox, Singh *et al*, ...

Neglect-zero effects on plain disjunction

Enriched meanings for disjunction

(29) Maria ate an apple or a banana $\rightsquigarrow$ $(\alpha \vee \beta)$

a. Scalar implicature: not both $\neg(\alpha \wedge \beta)$
b. Conjunctive interpretation: both $(\alpha \wedge \beta)$
c. Ignorance: speaker doesn't know which ?

Two components of ignorance: possibility vs uncertainty

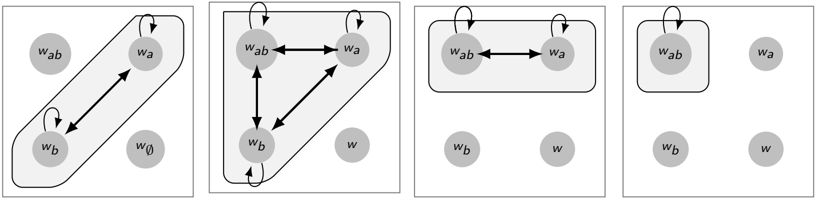
(30) Maria ate an apple or a banana $\rightsquigarrow$ speaker doesn't know which [Degano et al 2025][2]

a. Possibility: It is possible that M ate an apple and it is possible that M ate a banana $\Diamond_e \alpha \wedge \Diamond_e \beta$
b. Uncertainty: It is uncertain that M ate an apple and it is uncertain that M ate a banana $\neg\Box_e \alpha \wedge \neg\Box_e \beta$

[2] Degano, Marty, Ramotowska, MA, Breheny, Romoli, Sudo. "The ups and downs of ignorance." *Nat Lang Sem*, 2025.

Neglect-zero effects on disjunction: predictions of BSML

Many no-zero verifiers for enriched disjunction



(d) no-zero & scalar $\models \neg(a \wedge b)$ (e) no-z & non-scalar $\not\models \neg(a \wedge b)$ (f) no-z & no-uncertain $\not\models \neg\Box_e a$ (g) no-zero & no-split $\models (a \wedge b)$

Figure: Models for enriched $[a \vee b]^+$.

1. Neglect-zero enrichment derives **possibility**: $[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$
2. Neglect-zero enrichment does not derive scalar implicatures;
3. Neglect-zero enrichment does not derives uncertain inferences $\mapsto$ in contrast to standard neo-Gricean approach to ignorance $\Leftarrow$
4. **No-split** verifiers compatible with neglect-zero enrichments
   - **No-split** conjecture: only **no-split** verifiers accessible to 'conjunctive' pre-school children. [Klochowicz, Sbardolini, MA, SuB, 2025]

Two derivations of full ignorance

1. Standard neo-Gricean derivation [Sauerland 2004]

(i) Uncertainty derived through quantity reasoning

(31) $\alpha \vee \beta$ ASSERTION

(32) $\neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY (from QUANTITY)

(ii) Possibility derived from uncertainty and quality about assertion

(33) $\Box_e(\alpha \vee \beta)$ QUALITY ABOUT ASSERTION

(34) $\Rightarrow \Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY

2. Neglect-zero derivation

(i) Possibility derived as neglect-zero effect

(35) $\alpha \vee \beta$ ASSERTION

(36) $\Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY (from NEGLECT-ZERO)

(ii) Uncertainty derived from possibility and scalar reasoning

(37) $\neg(\alpha \wedge \beta)$ SCALAR IMPLICATURE

(38) $\Rightarrow \neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY

Neo-Gricean vs neglect-zero explanation

Contrasting predictions of competing accounts of ignorance

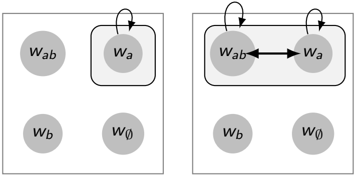
- Neo-Gricean: No possibility without uncertainty
- Neglect-zero: Possibility derived independently from uncertainty

Experimental findings [Degano *et al* 2025]

- Using adapted mystery box paradigm, compared conditions in which
  - both uncertainty and possibility are false [**zero-model**]
  - uncertainty false but possibility true [no-zero, no-uncertain model]
- Less acceptance when possibility is false (95% vs **44%**)

⇒ Evidence that possibility can arise without uncertainty

- A challenge for the traditional neo-Gricean approach



(a) **zero-model** (b) no-uncertain

Figure: Models for $(a \vee b)$

Conclusions

- FC, ignorance: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Nihil proposal: some non-classical inferences due to cognitive bias rather than Gricean reasoning
  - FC, possibility and related inferences as neglect-zero effects

    Literal meanings (classical fragment) + cognitive factor (NE) $\Rightarrow$ FC, possibility, etc

  - Conjunctive *or* as no-zero + no-split effect

    Literal meanings (classical fragment) + cognitive factors (NE, F) $\Rightarrow$ conjunctive *or*

- Implementation in (extensions of) BSML, a team-based modal logic
- Recent experiments provide some first tentative evidence in agreement with the neglect-zero hypothesis
- Appendix:
  - Experimenting with disjunction and quantifiers
  - Comparison via translation into Modal Information Logic

Collaborators & related (future) research


Anttila


Degano


Klochowicz


Knudstorp


Ramotowska


Zhou

& many more . . .

Logic

Proof theory (Anttila, Yang); expressive completeness (Anttila, Knudstorp); bimodal perspective (Knudstorp, Baltag, van Benthem, Bezhanishvili); qBSML (van Ormondt); dynamics (MA); typed BSML (Muskens); connexive logic (Knudstorp, Ziegler & MA); belief revision (Klochowicz)

Language

FC cancellations (Pinton, Hui); modified numerals (vOrmondt); attitude verbs (Yan); conditionals (Flachs, Ziegler); questions (Klochowicz); quantifiers (Klochowicz, Bott, Schlotterbeck); indefinites (Degano); homogeneity (Sbardolini); acquisition (Klochowicz, Sbardolini); experiments (Degano, Klochowicz, Ramotowska, Bott, Schlotterbeck, Marty, Breheny, Romoli, Sudo, Spychalska, Szymanik, Visser); . . .

Thank You![3]

[3] This work is supported by NWO OC project *Nothing is Logical* (grant no 406.21.CTW.023).

Selected References

Aloni, Maria (2022). "Logic and conversation: The case of free choice". In: Semantics and Pragmatics 15.5, pp. 1–60. DOI: 10.3765/sp.15.5.

Aloni, Maria, Aleksi Anttila, and Fan Yang (2024). "State-based Modal Logics for Free Choice". In: Notre Dame Journal of Formal Logic 65.4, pp. 367–413. DOI: https://doi.org/10.1215/00294527-2024-0027.

Aloni, Maria and Peter van Ormondt (2023). "Modified numerals and split disjunction: the first-order case". In: Journal of Logic, Language and Information 32.4, pp. 539–567. DOI: 10.1007/s10849-023-09399-w.

Bleotu, Camelia (2025). "Conjunction as a Default Meaning of Disjunction". Presented at TbiLLC 2025. URL: https://www.marialoni.org/resources/Bleotu_Kutaisi.pdf.

Bott, Oliver, Fabian Schlotterbeck, and Udo Klein (2019). "Empty-Set Effects in Quantifier Interpretation". In: Journal of Semantics 36 (1), pp. 99–163.

Degano, Marco et al. (2025). "The ups and downs of ignorance". In: Natural Language Semantics 33, pp. 1–41. DOI: 10.1007/s11050-024-09226-3.

Gotzner, Nicole, Jacopo Romoli, and Paolo Santorio (2020). "Choice and prohibition in non-monotonic contexts". In: Natural Language Semantics 28, pp. 141–174.

Grice, Herbert Paul (1975). "Logic and Conversation". In: Syntax and Semantics, Volume 3: Speech Acts. Ed. by Peter Cole and Jerry Morgan. Academic Pr, pp. 41–58.

— (1989). Studies in the Way of Words. Harvard University Press.

Johnson-Laird, Philip N. (1983). Mental Models. Cambridge University Press.

Kamp, Hans (1973). "Free Choice Permission". In: Proceedings of the Aristotelian Society 74, pp. 57–74.

Klochowicz, Tomasz, Giorgio Sbardolini, and Maria Aloni (2025). "Cognitive bias approach to the acquisition of disjunction". Presented at SuB 2025.

Klochowicz, Tomasz et al. (2025). "Neglect zero: evidence from priming across constructions". In: Proceedings of CogSci 2025. URL: https://escholarship.org/uc/item/36w6x7z9.

Selected References

Leahy, Brian P. and Susan E. Carey (2020). "The Acquisition of Modal Concepts". In: Trends in Cognitive Sciences 24.1, pp. 65–78. DOI: https://doi.org/10.1016/j.tics.2019.11.004.

Nieder, Andreas (2016). "Representing Something Out of Nothing: The Dawning of Zero". In: Trends in Cognitive Sciences 20 (11), pp. 830–842.

Redshaw, Jonathan and Thomas Suddendorf (2016). "Children's and Apes' Preparatory Responses to Two Mutually Exclusive Possibilities". In: Current Biology 26.13, pp. 1758–1762. DOI: https://doi.org/10.1016/j.cub.2016.04.062.

Wright, G.H. von (1968). An Essay on Deontic Logic and the Theory of Action. North Holland.

Zimmermann, Ede (2000). "Free Choice Disjunction and Epistemic Possibility". In: Natural Language Semantics 8, pp. 255–290.

Neglect-zero effects on quantifiers: Empty Set (ES) inferences

Predictions of qBSML$^{\rightarrow}$[4]

(39) Less than three squares are black $\mapsto \forall xyz((Sx \wedge Bx \wedge \ldots) \rightarrow (x = y \vee \ldots))$

a. Verifier: [■, □, ■]

b. Falsifier: [■, ■, ■]

c. Zero-models: [□, □, □]; [▲, ▲, ▲]; … $\rightsquigarrow_{nz}$ there are black squares

(40) Every square is black. $\mapsto \forall x(Sx \rightarrow Bx)$

a. Verifier: [■, ■, ■]

b. Falsifier: [■, □, ■]

c. Zero-models: [△, △, △]; [▲, ▲, ▲]; … $\rightsquigarrow_{nz}$ there are squares

(41) No squares are black. $\mapsto$ (i) $\forall x(Sx \rightarrow \neg Bx)$; (ii) $\neg\exists x(Sx \wedge Bx)$

a. Verifier: [□, □, □]

b. Falsifier: [■, □, □]

c. Zero-models for (i): [△, △, △]; [▲, ▲, ▲]; … $\rightsquigarrow_{nz}$ there are squares

d. Zero-models for (ii): none no neglect-zero effect

(42) Every square is red or white. $\mapsto \forall x(Sx \rightarrow (Rx \vee Wx))$

a. Verifier: [$\textcolor{red}{\blacksquare}$, □, $\textcolor{red}{\blacksquare}$]

b. Falsifier: [$\textcolor{red}{\blacksquare}$, □, ■]

c. Zero-models: [$\textcolor{red}{\blacksquare}$, $\textcolor{red}{\blacksquare}$, $\textcolor{red}{\blacksquare}$]; [□, □, □]; … $\rightsquigarrow_{nz}$ there are white & red squares

These predictions tested in Bott, Klochowicz, Schlotterbeck *et al* (2024, 2025)

[4] MA & vOrmondt, Modified numerals and split disjunction. *J of Log Lang and Inf* (2023).

Experimenting with quantifiers and disjunction

Four non-classical interpretations

(43) a. Some of the squares are black ⇒ not all of the squares are black [scalar UB]
b. Each square is red or white ⇒ there are white squares and red squares [DIST]
c. Less than 3 squares are black ⇒ there are some black squares [ES-scope]
d. Less than 3/every/no squares are black ⇒ there are some squares [ES-restrictor]

Three competing accounts

	UB	DIST	ES-scope	ES-restrictor
Alternative-based	implicature	implicature	implicature	implicature
Bott *et al*, 2019	—	—	neglect-zero	presupposition
Nihil	—	neglect-zero	neglect-zero	neglect-zero

Two experiments

- Exp 1: Answering questions about the emptyset (O. Bott *et al*, SuB 2024)
- Exp 2: Priming with zero-models (Klochowicz, Schlotterbeck *et al*, CogSci 2025, SuB 2025)

Three main conclusions

1. Evidence that ES-restrictor is a presupposition (Exp 1)
2. Evidence that UB differs from both ES-scope and DIST (Exp1 and Exp2)
3. Some evidence that ES-scope and DIST involve the same cognitive process (Exp 2)

Experimenting with quantifiers and disjunction

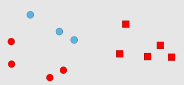
Non-classical interpretations

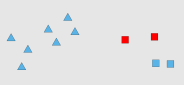
(44)
a. Some of the squares are black ⇒ not all of the squares are black [UB]
b. Each square is red or white ⇒ there are white squares and red squares [DIST]
c. Less than 3 squares are black ⇒ there are some black squares [ES-scope]
d. Less than 3/every/no squares are black ⇒ there are some squares [ES-restrictor]

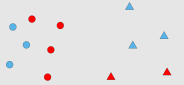
Exp1: Bott et al, SuB 2024

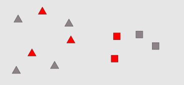
- Question-answer task:

(45) Ist jedes Dreieck entweder rot oder blau? Ja/Nein/Komische Frage
(Is every triangle either red or blue?) Yes/No/Odd question


empty restr


DIST target (zero-model)


control '*yes*'


control '*no*'

- Main results:
  1. Evidence that ES-restrictor is a presupposition: questions in empty restrictor models uniformly perceived as odd
  2. ES-scope (37%) and DIST (23%) unaffected by question environment; UB much less available (10%, while 40% when unembedded)
  3. Inconclusive evidence on whether ES-scope and DIST had the same source

Experimenting with quantifiers and disjunction

Non-classical interpretations

(46) a. Some of the squares are black ⇒ not all of the squares are black [UB/scalar]
b. Each square is red or white ⇒ there are white and red squares [DIST]
c. At most 2 squares are black ⇒ there are some black squares [ES-scope, sup]
d. Less than 3 squares are black ⇒ there are some black squares [ES-scope, comp]

Two competing accounts

	UB	DIST	ES-scope
Alternative-based	implicature	implicature	implicature
Nihil (qBSML$^{\to}$)	—	neglect-zero	neglect-zero

Exp2: Klochowicz, Schlotterbeck *et al*, CogSci 2025, SuB 2025

- Tested whether frequency of enrichment in (46-d) changed after participants were primed to suspend other enrichments in (46-a-c):
  - UB ⇒ ES-scope[c]; DIST ⇒ ES-scope[c]; ES-scope[s] ⇒ ES-scope[c]
- Results:
  1. Semantic priming between DIST and ES-scope (comp)
  2. No priming between UB and ES-scope (comp)
  3. No trial-to-trial priming from ES-scope (sup) to ES-scope (comp) but spill-over and adaptation effects
- Tentative conclusion: ES-scope and DIST (but not UB) involve the same cognitive process, as predicted by neglect-zero hypothesis

BSML & related systems: information states vs possible worlds

- Failure of bivalence in BSML

$$M, s \not\models p \ \& \ M, s \not\models \neg p, \text{ for some info state } s$$

- Info states: less determinate than possible worlds
  - just like truthmakers, situations, possibilities, ...
- Technically:
  - Truthmakers/possibilities: points in a partially ordered set
  - Info states: sets of possible worlds, also elements of a partially ordered set, the Boolean lattice $Pow(W)$
- Thus systems using these structures are closely connected, although might diverge in motivation:
  - Truthmaker & possibility semantics: description of ontological structures in the world
  - BSML & inquisitive semantics: explaining patterns in inferential & communicative human activities
- NEXT:
  - Comparison via translations in Modal Information Logic [vBenthem19]

BSML & related systems: comparisons via translation

- Modal Information Logic (MIL) (van Benthem, 1989, 2019):[5] common ground where related systems can be interpreted and their connections and differences can be explored
- Goal: translations into (extensions of) MIL of the following systems:
  - Truthmaker semantics (Fine)
  - Possibility semantics (Humberstone, Holliday)
  - BSML
  - Inquisitive semantics (Ciardelli, Groenendijk & Roelofsen)

  (cf. Gödel's (1933) translation of intuitionistic logic into modal logic)
- Here focus on propositional fragments
  - disjunction
  - negation
- (Based on work in progress with Søren B. Knudstorp, Nick Bezhanishvili, Johan van Benthem and Alexandru Baltag)

[5] Johan van Benthem (2019) Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic*.

Modal Information Logic (MIL)

Language

$$\phi \quad ::= \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \langle sup \rangle \phi\psi$$

where $p \in A$.

Models and interpretation

Formulas are interpreted on triples $M = (X, \leq, V)$ where $\leq$ is a partial order

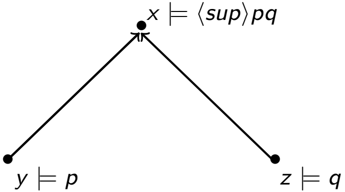
$$\begin{array}{rcl}
\mathcal{M}, x \models p & \text{iff} & x \in V(p) \\
\mathcal{M}, x \models \neg\phi & \text{iff} & \mathcal{M}, x \not\models \phi \\
\mathcal{M}, x \models \phi \wedge \psi & \text{iff} & \mathcal{M}, x \models \phi \text{ and } \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \phi \vee \psi & \text{iff} & \mathcal{M}, x \models \phi \text{ or } \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \langle sup \rangle \phi\psi & \text{iff} & \text{there are } y, z : x = sup_{\leq}(y, z) \ \& \ \mathcal{M}, y \models \phi \ \& \ \mathcal{M}, z \models \psi
\end{array}$$

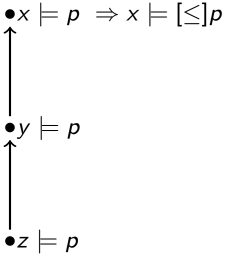
$$[\leq]\phi = \neg\langle sup \rangle(\neg\varphi)\top$$

$$\mathcal{M}, x \models [\leq]\phi \quad \text{iff} \quad \text{for all } y : y \leq x \ \Rightarrow \ \mathcal{M}, y \models \phi$$

Modal Information Logic (MIL)

Examples





Translations into Modal Information Logic

- Possibility semantics (Humberstone, Holliday)[6]

$$\begin{array}{rcl} & \vdots & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & [\leq]\langle\leq\rangle(tr(\phi) \vee tr(\psi)) \\ & \vdots & \end{array}$$

- Inquisitive semantics (Groenendijk, Roelofsen and Ciardelli)

$$\begin{array}{rcl} & \vdots & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & tr(\phi) \vee tr(\psi) \\ & \vdots & \end{array}$$

[6] Johan van Benthem, Nick Bezhanishvili, Wesley H. Holliday, A bimodal perspective on possibility semantics, *Journal of Logic and Computation*, Volume 27, Issue 5, July 2017, Pages 1353–1389.

Translations into Modal Information Logic

- Truthmaker semantics (Fine)[7]

$$\begin{array}{rcl} & \cdots & \\ (\neg\phi)^+ & = & (\phi)^- \\ (\neg\phi)^- & = & (\phi)^+ \\ (\phi \vee \psi)^+ & = & (\phi)^+ \vee (\psi)^+ \\ (\phi \vee \psi)^- & = & \langle sup \rangle (\phi)^- (\psi)^- \\ (\phi \wedge \psi)^+ & = & \langle sup \rangle (\phi)^+ (\psi)^+ \\ (\phi \wedge \psi)^- & = & (\phi)^- \vee (\psi)^- \end{array}$$

- BSML

$$\begin{array}{rcl} & \cdots & \\ (\neg\phi)^+ & = & (\phi)^- \\ (\neg\phi)^- & = & (\phi)^+ \\ (\phi \vee \psi)^+ & = & \langle sup \rangle (\phi)^+ (\psi)^+ \\ (\phi \vee \psi)^- & = & (\phi)^- \wedge (\psi)^- \\ (\phi \wedge \psi)^+ & = & (\phi)^+ \wedge (\psi)^+ \\ (\phi \wedge \psi)^- & = & \langle sup \rangle (\phi)^- (\psi)^- \\ & \cdots & \end{array}$$

[7] van Benthem, Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic* (2019).

Disjunction and Negation

- Three notions of disjunction expressible in MIL:
  - Boolean disjunction: $\phi \vee \psi$
    [classical logic, intuitionistic logic, inquisitive logic]
  - Lifted/tensor/split disjunction: $\langle sup \rangle \phi\psi$
    [BSML, dependence logic, team semantics, operational semantics for Positive R]
  - Cofinal disjunction: $[co](\phi \vee \psi)$ (where $[co]\phi =: [\leq]\langle\leq\rangle\phi$)
    [possibility semantics, dynamic semantics]
- Three notions of negation:
  - Boolean negation: $\neg\phi$
    [classical logic, ...]
  - Bilateral negation: $(\neg\phi)^+ = (\phi)^-$ & $(\neg\phi)^- = (\phi)^+$
    [truthmaker semantics, BSML, ...]
  - Intuitionistic-like negation: $[\leq]\neg\phi$
    [possibility semantics, inquisitive semantics, intuitionistic logic]
- Some combinations:
  - Boolean disjunction + boolean negation $\mapsto$ classical logic
  - Boolean notions in other combinations can generate non-classicality:
    - Boolean disjunction + intuitionistic negation $\mapsto$ intuitionistic/inquisitive logic[8]
  - Classicality also generated by non-boolean combinations:
    - Split disjunction + bilateral negation (classical fragm. BSML)
    - Cofinal disjunction and intuitionistic negation (possibility semantics)

[8] Inquisitive & intuitionistic logic: same connectives but different translations for the atoms.