N∅thing is Logical

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Slides: https://www.marialoni.org/resources/Frankfurt26.pdf

Frankfurt Semantics Colloquium
12 February 2026

N$\emptyset$thing is logical (Nihil)

- Goal of the project: a formal account of a class of natural language inferences which deviate from classical logic
- Common assumption: these deviations are not logical mistakes, but consequence of pragmatic enrichments (Grice)
- Strategy: develop *logics of conversation* which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency (a cognitive bias rather than a conversational principle) as crucial factor
- Main conclusion: deviations from classical logic consequence of enrichments albeit not (always) of the canonical Gricean kind

Nihil team


Anttila


Degano


Klochowicz


Knudstorp


Ramotowska


Zhou

& many more ...

Non-classical inferences

Free choice (FC)

(1) FC: $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [von Wright 1968]

(2) Deontic FC inference [Kamp 1973]
   a. You may go to the beach *or* to the cinema.
   b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(3) Epistemic FC inference [Zimmermann 2000]
   a. Mr. X might be in Victoria *or* in Brixton.
   b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

Ignorance

(4) The prize is either in the garden *or* in the attic $\rightsquigarrow$ The prize might be in the garden *and* might be in the attic [Grice 1989, p.45]

(5) ? I have two *or* three children.

- In the standard approach, ignorance is a conversational implicature
- Less consensus on FC inferences analysed as conversational implicatures; grammatical (scalar) implicatures; semantic entailments; . . .

The challenge of FC: adding FC to classical modal logic implies the equivalence of any two possibility claims

$$\Diamond a \;\Rightarrow_{\mathrm{CML}}\; \Diamond(a \vee b) \;\Rightarrow_{\mathrm{FC}}\; \Diamond b$$

Novel hypothesis: neglect-zero

- FC and ignorance inferences are [$\neq$ semantic entailments]
  - Not the result of Gricean reasoning [$\neq$ conversational implicatures]
  - Not the effect of applications of covert grammatical operators [$\neq$ grammatical (scalar) implicatures]
- They are rather a consequence of something else speakers do in conversation, namely,

  NEGLECT-ZERO
  when interpreting a sentence speakers construct models depicting reality (some verifying the sentence, some falsifying it) $\mapsto$ common assumption
  and in this process tend to neglect models that verify the sentence by virtue of an empty configuration (*zero-models*) $\mapsto$ novel hypothesis

- Tendency to neglect zero-models follows from the cognitive difficulty of:

  1. conceiving emptiness, the absence of things rather than their presence
  2. evaluating truths with respect to empty witness sets

  [Nieder 2016; O. Bott *et al* 2019]

Novel hypothesis: neglect-zero

Illustration

(6) Less than three squares are black.
  a. Verifier: [■, □, ■]
  b. Falsifier: [■, ■, ■]
  c. Zero-models: [□, □, □]; [■, ■, ■]; [△, △, △]; [▲, ▲, ▲]; . . .

Zero-models in (6-c) verify the sentence by virtue of an empty set of black squares

- Cognitive difficulty of zero-models confirmed by experimental findings and connected to / can be argued to explain:
  - the special status of 0 among the natural numbers [Nieder 2016]
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (*less* vs *more*) [O. Bott *et al* 2019]
- N-Z hypothesis: neglect-zero also at the origin of many common departures from classical reasoning
  1. FC and ignorance [MA 2022]
  2. Existential Import: every A is B ⇒ some A is B
  3. Aristotle's Thesis: if not A then A ⇒ ⊥
  4. Boethius' Thesis: if A then B & if A then not B ⇒ ⊥

  [Ziegler, Knudstorp & MA 2025]

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

(7) Maria ate an apple.
- a. Verifier: [🍎]
- b. Falsifiers: [🍌]; [🍐]; [ ]
- c. Zero-models: none

(8) Maria ate a banana.
- a. Verifier: [🍌]
- b. Falsifiers: [🍎]; [🍐]; [ ]
- c. Zero-models: none

(9) M ate an apple and a banana.
- a. Verifier: [🍎 🍌]
- b. Falsifiers: [🍐]; [ ]
- c. Zero-models: none

(10) M ate an apple or a banana.
- a. Verifier: ?
- b. Falsifiers: [🍐]; [ ]
- c. Zero-models: ?

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

(11) Maria ate an apple.
- a. Verifier: [🍎]
- b. Falsifiers: [🍌]; [🍐]; [ ]
- c. Zero-models: none

(12) Maria ate a banana.
- a. Verifier: [🍌]
- b. Falsifiers: [🍎]; [🍐]; [ ]
- c. Zero-models: none

(13) M ate an apple and a banana.
- a. Verifier: [🍎 🍌]
- b. Falsifiers: [🍐]; [ ]
- c. Zero-models: none

(14) M ate an apple or a banana.
- a. Verifier: ?
- b. Falsifiers: [🍐]; [ ]
- c. **Zero-models**: [🍎]; [🍌]

- Two **zero-models** in (14-c): verify the sentence by virtue of an empty witness for one of the disjuncts

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

(15) Maria ate an apple.
- a. Verifier: [🍎]
- b. Falsifiers: [🍌]; [🍐]; [ ]
- c. Zero-models: none

(16) Maria ate a banana.
- a. Verifier: [🍌]
- b. Falsifiers: [🍎]; [🍐]; [ ]
- c. Zero-models: none

(17) M ate an apple and a banana.
- a. Verifier: [🍎 🍌]
- b. Falsifiers: [🍐]; [ ]
- c. Zero-models: none

(18) M ate an apple or a banana.
- a. Verifier: [🍎 | 🍌] ⇐ 'split'
- b. Falsifiers: [🍐]; [ ]
- c. **Zero-models**: [🍎]; [🍌]

- Two **zero-models** in (18-c): sentence verified by virtue of an empty witness for one of the disjuncts
- **Split state** in (18-a): multiple (possibly conflicting) alternatives processed in a parallel fashion
- Neglect-zero hypothesis: Zero-models, where only one of the disjuncts is depicted, are cognitively taxing and therefore disregarded
- Deductive reasoning involves checking whether the conclusion of an argument is satisfied in all verifiers of the premises
- The exclusion of zero-models from the set of verifiers for a disjunction leads directly to an account of ignorance inferences and FC

A new conjecture: no-split

(19) Maria ate an apple or a banana.

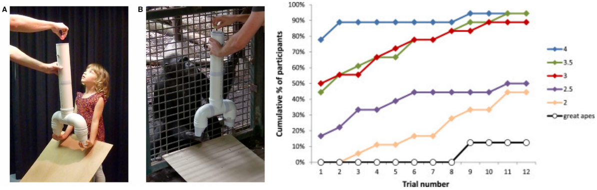
a. Verifier: [ |] [⇐ **split state**]

b. Falsifiers: []; [ ]

c. **Zero-models**: []; []

- **Split states**: multiple (possibly conflicting) alternatives processed in a parallel fashion ↦ also a cognitively taxing operation

NO-SPLIT CONJECTURE [Klochowicz, Sbardolini & MA, SuB 2025]
the ability to split states (entertain multiple possibilities) is developed late



Children have trouble conceiving multiple possibilities [Redshaw & Suddendorf 2016]

- Proposal: Combination of neglect-zero + no-split can explain non-classical interpretations of disjunction observed in pre-school children

No-split: conjunctive children

- Pre-school children sometimes (but systematically) interpret disjunctions conjunctively [Evans 93; Singh *et al* 16 (cf. Skordos *et al* 20); Cochard25; Bleotu *et al* 25]

(20) M ate an apple or a banana = M ate an apple and a banana
$\alpha \vee \beta \equiv \alpha \wedge \beta$

(21) M can eat an apple or a banana = M can eat an apple and a banana
$\Diamond(\alpha \vee \beta) \equiv \Diamond(\alpha \wedge \beta) \not\equiv \Diamond\alpha \wedge \Diamond\beta$

(22) M didn't eat an apple or a banana = M neither ate an apple nor a banana
$\neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta$

- Proposal: children have conjunctive readings as they (similarly to adults) neglect zero and, unlike adults, lack the ability to split

1. Deriving ignorance:

   Apple OR banana $\Rightarrow_{\mathrm{NZ}}$ 🍎 + 🍌 $\Rightarrow_{\mathrm{SPLIT}}$ [🍎 | 🍌]

   $\rightsquigarrow$ It might be an apple and it might be a banana (adults)

2. Deriving conjunctive reading:

   Apple OR banana $\Rightarrow_{\mathrm{NZ}}$ 🍎 + 🍌 $\Rightarrow_{\mathrm{NO\text{-}SPLIT}}$ [🍎 🍌]

   $\rightsquigarrow$ Both an apple and a banana (children)

3. In case of incompatible alternatives: [Leahy & Carey 2020]

   Left OR right $\Rightarrow_{\mathrm{NZ}}$ $\swarrow$ + $\searrow$ $\Rightarrow_{\mathrm{NO\text{-}SPLIT}}$ **contradiction** ($\perp$)

   $\rightsquigarrow$ Random singular guess (children)

No-split: homogeneity

- Ability to split a state not only required to interpret disjunction but also conjunction under negation. Neglect-zero + no-split can also derive homogeneus interpretations [Sbardolini, 2023]

(23) M did *not* eat an apple *and* a banana $\rightsquigarrow$ M *neither* ate an apple *nor* a banana $\neg(\alpha \wedge \beta) \rightsquigarrow \neg\alpha \wedge \neg\beta$

- New conjecture: also adults tend to avoid split-states in cognitively taxing environments, notably under negation

1. Deriving homogeneity:

   $\textsc{Not}$ [apple $\textsc{and}$ banana] $\Rightarrow_{\mathrm{NZ}}$ $\textsc{No}$ 🍎 + $\textsc{No}$ 🍌 $\Rightarrow_{\textsc{no-split}}$ [$\textsc{No}$ 🍎 $\textsc{No}$ 🍌]

   $\rightsquigarrow$ Neither an apple nor a banana (children & no-splitting adults)

2. Deriving ignorance:

   $\textsc{Not}$ [apple $\textsc{and}$ banana] $\Rightarrow_{\mathrm{NZ}}$ $\textsc{No}$ 🍎 + $\textsc{No}$ 🍌 $\Rightarrow_{\textsc{split}}$ [$\textsc{No}$ 🍎 | $\textsc{No}$ 🍌]

   $\rightsquigarrow$ Possibly not an apple and possibly not a banana (splitting adults)

- Testable predictions arising from these conjectures:
  - Increase of no-split interpretations under cognitive load in adults (testable via dual-task)
  - Correlation between failure in Redshaw & Suddendorf's task and conjunctive behaviour in children

Cognitive bias approach (NIHIL$^+$)

Common assumption: Reasoning and understanding of natural language involve the creation of mental models [Johnson-Laird 1983, Mascarenhas & Koralus 2013, a.o.]

- **Understanding** a sentence S means being able to mentally construct a model picturing the world which verifies S, and possibly also a model which falsifies it
- **Reasoning** depends on two main processes: first construct verifying models for the premises and then check the validity of the conclusion on these models

Novel hypothesis: biases can constrain the construction of these models and therefore impact both reasoning and interpretation:

- **Neglect-zero** prevents the constructions of zero-models;
- **No-split** expresses a dispreference for split-states.

Comparison with competing accounts

	Ignorance	**FC & DIST**	**ES-Quant**	**Scalar impl.**	**Conj *or* & homog.**
Neo-Gricean	reasoning	reasoning	reasoning	reasoning	—
Grammatical	debated	grammar	grammar	grammar	grammar
NIHIL$^+$	neglect-zero	neglect-zero	neglect-zero	—	negl-z + no-split

NEXT

- Logical modelling of biases in team semantics
- Experimental findings
  1. Degano *et al* (Nat Lang Sem, 2025): ignorance ⇐
  2. NIHIL *et al* (SuB24-25, CogSci25, XPRAG25): scalar, DIST & ES-Quant
  3. Bleotu *et al* (TbiLLC 2025, SuB25): on conjunctive *or*

Modelling biases in team semantics

General methodology

Natural language sentences translated into classical logic formulas interpreted in a team semantics which models both classical and enriched interpretations

$$\textsc{Natural Language} \quad \mapsto \quad \textsc{Logical Language} \quad \begin{array}{l} \nearrow \quad \alpha \quad \textbf{(classical)} \\ \\ \searrow \quad [\alpha]^{+\cdots} \quad \textbf{(enriched)} \end{array}$$

Back to FC challenge

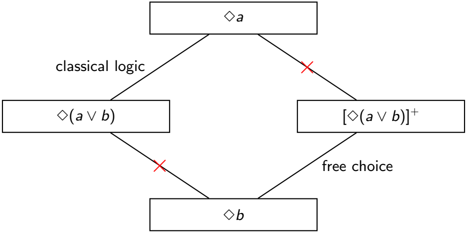


Figure: FC derived only for neglect-zero (NZ) enriched formulas

Modelling biases in team semantics

Team semantics

- Formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Hodges 1997; Väänänen 2007]
  - Classical modal logic: $[M = \langle W, R, V \rangle]$
    $$M, w \models \phi, \text{ where } w \in W$$
  - Team-based modal logic:
    $$M, t \models \phi, \text{ where } t \subseteq W$$
- Two crucial features
  - The empty set is among the possible teams ($\emptyset \subseteq W$) $\mapsto$ zero-models
  - Multi-membered teams can model parallel processing of alternatives $\mapsto$ split states

Neglect-zero & no-split

- Model-theoretic method: biases modelled by disallowing empty (neglect-zero) and multi-membered teams (no-split)
- Syntactic method: biases modelled via new logical atoms/operators
  - Neglect-zero: via **non-emptiness atom** NE which disallows empty teams as possible verifiers [Yang & Väänänen 2017]
    $$M, t \models \mathrm{NE} \text{ iff } t \neq \emptyset$$
  - No-split: via **flattening operator** F which induces pointwise evaluations and therefore mimics ban on parallel processing of alternatives
    $$M, t \models \mathrm{F}\phi \text{ iff for all } w \in t : M, \{w\} \models \phi$$

Modelling biases in team semantics

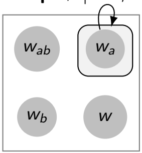
Classical and enriched interpretations

- $\alpha \Rightarrow$ empty team & split allowed $\mapsto$ **classical (literal) meaning**
- $[\alpha]^+ \Rightarrow$ empty team not allowed $\mapsto$ NZ-**enriched**
- $[\alpha]^{++} \Rightarrow$ neither empty team nor split allowed $\mapsto$ NZ+NS-**enriched**

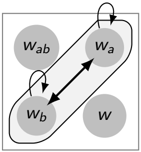
Disjunction in team semantics

- Team T supports a **disjunction** iff T is the union of two subteams, each supporting one of the disjuncts
  - $a \vee b$: empty team allowed $\mapsto$ subteams can be empty (**classical**)
  - $[a \vee b]^+$: empty team not allowed $\mapsto$ subteams cannot be empty (NZ-**enr**)
  - $[a \vee b]^{++}$: neither empty team nor split allowed $\mapsto$ no empty or distinct subteams allowed (NZ+NS -**enriched**)

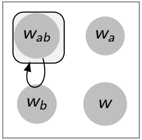
Empty Team Prop: $\emptyset \models \alpha$, for all classical α [**PROP**$= \{a, b\}$; $W = \{w_{ab}, w_a, w_b, w\}$]



$\models a \vee b$
$\not\models [a \vee b]^+$
$\not\models [a \vee b]^{++}$
$\mapsto$ **Zero-model**



$\models a \vee b$
$\models [a \vee b]^+$
$\not\models [a \vee b]^{++}$
$\mapsto$ **No-zero**, **split**



$\models a \vee b$
$\models [a \vee b]^+$
$\models [a \vee b]^{++}$
$\mapsto$ **No-zero**, **no-split**

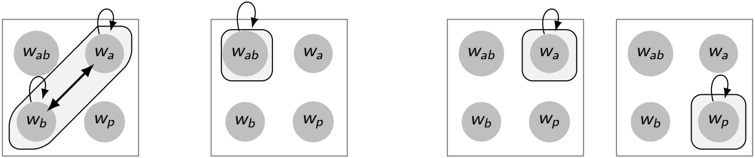
Modelling biases in team semantics

(24) Maria ate an apple or a banana. $[a \vee b]$

a. **No-zero**, **split** verifier: [🍎 | 🍌] ⇝ it might be apple, it might be banana

b. **No-zero**, **no-split** verifier: [🍎 🍌] ⇝ both apple and banana

c. **Zero-models**: [🍎] ; [🍌]   d. Falsifier: [🍐]



No-zero, **split** verifier   **No-zero**, **no-split** verifier   **Zero-model**   Falsifier

Predictions

We derive ignorance & conjunctive interpretations for nz- and ns-enriched formulas resp. with no undesirable effects under negation

(25) M ate an apple or a banana $\rightsquigarrow_{nz}$ It might be an apple and it might be a banana

$[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$ (where R is state-based)

(26) M ate an apple or a banana $\rightsquigarrow_{ns}$ M ate both apple and banana

$[\alpha \vee \beta]^{++} \models \alpha \wedge \beta$

(27) M didn't eat an apple or a banana $\rightsquigarrow_{(nz/ns)}$ M ate neither apple nor banana

$[\neg(\alpha \vee \beta)]^{(++)} \models \neg\alpha \wedge \neg\beta$

and more predictions including FC, homogeneity, ...

A closer look at these predictions

Many enrichments triggered by disjunction

(28) Maria ate an apple or a banana $(\alpha \vee \beta) \rightsquigarrow$

a. Ignorance: speaker doesn't know which $\Diamond_e\alpha \wedge \Diamond_e\beta$
b. Conjunctive interpretation: both $\alpha \wedge \beta$
c. Exclusive interpretation: not both $\neg(\alpha \wedge \beta)$

Two components of full ignorance: possibility vs uncertainty

(29) Maria ate an apple or a banana $\rightsquigarrow$ speaker doesn't know which
[Degano *et al* 2025][1]

a. Possibility: It is possible that M ate an apple and it is possible that M ate a banana $\Diamond_e\alpha \wedge \Diamond_e\beta$
b. Uncertainty: It is uncertain that M ate an apple and it is uncertain that M ate a banana $\neg\Box_e\alpha \wedge \neg\Box_e\beta$

Can we capture all these enrichments as nz- or ns-effects?

[1] Degano, Marty, Ramotowska, MA, Sudo, Romoli, Breheny. "The ups and downs of ignorance." *Natural Language Semantics*, 2025.

Neglect-zero effects on disjunction: predictions of $[\ ]^{+\cdots}$-enrichment

Many no-zero verifiers for nz-enriched disjunction

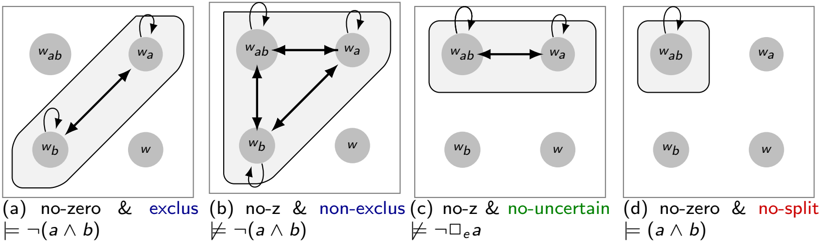


Figure: Models for enriched $[a \vee b]^+$.

1. Neglect-zero enrichment derives **possibility**: $[\alpha \vee \beta]^+ \models \Diamond_e\alpha \wedge \Diamond_e\beta$
2. Neglect-zero + no-split derives conjunctive *or*: $[\alpha \vee \beta]^{++} \models \alpha \wedge \beta$
3. Neglect-zero enrichment does not derive exclusive readings;
4. Neglect-zero enrichment does not derives uncertain inferences $\mapsto$ in contrast to standard neo-Gricean approach to ignorance $\Leftarrow$

Two derivations of full ignorance

1. Standard neo-Gricean derivation [Sauerland 2004]

   (i) Uncertainty derived through quantity reasoning

   (30) $\alpha \vee \beta$ ASSERTION

   (31) $\neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY (from QUANTITY)

   (ii) Possibility derived from uncertainty and quality about assertion

   (32) $\Box_e(\alpha \vee \beta)$ QUALITY ABOUT ASSERTION

   (33) $\Rightarrow \Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY

2. Neglect-zero derivation

   (i) Possibility derived as neglect-zero effect

   (34) $\alpha \vee \beta$ ASSERTION

   (35) $\Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY (from NEGLECT-ZERO)

   (ii) Uncertainty derived from possibility and scalar reasoning

   (36) $\neg(\alpha \wedge \beta)$ SCALAR IMPLICATURE

   (37) $\Rightarrow \neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY

The ups and downs of ignorance [Degano et al 2025]

Marco Degano

Paul Marty

Sonia Ramotowska

Yasu Sudo

Jacopo Romoli

Richard Breheny

Degano, M., Marty, P., Ramotowska, S., MA, Sudo, Y., Romoli, J., Breheny, R. The ups and downs of ignorance. *Nat Lang Semantics* 33, 1–41 (2025)

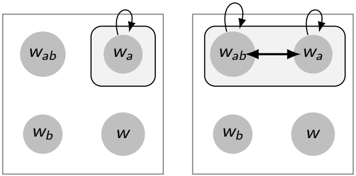
The ups and downs of ignorance [Degano et al 2025]

Contrasting predictions of competing accounts of ignorance

- Neo-Gricean: No possibility without uncertainty
- Neglect-zero: Possibility derived independently from uncertainty

Experimental findings

- Using adapted mystery box paradigm, compared conditions in which
  - both uncertainty and possibility are false [**zero-model**]
  - uncertainty false but possibility true [no-zero, no-uncertain model]
- Zero-models far more rejected (66%) than no-uncertain model (5%)
  - ⇒ Evidence that possibility can arise without uncertainty
- A challenge for the traditional neo-Gricean approach



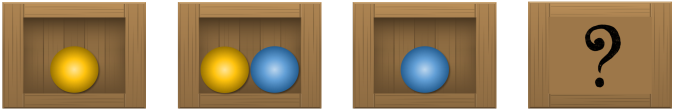
(a) **zero-model** (b) no-uncertain

Figure: Models for $(a \vee b)$

The mystery box paradigm[2]

The display

3 overt boxes containing colored balls and 1 mystery box.



The rule

The mystery box has the same contents as one of the overt boxes.

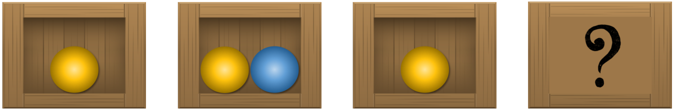
Examples

'The mystery box contains a yellow ball or a blue ball.'	$Y \vee B$	**True**
'The mystery box contains a yellow ball or a green ball.'	$Y \vee G$	**False**

[2] Adapted from Noveck 2001; see also Crnic et al. 2015

TARGET-1 (no-uncertain)

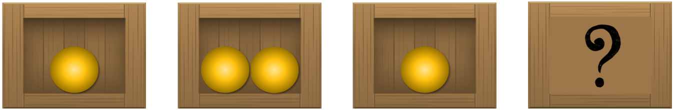
'The mystery box contains a yellow ball or a blue ball.' $Y \vee B$



UNCERTAINTY	$\neg\Box_e Y \wedge \neg\Box_e B$	**False**
POSSIBILITY	$\Diamond_e Y \wedge \Diamond_e B$	**True**

TARGET-2 (zero-model)

'The mystery box contains a yellow ball or a blue ball.' $Y \vee B$



UNCERTAINTY	$\neg\Box_e Y \wedge \neg\Box_e B$	**False**
POSSIBILITY	$\Diamond_e Y \wedge \Diamond_e B$	**False**

Hypothesis
If POSSIBILITY cannot arise without UNCERTAINTY, there should be no difference in responses between TARGET-1 and TARGET-2.
Results
Target-2 far more rejected (66%) than Target-1 (5%)

Conclusions

- Evidence that possibility can arise without uncertainty
- A challenge for the traditional neo-gricean approach

Conclusions

- FC, ignorance: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Nihil hypothesis: some strengthenings due to cognitive tendencies rather than Gricean reasoning
  - FC, possibility and related inferences as neglect-zero effects

    Literal meanings (classical fragment) + cognitive factor (NZ) $\Rightarrow$ FC, possibility, etc

  - Conjunctive *or*, homogeneity as no-zero + no-split effect

    Literal meanings (classical fragment) + cognitive factors (NZ, NS) $\Rightarrow$ conjunctive *or*, homogeneity

- Implementation in team-based modal logics
- Ups and downs of ignorance: a challenge for the neo-Gricean approach
- Appendix:
  - BSML; BSML$^{\rightarrow}$; qBSML$^{\rightarrow}$; Experimenting with zero-models.

THANK YOU![3]

[3] This work is supported by NWO OC project *Nothing is Logical* (grant no 406.21.CTW.023).

Appendix

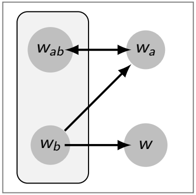
BSML: Classical ML + NE [focus on neglect-zero modelled syntactically]

Language

$$\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \mathrm{NE}$$

Bilateral team semantics

Given Kripke model $M = \langle W, R, V\rangle$ & teams/states $s, t, t' \subseteq W$



$M, s \models p$	iff	for all $w \in s : V(w, p) = 1$
$M, s \mathrel{=\!\!\mid} p$	iff	for all $w \in s : V(w, p) = 0$
$M, s \models \neg\phi$	iff	$M, s \mathrel{=\!\!\mid} \phi$
$M, s \mathrel{=\!\!\mid} \neg\phi$	iff	$M, s \models \phi$
$M, s \models \phi \vee \psi$	iff	there are $t, t' : t \cup t' = s$ & $M, t \models \phi$ & $M, t' \models \psi$ $\Leftarrow$
$M, s \mathrel{=\!\!\mid} \phi \vee \psi$	iff	$M, s \mathrel{=\!\!\mid} \phi$ & $M, s \mathrel{=\!\!\mid} \psi$
$M, s \models \phi \wedge \psi$	iff	$M, s \models \phi$ & $M, s \models \psi$
$M, s \mathrel{=\!\!\mid} \phi \wedge \psi$	iff	there are $t, t' : t \cup t' = s$ & $M, t \mathrel{=\!\!\mid} \phi$ & $M, t' \mathrel{=\!\!\mid} \psi$
$M, s \models \Diamond\phi$	iff	for all $w \in s : \exists t \subseteq R[w] : t \neq \emptyset$ & $M, t \models \phi$ $\Leftarrow$
$M, s \mathrel{=\!\!\mid} \Diamond\phi$	iff	for all $w \in s : M, R[w] \mathrel{=\!\!\mid} \phi$ [where $R[w] = \{v \in W \mid wRv\}$]
$M, s \models \mathrm{NE}$	iff	$s \neq \emptyset$
$M, s \mathrel{=\!\!\mid} \mathrm{NE}$	iff	$s = \emptyset$ [Empty Team Prop: $\emptyset \models \alpha$, for all classical α]

Entailment: $\phi_1, \ldots, \phi_n \models \psi$ iff for all M, s: $M, s \models \phi_1, \ldots, M, s \models \phi_n \Rightarrow M, s \models \psi$

Proof Theory: MA, Anttila & Yang (2024); Expressive completeness: Anttila & Knudstorp (2025); Comparisons via translation into Modal Information Logic: Knudstorp *et al* (2025)

Neglect-zero effects in BSML: neglect-zero enrichment

Neglect-zero modelled syntactically using NE

Non-emptiness

NE is supported in a state if and only if the state is not empty

$$\begin{array}{lll} M, s \models \mathrm{NE} & \text{iff} & s \neq \emptyset \\ M, s \dashv \mathrm{NE} & \text{iff} & s = \emptyset \end{array}$$

Neglect-zero enrichment function

For NE-free α, $[\alpha]^+$ defined as follows:

$$\begin{array}{rcl} [p]^+ & = & p \wedge \mathrm{NE} \\ [\neg\alpha]^+ & = & \neg[\alpha]^+ \wedge \mathrm{NE} \\ [\alpha \vee \beta]^+ & = & ([\alpha]^+ \vee [\beta]^+) \wedge \mathrm{NE} \\ [\alpha \wedge \beta]^+ & = & ([\alpha]^+ \wedge [\beta]^+) \wedge \mathrm{NE} \\ [\Diamond\alpha]^+ & = & \Diamond[\alpha]^+ \wedge \mathrm{NE} \end{array}$$

$[\ ]^+$ enriches formulas with the requirement to satisfy NE distributed along each of their subformulas

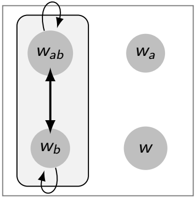
BSML$^{\rightarrow}$: adding implication

- A state s supports an implication $\phi \rightarrow \psi$ iff

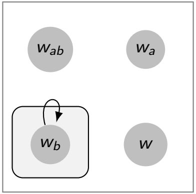
  (i) every subset of s that supports ϕ also supports ψ;
  (ii) there is a subset of s which supports ϕ.

  $M, s \models \phi \rightarrow \psi$ iff (i) for all $t \subseteq s: \; M, t \models \phi \;\Rightarrow\; M, t \models \psi$ & (ii) $\exists t \subseteq s: \; M, t \models \phi$

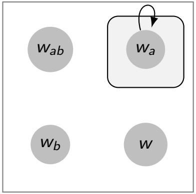
  Because of empty team property, condition (ii) trivial for classical $\alpha \rightarrow \beta$, but crucially non-trivial for enriched $[\alpha \rightarrow \beta]^{+} = ([\alpha]^{+} \rightarrow [\beta]^{+}) \wedge \text{NE}$



No-zero verifier
$\models a \rightarrow b$; $\models [a \rightarrow b]^{+}$



Zero-model
$\models a \rightarrow b$; $\not\models [a \rightarrow b]^{+}$



Falsifier
$=\!\mid a \rightarrow b$; $=\!\mid [a \rightarrow b]^{+}$

- An enriched implication requires the antecedent to be a live possibility:

(38) If it is raining, M will go home $\rightsquigarrow_{nz}$ it might be raining
$[\alpha \rightarrow \beta]^{+} \models \Diamond_e \alpha$ (where R is state-based)

qBSML$^{\rightarrow}$: adding quantifiers (MA & vOrtmondt 2023)[4]

- A quantified modal language interpreted in models $\mathcal{M} = \langle W, D, R, I \rangle$
- **Teams** defined as sets of world-assignment pairs
- A state s supports $\forall x \phi$ iff $s[x]$, the universal x-extension of s, supports ϕ:

$$\mathcal{M}, s \models \forall x \varphi \text{ iff } \mathcal{M}, s[x] \models \varphi$$

- The universal x-extension of $s \mapsto s$ updated with all possible values for x

$$s = \boxed{\langle w, \lambda \rangle} \mapsto s[x] = \begin{array}{|c|} \hline \langle w, x \rightarrow a \rangle \\ \hline \langle w, x \rightarrow b \rangle \\ \hline \langle w, x \rightarrow c \rangle \\ \hline \langle w, x \rightarrow d \rangle \\ \hline \ldots \\ \hline \end{array}$$

Figure: An initial max info state and its universal x-extension

- The NE-free fragment of qBSML$^{\rightarrow}$ is equivalent to classical quantified modal logic:

$$\alpha \models_{qBSML^{\rightarrow}} \beta \text{ iff } \alpha \models_{QML} \beta \qquad [\text{if } \alpha, \beta \text{ are NE-free}]$$

[4] MA & vOrmondt, Modified numerals and split disjunction. *J of Log Lang and Inf* (2023)

Neglect-zero effects on quantifiers

Predictions of qBSML$^{\rightarrow}$ (with max info)

(39) Less than three squares are black $\mapsto \forall xyz((Sx \wedge Bx \wedge \ldots) \rightarrow (x = y \vee \ldots))$

- a. Verifier: [■, □, ■]
- b. Falsifier: [■, ■, ■]
- c. Zero-models: [□, □, □]; [▲, ▲, ▲]; . . . $\rightsquigarrow_{nz}$ there are black squares

(40) Every square is black. $\mapsto \forall x(Sx \rightarrow Bx)$

- a. Verifier: [■, ■, ■]
- b. Falsifier: [■, □, ■]
- c. Zero-models: [△, △, △]; [▲, ▲, ▲]; . . . $\rightsquigarrow_{nz}$ there are squares

(41) No squares are black. $\mapsto$ (i) $\forall x(Sx \rightarrow \neg Bx)$; (ii) $\neg \exists x(Sx \wedge Bx)$

- a. Verifier: [□, □, □]
- b. Falsifier: [■, □, □]
- c. Zero-models for (i): [△, △, △]; [▲, ▲, ▲]; . . . $\rightsquigarrow_{nz}$ there are squares
- d. Zero-models for (ii): none no neglect-zero effect

(42) Every square is red or white. $\mapsto \forall x(Sx \rightarrow (Rx \vee Wx))$

- a. Verifier: [■, □, ■]
- b. Falsifier: [■, □, ■]
- c. Zero-models: [■, ■, ■]; [□, □, □] $\rightsquigarrow_{nz}$ there are white sqrs & red sqrs

These predictions tested in 3 recent experiments

Experimenting with zero-models

Three inferences

(43) a. Some of the squares are red ⇒ not all of the squares are red [UB]
b. Fewer than 3 squares are white ⇒ there are some white squares [ESQ]
c. Each square is white or red ⇒ there are white squares and red squares [DIST]

Two competing accounts

	UB	ESQ	DIST
Standard implicature view	alt-based	alt-based [?]	alt-based
NIHIL	—	neglect-zero	neglect-zero

Three experiments

- Exp 1: Answering questions about the empty set (O. Bott *et al*, SuB24)
- Exp 2: Priming with zero-models (Klochowicz *et al*, CogSci 2025, SuB25)
- Exp 3: Inferences under cognitive load (Ramotowska & MA, XPRAG25)

Three main conclusions

1. Evidence that DIST differs from UB (Exp 1, Exp 2, Exp 3)
2. Mixed results concerning ESQ (Exp 3)
3. Evidence that DIST & ESQ (but not UB) involve the same cognitive process (Exp 2)

Tentative conclusion While 1 in principle explainable by both views (via lexical access) and 2 by neither view, 3 is a serious challenge for the standard implicature approach

Experimenting with zero-models

Three inferences

(44)
a. Some of the squares are red ⇒ not all of the squares are red [UB]
b. Fewer than 3 squares are white ⇒ there are some white squares [ESQ]
c. Each square is white or red ⇒ there are white squares and red squares [DIST]

Two competing accounts

	UB	ESQ	DIST
Standard implicature view	alt-based	alt-based [?]	alt-based
NIHIL	—	neglect-zero	neglect-zero

Task

All experiments involved a picture verification task and compared:

- Target: literal reading ↦ **true** & enriched reading ↦ **false**

  [■, ■, ■, ■, ■]

- True-control: literal reading ↦ **true** & enriched reading ↦ **true**

  [■, ■, ■, □, □]

- False-control: literal reading ↦ **false** & enriched reading ↦ **false**

  [■, ■, □, □, □]

Exp2: Priming with zero-models (Klochowicz *et al*, CogSci 2025, SuB25)

Four inferences

(45)
a. Some of the squares are black ⇒ not all of the squares are black [UB]
b. Each square is red or white ⇒ there are white and red squares [DIST]
c. At most 2 squares are black ⇒ there are some black squares [**ESQ**-s]
d. Fewer than 3 squares are black ⇒ there are some black squares [ESQ]

Two competing accounts

	UB	ESQ(-s)	DIST
Standard implicature view	alt-based	alt-based [?]	alt-based
NIHIL	—	neglect-zero	neglect-zero

The study

- 3 sub-experiments tested whether frequency of ESQ changed after participants were primed to suspend other enrichments in (45-a)-(45-c)

  (1) UB ⇒ ESQ; (2) DIST ⇒ ESQ; (3) ESQ-s ⇒ ESQ
- Results:
  1. No priming between UB and ESQ
  2. Semantic priming between DIST and ESQ
  3. No trial-to-trial priming from ESQ-s to ESQ but spill-over and adaptation effects
- Tentative conclusion: ESQ and DIST (but not UB) involve the same cognitive process, as predicted by neglect-zero hypothesis

Exp2: Better-picture paradigm (adapted from L. Bott & Chemla 2016)

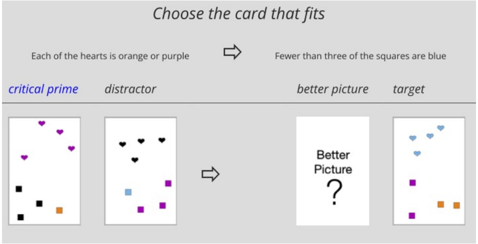


Figure: DIST $\Rightarrow$ ESQ

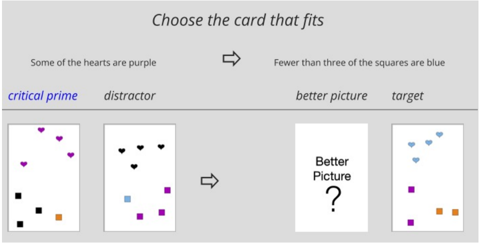


Figure: UB $\Rightarrow$ ESQ

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