N∅thing is Logical

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N∅thing is logical (Nihil)

- Goal of the project: a formal account of a class of natural language inferences which deviate from classical logic
- Common assumption: these deviations are not logical mistakes, but consequence of pragmatic enrichment
- Strategy: develop *logics of conversation* which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency as crucial pragmatic/cognitive factor
- Main conclusion: deviations from classical logic consequence of pragmatic enrichments albeit not of the canonical Gricean kind

Non-classical inferences

Free choice (FC)

(1) $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

(2) Deontic FC inference [Kamp 1973]

a. You may go to the beach *or* to the cinema.
b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(3) Epistemic FC inference [Zimmermann 2000]

a. Mr. X might be in Victoria *or* in Brixton.
b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

Ignorance

(4) The prize is in the attic *or* in the garden $\rightsquigarrow$ speaker doesn't know where
(5) ? I have two *or* three children. [Grice 1989]

- In the standard approach, ignorance inferences are conversational implicatures
- Less consensus on FC inferences analysed as conversational implicatures; grammatical scalar implicatures; semantic entailments; ...

Novel hypothesis: neglect-zero

- FC and ignorance inferences are [$\neq$ semantic entailments]
  - Not the result of Gricean reasoning [$\neq$ conversational implicatures]
  - Not the effect of applications of covert grammatical operators [$\neq$ scalar implicatures]
- But rather a consequence of something else speakers do in conversation, namely,

  NEGLECT-ZERO
  when interpreting a sentence speakers create structures representing reality[1] and in doing so they systematically neglect structures which verify the sentence by virtue of an empty configuration (*zero-models*)

- Tendency to neglect zero-models follows from the difficulty of the cognitive operation of evaluating truths with respect to empty witness sets [Nieder 2016, Bott et al, 2019]

[1] Johnson-Laird (1983) *Mental Models*. Cambridge University Press.

Novel hypothesis: neglect-zero

Illustrations

(6) Every square is black.

a. Verifier: [■, ■, ■]
b. Falsifier: [■, □, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]; [▲, ▲, ▲]; . . .

(7) Less than three squares are black.

a. Verifier: [■, □, ■]
b. Falsifier: [■, ■, ■]
c. Zero-models: [ ]; [□, □, □]; [△, △, △]; [◇, ▲, ◇]; [▲, ▲, ▲]; . . .

- Cognitive difficulty of zero-models confirmed by experimental findings from number cognition and has been argued to explain
  - the special status of 0 among the natural numbers [Nieder, 2016]
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (*less* vs *more*) [Bott et al., 2019]
  - existential import & connexive principles from Aristotle (*every A is B* ⇒ *some A is B*; *not (if A then not A)*) [MA & Knudstorp, 2024]
- Core idea: tendency to neglect zero-models, assumed to be operative in ordinary conversation, explains FC and related inferences

Novel hypothesis: neglect-zero

Illustrations

(8) It is raining.

- a. Verifier: [//// //// ////]
- b. Falsifier: [☼☼☼]
- c. Zero-models: none

(9) It is snowing.

- a. Verifier: [❄❄❄]
- b. Falsifier: [☼☼☼]; [//// //// ////]; . . .
- c. Zero-models: none

(10) It is raining or snowing.

- a. Verifier: [//// //// //// | ❄❄❄]
- b. Falsifier: [☼☼☼]
- c. Zero-models: [//// //// ////]; [❄❄❄]

- Two models in (10-c) are **zero-models** because they verify the sentence by virtue of an empty witness for one of the disjuncts
- Ignorance effects arise because such zero-models are cognitively taxing and therefore disregarded in everyday reasoning and conversation

A closer look at the disjunctive case

(11) It is raining or snowing.

a. Verifier: [//// //// //// | ❄❄❄] [$\Leftarrow$ "split" state]

b. Falsifier: [☼☼☼]

c. Zero-models: [//// //// ////]; [❄❄❄]

- The "split" verifier in (11-a) involves the entertainment of two alternatives $\mapsto$ arguably also a cognitively difficult operation

  NO-SPLIT CONJECTURE [Klochowicz, Sbardolini & MA 2024]
  the ability to split states (entertain multiple alternatives) is acquired late

- The combination of neglect-zero & no-split can explain non-classical inferences observed in pre-school children [Singh *et al* 2016]

  (12) The boy is holding an apple or a banana = The boy is holding an apple and a banana $(\alpha \vee \beta) \equiv (\alpha \wedge \beta)$

  (13) The boy is not holding an apple or a banana = The boy is neither holding an apple nor a banana $\neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta$

  (14) Every boy is holding an apple or a banana = Every boy is holding an apple and a banana $\forall x(\alpha \vee \beta) \equiv \forall x(\alpha \wedge \beta)$

BSML: teams and bilateralism

- Team semantics: formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Hodges 1997; Väänänen 2007]

Classical vs team-based modal logic

$[M = \langle W, R, V \rangle]$

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

Bilateral state-based modal logic (BSML)

- Teams $\mapsto$ information states [Dekker93; Groenendijk$^+$96; Ciardelli$^+$19]
- Assertion & rejection conditions modelled rather than truth

$$M, s \models \phi, \text{ “}\phi\text{ is assertable in } s\text{”}, \quad \text{with } s \subseteq W$$
$$M, s \;=\!\!|\; \phi, \text{ “}\phi\text{ is rejectable in } s\text{”}, \quad \text{with } s \subseteq W$$

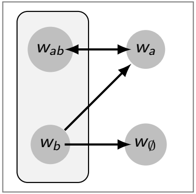
- Neglect-zero tendency modelled by NE [Yang & Väänänen 2017]
- BSML^{F}: No-split modelled via a flattening operator F

BSML: Classical Modal Logic + $\textsc{ne}$

Language

$$\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \textsc{ne}$$

Bilateral team semantics



Given a Kripke model $M = \langle W, R, V \rangle$ & states $s, t, t' \subseteq W$

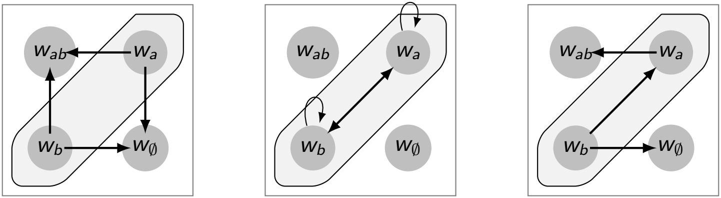
$$
\begin{array}{rll}
M, s \models p & \text{iff} & \text{for all } w \in s : V(w, p) = 1 \\
M, s \mathrel{=\!\!|} p & \text{iff} & \text{for all } w \in s : V(w, p) = 0 \\
M, s \models \neg\phi & \text{iff} & M, s \mathrel{=\!\!|} \phi \\
M, s \mathrel{=\!\!|} \neg\phi & \text{iff} & M, s \models \phi \\
M, s \models \phi \vee \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \;\&\; M, t \models \phi \;\&\; M, t' \models \psi \\
M, s \mathrel{=\!\!|} \phi \vee \psi & \text{iff} & M, s \mathrel{=\!\!|} \phi \;\&\; M, s \mathrel{=\!\!|} \psi \\
M, s \models \phi \wedge \psi & \text{iff} & M, s \models \phi \;\&\; M, s \models \psi \\
M, s \mathrel{=\!\!|} \phi \wedge \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \;\&\; M, t \mathrel{=\!\!|} \phi \;\&\; M, t' \mathrel{=\!\!|} \psi \\
M, s \models \Diamond\phi & \text{iff} & \text{for all } w \in s : \exists t \subseteq R[w] : t \neq \emptyset \;\&\; M, t \models \phi \\
M, s \mathrel{=\!\!|} \Diamond\phi & \text{iff} & \text{for all } w \in s : M, R[w] \mathrel{=\!\!|} \phi \\
M, s \models \textsc{ne} & \text{iff} & s \neq \emptyset \\
M, s \mathrel{=\!\!|} \textsc{ne} & \text{iff} & s = \emptyset \qquad\qquad [\text{where } R[w] = \{v \in W \mid wRv\}]
\end{array}
$$

Validity: $\phi_1, \ldots, \phi_n \models \psi$ iff for all M, s: $M, s \models \phi_1, \ldots, M, s \models \phi_n \Rightarrow M, s \models \psi$

Proof Theory: See Anttila 2021; Anttila et al. 2024.

Team-sensitive constraints on accessibility relation

- R is indisputable in (M, s) iff $\forall w, v \in s : R[w] = R[v]$
  $\mapsto$ all worlds in s access exactly the same set of worlds
- R is state-based in (M, s) iff $\forall w \in s : R[w] = s$
  $\mapsto$ all and only worlds in s are accessible within s



(a) indisputable

(b) state-base (& indisputable)

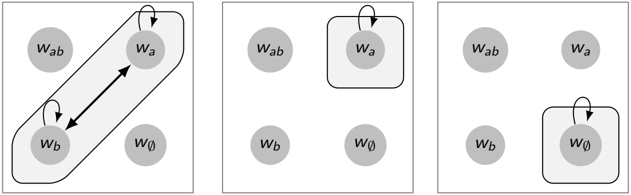
(c) neither

Deontic vs epistemic modals

- Difference deontic vs epistemic modals captured by different properties of accessibility relation:
  - Epistemics: R is state-based
  - Deontics: R is possibly indisputable (e.g. in performative uses)

Neglect-zero effects in BSML: split disjunction

- A state s supports a **disjunction** $(\phi \vee \psi)$ iff s is the union of two substates, each supporting one of the disjuncts



(d) Verifier (e) **Zero-model** (f) Falsifier

Figure: Models for $(a \vee b)$.

- $\{w_a\}$ verifies $(a \vee b)$ by virtue of an empty witness for the second disjunct, $\{w_a\} = \{w_a\} \cup \emptyset$ & $M, \emptyset \models b$ [$\mapsto$ **zero-model**]
- Main idea: define neglect-zero enrichments, $[\ ]^+$, whose core effect is to rule out such zero-models
- Implementation: $[\ ]^+$ defined using NE $(s \models \mathrm{NE}$ iff $s \neq \emptyset)$, which models neglect-zero in the logic

BSML: neglect-zero enrichment

Non-emptiness

$\textsc{ne}$ is supported in a state if and only if the state is not empty

$$M, s \models \textsc{ne} \text{ iff } s \neq \emptyset$$

Neglect-zero enrichment

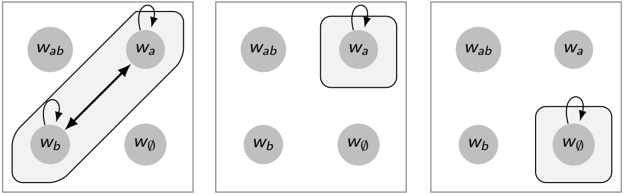
For $\textsc{ne}$-free α, $[\alpha]^+$ defined as follows:

$$\begin{array}{rcl}
[p]^+ & = & p \wedge \textsc{ne} \\
[\neg\alpha]^+ & = & \neg[\alpha]^+ \wedge \textsc{ne} \\
[\alpha \vee \beta]^+ & = & ([\alpha]^+ \vee [\beta]^+) \wedge \textsc{ne} \\
[\alpha \wedge \beta]^+ & = & ([\alpha]^+ \wedge [\beta]^+) \wedge \textsc{ne} \\
[\Diamond\alpha]^+ & = & \Diamond[\alpha]^+ \wedge \textsc{ne}
\end{array}$$

$[\ ]^+$ enriches formulas with the requirement to satisfy $\textsc{ne}$ distributed along each of their subformulas

Neglect-zero effects in BSML: enriched disjunction

- *s* supports an **enriched disjunction** $[\phi \vee \psi]^+$ iff *s* is the union of two non-empty substates, each supporting one of the disjuncts



(a) $\models [a \vee b]^+$ (b) $\not\models [a \vee b]^+$ (c) $\Dashv [a \vee b]^+$

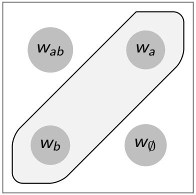
- An enriched disjunction requires both disjuncts to be live possibilities

(15) It is raining or snowing $\rightsquigarrow$ It might be raining and it might be snowing
$[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$ (where *R* is state-based)

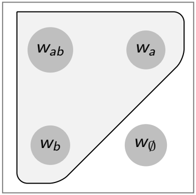
- Main result: in BSML $[\ ]^+$-enrichment has non-trivial effect only when applied to *positive* disjunctions[2]
  - $\mapsto$ we derive FC and related effects (for enriched formulas);
  - $\mapsto$ $[\ ]^+$-enrichment vacuous under single negation.

[2] MA (2022) Logic and Conversation: the case of free choice. *Semantics and Pragmatics* 15(5).

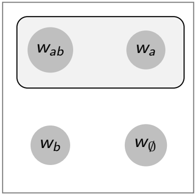
More no-zero verifiers for enriched disjunction



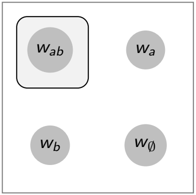
(d) no-zero & scalar $\models \neg(a \wedge b)$



(e) no-zero, non-scalar $\not\models \neg(a \wedge b)$



(f) no-zero, non-scalar & no-uncertain $\not\models \neg\Box_e a$

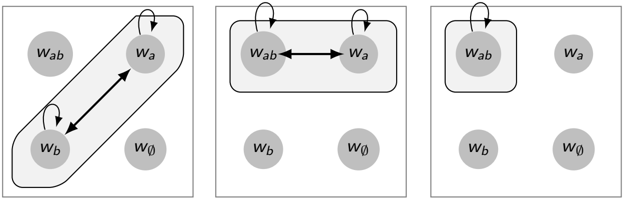


(g) no-zero, non-scalar, no-uncertain & no-split $\models (a \wedge b)$

Figure: Models for enriched $[a \vee b]^+$.

Neglect-zero and no-split

- More no-zero verifiers for $a \vee b$:



(a) scalar (b) no-uncertain (c) no-split

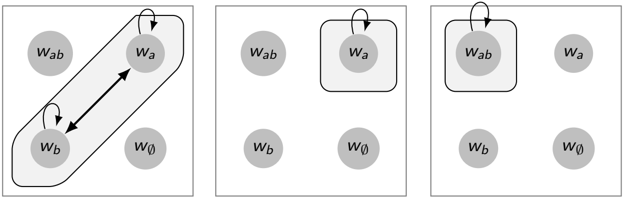
- $\{w_{ab}\}$ is a no-split verifier for the disjunction: no alternatives entertained;
- Conjecture: only no-split verifiers accessible to 'conjunctive' pre-school children [Klochowicz, Sbardolini, MA, 2024]
- Implementation: uses flattening operator F

$$M, s \models \mathrm{F}\phi \text{ iff for all } w \in s : M, \{w\} \models \phi$$

Flattening $\mapsto$ formulas always interpreted wrt to singleton substates
- Combination of no-split and **no-zero** yields conjunctive *or*:

$$\begin{array}{rcl} [\mathrm{F}(\alpha \vee \beta)]^+ & \equiv & \alpha \wedge \beta \\ [\neg\mathrm{F}(\alpha \vee \beta)]^+ & \equiv & \neg\alpha \wedge \neg\beta \end{array}$$

Illustration



(d) no-zero & split (e) zero & no-split (f) no-zero & no-split

Figure: Combination of no-split and no-zero yields conjunctive *or*

(16) It is raining or snowing.

a. No-zero & split: [|] [adult-like]

b. Zero & (no-)split: [ ] [logician]

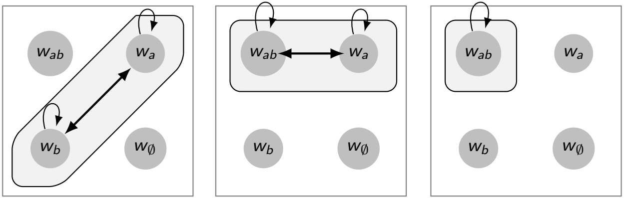
c. No-zero & no-split: [**&**] ['conjunctive' children]

Predicted inferences

- $[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$; $\not\models \alpha \wedge \beta$ [adult-like]
- $\mathrm{F}(\alpha \vee \beta) \not\models \Diamond_e \alpha \wedge \Diamond_e \beta$; $\not\models \alpha \wedge \beta$ [logician]
- $[\mathrm{F}(\alpha \vee \beta)]^+ \models \alpha \wedge \beta$ ['conjunctive' children]

Neglect-zero effects in BSML: possibility vs uncertainty

- More no-zero verifiers for $a \vee b$:



(a) scalar (b) no-uncertain (c) no-split

- Two components of full ignorance ('speaker doesn't know which'):[3]

(17) It is raining or it is snowing $(\alpha \vee \beta) \rightsquigarrow$

a. Uncertainty: $\neg\Box_e\alpha \wedge \neg\Box_e\beta$

b. Possibility: $\Diamond_e\alpha \wedge \Diamond_e\beta$ (equiv $\neg\Box_e\neg\alpha \wedge \neg\Box_e\neg\beta$)

- **Fact:** Only possibility derived as neglect-zero effect:
  - $[a \vee b]^+ \models \Diamond_e a \wedge \Diamond_e b$ (if R is state-based)
  - $\{w_{ab}, w_a\} \models [a \vee b]^+$, but $\not\models \neg\Box_e a$
  - $\{w_{ab}\} \models [a \vee b]^+$, but $\not\models \neg\Box_e a$; $\not\models \neg\Box_e b$

[3] Degano, Marty, Ramotowska, MA, Breheny, Romoli, Sudo. SuB & XPRAG, 2023.

Two derivations of full ignorance

1. Standard neo-Gricean derivation [Sauerland 2004]

   (i) Uncertainty derived through quantity reasoning

   (18) $\alpha \vee \beta$ ASSERTION

   (19) $\neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY (from QUANTITY)

   (ii) Possibility derived from uncertainty and quality about assertion

   (20) $\Box_e(\alpha \vee \beta)$ QUALITY ABOUT ASSERTION

   (21) $\Rightarrow \Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY

2. Neglect-zero derivation

   (i) Possibility derived as neglect-zero effect

   (22) $\alpha \vee \beta$ ASSERTION

   (23) $\Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY (from NEGLECT-ZERO)

   (ii) Uncertainty derived from possibility and scalar reasoning

   (24) $\neg(\alpha \wedge \beta)$ SCALAR IMPLICATURE

   (25) $\Rightarrow \neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY

Novel hypothesis: neglect-zero

Contrasting predictions of competing accounts of ignorance

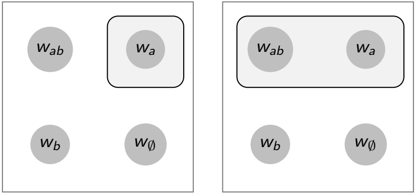
- Neo-Gricean: No possibility without uncertainty
- Neglect-zero: Possibility derived independently from uncertainty

Experimental findings [Degano *et al* 2023]

- Using adapted mystery box paradigm, compared conditions in which
  - both uncertainty and possibility are false [**zero-model**]
  - uncertainty false but possibility true [no-zero, no-uncertain model]
- Less acceptance when possibility is false (95% vs **44%**)

⇒ Evidence that possibility can arise without uncertainty

- A challenge for the traditional neo-gricean approach



(d) **zero-model** (e) no-uncertain

Novel hypothesis: neglect-zero

Comparison with competing accounts of FC inference

	NS FC	Dual Prohib	Universal FC	Double Neg	WS FC
Neo-Gricean	yes	yes	no	?	no
Grammatical	yes	yes*	yes	no*	no*
Semantic	yes	no*	yes	no*	no
Neglect-zero	yes	yes	yes	yes	yes

Argument in favor of neglect-zero hypothesis

- Empirical coverage: FC sentences give rise to a complex pattern of inferences

(26) a. $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Narrow Scope FC]
b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$ [Dual Prohibition]
c. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$ [Universal FC]
d. $\neg\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Double Negation FC]
e. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Wide Scope FC]

- Captured by neglect-zero approach implemented in BSML[4]
- Most other approaches need additional assumptions

[4] MA (2022). Logic and conversation: the case of FC. *Sem & Pra*, 15(5).

The data

(27) Dual Prohibition [Alonso-Ovalle 2006, Marty *et al.* 2021]

a. You are not allowed to eat the cake or the ice-cream.
$\rightsquigarrow$ You are not allowed to eat either one.

b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$

(28) Universal FC [Chemla 2009]

a. All of the boys may go to the beach or to the cinema.
$\rightsquigarrow$ All of the boys may go to the beach and all of the boys may go to the cinema.

b. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$

(29) Double Negation FC [Gotzner *et al.* 2020]

a. Exactly one girl cannot take Spanish or Calculus.
$\rightsquigarrow$ One girl can take neither of the two and each of the others can choose between them.

b. $\exists x(\neg\Diamond(\alpha(x) \vee \beta(x)) \wedge \forall y(y \neq x \rightarrow \neg\neg\Diamond(\alpha(y) \vee \beta(y)))) \rightsquigarrow$
$\exists x(\neg\Diamond\alpha(x) \wedge \neg\Diamond\beta(x) \wedge \forall y(y \neq x \rightarrow (\Diamond\alpha(y) \wedge \Diamond\beta(y))))$

(30) Wide Scope FC [Zimmermann 2000, Hoeks *et al.* 2017]

a. Detectives may go by bus or they may go by boat.
$\rightsquigarrow$ Detectives may go by bus and may go by boat.

b. Mr. X might be in Victoria or he might be in Brixton.
$\rightsquigarrow$ Mr. X might be in Victoria and might be in Brixton.

c. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

Neglect-zero effects in BSML: predictions on FC inferences

After enrichment

- We derive both wide and narrow scope FC inferences:
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Universal FC: $[\forall x \Diamond(\alpha \vee \beta)]^+ \models \forall x(\Diamond\alpha \wedge \Diamond\beta)$
  - Double negation FC: $[\neg\neg\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ (if R is indisputable)
- while no undesirable side effects obtain with other configurations:
  - Dual prohibition: $[\neg\Diamond(\alpha \vee \beta)]^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Before enrichment

- The NE-free fragment of BSML is equivalent to classical modal logic:

$$\alpha \models_{BSML^{\emptyset}} \beta \text{ iff } \alpha \models_{CML} \beta \qquad [\alpha, \beta \text{ are NE-free}]$$

- But we can capture the infelicity of epistemic contradictions [Yalcin, 2007] by putting team-based constraints on the accessibility relation:
  1. Epistemic contradiction: $\Diamond\alpha \wedge \neg\alpha \models \bot$ (if R is state-based)
  2. Non-factivity: $\Diamond\alpha \not\models \alpha$

BSML & related systems: information states vs possible worlds

- Failure of bivalence in BSML

$$M, s \not\models p \;\&\; M, s \not\models \neg p, \text{ for some info state } s$$

- Info states: less determinate than possible worlds
  - just like truthmakers, situations, possibilities, ...
- Technically:
  - Truthmakers/possibilities: points in a partially ordered set
  - Info states: sets of possible worlds, also elements of a partially ordered set, the Boolean lattice $Pow(W)$
- Thus systems using these structures are closely connected, although might diverge in motivation:
  - Truthmaker & possibility semantics: description of ontological structures in the world
  - BSML: explaining patterns in inferential & communicative human activities
- Next:
  - Comparison via translations in Modal Information Logic [vBenthem19]

BSML & related systems: comparisons via translation

- Modal Information Logic (MIL) (van Benthem, 1989, 2019):[5] common ground where related systems can be interpreted and their connections and differences can be explored
- Next: (simplified) translations into MIL of the following systems:
  - BSML
  - Truthmaker semantics (Fine)
  - Possibility semantics (Humberstone, Holliday)
  - Inquisitive semantics (Ciardelli, Groenendijk & Roelofsen)

  (cf. Gödel's (1933) translation of intuitionistic logic into modal logic)
- Focus on propositional fragments
  - disjunction
  - negation
- (Based on work in progress with Søren B. Knudstorp, Nick Bezhanishvili, Johan van Benthem and Alexandru Baltag)

[5] Johan van Benthem (2019) Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic*.

Modal Information Logic (MIL)

Language

$$\phi \quad ::= \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \langle sup \rangle \phi\psi$$

where $p \in A$.

Models and interpretation

Formulas are interpreted on triples $M = (X, \leq, V)$ where $\leq$ is a partial order

$$\begin{array}{rll}
\mathcal{M}, x \models p & \text{iff} & x \in V(p) \\
\mathcal{M}, x \models \neg\phi & \text{iff} & \mathcal{M}, x \not\models \phi \\
\mathcal{M}, x \models \phi \wedge \psi & \text{iff} & \mathcal{M}, x \models \phi \ \text{ and } \ \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \phi \vee \psi & \text{iff} & \mathcal{M}, x \models \phi \ \text{ or } \ \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \langle sup \rangle \phi\psi & \text{iff} & \text{there are } y, z : x = sup_{\leq}(y, z) \ \& \ \mathcal{M}, y \models \phi \ \& \ \mathcal{M}, z \models \psi
\end{array}$$

$$[\leq]\phi = \neg\langle sup \rangle(\neg\varphi)\top$$

$$\mathcal{M}, x \models [\leq]\phi \quad \text{iff} \quad \text{for all } y : y \leq x \ \Rightarrow \ \mathcal{M}, y \models \phi$$

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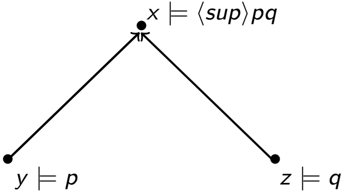
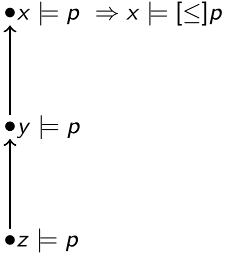
Examples





Translations into Modal Information Logic

- BSML (non-modal NE-free fragment): $\leq$ is subset relation $\subseteq$

$$\begin{array}{rcl} & \dots & \\ (\neg\phi)^+ & = & (\phi)^- \\ (\neg\phi)^- & = & (\phi)^+ \\ (\phi \vee \psi)^+ & = & \langle sup\rangle(\phi)^+(\psi)^+ \\ (\phi \vee \psi)^- & = & (\phi)^- \wedge (\psi)^- \\ (\phi \wedge \psi)^+ & = & (\phi)^+ \wedge (\psi)^+ \\ (\phi \wedge \psi)^- & = & \langle sup\rangle(\phi)^-(\psi)^- \\ & \dots & \end{array}$$

- Truthmaker semantics (Fine): $\leq$ is "part of" relation

$$\begin{array}{rcl} & \dots & \\ (\neg\phi)^+ & = & (\phi)^- \\ (\neg\phi)^- & = & (\phi)^+ \\ (\phi \vee \psi)^+ & = & (\phi)^+ \vee (\psi)^+ \\ (\phi \vee \psi)^- & = & \langle sup\rangle(\phi)^-(\psi)^- \\ (\phi \wedge \psi)^+ & = & \langle sup\rangle(\phi)^+(\psi)^+ \\ (\phi \wedge \psi)^- & = & (\phi)^- \vee (\psi)^- \\ & \dots & \end{array}$$

Translations into Modal Information Logic

- Possibility semantics (Humberstone, Holliday)

$$\begin{array}{rcl} & \vdots & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & [\leq]\langle\leq\rangle(tr(\phi) \vee tr(\psi)) \\ & \vdots & \end{array}$$

- Inquisitive semantics (Groenendijk, Roelofsen and Ciardelli)

$$\begin{array}{rcl} & \vdots & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & tr(\phi) \vee tr(\psi) \\ & \vdots & \end{array}$$

Disjunction and Negation

- Three notions of disjunction expressible in MIL:
  - Boolean disjunction: $\phi \vee \psi$
    [classical logic, intuitionistic logic, inquisitive logic]
  - Lifted/split disjunction: $\langle sup \rangle \phi\psi$
    [BSML, dependence logic, team semantics]
  - Cofinal disjunction: $[co](\phi \vee \psi)$ (where $[co]\phi =: [\leq]\langle\leq\rangle\phi$)
    [possibility semantics, dynamic semantics]
- Three notions of negation:
  - Boolean negation: $\neg\phi$
    [classical logic, ...]
  - Bilateral negation: $(\neg\phi)^+ = (\phi)^-$ & $(\neg\phi)^- = (\phi)^+$
    [truthmaker semantics, BSML, ...]
  - Intuitionistic-like negation: $[\leq]\neg\phi$
    [possibility semantics, inquisitive semantics, intuitionistic logic]
- Some combinations:
  - Boolean disjunction + boolean negation $\mapsto$ classical logic
  - Boolean notions in other combinations can generate non-classicality:
    - Boolean disjunction + intuitionistic negation $\mapsto$ intuitionistic logic
  - Classicality also generated by non-boolean combinations:
    - Split disjunction + bilateral negation (classical fragm. BSML)

Conclusions

- FC and ignorance: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Nihil proposal: non-classical inferences consequences of cognitive biases
  - FC and ignorance as neglect-zero effects

    Literal meanings (NE-free fragment) + cognitive factors (NE) $\Rightarrow$ FC & possibility inferences

  - Conjunctive *or* as no-zero + no-split effect

    Literal meanings (NE-free fragment) + cognitive factors (NE, F) $\Rightarrow$ conjunctive *or*

- Implementation in BSML^{F} (a team-based modal logic)
- Differences but also interesting connections with related systems
- MIL useful framework for comparisons via translations

Collaborators & related (future) research

Logic

Proof theory (Anttila, Yang); expressive completeness (Anttila, Knudstorp); bimodal perspective (Knudstorp, Baltag, van Benthem, Bezhanishvili); qBSML (van Ormondt); BiUS & qBiUS (MA); typed BSML (Muskens); connexive logic (Knudstorp & MA);. . .

Language

FC cancellations (Pinton, Hui); modified numerals (vOrmondt); attitude verbs (Yan); conditionals (Flachs); questions (Klochowicz); quantifiers (Klochowicz, Bott, Schlotterbeck); indefinites (Degano); homogeneity (Sbardolini); acquisition (Klochowicz, Sbardolini); experiments (Degano, Klochowicz, Ramotowska, Bott, Schlotterbeck, Marty, Breheny, Romoli, Sudo); . . .

Thank You![6]

[6] This work was supported by NWO OC project *Nothing is Logical* (grant no 406.21.CTW.023).