

Savitch's Algorithm

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Abstract

Savitch's algorithm is a clever divide and conquer solution to the graph reachability problem. The goal is not finding the shortest path from some node s to node t but establishing that there is a path that exists between these two nodes. We assume that the graph G in question is an $n \times n$ adjacency matrix which means that it takes exactly n^2 bits of memory. The key to this algorithm is recursion, so the information does not need to be presented in its entirety. The core idea, although somewhat trivial, is that if a path exists from u to v of length at most 2^i , then there must be a mid-point w . This can be modeled by the equation:

$$R(G, u, v, i) \iff (\exists w)[R(G, u, w, i - 1) \wedge R(G, w, v, i - 1)]$$

Algorithm 1 Savitch's Algorithm

```
1: if  $i = 0$  then
2:   if  $u = v$  then
3:     return True
4:   else if  $(u, v)$  is an edge then
5:     return True
6:   end if
7: else
8:   for every vertex  $w$  do
9:     if  $R(G, u, w, i - 1)$  and  $R(G, w, v, i - 1)$  then
10:      return True
11:    end if
12:  end for
13: end if
14: return False
```
