Problem A.2

a) $T(n) = 36T(n/6) + 2n$

This recurrence is solved with the master theorem.
Here: $a = 36$, $b = 6$, $f(n) = 2n$
$n^{\log_b a} = n^{\log_6 36} = n^2$

$f(n) = 2n$ grows slower than $n^{\log_b a} = n^2$ so it is Case 1 of the Master Theorem.
$f(n) = O(n^c)$ for $c < \log_b a$. Therefore:
$T(n) = \Theta(n^{\log_b a}) = \Theta(n^2)$

b) $T(n) = 5T(\frac{n}{3}) + 17n^{1.2}$

This recurrence is solved using the master theorem
Here: $a = 5$, $b = 3$ and $f(n) = 17n^{1.2}$
$n^{\log_b a} = n^{\log_3 5} = 1.465$

Since $1.2 < 1.465$, $f(n)$ grows slower than $n^{\log_b a}$ so we are in case 1 of the master theorem.
Therefore:

$T(n) = \Theta(n^{\log_3 5})$

c) $T(n) = 12T(n/2) + n^2 \lg n$

Here we use the Master Theorem.

$n^{\log_b a} = n^{\log_2 12} = n^{3.58}$

So $n^{\log_2 12}$ grows faster than $n^2 \log_2 n$ so:

$T(n) = \Theta(n^{\log_2 12})$

d) $T(n) = 3T(\frac{n}{5}) + T(\frac{n}{2}) + 2^n$

Here we don't use the Master Theorem since it dosen't fit but ~~for~~ from ~~standard~~

Standard analysis we can say that for larger "n" the exponential growth takes precedence, Therefore:

$T(n) = \Theta(2^n)$

e) $T(n) = T(2n/5) + T(3n/5) + \Theta(n)$

Here we use the Recursion Tree Analysis.

At root: the cost is cn

$c(2n/5) + c(3n/5) = cn$

This cost continues on every level since

$(2/5) + (3/5) = 1$

Now we analyze the height ~~we~~ where:

$(2/5)^h \cdot n \approx 1$

$n \approx (5/2)^h \Rightarrow h \approx \log_{5/2} n$

$cn \cdot \log_{5/2} n \Rightarrow \log_{5/2} n = \frac{\log n}{\log 5/2} \Rightarrow \Theta(n \log n)$

$T(n) = \Theta(n \log n)$