Lab Questions: Lab Session 10

Deadline: 01.11.2017 11:59pm SGT

Complete all assignments below. For those questions that are marked with an asterisk *, i.e. **Questions 6 and 10**, create the script files as requested. Once you are done with it, submit the file(s) via `iNTU`. Remember to put plenty of comments for all assignments, as this helps us to better understand your program (which might give you higher marks).

Important!!! Make sure your scripts work properly, as we give 0 marks otherwise. Please name the scripts according to the requirements, and upload each file separately and not in a Zip file or similar. Check that your files properly have been uploaded to `iNTU`. The submission system closes at the deadline. Hence after that, you will get no marks for your solution.

1. Test your knowledge and understanding of the set operations. Create two sets `v1,v2` filled with random integer elements and use the four set operators `union`, `intersect`, `difference`, `symmetric difference` to see if the outputs are as expected.

2. Create anonymous functions that calculate the areas of a rectangle, trapezoid, and sphere. Save all of the anonymous functions in a file called `my_functions.py`. Try to call the functions just created from another script. Do you get an error message? Then import the file `my_functions.py`, and try to call them again using the prefix `my_functions.`

   Solution:

   my_functions.py

```
import math
my_rect_area = lambda x,y: x*y
my_trap_area = lambda a,b,h: (a+b)/2 * h
my_sphe_area = lambda r: 4*r**2*math.pi
```

   anon.py

```
# example of utilization
import my_functions
print(my_functions.my_rect_area(3,4))
print(my_functions.my_trap_area(2,4,7))
print(my_functions.my_sphe_area(5))
```

3. The hyperbolic sine for an argument x is defined as:

$$hyperbolicsine(x) = (e^x - e^{-x})/2$$

   Write an anonymous function to implement this. Compare yours to the function **math.sinh** from the `math` module.

   Solution:

hyperbolicsine.py

```
import math
hyperbolicsine = lambda x: (math.exp(x) - math.exp(-x))/2

# example of utilization
print(hyperbolicsine(0.3))
print(math.sinh(0.3))
```

4. A vector can be represented by its rectangular coordinates x and y or by its polar coordinates r and θ. For positive values of x and y, the conversions from rectangular to polar coordinates in the range from 0 to 2π are $r = \sqrt{x^2 + y^2}$ and $\theta = arctan(y/x)$. The function for arctan is `math.atan`. Write a anonymous function `recpol` to receive as input arguments the rectangular coordinates and that returns the corresponding polar coordinates as a 2-element list.

   Solution:

recpol.py

```
import math
recpol = lambda x,y: [math.sqrt(x**2 + y**2),math.atan(y/x)]

# example of utilization
(r,theta) = recpol(2,4)
print(r)
print(theta)
```

5. A recursive definition of a^n where a is an integer and n is a non-negative integer is:

   $a^n = 1$ if $n == 0$

   $a * a^{n-1}$ if $n > 0$

   Write a recursive function called `mypower`, which receives a and n and returns the value of a^n by implementing the above definition. Note: The program should **not** use the exponentiation operator anywhere; this is to be done recursively instead! Test the function.

   Solution:

mypower.py

```
def mypower(a,n):
    ''' Own implementation of the power function
    Format: mypower(float,int)
    returns a float '''

    # the base case
    if 0 == n: return 1
    # otherwise, we just apply the recursive formula
    else: return a * mypower(a,n-1)
```

6. * Combinatorial coefficients `C(n,m)` can be defined recursively as follows:

```
C(n,m) = 1                              ,if m = 0 or m = n
       = C(n-1, m-1) + C(n-1, m)        ,otherwise
```

Write a recursive function `combi.py` to implement this definition. Return the error value -1 if the condition $0 \leq m \leq n$ is not respected.

Solution:

combi.py

```
def combi(n,m):
    '''function that computes the combinatorial coefficient
    for input values n and m.
    Format: combi(int,int)
    returns an integer value'''

    # check the error case
    if m<0 or m>n: return -1
    # check for the base cases
    elif m==0 or m==n: return 1
    # otherwise, just apply the recursive formula
    else: return combi(n-1,m-1) + combi(n-1,m)
```

7. A word is a palindrome, if it reads the same forward or reversed. For instance, 'rotator', 'rotor', 'radar', are all palindromes. Write a **recursive** function `isPalindrome` that checks if the input string is a palindrome. That is, the function takes as an input a string, and returns true (logical 1), if the input string is a palindrome, or false (logical 0) if it is not.
   Example of execution:

```
>>> isPalindrome( 'aba')
 True
>>> isPalindrome( 'taat')
 True
>>> isPalindrome( 'tata')
 False
```

Solution:

isPalindrome.py

```
def isPalindrome( x ):
    ''' Function the checks if the input string x
    is a palindrome.'''
    # base case
    if len(x) <= 1: return True
    # general case
    else:
        if x[0]==x[-1]:
            return isPalindrome(x[1:-1])
        else:
            return False
```

8. What does this function do?

mystery.py

```
def mystery(x,y):
    if y == 1: return x
    else: return x + mystery(x,y-1)
```

Explain what this mystery function does with its two arguments.

Solution:
mystery(x,y) = x*y

9. Write a recursive function `compute_gcd.py` that takes as input two positive integers a and b, and outputs the greatest common divisor of a and b (hint: the greatest common divisor of a and b is the same as the greatest common divisor of a and $b - a$). Return the error value -1 if any of the inputs is not a positive integer.

Solution:

compute_gcd.py

```
def compute_gcd(a,b):
    '''function that computes the greatest common divisor of a and b
    Format: compute_gcd(int,int)
    returns an integer value'''

    # check the error cases
    if a<=0 or b<=0 or int(a)!=a or int(b)!=b: return -1
    # check for the base cases
    elif a==b: return a
    # otherwise, just apply the recursive formula
    elif a<b: return compute_gcd(a,b-a)
    else: return compute_gcd(b,a-b)
```

10. * The Fibonacci numbers is a sequence of numbers F_i:

```
0 1 1 2 3 5 8 13 21 34 ...
```

where F_0 is 0, F_1 is 1, F_2 is 1, F_3 is 2, and in general $F_i = F_{i-1} + F_{i-2}$. That is, the sequence starts with 0 and 1, and all other Fibonacci numbers are obtained by adding the previous two Fibonacci numbers.

In a file `fibonacci.py`, write two independent functions that implement this definition: recursive function `r_fibonacci` and iterative function `i_fibonacci`. Each of the functions will receive one integer argument n, and will return one integer value, which is the n-th Fibonacci number F_n. To test if your functions are correct call them with the same input value.

Which of the functions is easier to write? Which one is faster (measure the time taken when calling each function for $n = 35$)? Draw the tree of the successive recursive calls to the function `r_fibonacci` (starting from F_4) and provide an explanation of the difference in timing (do not submit the drawing on NTU learn, just submit your explanations as comment in the python file submitted).

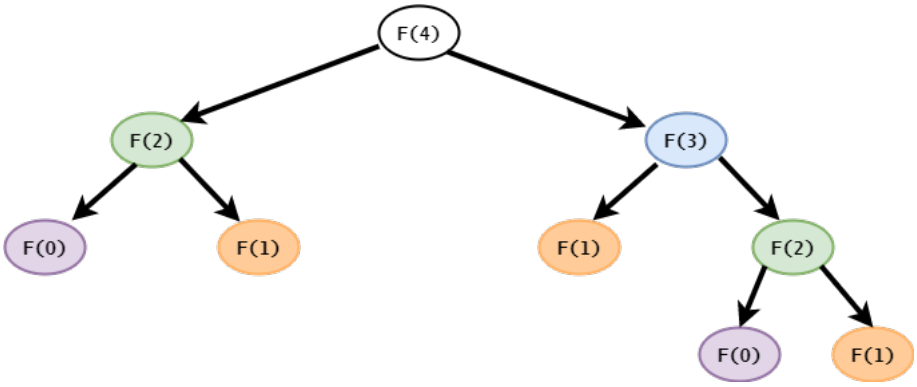
Hint: for the recursive function, in the definition there is one general case but two base cases.

Solution:

fibonacci.py

```
def r_fibonacci(n):
    ''' Recursive implementation of the Fibonacci sequence
    The function returns the n-th Fibonacci number in the sequence
    Format: r_fibonacci(int)
    returns an integer'''

    # the base cases
    if 0 == n: return 0
    elif 1 == n: return 1
    # otherwise just apply the recursive formula
    else: return r_fibonacci(n-1) + r_fibonacci(n-2)

def i_fibonacci(n):
    ''' Iterative implementation of the Fibonacci sequence
    The function returns the n-th Fibonacci number in the sequence
    Format: i_fibonacci(int)
    returns an integer'''

    # the base cases
    if 0 == n: return 0
    elif 1 == n: return 1
    # otherwise just unroll the recursive formula to make an
    # iterative process (the function i_fibonacci won't call itself)
    else:
        F_i_2 = 0
        F_i_1 = 1
        # apply the formula n-1 times to obtain the value F(n)
        for i in range(n-1):
            temp = F_i_1
            F_i_1 = F_i_1 + F_i_2
            F_i_2 = temp

        return F_i_1

import time

start = time.clock()
print(r_fibonacci(35))
print("Recursive function takes %.2f seconds" % (time.clock() - start)
    )

start = time.clock()
print(i_fibonacci(35))
print("Iterative function takes %.2f seconds" % (time.clock() - start)
    )
```

One can see in the call tree figure below, that there is a big overhead due to the fact that in the recursive case we are computing several times the function with the same input. This is the main reason of the overhead you can observe between the recursive and iterative versions of the algorithm.



11. The Ackermann-Peter function is defined as follows for non-negative integers m and n:

```
A(m,n) = n+1                          , if m = 0
       = A(m-1, 1)                    , if m>0 and n=0
       = A(m-1, A(m,n-1))             , if m>0 and n>0
```

In a file `ackermann.py`, write a recursive function `A` that implements this definition. Return the error value -1 if the condition that m and n are non-negative integers is not respected. Note: for testing, do not use values $m > 3$ as the function grows very quickly.

Solution:

ackermann.py

```
def A(m,n):
    '''function that computes the Ackermann-Peter
    function for input values m and n (positive integers).
    Format: A(int,int)
    returns an integer value'''

    # if we inserted negative or non-integer inputs, return -1
    if m<0 or n<0 or int(m)!=m or int(n)!=n: return -1
    # otherwise just apply the recursive formula
    elif m==0: return n+1
    elif n==0: return A(m-1,1)
    else: return A(m-1,A(m,n-1))
```

12. Write a recursive function `perm.py` that takes as input a string `S`, and that will output a list of strings containing of all possible permutations of the letter positions in S. Here are some execution examples:

```
>>> print(perm('bat'))
 ['bat', 'bta', 'abt', 'atb', 'tba', 'tab']
>>> print(perm('jean'))
 ['jean', 'jena', 'jaen', 'jane', 'jnea', 'jnae', 'ejan', 'ejna',
  'eajn', 'eanj', 'enja', 'enaj', 'ajen', 'ajne', 'aejn', 'aenj',
  'anje', 'anej', 'njea', 'njae', 'neja', 'neaj', 'naje', 'naej']
```

Hint: Assume that you have a function that generates the set S_n of all permutations of n elements, you can generate the set S_{n+1} of all permutations of $n+1$ elements by inserting the new element in every possible position of every member of S_n.

Solution:

perm.py

```
def perm(S):
    # if the input string is composed of only one or zero character
    # there is only one possible permutation: identity
    if len(S)<=1:
        return [S]
    # otherwise, we generate all possible permutation of all
    else:
        permutations = []
        # generate all possible permutations for the string
        # without its first character
        for x in perm(S[1:]):
            # place the charater in every possible position of this
            # permutation, and save this candidate into
            for i in range(len(S)):
                temp = x[:i] + S[0] + x[i:]
                permutations.append(temp)
        return permutations

# example of utilization
print(perm('bat'))
print(perm('jean'))
```