Tutorial (week 13)

All the answers can be given with pseudocode or with Python. Remember that most of the time, many solutions are possible.

1. What is the best way to multiply a chain of matrices with dimensions that are 10×5, 5×2, 2×20, 20×12 and 12×4 ? Show your work.
2. Design an efficient algorithm for the matrix chain multiplication problem that outputs a fully parenthesized expression for how to multiply the matrices in the chain using the minimum number of operations.
3. Suppose we define a probabilistic event so that $V(i, 0) = 1$ and $V(0, j) = 0$, for all i and j, and $V(i, j)$, for i, $j \geq 1$, is defined as $V(i, j) = 0.5V(i-1, j) + 0.5V(i, j-1)$. What is the probability of the event, $V(2, 2)$?
4. Show the longest common subsequence table, L, for the following two strings:

$$X = \texttt{"skullandbones"}$$
$$Y = \texttt{"lullabybabies"}.$$

   What is a longest common subsequence between these strings?

5. Binomial coefficients are a family of positive integers that have a number of useful properties and they can be defined in several ways. One way to define them is as an indexed recursive function, $C(n, k)$, where the C stands for "choice" or "combinations". In this case, the definition is as follows:

$$C(n, 0) = 1,$$
$$C(n, n) = 1,$$

   and, for $0 < k < n$,

$$C(n, k) = C(n-1, k-1) + C(n-1, k).$$

   a) Show that, if we don't use memoization, and n is even, then the running time for computing $C(n, n/2)$ is at least $2^{n/2}$.
   b) Describe a scheme for computing $C(n, k)$ using memoization. Give a big-oh characterization of the number of arithmetic operations needed for computing $C(n, \lceil n/2 \rceil)$ in this case.

6. Given a sequence $S = (x_0, x_1, x_2, \ldots, x_{n-1})$ of numbers, describe an $O(n^2)$-time algorithm for finding a longest subsequence $T = (x_{i_0}, x_{i_1}, x_{i_2}, \ldots, x_{i_{k-1}})$ of numbers, such that $i_j < i_{j+1}$ and $x_{i_j} > x_{i_{j+1}}$. That is, T is a longest decreasing subsequence of S.

Hints

- **Question 3: Just work through the recurrence equation.**
- **Question 4: Review the LCS algorithm.**
- **Question 5: Consider how Pascal's triangle applies to this problem.**
- **Question 6: Review the LCS algorithm.**