

Math 171  
Shahriari

Name: Forest Kobayashi

Ordered Number: 14

HW #: 9w

Day: Mon. Wed. Fri.

Date: 3/21/18

No.	Points	Acknowledgments
10.2.17		Cole K., Owen G.
10.3.9		
<i>10.3.11</i>		
11.1.2		kfldsafjksd
11.1.10		
<b>Total</b>		

This Homework is (check one):

☒

On Time

☐

2 day extension #1

☐

2 day extension # 2

☐

2 day extension #3

☐

2 day extension #4

☐

Brownie Point Extension

☐

Late



**Problem 10.2.17 (The alternating group of degree 5)**

Use problems **6.2.15** and **10.2.15** and Lagrange's theorem to prove that  $A_5$ , the alternating group of degree 5, has no non-trivial normal subgroups.

---

**Solution:**

**Problem 10.3.9**

Let  $G$  be a group, and let  $N \triangleleft G$ . Assume that  $|G : N| = m$ . Let  $x \in G$ . Prove that  $x^m \in N$ .

---

**Solution:**

**Problem 10.3.11**

Assume that  $N$  is a normal subgroup of a group  $G$ . Assume  $E$  is a subgroup of  $G/N$ . Thus  $E$  is a collection of right cosets of  $N$  in  $G$ . Let  $K$  be the union of all the elements of  $E$ . In other words,  $K$  is a subset of  $G$  consisting of all the elements in the right cosets in  $E$ . Prove that  $K$  is a subgroup of  $G$  that contains  $N$ . What is  $|K|$ ?

---

**Solution:**

**Problem 11.1.2**

Define  $\phi : (\mathbb{Z}/8\mathbb{Z}, +) \rightarrow (\mathbb{Z}/8\mathbb{Z}, +)$  by  $\phi(x) = 2x$ . Is  $\phi$  a homomorphism? If so, what is  $\phi^{-1}(\{0\})$ ? Answer the same questions for  $\theta : (\mathbb{Z}/8\mathbb{Z}, +) \rightarrow (\mathbb{Z}/8\mathbb{Z}, +)$  defined by  $\theta(x) = x^2$ .

---

**Solution:**

**Problem 11.1.10**

Let  $\phi : G \rightarrow H$  be an onto homomorphism.

- (a) Assume that  $G$  is abelian. Does this imply that  $H$  is abelian? What about the converse?
- (b) What if we replaced abelian by cyclic in the above question?

---

**Solution:**