Linear Algebra!

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Chapter 1

Introduction

1.1 What is Linear Algebra?

If you were to ask the average college student this question, they'd probably shudder, mumble something about boxes that you stick numbers in and do things to, then run away, screaming something about Linear Algebra being a scary & traumatic time. And who can blame them? Often, it's presented as a subject whose only discernable features are large volumes of mind-numbing arithmetic, tangled messes of theorems, and gratuitously-involved formulae to memorize. Little attention is typically given to the rich geometric intuition that underlies its core concepts. This is made all the more egregious by the fact that key results are often presented *without proof.* The horror, the horror!

I think this is a real shame. Linear Algebra is a pretty nifty subject, and (in my opinion) can be much more intuitive than something like Calculus. For the purposes of this document, you can think of Linear Algebra as the *mathematics of spaces*. In particular, Linear Algebra is all about abstracting the properties of some system into abstract *positions* in an appropriate space. This allows us to reduce complex, possibly-abstract systems to something visual and tangible. For instance, solutions to various systems of equations can be represented as *points* in a *solution space*. This has further applications in fields like Quantum Mechanics, we can use abstract vector spaces to represent the "state" of a system, and treat observable quantities such as momentum as *operators* that take our initial state and map it to some other.

Basically, Linear Algebra is pretty cool. At least, I think so. I hope you will too after reading this!

1.2 How to Read this Document

If this is still unwritten, and you'd like it to be uh... not unwritten, bother me on github, messenger, or some other platform for grievance voicing.

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