Seeing is Believing

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A Motivating Quote

'Algebra is but written geometry, and geometry is but figured algebra.' Marie-Sophie Germain

My perspective

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  - Algebra: abstract, opaque, uninspired
  - Geometry: tangible, intuitive, clear
- Today: learning to convert between the two

A sum identity

Theorem

Let $n \in \mathbb{N}$. Then

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Proof.

Induct on n

$$\frac{n(n+1)}{2} + (n+1) = \frac{(n^2+n)+(2n+2)}{2} = \frac{(n+1)(n+2)}{2}.$$

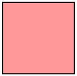
□

Evolution of perspective

- High School:
  - Didn't get induction
- Frosh year:
  - Finally got induction
  - Finally enjoyed math!
  - Wished somebody had told me math was so cool!
- Plan: write intuitive explanations
  - Some problems...
  - Can I do it without words?

Drawing a picture

Partial sums

1 

$S(1)=1$

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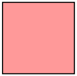
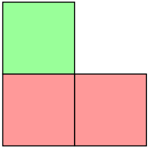
1



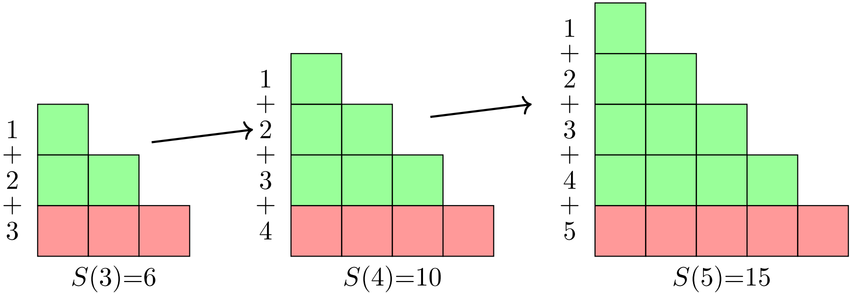
$S(1)=1$

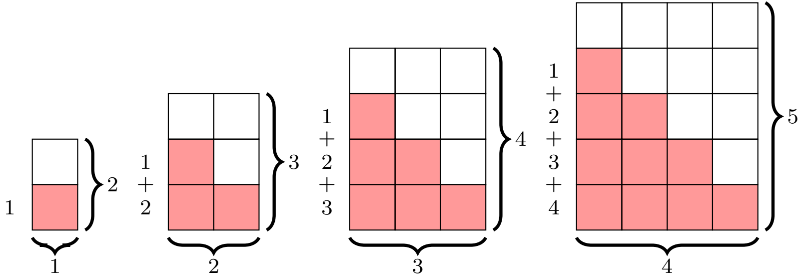
1
+
2



$S(2)=3$

Continuing

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4

Challenge

'Can you do it with Calculus? Because ***then*** *I'd be impressed.'*

My Neighbor

Starting simple

Theorem

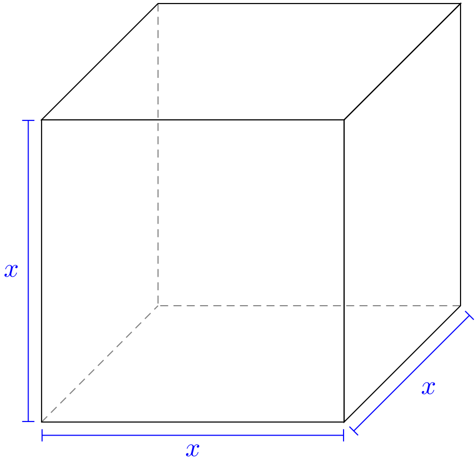
Let $n \in \mathbb{N}$. Then

$$\frac{dx^n}{dx} = nx^{n-1}.$$

Proof.

Take the difference quotient and apply the identity
$(x^n - y^n) = (x - y)(x^{n-1} + x^{n-2}y + \cdots + y^{n-1})$ $\square$

Special case: $n = 3$



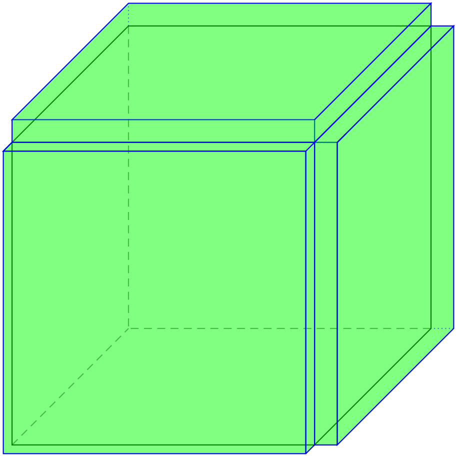


Figure: Cubes

References