Seeing is Believing

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A sum identity

Theorem

Let $n \in \mathbb{N}$. *Then*

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

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Proof.

Induct on n.

$$\frac{k(k+1)}{2} + (k+1) = \frac{(k^2+k)+(2k+2)}{2} = \frac{(k+1)(k+2)}{2}.$$

$\square$

The question

Why should you expect the formula to look like this?

Consternation

- Bothered me a lot in high school
  - Teachers: “because the induction works”
  - Eventually gave up on a deeper perspective
- College
  - Didn’t question the inductive proof
- Winter break

The challenge

'I think that I just don't work well in abstraction. I care about the tangible, and things that I can see.' My Neighbor

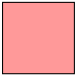
Desired explanation

- Brief
- Accessible
- Visualizable

Drawing a picture

Partial sums

1

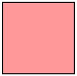


$S(1)=1$

Drawing a picture

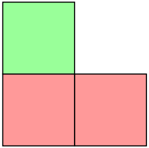
Partial sums

1



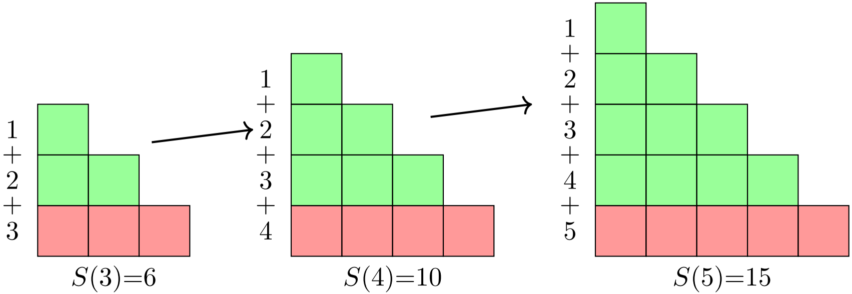
$S(1)$=1

1
+
2

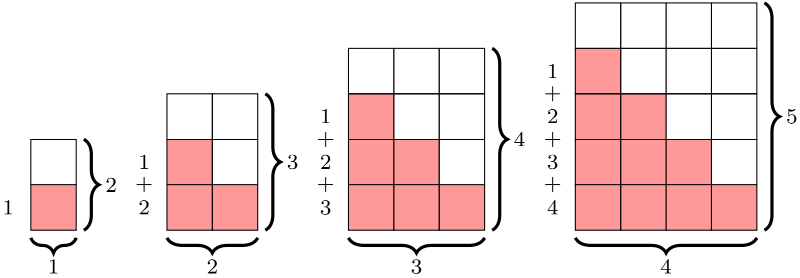


$S(2)$=3

Continuing



Calculating the shaded area



Challenge

'Ok, that's pretty cool. But can you do it with Calculus? Because then I'd be ***really*** *impressed. I didn't understand* ***any*** *of Calculus, and I mean it.'* My Neighbor

Starting simple

Theorem

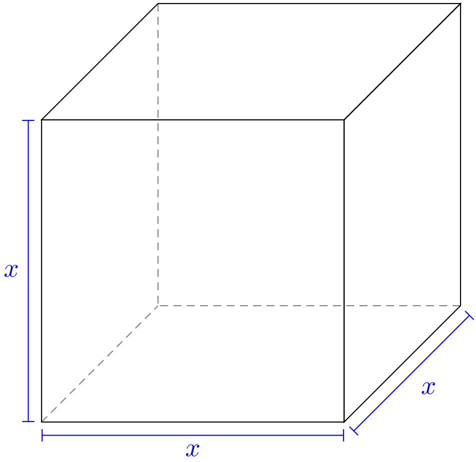
Let $n \in \mathbb{N}$. *Then*

$$\frac{dx^n}{dx} = nx^{n-1}.$$

Proof.

Take the difference quotient and apply the identity
$(x^n - y^n) = (x - y)(x^{n-1} + x^{n-2}y + \cdots + y^{n-1})$ $\square$

Special case: $n = 3$



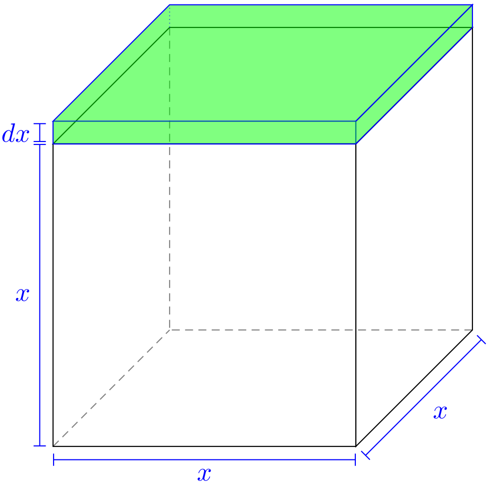


Figure: Extending one side by dx

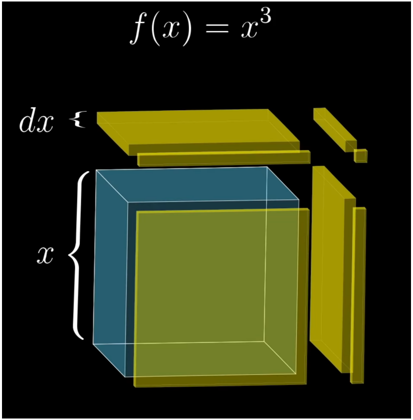


Figure: 3blue1brown's visualization ([3Bl17])

A final quote

'Algebra is but written geometry, and geometry is but figured algebra.' Marie-Sophie Germain

References

3Blue1Brown.

Derivative formulas through geometry | Essence of calculus, chapter 3, Apr 2017.

[Online; accessed 17. Feb. 2019].