Seeing is Believing

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High School

- Didn't like math
    - Focus on "algebra"
    - Never felt I understood why things worked

Example

Theorem

Let $n \in \mathbb{N}$. *Then*

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Proof.

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Theorem

Let $n \in \mathbb{N}$. Then

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Proof.

Induct on n.

$$\frac{k(k+1)}{2} + (k+1) = \frac{(k^2+k)+(2k+2)}{2} = \frac{(k+1)(k+2)}{2}.$$

$\square$

The question

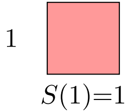
Why should I *expect* the formula to look like this?

Coming to college

- Forgot about the question for a while
- Transitioned to focusing on justifying an answer over "understanding" it (not a negative thing)
- Learned to love rigor and proof :)
- Wanted to share this!
  - Only problem: wasn't sure what had changed

Drawing a picture

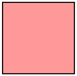
Partial sums



Drawing a picture

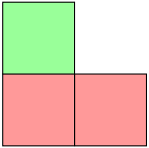
Partial sums

1



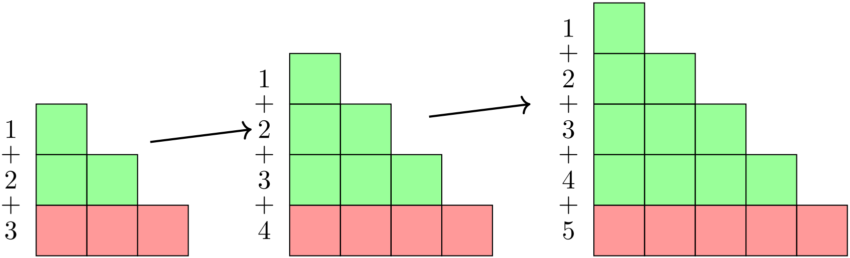
$S(1)=1$

1
$+$
2



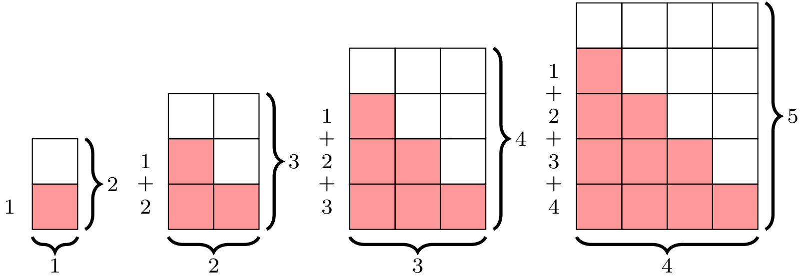
$S(2)=3$

Continuing



$S(3) = 6$ $S(4) = 10$ $S(5) = 15$

Calculating the shaded area



Challenge

'Ok, that's pretty cool. But can you do it with Calculus? Because then I'd be ***really*** *impressed.'* My Neighbor

Starting simple

Theorem

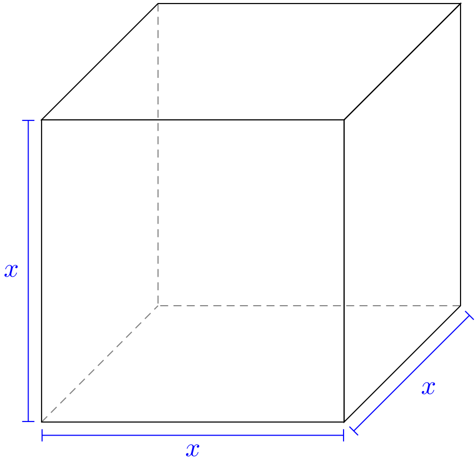
Let $n \in \mathbb{N}$*. Then*

$$\frac{dx^n}{dx} = nx^{n-1}.$$

Proof.

Take the difference quotient and apply the identity
$(x^n - y^n) = (x - y)(x^{n-1} + x^{n-2}y + \cdots + y^{n-1})$ $\square$

Special case: $n = 3$



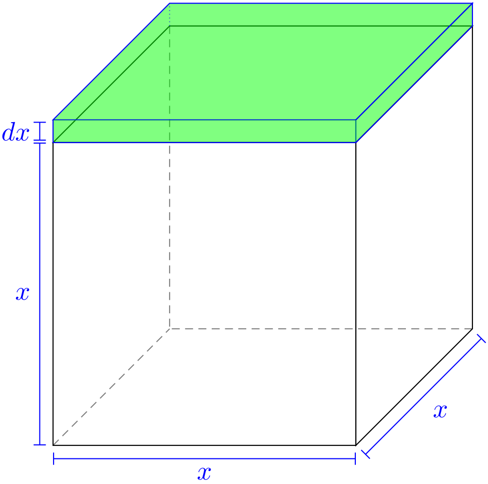


Figure: Extending one side by dx

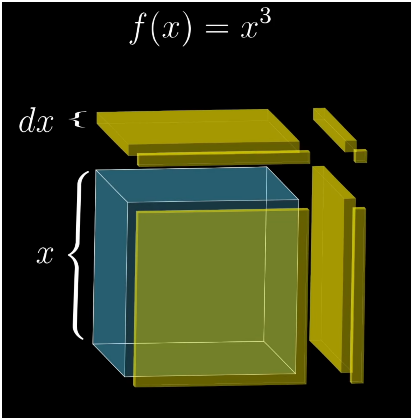


Figure: 3blue1brown's visualization ([3Bl17])

A final quote

'Algebra is but written geometry, and geometry is but figured algebra.' Marie-Sophie Germain

References

3Blue1Brown.

Derivative formulas through geometry | Essence of calculus, chapter 3, Apr 2017.

[Online; accessed 17. Feb. 2019].