The Euler-Lagrange Equation

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Airlines

- Time is fuel. Time is

Airline

Figure: Flight 1 (UPS9859), Saturday 03/23/2019

Airline, cont.

Figure: Flight 2 (FDX50252), Friday 03/22/2019

Some statistics:

- Total flight distance:
  - Flight 1: 4551km
  - Flight 2: 4670km
- Total flight time:
  - Flight 1: 5h 30m 7s
  - Flight 2: 5h 14m 6s

The difference:

The difference:

Figure: Wind patterns at 70hPa during Flight 1

The difference:

Figure: Wind patterns at 70hPa during Flight 2

Abstracting

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Overview

- We want to find some optimal path $\boldsymbol{q}(t)$ satisfying

Wind

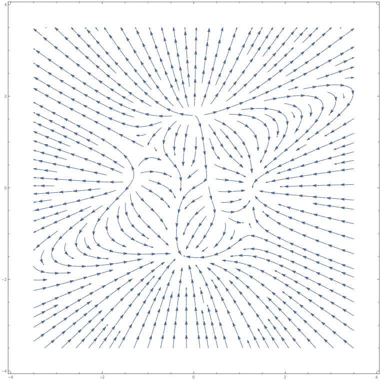
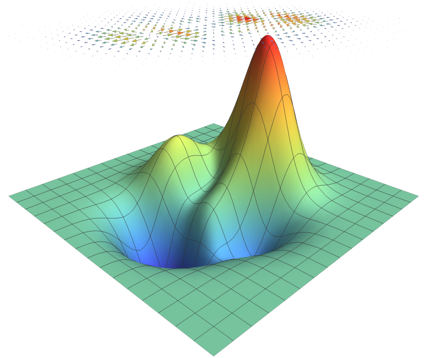
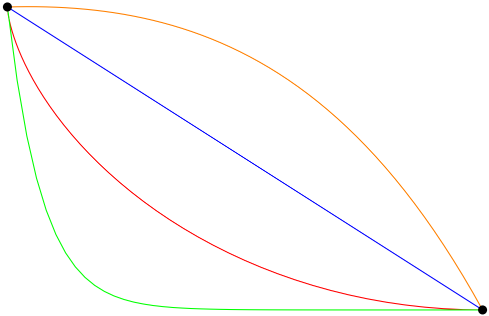


Figure: Wind Vector Field

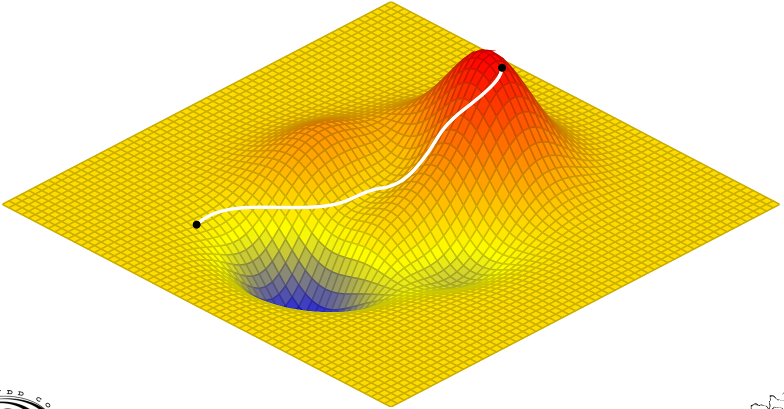
"Cost" function



Shortest Time Path



Shortest Path



The statement

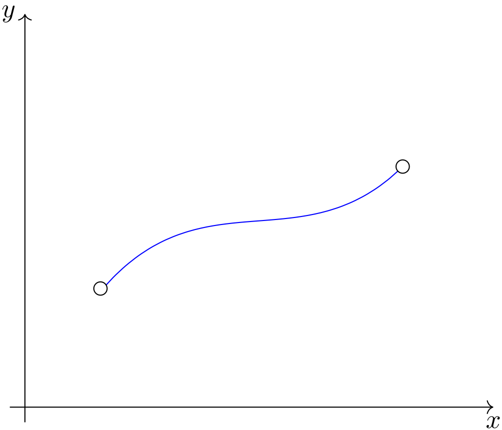
Theorem (Euler-Lagrange)

Let $\boldsymbol{q}(t) : \mathbb{R} \to \mathbb{R}^n$ be a path. Then if $\boldsymbol{q}(t)$ is an extreme value of the functional

$$S(\boldsymbol{q}) = \int_a^b \mathcal{L}(t, \boldsymbol{q}(t), \dot{\boldsymbol{q}}(t)) \, \mathrm{d}t$$

then $\boldsymbol{q}$ is a solution to the differential equation

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{q}} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{q}}}\right) = 0$$


y
x