The Euler-Lagrange Equation

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Wind

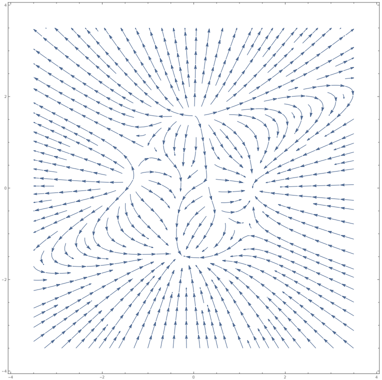
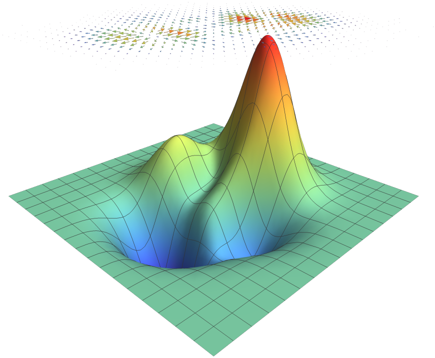
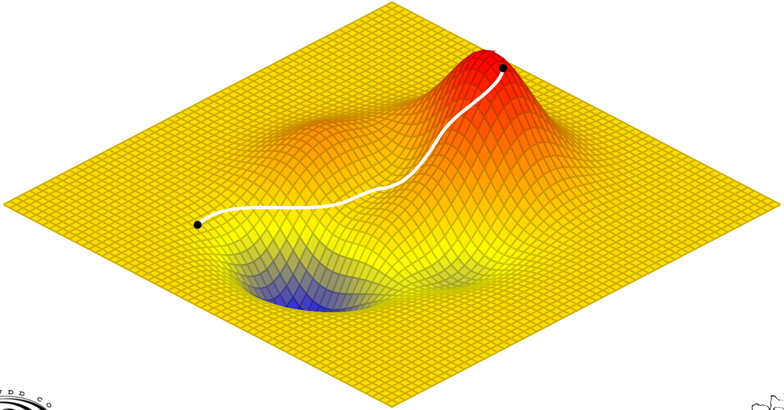


Figure: Wind Vector Field

Underlying Scalar Function



Shortest Path



The statement

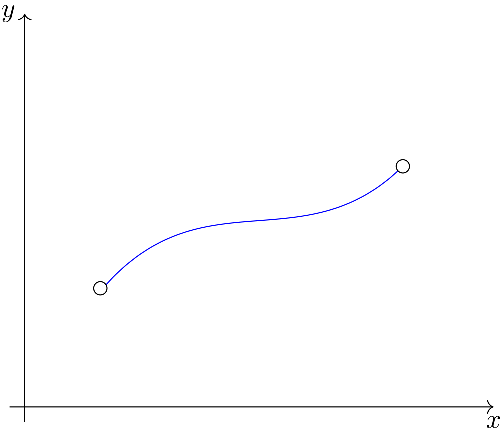
Theorem (Euler-Lagrange)

Let $\boldsymbol{q}(t) : \mathbb{R} \to \mathbb{R}^n$ *be a path. Then if* $\boldsymbol{q}(t)$ *is an extreme value of the functional*

$$S(\boldsymbol{q}) = \int_a^b \mathcal{L}(t, \boldsymbol{q}(t), \dot{\boldsymbol{q}}(t)) \ \mathrm{d}t$$

then $\boldsymbol{q}$ *is a solution to the differential equation*

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{q}} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{q}}}\right) = 0$$


y
x