

The Euler-Lagrange Equation

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Wind

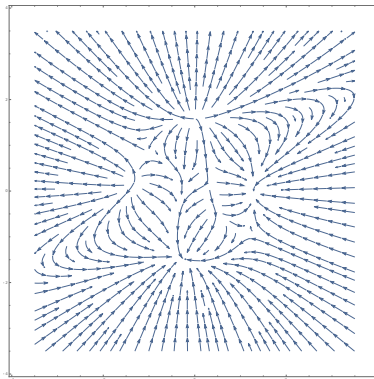
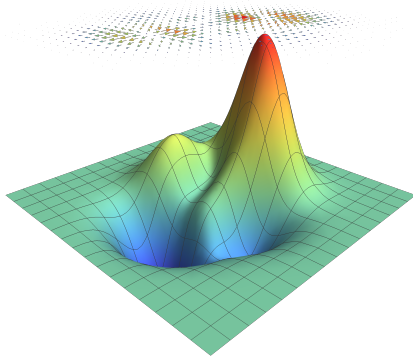


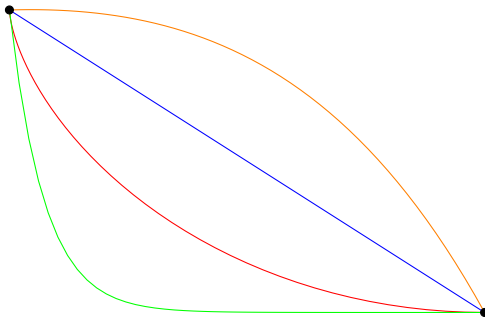
Figure: Wind Vector Field



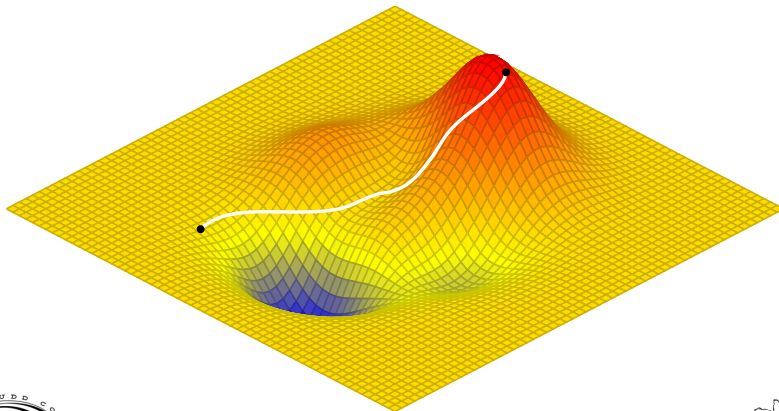
Underlying Scalar Function



Shortest Time Path



Shortest Path



The statement

Theorem (Euler-Lagrange)

Let $\mathbf{q}(t) : \mathbb{R} \rightarrow \mathbb{R}^n$ be a path. Then if $\mathbf{q}(t)$ is an extreme value of the functional

$$S(\mathbf{q}) = \int_a^b \mathcal{L}(t, \mathbf{q}(t), \dot{\mathbf{q}}(t)) \, dt$$

then \mathbf{q} is a solution to the differential equation

$$\frac{\partial \mathcal{L}}{\partial \mathbf{q}} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}} \right) = 0$$



