# Seeing is Believing

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February 18th, 2018


# A sum identity

## Theorem

*Let $n \in \mathbb{N}$. Then*

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$


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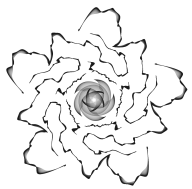
## Proof.

Induct on $n$.

$$\frac{k(k+1)}{2} + (k+1) = \frac{(k^2+k)+(2k+2)}{2} = \frac{(k+1)(k+2)}{2}.$$

□

# The question

Why should you expect the formula to look like this?


# Consternation

- Bothered me a lot in high school
  - Teachers: “because the induction works”
  - Eventually gave up on a deeper perspective
- College
  - Didn’t question the inductive proof
- Winter break


# The challenge

*'I think that I just don't work well in abstraction. I care about the tangible, and things that I can see.'* My Neighbor

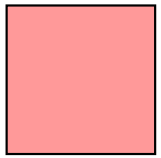

# Desired explanation

- Brief
- Accessible
- Visualizable


# Drawing a picture

Partial sums

1



$S(1)=1$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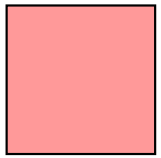
Partial sums

1

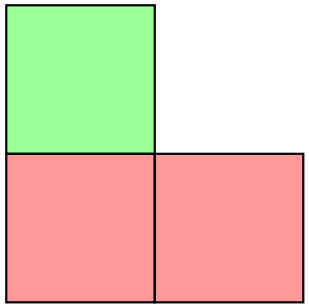


$S(1)=1$

1

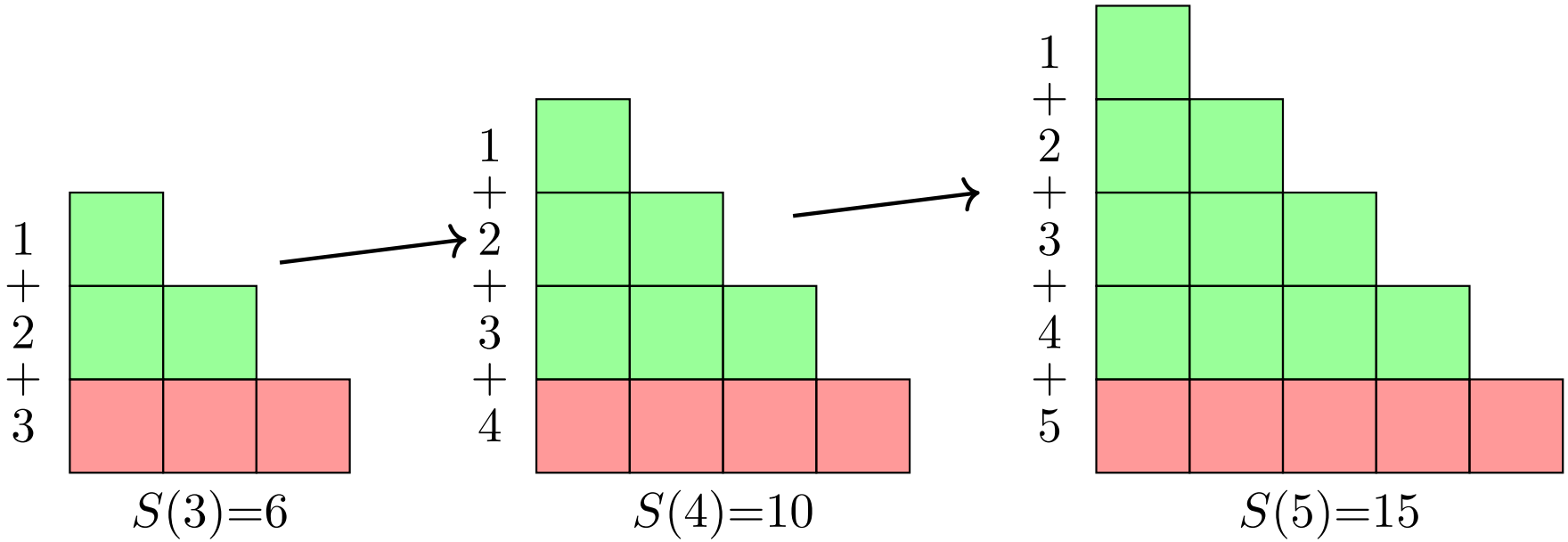
+

2



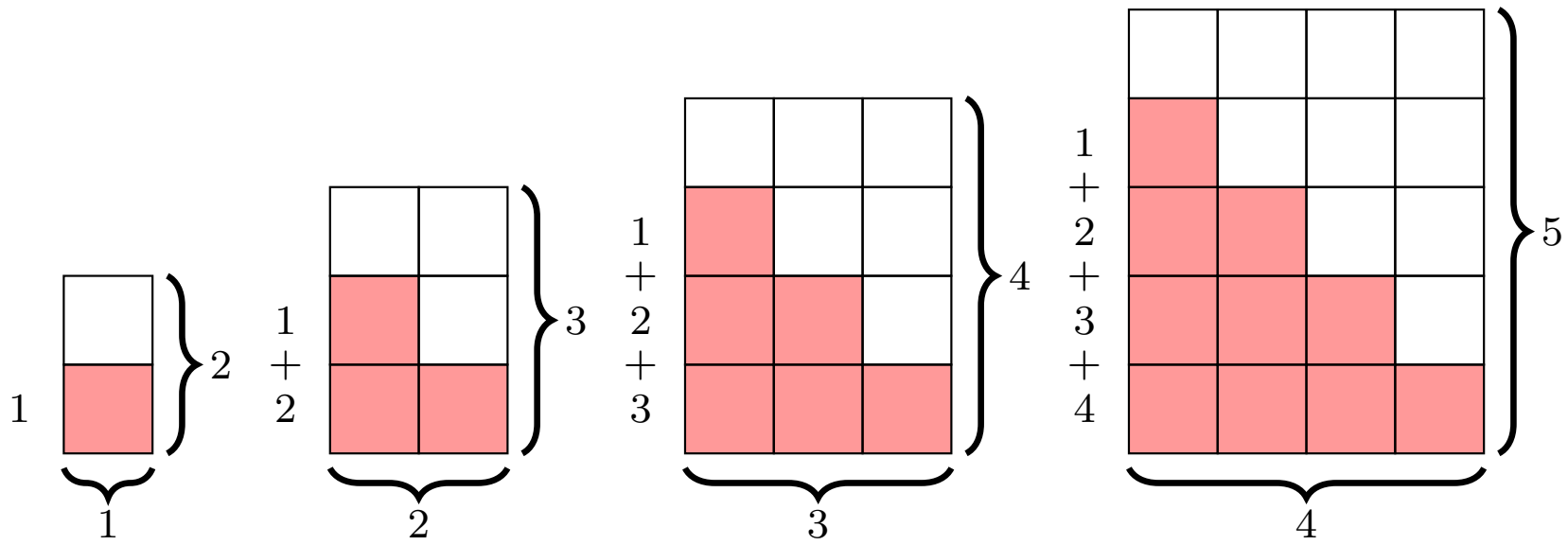
$S(2)=3$


# Continuing





# Calculating the shaded area



# Challenge

*'Ok, that's pretty cool. But can you do it with Calculus? Because then I'd be **really** impressed. I didn't understand **any** of Calculus, and I mean it.'* My Neighbor


# Starting simple

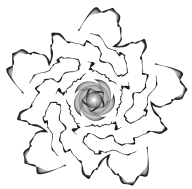
## Theorem

*Let* $n \in \mathbb{N}$. *Then*

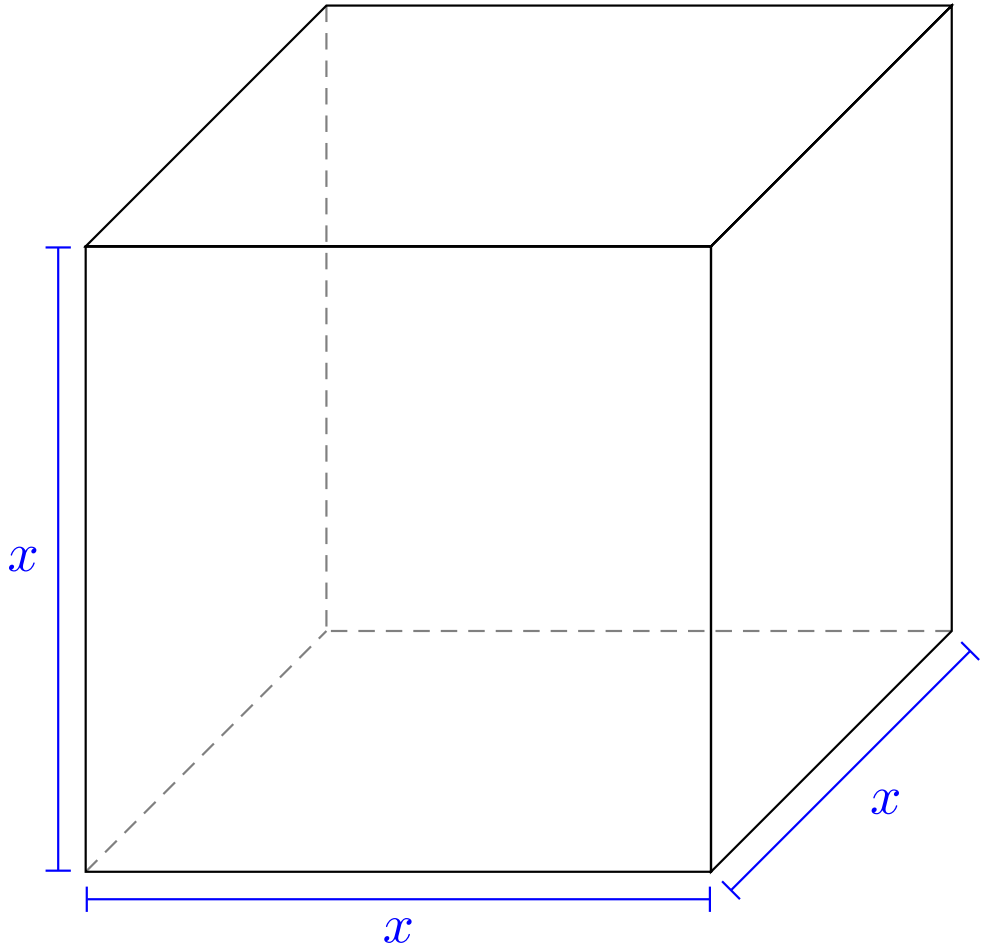
$$\frac{dx^n}{dx} = nx^{n-1}.$$

## Proof.

Take the difference quotient and apply the identity $(x^n - y^n) = (x - y)(x^{n-1} + x^{n-2}y + \cdots + y^{n-1})$ $\square$

# Special case: $n = 3$



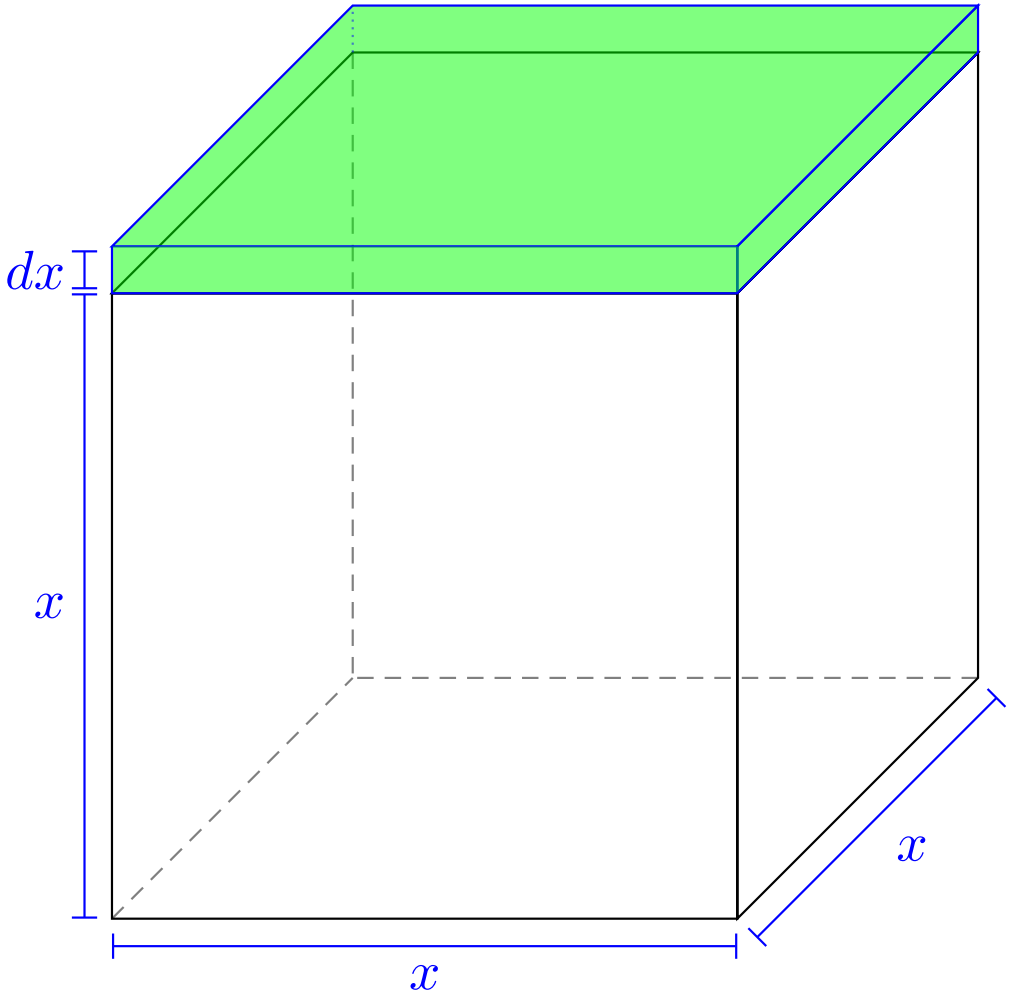



Figure: Extending one side by $dx$

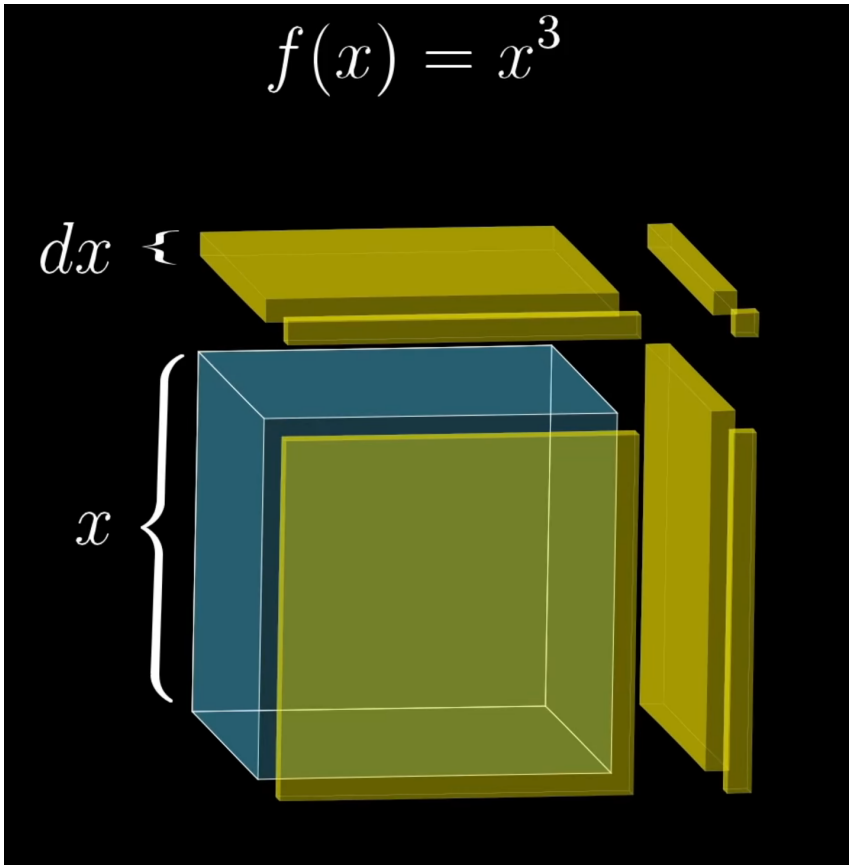




Figure: 3blue1brown's visualization ([**?**])

# A final quote

*'Algebra is but written geometry, and geometry is but figured algebra.'*
Marie-Sophie Germain


# References


3Blue1Brown.

Derivative formulas through geometry | Essence of calculus, chapter 3, Apr 2017.

[Online; accessed 17. Feb. 2019].