Problem	5.29	5.32	6.6	6.11	6.18	Total
Points						

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Math 147
HW 6 Solutions
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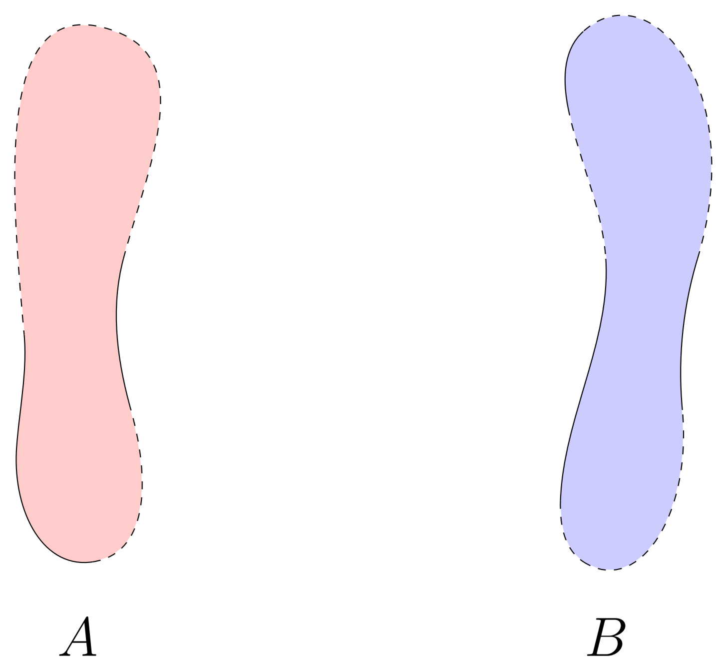
5.29 (The Normality Lemma). Let A and B be subsets of a topological space X and let $\{U_i\}_{i\in\mathbb{N}}$ and $\{V_i\}_{i\in\mathbb{N}}$ be two collections of open sets such that

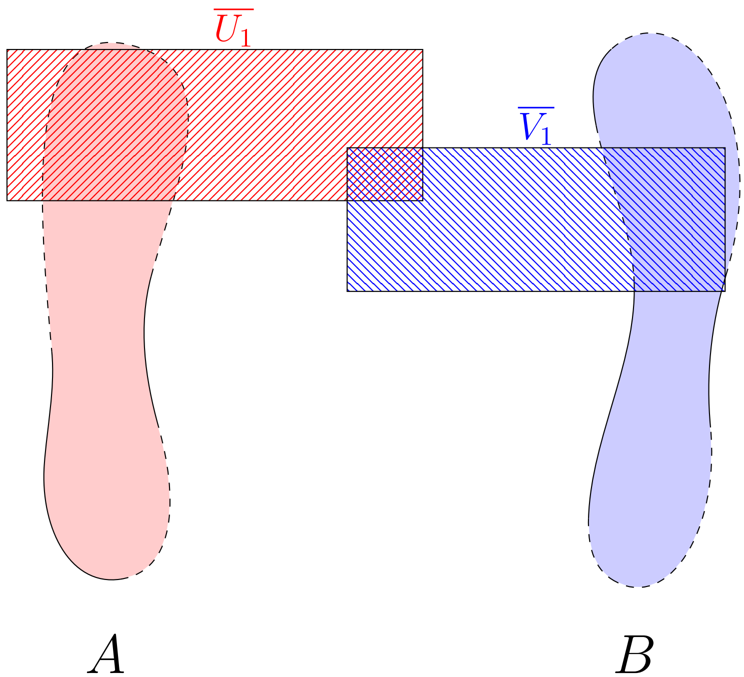
(1) $A \subset \bigcup_{i\in\mathbb{N}} U_i$
(2) $B \subset \bigcup_{i\in\mathbb{N}} V_i$
(3) For each $i \in \mathbb{N}$, $\overline{U_i} \cap B = \varnothing$ and $\overline{V_i} \cap A = \varnothing$.

Then there exist open sets U and V such that $A \subset U$, $B \subset V$, and $U \cap V = \varnothing$.

Before a solution, I'll give some intuition on how we might arrive at the candidate U, V that work. Depict our two sets A, B as follows:



To make the TikZ easier, I'll draw our covers with boxes — but note, in general they could be blobby. Anyways, we now draw $\overline{U_1}, \overline{V_1}$:



We need to be sure that in our construction of U, we don't include any of the V_i. We want to use the U_i somehow, but as we can see, they might intersect the V_i.

Solution.

■

5.32. Suppose a space X is regular and has a countable basis. Then X is normal.

Solution. Let A, B be disjoint closed sets. ■

6.6. The space $2^{\mathbb{R}}$ is separable.

Solution. ■

6.11. Every uncountable set in a 2^{nd} countable space has a limit point.

Solution. ■

6.18. Suppose x is a limit point of the set A in a 1^{st} countable space X. Then there is a sequence of points $\{a_i\}_{i\in\mathbb{N}}$ that converges to x.

Solution. ■