Forest Kobayashi
Topology
HW 2 Comments
02/16/2019

1 Writing Style Comments

A1. When defining something, put it on the left side of the equals sign. Do not write something like

> Let
> $$\bigcup_{\substack{U \in \mathfrak{T} \\ U \subset A}} U = A^\circ$$

to define A°. Instead, say

> Let
> $$A^\circ = \bigcup_{\substack{U \in \mathfrak{T} \\ U \subset A}} U$$

A2. While Prof. Su's book does define a *neighborhood* of a point x to be some open set containing x, in an extended proof it's probably best to avoid this term. Reasons:

(a) If you're using the same neighborhood repeatedly throughout your proof, it's much shorter to make the declaration

> Let $U \in \mathfrak{T}$ such that $x \in U$

and use U for the rest of the proof instead of saying "the neighborhood" repeatedly.

(b) It's easy to forget to specify what the neighborhood is *of*. Remember, we've defined a neighborhood *of* x to be an open set containing x. Without stating what x we're looking at, a neighborhood has no meaning. I saw a few situations on the homework where people said things along the lines of

> "[...] so for each x, there exists q such that [...]. Then taking any neighborhood we see that [...]"

it's not clear here whether we're taking a neighborhood of x or of q. Don't leave this for the reader to figure out!

(c) Finally, the definition of a neighborhood isn't universally standardized. Some mathematicians draw a distinction between a *neighborhood* and an *open neighborhood*:

Definition 1.1. Let $(X, \mathfrak{T})$ be a topological space. Let $x \in X$, and $N \subset X$. Then N is called a *neighborhood* of x iff there exists $U \in \mathfrak{T}$ such that $x \in U \subset N$. If $N \in \mathfrak{T}$, we call N an *open neighborhood* of x.

Thus, saying "neighborhood" has the potential to confuse your reader.

For these reasons, it's probably best to use "neighborhood" sparingly. As a general rule of thumb, any time you'll need to use it more than once, you should define some $U \in \mathfrak{T}$ instead.

A3. Don't say "the set A" if it's already implicitly clear that A is a set. Similarly with things like "the point p."

A4. If you define your variables carefully at the start of a proof, you can cut out a lot of redundant phrases from the end of your proof. For instance, if you're trying to show a claim holds for all open sets U, then you need only start your proof with

> Let $U \in \mathfrak{T}$ be arbitrary. Then [...]

and conclude with

> Since U was arbitrarily chosen, [...].

When you have more than two or three variables at play, this makes a big difference:

> Since U, V, and p were arbitrarily chosen, [...]

A5. Try not to implicitly employ contradiction, unless the claim is *very* simple. I saw a lot of things like the following:

> "We have A. We must have B, or else $\neg C$, but A" (where the reader is left to infer that $A \implies C$).

In general, the only time this is acceptable is when C is a premise we have already established. That is, things like

> "We have A. Then [...], and so C. Note that $\neg B \implies \neg C$, hence B"

can sometimes be acceptable. Here're the general guidelines:

- If the proof of $A \implies C$ is ≈ 1 step, then use the format above.
- If the proof of $A \implies C$ is $\approx$ 1-2 steps, you can put the proof of $A \implies C$ in a footnote or a parenthetical. If it's on the longer side, definitely use a footnote.
- If the proof of $A \implies C$ is $\approx$ 3+ steps, you should probably do a full proof by contradiction.

A6. If you're claiming something is a topology, don't refer to it as "the topology" until you've finished proving that it satisfies all the topological axioms. Similarly with claiming something is open, closed, etc.

A8. In an iff proof (or analogous), you should usually define any global variables before you begin each arm of the proof.

A9. For the purposes of our psets, if you find yourself saying "in other words," it's often worth checking your previous sentence to see if there's a better way you could phrase things that'd eliminate this redundancy.

2 Correctness

2.1 Exercise 3.30

B1. Make sure to treat the special case where $p \in \{x_i\}_{i\in\mathbb{N}}$. For instance, if you say the following

> Let $\{x_i\}_{i\in\mathbb{N}} \subset A$, and suppose $x_i \to p$. Let $U \in \mathfrak{T}$ such that $p \in U$. Then by definition of convergence, $\exists N \in \mathbb{N}$ such that for all $i > N$, $x_i \in U$. Thus, $\{x_i\}_{i>N} \subset (U - \{p\}) \cap A$, hence [...].

this doesn't hold if for all $i > N$, $x_i = p$.

2.2 Exercise 4.3

B2. When proving that $\mathcal{B}$ generates a topology $\mathcal{T}$, in verifying the topological axioms, be sure you show that that $\mathcal{T}$ is closed under finite intersection, not that $\mathcal{B}$ is (although this is also true). To do so, you should define arbitrary $U, V \in \mathcal{T}$, and show that $U \cap V \in \mathcal{T}$. Be sure not to accidentally pick $U, V \in \mathcal{B}$!

3 Comments on TeX / Notation

C1. Usually, it's easier to read something like

$$A^\circ = \bigcup_{\substack{U \in \mathcal{T} \\ U \subset A}} U$$

over

$$A^\circ = \bigcup_{U \in \mathcal{T}, U \subset A} U$$

To do multi-line subscripts for things like `\bigcup` in LaTeX, you'll want to use `\substack` (requires the `amsmath` package):

```
\[
  A^\circ = \bigcup_{\substack{U \in \ms T \\ U \subset A}} U
\]
```

If the subscript is still really long even when using `\substack` and you don't like the whitespace you're getting, e.g.

$$A = \bigcup_{\substack{\text{look} \\ \text{how} \\ \text{ultra}-\text{super}-\text{wide} \\ \text{this} \\ \text{subscript} \\ \text{is}}} U$$

you can close the gap by enclosing your subscript with `\mathclap`:

$$A = \bigcup_{\mathclap{\substack{\text{look} \\ \text{how} \\ \text{ultra}-\text{super}-\text{wide} \\ \text{this} \\ \text{subscript} \\ \text{is}}}} U$$

or, applied to our earlier example,

$$A^\circ = \bigcup_{\mathclap{U \in \mathcal{T}, U \subset A}} U.$$

this requires the `mathtools` package, and the code is as follows:

```
\[
  A^\circ = \bigcup_{\mathclap{U \in \mathscr T, U \subset A}} U.
\]
```

of course, you can put a `\substack` inside a `\mathclap` to combine the effects.