Building and effective field theory for neutron-proton scattering

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We build an effective field theory (EFT) to describe neutron-proton scattering. We take the underlying theory to be a sum of three Yukawa potentials with empirically derived parameters (the potential model from the previous project). By fitting to the low energy phase shifts (δ_0) calculated from solving the Lippman-Schwinger equation, we determine the coefficients for the contact interactions in the EFT potential. We report reasonable agreement (errors $< 10\%$) with our underlying theory up to lab energies around 10 MeV. We find the breakdown scale for our pionless EFT to be of order m_π and roughly 3 fm^{-1} for our one pion EFT.

INTRODUCTION

Measuring the final states of two nucleons after they interact provides information about the nature of their interaction. One way to summarize the information we get from these scattering experiments is through the phase shift - which can be roughly described as a shift in the scattered wavefunction relative to the incoming one. In this project, we create an effective field theory to describe the nucleon scattering processes.

Effective field theories attempt to describe the low energy physics of some interaction without exactly treating the high energy processes that act over short distances. We describe here a simple Effective Field Theory (EFT) for neutron-proton scattering that does not incorporate spin or isospin degrees of freedom. One key component of an EFT is the cutoff beyond which high momentum states that are sensitive to the unknown short-range dynamics are excluded [2]. In the first version of our EFT, we treat only the physics described by the effective range expansion. As such, we only expect our model to accurately calculate the phase shifts for lab energies up to about 10 MeV. In the second version, we incorporate information about the long range nuclear potential into our model in an effort to improve agreement at higher energies with our underlying theory.

We organize this report as follows: first we summarize the forms for the EFT potentials we use to numerically calculate the phase shifts. We then describe the framework we used to carry out our calculations. Next, we present the results of our calcuations, and finally, we summarize our findings and offer some comments on the work.

THEORY AND ALGORITHMS

Pionless EFT

To begin building our pionless effective field theory, we take our underlying theory, to be the three-Yukawa np model

$$V_{np}(r) = V_a \frac{e^{-ax}}{x} + V_b \frac{e^{-bx}}{x} + V_c \frac{e^{-cx}}{x}, \tag{1}$$

with $x = \mu r$, $\mu = 0.7$ fm^{-1}, $V_a = -10.463$ MeV and $a = 1$, $V_b = -1650.6$ MeV and $b = 4$ and $V_c = 6484.3$ MeV and $c = 7$. This potential is spin- and isospin-independent, so the leading order term in the EFT potential has the simple form

$$V^{\mathrm{LO}}_{^1S_0}(p, p') = C_0.$$

To avoid UV divergences when solving the Lippman-Schwinger equation with this potential, we use a smooth regulator function

$$f_{\Lambda_c}(p) = \exp\left[-p^4/\Lambda_c^4\right]$$

so that the potential we feed into our LS solver has the form

$$V(p, p') \Rightarrow f_{\Lambda_c}(p') V(p, p') f_{\Lambda_c}(p).$$

The next-to leading order (NLO) and next-to-next-to leading order (NNLO) terms:

$$V^{\mathrm{NLO}}_{^1S_0}(p, p') = C_2(p^2 + p'^2)$$
$$V^{\mathrm{NNLO}}_{^1S_0}(p, p') = C_4(p^4 + p'^4) + C_4 p^2 p'^2$$

so that $V(p, p')$ up to some desired order is a sum of these terms.

We set the cutoff for our regulator (Λ_c) to be the pion mass ($m_\pi \approx 0.71$ fm^{-1}) since this is sufficiently above the range of validity for the effective range expansion.

Next, we add a simple one-pion exchange term in the potential to improve the EFT at higher energies. This is the long range Yukawa potential

$$V(r) = V_a \frac{e^{-\mu r}}{\mu r}$$

with $V_a = -10.463$ MeV and $\mu = 0.7$ fm^{-1}. This term gets added to the sum of the terms defined above to give our one-pion EFT potential. We take the cutoff for this version of the EFT potential to be $\Lambda_c = 2.8$ fm^{-1} because this is the scale of the next Yukawa term that we omit from our EFT potential.

METHODS

We determine our C_i parameters by fitting to the low-energy phase shifts calculated using the three-Yukawa empirical model (eq. 1). To calculate phase shifts from the *np* model and from our EFT potential, we implemented a numerical solution to the Lippman-Schwinger equation as a `C++` class. This class consists of three two-dimensional data structures to hold the V, A and R matrices, two one-dimensional data structures to hold the weights and momentum mesh points that define the integration domain and several ancillary variables that specify the nucleon mass and other book-keeping parameters. Application of the algorithms described in our previous report is carried out in a series of member functions that (a) set up the mesh points and weights (b) set up the potential matrix (c) set up the A matrix (d) invert A to get R then extract δ_0.

This class is defined in the source code files `NucleonScattering.cpp` and `NucleonScattering.hh`. An instance of the class is created and the correct sequence of methods is called in the `main` function defined in `main.cpp`. In the `src` directory, we include several versions of `main.cpp` that call the correct series of functions to compute phase shifts using the different models and generate text files comparing the *np* phase shifts to those generated from the EFT. We use the text files as input to the functions defined in `eftPlots.C` which plot our results using the ROOT data analysis framework [1].

Fitting EFT coefficients

In order to determine values for the C_i parameters, we fit our EFT phase shifts to those calculated from the *np* model. According to Lepage [2] we use the lowest energy pseudo data when fitting the coefficients; we compare phase shifts for lab energies of 1,2,3,4,5 keV with the 1 keV point weighted the heaviest. We define a χ^2 as a measure of agreement between the phase shifts calculated from the two different models.

$$\chi^2 = \sum_{n=1}^{5} \frac{\delta_{np} - \delta^2_{\mathrm{EFT}}}{w_i} \tag{2}$$

where $w_i = 1, 2, 4, 8, 16$.

The general procedure we used to determine the EFT coefficients involved a grid search of the parameter space to identify the region of the χ^2 minimum, then we adjusted the parameters by hand until we saw good agreement with the *np* phase shifts.

RESULTS AND DISCUSSION

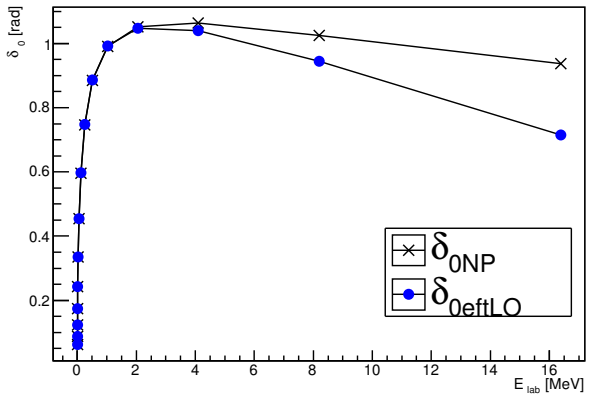
Pionless EFT

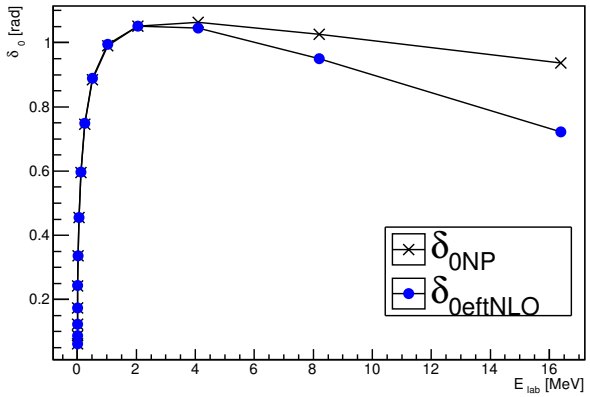
We compare the phase shifts calculated using the *np* model to those from our EFT in FIG. 1. We note reasonable agreement with the *np* model at all orders up to roughly 4 MeV. There is not a noticable difference between the EFT orders so we plot $\log|\delta_{\mathrm{np}} - \delta_{\mathrm{EFT}}/\delta_{\mathrm{np}}|$ versus the lab energy (a Lepage error plot) in FIG. 2. In this figure, we see several kinks which are due to the error changing sign, however, in general the NLO and NNLO calculations better reproduce the pseudo data than the LO. We do not see a substantial improvement going from NLO to NNLO. We attribute this to not fitting the NNLO coefficients rigorously enough. The three dimensional parameter space was the most difficult to search for a χ^2 minimum. We found for the NLO case (a two-dimensional parameter space) that there were large regions of low χ^2 where changes in χ^2 are very small so any set of parameters in this region would resonably reproduce the *np* model phase shifts. We expect a similar problem for NNLO case but with one extra dimension, so we may not have perfectly optimized the NNLO coefficients.

We next adjusted the cutoff momentum to examine its effect on the NLO model's ability to reproduce the underlying theory. The results are shown in FIG. 3. We note that our cutoff value at the pion mass gives the best agreement; this makes sense because the pion mass roughly corresponds to the "high" energy physics that we exclude in this first version of our EFT. Varying Λ_c around this value worsens agreement with the underlying model. Placing the cutoff too high incorporates high momentum states to which our low energy model isn't applicable. Making the cutoff too low restricts the number of states available for the calculation. Therefore, we estimate the breakdown scale for our pionless EFT to be on the order of the pion mass. We also note that the coefficents vary with the value of the cutoff (see TABLE I).

Including One-Pion Exchange

We compare our phase shifts calculated with our one pion EFT to the *np* model in FIG. 4. Note the better agreement with the pseudo data compared to the pionless case in FIG. 1. Here we have set $\Lambda_c = 2.8$ fm^{-1} since this is the range of second Yukawa term representing the high energy physics that we're excluding from our EFT. We show a new Lepage error plot in FIG. 5. Where we





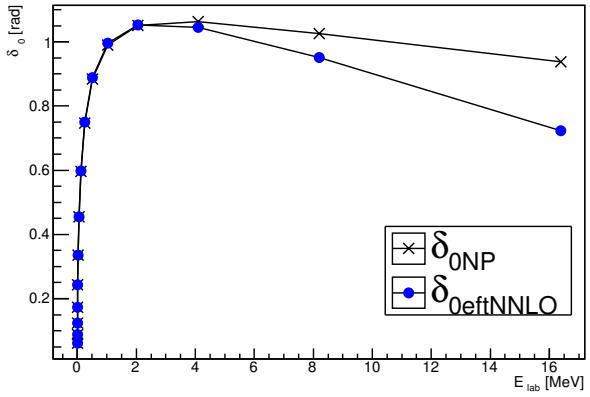


FIG. 1. Comparison between the three-Yukawa np model $l = 0$ phase shifts (black X) and the phase shifts calculated from the LO, NLO, NNLO EFT potentials (blue dots).

see features similar to those described in the previous section. However, for the one-pion EFT, we see a more noticable improvement from LO to NLO and from NLO to NNLO. Finally, we examine the effect of changing Λ_c in FIG. 6; TABLE II lists the coupling constants we used for each value of Λ_c. We see a similar trend in the errors

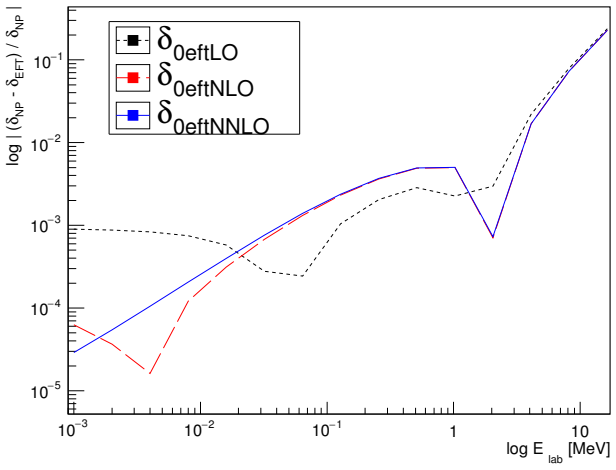


FIG. 2. Lepage error plot comparing the LO, NLO, NNLO EFT potentials to the np model pseudo data. The kinks in the lines are where the sign of the error changes. The NLO and NNLO errors are the same above lab energies of 100 keV.

Λ_c	C_0	C_2
0.07	-2.5	180
0.35	-0.9	-0.67
0.71	-0.86	3.6
1.4	-0.3	0.017
2.1	-0.196	4e-4
7.1	-0.01	0.01

TABLE I. Pionless EFT parameters for various values of the cutoff Λ_c. We note that the values for C_0 and C_2 are very cutoff dependent.

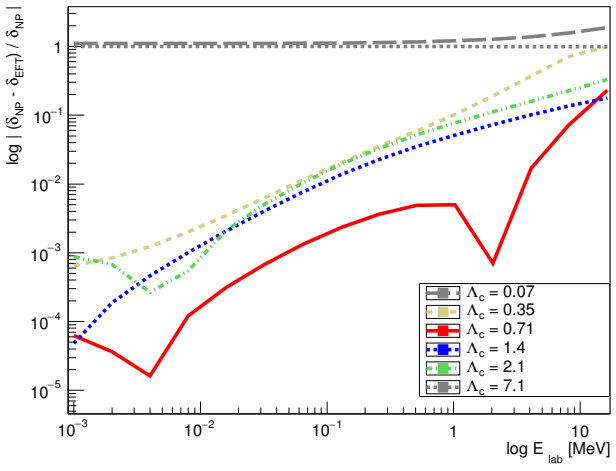


FIG. 3. Lepage error plot for NLO EFT potentials with various cutoffs Λ_c. From these results, we deduce that $\Lambda_c = 0.71$ (the pion mass) is roughly the breakdown scale for which setting Λ_c too far below or above this value worsens the agreement with the underlying theory.

Λ_c	C_0	C_2
1.4	-0.14	0.1
2.8	-0.13	0.93
5.6	0.075	0.02
10.	-0.048	0.002

TABLE II. One-pion EFT parameters for various values of the cutoff Λ_c. We note that the values for C_0 and C_2 are very cutoff dependent.

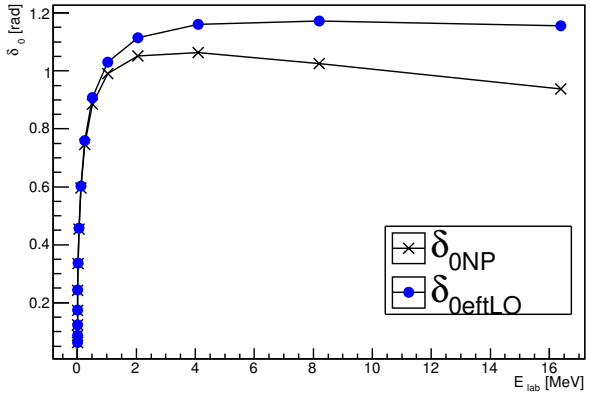
for different values of the cutoff although the one pion EFT appears to be more stable in that doubling Λ_c still produces as good an agreement.

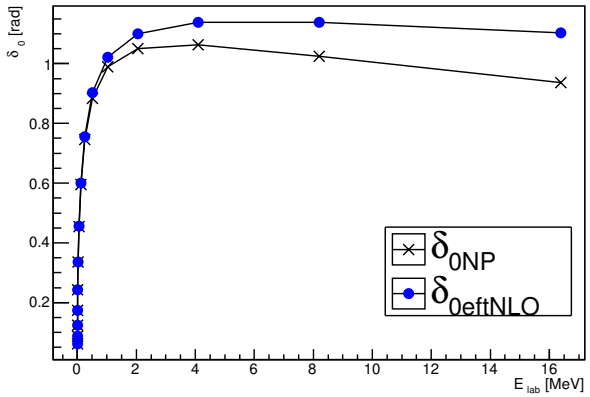
CONCLUSIONS

We have crafted two versions of an effective field theory that approximates the physics of neutron-proton scattering as it's described by the three-Yukawa potential model of the previous project. By fitting to the low energy phase shifts of our underlying theory, we extracted coupling constants for a pionless and one pion exchange version of an EFT. We note that these were not rigorous fits and expect that improving them would improve the agreement of our EFT with the phase shifts calculated using the underlying theory. In each case we varied the cutoff momenta to find the energy scale at which the EFT breaks down.

[1] R. Brun and F. Rademakers. ROOT: An object oriented data analysis framework. *Nucl. Instrum. Meth.*, A389:81–86, 1997. doi:10.1016/S0168-9002(97)00048-X.

[2] P. Lepage. How to renormalize the schrodinger equation. url = https://arxiv.org/abs/nucl-th/9706029, June 1997.





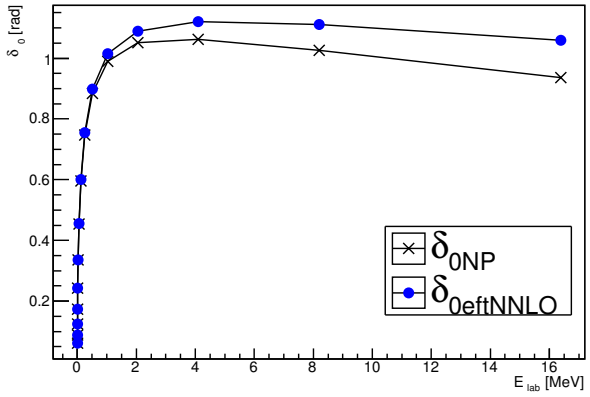


FIG. 4. Comparison between the three-Yukawa *np* model $l = 0$ phase shifts (black X) and the phase shifts calculated from the LO, NLO, NNLO one-pion exchange EFT potentials (blue dots).

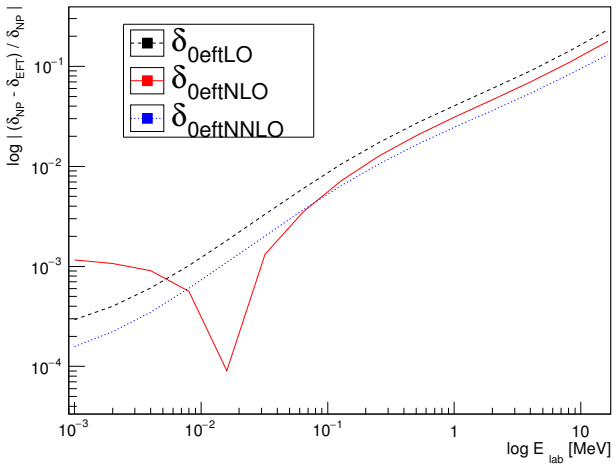


FIG. 5.

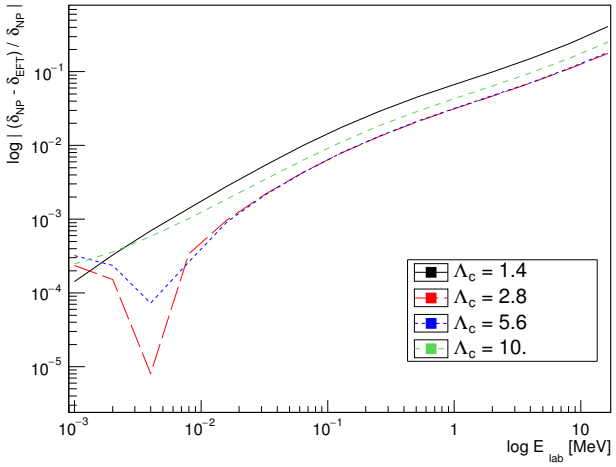


FIG. 6. Lepage error plot comparing the *np* model and EFT $\pi-$NLO with for various cutoff values Λ_c.