TUTORIAL LECTURE → 2021/12/13

$$\begin{cases} Au_1 = \lambda_1 u_1 & Au_2 = \lambda_2 v_2 \\ Av_1 = \lambda_1 v_1 & Av_2 = \lambda_2 v_2 \end{cases}$$

i) Suppose $v_1 \neq -v_2$. is $v_1 + v_2$ an eigenvector of A?

$$A(v_1 + v_2) = Av_1 + Av_2$$
$$\stackrel{?}{=} \lambda_1 v_1 + \lambda_2 v_2$$

$v_1 + v_2$ is not always a multiple of $\lambda_1 v_1 + \lambda_2 v_2$

↳ **No.**

ii) Suppose $u_1 \neq -v_1$ is $u_1 + v_1$ an eigenvector of A?

$$A(u_1 + v_1) = Au_1 + Av_1$$
$$\lambda_1(u_1 + v_1) = \lambda_1 u_1 + \lambda_1 v_1$$

→ an eigenvector → **Yes.**

iii) Is λ_1^{-1} an eigenvalue of A^{-1}?

$$Av_1 = \lambda_1 v_1$$
$$A^{-1}Av_1 = A^{-1}\lambda_1 v_1$$
$$Iv_1 = \lambda_1 A^{-1} v_1$$

// can multiply the scalar by either matrix

$$\lambda_1^{-1} v_1 = \lambda_1^{-1} \lambda_1 A^{-1} v_1$$
$$\lambda_1^{-1} v_1 = A^{-1} v_1$$

→ **Yes**

iv) is v_1^{-1} an eigenvector of A^{-1}?

the inverse of a vector isn't really a thing, so **No**

v) is Kv_1 an eigenvector of A for any $K \in \mathbb{R}\{0\}$

$$A(\underbrace{Kv_1}_{l}) = KAv_1 = K\lambda_1 v_1 = \lambda_1 (\underbrace{Kv_1}_{l})$$

$$Al = \lambda_1 l$$

$\therefore$ **Yes**

vi) is $Ku_1 + K'v_1$ an eigenvector of A for any $K, K' \in \mathbb{R} \setminus \{0\}$

$$A(Ku_1 + K'v_1) = AKu_1 + AK'v_1 = KAu_1 + K'Av_1$$
$$= K\lambda_1 u_1 + K'\lambda_1 v_1$$
$$A(Ku_1 + K'v_1) = \lambda_1 (Ku_1 + K'v_1)$$

however, even if $K + K' \neq 0$, $Ku_1 + K'v_1$ could be a zero vector, so **No**

Let $A = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$

Find integers m and n such that $A^4 = mA^2 + nI$

$|A - \lambda I|$ → characteristic polynomial

$$\begin{pmatrix} 2-\lambda & 4 \\ 1 & 2-\lambda \end{pmatrix}$$

$$(2-\lambda)(2-\lambda) - 1 \cdot 4 = \lambda^2 - 4\lambda + 4 - 4 = \lambda^2 - 4\lambda$$

// Kayley-hamilton theorem

$$A^2 - 4A = 0$$
$$A^2 = 4A$$
$$A^4 = (4A)^2 = 16A^2$$
$$A^4 = 16A^2 + 0I$$

* LOOK OVER KAYLEY-HAMILTON *