MA 190 **LÉONARD EULER** - phi Function

Coprime integers - two integers m, n are coprime if $hcf(m,n) = 1$

Definition :

Let $\varphi(n)$ denote the number of integers in the range $1 \leq x \leq n$ that are coprime to n.

e.g. $\varphi(8) \Rightarrow \{1, 3, 5, 7\}$ $\quad$ $\varphi(6) \rightarrow 1, 5$ $\quad$ $\varphi(107) = 106$

$\varphi(8) = 4$ $\quad$ $\varphi(6) = 2$ $\quad$ ↳ prime

$\varphi(19) = 18$

We call $\varphi(n)$ **EULER'S PHI FUNCTION** or **EULER'S TOTIENT FUNCTION**

THEOREM : a proposition that's arbitrarily more important

PROPOSITION 1

→ if p is a prime integer, then $\varphi(p) = p - 1$.

PROPOSITION 2

$\varphi(2^2) = \varphi(4) = 2 = 2^2 - 2^1$

$\varphi(3^2) = \varphi(9) = 6 = 3^2 - 3^1$

$\varphi(2^4) = \varphi(16) = 8 = 2^4 - 2^3$

→ if p is prime, then $\varphi(p^n) = p^n - p^{n-1}$

$\varphi(3 \cdot 5) = \varphi(15) = 8$

$\varphi(3) = 2$ $\quad$ $\varphi(5) = 4$

$$\left\{\begin{array}{l} \varphi(4 \cdot 6) = \varphi(24) \\ \varphi(4) \cdot \varphi(6) = \varphi(24) \\ 2 \cdot 2 \neq 8 \end{array}\right\}$$

PROPOSITION 3

→ $\varphi(m \cdot n) = \varphi(m) \cdot \varphi(n)$

↳ if m + n are coprime

Example

$\varphi(220)$

↳ $2 \cdot 2 \cdot 5 \cdot 11 \Rightarrow \varphi(2^2 \cdot 5 \cdot 11)$ (all primes)

$= \varphi(2^2) \cdot \varphi(5) \cdot \varphi(11)$ (... proposition 3)

$= (2^2 - 2^1) \cdot \varphi(5) \cdot \varphi(11)$ (... proposition 2)

$= 2 \cdot 4 \cdot 10$ (... proposition 1)

$= 80$

Look at lecture notes for proofs