5a. $\int x\sin(2x)dx \qquad u=x \quad du=dx \quad dv=\sin(2x)\,dx \quad v=-\frac{1}{2}\cos(2x)$

$= -\frac{1}{2}x\cos(2x) - \int -\frac{1}{2}\cos(2x)\,dx$

$= -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C$

b. $\int xe^{x^2}dx \qquad u=x^2 \Rightarrow x=\sqrt{u} \Rightarrow 1=\frac{1}{2}u^{-\frac{1}{2}}\frac{du}{dx} \Rightarrow dx = \frac{1}{2}u^{-\frac{1}{2}}du$

$= \int \frac{1}{2}u^{\frac{1}{2}}u^{-\frac{1}{2}}e^{u}du = \int \frac{1}{2}e^{u}du = \frac{1}{2}e^{u}+c$

$= \frac{1}{2}e^{x^2}+C$

c. $\int xe^{x}dx \qquad u=x \quad du=dx \quad dv=e^{x}dx \quad v=e^{x}$

$= xe^{x} - \int e^{x}dx$

$= xe^{x} - e^{x} + C$

d. $\int e^{x^2}dx = \int 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \ldots\, dx$

$= x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42} + \ldots + C$

e. $\int x\sqrt{1+x}\,dx \qquad u=x \quad du=dx \quad dv=(1+x)^{\frac{1}{2}}dx \quad v=\frac{2}{3}(1+x)^{\frac{3}{2}}$

$= \frac{2}{3}x(1+x)^{\frac{3}{2}} - \int \frac{2}{3}(1+x)^{\frac{3}{2}}dx$

$= \frac{2}{3}x(1+x)^{\frac{3}{2}} - \frac{4}{15}(1+x)^{\frac{5}{2}} + C$

f. $\int \sec\theta\, d\theta = \int \frac{\sec\theta(\tan\theta+\sec\theta)}{\tan\theta+\sec\theta}d\theta = \int \frac{\sec\theta\tan\theta+\sec^2\theta}{\tan\theta+\sec\theta}d\theta$

$= \ln|\tan\theta+\sec\theta| + c$

g. $\int \sec^2\theta\, d\theta = \tan\theta + c$

h. $\int \operatorname{sech}^2\theta\, d\theta = \tanh\theta + c$

i. $\int \frac{x^2+2}{7-x^2}dx = -\int \frac{x^2+2}{x^2-7}dx = -\int \frac{x^2-7}{x^2-7} + \frac{9}{x^2-7}dx$

$= -\int 1 + \frac{9}{x^2-7}dx = -x - 9\int \frac{1}{x^2-7}dx$

$= -x - 9\int \frac{1}{(x-\sqrt{7})(x+\sqrt{7})}dx \qquad \frac{A}{x-\sqrt{7}} + \frac{B}{x+\sqrt{7}} = \frac{1}{(x-\sqrt{7})(x+\sqrt{7})}$

$A(x+\sqrt{7}) + B(x-\sqrt{7}) = 1$

$A = \frac{1}{2\sqrt{7}} \qquad B = \frac{1}{-2\sqrt{7}}$

$= -x - \frac{9}{2\sqrt{7}}\int \frac{1}{x-\sqrt{7}} - \frac{1}{x+\sqrt{7}}dx$

$= -x - \frac{9}{2\sqrt{7}}\left(\ln|x-\sqrt{7}| - \ln|x+\sqrt{7}|\right) + c$

j. $\int \frac{1}{ap-bp^2}dp = \int \frac{1}{p(a-bp)}dp \qquad \frac{A}{p} + \frac{B}{a-bp} = \frac{1}{p(a-bp)}$

$= \frac{1}{a}\int \frac{1}{p} + \frac{b}{a-bp}dp \qquad A(a-bp) + Bp = 1$

$= \frac{1}{a}\left(\ln|p| - \ln|a-bp|\right) + c \qquad A=\frac{1}{a} \quad B=\frac{b}{a}$