

Homework 3: Fundamentals and Discrete Distributions

- (1) It is believed that 20% of Americans do not have any health insurance. Let X equal the number with no health insurance in a random sample of $n = 15$ Americans.
 - (a) How is X distributed?
 - (b) Find the mean and variance of X .
 - (c) $P(X \geq 2)$.

- (2) Consider a random experiment of casting a pair of unbiased six-sided dice and let the r.v. X equal the *smaller* of the outcomes if they are different and the common value if they are equal.
 - (a) Find the p.d.f. of r.v. X .
 - (b) Draw a probability histogram.
 - (c) Find the expectation and variance of r.v. X .

- (3) In a lottery, a 3-digit integer is selected at random from 000 to 999, inclusive. Let X be the integer selected on a particular day.
 - (a) Find the p.m.f. (p.d.f.) of the r.v. X .
 - (b) Find the mean of the r.v. X .
 - (c) Find variance of the r.v. X .

- (4) Let the r.v. X have a Poisson distribution with the p.d.f. $f(x) = \lambda^x e^{-\lambda} / x!$, $x = 0, 1, 2, \dots, \infty$, where $\lambda > 0$ is a known parameter.
 - (a) Find the mean, $E(X)$.
 - (b) Find the variance, $\text{Var}(X)$.
 - (c) Find the mode of the probability density function f .

- (5) Consider a *binomial distribution* $X \sim b(n, p)$, draw the bar chart for each density function described below.

- (a) Plot the density function $X \sim b(10, 0.6)$.
 - (b) Plot the density function $X \sim b(9, 0.6)$.
 - (c) What are the modes of **(a)** and **(b)**, respectively?
- (6)** Let $Y \sim Poisson(\lambda)$ be a Poisson distribution with mean λ .
- (a) Plot the density function $Y \sim Poisson(4)$.
 - (b) Plot the density function $Y \sim Poisson(7)$.
 - (c) What are the modes of **(a)** and **(b)**, respectively?