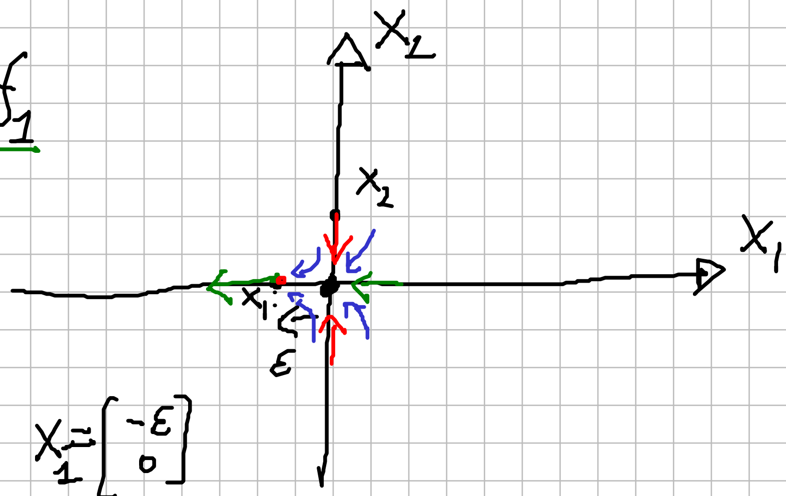
$$\dot{x}_1 = \mu - x_1^2 = f_1$$

$$\dot{x}_2 = -x_2 = f_2$$



$\mu = 0$

$$f(x_1) = \begin{bmatrix} -\varepsilon^2 \\ 0 \end{bmatrix} \qquad x_1 = \begin{bmatrix} -\varepsilon \\ 0 \end{bmatrix}$$

$$f(x_2) = \begin{bmatrix} 0 \\ -\alpha \end{bmatrix} \qquad x_2 = \begin{bmatrix} 0 \\ \alpha \end{bmatrix}$$

$$A = \left[\begin{array}{cc|c} -2 & 1 & 0 \\ 0 & -2 & 0 \\ \hline 0 & 0 & -2 \end{array}\right] \qquad A = J = \mathrm{diag}(J_1, J_2)$$

$\lambda_1 = -2$, $\sigma(A) = \{2\}$

$m_1 = 3$

$\mu_1 = 2$

$\varepsilon > 0 \;\; \exists \delta_\varepsilon > 0 :$ se $\boxed{\|x(0)\| < \delta} \Longrightarrow \|x(t)\| < \varepsilon \;\; \forall t \geq 0$

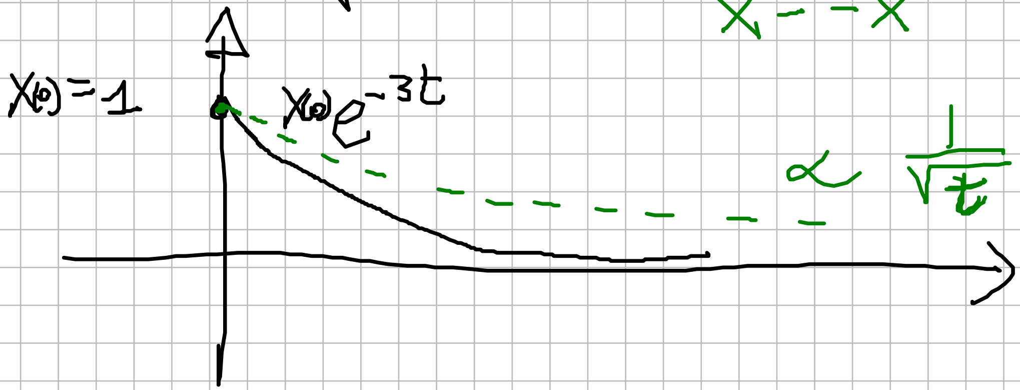
$$\|x(t)\| \leq K \|x(0)\| e^{-\lambda_{min} t}$$

$$\| \cancel{K} \|x(0)\| e^{-\lambda_{min} t} \| < \varepsilon$$

$$\boxed{\|x(0)\| \leq \frac{1}{\cancel{K}} \cdot \frac{1}{1+\gamma} \cdot \varepsilon} \qquad \varepsilon > 0$$

$$\delta_\varepsilon = \frac{\varepsilon}{K(1+\gamma)}, \; \gamma > 0 \qquad \left\| \underbrace{\frac{e^{-\lambda_m t}}{1+\gamma}}_{<1} \varepsilon \right\| < \varepsilon \qquad \gamma > 0$$

$\dot{x} = -3x$ $\quad\quad$ $\dot{x} = -x^3$



$f(x)$, $f(0) = 0$, f smooth $(\in C^\infty)$

$f(x) = Ax + \overline{\sigma(x)x} \rightarrow f(x) = x^3$

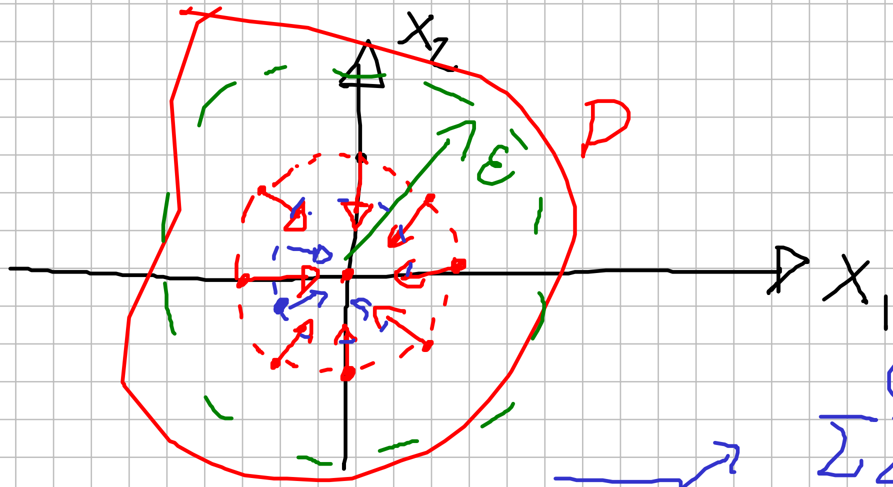
$A = J f(x)\big|_{x=0}$ $\quad\quad$ $f(x) = Ax + (x^3 - Ax)$

$= Ax + (x^2 - A)x$

$a = 0$, $b = \sigma + a$

$f \in C^1$, $[a, b]$ $\quad\quad$ $f(\sigma) - f(0) = f'(\xi)\sigma$

$\exists\, \xi \in [a, b]$: $\quad\quad$ $\frac{f(b) - f(a)}{b - a} = f'(\xi)\sigma$, $f(a) = 0$, $b - a = \sigma - 0$

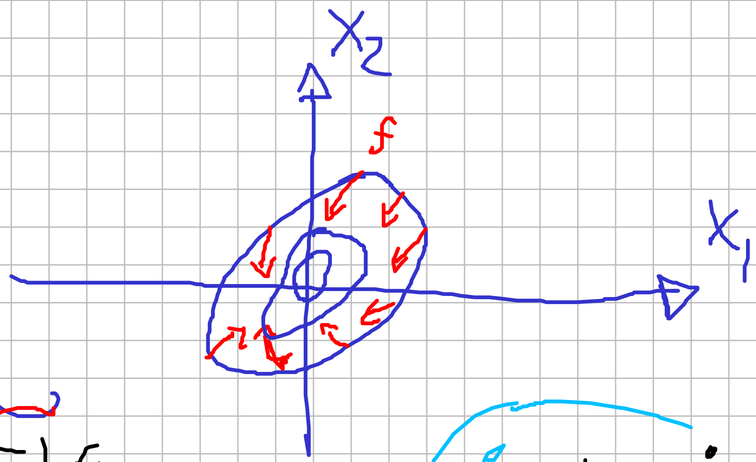


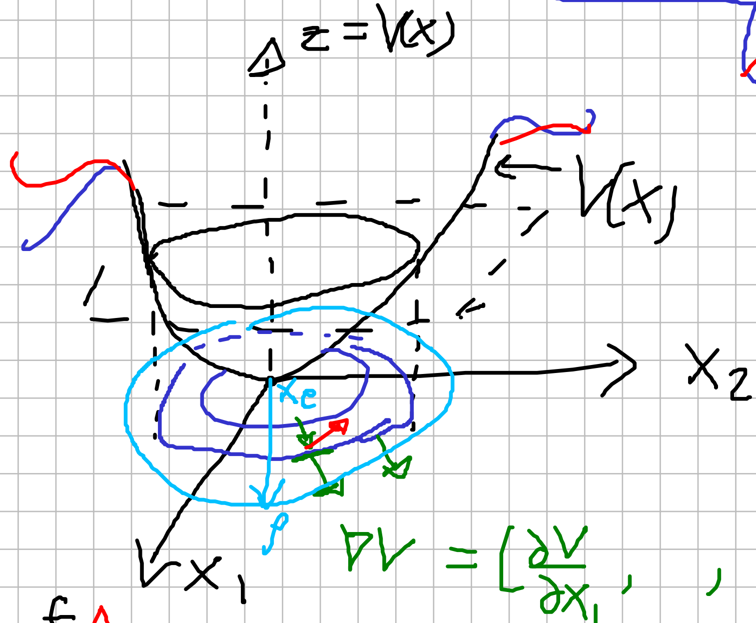
$$\sum \sum_k \boxed{t^{k-1} e^{\lambda_i t}} R_{ik}$$

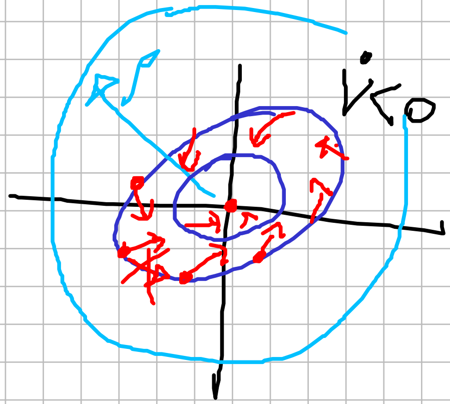
$$x(t) = e^{At} x_0 \to T^{-1} \boxed{e^{Jt}} T x_0$$

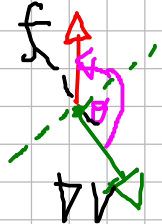
$$\| x(t) \| < \varepsilon$$

$$\dot{x} = \boxed{f(t)},\ x_0$$









$$\nabla V = \left[\frac{\partial V}{\partial x_1},\ ,\ \frac{\partial V}{\partial x_2} \right]$$

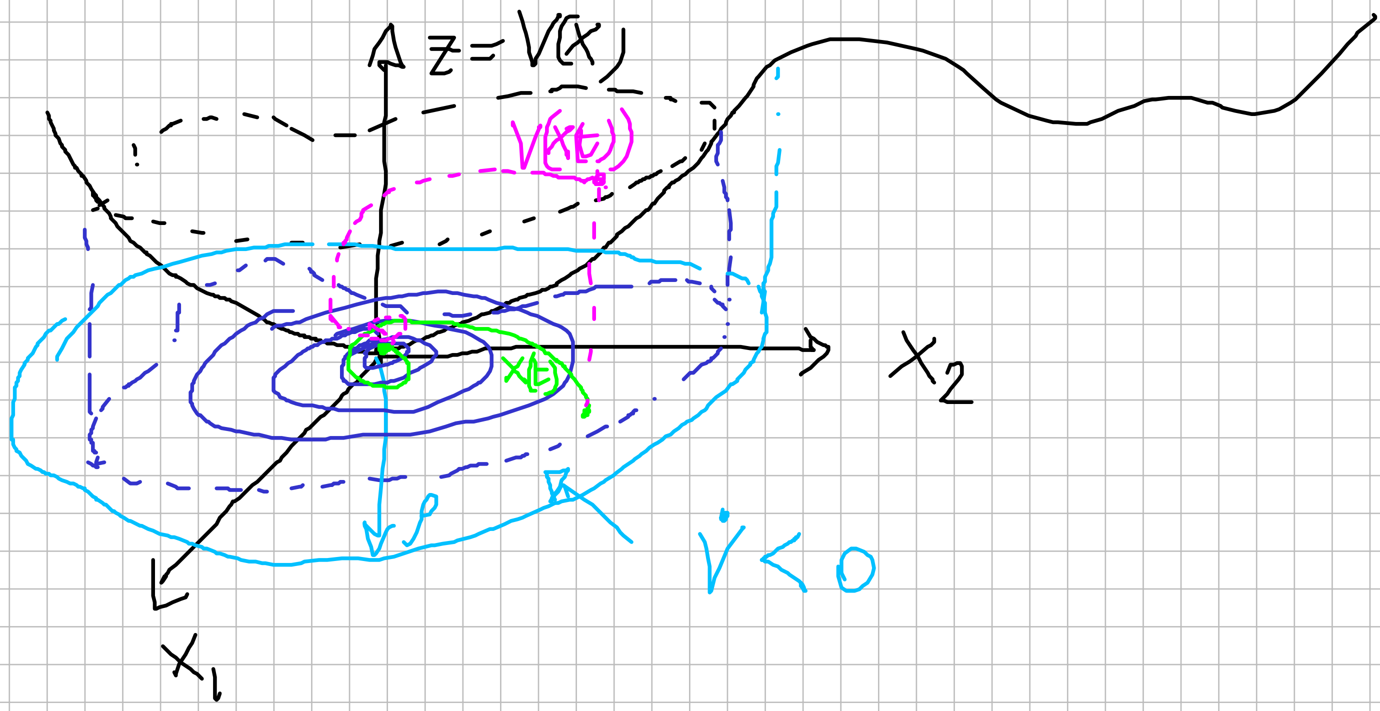
$$\langle \nabla V, f \rangle = \|f\| \, \|\nabla V\| \cdot \cos(\theta) < 0$$

$$\theta > \pi/2$$

$$\pi > \theta > 0$$

$$\dot{V} = \frac{\partial V}{\partial x} f(x) < 0 \qquad \forall x \in B_{x_e}(\rho), \exists \rho > 0$$

$\Rightarrow$ A.S $\qquad \forall \rho > 0$ GAS



z = V(x)
V(x(t))
x(t)
x_2
x_1
ρ
V̇ < 0