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Assignment 7

Considera il modello del pendolo inverso su un cart descritto dalle equazioni:

$$M\ddot{s}+F\dot{s}-\mu=d_1 \quad \ddot{\phi}=\frac{g}{L}\sin(\phi)+\frac{1}{L}\ddot{s}\cos(\phi)=0$$

con $M=1\,\mathrm{kg}, L=1m, F=1\,\mathrm{kg}\,s^{-1}, g=9.81ms^{-1}$.
A1)Calcolare tutte i punti di equilibrio del sistema per $\mu=d_1(t)=0$,

(%i 1) dx[1]:x[2]

(%o1) x_2

(%i 2) dx[2]:(1/M)*(-F*x[2]+u[1]+u[2])

(%o2) $\frac{-x_2F+u_2+u_1}{M}$

(%i 3) dx[3]:x[4]

(%o3) x_4

(%i 4) dx[4]:expand((1/L)*(g*sin(x[3])-dx[2]*cos(x[3])))

(%o4) $\frac{\sin(x_3)g}{L}+\frac{x_2\cos(x_3)F}{LM}-\frac{u_2\cos(x_3)}{LM}-\frac{u_1\cos(x_3)}{LM}$ A2) Scrivere le equazioni del sistema linearizzato attorno al punto di equilibrio $\phi=s=\dot{\phi}=0$

```
(%i 5) diffx(dx):=block(
   res:zeromatrix(4,4)
,
for i:1 thru 4 do (
for j:1 thru 4 do (
res[i,j]:diff(dx[i],x[j])
)
),
return(res)
)$
```

```
(%i 6) diffu(dx):=block(
   res:zeromatrix(4,2)
,
for i:1 thru 4 do (
for j:1 thru 2 do (
res[i,j]:diff(dx[i],u[j])
)
),
return(res)
)$
```

(%i 7) dx:[dx[1],dx[2],dx[3],dx[4]]$

(%i 8) nablax:diffx(dx)

(%o8) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{F}{M} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{\cos(x_3)F}{LM} & \frac{\cos(x_3)g}{L}-\frac{x_2\sin(x_3)F}{LM}+\frac{u_2\sin(x_3)}{LM}+\frac{u_1\sin(x_3)}{LM} & 0 \end{pmatrix}$

(%i 9) nablau:diffu(dx)

(%o9) $\begin{pmatrix} 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ 0 & 0 \\ -\frac{\cos(x_3)}{LM} & -\frac{\cos(x_3)}{LM} \end{pmatrix}$

(%i 10) sub:[x[1]=0,x[2]=0,x[3]=0,x[4]=0,u[1]=0,u[2]=0]$

(%i 11) `A:subst(sub,nablax)`

(%o11)
$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{F}{M} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{F}{LM} & \frac{g}{L} & 0 \end{pmatrix}$$

(%i 12) `B:subst(sub,nablau)`

(%o12)
$$\begin{pmatrix} 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ 0 & 0 \\ -\frac{1}{LM} & -\frac{1}{LM} \end{pmatrix}$$
A3) Mostra che la coppia (A, B) è controllabile

(%i 13) `P:addcol(col(B,2),matrix([0],[0],[0],[0]),matrix([0],[0],[0],[0]))`

(%o13)
$$\begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{M} & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{1}{LM} & 0 & 0 \end{pmatrix}$$

(%i 14) `B:col(B,1)`

(%o14)
$$\begin{pmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{LM} \end{pmatrix}$$

(%i 15) `C:matrix([1,0,0,0],[0,0,1,0])`

(%o15)
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$
A4)Mostra che la coppia (A, B) è controllabile.

(%i 16) `A1B:A.B`

(%o16)
$$\begin{pmatrix} \frac{1}{M} \\ -\frac{F}{M^2} \\ -\frac{1}{LM} \\ \frac{F}{LM^2} \end{pmatrix}$$

(%i 17) `A2B:A.A1B`

(%o17)
$$\begin{pmatrix} -\frac{F}{M^2} \\ \frac{F^2}{M^3} \\ \frac{F}{LM^2} \\ -\frac{g}{L^2 M} - \frac{F^2}{LM^3} \end{pmatrix}$$

(%i 18) `A3B:A.A2B`

(%o18)
$$\begin{pmatrix} \frac{F^2}{M^3} \\ -\frac{F^3}{M^4} \\ -\frac{g}{L^2 M} - \frac{F^2}{LM^3} \\ \frac{Fg}{L^2 M^2} + \frac{F^3}{LM^4} \end{pmatrix}$$

(%i 19) `R:addcol(B,A1B,A2B,A3B)`

(%o19)
$$\begin{pmatrix} 0 & \frac{1}{M} & -\frac{F}{M^2} & \frac{F^2}{M^3} \\ \frac{1}{M} & -\frac{F}{M^2} & \frac{F^2}{M^3} & -\frac{F^3}{M^4} \\ 0 & -\frac{1}{LM} & \frac{F}{LM^2} & -\frac{g}{L^2 M} - \frac{F^2}{LM^3} \\ -\frac{1}{LM} & \frac{F}{LM^2} & -\frac{g}{L^2 M} - \frac{F^2}{LM^3} & \frac{Fg}{L^2 M^2} + \frac{F^3}{LM^4} \end{pmatrix}$$

(%i 20) `rank(R)`

(%o20) 4 A5)Considera il sistema lineare A3). Supponi che d_1 non sia nota. La legge di controllo deve essere tale che l'effetto del disturbo d_1 sia asintoticamente respindo e la prima uscita $s(t)$ insegua asintoticamente il segnale di riferimento $d_2 = \alpha \sin(\omega t)$. Struttura il problema come un problema di regolazione.

```
(%i 21) S:matrix([0,0,0],[0,0,omega],[0,-omega,0])
```

(%o21) $$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \omega \\ 0 & -\omega & 0 \end{pmatrix}$$

```
(%i 22) Q:matrix([0,-1,0])
```

(%o22) $\begin{pmatrix} 0 & -1 & 0 \end{pmatrix}$

```
(%i 23) Ce:row(C,1)
```

(%o23) $\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$ A6)Considera il problema di regolazione determinato in A5) e mostra che il problema è risolubile tramite la legge di controllo a full information Lemma di Hautus:

```
(%i 24) H:addcol(s*ident(4)-A,B)
```

(%o24) $$\begin{pmatrix} s & -1 & 0 & 0 & 0 \\ 0 & s+\frac{F}{M} & 0 & 0 & \frac{1}{M} \\ 0 & 0 & s & -1 & 0 \\ 0 & -\frac{F}{LM} & -\frac{g}{L} & s & -\frac{1}{LM} \end{pmatrix}$$

```
(%i 25) H:addrow(H,addcol(row(C,1),matrix([0])))
```

(%o25) $$\begin{pmatrix} s & -1 & 0 & 0 & 0 \\ 0 & s+\frac{F}{M} & 0 & 0 & \frac{1}{M} \\ 0 & 0 & s & -1 & 0 \\ 0 & -\frac{F}{LM} & -\frac{g}{L} & s & -\frac{1}{LM} \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
(%i 26) rank(H)
```

(%o26) 5

```
(%i 27) rank(subst(s=0,H))
```

(%o27) 5

```
(%i 28) rank(subst(s=%i*omega,H))
```

(%o28) 5

```
(%i 29) rank(subst(s=-%i*omega,H))
```

(%o29) 5 A7)Considera il problema di regolazione determinato in A5). Mostra che il problema è risolubi tramite una legge di controllo in feedback dall'errore.

$$\begin{bmatrix} \dot{x} \\ \dot{d} \end{bmatrix} = \begin{bmatrix} A & P \\ 0 & S \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} \qquad e = \begin{bmatrix} C & Q \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix}$$

deve essere osservabile.

```
(%i 30) Ao:addcol(A,P)
```

(%o30) $$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{F}{M} & 0 & 0 & \frac{1}{M} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{F}{LM} & \frac{g}{L} & 0 & -\frac{1}{LM} & 0 & 0 \end{pmatrix}$$

```
(%i 31) Ao:addrow(Ao,addcol(zeromatrix(3,4),S))
```

(%o31) $$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{F}{M} & 0 & 0 & \frac{1}{M} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{F}{LM} & \frac{g}{L} & 0 & -\frac{1}{LM} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \omega \\ 0 & 0 & 0 & 0 & 0 & -\omega & 0 \end{pmatrix}$$

(%i 32)
```
Co:addcol(C,addrow(Q,zeromatrix(1,3)))
```

(%o32) $$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(%i 33)
```
O:Co;
```

(%o33) $$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(%i 34)
```
rank(O)
```

(%o34) 2

(%i 35)
```
CA1:Co.Ao$
```

(%i 36)
```
O:addrow(O,CA1)
```

(%o36) $$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\omega \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

(%i 37)
```
rank(O)
```

(%o37) 4

(%i 38)
```
CA2:CA1.Ao$
O:addrow(O,CA2);
rank(O);
```

(%o39) $$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\omega \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{F}{M} & 0 & 0 & \frac{1}{M} & \omega^2 & 0 \\ 0 & \frac{F}{LM} & \frac{g}{L} & 0 & -\frac{1}{LM} & 0 & 0 \end{pmatrix}$$

(%o40) 6

(%i 41)
```
CA3:CA2.Ao$
O:addrow(O,CA3);
rank(O);
```

(%o42) $$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\omega \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{F}{M} & 0 & 0 & \frac{1}{M} & \omega^2 & 0 \\ 0 & \frac{F}{LM} & \frac{g}{L} & 0 & -\frac{1}{LM} & 0 & 0 \\ 0 & \frac{F^2}{M^2} & 0 & 0 & -\frac{F}{M^2} & 0 & \omega^3 \\ 0 & -\frac{F^2}{LM^2} & 0 & \frac{g}{L} & \frac{F}{LM^2} & 0 & 0 \end{pmatrix}$$

(%o43) 7

(%i 44)
```
Ao2:Ao
```

(%o44) $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{F}{M} & 0 & 0 & \frac{1}{M} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{F}{LM} & \frac{g}{L} & 0 & -\frac{1}{LM} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \omega \\ 0 & 0 & 0 & 0 & 0 & -\omega & 0 \end{pmatrix}$

(%i 45) `Co2:addcol(Ce,Q)`

(%o45) $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$

(%i 46) `O2:Co2;`
`rank(O2)`

(%o46) $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$
(%o47) 1

(%i 48) `CA1_2:Co2.Ao2$`
`O2:addrow(O2,CA1_2);`
`rank(O2);`

(%o49) $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\omega \end{pmatrix}$
(%o50) 2

(%i 51) `CA2_2:CA1_2.Ao2$`
`O2:addrow(O2,CA2_2);`
`rank(O2);`

(%o52) $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\omega \\ 0 & -\frac{F}{M} & 0 & 0 & \frac{1}{M} & \omega^2 & 0 \end{pmatrix}$
(%o53) 3

(%i 54) `CA3_2:CA2_2.Ao2$`
`O2:addrow(O2,CA3_2);`
`rank(O2);`

(%o55) $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\omega \\ 0 & -\frac{F}{M} & 0 & 0 & \frac{1}{M} & \omega^2 & 0 \\ 0 & \frac{F^2}{M^2} & 0 & 0 & -\frac{F}{M^2} & 0 & \omega^3 \end{pmatrix}$
(%o56) 4

(%i 57) `CA4_2:CA3_2.Ao2$`
`O2:addrow(O2,CA4_2);`
`rank(O2);`

(%o58) $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\omega \\ 0 & -\frac{F}{M} & 0 & 0 & \frac{1}{M} & \omega^2 & 0 \\ 0 & \frac{F^2}{M^2} & 0 & 0 & -\frac{F}{M^2} & 0 & \omega^3 \\ 0 & -\frac{F^3}{M^3} & 0 & 0 & \frac{F^2}{M^3} & -\omega^4 & 0 \end{pmatrix}$

(%o59) 5

(%i 60) `CA5_2:CA4_2.Ao2$`
`O2:addrow(O2,CA5_2);`
`rank(O2);`

(%o61) $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\omega \\ 0 & -\frac{F}{M} & 0 & 0 & \frac{1}{M} & \omega^2 & 0 \\ 0 & \frac{F^2}{M^2} & 0 & 0 & -\frac{F}{M^2} & 0 & \omega^3 \\ 0 & -\frac{F^3}{M^3} & 0 & 0 & \frac{F^2}{M^3} & -\omega^4 & 0 \\ 0 & \frac{F^4}{M^4} & 0 & 0 & -\frac{F^3}{M^4} & 0 & -\omega^5 \end{pmatrix}$

(%o62) 5

(%i 63)
```
CA6_2:CA5_2.Ao2$
O2:addrow(O2,CA6_2);
rank(O2);
```

(%o64)
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\omega \\ 0 & -\frac{F}{M} & 0 & 0 & \frac{1}{M} & \omega^2 & 0 \\ 0 & \frac{F^2}{M^2} & 0 & 0 & -\frac{F}{M^2} & 0 & \omega^3 \\ 0 & -\frac{F^3}{M^3} & 0 & 0 & \frac{F^2}{M^3} & -\omega^4 & 0 \\ 0 & \frac{F^4}{M^4} & 0 & 0 & -\frac{F^3}{M^4} & 0 & -\omega^5 \\ 0 & -\frac{F^5}{M^5} & 0 & 0 & \frac{F^4}{M^5} & \omega^6 & 0 \end{pmatrix}$$

(%o65) 5 B1)Sia $d_1(t)$ un'onda quadra di ampiezza 0.5 e periodo 50s,$\alpha = 1, \omega = 0.1$. Progetta una legge di controllo a full information che risolve A5).

$$\Pi S = A\Pi + B\Gamma + P \quad 0 = C\Pi + Q$$

(%i 66)
```
Pi:matrix([p[1,1],p[1,2],p[1,3]],
   p[2,1
,p[2,2],p[2,3]],
   p[3,1
,p[3,2],p[3,3]],
   p[4,1
,p[4,2],p[4,3]])
```

(%o66)
$$\begin{pmatrix} p_{1,1} & p_{1,2} & p_{1,3} \\ p_{2,1} & p_{2,2} & p_{2,3} \\ p_{3,1} & p_{3,2} & p_{3,3} \\ p_{4,1} & p_{4,2} & p_{4,3} \end{pmatrix}$$

(%i 67)
```
Gamma:matrix([g[1,1],g[1,2],g[1,3]])
```

(%o67) $\begin{pmatrix} g_{1,1} & g_{1,2} & g_{1,3} \end{pmatrix}$

(%i 68)
```
expr1:Pi.S-A.Pi-B.Gamma-P
```

(%o68)
$$\begin{pmatrix} -p_{2,1} & -p_{1,3}\omega - p_{2,2} & p_{1,2}\omega - p_{2,3} \\ \frac{p_{2,1}F}{M} - \frac{g_{1,1}}{M} - \frac{1}{M} & -p_{2,3}\omega + \frac{p_{2,2}F}{M} - \frac{g_{1,2}}{M} & p_{2,2}\omega + \frac{p_{2,3}F}{M} - \frac{g_{1,3}}{M} \\ -p_{4,1} & -p_{3,3}\omega - p_{4,2} & p_{3,2}\omega - p_{4,3} \\ -\frac{p_{3,1}g}{L} - \frac{p_{2,1}F}{LM} + \frac{g_{1,1}}{LM} + \frac{1}{LM} & -p_{4,3}\omega - \frac{p_{3,2}g}{L} - \frac{p_{2,2}F}{LM} + \frac{g_{1,2}}{LM} & p_{4,2}\omega - \frac{p_{3,3}g}{L} - \frac{p_{2,3}F}{LM} + \frac{g_{1,3}}{LM} \end{pmatrix}$$

(%i 69)
```
expr2:Ce.Pi+Q
```

(%o69) $\begin{pmatrix} p_{1,1} & p_{1,2}-1 & p_{1,3} \end{pmatrix}$

(%i 70)
```
expr1
```

(%o70)
$$\begin{pmatrix} -p_{2,1} & -p_{1,3}\omega - p_{2,2} & p_{1,2}\omega - p_{2,3} \\ \frac{p_{2,1}F}{M} - \frac{g_{1,1}}{M} - \frac{1}{M} & -p_{2,3}\omega + \frac{p_{2,2}F}{M} - \frac{g_{1,2}}{M} & p_{2,2}\omega + \frac{p_{2,3}F}{M} - \frac{g_{1,3}}{M} \\ -p_{4,1} & -p_{3,3}\omega - p_{4,2} & p_{3,2}\omega - p_{4,3} \\ -\frac{p_{3,1}g}{L} - \frac{p_{2,1}F}{LM} + \frac{g_{1,1}}{LM} + \frac{1}{LM} & -p_{4,3}\omega - \frac{p_{3,2}g}{L} - \frac{p_{2,2}F}{LM} + \frac{g_{1,2}}{LM} & p_{4,2}\omega - \frac{p_{3,3}g}{L} - \frac{p_{2,3}F}{LM} + \frac{g_{1,3}}{LM} \end{pmatrix}$$

(%i 71)
```
transpose(flatten(args(expr1)=0))
```

(%o71) $\mathrm{transpose}\left(\left[\left[-p_{2,1}, -p_{1,3}\omega - p_{2,2}, p_{1,2}\omega - p_{2,3}\right], \left[\frac{p_{2,1}F}{M} - \frac{g_{1,1}}{M} - \frac{1}{M}, -p_{2,3}\omega + \frac{p_{2,2}F}{M} - \frac{g_{1,2}}{M}, p_{2,2}\omega + \frac{p_{2,3}F}{M} - \frac{g_{1,3}}{M}\right], \left[-p_{4,1}, -\right.\right.\right.$

(%i 72)
```
transpose(flatten(args(expr2)))
```

(%o72)
$$\begin{pmatrix} p_{1,1} \\ p_{1,2}-1 \\ p_{1,3} \end{pmatrix}$$

(%i 73)
```
toSubst:[p[1,1]=0,p[1,3]=0,p[1,2]=1,p[4,1]=0,p[2,1]=0]
```

(%o73) $[p_{1,1} = 0, p_{1,3} = 0, p_{1,2} = 1, p_{4,1} = 0, p_{2,1} = 0]$

(%i 74) `expr1:subst(toSubst,expr1)`

$$(\%o74)\quad \begin{pmatrix} 0 & -p_{2,2} & \omega - p_{2,3} \\ -\frac{g_{1,1}}{M} - \frac{1}{M} & -p_{2,3}\omega + \frac{p_{2,2}F}{M} - \frac{g_{1,2}}{M} & p_{2,2}\omega + \frac{p_{2,3}F}{M} - \frac{g_{1,3}}{M} \\ 0 & -p_{3,3}\omega - p_{4,2} & p_{3,2}\omega - p_{4,3} \\ -\frac{p_{3,1}g}{L} + \frac{g_{1,1}}{LM} + \frac{1}{LM} & -p_{4,3}\omega - \frac{p_{3,2}g}{L} - \frac{p_{2,2}F}{LM} + \frac{g_{1,2}}{LM} & p_{4,2}\omega - \frac{p_{3,3}g}{L} - \frac{p_{2,3}F}{LM} + \frac{g_{1,3}}{LM} \end{pmatrix}$$

(%i 75) `expr2:subst(toSubst,expr2)`

$$(\%o75)\quad \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

(%i 76) `transpose(flatten(args(expr1)))`

$$(\%o76)\quad \begin{pmatrix} 0 \\ -p_{2,2} \\ \omega - p_{2,3} \\ -\frac{g_{1,1}}{M} - \frac{1}{M} \\ -p_{2,3}\omega + \frac{p_{2,2}F}{M} - \frac{g_{1,2}}{M} \\ p_{2,2}\omega + \frac{p_{2,3}F}{M} - \frac{g_{1,3}}{M} \\ 0 \\ -p_{3,3}\omega - p_{4,2} \\ p_{3,2}\omega - p_{4,3} \\ -\frac{p_{3,1}g}{L} + \frac{g_{1,1}}{LM} + \frac{1}{LM} \\ -p_{4,3}\omega - \frac{p_{3,2}g}{L} - \frac{p_{2,2}F}{LM} + \frac{g_{1,2}}{LM} \\ p_{4,2}\omega - \frac{p_{3,3}g}{L} - \frac{p_{2,3}F}{LM} + \frac{g_{1,3}}{LM} \end{pmatrix}$$

(%i 77) `toSubst:append(toSubst,[p[2,2]=0,p[2,3]=omega,g[1,1]=-1])`

$$(\%o77)\quad [p_{1,1} = 0, p_{1,3} = 0, p_{1,2} = 1, p_{4,1} = 0, p_{2,1} = 0, p_{2,2} = 0, p_{2,3} = \omega, g_{1,1} = -1]$$

(%i 78) `expr1:subst(toSubst,expr1)`

$$(\%o78)\quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\omega^2 - \frac{g_{1,2}}{M} & \frac{F\omega}{M} - \frac{g_{1,3}}{M} \\ 0 & -p_{3,3}\omega - p_{4,2} & p_{3,2}\omega - p_{4,3} \\ -\frac{p_{3,1}g}{L} & -p_{4,3}\omega - \frac{p_{3,2}g}{L} + \frac{g_{1,2}}{LM} & -\frac{F\omega}{LM} + p_{4,2}\omega - \frac{p_{3,3}g}{L} + \frac{g_{1,3}}{LM} \end{pmatrix}$$

(%i 79) `toSubst:append(toSubst,[g[1,2]=-M*omega^2,g[1,3]=F*omega])`

$$(\%o79)\quad [p_{1,1} = 0, p_{1,3} = 0, p_{1,2} = 1, p_{4,1} = 0, p_{2,1} = 0, p_{2,2} = 0, p_{2,3} = \omega, g_{1,1} = -1, g_{1,2} = -M\omega^2, g_{1,3} = F\omega]$$

(%i 80) `expr1:subst(toSubst,expr1)`

$$(\%o80)\quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -p_{3,3}\omega - p_{4,2} & p_{3,2}\omega - p_{4,3} \\ -\frac{p_{3,1}g}{L} & -\frac{\omega^2}{L} - p_{4,3}\omega - \frac{p_{3,2}g}{L} & p_{4,2}\omega - \frac{p_{3,3}g}{L} \end{pmatrix}$$

(%i 81) `toSubst:append(toSubst,[p[3,1]=0])`

$$(\%o81)\quad [p_{1,1} = 0, p_{1,3} = 0, p_{1,2} = 1, p_{4,1} = 0, p_{2,1} = 0, p_{2,2} = 0, p_{2,3} = \omega, g_{1,1} = -1, g_{1,2} = -M\omega^2, g_{1,3} = F\omega, p_{3,1} = 0]$$

(%i 82) `expr1:subst(toSubst,expr1)`

$$(\%o82)\quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -p_{3,3}\omega - p_{4,2} & p_{3,2}\omega - p_{4,3} \\ 0 & -\frac{\omega^2}{L} - p_{4,3}\omega - \frac{p_{3,2}g}{L} & p_{4,2}\omega - \frac{p_{3,3}g}{L} \end{pmatrix}$$

(%i 83) `linsol:transpose(flatten(args(expr1)))`

(%o83) $$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -p_{3,3}\omega - p_{4,2} \\ p_{3,2}\omega - p_{4,3} \\ 0 \\ -\frac{\omega^2}{L} - p_{4,3}\omega - \frac{p_{3,2}g}{L} \\ p_{4,2}\omega - \frac{p_{3,3}g}{L} \end{pmatrix}$$

(%i 84) `temp:[p[4,2]=-p[3,3]*omega,p[4,3]=p[3,2]*omega]`

(%o84) $[p_{4,2} = -p_{3,3}\omega, p_{4,3} = p_{3,2}\omega]$

(%i 85) `tmp:subst(temp,expr1)`

(%o85) $$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{\omega^2}{L} - p_{3,2}\omega^2 - \frac{p_{3,2}g}{L} & -p_{3,3}\omega^2 - \frac{p_{3,3}g}{L} \end{pmatrix}$$

(%i 86) `solve(tmp[4,3]=0,p[3,3])`

(%o86) $[p_{3,3} = 0]$

(%i 87) `solve(tmp[4,2]=0,p[3,2])`

(%o87) $\left[p_{3,2} = -\frac{\omega^2}{L\omega^2+g}\right]$

(%i 88) `toSubst:append(toSubst,[p[3,3]=0,p[3,2]=-omega^2/(L*omega^2+g),p[4,2]=0,p[4,3]=-omega^2/(L*omega^2`

(%o88) $\left[p_{1,1} = 0, p_{1,3} = 0, p_{1,2} = 1, p_{4,1} = 0, p_{2,1} = 0, p_{2,2} = 0, p_{2,3} = \omega, g_{1,1} = -1, g_{1,2} = -M\omega^2, g_{1,3} = F\omega, p_{3,1} = 0, p_{3,3} = 0, p\right.$

(%i 90) `ratsimp(subst(toSubst,expr1))`

(%o90) $$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(%i 91) `solPi:subst(toSubst,Pi)`

(%o91) $$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega \\ 0 & -\frac{\omega^2}{L\omega^2+g} & 0 \\ 0 & 0 & -\frac{\omega^3}{L\omega^2+g} \end{pmatrix}$$

(%i 92) `solGamma:subst(toSubst,Gamma)`

(%o92) $\begin{pmatrix} -1 & -M\omega^2 & F\omega \end{pmatrix}$

(%i 93) `ratsimp(solPi.S-A.solPi-B.solGamma-P)`

(%o93) $$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(%i 94) `ratsimp(Ce.solPi+Q)`

(%o94) $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$

(%i 97) `matsize(M):=[length(M),length(transpose(M))]$`

(%i 98)
```
fbisol(A,B,C,S,P,Q):=block(
   dimA,dimS,dimB,XPi,XGamma,vars,eq1,eq2,exprs
 ,
 dimA:matsize(A),
 dimS:matsize(S),
 dimB:matsize(B),
 XPi:zeromatrix(dimA[1],dimS[1]),
 XGamma:zeromatrix(dimB[2],dimS[1]),
 vars:[],
 for r:1 thru dimA[1] do(
 for c:1 thru dimS[1] do(
 XPi[r,c]:p[r,c],
 vars:append(vars,[p[r,c]])
 )
 ),
 for r:1 thru dimB[2] do(
 for c:1 thru dimS[1] do(
 XGamma[r,c]:g[r,c],
 vars:append(vars,[g[r,c]])
 )
 ),
 eq1:XPi.S-A.XPi-B.XGamma-P,
 eq2:C.XPi+Q,
 exprs:[],
 for r:1 thru dimA[1] do(
 for c:1 thru dimS[1] do(
 exprs:append(exprs,[eq1[r,c]])
 )
 ),
 for r:1 thru dimB[2] do(
 for c:1 thru dimS[1] do(
 exprs:append(exprs,[eq2[r,c]])
 )
 ),
 sol:solve(exprs,vars),
 return([subst(sol[1],XPi),subst(sol[1],XGamma)])
 )$
```

(%i 99) `sol:fbisol(A,B,Ce,S,P,Q)`

(%o99)
$$\left[\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega \\ 0 & -\frac{\omega^2}{L\omega^2+g} & 0 \\ 0 & 0 & -\frac{\omega^3}{L\omega^2+g} \end{pmatrix}, \begin{pmatrix} -1 & -M\omega^2 & F\omega \end{pmatrix}\right]$$

(%i 100) `sol[1].S`

(%o100)
$$\begin{pmatrix} 0 & 0 & \omega \\ 0 & -\omega^2 & 0 \\ 0 & 0 & -\frac{\omega^3}{L\omega^2+g} \\ 0 & \frac{\omega^4}{L\omega^2+g} & 0 \end{pmatrix}$$

(%i 101) `A.sol[1]`

(%o101)
$$\begin{pmatrix} 0 & 0 & \omega \\ 0 & 0 & -\frac{F\omega}{M} \\ 0 & 0 & -\frac{\omega^3}{L\omega^2+g} \\ 0 & -\frac{g\omega^2}{L(L\omega^2+g)} & \frac{F\omega}{LM} \end{pmatrix}$$

(%i 102) `B.sol[2]`

(%o102)
$$\begin{pmatrix} 0 & 0 & 0 \\ -\frac{1}{M} & -\omega^2 & \frac{F\omega}{M} \\ 0 & 0 & 0 \\ \frac{1}{LM} & \frac{\omega^2}{L} & -\frac{F\omega}{LM} \end{pmatrix}$$

(%i 103) Ce.sol[1]

(%o103) $\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$

(%i 104)