$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x \qquad P_A(\lambda) = \lambda^2 + 1 \longrightarrow \lambda_{1,2} = \pm j$$

$$\begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix}$$

$$\Sigma \begin{cases} \dot{x} = f(\cancel{t}, x, u), x_0 \\ y = h(\cancel{t}, x, u) \end{cases}$$

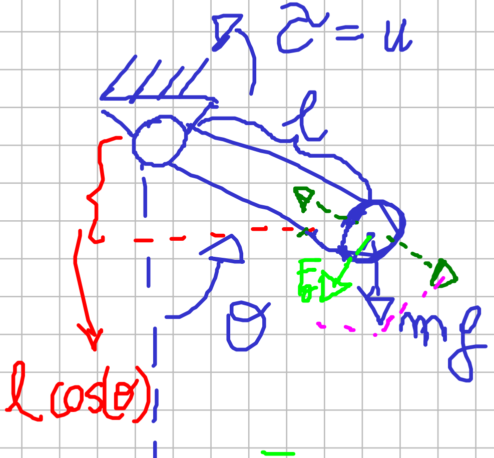
$$u \longrightarrow \boxed{x} \longrightarrow y$$

$$(x_e, u_e) \longrightarrow \begin{array}{l} 0 = f(x_e, u_e) \\ y_e = h(x_e, u_e) \end{array}$$

$$\left.\begin{array}{l} \tilde{x} = x - x_e \\ \tilde{y} = y - y_e \end{array}\right\} \longrightarrow \dot{\tilde{x}} = \frac{\partial f}{\partial x}(x,u)\Big|_{x_e, u_e} \overbrace{(x - x_e)}^{\tilde{x}} + \cancel{(\ )} + \frac{\partial f}{\partial u}(x,u)\Big|_{x_e, u_e} \overbrace{(u - u_e)}^{\tilde{u}}$$

$$\tilde{u} = u - u_e$$

$$\begin{cases} \dot{\tilde{x}} \simeq A\tilde{x} + B\tilde{u} \\ \tilde{y} \simeq \frac{\partial h}{\partial x}(x,u)\Big|_{x_e u_e} \tilde{x} + \frac{\partial h}{\partial u}(x,u)\tilde{u} \end{cases} \qquad \Sigma_{\text{lin in }(x_e, u_e)}$$



$$F_\theta = mg\sin(\theta)$$

$$\tau_\theta = mg\sin(\theta)\,\ell$$

$$I\ddot{\theta} = -mg\ell\sin(\theta) - c\dot{\theta} + \tau$$

$$x_1 = \theta$$

$$x_2 = \dot{\theta}$$

$$\begin{cases} \dot{x}_1 = x_2 & = f_1(x,u) \\ \dot{x}_2 = -\underbrace{\frac{mg\ell}{I}}_{=1}\sin(x_1) - \underbrace{\frac{c}{I}}_{=2}x_2 + \underbrace{\frac{u}{I}}_{1} \end{cases}$$

$$\dot{x}_2 = f_2(x,u)$$

$(x_e, u_e) \rightarrow u_e = 0 = \tilde{\tau}_e$

$\downarrow$?

$$0 = x_{2e}$$

$$0 = -\frac{mlg}{I}\sin(x_{1e}) - \frac{c}{I}x_{2e} + \frac{1}{I}\cdot \underset{0}{\underset{\parallel}{\tilde{\tau}_e}}$$

$$\rightarrow x_e = \begin{bmatrix} k\pi \\ 0 \end{bmatrix}$$

$k \in \mathbb{Z}$

$$\tilde{x} = x - x_e$$

$$\dot{\tilde{x}} = \begin{bmatrix} 0 & 1 \\ -\cos(x_{1e}) & -2 \end{bmatrix}\tilde{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\tilde{u}$$

$$x_e = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\dot{\tilde{x}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}}_{A}\tilde{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\tilde{u} \rightarrow \det(\lambda I - A) = \begin{vmatrix} \lambda & -1 \\ -1 & \lambda+2 \end{vmatrix} = \lambda(\lambda+2) - 1$$

$$\downarrow$$

$$p_A(\lambda) = \underline{\lambda^2 + 2\lambda + 1}$$

A è Hurwitz $\Longleftrightarrow$ $p_A(\lambda)$ è Hurwitz $\Longleftrightarrow$ $\forall \lambda_i \in \sigma(A), Re(\lambda_i) < 0$

SHURR SHURR $|\lambda_i| < 1$

Σ_{lin} in $\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, 0\right)$ è A.S.

$$\dot{\tilde{x}} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}\tilde{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\tilde{u} \rightarrow \Sigma_{lin} \text{ in } \left(\begin{bmatrix} \pi \\ 0 \end{bmatrix}, 0\right) \text{ è } (*)$$

in $x_e \begin{bmatrix} \pi \\ 0 \end{bmatrix}$

$$\downarrow p_A(\lambda) = \lambda^2 + 2\lambda - 1 \rightarrow$$ Cartesio $p_A(\lambda)$ NON è di Hurwitz quindi A NON è di Hurwitz quindi $(*)$ è INSTABILE

(condiz. necessaria)

$P(\lambda)$ è di HURWITZ $\Longrightarrow$ tutti i coff. di $P(\lambda)$ hanno lo stesso segno

tutte $\lambda_i \in \mathbb{C}^-$ stretto $\Longleftrightarrow$ grad$(P(\lambda)) = 2$ (Regola di Cartesio)

$\Updownarrow$

Genera una tabella di Routh regolare con gli elementi della I colonna aventi lo stesso segno

$$(\lambda + p_1)(\lambda + p_2) = \overset{>0}{1}\lambda^2 + \overbrace{(p_1+p_2)}^{>0}\lambda + \underbrace{p_1 p_2}_{>0}$$

$\lambda_1 = -p_1$, $\lambda_2 = -p_2$

se $(p_1, p_2) > 0$

$p_1 < 0$, $p_2 > 0$

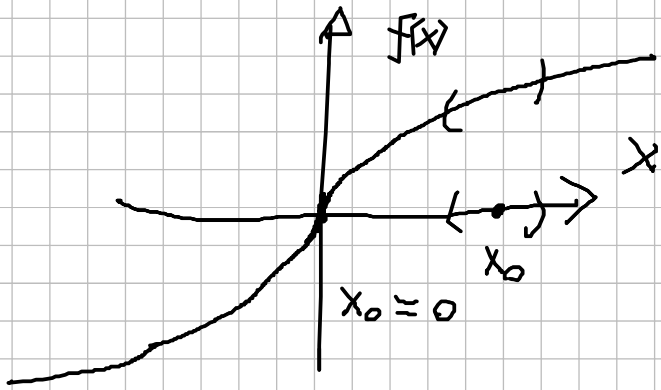
$\lambda^3 + \lambda + 1$ NON è HURWITZ

$\lambda^3 + 4\lambda^2 + 10 + \lambda$ potrebbe $\rightarrow$

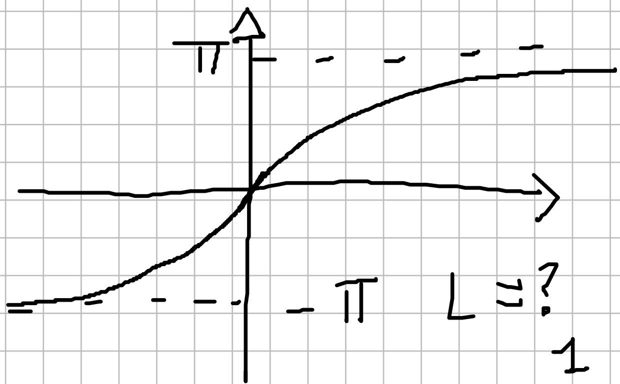
$$(\lambda + 0.5)(\lambda - 0.5) = \underbrace{\lambda^2 + 0 \cdot \lambda - 0.25}$$

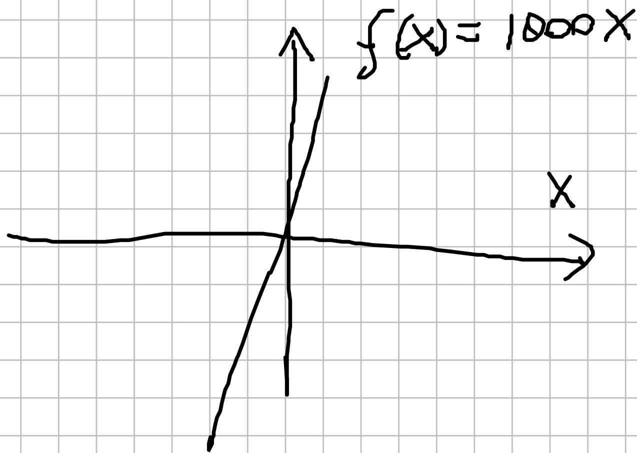
No segno coeff. e T.D.

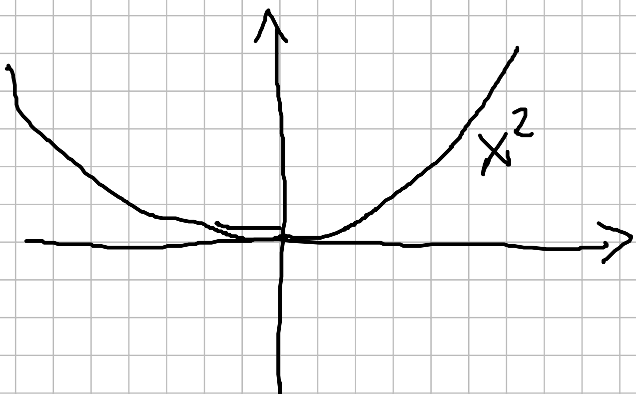
$$f(x) \triangleq \begin{cases} \sqrt{x} & \text{se } x > 0 \\ -\sqrt{-x} & \text{se } x \leq 0 \end{cases}$$



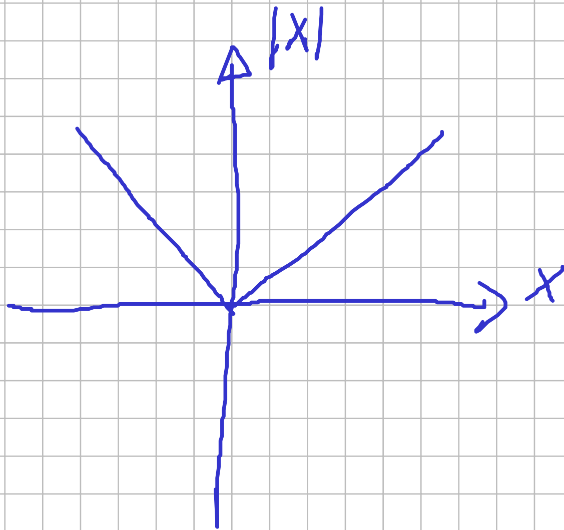
$L = 1000$

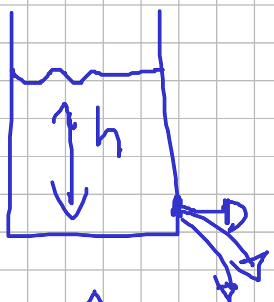






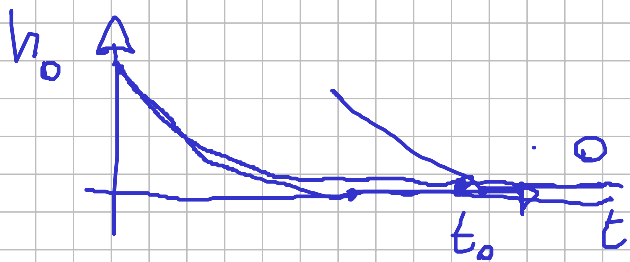
$\exists! L \ \forall x_0$

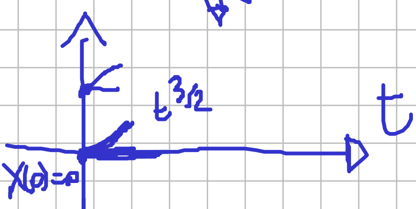


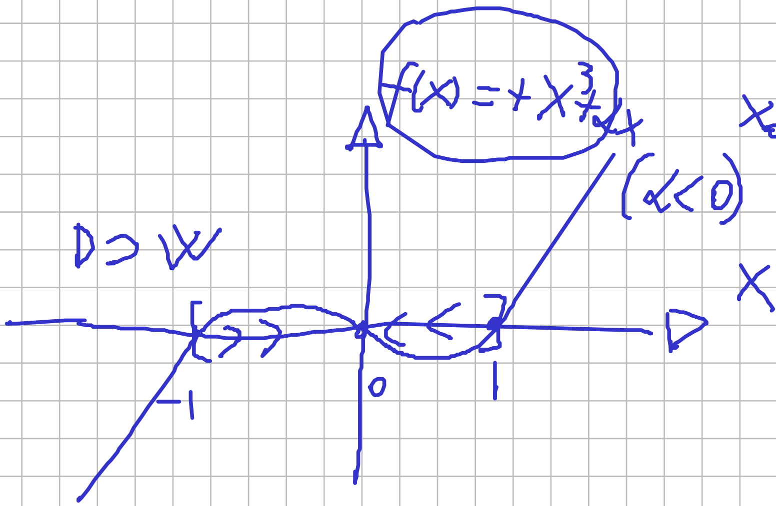


$$\dot{h} = c\sqrt{h}$$

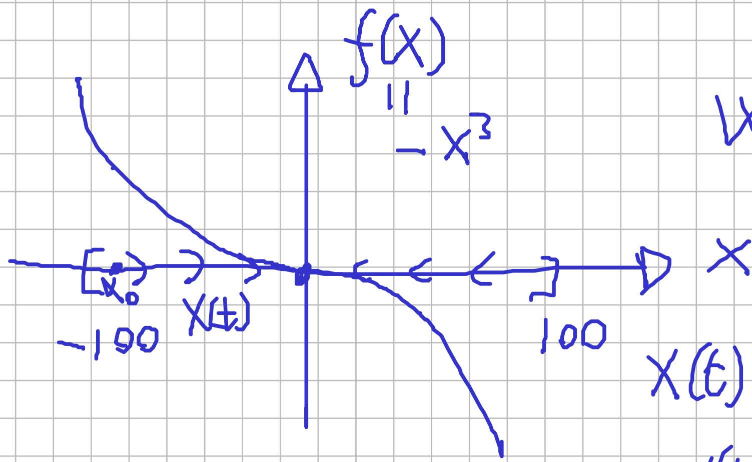
$$h_0 = h(t_0)$$







$x_0 \in W \triangleq \{x \in \mathbb{R} : |x| \leq 1\}$



$W \triangleq \{x \in \mathbb{R} : |x| < 100\}$

$x(t) \in W \;\; \forall t \geq t_0$ se

$x(t_0) \in W \equiv \mathbb{R}$

$\underset{=0}{\dot{x}_e^+} = \boxed{A x_e}$

$I x_e = A x_e$