$$\dot V \leq -\vartheta c_3\|x\|^2 + c_4\delta\|x\| =$$

$$= -\underbrace{(\vartheta c_3\|x\| - c_4\delta)}_{>0}\|x\|$$

$$\|x\| > \frac{c_4\delta}{\vartheta c_3}$$

$$\dot x = \underbrace{\overset{A}{\begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}}}\, x + \begin{bmatrix} 0 \\ g(x,t) \end{bmatrix}$$

$P_A(\lambda) = \lambda^2 + 2\lambda + 4$

λ_1, λ_2 distinti $\Rightarrow$ A è diagon. T

$z = Tx$

$\dot x = -x + ux^3 = f(x,u) \longrightarrow \boxed{ISS}$? $\beta(\cdot,\cdot)$, $\gamma(\cdot)$

$u = 0$

GES $\leftarrow f(x,0)$

$u = 1$

$\dot x = -x + x^3 \longrightarrow$ LES

$+\,o(x)$

$\dot{\tilde x} = -\tilde x \Rightarrow$ E.S. $\rightarrow$

$V(x) = \frac{1}{2}\|x\|^2$; $\dot V = x \cdot (-x + x^3)$

$= -x^2 + x^4 = -(1-x^2)x^2$

$\dot V(x) \leq -(1-0.5)x^2$ $\quad |x| < \sqrt{0.5}$

$\boxed{\dot V(x) \le -\frac{1}{2}\|x\|^2}$ in $B_r(0)$, $r = \sqrt{0.5}$

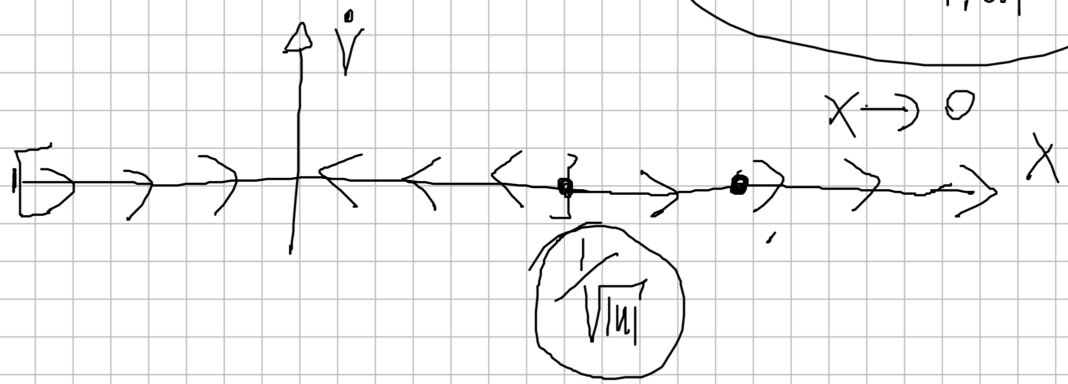
$\dot x = -x + u x^3$ $\boxed{\text{NON È ISS}}$

$V(x) = \frac{1}{2} x^2$, $c_1 x^2 \le V(x) \le c_2 x^2$

$\dot V(x) \le -x^2$ $u = 0$

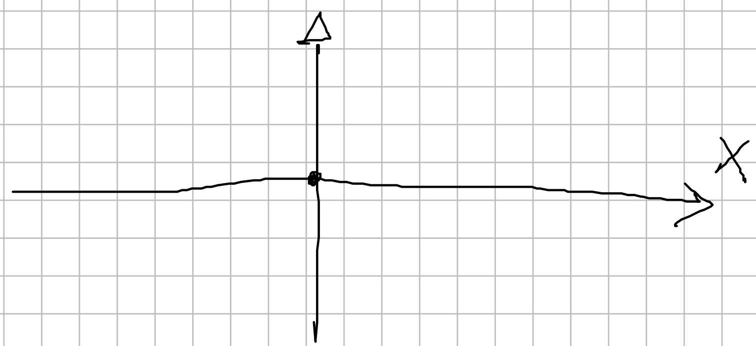
$\dot V(x) \le -x^2 + |u| x^4 = -\overset{>0}{(1 - |u| x^2)}\, x^2 < 0$

$1 - |u| x^2 > 0 \Rightarrow |x| < \frac{1}{\sqrt{|u|}}$



$\dot x = x - |u| x^3$

NON È ISS $u = 0$



$\dot x = -x^\alpha + u$ ISS

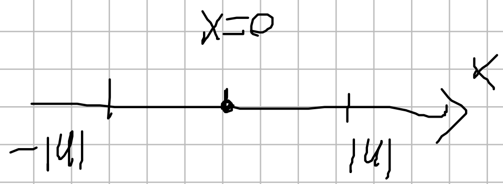
$\alpha \in (1, 3, \dots)$ $\alpha = 1$

$+|u||x|$

$V(x) = \frac{1}{2} x^2 \to \dot V(x) = -x^2 + ux$

$\le -|x|(-|u| + |x|) < 0$

$\dot{V} < 0$ se $\underline{|x| < |u|}$



$\gamma(\cdot) = ?$

$$|x(t)| \le \underbrace{\beta(|x_0|, t-t_0)}_{u=0} + \sup_{\tau \in [t, t_0]} |u(\tau)|$$

$\gamma(|s|) = |s|$

$u = 0$

$$\dot{x} = -x + u$$

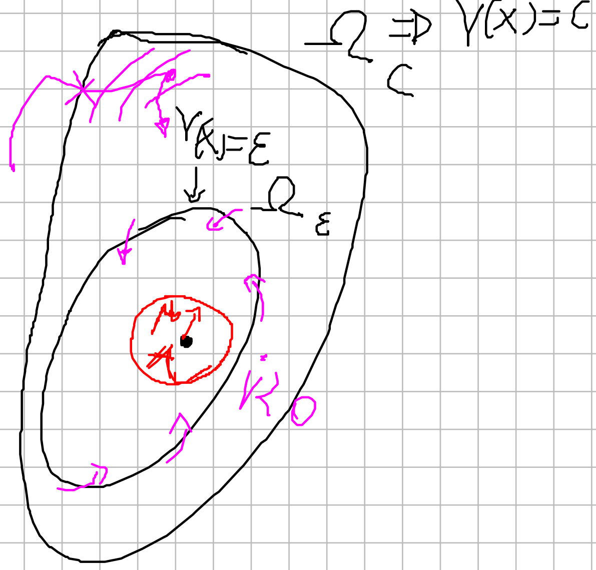
$V(x) = \frac{1}{2} x^2 \qquad u = 0 \qquad c_1 \|x\|^2 \le V(x) \le c_2 \|x\|^2$

$\boxed{\dot{V}(x) = -x^2} \rightarrow \boxed{\dot{V}(x) \le -2V(x)}$

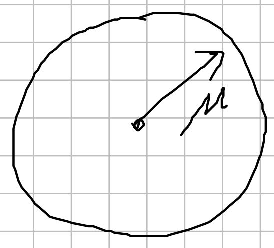
$|x(t)| = (2V(x))^{\frac{1}{2}} \qquad V(x(t)) \le V(x_0)\, e^{-2(t-t_0)}$, $\quad V(x_0) = \frac{1}{2}|x_0^2|$

$$|x(t)| \le \left(2V(x_0)\, e^{-2(t-t_0)}\right)^{\frac{1}{2}}$$

$$\underline{|x(t)| \le |x_0|\, e^{-(t-t_0)}} \triangleq \beta(|x_0|, t-t_0)$$



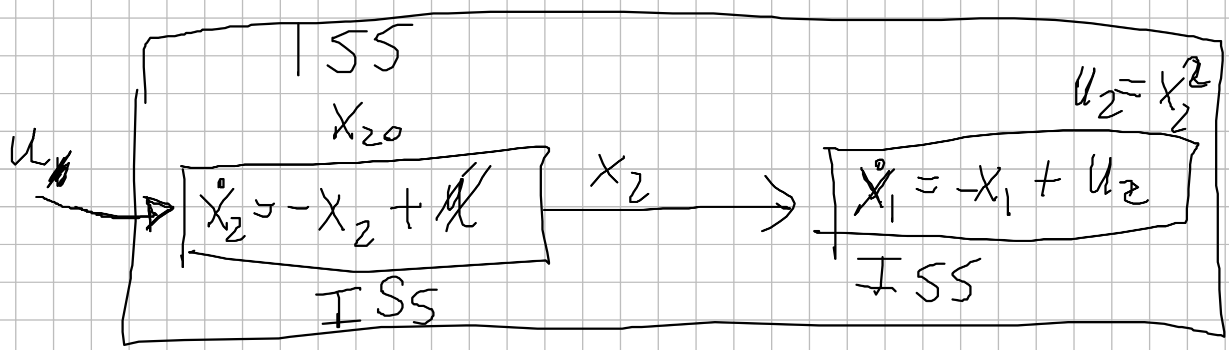
$\varepsilon \le V(x) \le c$



$$-\theta x^4 + xu \le -(|x|^3\theta - |u|)|x| < 0$$

$\le |x||u|$

$$|x|^3 > \frac{|u|}{\theta}$$

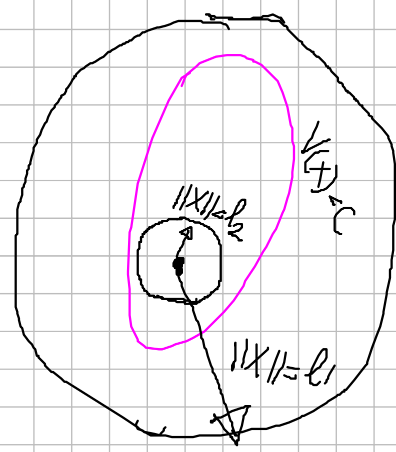


u è limitat. $\Rightarrow \left\|\begin{matrix} x_1 \\ x_2 \end{matrix}\right\|$ $\|x\|$ è limitat.

$$\underline{\alpha_1}(\|x\|) \le \underline{V(x)} \le \underline{\alpha_2}(\|x\|)$$

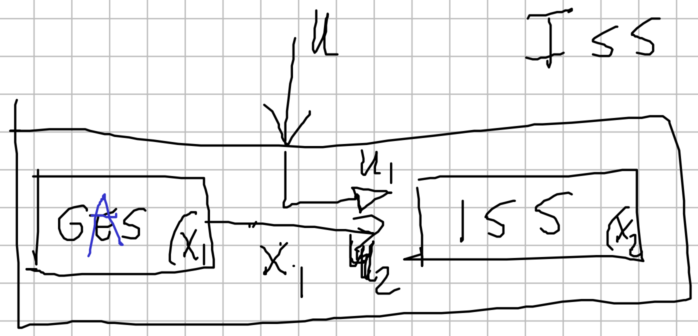
$\dot V(x) \le -W_3(x)$ se $\|x\| > \mu$

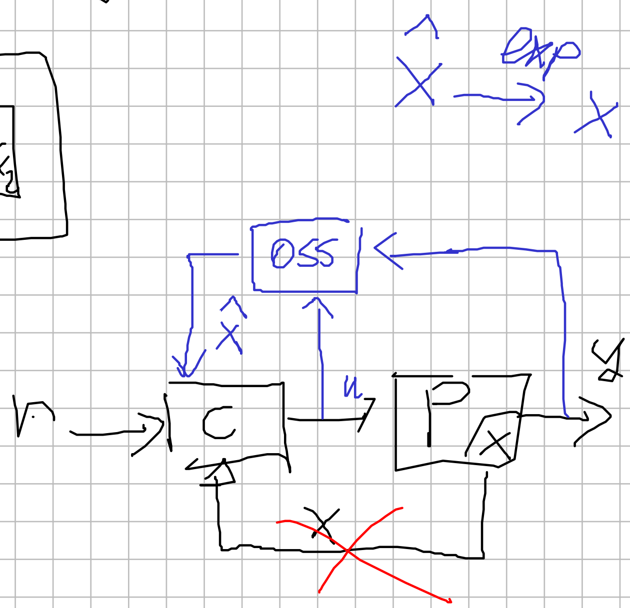
$$\|x(t)\| \le \beta(\ \ ) + \gamma(\sup \|u\|)$$

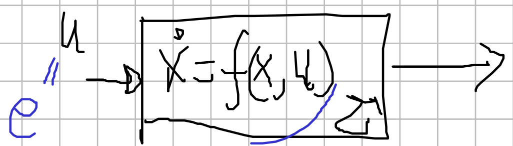


$$\|x(t)\| \leq \alpha_1^{-1}(V(x)) \leq \alpha_1^{-1}(\alpha_2(\mu)) \triangleq \gamma(\mu)$$

ISS ?







se $u = 0$

x_e è G.A.S.

se $u \xrightarrow[exp]{} 0$ $\quad$ $x \to$ è F.E.T.

$C(\underbrace{\hat{x}+e}_{x})$

$x = \hat{x} + e$