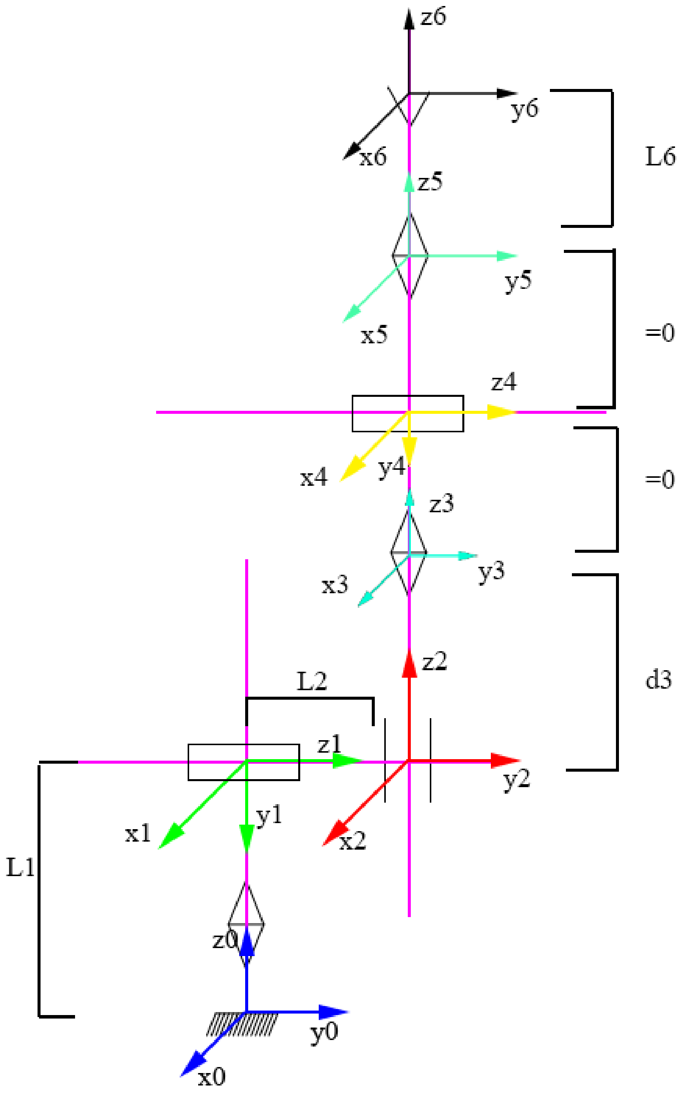
Stanford completo (robot sferico II tipo (stanford) + polso sferico)

N.B.: **le grandezze diverse da quelle di giunto q_i sono L_i, D_i. Esse sono rispettivamente la distanza tra i sistemi di riferimento R_i e R_{i+1} nelle operazioni della matrice avvitamento $A_z(\theta, d)$ e $A_x(\alpha, a)$.**



	ϑ	d	α	a
1	q_1	L_1	$-\frac{\pi}{2}$	0
2	q_2	L_2	$\frac{\pi}{2}$	0
3	0	q_3	0	0
4	q_4	0	$-\frac{\pi}{2}$	0
5	q_5	0	$\frac{\pi}{2}$	0
6	q_6	L_6	0	0

Tabella 1.

Funzioni ausiliarie:

```
(%i1) inverseLaplace(SI,theta):=block([res],
                                M:SI,
                                MC:SI,
                                for i:1 thru 3 do(
                                  for j:1 thru 3 do
                                     (
                                       aC:M[i,j],
                                       b:ilt(aC,s,theta),
                                       MC[i,j]:b
                                      )
                                   ),
                                res:MC

                             )
```

(%o1) inverseLaplace(SI, ϑ) := **block** ([res], M: SI, MC: SI, **for** i **thru** 3 **do for** j **thru** 3 **do** (aC: $M_{i,j}$, b: ilt(aC, s, ϑ), $\mathrm{MC}_{i,j}$: b), res: MC)

```
(%i2) rotLaplace(k,theta):=block([res],
                            S:ident(3),
                            I:ident(3),
                          for i:1 thru 3 do
                            (
                            for j:1 thru 3 do
                               (
                                 if i=j
                                    then S[i][j]:0
                                 elseif j>i
                                    then (
                                   temp:(-1)^(j-i)*k[3-remainder(i+j,3)],
                                          S[i][j]:temp,
                                          S[j][i]:-temp

                                           )
                                 )
                             ),
                            res:inverseLaplace(invert(s*I-S),theta)


                          )
```

(%o2) rotLaplace(k,ϑ) := **block** ([res], S: ident(3), I: ident(3), **for** i **thru** 3 **do for** j **thru** 3 **do if** $i=j$ **then** $(S_i)_j$: 0 **elseif** $j>i$ **then** (temp: $(-1)^{j-i}\,k_{3-\mathrm{remainder}(i+j,3)}$, $(S_i)_j$: temp, $(S_j)_i$: $-$temp), res: inverseLaplace(invert($s\,I-S$), ϑ))

```
(%i3) Av(v,theta,d):=block([res],
                            Trot:rotLaplace(v,theta),
                            row:matrix([0,0,0,1]),
                            Atemp:addcol(Trot,d*transpose(v)),
                            A:addrow(Atemp,row),
                            res:A
                            )
```

(%o3) Av(v, ϑ, d) := **block** ([res], Trot: rotLaplace(v, ϑ), row: $\begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$, Atemp: addcol(Trot, d transpose(v)), A: addrow(Atemp, row), res: A)

```
(%i4) Q(theta,d,alpha,a):=block([res],
                                tempMat:Av([0,0,1],theta,d).Av([1,0,0],alpha,a),
                                Qtrasf:zeromatrix(4,4),
                                for i:1 thru 4 do
                            (
                            for j:1 thru 4 do
                               (
                                Qtrasf[i][j]:trigreduce(tempMat[i][j])
                                          )
                              ),

                                res:Qtrasf
                                )
```

(%o4) $Q(\vartheta, d, \alpha, a) := \textbf{block}\ ([\mathrm{res}], \mathrm{tempMat}\colon \mathrm{Av}([0,0,1], \vartheta, d) \cdot \mathrm{Av}([1,0,0], \alpha, a), \mathrm{Qtrasf}\colon \mathrm{zeromatrix}(4,4), \textbf{for}\ i\ \textbf{thru}\ 4\ \textbf{do}\ \textbf{for}\ j\ \textbf{thru}\ 4\ \textbf{do}\ (\mathrm{Qtrasf}_i)_j\colon \mathrm{trigreduce}((\mathrm{tempMat}_i)_j), \mathrm{res}\colon \mathrm{Qtrasf})$

```
(%i5) let(sin(q[1]), s[1]);
```

(%o5) $\sin(q_1) \longrightarrow s_1$

```
(%i6) let(sin(q[2]), s[2]);
```

(%o6) $\sin(q_2) \longrightarrow s_2$

```
(%i7) let(cos(q[1]),c[1]);
```

(%o7) $\cos(q_1) \longrightarrow c_1$

```
(%i8) let(cos(q[2]),c[2]);
```

(%o8) $\cos(q_2) \longrightarrow c_2$

```
(%i9) let(sin(q[1]+q[2]), s[12]);
```

(%o9) $\sin(q_2 + q_1) \longrightarrow s_{12}$

```
(%i10) let(cos(q[1]+q[2]),c[12]);
```

(%o10) $\cos(q_2 + q_1) \longrightarrow c_{12}$

```
(%i11) let(sin(q[2]+q[3]),s[23]);
```

(%o11) $\sin(q_3 + q_2) \longrightarrow s_{23}$

```
(%i12) let(cos(q[2]+q[3]),c[23]);
```

(%o12) $\cos(q_3 + q_2) \longrightarrow c_{23}$

```
(%i13) let(sin(q[1]+q[3]),s[23]);
```

(%o13) $\sin(q_3 + q_1) \longrightarrow s_{23}$

```
(%i14) let(cos(q[1]+q[3]),c[13]);
```

(%o14) $\cos(q_3 + q_1) \longrightarrow c_{13}$

```
(%i15) let(sin(q[3]),s[3]);
```

(%o15) $\sin(q_3) \longrightarrow s_3$

```
(%i16) let(cos(q[3]),c[3]);
```

(%o16) $\cos(q_3) \longrightarrow c_3$

```
(%i17) let(sin(q[4]),s[4]);
```

(%o17) $\sin(q_4) \longrightarrow s_4$

(%i18) `let(cos(q[4]),c[4]);`

(%o18) $\cos\left(q_4\right) \longrightarrow c_4$

(%i19) `let(sin(q[5]),s[5]);`

(%o19) $\sin\left(q_5\right) \longrightarrow s_5$

(%i20) `let(cos(q[5]),c[5]);`

(%o20) $\cos\left(q_5\right) \longrightarrow c_5$

(%i21) `let(sin(q[6]),s[6]);`

(%o21) $\sin\left(q_6\right) \longrightarrow s_6$

(%i22) `let(cos(q[6]),c[6]);`

(%o22) $\cos\left(q_6\right) \longrightarrow c_6$

(%i23)

Cinematica diretta:

(%i26)
```
Q[stanford6DOF](q1,q2,q3,q4,q5,q6,L1,L2,L6):=
                    Q(q1,L1,-%pi/2,0).
                    Q(q2,L2,%pi/2,0).
                    Q(0,q3,0,0).
                    Q(q4,0,-%pi/2,0).
                    Q(q5,0,%pi/2,0).
                    Q(q6,L6,0,0);
```

(%o26) $Q_{\mathrm{stanford6DOF}}(q1, q2, q3, q4, q5, q6, L1, L2, L6) := Q\left(q1, L1, \frac{-\pi}{2}, 0\right) \cdot Q\left(q2, L2, \frac{\pi}{2}, 0\right) \cdot Q(0, q3, 0, 0) \cdot Q\left(q4, 0, \frac{-\pi}{2}, 0\right) \cdot Q\left(q5, 0, \frac{\pi}{2}, 0\right) \cdot Q(q6, L6, 0, 0)$

(%i27)
```
Qstanford6DOF:Q[standford6DOF](q[1],q[2],q[3],q[4],q[5],q[6],L[1],L[2],
L[6]);
```

(%o27) $(\cos(q_1)(\cos(q_2)(\cos(q_4)\cos(q_5)\cos(q_6) - \sin(q_4)\sin(q_6)) - \sin(q_2)\sin(q_5)\cos(q_6)) - \sin(q_1)(\cos(q_4)\sin(q_6) + \sin(q_4)\cos(q_5)\cos(q_6)), \cos(q_1)(\cos(q_2)(-\cos(q_4)\cos(q_5)\sin(q_6) - \sin(q_4)\cos(q_6)) + \sin(q_2)\sin(q_5)\sin(q_6)) - \sin(q_1)(\cos(q_4)\cos(q_6) - \sin(q_4)\cos(q_5)\sin(q_6)),$
$\cos(q_1)(\cos(q_2)\cos(q_4)\sin(q_5) + \sin(q_2)\cos(q_5)) - \sin(q_1)\sin(q_4)\sin(q_5),$
$\cos(q_1)(L_6\cos(q_2)\cos(q_4)\sin(q_5) + \sin(q_2)(L_6\cos(q_5) + q_3)) - \sin(q_1)(L_6\sin(q_4)\sin(q_5) + L_2);$
$\sin(q_1)(\cos(q_2)(\cos(q_4)\cos(q_5)\cos(q_6) - \sin(q_4)\sin(q_6)) - \sin(q_2)\sin(q_5)\cos(q_6)) +$
$\cos(q_1)(\cos(q_4)\sin(q_6) + \sin(q_4)\cos(q_5)\cos(q_6)), \sin(q_1)(\cos(q_2)(-\cos(q_4)\cos(q_5)\sin(q_6) - \sin(q_4)\cos(q_6)) + \sin(q_2)\sin(q_5)\sin(q_6)) + \cos(q_1)(\cos(q_4)\cos(q_6) - \sin(q_4)\cos(q_5)\sin(q_6)),$
$\sin(q_1)(\cos(q_2)\cos(q_4)\sin(q_5) + \sin(q_2)\cos(q_5)) + \cos(q_1)\sin(q_4)\sin(q_5),$
$\cos(q_1)(L_6\sin(q_4)\sin(q_5) + L_2) + \sin(q_1)(L_6\cos(q_2)\cos(q_4)\sin(q_5) + \sin(q_2)(L_6\cos(q_5) + q_3));$
$-\sin(q_2)(\cos(q_4)\cos(q_5)\cos(q_6) - \sin(q_4)\sin(q_6)) - \cos(q_2)\sin(q_5)\cos(q_6),$
$\cos(q_2)\sin(q_5)\sin(q_6) - \sin(q_2)(-\cos(q_4)\cos(q_5)\sin(q_6) - \sin(q_4)\cos(q_6)), \cos(q_2)\cos(q_5) - \sin(q_2)\cos(q_4)\sin(q_5), -L_6\sin(q_2)\cos(q_4)\sin(q_5) + \cos(q_2)(L_6\cos(q_5) + q_3) + L_1; 0, 0, 0, 1)$

(%i28) `letsimp(Qstanford6DOF);`

(%o28) $(-c_1\,c_2\,s_4\,s_6 - s_1\,c_4\,s_6 - c_1\,s_2\,s_5\,c_6 - s_1\,s_4\,c_5\,c_6 + c_1\,c_2\,c_4\,c_5\,c_6, c_1\,s_2\,s_5\,s_6 + s_1\,s_4\,c_5\,s_6 - c_1\,c_2\,c_4\,c_5\,s_6 - c_1\,c_2\,s_4\,c_6 - s_1\,c_4\,c_6, -s_1\,s_4\,s_5 + c_1\,c_2\,c_4\,s_5 + c_1\,s_2\,c_5, -s_1\,s_4\,s_5\,L_6 + c_1\,c_2\,c_4\,s_5\,L_6 + c_1\,s_2\,c_5\,L_6 + c_1\,s_2\,q_3 - s_1\,L_2; -s_1\,c_2\,s_4\,s_6 + c_1\,c_4\,s_6 - s_1\,s_2\,s_5\,c_6 + c_1\,s_4\,c_5\,c_6 + s_1\,c_2\,c_4\,c_5\,c_6, s_1\,s_2\,s_5\,s_6 - c_1\,s_4\,c_5\,s_6 - s_1\,c_2\,c_4\,c_5\,s_6 - s_1\,c_2\,s_4\,c_6 + c_1\,c_4\,c_6, c_1\,s_4\,s_5 + s_1\,c_2\,c_4\,s_5 + s_1\,s_2\,c_5, c_1\,s_4\,s_5\,L_6 + s_1\,c_2\,c_4\,s_5\,L_6 + s_1\,s_2\,c_5\,L_6 + s_1\,s_2\,q_3 + c_1\,L_2; s_2\,s_4\,s_6 - c_2\,s_5\,c_6 - s_2\,c_4\,c_5\,c_6, c_2\,s_5\,s_6 + s_2\,c_4\,c_5\,s_6 + s_2\,s_4\,c_6, c_2\,c_5 - s_2\,c_4\,s_5, -s_2\,c_4\,s_5\,L_6 + c_2\,c_5\,L_6 + c_2\,q_3 + L_1; 0, 0, 0, 1)$

Cinematica Inversa Robot Stanford

Al fine di risolvere il problema di cinematica inversa del robot Stanford occorre risolvere il problema di posizione ed orientamente inverso.Inizialmente occorre verificare la condizione di disaccoppiamento, individuare lo spazio di lavoro, le soluzioni generiche, singolari ed infine le variabili di giunto q_i ed in seguito determinare l'orientamento del robot.

Poiché il robot Stanford è un robot 6 DOF (6 gradi di libertà), occorre disaccoppiare la struttura portante dal suo polso. Ciò è possibile se:

$$d_{36}(q_b) = R_{36}(q_6)\, d_1 + d_0$$

In particolare:

$$Q_{03} = \begin{pmatrix} s_1 & c_1c_2 & c_1s_2 & q_3c_1s_2 - L_2s_1 \\ -c_1 & s_1c_2 & s_1s_2 & q_3s_1s_2 + L_2\, c_1 \\ 0 & -s_2 & c_2 & L_1 + q_3c_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Q_{36} = \begin{pmatrix} c_4\, c_5\, c_6 - s_4\, s_6 & -c_4\, c_5\, s_6 - s_4\, c_6 & c_4\, s_5 & c_4\, s_5\, L_6 \\ c_4\, s_6 + s_4\, c_5\, c_6 & c_4\, c_6 - s_4\, c_5\, s_6 & s_4\, s_5 & s_4\, s_5\, L_6 \\ -s_5\, c_6 & s_5\, s_6 & c_5 & c_5\, L_6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} c_4\, s_5\, L_6 \\ s_4\, s_5\, L_6 \\ c_5\, L_6 \end{pmatrix} = \begin{pmatrix} c_4\, c_5\, c_6 - s_4\, s_6 & -c_4\, c_5\, s_6 - s_4\, c_6 & c_4\, s_5 \\ c_4\, s_6 + s_4\, c_5\, c_6 & c_4\, c_6 - s_4\, c_5\, s_6 & s_4\, s_5 \\ -s_5\, c_6 & s_5\, s_6 & c_5 \end{pmatrix} d_1 + d_0$$

Ottenendo:

$$d_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} L_6 \quad d_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Quindi:

$$\begin{cases} R = R_{03}\, R_{36} \\ P = R_{03}\, d_{36} + d_{03} = R_{03}(R_{36} d_1 + d_0) + d_{03} = R\, d_1 + d_{03} \end{cases}$$

$$P - R\, d_1 = \hat{P} = d_{03} \longrightarrow \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} = \begin{pmatrix} q_3c_1s_2 - L_2s_1 \\ q_3s_1s_2 + L_2\, c_1 \\ L_1 + q_3c_2 \end{pmatrix}$$

$$\begin{cases} \hat{x} = q_3c_1s_2 - L_2s_1 \\ \hat{y} = q_3s_1s_2 + L_2\, c_1 \\ \hat{z} = L_1 + q_3c_2 \end{cases} \rightarrow \begin{cases} \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 \\ s_1 & c_1 \end{pmatrix} \begin{pmatrix} q_3s_2 \\ L_2 \end{pmatrix} \\ (\hat{z} - L_1)^2 = q_3^2 c_2^2 \end{cases}$$

Poiché $\begin{pmatrix} c_1 & -s_1 \\ s_1 & c_1 \end{pmatrix}$ è una matrice di rotazione, $\begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix}$ e $\begin{pmatrix} q_3s_2 \\ L_2 \end{pmatrix}$ devono avere stessa norma:

$$\begin{cases} \hat{x}^2 + \hat{y}^2 = q_3^2 s_2^2 + L_2^2 \\ (\hat{z} - L_1)^2 = q_3^2 c_2^2 \end{cases} \rightarrow \hat{x}^2 + \hat{y}^2 + (\hat{z} - L_1)^2 - L_2^2 = q_3^2(s_2^2 + c_2^2)$$

Ottenendo:

$$\hat{x}^2 + \hat{y}^2 + (\hat{z} - L_1)^2 - L_2^2 = q_3^2$$

$$q_3 = \pm\sqrt{\hat{x}^2 + \hat{y}^2 + (\hat{z} - L_1)^2 - L_2^2}$$

Poich $q_3 \neq 0$, è possibile ottente c_2:

$$c_2 = \frac{(\hat{z} - L_1)}{q_3}, s_2 = \pm\sqrt{1 - c_2^2} \longrightarrow q_2 = \operatorname{atan2}\left(\pm\sqrt{1 - \frac{(\hat{z} - L_1)^2}{q_3^2}}, \frac{(\hat{z} - L_1)^2}{q_3^2}\right)$$

A questo punto, la quantità $\left(\begin{smallmatrix} q_3 s_2 \\ L_2 \end{smallmatrix}\right)$ è nota, quindi è possibile determinare la variabile di giunto q_1:

$$\begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 \\ s_1 & c_1 \end{pmatrix} \begin{pmatrix} q_3 s_2 \\ L_2 \end{pmatrix} \rightarrow \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} q_3 s_2 & -L_2 \\ L_2 & q_3 s_2 \end{pmatrix} \begin{pmatrix} c_1 \\ s_1 \end{pmatrix}$$

Poiché $\det\begin{pmatrix} q_3 s_2 & -L_2 \\ L_2 & q_3 s_2 \end{pmatrix} = q_3^2 s_2^2 + L_2^2 \neq 0$, è possibile effettuare l'inversa:

$$\begin{pmatrix} c_1 \\ s_1 \end{pmatrix} = \frac{1}{q_3^2 s_2^2 + L_2^2} \begin{pmatrix} q_3 s_2 & L_2 \\ -L_2 & q_3 s_2 \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} q_3 s_2 \hat{x} + L_2 \hat{y} \\ -L_2 \hat{x} + q_3 s_2 \hat{y} \end{pmatrix}$$

$$q_1 = \operatorname{atan2}(-L_2 \hat{x} + q_3 s_2 \hat{y}, q_3 s_2 \hat{x} + L_2 \hat{y})$$

Riassumendo:

$$\begin{pmatrix} q_{3+} \\ q_{2+} \\ q_1 \end{pmatrix}, \begin{pmatrix} q_{3+} \\ q_{2-} \\ q_1 \end{pmatrix}, \begin{pmatrix} q_{3-} \\ q_{2+} \\ q_1 \end{pmatrix}, \begin{pmatrix} q_{3-} \\ q_{2-} \\ q_1 \end{pmatrix}$$

Orientamento inverso

$$\hat{R} = R_{03}^T R$$

In cui:

$$R_{03} = \begin{pmatrix} s_1 & c_1 c_2 & c_1 s_2 \\ -c_1 & s_1 c_2 & s_1 s_2 \\ 0 & -s_2 & c_2 \end{pmatrix}, R = R_{\mathrm{zyz}} = \begin{pmatrix} c_\alpha c_\beta c_\gamma - s_\alpha s_\gamma & -c_\alpha c_\beta s_\gamma - s_\alpha c_\gamma & c_\alpha s_\beta \\ c_\alpha s_\gamma + s_\alpha c_\beta c_\gamma & c_\alpha c_\gamma - s_\alpha c_\beta s_\gamma & s_\alpha s_\beta \\ -s_\beta c_\gamma & s_\beta s_\gamma & c_\beta \end{pmatrix}$$

$$\hat{R} = \begin{pmatrix} \hat{r}_{1,1} & \hat{r}_{1,2} & \hat{r}_{1,3} \\ \hat{r}_{2,1} & \hat{r}_{2,2} & \hat{r}_{2,3} \\ \hat{r}_{3,1} & \hat{r}_{3,2} & \hat{r}_{3,3} \end{pmatrix}$$

$$c_5 = \hat{r}_{3,3}$$

$$s_5 = \pm\sqrt{1 - \hat{r}_{3,3}^2}$$

$$q_5 = \operatorname{atan}\left(\pm\sqrt{1 - \hat{r}_{3,3}^2}, \hat{r}_{3,3}\right)$$

$$\begin{cases} s_5 s_6 = \hat{r}_{3,2} \\ -s_5 c_6 = \hat{r}_{3,1} \end{cases} \rightarrow \begin{cases} s_6 = \pm\frac{\hat{r}_{3,2}}{s_5} \\ c_6 = \mp\frac{\hat{r}_{3,1}}{s_5} \end{cases} \rightarrow q_6 = \operatorname{atan2}\left(\pm\frac{\hat{r}_{3,2}}{s_5}, \mp\frac{\hat{r}_{3,1}}{s_5}\right)$$

$$\begin{cases} c_4 s_5 = \hat{r}_{1,3} \\ s_4 s_5 = \hat{r}_{2,3} \end{cases} \rightarrow \begin{cases} c_4 = \pm\frac{\hat{r}_{1,3}}{s_5} \\ s_4 = \pm\frac{\hat{r}_{2,3}}{s_5} \end{cases} \rightarrow q_4 = \operatorname{atan2}\left(\pm, \frac{\hat{r}_{2,3}}{s_5}, \pm\frac{\hat{r}_{1,3}}{s_5}\right)$$

Riassumendo:

$$\begin{pmatrix} q_{3+} \\ q_{2+} \\ q_1 \end{pmatrix} \rightarrow R_{03,1} = \begin{pmatrix} s_1 & c_1\,c_2 & c_1\,s_2 \\ -c_1 & s_1\,c_2 & s_1\,s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \rightarrow \begin{pmatrix} q_4 \\ q_5 \\ q_6 \end{pmatrix}, \begin{pmatrix} q_4 \\ q_5 \\ q_6 \end{pmatrix}$$

$$\begin{pmatrix} q_{3-} \\ q_{2+} \\ q_1 \end{pmatrix} \rightarrow R_{03,3} = \begin{pmatrix} s_1 & c_1\,c_2 & c_1\,s_2 \\ -c_1 & s_1\,c_2 & s_1\,s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \rightarrow \begin{pmatrix} q_4 \\ q_5 \\ q_6 \end{pmatrix}, \begin{pmatrix} q_4 \\ q_5 \\ q_6 \end{pmatrix}$$

$$\begin{pmatrix} q_{3+} \\ q_{2-} \\ q_1 \end{pmatrix} \rightarrow R_{03,2} = \begin{pmatrix} s_1 & c_1\,c_2 & c_1\,s_2 \\ -c_1 & s_1\,c_2 & s_1\,s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \rightarrow \begin{pmatrix} q_4 \\ q_5 \\ q_6 \end{pmatrix}, \begin{pmatrix} q_4 \\ q_5 \\ q_6 \end{pmatrix}$$

$$\begin{pmatrix} q_{3-} \\ q_{2-} \\ q_1 \end{pmatrix} \rightarrow R_{03,4} = \begin{pmatrix} s_1 & c_1\,c_2 & c_1\,s_2 \\ -c_1 & s_1\,c_2 & s_1\,s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \rightarrow \begin{pmatrix} q_4 \\ q_5 \\ q_6 \end{pmatrix}, \begin{pmatrix} q_4 \\ q_5 \\ q_6 \end{pmatrix}$$

```
(%i23) skewMatrix(x):=block([res],
                            S:ident(3),
                            for i:1 thru 3 do
                            (
                            for j:1 thru 3 do
                               (
                                 if i=j
                                    then S[i][j]:0
                                 elseif j>i
                                    then (
                                   temp:(-1)^(j-i)*x[3-remainder(i+j,3)],
                                          S[i][j]:temp,
                                          S[j][i]:-temp

                                           )
                                 )
                             ),
                             res:S
                             )
```

(%o23) $\mathrm{skewMatrix}(x) := \textbf{block}\ ([\mathrm{res}], S{:}\ \mathrm{ident}(3), \textbf{for}\ i\ \textbf{thru}\ 3\ \textbf{do}\ \textbf{for}\ j\ \textbf{thru}\ 3\ \textbf{do}\ \textbf{if}\ i = j\ \textbf{then}\ (S_i)_j{:}\ 0\ \textbf{elseif}\ j > i\ \textbf{then}\ (\mathrm{temp}{:}\ (-1)^{j-i}\, x_{3-\mathrm{remainder}(i+j,3)}, (S_i)_j{:}\ \mathrm{temp}, (S_j)_i{:}\ -\mathrm{temp}), \mathrm{res}{:}$

$S)$

(%i24)
```
rodriguez(y,arg):=block([res],
                        I:ident(3),
                        S:skewMatrix(y),
                        res:I+S.S*(1-cos(arg))+S*sin(arg)
                        )
```

(%o24) $\mathrm{rodriguez}(y, \mathrm{arg}) := \textbf{block}\ ([\mathrm{res}], I\colon \mathrm{ident}(3), S\colon \mathrm{skewMatrix}(y), \mathrm{res}\colon I + S \cdot S\, (1 - \cos(\mathrm{arg})) + S \sin(\mathrm{arg}))$

(%i39)
```
posInversa(x,y,z,L1,L2):=block(
               [valueq3,valueq2,q3,q2,s1,c1,q1,res],
               if x^(2)+y^(2)+(z-L1)^2-L2^(2)<=0 then error("singolare!"),
                valueq3:sqrt(x^(2)+y^(2)+(z-L1)^2-L2^(2)),

                q3:[valueq3,-valueq3],
                valueq2:((z-L1)^2)/q3^(2),
                q2:[atan2(sqrt(1-valueq2),valueq2),
                    atan2(-sqrt(1-valueq2),valueq2)],

                s1:[-L2*x+q3[1]*sin(q2[1][1][1])*y,
                    -L2*x+q3[1]*sin(q2[1][1][2])*y,
                    -L2*x+q3[2]*sin(q2[1][1][1])*y,
                    -L2*x+q3[2]*sin(q2[1][1][2])*y],

                c1:[q3[1]*sin(q2[1][1][1])*x+L2*y,
                    q3[1]*sin(q2[1][1][2])*x+L2*y,
                    q3[2]*sin(q2[1][1][1])*x+L2*y,
                    q3[2]*sin(q2[1][1][2])*x+L2*y],
                q1:[atan2(s1[1],c1[1]),
                    atan2(s1[2],c1[2]),
                    atan2(s1[3],c1[3]),
                    atan2(s1[4],c1[4])],
                 res:[[q1[1],q2[1][1][1],q3[1]],
                      [q1[2],q2[1][1][2],q3[1]],
                      [q1[3],q2[1][1][1],q3[2]],
                      [q1[4],q2[1][1][2],q3[2]]]
                )
```

(%o39) $\mathrm{posInversa}(x, y, z, L1, L2) := \textbf{block}\Big([\mathrm{valueq3}, \mathrm{valueq2}, q3, q2, s1, c1, q1, \mathrm{res}], \textbf{if}\ x^2 + y^2 + (z - L1)^2 - L2^2 \le 0\ \textbf{then}\ \mathrm{error}(\mathrm{singolare!}\ ), \mathrm{valueq3}\colon \sqrt{x^2 + y^2 + (z - L1)^2 - L2^2}, q3\colon [\mathrm{valueq3}, -\mathrm{valueq3}], \mathrm{valueq2}\colon \frac{(z - L1)^2}{q3^2}, q2\colon \big[\mathrm{atan2}\big(\sqrt{1 - \mathrm{valueq2}}, \mathrm{valueq2}\big), \mathrm{atan2}\big(-\sqrt{1 - \mathrm{valueq2}}, \mathrm{valueq2}\big)\big], s1\colon [(-L2)\, x + q3_1 \sin(((q2_1)_1)_1)\, y, (-L2)\, x + q3_1 \sin(((q2_1)_1)_2)\, y, (-L2)\, x + q3_2 \sin(((q2_1)_1)_1)\, y, (-L2)\, x + q3_2 \sin(((q2_1)_1)_2)\, y], c1\colon [q3_1 \sin(((q2_1)_1)_1)\, x + L2\, y, q3_1 \sin(((q2_1)_1)_2)\, x + L2\, y, q3_2 \sin(((q2_1)_1)_1)\, x + L2\, y, q3_2 \sin(((q2_1)_1)_2)\, x + L2\, y], q1\colon [\mathrm{atan2}(s1_1, c1_1), \mathrm{atan2}(s1_2, c1_2), \mathrm{atan2}(s1_3, c1_3), \mathrm{atan2}(s1_4, c1_4)], \mathrm{res}\colon [[q1_1, ((q2_1)_1)_1, q3_1], [q1_2, ((q2_1)_1)_2, q3_1],$

$[q1_3, ((q2_1)_1)_1, q3_2], [q1_4, ((q2_1)_1)_2, q3_2]]\Big)$

(%i58)
```
orienInversa(R,sol):=block([R03,Rcap,q4,q5,q6,res],
          Rcap:[0,0,0,0],

          for i:1 thru 4 do (

            R03:matrix([sin(sol[i][1]),cos(sol[i][1])*cos(sol[i][2]),
cos(sol[i][1])*cos(sol[i][2])],
                      [-cos(sol[i][1]),sin(sol[i][1])*cos(sol[i][2]),
sin(sol[i][1])*sin(sol[i][2])],
                                          [0,-sin(sol[i][2]),
cos(sol[i][2])]),


           Rcap[i]:transpose(R03).R

           ),

           for i:1 thru 4 do (

                             c5:Rcap[i][3][3],
                             s5:sqrt(1-c5^2),

                             q5plus:atan2(s5,c5),
                             q5minus:atan2(-s5,c5),

                             s6:abs(Rcap[i][3][2]/s5),
                             c6:abs(Rcap[i][3][1]/s5),

                             q6plus:atan2(s6,-c6),
                             q6minus:atan2(-s6,c6),
                             c4:Rcap[i][1][3]/s5,
                             s4:Rcap[i][2][3]/s5,

                             q4plus:atan2(s4,c4),
                             q4minus:atan2(-s4,-c4),
                             first:[q4plus,q5plus,q6plus],
                             second:[q4minus,q5minus,q6minus],
                             res[i]:[first,second]
                             ),
                              res)
```

(%o58) $\mathrm{orienInversa}(R,\mathrm{sol}) := \mathbf{block}\Bigg([R03, \mathrm{Rcap}, q4, q5, q6, \mathrm{res}], \mathrm{Rcap}{:}\, [0, 0, 0, 0],$

$$\mathbf{for}\ i\ \mathbf{thru}\ 4\ \mathbf{do}\left(R03{:}\begin{pmatrix} \sin((\mathrm{sol}_i)_1) & \cos((\mathrm{sol}_i)_1)\cos((\mathrm{sol}_i)_2) & \cos((\mathrm{sol}_i)_1)\cos((\mathrm{sol}_i)_2) \\ -\cos((\mathrm{sol}_i)_1) & \sin((\mathrm{sol}_i)_1)\cos((\mathrm{sol}_i)_2) & \sin((\mathrm{sol}_i)_1)\sin((\mathrm{sol}_i)_2) \\ 0 & -\sin((\mathrm{sol}_i)_2) & \cos((\mathrm{sol}_i)_2) \end{pmatrix},\right.$$

$\text{Rcap}_i\text{: transpose}(R03)\cdot R\Big)$, **for** i **thru** 4 **do** $\Big(c5\text{: }((\text{Rcap}_i)_3)_3,\ s5\text{: }\sqrt{1-c5^2},\ q5\text{plus: atan2}(s5, c5),$ $q5\text{minus: atan2}(-s5, c5),\ s6\text{: }\left|\frac{((\text{Rcap}_i)_3)_2}{s5}\right|,\ c6\text{: }\left|\frac{((\text{Rcap}_i)_3)_1}{s5}\right|,\ q6\text{plus: atan2}(s6, -c6),\ q6\text{minus: atan2}(-s6, c6),\ c4\text{: }\frac{((\text{Rcap}_i)_1)_3}{s5},\ s4\text{: }\frac{((\text{Rcap}_i)_2)_3}{s5},\ q4\text{plus: atan2}(s4, c4),\ q4\text{minus: atan2}(-s4, -c4),$ $\text{first: }[q4\text{plus}, q5\text{plus}, q6\text{plus}],\ \text{second: }[q4\text{minus}, q5\text{minus}, q6\text{minus}],\ \text{res}_i\text{: [first, second]}\Big), \text{res}\Big)$

```
(%i67) invSTANFORD(x,y,z,L1,L2,L6,alpha,beta,gamma):=block(
                [R,pos,orien,res],
                 R:rodriguez([0,0,1],alpha).
                                   rodriguez([0,1,0],beta).
                                   rodriguez([0,0,1],gamma),
                                coordPolso:R.matrix([0],[0],[L6]),
                                xCap:x-coordPolso[1],
                                yCap:y-coordPolso[2],
                                zCap:z-coordPolso[3],
                                pos:posInversa(xCap[1],yCap[1],zCap[1],L1,L2),

                                orien:orienInversa(R,pos),

                                res:[
                                   [pos[1][1],pos[1][2],pos[1][3],
       orien[1][1][1],orien[1][1][2],orien[1][1][3]],
                                   [pos[1][1],pos[1][2],pos[1][3],
       orien[1][2][1],orien[1][2][2],orien[1][2][3]],
                                   [pos[2][1],pos[2][2],pos[2][3],
       orien[2][1][1],orien[2][1][2],orien[2][1][3]],
                                   [pos[2][1],pos[2][2],pos[2][3],
       orien[2][2][1],orien[2][2][2],orien[2][2][3]],
                                   [pos[3][1],pos[3][2],pos[3][3],
       orien[3][1][1],orien[3][1][2],orien[3][1][3]],
                                   [pos[3][1],pos[3][2],pos[3][3],
       orien[3][2][1],orien[3][2][2],orien[3][2][3]],
                                   [pos[4][1],pos[4][2],pos[4][3],
       orien[4][1][1],orien[4][1][2],orien[4][1][3]],
                                   [pos[4][1],pos[4][2],pos[4][3],
       orien[4][2][1],orien[4][2][2],orien[4][2][3]]
                                ])
```

(%o67) $\text{invSTANFORD}(x, y, z, L1, L2, L6, \alpha, \beta, \gamma) := \textbf{block}\Big([R, \text{pos}, \text{orien}, \text{res}],\ R\text{: rodriguez}([0, 0, 1], \alpha)\cdot\text{rodriguez}([0, 1, 0], \beta)\cdot\text{rodriguez}([0, 0, 1], \gamma),\ \text{coordPolso: }R\cdot\begin{pmatrix}0\\0\\L6\end{pmatrix}, \text{xCap: }x-$ $\text{coordPolso}_1,\ \text{yCap: }y-\text{coordPolso}_2,\ \text{zCap: }z-\text{coordPolso}_3,\ \text{pos: posInversa}(\text{xCap}_1, \text{yCap}_1, \text{zCap}_1, L1, L2),\ \text{orien: orienInversa}(R, \text{pos}),\ \text{res: }[[(\text{pos}_1)_1, (\text{pos}_1)_2, (\text{pos}_1)_3, ((\text{orien}_1)_1)_1, ((\text{orien}_1)_1)_2, ((\text{orien}_1)_1)_3], [(\text{pos}_1)_1, (\text{pos}_1)_2, (\text{pos}_1)_3, ((\text{orien}_1)_2)_1, ((\text{orien}_1)_2)_2, ((\text{orien}_1)_2)_3], [(\text{pos}_2)_1, (\text{pos}_2)_2, (\text{pos}_2)_3, ((\text{orien}_2)_1)_1, ((\text{orien}_2)_1)_2, ((\text{orien}_2)_1)_3], [(\text{pos}_2)_1, (\text{pos}_2)_2, (\text{pos}_2)_3, ((\text{orien}_2)_2)_1, ((\text{orien}_2)_2)_2, ((\text{orien}_2)_2)_3], [(\text{pos}_3)_1, (\text{pos}_3)_2, (\text{pos}_3)_3, ((\text{orien}_3)_1)_1, ((\text{orien}_3)_1)_2, ((\text{orien}_3)_1)_3], [(\text{pos}_3)_1, (\text{pos}_3)_2, (\text{pos}_3)_3, ((\text{orien}_3)_2)_1, ((\text{orien}_3)_2)_2, ((\text{orien}_3)_2)_3], [(\text{pos}_4)_1, (\text{pos}_4)_2, (\text{pos}_4)_3, ((\text{orien}_4)_1)_1, ((\text{orien}_4)_1)_2,$

$$\left. ((\mathrm{orien}_4)_1)_3], [(\mathrm{pos}_4)_1, (\mathrm{pos}_4)_2, (\mathrm{pos}_4)_3, ((\mathrm{orien}_4)_2)_1, ((\mathrm{orien}_4)_2)_2, ((\mathrm{orien}_4)_2)_3]] \right)$$

```
(%i74) invSTANFORD(10,1,1,0.412,0.154,0.263,%pi,%pi/2,%pi/4);
```

(%o74) $[[0.08221974213827725, 1.567549250542036, 10.32720664071364, -3.102209469265483, 1.57403243393346, \pi - 0.7458814168204394], [0.08221974213827725, 1.567549250542036, 10.32720\backslash 664071364, 0.03938318432431065, -1.57403243393346, -0.7458814168204394], [0.0822197421382\backslash 7725, 1.567549250542036, 10.32720664071364, -3.102209469265483, 1.57403243393346, \pi - 0.745\backslash 8814168204394], [0.08221974213827725, 1.567549250542036, 10.32720664071364, 0.0393831843243\backslash 1065, -1.57403243393346, -0.7458814168204394], [-3.029550830113841, 1.567549250542036, -10.32720664071364, 0.02885150023036301, 1.567569610176263, \pi - 0.814431758491189], [-3.029\backslash 550830113841, 1.567549250542036, -10.32720664071364, -3.11274115335943, -1.56756961017\backslash 6263, -0.814431758491189], [-3.029550830113841, 1.567549250542036, -10.32720664071364, 0.02\backslash 885150023036301, 1.567569610176263, \pi - 0.814431758491189], [-3.029550830113841, 1.56754925\backslash 0542036, -10.32720664071364, -3.11274115335943, -1.567569610176263, -0.814431758491189]]$

```
(%i75)
```