Lezione 1

lunedì 21 settembre 2020 08:10

http://maxima.sourceforge.net/

https://www.texmacs.org/tmweb/home/welcome.en.html

https://processing.org/

$x \in \mathbb{R}^2$ $\quad x \in \mathbb{R}^3$

$a \in \mathbb{R}^3 = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$ $\quad a_h \in \mathbb{R}$ $\quad b = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$

$a = a_x e_x + a_y e_y + a_z e_z$

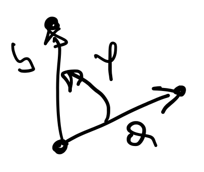
$e_x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad e_y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad e_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$a \cdot b = a^T b = \underline{[a_x \ a_y \ a_z]} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$ PRODOTTO SCALARE

$1 \times 3 \quad 3 \times 1$

1×1 SCALARE

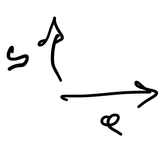
$a \cdot b = \|a\| \, \|b\| \, \cos(\varphi)$ $\qquad \cos(-\varphi) = \cos(\varphi)$ PARI



$a \cdot b = b \cdot a$

$a^T b = (a^T b)^T = b^T a = b \cdot a$

$\varphi = \pm \pi/2 \quad \cos(\varphi) = 0$



$a \cdot b = 0$

$\boxed{a \perp b \iff a \cdot b = 0}$

$a^T b \overset{?}{=} 0$ SI $\perp$ / NO $\not\perp$

$$a \times b \equiv \det \begin{bmatrix} e_x & e_y & e_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix} = e_x \cdot (a_y b_z - a_z b_y) + e_y \cdot (a_z b_x - a_x b_z) + e_z \cdot (a_x b_y - a_y b_x)$$

$$a \times b = \begin{bmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{bmatrix}$$

$e_x \cdot e_y = 0$

$e_x \cdot e_y = [1 \ 0 \ 0] \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 1 * 0 + 0 * 1 + 0 * 0 = 0$

$e_x \times e_y = e_z$!

$= \det \begin{bmatrix} e_x & e_y & e_z \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \boxed{e_z}$

$x\,y\,z \to x\,z\,y \to z\,x\,y \to z\,y\,x$

DISPARI PARI DISPARI

$x\,y\,z$ / $z\,y\,x$ 3 DISPARI PERMUTAZIONE DISPARI

$x\,y\,x$ NO PERMUTAZIONE

$$c_{hke} = \begin{cases} hke & \text{PARI} & 1 \\ hke & \text{DISPARI} & -1 \\ & \text{ALTRIMENTI} & 0 \end{cases}$$

$$\boxed{e_h \times e_k = c_{hke} \, e_e}$$

$e_x \times e_y = + e_z$ $\qquad$ $x\,y\,z$ / $x\,y\,z$ 0 PARI PARI

$e_y \times e_z = + e_x$ $\qquad \to$ $x\,y\,z$ / $y\,z\,x$

2 PARI

$e_y \times e_y = 0$

$\boxed{a \parallel b \iff a \times b = 0}$

$a \parallel b \iff a = \alpha b \to a^T = \alpha b^T$ (α SCALARE)

$$a \times b = \det \begin{bmatrix} e_x & e_y & e_z \\ - & a^T & - \\ - & b^T & - \end{bmatrix} = \det \begin{bmatrix} e_x & e_y & e_z \\ & \alpha b^T & \\ & b^T & \end{bmatrix} = \alpha \det \begin{bmatrix} e_x & e_y & e_z \\ & b^T & \\ & b^T & \end{bmatrix} = \alpha \cdot 0 = 0$$

$\boxed{a \cdot b = 0 \iff a \perp b \qquad a \times b = 0 \iff a \parallel b}$

SCALARE $\qquad$ VETTORE

$a \times b \neq b \times a$

$\boxed{a \times b = - b \times a}$

$a \times b = \det \begin{bmatrix} e_x & e_y & e_z \\ & a^T & \\ & b^T & \end{bmatrix}$

$b \times a = \det \begin{bmatrix} e_x & e_y & e_z \\ & b^T & \\ & a^T & \end{bmatrix}$

$\boxed{a \times b \times c}$ $\quad (a \times b) \times c \ne \quad a \times (b \times c) \ne$

$\cancel{a \times b \times c}$

$e_x \times (e_y \times e_y) = e_x \times 0 = \boxed{0}$

$(e_x \times e_y) \times e_y = e_z \times e_y$

$x\,y\,z$ / $z\,y\,x$

$e_x \times e_y = + e_z$

$e_z \times e_y = - e_x$ $\neq$ 0