Interferometro

(MISURA LA DENSITÀ)

Ing. Michela Gelfusa
Email: gelfusa@ing.uniroma2.it
Ufficio 3° Piano – Ingegneria Industriale
Tel.: 06 7259 7210

Introduction to Interferometry

Wave Optics

Wave optics is a study concerned with phenomena that cannot be adequately explained by geometric (ray) optics.

These phenomena include:

– Interference → onde che si ricombinano

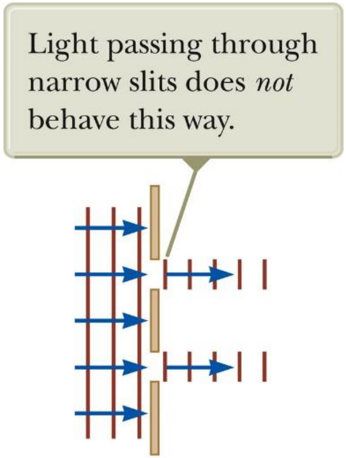
– Diffraction → onda che "sbatte"

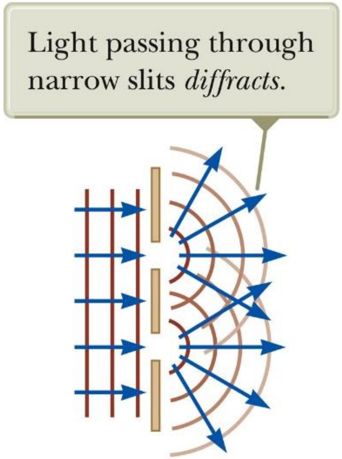
– Polarization → già vista

Diffraction

If the light traveled in a straight line after passing through the slits, no interference pattern would be observed.

- From Huygens's principle we know the waves spread out from the slits.
- This divergence of light from its initial line of travel is called **diffraction**.





Conditions for Interference

To observe interference in light waves, the following two conditions must be met:

mantengono costante la fase

- The sources must be **coherent**.
  - They must maintain a constant phase with respect to each other.

non cambiano lunghezza d'onda

- The sources should be **monochromatic**.
  - Monochromatic means they have a single wavelength.

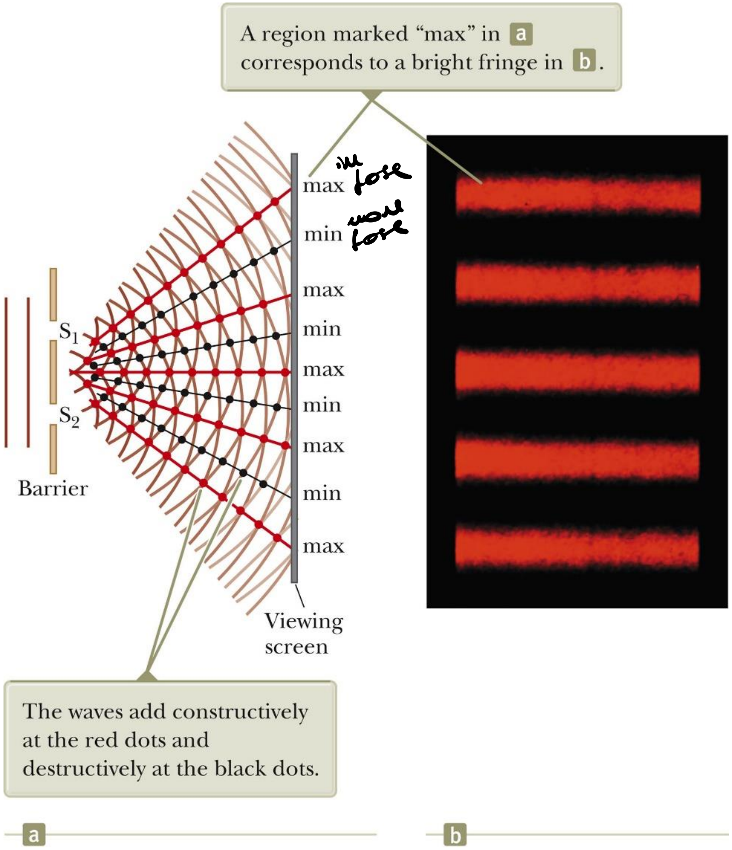
Interference

- In *constructive interference* the amplitude of the resultant wave is greater than that of either individual wave.
- In *destructive interference* the amplitude of the resultant wave is less than that of either individual wave.

All interference associated with light waves arises when the electromagnetic fields that constitute the individual waves combine.

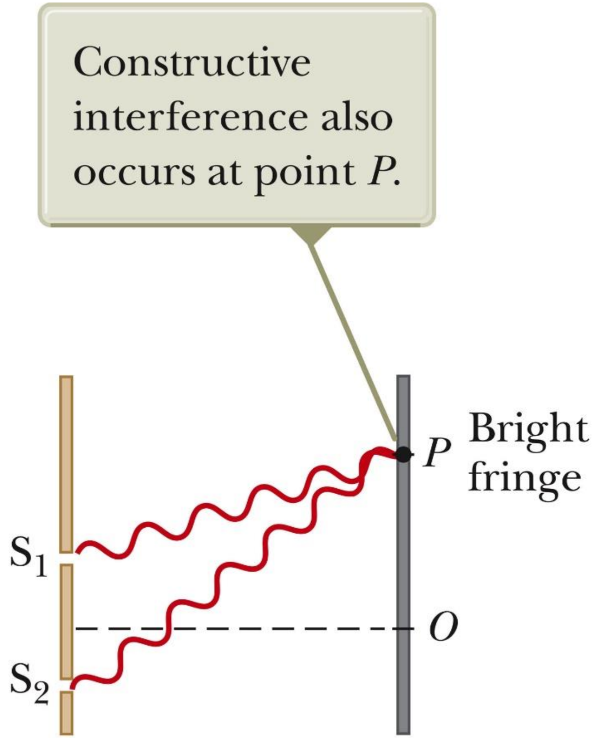
Resulting Interference Pattern

- Thomas Young first demonstrated interference in light waves from two sources in 1801 (*Young's Double-Slit Experiment*).
- The light from the two slits forms a visible pattern on a screen.
- The pattern consists of a series of bright and dark parallel bands called *fringes.*
- *Constructive interference* occurs where a bright fringe occurs.
- *Destructive interference* results in a dark fringe.



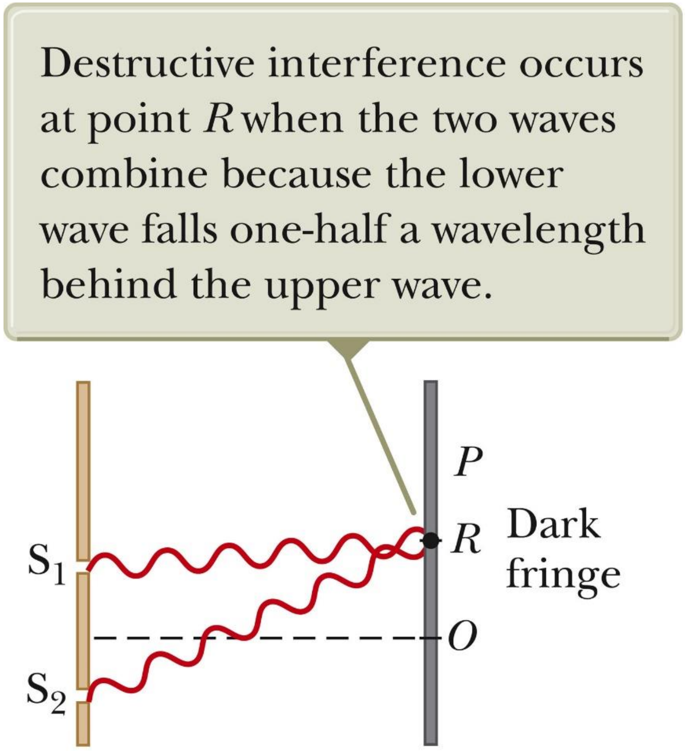
Interference Patterns

- The lower wave has to travel farther than the upper wave to reach point *P*.
- The lower wave travels one wavelength farther.
  - Therefore, the waves arrive in phase
- A second bright fringe occurs at this position.



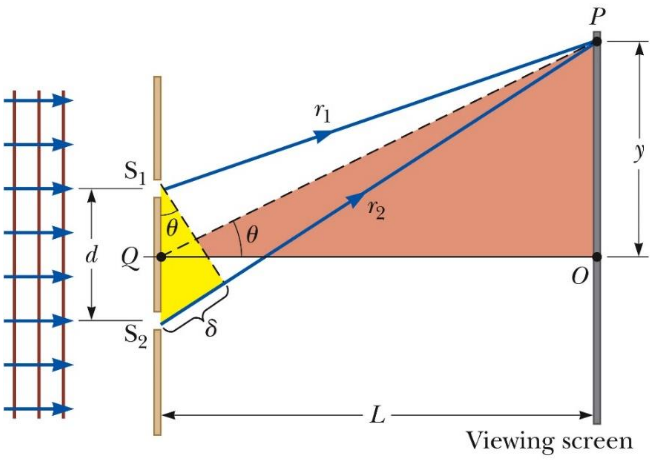
Interference Patterns

- The upper wave travels one-half of a wavelength farther than the lower wave to reach point *R.*
- The trough of the upper wave overlaps the crest of the lower wave.
- This is destructive interference.
  - A dark fringe occurs.



Interference Equations

For a **bright fringe** produced by constructive interference, the path difference must be either zero or some integer multiple of the wavelength.



$$\delta = d \sin \theta_{bright} = m\lambda$$

m = 0, ±1, ±2, …

- m is called the order number
  - When m = 0, it is the zeroth-order maximum
  - When m = ±1, it is called the first-order maximum

When **destructive interference** occurs, a dark fringe is observed. This needs a path difference of an odd half wavelength.

$$\delta = d \sin \theta_{dark} = (m + ½)\lambda$$

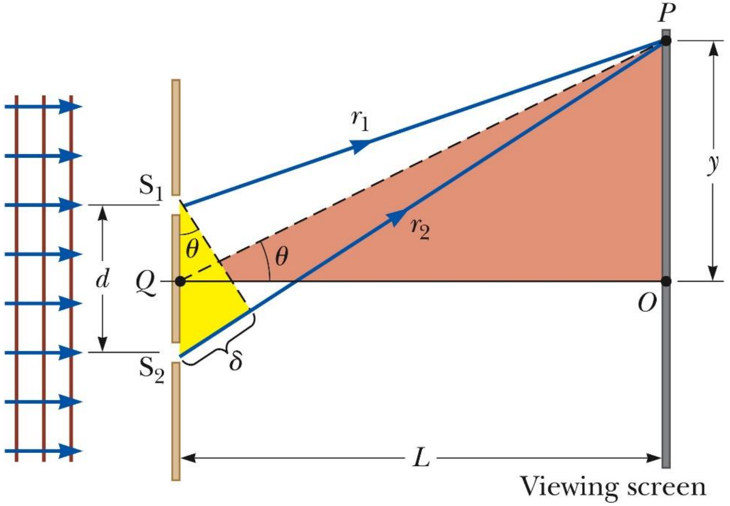
m = 0, ±1, ±2, …

Interference Equations

The positions of the fringes can be measured vertically from the zeroth-order maximum.

Using the large triangle (see figure)

- $y_{bright} = L \tan \theta_{bright}$
- $y_{dark} = L \tan \theta_{dark}$

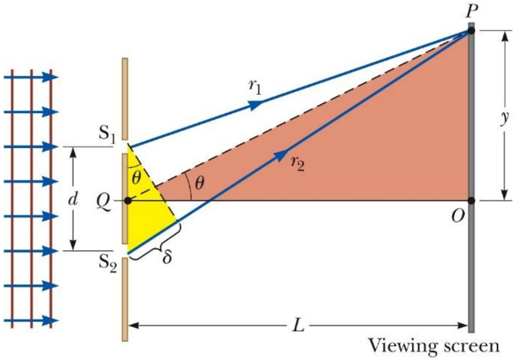


Interference Equations

Assumptions in a Young's Double Slit Experiment:

L >> d

d >> λ



Approximation:

- θ is small and therefore the small angle approximation tan θ ~ sin θ can be used
- y = L tan θ ≈ L sin θ

For small angles, we have:

$$y_{\text{bright}} = L\frac{m\lambda}{d} \text{ and } y_{\text{dark}} = L\frac{(m+\frac{1}{2})\lambda}{d}$$

m is called the order number

Intensity Distribution, Electric Fields

The equations developed give the location of only the centers of the bright and dark fringes.

We can calculate the distribution of light intensity associated with the double-slit interference pattern given the following assumptions:

1. The two slits represent coherent sources of sinusoidal waves.
2. The waves from the slits have the same angular frequency, ω.
3. The waves have a constant phase difference, ϕ.

The total magnitude of the electric field at any point on the screen is the superposition of the two waves.

The magnitude of each wave at point *P* on a screen can be found.

$$E_1 = E_o \sin \omega t$$

$$E_2 = E_o \sin (\omega t + \varphi)$$

- Both waves have the same amplitude, E_o.

Intensity Distribution, Resultant Field

The magnitude of the resultant electric field comes from the superposition principle.

$$E_P = E_1 + E_2 = E_o[\sin \omega t + \sin (\omega t + \varphi)]$$

This can also be expressed as:

$$E_P = 2E_o \cos\left(\frac{\varphi}{2}\right)\sin\left(\omega t + \frac{\varphi}{2}\right)$$

- E_P has the same frequency as the light at the slits.
- The magnitude of the field is multiplied by the factor 2 cos (φ / 2).

Intensity Distribution, Phase Relationships

The phase difference between the two waves at P depends on their path difference.

$$\boxed{\delta = r_2 - r_1 = d \sin \theta}$$

A path difference of λ (for constructive interference) corresponds to a phase difference of 2π rad.

A path difference of δ is the same fraction of λ as the phase difference φ is of 2π ($\delta : \lambda = \theta : 2\pi$).

This gives $$\varphi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda} d \sin \theta$$

Intensity Distribution

The expression for the intensity comes from the fact that the *intensity of a wave is proportional to the square of the resultant electric field magnitude at that point.*

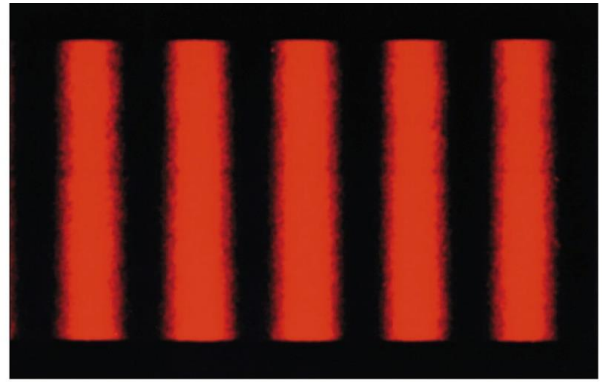
The intensity therefore is

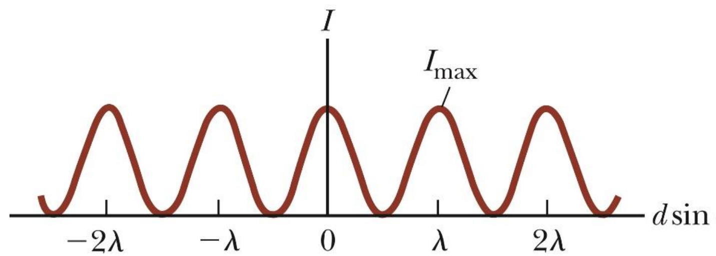
$$I = I_{max} \cos^2\left(\frac{\pi d \ \sin\theta}{\lambda}\right) \approx I_{max} \cos^2\left(\frac{\pi d}{\lambda L}\, y\right) \ (small\ angles)$$

$$y = L \tan\theta \simeq L \, sen\,\theta$$

$$sen\,\theta \approx {}^{y}\!/_{L}$$

Light Intensity: Graph





The interference pattern consists of equally spaced fringes of equal intensity

Michelson Interferometer

- The interferometer was invented by an American physicist, A. A. Michelson.
- The interferometer splits light into two parts and then recombines the parts to form an interference pattern.
- The device can be used to measure wavelengths or other lengths with great precision.

Michelson Interferometer

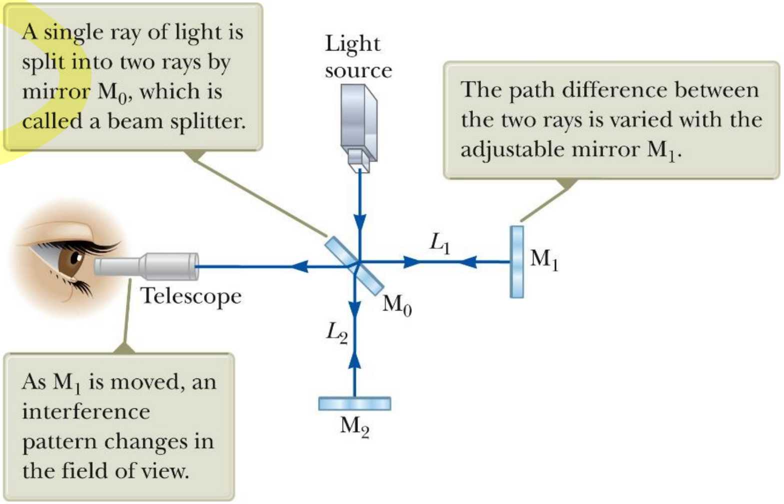
A ray of light is split into two rays by the mirror M_o.

- The mirror is at 45° to the incident beam.
  - The mirror is called a *beam splitter.*
  - It transmits half the light and reflects the rest.

The reflected ray goes toward mirror M_1.
The transmitted ray goes toward mirror M_2.

The two rays travel separate paths L_1 and L_2.

After reflecting from M_1 and M_2, the rays eventually recombine at M_o and form an interference pattern.



Interferometer in fusion devices

Optical properties of magnetically confined plasmas

Refractive index

The refractive index (or index of refraction) of a medium is a measure of how much the speed of light is reduced inside the medium

Optical activity

The property of a medium to rotate the polarisation plane of a polarised light beam that propagates through that medium.
In the presence of magnetic fields all molecules have optical activity.

Birefringence (double refraction)

A medium is called birefringent if has two different indices of refraction in different directions.
A light beam passing through this medium can be divided into two components (an "ordinary" and an "extraordinary ray") that travel at different speeds through that medium.

TOR VERGATA
UNIVERSITY OF ROME

Why a FIR plasma diagnostic?

frequenze nel
lontano infrarosso

Diagnostics using FIR beams are non-invasive as FIR wavelengths (100μm equiv. with freq. of **3 THz**) are far away from the plasma frequencies(GHz region) and the beams do not disturb the plasma.

Plasma density and Magnetic fields inside a plasma via interferometry and polarimetry techniques

⬇

Essential for safety of the plant, plasma performances and real-time control of the plasma

Target

Optical interferometers are normally used in magnetically confined plasmas to measure the refractive index of the plasma by comparing the phase shift variation between a reference and a probe laser beam, from which the line-integrated electron density can be derived. Its major advantages above other electron density diagnostics are:

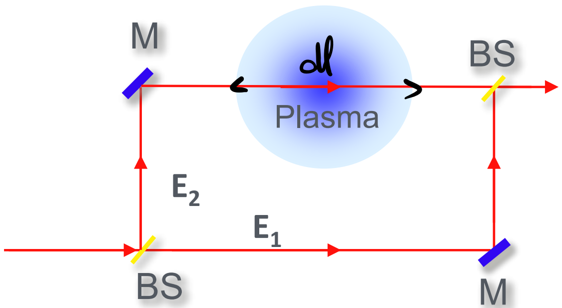
(1) that the temporal evolution of the electron density can be followed (in contrast to Thomson scattering)

(2) that no absolute calibration of the instrumentation is required

(3) that no additional information is needed from other diagnostics (as in the case with spectroscopic measurement of the density).

Interferometry principle

Interferometro di Mach-Zender



An interferometer is sensible to relative variation of "optical path" between the two arms.

corda nel plasma

Phase lag in the plasma arm

$$\varphi = \int k \cdot dl \int_l n \cdot \frac{\omega}{c} dl$$

n : refractive index

Signal at detector

$$\boxed{\begin{aligned} V &= |E_1|^2 + |E_2|^2 + 2E_1E_2 \operatorname{Re} e^{j\Delta\varphi} = \\ &= A_o + A_1 \cos\Delta\varphi \end{aligned}}$$

where

$$\Delta\varphi = \int \left(k_{plasma} - k_o\right) dl = \int (n-1)\frac{\omega}{c} dl$$

$$\left\{ \begin{aligned} k &= \frac{2\pi}{\lambda} \\ \omega &= \frac{2\pi c}{\lambda} \end{aligned} \right.$$

Plasma refractive index

The measurement of the interferometer phase shift $\Delta\varphi$ thus provides us with a measure of the mean refractive index along the line of the interferometer beam through the plasma.

The plasma refractive index is given by the equation:

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2} = \frac{n_e}{n_c}$$

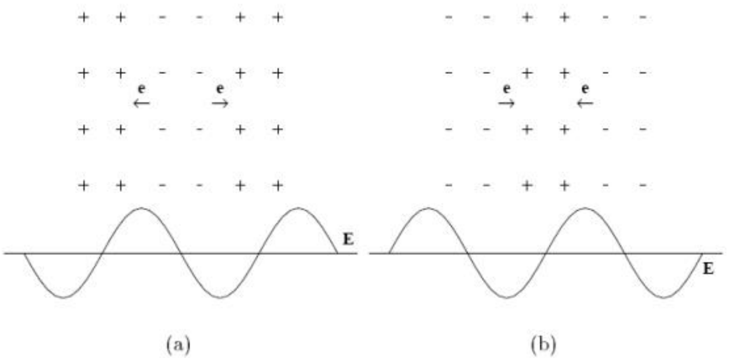
n_e is the electron density
n_c is the cutoff density

ω_p is the plasma freq.

Plasma frequency

A plasma has a characteristic frequency which can be understood by considering a displaced sheet of electrons and the resulting electric field as shown below.

It corresponds to the typical electrostatic oscillation frequency of a given species in response to a small charge separation.



Description of the origins of the plasma frequency, (a) displaced electrons give rise to a restoring force and acceleration, (b) after half a cycle the charges are interchanged.

The resulting motion of the electron constitutes plasma oscillations with a characteristic plasma frequency ω_p

$$\omega_p = \sqrt{\frac{n_e\, e^2}{\varepsilon_0 m_e}} \approx \quad GHz \quad range$$

where n_e is the electron density,
m_e is the electron mass
e the elementary charge
e_0 dielectric constant of a vacuum

Cuf-off plasma density

For a fixed probing frequency ***w*** the critical (cut-off) density n_c is defined as the density which the probing frequency equals the plasma frequency and is given by

$$n_c = \frac{\varepsilon_0 m_e \omega^2}{e^2}$$

m_e is the electron mass
e the elementary charge
e_0 dielectric constant of a vacuum

If the plasma density is sufficiently small, $n_e < n_c$, then an approximate expression is

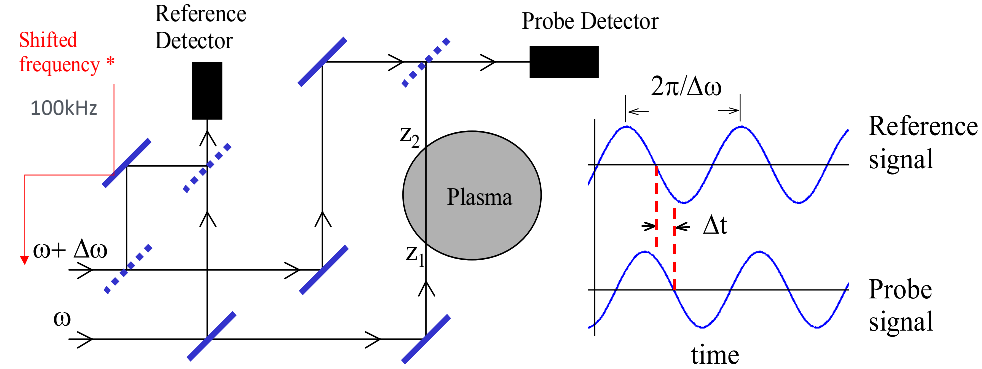
$$n^2 \approx 1 - \frac{1}{2}\frac{n_e}{n_c}$$

Phase shift

$$\Delta\varphi = \frac{\omega}{2 \cdot c \cdot n_c} \int n_e dl$$

If the density exceeds the cutoff value, the wave is no longer propagating but evanescent, falling off exponentially with distance. The result is that, normally, very little power is transmitted through the plasma and the interferometer ceases to function.

JET Interferometer



Simple schematic of an interferometer

$$S_{probe} \propto \cos(\omega t - \varphi(t))$$

$$S_{reference} \propto \cos\left(\left(\omega + \Delta\omega\right) \cdot t\right)$$

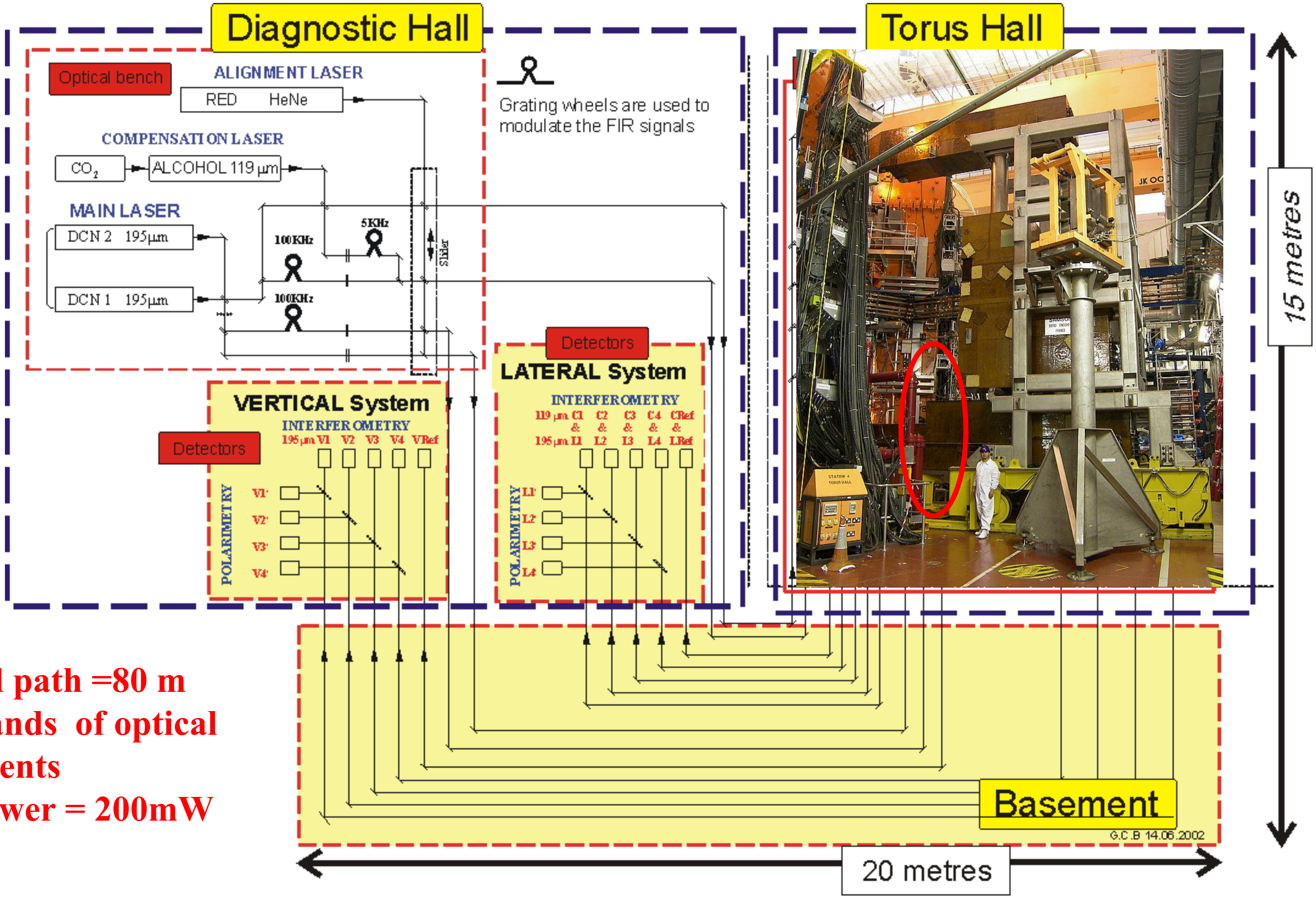
* the shift in frequency of the laser beam can obtained by using a Veron grating wheel or two laser cavities with different lengths for example.

The probe laser beam that pass through the plasma suffers a phase shift variation in time. By subtracting the phase shift of the reference beam (due to vibrations) one can obtain the phase shift due only to the plasma effects.

This ***phase shift*** is proportional to the ***line-integrated electron density*** n_e along the propagation direction inside the plasma. The phase change is usually many multiples of 2π. What the diagnostic delivers is the number of fringes F of interference (1 fringe represents 2π phase changes) .

$$F = \frac{\phi}{2\pi} = C\,\lambda \int_{z_1}^{z_2} n_e(z)\,dz$$

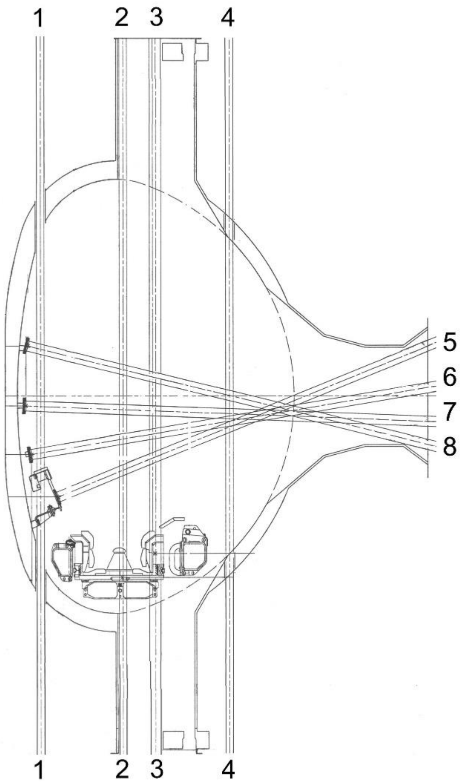
JET FIR Interferometer / Polarimeter



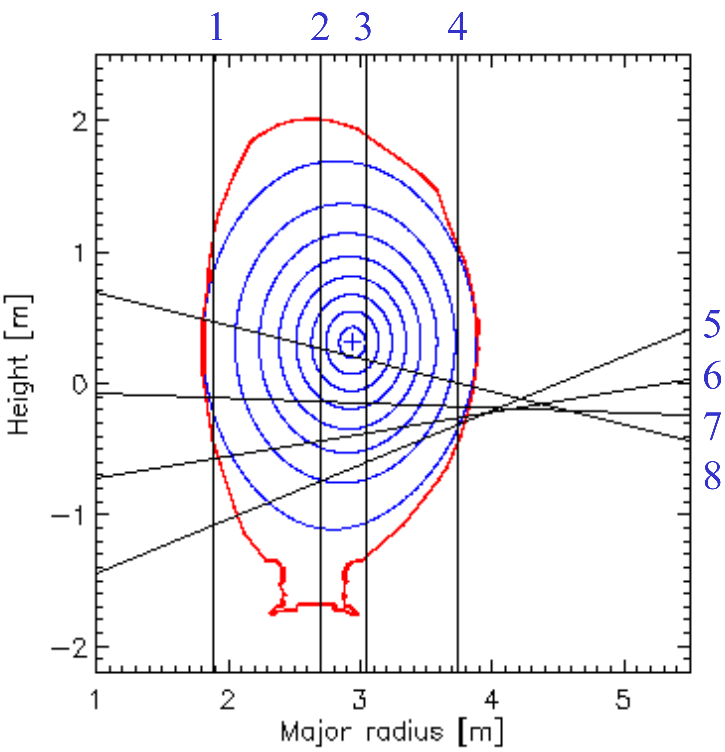
- **Optical path =80 m**
- **Thousands of optical components**
- **FIR power = 200mW**

JET FIR Channels

Vacuum vessel



Magnetic flux distribution



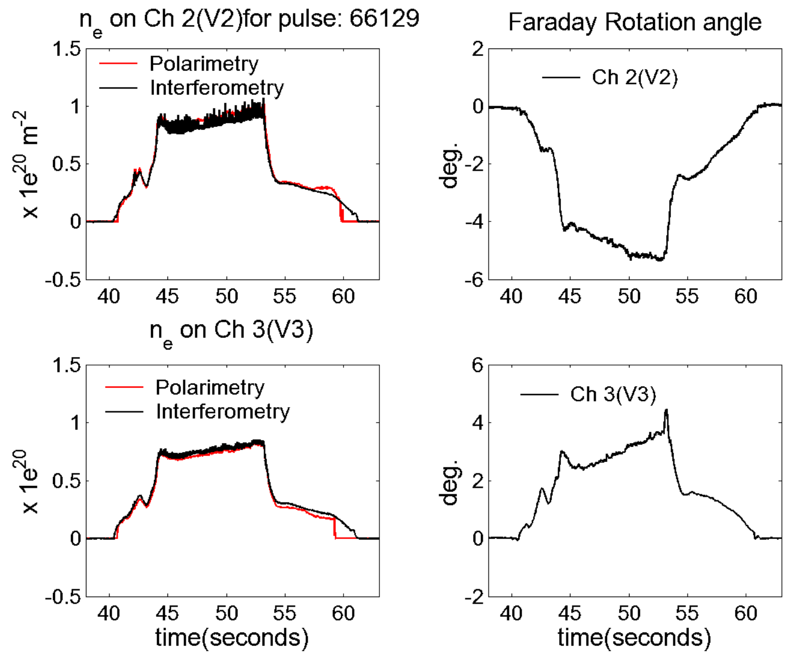
Measurements at JET

FIR interferometer/polarimeter main parameters

Laser wavelengths: 195µm and 119 µm
FIR power: 200mW and 120mW
No channels: 8 (4 vertical, 4 lateral)
Time on: 16h/day during campaigns

Interferometer: Range: 10^{18} - 4×10^{22} m^{-2}
(n_e) Accuracy: 3×10^{17} m^{-2}
Time resolution : 1ms
10 µs(new)

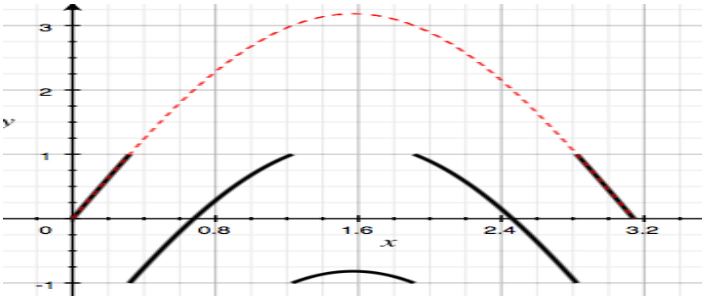
Polarimeter: Range: 0-70 deg
(FAR) Accuracy:0.2 deg
Time resolution: 1 ms

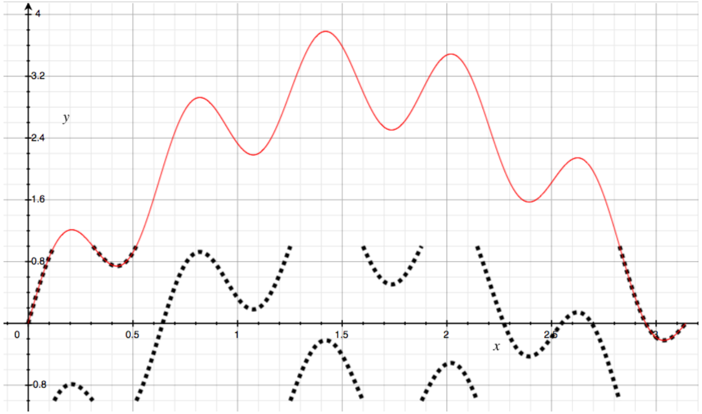


Example of line-integrated density (n_e) and Faraday rotation angle measurements at JET

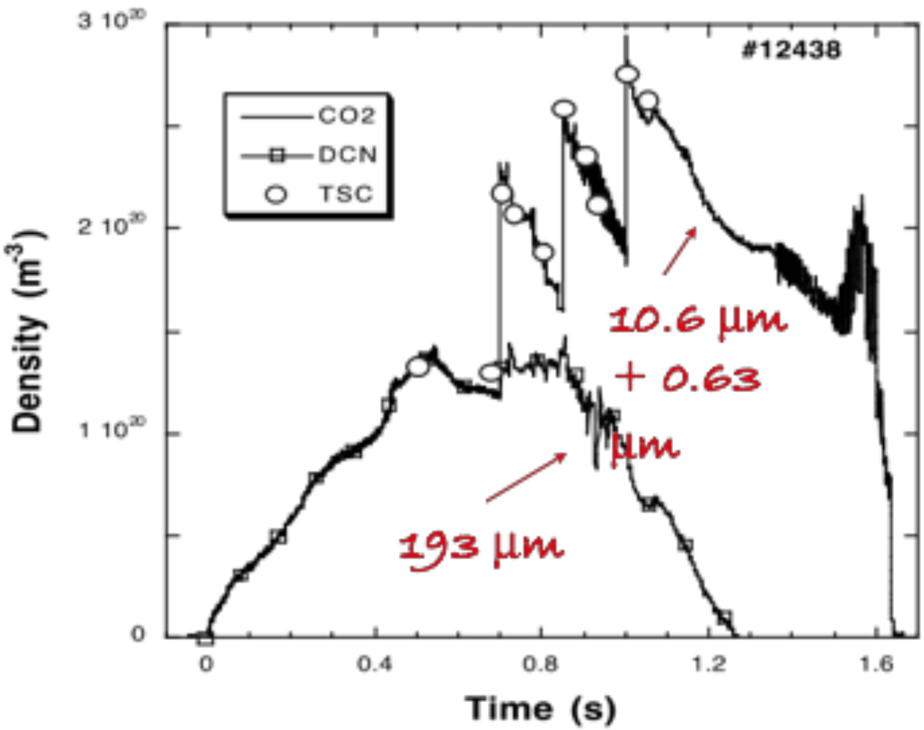
Main problems of interferometry: Fringe Jumps

Unfortunately, interferometric measurements are affected by fringe jumps, which are basically the erroneous phase difference determination due to the loss of signals or a phase difference bigger than 2π. The multiple causes include refraction, wavelength shift of the laser radiation used, changing in sensitivity and time resolution of the measurements.





Main problems of interferometry: Fringe Jumps



Components of a FIR system

- Wavelength
- Lasers
- Optics
- Modulators
- Detectors
- Mechanical structure
- Atmosphere control

Laser wavelength

Long wavelength

Pros
Better resolution of measurements

Cons
Refraction ($\sim\lambda^2$)

Short wavelength

Pros
Low refraction

Cons
Mechanical Vibrations issues
Low resolution

Example $\lambda = 195\ \mu m$

Faraday = 5-20 deg (core ch.) (accuracy 0.2 deg)
Cotton-Mouton = 5-20 deg (core ch.)
Refraction of few cm in worst case
Density 1 fringe(360deg phase) = $1x10^{19}/m^2$

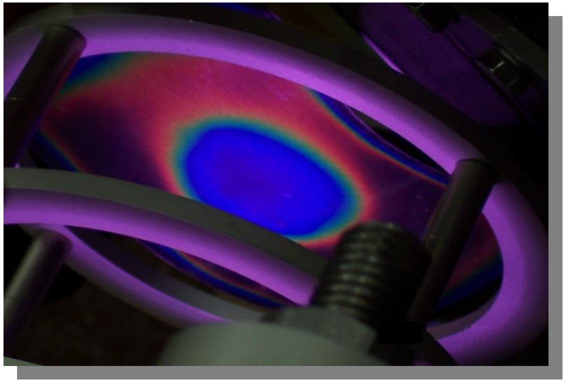
$\lambda = 119\ \mu m$

Faraday = 1.31 - 3.915 deg (core ch.)
Cotton-Mouton = 0.6-2.7 deg (core ch.)
Refraction of 1-3 mm (good for interferometry)
BAD for JET polarimeter, well suited for ITER
Density 1 fringe(360deg phase) = $1x10^{19}/m^2$

$\lambda = 10\ \mu m$ (CO_2 laser)

No refraction –use for machine protection
Density 1 fringe(360deg phase) = $1x10^{21}/m^2$

FIR lasers



- The choose of the FIR lasers depends with the wavelength to be used
- Very good stability required as these devices must operate for long plasma pulses (laser technologies developed extraordinarily during last 2 decades)
- Considerations at the design phase regarding *initial implementation and installation* versus the long term *operation and maintenance*

- 119mm FIR methanol laser is a laser with optical pumping that implies controlling two laser cavities (FIR and pumping), careful optical coupling.
- 195mm FIR DCN laser needs regular and lengthy maintenance

Input and Collection Optics

Must operate in high ambient temperatures, radiation, neutron fluxes and magnetic fields

New materials needed
(at present for development of new mirrors there are more than 10 institutions in the fusion community involved)
No magnetic materials employed
Extra shielding required.

Very low or zero access for handling to some parts of diagnostics

Different alignment schemes.
Remote handling scenarios.
Proper evaluation of risk of failures.

TOR VERGATA
UNIVERSITY OF ROME

Input and Collection Optics

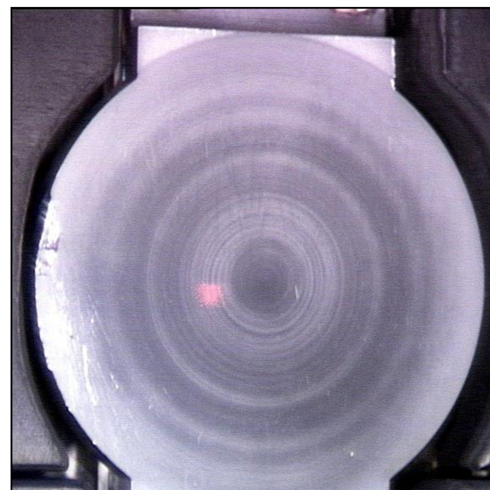
- Windows need to be radiation resistant

- Wire-grids (wire diameter is around 10 mm) that are widely used for the FIR devices as polarisers and beam splitters must be away from the heat-sources

- Very high <u>sputtering</u> on the in-vessel components changes the optical properties (reflection, transmission) and causes damage

JET FIR interferometer in-vessel mirror (new)

4 years later



Beam modulators

- FIR beams are amplitude or frequency modulated to facilitate detection.
- The higher the modulation, the better the temporal resolution
- There are different modulation techniques

Frequency controlled choppers

Pros
•Very cheap
•Easy to implement

Cons
•Low modulation frequency

Diffraction wheels

Pros
•High modulation (300kHz)
•Very stable

Cons
Difficult to make
Break points for the alignment

Rotator stages with air bearing

Pros
•Medium modulation (30kHz)
•Accurate modulation

Cons
• Vibration and air-leak control needed

Twin-cavity length modulation

Pros
•Very high modulation (MHz region)

Cons
• Difficult to maintain the system stability

TOR VERGATA
UNIVERSITY OF ROME

Detectors

- The choice depends strongly on the modulation frequency and the power of the laser beams as well as on required accuracy of the measurements

- Pyro-detectors are adequate for low temporal resolution measurements

- At very high modulation frequency or very low beam power the use of cryogenic detectors becomes essential as they have very low NEP (system noise equivalent power) of 10^{-11}WxHz $^{-1/2}$

  - At RFX polarimeter the pyro-detectors are involved as there are only 6 channels (200mW FIR power) and 3kHz modulation via a chopper.

  - At Jet for example, the main FIR DCN with 200mW of laser power is divided in 16 optical branches and the power level of the FIR beams that reach the detectors are of the order of few μW.

Mechanical structure

- FIR diagnostics require vibration level of the order of 1/10th of the wavelength (structures must have large mass)

- For short wavelength (10-50 μm) the system needs active compensation

- At longer wavelength (100 -400 μm) the compensation is done generally with reference channels for interferometer schemes for example.

- In large FIR devices the design of mechanical structures becomes very complicated due to space constrains limitation and shared area between different diagnostics.

Atmosphere control

FIR beams are strongly absorbed by the ambient air due to the presence of water in particular (a 200mW 119μm FIR beam is completely absorbed in normal air after few meters of free propagation).

Different gasses for purging a sealed optical system are available (dry-air, nitrogen, argon etc)

Operation and Maintenance

- Alignment/optimisation of the FIR beams is needed from time to time(every 1-2 years) and often visible beams are used for this task.
On ITER visible beams cannot be used due to no-access areas

- Larger optics and space for temporary optics used for beams alignment need to be allocated straight from the design phase as well as a clever strategy for accessing some components for easy repair/removal.

- Duplication of components that needs often and/or lengthy maintenance

Management of Operation and Maintenance of a FIR system have big implications in long term and sometimes this is not considered properly at the original design phase

Conclusions

- The plasma has optical properties that can be used to measure key parameters of the magnetically confined plasmas.

- The FIR diagnostics technologies are now mature to be used in ITER-like machines.

- Reliable operation of FIR diagnostics will be an important requirement on the path to the first commercial fusion reactor.