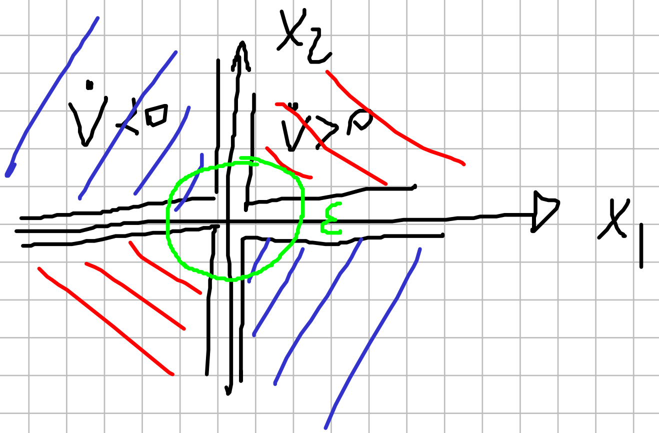
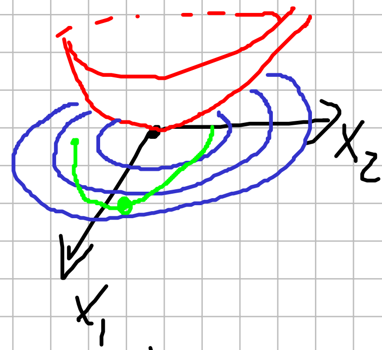
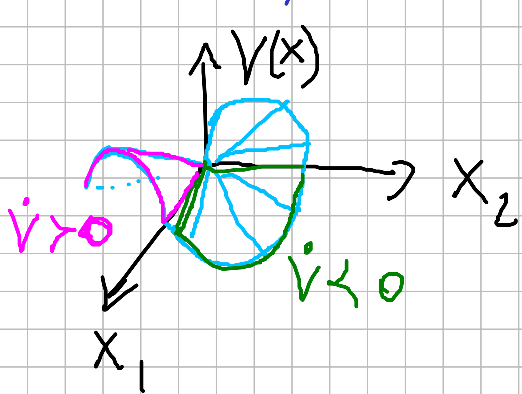
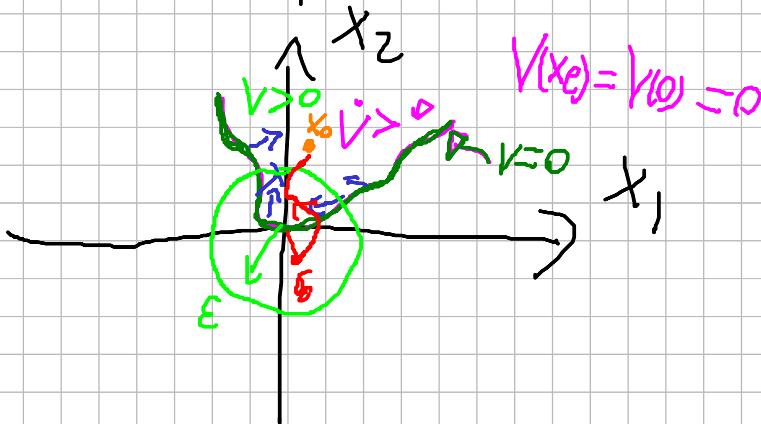
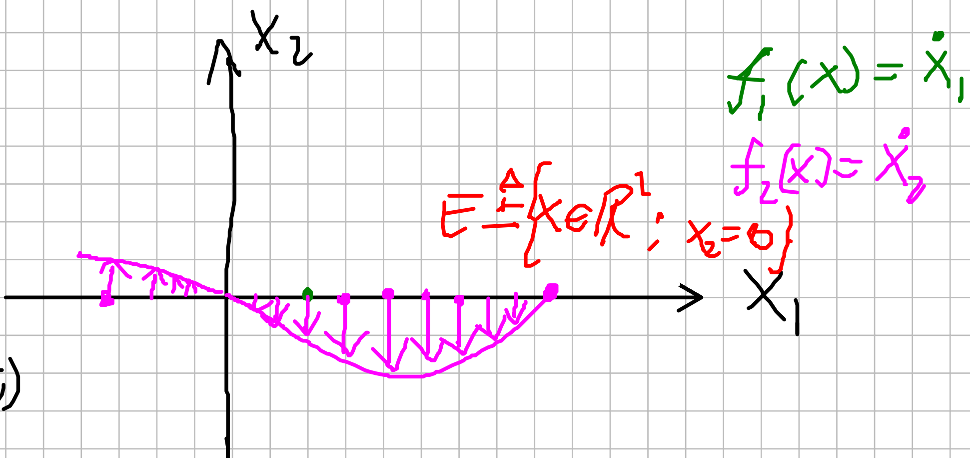
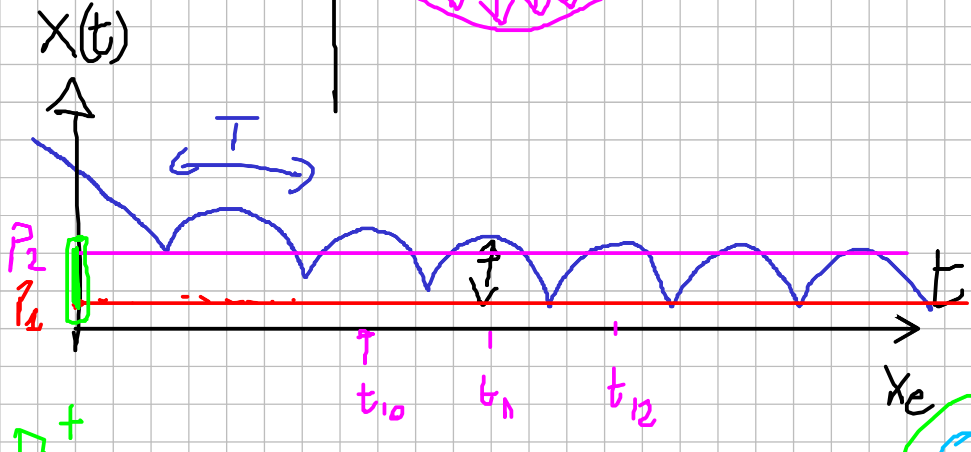

X2
X1
V̇<0
V̇>0
ε


X2
X1


V(x)
X2
X1
V̇>0
V̇<0

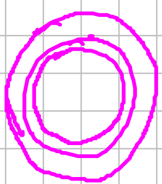

X2
V(xe)=V(0)=0
V̇>0
V̇>0
x0
V=0
X1
ε
δ

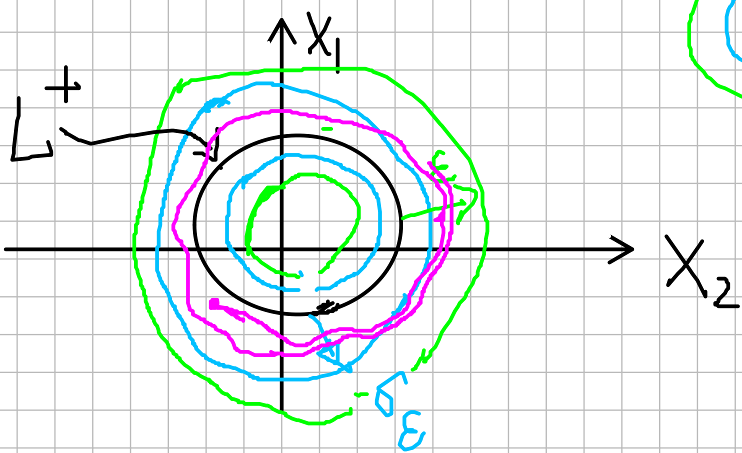

X2
f1(x)=ẋ1
f2(x)=ẋ2
E≜{x∈R1; x2=0}
X1


X(t)
T
P2
P1
t
t10
t11
t12

Γ^+


Xe
δε
ε



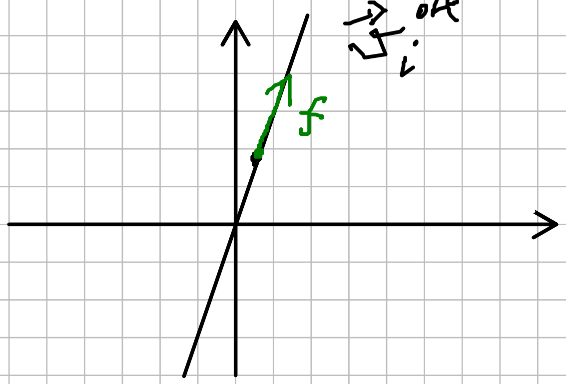

X1
L+
ε
X2
δε

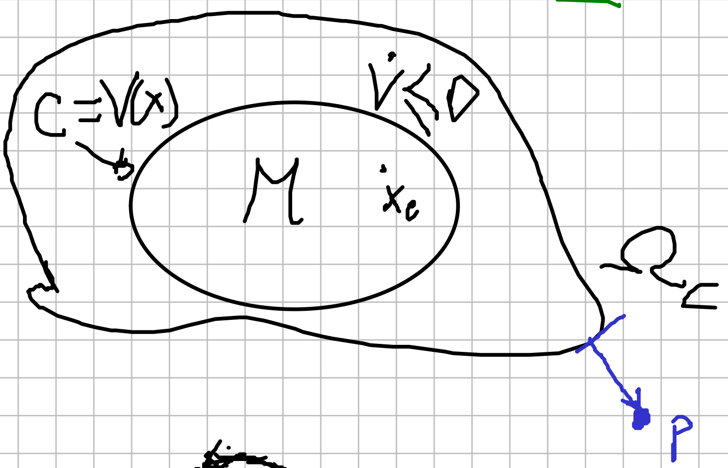
$\dot{X} = AX$, $\exists M$? Come li trovo $\quad v_i^{sx} \triangleq v_i^{sx} A = v_i^{sx} \lambda_i$

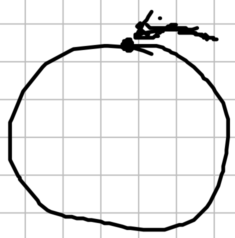
$M \triangleq ?$ $\quad v_i^{dx} \triangleq \rightarrow A v_i^{dx} = \lambda_i v_i^{dx}$

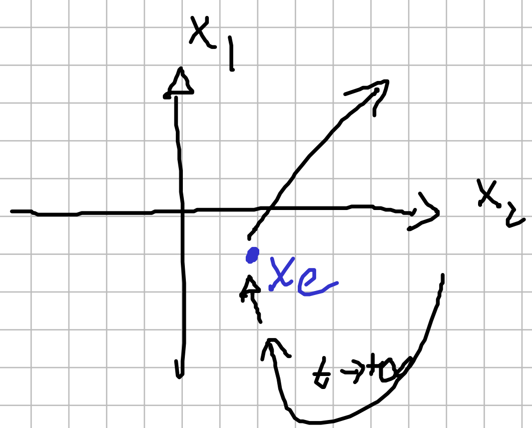
$X_0 = \alpha v_i^{dx}, \ \alpha \in \mathbb{R}$

$\dot{X} = AX \overset{x_0 \triangleq}{=} \lambda_i \alpha v_i^{dx} = f$





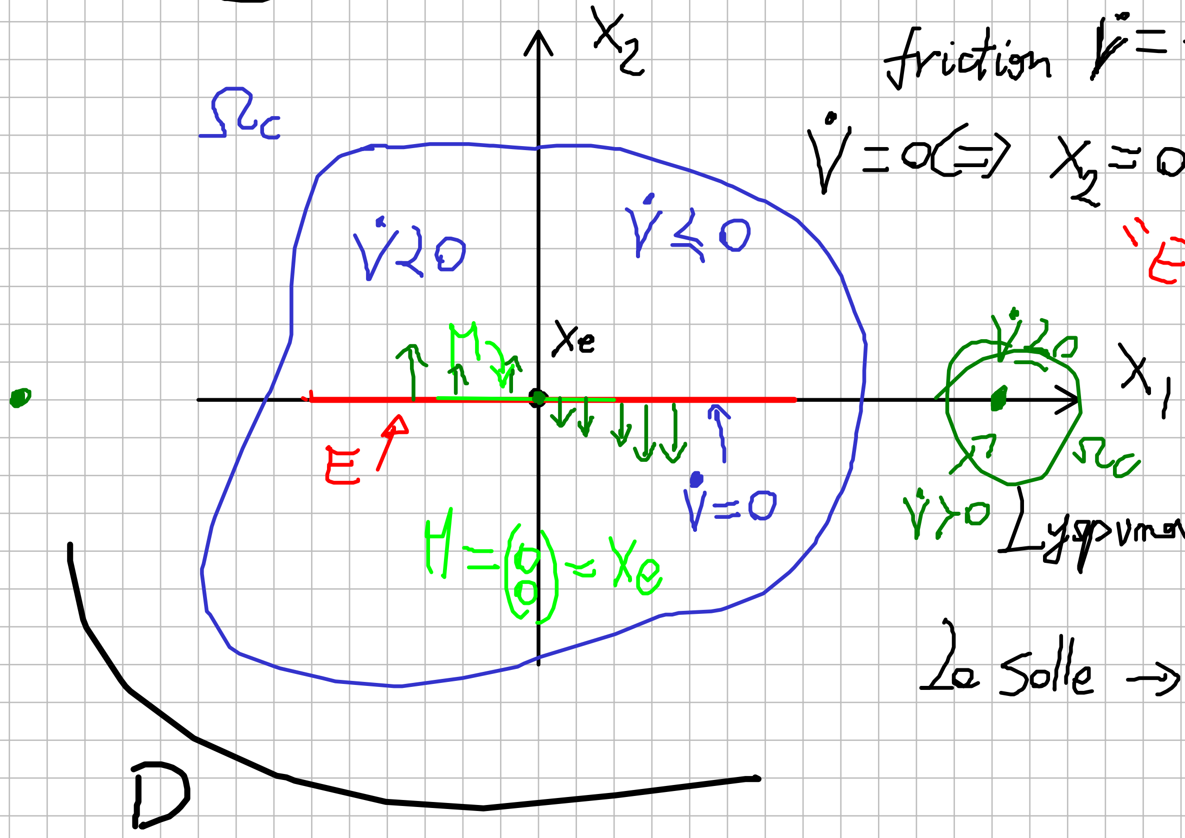




friction $\dot V = -b x_2^2 \le 0$ ovunque

$\dot V = 0 \iff x_2 = 0$

"E"



Lyapunov $\Rightarrow X_e$ STAB

La Salle $\rightarrow X_e$ è A.S.

$$\dot{X} = \begin{bmatrix} -X_2 - X_1 \\ X_1 + X_2 u \end{bmatrix}$$

$u(x)$

$\frac{1}{x^2} \to 0$

$$V(x) = \frac{X_1^2}{2} + \frac{X_2^2}{2} > 0 \text{ in } X_e \qquad P = I \cdot \frac{1}{2}$$

$X' P X \to \sqrt{X'X} \triangleq \|X\|$ $P = I$

$$\dot{V} = -X_1^2 - \cancel{X_1 X_2} + \cancel{X_1 X_2} + X_2^2 u$$
$$= -X_1^2 + X_2^2 u$$

$u(x) = \cancel{-\frac{X_2^2}{2}}$ -1

$$\dot{V} = -X_1^2 - X_2^2 < 0 \text{ in } X_e \qquad \forall x \in \mathbb{R}^n$$

$$\dot{V}(x) = -\overset{\alpha}{2}\, V(x), \quad V(X(0))$$

$V(X(t)) = ?$ $\quad V$

$u = -1$

X_e è G.E.S.

$V(x) = z$

$\dot{z} = -2z, \; z_0$

$z(t) = e^{-2t} z_0$

$V(X(t)) = V(X(0))\, e^{-2t}$

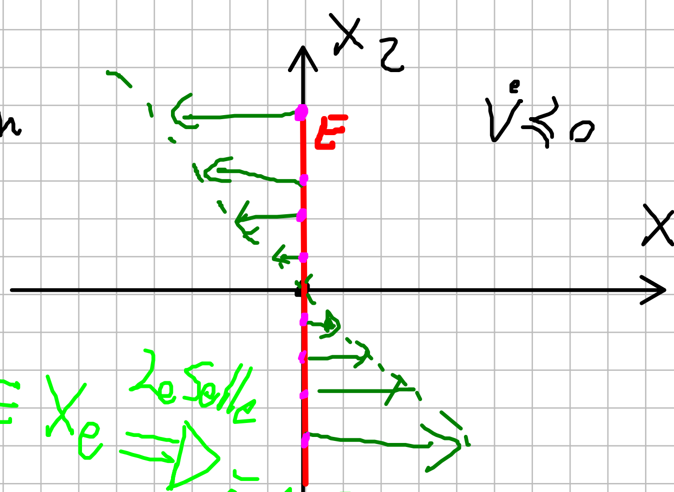
$\frac{1}{2} \|X(t)\|^2 = \frac{\|X(0)\|^2}{2} e^{-2t}$

$$\|X(t)\| = \|X(0)\|\, e^{-t}$$

$u = 0$

$$\dot{V} = -X_1^2 \leq 0 \quad \forall x \in \mathbb{R}^n$$

$$\dot{X} = \begin{bmatrix} -X_1 - X_2 \\ X_1 \end{bmatrix} \begin{matrix} f_1 \\ f_2 \end{matrix}$$



$\dot{V} \leq 0$

$M \equiv X_e$ 2e Sollé $\Rightarrow$ è A.S.

$$Q = Q' \geq 0 \qquad \nexists\, \alpha > 0$$

$$\dot V = -\begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq -\underbrace{x' P x}_{V(x) > 0} \cdot \alpha = -\alpha V(x)$$

$$\dot V(x) \leq \underbrace{-\alpha V(x)}_{=}, \quad V(x_0) = V_0 \qquad z_0 = c V_0$$

$$V(x(t)) \leq e^{-\alpha t} V_0 \qquad z_0 \geq V_0 \qquad c \geq 1$$

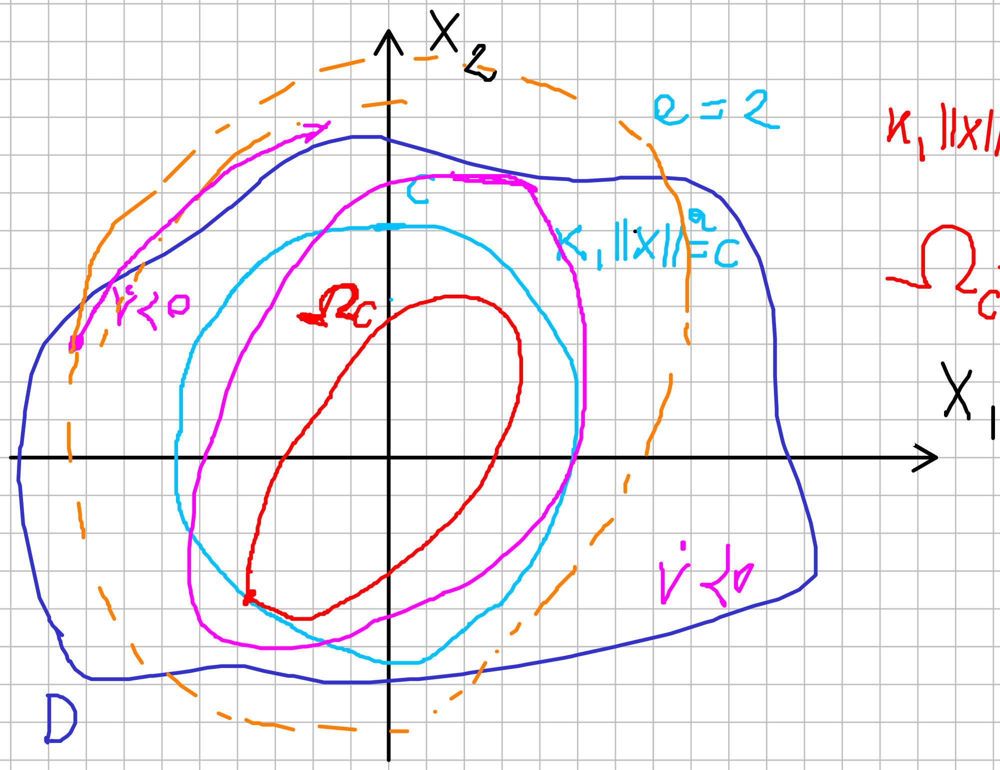
$$\dot z = -\alpha z$$

$$V(x(t)) \leq z(t) = e^{-\alpha t} c V_0 \qquad z(t) \leq e^{-\alpha t} z_0$$

$$x = \begin{bmatrix} e \\ \dot e \end{bmatrix} \rightarrow \qquad V(x) = x' \overbrace{\begin{bmatrix} \frac{1}{2} M(q) & 0 \\ 0 & \frac{1}{2} K_D \end{bmatrix}}^{P(t)} x$$

$$M(q)\ddot q = \underbrace{-C(q,\dot q)\dot q - D\dot q - g(q)}_{} + u$$

$$u = -(\;) + M(q)\ddot q^{*} + PD\text{-like}$$



$$K_1\|x\|^e \le V(x) \le K_2\|x\|^e$$

$$\Omega_c \triangleq \{x\in\mathbb{R}^n : V(x)\le c\}$$

$$\underbrace{K_1\|x\|^e \le V(x)}\ \underline{\le K_2\|x\|^e}$$

$$-K_2\|x\|^e \le -V(x) \le -K_1\|x\|^e$$

$$\dot V \le -K_3\|x\|^e \le -\overbrace{\frac{K_3}{K_2}}^{\alpha} V.$$

$$V(x(t)) \le V_0\, e^{-\alpha t} = V(x(0))\, e^{-\frac{K_3}{K_2}t} \le K_2\|x(0)\|^e e^{-\frac{K_3}{K_2}t}$$

$$K_1\|x\|^e \le V(x(t)) \le V_0\, e^{-\frac{K_3}{K_2}t} \le K_2\|x(0)\|^e\, e^{-\frac{K_3}{K_2}t}$$

$$\|x(t)\|^e \le \frac{K_2}{K_1}\|x(0)\|^e\, e^{-\frac{K_3}{K_2}t}$$

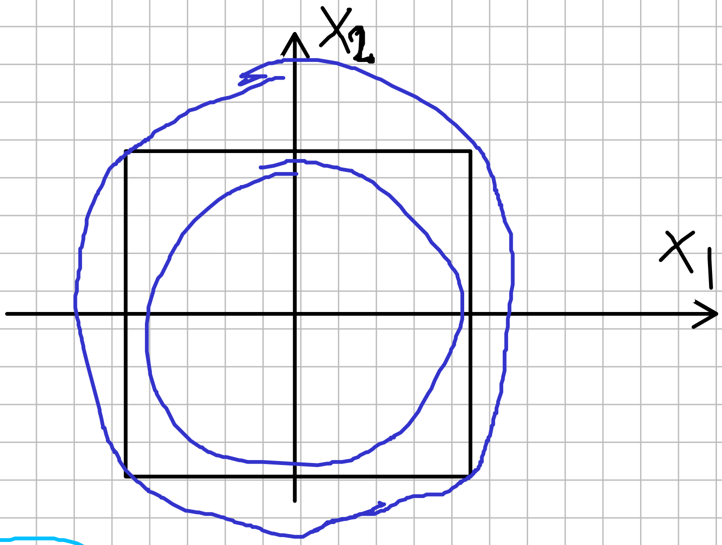
$\|x\|^2$ $\quad e=2$

$$m_1\|x\|^e \le \|x\|^2 \le M_1\|x\|^e \qquad x\in I$$

$\|x\| \to e=1$

$$\boxed{\|x(t)\| \le \left(\frac{K_2}{K_1}\right)^{1/e}\|x(0)\|\, e^{-\frac{K_3 t}{K_2 e}}}$$

$$c_1\|x\|_p \le \|x\|_q \le c_2\|x\|_p$$



$$\leq c_2 x_1^2$$

$$\boxed{\begin{matrix} x' x \cdot \lambda_{min}(P) \\ \|x\|^2 \end{matrix}} \leq V(x) = \underline{x' P x} + \int_0^{x_1} h(y)\,dy \leq x' P_1 x$$

$$h(y)dy \qquad \leq c_3 \|x\|^2$$

$$P_1 = \begin{bmatrix} P_{11} + c_2 & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \qquad c_1 y^2 \leq h(y) y$$

$$y\,dy$$

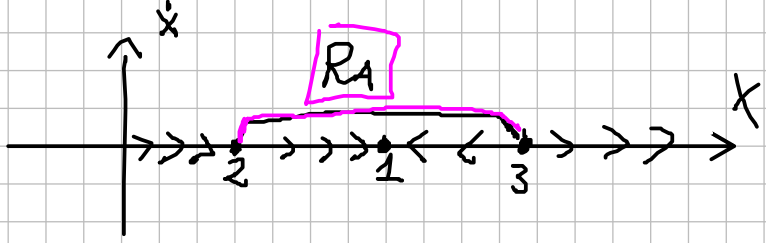
$$y^2/2$$

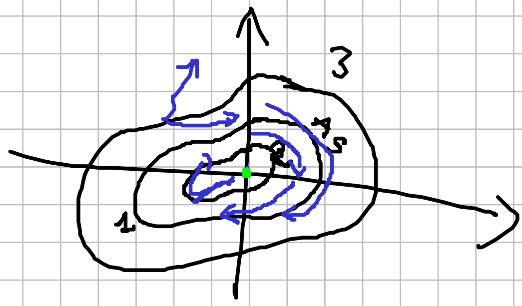
$$\|x\|^2 \lambda_{min}(P) \leq V(x) \leq \|x\|^2 \lambda_{max}(P_1)$$

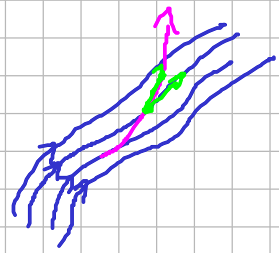
$$\dot{V} \leq -c_1 x_1^2 - x_2^2 \overset{?}{\leq} -K_3 \|x\|^2$$

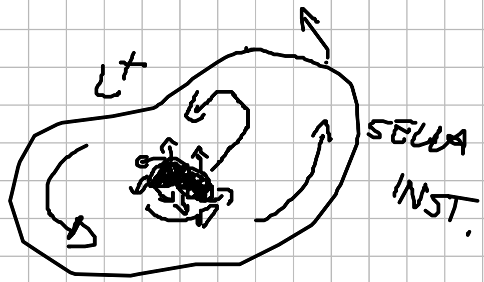
$$\|x\|^2 = x_1^2 + x_2^2 \qquad K_3 \triangleq \min\{c_1, 1\} \qquad -K_3 x_1^2 - K_3 x_2^2$$

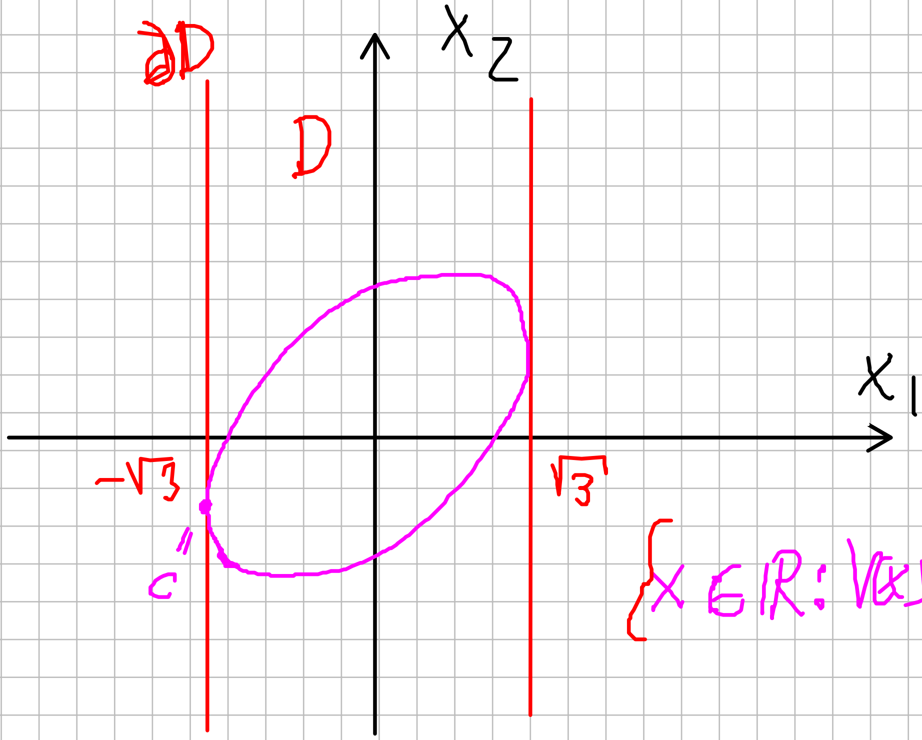
$$\leq -100 x_1^2 - 0.1 x_2^2 \leq (-x_1^2 - x_2^2)\, 0.1$$











$\{x \in \mathbb{R} : V(x) < c\} = \Omega_c$