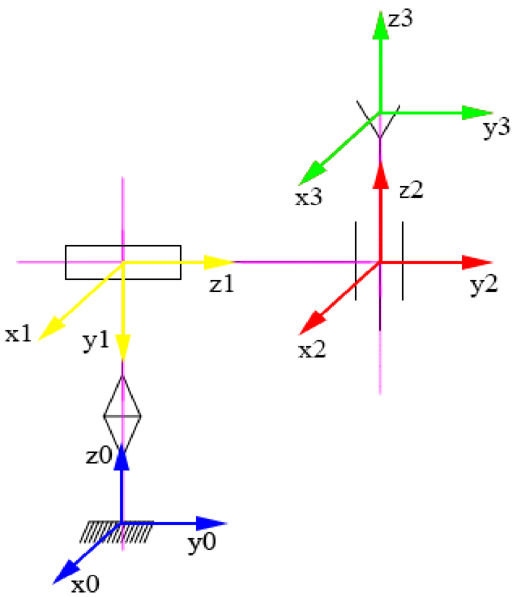
Cinematica diretta Robot Stanford

***N.B.:* le grandezze diverse da quelle di giunto q_i sono L_i, D_i. Esse sono rispettivamente la distanza tra i sistemi di riferimento R_i e R_{i+1} nelle operazioni della matrice avvitamento $A_z(\theta, d)$ e $A_x(\alpha, a)$.**



	ϑ	d	α	a
1	q_1	L_1	$-\frac{\pi}{2}$	0
2	q_2	L_2	$\frac{\pi}{2}$	0
3	$-\frac{\pi}{2}$	q_3	0	0

Tabella 1.

Funzioni ausiliarie:

```
(%i1) inverseLaplace(SI,theta):=block([res],
                              M:SI,
                              MC:SI,
                              for i:1 thru 3 do(
                                for j:1 thru 3 do
                                    (
                                      aC:M[i,j],
                                      b:ilt(aC,s,theta),
                                      MC[i,j]:b
                                     )
                                  ),
                              res:MC

                           )
```

(%o1) inverseLaplace(SI, ϑ) := **block** ([res], M: SI, MC: SI, **for** i **thru** 3 **do for** j **thru** 3 **do** (aC: $M_{i,j}$, b: ilt(aC, s, ϑ), MC$_{i,j}$: b), res: MC)

```
(%i2) rotLaplace(k,theta):=block([res],
                          S:ident(3),
                          I:ident(3),
                        for i:1 thru 3 do
                          (
                          for j:1 thru 3 do
                              (
                                if i=j
                                   then S[i][j]:0
                                elseif j>i
                                   then (
                                  temp:(-1)^(j-i)*k[3-remainder(i+j,3)],
                                         S[i][j]:temp,
                                         S[j][i]:-temp

                                          )
                                )
                           ),
                          res:inverseLaplace(invert(s*I-S),theta)


                         )
```

(%o2) $\mathrm{rotLaplace}(k,\vartheta) := \textbf{block}\ ([\mathrm{res}], S\colon \mathrm{ident}(3), I\colon \mathrm{ident}(3),$ $\textbf{for}\ i\ \textbf{thru}\ 3\ \textbf{do}\ \textbf{for}\ j\ \textbf{thru}\ 3\ \textbf{do}\ \textbf{if}\ i=j\ \textbf{then}\ (S_i)_j\colon 0\ \textbf{elseif}\ j>i\ \textbf{then}\ (\mathrm{temp}\colon$ $(-1)^{j-i}\,k_{3-\mathrm{remainder}(i+j,3)}, (S_i)_j\colon \mathrm{temp}, (S_j)_i\colon -\mathrm{temp}), \mathrm{res}\colon \mathrm{inverseLaplace}(\mathrm{invert}(s\,I-S),\vartheta))$

```
(%i3) Av(v,theta,d):=block([res],
                          Trot:rotLaplace(v,theta),
                          row:matrix([0,0,0,1]),
                          Atemp:addcol(Trot,d*transpose(v)),
                          A:addrow(Atemp,row),
                          res:A
                          )
```

(%o3) $\mathrm{Av}(v,\vartheta,d) := \textbf{block}\ ([\mathrm{res}], \mathrm{Trot}\colon \mathrm{rotLaplace}(v,\vartheta), \mathrm{row}\colon \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{Atemp}\colon \mathrm{addcol}(\mathrm{Trot},$ $d\,\mathrm{transpose}(v)), A\colon \mathrm{addrow}(\mathrm{Atemp},\mathrm{row}), \mathrm{res}\colon A)$

```
(%i4) Q(theta,d,alpha,a):=block([res],
                              tempMat:Av([0,0,1],theta,d).Av([1,0,0],alpha,a),
                              Qtrasf:zeromatrix(4,4),
                              for i:1 thru 4 do
                          (
                          for j:1 thru 4 do
                              (
                                Qtrasf[i][j]:trigreduce(tempMat[i][j])
                                          )
                              ),

                                res:Qtrasf
                                )
```

(%o4) $Q(\vartheta,d,\alpha,a) := \textbf{block}\ ([\mathrm{res}], \mathrm{tempMat}\colon \mathrm{Av}([0,0,1],\vartheta,d)\cdot\mathrm{Av}([1,0,0],\alpha,a), \mathrm{Qtrasf}\colon$ $\mathrm{zeromatrix}(4,4), \textbf{for}\ i\ \textbf{thru}\ 4\ \textbf{do}\ \textbf{for}\ j\ \textbf{thru}\ 4\ \textbf{do}\ (\mathrm{Qtrasf}_i)_j\colon \mathrm{trigreduce}((\mathrm{tempMat}_i)_j), \mathrm{res}\colon \mathrm{Qtrasf})$

```
(%i5) let(sin(q[1]), s[1]);
```

(%o5) $\sin(q_1) \longrightarrow s_1$

```
(%i6) let(sin(q[2]), s[2]);
```
(%o6) $\sin(q_2) \longrightarrow s_2$

```
(%i7) let(cos(q[1]),c[1]);
```
(%o7) $\cos(q_1) \longrightarrow c_1$

```
(%i8) let(cos(q[2]),c[2]);
```
(%o8) $\cos(q_2) \longrightarrow c_2$

```
(%i9) let(sin(q[1]+q[2]), s[12]);
```
(%o9) $\sin(q_2 + q_1) \longrightarrow s_{12}$

```
(%i10) let(cos(q[1]+q[2]),c[12]);
```
(%o10) $\cos(q_2 + q_1) \longrightarrow c_{12}$

```
(%i11) let(sin(q[2]+q[3]),s[23]);
```
(%o11) $\sin(q_3 + q_2) \longrightarrow s_{23}$

```
(%i12) let(cos(q[2]+q[3]),c[23]);
```
(%o12) $\cos(q_3 + q_2) \longrightarrow c_{23}$

```
(%i13) let(sin(q[1]+q[3]),s[23]);
```
(%o13) $\sin(q_3 + q_1) \longrightarrow s_{23}$

```
(%i14) let(cos(q[1]+q[3]),c[13]);
```
(%o14) $\cos(q_3 + q_1) \longrightarrow c_{13}$

```
(%i15) let(sin(q[3]),s[3]);
```
(%o15) $\sin(q_3) \longrightarrow s_3$

```
(%i16) let(cos(q[3]),q[3]);
```
(%o16) $\cos(q_3) \longrightarrow q_3$

```
(%i17)
```

Cinematica diretta:

```
(%i17) Q[stanford](q1,q2,q3,L1,L2):=
                                   Q(q1,L1,-%pi/2,0).
                                   Q(q2,L2,%pi/2,0).
                                   Q(-%pi/2,q3,0,0)
                                   ;
```
(%o17) $Q_{\mathrm{stanford}}(q1, q2, q3, L1, L2) := Q\left(q1, L1, \frac{-\pi}{2}, 0\right) \cdot Q\left(q2, L2, \frac{\pi}{2}, 0\right) \cdot Q\left(\frac{-\pi}{2}, q3, 0, 0\right)$

```
(%i18) Qstanford:Q[stanford](q[1],q[2],q[3],L[1],L[2]);
```

(%o18) $$\begin{pmatrix} \sin(q_1) & \cos(q_1)\cos(q_2) & \cos(q_1)\sin(q_2) & q_3\cos(q_1)\sin(q_2) - L_2\sin(q_1) \\ -\cos(q_1) & \sin(q_1)\cos(q_2) & \sin(q_1)\sin(q_2) & q_3\sin(q_1)\sin(q_2) + L_2\cos(q_1) \\ 0 & -\sin(q_2) & \cos(q_2) & q_3\cos(q_2) + L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
(%i19) letsimp(Qstanford);
```

$$\text{(\%o19)} \quad \begin{pmatrix} s_1 & c_1\,c_2 & c_1\,s_2 & c_1\,s_2\,q_3 - s_1\,L_2 \\ -c_1 & s_1\,c_2 & s_1\,s_2 & s_1\,s_2\,q_3 + c_1\,L_2 \\ 0 & -s_2 & c_2 & c_2\,q_3 + L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
(%i20)
```

Cinematica inversa robot Stanford

Al fine di risolvere il problema di cinematica inversa del robot sferico di II tipo (Stanford) occorre risolevere il problema di posizione ed orientamente inverso.Inizialmente individuare lo spazio di lavoro, le soluzioni generiche, singolari ed infine le variabili di giunto q_i ed in seguito determinare l'orientamento del robot.

Dalla cinematica diretta sappiamo che:

$$Q_{\text{Stanford}} = \begin{pmatrix} s_1 & c_1\,c_2 & c_1\,s_2 & c_1\,s_2\,q_3 - s_1\,L_2 \\ -c_1 & s_1\,c_2 & s_1\,s_2 & s_1\,s_2\,q_3 + c_1\,L_2 \\ 0 & -s_2 & c_2 & c_2\,q_3 + L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Cinematica inversa di posizione

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c_1\,s_2\,q_3 - s_1\,L_2 \\ s_1\,s_2\,q_3 + c_1\,L_2 \\ c_2\,q_3 + L_1 \end{pmatrix}$$

$$\begin{cases} x = c_1\,s_2\,q_3 - s_1\,L_2 \\ y = s_1\,s_2\,q_3 + c_1\,L_2 \end{cases}$$

$$x^2 + y^2 = c_1^2 s_2^2 q_3^2 + s_1^2 L_2^2 - 2 c_1 s_2 q_3 s_1 c_1 + s_1^2 s_2^2 q_3^2 + c_1^2 L_2^2 + 2 s_1\,s_2\,q_3 c_1\,L_2$$

$$\begin{cases} x^2 + y^2 = s_2^2 q_3^2 + L_2^2 \\ (z - L_1)^2 = c_2^2 q_3^2 \end{cases} \rightarrow \quad x^2 + y^2 + (z - L_1)^2 = q_3^2 + L_2^2$$

L'equazione $x^2 + y^2 + (z - L_1)^2 - L_2^2 = q_3^2$ descrive lo spazio operativo del robot Stanford. In particolare, rappresenta una sfera cava di centro $\begin{pmatrix} 0 \\ 0 \\ L_1 \end{pmatrix}$ e raggio r$=\sqrt{L_2^2 + q_3^3}$. Il raggio interno si ottiene con $q_3 = 0 \rightarrow r = L_2$ e il raggio esterno con $q_3 = \pm\infty \rightarrow r = +\infty$.

Per determinare lo spazio operativo, occorre determinare i punti di singolarità e le soluzioni generiche:

$$q_3 = \pm\sqrt{x^2 + y^2 + (z - L_1)^2 - L_2^2}$$

Si ha una singolarità se: $x^2 + y^2 + (z - L_1)^2 - L_2^2 = 0$.

Inoltre, la variabile di giunto $q_3 \neq 0$. Quindi:

$$c_2 = \frac{z - L_1}{q_3}$$

$$s_2 = \pm\sqrt{\frac{x^2 + y^2 - L_2^2}{q_3^2}}$$

Affinche s_2 sia definito deve essere valida la relazione $x^2 + y^2 \geqslant L_2^2$. Quindi si hanno due soluzioni generiche ed una singolarità per $x^2 + y^2 = L_2^2$ che ci permette di definire lo spazio operativo finale. Esso è una sfera attraversata da un cilindro cavo di raggio L_2.

In aggiunta:

$$q_2 = \text{atan2}\left(\pm\sqrt{\frac{x^2 + y^2 - L_2^2}{q_3^2}}, \frac{z - L_1}{q_3}\right)$$

Al fine di determinare l'espressione della variabile di giunto q_1, si impone:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 \\ s_1 & c_1 \end{pmatrix} \begin{pmatrix} q_3 s_2 \\ L_2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} q_3 s_2 & -L_2 \\ L_2 & q_3 s_2 \end{pmatrix} \begin{pmatrix} c_1 \\ s_1 \end{pmatrix}$$

Poiché $\det\begin{pmatrix} q_3 s_2 & -L_2 \\ L_2 & q_3 s_2 \end{pmatrix} = q_3^2 s_2^2 + L_2^2 \neq 0$, è possibile effettuare l'inversa ed ottenere:

$$\begin{pmatrix} c_1 \\ s_1 \end{pmatrix} = \frac{1}{q_3^2 s_2^2 + L_2^2} \begin{pmatrix} q_3 s_2 & L_2 \\ -L_2 & q_3 s_2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{cases} c_1 = \frac{q_3 s_2 x + L_2 y}{q_3^2 s_2^2 + L_2^2} \\ s_1 = \frac{-L_2 x + q_3 s_2 y}{q_3^2 s_2^2 + L_2^2} \end{cases}$$

$$q_1 = \text{atan2}\left(\frac{-L_2 x + q_3 s_2 y}{q_3^2 s_2^2 + L_2^2}, \frac{q_3 s_2 x + L_2 y}{q_3^2 s_2^2 + L_2^2}\right) = \text{atan2}(-L_2 x + q_3 s_2 y, q_3 s_2 x + L_2 y)$$

Orientamento inverso

La risoluzione del problema di orientamento inverso si basa sulla scelta di una terna di Eulero o nautica in condizione non singolari, se possibile.

$$R_{\text{Stanford}} = \begin{pmatrix} s_1 & c_1\,c_2 & c_1\,s_2 \\ -c_1 & s_1\,c_2 & s_1\,s_2 \\ 0 & -s_2 & c_2 \end{pmatrix}$$

$$R_{\text{zyx}} = \begin{pmatrix} c_y c_z & \cdots\cdots\cdots & \\ c_y s_z & \cdots\cdots\cdots & \\ -s_y & s_x c_y & c_x c_y \end{pmatrix}$$

$$s_y = 0 \rightarrow c_y = \pm 1 \qquad \rightarrow \phi_y = \text{atan2}(0, \pm 1) = \begin{cases} 0 \\ \pi \end{cases}$$

$$\begin{cases} s_x\,c_y = -s_2 \\ c_x\,c_y = c_2 \end{cases} \rightarrow \begin{cases} \pm s_x = s_2 \\ \pm c_{\text{yx}} = c_2 \end{cases} \rightarrow \begin{cases} s_{\text{yx}} = \mp s_2 \\ c_x = \pm c_2 \end{cases} \rightarrow \phi_y = \text{atan2}(\mp s_2, \pm c_2) = \begin{cases} -q_2 \\ -q_2 + \pi \end{cases}$$

$$\begin{cases} c_y\,c_z = s_1 \\ c_y\,s_z = -c_1 \end{cases} \rightarrow \begin{cases} \pm c_z = s_1 \\ \pm s_z = -c_1 \end{cases} \rightarrow \begin{cases} c_z = \pm s_1 \\ s_z = \mp c_1 \end{cases} \rightarrow \phi_z = \text{atan2}(\mp c_1, \pm c_1) = \begin{cases} q_1 - \frac{\pi}{2} \\ q_1 + \frac{\pi}{2} \end{cases}$$

Riassumendo:

$$\begin{pmatrix} -q_2 \\ 0 \\ q_1 - \frac{\pi}{2} \end{pmatrix}, \begin{pmatrix} -q_2 + \pi \\ \pi \\ q_1 + \frac{\pi}{2} \end{pmatrix}$$

In alternativa, tramite una scelta di una terna di Eulero:

$$R_{\text{zyz}} = \begin{pmatrix} \cdots & \cdots & \cos(\alpha)\sin(\beta) \\ \cdots & \cdots & \sin(\alpha)\sin(\beta) \\ -\sin(\beta)\cos(\gamma) & \sin(\beta)\sin(\gamma) & \cos(\beta) \end{pmatrix}$$

$$\cos(\beta) = c_2 \neq \pm 1 \rightarrow q_2 \neq \begin{cases} 0 \\ \pi \end{cases}$$

Supponiamo che $q_2 \neq \begin{cases} 0 \\ \pi \end{cases} \rightarrow \cos(\beta) \neq \pm 1 \rightarrow \sin(\beta) \neq 0$:

$$\sin(\beta) = \pm\sqrt{1 - \cos(\beta)^2} = \pm\sqrt{1 - \cos(q_2)^2} = \pm\sin(q_2)$$

$$\beta = \text{atan2}(\pm\sin(q_2), \cos(q_2)) = \begin{cases} q_2 \\ -q_2 \end{cases}$$

$$\begin{cases} \sin(\beta)\sin(\gamma) = 0 \\ -\sin(\beta)\cos(\gamma) = -s_2 \end{cases} \rightarrow \begin{cases} \sin(\gamma) = 0 \\ \cos(\gamma) = \pm 1 \end{cases} \rightarrow \gamma = \text{atan2}(0, \pm 1) = \begin{cases} 0 \\ \pi \end{cases}$$

$$\begin{cases} \cos(\alpha)\sin(\beta) = c_1 s_2 \\ \sin(\alpha)\sin(\beta) = s_1 s_2 \end{cases} \rightarrow \begin{cases} \cos(\alpha) = \pm c_1 \\ \sin(\alpha) = \pm s_1 \end{cases} \rightarrow \alpha = \text{atan2}(\pm s_1, \pm c_1) = \begin{cases} q_1 \\ q_1 + \pi \end{cases}$$

Riassumendo:

$$\begin{pmatrix} q_1 \\ q_2 \\ 0 \end{pmatrix}, \begin{pmatrix} q_1 + \pi \\ -q_2 \\ \pi \end{pmatrix}$$

```
(%i20) isRotation(M):=block([MC,res],
                         I:ident(3),
                         MC:ident(3),
                         for i:1 thru 3 do
                         (
                         for j:1 thru 3 do
                            (
                               MC[i][j]:M[i][j]
                               )
                            ),
                         MMT:trigsimp(expand(MC.transpose(MC))),
                         detM:trigsimp(expand(determinant(MC))),

                         if MMT=I  and detM=1
                            then(

                                 return(res:1)
                                 )

                         else(

                               res: "R is not rotation matrix"
                               )
                         )
```

(%o20) $\mathrm{isRotation}(M) := \mathbf{block}\ ([\mathrm{MC}, \mathrm{res}], I\colon \mathrm{ident}(3), \mathrm{MC}\colon \mathrm{ident}(3),$
$\mathbf{for}\ i\ \mathbf{thru}\ 3\ \mathbf{do}\ \mathbf{for}\ j\ \mathbf{thru}\ 3\ \mathbf{do}\ (\mathrm{MC}_i)_j\colon (M_i)_j, \mathrm{MMT}\colon \mathrm{trigsimp}(\mathrm{expand}(\mathrm{MC}\cdot \mathrm{transpose}(\mathrm{MC}))),$
$\mathrm{detM}\colon \mathrm{trigsimp}(\mathrm{expand}(\mathrm{determinant}(\mathrm{MC}))), \mathbf{if}\ \mathrm{MMT} = I \wedge \mathrm{detM} = 1\ \mathbf{then}\ \mathrm{return}(\mathrm{res}\colon 1)\ \mathbf{else}\ \mathrm{res}\colon \mathrm{R}$
is not rotation matrix)

$-L_2x + q_3s_2y, q_3s_2x + L_2y$

```
(%i27) calculate(x,y,L1,L2,z):=block(
       [q3plus,q3minus,q2plus,q2minus,q1plus,q1minus,res],
       if x^(2)+y^(2)+(z-L1)^2=L2^(2) then print("La soluzione è singolare")
       else
       (
       q3value:sqrt(trigreduce(trigexpand(ratsimp(x^(2)+y^(2)+(z-L1)^(2)-
       L2^(2))))),
       q3plus:trigreduce(trigexpand(ratsimp(q3value))),
       q3minus:-trigreduce(trigexpand(ratsimp(q3value))),
       s2plus:trigreduce(trigexpand(ratsimp(sqrt((x^(2)+y^(2)-L2^(2))/
       q3value^2)))),
       s2minus:-s2plus,
       c2plus:trigreduce(trigexpand(ratsimp((z-L1)/q3plus))),
       c2minus:trigreduce(trigexpand(ratsimp((z-L1)/q3minus))),
       q2plus:atan2(s2plus,c2plus),
       q2minus:atan2(s2minus,c2minus),
       s1plus:trigreduce(trigexpand(ratsimp(-L2*x+q3plus*s2plus*y))),
       s1minus:trigreduce(trigexpand(ratsimp(-L2*x+q3minus*s2minus*y))),
       c1plus:trigreduce(trigexpand(ratsimp(q3plus*s2plus*x+L2*y))),
       c1minus:trigreduce(trigexpand(ratsimp(q3minus*s2minus*x+L2*y))),
       q1plus:atan2(s1plus,c1plus),
       q1minus:atan2(s1minus,c1minus),
       res:[[q1plus,q2plus,q3plus],[q1minus,q2minus,q3minus]]
       )
       )
```

(%o27) $\mathrm{calculate}(x, y, L1, L2, z) := \textbf{block}\Bigg([q3\mathrm{plus}, q3\mathrm{minus}, q2\mathrm{plus}, q2\mathrm{minus}, q1\mathrm{plus}, q1\mathrm{minus}, \mathrm{res}], \textbf{if } x^2+y^2+(z-L1)^2 = L2^2 \textbf{ then } \mathrm{print}(\text{La soluzione è singolare }) \textbf{ else } \Bigg(q3\mathrm{value}: \sqrt{\mathrm{trigreduce}(\mathrm{trigexpand}(\mathrm{ratsimp}(x^2+y^2+(z-L1)^2-L2^2)))}, q3\mathrm{plus}: \mathrm{trigreduce}(\mathrm{trigexpand}(\mathrm{ratsimp}(q3\mathrm{value}))), q3\mathrm{minus}: -\mathrm{trigreduce}(\mathrm{trigexpand}(\mathrm{ratsimp}(q3\mathrm{value}))), s2\mathrm{plus}: \mathrm{trigreduce}\left(\mathrm{trigexpand}\left(\mathrm{ratsimp}\left(\sqrt{\frac{x^2+y^2-L2^2}{q3\mathrm{value}^2}}\right)\right)\right), s2\mathrm{minus}: -s2\mathrm{plus}, c2\mathrm{plus}: \mathrm{trigreduce}\left(\mathrm{trigexpand}\left(\mathrm{ratsimp}\left(\frac{z-L1}{q3\mathrm{plus}}\right)\right)\right), c2\mathrm{minus}: \mathrm{trigreduce}\left(\mathrm{trigexpand}\left(\mathrm{ratsimp}\left(\frac{z-L1}{q3\mathrm{minus}}\right)\right)\right), q2\mathrm{plus}: \mathrm{atan2}(s2\mathrm{plus}, c2\mathrm{plus}), q2\mathrm{minus}: \mathrm{atan2}(s2\mathrm{minus}, c2\mathrm{minus}), s1\mathrm{plus}: \mathrm{trigreduce}(\mathrm{trigexpand}(\mathrm{ratsimp}((-L2)\,x + q3\mathrm{plus}\, s2\mathrm{plus}\, y))), s1\mathrm{minus}: \mathrm{trigreduce}(\mathrm{trigexpand}(\mathrm{ratsimp}((-L2)\,x + q3\mathrm{minus}\, s2\mathrm{minus}\, y))), c1\mathrm{plus}: \mathrm{trigreduce}(\mathrm{trigexpand}(\mathrm{ratsimp}(q3\mathrm{plus}\, s2\mathrm{plus}\, x + L2\, y))), c1\mathrm{minus}: \mathrm{trigreduce}(\mathrm{trigexpand}(\mathrm{ratsimp}(q3\mathrm{minus}\, s2\mathrm{minus}\, x + L2\, y))), q1\mathrm{plus}: \mathrm{atan2}(s1\mathrm{plus}, c1\mathrm{plus}), q1\mathrm{minus}: \mathrm{atan2}(s1\mathrm{minus}, c1\mathrm{minus}), \mathrm{res}: [[q1\mathrm{plus}, q2\mathrm{plus}, q3\mathrm{plus}], [q1\mathrm{minus}, q2\mathrm{minus}, q3\mathrm{minus}]]\Bigg)\Bigg)$

(%i77)
```
orientation(Qdiretta):=block([sx,cx,sy,cy,phiy1,phiy2,phiz1,phiz2,phix1,
phix2,sz,sxfirst,second,res],

                                    rotation:isRotation(Qdiretta),
                                    if rotation=1 then(
                                    sy:Qdiretta[3][1],

                                    if sy=1 or sy=-1 then "soluzione singolare"
                                    else(
                                    cy:sqrt(1-sy^2),
                                    phiy1:atan2(-sy,cy),
                                    phiy2:atan2(-sy,-cy),

                                    sx:Qdiretta[3][2]/cy,
                                    cx:Qdiretta[3][3]/cy,
                                    phix1:atan2(sx,cx),
                                    phix2:phix1+%pi,
                                    cz:Qdiretta[1][1]/cy,
                                    sz:Qdiretta[2][1]/cy,
                                    phiz1:atan2(sz,cz)-%pi/2,
                                    print(phiz1),
                                    phiz2:phiz1+(%pi/2),
                                    first:[phix1,phiy1,phiz1],
                                    second:[phix2,phiy2,phiz2],

                                    res:[first,second])
                                    )
                                    else res:rotation

);
```

(%o77) $\mathrm{orientation}(\mathrm{Qdiretta}) := \textbf{block}\Big([\mathrm{sx}, \mathrm{cx}, \mathrm{sy}, \mathrm{cy}, \mathrm{phiy1}, \mathrm{phiy2}, \mathrm{phiz1}, \mathrm{phiz2}, \mathrm{phix1}, \mathrm{phix2}, \mathrm{sz}, \mathrm{sxfirst}, \mathrm{second}, \mathrm{res}], \mathrm{rotation}: \mathrm{isRotation}(\mathrm{Qdiretta}), \textbf{if } \mathrm{rotation} = 1 \textbf{ then } \Big(\mathrm{sy}: (\mathrm{Qdiretta}_3)_1, \textbf{if } \mathrm{sy} =$

$1 \vee \mathrm{sy} = -1$ **then** soluzione singolare **else** $\Big($cy: $\sqrt{1-\mathrm{sy}^2}$, phiy1: atan2($-$sy, cy), phiy2: atan2($-$sy, $-$cy), sx: $\dfrac{(\mathrm{Qdiretta}_3)_2}{\mathrm{cy}}$, cx: $\dfrac{(\mathrm{Qdiretta}_3)_3}{\mathrm{cy}}$, phix1: atan2(sx, cx), phix2: phix1 $+\,\pi$, cz: $\dfrac{(\mathrm{Qdiretta}_1)_1}{\mathrm{cy}}$, sz: $\dfrac{(\mathrm{Qdiretta}_2)_1}{\mathrm{cy}}$, phiz1: atan2(sz, cz) $-\dfrac{\pi}{2}$, print(phiz1), phiz2: phiz1 $+\dfrac{\pi}{2}$, first: [phix1, phiy1, phiz1], second: [phix2, phiy2, phiz2], res: [first, second]$\Big)\Big)$ **else** res: rotation$\Big)$

```
(%i78) invStanford(Qdiretta,L1,L2):=block(
                              [x,y,z,pos1,pos2,orien1,orien2,res],
                              x:Qdiretta[1][4],
                                   y:Qdiretta[2][4],
                                   z:Qdiretta[3][4],
                                   pos:calculate(x,y,L1,L2,z),
                                   orien:orientation(Qdiretta),
                                   orien1:orien[1],
                                   orien2:orien[2],
                                   pos1:pos[1],
                                   pos2:pos[2],
                                   res:[pos1,pos2,orien1,orien2]
                                   );
```

(%o78) invStanford(Qdiretta, $L1$, $L2$) := **block** ([x, y, z, pos1, pos2, orien1, orien2, res], x: $(\mathrm{Qdiretta}_1)_4$, y: $(\mathrm{Qdiretta}_2)_4$, z: $(\mathrm{Qdiretta}_3)_4$, pos: calculate(x, y, $L1$, $L2$, z), orien: orientation(Qdiretta), orien1: orien_1, orien2: orien_2, pos1: pos_1, pos2: pos_2, res: [pos1, pos2, orien1, orien2])

```
(%i79) Qstanford:Q[stanford](%pi/3,%pi/3,5,10,10);
```

(%o79)
$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{4} & \frac{\sqrt{3}}{4} & -\frac{5\,3^{\frac{3}{2}}}{4} \\ -\frac{1}{2} & \frac{\sqrt{3}}{4} & \frac{3}{4} & \frac{35}{4} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{25}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
(%i80) invStanford(Qstanford,10,10);
```

$$-\frac{2\,\pi}{3}$$

(%o80)
$$\left[\left[\frac{\pi}{3}, \frac{\pi}{3}, 5\right], \left[\frac{\pi}{3}, -\frac{2\,\pi}{3}, -5\right], \left[-\frac{\pi}{3}, 0, -\frac{2\,\pi}{3}\right], \left[\frac{2\,\pi}{3}, \pi, -\frac{\pi}{6}\right]\right]$$

```
(%i81) Qstanford:Q[stanford](q[1],q[2],q[3],L[1],L[2]);
```

(%o81)
$$\begin{pmatrix} \sin(q_1) & \cos(q_1)\cos(q_2) & \cos(q_1)\sin(q_2) & q_3\cos(q_1)\sin(q_2) - L_2\sin(q_1) \\ -\cos(q_1) & \sin(q_1)\cos(q_2) & \sin(q_1)\sin(q_2) & q_3\sin(q_1)\sin(q_2) + L_2\cos(q_1) \\ 0 & -\sin(q_2) & \cos(q_2) & q_3\cos(q_2) + L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
(%i39) invStanford(Qstanford,L[1],L[2]);
```

(%o39)
$$\left[\left[-\mathrm{atan2}\left(\frac{L_2\,q_3\sin(q_2+q_1)}{2} + \frac{q_3\sqrt{q_3^2 - q_3^2\cos(2\,q_2)}\cos(q_2+q_1)}{2^{\frac{3}{2}}} + \frac{L_2\,q_3\sin(q_2-q_1)}{2} - \frac{q_3\sqrt{q_3^2 - q_3^2\cos(2\,q_2)}\cos(q_2-q_1)}{2^{\frac{3}{2}}} - \frac{L_2\cos(q_1)\sqrt{q_3^2 - q_3^2\cos(2\,q_2)}}{\sqrt{2}} - L_2^2\sin(q_1),\right.\right.\right.$$

$$\frac{q_3\sqrt{q_3^2-q_3^2\cos(2\,q_2)}\,\sin(q_2+q_1)}{2^{\frac{3}{2}}}-\frac{L_2\,q_3\cos(q_2+q_1)}{2}+\frac{q_3\sqrt{q_3^2-q_3^2\cos(2\,q_2)}\,\sin(q_2-q_1)}{2^{\frac{3}{2}}}+$$
$$\left.\frac{L_2\,q_3\cos(q_2-q_1)}{2}-\frac{L_2\sin(q_1)\sqrt{q_3^2-q_3^2\cos(2\,q_2)}}{\sqrt{2}}+L_2^2\cos(q_1)\right),\operatorname{atan2}\left(\frac{\sqrt{q_3^2-q_3^2\cos(2\,q_2)}}{\sqrt{2}\,|q_3|},\right.$$
$$\left.\left.\frac{q_3\cos(q_2)}{|q_3|}\right),|q_3|\right],\left[-\operatorname{atan2}\left(\frac{L_2\,q_3\sin(q_2+q_1)}{2}+\frac{q_3\sqrt{q_3^2-q_3^2\cos(2\,q_2)}\,\cos(q_2+q_1)}{2^{\frac{3}{2}}}+\right.\right.$$
$$\frac{L_2\,q_3\sin(q_2-q_1)}{2}-\frac{q_3\sqrt{q_3^2-q_3^2\cos(2\,q_2)}\,\cos(q_2-q_1)}{2^{\frac{3}{2}}}-\frac{L_2\cos(q_1)\sqrt{q_3^2-q_3^2\cos(2\,q_2)}}{\sqrt{2}}-$$
$$L_2^2\sin(q_1),\frac{q_3\sqrt{q_3^2-q_3^2\cos(2\,q_2)}\,\sin(q_2+q_1)}{2^{\frac{3}{2}}}-\frac{L_2\,q_3\cos(q_2+q_1)}{2}+$$
$$\frac{q_3\sqrt{q_3^2-q_3^2\cos(2\,q_2)}\,\sin(q_2-q_1)}{2^{\frac{3}{2}}}+\frac{L_2\,q_3\cos(q_2-q_1)}{2}-\frac{L_2\sin(q_1)\sqrt{q_3^2-q_3^2\cos(2\,q_2)}}{\sqrt{2}}+$$
$$\left.\left.L_2^2\cos(q_1)\right),-\operatorname{atan2}\left(\frac{\sqrt{q_3^2-q_3^2\cos(2\,q_2)}}{\sqrt{2}\,|q_3|},-\frac{q_3\cos(q_2)}{|q_3|}\right),-|q_3|\right],[\operatorname{atan2}(\sin(q_2),\cos(q_2)),0,$$
$$\left.-\operatorname{atan2}(\sin(q_1),-\cos(q_1))],[-\operatorname{atan2}(\sin(q_2),-\cos(q_2)),\pi,\operatorname{atan2}(\sin(q_1),\cos(q_1))]\right]$$

(%i40)

Singolarità di velocità

```
Maxima 5.44.0 http://maxima.sourceforge.net
using Lisp SBCL 2.0.0
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
```

(%i1)

(%i1) `x:q[3]*cos (q[1])*sin (q[2])-L[2]*sin (q[1]);`

(%o1) $q_3\cos(q_1)\sin(q_2)-L_2\sin(q_1)$

(%i2) `y:q[3]*sin (q[1])*sin (q[2])+L[2]*cos (q[1]);`

(%o2) $q_3\sin(q_1)\sin(q_2)+L_2\cos(q_1)$

(%i3) `z:q[3]*cos (q[2])+L[1];`

(%o3) $q_3\cos(q_2)+L_1$

(%i23)
```
J:J:matrix([diff(x,q[1]),diff(x,q[2]),diff(x,q[3])],
           [diff(y,q[1]),diff(y,q[2]),diff(y,q[3])],
           [diff(z,q[1]),diff(z,q[2]),diff(z,q[3])]);
```

(%o23) $$\begin{pmatrix} -q_3\sin(q_1)\sin(q_2)-L_2\cos(q_1) & q_3\cos(q_1)\cos(q_2) & \cos(q_1)\sin(q_2) \\ q_3\cos(q_1)\sin(q_2)-L_2\sin(q_1) & q_3\sin(q_1)\cos(q_2) & \sin(q_1)\sin(q_2) \\ 0 & -q_3\sin(q_2) & \cos(q_2) \end{pmatrix}$$

```
(%i5) dJ:trigsimp(expand(determinant(J)));
```

(%o5) $-q_3^2\sin(q_2)$

Si hanno singolarità per:

$$q_3=0 \vee q_2=0$$

Caso $q_3=0$:

```
(%i6) Jq3:subst(q[3]=0,J);
```

(%o6) $\begin{pmatrix} -L_2\cos(q_1) & 0 & \cos(q_1)\sin(q_2) \\ -L_2\sin(q_1) & 0 & \sin(q_1)\sin(q_2) \\ 0 & 0 & \cos(q_2) \end{pmatrix}$

```
(%i7) nullspace(Jq3)
```

Proviso: $\text{notequal}(-L_2\cos(q_1),0) \wedge \text{notequal}(-L_2\cos(q_1)\cos(q_2),0)$

(%o7) $\text{span}\left(\begin{pmatrix} 0 \\ -L_2\cos(q_1)\cos(q_2) \\ 0 \end{pmatrix}\right)$

$\ker(J(q_3=0)) = \Im m\left\{\begin{pmatrix} 0 \\ -L_2\cos(q_1)\cos(q_2) \\ 0 \end{pmatrix}\right\} \to v = k\begin{pmatrix} 0 \\ -L_2\cos(q_1)\cos(q_2) \\ 0 \end{pmatrix} \forall k,\, q_1,\, q2 \neq \begin{cases} \frac{\pi}{2} \\ \frac{3\pi}{2} \end{cases} \to w = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Se $q_1=\frac{\pi}{2} \wedge q_2 \neq \frac{\pi}{2}$:

```
(%i8) Jq31:subst(q[1]=%pi/2,Jq3);
```

(%o8) $\begin{pmatrix} 0 & 0 & 0 \\ -L_2 & 0 & \sin(q_2) \\ 0 & 0 & \cos(q_2) \end{pmatrix}$

```
(%i9) nullspace(Jq31);
```

Proviso: $\text{notequal}(-L_2,0) \wedge \text{notequal}(-L_2\cos(q_2),0)$

(%o9) $\text{span}\left(\begin{pmatrix} 0 \\ -L_2\cos(q_2) \\ 0 \end{pmatrix}\right)$

Si hanno singolarità di velocità se $v \in \text{Im}\left\{\begin{pmatrix} 0 \\ -L_2\cos(q_2) \\ 0 \end{pmatrix}\right\}$.

Se $q_1 \neq \frac{\pi}{2} \wedge q_2=\frac{\pi}{2}$:

```
(%i10) Jq32:subst(q[2]=%pi/2,Jq3);
```

(%o10) $\begin{pmatrix} -L_2\cos(q_1) & 0 & \cos(q_1) \\ -L_2\sin(q_1) & 0 & \sin(q_1) \\ 0 & 0 & 0 \end{pmatrix}$

```
(%i11) nullspace(Jq32);
```

Proviso: $\text{notequal}(\cos(q_1),0)$

(%o11) $\text{span}\left(\begin{pmatrix} 0 \\ \cos(q_1) \\ 0 \end{pmatrix}, \begin{pmatrix} \cos(q_1) \\ 0 \\ L_2\cos(q_1) \end{pmatrix}\right)$

Si hanno singolarità di velocità se $v \in \text{Im}\left\{\begin{pmatrix} 0 \\ \cos(q_1) \\ 0 \end{pmatrix}, \begin{pmatrix} \cos(q_1) \\ 0 \\ L_2\cos(q_1) \end{pmatrix}\right\}$.

Se $q_1=\frac{\pi}{2} \wedge q_2=\frac{\pi}{2}$:

(%i12) `Jq32:subst([q[1]=%pi/2,q[2]=%pi/2],Jq3);`

(%o12) $$\begin{pmatrix} 0 & 0 & 0 \\ -L_2 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

(%i13) `nullspace(Jq32);`

(%o13) $$\mathrm{span}\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ L_2 \end{pmatrix}\right)$$

Si hanno singolarità di velocità se $v \in \mathrm{Im}\left\{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ L_2 \end{pmatrix}\right\}$.

Inoltre, se $q_2=0$:

(%i19) `Jq2:subst(q[2]=0,J);`

(%o19) $$\begin{pmatrix} -L_2\cos(q_1) & q_3\cos(q_1) & 0 \\ -L_2\sin(q_1) & q_3\sin(q_1) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(%i20) `nullspace(Jq2);`

Proviso: notequal$(-L_2\cos(q_1), 0) \wedge$ notequal$(-L_2\cos(q_1), 0)$

(%o20) $$\mathrm{span}\left(\begin{pmatrix} -q_3\cos(q_1) \\ -L_2\cos(q_1) \\ 0 \end{pmatrix}\right)$$

$\ker(J(q_2=0)) = \Im m\left\{\begin{pmatrix} -q_3\cos(q_1) \\ -L_2\cos(q_1) \\ 0 \end{pmatrix}\right\} \to v = k\begin{pmatrix} -q_3\cos(q_1) \\ -L_2\cos(q_1) \\ 0 \end{pmatrix} \forall k, q_{3;} \quad q_1 \neq \frac{\pi}{2} \to w = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Se $q_1=\frac{\pi}{2}$:

(%i21) `Jq21:subst(q[1]=%pi/2,Jq2);`

(%o21) $$\begin{pmatrix} 0 & 0 & 0 \\ -L_2 & q_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(%i22) `nullspace(Jq21);`

Proviso: notequal$(-L_2, 0) \wedge$ notequal$(-L_2, 0)$

(%o22) $$\mathrm{span}\left(\begin{pmatrix} -q_3 \\ -L_2 \\ 0 \end{pmatrix}\right)$$

Si hanno singolarità di velocità se $v \in \mathrm{Im}\left\{\begin{pmatrix} -q_3 \\ -L_2 \\ 0 \end{pmatrix}\right\}$.

Singolarità di forza

(%i24) `J:-transpose(J)`

(%o24) $$\begin{pmatrix} q_3\sin(q_1)\sin(q_2)+L_2\cos(q_1) & L_2\sin(q_1)-q_3\cos(q_1)\sin(q_2) & 0 \\ -q_3\cos(q_1)\cos(q_2) & -q_3\sin(q_1)\cos(q_2) & q_3\sin(q_2) \\ -\cos(q_1)\sin(q_2) & -\sin(q_1)\sin(q_2) & -\cos(q_2) \end{pmatrix}$$

(%i26) `dJ:trigsimp(expand(determinant(J)));`

(%o26) $q_3^2\sin(q_2)$

Si hanno singolarità di forza per $q_3=0 \vee q_2=0$.

Se $q_3=0$:

```
(%i27) Jq3:subst(q[3]=0,J);
```

$$\text{(\%o27)}\quad \begin{pmatrix} L_2\cos(q_1) & L_2\sin(q_1) & 0 \\ 0 & 0 & 0 \\ -\cos(q_1)\sin(q_2) & -\sin(q_1)\sin(q_2) & -\cos(q_2) \end{pmatrix}$$

```
(%i28) nullspace(Jq3);
```

Proviso: $\mathrm{notequal}(L_2\cos(q_1),0)\wedge\mathrm{notequal}(-L_2\cos(q_1)\cos(q_2),0)$

$$\text{(\%o28)}\quad \mathrm{span}\left(\begin{pmatrix} L_2\sin(q_1)\cos(q_2) \\ -L_2\cos(q_1)\cos(q_2) \\ 0 \end{pmatrix}\right)$$

Se $q_1=\frac{\pi}{2}\wedge q_2\neq\frac{\pi}{2}$:

```
(%i29) Jq31:subst(q[1]=%pi/2,Jq3);
```

$$\text{(\%o29)}\quad \begin{pmatrix} 0 & L_2 & 0 \\ 0 & 0 & 0 \\ 0 & -\sin(q_2) & -\cos(q_2) \end{pmatrix}$$

```
(%i30) nullspace(Jq31);
```

Proviso: $\mathrm{notequal}(L_2,0)\wedge\mathrm{notequal}(-L_2\cos(q_2),0)$

$$\text{(\%o30)}\quad \mathrm{span}\left(\begin{pmatrix} -L_2\cos(q_2) \\ 0 \\ 0 \end{pmatrix}\right)$$

Si hanno singolarità di forza se $\tau\in\mathrm{Im}\left\{\begin{pmatrix} -L_2\cos(q_2) \\ 0 \\ 0 \end{pmatrix}\right\}$.

Se $q_2=\frac{\pi}{2}\wedge q_1\neq\frac{\pi}{2}$:

```
(%i31) Jq32:subst(q[2]=%pi/2,Jq3);
```

$$\text{(\%o31)}\quad \begin{pmatrix} L_2\cos(q_1) & L_2\sin(q_1) & 0 \\ 0 & 0 & 0 \\ -\cos(q_1) & -\sin(q_1) & 0 \end{pmatrix}$$

```
(%i32) nullspace(Jq32);
```

Proviso: $\mathrm{notequal}(L_2\cos(q_1),0)$

$$\text{(\%o32)}\quad \mathrm{span}\left(\begin{pmatrix} 0 \\ 0 \\ -L_2\sin(q_1) \end{pmatrix},\begin{pmatrix} -L_2\sin(q_1) \\ L_2\cos(q_1) \\ 0 \end{pmatrix}\right)$$

Si hanno singolarità di forza se $\tau\in\mathrm{Im}\left\{\begin{pmatrix} 0 \\ 0 \\ -L_2\sin(q_1) \end{pmatrix},\begin{pmatrix} -L_2\sin(q_1) \\ L_2\cos(q_1) \\ 0 \end{pmatrix}\right\}$.

Se $q_2=\frac{\pi}{2}\wedge q_1=\frac{\pi}{2}$:

```
(%i33) Jq32:subst([q[2]=%pi/2,q[1]=%pi/2],Jq3);
```

$$\text{(\%o33)}\quad \begin{pmatrix} 0 & L_2 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

```
(%i34) nullspace(Jq32);
```

Proviso: $\mathrm{notequal}(L_2,0)$

$$\text{(\%o34)}\quad \mathrm{span}\left(\begin{pmatrix} 0 \\ 0 \\ L_2 \end{pmatrix},\begin{pmatrix} L_2 \\ 0 \\ 0 \end{pmatrix}\right)$$

Si hanno singolarità di forza se $\tau \in \mathrm{Im}\left\{ \begin{pmatrix} 0 \\ 0 \\ L_2 \end{pmatrix}, \begin{pmatrix} L_2 \\ 0 \\ 0 \end{pmatrix} \right\}$

(%i35)