$$R(t) = e^{St}$$

Laplace

Autovalori, Autovettori

$$e^{At}$$

```
Maxima 5.36.1 http://maxima.sourceforge.net
using Lisp SBCL 1.2.10
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Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
STYLE-WARNING: redefining MAXIMA::TEX-MATRIX in DEFUN
```

(%i1) `R:matrix([cos(theta),-sin(theta)],[sin(theta),cos(theta)])`

(%o1) $$\begin{pmatrix} \cos(\vartheta) & -\sin(\vartheta) \\ \sin(\vartheta) & \cos(\vartheta) \end{pmatrix}$$

(%i2) `laplace(1,t,s)`

(%o2) $$\frac{1}{s}$$

(%i3) `laplace(exp(alpha*t),t,s)`

(%o3) $$\frac{1}{s-\alpha}$$

(%i4) `laplace(sin(omega*t),t,s)`

(%o4) $$\frac{\omega}{s^2+\omega^2}$$

(%i5) `laplace(cos(omega*t),t,s)`

(%o5) $$\frac{s}{s^2+\omega^2}$$

(%i6) `ilt(1/s,s,t)`

(%o6) 1

(%i7) `ilt(1/((s-a)*(s-b)*(s-c)),s,t)`

(%o7) $$\frac{\mathrm{e}^{ct}}{c^2+(-b-a)\,c+a\,b}-\frac{\mathrm{e}^{bt}}{(b-a)\,c-b^2+a\,b}+\frac{\mathrm{e}^{at}}{(b-a)\,c-a\,b+a^2}$$

(%i8) `ilt(1/(s^2+1),s,t)`

(%o8) $\sin(t)$

(%i9) `ilt(s/(s^2+1),s,t)`

(%o9) $\cos(t)$

(%i10) `ilt(1/(s^2+s+1),s,t)`

(%o10) $$\frac{2\,\mathrm{e}^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}\,t}{2}\right)}{\sqrt{3}}$$

```
(%i11) ilt(1/((s+1)^2*(s+2)),s,t)
```

(%o11) $t\,\mathrm{e}^{-t}-\mathrm{e}^{-t}+\mathrm{e}^{-2t}$

```
(%i12)
```

```
(%i1) A:matrix([1,1,1],[0,-1,2],[0,0,2])
```

(%o1) $$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$$

```
(%i2) II:matrix([1,0,0],[0,1,0],[0,0,1])
```

(%o2) $$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
(%i3) sA:s*II-A
```

(%o3) $$\begin{pmatrix} s-1 & -1 & -1 \\ 0 & s+1 & -2 \\ 0 & 0 & s-2 \end{pmatrix}$$

```
(%i4) sAi:invert(sA)
```

(%o4) $$\begin{pmatrix} \frac{1}{s-1} & \frac{1}{(s-1)\,(s+1)} & \frac{s+3}{(s-2)\,(s-1)\,(s+1)} \\ 0 & \frac{1}{s+1} & \frac{2}{(s-2)\,(s+1)} \\ 0 & 0 & \frac{1}{s-2} \end{pmatrix}$$

```
(%i5) ilt(sAi[1,1],s,t)
```

(%o5) e^{t}

```
(%i6) matrix([ilt(sAi[1,1],s,t),ilt(sAi[1,2],s,t),ilt(sAi[1,3],s,t)],
             [ilt(sAi[2,1],s,t),ilt(sAi[2,2],s,t),ilt(sAi[2,3],s,t)],
             [ilt(sAi[3,1],s,t),ilt(sAi[3,2],s,t),ilt(sAi[3,3],s,t)])
```

(%o6) $$\begin{pmatrix} \mathrm{e}^{t} & \frac{\mathrm{e}^{t}}{2}-\frac{\mathrm{e}^{-t}}{2} & \frac{5\,\mathrm{e}^{2t}}{3}-2\,\mathrm{e}^{t}+\frac{\mathrm{e}^{-t}}{3} \\ 0 & \mathrm{e}^{-t} & \frac{2\,\mathrm{e}^{2t}}{3}-\frac{2\,\mathrm{e}^{-t}}{3} \\ 0 & 0 & \mathrm{e}^{2t} \end{pmatrix}$$

```
(%i7) sAi
```

(%o7) $$\begin{pmatrix} \frac{1}{s-1} & \frac{1}{(s-1)\,(s+1)} & \frac{s+3}{(s-2)\,(s-1)\,(s+1)} \\ 0 & \frac{1}{s+1} & \frac{2}{(s-2)\,(s+1)} \\ 0 & 0 & \frac{1}{s-2} \end{pmatrix}$$

```
(%i8) sAiC:sAi
```

(%o8) $$\begin{pmatrix} \frac{1}{s-1} & \frac{1}{(s-1)\,(s+1)} & \frac{s+3}{(s-2)\,(s-1)\,(s+1)} \\ 0 & \frac{1}{s+1} & \frac{2}{(s-2)\,(s+1)} \\ 0 & 0 & \frac{1}{s-2} \end{pmatrix}$$

```
(%i9) for i:1 thru 3 do
      for j:1 thru 3 do
      (
      a:sAi[i,j],
      b:ilt(a,s,t),
      sAiC[i,j]:b
      )
```

(%o9) **done**

```
(%i10) sAiC
```

$$(\%o10)\quad \begin{pmatrix} e^{t} & \frac{e^{t}}{2}-\frac{e^{-t}}{2} & \frac{5\,e^{2t}}{3}-2\,e^{t}+\frac{e^{-t}}{3} \\ 0 & e^{-t} & \frac{2\,e^{2t}}{3}-\frac{2\,e^{-t}}{3} \\ 0 & 0 & e^{2t} \end{pmatrix}$$

```
(%i11) matrix([%e^t, ((%e^t)/2)-((%e^(-t))/2), ((5*%e^(2*t))/3)-2*%e^t+((%e^(-t))/
       3)], [0, %e^(-t), ((2*%e^(2*t))/3)-((2*%e^(-t))/3)], [0, 0, %e^(2*t)])
```

$$(\%o11)\quad \begin{pmatrix} e^{t} & \frac{e^{t}}{2}-\frac{e^{-t}}{2} & \frac{5\,e^{2t}}{3}-2\,e^{t}+\frac{e^{-t}}{3} \\ 0 & e^{-t} & \frac{2\,e^{2t}}{3}-\frac{2\,e^{-t}}{3} \\ 0 & 0 & e^{2t} \end{pmatrix}$$

```
(%i12) A
```

$$(\%o12)\quad \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$$

```
(%i13) eigenvalues(A)
```

$$(\%o13)\quad [[-1,1,2],[1,1,1]]$$

```
(%i15) B:eigenvectors(A)
```

$$(\%o15)\quad \left[[[-1,1,2],[1,1,1]],\left[[[1,-2,0]],[[1,0,0]],\left[\left[1,\frac{2}{5},\frac{3}{5}\right]\right]\right]\right]$$

```
(%i21) v1:transpose(matrix(B[2][1][1]))
```

$$(\%o21)\quad \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$$

```
(%i24) v2:transpose(matrix(B[2][2][1]))
```

$$(\%o24)\quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

```
(%i25) v3:transpose(matrix(B[2][3][1]))
```

$$(\%o25)\quad \begin{pmatrix} 1 \\ \frac{2}{5} \\ \frac{3}{5} \end{pmatrix}$$

```
(%i26) V:addcol(v1,v2,v3)
```

$$(\%o26)\quad \begin{pmatrix} 1 & 1 & 1 \\ -2 & 0 & \frac{2}{5} \\ 0 & 0 & \frac{3}{5} \end{pmatrix}$$

```
(%i27) Vi:invert(V)
```

$$(\%o27)\quad \begin{pmatrix} 0 & -\frac{1}{2} & \frac{1}{3} \\ 1 & \frac{1}{2} & -2 \\ 0 & 0 & \frac{5}{3} \end{pmatrix}$$

```
(%i29) D:Vi.A.V
```

$$(\%o29)\quad \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

```
(%i30) eD:D
```

$$(\%o30)\quad \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

```
(%i31) for i:1 thru 3 do
       eD[i,i]:exp(eD[i,i]*t)
```

(%o31) **done**

```
(%i32) eD
```

$$(\%o32)\quad \begin{pmatrix} \mathrm{e}^{-t} & 0 & 0 \\ 0 & \mathrm{e}^{t} & 0 \\ 0 & 0 & \mathrm{e}^{2t} \end{pmatrix}$$

```
(%i33) V.eD.Vi
```

$$(\%o33)\quad \begin{pmatrix} \mathrm{e}^{t} & \frac{\mathrm{e}^{t}}{2}-\frac{\mathrm{e}^{-t}}{2} & \frac{5\,\mathrm{e}^{2t}}{3}-2\,\mathrm{e}^{t}+\frac{\mathrm{e}^{-t}}{3} \\ 0 & \mathrm{e}^{-t} & \frac{2\,\mathrm{e}^{2t}}{3}-\frac{2\,\mathrm{e}^{-t}}{3} \\ 0 & 0 & \mathrm{e}^{2t} \end{pmatrix}$$

```
(%i34) matrix([%e^t, ((%e^t)/2)-((%e^(-t))/2), ((5*%e^(2*t))/3)-2*%e^t+((%e^(-t))/
       3)], [0, %e^(-t), ((2*%e^(2*t))/3)-((2*%e^(-t))/3)], [0, 0, %e^(2*t)])
```

$$(\%o34)\quad \begin{pmatrix} \mathrm{e}^{t} & \frac{\mathrm{e}^{t}}{2}-\frac{\mathrm{e}^{-t}}{2} & \frac{5\,\mathrm{e}^{2t}}{3}-2\,\mathrm{e}^{t}+\frac{\mathrm{e}^{-t}}{3} \\ 0 & \mathrm{e}^{-t} & \frac{2\,\mathrm{e}^{2t}}{3}-\frac{2\,\mathrm{e}^{-t}}{3} \\ 0 & 0 & \mathrm{e}^{2t} \end{pmatrix}$$

```
(%i37) expand(matrixexp(A*t))
Proviso: assuming 36*t # 0
```

$$(\%o37)\quad \begin{pmatrix} \mathrm{e}^{t} & \frac{\mathrm{e}^{t}}{2}-\frac{\mathrm{e}^{-t}}{2} & \frac{5\,\mathrm{e}^{2t}}{3}-2\,\mathrm{e}^{t}+\frac{\mathrm{e}^{-t}}{3} \\ 0 & \mathrm{e}^{-t} & \frac{2\,\mathrm{e}^{2t}}{3}-\frac{2\,\mathrm{e}^{-t}}{3} \\ 0 & 0 & \mathrm{e}^{2t} \end{pmatrix}$$

```
(%i38)
```

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} s-1 & -1 & -1 \\ 0 & s+1 & -2 \\ 0 & 0 & s-2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & -1 \\ 0 & 2 & -2 \\ 0 & 0 & -1 \end{pmatrix} . \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & -1 & -1 \\ 0 & 0 & -2 \\ 0 & 0 & -3 \end{pmatrix} . \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{pmatrix} . \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$sI - A = \begin{pmatrix} s-1 & -1 & -1 \\ 0 & s+1 & -2 \\ 0 & 0 & s-2 \end{pmatrix}$$

$$\operatorname{adj}(sI - A) = \begin{pmatrix} (s-2)\,(s+1) & s-2 & s+3 \\ 0 & (s-2)\,(s-1) & 2\,(s-1) \\ 0 & 0 & (s-1)\,(s+1) \end{pmatrix}$$

$$(\operatorname{adj}(sI - A))_{s=1} = \begin{pmatrix} -2 & -1 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(\operatorname{adj}(sI - A))_{s=-1} = \begin{pmatrix} 0 & -3 & 2 \\ 0 & 6 & -4 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(\operatorname{adj}(sI - A))_{s=2} = \begin{pmatrix} 0 & 0 & 5 \\ 0 & 0 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

```
(%i1) S:matrix([0,-omega],[omega,0])
```

(%o1) $\begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix}$

```
(%i9) eS:matrixexp(S*t)
```

```
Proviso: assuming 4*omega*t # 0
```
(%o9) $\begin{pmatrix} \frac{e^{-i\omega t}\,(e^{2i\omega t}+1)}{2} & \frac{e^{-i\omega t}\,(i\,e^{2i\omega t}-i)}{2} \\ -\frac{e^{-i\omega t}\,(i\,e^{2i\omega t}-i)}{2} & \frac{e^{-i\omega t}\,(e^{2i\omega t}+1)}{2} \end{pmatrix}$

```
(%i10) eS:expand(demoivre(expand(eS)))
```

$$(\%o10)\ \begin{pmatrix} \cos(\omega\, t) & -\sin(\omega\, t) \\ \sin(\omega\, t) & \cos(\omega\, t) \end{pmatrix}$$

```
(%i23) sS:lambda*matrix([1,0],[0,1])-S
```

$$(\%o23)\ \begin{pmatrix} \lambda & \omega \\ -\omega & \lambda \end{pmatrix}$$

```
(%i24) sSi:invert(sS)
```

$$(\%o24)\ \begin{pmatrix} \frac{\lambda}{\lambda^2+\omega^2} & -\frac{\omega}{\lambda^2+\omega^2} \\ \frac{\omega}{\lambda^2+\omega^2} & \frac{\lambda}{\lambda^2+\omega^2} \end{pmatrix}$$

```
(%i25) sSiC:sSi
```

$$(\%o25)\ \begin{pmatrix} \frac{\lambda}{\lambda^2+\omega^2} & -\frac{\omega}{\lambda^2+\omega^2} \\ \frac{\omega}{\lambda^2+\omega^2} & \frac{\lambda}{\lambda^2+\omega^2} \end{pmatrix}$$

```
(%i26) assume(omega>0)
```

$$(\%o26)\ [\omega > 0]$$

```
(%i27) for i:1 thru 2 do
       for j:1 thru 2 do
       (
       a:sSi[i,j],
       b:ilt(a,lambda,t),
       sSiC[i,j]:b
       )
```

$$(\%o27)\ \textbf{done}$$

```
(%i28) sSiC
```

$$(\%o28)\ \begin{pmatrix} \cos(\omega\, t) & -\sin(\omega\, t) \\ \sin(\omega\, t) & \cos(\omega\, t) \end{pmatrix}$$

```
(%i33) expand(trigreduce(sin(alpha)^2+cos(alpha)^2-1))
```

$$(\%o33)\ 0$$

```
(%i34) cos(q[1])
```

$$(\%o34)\ \cos(q_1)$$

```
(%i35) c[1]
```

$$(\%o35)\ c_1$$

```
(%i1) let(cos(q[1]),c[1])
```

$$(\%o1)\ \cos(q_1) \longrightarrow c_1$$

```
(%i2) let(sin(q[1]),s[1])
```

$$(\%o2)\ \sin(q_1) \longrightarrow s_1$$

```
(%i4) expr:sin(q[1])*cos(q[1])-sin(q[1])^2
```

(%o4) $\cos(q_1)\sin(q_1) - \sin(q_1)^2$

```
(%i5) letsimp(expr)
```

(%o5) $c_1\, s_1 - s_1^2$

```
(%i7) diff(letsimp(expr),q[1])
```

(%o7) 0