Misura della topologia magnetica

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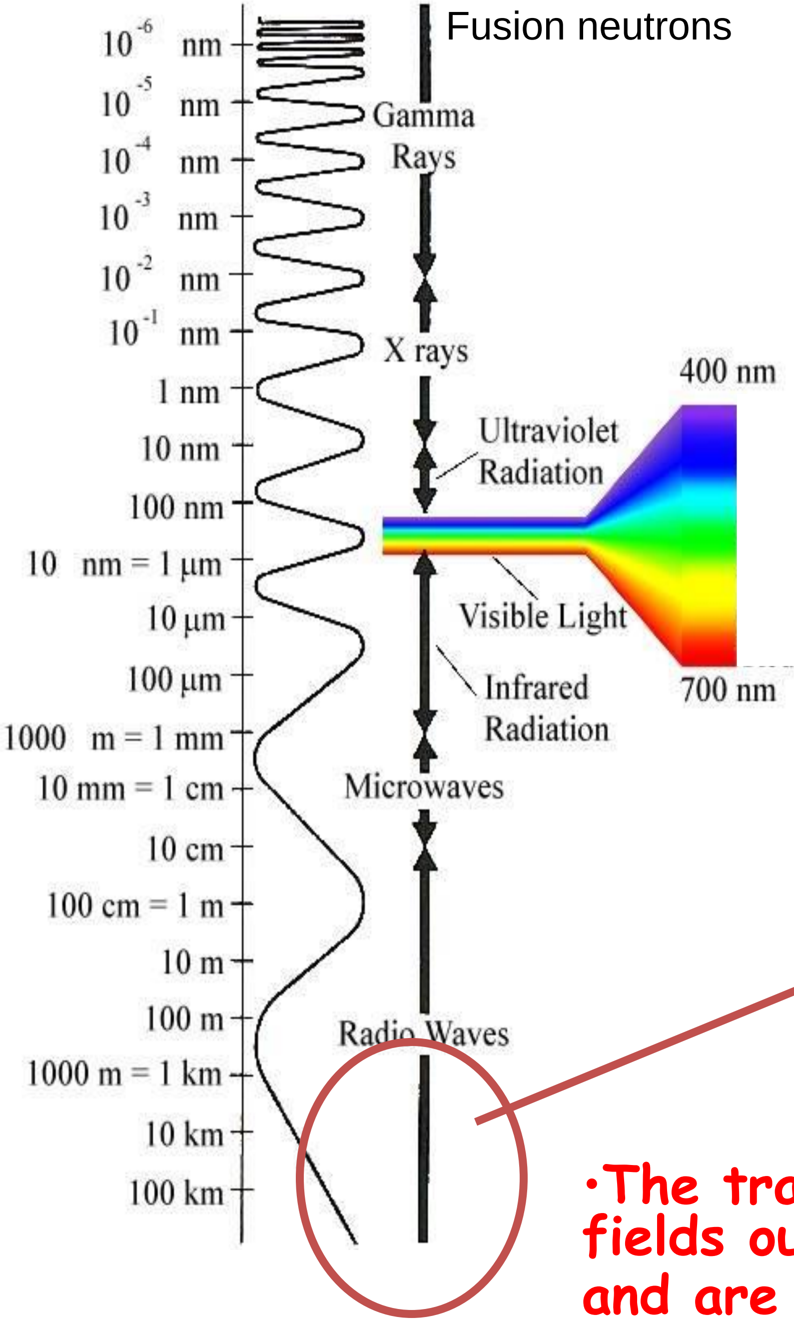
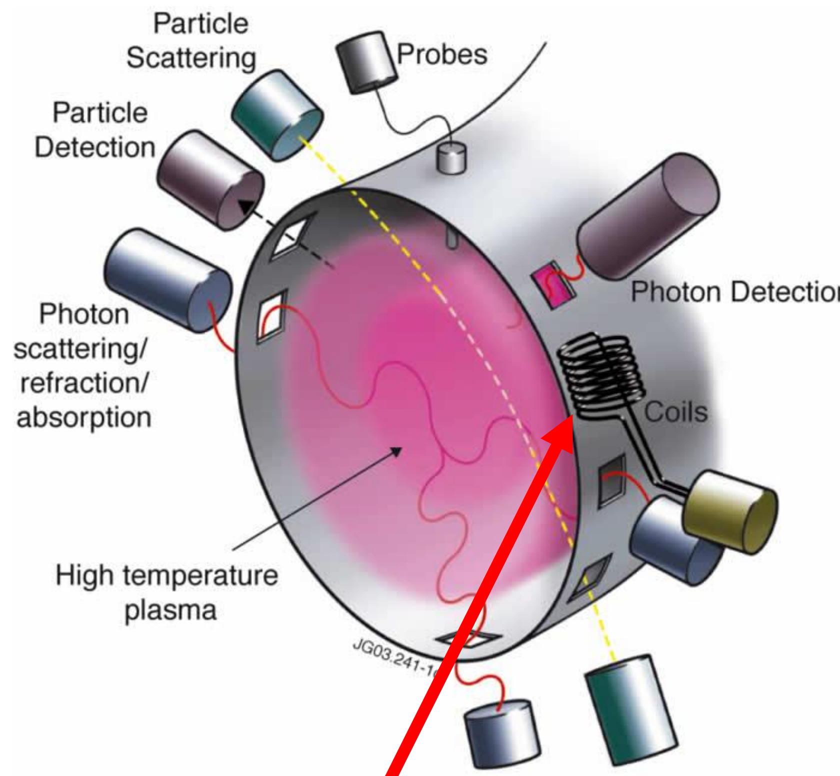




Coil systems

$$V = -\frac{d\phi}{dt}$$

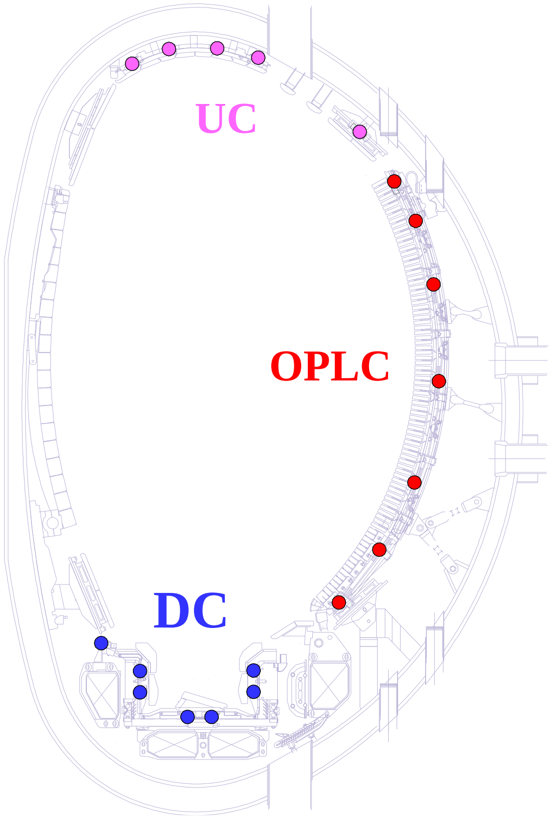
- The traditional measurements of the magnetic fields outside the plasma are performed with coils and are based on induction.

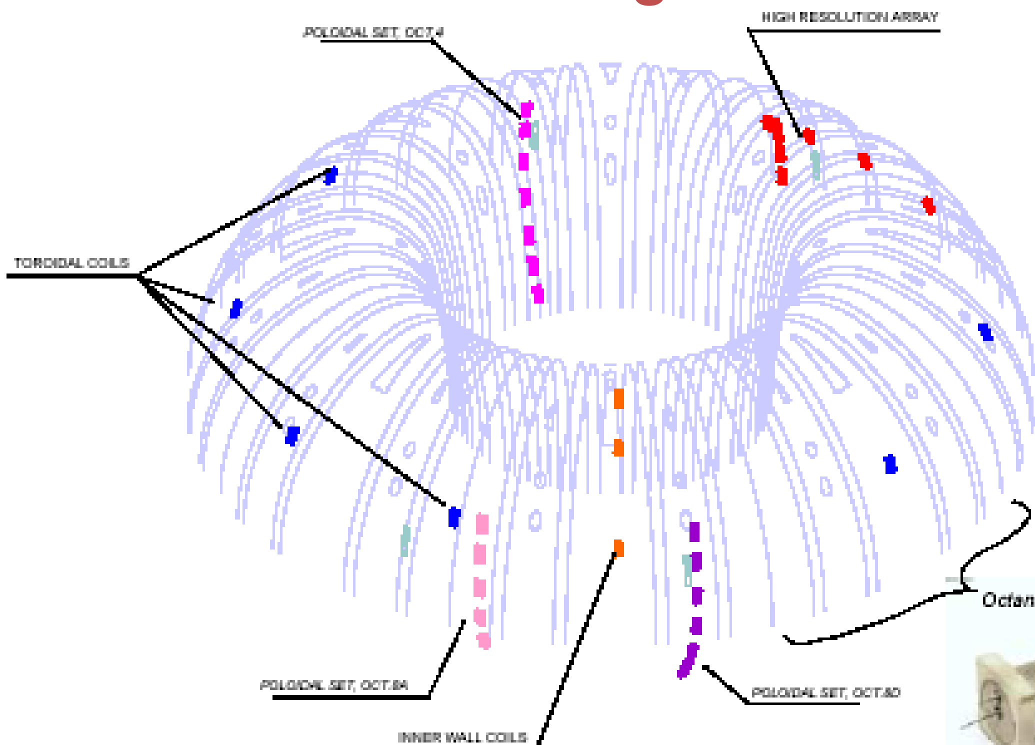
Location of the pick-up coils

Hundreds of coils of various nature are typically located around the vacuum vessel of a Fusion device and some inside.

Various methods based on Classical electrodynamics (vacuum) are used to derive the plasma boundary from the external magnetic fields

Poloidal cross section





High Resolution Magnetic Coils

Equilibrium reconstruction

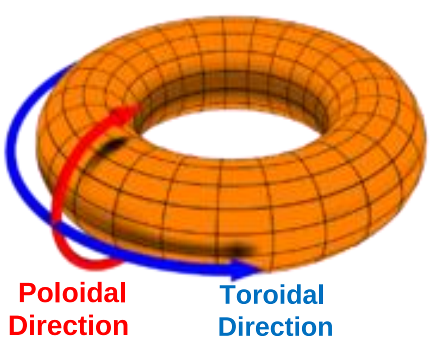
Magnetic Confinement Fusion in a nutshell

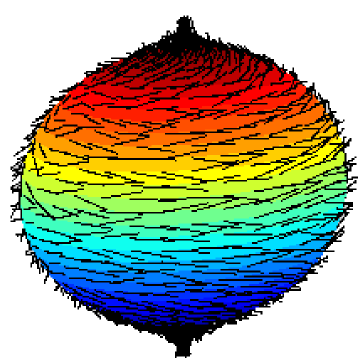
Two theorems provide constraints on the possible magnetic configurations for confining a fusion plasma:

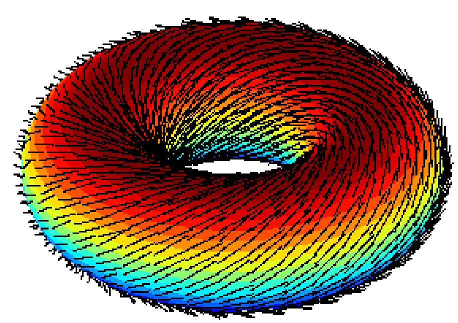
1. The **Virial Theorem** provides a relationship between the total kinetic and potential energies of a system of particles. A consequence of this is that **a plasma will always expand in the absence of external forces**. Therefore a **confining magnetic field cannot be entirely self generated through the dynamics of the ionised particles**, and **must be in some part produced** (via external coils).

2. The second theorem, '**The Hairy Ball' Theorem** **of algebraic topology** , states that **a sphere's surface cannot have a non-vanishing continuous tangent vector field**. This means that it is impossible **to comb a hairy ball flat without a creating a cow lick**. This means that **all magnetic geometries topologically equivalent to a sphere necessarily have at least one null point through which a plasma would be able to escape** and therefore cannot be used for confinement.

One convenient magnetic topology is a torus (it is possible to comb the hair flat on a hairy doughnut), and for this reason the majority of magnetic confinement devices utilize this geometry, with the **magnetic field predominantly acting in the toroidal direction addition of a poloidal magnetic field component in order to compensate the slowly particle drift across the magnetic field**.





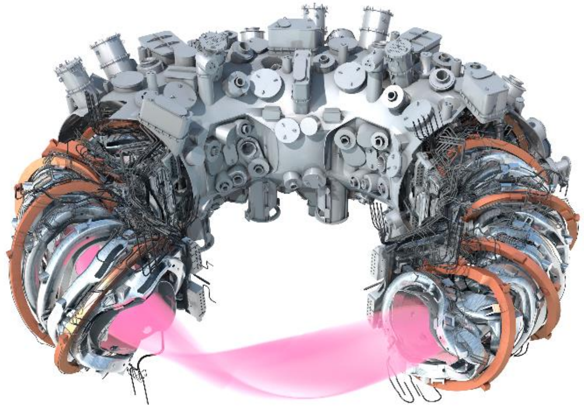


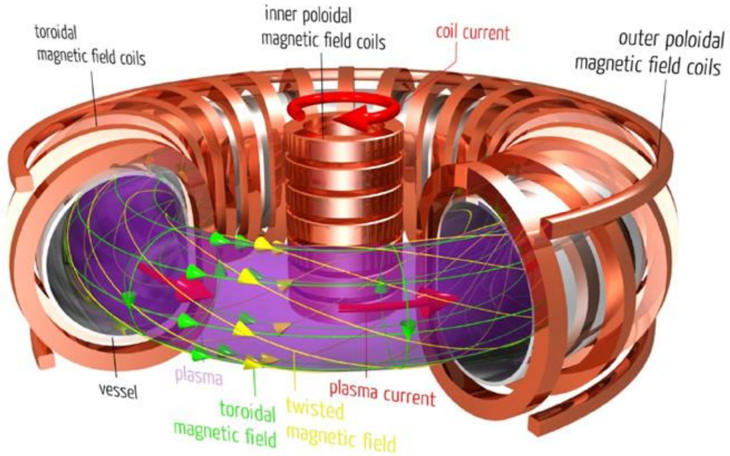
Magnetic Confinement Fusion in a nutshell

Whilst **the toroidal component** of the magnetic field **is generally produced** by passing currents through a number of **external coils that wrap around the plasma**, **the poloidal component can be generated in two different ways** (corresponding to two classes of device):

1. Stellarator: **the poloidal magnetic field component is generated externally from the plasma**, either through additional poloidal field coils, or as in more recent machines, by producing both components of the magnetic field from one set of coils. It can be seen that the coils take highly contorted shapes, which make stellarators **very difficult to design, manufacture and maintain but generally "more stable".**

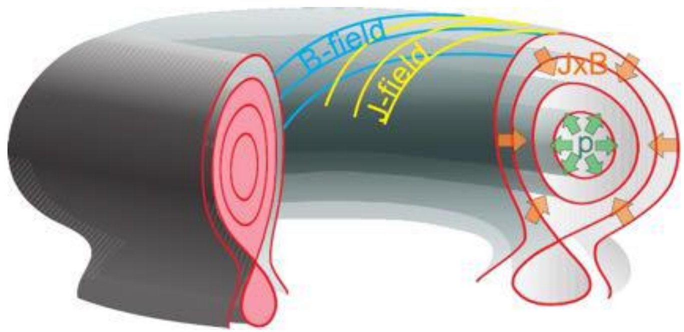
2. Tokamak: **the poloidal magnetic field component is generated by toroidal current driven through the plasma itself** via transformer action. A current is ramped up through a central solenoid located in the middle of the device, which acts as the primary winding of the transformer. It is noted however that the current through the solenoid cannot be increased indefinitely, and therefore tokamaks are **inherently pulsed devices, more simple and economic "less stable"**.



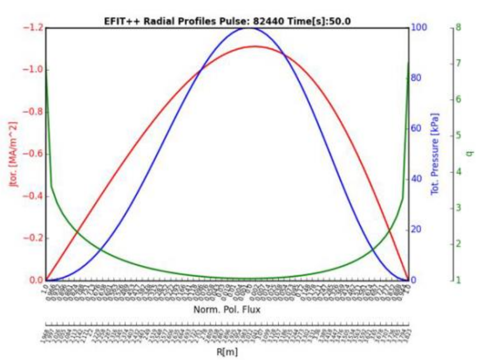


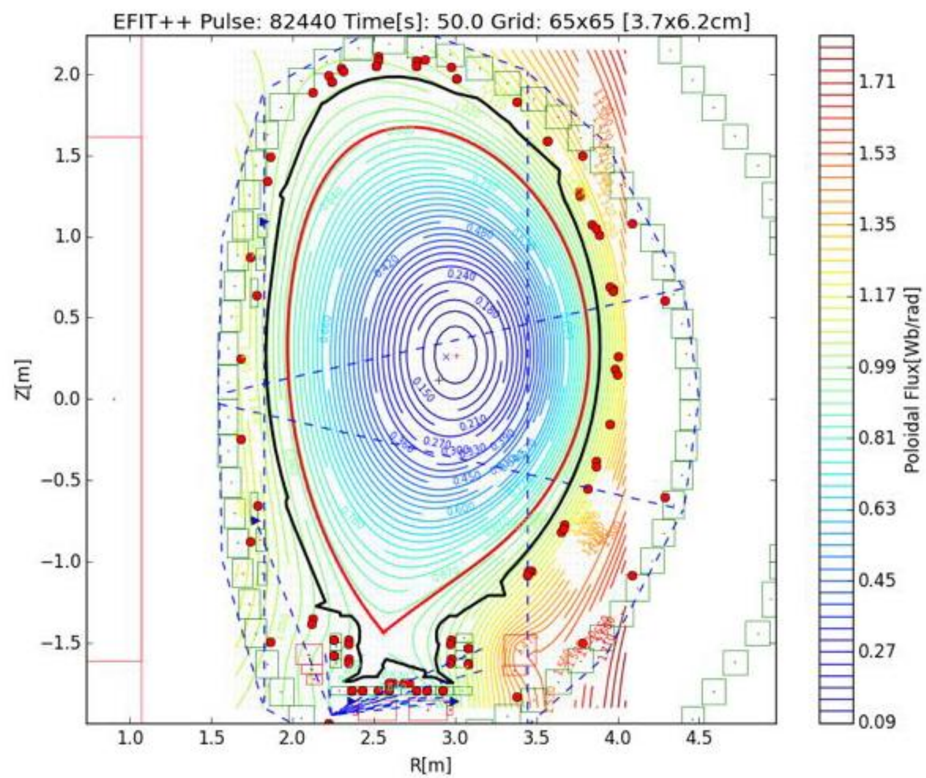
Machine-Independent Approach to the Equilibrium Identification (in Tokamaks)

Plasma “exists” in a tokamak if there is equilibrium between the magnetic force and the force due to the plasma pressure.



Thus for a plasma in equilibrium the magnetic field lines and current lines lie on isobaric nested magnetic surfaces and with toroidal topology (p=const.) (The innermost torus degenerates into a curve called magnetic axis).





Machine-Independent Approach to the Equilibrium Identification (in Tokamaks)

For several purposes **it is required to know the equilibrium configurations of tokamak plasmas**, i.e., the **current profile**, the **pressure** profile, and the **positions and shapes of the flux surfaces** (or, alternatively, the direction and magnitude of the poloidal magnetic field):

1. **Fast reconstruction during the discharge (required for control) or inter-discharge**. This requires **good modelling of the vacuum conditions** (iron and passive structures), the coils. The **modelling of the plasma equilibrium itself need not be as accurate as in other applications** (crude assumptions about the plasma current and pressure profiles may be used).

2. **Interpretation of some plasma diagnostic data**. **Some tokamak diagnostics measure quantities inside the plasma integrated over a line of sight** and frequently we have to combine different plasma measured quantities (density and temperature in order to obtain the pressure->**Remapping**). For such a profile reconstruction one **needs to know both the positions of the lines of sight and the shapes and positions of the flux surfaces**. Since the problem is entirely geometrical, **accuracy of the pressure and current profiles in the Grad-Shafranov equation is only important insofar the shapes of the flux sur- faces are concerned**.

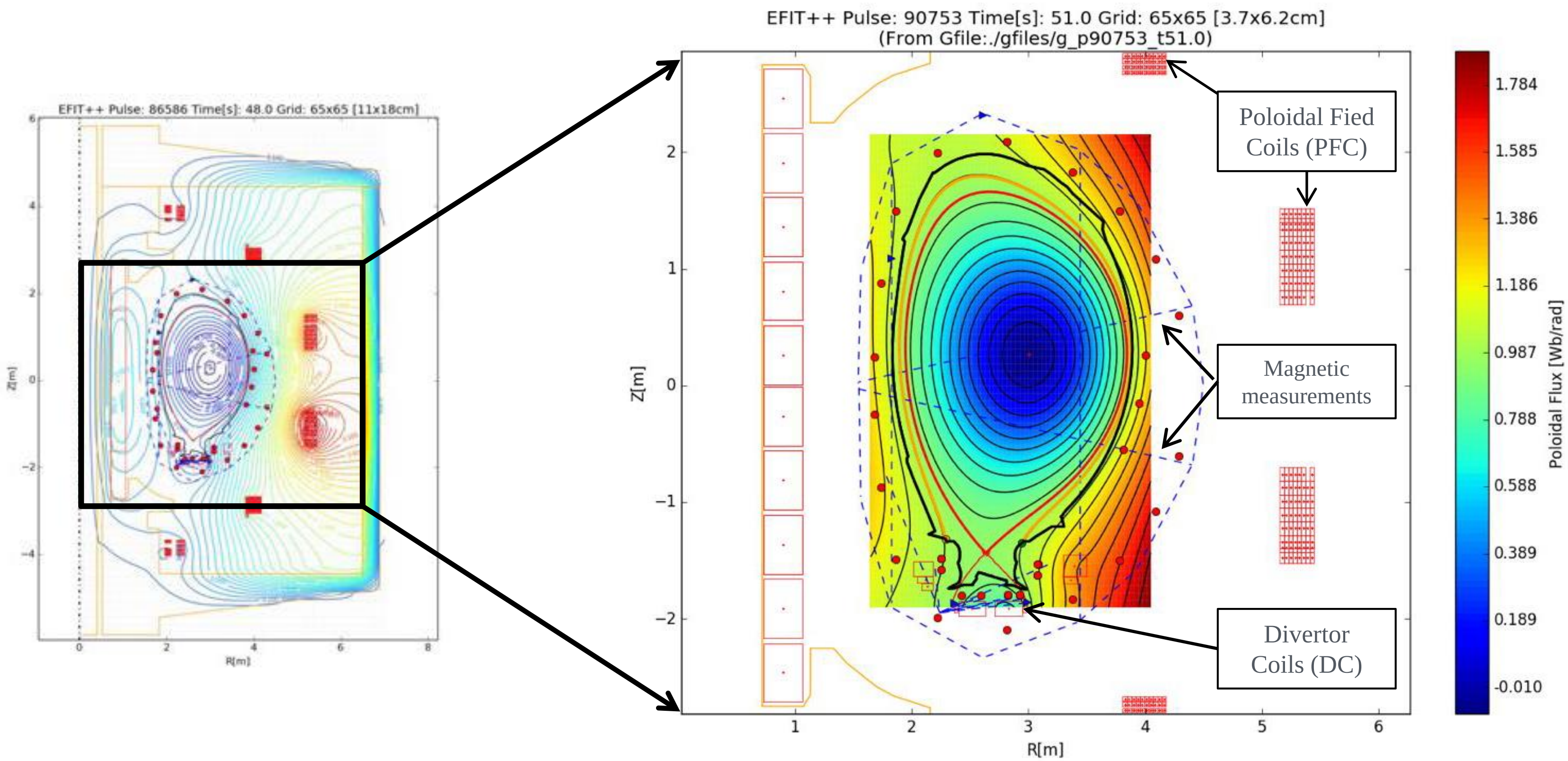
3. **Plasma energy, heating and transport simulations**. To study transport rates and **local power balance in the plasma it is important to get optimal information about the plasma pressure other measurements**. For correct interpretation of the data, the presence of **impurity particle species and non-thermal particle populations** may have to be accounted for.

Stability analysis. In order to analyze the stability of plasmas (experimentally), knowledge of the current density (qprofile) and local pressure gradients is important. **For many numerical stability calculations, a precise numerical solution of the Grad-Shafranov equation is importan**t.

Machine-Independent Approach to the Equilibrium Identification (in Tokamaks)

For several purposes it is required to know the equilibrium configurations of tokamak plasmas, i.e., the current profile, the pressure profile, and the positions and shapes of the flux surfaces (or, alternatively, the direction and magnitude of the poloidal magnetic field).

Primarily, **this information has to be deduced from the known coil currents and magnetic measurements outside the plasma** (typically near the coils or around the vacuum vessel).



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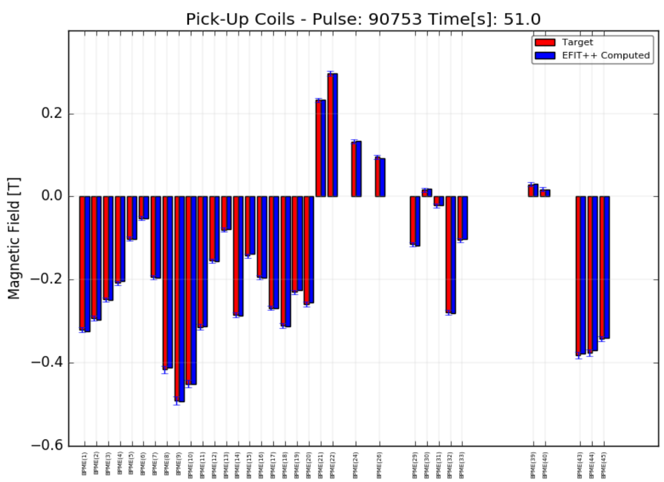
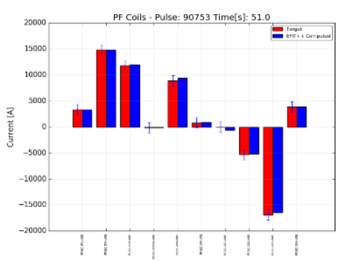
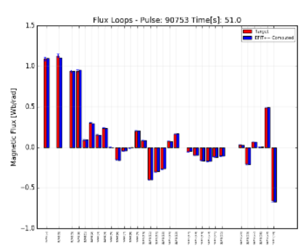
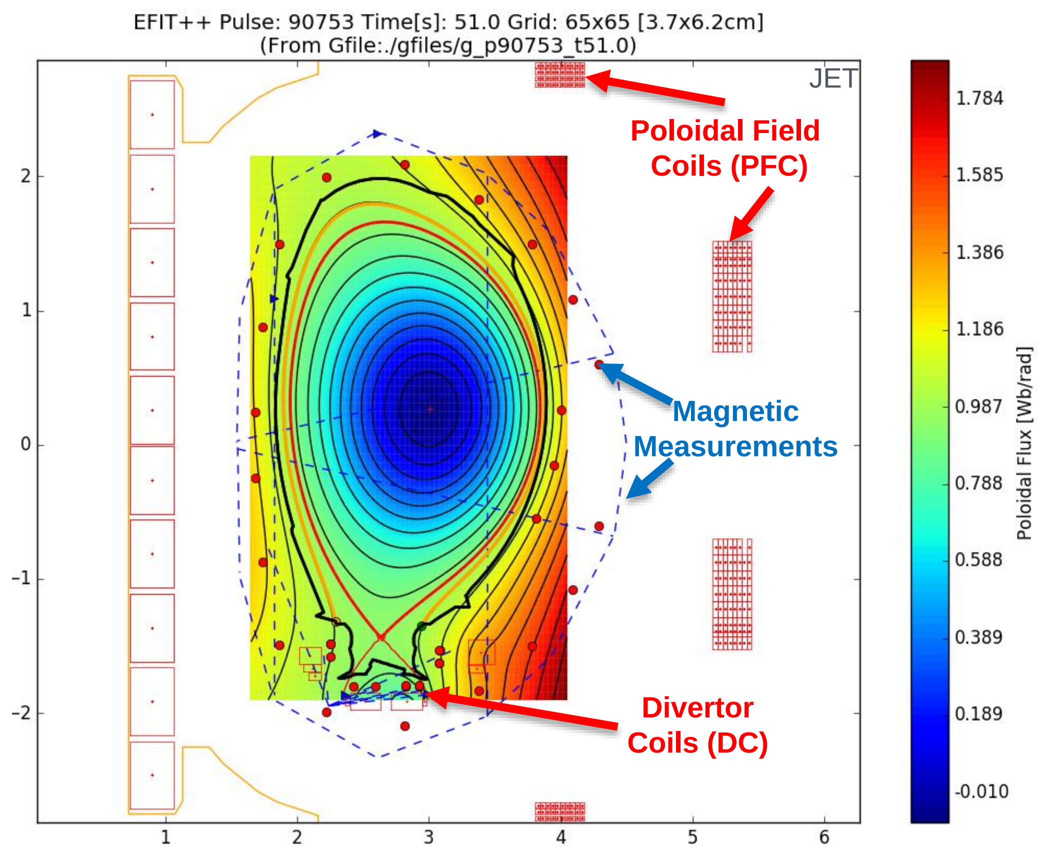
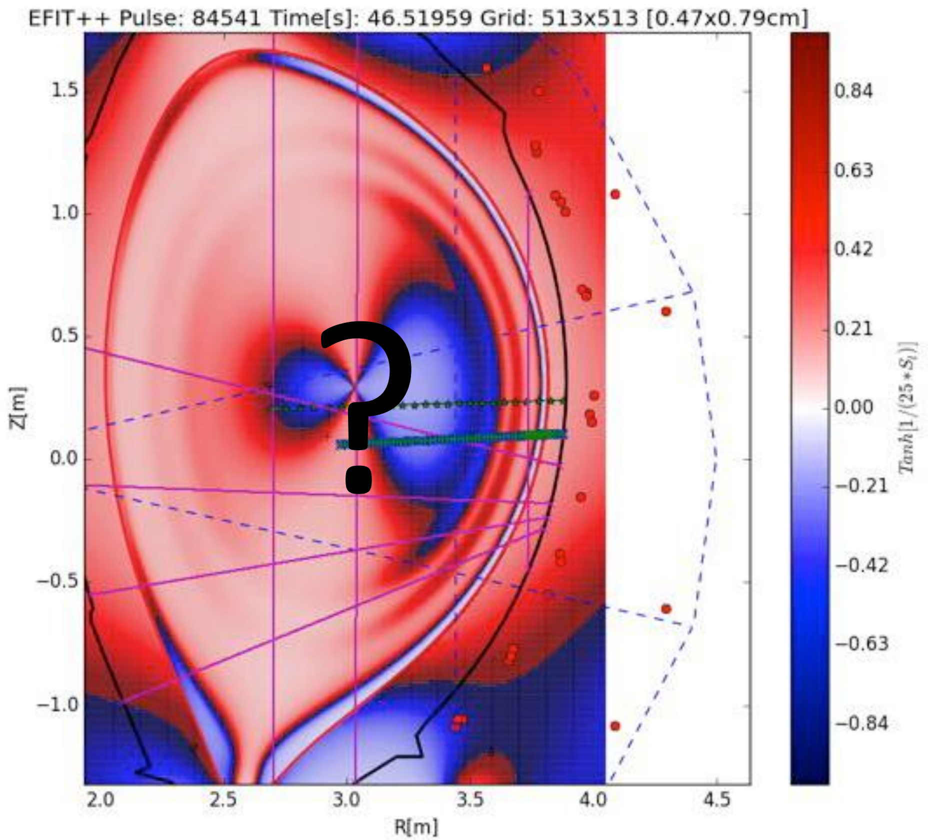








Machine-Independent Approach to the Equilibrium Identification (in Tokamaks)

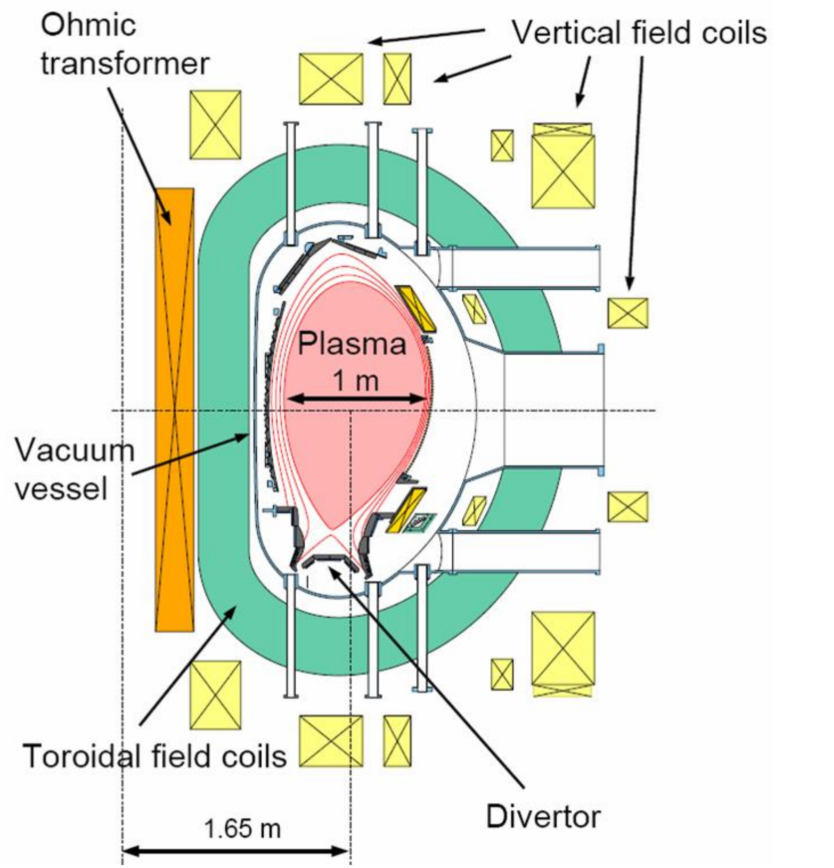


The problem is to find a smooth solution of the Grad-Shafranov equation that best fits the measurements (Neumann conditions on a closed boundary).

Assumptions on ”reasonable” profiles is necessary because **the reconstruction problem is inherently ill-posed: small changes (errors, inconsistencies) in the magnetic measurements outside the plasma cause changes in the solution that inflate dramatically further inward**.

Tokamak: plasma

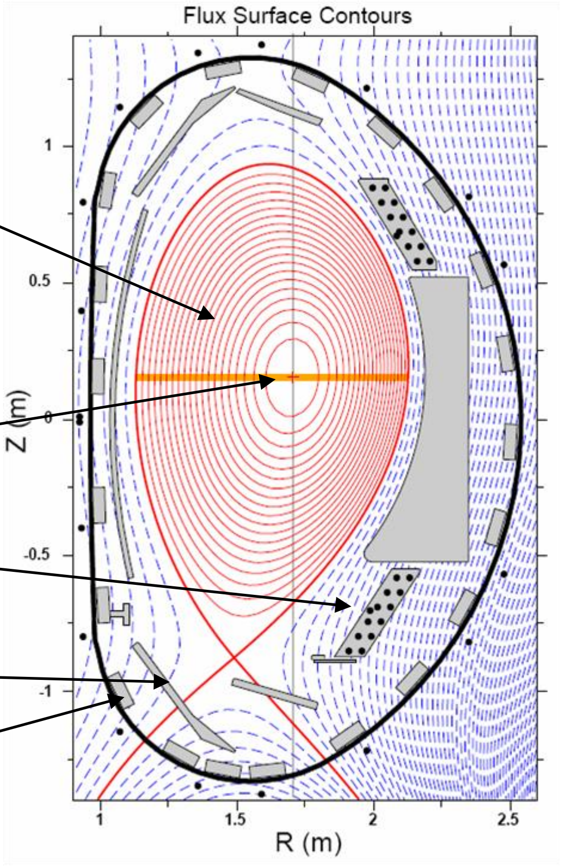
- Plasma (purple) Notice the shape
- Surrounded by plates
- Vessel (pumps)
- Coils mostly outside vessel (finite reaction time)
- Ohmic transformer / toroidal field coils (green)



Schematic Drawing of the poloidal cross section of the ASDEX Upgrade tokamak

Tokamak: plasma

- Magnetic surfaces are the surfaces traced out by the magnetic field
- They are nested (best confinement)
- Centre is shifted outward
- Large passive coils
- Magnetic field ends on a set of plates
- Large set of small coils for diagnostic purposes



Schematic Drawing of the poloidal cross section of the ASDEX Upgrade tokamak

Pitch of the field line: safety factor

The rotational transform (or field line pitch) *ι/2π* is defined as the number of poloidal transits per single toroidal transit of a field line on a toroidal flux surface.

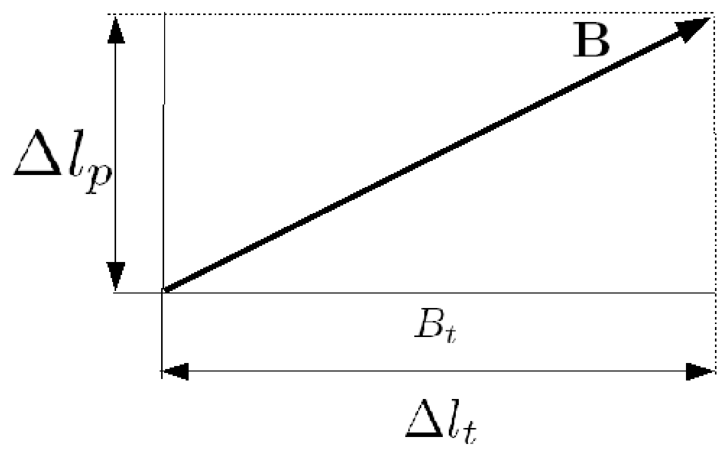
- Along the magnetic field

$$\frac{\Delta l_t}{\Delta l_p} = \frac{B_t}{B_p}$$

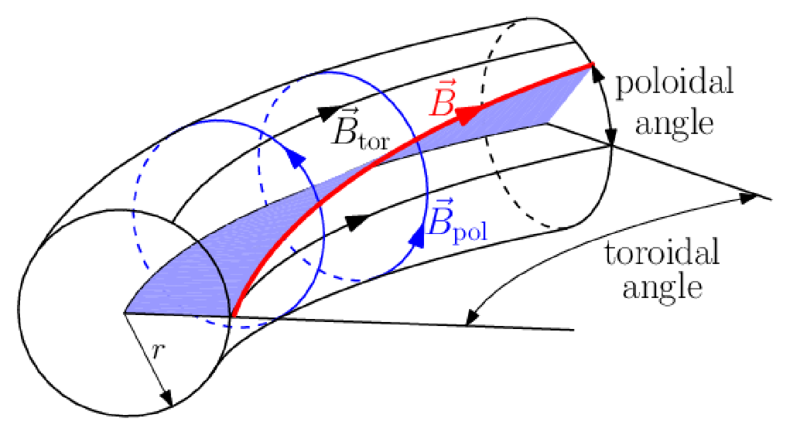
$$\mathrm{d}l_t = \frac{B_t}{B_p}\mathrm{d}l_p$$

- Consequently the length of the field line in toroidal direction is

$$l_t = \int \mathrm{d}l_t = \int \frac{B_t}{B_p}\mathrm{d}l_p$$



Pitch of the field line



Pitch of the magnetic field

- Length of the field

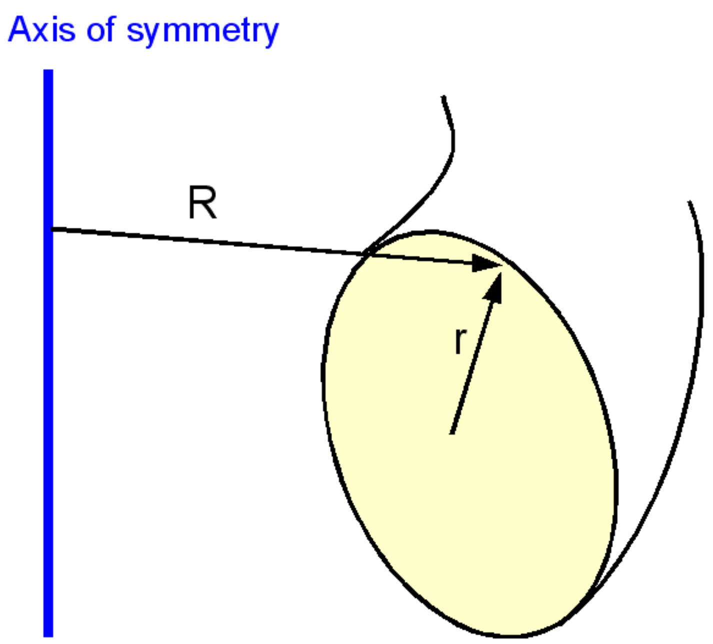
$$l_t = \int \mathrm{d}l_t = \int \frac{B_t}{B_p} \mathrm{d}l_p$$

- In one poloidal turn

$$l_t = 2\pi r \frac{B_t}{B_p}$$

- Number of toroidal turns in one poloidal turn (**safety factor q**)

$$q \equiv \frac{l_t}{2\pi R} = \frac{r B_t}{R B_p}$$



Definition of the minor r and major R radius

Ratio of poloidal and toroidal

- From the safety factor it follows

$$q = \frac{rB_t}{RB_p} = 3 \qquad \frac{B_p}{B_t} = \frac{r}{3R} \approx 0.1$$

- Relation with the current (Ampere's law)

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$2\pi r B_p = \mu_0 I$$

$$q = \frac{2\pi r^2 B_t}{\mu_0 R I} = \frac{2AB_t}{\mu_0 R I}$$

- For stable operation the safety factor at the edge is chosen q > 3. This means a maximum current!!!

Pressure and current

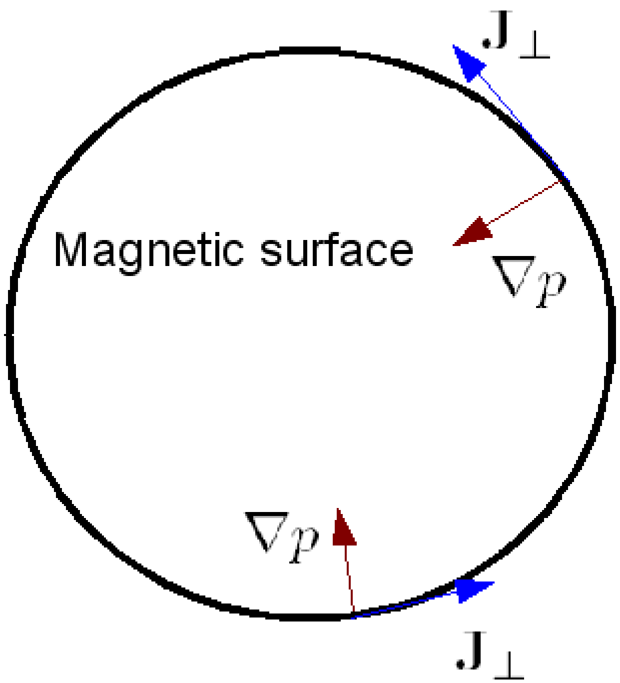
- From the force balance

$$\mathbf{J} \times \mathbf{B} = \nabla p$$

- Taking the inner product with the magnetic field

$$\mathbf{B} \cdot \nabla p = 0$$

- The pressure gradient is perpendicular to the surface
- Pressure is constant on a surface



Pressure is constant on the magnetic surface, and the current lies inside the surface

Pressure and current

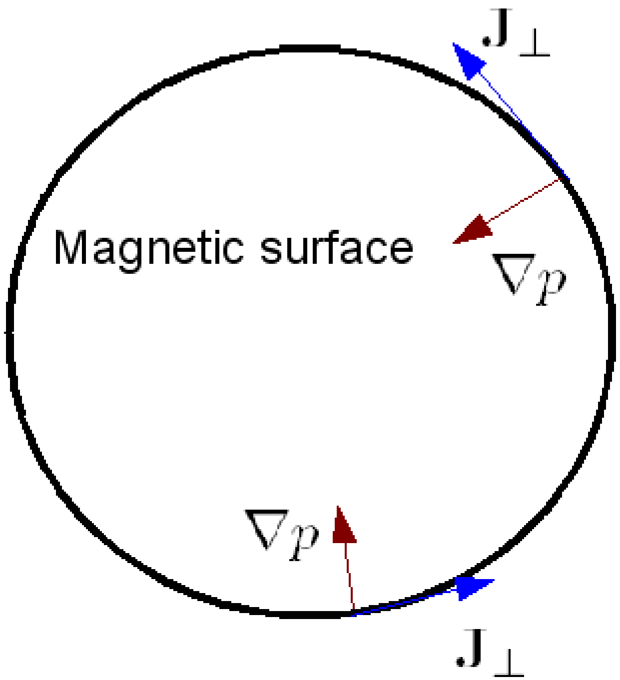
- Again using the force balance

$$\mathbf{J} \times \mathbf{B} = \nabla p$$

- Taking the cross product with the magnetic field

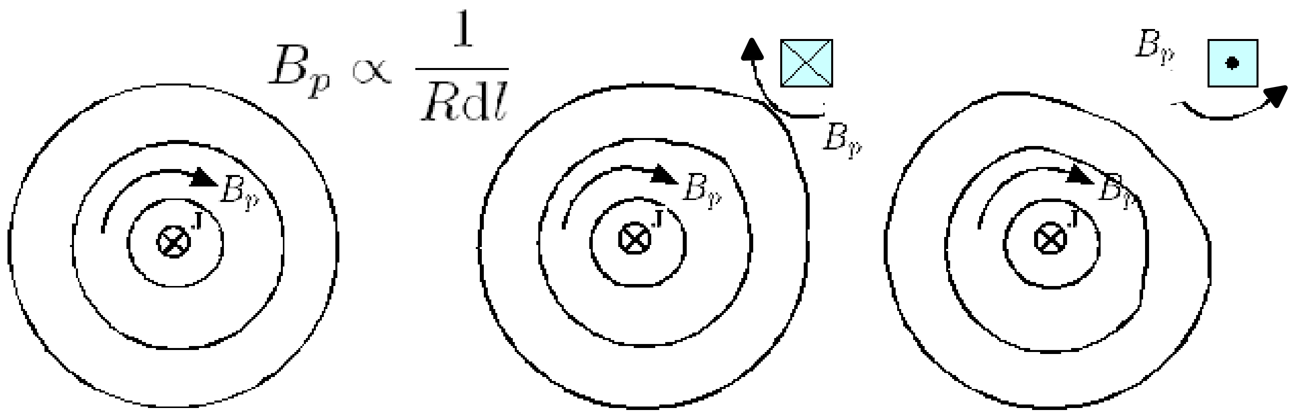
$$\mathbf{J} \cdot \nabla p = 0$$

- Since the pressure gradient is perpendicular to the surface the current lies inside the surface



Pressure is constant on the magnetic surface, and the current lies inside the surface

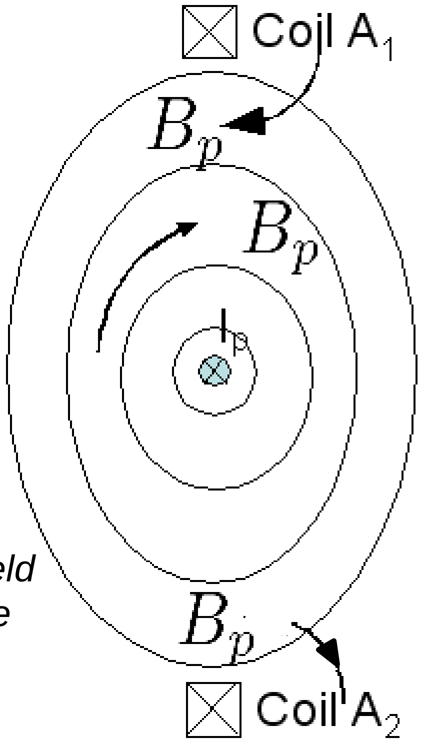
Plasma shaping



- Can be understood from the relation between poloidal field and distance between the surfaces
- A current in a coil outside the plasma will change the poloidal field
- If it weakens the poloidal field of the current the distance between the surfaces increases
- If it enhances the field the distance decreases

Elongation

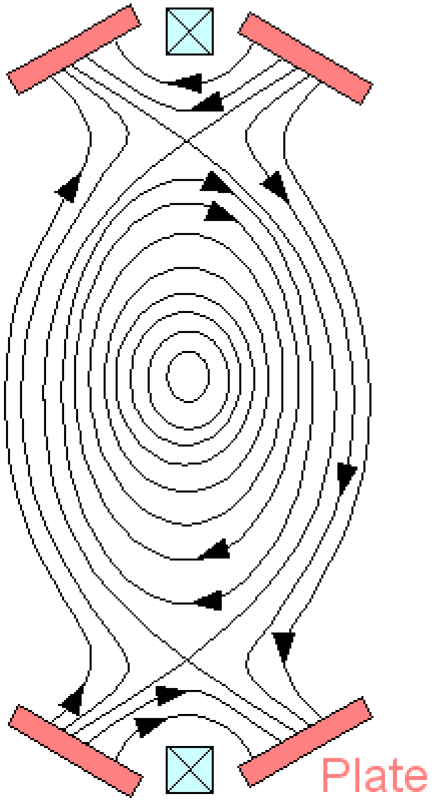
- Dominant shaping is the elongation of the plasma.
- This is achieved by two coils on the top and bottom of the plasma with a current in the direction of the plasma current.



Elongation is generated by two field coils at the top and bottom of the plasma

Reason 1 for plasma elongation

- Plasma can be diverted onto a set of plates
- Close to the coils the field of the coils dominates
- In between the field is zero resulting in a purely toroidal field line
- This shows up as an X-point in the figure of the magnetic surfaces
- Surfaces outside the one with the X-point are not close with the field ending on the plates



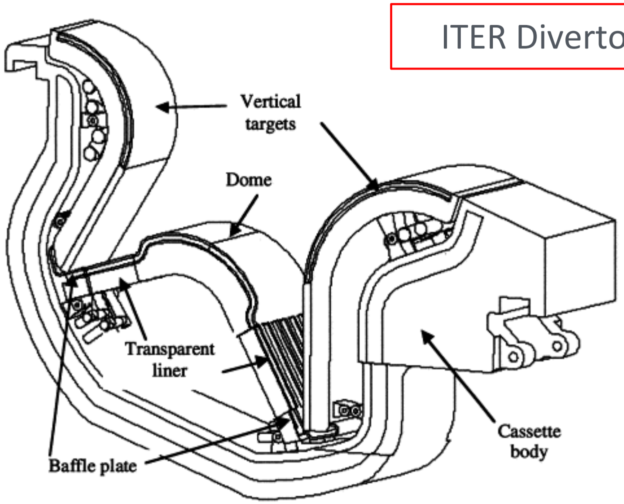
Shaping coils allow for plasma to be diverted onto the divertor-plates

TOR VERGATA
UNIVERSITY OF ROME

Reason 1: Divertor

ITER Divertor

- A modern divertor design looks something like this
- Note that it has, as far as possible a closed structure. This to allow the efficient pumping of the neutral particles.
- Note also that the angle between the magnetic field and the plate is as small as possible. This makes that the energy carried by the particles to the plate is distributed over the largest possible are.



Reason II : Plasma elongation

- Distance to go around poloidally is larger

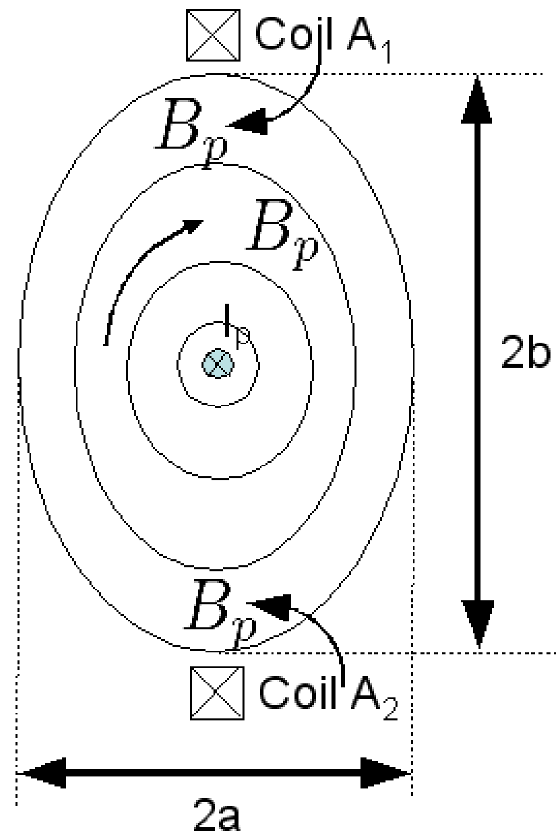
$$q = \frac{2\pi r^2 B_t}{\mu_0 R I} = \frac{2 A B_t}{\mu_0 R I}$$

$$A = \pi a b = \pi a^2 \kappa \qquad \kappa = \frac{b}{a}$$

For the same plasma current

$$\boxed{q_{\mathrm{elip}} = q_{\mathrm{circ}} \kappa}$$

- If q = 3 is the limit of operation one can run a larger current in an elliptically shaped plasma



Reason II : Plasma elongation

Elongation allows for a larger plasma current.

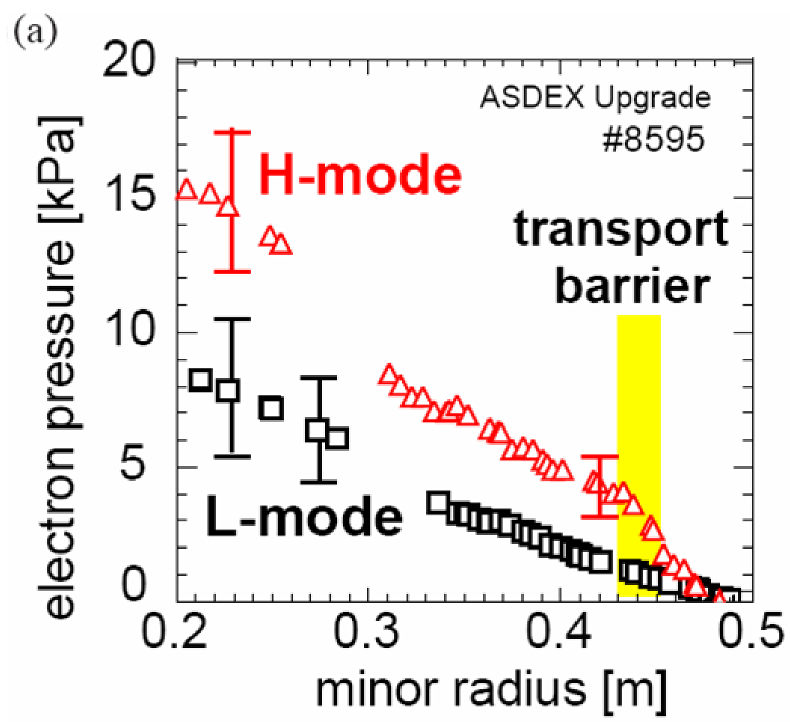
The larger current leads to a strong magnetic field.

The larger poloidal field allows for a higher kinetic pressure.

Better confinement

Reason III : Plasma elongation

- A transition phenomenon is observed in Divertor plasmas known as the L (low) to H (high confinement) transition
- In this transition a steep pressure profile is generated at the plasma edge
- Not very well understood
- Confinement improvement is roughly a factor 2 !!!!



Equilibrium / Vertical instability

- Magnetic field due to the coil follows form

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

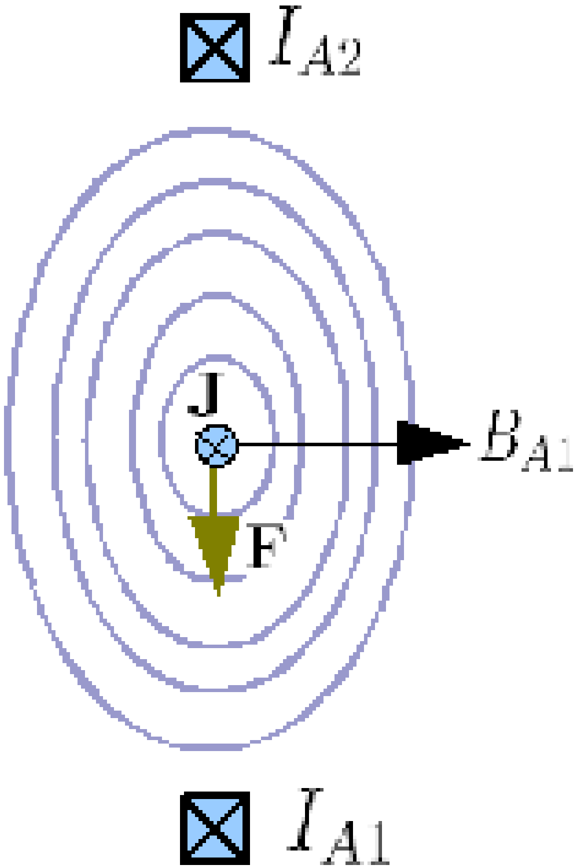
- Assume d<<R one finds

$$2\pi d B_{A1} = \mu_0 I_{A1}$$

$$B_{A1} = \frac{\mu_0 I_{A1}}{2\pi d}$$

- This leads to a force on the plasma

$$\mathbf{F} = \mathbf{J} \times \mathbf{B}$$



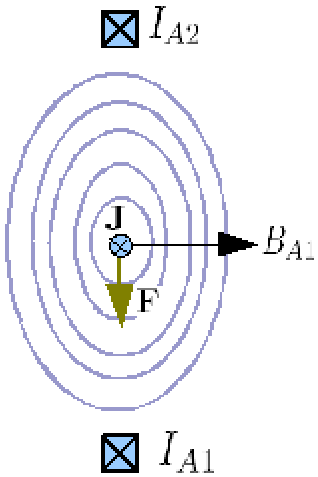
Vertical stability

- Integrating the force

$$\mathbf{F}_T = \int \mathrm{d}^3 V\, \mathbf{J} \times \mathbf{B}$$
$$= -2\pi R \int \mathrm{d}^2 A\, J B_{A1} \mathbf{e}_z$$
$$= -2\pi R \frac{\mu_0 I_{A1}}{2\pi d} I_p\, \mathbf{e}_z$$

- Thus

$$\mathbf{F}_{T1} = -\frac{\mu_0 R I_{A1} I_p}{d} \mathbf{e}_z$$
$$\mathbf{F}_{T2} = \frac{\mu_0 R I_{A2} I_p}{d} \mathbf{e}_z$$



Vertical stability

- Forces

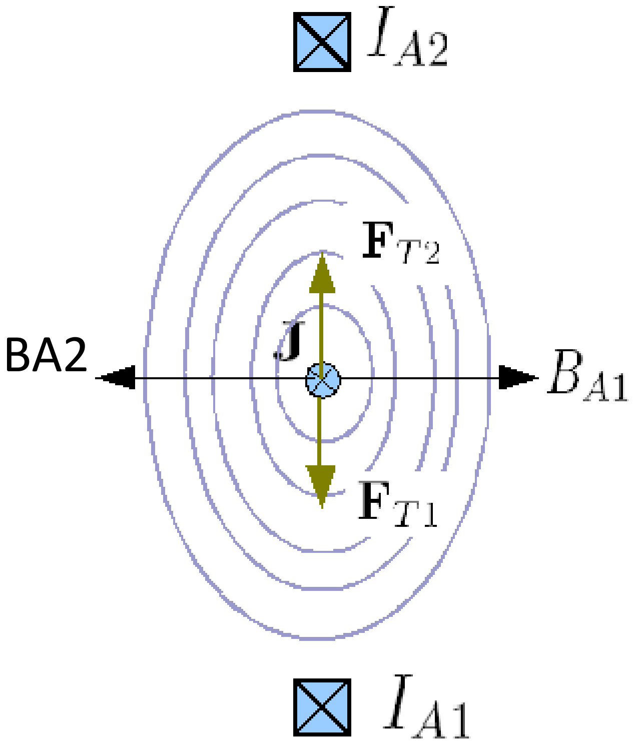
$$\mathbf{F}_{T1} = -\frac{\mu_0 R I_{A1} I_p}{d}\mathbf{e}_z$$

$$\mathbf{F}_{T2} = \frac{\mu_0 R I_{A2} I_p}{d}\mathbf{e}_z$$

- Equilibrium requires

$$I_{A1} = I_{A2} = I_A$$

- Such that the forces balance



Vertical stability

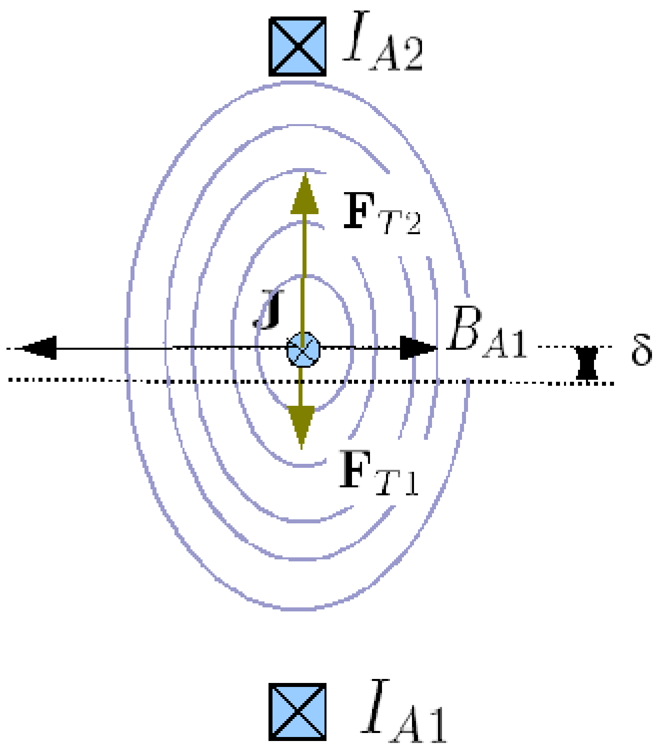
- The forces

$$\mathbf{F}_{T1} = -\frac{\mu_0 R I_{A1} I_p}{d}\mathbf{e}_z$$

$$\mathbf{F}_{T2} = \frac{\mu_0 R I_{A2} I_p}{d}\mathbf{e}_z$$

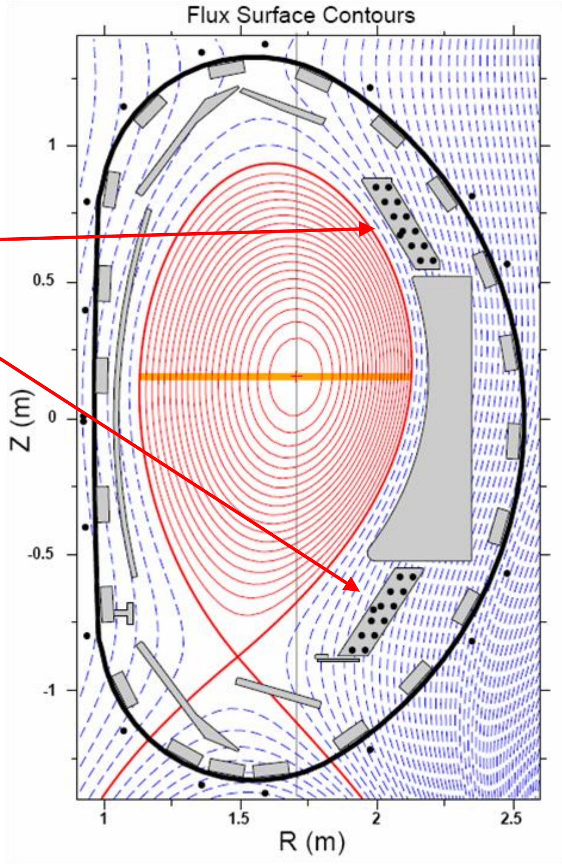
- Are in equilbrium when the coil currents are the same.
- But when the plasma is shifted upward by a small amount δ

$$\mathbf{F}_T = \mu_0 R I_A I_p \left[\frac{1}{d-\delta} - \frac{1}{d+\delta}\right]\mathbf{e}_z$$



Plasma

- Plasma vertical instability with high growth rates
- For this reason the passive coils have been placed in the plasma
- When the plasma moves it changes the flux through the coils which generates a current that pushes the plasma back
- Growth rate is reduced to the decay time of the current in the coils (ms)



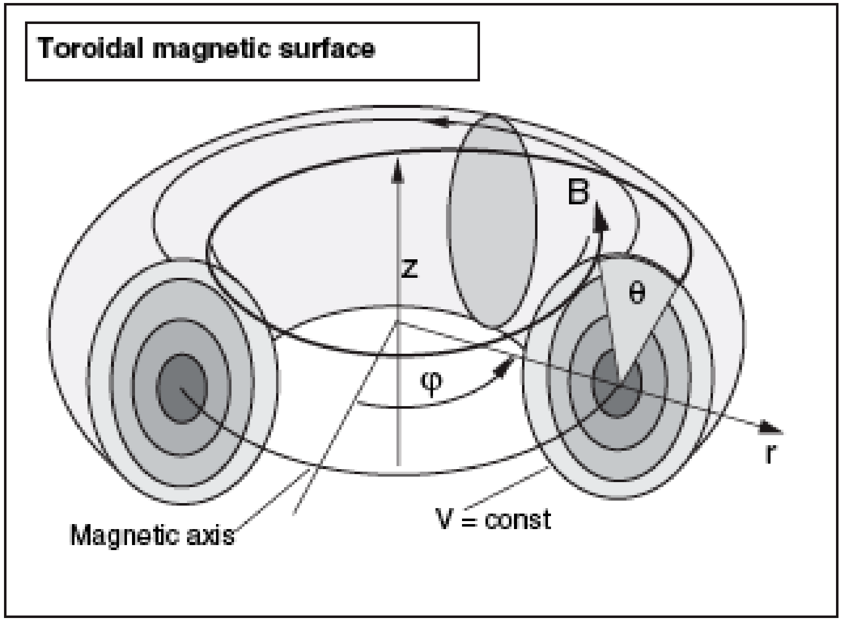
Equilibrium reconstruction code

Plasma coordinates

Coordinate cilindriche:
ϕ : angolo toroidale
R : raggio maggiore
z : asse verticale

Coordinate toroidale:
ϕ : angolo toroidale
θ : angolo poloidale
r=R-Ro : raggio

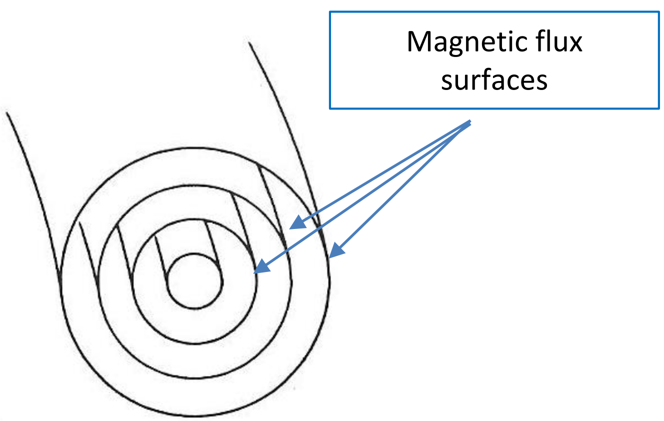
Coordinate Toroidali di flusso:
ϕ : toroidal angle
θ : poloidal angle
ψ : radial coordinate



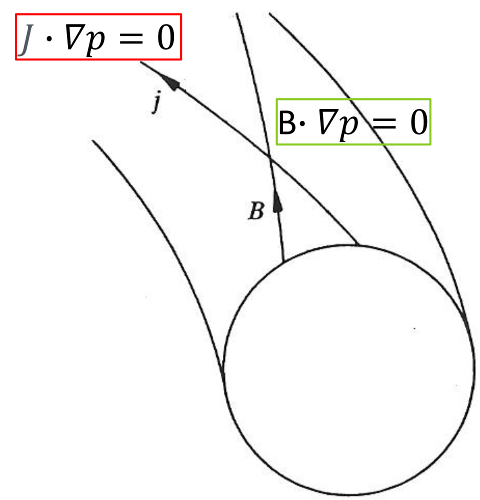
Axisymmetric magnetic surface in cylinder coordinates

Field line and equilibrium

- The basic condition for equilibrium is that the magnetic force balances the force due to the plasma pressure. $\boldsymbol{j} \times \boldsymbol{B} = \boldsymbol{\nabla} p$
- In studying tokamak equilibria, it is convenient to introduce the poloidal magnetic flux function ψ. The ψ satisies $\boldsymbol{B}\ \boldsymbol{\nabla}\psi = 0$



“nested surfaces”



Magnetic field lines and $\mathbf{J}$ lie on the magnetic flux surfaces (but can not overlap otherwise the pressure gradient would be zero!)

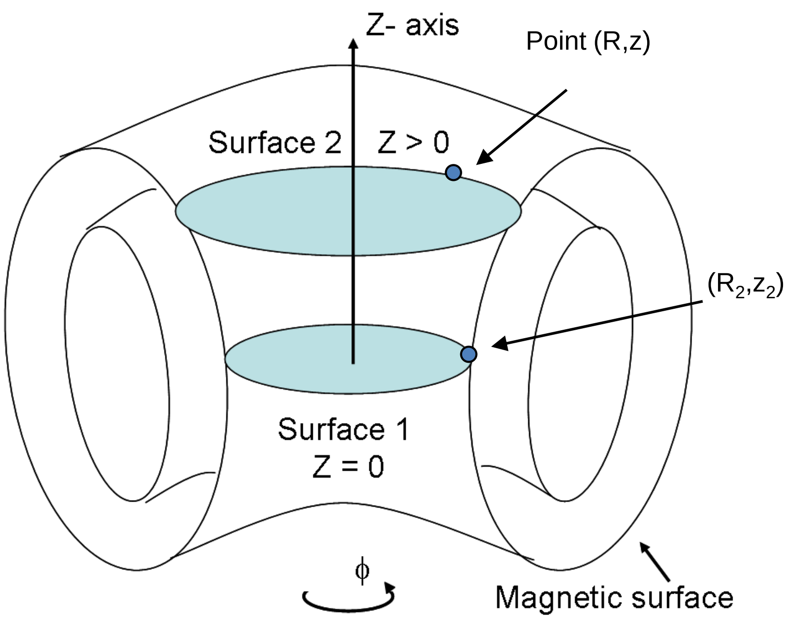
Poloidal flux

- The poloidal flux ψ(R,z) is the flux through the circle with its centre at R = 0 lying in the z-plane and having (R,z) lying on its boundary

$$\nabla \cdot \mathbf{B} = 0$$

- Integrated over a volume enclosed by two of these circles and the magnetic surface yields

$$\psi(R,z) = \psi(R_2,z_2)$$



The poloidal flux is the flux through the blue areas. It is constant on a magnetic surface

Magnetic surfaces

- Traced out by the magnetic field
- The pressure is constant on the surface
- The current lies inside the surface
- The poloidal flux is constant on a surface. The surfaces are therefore also called flux-surfaces

Distance between the surfaces

- Magnetic field is divergence free

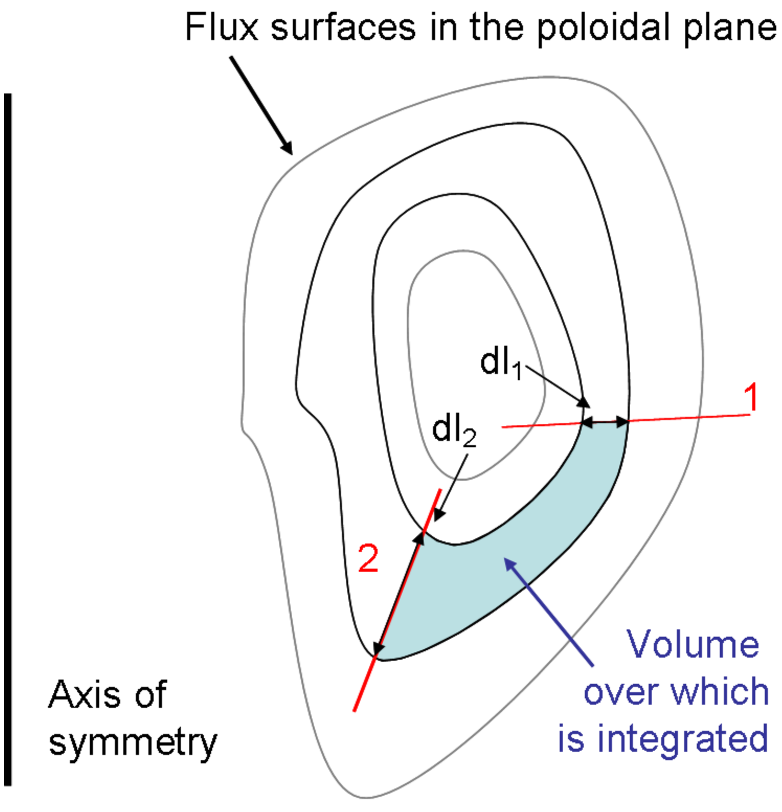
$$\nabla \cdot \mathbf{B} = 0$$

- Integrating over the indicated volume gives

$$-B_{p1} 2\pi R_1 \mathrm{d}l_1$$
$$+B_{p2} 2\pi R_2 \mathrm{d}l_2 = 0$$

- Inside the surface

$$B_{p2} = B_{p1} \frac{R_1}{R_2} \frac{\mathrm{d}l_1}{\mathrm{d}l_2} \rightarrow B_p \propto \frac{1}{R \mathrm{d}l}$$



Relation with the poloidal

- The poloidal flux is constant on each of the surfaces

$$\delta\psi = |\nabla\psi(1)|\mathrm{d}l_1 = |\nabla\psi(2)|\mathrm{d}l_2 \longrightarrow \frac{\mathrm{d}l_1}{\mathrm{d}l_2} = \frac{|\nabla\psi(2)|}{|\nabla\psi(1)|}$$

- This yields for the poloidal field

$$B_{p2} = B_{p1}\frac{R_1}{R_2}\frac{\mathrm{d}l_1}{\mathrm{d}l_2} \longrightarrow B_{p2} = B_{p1}\frac{R_1|\nabla\psi(2)|}{R_2|\nabla\psi(1)|} \longrightarrow B_p \propto \frac{1}{R}|\nabla\psi|$$

Plasma equilibrium

The behavior of a plasma immersed in a magnetic field is described by MHD equations (MagnetoHydroDynamics).
These equations, once constrained at the equilibrium condition of the plasma, are reduced to the solution of an equation differential, called the Grad-Shafranov equation.

The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. The set of equations that describe MHD are a combination of the Navier–Stokes equations of fluid dynamics and Maxwell's equations of electro-magnetism.

The simplest form of MHD, Ideal MHD, assumes that the fluid has so little resistivity that it can be treated as a perfect conductor.

Ideal MHD equations

The main quantities which characterize the electrically conducting fluid are the bulk plasma velocity field **v**, the current density **J**, the mass density *ρ*, and the plasma pressure *p*. The flowing electric charge in the plasma is the source of a magnetic field **B** and electric field **E**. All quantities generally vary with time *t*. Vector operator notation will be used, in particular ∇ is gradient, ∇ · is divergence, and ∇ × is curl.

1. $\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$
2. $\rho \frac{\partial v}{\partial t} = j \times B - \nabla p$
3. $\frac{\partial B}{\partial t} = -\nabla \times E$
4. $\nabla \times B = \mu_0 \cdot j$
5. $\nabla B = 0$

- j is the current density ;
- μ_0 is the magnetic permeability è la permeabilità magnetica del vuoto;
- B is the magnetic field;
- p is the plasma pressure;
- j × B is the Lorentz force agente sul volume elementare di plasma;
- Equation 3 is the Faraday law
- Equation 4 is the low-frequency Ampere’s law
- Equation 5 is the Gauss Law

Derivation of Grad-Shafranov equation

MHD equilibrium

The the Grad-Shafranov equation is derived from the MHD equilibrium equations given by

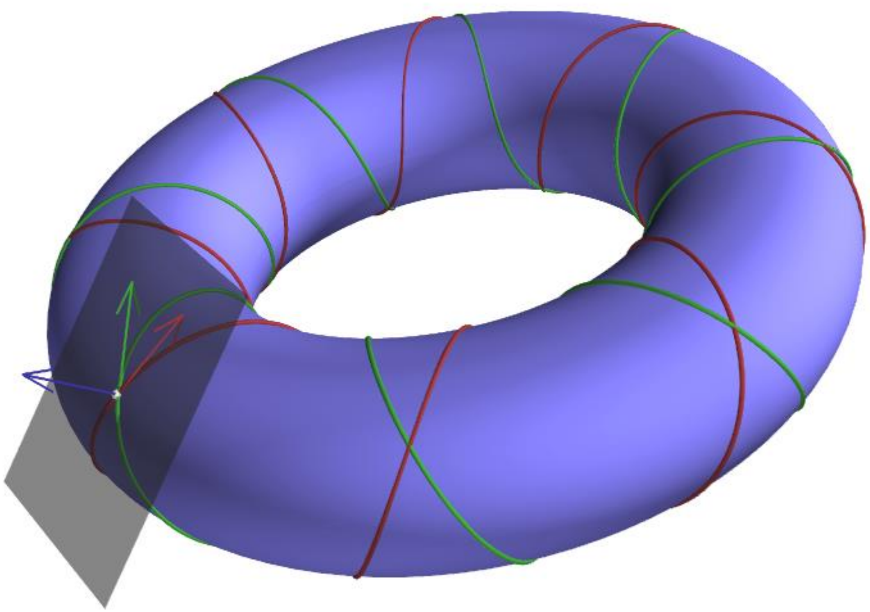
(1) $\nabla p = J \times B$,
(2) $\nabla \times B = \mu_0 J$
(3) $\nabla \cdot B = 0$

Eq.1 is the momentum equation for a plasma in equilibrium: no time variation,
Eq.2 is Ampère's law, relating the current density **J** to the curl of the magnetic flux density, **B**.
Eq.3 is the divergence free postulate, stating that there are no sources of magnetic flux, that is, no magnetic monopoles.

Derivation of Grad-Shafranov equation

$$\nabla \cdot p = J \times B$$

This equation establishes the force balance needed for equilibrium: the pressure gradient (expansion force) needs to be equal to the magnetic force (confinement force). In this way the plasma is in equilibrium. Another important point to note is that the plane defined by **J** and **B** is perpendicular to ∇p. That is, J and B are everywhere tangent to the isosurfaces of pressure. Recall that ∇p is perpendicular to the isosurfaces of p.



The light blue surface depicts an isosurface of pressure, having a toroidal shape. Both the magnetic flux lines and the magnetic flux vector at the point P are represented in green. The current density lines and the current density vector at the point P are represented in red. The vector representing the gradient of pressure, ∇p, is depicted in blue. The tangent plane containing both J and B is represented by a transparent grey surface.

Grad-Shafranov main derivation steps

The Equilibrium code solves the equation of equilibrium force balance in a tokamak assuming axisymmetry in the presence of finite toroidal rotation , *ω*, and zero poloidal rotation:

$$\mathbf{j}\times\mathbf{B} = \nabla p + \rho_m \frac{d\mathbf{v}}{dt} \approx \nabla p - \cancel{\rho_m \omega^2 R \mathbf{e}_R}$$

In cylindrical coordinates and assuming toroidal symmetry (d/dphi=0):

$$j_\phi B_z - j_z B_\phi = \frac{\partial p}{\partial R}$$

The quantities J, p and B can express in terms of poloidal flux (psi).

Grad-Shafranov main derivation steps

From $\nabla \cdot \mathbf{B} = 0$, we obtain two components of the magnetic field

$$\left.\begin{aligned} B_R &= -\frac{1}{2\pi R}\frac{\partial \psi}{\partial z} \\ B_z &= \frac{1}{2\pi R}\frac{\partial \psi}{\partial R} \end{aligned}\right\} B_p$$

From the Ampere's law, we have the third component of the magnetic field:

$$B_\phi = \frac{\mu_0 I}{2\pi R} = B_T$$

Again from the ampere's law:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \quad \Rightarrow \quad j_z = \frac{1}{\mu_0}\frac{\partial B_\phi}{\partial R} = \frac{1}{2\pi R}\frac{\partial I(\psi)}{\partial R}$$

$$j_\phi = \frac{1}{\mu_0}\left(\frac{\partial B_R}{\partial z} - \frac{\partial B_z}{\partial R}\right) = -\frac{1}{2\pi\mu_0}\left(\frac{1}{R}\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial}{\partial R}\frac{1}{R}\frac{\partial \psi}{\partial R}\right) = -\frac{\Delta^* \psi}{2\pi\mu_0 R}$$

Grad-Shafranov main derivation steps

Now we can express the force balance equation in terme of quantities all depending on ψ:

$$j_\phi B_z - j_z B_\phi = \frac{\partial p}{\partial R}$$

Substituting J_φ and J_z

$$\boxed{\Delta^* \psi + \mu_0^2 I\, I' + 4\pi^2 \mu_0 R^2 p' = 0}$$

Where p' is the derivative with respect to ψ

This is the basic equition which is solved by the equilibirum codes in the main devices, for example on JET EFIT++.

EFIT at JET

1985

EFIT *algorithm published by Lang Lao: Lao, L L et al, Nuclear Fusion 25 (1985) 1611*.

1994

EFIT *code ported to JET as* ***EFITJ***

2008

EFIT-F90 *replaces* ***EFITJ*** *for CHAIN-1;* ***EFITJ*** *retained for detailed special purpose reconstructions*

2015

EFIT++ replaces EFITJ and EFIT-F90[1]

[1] https://users.euro-fusion.org/cgi-dvcm/DVCMMeetingPage.perl?date=23-Nov-2015

Features of EFIT++

- EFIT++ is machine-agnostic
  - Runs on JET, MAST, ITER (simulated), COMPASS (Prague), FTU
  - Enable synergetic code development by multiple project teams.
- Composition of EFIT++:
  - Comprehensive documentation,
  - Test suite
  - Induced current and iron models.
  - Input controlled by XML files
  - Output in self-documented netCDF4 files (plus capability to write PPFs!)
  - Comprehensive graphics facilities

Grad Shafranov equation (1)

EFIT++ solves the equation of equilibrium force balance in a tokamak assuming axisymmetry in the presence of finite toroidal rotation , *ω*, and zero poloidal rotation:

$$\mathbf{j}\times\mathbf{B} = \nabla p + \rho_m \frac{d\mathbf{v}}{dt} \approx \nabla p - \rho_m \omega^2 R\mathbf{e}_R$$

Resulting *Grad-Shafranov* equation is[1]

$$\Delta^* \psi_p = -\mu_0 R J_\phi(R,Z)$$

where

$$J_\varphi(R,Z) = \frac{1}{\mu_o R} ff'(\psi_p) + Rp'(\psi_p, R) \quad \text{and} \quad \Delta^* \psi_p = R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi_p}{\partial R}\right) + \frac{\partial^2 \psi_p}{\partial Z^2}$$

in which

$$p(\psi_p, R) = p_o(\psi_p)\exp\left[\frac{TM^2}{2}\right] \quad \text{and} \quad f(\psi_p) = RB_\varphi$$

and, $c_s = \sqrt{kT_i(\psi_p)/m_i}$ $\quad T = T_i/(T_i + T_e)$ $\quad v_\phi = \omega R$ $\quad M = v_\phi / c_s$

[1]*Thyaragaraja, A and McClements, K.G., Phys of Plasmas* ***13*** *026502 (2006)*

Grad Shafranov equation (2)

In the limit $\omega/c_s \ll 1$

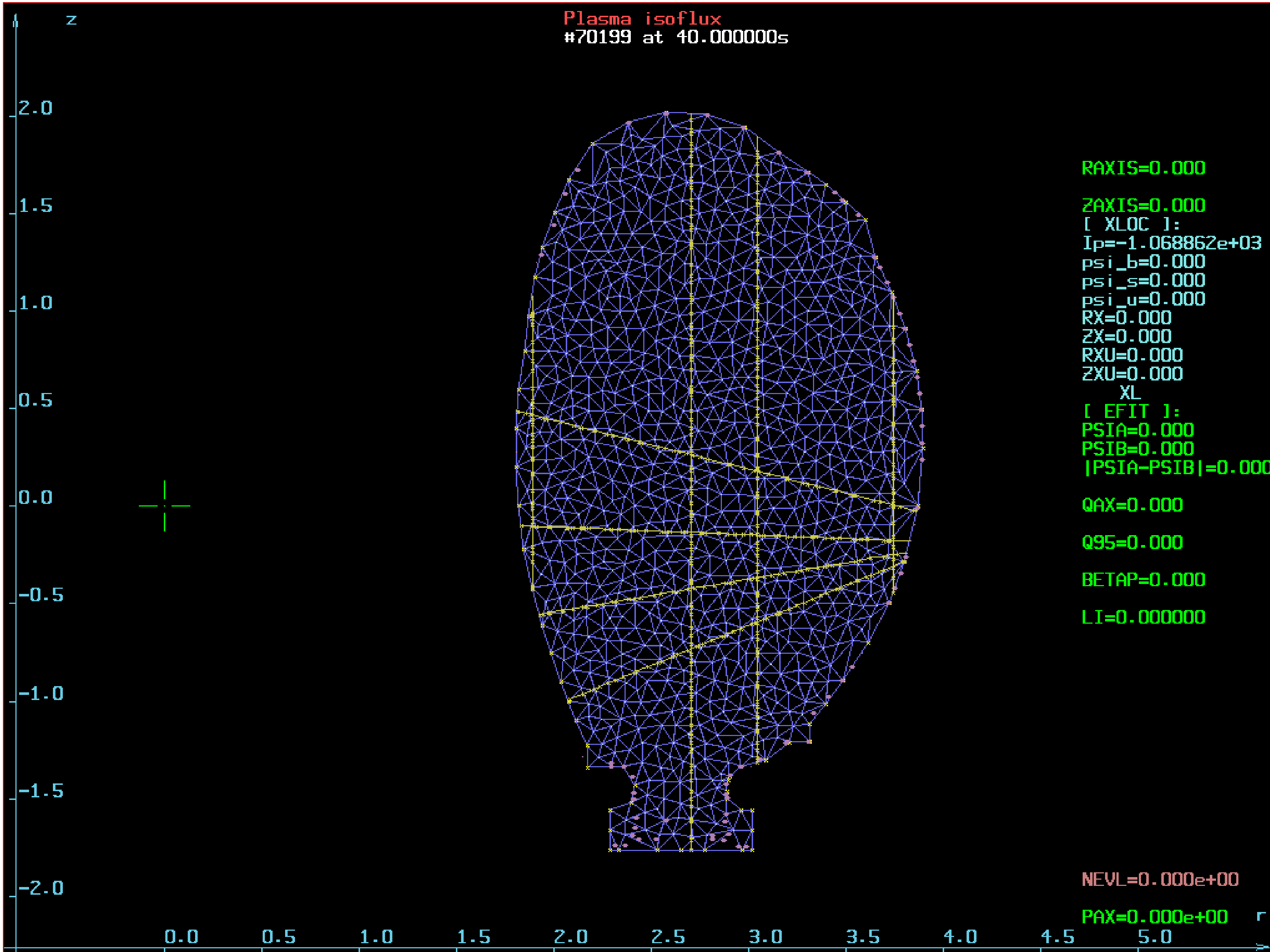
$$p \approx p_o(\psi_p) + \left(\frac{R^2 - R_T^2}{R_T^2}\right) p_w(\psi_p)$$

where

$$p_w(\psi_p) = \frac{TM^2}{2} p_o(\psi_p)$$

yielding

$$J_\varphi(R,Z) = \frac{1}{\mu_o R} ff'(\psi_p) + Rp'(\psi_p) + \left(\frac{R^2 - R_T^2}{R_T^2}\right) Rp'_w(\psi_p)$$



Plasma isoflux
#70199 at 40.000000s
z
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
r
RAXIS=0.000
ZAXIS=0.000
[XLOC]:
Ip=-1.068862e+03
psi_b=0.000
psi_s=0.000
psi_u=0.000
RX=0.000
ZX=0.000
RXU=0.000
ZXU=0.000
XL
[EFIT]:
PSIA=0.000
PSIB=0.000
|PSIA-PSIB|=0.000
QAX=0.000
Q95=0.000
BETAP=0.000
LI=0.000000
NEVL=0.000e+00
PAX=0.000e+00

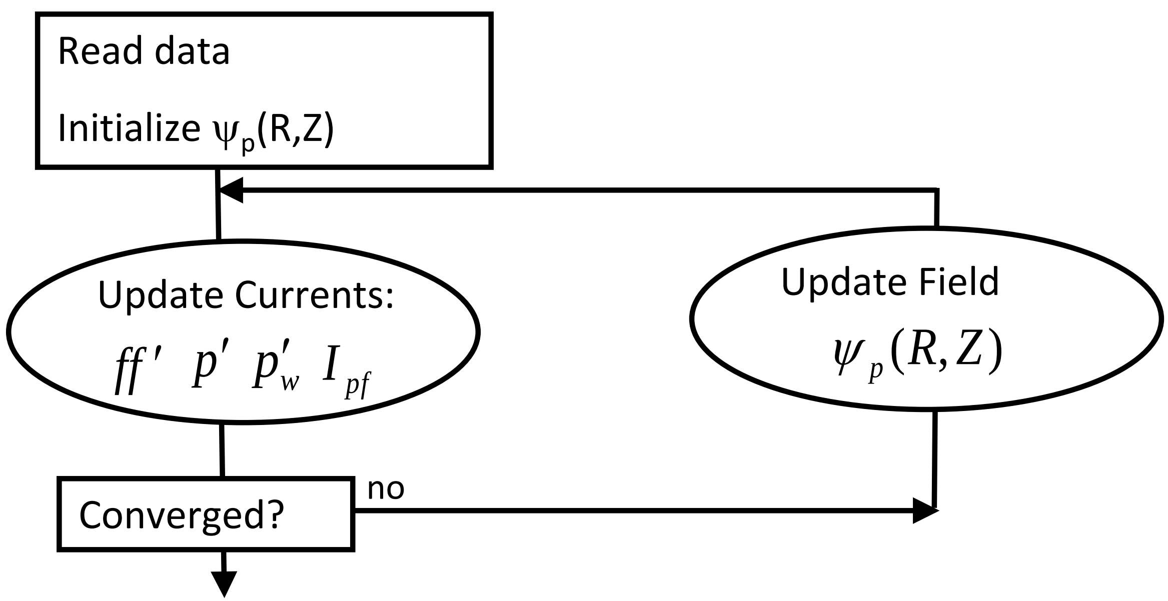
The EFIT++ algorithm

EFIT++ solves a 2-D nonlinear elliptic equation of the form

$$\Delta^* \psi_p = F(ff'(\psi_p), p'(\psi_p), p'_w(\psi_p), R, \{I_{pf}\})$$

Algorithm uses a Picard's iteration scheme. Each iteration solves two separate problems:

- Given $\psi_p(R,Z)$ obtain $ff', p', p'_w, R, \{I_{pf}\})$.
- Given $ff', p', p'_w, R, \{I_{pf}\})$ obtain $\psi_p(R,Z)$



Picard Iteration (1)

1) Given $\psi_p(R,Z)$ obtain $ff', p', p'_w, R, \{I_{pf}\})$.

- *Obtain chi-squared fit to spline coefficients and poloidal currents*

$$\chi^2 = \left| \sum_{i=1}^{N} \chi_i^2 \right|_{Min}$$

$$\chi_i^{\,2} = \left(\frac{M_i - P_i}{\sigma_i} \right)^2$$

M_i is measured value of constraint i *input by user*

ε_{abs} *(absolute error)* and ε_{rel} *(relative error)* *input by user*

W_i *(weight)* *input by user*

Form linearised constraint equations, eg. for magnetic detectors:

$$\sigma_i = \frac{\left| \varepsilon_{abs}, \varepsilon_{rel} M_i \right|_{MAX}}{W_i}$$

Currents and Flux function coefficients

$$P_i(r_i) = \sum_j G(r_i; r_{pf(j)}) I_{pf(j)}^{(m+1)} + \int_{plasma} G(r_i; R, Z) J_\phi(R, \psi_p^{(m)}, \alpha_{1\ldots p}^{(m+1)}, \beta_{1\ldots q}^{(m+1)}, \gamma_{1\ldots r}^{(m+1)}) dRdZ$$

Picard Iteration (2)

2) Given $ff', p', p'_w, R, \{I_{pf}\})$ obtain $\psi_p(R,Z)$.

- *Field solution obtained by solving linearised Grad-Shafranov equation*

$$\Delta^* \psi_p = -\mu_o R J_\varphi$$

using Finite Difference Grid with boundary-integral to represent external fields.

EFIT++ Flux Functions

- The functions: p', p'_w, ff' are 1-D functions of poloidal flux, ψ_p
- Users can select the basis function representation of each flux function
  - Polynomial
  - Tension spline
  - Chebyshev
  - Sine

Polynomial functions

Flux functions written as

$$g(\tilde{\psi}) = \sum_{i=1}^{i=n} \alpha_i \tilde{\psi}^{i-1} - H\tilde{\psi}^n \sum_{i=1}^{i=n} \alpha_i$$

$$\tilde{\psi} = (\psi_p - \psi_p^{axis})/(\psi_p^{LCFS} - \psi_p^{axis})$$

- Polynomial functions are easy to use!
- User defines:
  - Order of polynomial
  - Value of H (putting H=1 provides a simple way to guarantee J_ϕ^{lcfs}=0)
  - *Relational constraints* permit users to relate $\{\alpha_i\}$ coefficients. Enables users to impose some restrictions on the flux functions.
    - Eg to enforce $g(\tilde{\psi}=0)=0$ put $\alpha_1 = 0$.

Tension splines

- Tension splines[1] overcome oscillations present in cubic spline functions by providing a *tension* parameter.
- Users specify a set of knot positions: $\{\tilde{\psi}_i\}$ and a tension parameter *T.*

$$g(\tilde{\psi}) = \frac{g''_i}{T^2}\left[\frac{\sinh\left(T\left(\tilde{\psi}_{i+1}-\tilde{\psi}\right)\right)}{\sinh\left(Th_i\right)} - \frac{1}{h_i}\left(\tilde{\psi}_{i+1}-\tilde{\psi}\right)\right] + \frac{g_i}{h_i}\left(\tilde{\psi}_{i+1}-\tilde{\psi}\right) +$$

$$\frac{g''_{i+1}}{T^2}\left[\frac{\sinh\left(T\left(\tilde{\psi}-\tilde{\psi}_i\right)\right)}{\sinh\left(Th_i\right)} - \frac{1}{h_i}\left(\tilde{\psi}-\tilde{\psi}_i\right)\right] + \frac{g_{i+1}}{h_i}\left(\tilde{\psi}-\tilde{\psi}_i\right)$$

$$h_i = \tilde{\psi}_{i+1} - \tilde{\psi}_i$$

$$\tilde{\psi}_i \le \tilde{\psi} \le \tilde{\psi}_{i+1}$$

- $g(\tilde{\psi})$ *has C^0 continuity automatically enforced at each internal knot.*
- *Tension spline requires* $g''(\tilde{\psi}) - T^2 g(\tilde{\psi})$ *to vary linearly between knots.*
  - ❖ *T=0 =>* $g''(\tilde{\psi})$ *is continuous and varies linearly between knots (cubic spline)*
  - ❖ *T=∞ =>* $g(\tilde{\psi})$ *varies linearly between knots (linear spline)*
- *RelationalConstraints can prescribe* $g(\tilde{\psi})$ *or* $g''(\tilde{\psi})$ *at any knot location.*
  - ❖ *Usually apply natural boundary conditions at end points:* $g''(0) = g''(1) = 0$

[1]*Cline, A,,K, ACM **17** (1974) 218*

Comparing polynomial and splines

The flux functions in EFIT++ can be written as the product of coefficients $\{\alpha_i\},\{\beta_i\},\{\gamma_i\}$ and basis functions $\{a_i\},\{b_i\},\{c_i\}$:

$$p'(\tilde{\psi}) = \sum_{i=1}^{i=n_\alpha} \alpha_i a_i(\tilde{\psi}) \qquad ff'(\tilde{\psi}) = \sum_{i=1}^{i=n_\beta} \beta_i b_i(\tilde{\psi}) \qquad p'_w(\tilde{\psi}) = \sum_{i=1}^{i=n_\gamma} \gamma_i c_i(\tilde{\psi})$$

Polynomials

- the basis functions are: $\begin{vmatrix} a_i \\ b_i \\ c_i \end{vmatrix} = \tilde{\psi}^{i-1} - H \begin{vmatrix} \tilde{\psi}^{n_\alpha} \\ \tilde{\psi}^{n_\beta} \\ \tilde{\psi}^{n_\gamma} \end{vmatrix}$
- Function values at any arbitrary ψ is expressed using all n basis functions.
- The coefficients $\{\alpha_i\},\{\beta_i\},\{\gamma_i\}$ have no physical meaning.
- Difficult to apply constraints on scale-length variations.

Tension Splines

- the coefficients are the values of f and f'' at knot locations (number of knots always even!)
- Function values at any arbitrary ψ is expressed in terms of 4 non-zero terms.
- Storage order of coefficients is:

  $\{f_1, \dots, f_n\} = \{f(\psi_1), f''(\psi_1), \dots, f(\psi_{n/2}), f''(\psi_{n/2})\}$
- Generally use:
  - ✓ natural boundary conditions: $f''(\psi_1) = f''(\psi_{n/2}) = 0$
  - ✓ regularisations: $\{f(\psi_i) - f(\psi_{i+2})\} = 0$