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Q1 Prove that a Pedersen Commitment is homomorphic

Q2 How is the private key SK of an user named “bob” constructed in the Boneh-Franklin’s Identity Based Encryption scheme? (notation: s, g^s : PKG key pair; H(): hash function which maps string into EC point)

- ☐ a) $SK = g^H(\text{bob})$
- ☐ b) $SK = H(\text{bob})^s$
- ☐ c) $SK = \text{bob}^s$
- ☐ d) $SK = g^s \times H(\text{bob})$

Q3 In ECDSA, the private/public key pair is...

- ☐ a) A pair of EC points
- ☐ b) A pair of modular integers
- ☐ c) the private key is a modular integer whereas the public key is an EC point
- ☐ d) the private key is an EC point whereas the public key is a modular integer

Q4 A Secret Sharing scheme is ideal if...

- ☐ a) Each party receives exactly one share
- ☐ b) The total number of participating parties n is equal to the minimum number of parties t which can reconstruct the secret
- ☐ c) the size of each share is an integer value
- ☐ d) none of the above answers

Q5 Describe the RSA common modulus attack

Q6 Determine the access control matrix that implements the policy: $P = \mathbf{A} \text{ AND } \mathbf{B} \text{ AND } (\mathbf{C} \text{ OR } (\mathbf{D} \text{ AND } \mathbf{E}))$

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E1 Consider the Elliptic curve $y^2 = x^3 + x + 1$ defined over the modular integer field Z_7 .

A. find all the points $EC(Z_7)$

$$P = (x_1, y_1)$$

$$Q = (x_2, y_2)$$

$$R = P + Q = (x_3, y_3)$$

$$x_3 = \lambda^2 - x_1 - x_2$$

$$y_3 = \lambda(x_1 - x_3) - y_1$$

B. State what is the order of the corresponding group

$$\lambda = \begin{cases} \frac{y_2 - y_1}{x_2 - x_1} & P \neq Q \\ \frac{3x_1^2 + a}{2y_1} & P = Q \end{cases}$$

C. Compute $[3](2,2)$

[HELP: possibly useful mnemonic hints reported here on the right;

MUST-DO: show step-by-step detailed computations]

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E2 Assume arithmetic modulus 101. A Linear secret sharing scheme involving 4 parties is described by the following access control matrix:

| | | | |
|----|---|---|----|
| A: | 1 | 1 | 0 |
| B: | 0 | 1 | -1 |
| C: | 0 | 0 | -1 |
| D: | 0 | 1 | 1 |

A. Assume that the following shares are revealed:

A \rightarrow 23

B \rightarrow 88

C \rightarrow 57

What is the secret? (explain how you arrived to the result, otherwise the answer is not considered valid)

B. [optional, extra] Assume that the following shares are revealed:

A \rightarrow 79

B \rightarrow 20

D \rightarrow 7

What is the secret? (explain how you arrived to the result, otherwise the answer is not considered valid)

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E3 – part 1 – El Gamal Encryption, $g=29$, $p=83$:

1. Review El Gamal encryption
2. Assume operations are modulo $p=83$: is $g=29$ a generator of the \mathbb{Z}_{83}^* multiplicative group?
[you must respond to this question by performing a single “test”! Trying all possible values in the range is not considered a valid answer]
3. Using $g=29$ and $p=83$, encrypt message $M=37$ for an user whose private key is $sk=7$ and whose public key is $pk=4$ – if you need an ephemeral value, use $r=13$.

E3 – part 2 – Threshold El Gamal Decryption.

If you have not solved the previous part, solve the exercise by using as ciphertext the pair {41,25} *[note: on purpose different from the solution of the previous exercise!]*

The ciphertext produced at the end of the previous part is now sent for threshold decryption to a (2,3) group. The group has been built by sharing the secret key via a (2,3) Shamir Secret Sharing scheme, prime modulus 41.

The three participating parties P_1 , P_2 , P_3 , use standard x-coordinates $x_i = \{1,2,3\}$.

The message is received by parties P_1 and P_3 which have, shares $\sigma_1=26$ and $\sigma_3=23$, respectively

- compute the Lagrange interpolation coefficients for parties 1 and 3;
- Assuming that P_1 and P_3 directly exchange their shares, reconstruct the original secret key
- Assuming, instead, that P_1 and P_3 do NOT explicitly exchange their shares: show how P_1 and P_3 can still cooperate to decrypt the previous El Gamal encrypted message (and numerically compute the result, showing the step-by-step operations).