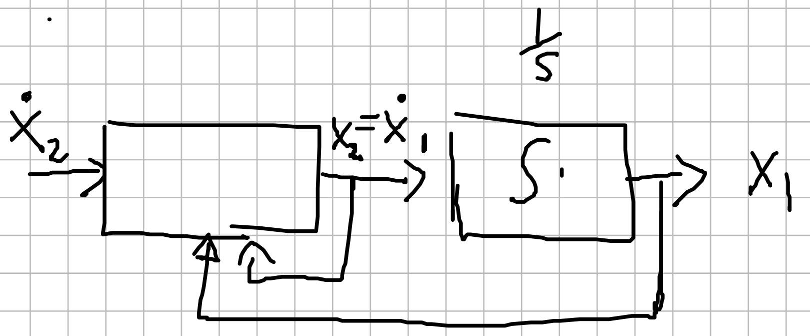
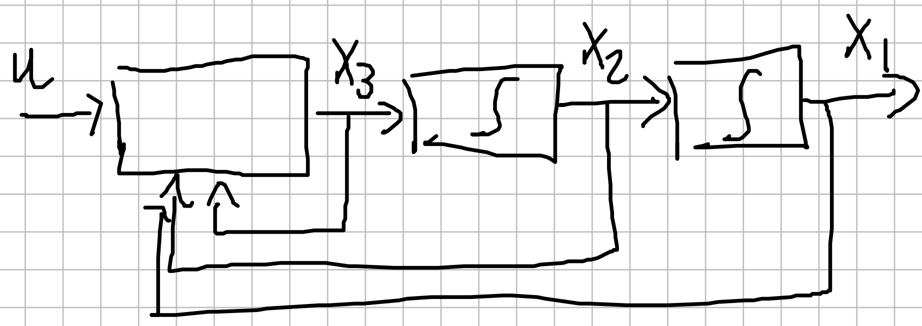


$\dot{x}_1 = x_2$

$\dot{x}_2 = f_1(x_2, x_1)$



$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = f(x, u)$

$$c_1\|x\|^2 \leq V(x) \leq c_2\|x\|^2$$

$$V(x) = x'Px \quad , \quad D_0 := \{x \in \mathbb{R}^n : \|x\| < r_0\}$$

$$f(x) = \underline{Ax + \sigma(x)x}$$

$$\dot V = \dot x' P x + x' P \dot x = (x'A' + x'\sigma'(x))Px +$$

$$+ x'P(Ax + \sigma(x)x)$$

$-Q = Q' > 0$

$$= \underbrace{x'(A'P + PA)x}_{= -Q} + \underbrace{x'\sigma'(x)Px + x'P\sigma(x)x}_{\Delta}$$

$P = P' > 0 : A'P + PA = -Q \;\Leftrightarrow\; \exists Q$

A è HURWITZ

$P(Q)$

$$= -x'Qx + \Delta < 0 \text{ in } 0 \; (\text{se } \|x\| < r_1)$$

$$\dot V \leq -x'Qx + \underbrace{x'(\sigma'(x)P + P\sigma(x))x}_{\leq 2\|x\|^2 \lambda_{max}(P)\cdot L}$$

$$\leq -\lambda_{min}(Q)\|x\|^2 + \underbrace{}$$

$$\leq \underbrace{-\|x\|^2 \underbrace{(\lambda_{min}(Q) - 2\lambda_{max}(P)\cdot L)}_{}}_{>0} < 0$$

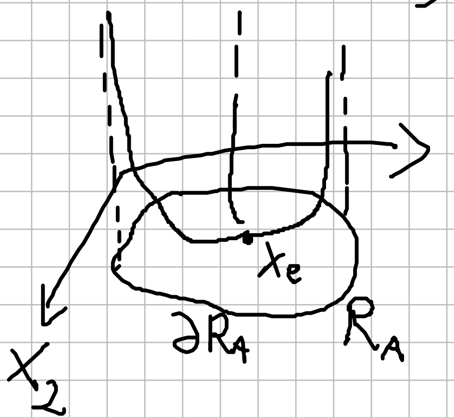
$$L = \sup_{x \in B_{r_1}(0)} \|\sigma(x)\|$$

$k_3 > 0$ in $B_{r_1}(0)$

$$\dot V \leq -k_3\|x\|^2$$

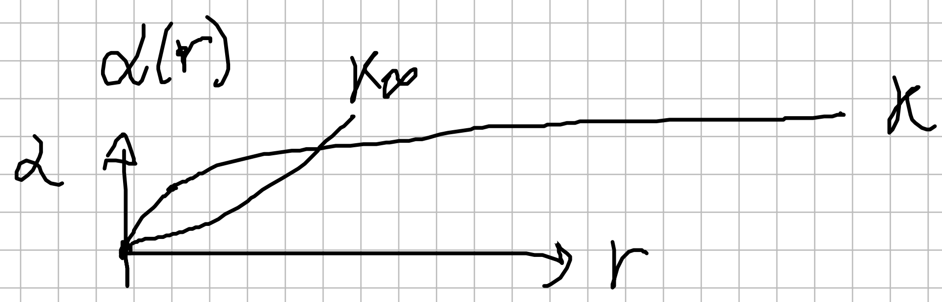
$$\|x(t)\| \le \delta_1 \|x_0\| e^{-\underbrace{\frac{k_3}{2c_2}}_{\lambda>0} t} = \delta_1 \|x_0\| e^{-\lambda t}$$

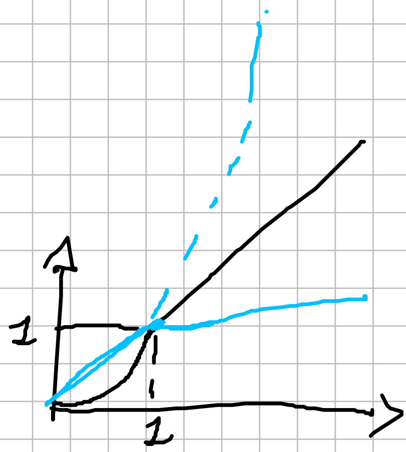
$$\frac{\partial V}{\partial x} f(x) \le \underbrace{-10\,\|x\|}_{W(x)} \qquad \forall x \in R_A$$



f locally Lyp.

$r \in \mathbb{R}$





$\alpha(\|x\|) \triangleq \|x\|$

$r = \|x\|$

$$\|x(t)\| \le \underbrace{c_0 \|x_0\| e^{-\lambda t}}_{\rhd\, KL} \qquad \lambda > 0$$

$$\beta(\underbrace{\|x_0\|}_{r}, \underbrace{t}_{s}) \qquad t \to +\infty$$