Lezione 10

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$$a \to S(a) \to R = (I+S)(I-S)^{-1}$$

$$a = \frac{a}{\|a\|} \cdot \|a\|$$ — NON È L'ANGOLO DI ROTAZIONE (ANGOLI PICCOLI SÌ)

versore

$$\boxed{\|a\| = \frac{s}{c+1}} \qquad \begin{cases} s = \sin(\theta) \\ c = \cos(\theta) \end{cases}$$

θ piccolo $\quad s \approx \theta \quad c \simeq 1 \quad \approx 2\theta$

$a \to S(a) \to R(a)$ rotazione

$$R \text{ rotazione} \begin{cases} RR^T = I \\ R^T R = I \\ \det(R) = 1 \end{cases} \Rightarrow \boxed{S = (R+I)^{-1}(R-I)}$$

1) $R = (I+S)(I-S)^{-1}$

2) S è antisimmetrica

$R = (I+S)(I-S)^{-1} \qquad R(I-S) = I+S$

$R - RS = I+S \qquad R - I = RS + S = (R+I)S$

$$\boxed{S = (R+I)^{-1}(R-I)}$$

R rotazione $\quad R^T R = I$

$R = (I+S)(I-S)^{-1}$

$R^T = (I-S)^{-T}(I+S)^T$

$((I-S)^T)^{-1}$

$(I-S^T)^{-1}(I+S^T)$

$$(I-S^T)^{-1}(I+S^T)(I+S)(I-S)^{-1} = I$$

$$(I+S^T)(I+S)(I-S)^{-1} = (I-S^T)$$

$$(I+S^T)(I+S) = (I-S^T)(I-S)$$

$$\cancel{I} + S^T + S + \cancel{S^T S} = \cancel{I} - S^T - S + \cancel{S^T S}$$

$$(S^T + S) = -(S^T + S)$$

$$(S^T + S) + (S^T + S) = 0$$

$$\cancel{2}(S^T + S) = 0$$

$$S^T + S = 0 \Rightarrow S^T = -S \quad \text{ANTISIMMETRICA!}$$

$\to S \to \quad R = (I+S)(I-S)^{-1} \quad$ ROTAZIONE

$\to R \to \quad S = (R+I)^{-1}(R-I) \quad$ ANTI-SIMMETRICA

$\det(R+I) = 0$

$-1 \in \sigma(R) \qquad \mathbb{R}^3$

$R = (I+S)(I-S)^{-1}$

$$\hat{R} R \hat{R}^T = \hat{R}(I+S)(I-S)^{-1}\hat{R}^T$$

$I = \hat{R}^T \hat{R} \qquad \hat{R}$ rotazione!

$$\hat{R} R \hat{R}^T = \hat{R}(I+S)\hat{R}^T \cdot \hat{R}(I-S)^{-1}\hat{R}^T$$

$$\hat{R}\hat{R}^T + \hat{R} S(a) \hat{R}^T = I + S(\hat{R}a)$$

$$\hat{R}(I-S)^{-1}\hat{R}^T = \left(\hat{R}(I-S)\hat{R}^T\right)^{-1} = \left(\hat{R}\hat{R}^T - \hat{R}S(a)\hat{R}^T\right)^{-1}$$

$\hat{R}(I-S)^{-1}\hat{R}^T$

$$= \left(I - S(\hat{R}a)\right)^{-1}$$

$$\boxed{\hat{R} R \hat{R}^T = (I + S(\hat{R}a))(I - S(\hat{R}a))^{-1}}$$

$R = R_x(\theta)$

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{bmatrix} \qquad S = (R_x + I)^{-1}(R_x - I)$$

$$R_x + I = \begin{bmatrix} 2 & 0 & 0 \\ 0 & c+1 & -s \\ 0 & s & c+1 \end{bmatrix} \qquad \begin{bmatrix} c+1 & -s \\ s & c+1 \end{bmatrix}^{-1} = \frac{1}{d}\begin{bmatrix} c+1 & s \\ -s & c+1 \end{bmatrix}$$

$$(R_x + I)^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & (\;\; &)^{-1} \\ 0 & & \end{bmatrix}$$

$(c+1)^2 + s^2$

$= c^2 + 2c + 1 + s^2$

$= 2c + 2$

$= 2(c+1) = d$

$$(R_x + I)^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{c+1}{d} & \frac{s}{d} \\ 0 & -\frac{s}{d} & \frac{c+1}{d} \end{bmatrix} \qquad (R_x - I) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c-1 & -s \\ 0 & s & c-1 \end{bmatrix}$$

$$\frac{1}{d}\begin{bmatrix} c+1 & s \\ -s & c+1 \end{bmatrix}\begin{bmatrix} c-1 & -s \\ s & c-1 \end{bmatrix}$$

$-s \quad c-1$

$$= \frac{1}{d}\begin{bmatrix} \cancel{c^2-1+s^2} \;\; 0 & \cancel{-cs} - s + \cancel{cs} - s = -2s \\ \cancel{-cs} + s + \cancel{cs} + s = 2s & s^2 + c^2 - 1 = 0 \end{bmatrix}$$

$$= \frac{1}{d}\begin{bmatrix} 0 & -2s \\ 2s & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{2s}{d} \\ 0 & \frac{2s}{d} & 0 \end{bmatrix}$$

$R_x(\theta)$

$$a = \begin{bmatrix} \frac{2s}{d} \\ 0 \\ 0 \end{bmatrix} = \frac{2s}{d} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

e_x

$$a = \frac{\cancel{2}s}{\cancel{2}(c+1)}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{s}{c+1}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$R \to S \to a = \frac{a}{\|a\|} \cdot \|a\| \qquad \boxed{\|a\| = \frac{s}{c+1}} \quad \alpha$$

ASSE DI ROTAZIONE

ANGOLO DI ROTAZIONE

$$\alpha = \frac{s}{c+1} \to \alpha^2 = \frac{s^2}{(c+1)^2} = \frac{1-c^2}{(c+1)^2} = \frac{(1-c)\cancel{(1+c)}}{(1+c)^{\cancel{2}}}$$

$$\alpha^2 = \frac{1-c}{1+c} \Rightarrow$$ CALCOLO IL COSENO

$(1+c)\alpha^2 = 1-c \qquad \alpha^2 + c\alpha^2 = 1-c$

$\alpha^2 - 1 = -(1+\alpha^2)c \qquad c = \frac{1-\alpha^2}{1+\alpha^2}$

$$\cos(\theta) = \frac{1 - \|a\|^2}{1 + \|a\|^2}$$

$$s^2 = 1 - c^2 = 1 - \frac{(1-\alpha^2)^2}{(1+\alpha^2)^2} = 1 - \frac{1+\alpha^4-2\alpha^2}{1+\alpha^4+2\alpha^2}$$

$$s^2 = \frac{\cancel{1+\alpha^4}+2\alpha^2 \cancel{-1-\alpha^4}+2\alpha^2}{1+\alpha^4+2\alpha^2} = \frac{4\alpha^2}{(1+\alpha^2)^2}$$

$$\sin(\theta) = \pm\frac{2\alpha}{1+\alpha^2} \qquad \frac{s}{1+c} \text{ POSITIVO}$$

$$\theta = \operatorname{atan2}\left(\pm\frac{2\alpha}{1+\alpha^2}, \frac{1-\alpha^2}{1+\alpha^2}\right)$$