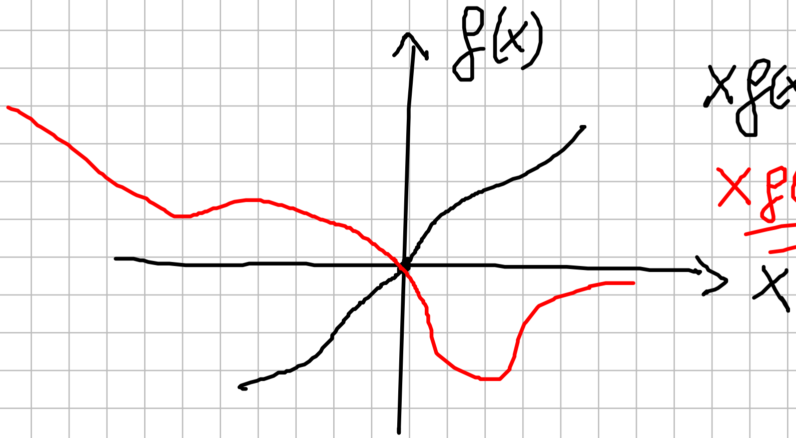


$x f(x) > 0$ se $x \neq 0$

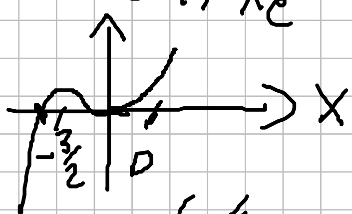
$\underline{x f(x) < 0}$

$\boxed{\dot{x} = -2x^3} = -2x\,x^2 = -2x f(x) \Big|_{f(x) = x^2}$

$V(x) = \int_0^x f(\tau)\,d\tau = \int_0^x \tau^2\,d\tau = \frac{\tau^3}{3}\Big|_{\tau=0}^{\tau=x} = \frac{x^3}{3}$ $x > -\frac{3}{2}$

$V(x) > 0$ in x_e $\exists \rho > 0 : \forall x \in B_\rho(x_e)$ $V(x) > 0$ e $x \neq x_e$ $V(x) = x^2\left(\frac{1}{2} + \frac{x}{3}\right) > 0$

$V(x) = \frac{x^2}{2} + \frac{x^3}{3}$ se $\underline{|x| < 1}$



$\dot{V} = -2x^4 - 2x^5 = -2(x^4 + x^5) \leq -2(x^4 - |x|^5) = -2x^4(1 - |x|) < 0$

$V(x) = \frac{x^2}{2}$ $\dot{x} = -2x^3$ $V(x) > 0$ in (0)

$V(x)$ è rad.-ill.

$\dot{V}(x) = x\dot{x} = -2x^4 < 0 \ \forall x \in \mathbb{R},\ x \neq (0) \triangleq x_e$

$x = 0$ è G.A.S.

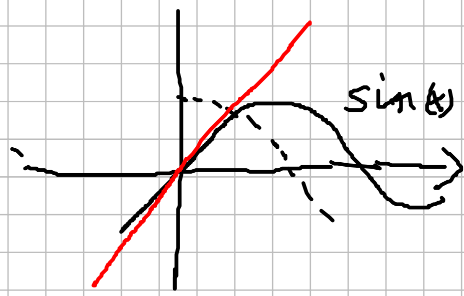
$$\dot{X} = \begin{bmatrix}\dot X_1\\ \dot X_2\end{bmatrix} = \begin{bmatrix} +\sin(X_1) - 2X_1 \\ X_1^2 - X_2^3\end{bmatrix} \qquad V(X) = X'\underbrace{\begin{bmatrix}\frac12 & 0\\ 0 & \frac{c_2}{2}\end{bmatrix}}_{P} X$$

$$V(X) = \frac{X_1^2}{2} + c_2\frac{X_2^2}{2}, \quad c_2 > 0, \quad V(X) > 0 \text{ in } X_e \text{ è rad. ill.}$$

$$\dot V = X_1(\sin(x_1) - 2X_1) + c_2 X_2 X_1^2 - c_2 X_2^4$$

$$= X_1 \underbrace{\sin(x_1)}_{(\cdot) \le |x_1|} - 2X_1^2 + c_2 X_2 X_1^2 - c_2 X_2^4$$

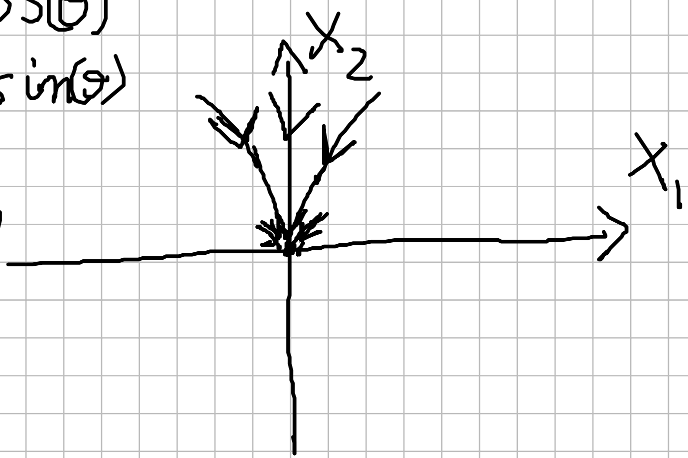
$$\le \cancel{X_1^2} - \cancel{2}X_1^2 + \underbrace{\underline{c_2 X_2 X_1^2} - c_2 X_2^4}_{HOT}$$



$$\frac{\partial \sin(x)}{\partial x} \le 1$$

$$\frac{\partial x}{\partial x} = 1$$

$$X_1 = \rho\cos(\theta)$$
$$X_2 = \rho\sin(\theta)$$
$$\rho \to 0 \quad \forall\theta$$



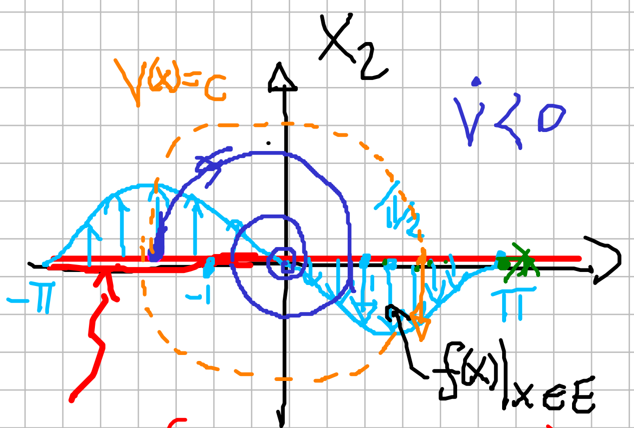
$$\dot X = \begin{bmatrix} \sin(x_1) - 2X_1 \\ X_1^2 - X_2^3 - X_2 \end{bmatrix}$$

$$V(X) = \frac{X_1^2}{2} + c_2\frac{X_2^2}{2}$$

$$\dot V \le -X_1^2 + \underbrace{c_2 X_2 X_1^2 - X_2^4} - X_2^2$$

$$\le \underbrace{\underbrace{-X_1^2 - X_2^2}_{\substack{-\|X\|^2 \\ < 0}} + \underbrace{\qquad\qquad}}_{\substack{< 0 \\ X \in \Omega_\beta}} \Rightarrow \text{Bacino di attraz.}$$

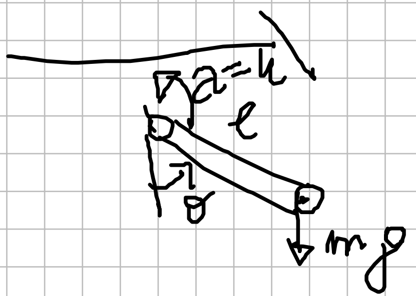
è A.S.



$$\dot V = -b x_2^2 \implies x_e \text{ è STAB.}$$

$$\dot x_1 = x_2$$
$$\dot x_2 = -\underset{\geq 0}{e}\sin(x_1) - b x_2$$

$$E = \left\{ x \in \mathbb{R}^2 : \begin{array}{l} x_2 = 0 \\ \dot V = 0 \end{array} \right\}$$



$\theta = x_1$

$$\dot x_1 = x_2$$
$$\dot x_2 = -e\sin(x_1) - b x_2 + \underline{\underline{u}}$$

$u = 0$ quando $x = 0 = x_e$

$$V(x) = \frac{x_1^2}{2} + x_1 x_2 c + \frac{x_2^2}{2} = x' \underbrace{\begin{bmatrix} \frac{1}{2} & \frac{c}{2} \\ \frac{c}{2} & \frac{1}{2} \end{bmatrix}}_{=P} x > 0$$

$u(x) = 0$ $x = 0$ in x_e

$$P > 0 \iff \frac{1}{2} > 0, \quad \frac{1}{4} - \frac{c^2}{4} > 0 \quad -1 < c < 1 \qquad c = P_{12}$$

$$\dot V = \cancel{x_1} x_2 + x_2^2 c + (c\,x_1 + x_2)(-e\sin(x_1) - b\cancel{x_2} + u)$$

$$-1 < c = 1/6 < 1$$

$$= \boxed{x_2^2 c - b\, x_2^2} - \underbrace{c\, e \sin(x_1) x_1}_{<0} + x_2 e \sin(x_1)$$
$$\leq -\varepsilon x_2^2$$
$$+ (c\, x_1 + x_2)\, u$$

$$u = b x_2 - a\sin(x_1) + u_1$$

$$c\,x_1 = -x_2$$

$$\dot V = x_1 x_2 + x_2^2 c + (c\,x_1 + x_2)(u_1) < 0$$

$$V(x)|_u \to \dot V < 0$$

$$\dot{X} = \begin{bmatrix} X_2 \\ -a\sin(X_1) - bX_2 + u(x) \end{bmatrix} \qquad u(x) = a\sin(X_1) - \underline{\underline{K_1 X_1}}$$

$$= \begin{bmatrix} X_2 \\ -KX_1 - bX_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -K_1 & -b \end{bmatrix}}_{= A_{cl}} X \longrightarrow \lambda^2 + b\lambda + K_1 \qquad \dot{V} < 0 \text{ in } X_e$$

$$\dot{X} = AX, \quad X_e \text{ è A.S.} \Longrightarrow X_e = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ è GES}$$

G. (Vrad. ill)

$$\|X(t) - X_e\| \le c\,\|X(0) - X_e\|\, e^{-\lambda t}$$

$\exists (c, \lambda) > 0$

$$V(x) = x' P x, \quad P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = P', \quad \underline{P_{11} > 0, \ P_{11}P_{22} - P_{12}^2 > 0 \Leftrightarrow P > 0}$$

$V(x) > 0$ in X_e.

$$\dot{V} = \dot{x}' P x + x' P \dot{x} = x' A' P x + x' P A x = x' \underbrace{(A'P + PA)}_{= -Q} x$$

$$Q = Q' > 0 \qquad Q = \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix}; \Leftrightarrow (q_{11}, q_{22}) > 0$$

$$\begin{bmatrix} 0 & -K_1 \\ 1 & -b \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -K_1 & -b \end{bmatrix} = -\begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix}$$

$$\begin{bmatrix} -K_1 P_{21} & -K_1 P_{22} \\ P_{11} - bP_{21} & P_{12} - bP_{22} \end{bmatrix} + \begin{bmatrix} -K_1 P_{12} & P_{11} - P_{12} b \\ -K_1 P_{22} & P_{21} - bP_{22} \end{bmatrix} = \begin{bmatrix} -q_{11} & 0 \\ 0 & -q_{22} \end{bmatrix}$$

$$\begin{bmatrix} -2K_1 P_{21} & P_{11} - P_{12} b - K_1 P_{22} \\ P_{11} - P_{12} b - K_1 P_{22} & 2(P_{12} - bP_{22}) \end{bmatrix} = \begin{bmatrix} -q_{11} & 0 \\ 0 & -q_{22} \end{bmatrix}$$

$$P_{12} = P_{21} = +\frac{q_{11}}{2K_1}$$

$$2\left(\frac{q_{11}}{2K_1} - bP_{22}\right) = -q_{22} \Longleftrightarrow P_{22} = \frac{1}{b}\left(\frac{q_{22}}{2} + \frac{q_{11}}{2K_1}\right)$$

$$P_{11} = P_{12}b + K_1P_{22} = \underbrace{\frac{q_{11}b}{2K_1} + \frac{K_1}{b2}q_{22} + \frac{q_{11}}{2b}}_{>0}$$

$\Rightarrow$ G.E.S. $\Rightarrow$ $P_{11} > 0 \Longleftrightarrow$ $P_{11}P_{22} - P_{21}^2 > 0$ $\Longleftrightarrow (K_1, b) > 0$ $\quad K_{11} >> \emptyset$

$$\boxed{x^+ = Ax}$$

$V = x'Px \rightarrow \underline{\delta V(x) = V(x^+) - V(x)}$

x_e è A.S. (GES) $\Longleftrightarrow \delta V < 0$ in x_e

STAB $\Longleftrightarrow \delta V \leq 0$ in x_e

INST. $\Longleftrightarrow \delta V > 0$

$$\delta V = V(x^+) - V(x) = (x^+)'Px^+ - x'Px =$$

$$= x'A'PAx - x'Px = x'\underbrace{(A'PA - P)}_{=-Q}x \qquad \exists Q = Q' > 0$$

$\delta V = -x'Qx$ con $Q > 0 \Longleftrightarrow x_e$ è GES

$Q \geq 0 \Longleftrightarrow x_e$ è STAB.

$Q < 0 \Longleftrightarrow x_e$ è INST.

$$\boxed{A'PA - P = -Q}$$

$A'PA - P = -Q = -I$

$x^+ = \begin{bmatrix} 0.1 & 1 \\ 0 & 0.5 \end{bmatrix} x \rightarrow$ $\exists P?:$ $\begin{bmatrix} 0.1 & 0 \\ 1 & 0.5 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 0.1 & 1 \\ 0 & 0.5 \end{bmatrix} - \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$x^+ = \begin{bmatrix} -\sin(x_1)a \\ x_1^2 - \frac{x_2}{2} \end{bmatrix}$; $x_e = ?$

$x_{1e} = -\sin(x_{1e}) \Leftrightarrow x_{1e} = 0$

$x_{2e} = -x_{1e}^2 - x_{2e}$ $\quad x_{2e} = 0$

$x_e = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; $V(x) = x_1^2 + x_2^2 > 0$ in x_e, rad ill.

$\delta V = x_1^{+2} + x_2^{+2} - x_1^2 - x_2^2$

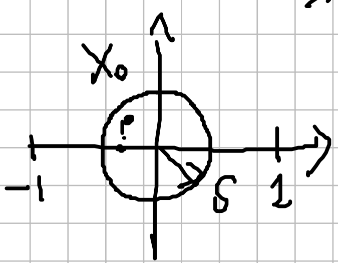
$= a\sin(x_1)^2 + \left(x_1^2 - \frac{x_2}{2}\right)^2 - x_1^2 - x_2^2$, $0 < a < 1$

$\varepsilon = 1 - a$

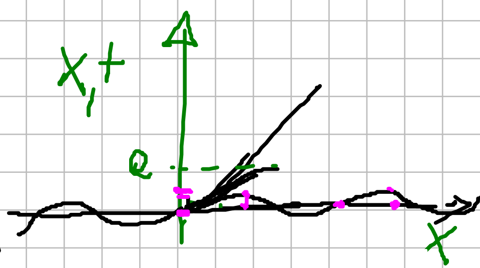
$\leq -x_2^2 + x_1^4 + x_1^2 x_2 + \frac{x_2^2}{4} - \varepsilon x_1^2$

$\leq -\frac{3}{4}x_2^2 + x_1^4 + x_1^2 x_2 \leq 0$ in x_e in $\exists B_\rho(0), \rho > 0$

$|\sin(x_1)| < |x_1|$ $\quad x_1 \neq 0$



$x_1^+ = -a\sin(x_1) \longrightarrow x_1(k) \xrightarrow{k \to +\infty} 0$



$\boxed{x_1^+ = -a\sin(x_1)} \xrightarrow{x_1} \boxed{x_2^+ = -\frac{x_2}{2} + x_1^2} \longrightarrow x_2$

x_{01}

$x_2^+ = \left(-\frac{1}{2}\right)x_2 + u$

$x_2(k) \xrightarrow{k \to +\infty} 0$

$|x(1)| \leq a$

$\tilde{x} = x - x_e = x$

$$\tilde{x}^+ = \begin{bmatrix} -\alpha\cos(x_1) & 0 \\ 2x_1 & -\frac{1}{2} \end{bmatrix}\Bigg|_{\substack{x_1 = x_{1e} = 0 \\ x_2 = x_{2e} = 0}} \tilde{x}$$

$$= \underbrace{\begin{bmatrix} -\alpha & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}}_{\text{SHURR}} \tilde{x} \qquad 0 < \alpha < 1$$

SHURR $\rightarrow$ $\tilde{x} \rightarrow 0$ exp. (locale)

GAS + LES

$\dot{x} = f(x)$ $\qquad \boxed{\dot{\tilde{x}}(t) \simeq A\tilde{x}}$

$$\dot{\tilde{x}}(t) = A\tilde{x} + \underbrace{\sigma(x)\tilde{x}}_{} \rightarrow 0$$

se A è HURWITZ SHURR $\Rightarrow$ x_e è LES

$x \rightarrow 0$

$P = P' > 0$

$V(x) = \underline{x' P x} > 0$ in $x_e = 0$

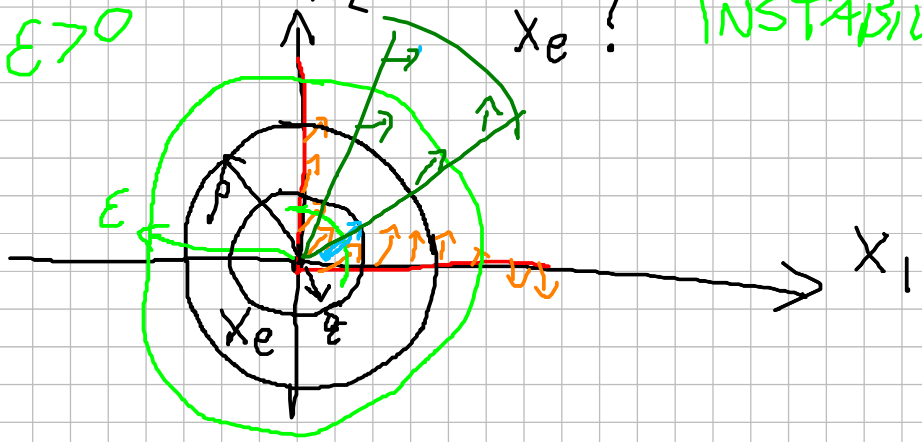
$$\dot{V} = x'\underbrace{(A'P + PA)}_{-Q}x + \cancel{(\ldots)} < 0 \text{ in } x_e$$

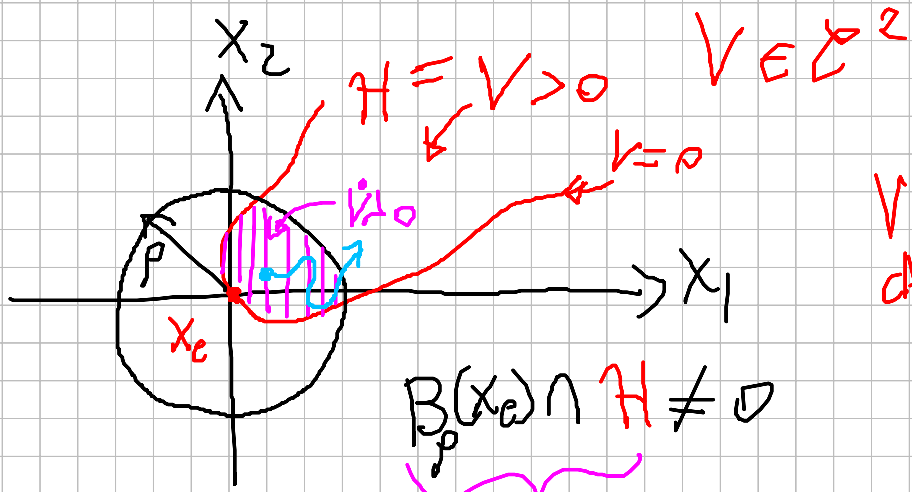
(NON è VERA GLOB.)

$\exists Q : Q' > 0$ $\Rightarrow$ $x_e = 0$ è A.S. $\Rightarrow$ L.E.S.

STUDIO LOCALE

$\varepsilon > 0$ $\qquad$ x_e ? $\qquad$ INSTABILE





$\underbrace{B_\rho(x_e) \cap H}_{\mathcal{N}} \neq \emptyset$

$\mathcal{N}$, $\dot V > 0$ in $\mathcal{N}$

V è la funz. che vi permette

$\Downarrow$

x_e è INST.