PROGETTO RETI

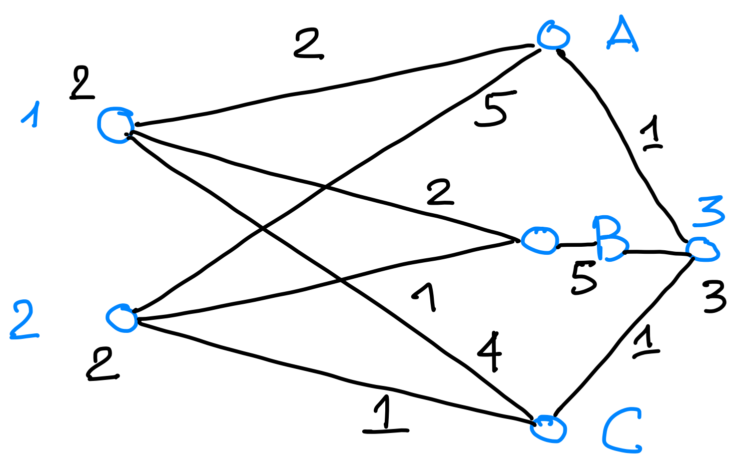
#2 e 3

$N = \{A, B, C\}$

$F = \{1, 2\}$; $f_1 = f_2 = 2$

$f_3 = 3$

$F = \{1, 2, 3\}$



$c(\emptyset) = 0$	0	
$c(\{A\}) = 4$	4	
$c(\{B\}) = 3$	3	
$c(\{C\}) = 3$	3	
$c(\{A,B\}) = 6$	6	
$c(\{B,C\}) = 4$	4	
$c(\{A,C\}) = 7$	5	
$c(N) = 8$	8	

$\alpha_A = 4$ $\quad$ $\alpha_A = 4$

$\alpha_B = 1$ $\quad$ $\alpha_B = 2$

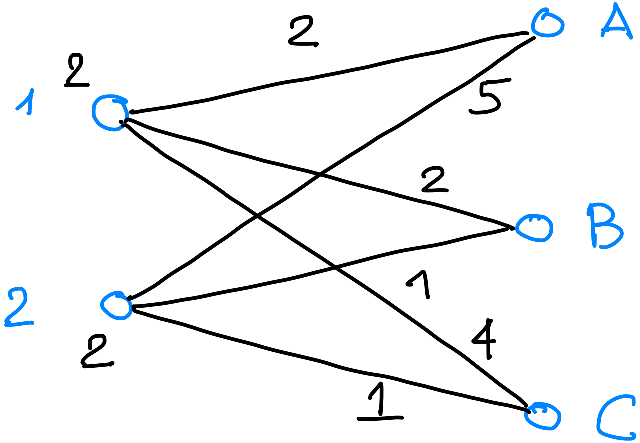
$\alpha_C = 3$ $\quad$ $\alpha_C = 2$ $\quad$ ...

NUCLEO
$$\begin{cases} \alpha_A \le 4 & 0 \\ \alpha_B \le 3 & 0 \\ \alpha_C \le 3 & 0 \\ \alpha_A + \alpha_B \le 6 & \frac{1}{2} \\ \alpha_B + \alpha_C \le 4 & \frac{1}{2} \\ \alpha_A + \alpha_C \le 5 & \frac{1}{2} \\ \alpha_A + \alpha_B + \alpha_C = 8 \end{cases}$$

VETTORE COST SHARING

3.5 $\quad$ 2.5 $\quad$ 1.5

$$\alpha_A + \alpha_B + \alpha_C \le \frac{15}{2}$$



$$Ax \leq b$$

$$A^T y \geq c$$

$$z^* = \max\ 4x_1 + x_2 + 5x_3 + 3x_4 \qquad = cx$$

$$\begin{array}{llll}
y_1 & 0 & 0 & x_1 - x_2 - x_3 + 3x_4 \cancel{\le} 1 \quad (=) \\
y_2 & 1 & 5/3 & 5x_1 + x_2 + 3x_3 + 8x_4 \le 55 \\
y_3 & 1 & 0 & -x_1 + 2x_2 + 3x_3 - 5x_4 \le 3
\end{array}$$

$$x_1, x_2, x_3, x_4 \ge 0$$

$$\max cx \qquad Ax \le b$$

$$\boxed{\begin{array}{l} \min\ y_1 + 55y_2 + 3y_3 \\ \left\{ \dots A^T y \ge c \right. \\ y_1, y_2, y_3 \ge 0 \end{array}}$$

$$\frac{25}{3}x_1 + \frac{5}{3}x_2 + 5x_3 + \frac{40}{3}x_4 \le \frac{275}{3} \longrightarrow z^* \le \frac{275}{3}$$

$$4x_1 + 3x_2 + 6x_3 + 3x_4 \le 58 \longrightarrow z^* \le 58$$

$$\left\{\begin{array}{l}
1\cdot y_1 + 5y_2 - y_3 \ge 4 \\
-y_1 + y_2 + 2y_3 \ge 1 \\
-y_1 + 3y_2 + 3y_3 \ge 5 \\
3y_1 + 8y_2 - 5y_3 \ge 3
\end{array}\right. \longrightarrow z^* \le y_1 + 55y_2 + 3y_3$$

$$y_1, y_2, y_3 \ge 0$$

$x \in \mathbb{R}^n_+$ è nel nucleo γ-approssimato (γ-nucleo) di un gioco cooperativo (N, c) se:

$$\sum_{j \in S} x_j \leq c(S) \quad \forall S \subseteq N$$

$$\sum_{j \in N} x_j \geq \gamma \cdot c(N)$$

$x = (3.5, 2.5, 1.5) \in \frac{15}{16}$ nucleo

PER DEF $\gamma^* \leq 1$

TROVARE $\gamma^* = \max \gamma$: γ NUCLEO $\neq \emptyset$

- come individuare un vettore α che ci permette di ripartire la frazione γ^* di $C(N)$

$$\equiv$$

$$\text{''} \qquad \text{''} \qquad \alpha \in \gamma^*\text{-NUCLEO}$$

TEORIA DEI GIOCHI

$$c(N) = \min \sum_{i \in F} f_i \cdot y_i + \sum_{j \in N} \sum_{i \in F} d_{ij} x_{ij}$$

$$\max \sum_{j \in N} \alpha_j$$

QUANTO PAGA J

$\alpha_j \qquad \sum_{i \in F} x_{ij} \geq 1 \quad \forall j \in N$

$y_i \qquad \sum_{j \in N} \beta_{ij} \leq f_i \quad i \in F$

$\beta_{ij} \qquad y_i - x_{ij} \geq 0 \quad \forall j \in N, i \in F$

$x_{ij} \qquad \alpha_j \leq \beta_{ij} + d_{ij} \quad i \in F, \; j \in N$

CON QUANTO CONTRIBUISCE J ALL'APERTURA DELLA FACILITY i

$\cancel{y_i, x_{ij} \in \{0,1\}}$

$y_i \geq 0$

$x_{ij} \geq 0$

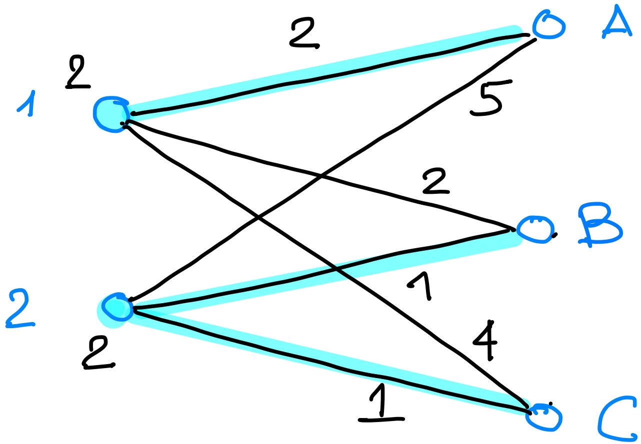
$\alpha_j \geq 0 \quad j \in N$

$\beta_{ij} \geq 0 \quad j \in N, i \in F$

z^*_{RL} : valore soluzione ottima rilassamento lineare

z^*_I : valore " " problema intero $= c(N)$

$$w^*_D = z^*_{RL} \leq z^*_I$$



$$\min \sum_{i\in F} f_i \cdot y_i + \sum_{j\in N}\sum_{i\in F} d_{ij}\, x_{ij}$$

$$\sum_{i\in F} x_{ij} \geq 1 \quad \forall j\in N$$

$$y_i - x_{ij} \geq 0 \quad \forall j\in N, i\in F$$

$$y_i, x_{ij} \geq 0$$

$z_I^* = 8$

$z_{RL}^* = 8;\ y_1 = y_2 = 1$

$x_{1A} = x_{2B} = x_{2C} = 1$ else 0

$w_D^* = 8;\ \alpha_A = 4;\ \alpha_B = 2;\ \alpha_C = 2$

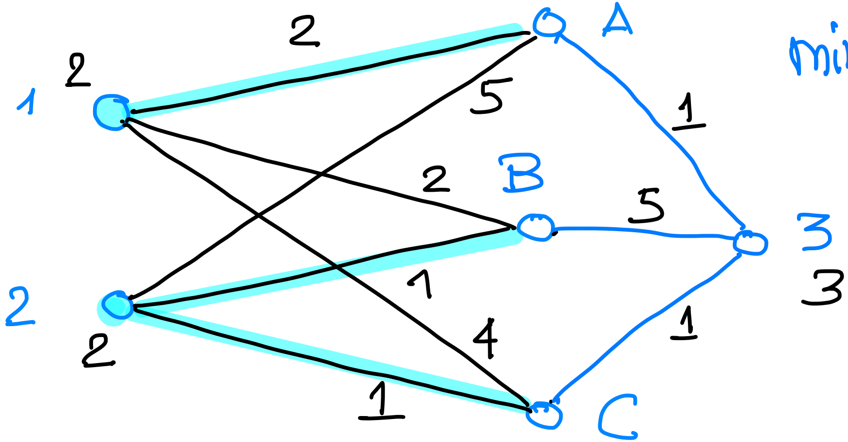
$\beta_{1A} = 2;\ \beta_{2B} = 1;\ \beta_{2C} = 1$

$$\max \sum_{j\in N} \alpha_j$$

$$\sum_{j\in N} \beta_{ij} \leq f_i \quad i\in F$$

$$\alpha_j \leq \beta_{ij} + d_{ij} \quad i\in F,\ j\in N$$

$$\alpha_j, \beta_{ij} \geq 0$$



$$\min \sum_{i\in F} f_i \cdot y_i + \sum_{j\in N}\sum_{i\in F} d_{ij}\, x_{ij}$$

$$\sum_{i\in F} x_{ij} \geq 1 \quad \forall j\in N$$

$$y_i - x_{ij} \geq 0 \quad \forall j\in N, i\in F$$

$$y_i, x_{ij} \geq 0$$

$z_I^* = 8;$

$z_{RL}^* = \frac{15}{2}$

$y_1 = y_2 = y_3 = 1/2$

$x_{C2} = x_{C3} = 1/2$

$x_{B1} = x_{B2} = 1/2$

$x_{A1} = x_{A3} = 1/2$

$w_D^* = \frac{15}{2}$ → DA TROVARE

$$\max \sum_{j\in N} \alpha_j$$

$$\sum_{j\in N} \beta_{ij} \leq f_i \quad i\in F$$

$$\alpha_j \leq \beta_{ij} + d_{ij} \quad i\in F,\ j\in N$$

$$\alpha_j, \beta_{ij} \geq 0$$