Matrice di Inerzia

Definiamo P:= coordinate di un ponto generico solidale al copro ($\hat{P}$):

$$\mathbb{I} = \rho \int_V S^T(P) S(P) \partial V \quad \text{con} \, V = \int_V \partial V \, , \, \rho = \frac{M}{V}$$

Ipotizziamo che il punto $P = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ abbia x,y,z sui piani di simmetria del corpo. Allora:

$$S^T S = \begin{pmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{pmatrix}$$

Se la densità del corpo ϱ=cost:

$$\mathbb{I} = \frac{M}{V} \iiint \begin{pmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{pmatrix} \partial x \partial y \partial z$$

In alternativa:

$$\mathbb{I} = \iiint \rho(x,y,z) \begin{pmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{pmatrix} \partial x \partial y \partial z$$

Parallelepipedo di lati A,B,C senza tappi

$$x \in \left[-\frac{A}{2}, \frac{A}{2} \right], y \in \left[-\frac{B}{2}, \frac{B}{2} \right], z \in \left[-\frac{C}{2}, \frac{C}{2} \right]$$

$$\mathbb{I} = \frac{M}{V} \iiint \begin{pmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{pmatrix} \partial x \partial y \partial z$$

$$x^2 \longrightarrow \int_{-\frac{C}{2}}^{\frac{C}{2}} \int_{-\frac{B}{2}}^{\frac{B}{2}} \int_{-\frac{A}{2}}^{\frac{A}{2}} x^2 \partial x \partial y \partial z = \int_{-\frac{C}{2}}^{\frac{C}{2}} \int_{-\frac{B}{2}}^{\frac{B}{2}} \left[\frac{x^3}{3} \right]_{-\frac{A}{2}}^{\frac{A}{2}} = \int_{-\frac{C}{2}}^{\frac{C}{2}} \int_{-\frac{B}{2}}^{\frac{B}{2}} \frac{A^3}{12} \partial y \partial z = \frac{A^3}{12} \mathrm{BC}$$

$$\rho x^2 \Rightarrow \frac{M}{\mathrm{ABC}} \frac{A^3}{12} \mathrm{BC} = \frac{\mathrm{MA}^2}{12}$$

$$y^2 \longrightarrow \frac{\mathrm{MB}^2}{12}; z^2 \longrightarrow \frac{\mathrm{MC}^2}{12}$$

$$\mathrm{xy} \longrightarrow \int_{-\frac{C}{2}}^{\frac{C}{2}} \int_{-\frac{B}{2}}^{\frac{B}{2}} \int_{-\frac{A}{2}}^{\frac{A}{2}} \mathrm{xy} \partial x \partial y \partial z = \int_{-\frac{C}{2}}^{\frac{C}{2}} \int_{-\frac{B}{2}}^{\frac{B}{2}} \left[\frac{x^2}{2} y \right]_{-\frac{A}{2}}^{\frac{A}{2}} \partial y \partial z = 0$$

$$\mathrm{xz} \longrightarrow 0; \mathrm{yz} \longrightarrow 0$$

Quindi:

$$\mathbb{I} = \frac{M}{12}\begin{pmatrix} B^2+C^2 & 0 & 0 \\ 0 & A^2+C^2 & 0 \\ 0 & 0 & A^2+B^2 \end{pmatrix}$$

```
Maxima 5.44.0 http://maxima.sourceforge.net
using Lisp SBCL 2.0.0
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
```

```
(%i1) SSt:matrix([y^2+z^2,-y*x,-x*z],[-y*z,x^2+z^2,-y*z],[-x*z,-y*z,x^2+y^2]);
```

(%o1) $$\begin{pmatrix} z^2+y^2 & -x\,y & -x\,z \\ -y\,z & z^2+x^2 & -y\,z \\ -x\,z & -y\,z & y^2+x^2 \end{pmatrix}$$

```
(%i1) paral(A,B,C,M):=block([x2,y2,z2,xy,xz,yz,rho,I],
                rho:M/(A*B*C),
                x2:integrate(integrate(integrate(x^2,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                y2:integrate(integrate(integrate(y^2,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                z2:integrate(integrate(integrate(z^2,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                xy:integrate(integrate(integrate(x*y,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                xz:integrate(integrate(integrate(x*z,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                yz:integrate(integrate(integrate(y*z,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                I:zeromatrix(3,3),
                I[1][1]:y2+z2,I[1][2]:-xy,I[1][3]:-xz,
                I[2][1]:I[1][2],I[2][2]:x2+z2,I[2][3]:-yz,
                I[3][1]:I[1][3],I[3][2]:I[2][3],I[3][3]:x2+y2,
                I:ratsimp(rho*I),
                print("Matrice di Ineriza"),
                print(I)


                              );
```

(%o2) $\text{paral}(A,B,C,M) := \textbf{block}\left([x2, y2, z2, \text{xy}, \text{xz}, \text{yz}, \rho, I], \rho: \frac{M}{A\,B\,C}, x2:\right.$
$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(x^2, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), y2:$
$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(y^2, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), z2:$
$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(z^2, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), \text{xy}:$
$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(x\,y, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), \text{xz}:$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(x\,z, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), \text{yz}:$$
$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(y\,z, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), I: \text{zeromatrix}(3,3), (I_1)_1:$$
$$y2+z2, (I_1)_2: -\text{xy}, (I_1)_3: -\text{xz}, (I_2)_1: (I_1)_2, (I_2)_2: x2+z2, (I_2)_3: -\text{yz}, (I_3)_1: (I_1)_3, (I_3)_2: (I_2)_3, (I_3)_3:$$
$$x2+y2, I: \text{ratsimp}(\rho\, I), \text{print}(\text{Matrice di Ineriza }), \text{print}(I)\Big)$$

```
(%i3) paral(A,B,C,M);
```

Matrice di Ineriza

$$\begin{pmatrix} \frac{(C^2+B^2)\,M}{12} & 0 & 0 \\ 0 & \frac{(C^2+A^2)\,M}{12} & 0 \\ 0 & 0 & \frac{(B^2+A^2)\,M}{12} \end{pmatrix}$$

$$\text{(\%o3)} \begin{pmatrix} \frac{(C^2+B^2)\,M}{12} & 0 & 0 \\ 0 & \frac{(C^2+A^2)\,M}{12} & 0 \\ 0 & 0 & \frac{(B^2+A^2)\,M}{12} \end{pmatrix}$$

```
(%i4)
```

Parallelepipedo di lato A,B,C con tappi A',B',C'

```
(%i1) paral(A,B,C,M):=block([x2,y2,z2,xy,xz,yz,rho,I],
                rho:M/(A*B*C),
                x2:integrate(integrate(integrate(x^2,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                y2:integrate(integrate(integrate(y^2,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                z2:integrate(integrate(integrate(z^2,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                xy:integrate(integrate(integrate(x*y,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                xz:integrate(integrate(integrate(x*z,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                yz:integrate(integrate(integrate(y*z,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                I:zeromatrix(3,3),
                I[1][1]:y2+z2,I[1][2]:-xy,I[1][3]:-xz,
                I[2][1]:I[1][2],I[2][2]:x2+z2,I[2][3]:-yz,
                I[3][1]:I[1][3],I[3][2]:I[2][3],I[3][3]:x2+y2,
                I:ratsimp(rho*I),
                print("Matrice di Ineriza"),
                print(I)



                        );
```

$$\text{(\%o1)}\ \text{paral}(A,B,C,M) := \textbf{block}\left([x2, y2, z2, \text{xy}, \text{xz}, \text{yz}, \rho, I], \rho: \frac{M}{A\,B\,C}, x2:\right.$$
$$\left.\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(x^2, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), y2:\right.$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(y^2,x,\frac{-A}{2},\frac{A}{2}\right),y,\frac{-B}{2},\frac{B}{2}\right),z,\frac{-C}{2},\frac{C}{2}\right),z2:$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(z^2,x,\frac{-A}{2},\frac{A}{2}\right),y,\frac{-B}{2},\frac{B}{2}\right),z,\frac{-C}{2},\frac{C}{2}\right),\text{xy}:$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(x\,y,x,\frac{-A}{2},\frac{A}{2}\right),y,\frac{-B}{2},\frac{B}{2}\right),z,\frac{-C}{2},\frac{C}{2}\right),\text{xz}:$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(x\,z,x,\frac{-A}{2},\frac{A}{2}\right),y,\frac{-B}{2},\frac{B}{2}\right),z,\frac{-C}{2},\frac{C}{2}\right),\text{yz}:$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(y\,z,x,\frac{-A}{2},\frac{A}{2}\right),y,\frac{-B}{2},\frac{B}{2}\right),z,\frac{-C}{2},\frac{C}{2}\right),I:\text{zeromatrix}(3,3),(I_1)_1:$$
$$y2+z2,(I_1)_2:-\text{xy},(I_1)_3:-\text{xz},(I_2)_1:(I_1)_2,(I_2)_2:x2+z2,(I_2)_3:-\text{yz},(I_3)_1:(I_1)_3,(I_3)_2:(I_2)_3,(I_3)_3:$$
$$x2+y2,I:\text{ratsimp}(\rho\,I),\text{print}(\text{Matrice di Ineriza }),\text{print}(I)\Bigg)$$

```
(%i2) paralTappi(A,B,C,A1,B1,C1,M):=block(
                V:A*B*C-A1*B1*C1,
                rho:M/V,
                x2:integrate(integrate(integrate(x^2,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                y2:integrate(integrate(integrate(y^2,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                z2:integrate(integrate(integrate(z^2,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                xy:integrate(integrate(integrate(x*y,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                xz:integrate(integrate(integrate(x*z,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                yz:integrate(integrate(integrate(y*z,x,-A/2,A/2),y,-B/2,B/2),z,-C/
      2,C/2),
                x21:integrate(integrate(integrate(x^2,x,-A1/2,A1/2),y,-B1/2,B1/2),
      z,-C1/2,C1/2),
                y21:integrate(integrate(integrate(y^2,x,-A1/2,A1/2),y,-B1/2,B1/2),
      z,-C1/2,C1/2),
                z21:integrate(integrate(integrate(z^2,x,-A1/2,A1/2),y,-B1/2,B1/2),
      z,-C1/2,C1/2),
                xy1:integrate(integrate(integrate(x*y,x,-A1/2,A1/2),y,-B1/2,B1/2),
      z,-C1/2,C1/2),
                xz1:integrate(integrate(integrate(x*z,x,-A1/2,A1/2),y,-B1/2,B1/2),
      z,-C1/2,C1/2),
                yz1:integrate(integrate(integrate(y*z,x,-A1/2,A1/2),y,-B1/2,B1/2),
      z,-C1/2,C1/2),
                x2:x2-x21,y2:y2-y21,z2:z2-z21,
                xy:xy-xy1,xz:xz-xz1,yz:yz-yz1,
                I:zeromatrix(3,3),
                I[1][1]:y2+z2,I[1][2]:-xy,I[1][3]:-xz,
                I[2][1]:I[1][2],I[2][2]:x2+z2,I[2][3]:-yz,
                I[3][1]:I[1][3],I[3][2]:I[2][3],I[3][3]:x2+y2,
                I:ratsimp(expand(rho*I)),
                print("Matrice di Ineriza"),
                print(I));
```

$$\text{(\%o2)}\quad \text{paralTappi}(A,B,C,A1,B1,C1,M):=\textbf{block}\left(V:A\,B\,C-A1\,B1\,C1,\rho:\frac{M}{V},x2:\right.$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(x^2, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), y2:$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(y^2, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), z2:$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(z^2, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), \text{xy:}$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(x\,y, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), \text{xz:}$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(x\,z, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), \text{yz:}$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(y\,z, x, \frac{-A}{2}, \frac{A}{2}\right), y, \frac{-B}{2}, \frac{B}{2}\right), z, \frac{-C}{2}, \frac{C}{2}\right), x21:$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(x^2, x, \frac{-A1}{2}, \frac{A1}{2}\right), y, \frac{-B1}{2}, \frac{B1}{2}\right), z, \frac{-C1}{2}, \frac{C1}{2}\right), y21:$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(y^2, x, \frac{-A1}{2}, \frac{A1}{2}\right), y, \frac{-B1}{2}, \frac{B1}{2}\right), z, \frac{-C1}{2}, \frac{C1}{2}\right), z21:$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(z^2, x, \frac{-A1}{2}, \frac{A1}{2}\right), y, \frac{-B1}{2}, \frac{B1}{2}\right), z, \frac{-C1}{2}, \frac{C1}{2}\right), \text{xy1:}$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(x\,y, x, \frac{-A1}{2}, \frac{A1}{2}\right), y, \frac{-B1}{2}, \frac{B1}{2}\right), z, \frac{-C1}{2}, \frac{C1}{2}\right), \text{xz1:}$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(x\,z, x, \frac{-A1}{2}, \frac{A1}{2}\right), y, \frac{-B1}{2}, \frac{B1}{2}\right), z, \frac{-C1}{2}, \frac{C1}{2}\right), \text{yz1:}$$

$$\text{integrate}\left(\text{integrate}\left(\text{integrate}\left(y\,z, x, \frac{-A1}{2}, \frac{A1}{2}\right), y, \frac{-B1}{2}, \frac{B1}{2}\right), z, \frac{-C1}{2}, \frac{C1}{2}\right), x2: x2 - x21, y2:$$

$y2 - y21$, $z2$: $z2 - z21$, xy: xy $-$ xy1, xz: xz $-$ xz1, yz: yz $-$ yz1, I: zeromatrix(3, 3), $(I_1)_1$: $y2 + z2$, $(I_1)_2$: $-$xy, $(I_1)_3$: $-$xz, $(I_2)_1$: $(I_1)_2$, $(I_2)_2$: $x2 + z2$, $(I_2)_3$: $-$yz, $(I_3)_1$: $(I_1)_3$, $(I_3)_2$: $(I_2)_3$, $(I_3)_3$: $x2 + y2$, I:

$$\text{ratsimp}(\text{expand}(\rho\, I)), \text{print}(\text{Matrice di Ineriza }), \text{print}(I)\Big)$$

Parallelepipedo con tappi

```
(%i3) paralTappi(A,B,C,A1,B1,C1,M);
```

Matrice di Ineriza

$$\left(\frac{(A1\,B1\,C1^3 + A1\,B1^3\,C1 - A\,B\,C^3 - A\,B^3\,C)\,M}{12\,A1\,B1\,C1 - 12\,A\,B\,C}, 0, 0; 0, \frac{(A1\,B1\,C1^3 + A1^3\,B1\,C1 - A\,B\,C^3 - A^3\,B\,C)\,M}{12\,A1\,B1\,C1 - 12\,A\,B\,C}, 0; 0, 0,\right.$$
$$\left.\frac{((A1\,B1^3 + A1^3\,B1)\,C1 + (-A\,B^3 - A^3\,B)\,C)\,M}{12\,A1\,B1\,C1 - 12\,A\,B\,C}\right)$$

```
(%o3)
```

$$\left(\frac{(A1\,B1\,C1^3 + A1\,B1^3\,C1 - A\,B\,C^3 - A\,B^3\,C)\,M}{12\,A1\,B1\,C1 - 12\,A\,B\,C}, 0, 0; 0, \frac{(A1\,B1\,C1^3 + A1^3\,B1\,C1 - A\,B\,C^3 - A^3\,B\,C)\,M}{12\,A1\,B1\,C1 - 12\,A\,B\,C}, 0; 0,\right.$$
$$\left.0, \frac{((A1\,B1^3 + A1^3\,B1)\,C1 + (-A\,B^3 - A^3\,B)\,C)\,M}{12\,A1\,B1\,C1 - 12\,A\,B\,C}\right)$$

Parallelepipedo senza tappi

```
(%i4) paralTappi(A,B,C,A,B1,C1,M);
```

Matrice di Ineriza

$$\left(\frac{(B1\,C1^3+B1^3\,C1-B\,C^3-B^3\,C)\,M}{12\,B1\,C1-12\,B\,C},0,0;0,\frac{(B1\,C1^3+A^2\,B1\,C1-B\,C^3-A^2\,B\,C)\,M}{12\,B1\,C1-12\,B\,C},0;0,0,\right.$$
$$\left.\frac{((B1^3+A^2\,B1)\,C1+(-B^3-A^2\,B)\,C)\,M}{12\,B1\,C1-12\,B\,C}\right)$$

(%o4) $$\left(\frac{(B1\,C1^3+B1^3\,C1-B\,C^3-B^3\,C)\,M}{12\,B1\,C1-12\,B\,C},0,0;0,\frac{(B1\,C1^3+A^2\,B1\,C1-B\,C^3-A^2\,B\,C)\,M}{12\,B1\,C1-12\,B\,C},0;0,0,\right.$$
$$\left.\frac{((B1^3+A^2\,B1)\,C1+(-B^3-A^2\,B)\,C)\,M}{12\,B1\,C1-12\,B\,C}\right)$$

```
(%i5)
```

Cilindro di raggio R ed altezza H

$$\mathbb{I}=\frac{M}{V}\iiint\begin{pmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{pmatrix}\partial x\partial y\partial z$$

$$V=A_b\,H=\pi R^2 H$$

$$\rho=\frac{M}{\pi R^2 H}$$

Al fine di svolgere l'integrale, una delle scelte è quella di ricorrere alle coordinate cilindriche. In particolare:

$$\begin{cases} x=\rho\cos(\theta) \\ y=\rho\sin(\theta) \\ z=z \end{cases} \longrightarrow \theta\in[0,2\pi],\,\rho\in[0,R],\,z\in\left[-\frac{H}{2},\frac{H}{2}\right]$$

```
(%i23) cilindro(R,H):=block([x,y,z,a,b,c,d],
                            x:rho*cos(theta),
                            y:rho*sin(theta),
                            z:z,
                             V:%pi*R^2*H,
                    d:M/V,
                    x2:integrate(integrate(integrate(rho*x^2,theta,0,2*%pi),rho,0,R),
       z,-H/2,H/2),
                    y2:integrate(integrate(integrate(rho*y^2,theta,0,2*%pi),rho,0,R),
       z,-H/2,H/2),
                    z2:integrate(integrate(integrate(rho*z^2,theta,0,2*%pi),rho,0,R),
       z,-H/2,H/2),
                    xy:integrate(integrate(integrate(rho*x*y,theta,0,2*%pi),rho,0,R),
       z,-H/2,H/2),
                    xz:integrate(integrate(integrate(rho*x*z,theta,0,2*%pi),rho,0,R),
       z,-H/2,H/2),
                    yz:integrate(integrate(integrate(rho*y*z,theta,0,2*%pi),rho,0,R),
       z,-H/2,H/2),

                    I:zeromatrix(3,3),
                    I[1][1]:y2+z2,I[1][2]:-xy,I[1][3]:-xz,
                    I[2][1]:I[1][2],I[2][2]:x2+z2,I[2][3]:-yz,
                    I[3][1]:I[1][3],I[3][2]:I[2][3],I[3][3]:x2+y2,
                    I:ratsimp((d*I)),
                    print("Matrice di Ineriza"),
                    print(I))
```

(%o23) $\text{cilindro}(R, H) := \textbf{block}\left([x, y, z, a, b, c, d], x\text{: } \rho\cos(\vartheta), y\text{: } \rho\sin(\vartheta), z\text{: } z, V\text{: } \pi R^2 H, d\text{: } \frac{M}{V}, x2\text{: integrate}\left(\text{integrate}(\text{integrate}(\rho x^2, \vartheta, 0, 2\pi), \rho, 0, R), z, \frac{-H}{2}, \frac{H}{2}\right), y2\text{: integrate}\left(\text{integrate}(\text{integrate}(\rho y^2, \vartheta, 0, 2\pi), \rho, 0, R), z, \frac{-H}{2}, \frac{H}{2}\right), z2\text{: integrate}\left(\text{integrate}(\text{integrate}(\rho z^2, \vartheta, 0, 2\pi), \rho, 0, R), z, \frac{-H}{2}, \frac{H}{2}\right), \text{xy: integrate}\left(\text{integrate}(\text{integrate}(\rho x y, \vartheta, 0, 2\pi), \rho, 0, R), z, \frac{-H}{2}, \frac{H}{2}\right), \text{xz: integrate}\left(\text{integrate}(\text{integrate}(\rho x z, \vartheta, 0, 2\pi), \rho, 0, R), z, \frac{-H}{2}, \frac{H}{2}\right), \text{yz: integrate}\left(\text{integrate}(\text{integrate}(\rho y z, \vartheta, 0, 2\pi), \rho, 0, R), z, \frac{-H}{2}, \frac{H}{2}\right), I\text{: zeromatrix}(3, 3), (I_1)_1\text{: } y2 + z2, (I_1)_2\text{: } -\text{xy}, (I_1)_3\text{: } -\text{xz}, (I_2)_1\text{: } (I_1)_2, (I_2)_2\text{: } x2 + z2, (I_2)_3\text{: } -\text{yz}, (I_3)_1\text{: } (I_1)_3, (I_3)_2\text{: } (I_2)_3, (I_3)_3\text{: } x2 + y2, I\text{: ratsimp}(d\, I), \text{print}(\text{Matrice di Ineriza }), \text{print}(I)\right)$

```
(%i24) cilindro(R,H);
```

Matrice di Ineriza

$$\begin{pmatrix} \frac{3\,M\,R^2+H^2\,M}{12} & 0 & 0 \\ 0 & \frac{3\,M\,R^2+H^2\,M}{12} & 0 \\ 0 & 0 & \frac{M\,R^2}{2} \end{pmatrix}$$

(%o24)
$$\begin{pmatrix} \frac{3\,M\,R^2+H^2\,M}{12} & 0 & 0 \\ 0 & \frac{3\,M\,R^2+H^2\,M}{12} & 0 \\ 0 & 0 & \frac{M\,R^2}{2} \end{pmatrix}$$

(%i25)

Cilindro di raggio R, altezza H cavo con cilindro R',H senza tappi

```
(%i40) cilindro(R,H,R1):=block([x,y,z,a,b,c,d],
                              x:rho*cos(theta),
                              y:rho*sin(theta),
                              z:z,
                               V:%pi*(R)^2*H-%pi*(R1)^2*H,
                    d:M/V,
                    x2:integrate(integrate(integrate(rho*x^2,theta,0,2*%pi),rho,R1,
          R),z,0,H),
                    y2:integrate(integrate(integrate(rho*y^2,theta,0,2*%pi),rho,R1,
          R),z,0,H),
                    z2:integrate(integrate(integrate(rho*z^2,theta,0,2*%pi),rho,R1,
          R),z,0,H),
                    xy:integrate(integrate(integrate(rho*x*y,theta,0,2*%pi),rho,R1,
          R),z,0,H),
                    xz:integrate(integrate(integrate(rho*x*z,theta,0,2*%pi),rho,R1,
          R),z,0,H),
                    yz:integrate(integrate(integrate(rho*y*z,theta,0,2*%pi),rho,R1,
          R),z,0,H),

                    I:zeromatrix(3,3),
                    I[1][1]:y2+z2,I[1][2]:-xy,I[1][3]:-xz,
                    I[2][1]:I[1][2],I[2][2]:x2+z2,I[2][3]:-yz,
                    I[3][1]:I[1][3],I[3][2]:I[2][3],I[3][3]:x2+y2,
                    I:ratsimp(expand(d*I)),
                    print("Matrice di Ineriza"),
                    print(I))
```

(%o40) cilindro$(R, H, R1) :=$ **block** $\Big([x, y, z, a, b, c, d], x\colon \rho\cos(\vartheta), y\colon \rho\sin(\vartheta), z\colon z, V\colon \pi\,R^2\,H - \pi\,R1^2\,H, d\colon \frac{M}{V}, x2\colon \text{integrate}(\text{integrate}(\text{integrate}(\rho\,x^2, \vartheta, 0, 2\,\pi), \rho, R1, R), z, 0, H), y2\colon \text{integrate}(\text{integrate}(\text{integrate}(\rho\,y^2, \vartheta, 0, 2\,\pi), \rho, R1, R), z, 0, H), z2\colon \text{integrate}(\text{integrate}(\text{integrate}(\rho\,z^2, \vartheta, 0, 2\,\pi), \rho, R1, R), z, 0, H), \text{xy}\colon \text{integrate}(\text{integrate}(\text{integrate}(\rho\,x\,y, \vartheta, 0, 2\,\pi), \rho, R1, R), z, 0, H), \text{xz}\colon \text{integrate}(\text{integrate}(\text{integrate}(\rho\,x\,z, \vartheta, 0, 2\,\pi), \rho, R1, R), z, 0, H), \text{yz}\colon \text{integrate}(\text{integrate}(\text{integrate}(\rho\,y\,z, \vartheta, 0, 2\,\pi), \rho, R1, R), z, 0, H), I\colon \text{zeromatrix}(3, 3), (I_1)_1\colon y2 + z2, (I_1)_2\colon -\text{xy}, (I_1)_3\colon -\text{xz}, (I_2)_1\colon (I_1)_2, (I_2)_2\colon x2 + z2, (I_2)_3\colon -\text{yz}, (I_3)_1\colon (I_1)_3, (I_3)_2\colon (I_2)_3, (I_3)_3\colon x2 + y2, I\colon \text{ratsimp}(\text{expand}(d\,I)), \text{print}(\text{Matrice di Ineriza }), \text{print}(I)\Big)$

```
(%i41) cilindro(R,H,R[1]);
```

Matrice di Ineriza

$$\begin{pmatrix} \frac{3\,M\,R^2+(4\,H^2+3\,R_1^2)\,M}{12} & 0 & 0 \\ 0 & \frac{3\,M\,R^2+(4\,H^2+3\,R_1^2)\,M}{12} & 0 \\ 0 & 0 & \frac{M\,R^2+R_1^2\,M}{2} \end{pmatrix}$$

(%o41)
$$\begin{pmatrix} \frac{3\,M\,R^2+(4\,H^2+3\,R_1^2)\,M}{12} & 0 & 0 \\ 0 & \frac{3\,M\,R^2+(4\,H^2+3\,R_1^2)\,M}{12} & 0 \\ 0 & 0 & \frac{M\,R^2+R_1^2\,M}{2} \end{pmatrix}$$

Cilindro di raggio R, altezza H cavo con cilindro R',H' con tappi

```
(%i52) cilindro(R,H):=block([x,y,z,a,b,c,d],
                            x:rho*cos(theta),
                            y:rho*sin(theta),
                            z:z,

                  x2:integrate(integrate(integrate(rho*x^2,theta,0,2*%pi),rho,0,R),
       z,-H/2,H/2),
                  y2:integrate(integrate(integrate(rho*y^2,theta,0,2*%pi),rho,0,R),
       z,-H/2,H/2),
                  z2:integrate(integrate(integrate(rho*z^2,theta,0,2*%pi),rho,0,R),
       z,-H/2,H/2),
                  xy:integrate(integrate(integrate(rho*x*y,theta,0,2*%pi),rho,0,R),
       z,-H/2,H/2),
                  xz:integrate(integrate(integrate(rho*x*z,theta,0,2*%pi),rho,0,R),
       z,-H/2,H/2),
                  yz:integrate(integrate(integrate(rho*y*z,theta,0,2*%pi),rho,0,R),
       z,-H/2,H/2),

                  I:zeromatrix(3,3),
                  I[1][1]:y2+z2,I[1][2]:-xy,I[1][3]:-xz,
                  I[2][1]:I[1][2],I[2][2]:x2+z2,I[2][3]:-yz,
                  I[3][1]:I[1][3],I[3][2]:I[2][3],I[3][3]:x2+y2,
                  I:ratsimp(I)
                  )
```

(%o52) cilindro$(R,H) :=$ **block**$\Big([x,y,z,a,b,c,d], x$: $\rho\cos(\vartheta), y$: $\rho\sin(\vartheta), z$: $z, x2$:
integrate$\Big($integrate$($integrate$(\rho\,x^2,\vartheta,0,2\,\pi),\rho,0,R),z,\frac{-H}{2},\frac{H}{2}\Big), y2$:
integrate$\Big($integrate$($integrate$(\rho\,y^2,\vartheta,0,2\,\pi),\rho,0,R),z,\frac{-H}{2},\frac{H}{2}\Big), z2$:
integrate$\Big($integrate$($integrate$(\rho\,z^2,\vartheta,0,2\,\pi),\rho,0,R),z,\frac{-H}{2},\frac{H}{2}\Big),$ xy:
integrate$\Big($integrate$($integrate$(\rho\,x\,y,\vartheta,0,2\,\pi),\rho,0,R),z,\frac{-H}{2},\frac{H}{2}\Big),$ xz:
integrate$\Big($integrate$($integrate$(\rho\,x\,z,\vartheta,0,2\,\pi),\rho,0,R),z,\frac{-H}{2},\frac{H}{2}\Big),$ yz:
integrate$\Big($integrate$($integrate$(\rho\,y\,z,\vartheta,0,2\,\pi),\rho,0,R),z,\frac{-H}{2},\frac{H}{2}\Big), I$: zeromatrix$(3,3), (I_1)_1$: $y2+$

$z2, (I_1)_2: -\mathrm{xy}, (I_1)_3: -\mathrm{xz}, (I_2)_1: (I_1)_2, (I_2)_2: x2 + z2, (I_2)_3: -\mathrm{yz}, (I_3)_1: (I_1)_3, (I_3)_2: (I_2)_3, (I_3)_3: x2 + y2, I: \mathrm{ratsimp}(I)\Big)$

```
(%i53) cilindroSV(R,H,H1,R1):=block([x,y,z,a,b,c,d],
                    V:%pi*R^2*H-%pi*R1^2*H1,
                  d:M/V,
                  I:ratsimp(expand(cilindro(R,H))),
                  Icavo:ratsimp(expand(cilindro(R1,H1))),
                  print("Matrice di Ineriza"),
                  print(ratsimp(d*I)))
```

(%o53) $\mathrm{cilindroSV}(R, H, H1, R1) := \textbf{block}\left([x, y, z, a, b, c, d], V: \pi\, R^2\, H - \pi\, R1^2\, H1, d: \frac{M}{V}, I: \mathrm{ratsimp}(\mathrm{expand}(\mathrm{cilindro}(R, H))), \mathrm{Icavo}: \mathrm{ratsimp}(\mathrm{expand}(\mathrm{cilindro}(R1, H1))), \mathrm{print}(\mathrm{Matrice\ di\ Ineriza\ }), \mathrm{print}(\mathrm{ratsimp}(d\, I))\right)$

```
(%i54) cilindroSV(R,H,H[1],R[1]);
```

Matrice di Ineriza

$$\begin{pmatrix} \frac{(3\,H_1\,R_1^4 + H_1^3\,R_1^2)\,M}{12\,H\,R^2 - 12\,H_1\,R_1^2} & 0 & 0 \\ 0 & \frac{(3\,H_1\,R_1^4 + H_1^3\,R_1^2)\,M}{12\,H\,R^2 - 12\,H_1\,R_1^2} & 0 \\ 0 & 0 & \frac{H_1\,R_1^4\,M}{2\,H\,R^2 - 2\,H_1\,R_1^2} \end{pmatrix}$$

(%o54)
$$\begin{pmatrix} \frac{(3\,H_1\,R_1^4 + H_1^3\,R_1^2)\,M}{12\,H\,R^2 - 12\,H_1\,R_1^2} & 0 & 0 \\ 0 & \frac{(3\,H_1\,R_1^4 + H_1^3\,R_1^2)\,M}{12\,H\,R^2 - 12\,H_1\,R_1^2} & 0 \\ 0 & 0 & \frac{H_1\,R_1^4\,M}{2\,H\,R^2 - 2\,H_1\,R_1^2} \end{pmatrix}$$

```
(%i55)
```

Sfera di raggio R

$$\mathbb{I} = \frac{M}{V}\iiint \begin{pmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{pmatrix} \partial x \partial y \partial z$$

$$V = \frac{4}{3}\pi R^3$$

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

Al fine di svolgere l'integrale, una delle scelte è quella di ricorrere alle coordinate sferiche. In particolare:

$$\begin{cases} x = \rho\sin(\phi)\cos(\theta) \\ y = \rho\sin(\phi)\sin(\theta) \\ z = \rho\cos(\phi) \end{cases} \longrightarrow \theta \in [0; 2\pi],\ \rho \in \left[-\frac{R}{2}, \frac{R}{2}\right],\ \phi \in [0; 2\pi]$$

```
(%i57) sfera(R):=block([x,y,z,a,b,c,d],
                          x:rho*cos(theta)*sin(phi),
                          y:rho*sin(theta)*sin(phi),
                          z:rho*cos(phi),
                           V:(4/3)*%pi*(R)^3,
                 d:M/V,
                 x2:integrate(integrate(integrate(rho^2*sin(phi)*x^2,theta,0,
       2*%pi),rho,0,R),phi,0,%pi),
                 y2:integrate(integrate(integrate(rho^2*sin(phi)*y^2,theta,0,
       2*%pi),rho,0,R),phi,0,%pi),
                 z2:integrate(integrate(integrate(rho^2*sin(phi)*z^2,theta,0,
       2*%pi),rho,0,R),phi,0,%pi),
                 xy:integrate(integrate(integrate(rho^2*sin(phi)*x*y,theta,0,
       2*%pi),rho,0,R),phi,0,%pi),
                 xz:integrate(integrate(integrate(rho^2*sin(phi)*x*z,theta,0,
       2*%pi),rho,0,R),phi,0,%pi),
                 yz:integrate(integrate(integrate(rho^2*sin(phi)*y*z,theta,0,
       2*%pi),rho,0,R),phi,0,%pi),

                 I:zeromatrix(3,3),
                 I[1][1]:y2+z2,I[1][2]:-xy,I[1][3]:-xz,
                 I[2][1]:I[1][2],I[2][2]:x2+z2,I[2][3]:-yz,
                 I[3][1]:I[1][3],I[3][2]:I[2][3],I[3][3]:x2+y2,
                 I:ratsimp(expand(d*I)),
                 print("Matrice di Ineriza"),
                 print(I))
```

(%o57) $\text{sfera}(R) := \textbf{block}\left([x, y, z, a, b, c, d], x\text{: } \rho\cos(\vartheta)\sin(\varphi), y\text{: } \rho\sin(\vartheta)\sin(\varphi), z\text{: } \rho\cos(\varphi), V\text{: } \frac{4}{3}\pi R^3, d\text{: } \frac{M}{V}, x2\text{: integrate(integrate(integrate(}\rho^2\sin(\varphi)\,x^2, \vartheta, 0, 2\pi), \rho, 0, R), \varphi, 0, \pi), y2\text{: integrate(integrate(integrate(}\rho^2\sin(\varphi)\,y^2, \vartheta, 0, 2\pi), \rho, 0, R), \varphi, 0, \pi), z2\text{: integrate(integrate(integrate(}\rho^2\sin(\varphi)\,z^2, \vartheta, 0, 2\pi), \rho, 0, R), \varphi, 0, \pi), \text{xy: integrate(integrate(integrate(}\rho^2\sin(\varphi)\,x\,y, \vartheta, 0, 2\pi), \rho, 0, R), \varphi, 0, \pi), \text{xz: integrate(integrate(integrate(}\rho^2\sin(\varphi)\,x\,z, \vartheta, 0, 2\pi), \rho, 0, R), \varphi, 0, \pi), \text{yz: integrate(integrate(integrate(}\rho^2\sin(\varphi)\,y\,z, \vartheta, 0, 2\pi), \rho, 0, R), \varphi, 0, \pi), I\text{: zeromatrix}(3, 3), (I_1)_1\text{: } y2 + z2, (I_1)_2\text{: } -\text{xy}, (I_1)_3\text{: } -\text{xz}, (I_2)_1\text{: } (I_1)_2, (I_2)_2\text{: } x2 + z2, (I_2)_3\text{: } -\text{yz}, (I_3)_1\text{: } (I_1)_3, (I_3)_2\text{: } (I_2)_3, (I_3)_3\text{: } x2 + y2, I\text{: ratsimp(expand}(d\,I)), \text{print(Matrice di Ineriza)}, \text{print}(I)\right)$

```
(%i58) sfera(R);
```

Matrice di Ineriza

$$\begin{pmatrix} \frac{2\,M\,R^2}{5} & 0 & 0 \\ 0 & \frac{2\,M\,R^2}{5} & 0 \\ 0 & 0 & \frac{2\,M\,R^2}{5} \end{pmatrix}$$

(%o58)
$$\begin{pmatrix} \frac{2\,M\,R^2}{5} & 0 & 0 \\ 0 & \frac{2\,M\,R^2}{5} & 0 \\ 0 & 0 & \frac{2\,M\,R^2}{5} \end{pmatrix}$$

```
(%i59)
```

Sfera cava di raggio R e R'

```
Maxima 5.44.0 http://maxima.sourceforge.net
using Lisp SBCL 2.0.0
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
```

```
(%i61) sfera(R,R1):=block([x,y,z,a,b,c,d],
                           x:rho*cos(theta)*sin(phi),
                           y:rho*sin(theta)*sin(phi),
                           z:rho*cos(phi),
                            V:(4/3)*%pi*(R)^3-(4/3)*%pi*(R1)^3,
                d:M/V,
                x2:integrate(integrate(integrate(rho^2*sin(phi)*x^2,theta,0,
       2*%pi),rho,R1,R),phi,0,%pi),
                y2:integrate(integrate(integrate(rho^2*sin(phi)*y^2,theta,0,
       2*%pi),rho,R1,R),phi,0,%pi),
                z2:integrate(integrate(integrate(rho^2*sin(phi)*z^2,theta,0,
       2*%pi),rho,R1,R),phi,0,%pi),
                xy:integrate(integrate(integrate(rho^2*sin(phi)*x*y,theta,0,
       2*%pi),rho,R1,R),phi,0,%pi),
                xz:integrate(integrate(integrate(rho^2*sin(phi)*x*z,theta,0,
       2*%pi),rho,R1,R),phi,0,%pi),
                yz:integrate(integrate(integrate(rho^2*sin(phi)*y*z,theta,0,
       2*%pi),rho,R1,R),phi,0,%pi),

                I:zeromatrix(3,3),
                I[1][1]:y2+z2,I[1][2]:-xy,I[1][3]:-xz,
                I[2][1]:I[1][2],I[2][2]:x2+z2,I[2][3]:-yz,
                I[3][1]:I[1][3],I[3][2]:I[2][3],I[3][3]:x2+y2,
                I:ratsimp(expand(d*I)),
                print("Matrice di Ineriza"),
                print(I))
```

(%o61) $\mathrm{sfera}(R, R1) := \textbf{block}\left([x, y, z, a, b, c, d], x\text{: } \rho\cos(\vartheta)\sin(\varphi), y\text{: } \rho\sin(\vartheta)\sin(\varphi), z\text{: } \rho\cos(\varphi), V\text{: } \frac{4}{3}\pi R^3 - \frac{4}{3}\pi R1^3, d\text{: } \frac{M}{V}, x2\text{: } \mathrm{integrate}(\mathrm{integrate}(\mathrm{integrate}(\rho^2\sin(\varphi)\,x^2, \vartheta, 0, 2\pi), \rho, R1, R), \varphi, 0, \pi), y2\text{: } \mathrm{integrate}(\mathrm{integrate}(\mathrm{integrate}(\rho^2\sin(\varphi)\,y^2, \vartheta, 0, 2\pi), \rho, R1, R), \varphi, 0, \pi), z2\text{: } \mathrm{integrate}(\mathrm{integrate}(\mathrm{integrate}(\rho^2\sin(\varphi)\,z^2, \vartheta, 0, 2\pi), \rho, R1, R), \varphi, 0, \pi), \mathrm{xy}\text{: } \mathrm{integrate}(\mathrm{integrate}(\mathrm{integrate}(\rho^2\sin(\varphi)\,x\,y, \vartheta, 0, 2\pi), \rho, R1, R), \varphi, 0, \pi), \mathrm{xz}\text{: } \mathrm{integrate}(\mathrm{integrate}(\mathrm{integrate}(\rho^2\sin(\varphi)\,x\,z, \vartheta, 0, 2\pi), \rho, R1, R), \varphi, 0, \pi), \mathrm{yz}\text{: } \mathrm{integrate}(\mathrm{integrate}(\mathrm{integrate}(\rho^2\sin(\varphi)\,y\,z, \vartheta, 0, 2\pi), \rho, R1, R), \varphi, 0, \pi), I\text{: } \mathrm{zeromatrix}(3, 3), (I_1)_1\text{: } y2 + z2, (I_1)_2\text{: } -\mathrm{xy}, (I_1)_3\text{: } -\mathrm{xz}, (I_2)_1\text{: } (I_1)_2, (I_2)_2\text{: } x2 + z2, (I_2)_3\text{: } -\mathrm{yz}, (I_3)_1\text{: } (I_1)_3, (I_3)_2\text{: } (I_2)_3, (I_3)_3\text{: } x2 + y2, I\text{: } \mathrm{ratsimp}(\mathrm{expand}(d\,I)), \mathrm{print}(\text{Matrice di Ineriza }), \mathrm{print}(I)\right)$

```
(%i62) sfera(R,R[1])
```

Matrice di Ineriza

$$\left(\frac{2\,M\,R^4+2\,R_1\,M\,R^3+2\,R_1^2\,M\,R^2+2\,R_1^3\,M\,R+2\,R_1^4\,M}{5\,R^2+5\,R_1\,R+5\,R_1^2}, 0, 0; 0, \frac{2\,M\,R^4+2\,R_1\,M\,R^3+2\,R_1^2\,M\,R^2+2\,R_1^3\,M\,R+2\,R_1^4\,M}{5\,R^2+5\,R_1\,R+5\,R_1^2}, 0; 0, 0, \frac{2\,M\,R^4+2\,R_1\,M\,R^3+2\,R_1^2\,M\,R^2+2\,R_1^3\,M\,R+2\,R_1^4\,M}{5\,R^2+5\,R_1\,R+5\,R_1^2}\right)$$

(%o62) $$\left(\frac{2\,M\,R^4+2\,R_1\,M\,R^3+2\,R_1^2\,M\,R^2+2\,R_1^3\,M\,R+2\,R_1^4\,M}{5\,R^2+5\,R_1\,R+5\,R_1^2},0,0;0,\right.$$
$$\left.\frac{2\,M\,R^4+2\,R_1\,M\,R^3+2\,R_1^2\,M\,R^2+2\,R_1^3\,M\,R+2\,R_1^4\,M}{5\,R^2+5\,R_1\,R+5\,R_1^2},0;0,0,\frac{2\,M\,R^4+2\,R_1\,M\,R^3+2\,R_1^2\,M\,R^2+2\,R_1^3\,M\,R+2\,R_1^4\,M}{5\,R^2+5\,R_1\,R+5\,R_1^2}\right)$$

(%i63)