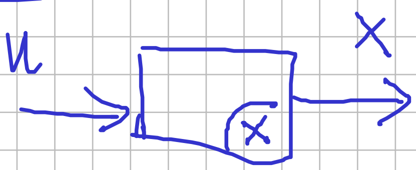
$$\dot{x} = Ax + Bu \qquad x \in \mathbb{R}^n \qquad\qquad x^+ = Ax + Bu$$

$$x_e = Ax_e + Bu_e$$



$$(x_e, u_e) \to \quad 0 = Ax_e + Bu_e$$

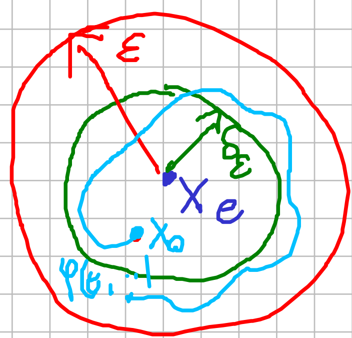
$$\Phi(t, t_0) = e^{A(t-t_0)}$$

$$x(t) = \varphi(t, t_0, x_0, u(\cdot)) = \underbrace{e^{A(t-t_0)} x_0}_{\text{evol. libera}} + \underbrace{\cancel{\int_{t_0}^{t} e^{A(t-\tau)} B\, u(\tau)\, d\tau}}_{\text{risp. stato forzata}}$$

$(x_e, u_e) \to x_e$ è p.to di equilibrio stabile

se $\forall \varepsilon > 0, \exists \delta_\varepsilon > 0 : \forall x_0 : \|\underset{x_0}{x(t_0)} - x_e\| \le \delta_\varepsilon \Rightarrow$

$$\|\varphi(t, t_0, x_0, u_e) - x_e\| < \varepsilon$$

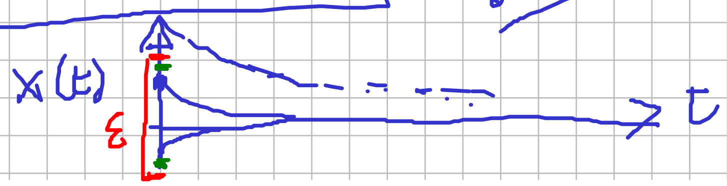


$$\dot{x} = -2x + 4u \to (x_e, u_e) \qquad \boxed{x_e = 2u_e}$$

$$u_e = 0 \Rightarrow x_e = 0$$

$$A = -2 \to \sigma(A) = \{-2\}, \quad t_0 = 0$$

$$x(t) = e^{-2t} x_0 + \cancel{\int_0^t e^{-2(t-\tau)} B\, u(\tau)\, d\tau} \qquad u = u_e = 0$$



$\dot{X} = AX$ sist. Σ e T. C.

Σ è **A. S.** se

$\forall \lambda_i \in \sigma(A) : \mathrm{Re}(\lambda_i) < 0$

Σ è **stabile** se

$\forall \lambda_i \in \sigma(A) : \mathrm{Re}(\lambda_i) \leq 0$ e

$\forall \lambda_i : \mathrm{Re}(\lambda_i) = 0 \Rightarrow \underset{\substack{\text{molt.}\\ \text{algebrica}\\ \text{di } \lambda_i}}{m_i} = \underset{\substack{\text{molt.}\\ \text{geom.}}}{\mu_i}$

di λ_i $\longrightarrow$ quante volte λ_i è radice di $P_A(\lambda) = \det(\lambda I - A)$

$\exists\, v_i \neq 0$: se $\lambda_i \in \sigma(A)$

$(A - \lambda_i I)\, v_i = 0$

$\dim(\ker(A - \lambda_i I)) = \mu_i$

Σ è inst. altrimenti

$\dot{x} = -2x \Rightarrow$ A.S

$\dot{x} = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix} x \Rightarrow \sigma(A) = \{0, -1\} \longrightarrow$ S.

$m = 1 \Rightarrow \mu = 1$

$\det(A - \lambda I) = \lambda(\lambda + 1) = P_A(\lambda)$

$\dot{x} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} x$, $\sigma(A) = \{-1\}$, $m_1 = 2 \Rightarrow \mu_1 = 1$

$P_A(\lambda) = (\lambda + 1)^2$

$(A - \lambda_1 I)\, v_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} v_1 = 0 \iff v_{12} = 0$

$\begin{bmatrix} v_{11} \\ \cancel{v_{12}} \end{bmatrix}$ $\quad v_1 = \begin{bmatrix} \alpha \\ 0 \end{bmatrix}$

$X(t) = \boxed{e^{At}} X_0 = \boxed{\mathcal{L}^{-1}\left((sI - A)^{-1}\right)(t)} X_0$

$\dot{x} = Ax$

$\downarrow \mathcal{L}$

$sX(s) - X_0 = AX(s)$

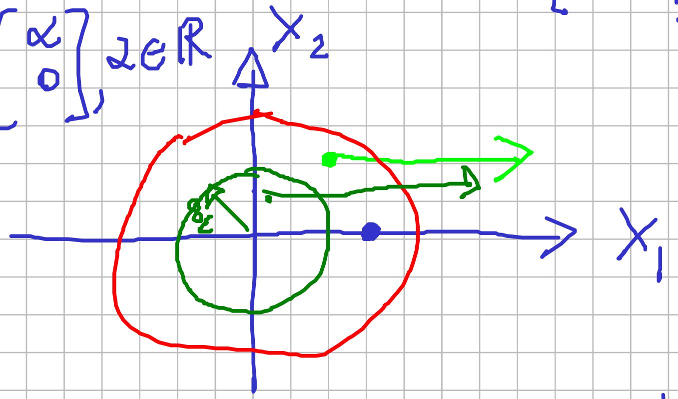
$X(s) = (sI - A)^{-1} X_0$

$z = TX \Rightarrow \dot{z} = T\dot{x} = TAX = \underline{TAT^{-1}}\, z$

$$x(t) = \begin{bmatrix} e^{-t} & te^{-t} \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x \longrightarrow x(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{matrix} x_{10} + t x_{20} \\ x_{20} \end{matrix}$$

$x_e = \begin{bmatrix} \alpha \\ 0 \end{bmatrix}, \alpha \in \mathbb{R}$



$Re(\lambda_1) \leq 0$ INST. (Poli...)

$= 0$

$m_1 \neq \mu_1$

$$\dot{x} = \begin{bmatrix} -1 & 1 & & & & \\ 0 & -1 & & & & \\ & & 0 & & & \\ & & & 3 & & \\ & & & & -2 & 1 \\ & & & & 0 & -2 \end{bmatrix} x$$

$\lambda_1 = -1,\ m_1 = 2,\ \mu_1 = 1$

$\lambda_2 = 0,\ m_2 = 1 = \mu_2$

$\lambda_3 = 3,\ m_3 = 1 = \mu_3$

$\lambda_4 = -2,\ m_4 < \mu_4$

INST.

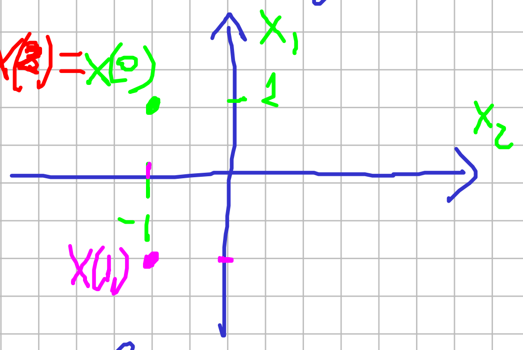
$\Sigma \quad x^+ = Ax, \quad t \in \mathbb{Z}$

Σ è A.S. se $|\lambda_i| < 1 \quad \forall \lambda_i \in \sigma(A)$

Σ è S. se $|\lambda_i| \leq 1$ " e se $|\lambda_i| = 1$ allora $m_i = \mu_i$

Σ instabile altrimenti

$$\begin{matrix} x_1^+ = x_2 \\ x_2^+ = -x_1 \end{matrix} \longrightarrow x^+ = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x$$



$P_A(\lambda) = \det(\lambda I - A) = \begin{vmatrix} \lambda & -1 \\ 1 & \lambda \end{vmatrix} = \lambda^2 + 1 = 0$

$x(2) = Ax(1)$

$\lambda_{1,2} = \pm j$

$\sigma(A) = \{j, -j\} \rightarrow |\lambda_1| = 1,\ |\lambda_2| = 1 \longrightarrow$ STABILE

$$u(s) \longrightarrow \boxed{P(s)} \longrightarrow y(s)$$

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$y(s) = \underbrace{\left(C(sI-A)^{-1}B + D\right)}_{=P(s)} u(s)$$

$x_0 = 0$

$P(s)$ è A.S. $\Longleftrightarrow$ tutti i poli di $P(s) \in \mathbb{C}^-$

S. $\Longleftrightarrow$ " " " $Re(\lambda_i) \leq 0$ e i poli con $Re(\lambda_i) = 0$ sono semplici

INST. altrim.

$$P_1(s) = \frac{*}{(s+2)(s+4)^2} \longrightarrow p_1 = -1 \text{ semplice}, \; p_2 = -4 \text{ doppio} \longrightarrow \text{S.A.}$$

$$P_2(s) = \frac{*}{s(s+1)^2} \longrightarrow \text{S.}$$

$$P_3(s) = \frac{*}{(s+4-j)(s+4+j)\underbrace{s^2}} \longrightarrow \text{INS.}$$

$u(s) = 1 \quad (u(t) = \delta_0(t))$

$$y(t) = \mathcal{L}^{-1}\left(P_3(s)\,u(s)\right) = \mathcal{L}^{-1}\left(\underbrace{\frac{R_1}{s+4-j} + \frac{R_1^*}{s+4+j}} + \frac{R_2}{s} + \frac{R_3}{s^2}\right)(t)$$

$$= 2|R_1|\cos(t + \angle R_1) \cdot e^{-4t} + R_2\,\delta_{-1}(t) + R_3\,t\,\delta_{-1}(t)$$

$P(z) \rightarrow$ A.S. $\Longleftrightarrow$ tutti i poli $\in$ cerchio unitario stretto $(|p_i| < 1)$

S. $\Longleftrightarrow$ $|p_i| \leq 1 \;\forall p_i$ e se $|p_i| = 1$ allora deve essere un polo semplice