Procedura 26

Scrivere una procedura Maxima che, prendendo in ingresso la tabella di Denavit-Hartenberg e le informazioni necessarie per individuare i baricentri dei link, restituisca: energia cinetica dovuta alla rotazione ed energia cinetica dovuta alla traslazione per ogni link e per l'intero robot; la matrice delle inerzie generalizzate per ogni link e per l'intero robot.

```
(%i1) kill(all);
```

(%o0) **done**

Procedure ausiliarie per il calcolo della cinemarica diretta in accordo a Denavit-Hartenberg

```
(%i1) inverseLaplace(SI,theta):=block([res,M,MC,aC,b],
                              M:SI,
                              MC:SI,
                              for i:1 thru 3 do(
                                for j:1 thru 3 do
                                   (
                                     aC:M[i,j],
                                     b:ilt(aC,s,theta),
                                     MC[i,j]:b
                                    )
                                 ),
                              res:MC

                           )
```

(%o1) $\mathrm{inverseLaplace}(\mathrm{SI},\vartheta):=\mathbf{block}\ ([\mathrm{res},M,\mathrm{MC},\mathrm{aC},b],M\colon \mathrm{SI},\mathrm{MC}\colon \mathrm{SI},$ $\mathbf{for}\ i\ \mathbf{thru}\ 3\ \mathbf{do}\ \mathbf{for}\ j\ \mathbf{thru}\ 3\ \mathbf{do}\ (\mathrm{aC}\colon M_{i,j},b\colon \mathrm{ilt}(\mathrm{aC},s,\vartheta),\mathrm{MC}_{i,j}\colon b),\mathrm{res}\colon \mathrm{MC})$

```
(%i2) rotLaplace(k,theta):=block([res,S,I,temp],
                          S:ident(3),
                          I:ident(3),
                        for i:1 thru 3 do
                          (
                          for j:1 thru 3 do
                              (
                                if i=j
                                   then S[i][j]:0
                                elseif j>i
                                   then (
                                  temp:(-1)^(j-i)*k[3-remainder(i+j,3)],
                                         S[i][j]:temp,
                                         S[j][i]:-temp

                                          )
                                )
                           ),
                          res:inverseLaplace(invert(s*I-S),theta)

                        )
```

(%o2) $\mathrm{rotLaplace}(k,\vartheta):=\mathbf{block}\ ([\mathrm{res},S,I,\mathrm{temp}],S\colon \mathrm{ident}(3),I\colon \mathrm{ident}(3),$ $\mathbf{for}\ i\ \mathbf{thru}\ 3\ \mathbf{do}\ \mathbf{for}\ j\ \mathbf{thru}\ 3\ \mathbf{do}\ \mathbf{if}\ i=j\ \mathbf{then}\ (S_i)_j\colon 0\ \mathbf{elseif}\ j>i\ \mathbf{then}\ (\mathrm{temp}\colon$

$(-1)^{j-i}\,k_{3-\mathrm{remainder}(i+j,3)}, (S_i)_j\colon \mathrm{temp}, (S_j)_i\colon -\mathrm{temp}), \mathrm{res}\colon \mathrm{inverseLaplace}(\mathrm{invert}(s\,I-S), \vartheta))$

```
(%i3) Av(v,theta,d):=block([res,Trot,row,Atemp,A],
                          Trot:rotLaplace(v,theta),
                          row:matrix([0,0,0,1]),
                          Atemp:addcol(Trot,d*transpose(v)),
                          A:addrow(Atemp,row),
                          res:A
                          )$
(%i4) Q(theta,d,alpha,a):=block([res,tempMat,Qtrasf],
                               tempMat:Av([0,0,1],theta,d).Av([1,0,0],alpha,a),
                               Qtrasf:zeromatrix(4,4),
                               for i:1 thru 4 do
                          (
                          for j:1 thru 4 do
                             (
                               Qtrasf[i][j]:trigreduce(tempMat[i][j])
                                         )
                             ),

                               res:Qtrasf
                               )
```

(%o4) $Q(\vartheta, d, \alpha, a) := \mathbf{block}\ ([\mathrm{res}, \mathrm{tempMat}, \mathrm{Qtrasf}], \mathrm{tempMat}\colon \mathrm{Av}([0,0,1], \vartheta, d) \cdot \mathrm{Av}([1,0,0], \alpha, a), \mathrm{Qtrasf}\colon \mathrm{zeromatrix}(4,4), \mathbf{for}\ i\ \mathbf{thru}\ 4\ \mathbf{do}\ \mathbf{for}\ j\ \mathbf{thru}\ 4\ \mathbf{do}\ (\mathrm{Qtrasf}_i)_j\colon \mathrm{trigreduce}((\mathrm{tempMat}_i)_j), \mathrm{res}\colon \mathrm{Qtrasf})$

```
(%i5) Qdirect(DH):=block([res,Q,Qtemp],
                        Q:[Q(DH[1][1],DH[1][2],DH[1][3],DH[1][4])],
                        for i:2 thru length(DH) do(

                             Qtemp:Q(DH[i][1],DH[i][2],DH[i][3],DH[i][4]),

                             Q:append(Q,[trigsimp(trigreduce(trigexpand(Q[i-
      1].Qtemp)))])


                            ),
                         res:Q)
```

(%o5) $\mathrm{Qdirect}(\mathrm{DH}) := \mathbf{block}\ ([\mathrm{res}, Q, \mathrm{Qtemp}], Q\colon [Q((\mathrm{DH}_1)_1, (\mathrm{DH}_1)_2, (\mathrm{DH}_1)_3, (\mathrm{DH}_1)_4)], \mathbf{for}\ i\ \mathbf{from}\ 2\ \mathbf{thru}\ \mathrm{length}(\mathrm{DH})\ \mathbf{do}\ (\mathrm{Qtemp}\colon Q((\mathrm{DH}_i)_1, (\mathrm{DH}_i)_2, (\mathrm{DH}_i)_3, (\mathrm{DH}_i)_4), Q\colon \mathrm{append}(Q, [\mathrm{trigsimp}(\mathrm{trigreduce}(\mathrm{trigexpand}(Q_{i-1} \cdot \mathrm{Qtemp})))])), \mathrm{res}\colon Q)$

Qbc(Q,bc):=prende in ingresso la matrice Q della cinematica diretta ed applica la traslazione necessaria a portare il sistema di riferimento nel baricentro.

```
(%i6) Qbc(Q,bc,dist):=block([traslBC,Qcap],
                     Qcap:[],
                     ex:matrix([1],[0],[0]), ez:matrix([0],[0],[1]),
                     for j:1 thru length(Q) do(
                              traslBC:addrow(addcol(ident(3),dist[j]),[0,0,0,1]),
                              Qcap:append(Qcap,[trigsimp(Q[j].traslBC)])

                              ),
                     Qcap
                     )
```

(%o6) $\mathrm{Qbc}(Q,\mathrm{bc},\mathrm{dist}) := \mathbf{block}\left([\mathrm{traslBC},\mathrm{Qcap}],\mathrm{Qcap}:[],\mathrm{ex}:\begin{pmatrix}1\\0\\0\end{pmatrix},\mathrm{ez}:\begin{pmatrix}0\\0\\1\end{pmatrix},\right.$
$\mathbf{for}\ j\ \mathbf{thru}\ \mathrm{length}(Q)\ \mathbf{do}\ (\mathrm{traslBC}:\mathrm{addrow}(\mathrm{addcol}(\mathrm{ident}(3),\mathrm{dist}_j),[0,0,0,1]),\mathrm{Qcap}:\mathrm{append}(\mathrm{Qcap},$
$\left.[\mathrm{trigsimp}(Q_j\cdot\mathrm{traslBC})])),\mathrm{Qcap}\right)$

inerzia(j):= funzione che associa al link j-esimo la corrispettiva matrice di interzia.

```
(%i7) inerzia(j):=block(
      [II],
      II:matrix([alpha[xx][i],alpha[xy][i],alpha[xz][i]],
                [alpha[xy][i],alpha[yy][i],alpha[yz][i]],
                [alpha[xz][i],alpha[yz][i],alpha[zz][i]]),
      II:subst(j, i, II),
      return(II)
      )
```

(%o7) $\mathrm{inerzia}(j) := \mathbf{block}\left([\mathrm{II}],\mathrm{II}:\begin{pmatrix}(\alpha_{\mathrm{xx}})_i & (\alpha_{\mathrm{xy}})_i & (\alpha_{\mathrm{xz}})_i\\(\alpha_{\mathrm{xy}})_i & (\alpha_{\mathrm{yy}})_i & (\alpha_{\mathrm{yz}})_i\\(\alpha_{\mathrm{xz}})_i & (\alpha_{\mathrm{yz}})_i & (\alpha_{\mathrm{zz}})_i\end{pmatrix},\mathrm{II}:\mathrm{subst}(j,i,\mathrm{II}),\mathrm{return}(\mathrm{II})\right)$

```
(%i8) massa(k):=M[k];
```

(%o8) $\mathrm{massa}(k) := M_k$

ek(DH):=funzione responsabile del calcolo dell'energia cinetica dell'intero robot.

$$\mathrm{DH}=\begin{pmatrix}\theta & d & \alpha & a\\ \vdots & \vdots & \vdots & \vdots\end{pmatrix}$$

Calcolo cinematica diretta in accordo all'algoritmo di Denavit-Hartenberg:

$$Q_{0,n}=Q_{01}Q_{12}\dots Q_{n-1,n}\quad \text{con}\, n\equiv\#\mathrm{DOF}$$

Applico traslazioni necessarie per portare il sistema il $\mathrm{SR}_{i-1,i}$ con l'origine coincidente con il baricentro del link:

$$\hat{Q}_{01}=Q_{01}\begin{pmatrix}I & d\\0 & 1\end{pmatrix},\hat{Q}_{12}\dots\hat{Q}_{n-1,n}\quad \text{in cui}\, d\,\text{sono le coordinate del baricentro del link} -\frac{L_i}{2}$$

A questo punto, l'energia cinetica del link i-esimo:

$$T_i=T_{i_a}+T_{i_b}=\frac{1}{2}\omega_i^T R_i \mathbb{I}_i R_i^T \omega_I+\frac{1}{2}M_i \dot{d}_i^T \dot{d}_i$$

In cui:

$$\omega_i \equiv \dot{q}_i \, e_k \qquad \text{con}\, k \in \{x, z\} \,\text{in base all'asse su cui avviene la rotazione}$$

In particolare:

$$\omega_i = \omega = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \text{ottenuto da}\, S(\omega) = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ \omega_y & \omega_x & 0 \end{pmatrix} = \dot{R}R^T$$

$$R := \text{ matrice di rotazione associata}\, a\, \hat{Q}_{i-1,i}$$

$$\mathbb{I} := \text{matrice di inerzia del link}\, i - \text{esimo}$$

$$M_i := \text{massa dell'}i - \text{esimo link}$$

$$d_i := \text{ coordinate di posizione associata}\, a\, \hat{Q}_{i-1,i}$$

$$T_a := \text{ energia cinetica associata alla rotazione}$$

$$T_b := \text{ energia cinetica associata alla traslazione}$$

Inoltre si definisce, la matrice delle inerzie generalizzate B_i, nel seguente modo:

$$T_i = T_{i_a} + T_{i_b} = \frac{1}{2}(\ \dot{q}_1 \ldots \dot{q}_n\ ) B_i \begin{pmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_n \end{pmatrix}$$

In altri termini è la forma quadratica corrispondende all'energia totale del link i-esimo. In particolare, per l'intero robot la matrice delle interzie generalizzate:

$$B = B_1 + \cdots + B_n$$

(%i9) ek(DH,dist):=block([Q,Qcap,I,wtemp,w,Si,Tatemp,Ta,Tbtemp,Tb,d,dd,Qend,B,
Btemp,T,Tot,Btot,res],

```
                    I:[],w:[],Ta:[],Tb:[],B:[],T:[],Ttot:0,
                    depends([q,omega],t),
                    Q:Qdirect(DH),

                    Qcap:Qbc(Q,DH,dist),

                   for i:1 thru length(Qcap) do( I:append(I,[inerzia(i)]),
                        R:matrix([Qcap[i][1][1],Qcap[i][1][2],
Qcap[i][1][3]],[Qcap[i][2][1],Qcap[i][2][2],Qcap[i][2][3]],[Qcap[i][3][1],
Qcap[i][3][2],Qcap[i][3][3]]),
                       dR:diff(R,t),
                       for j:1 thru length(DH) do(
                           dR:subst('diff(q[j],t)=omega[j],dR)),
                       Sw:dR.transpose(R),
                       wtemp:matrix([Sw[3][2]],[Sw[1][3]],[Sw[2][1]]),
                       w:append(w,[trigreduce(expand(wtemp))]),
                  Tatemp:(1/2)*transpose(wtemp).R.I[i].transpose(R).wtemp,
                  Tatemp:trigsimp(trigreduce(trigexpand(Tatemp))),
                   Ta:append(Ta,[Tatemp]),
                   d:matrix([Qcap[i][1][4]],[Qcap[i][2][4]],[Q[i][3][4]]),
                   dd:diff(d,t),
                   for j:1 thru length(DH)
do(dd:subst('diff(q[j],t)=omega[j],dd)),Tbtemp:(massa(i)/
2)*trigsimp(trigreduce(trigexpand(transpose(dd).dd))),
                   Tb:append(Tb,[Tbtemp]),
                   T:append(T,[trigreduce(Tatemp+Tbtemp)])),
                   for i:1 thru length(DH) do(
                     Ttot:T[i]+Ttot
                   ),
                   B:zeromatrix(length(DH),length(DH)),
                   for i:1 thru length(DH) do(
                     B[i][i]:coeff(collectterms(expand(2*Ttot),omega[i]^2),
omega[i],2),
                     for j:1 thru length(DH) do(
                     if i#j then
                        (
                          B[i][j]:ratsimp(coeff(coeff(expand(Ttot*(1/2)),
omega[i],1),omega[j],1))
                         )
                   )),
                   if length(DH)#2 then(
                   res:[[Ta[1],Tb[1],T[1]],
                        [Ta[2],Tb[2],T[2]],
                        [Ta[3],Tb[3],T[3]],[B]])
                   else (res:[[Ta[1],Tb[1],T[1]],
                        [Ta[2],Tb[2],T[2]],[B]])
                    )
```

(%o9) $\mathrm{ek}(\mathrm{DH},\mathrm{dist}) := \textbf{block}\Bigg([Q,\mathrm{Qcap},I,\mathrm{wtemp},w,\mathrm{Si},\mathrm{Tatemp},\mathrm{Ta},\mathrm{Tbtemp},\mathrm{Tb},d,\mathrm{dd},\mathrm{Qend},B,$

$\mathrm{Btemp},T,\mathrm{Tot},\mathrm{Btot},\mathrm{res}], I\colon[], w\colon[], \mathrm{Ta}\colon[], \mathrm{Tb}\colon[], B\colon[], T\colon[], \mathrm{Ttot}\colon 0, \mathrm{depends}([q,\omega],t), Q\colon$

Qdirect(DH), Qcap: Qbc(Q, DH, dist), **for** i **thru** length(Qcap) **do** $\Bigg($ I: append(I, [inerzia(i)]), R: $\begin{pmatrix} ((\mathrm{Qcap}_i)_1)_1 & ((\mathrm{Qcap}_i)_1)_2 & ((\mathrm{Qcap}_i)_1)_3 \\ ((\mathrm{Qcap}_i)_2)_1 & ((\mathrm{Qcap}_i)_2)_2 & ((\mathrm{Qcap}_i)_2)_3 \\ ((\mathrm{Qcap}_i)_3)_1 & ((\mathrm{Qcap}_i)_3)_2 & ((\mathrm{Qcap}_i)_3)_3 \end{pmatrix}$, dR: diff($R, t$), **for** j **thru** length(DH) **do** dR: subst$\left(\frac{1}{\mathrm{mtimes}()}\, q_j = \omega_j, \mathrm{dR}\right)$, Sw: dR $\cdot$ transpose(R), wtemp: $\begin{pmatrix} (\mathrm{Sw}_3)_2 \\ (\mathrm{Sw}_1)_3 \\ (\mathrm{Sw}_2)_1 \end{pmatrix}$, w: append(w, [trigreduce(expand(wtemp))]), Tatemp: $\frac{1}{2}$ transpose(wtemp) $\cdot R \cdot I_i \cdot$ transpose(R) $\cdot$ wtemp, Tatemp: trigsimp(trigreduce(trigexpand(Tatemp))), Ta: append(Ta, [Tatemp]), d: $\begin{pmatrix} ((\mathrm{Qcap}_i)_1)_4 \\ ((\mathrm{Qcap}_i)_2)_4 \\ ((Q_i)_3)_4 \end{pmatrix}$, dd: diff($d, t$), **for** j **thru** length(DH) **do** dd: subst$\left(\frac{1}{\mathrm{mtimes}()}\, q_j = \omega_j, \mathrm{dd}\right)$, Tbtemp: $\frac{\mathrm{massa}(i)}{2}$ trigsimp(trigreduce(trigexpand(transpose(dd) $\cdot$ dd))), Tb: append(Tb, [Tbtemp]), T: append(T, [trigreduce(Tatemp + Tbtemp)]) $\Bigg)$, **for** i **thru** length(DH) **do** Ttot: T_i + Ttot, B: zeromatrix(length(DH), length(DH)), **for** i **thru** length(DH) **do** $\Bigg($ $(B_i)_i$: coeff(collectterms(expand(2 Ttot), ω_i^2), ω_i, 2), **for** j **thru** length(DH) **do if** $i \neq j$ **then** $(B_i)_j$: ratsimp$\left(\mathrm{coeff}\left(\mathrm{coeff}\left(\mathrm{expand}\left(\mathrm{Ttot}\ \left(\frac{1}{2}\right)\right), \omega_i, 1\right), \omega_j, 1\right)\right)$, **if** length(DH) $\neq 2$ **then** res: [[Ta$_1$, Tb$_1$, T_1], [Ta$_2$, Tb$_2$, T_2], [Ta$_3$, Tb$_3$, T_3], [B]] **else** res: [[Ta$_1$, Tb$_1$, T_1], [Ta$_2$, Tb$_2$, T_2], [B]] $\Bigg)$

ep(DH):=funzione responsabile del calcolo dell'energia potenziale a cui è soggetto il robot.

$$\mathrm{DH} = \begin{pmatrix} \theta & d & \alpha & a \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

Calcolo cinematica diretta in accordo all'algoritmo di Denavit-Hartenberg:

$$Q_{0,n} = Q_{01} Q_{12} \ldots Q_{n-1,n} \quad \text{con}\, n \equiv \#\mathrm{DOF}$$

Applico traslazioni necessarie per portare il sistema il $\mathrm{SR}_{i-1,i}$ con l'origine coincidente con il baricentro del link:

$\hat{Q}_{01} = Q_{01} \begin{pmatrix} I & d \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \hat{R} & \hat{d} \\ 0 & 1 \end{pmatrix}, \hat{Q}_{12} \ldots \hat{Q}_{n-1,n}$ in cui d sono le coordinate del baricentro del link $-\frac{L_i}{2}$

L'energia potenziale per il link i-esimo ha la seguente forma:

$$U_i = -M\, g^T d \quad \text{con}\ g = 9,8\, e_z \simeq 10\, e_z, d = \hat{d} := \text{coordinate nel baricentro}$$

Per l'intero robot:

$$U = \sum_{i=1}^{n} U_i = -\sum_{i=1}^{n} M_i\, g^T d_i$$

```
(%i10) ep(DH,dist):=block([Q,Qcap,g,U,Utemp,dTemp,prev,Utot],
                         Q:[], Qcap:[],U:[],Utot:zeromatrix(3,3),Utot:0,
                          depends([q,omega],t),
                         g:10*matrix([0],[0],[1]),
                         prev:ident(4),
                         Q:Qdirect(DH),
                          Qcap:Qbc(Q,DH,dist),

                            for i:1 thru length(Qcap) do(
                             print("Energia gravitazionale link",i),
                             dTemp:matrix([Qcap[i][1][4]],[Qcap[i][2][4]],
                                           [Qcap[i][3][4]]),

                             Utemp:M[i]*transpose(g).dTemp,
                             U:append(U,[Utemp]),
                             print("U[",i,"]=",Utemp)
                           ),
                            for i:1 thru length(U) do(
                                Utot:Utot+U[i]
                            ),
                            print("Energia gravitazionale totale=",
       ratsimp(trigsimp(trigreduce(trigexpand(Utot))))
                            );
```

(%o10) $\mathrm{ep}(\mathrm{DH},\mathrm{dist}) := \textbf{block}\Bigg([Q,\mathrm{Qcap},g,U,\mathrm{Utemp},\mathrm{dTemp},\mathrm{prev},\mathrm{Utot}], Q\colon [], \mathrm{Qcap}\colon [], U\colon [],$
$\mathrm{Utot}\colon \mathrm{zeromatrix}(3,3), \mathrm{Utot}\colon 0, \mathrm{depends}([q,\omega],t), g\colon 10\begin{pmatrix}0\\0\\1\end{pmatrix}, \mathrm{prev}\colon \mathrm{ident}(4), Q\colon \mathrm{Qdirect}(\mathrm{DH}),$
$\mathrm{Qcap}\colon \mathrm{Qbc}(Q,\mathrm{DH},\mathrm{dist}), \textbf{for } i \textbf{ thru } \mathrm{length}(\mathrm{Qcap}) \textbf{ do } \Bigg(\mathrm{print}(\text{Energia gravitazionale link }, i),$
$\mathrm{dTemp}\colon \begin{pmatrix}((\mathrm{Qcap}_i)_1)_4\\((\mathrm{Qcap}_i)_2)_4\\((\mathrm{Qcap}_i)_3)_4\end{pmatrix}, \mathrm{Utemp}\colon M_i\,\mathrm{transpose}(g)\cdot\mathrm{dTemp}, U\colon \mathrm{append}(U,[\mathrm{Utemp}]), \mathrm{print}(\mathrm{U}[\ ,i,]=$
$,\mathrm{Utemp})\Bigg), \textbf{for } i \textbf{ thru } \mathrm{length}(U) \textbf{ do } \mathrm{Utot}\colon \mathrm{Utot}+U_i, \mathrm{print}(\text{Energia gravitazionale totale=}\ ,$
$\mathrm{ratsimp}(\mathrm{trigsimp}(\mathrm{trigreduce}(\mathrm{trigexpand}(\mathrm{Utot}))))\Bigg)$

```
(%i11) dinamica(DH,dist):=block([T],
                              T:ek(DH,dist),
                              for i:1 thru length(T)-1 do(
                                 print("Energia cinetica link", i),
                                  print("Energia cinetica rotazione Ta=",
       T[i][1]),
                                  print("Energia cinetica traslazione Tb=",
       T[i][2]),print("Energia cinetica totale T=",T[i][3])

                                ),print("Matrice inerzie generalizzate B=",
       T[length(T)]),ep(DH,dist));
```

(%o11) $\mathrm{dinamica}(\mathrm{DH},\mathrm{dist}) := \textbf{block}\ ([T], T\colon \mathrm{ek}(\mathrm{DH},\mathrm{dist}), \textbf{for } i \textbf{ thru } \mathrm{length}(T) -$
1 **do** $(\mathrm{print}(\text{Energia cinetica link }, i), \mathrm{print}(\text{Energia cinetica rotazione Ta=}\ , (T_i)_1), \mathrm{print}(\text{Energia}$
$\text{cinetica traslazione Tb=}\ , (T_i)_2), \mathrm{print}(\text{Energia cinetica totale T=}\ , (T_i)_3)), \mathrm{print}(\text{Matrice inerzie}$

generalizzate B= $, T_{\mathrm{length}(T)}), \mathrm{ep}(\mathrm{DH}, \mathrm{dist}))$

2DOF PLANARE

```
(%i12) DH:[[q[1],0,0,L[1]],[q[2],0,0,L[2]]];
```

(%o12) $[[q_1, 0, 0, L_1], [q_2, 0, 0, L_2]]$

```
(%i13) distance:[matrix([-L[1]/2],[0],[0]),matrix([-L[2]/2],[0],[0])];
```

$$\text{(\%o13)}\quad \left[\begin{pmatrix} -\frac{L_1}{2} \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{L_2}{2} \\ 0 \\ 0 \end{pmatrix}\right]$$

```
(%i14) dinamica(DH,distance)
```

Energia cinetica link 1

Energia cinetica rotazione Ta= $\frac{\omega_1^2\,(\alpha_{zz})_1}{2}$

Energia cinetica traslazione Tb= $\frac{L_1^2\,M_1\,\omega_1^2}{8}$

Energia cinetica totale T= $\frac{\omega_1^2\,(\alpha_{zz})_1}{2} + \frac{L_1^2\,M_1\,\omega_1^2}{8}$

Energia cinetica link 2

Energia cinetica rotazione Ta= $\frac{(\omega_2^2 + 2\,\omega_1\,\omega_2 + \omega_1^2)\,(\alpha_{zz})_2}{2}$

Energia cinetica traslazione Tb=

$$\frac{M_2\,((4\,L_1\,\omega_1\,L_2\,\omega_2 + 4\,L_1\,\omega_1^2\,L_2)\cos(q_2) + L_2^2\,\omega_2^2 + 2\,\omega_1\,L_2^2\,\omega_2 + \omega_1^2\,L_2^2 + 4\,L_1^2\,\omega_1^2)}{8}$$

Energia cinetica totale T= $(4\,L_1\,\omega_1\,L_2\,M_2\,\omega_2\cos(q_2) + 4\,L_1\,\omega_1^2\,L_2\,M_2\cos(q_2) + L_2^2\,M_2\,\omega_2^2 + 2\,\omega_1\,L_2^2\,M_2\,\omega_2 + \omega_1^2\,L_2^2\,M_2 + 4\,L_1^2\,\omega_1^2\,M_2)/8 + \frac{\omega_2^2\,(\alpha_{zz})_2 + 2\,\omega_1\,\omega_2\,(\alpha_{zz})_2 + \omega_1^2\,(\alpha_{zz})_1}{2}$

Matrice inerzie generalizzate B= $\left[\left(L_1\,L_2\,M_2\cos(q_2) + (\alpha_{zz})_2 + \frac{L_2^2\,M_2}{4} + L_1^2\,M_2 + (\alpha_{zz})_1 + \frac{L_1^2\,M_1}{4}, \frac{2\,L_1\,L_2\,M_2\cos(q_2) + 4\,(\alpha_{zz})_2 + L_2^2\,M_2}{8}; \frac{2\,L_1\,L_2\,M_2\cos(q_2) + 4\,(\alpha_{zz})_2 + L_2^2\,M_2}{8}, (\alpha_{zz})_2 + \frac{L_2^2\,M_2}{4}\right)\right]$

Energia gravitazionale link 1

U[1]= 0

Energia gravitazionale link 2

U[2]= 0

Energia gravitazionale totale= 0

(%o14) 0

Robot Cartesiano

```
(%i15) DH:[[0,q[1],-%pi/2,0],[-%pi/2,q[2],-%pi/2,0],[0,q[3],0,0]];
```

$$\text{(\%o15)}\quad \left[\left[0, q_1, -\frac{\pi}{2}, 0\right], \left[-\frac{\pi}{2}, q_2, -\frac{\pi}{2}, 0\right], [0, q_3, 0, 0]\right]$$

```
(%i16) distance:[matrix([0],[-L[1]/2],[0]),matrix([0],[-L[2]/2],[0]),matrix([0],
       [0],[-L[3]/2])];
```

$$\text{(\%o16)}\quad \left[\begin{pmatrix} 0 \\ -\frac{L_1}{2} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -\frac{L_2}{2} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -\frac{L_3}{2} \end{pmatrix}\right]$$

```
(%i17) dinamica(DH,distance);
```

Energia cinetica link 1
Energia cinetica rotazione Ta$= 0$

Energia cinetica traslazione Tb$= \frac{M_1\,\omega_1^2}{2}$

Energia cinetica totale T$= \frac{M_1\,\omega_1^2}{2}$
Energia cinetica link 2

Energia cinetica rotazione Ta$= 0$
Energia cinetica traslazione Tb$= \frac{M_2\,(\omega_2^2+\omega_1^2)}{2}$

Energia cinetica totale T$= \frac{M_2\,\omega_2^2+\omega_1^2\,M_2}{2}$

Energia cinetica link 3
Energia cinetica rotazione Ta$= 0$

Energia cinetica traslazione Tb$= \frac{M_3\,(\omega_3^2+\omega_2^2+\omega_1^2)}{2}$
Energia cinetica totale T$= \frac{M_3\,\omega_3^2+\omega_2^2\,M_3+\omega_1^2\,M_3}{2}$

Matrice inerzie generalizzate B$= \left[\begin{pmatrix} M_3+M_2+M_1 & 0 & 0 \\ 0 & M_3+M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}\right]$
Energia gravitazionale link 1

U[1]$= 5\,M_1\,(2\,q_1+L_1)$

Energia gravitazionale link 2
U[2]$= 10\,q_1\,M_2$

Energia gravitazionale link 3
U[3]$= 10\,q_1\,M_3$

Energia gravitazionale totale$= 10\,q_1\,M_3+10\,q_1\,M_2+10\,M_1\,q_1+5\,L_1\,M_1$

(%o17) $10\,q_1\,M_3+10\,q_1\,M_2+10\,M_1\,q_1+5\,L_1\,M_1$

Robot Cilindrico

```
(%i18) DH:[[q[1],L[1],0,0],[0,q[2],-%pi/2,0],[0,q[3],0,0]];
```

(%o18) $\left[[q_1,L_1,0,0],\left[0,q_2,-\frac{\pi}{2},0\right],[0,q_3,0,0]\right]$

```
(%i19) distance:[matrix([0],[0],[-L[1]/2]),matrix([0],[-L[2]/2],[0]),matrix([0],
       [0],[-L[3]/2])];
```

(%o19) $\left[\begin{pmatrix} 0 \\ 0 \\ -\frac{L_1}{2} \end{pmatrix},\begin{pmatrix} 0 \\ -\frac{L_2}{2} \\ 0 \end{pmatrix},\begin{pmatrix} 0 \\ 0 \\ -\frac{L_3}{2} \end{pmatrix}\right]$

```
(%i20) dinamica(DH,distance);
```

Energia cinetica link 1
Energia cinetica rotazione Ta$= \frac{\omega_1^2\,(\alpha_{zz})_1}{2}$

Energia cinetica traslazione Tb= 0

Energia cinetica totale T= $\frac{\omega_1^2\,(\alpha_{zz})_1}{2}$

Energia cinetica link 2

Energia cinetica rotazione Ta= $\frac{\omega_1^2\,(\alpha_{yy})_2}{2}$

Energia cinetica traslazione Tb= $\frac{M_2\,\omega_2^2}{2}$

Energia cinetica totale T= $\frac{\omega_1^2\,(\alpha_{yy})_2}{2}+\frac{M_2\,\omega_2^2}{2}$

Energia cinetica link 3

Energia cinetica rotazione Ta= $\frac{\omega_1^2\,(\alpha_{yy})_3}{2}$

Energia cinetica traslazione Tb= $\frac{M_3\,(4\,\omega_1^2\,q_3^2-4\,\omega_1^2\,L_3\,q_3+4\,\omega_3^2+\omega_1^2\,L_3^2+4\,\omega_2^2)}{8}$

Energia cinetica totale T= $\frac{\omega_1^2\,(\alpha_{yy})_3}{2}+\frac{4\,\omega_1^2\,M_3\,q_3^2-4\,\omega_1^2\,L_3\,M_3\,q_3+4\,M_3\,\omega_3^2+\omega_1^2\,L_3^2\,M_3+4\,\omega_2^2\,M_3}{8}$

Matrice inerzie generalizzate B=

$$\left[\begin{pmatrix} (\alpha_{yy})_3+M_3\,q_3^2-L_3\,M_3\,q_3+\frac{L_3^2\,M_3}{4}+(\alpha_{yy})_2+(\alpha_{zz})_1 & 0 & 0 \\ 0 & M_3+M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}\right]$$

Energia gravitazionale link 1

U[1]= $5\,L_1\,M_1$

Energia gravitazionale link 2

U[2]= $5\,M_2\,(2\,q_2+L_2+2\,L_1)$

Energia gravitazionale link 3

U[3]= $10\,(q_2+L_1)\,M_3$

Energia gravitazionale totale= $(10\,q_2+10\,L_1)\,M_3+10\,M_2\,q_2+(5\,L_2+10\,L_1)\,M_2+5\,L_1\,M_1$

(%o20) $(10\,q_2+10\,L_1)\,M_3+10\,M_2\,q_2+(5\,L_2+10\,L_1)\,M_2+5\,L_1\,M_1$

Robot SCARA

```
(%i21) DH:[[q[1],L[1],0,D[1]],[q[2],0,0,0],[0,q[3],0,0]];
```

(%o21) $[[q_1,L_1,0,D_1],[q_2,0,0,0],[0,q_3,0,0]]$

```
(%i22) distance:[matrix([-D[1]/2],[0],[0]),matrix([-D[2]/2],[0],[0]),matrix([0],
       [0],[-L[3]/2])];
```

(%o22) $$\left[\begin{pmatrix} -\frac{D_1}{2} \\ 0 \\ 0 \end{pmatrix},\begin{pmatrix} -\frac{D_2}{2} \\ 0 \\ 0 \end{pmatrix},\begin{pmatrix} 0 \\ 0 \\ -\frac{L_3}{2} \end{pmatrix}\right]$$

```
(%i23) dinamica(DH,distance);
```

Energia cinetica link 1

Energia cinetica rotazione Ta= $\frac{\omega_1^2\,(\alpha_{zz})_1}{2}$

Energia cinetica traslazione Tb= $\frac{D_1^2\,M_1\,\omega_1^2}{8}$

Energia cinetica totale T= $\frac{\omega_1^2\,(\alpha_{zz})_1}{2}+\frac{D_1^2\,M_1\,\omega_1^2}{8}$

Energia cinetica link 2

Energia cinetica rotazione Ta= $\dfrac{(\omega_2^2+2\,\omega_1\,\omega_2+\omega_1^2)\,(\alpha_{zz})_2}{2}$
Energia cinetica traslazione Tb= $-M_2\,((4\,D_1\,\omega_1\,D_2\,\omega_2+4\,D_1\,\omega_1^2\,D_2)\cos(q_2)-D_2^2\,\omega_2^2-2\,\omega_1\,D_2^2\,\omega_2-\omega_1^2\,D_2^2-4\,D_1^2\,\omega_1^2)/8$
Energia cinetica totale T= $\dfrac{\omega_2^2\,(\alpha_{zz})_2+2\,\omega_1\,\omega_2\,(\alpha_{zz})_2+\omega_1^2\,(\alpha_{zz})_2}{2}-(4\,D_1\,\omega_1\,D_2\,M_2\,\omega_2\cos(q_2)+4\,D_1\,\omega_1^2\,D_2\,M_2\cos(q_2)-D_2^2\,M_2\,\omega_2^2-2\,\omega_1\,D_2^2\,M_2\,\omega_2-\omega_1^2\,D_2^2\,M_2-4\,D_1^2\,\omega_1^2\,M_2)/8$

Energia cinetica link 3
Energia cinetica rotazione Ta= $\dfrac{(\omega_2^2+2\,\omega_1\,\omega_2+\omega_1^2)\,(\alpha_{zz})_3}{2}$
Energia cinetica traslazione Tb= $\dfrac{M_3\,(\omega_3^2+D_1^2\,\omega_1^2)}{2}$
Energia cinetica totale T= $\dfrac{\omega_2^2\,(\alpha_{zz})_3+2\,\omega_1\,\omega_2\,(\alpha_{zz})_3+\omega_1^2\,(\alpha_{zz})_3}{2}+\dfrac{M_3\,\omega_3^2+D_1^2\,\omega_1^2\,M_3}{2}$
Matrice inerzie generalizzate B= $\left[\left(-D_1\,D_2\,M_2\cos(q_2)+(\alpha_{zz})_3+D_1^2\,M_3+(\alpha_{zz})_2+\frac{D_2^2\,M_2}{4}+D_1^2\,M_2+(\alpha_{zz})_1+\frac{D_1^2\,M_1}{4},-\frac{2\,D_1\,D_2\,M_2\cos(q_2)-4\,(\alpha_{zz})_3-4\,(\alpha_{zz})_2-D_2^2\,M_2}{8},0;-\frac{2\,D_1\,D_2\,M_2\cos(q_2)-4\,(\alpha_{zz})_3-4\,(\alpha_{zz})_2-D_2^2\,M_2}{8},(\alpha_{zz})_3+(\alpha_{zz})_2+\frac{D_2^2\,M_2}{4},0;0,0,M_3\right)\right]$
Energia gravitazionale link 1

U[1]= $10\,L_1\,M_1$
Energia gravitazionale link 2

U[2]= $10\,L_1\,M_2$
Energia gravitazionale link 3

U[3]= $5\,M_3\,(2\,q_3-L_3+2\,L_1)$
Energia gravitazionale totale= $10\,M_3\,q_3+(10\,L_1-5\,L_3)\,M_3+10\,L_1\,M_2+10\,L_1\,M_1$
(%o23) $10\,M_3\,q_3+(10\,L_1-5\,L_3)\,M_3+10\,L_1\,M_2+10\,L_1\,M_1$

Robot Sferico Tipo 1

```
(%i24) DH:[[q[1],L[1],%pi/2,0],[q[2],0,%pi/2,L[2]],[0,q[3],0,0]];
```

(%o24) $\left[\left[q_1,L_1,\frac{\pi}{2},0\right],\left[q_2,0,\frac{\pi}{2},L_2\right],[0,q_3,0,0]\right]$

```
(%i25) distance:[matrix([0],[-L[1]/2],[0]),matrix([-L[2]/2],[0],[0]),matrix([0],
       [0],[-L[3]/2])];
```

(%o25) $\left[\begin{pmatrix}0\\-\frac{L_1}{2}\\0\end{pmatrix},\begin{pmatrix}-\frac{L_2}{2}\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\-\frac{L_3}{2}\end{pmatrix}\right]$

```
(%i26) sfericoI:dinamica(DH,distance);
```

Energia cinetica link 1
Energia cinetica rotazione Ta= $\dfrac{\omega_1^2\,(\alpha_{yy})_1}{2}$

Energia cinetica traslazione Tb= 0
Energia cinetica totale T= $\dfrac{\omega_1^2\,(\alpha_{yy})_1}{2}$

Energia cinetica link 2
Energia cinetica rotazione Ta= $-(2\,\omega_1^2\,(\alpha_{xz})_2\sin(2\,q_2)+(\omega_1^2\,(\alpha_{xx})_2-\omega_1^2\,(\alpha_{zz})_2)\cos(2\,q_2)-4\,\omega_1\,\omega_2\,(\alpha_{xy})_2\sin(q_2)+4\,\omega_1\,\omega_2\,(\alpha_{yz})_2\cos(q_2)-\omega_1^2\,(\alpha_{zz})_2-2\,\omega_2^2\,(\alpha_{yy})_2-\omega_1^2\,(\alpha_{xx})_2)/4$

Energia cinetica traslazione Tb= $\frac{M_2\left(\left(3 L_2^2 \omega_2^2+\omega_1^2 L_2^2\right)\cos(2q_2)+5 L_2^2\omega_2^2+\omega_1^2 L_2^2\right)}{16}$
Energia cinetica totale T= $\frac{3 L_2^2 M_2\omega_2^2\cos(2q_2)+\omega_1^2 L_2^2 M_2\cos(2q_2)+5L_2^2M_2\omega_2^2+\omega_1^2L_2^2M_2}{16} - (2\omega_1^2(\alpha_{xz})_2\sin(2q_2)-\omega_1^2(\alpha_{zz})_2\cos(2q_2)+\omega_1^2(\alpha_{xx})_2\cos(2q_2)-4\omega_1\omega_2(\alpha_{xy})_2\sin(q_2)+4\omega_1\omega_2(\alpha_{yz})_2\cos(q_2)-\omega_1^2(\alpha_{zz})_2-2\omega_2^2(\alpha_{yy})_2-\omega_1^2(\alpha_{xx})_2)/4$
Energia cinetica link 3

Energia cinetica rotazione Ta= $-(2\omega_1^2(\alpha_{xz})_3\sin(2q_2)+(\omega_1^2(\alpha_{xx})_3-\omega_1^2(\alpha_{zz})_3)\cos(2q_2)-4\omega_1\omega_2(\alpha_{xy})_3\sin(q_2)+4\omega_1\omega_2(\alpha_{yz})_3\cos(q_2)-\omega_1^2(\alpha_{zz})_3-2\omega_2^2(\alpha_{yy})_3-\omega_1^2(\alpha_{xx})_3)/4$
Energia cinetica traslazione Tb= $M_3(((8\omega_2\omega_3-8L_2\omega_2^2+8\omega_1^2L_2)q_3-4\omega_2L_3\omega_3+(4L_2\omega_2^2-4\omega_1^2L_2)L_3)\sin(2q_2)+((4\omega_2^2-4\omega_1^2)q_3^2+(4\omega_1^2-4\omega_2^2)L_3q_3-4\omega_3^2+8L_2\omega_2\omega_3+(\omega_2^2-\omega_1^2)L_3^2-4L_2^2\omega_2^2+4\omega_1^2L_2^2)\cos(2q_2)+(16L_2\omega_2^2-16\omega_2\omega_3)q_3\cos(q_2)\sin(q_2)+(-8\omega_2^2q_3^2+8\omega_3^2-16L_2\omega_2\omega_3+8L_2^2\omega_2^2)\cos(q_2)^2+(12\omega_2^2+4\omega_1^2)q_3^2+(-4\omega_2^2-4\omega_1^2)L_3q_3+4\omega_3^2-8L_2\omega_2\omega_3+(\omega_2^2+\omega_1^2)L_3^2+4L_2^2\omega_2^2+4\omega_1^2L_2^2)/16$
Energia cinetica totale T= $(8\omega_1^2L_2M_3q_3\sin(2q_2)-4\omega_2L_3M_3\omega_3\sin(2q_2)+4L_2\omega_2^2L_3M_3\sin(2q_2)-4\omega_1^2L_2L_3M_3\sin(2q_2)-4\omega_1^2M_3q_3^2\cos(2q_2)-4\omega_2^2L_3M_3q_3\cos(2q_2)+4\omega_1^2L_3M_3q_3\cos(2q_2)+\omega_2^2L_3^2M_3\cos(2q_2)-\omega_1^2L_3^2M_3\cos(2q_2)+4\omega_1^2L_2^2M_3\cos(2q_2)+8\omega_2^2M_3q_3^2+4\omega_1^2M_3q_3^2-4\omega_2^2L_3M_3q_3-4\omega_1^2L_3M_3q_3+8M_3\omega_3^2-16L_2\omega_2M_3\omega_3+\omega_2^2L_3^2M_3+\omega_1^2L_3^2M_3+8L_2^2\omega_2^2M_3+4\omega_1^2L_2^2M_3)/16-(2\omega_1^2(\alpha_{xz})_3\sin(2q_2)-\omega_1^2(\alpha_{zz})_3\cos(2q_2)+\omega_1^2(\alpha_{xx})_3\cos(2q_2)-4\omega_1\omega_2(\alpha_{xy})_3\sin(q_2)+4\omega_1\omega_2(\alpha_{yz})_3\cos(q_2)-\omega_1^2(\alpha_{zz})_3-2\omega_2^2(\alpha_{yy})_3-\omega_1^2(\alpha_{xx})_3)/4$

Matrice inerzie generalizzate B= $\left[\left(-(\alpha_{xz})_3\sin(2q_2)+L_2M_3q_3\sin(2q_2)-\frac{L_2L_3M_3\sin(2q_2)}{2}-(\alpha_{xz})_2\sin(2q_2)+\frac{(\alpha_{zz})_3\cos(2q_2)}{2}-\frac{(\alpha_{xx})_3\cos(2q_2)}{2}-\frac{M_3q_3^2\cos(2q_2)}{2}+\frac{L_3M_3q_3\cos(2q_2)}{2}-\frac{L_3^2M_3\cos(2q_2)}{8}+\frac{L_2^2M_3\cos(2q_2)}{2}+\frac{(\alpha_{zz})_2\cos(2q_2)}{2}-\frac{(\alpha_{xx})_2\cos(2q_2)}{2}+\frac{L_2^2M_2\cos(2q_2)}{8}+\frac{(\alpha_{zz})_3}{2}+\frac{(\alpha_{xx})_3}{2}+\frac{M_3q_3^2}{2}-\frac{L_3M_3q_3}{2}+\frac{L_3^2M_3}{8}+\frac{L_2^2M_3}{2}+\frac{(\alpha_{zz})_2}{2}+\frac{(\alpha_{xx})_2}{2}+\frac{L_2^2M_2}{8}+(\alpha_{yy})_1, \frac{((\alpha_{xy})_3+(\alpha_{xy})_2)\sin(q_2)+(-(\alpha_{yz})_3-(\alpha_{yz})_2)\cos(q_2)}{2}, 0; \frac{((\alpha_{xy})_3+(\alpha_{xy})_2)\sin(q_2)+(-(\alpha_{yz})_3-(\alpha_{yz})_2)\cos(q_2)}{2}, \frac{L_2L_3M_3\sin(2q_2)}{2}-\frac{L_3M_3q_3\cos(2q_2)}{2}+\frac{L_3^2M_3\cos(2q_2)}{8}+\frac{3L_2^2M_2\cos(2q_2)}{8}+(\alpha_{yy})_3+M_3q_3^2-\frac{L_3M_3q_3}{2}+\frac{L_3^2M_3}{8}+L_2^2M_3+(\alpha_{yy})_2+\frac{5L_2^2M_2}{8}, -\frac{L_3M_3\sin(2q_2)+4L_2M_3}{8}; 0, -\frac{L_3M_3\sin(2q_2)+4L_2M_3}{8}, M_3\right)\right]$
Energia gravitazionale link 1

U[1]= $5L_1M_1$
Energia gravitazionale link 2

U[2]= $5M_2(L_2\sin(q_2)+2L_1)$
Energia gravitazionale link 3

U[3]= $5M_3(2L_2\sin(q_2)+(L_3-2q_3)\cos(q_2)+2L_1)$
Energia gravitazionale totale= $(10L_2M_3+5L_2M_2)\sin(q_2)+(5L_3M_3-10M_3q_3)\cos(q_2)+10L_1M_3+10L_1M_2+5L_1M_1$

(%o26) $(10L_2M_3+5L_2M_2)\sin(q_2)+(5L_3M_3-10M_3q_3)\cos(q_2)+10L_1M_3+10L_1M_2+5L_1M_1$

Robot Sferico II
```
(%i27) DH:[[q[1],L[1],-%pi/2,0],[q[2],L[2],%pi/2,0],[0,q[3],0,0]];
```

(%o27) $\left[\left[q_1, L_1, -\frac{\pi}{2}, 0\right], \left[q_2, L_2, \frac{\pi}{2}, 0\right], [0, q_3, 0, 0]\right]$

(%i28) `distance:[matrix([0],[-L[1]/2],[0]),matrix([0],[-L[2]/2],[0]),matrix([0],[0],[-L[3]/2])];`

(%o28) $$\left[\begin{pmatrix}0\\-\frac{L_1}{2}\\0\end{pmatrix},\begin{pmatrix}0\\-\frac{L_2}{2}\\0\end{pmatrix},\begin{pmatrix}0\\0\\-\frac{L_3}{2}\end{pmatrix}\right]$$

(%i29) `dinamica(DH,distance);`

Energia cinetica link 1

Energia cinetica rotazione Ta= $\frac{\omega_1^2\,(\alpha_{yy})_1}{2}$

Energia cinetica traslazione Tb= 0

Energia cinetica totale T= $\frac{\omega_1^2\,(\alpha_{yy})_1}{2}$

Energia cinetica link 2

Energia cinetica rotazione Ta= $-(2\,\omega_1^2\,(\alpha_{xz})_2\sin(2\,q_2)+(\omega_1^2\,(\alpha_{xx})_2-\omega_1^2\,(\alpha_{zz})_2)\cos(2\,q_2)+4\,\omega_1\,\omega_2\,(\alpha_{xy})_2\sin(q_2)-4\,\omega_1\,\omega_2\,(\alpha_{yz})_2\cos(q_2)-\omega_1^2\,(\alpha_{zz})_2-2\,\omega_2^2\,(\alpha_{yy})_2-\omega_1^2\,(\alpha_{xx})_2)/4$

Energia cinetica traslazione Tb= $\frac{\omega_1^2\,L_2^2\,M_2}{8}$

Energia cinetica totale T= $\frac{\omega_1^2\,L_2^2\,M_2}{8}-(2\,\omega_1^2\,(\alpha_{xz})_2\sin(2\,q_2)-\omega_1^2\,(\alpha_{zz})_2\cos(2\,q_2)+\omega_1^2\,(\alpha_{xx})_2\cos(2\,q_2)+4\,\omega_1\,\omega_2\,(\alpha_{xy})_2\sin(q_2)-4\,\omega_1\,\omega_2\,(\alpha_{yz})_2\cos(q_2)-\omega_1^2\,(\alpha_{zz})_2-2\,\omega_2^2\,(\alpha_{yy})_2-\omega_1^2\,(\alpha_{xx})_2)/4$

Energia cinetica link 3

Energia cinetica rotazione Ta= $-(2\,\omega_1^2\,(\alpha_{xz})_3\sin(2\,q_2)+(\omega_1^2\,(\alpha_{xx})_3-\omega_1^2\,(\alpha_{zz})_3)\cos(2\,q_2)+4\,\omega_1\,\omega_2\,(\alpha_{xy})_3\sin(q_2)-4\,\omega_1\,\omega_2\,(\alpha_{yz})_3\cos(q_2)-\omega_1^2\,(\alpha_{zz})_3-2\,\omega_2^2\,(\alpha_{yy})_3-\omega_1^2\,(\alpha_{xx})_3)/4$

Energia cinetica traslazione Tb= $M_3\,((8\,\omega_2\,\omega_3\,q_3-4\,\omega_2\,L_3\,\omega_3)\sin(2\,q_2)+((4\,\omega_2^2-4\,\omega_1^2)\,q_3^2+(4\,\omega_1^2-4\,\omega_2^2)\,L_3\,q_3-4\,\omega_3^2+(\omega_2^2-\omega_1^2)\,L_3^2)\cos(2\,q_2)+(-16\,\omega_2\,\omega_3\,q_3\cos(q_2)-16\,\omega_1\,L_2\,\omega_3)\sin(q_2)+(8\,\omega_3^2-8\,\omega_2^2\,q_3^2)\cos(q_2)^2+(8\,\omega_1\,L_2\,\omega_2\,L_3-16\,\omega_1\,L_2\,\omega_2\,q_3)\cos(q_2)+(12\,\omega_2^2+4\,\omega_1^2)\,q_3^2+(-4\,\omega_2^2-4\,\omega_1^2)\,L_3\,q_3+4\,\omega_3^2+(\omega_2^2+\omega_1^2)\,L_3^2+8\,\omega_1^2\,L_2^2)/16$

Energia cinetica totale T= $(-4\,\omega_2\,L_3\,M_3\,\omega_3\sin(2\,q_2)-4\,\omega_1^2\,M_3\,q_3^2\cos(2\,q_2)-4\,\omega_2^2\,L_3\,M_3\,q_3\cos(2\,q_2)+4\,\omega_1^2\,L_3\,M_3\,q_3\cos(2\,q_2)+\omega_2^2\,L_3^2\,M_3\cos(2\,q_2)-\omega_1^2\,L_3^2\,M_3\cos(2\,q_2)-16\,\omega_1\,L_2\,M_3\,\omega_3\sin(q_2)-16\,\omega_1\,L_2\,\omega_2\,M_3\,q_3\cos(q_2)+8\,\omega_1\,L_2\,\omega_2\,L_3\,M_3\cos(q_2)+8\,\omega_2^2\,M_3\,q_3^2+4\,\omega_1^2\,M_3\,q_3^2-4\,\omega_2^2\,L_3\,M_3\,q_3-4\,\omega_1^2\,L_3\,M_3\,q_3+8\,M_3\,\omega_3^2+\omega_2^2\,L_3^2\,M_3+\omega_1^2\,L_3^2\,M_3+8\,\omega_1^2\,L_2^2\,M_3)/16-(2\,\omega_1^2\,(\alpha_{xz})_3\sin(2\,q_2)-\omega_1^2\,(\alpha_{zz})_3\cos(2\,q_2)+\omega_1^2\,(\alpha_{xx})_3\cos(2\,q_2)+4\,\omega_1\,\omega_2\,(\alpha_{xy})_3\sin(q_2)-4\,\omega_1\,\omega_2\,(\alpha_{yz})_3\cos(q_2)-\omega_1^2\,(\alpha_{zz})_3-2\,\omega_2^2\,(\alpha_{yy})_3-\omega_1^2\,(\alpha_{xx})_3)/4$

Matrice inerzie generalizzate B= $\Big[\Big(-(\alpha_{xz})_3\sin(2\,q_2)-(\alpha_{xz})_2\sin(2\,q_2)+\frac{(\alpha_{zz})_3\cos(2\,q_2)}{2}-\frac{(\alpha_{xx})_3\cos(2\,q_2)}{2}-\frac{M_3\,q_3^2\cos(2\,q_2)}{2}+\frac{L_3\,M_3\,q_3\cos(2\,q_2)}{2}-\frac{L_3^2\,M_3\cos(2\,q_2)}{8}+\frac{(\alpha_{zz})_2\cos(2\,q_2)}{2}-\frac{(\alpha_{xx})_2\cos(2\,q_2)}{2}+\frac{(\alpha_{zz})_3}{2}+\frac{(\alpha_{xx})_3}{2}+\frac{M_3\,q_3^2}{2}-\frac{L_3\,M_3\,q_3}{2}+\frac{L_3^2\,M_3}{8}+L_2^2\,M_3+\frac{(\alpha_{zz})_2}{2}+\frac{(\alpha_{xx})_2}{2}+\frac{L_2^2\,M_2}{4}+(\alpha_{yy})_1,$
$-\frac{(2\,(\alpha_{xy})_3+2\,(\alpha_{xy})_2)\sin(q_2)+(-2\,(\alpha_{yz})_3+2\,L_2\,M_3\,q_3-L_2\,L_3\,M_3-2\,(\alpha_{yz})_2)\cos(q_2)}{4},-\frac{L_2\,M_3\sin(q_2)}{2};$
$-\frac{(2\,(\alpha_{xy})_3+2\,(\alpha_{xy})_2)\sin(q_2)+(-2\,(\alpha_{yz})_3+2\,L_2\,M_3\,q_3-L_2\,L_3\,M_3-2\,(\alpha_{yz})_2)\cos(q_2)}{4},-\frac{L_3\,M_3\,q_3\cos(2\,q_2)}{2}+\frac{L_3^2\,M_3\cos(2\,q_2)}{8}+(\alpha_{yy})_3+M_3\,q_3^2-\frac{L_3\,M_3\,q_3}{2}+\frac{L_3^2\,M_3}{8}+(\alpha_{yy})_2,-\frac{L_3\,M_3\sin(2\,q_2)}{8};-\frac{L_2\,M_3\sin(q_2)}{2},$
$-\frac{L_3\,M_3\sin(2\,q_2)}{8},M_3\Big)\Big]$

Energia gravitazionale link 1

U[1]= $15\,L_1\,M_1$

Energia gravitazionale link 2

U[2]= $10\,L_1\,M_2$
Energia gravitazionale link 3

U[3]= $5\,M_3\,((2\,q_3-L_3)\cos(q_2)+2\,L_1)$
Energia gravitazionale totale= $(10\,M_3\,q_3-5\,L_3\,M_3)\cos(q_2)+10\,L_1\,M_3+10\,L_1\,M_2+15\,L_1\,M_1$

(%o29) $(10\,M_3\,q_3-5\,L_3\,M_3)\cos(q_2)+10\,L_1\,M_3+10\,L_1\,M_2+15\,L_1\,M_1$

Robot Antropomorfo

```
(%i30) DH:[[q[1],L[1],%pi/2,0],[q[2],0,0,D[3]],[q[3],0,0,D[3]]];
```

(%o30) $\left[\left[q_1,L_1,\frac{\pi}{2},0\right],[q_2,0,0,D_3],[q_3,0,0,D_3]\right]$

```
(%i31) distance:[matrix([0],[-L[1]/2],[0]),matrix([-L[2]/2],[0],[0]),matrix([-
L[3]/2],[0],[0])];
```

(%o31) $\left[\begin{pmatrix}0\\-\frac{L_1}{2}\\0\end{pmatrix},\begin{pmatrix}-\frac{L_2}{2}\\0\\0\end{pmatrix},\begin{pmatrix}-\frac{L_3}{2}\\0\\0\end{pmatrix}\right]$

```
(%i32) dinamica(DH,distance);
```

Energia cinetica link 1
Energia cinetica rotazione Ta= $\frac{\omega_1^2\,(\alpha_{yy})_1}{2}$

Energia cinetica traslazione Tb= 0
Energia cinetica totale T= $\frac{\omega_1^2\,(\alpha_{yy})_1}{2}$

Energia cinetica link 2
Energia cinetica rotazione Ta= $(2\,\omega_1^2\,(\alpha_{xy})_2\sin(2\,q_2)+(\omega_1^2\,(\alpha_{yy})_2-\omega_1^2\,(\alpha_{xx})_2)\cos(2\,q_2)+4\,\omega_1\,\omega_2\,(\alpha_{xz})_2\sin(q_2)+4\,\omega_1\,\omega_2\,(\alpha_{yz})_2\cos(q_2)+2\,\omega_2^2\,(\alpha_{zz})_2+\omega_1^2\,(\alpha_{yy})_2+\omega_1^2\,(\alpha_{xx})_2)/4$
Energia cinetica traslazione Tb= $M_2\,((4\,\omega_1^2\,D_3^2+(4\,L_2\,\omega_2^2-4\,\omega_1^2\,L_2)\,D_3-L_2^2\,\omega_2^2+\omega_1^2\,L_2^2)\cos(2\,q_2)+(8\,\omega_2^2+4\,\omega_1^2)\,D_3^2+(-4\,L_2\,\omega_2^2-4\,\omega_1^2\,L_2)\,D_3+L_2^2\,\omega_2^2+\omega_1^2\,L_2^2)/16$
Energia cinetica totale T= $(2\,\omega_1^2\,(\alpha_{xy})_2\sin(2\,q_2)+\omega_1^2\,(\alpha_{yy})_2\cos(2\,q_2)-\omega_1^2\,(\alpha_{xx})_2\cos(2\,q_2)+4\,\omega_1\,\omega_2\,(\alpha_{xz})_2\sin(q_2)+4\,\omega_1\,\omega_2\,(\alpha_{yz})_2\cos(q_2)+2\,\omega_2^2\,(\alpha_{zz})_2+\omega_1^2\,(\alpha_{yy})_2+\omega_1^2\,(\alpha_{xx})_2)/4+(4\,\omega_1^2\,M_2\,D_3^2\cos(2\,q_2)+4\,L_2\,M_2\,\omega_2^2\,D_3\cos(2\,q_2)-4\,\omega_1^2\,L_2\,M_2\,D_3\cos(2\,q_2)-L_2^2\,M_2\,\omega_2^2\cos(2\,q_2)+\omega_1^2\,L_2^2\,M_2\cos(2\,q_2)+8\,M_2\,\omega_2^2\,D_3^2+4\,\omega_1^2\,M_2\,D_3^2-4\,L_2\,M_2\,\omega_2^2\,D_3-4\,\omega_1^2\,L_2\,M_2\,D_3+L_2^2\,M_2\,\omega_2^2+\omega_1^2\,L_2^2\,M_2)/16$
Energia cinetica link 3

Energia cinetica rotazione Ta= $(2\,\omega_1^2\,(\alpha_{xy})_3\sin(2\,q_3+2\,q_2)+(\omega_1^2\,(\alpha_{yy})_3-\omega_1^2\,(\alpha_{xx})_3)\cos(2\,q_3+2\,q_2)+(4\,\omega_1\,\omega_3+4\,\omega_1\,\omega_2)\,(\alpha_{xz})_3\sin(q_3+q_2)+(4\,\omega_1\,\omega_3+4\,\omega_1\,\omega_2)\,(\alpha_{yz})_3\cos(q_3+q_2)+(2\,\omega_3^2+4\,\omega_2\,\omega_3+2\,\omega_2^2)\,(\alpha_{zz})_3+\omega_1^2\,(\alpha_{yy})_3+\omega_1^2\,(\alpha_{xx})_3)/4$
Energia cinetica traslazione Tb= $-M_3\,(((L_3^2-4\,D_3\,L_3+4\,D_3^2)\,\omega_3^2+(2\,\omega_2\,L_3^2-8\,\omega_2\,D_3\,L_3+8\,\omega_2\,D_3^2)\,\omega_3+(\omega_2^2-\omega_1^2)\,L_3^2+(4\,\omega_1^2-4\,\omega_2^2)\,D_3\,L_3+(4\,\omega_2^2-4\,\omega_1^2)\,D_3^2)\cos(2\,q_3+2\,q_2)+((8\,\omega_2\,D_3^2-4\,\omega_2\,D_3\,L_3)\,\omega_3+(4\,\omega_1^2-4\,\omega_2^2)\,D_3\,L_3+(8\,\omega_2^2-8\,\omega_1^2)\,D_3^2)\cos(q_3+2\,q_2)+(-8\,D_3^2\,\omega_3^2-16\,\omega_2\,D_3^2\,\omega_3-8\,\omega_2^2\,D_3^2)\cos(q_3+q_2)^2+(-16\,\omega_2\,D_3^2\,\omega_3-16\,\omega_2^2\,D_3^2)\cos(q_2)\cos(q_3+q_2)+((4\,\omega_2\,D_3\,L_3-8\,\omega_2\,D_3^2)\,\omega_3+(4\,\omega_2^2+4\,\omega_1^2)\,D_3\,L_3+(-8\,\omega_2^2-8\,\omega_1^2)\,D_3^2)\cos(q_3)+(4\,\omega_2^2-4\,\omega_1^2)\,D_3^2\cos(2\,q_2)+8\,\omega_2^2\,D_3^2\sin(q_2)^2+(-L_3^2+4\,D_3\,L_3-4\,D_3^2)\,\omega_3^2+(-2\,\omega_2\,L_3^2+8\,\omega_2\,D_3\,L_3-8\,\omega_2\,D_3^2)\,\omega_3+(-\omega_2^2-\omega_1^2)\,L_3^2+(4\,\omega_2^2+4\,\omega_1^2)\,D_3\,L_3+(-16\,\omega_2^2-8\,\omega_1^2)\,D_3^2)/16$

Energia cinetica totale T= $(2\,\omega_1^2\,(\alpha_{xy})_3 \sin(2\,q_3+2\,q_2)+\omega_1^2\,(\alpha_{yy})_3\cos(2\,q_3+2\,q_2)-\omega_1^2\,(\alpha_{xx})_3\cos(2\,q_3+2\,q_2)+4\,\omega_1\,\omega_3\,(\alpha_{xz})_3\sin(q_3+q_2)+4\,\omega_1\,\omega_2\,(\alpha_{xz})_3\sin(q_3+q_2)+4\,\omega_1\,\omega_3\,(\alpha_{yz})_3\cos(q_3+q_2)+4\,\omega_1\,\omega_2\,(\alpha_{yz})_3\cos(q_3+q_2)+2\,\omega_3^2\,(\alpha_{zz})_3+4\,\omega_2\,\omega_3\,(\alpha_{zz})_3+2\,\omega_2^2\,(\alpha_{zz})_3+\omega_1^2\,(\alpha_{yy})_3+\omega_1^2\,(\alpha_{xx})_3)/4-(L_3^2\,M_3\,\omega_3^2\cos(2\,q_3+2\,q_2)-4\,D_3\,L_3\,M_3\,\omega_3^2\cos(2\,q_3+2\,q_2)+2\,\omega_2\,L_3^2\,M_3\,\omega_3\cos(2\,q_3+2\,q_2)-8\,\omega_2\,D_3\,L_3\,M_3\,\omega_3\cos(2\,q_3+2\,q_2)+\omega_2^2\,L_3^2\,M_3\cos(2\,q_3+2\,q_2)-\omega_1^2\,L_3^2\,M_3\cos(2\,q_3+2\,q_2)-4\,\omega_2^2\,D_3\,L_3\,M_3\cos(2\,q_3+2\,q_2)+4\,\omega_1^2\,D_3\,L_3\,M_3\cos(2\,q_3+2\,q_2)-4\,\omega_1^2\,D_3^2\,M_3\cos(2\,q_3+2\,q_2)-4\,\omega_2\,D_3\,L_3\,M_3\,\omega_3\cos(q_3+2\,q_2)-4\,\omega_2^2\,D_3\,L_3\,M_3\cos(q_3+2\,q_2)+4\,\omega_1^2\,D_3\,L_3\,M_3\cos(q_3+2\,q_2)-8\,\omega_1^2\,D_3^2\,M_3\cos(q_3+2\,q_2)+4\,\omega_2\,D_3\,L_3\,M_3\,\omega_3\cos(q_3)-16\,\omega_2\,D_3^2\,M_3\,\omega_3\cos(q_3)+4\,\omega_2^2\,D_3\,L_3\,M_3\cos(q_3)+4\,\omega_1^2\,D_3\,L_3\,M_3\cos(q_3)-16\,\omega_2^2\,D_3^2\,M_3\cos(q_3)-8\,\omega_1^2\,D_3^2\,M_3\cos(q_3)-4\,\omega_1^2\,D_3^2\,M_3\cos(2\,q_2)-L_3^2\,M_3\,\omega_3^2+4\,D_3\,L_3\,M_3\,\omega_3^2-8\,D_3^2\,M_3\,\omega_3^2-2\,\omega_2\,L_3^2\,M_3\,\omega_3+8\,\omega_2\,D_3\,L_3\,M_3\,\omega_3-16\,\omega_2\,D_3^2\,M_3\,\omega_3-\omega_2^2\,L_3^2\,M_3-\omega_1^2\,L_3^2\,M_3+4\,\omega_2^2\,D_3\,L_3\,M_3+4\,\omega_1^2\,D_3\,L_3\,M_3-16\,\omega_2^2\,D_3^2\,M_3-8\,\omega_1^2\,D_3^2\,M_3)/16$

Matrice inerzie generalizzate B= $\Big[\Big((\alpha_{xy})_3\sin(2\,q_3+2\,q_2)+\frac{(\alpha_{yy})_3\cos(2\,q_3+2\,q_2)}{2}-\frac{(\alpha_{xx})_3\cos(2\,q_3+2\,q_2)}{2}+\frac{L_3^2\,M_3\cos(2\,q_3+2\,q_2)}{8}-\frac{D_3\,L_3\,M_3\cos(2\,q_3+2\,q_2)}{2}+\frac{D_3^2\,M_3\cos(2\,q_3+2\,q_2)}{2}-\frac{D_3\,L_3\,M_3\cos(q_3+2\,q_2)}{2}+D_3^2\,M_3\cos(q_3+2\,q_2)-\frac{D_3\,L_3\,M_3\cos(q_3)}{2}+D_3^2\,M_3\cos(q_3)+(\alpha_{xy})_2\sin(2\,q_2)+\frac{D_3^2\,M_3\cos(2\,q_2)}{2}+\frac{M_2\,D_3^2\cos(2\,q_2)}{2}-\frac{L_2\,M_2\,D_3\cos(2\,q_2)}{2}+\frac{(\alpha_{yy})_2\cos(2\,q_2)}{2}-\frac{(\alpha_{xx})_2\cos(2\,q_2)}{2}+\frac{L_2^2\,M_2\cos(2\,q_2)}{8}+\frac{(\alpha_{yy})_3}{2}+\frac{(\alpha_{xx})_3}{2}+\frac{L_3^2\,M_3}{8}-\frac{D_3\,L_3\,M_3}{2}+D_3^2\,M_3+\frac{M_2\,D_3^2}{2}-\frac{L_2\,M_2\,D_3}{2}+\frac{(\alpha_{yy})_2}{2}+\frac{(\alpha_{xx})_2}{2}+\frac{L_2^2\,M_2}{8}+(\alpha_{yy})_1,\frac{(\alpha_{xz})_3\sin(q_3+q_2)+(\alpha_{yz})_3\cos(q_3+q_2)+(\alpha_{xz})_2\sin(q_2)+(\alpha_{yz})_2\cos(q_2)}{2},\frac{(\alpha_{xz})_3\sin(q_3+q_2)+(\alpha_{yz})_3\cos(q_3+q_2)}{2};\frac{(\alpha_{xz})_3\sin(q_3+q_2)+(\alpha_{yz})_3\cos(q_3+q_2)+(\alpha_{xz})_2\sin(q_2)+(\alpha_{yz})_2\cos(q_2)}{2},-\frac{L_3^2\,M_3\cos(2\,q_3+2\,q_2)}{8}+\frac{D_3\,L_3\,M_3\cos(2\,q_3+2\,q_2)}{2}+\frac{D_3\,L_3\,M_3\cos(q_3+2\,q_2)}{2}-\frac{D_3\,L_3\,M_3\cos(q_3)}{2}+2\,D_3^2\,M_3\cos(q_3)+\frac{L_2\,M_2\,D_3\cos(2\,q_2)}{2}-\frac{L_2^2\,M_2\cos(2\,q_2)}{8}+(\alpha_{zz})_3+\frac{L_3^2\,M_3}{8}-\frac{D_3\,L_3\,M_3}{2}+2\,D_3^2\,M_3+M_2\,D_3^2-\frac{L_2\,M_2\,D_3}{2}+(\alpha_{zz})_2+\frac{L_2^2\,M_2}{8},-((L_3^2-4\,D_3\,L_3)\,M_3\cos(2\,q_3+2\,q_2)-2\,D_3\,L_3\,M_3\cos(q_3+2\,q_2)+(2\,D_3\,L_3-8\,D_3^2)\,M_3\cos(q_3)-8\,(\alpha_{zz})_3+(-L_3^2+4\,D_3\,L_3-8\,D_3^2)\,M_3)/16;\frac{(\alpha_{xz})_3\sin(q_3+q_2)+(\alpha_{yz})_3\cos(q_3+q_2)}{2},-((L_3^2-4\,D_3\,L_3)\,M_3\cos(2\,q_3+2\,q_2)-2\,D_3\,L_3\,M_3\cos(q_3+2\,q_2)+(2\,D_3\,L_3-8\,D_3^2)\,M_3\cos(q_3)-8\,(\alpha_{zz})_3+(-L_3^2+4\,D_3\,L_3-8\,D_3^2)\,M_3)/16,-\frac{L_3^2\,M_3\cos(2\,q_3+2\,q_2)}{8}+\frac{D_3\,L_3\,M_3\cos(2\,q_3+2\,q_2)}{2}+(\alpha_{zz})_3+\frac{L_3^2\,M_3}{8}-\frac{D_3\,L_3\,M_3}{2}+D_3^2\,M_3\Big)\Big]$

Energia gravitazionale link 1

U[1]= $5\,L_1\,M_1$

Energia gravitazionale link 2

U[2]= $5\,M_2\,((2\,D_3-L_2)\sin(q_2)+2\,L_1)$

Energia gravitazionale link 3

U[3]= $-5\,M_3\,((L_3-2\,D_3)\sin(q_3+q_2)-2\,D_3\sin(q_2)-2\,L_1)$

Energia gravitazionale totale= $(10\,D_3-5\,L_3)\,M_3\sin(q_3+q_2)+(10\,D_3\,M_3+10\,M_2\,D_3-5\,L_2\,M_2)\sin(q_2)+10\,L_1\,M_3+10\,L_1\,M_2+5\,L_1\,M_1$

(%o32) $(10\,D_3-5\,L_3)\,M_3\sin(q_3+q_2)+(10\,D_3\,M_3+10\,M_2\,D_3-5\,L_2\,M_2)\sin(q_2)+10\,L_1\,M_3+10\,L_1\,M_2+5\,L_1\,M_1$

(%i33)