1 Problems

Problem Solve $\frac{dy}{dx} = \frac{1}{xy}$ numerically with the Runga-Kutta method. Assume that $x_0 = 1, y_0 = 1, \delta x = 0.1, x_{end} = 2$
. The analytical solution is $\sqrt{1 + 2\log x}$

Problem Mass hanging on a spring: We can use RK4 for coupled ODEs to solve for position and velocity of a mass hanging from a spring as a function of time.

The spring obeys Hooke's law: $F_x = -kx$, where x is the displacement of the spring and F_s is the force exerted by the spring.

The force due to gravity can be written as: $F_g = -mg$, where m is the mass of the weight and g is the acceleration due to gravity.

By force balance, we obtain: $m\frac{dv}{dt} = -kx - mg$, where v is the velocity of the mass. This leads to: $\frac{dv}{dt} = -\frac{k}{m}x - g$.

Also note that by defintion $\frac{dx}{dt} = v$. These are our coupled ODEs.

Solve for the position and velocity of the mass as a function of time, assuming that the mass is initially at rest. Use $g = 1$, $\frac{k}{m} = 1$, $\delta t = 0.1$, and $t_{end} = 30$

Problem A long thin rod connects two bodies with different temperatures. At steady state, the temperature profile in a long thin rod is governed by the following equation: $\frac{d^2\tau}{dx^2} = -K(\tau - \tau_{ext})$,

τ is a non-dimensional temperature, τ_{ext} is the temperature of the environment to which heat is being lost. K is a ratio of the heat transfer to the environment versus the conduction along the rod.

The heat flux, H is given by the following relationship: $H = -\frac{d\tau}{dx}$.

We can rewrite the system into two 1st order ODEs by using heat flux H.

At first we need to change the first equation to involve H: $\frac{d^2\tau}{dx^2} = \frac{d}{dx}\left(\frac{dx}{d\tau}\right)$, $\quad \frac{d\tau}{dx} = -H$.

Therefore, $\frac{d(-H)}{dx} = -K(\tau - \tau_{ext}) \Rightarrow \frac{dH}{dx} = K(\tau - \tau_{ext})$.

Assuming $H_0 = 10$, $\tau_0 = 100$, $\tau_{ext} = 0$, $\delta x = 0.01$, and $K = 1$, we can calculate the temperature and heat flux out of the other end of the rod of length 1. What happens if we change the external temperature between 0 and 100 C?