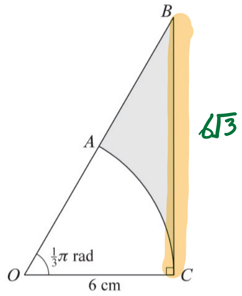


$\tan\left(\frac{\pi}{3}\right) = \frac{BC}{6}$

$\sqrt{3} = \frac{BC}{6}$

$BC = 6\sqrt{3}$

In the diagram, AC is an arc of a circle, centre O and radius 6 cm. The line BC is perpendicular to OC and OAB is a straight line. Angle $AOC = \frac{1}{3}\pi$ radians. Find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [5]

Shaded Area = Triangle − Sector

$= \frac{1}{2}(OC)(CB) - \frac{1}{2}r^2\theta$

$= \frac{1}{2}(6)(6\sqrt{3}) - \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right)$

$= \boxed{18\sqrt{3} - 6\pi}$

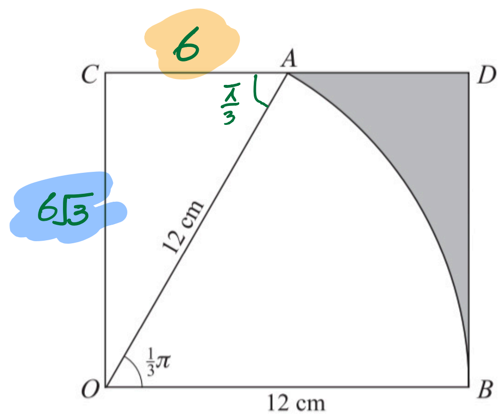
6

$\sin\theta = \frac{P}{H}$

$\sin\frac{\pi}{3} = \frac{OC}{12}$

$\frac{\sqrt{3}}{2} = \frac{OC}{12}$

$OC = 6\sqrt{3}$



$\cos\theta = \frac{B}{H}$

$\cos\frac{\pi}{3} = \frac{AC}{12}$

$\frac{1}{2} = \frac{AC}{12}$

$\boxed{AC = 6}$

In the diagram, AOB is a sector of a circle with centre O and radius 12 cm. The point A lies on the side CD of the rectangle $OCDB$. Angle $AOB = \frac{1}{3}\pi$ radians. Express the area of the shaded region in the form $a(\sqrt{3}) - b\pi$, stating the values of the integers a and b. [6]

Shaded region = Rectangle − Sector − Triangle.

$= (OC)(OB) - \frac{1}{2}r^2\theta - \frac{1}{2}(AC)(OC)$

$= (6\sqrt{3})(12) - \frac{1}{2}(12)^2\left(\frac{\pi}{3}\right) - \frac{1}{2}(6\sqrt{3})(6)$

$= 72\sqrt{3} - 24\pi - 18\sqrt{3}$

$= 72\sqrt{3} - 18\sqrt{3} - 24\pi$

$= 54\sqrt{3} - 24\pi$

$a\sqrt{3} - b\pi$

$\boxed{a = 54}$ $\quad -b = -24$

$\boxed{b = 24}$

$OX^2 = 6^2 + (6\sqrt{3})^2$

$OX^2 = 36 + 36(3)$

$OX^2 = 144$

$OX = 12$

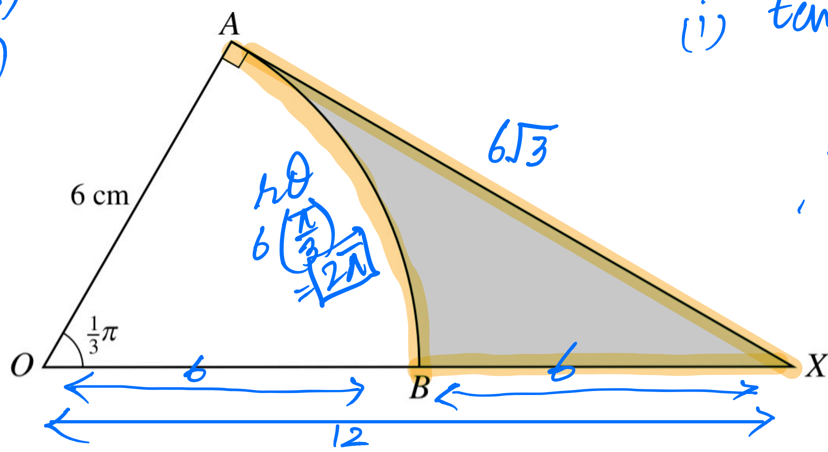
$BX = 12 - 6$

$= 6$

(i) $\tan\left(\frac{\pi}{3}\right) = \frac{AX}{6}$

$\sqrt{3} = \frac{AX}{6}$

$AX = 6\sqrt{3}$



In the diagram, AB is an arc of a circle, centre O and radius 6 cm, and angle $AOB = \frac{1}{3}\pi$ radians. The line AX is a tangent to the circle at A, and OBX is a straight line.

(i) Show that the exact length of AX is $6\sqrt{3}$ cm. [1]

Find, in terms of π and $\sqrt{3}$,

(ii) the area of the shaded region, [3]

(iii) the perimeter of the shaded region. [4]

(ii) Area of shaded = TRIANGLE − SECTOR

$\frac{1}{2}(OA)(AX) - \frac{1}{2}r^2\theta$

$\frac{1}{2}(6)(6\sqrt{3}) - \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right)$

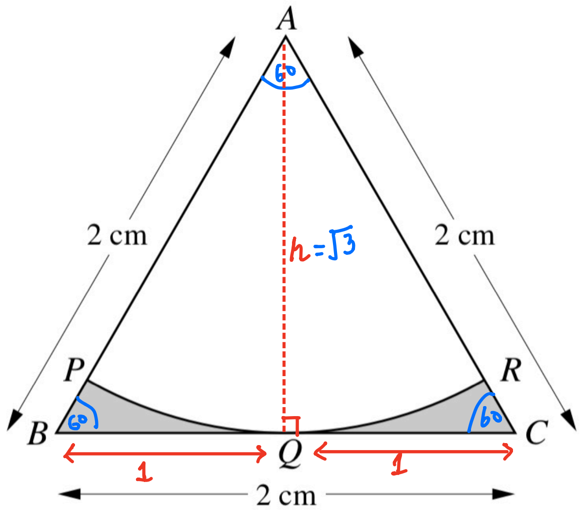
$\boxed{18\sqrt{3} - 6\pi}$

(iii) Perimeter $= AX + BX + AB\,(\text{arc})$

$= 6\sqrt{3} + (BX) + r\theta$

$= 6\sqrt{3} + 6 + (6)\left(\frac{\pi}{3}\right)$

$= \boxed{6\sqrt{3} + 6 + 2\pi}$



Pythagoras.

$2^2 = h^2 + 1^2$

$4 = h^2 + 1$

$h^2 = 3$

$h = \sqrt{3}$

In the diagram, ABC is an equilateral triangle of side 2 cm. The mid-point of BC is Q. An arc of a circle with centre A touches BC at Q, and meets AB at P and AC at R. Find the total area of the shaded regions, giving your answer in terms of π and $\sqrt{3}$. [5]

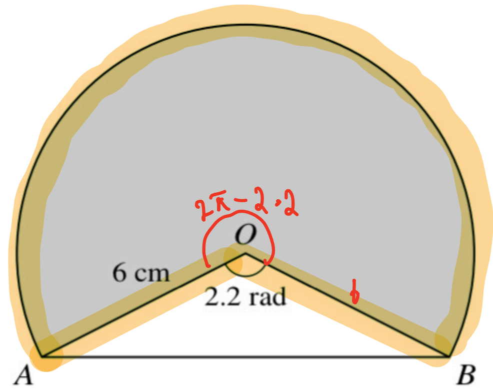
Shaded region = Triangle − Sector.

$$= \frac{1}{2}\boxed{}\,\boxed{}\sin\bigcirc - \frac{\theta}{360}\times\pi h^2$$

$$= \frac{1}{2}(2)(2)\sin 60 - \frac{60}{360}\times\pi(\sqrt{3})^2$$

$$= \frac{1}{\cancel{2}}(\cancel{2})(\cancel{2})\left(\frac{\sqrt{3}}{\cancel{2}}\right) - \frac{\cancel{60}}{\cancel{360}_{\cancel{6}\,2}}\times\pi(\cancel{3})$$

$$= \sqrt{3} - \frac{\pi}{2}$$



The diagram shows part of a circle with centre O and radius 6 cm. The chord AB is such that angle $AOB = 2.2$ radians. Calculate

(i) the perimeter of the shaded region, [3]

(ii) the ratio of the area of the shaded region to the area of the triangle AOB, giving your answer in the form $k : 1$. [3]

Arc length - $\boxed{r\theta}$

(i) Perimeter $= 6 + 6 + (6)(2\pi - 2.2) = 36.499.$

(ii) Shaded region $=$ Sector $= \frac{1}{2} r^2 \theta = \frac{1}{2}(6)^2(2\pi - 2.2) = 73.497$

Triangle $= \frac{1}{2}(6)(6)\sin 2.2 = 14.55$

Ratio Shaded : Triangle

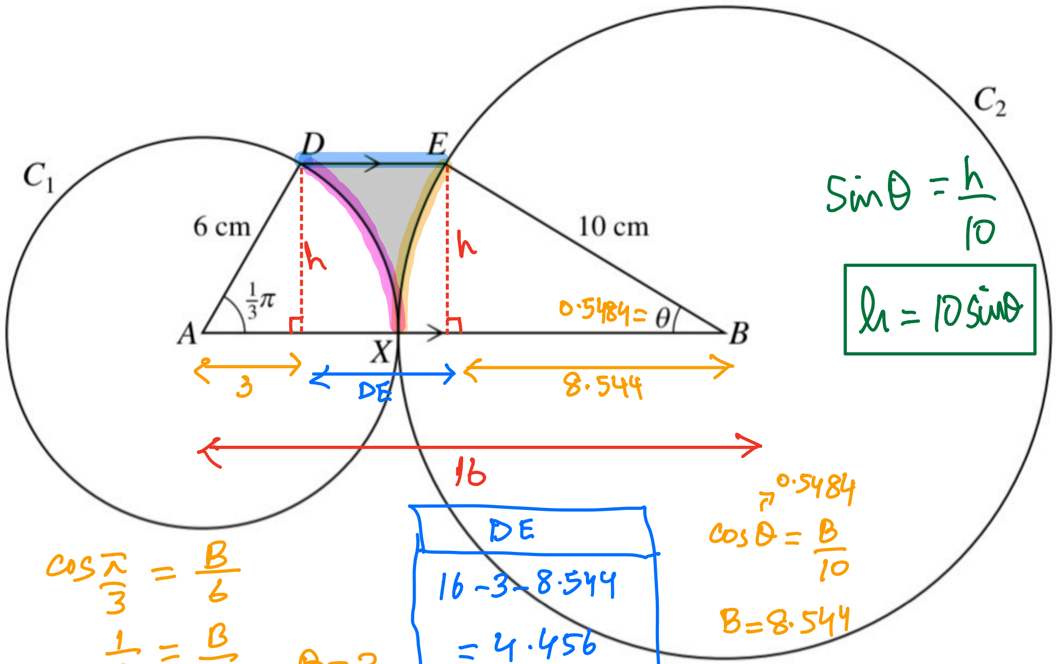
$73.497 : 14.55$

$= 5.05 : 1$

$\sin\frac{\pi}{3} = \frac{h}{6}$

$\frac{\sqrt{3}}{2} = \frac{h}{6}$

$h = 3\sqrt{3}$



$\cos\frac{\pi}{3} = \frac{B}{6}$

$\frac{1}{2} = \frac{B}{6}$ $\quad B = 3$

DE
$16 - 3 - 8.544$
$= 4.456$

$\cos\theta = \frac{B}{10}$ ($\theta = 0.5484$)

$B = 8.544$

The diagram shows a circle C_1 touching a circle C_2 at a point X. Circle C_1 has centre A and radius 6 cm, and circle C_2 has centre B and radius 10 cm. Points D and E lie on C_1 and C_2 respectively and DE is parallel to AB. Angle $DAX = \frac{1}{3}\pi$ radians and angle $EBX = \theta$ radians.

(i) By considering the perpendicular distances of D and E from AB, show that the exact value of θ is $\sin^{-1}\left(\frac{3\sqrt{3}}{10}\right)$. [3]

(ii) Find the perimeter of the shaded region, correct to 4 significant figures. [5]

$h = 3\sqrt{3} \qquad h = 10\sin\theta$

$10\sin\theta = 3\sqrt{3}$

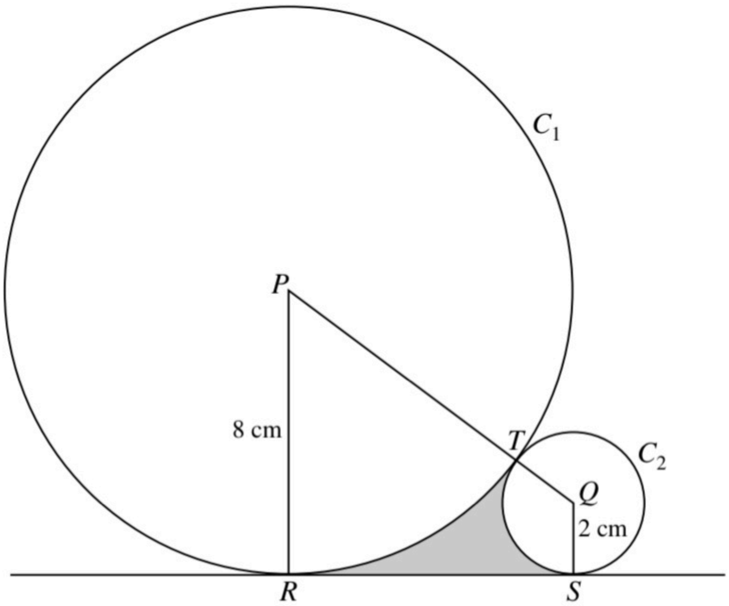
$\sin\theta = \frac{3\sqrt{3}}{10}$

$\theta = \sin^{-1}\left(\frac{3\sqrt{3}}{10}\right)$

Perimeter = Arc XD + Arc XE + LINE DE.

$r\theta \qquad r\theta$

$(6)\left(\frac{\pi}{3}\right) + (10)\left(\sin^{-1}\left(\frac{3\sqrt{3}}{10}\right)\right) + 4.456 = \boxed{}$



The diagram shows two circles, C_1 and C_2, touching at the point T. Circle C_1 has centre P and radius 8 cm; circle C_2 has centre Q and radius 2 cm. Points R and S lie on C_1 and C_2 respectively, and RS is a tangent to both circles.

(i) Show that $RS = 8$ cm. [2]

(ii) Find angle RPQ in radians correct to 4 significant figures. [2]

(iii) Find the area of the shaded region. [4]

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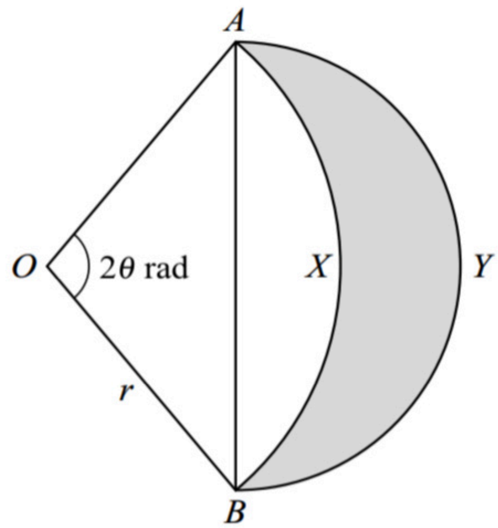

In the diagram, AYB is a semicircle with AB as diameter and $OAXB$ is a sector of a circle with centre O and radius r. Angle $AOB = 2\theta$ radians. Find an expression, in terms of r and θ, for the area of the shaded region. [4]