P1 ⟶ 30% of A Levels

(75 Marks) (1h 50 min)

11 QUESTIONS.

COORDINATE GEOMETRY

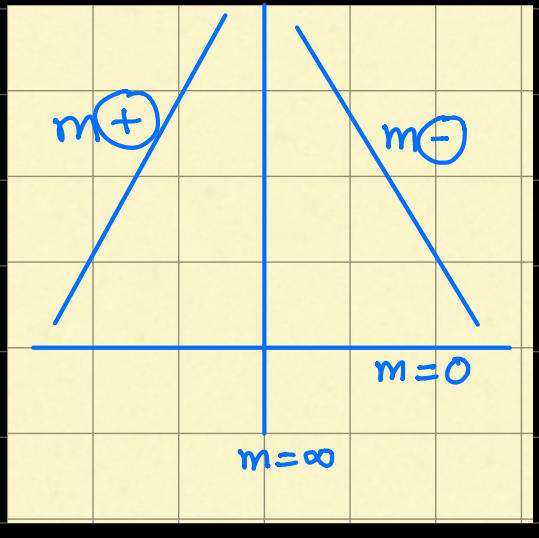
(7 + 4 Marks)

1. DISTANCE $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. MIDPOINT $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

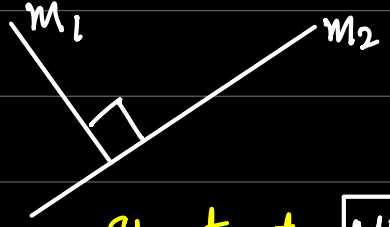
3. GRADIENT (SLOPE) $= m = \frac{y_2 - y_1}{x_2 - x_1}$

GRADIENTS:



PARALLEL LINES SAME GRADIENT

PERPENDICULAR LINES (90°)



Memorize:

$m_1 \times m_2 = -1$

Shortcut: NEGATIVE RECIPROCAL

$m_1 = 3 \longrightarrow m_2 = -\frac{1}{3}$

$m_1 = -\frac{3}{5} \longrightarrow m_2 = \frac{5}{3}$

EQUATION OF A LINE

MOST IMPORTANT.

$y = mx + c$	$y - y_1 = m(x - x_1)$	$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$
Given: m = slope, c = y-intercept	Given: m = slope, (x_1, y_1) Point on line	Given: Two points on line (x_1, y_1) (x_2, y_2)
Q: Find equation of line that has slope 5 and cuts y-axis at 3	Q: Find equation of line with gradient 2 and passes through (2,5)	Q: Find equation of line that passes through (1,3) & (4,8)
$m = 5$, $c = 3$	$m = 2$ Point $(2, 5)$ x_1 y_1	$(1, 3)$ x_1 y_1 $(4, 8)$ x_2 y_2
$y = mx + c$	$y - y_1 = m(x - x_1)$	$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$
$\boxed{y = 5x + 3}$	$y - 5 = 2(x - 2)$	$\frac{y - 3}{8 - 3} = \frac{x - 1}{4 - 1}$
	$y - 5 = 2x - 4$	$\frac{y - 3}{5} = \frac{x - 1}{3}$
	$\boxed{y = 2x + 1}$	$3y - 9 = 5x - 5$
		$\boxed{3y = 5x + 4}$

HOW TO FIND GRADIENT OF LINE FROM EQUATION.

$3x + 4y = 12$ gradient = ?

STEP 1: Make y - subject.

STEP 2: Compare with $y = mx + c$

$$4y = -3x + 12$$

$$y = \frac{-3x}{4} + \frac{12}{4}$$

$$y = mx + c$$

$$\text{grad} = -\frac{3}{4}$$

$$y\text{-int} = 3.$$

IF A POINT LIES ON A LINE, IT MUST SATISFY ITS EQUATION

Q. Check if $(1, 5)$ (x y) lies on line $y = 4x + 1$ or not.

$y = 4x + 1$

$5 = 4(1) + 1$

$5 = 5$

Yes the point lies on the line.

Q. Given that $(9, k)$ lies on $3x + 4y = 27$, find value of k.

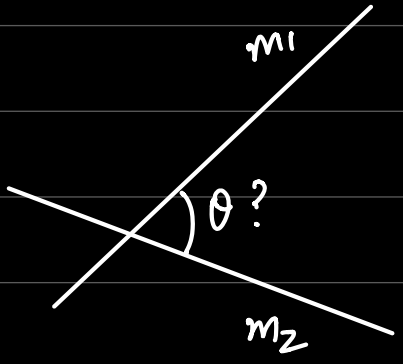
$3x + 4y = 27$ $\quad x = 9, y = k$

$3(9) + 4k = 27$

$27 + 4k = 27$

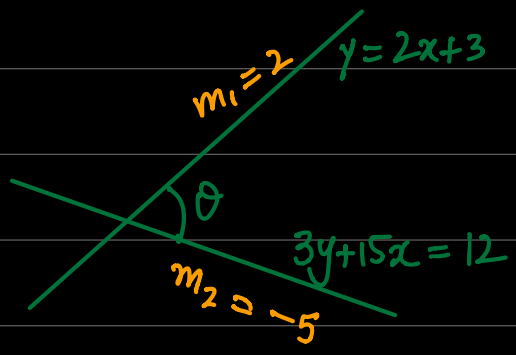
$k = 0$

ANGLE BETWEEN TWO LINES



$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

Q.



$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$\tan\theta = \frac{-5 - 2}{1 + (-5)(2)}$$

$$\tan\theta = \frac{-7}{-9}$$

$$\tan\theta = \frac{7}{9}$$

$$\theta = \tan^{-1}\left(\frac{7}{9}\right)$$

$$\theta = 37.875$$

while taking inverse, ignore any negative signs.

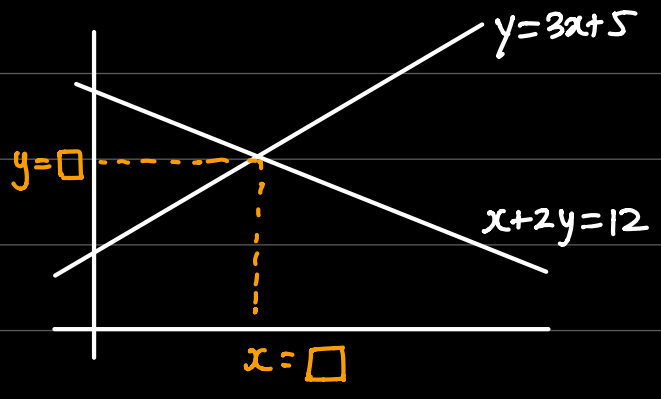
Concept in TRIG

POINTS OF INTERSECTION (4 MARKS)

SIMULTANEOUS EQUATIONS.

$x + 2y = 12$ $\quad$ $y = 3x + 5$

$x = \square$, $y = \square$



ELIMINATION WORKS ONLY IF BOTH GRAPHS ARE STRAIGHT LINES

WE WILL USE SUBSTITUTION ALL THE TIME.

make y subject and put both equations equal.

LINEAR	NON-LINEAR
STRAIGHT LINE	CURVE
$2x+3y=5$	$y=x^2+3x+5$

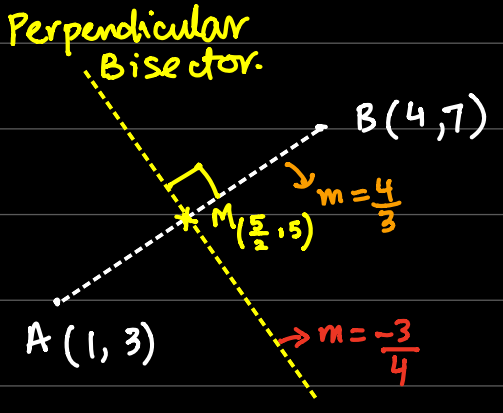
FOR LINEAR, MAKE y-SUBJECT AND THEN POWERS OF x and y SHOULD BE 1.

$$xy=12$$

$$y=\frac{12}{x}$$

$$y=12x^{-1} \text{ (curve)}$$

PERPENDICULAR BISECTOR (4 MARKS)



STEP 1: Find gradients.

$$m_{AB}=\frac{7-3}{4-1}=\frac{4}{3}$$

$$m_{\perp}=-\frac{3}{4}$$

STEP 2: Find midpoint M.

$$M=\left(\frac{1+4}{2},\frac{7+3}{2}\right)=\left(\frac{5}{2},5\right)$$

STEP 3: Find equation of line.

$$m=-\frac{3}{4}, \quad M\left(\underset{x_1}{\frac{5}{2}},\underset{y_1}{5}\right)$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{3}{4}\left(x - \frac{5}{2}\right)$$

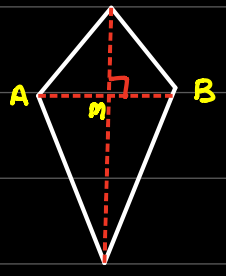
$$4y - 20 = -3x + \frac{15}{2}$$

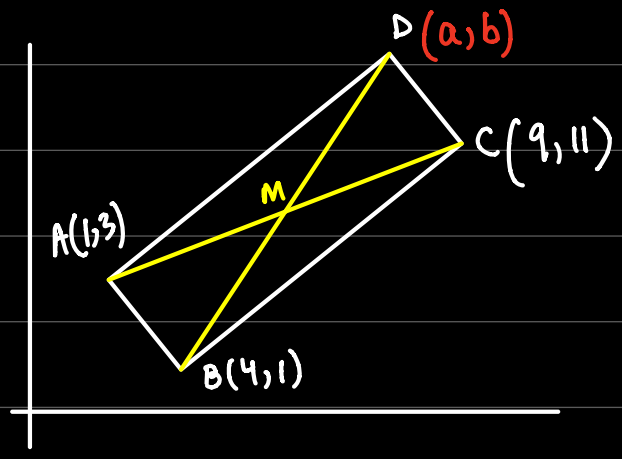
$$8y - 40 = -6x + 15$$

$$\boxed{8y = -6x + 55}$$

SPECIAL SHAPES

DIAGONALS MEET AT MID POINT	DIAGONALS MEET AT 90°
SQUARE	SQUARE
RHOMBUS	RHOMBUS
RECTANGLE	KITE
PARALLELOGRAM.	





ABCD is rectangle.

Find coordinates of D

(1 mark)

midpoint of AC = midpoint of BD

$$\left(\frac{1+9}{2}, \frac{3+11}{2}\right) = \left(\frac{a+4}{2}, \frac{b+1}{2}\right)$$

$$\frac{10}{2} = \frac{a+4}{2}$$

$$10 = a+4$$

$$a = 6$$

$$\frac{14}{2} = \frac{b+1}{2}$$

$$14 = b+1$$

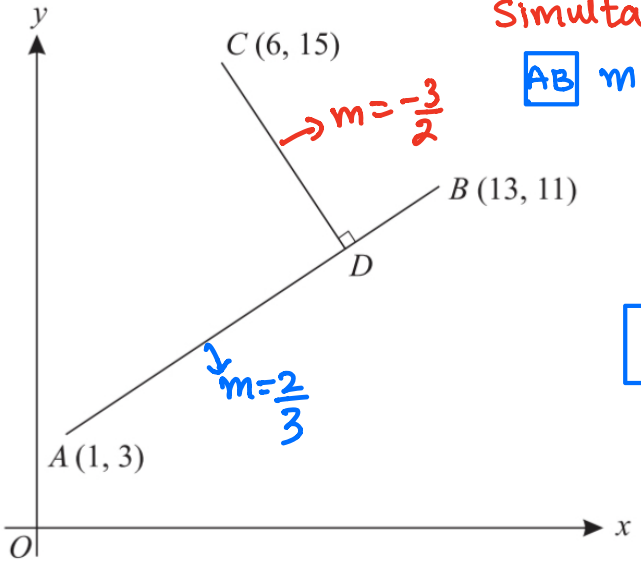
$$b = 13$$

D (6, 13)

8

D POINT OF INTERSECTION

Simultaneously solve.



(i) $m_{AB} = \frac{11-3}{13-1} = \frac{8}{12} = \frac{2}{3}$

$m_{CD} = -\frac{3}{2}$

CD $m = -\frac{3}{2}$, C(6,15)

$y - y_1 = m(x - x_1)$

$y - 15 = -\frac{3}{2}(x - 6)$

$2y - 30 = -3x + 18$

$2y = -3x + 48$

AB $m = \frac{2}{3}$ A(1,3)

$y - 3 = \frac{2}{3}(x - 1)$

$3y - 9 = 2x - 2$

$3y = 2x + 7$

The three points A (1, 3), B (13, 11) and C (6, 15) are shown in the diagram. The perpendicular from C to AB meets AB at the point D. Find

(i) the equation of CD, [3]

(ii) the coordinates of D. [4]

AB $3y = 2x + 7$ CD $2y = -3x + 48$

$y = \frac{2x+7}{3}$ $\quad$ $y = \frac{-3x+48}{2}$

$$\frac{2x+7}{3} = \frac{-3x+48}{2}$$

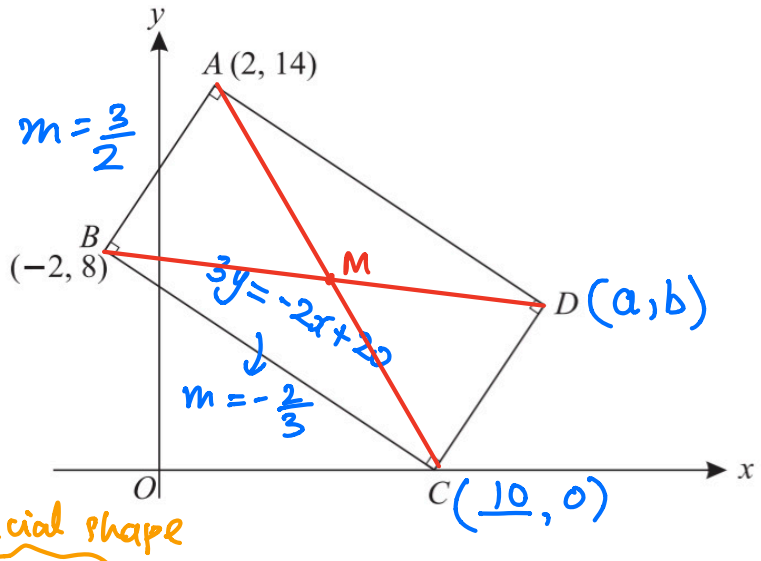
$$4x + 14 = -9x + 144$$

$$13x = 130$$

$$x = 10$$

$$y = \frac{2(10)+7}{3} = 9 \qquad D\,(10, 9)$$

9



Special shape

The diagram shows a rectangle $ABCD$. The point A is (2, 14), B is (−2, 8) and C lies on the x-axis. Find

(i) the equation of BC, [4]

(ii) the coordinates of C and D. [3]

$$m_{AB} = \frac{14-8}{2-(-2)} = \frac{6}{4} = \frac{3}{2}$$

BC $\quad m_{BC} = \frac{-2}{3}$, $B\,(-2, 8)$

$$y - 8 = -\frac{2}{3}\left(x - (-2)\right)$$

(ii)

C $\quad 3y = -2x + 20$

$y = 0$

$3(0) = -2x + 20$

$2x = 20$

$x = 10$

$3y - 24 = -2x - 4$

$\boxed{3y = -2x + 20}$

C(10, 0)

D

midpoint of AC = midpoint of BD

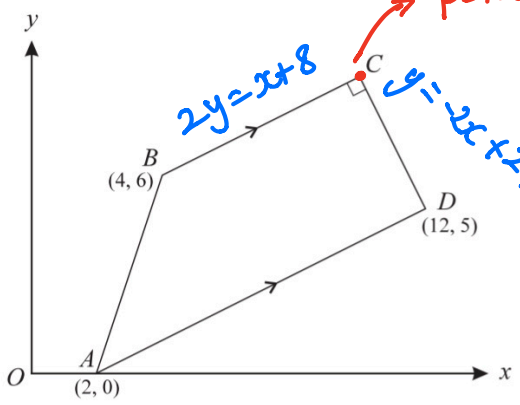
$\left(\frac{2+10}{2}, \frac{14+0}{2}\right) = \left(\frac{-2+a}{2}, \frac{8+b}{2}\right)$

$\frac{12}{2} = \frac{-2+a}{2}$, $\frac{14}{2} = \frac{8+b}{2}$

$a = 14$ $\quad b = 6$

D (14, 6)

3



point of intersection. → Simultaneously Solve.

$2y = x + 8$

$y = -2x + 29$

The diagram shows a trapezium $ABCD$ in which BC is parallel to AD and angle $BCD = 90°$. The coordinates of A, B and D are (2, 0), (4, 6) and (12, 5) respectively.

(i) Find the equations of BC and CD. [5]

(ii) Calculate the coordinates of C. [2]

$m_{AD} = \frac{5-0}{12-2} = \frac{5}{10} = \frac{1}{2}$

BC $m_{BC} = \frac{1}{2}$, $B(4,6)$

$y - 6 = \frac{1}{2}(x - 4)$

$2y - 12 = x - 4$

$\boxed{2y = x + 8}$

CD $m_{CD} = -2$, $D(12,5)$

$y - 5 = -2(x - 12)$

$y - 5 = -2x + 24$

$\boxed{y = -2x + 29}$

(ii) C $\quad y = \frac{x+8}{2} \qquad y = -2x + 29$

$$\frac{x+8}{2} = -2x + 29$$

$$x + 8 = -4x + 58$$

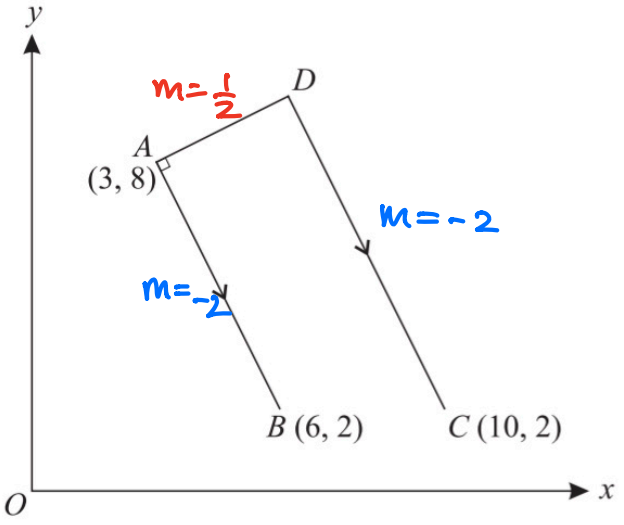
$$5x = 50$$

$$x = 10$$

$$y = -2(10) + 29 = 9$$

$$C(10, 9)$$

10



The three points A (3, 8), B (6, 2) and C (10, 2) are shown in the diagram. The point D is such that the line DA is perpendicular to AB and DC is parallel to AB. Calculate the coordinates of D. [7]

$$m_{AB} = \frac{2-8}{6-3} = \frac{-6}{3} = -2$$

CD $\quad m_{CD} = -2 \qquad C(10, 2)$

$$y - 2 = -2(x - 10)$$

$$y - 2 = -2x + 20$$

AD $\quad m_{AD} = \frac{1}{2}$, $A(3, 8)$

$$y - 8 = \frac{1}{2}(x - 3)$$

$y = -2x + 22$

$2y - 16 = x - 3$

$2y = x + 13$

D point of intersection

$y = -2x + 22$ $\qquad$ $y = \frac{x+13}{2}$

$$-2x + 22 = \frac{x+13}{2}$$

$$-4x + 44 = x + 13$$

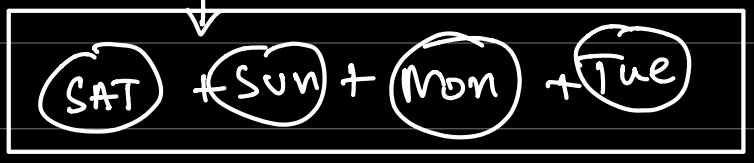
$$31 = 5x$$

$$x = \frac{31}{5} = 6.2$$

$$y = \frac{6.2 + 13}{2} = 9.6$$

$$D = (6.2, 9.6)$$

Sheet = 40 Questions. ←



AS only.
15 Q/day. for Maths.