FUNCTIONS (P1)

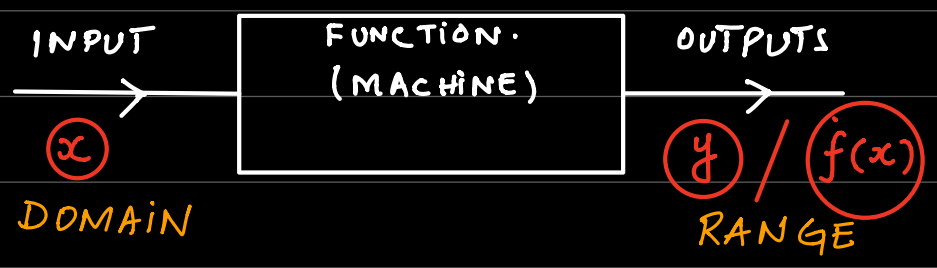
(10 MARKS)

FUNCTIONS ALONE	WITH QUADRATICS	WITH TRIG	WITH DIFFERENTIATION

VERY LONG BUT EASY CHAPTER.

MEMORIZE EVERY DETAIL!

FUNCTIONS ARE NUMBER MACHINES.



$f(x) = 2x + 3$ $\qquad$ $g(x) = x^2 - 5$

Name of function $\qquad$ value of input (x)

$f(1) = 2(1) + 3 = 5$

$f(4) = 2(4) + 3 = 11$

$g(3) = (3)^2 - 5 = 4$

$f(K) = 2K + 3$

$g(t-1) = (t-1)^2 - 5$

INVERSE OF A FUNCTION:

(2 marks in O Levels) (3 MARKS IN A LEVELS)

THERE ARE ADDITIONAL STEPS. BE CAREFUL!

$$f(x) = 2x - 5$$

STEP 1: $f(x) = y$

$$y = 2x - 5$$

$$y + 5 = 2x$$

STEP 2: $x = \dfrac{y+5}{2}$

STEP 3: $f^{-1}(y) = x$

$f(x) = y$

$f^{-1}(y) = x$

STEP 4: $f^{-1}(y) = \dfrac{y+5}{2}$

$$f^{-1}(x) = \frac{x+5}{2}$$

Q: $f(x) = 2x + 8$

$$f(x) = y$$
$$y = 2x + 8$$
$$y - 8 = 2x$$
$$x = \frac{y-8}{2}$$

$$f^{-1}(y) = x$$

$$f^{-1}(y) = \frac{y-8}{2}$$

$$f^{-1}(x) = \frac{x-8}{2}$$

Q: $g(x) = x^3 - 8$

$$g(x) = y$$
$$y = x^3 - 8$$
$$y + 8 = x^3$$
$$x^3 = y + 8$$
$$x = \sqrt[3]{y+8}$$
$$g^{-1}(y) = x$$

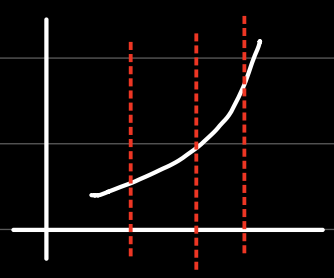
$$g^{-1}(y) = \sqrt[3]{y+8}$$

$$\boxed{g^{-1}(x) = \sqrt[3]{x+8}}$$

TEST

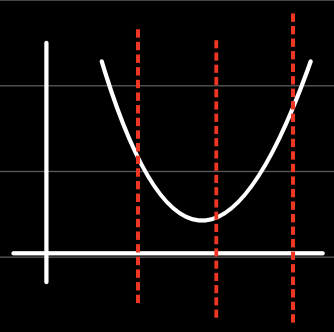
VERTICAL LINE TEST

CHECKS WHETER A GRAPH IS A FUNCTION OR NOT?



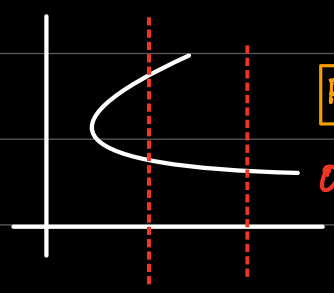
PASS IT IS FUNCTION

ONE-ONE FUNCTION



PASS IT IS FUNCTION

MANY-ONE FUNCTION

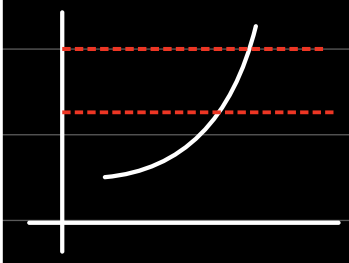


FAIL NOT A FUNCTION

ONE-MANY.

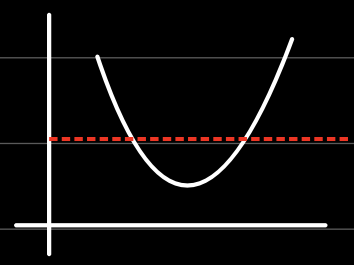
HORIZONTAL LINE TEST

CHECKS IF THE INVERSE OF A FUNCTION EXISTS OR NOT?



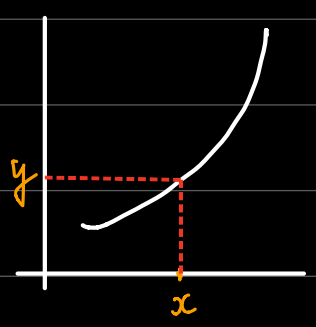
PASS INVERSE EXISTS

ONE-ONE FUNCTION



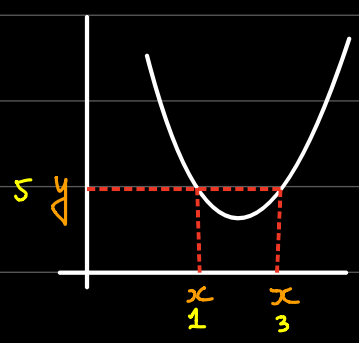
FAIL INVERSE DOES NOT EXIST.

MANY-ONE FUNCTION



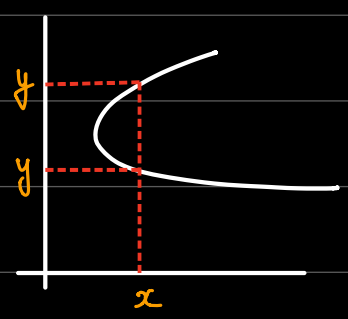
ONE-ONE FUNCTION

For one input you will get one output.



MANY TO ONE FUNCTION

For Many inputs you can get one output.



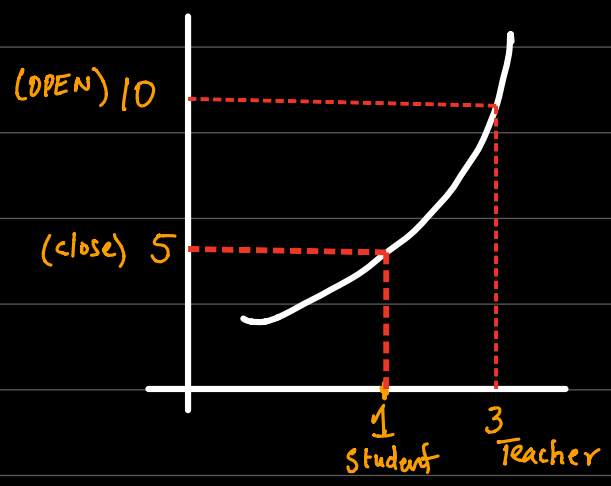
ONE TO MANY

one input can get you many outputs.

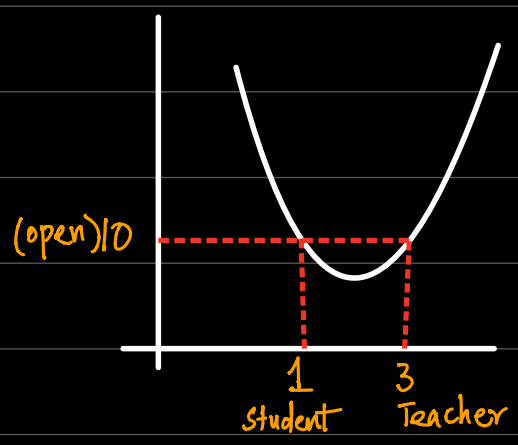
ACCESS POINTS

INPUT = 1 (STUDENT)
= 3 (TEACHER)

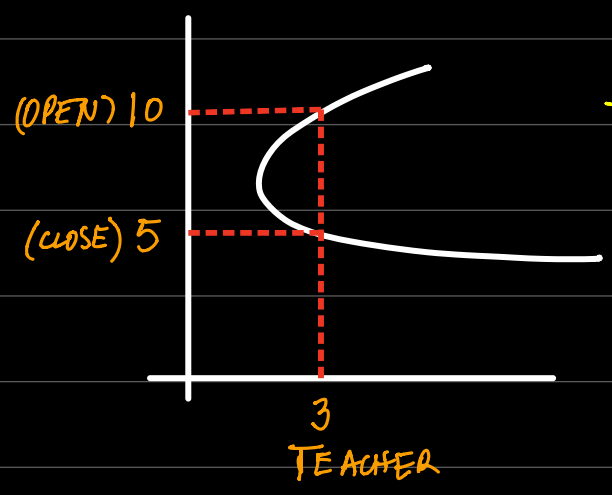
OUTPUT = 5 (CLOSE)
= 10 (OPEN)



→ This would be useful at STAFF ROOM DOOR.



→ THIS WOULD BE USEFUL AT SCHOOL MAIN GATE etc.



→ THIS is NOT A USEFUL MACHINE.

ONE-TO-MANY is NOT A FUNCTION.

COMPOSITE FUNCTIONS

(FUNCTION WITHIN A FUCTION)

$f(x) = 2x + 5$ $\qquad$ $g(x) = x^2 + 3$

$$fg(x) = f(g(x)) = 2\,g(x) + 5$$
$$= 2(x^2+3) + 5$$
$$= 2x^2 + 6 + 5$$
$$= 2x^2 + 11$$

ESSENTIAL WORKING.

$$gf(x) = (2x+5)^2 + 3$$

$$ff(x) = 2(2x+5) + 5$$

$$gg(x) = (x^2+3)^2 + 3$$

STEP 1: Write the first function with ( ) instead of x.

STEP 2: Write value of second function inside the bracket.

Q: $f(x) = 2x + 3$

Given that $ff(t) = 21$, find t.

$$ff(x) = 2(2x+3) + 3 = 4x + 6 + 3 = 4x + 9$$

$$ff(x) = 4x + 9$$

$$ff(t) = 4t + 9$$

$$21 = 4t + 9$$

$4t = 12$

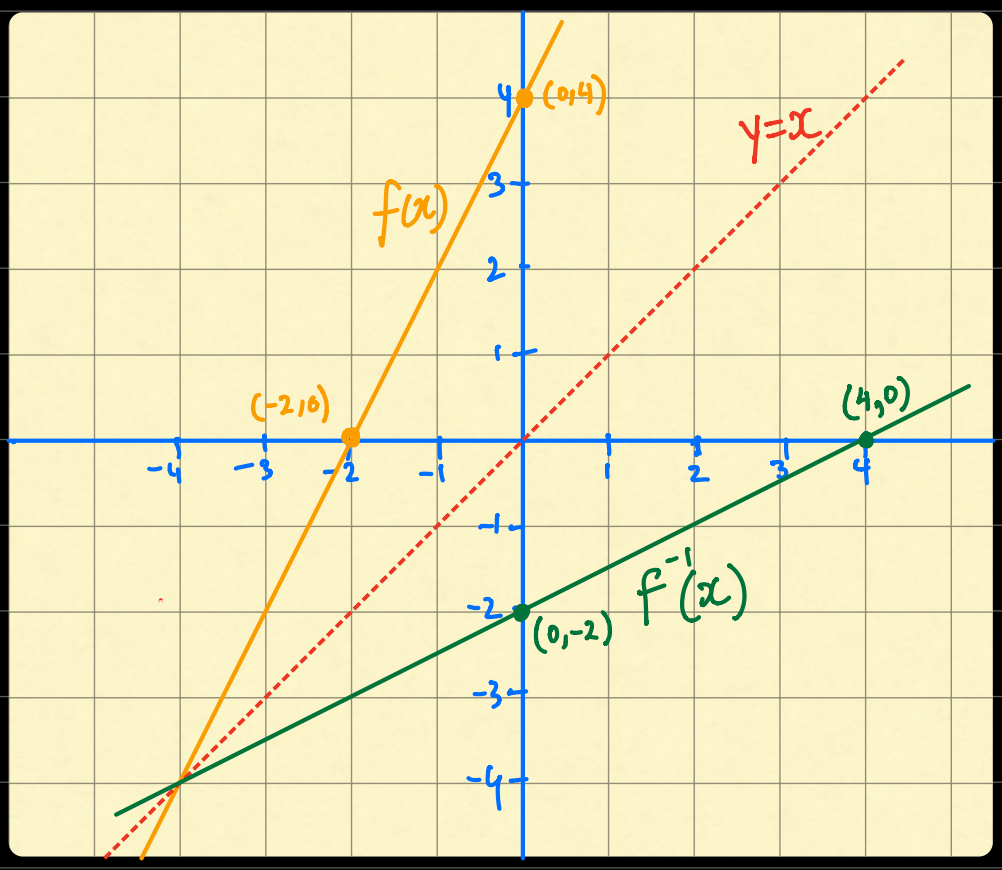
$t = 3$

SKETCH OF $f(x)$ and $f^{-1}(x)$ ON SAME DIAGRAM.

Q: $f(x) = 2x + 4$

Sketch $f(x)$ and $f^{-1}(x)$ on same diagram making clear the relationship between them. (3 marks).

IMP: THIS HAS TO BE DRAWN TO-SCALE.



$f(x) = 2x + 4$

$y = 2x + 4$

STRAIGHT LINE.

<u>x-intercept</u> $y = 0$

$0 = 2x + 4$

$x = -2$

<u>y-intercept</u> $x = 0$

$y = 2(0) + 4$

$y = 4$

$f^{-1}(x)$ is reflection of $f(x)$ in line $y = x$.

Q: $f(x) = 2x^2 - 8x + 14$

(a) Express $f(x)$ in form $a(x-b)^2 + c$

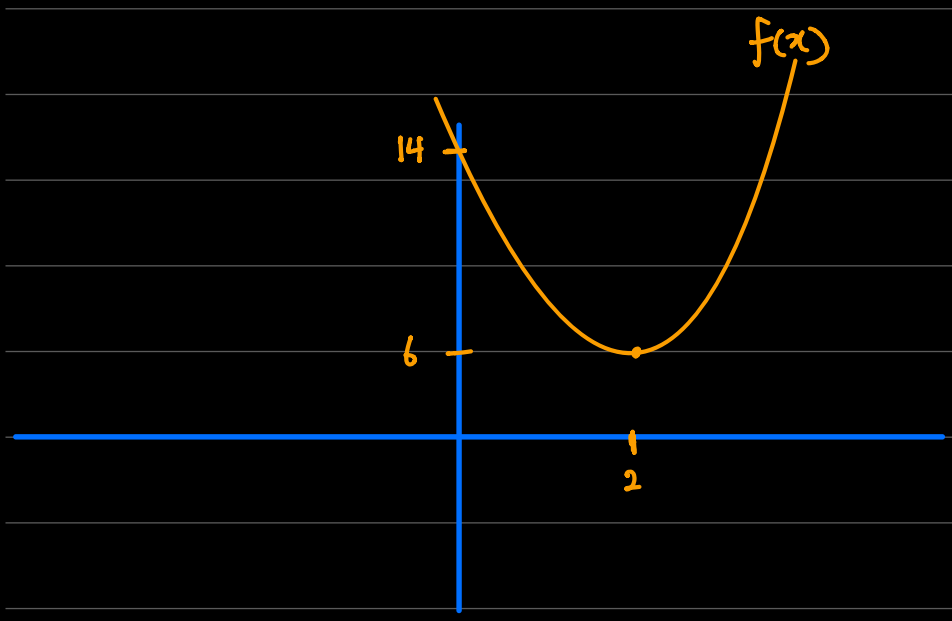
$$2\left[x^2 - 4x + (2)^2 - (2)^2 + 7\right]$$

$$2\left[(x-2)^2 - 4 + 7\right]$$

$$2\left[(x-2)^2 + 3\right]$$

$$2(x-2)^2 + 6$$

(b) Sketch $f(x)$.



$f(x) = 2x^2 - 8x + 14$

Shape

y-intercept

$f(x) = 2(x-2)^2 + 6$

Shape

$x - 2 = 0$

Turning pt: $\boxed{x=2}$ $\boxed{y=6}$

(c) State with a reason whether $f^{-1}(x)$ exists or not?

No, inverse does not exist because it is not a one-one function.

DOMAIN VALUES OF X FOR WHICH A FUNCTION DOES NOT GO CRAZY.

TYPE 1

$f(x) = \sqrt{x}$

Domain:
$x \geq 0$
$x \in R$

$f(x) = \sqrt{x-2}$

Domain:
$x \geq 2$
$x \in R$

GENERAL RULE

$f(x) = \sqrt{\square}$

$x \in R$
x belongs to Real Numbers
Any no.

Domain:
$\square \geq 0$ and solve.
$x \in R$

$f(x) = \sqrt{2x+3}$

Domain: $2x + 3 \geq 0$

$2x \geq -3$

$x \geq -\frac{3}{2}$ and $x \in R$

TYPE 2

$f(x) = \frac{3}{x}$

Domain: $x \neq 0$
$x \in R$

$f(x) = \frac{3}{x-2}$

Domain: $x \neq 2$
$x \in R$

GENERAL RULE.

$f(x) = \frac{\text{Anything}}{\square}$

Domain: $\square \neq 0$ solve
$x \in R$

$$f(x) = \frac{2x+5}{\boxed{3x-1}}$$

Domain:

$3x - 1 \neq 0$

$3x \neq 1$

$$\boxed{x \neq \frac{1}{3} \quad \text{and} \quad x \in R}$$

RANGE: VALUES OF y THAT GRAPH OF $f(x)$ COVERS.

GOLDEN RULE: NEVER TELL RANGE OF A FUNCTION WITHOUT LOOKING AT ITS GRAPH

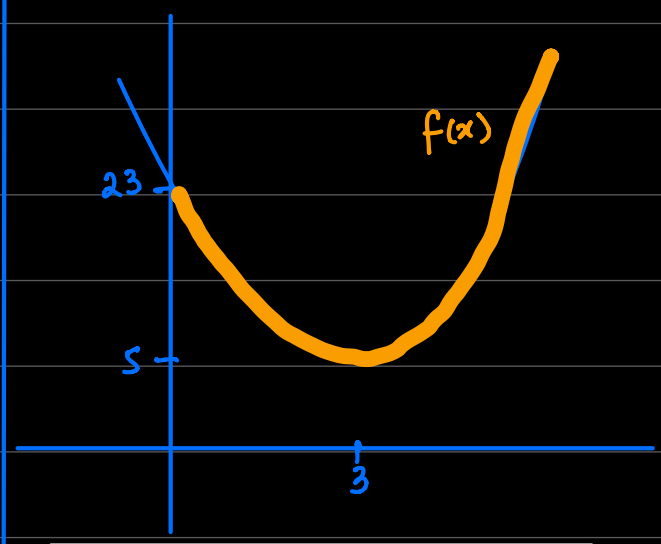
FIND RANGE OF FOLLOWING FUNCTIONS:

(a) $f(x) = 2(x-3)^2 + 5 \quad \underbrace{x \in R}_{\text{you are allowed all values of } x.}$

Range: $y \geq 5$

$f(x) \geq 5$

(b) $f(x) = 2(x-3)^2 + 5 \quad x > 0$



Range: $y \geq 5$

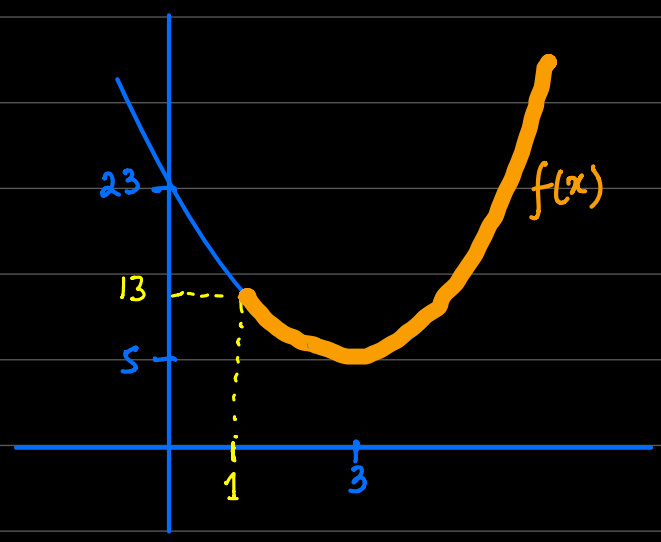
$f(x) \geq 5$

Range is usually written by name of function.

(c) $f(x) = 2(x-3)^2 + 5 \quad x \geq 1$

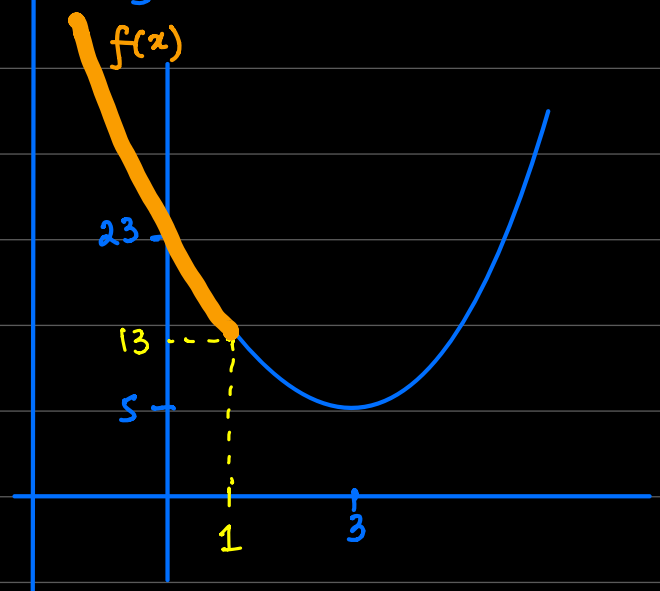
Put $x = 1$, $y = 2(1-3)^2 + 5$

$y = 13$



$$\boxed{\text{Range: } \begin{array}{l} y \geq 5 \\ f(x) \geq 5 \end{array}}$$

(d) $f(x) = 2(x-3)^2 + 5 \quad x < 1$

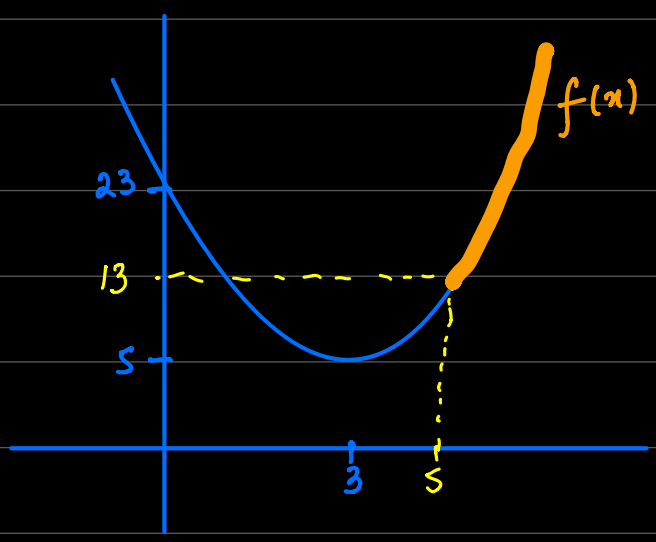


$$\boxed{\text{Range: } \begin{array}{l} y > 13 \\ f(x) > 13 \end{array}}$$

(e) $f(x) = 2(x-3)^2 + 5 \quad x \geq 5$

Put $x = 5$, $y = 2(5-3)^2 + 5$

$y = 13$



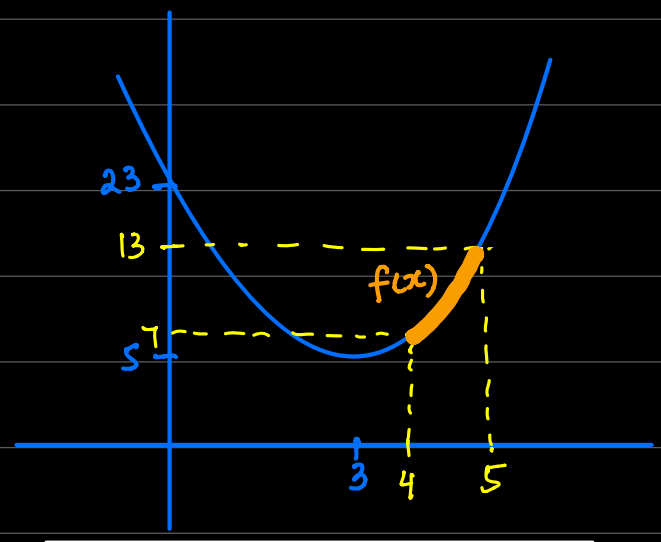
Range: $\boxed{\begin{array}{l} y \geq 13 \\ f(x) \geq 13 \end{array}}$

(f)

$f(x) = 2(x-3)^2 + 5 \quad 4 < x \leq 5$

put $x = 4$, $y = 2(4-3)^2 + 5 =$

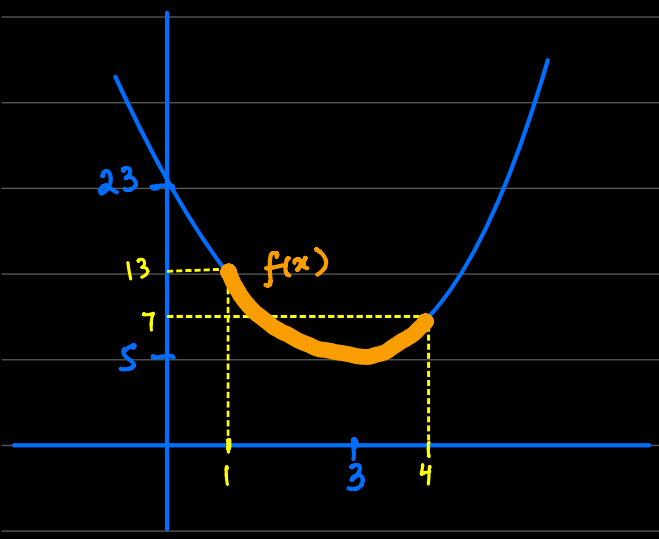
$y = 7$



$$\boxed{\text{Range: } \begin{array}{l} 7 < y \leq 13 \\ 7 < f(x) \leq 13 \end{array}}$$

(g)

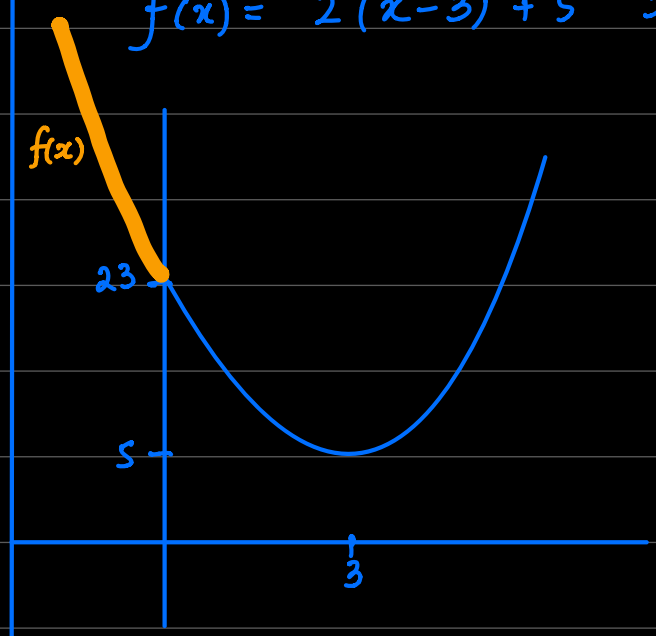
$f(x) = 2(x-3)^2 + 5 \qquad 1 < x < 4$



$$\boxed{\begin{array}{ll} \text{Range:} & 5 \leq y < 13 \\ & 5 \leq f(x) < 13 \end{array}}$$

(h)

$f(x) = 2(x-3)^2 + 5 \qquad x < 0$



$$\boxed{\begin{array}{ll} \text{Range:} & y > 23 \\ & f(x) > 23 \end{array}}$$