FRICTION

Resistance to motion on a surface.

$$\boxed{F = \mu R}$$

MAX FRICTION

THIS FORMULA CAN ONLY BE APPLIED IF BODY:

1) IS ABOUT TO MOVE.
2) MOVING.

Cofficient of Friction $\boxed{\mu}$

(Property of surfae).

Type of surface.

Rough Surface, $\mu\uparrow$

Smoother surface, $\mu\downarrow$

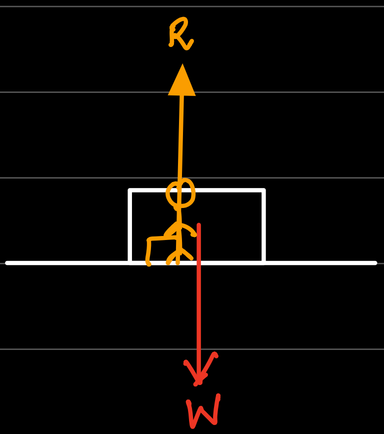
(90°) NORMAL FORCE
REACTION FORCE
CONTACT FORCE

$\boxed{R}$

DIRECTION: TOWARDS HEAD OF M or M1.

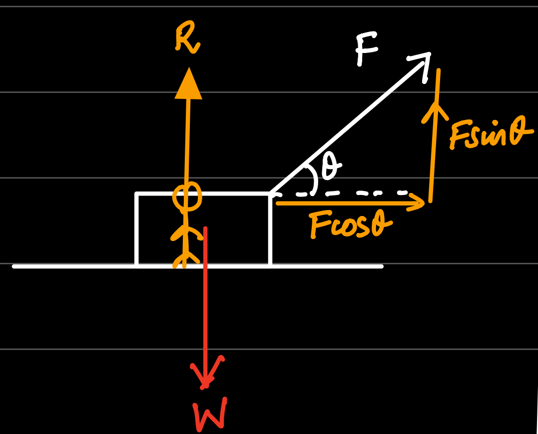
ALWAYS FIND R using

$\boxed{UP = DOWN}$



up = down

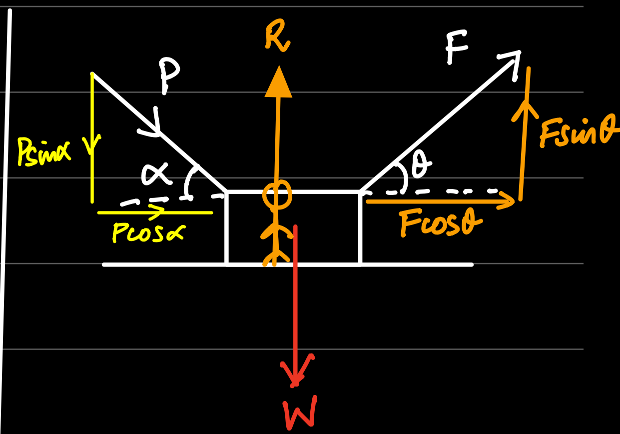
$R = W$



up = down

$R + F\sin\theta = W$

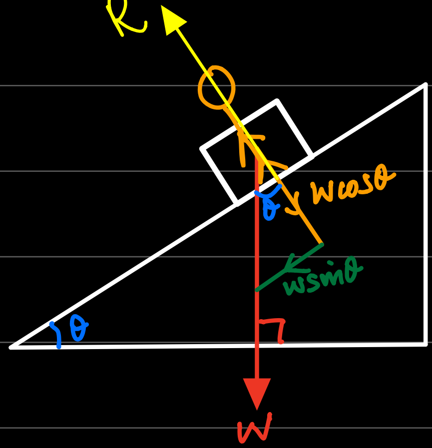
$R = W - F\sin\theta$



up = down

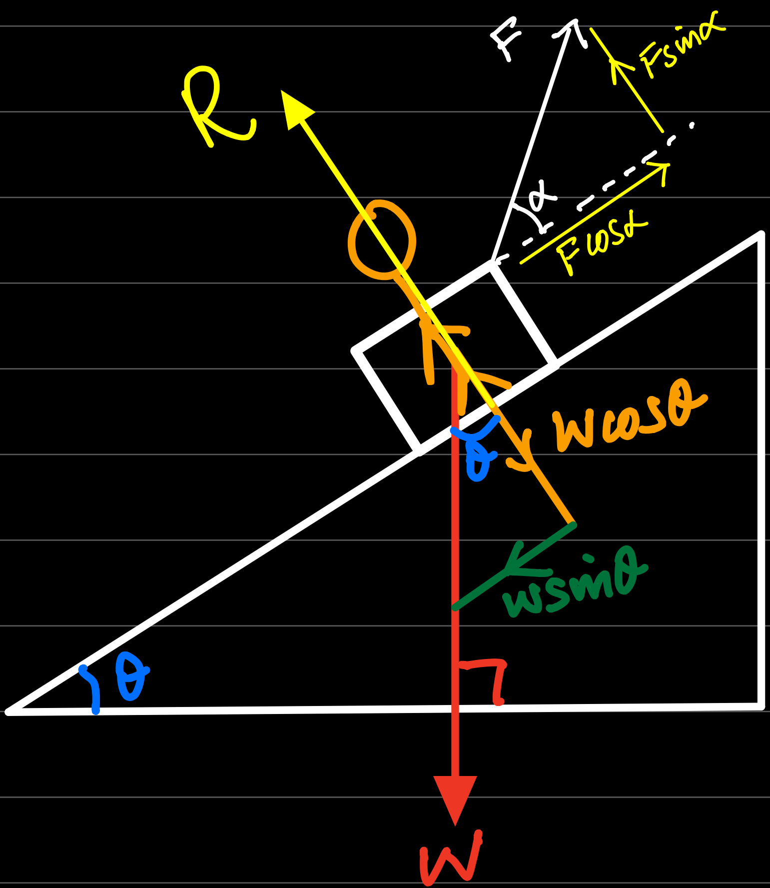
$R + F\sin\theta = W + P\sin\alpha$

$R = W + P\sin\alpha - F\sin\theta$



$$up = down$$

$$R = W\cos\theta$$



$$up = down$$

$$R + F\sin\alpha = W\cos\theta$$

$$R = W\cos\theta - F\sin\alpha.$$

THE MAXIMUM FRICTION BETWEEN BOX AND GROUND IS 15N.

REST

REST

FRICTION. (cannot use μR).

ABOUT TO MOVE

MAX FRICTION

$$\boxed{F = \mu R}$$

MOVE

MAX FRICTION

Example BOX IS AT REST. FIND FRICTION.

Left = Right.

3 + Friction = 14

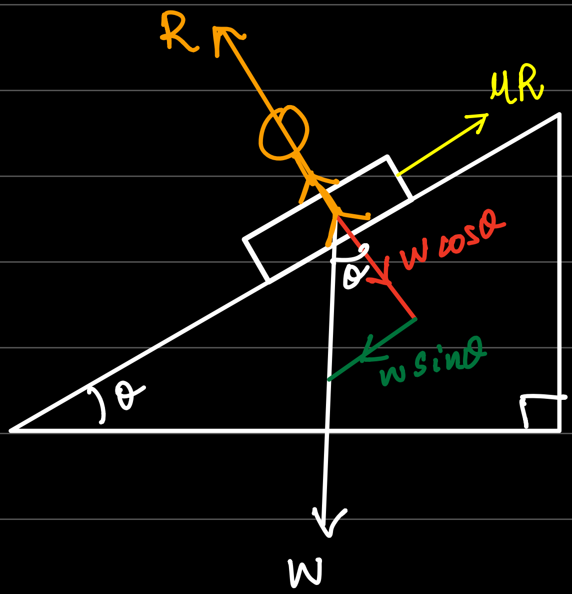
Friction = 11 N.

LIMITING EQUILIBRIUM
(BODY IS ABOUT TO MOVE)

STEP 1: MARK ON DIAGRAM THE MAX FRICTION (μR)
(Direction: Against direction of motion).

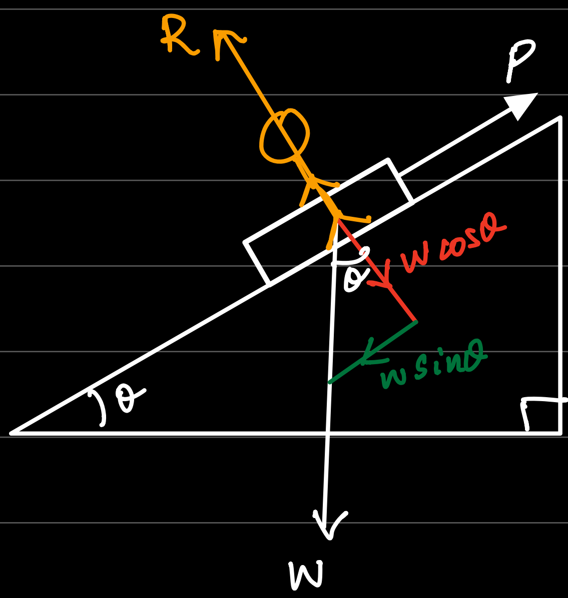
STEP 2: up = down
Left = Right.

BOX IS ABOUT TO SLIDE



We are sure that in this case box will :

1) Either stay at rest
2) move downwards.



A force P pulls the box upwards. The box is about to slide.

$\boxed{\text{V. IMP}}$ We cannot determine direction of Friction here.

NOW THERE ARE 2 TYPES OF QUESTIONS :

BASIC : QUESTION TELLS DIRECTION OF MOTION

ADVANCED : Find two values of P for which box is about to slide. (8 marks)

slide up

↓

Mark friction

↓

up = down

Left = Right.

slide down

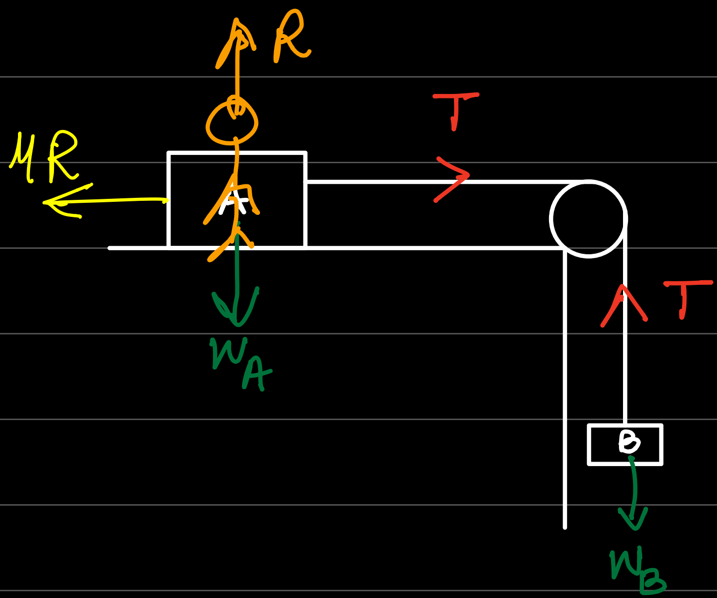
↓

Mark friction

↓

up = down

Left = Right.

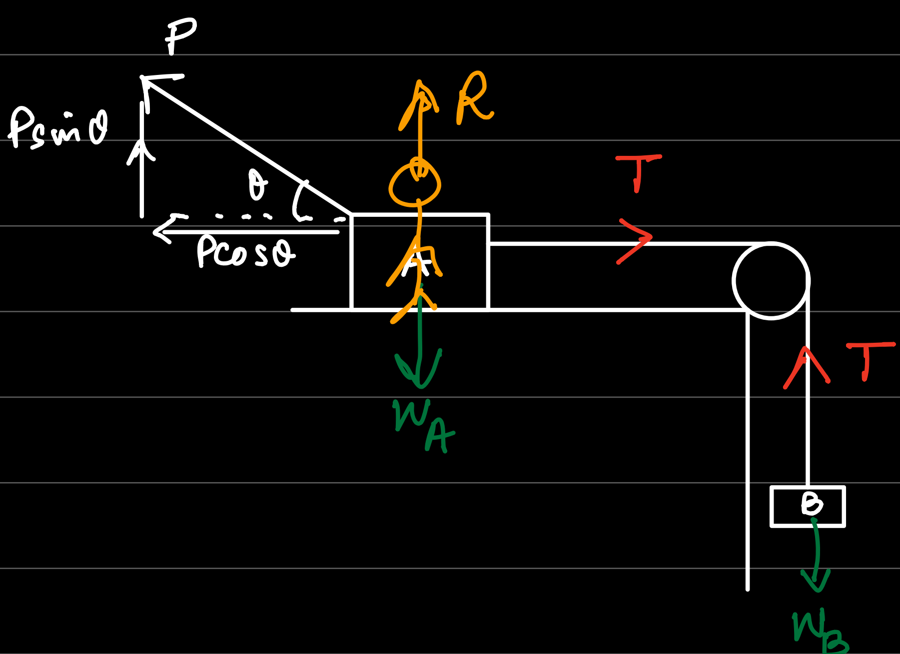


Box A is about to slide.

In this case we are sure that Box A will :

1) either stay at rest

2) or move to right



Box A is about to move.

$\boxed{\text{IMP}}$ We cannot determine the direction of Motion.

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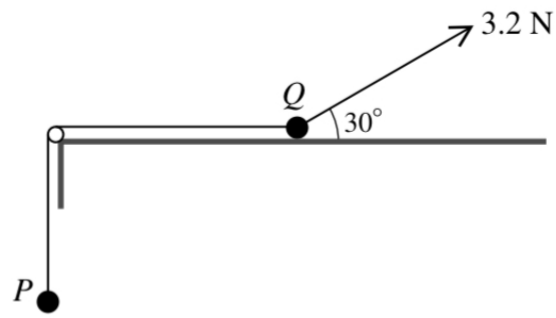


2N 5N

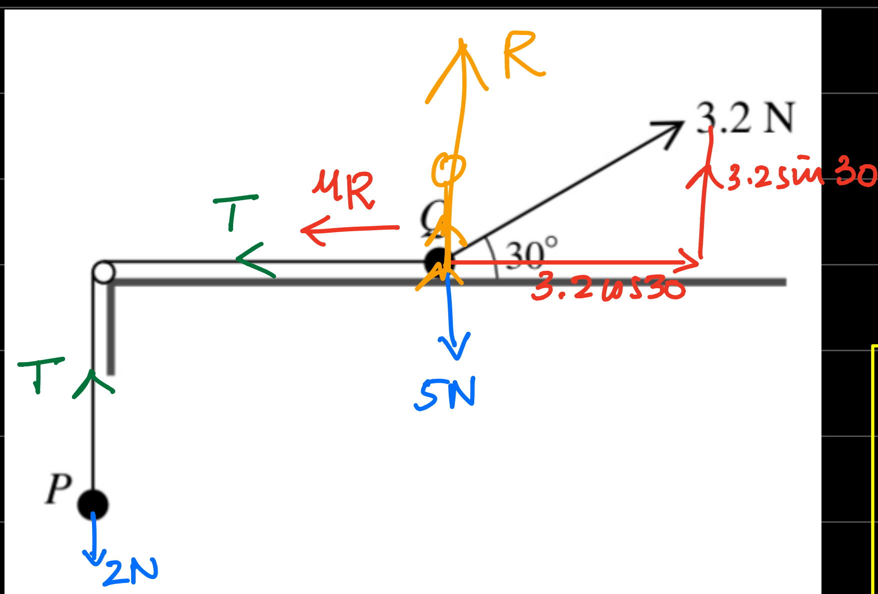
Particles P and Q, of masses 0.2 kg and 0.5 kg respectively, are connected by a ~~light inextensible~~ string. The string passes over a smooth pulley at the edge of a rough horizontal table. P hangs freely and Q is in contact with the table. A force of magnitude 3.2 N acts on Q, upwards and away from the pulley, at an angle of 30° to the horizontal (see diagram).

about to move

(i) The system is in limiting equilibrium with P about to move upwards. Find the coefficient of friction between Q and the table. [6]

Basic

Q is about to move to right.



$\boxed{P}$ up = down

$$\boxed{T = 2}$$

NOTE: Forces on one side of pulley cannot be used on other side of pulley.

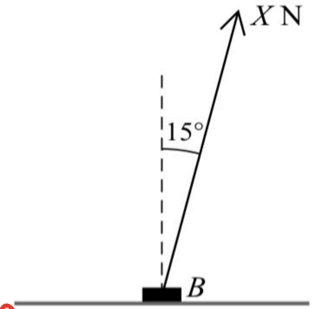
$\boxed{Q}$ up = down

$$R + 3.2\sin 30 = 5$$
$$R = 5 - 3.2\sin 30$$
$$R = 3.4$$

Left = Right.

$$T + \mu R = 3.2\cos 30$$
$$2 + \mu(3.4) = 3.2\cos 30$$
$$\mu = \frac{3.2\cos 30 - 2}{3.4}$$
$$\boxed{\mu = 0.2268}$$

71



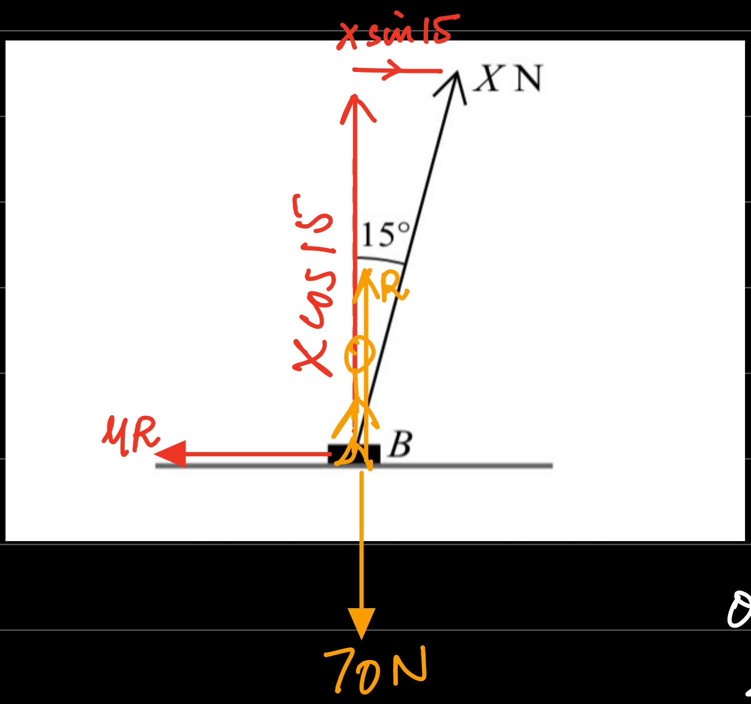
cannot use μR.

A block B of mass 7 kg is at rest on rough horizontal ground. A force of magnitude X N acts on B at an angle of 15° to the upward vertical (see diagram).

(i) Given that B is in equilibrium find, in terms of X, the normal (R) component of the force exerted on B by the ground. [2]

$\mu = 0.4$

(ii) The coefficient of friction between B and the ground is 0.4. Find the value of X for which B is in limiting equilibrium. : Friction $= \mu R$. [3]



(i) up = down

$$X\cos 15 + R = 70$$

$$\boxed{R = 70 - X\cos 15}$$

(ii) Left = Right

$$\mu R = X\sin 15$$

$$0.4(70 - X\cos 15) = X\sin 15$$

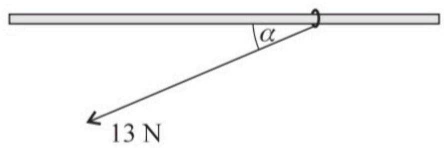
$$28 - 0.4X\cos 15 = X\sin 15$$

$$28 = X(0.4\cos 15 + \sin 15)$$

$$X = \frac{28}{0.4\cos 15 + \sin 15}$$

$$X = \boxed{43.398}$$

1



11N

A ring of mass 1.1 kg is threaded on a fixed rough horizontal rod. A light string is attached to the ring and the string is pulled with a force of magnitude 13 N at an angle α below the horizontal, where $\tan \alpha = \frac{5}{12}$ (see diagram). The ring is in equilibrium.

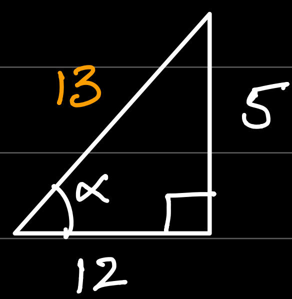
(i) Find the frictional component of the contact force on the ring. [2]

(ii) Find the normal component of the contact force on the ring. [2]

(iii) Given that the equilibrium of the ring is limiting, find the coefficient of friction between the ring and the rod. [1]

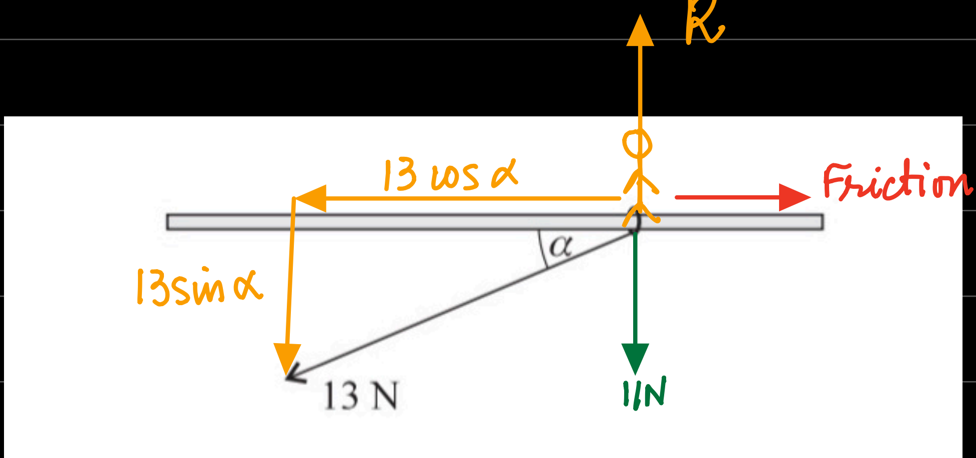
Friction = μR

$\tan \alpha = \frac{5}{12}$ P B



$\sin \alpha = \frac{5}{13}$

$\cos \alpha = \frac{12}{13}$



(i) Left = Right

$13 \cos \alpha = \text{Friction}$

$13\left(\frac{12}{13}\right) = \text{Friction}$

$\boxed{\text{Friction} = 12\,N}$

(ii) up = down

$R = 11 + 13 \sin \alpha$

$R = 11 + 13\left(\frac{5}{13}\right)$

$\boxed{R = 16\,N}$

(iii) $F = \mu R$

$12 = \mu(16)$

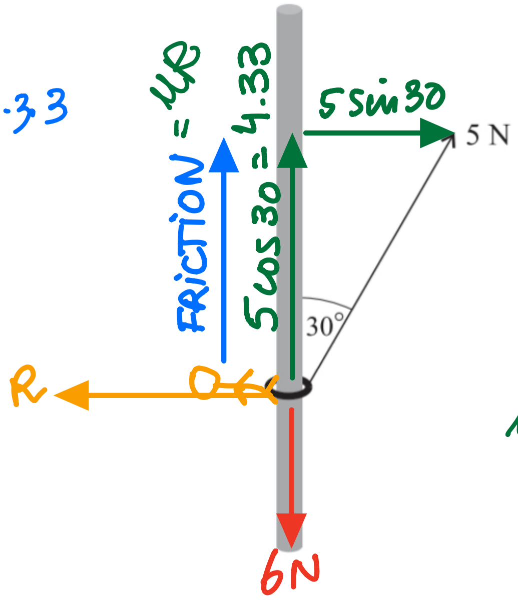
$\boxed{\mu = \frac{3}{4}}$

9

(i) Left = Right

$6 = \text{friction} + 4.33$

Friction = 1.67



(ii) up = down

$R = 5\sin 30$

$\boxed{R = 2.5}$

Left = Right

$\mu R + 4.33 = 6$

$\mu(2.5) = 1.67$

$\mu = \frac{1.67}{2.5} = \boxed{0.67}$

W = 6N

A small ring of mass 0.6 kg is threaded on a rough rod which is fixed vertically. The ring is in equilibrium, acted on by a force of magnitude 5 N pulling upwards at 30° to the vertical (see diagram).

(i) Show that the frictional force acting on the ring has magnitude 1.67 N, correct to 3 significant figures. [2]

(ii) The ring is on the point of sliding down the rod. Find the coefficient of friction between the ring and the rod. [3]

MOST ADVANCED VARIATIONS.

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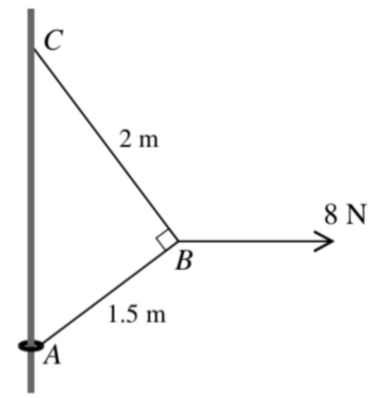


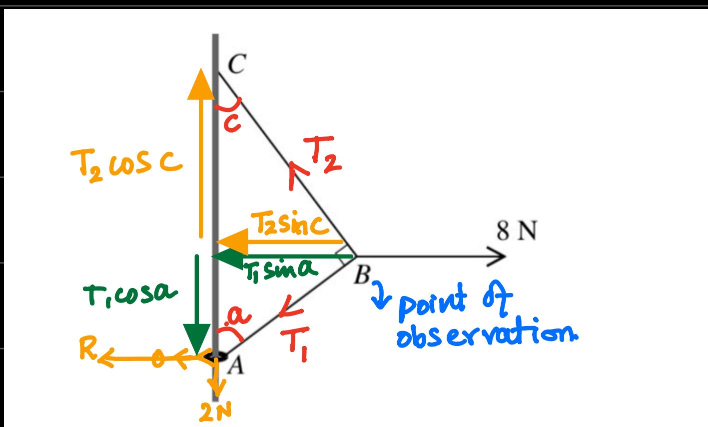
A small ring of mass 0.2 kg is threaded on a fixed vertical rod. The end A of a ~~light inextensible~~ string is attached to the ring. The other end C of the string is attached to a fixed point of the rod above A. A horizontal force of magnitude 8 N is applied to the point B of the string, where $AB = 1.5$ m and $BC = 2$ m. The system is in equilibrium with the string taut and AB at right angles to BC (see diagram).

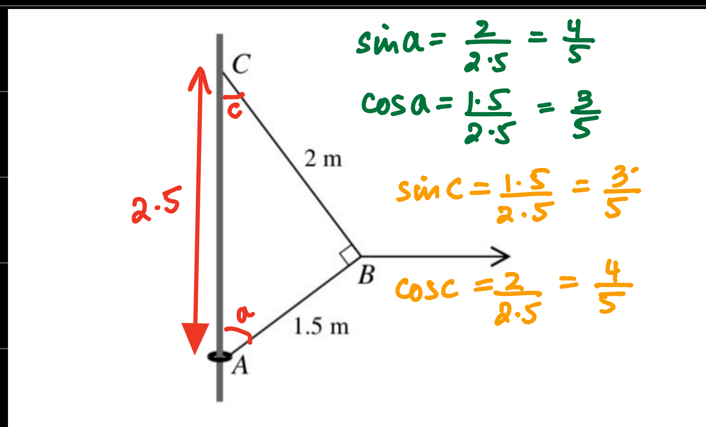
(i) Find the tension in the part AB of the string and the tension in the part BC of the string. [5]

New point of observation.

The equilibrium is limiting with the ring on the point of sliding up the rod.

(ii) Find the coefficient of friction between the ring and the rod. [5]





weight is not included because it is not acting on point of observation

up = down	Left = Right.
$T_2 \cos c = T_1 \cos a$	$T_2 \sin c + T_1 \sin a = 8$
$T_2\left(\frac{4}{5}\right) = T_1\left(\frac{3}{5}\right)$	$T_2\left(\frac{3}{5}\right) + T_1\left(\frac{4}{5}\right) = 8$
$\boxed{T_1 = \frac{4}{3} T_2}$	$\frac{3T_2 + 4T_1}{5} = 8$

$$\boxed{3T_2 + 4T_1 = 40}$$

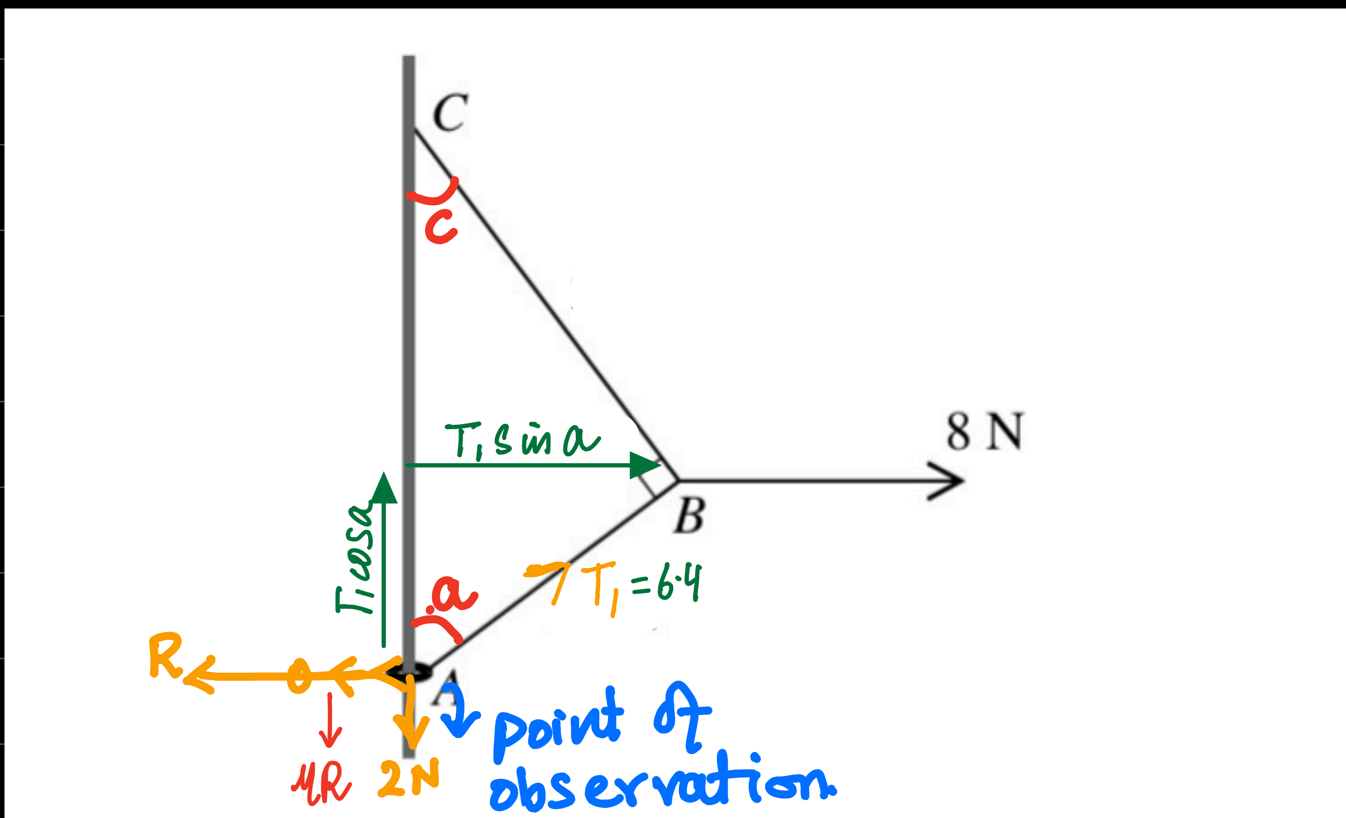
$$3T_2 + 4\left(\frac{4}{3}T_2\right) = 40$$

$$\frac{25T_2}{3} = 40$$

$$T_2 = 4.8$$

$$T_1 = \frac{4}{3} \times 4.8$$

$$\boxed{T_1 = 6.4}$$



up = down

$$R = T_1 \sin a$$

$$R = 6.4\left(\frac{4}{5}\right)$$

$$R = 5.12$$

Left = Right

$$2 + \mu R = T_1 \cos a$$

$$2 + \mu(5.12) = 6.4\left(\frac{3}{5}\right)$$

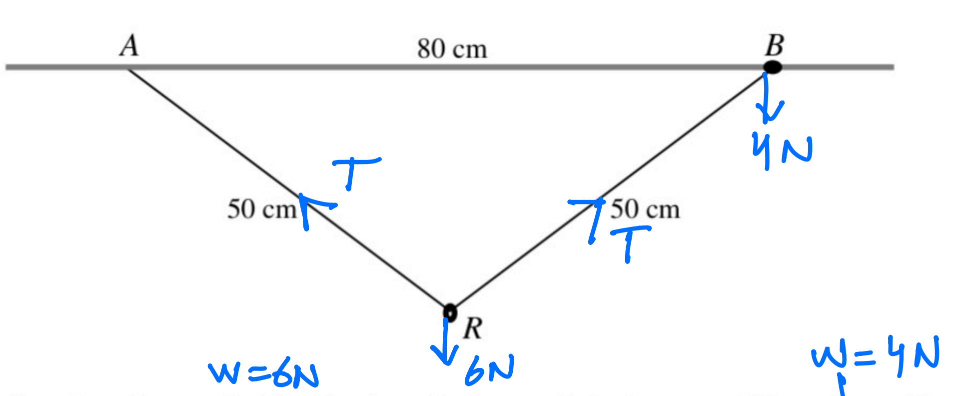
$$\boxed{\mu = 0.3594}$$

Balanced forces.
EQUILIBRIUM (Rest)
LIMITING EQUILIBRIUM (ABOUT TO MOVE)

UNBALANCED FORCES
RESULTANT FORCE.
KINEMATICS
MOMENTUM.

WORK, POWER, ENERGY.

31



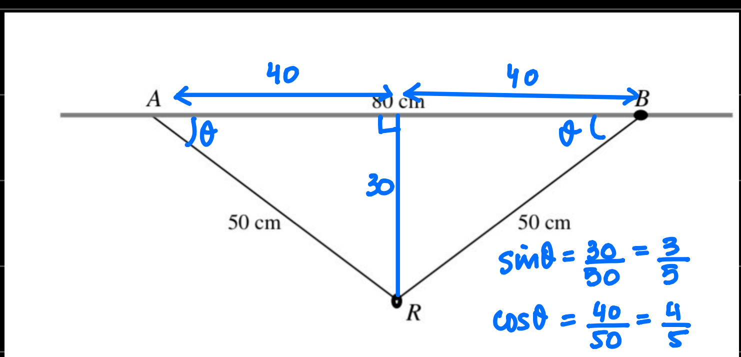
A small smooth ring R, of mass 0.6 kg, is threaded on a light inextensible string of length 100 cm. One end of the string is attached to a fixed point A. A small bead B of mass 0.4 kg is attached to the other end of the string, and is threaded on a fixed rough horizontal rod which passes through A. The system is in equilibrium with B at a distance of 80 cm from A (see diagram).

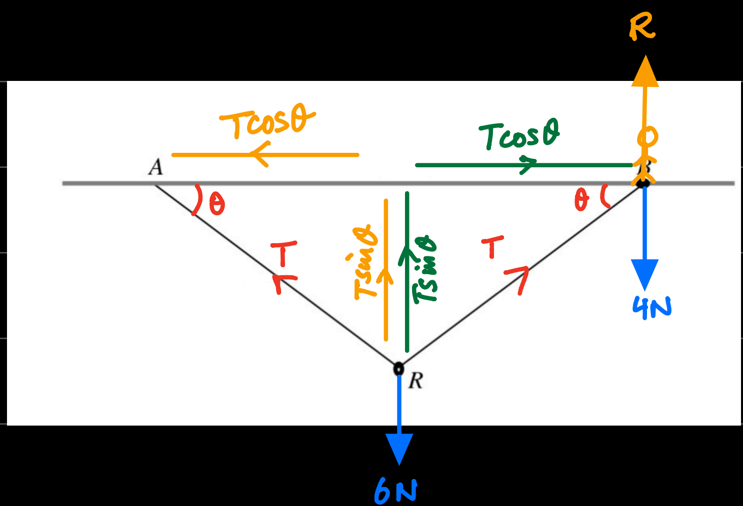
(i) Find the tension in the string. (Take R as point of observation). [3]

(ii) Find the frictional and normal components of the contact force acting on B. [4]

(iii) Given that the equilibrium is limiting, find the coefficient of friction between the bead and the rod. [2]

$F = \mu R$





(i) up = down

$T\sin\theta + T\sin\theta = 6$

$2T\sin\theta = 6$

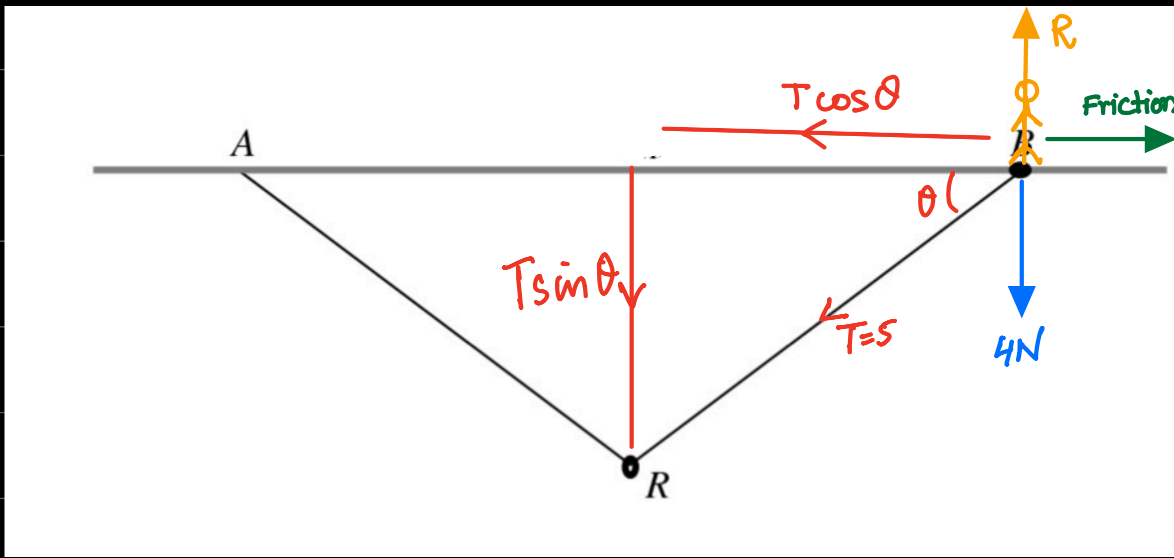
$2T\left(\frac{3}{5}\right) = 6$

$T = 5N$

Left = Right

$T\cos\theta = T\cos\theta$.

(ii) Point of Observation = B.



up = down

$R = 4 + T\sin\theta$

$R = 4 + 5\left(\frac{3}{5}\right)$

$R = 7$

Left = Right.

$T\cos\theta$ = Friction

$5\left(\frac{4}{5}\right)$ = Friction

friction = 4

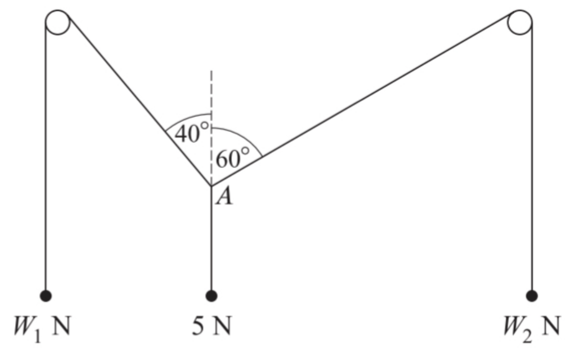
(iii)

$F = \mu R$

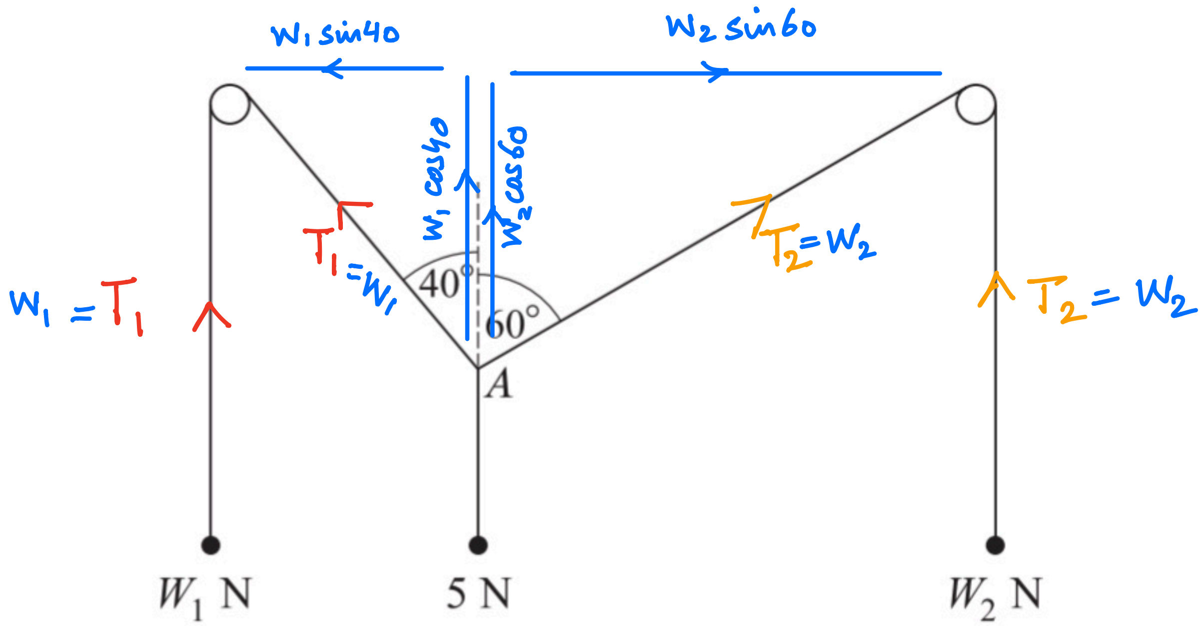
$4 = \mu(7)$

$\mu = \frac{4}{7}$

4



Each of three ~~light~~ strings has a particle attached to one of its ends. The other ends of the strings are tied together at a point A. The strings are in equilibrium with two of them passing over fi ed smooth horizontal pegs, and with the particles hanging freely. The weights of the particles, and the angles between the sloping parts of the strings and the vertical, are as shown in the diagram. Find the values of W_1 and W_2. [6]



At A

up = down

$\boxed{T_1 = W_1}$

up = down

$\boxed{W_1 \cos 40 + W_2 \cos 60 = 5}$

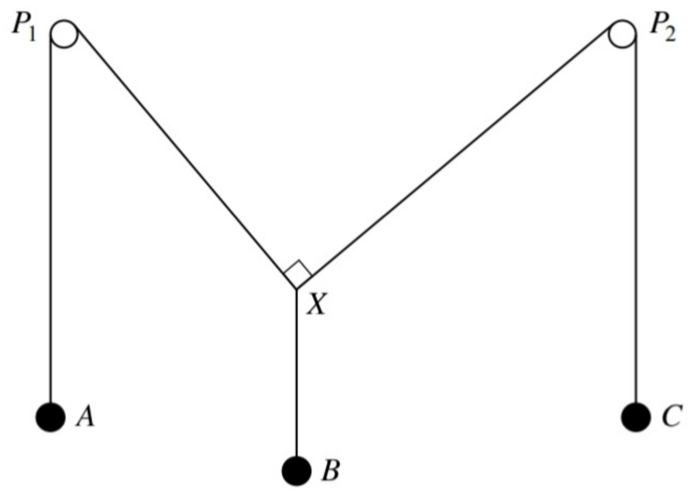
Left = Right

$\boxed{W_1 \sin 40 = W_2 \sin 60}$

up = down

$T_2 = W_2$

15



The diagram shows three particles A, B and C hanging freely in equilibrium, each being attached to the end of a string. The other ends of the three strings are tied together and are at the point X. The strings carrying A and C pass over smooth f xed horizontal pegs P_1 and P_2 respectively. The weights of A, B and C are 5.5 N, 7.3 N and W N respectively, and the angle P_1XP_2 is a right angle. Find the angle AP_1X and the value of W. [5]