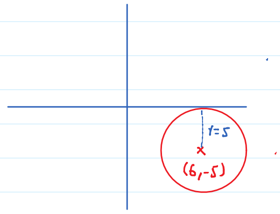
6 Find the equation of the circle that touches the x-axis and whose centre is $(6, -5)$.

$(x-a)^2 + (y-b)^2 = r^2$

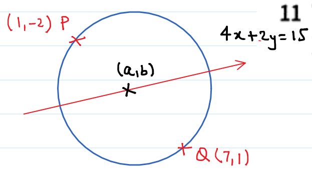
$(x-6)^2 + (y+5)^2 = 5^2$

$(x-6)^2 + (y+5)^2 = 25$



7 The points $P(1, -2)$ and $Q(7, 1)$ lie on the circumference of a circle.

Show that the centre of the circle lies on the line $4x + 2y = 15$.



11 The equation of a circle is $(x-3)^2 + (y+2)^2 = 25$. Show that the point $A(6, -6)$ lies on the circle and find the equation of the tangent to the circle at the point A.

Sol

$(x-a)^2 + (y-b)^2 = r^2$

P: $(1-a)^2 + (-2-b)^2 = r^2$ — 1

Q: $(7-a)^2 + (1-b)^2 = r^2$ — 2

$(1-a)^2 + (-2-b)^2 = (7-a)^2 + (1-b)^2$

$\cancel{1} - 2a + \cancel{a^2} + 4 + 4b + \quad = 49 - 14a + \cancel{a^2} + \cancel{1} - 2b + \cancel{b^2}$

$-2a + 14a + 4 = 9 - 2b - 4b$

$12a = -6b + 45$

$4a = -2b + 15$ — 3

11 The equation of a circle is $(x-3)^2 + (y+2)^2 = 25$. Show that the point $A(6, -6)$ lies on the circle and find the equation of the tangent to the circle at the point A.

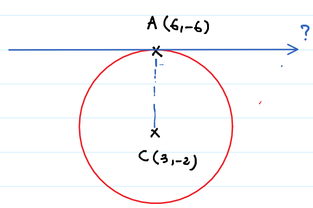
$(x-3)^2 + (y+2)^2 = 25$

$(6-3)^2 + (-6+2)^2 = 25$

$9 + 16 = 25$

$25 = 25$

A lies on the circle.



$m_{AC} = \dfrac{-6+2}{6-3} = -\dfrac{4}{3}$

$m_T = 3/4$

$y - y_1 = m(x - x_1)$

$y - (-6) = \frac{3}{4}(x-6)$

$4y + 24 = 3x - 18$

$4y = 3x - 42$

$\boxed{4y - 3x + 42 = 0}$ Ans

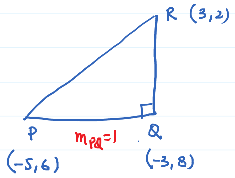
13 The points $P(-5, 6)$, $Q(-3, 8)$ and $R(3, 2)$ are joined to form a triangle.

a Show that angle PQR is a right angle.

b Find the equation of the circle that passes through the points P, Q and R.

a) $m_{PQ} = \frac{8-6}{-3+5} = \frac{2}{2} = 1$

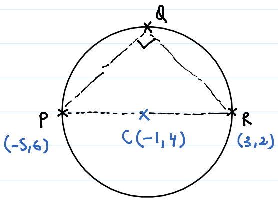
$m_{RQ} = \frac{8-2}{-3-3} = \frac{6}{-6} = -1$



$\because \; m_{PQ} \times m_{RQ} = -1$

So $P\hat{Q}R$ is right angle.

b) $r = CP = CQ = CR$



$r = \sqrt{(-1+5)^2 + (4-6)^2}$

$r = \sqrt{16+4}$

$r = \sqrt{20}$

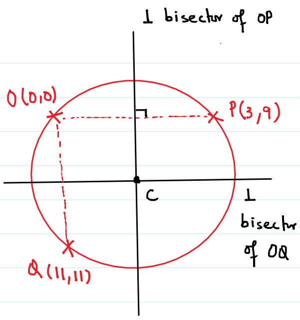
$(x-a)^2 + (y-b)^2 = r^2$

$(x+1)^2 + (y-4)^2 = (\sqrt{20})^2$

$\boxed{(x+1)^2 + (y-4)^2 = 20}$ Ans

15 A circle passes through the points $O(0, 0)$, $P(3, 9)$ and $Q(11, 11)$.

Find the equation of the circle.



Sol

⊥ bisector of OP:

$M = \left(\frac{3}{2}, \frac{9}{2}\right)$

$m_{OP} = \frac{9}{3} = 3$

$m' = -\frac{1}{3}$

$y - \frac{9}{2} = -\frac{1}{3}\left(x - \frac{3}{2}\right)$

$\frac{2y-9}{\cancel{2}} = -\frac{1}{3}\left(\frac{2x-3}{\cancel{2}}\right)$

$6y - 27 = -2x + 3$

$2x + 6y = 30$

$x + 3y = 15$ —— (1)

⊥ bisector of OQ:

$M = \left(\frac{11}{2}, \frac{11}{2}\right)$

$m_{OQ} = \frac{11}{11} = 1$

$m' = -1$

$y - \frac{11}{2} = -1\left(x - \frac{11}{2}\right)$

$\frac{2y-11}{2} = -x + \frac{11}{2}$

$2y - 11 = -2x + 11$

$2x + 2y = 22$

$x + y = 11$ —— (2)

Solving (1) & (2) Centre : $C(9,2)$

$r = OC = PC = QC$

$r = \sqrt{(9-0)^2 + (2-0)^2}$

$r = \sqrt{85}$

$(x-a)^2 + (y-b)^2 = r^2$

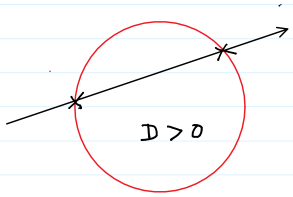
$(x-9)^2 + (y-2)^2 = (\sqrt{85})^2$

$\boxed{(x-9)^2 + (y-2)^2 = 85}$ Am.

EX. 3E

4 Find the set of values of m for which the line $y = mx + 1$ intersects the circle $(x-7)^2 + (y-5)^2 = 20$ at two distinct points.

Sol $(x-7)^2 + (mx+1-5)^2 = 20$



$(x-7)^2 + (mx-4)^2 = 20$

$x^2 - 14x + 49 + m^2x^2 - 8mx + 16 - 20 = 0$

$x^2 + m^2x^2 - 14x - 8mx + 45 = 0$

$(1+m^2)x^2 - 2(7+4m)x + 45 = 0$

$a = 1 + m^2, \quad b = -2(7+4m), \quad c = 45$

$D > 0$

$[-2(7+4m)]^2 - 4(1+m^2)(45) > 0$

$4(49 + 56m + 16m^2) - 180 - 180m^2 > 0$

$196 + 224m + 64m^2 - 180 - 180m^2 > 0$

$-116m^2 + 224m + 16 > 0$

$116m^2 - 224m - 16 < 0$

$\boxed{-\frac{2}{29} < m < 2}$ Am.

5 The line $2y - x = 12$ intersects the circle $x^2 + y^2 - 10x - 12y + 36 = 0$ at the points A and B.

a Find the coordinates of the points A and B.

Sol $x = 2y - 12$ — ①

$(2y-12)^2 + y^2 - 10(2y-12) - 12y + 36 = 0$

$4y^2 - 48y + 144 + y^2 - 20y + 120 - 12y + 36 = 0$

$5y^2 - 80y + 300 = 0$

$y^2 - 16y + 60 = 0$

$y = 10$, $y = 6$

$x = 2(10) - 12$ $\quad$ $x = 2(6) - 12$

$x = 8$ $\quad$ $x = 0$

$\boxed{A(8, 10) \quad B(0, 6)}$ Ans

b Find the equation of the perpendicular bisector of AB.

$A(8, 10)$, $B(0, 6)$

$M = \left(\frac{8+0}{2}, \frac{10+6}{2}\right) = (4, 8)$

$m_{AB} = \frac{6-10}{6-8} = \frac{-4}{-8} = \frac{1}{2}$

$m' = -2$

$y - 8 = -2(x - 4)$

$y = -2x + 8 + 8$

$\boxed{y = -2x + 16}$ Ans

c The perpendicular bisector of AB intersects the circle at the points P and Q.

Find the exact coordinates of P and Q.

$x^2 + (16-2x)^2 - 10x - 12(16-2x) + 36 = 0$

$x^2 + 256 - 64x + 4x^2 - 10x - 192 + 24x + 36 = 0$

$5x^2 - 50x + 100 = 0$

$x^2 - 10x + 20 = 0$

$x = 5 + \sqrt{5}$, $x = 5 - \sqrt{5}$

$y = -2(5+\sqrt{5}) + 16$, $y = -2(5-\sqrt{5}) + 16$

$y = -10 - 2\sqrt{5} + 16$, $y = -10 + 2\sqrt{5} + 16$

$y = 6 - 2\sqrt{5}$ $\quad$ $y = 6 + 2\sqrt{5}$

$y = 6 - 2\sqrt{5}$ $\quad\quad$ $y = 6 + 2\sqrt{5}$

$\boxed{P\,(5+\sqrt{5},\, 6-2\sqrt{5}) \;,\; Q\,(5-\sqrt{5},\, 6+2\sqrt{5})}$ Am

d Find the exact area of quadrilateral *APBQ*.

$$A = \frac{1}{2}\left| \begin{matrix} 8 & 5+\sqrt{5} & 0 & 5-\sqrt{5} & 8 \\ 10 & 6-2\sqrt{5} & 6 & 6+2\sqrt{5} & 10 \end{matrix} \right|$$

$$A = \frac{1}{2}\ (48 - 16\sqrt{5} + 30 + 6\sqrt{5} + 50 - 10\sqrt{5}) - (48 + 16\sqrt{5} + 30 - 6\sqrt{5} + 50 + 10\sqrt{5})$$

$$A = \frac{1}{2}\left| 12\cancel{8} - 20\sqrt{5} - 12\cancel{8} - 20\sqrt{5} \right|$$

$$A = \frac{1}{2}\left| -40\sqrt{5} \right|$$

$\boxed{A = 20\sqrt{5}\ \text{unit}^2}$ Am.