QUADRATICS :-

ROOTS :- (x- INTERCEPTS) (y=0)

$y = 2x^2 + 6x + 9$

$0 = 2x^2 + 6x + 9$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

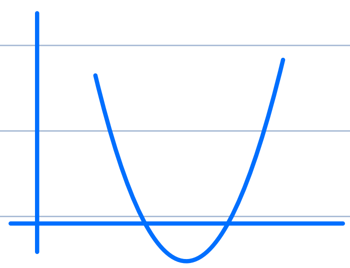
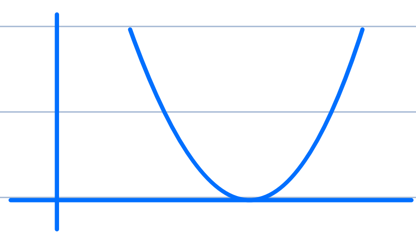
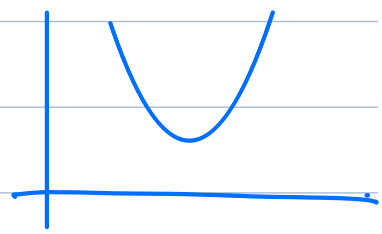
→ Positive (2 Ans)

→ Zero (1 Ans)

→ Negative (No Ans)

DISCRIMINANT

$\boxed{b^2 - 4ac}$

Positive (2 Ans)	Zero (1 Ans)	NEGATIVE (NO Ans).
		
$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
TWO DISTINCT REAL ROOTS	TWO EQUAL/ REPEAT REAL ROOTS	NO REAL ROOTS.

INTERSECTION OF LINE AND QUADRATIC CURVE.

$y = 2x^2 - 9x + 5$ $y = 2x + 3$

$$2x^2 - 9x + 5 = 2x + 3$$

$$2x^2 - 11x + 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac \to$ Positive (2 Ans)

$b^2 - 4ac \to$ Zero (1 Ans)

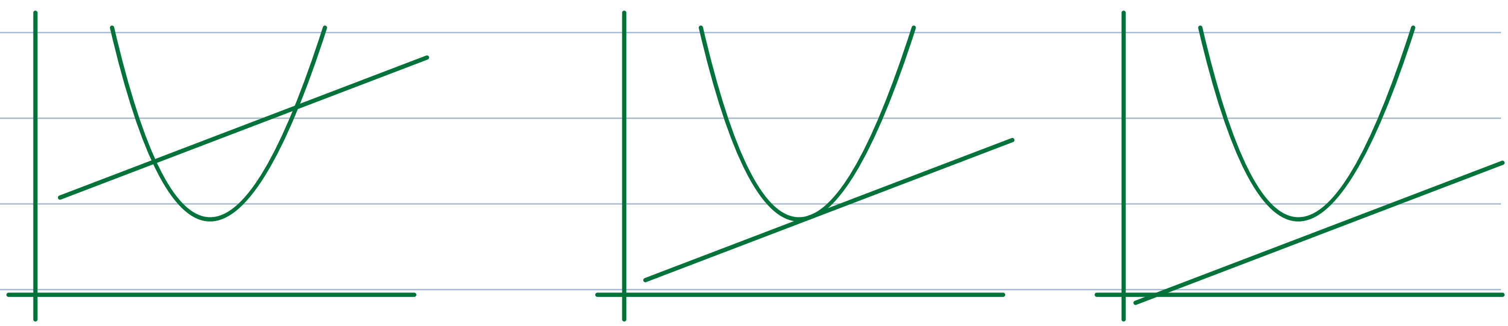
$b^2 - 4ac \to$ Negative (No Ans)

FOR LINE & CURVE:

STEP 1: EQUATE AND BRING TO STANDARD FORM.

$$\boxed{b^2 - 4ac}$$

$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$



TWO POINTS OF INTERSECTION	LINE IS TANGENT TO CURVE	LINE DOES NOT MEET CURVE.

7 The equation of a curve is $y^2 + 2x = 13$ and the equation of a line is $2y + x = k$, where k is a constant.

(i) In the case where $k = 8$, find the coordinates of the points of intersection of the line and the curve. [4]

(ii) Find the value of k for which the line is a tangent to the curve. [3]

(i) $y^2 + 2x = 13$ $\qquad$ $2y + x = 8$

$y = \sqrt{13-2x}$ $\qquad$ $y = \dfrac{8-x}{2}$

$$\left(\sqrt{13-2x}\right)^2 = \left(\frac{8-x}{2}\right)^2$$

$$13 - 2x = \frac{64 - 16x + x^2}{4}$$

$$52 - 8x = 64 - 16x + x^2$$

$$0 = x^2 - 16x + 8x + 64 - 52$$

$$0 = x^2 - 8x + 12$$

$$0 = x^2 - 6x - 2x + 12$$

$$0 = x(x-6) - 2(x-6)$$

$$0 = (x-2)(x-6)$$

$x - 2 = 0$ $\qquad$ $x - 6 = 0$

$x = 2$ $\qquad$ $x = 6$

$y = \dfrac{8-2}{2}$ $\qquad$ $y = \dfrac{8-6}{2}$

$y = 3$ $\qquad$ $y = 1$

$(2, 3)$ $\qquad$ $(6, 1)$

(11) $y^2 + 2x = 13$ $\qquad$ $2y + x = K$

STEP1 : EQUATE.

$y = \sqrt{13-2x}$ $\qquad$ $y = \dfrac{K-x}{2}$

$$\left(\sqrt{13-2x}\right)^2 = \left(\frac{K-x}{2}\right)^2$$

$$13-2x = \frac{K^2 - 2Kx + x^2}{4}$$

$$52 - 8x = K^2 - 2Kx + x^2$$

$$0 = x^2 + 8x - 2Kx + K^2 - 52$$

$$0 = x^2 + (8-2K)x + K^2 - 52$$

$$0 = ax^2 + b\,x + c$$

$a = 1$, $b = 8-2K$, $c = K^2 - 52$

$b^2 - 4ac = 0$ (Tangent)

$$(8-2K)^2 - 4(1)(K^2-52) = 0$$

$$64 - 32K + \cancel{4K^2} - \cancel{4K^2} + 208 = 0$$

$$272 - 32K = 0$$

$$\boxed{K = 8.5}$$

6 A line has equation $y = kx + 6$ and a curve has equation $y = x^2 + 3x + 2k$, where k is a constant.

(i) For the case where $k = 2$, the line and the curve intersect at points A and B. Find the distance AB and the coordinates of the mid-point of AB. [5]

(ii) Find the two values of k for which the line is a tangent to the curve. [4]

(ii) $y = Kx + 6$ $\quad y = x^2 + 3x + 2K$

$x^2 + 3x + 2K = Kx + 6$

$x^2 + 3x - Kx + 2K - 6 = 0$

$x^2 + (3 - K)x + 2K - 6 = 0$

$a = 1, \quad b = 3 - K, \quad c = 2K - 6$

$b^2 - 4ac = 0$

$(3 - K)^2 - 4(1)(2K - 6) = 0$

$9 - 6K + K^2 - 8K + 24 = 0$

$K^2 - 14K + 33 = 0$

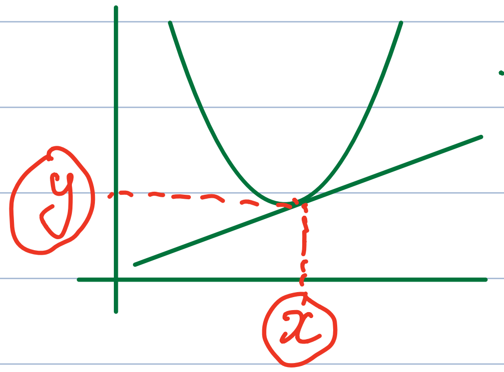
$K^2 - 3K - 11K + 33 = 0$

$K(K - 3) - 11(K - 3) = 0$

$(K - 3)(K - 11) = 0$

$K - 3 = 0, \quad K - 11 = 0$

$\boxed{K = 3}, \quad \boxed{K = 11}$



INEQUALITIES

LINEAR

1) $2x - 3 < 7$

$2x < 10$

$x < 5$

2) $-2x < 6$

$x > \frac{6}{-2}$

$x > -3$

3) $\frac{x}{-3} < 6$

$x > (6)(-3)$

$x > -18$

$x^2 < 16$

YOU ARE NOT ALLOWED TO TAKE SQUARE ROOT ON AN INEQUALITY SIGN.

$(2x+1)(x-3) > 0$

YOU CANNOT DO EITHER $2x+1 > 0$ OR $x-3 > 0$ ON AN INEQUALITY SIGN.

QUADRATIC INEQUALITIES.

(P1, P3, P4)

STEP1 :- FACTORIZE

STEP2: SKETCH

(a) Shape

(b) x-intercept.

Step3: Colour the relevant region.

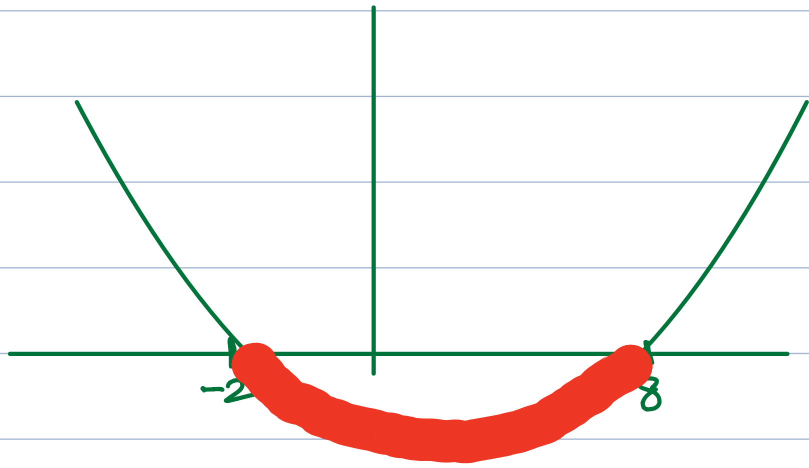
Step4: Write down inequality for x-axis values of coloured region only.

1) $x^2 - 6x - 16 < 0$

$x^2 - 8x + 2x - 16 < 0$

$x(x-8) + 2(x-8) < 0$

$\underbrace{(x+2)(x-8)}_{y} < 0$ (below x axis)



$$\boxed{-2 < x < 8}$$

STEP1 :- FACTORIZE

STEP2 : SKETCH
(a) Shape
(b) x-intercept.

Step3: Colour the relevant region.

Step4: Write down inequality for x-axis values of coloured region only.

$\boxed{2}$

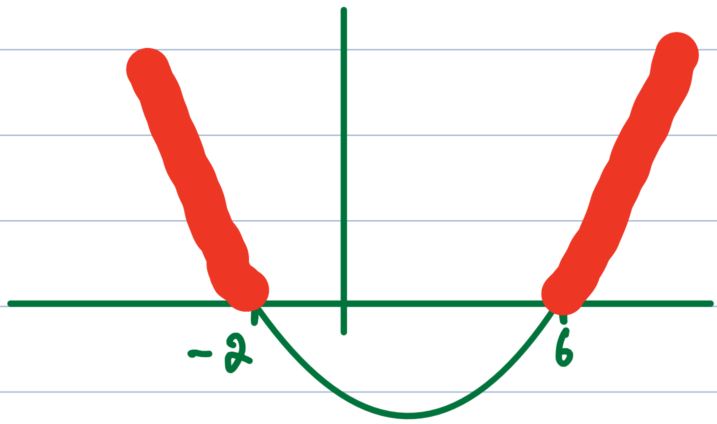
$$x^2 - 4x - 12 > 0$$

$$x^2 - 6x + 2x - 12 > 0$$

$$x(x-6) + 2(x-6) > 0$$

$$(x-6)(x+2) > 0$$

$y > 0$ (above x-axis)



$\boxed{x < -2}$ or $\boxed{x > 6}$

STEP1 :- FACTORIZE

STEP2 : SKETCH
(a) Shape
(b) x - intercept.

Step3 : Colour the relevant region.

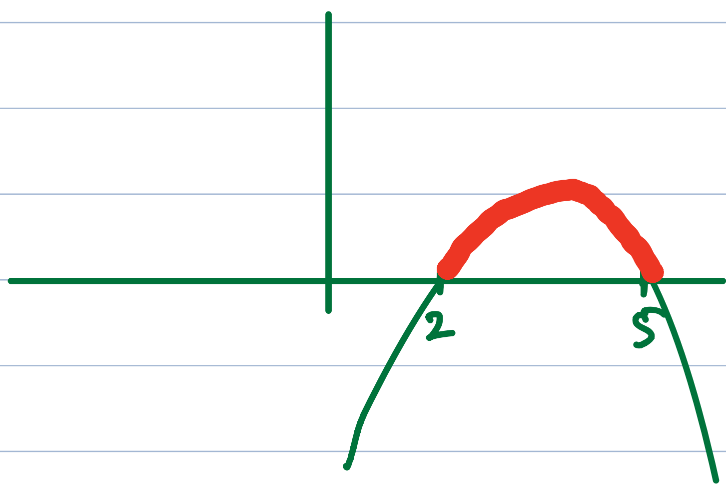
Step4 : Write down inequality for x-axis values of coloured region only.

$\boxed{3}$

sad face.

$$(2-x)(x-5) > 0$$

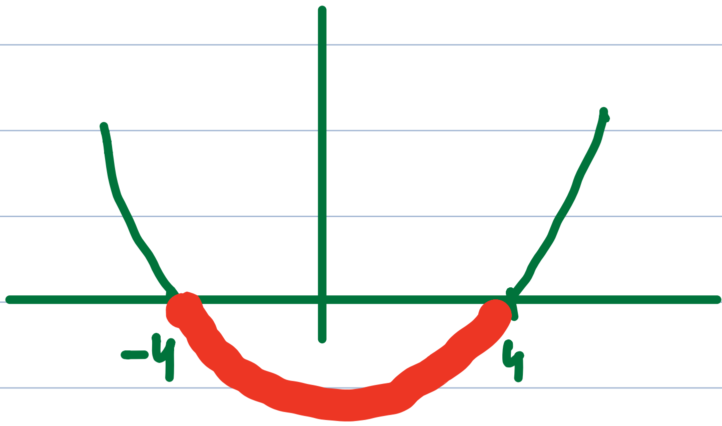
$y > 0$ (above x-axis)



$$\boxed{2 < x < 5}$$

$\boxed{4}$

$$x^2 < 16$$
$$x^2 - 16 < 0$$
$$(x)^2 - (4)^2 < 0$$
$$(x+4)(x-4) < 0$$

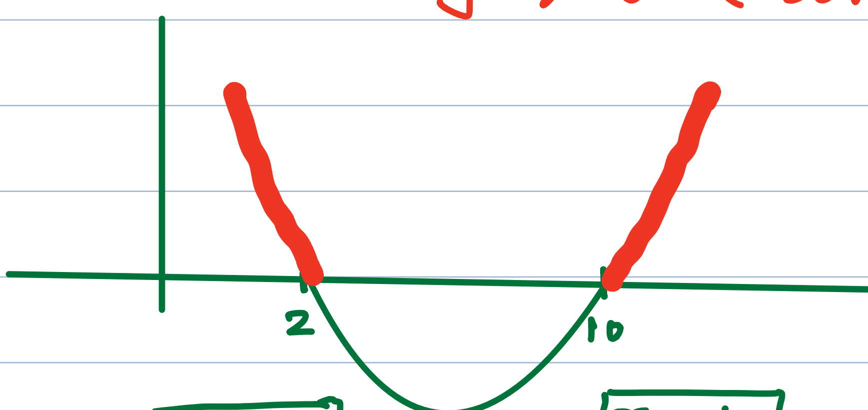


$$\boxed{-4 < x < 4}$$

$\boxed{5}$

$$x^2 - 12x + 20 > 0$$
$$x^2 - 2x - 10x + 20 > 0$$
$$x(x-2) - 10(x-2) > 0$$
$$(x-10)(x-2) > 0$$

$y > 0$ (above x-axis)



$\boxed{x < 2}$ or $\boxed{x > 10}$

10 The equation of a line is $2y + x = k$, where k is a constant, and the equation of a curve is $xy = 6$.

(i) In the case where $k = 8$, the line intersects the curve at the points A and B. Find the equation of the perpendicular bisector of the line AB. [6]

(ii) Find the set of values of k for which the line $2y + x = k$ intersects the curve $xy = 6$ at two distinct points. [3]

inequality.

(i) $2y + x = 8$ $\quad$ $xy = 6$

$y = \frac{8-x}{2}$ $\quad$ $y = \frac{6}{x}$

$\frac{8-x}{2} = \frac{6}{x}$

$8x - x^2 = 12$

$0 = x^2 - 8x + 12$

$0 = x^2 - 2x - 6x + 12$

$0 = x(x-2) - 6(x-2)$

$0 = (x-2)(x-6)$

$x - 2 = 0$ $\quad$ $x - 6 = 0$

$\boxed{x = 2}$ $\quad$ $\boxed{x = 6}$

$y = \frac{6}{2}$ $\quad$ $y = \frac{6}{6}$

$\boxed{y = 3}$ $\quad$ $\boxed{y = 1}$

$A(2,3)$ $\quad$ $B(6,1)$

$A(2,3)$ $\quad$ $B(6,1)$

$m_{AB} = \frac{1-3}{6-2} = \frac{-2}{4} = -\frac{1}{2}$

$m_{\perp} = 2$, Mid $= \left(\frac{2+6}{2}, \frac{3+1}{2}\right)$

$M(4,2)$

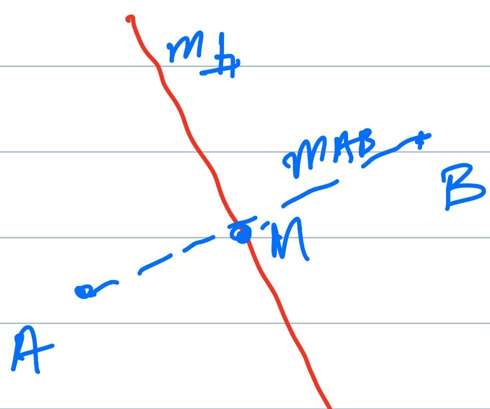
$m = 2$, $M(4,2)$

$y - y_1 = m(x - x_1)$

$y - 2 = 2(x - 4)$

$y = 2x - 8 + 2$

$\boxed{y = 2x - 6}$



(ii)

$2y + x = K \qquad\qquad xy = 6$

$y = \dfrac{K - x}{2} \qquad\qquad y = \dfrac{6}{x}$

$$\frac{K - x}{2} = \frac{6}{x}$$

$$Kx - x^2 = 12$$

$$0 = x^2 - Kx + 12$$

$$b^2 - 4ac > 0$$

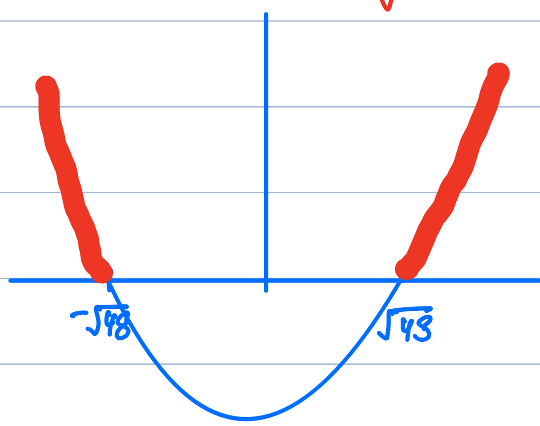
$$(-K)^2 - 4(1)(12) > 0$$

$$K^2 - 48 > 0$$

$$(K)^2 - (\sqrt{48})^2 > 0$$

$$(K + \sqrt{48})(K - \sqrt{48}) > 0$$

$y > 0$ (above)



$\boxed{K < -\sqrt{48}}$ or $\boxed{K > \sqrt{48}}$

5 Find the set of values of m for which the line $y = mx + 4$ intersects the curve $y = 3x^2 - 4x + 7$ at two distinct points. [5]

$3x^2 - 4x + 7 = mx + 4$

$3x^2 - 4x - mx + 3 = 0$

$3x^2 - (4+m)x + 3 = 0$

$a = 3, \quad b = -(4+m), \quad c = 3$

$b^2 - 4ac > 0$

$[-(4+m)]^2 - 4(3)(3) > 0$

$+(4+m)^2 - 36 > 0$

$(4+m)^2 - (6)^2 > 0$

$(4+m+6)(4+m-6) > 0$

$(m+10)(m-2) > 0$

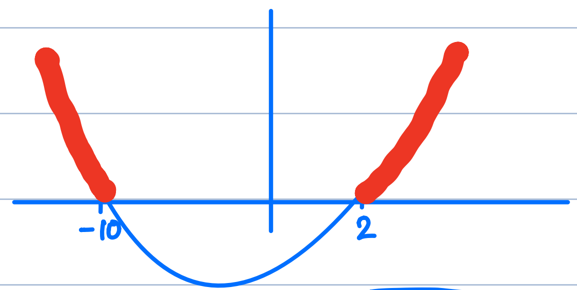
$b = -4 - m$

$b^2 = (-4-m)^2$

you cannot use $(a-b)^2$.

$[-(4+m)]^2$

$y > 0$ (above x axis)



$\boxed{m < -10}$ or $\boxed{m > 2}$

16 **(i)** Express $4x^2 - 12x$ in the form $(2x + a)^2 + b$. [2]

(ii) Hence, or otherwise, find the set of values of x satisfying $4x^2 - 12x > 7$. [2]

(i) $4\left[x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right]$

$4\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right]$

COMPLETED SQUARE FORM.

$4\left(x - \frac{3}{2}\right)^2 - 9$

$2^2\left(x - \frac{3}{2}\right)^2 - 9$

$\left[2\left(x - \frac{3}{2}\right)\right]^2 - 9$

$(2x - 3)^2 - 9$

$(2x + a)^2 + b$

$\boxed{a = -3}$, $\boxed{b = -9}$

(ii) $4x^2 - 12x > 7$

$(2x-3)^2 - 9 > 7$

$(2x-3)^2 - 9 - 7 > 0$

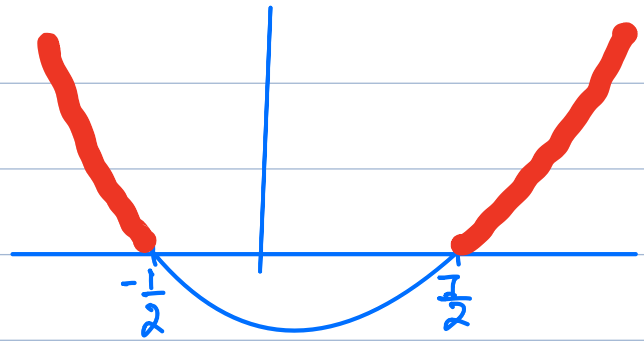
$$(2x-3)^2 - 16 > 0$$

$$(2x-3)^2 - (4)^2 > 0$$

$$(2x-3+4)(2x-3-4) > 0$$

$$(2x+1)(2x-7) > 0$$

$y > 0$



$\boxed{x < -\frac{1}{2}}$ or $\boxed{x > \frac{7}{2}}$