BINOMIAL P1

Saturday, November 28, 2020 7:16 PM

BINOMIAL P1
(4-5 Marks)

OLevels: $(a+b)^2 = a^2 + 2ab + b^2$

WHAT IF: $(a+b)^7 = ???$

ALEVELS (P1)

$$(a+b)^n = a^n + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 \dots$$

^nC_r → $\boxed{\text{SHIFT}}\ \boxed{\div}$ → BUTTON BLEOW $\boxed{\text{ALPHA}}$

$7 \boxed{nCr} 1 = 7$

$4 \boxed{nCr} 2 = 6$

For now just learn to use button.
Detailed concept is in S1.

$$(a+b)^n = a^n + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 \dots$$

EXPAND FIRST THREE TERMS:-

1. $$(2+3x)^6 = 2^6 + {}^6C_1 (2)^{6-1}(+3x)^1 + {}^6C_2 (2)^{6-2}(+3x)^2$$
$$= 64 + (6)(32)(3x) + (15)(16)(9x^2)$$
$$= 64 + 576x + 2160x^2$$

2. $$(3-2x)^4 = (3)^4 + {}^4C_1 (3)^{4-1}(-2x)^1 + {}^4C_2 (3)^{4-2}(-2x)^2$$
$$= 81 + (4)(27)(-2x) + (6)(9)(+4x^2)$$
$$= 81 - 216x + 216x^2$$

V.V.V. IMP THING TO REMEMBER!

$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(x-3)^2 = (x)^2 - 2(x)(3) + (3)^2$$

$$(a-b)^2 = (a)^2 + {}^2C_1 (a)^{2-1}(-b)^1 + {}^2C_2 (a)^{2-2}(-b)^2$$
$$a^2 + 2(a)(-b) + 1(1)(+b^2)$$
$$a^2 - 2ab + b^2$$

V.IMP

1) WE TAKE -ve sign of b in BINOMIAL FORMULA.

1) WE TAKE -ve sign of b in BINOMIAL FORMULA.

2) WE DONOT TAKE -ve sign of b in

$(a-b)^2 = a^2 - 2ab + b^2$.

TYPE 1 :-

Q. (i) Expand first three terms in expansion of $(2-x)^6$.

$$(2)^6 + {}^6C_1 (2)^{6-1}(-x)^1 + {}^6C_2 (2)^{6-2}(-x)^2$$

$$64 + (6)(32)(-x) + (15)(16)(+x^2)$$

$$(2-x)^6 = 64 - 192x + 240x^2$$

(ii) Hence find cofficient of x^2 in expansion

$$\underbrace{(3+4x)}_{2\,terms}\underbrace{(2-x)^6}_{7\,terms} \longrightarrow x^2$$

$$3 \times 240x^2 = 720\,x^2$$

$$+4x \times -192x = -768\,x^2$$

$$-48\,x^2$$

cofficient of $x^2 = -48$

$(2-x)^6 = 64 - 192x + 240x^2$

(iii) Hence find cofficient of x^2 in expansion

$$(3x - 2 + 4x^2)(2-x)^6.$$

$$3x \times -192x = -576\,x^2$$

$$-2 \times 240x^2 = -480\,x^2$$

$$+4x^2 \times 64 = 256\,x^2$$

$$-800\,x^2$$

$\boxed{(2-x)^6 = 64 - 192x + 240x^2} \rightarrow$ from first part.

(IV) Hence find value of a for which cofficient of x is 45 in expansion

Final Ans

$$(2+ax)(2-x)^6.$$

$$2 \times -192x = -384\,x$$

$$+ax \times 64 = 64a\,x$$

$$45\,x$$

$$-384 + 64a = 45$$

$$64a = 45 + 384$$

$$64a = 429$$

$$a = \frac{429}{64} = 6.7$$

$$(2-x)^6 = 64 - 192x + 240x^2$$

(V) Hence find K for which cofficient of x^2 is -20 in expansion

$$(3 - 2x + 7Kx^2)(2-x)^6$$

$$\begin{aligned} 3 &\times 240x^2 = 720\ x^2 \\ -2x &\times -192x = 384\ x^2 \\ +7Kx^2 &\times 64 = 448K\ x^2 \\ & \hline \qquad\qquad -20\ x^2 \end{aligned}$$

$$720 + 384 + 448K = -20$$

$$448K = -20 - 720 - 384$$

$$\boxed{K = -2.5089}$$

$$(2-x)^6 = 64 - 192x + 240x^2$$

(VI) Hence find K for which there is no term in x^2 in expansion

Final Ans $= 0x^2$

$$(3K + 2x)(2-x)^6$$

$$\begin{aligned} 3K &\times 240x^2 = 720K\ x^2 \\ +2x &\times -192x = -384\ x^2 \\ & \hline \qquad\qquad 0\ x^2 \end{aligned}$$

$$720K - 384 = 0$$

$$720K = 384$$

$$K = \frac{8}{15}$$

NEW VOCAB

No Term in x^2 $\xrightarrow{\text{means}}$ final Ans is $0x^2$

No term in x $\xrightarrow{\text{means}}$ final Ans is $0x^1$

TYPE 2

WHEN YOU NEED A PARTICULAR TERM FROM ONE EXPANSION

$$\boxed{T_{r+1} = {}^nC_r \, a^{n-r} \, b^r}$$

(TERM NO)

Q Find the fourth term (T_4) in expansion of $(2+3x)^8$

$T_{r+1} = {}^8C_r (2)^{8-r} (3x)^r$

$\boxed{r=3}$

$T_4 = {}^8C_3 (2)^{8-3} (3x)^3$

$= (56)(32)(27x^3)$

$T_4 = 48384x^3$

(POWER OF X) (eg x^2 etc)

Q Find the x^2 term in expansion $(2x+3)^7$.

STEPS:

1) Apply formula
2) expand all powers and isolate x terms.
3) Equate isolated x term(s) to your required power.
4) Find r and put it in first step.

$T_{r+1} = {}^7C_r (2x)^{7-r} (3)^r$

${}^7C_r \cdot 2^{7-r} \cdot x^{7-r} \cdot 3^r$

$x^{7-r} = x^2$

$7 - r = 2$

$\boxed{r=5}$

$T_{5+1} = {}^7C_5 (2x)^{7-5} (3)^5$

$(21)(2x)^2(243)$

$(21)(4x^2)(243)$

$T_6 = 20412x^2$

Q Find x^2 term in expansion of $\left(3x + \frac{2}{x}\right)^8$

$T_{r+1} = {}^8C_r (3x)^{8-r} \left(\frac{2}{x}\right)^r$ | $T \ \ 8 \ (\ )^{8-3} (\ )^3$

$$T_{r+1} = {}^8C_r\,(3x)^{8-r}\left(\frac{2}{x}\right)^r$$

$${}^8C_r \cdot 3^{8-r} \cdot x^{8-r} \cdot \frac{2^r}{x^r}$$

isolate x terms

$$\frac{x^{8-r}}{x^r} = x^2$$

$$x^{8-r-r} = x^2$$

$$8 - 2r = 2$$

$$6 = 2r$$

$$\boxed{r = 3}$$

$$T_{3+1} = {}^8C_3\,(3x)^{8-3}\left(\frac{2}{x}\right)^3$$

$$= 56\,(3x)^5\left(\frac{2}{x}\right)^3$$

$$= 56\,(243\,\cancel{x^5}^{\,2})\left(\frac{8}{\cancel{x^3}}\right)$$

$$T_4 = \boxed{108864\,x^2}$$

NEW VOCAB

TERM INDEPENDENT OF X $\longrightarrow$ Find x^0 term using $T_{r+1} = {}^nC_r\,a^{n-r}b^r$

Q Find term independent of x (x^0 term) in expansion $\left(x - \frac{3}{x}\right)^8$

$$T_{r+1} = {}^8C_r\,(x)^{8-r}\cdot\left(\frac{-3}{x}\right)^r$$

$$= {}^8C_r \cdot x^{8-r} \cdot \frac{(-3)^r}{x^r}$$

Isolate x-terms

$$\frac{x^{8-r}}{x^r} = x^0$$

$$x^{8-r-r} = x^0$$

$$8 - 2r = 0$$

$$\boxed{r = 4}$$

$$T_{4+1} = {}^8C_4\,(x)^{8-4}\left(\frac{-3}{x}\right)^4$$

$$= (70)(\cancel{x^4})\left(\frac{+81}{\cancel{x^4}}\right)$$

$$T_5 = 5670$$

(term independent of x)