1. Physical Quantities & Units

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1.1 PHYSICAL QUANTITIES & UNITS

1.1.1 PHYSICAL QUANTITIES

What is a Physical Quantity?

- Speed and velocity are examples of physical quantities; both can be measured
- All physical quantities consist of a numerical magnitude and a unit
- In physics, every letter of the alphabet (and most of the Greek alphabet) is used to represent these physical quantities
- These letters, without any context, are meaningless
- To represent a physical quantity, it must contain both a numerical value and the **unit** in which it was measured
- The letter *v* be used to represent the physical quantities of velocity, volume or voltage
- The units provide the context as to what *v* refers to
  - If *v* represents velocity, the unit would be $\mathbf{m\ s^{-1}}$
  - If *v* represents volume, the unit would be $\mathbf{m^3}$
  - If *v* represents voltage, the unit would be **V**

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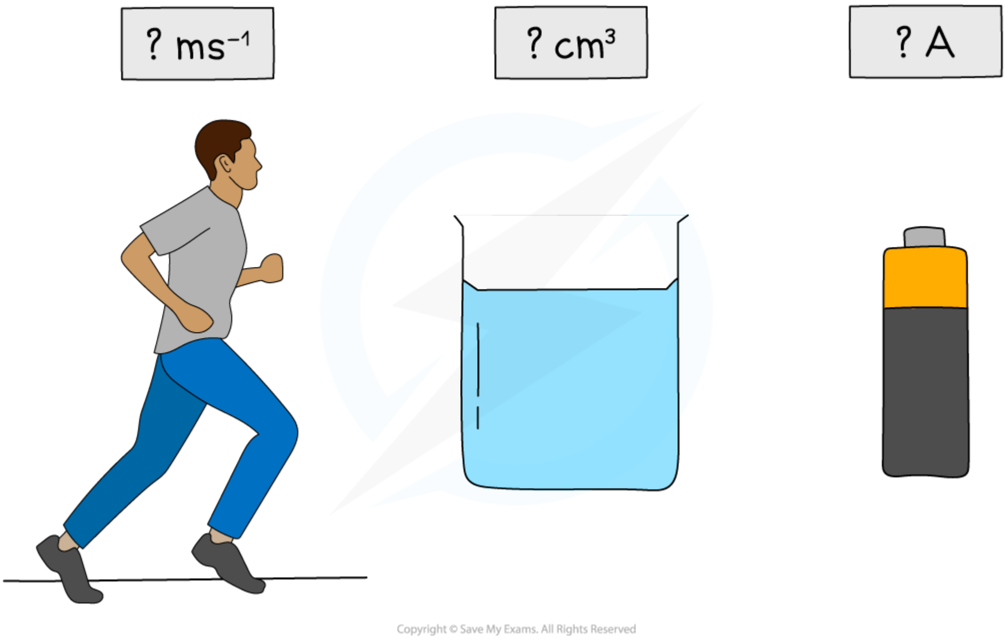

All physical quantities must have a numerical magnitude and a unit

Estimating Physical Quantities

- There are important physical quantities to learn in physics
- It is useful to know these physical quantities, they are particularly useful when making estimates
- A few examples of useful quantities to memorise are given in the table below (this is by no means an exhaustive list)

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Estimating Physical Quantities Table

QUANTITY	SIZE
DIAMETER OF AN ATOM	10^{-10} m
WAVELENGTH OF UV LIGHT	10 nm
HEIGHT OF AN ADULT HUMAN	2 m
DISTANCE BETWEEN THE EARTH AND THE SUN (1 AU)	1.5×10^{8} m
MASS OF A HYDROGEN ATOM	10^{-27} kg
MASS OF AN ADULT HUMAN	70 kg
MASS OF A CAR	1000 kg
SECONDS IN A DAY	90000 s
SECONDS IN A YEAR	3×10^{7} s
SPEED OF SOUND IN AIR	300 ms^{-1}
POWER OF A LIGHTBULB	60W
ATMOSPHERIC PRESSURE	1×10^{5} Pa

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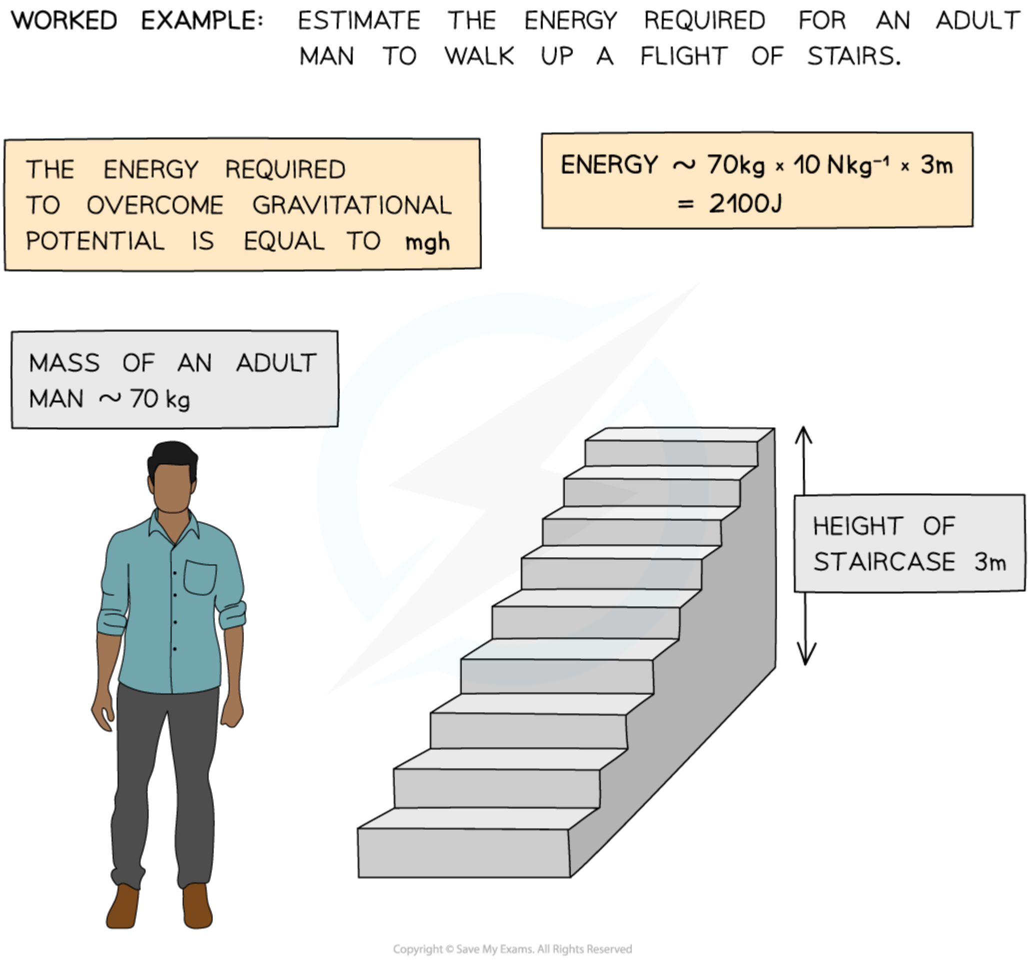
Worked Example



Worked example: estimating gravitational potential energy

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1.1.2 SI UNITS

SI Base Quantities

- There are a seemingly endless number of units in Physics
- These can all be reduced to seven base units from which every other unit can be derived
- These seven units are referred to as the SI Base Units; this is the only system of measurement that is officially used in almost every country around the world

SI Base Quantities Table

QUANTITY	SI BASE UNIT	SYMBOL
MASS	KILOGRAM	kg
LENGTH	METRE	m
TIME	SECOND	s
CURRENT	AMPERE	A
TEMPERATURE	KELVIN	K
AMOUNT OF SUBSTANCE	MOLE	mol

Exam Tip

You will only be required to use the first five SI base units in this course, so make sure you know them!

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Derived Units

- Derived units are derived from the seven SI Base units
- The base units of physical quantities such as:
  - Newtons, **N**
  - Joules, **J**
  - Pascals, **Pa,** can be deduced
- To deduce the base units, it is necessary to use the definition of the quantity
- The Newton (N), the unit of force, is defined by the equation:
  - Force = mass × acceleration
  - N = kg × m s^{-2} = kg m s^{-2}
  - Therefore, the Newton (N) in SI base units is **kg m s^{-2}**
- The Joule (J), the unit of energy, is defined by the equation:
  - Energy = ½ × mass × $velocity^2$
  - J = kg × $(m\ s^{-1})^2$ = kg m^2 s^{-2}
  - Therefore, the Joule (J) in SI base units is **kg m^2 s^{-2}**
- The Pascal (Pa), the unit of pressure, is defined by the equation:
  - Pressure = force ÷ area
  - Pa = N ÷ m^2 = (kg m s^{-2}) ÷ m^2 = kg m^{-1} s^{-2}
  - Therefore, the Pascal (Pa) in SI base units is **kg m^{-1} s^{-2}**

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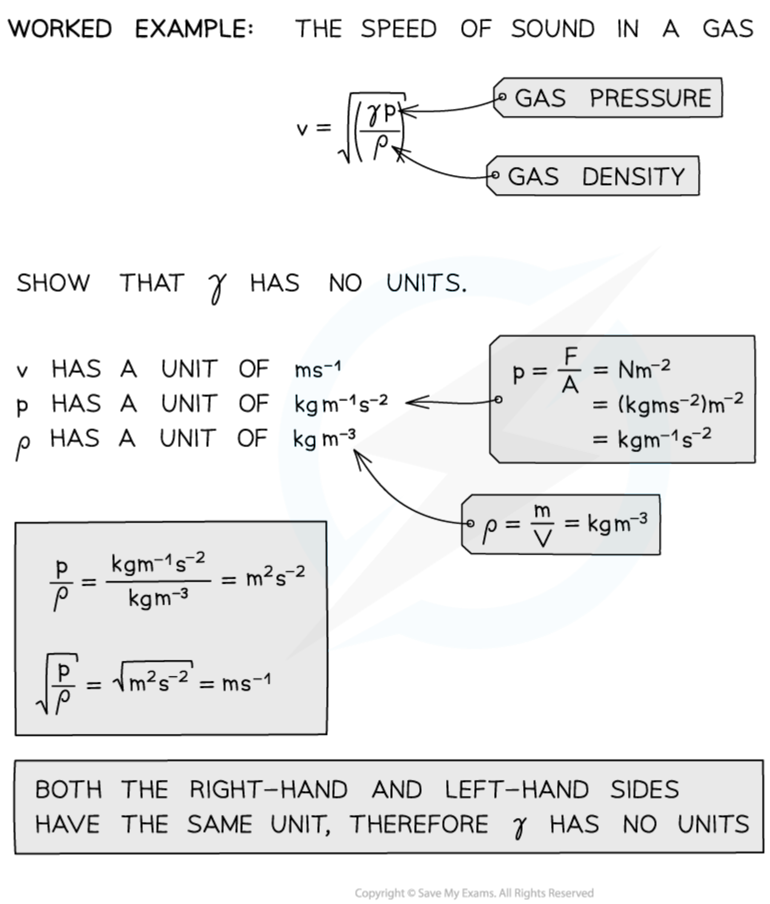
1.1.3 HOMOGENEITY OF PHYSICAL EQUATIONS & POWERS OF TEN

Homogeneity of Physical Equations

- An important skill is to be able to check the homogeneity of physical equations using the SI base units
- The units on either side of the equation should be the same
- To check the homogeneity of physical equations:
    - Check the units on both sides of an equation
    - Determine if they are equal
    - If they do not match, the equation will need to be adjusted



How to check the homogeneity of physical equations

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Powers of Ten

- Physical quantities can span a huge range of values
- For example, the diameter of an atom is about 10^{-10} m (0.0000000001 m), whereas the width of a galaxy may be about 10^{21} m (1000000000000000000000 m)
- This is a difference of 31 powers of ten
- Powers of ten are numbers that can be achieved by multiplying 10 times itself
- It is useful to know the prefixes for certain powers of ten

Powers of Ten Table

PREFIX	ABBREVIATION	POWER OF TEN
TERA–	T	10^{12}
GIGA–	G	10^{9}
MEGA–	M	10^{6}
KILO–	k	10^{3}
CENTI–	c	10^{-2}
MILLI–	m	10^{-3}
MICRO–	μ	10^{-6}
NANO–	n	10^{-9}
PICO–	p	10^{-12}

Exam Tip

You will often see very large or very small numbers categorised by powers of ten, so it is very important you become familiar with these as getting these prefixes wrong is a very common exam mistake!

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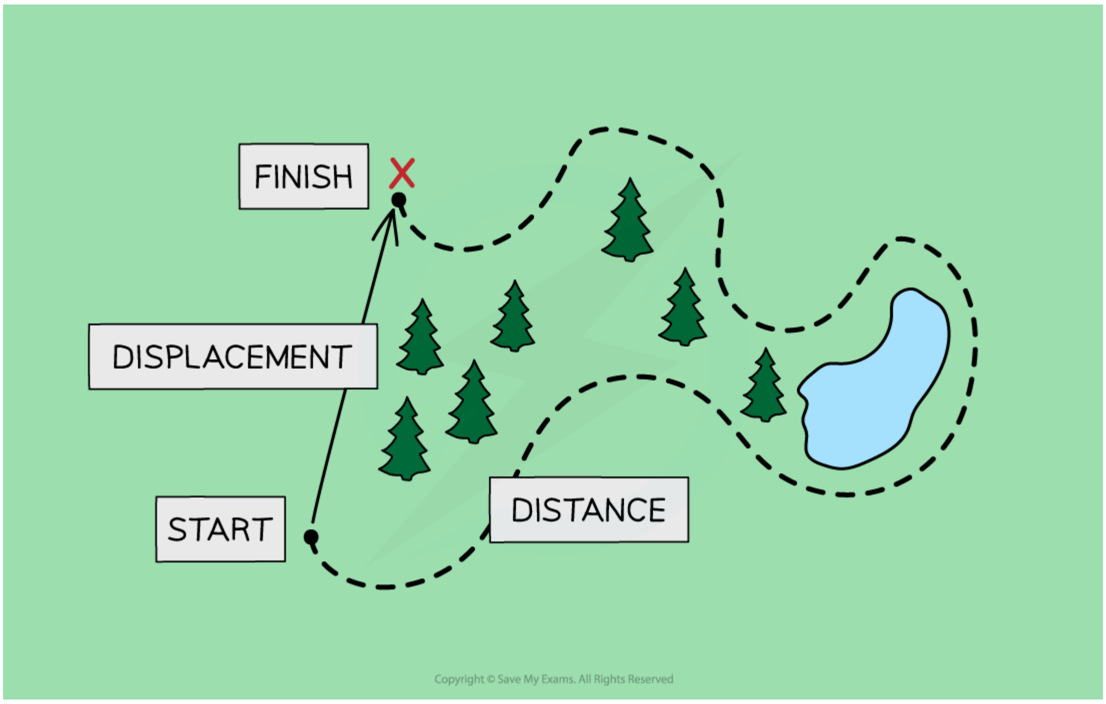
1.1.4 SCALARS & VECTORS

What are Scalar & Vector Quantities?

- A **scalar** is a quantity which **only** has a magnitude (size)
- A **vector** is a quantity which has **both** a magnitude and a direction
- For example, if a person goes on a hike in the woods to a location which is a couple of miles from their starting point
    - As the crow flies, their **displacement** will only be a few miles but the **distance** they walked will be much longer



Displacement is a vector while distance is a scalar quantity

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- **Distance** is a scalar quantity because it describes how an object has travelled overall, but not the direction it has travelled in
- **Displacement** is a vector quantity because it describes how far an object is from where it started and in what direction
- There are a number of common scalar and vector quantities

Scalars and Vectors Table

SCALARS	VECTORS
DISTANCE	DISPLACEMENT
SPEED	VELOCITY
MASS	ACCELERATION
TIME	FORCE
ENERGY	MOMENTUM
VOLUME	
DENSITY	
PRESSURE	
ELECTRIC CHARGE	
TEMPERATURE	

Exam Tip

Do you have trouble figuring out if a quantity is a vector or a scalar? Just think – can this quantity have a minus sign? For example – can you have negative energy? No. Can you have negative displacement? Yes!

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Combining Vectors

- **Vectors** are represented by an arrow
  - The arrowhead indicates the **direction** of the vector
  - The length of the arrow represents the **magnitude**
- Vectors can be combined by **adding** or **subtracting** them from each other
- There are two methods that can be used to combine vectors: the **triangle method** and the **parallelogram method**
- To combine vectors using the triangle method:
  - **Step 1:** link the vectors head-to-tail
  - **Step 2:** the resultant vector is formed by connecting the tail of the first vector to the head of the second vector
- To combine vectors using the parallelogram method:
  - **Step 1:** link the vectors tail-to-tail
  - **Step 2:** complete the resulting parallelogram
  - **Step 3:** the resultant vector is the diagonal of the parallelogram
- When two or more vectors are added together (or one is subtracted from the other), a single vector is formed and is known as the **resultant** vector

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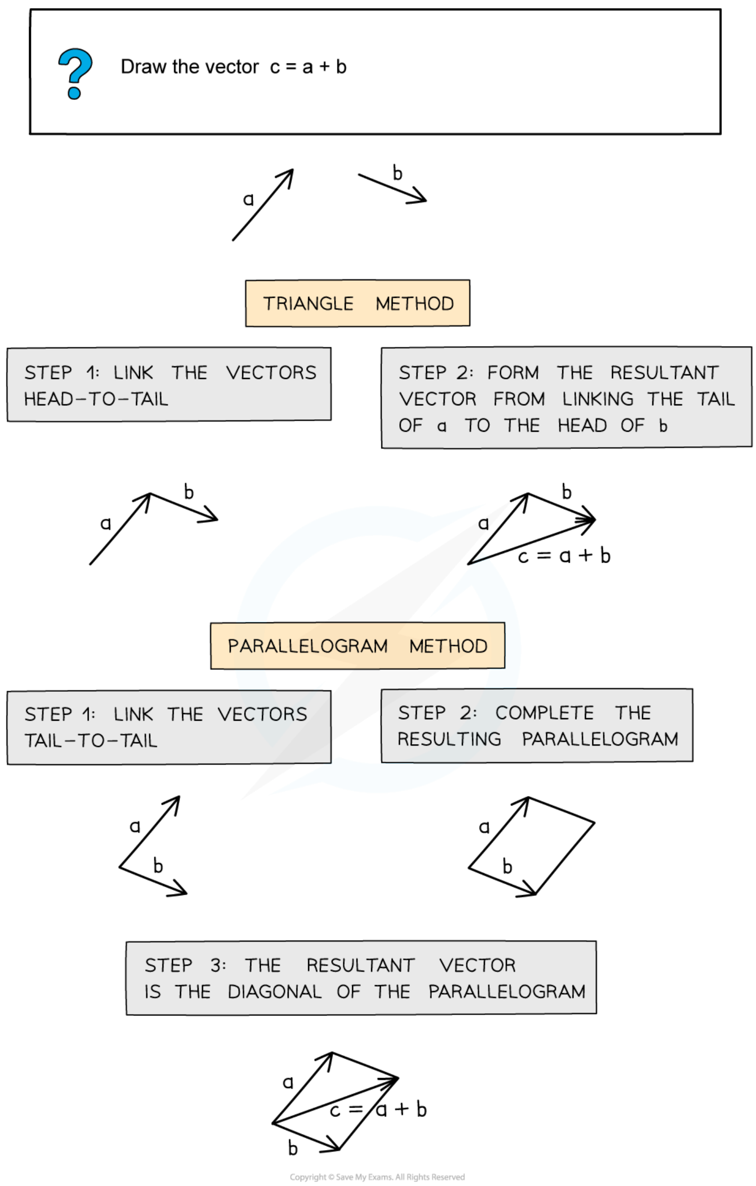
Vector Addition



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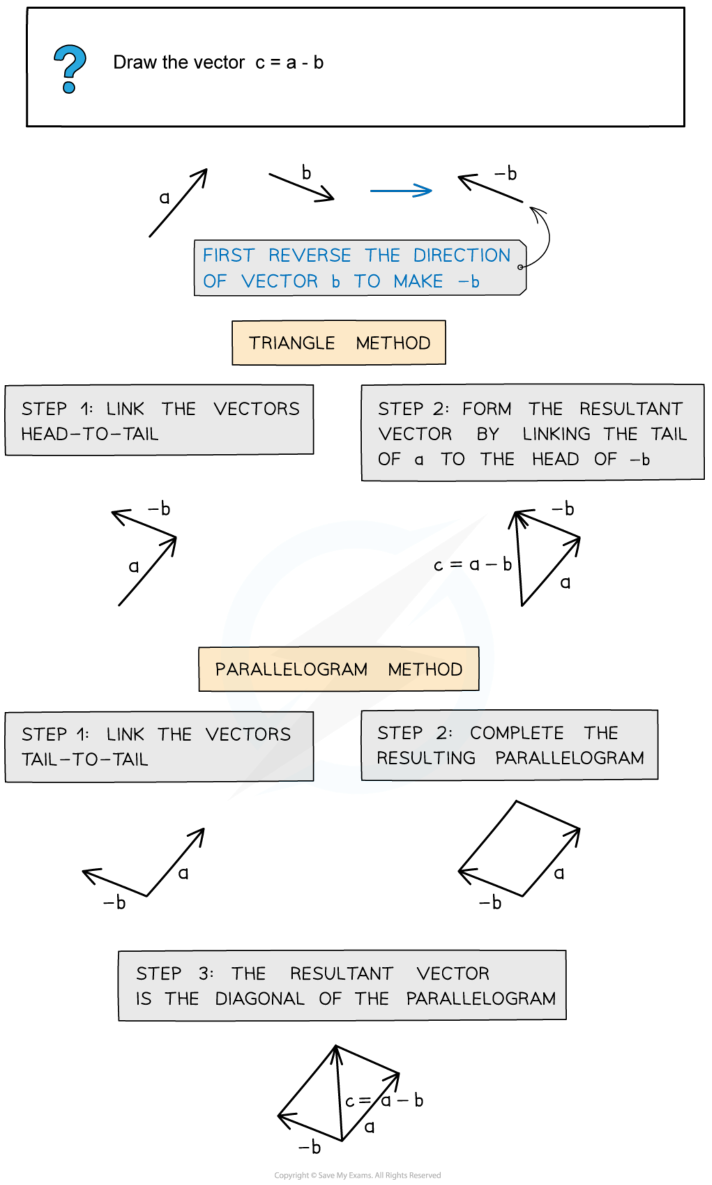
Vector Subtraction



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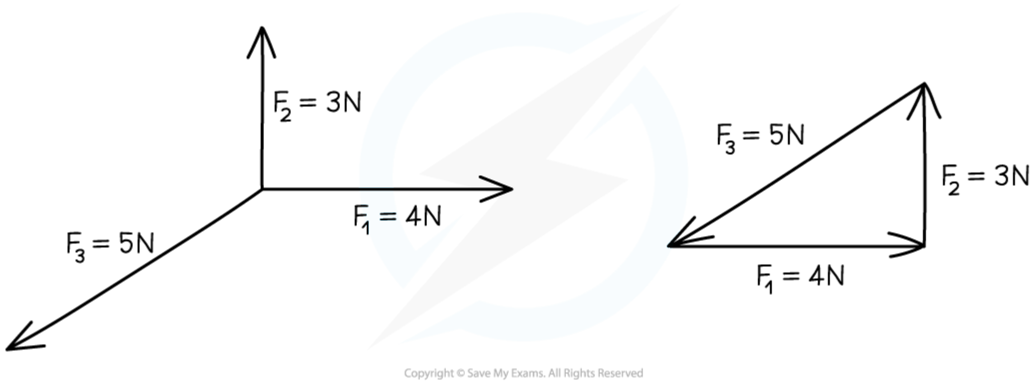
Condition for Equilibrium

- Coplanar forces can be represented by vector triangles
- In equilibrium, these are **closed** vector triangles. The vectors, when joined together, form a closed path



If three forces acting on an object are in equilibrium; they form a closed triangle

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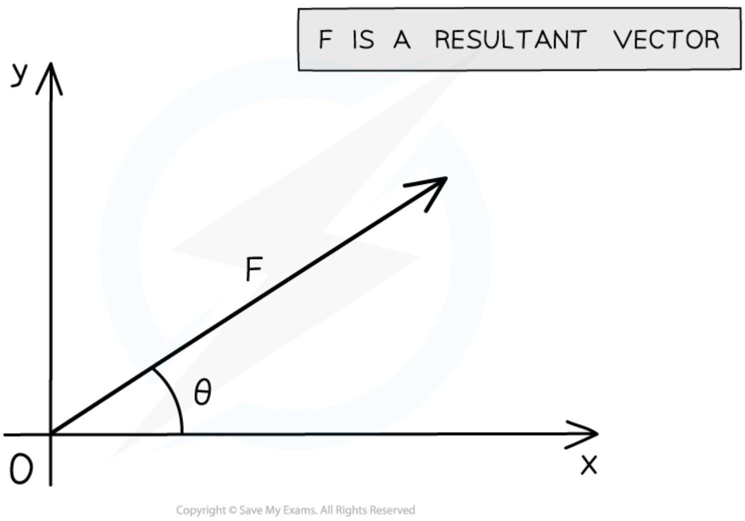
Resolving Vectors

- Two vectors can be represented by a single **resultant vector** that has the same effect
- A single resultant vector can be resolved and represented by **two** vectors, which in combination have the same effect as the original one
- When a single resultant vector is broken down into its **parts**, those **parts** are called components
- For example, a force vector of magnitude *F* and an angle of *θ* to the horizontal is shown below



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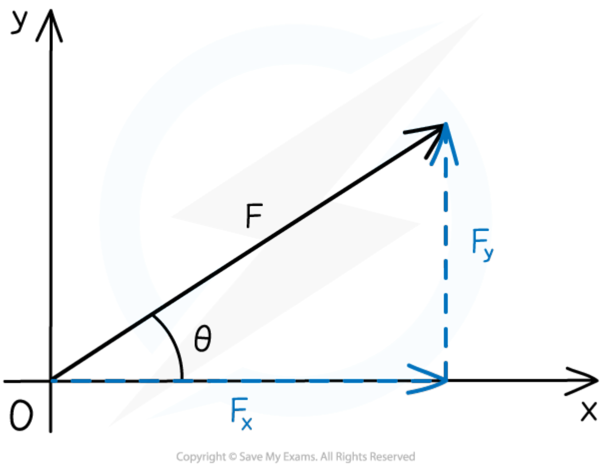
- It is possible to **resolve** this vector into its **horizontal** and **vertical** components using trigonometry



- For the **horizontal component**, $F_x = F\cos\theta$
- For the **vertical component**, $F_y = F\sin\theta$

Exam Question: Easy

Which of the following is an SI base unit?

A current

B gram

C Kelvin

D volt

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Exam Question: Medium

The average kinetic energy *E* of a gas molecule is given by the equation

$$E = \frac{3}{2} kT$$

where *T* is the absolute (kelvin) temperature.

What are the SI base units of *k*?

A $kg^{-1}\, m^{-1}\, s^{2}\, K$

B $kg^{-1}\, m^{-2}\, s^{2}\, K$

C $kg\, m\, s^{-2}\, K^{-1}$

D $kg\, m^{2}\, s^{-2}\, K^{-1}$

Exam Question: Hard

The speed *v* of a liquid leaving a tube depends on the change in pressure ΔP and the density ρ of the liquid. The speed is given by the equation

$$v = k\left(\frac{\Delta P}{\rho}\right)^{n}$$

where *k* is a constant that has no units.

What is the value of *n*?

A $\frac{1}{2}$ **B** 1 **C** $\frac{3}{2}$ **D** 2

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1.2 MEASUREMENTS & ERRORS

1.2.1 ERRORS & UNCERTAINTIES

Random & Systematic Errors

- Measurements of quantities are made with the aim of finding the true value of that quantity
- In reality, it is impossible to obtain the true value of any quantity, there will always be a degree of uncertainty
- The uncertainty is an estimate of the difference between a measurement reading and the true value
- Random and systematic errors are two types of measurement errors which lead to uncertainty

Random error

- Random errors cause unpredictable fluctuations in an instrument's readings as a result of uncontrollable factors, such as environmental conditions
- This affects the **precision** of the measurements taken**,** causing a wider spread of results about the mean value
- To **reduce** random error: **repeat** measurements several times and calculate an average from them

Systematic error

- Systematic errors arise from the use of faulty instruments used or from flaws in the experimental method
- This type of error is repeated every time the instrument is used or the method is followed, which affects the **accuracy** of all readings obtained
- To **reduce** systematic errors: instruments should be **recalibrated** or the technique being used should be corrected or adjusted

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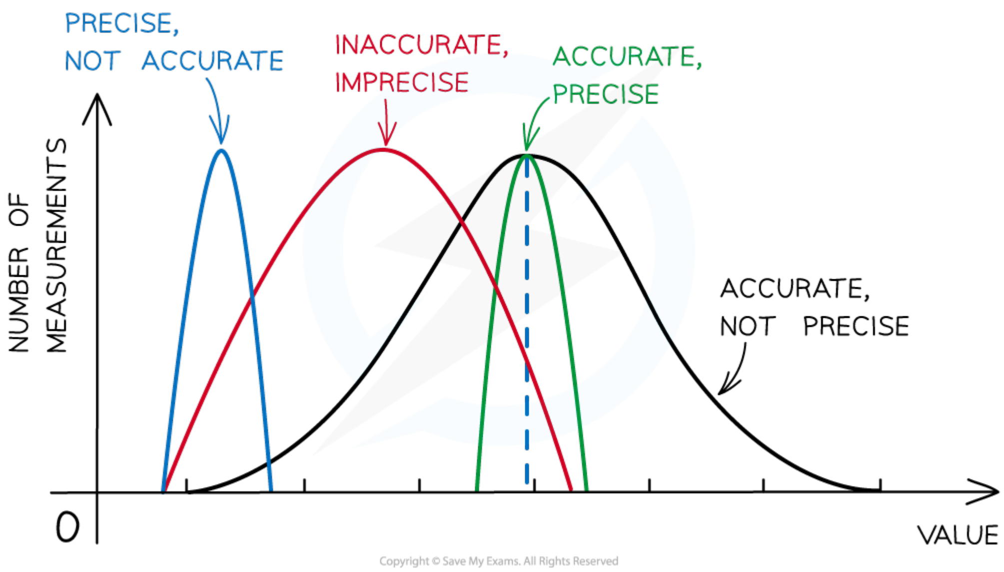

Representing precision and accuracy on a graph

Zero error

- This is a type of systematic error which occurs when an instrument gives a reading when the **true reading is zero**
- This introduces a fixed error into readings which must be accounted for when the results are recorded

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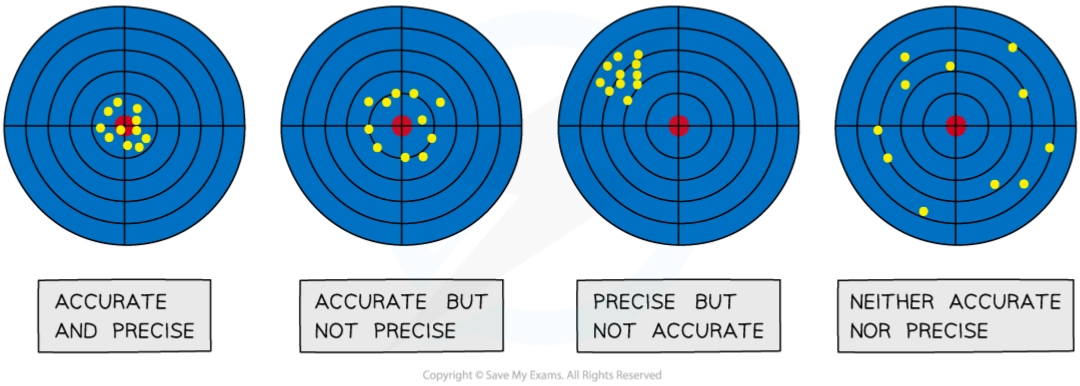
Precision & Accuracy

- **Precision of a measurement:** this is how close the measured values are to each other; if a measurement is repeated several times, then they can be described as precise when the values are very similar to, or the same as, each other
- The precision of a measurement is reflected in the values recorded – measurements to a greater number of decimal places are said to be more **precise** than those to a whole number
- **Accuracy**: this is how close a measured value is to the true value; the accuracy can be increased by repeating measurements and finding a mean average



The difference between precise and accurate results

Exam Tip

It is very common for students to confuse precision with accuracy – measurements can be precise but not accurate if each measurement reading has the same error. Precision refers to the ability to take multiple readings with an instrument that are close to each other, whereas accuracy is the closeness of those measurements to the true value.

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1.2.2 CALCULATING UNCERTAINTIES

Calculating Uncertainty

- There is always a degree of uncertainty when measurements are taken; the uncertainty can be thought of as the difference between the **actual** reading taken (caused by the equipment or techniques used) and the **true value**
- Uncertainties are not the same as errors
  - Errors can be thought of as issues with equipment or methodology that cause a reading to be different from the true value
  - The uncertainty is a range of values around a measurement within which the true value is expected to lie, and is an **estimate**
- For example, if the true value of the mass of a box is 950 g, but a systematic error with a balance gives an actual reading of 952 g, the uncertainty is ±2 g
- These uncertainties can be represented in a number of ways:
  - **Absolute Uncertainty:** where uncertainty is given as a fixed quantity
  - **Fractional Uncertainty:** where uncertainty is given as a fraction of the measurement
  - **Percentage Uncertainty:** where uncertainty is given as a percentage of the measurement

$$percentage\ uncertainty = \frac{uncertainty}{measured\ value}\ x\ 100\%$$

- To find uncertainties in different situations:
- **The uncertainty in a reading:** ± half the smallest division
- **The uncertainty in a measurement:** at least ±1 smallest division
- **The uncertainty in repeated data:** half the range i.e. ± ½ (largest – smallest value)
- **The uncertainty in digital readings:** ± the last significant digit unless otherwise quoted

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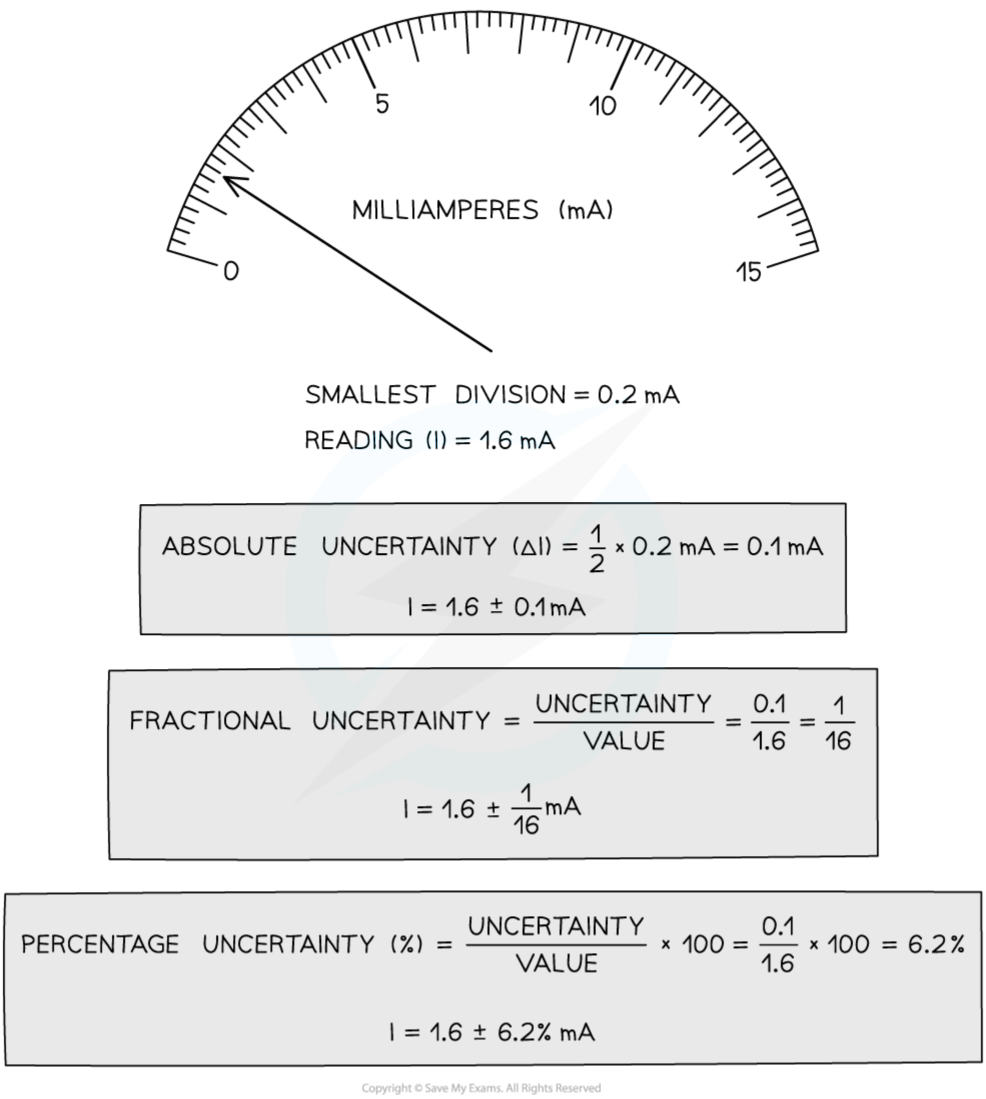

How to calculate absolute, fractional and percentage uncertainty

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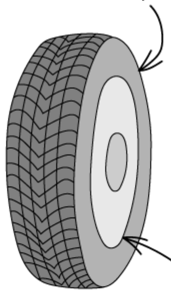
Combining Uncertainties

- The rules to follow
- Adding / subtracting data – add the absolute uncertainties

ADDING / SUBTRACTING DATA

DIAMETER OF TYRE $(d_1) = 55.0 \pm 0.5$ cm



DIAMETER OF INNER TYRE $(d_2) = 21.0 \pm 0.7$ cm

DIFFERENCE IN DIAMETERS $(d_1 - d_2) = 55.0 - 21.0 = 34.0$ cm

UNCERTAINTY IN DIFFERENCE $= \pm(0.5 + 0.7) = \pm 1.2$ cm

$d_1 - d_2 = 34.0 \pm 1.2$ cm

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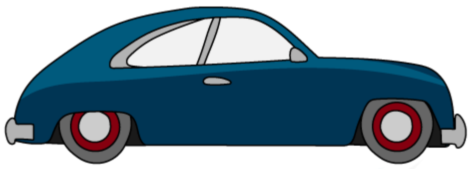
- Multiplying / dividing data – add the percentage uncertainties

MULTIPLYING / DIVIDING DATA



DISTANCE = 50.0 ± 0.1 m

TIME = 5.00 ± 0.05 s

$$\text{SPEED } (v) = \frac{\text{DISTANCE } (s)}{\text{TIME } (t)}$$

$$V = \frac{50.0}{5.0} = 10.0 \text{ ms}^{-1}$$

$$\frac{\Delta v}{v} = \frac{\Delta s}{s} + \frac{\Delta t}{t} = \frac{0.1}{50.0} + \frac{0.05}{5.00} = 0.002 + 0.01 = 0.012$$

ABSOLUTE UNCERTAINTY $(\Delta v) = 10.0 \times 0.012 = \pm 0.12 \text{ ms}^{-1}$

$v = 10.0 \pm 0.12 \text{ ms}^{-1}$

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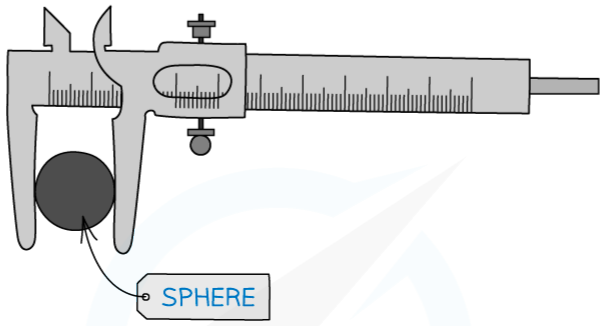
- Raising to a power – multiply the uncertainty by the power

RAISING TO A POWER



$$V = \frac{4}{3}\pi r^3$$

$$r = 2.50 \pm 0.02 \text{ cm}$$

$$V = \frac{4}{3}\pi (2.50)^3 = 65.5 \text{ cm}^3$$

$$\frac{\Delta V}{V} = 3 \times \frac{\Delta r}{r} = 3 \times \frac{0.02}{2.50} = 0.024$$

ABSOLUTELY UNCERTAINTY $(\Delta V) = 65.5 \times 0.024 = 1.57 \text{cm}^3$

PERCENTAGE UNCERTAINTY $(\% \Delta V) = 100 \times 0.024 = 2.4\%$

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1.2.3 MEASUREMENT TECHNIQUES

Measurement Techniques

- Common instruments used in Physics are:
  - Metre rules – to measure distance and length
  - Balances – to measure mass
  - Protractors – to measure angles
  - Stopwatches – to measure time
  - Ammeters – to measure current
  - Voltmeters – to measure potential difference
- More complicated instruments such as the micrometer screw gauge and Vernier calipers can be used to more accurately measure length

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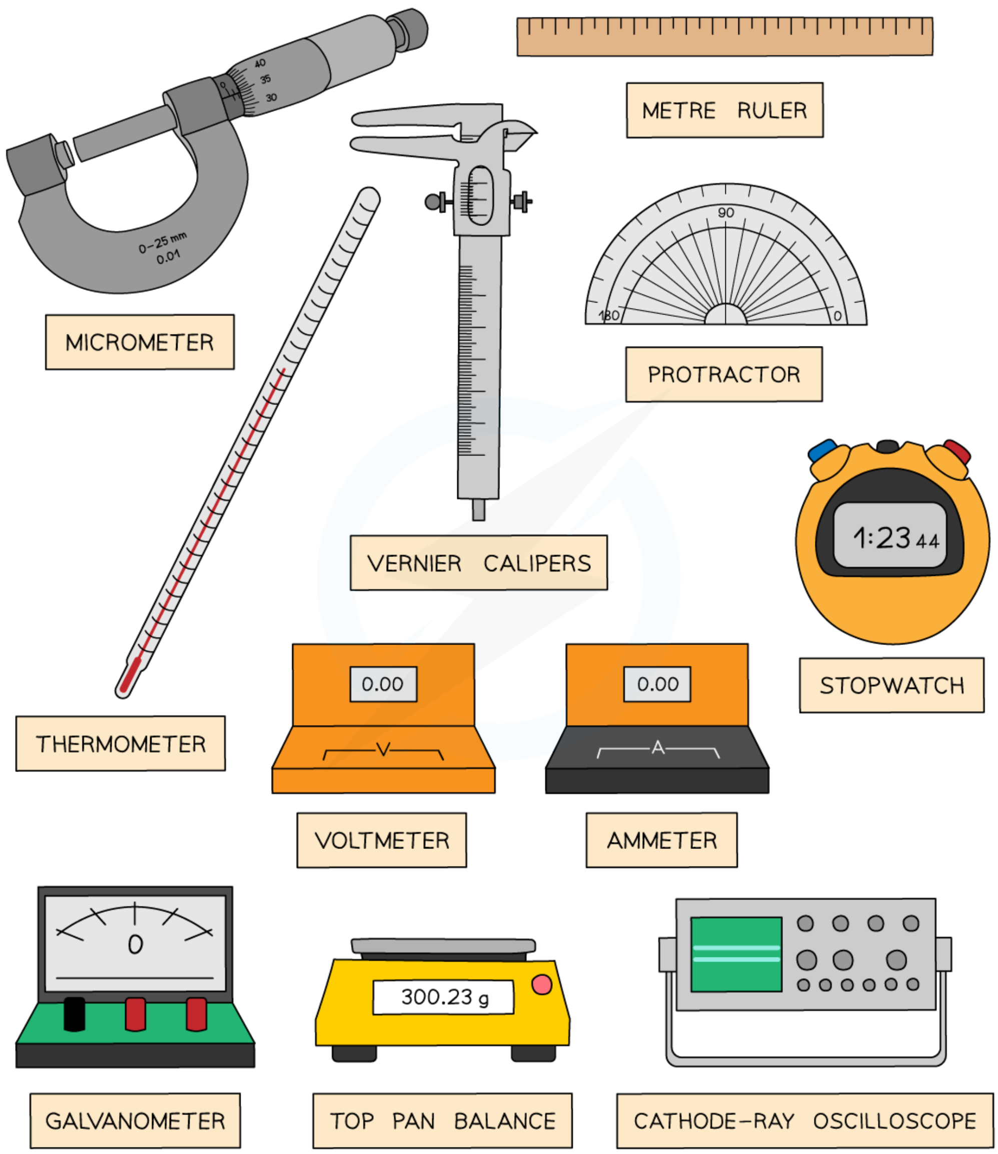

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- When using measuring instruments like these you need to ensure that you are fully aware of what each division on a scale represents
    - This is known as the **resolution**
- The resolution is the smallest change in the physical quantity being measured that results in a change in the reading given by the measuring instrument
- The smaller the change that can be measured by the instrument, the greater the degree of resolution
- For example, a standard mercury thermometer has a resolution of 1°C whereas a typical digital thermometer will have a resolution of 0.1°C
    - The digital thermometer has a higher resolution than the mercury thermometer

Measuring Instruments Table

Quantity	Instrument	Typical Resolution
Length	Metre rule	1 mm
Length	Vernier Calipers	0.05 mm
Length	Micrometer	0.001 mm
Mass	Top-pan Balance	0.01 g
Angle	Protractor	1°
Time	Stopwatch	0.01 s
Temperature	Thermometer	1 °C
Potential Difference	Voltmeter	1 mV – 0.1 V
Current	Ammeter	1 mA – 0.1 A

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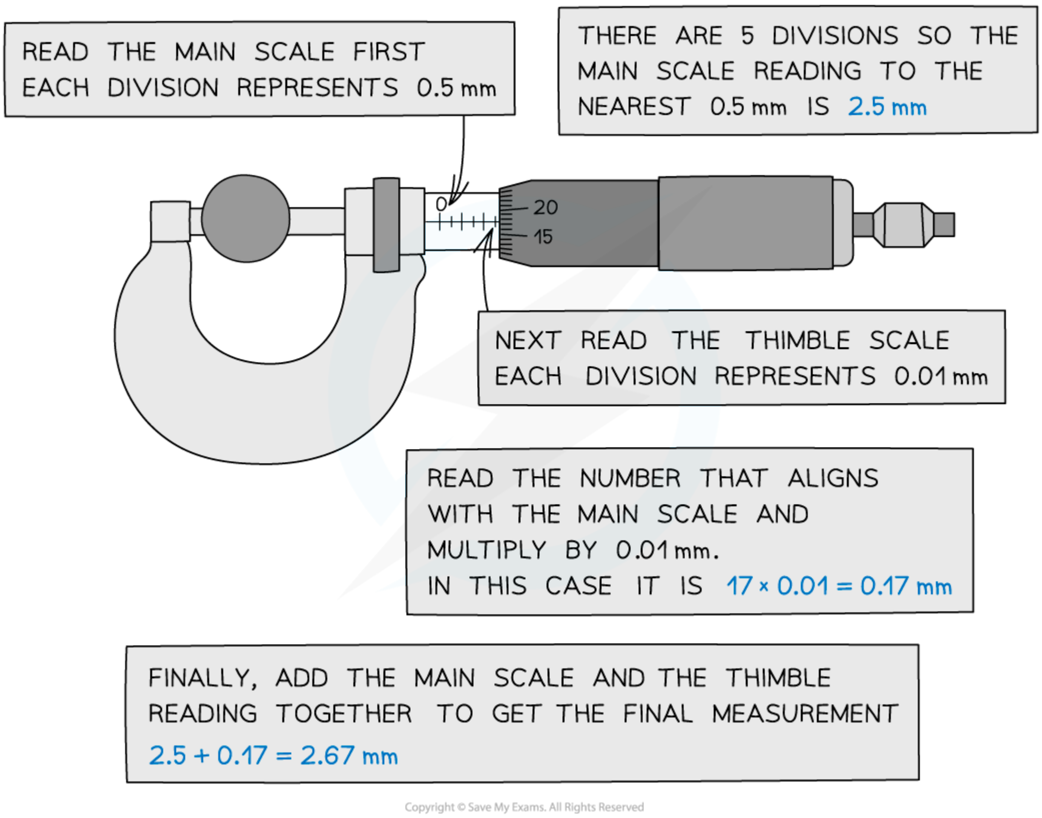
Micrometer Screw Gauge



How to operate a micrometer

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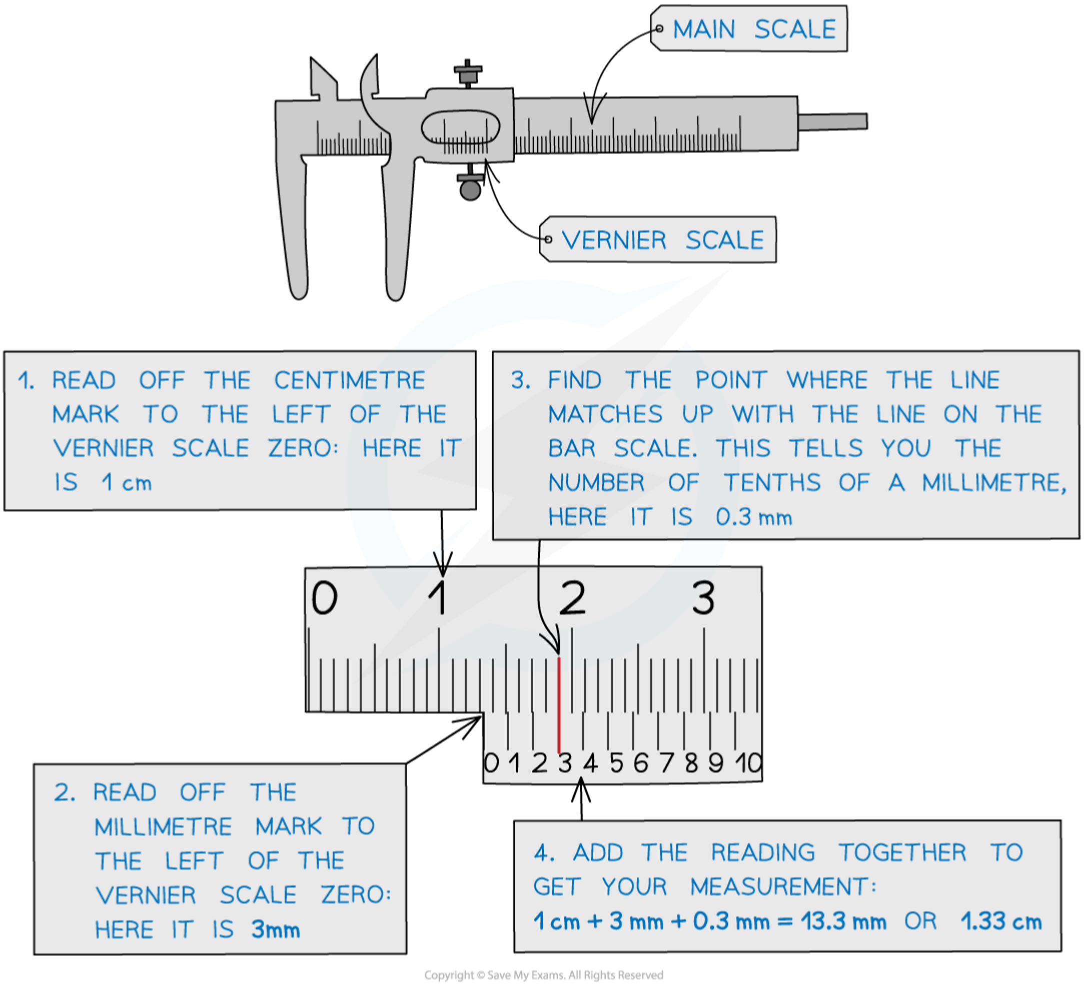
Vernier Calipers



How to operate vernier calipers

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Exam Question: Easy

The density of the material of a coil of thin wire is to be found.

Which set of instruments could be used to do this most accurately?

A metre rule, protractor, spring balance

B micrometer, metre rule, top-pan balance

C stopwatch, newton-meter, vernier calipers

D tape measure, vernier calipers, lever balance

Exam Question: Medium

The measurement of a physical quantity may be subject to random errors and systematic errors.

Which statement is correct?

A random errors can be reduced by taking the average of several measurements

B random errors are always caused by the person taking the measurement

C a systematic error cannot be reduced by adjusting the apparatus

D a systematic error results in a different reading each time the measurement is taken

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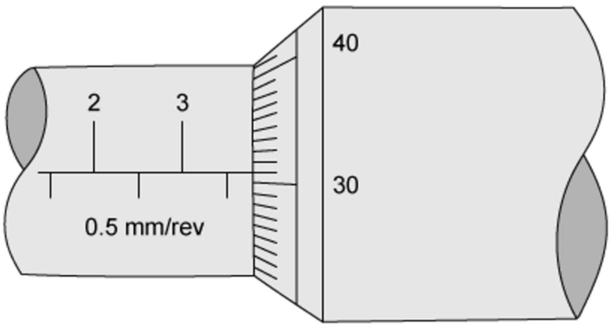
Exam Question: Hard

The diameter of a cylindrical metal rod is measured using a micrometer screw gauge.

The diagram below shows an enlargement of the scale on the micrometer screw gauge when taking the measurement.



What is the cross-sectional area of the rod?

A 3.81mm^2 **B** 11.4mm^2 **C** 22.8mm^2 **D** 45.6mm^2

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2. Kinematics

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2.1 EQUATIONS OF MOTION

2.1.1 DISPLACEMENT, VELOCITY & ACCELERATION

Defining Displacement, Velocity & Acceleration

Scalar quantities

- Remember scalar quantities only have a magnitude (size)
  - **Distance:** the total length between two points
  - **Speed:** the total distance travelled per unit of time

Vector quantities

- Remember vector quantities have both magnitude and direction
  - **Displacement:** the distance of an object from a fixed point in a specified direction
  - **Velocity:** the rate of change of displacement of an object
  - **Acceleration:** the rate of change of velocity of an object

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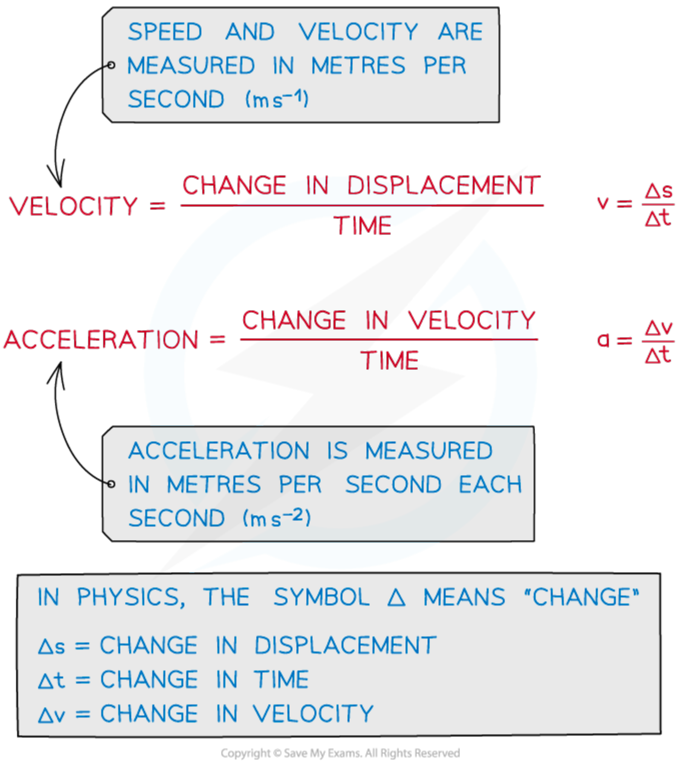
Equations



Equations linking displacement, velocity and acceleration

2. Kinematics

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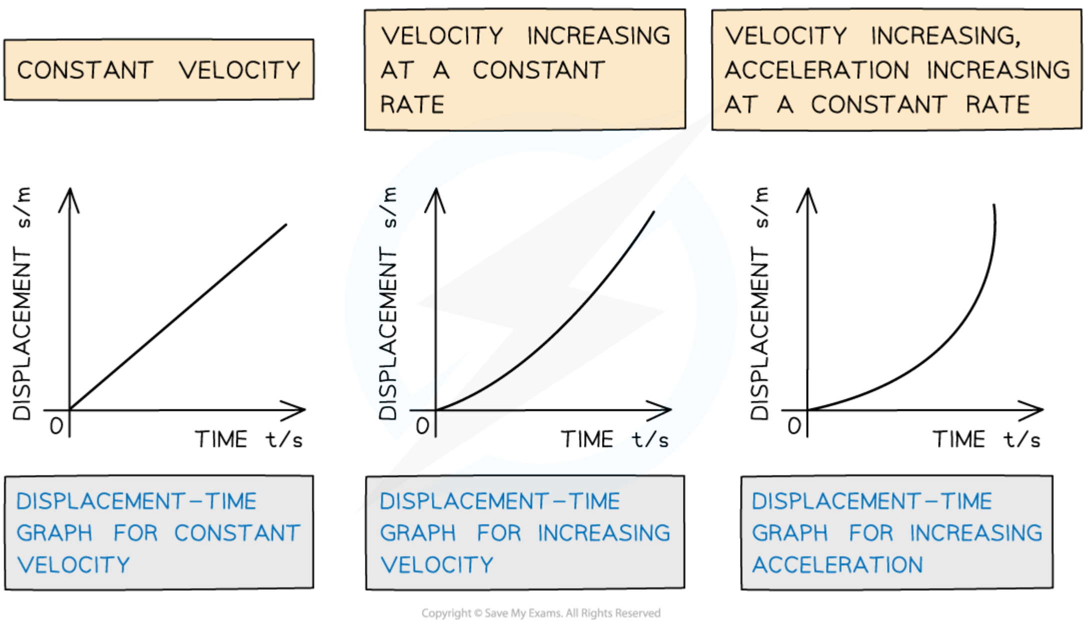
2.1.2 MOTION GRAPHS

Motion Graphs

- Three types of graph that can represent motion are **displacement-time** graphs, **velocity-time** graphs and **acceleration-time** graphs
- On a **displacement-time graph...**
    - **slope** equals **velocity**
    - the **y-intercept** equals the **initial displacement**
    - a **straight** line represents a **constant** velocity
    - a **curved** line represents an **acceleration**
    - a **positive slope** represents motion in the **positive direction**
    - a **negative slope** represents motion in the **negative direction**
    - a **zero** slope (horizontal line) represents a state of **rest**
    - the area under the curve is meaningless



2. Kinematics

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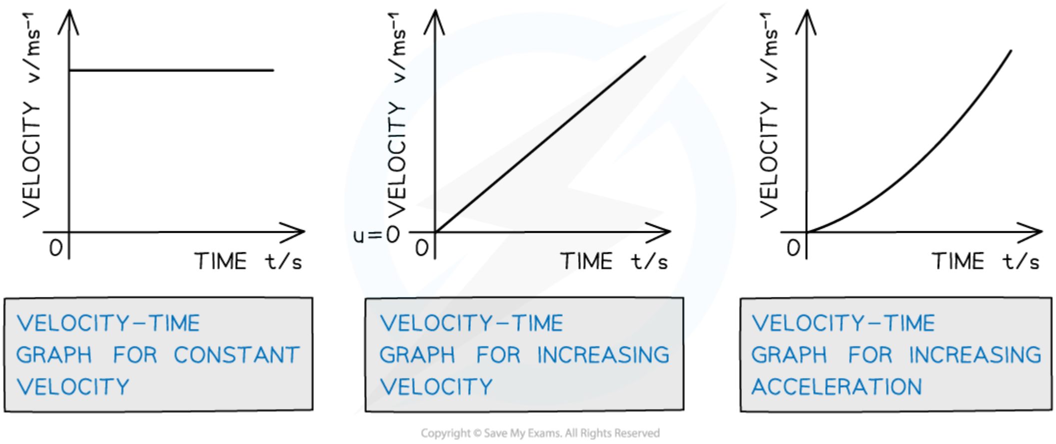
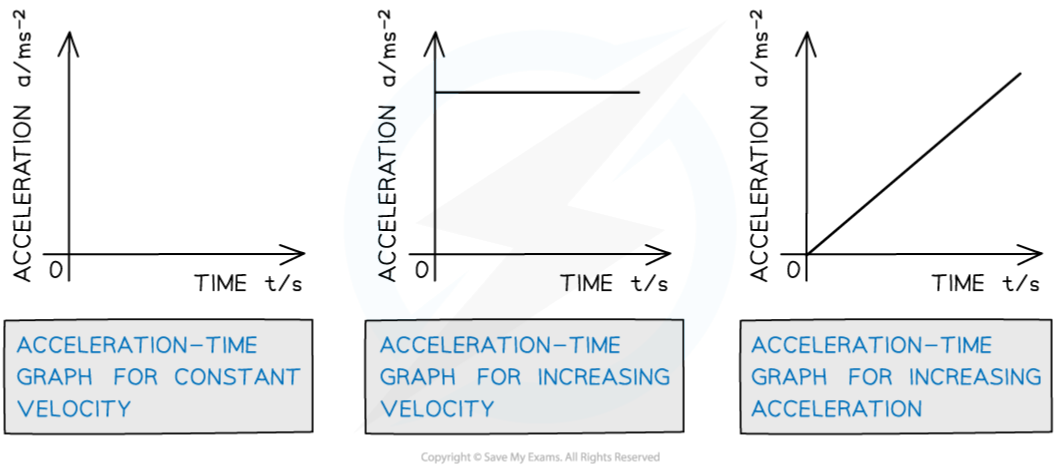
- On a **velocity-time graph…**
  - **slope** equals **acceleration**
  - the **y-intercept** equals the **initial velocity**
  - a **straight** line represents **uniform** acceleration
  - a **curved** line represents **non-uniform acceleration**
  - a **positive** slope represents an **increase** in **velocity** in the **positive direction**
  - a **negative** slope represents an **increase** in **velocity** in the **negative direction**
  - a **zero** slope (horizontal line) represents motion with **constant velocity**
  - the **area** under the curve equals the **change** in **displacement**



2. Kinematics

YOUR NOTES

- On an **acceleration-time graph...**
  - slope is meaningless
  - the y-intercept equals the initial acceleration
  - a zero slope (horizontal line) represents an object undergoing constant acceleration
  - the area under the curve equals the change in velocity



How displacement, velocity and acceleration graphs relate to each other

2. Kinematics

YOUR NOTES
⬇

2.1.3 AREA UNDER A VELOCITY-TIME GRAPH

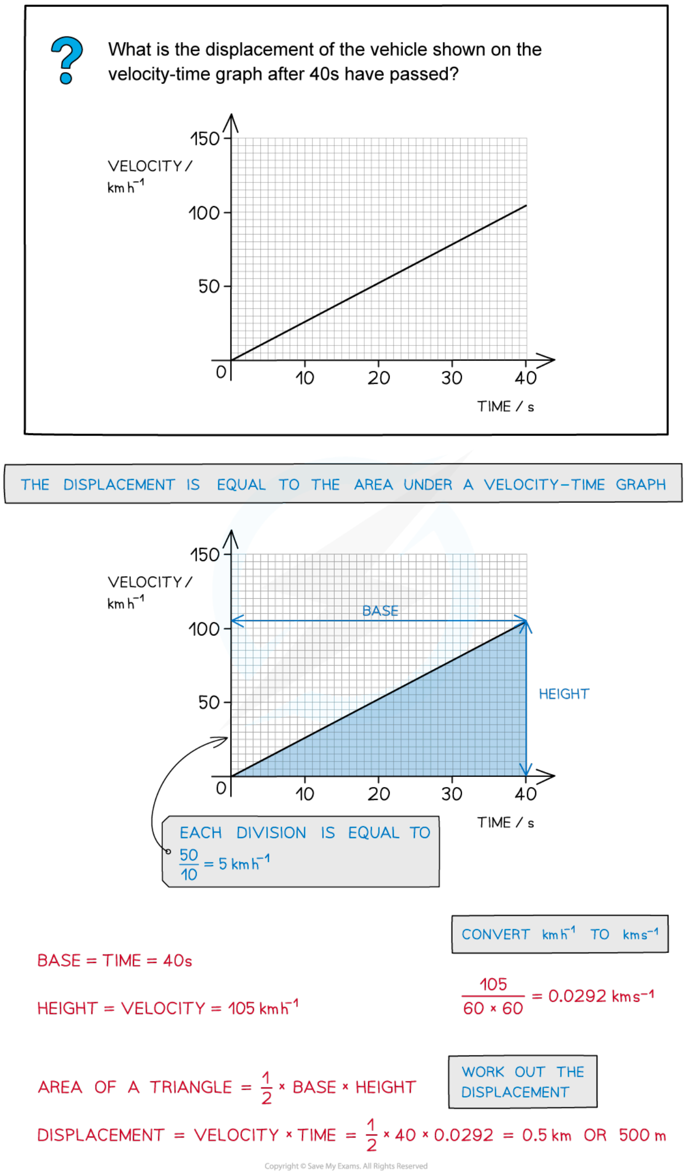
Area under a Velocity-Time Graph

- Velocity-time graphs show the speed and direction of an object in motion over a specific period of **time**
- The area under a velocity-time graph is equal to the **displacement** of a moving object

displacement = area under a velocity-time graph

2. Kinematics

YOUR NOTES
⬇



How to determine the area under a velocity-time graph

Exam Tip

Always check the values given on the y-axis of a motion graph – students often confuse displacement-time graphs and velocity-time graphs.

The area under the graph can often be broken down into triangles, squares and rectangles, so make sure you are comfortable with calculating area!

2. Kinematics

YOUR NOTES

2.1.4 GRADIENT OF A DISPLACEMENT-TIME GRAPH

Gradient of a Displacement-Time Graph

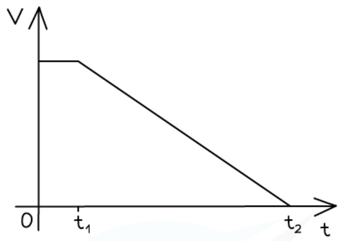
- Displacement-time graphs show the changing position of an object in motion
- They also show whether an object is moving forwards (positive displacement) or backwards (negative displacement)
  - A negative gradient = a negative velocity (the object is moving backwards)
- The gradient (slope) of a displacement-time graph is equal to **velocity**
  - The greater the slope, the greater the velocity

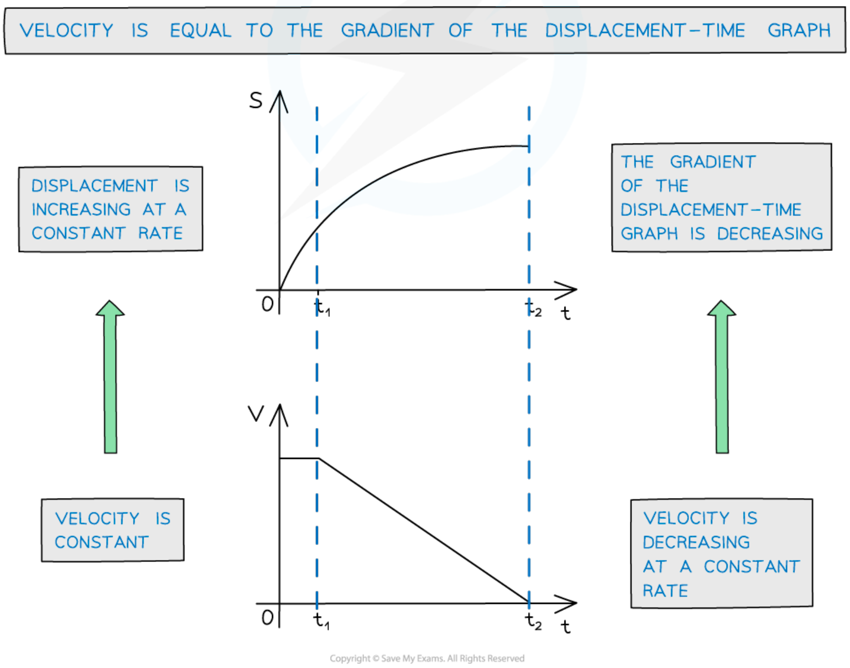
2. Kinematics

YOUR NOTES

A car driver sees a hazard ahead and applies the brakes to bring the car to rest.

What does the displacement-time graph look like?





How to determine the slope of a displacement-time graph

Exam Tip

Don’t forget that velocity is a vector quantity; it has a size and a direction. If velocity is initially positive and then becomes negative, then the object has changed direction.

2. Kinematics

YOUR NOTES
⬇

2.1.5 GRADIENT OF A VELOCITY-TIME GRAPH

Gradient of a Velocity-Time Graph

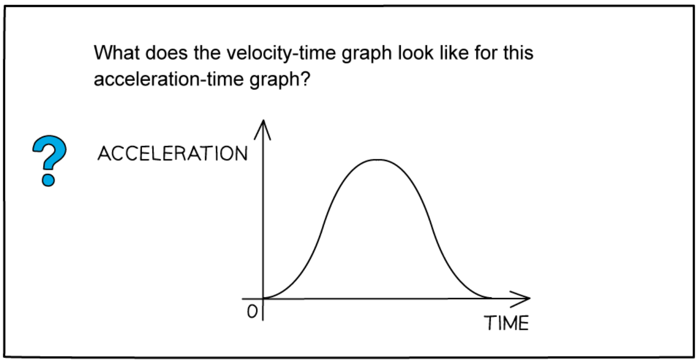
- **Acceleration** is any change in the velocity of an object in a given time

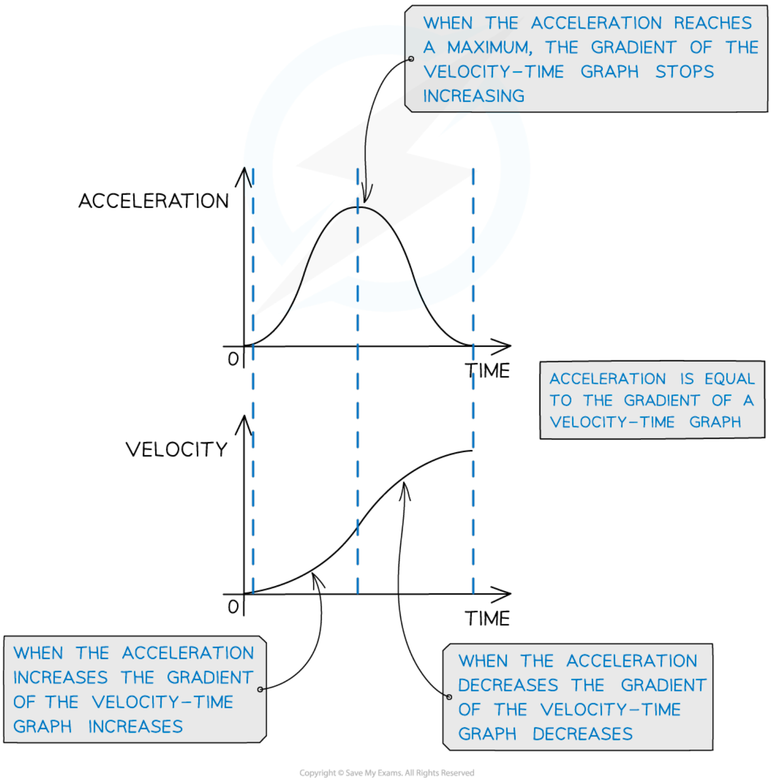
$$acceleration = \frac{change\ in\ velocity}{time} = \frac{(v - u)}{t}$$

- As velocity is a vector quantity, this means that if the **speed** of an object **changes**, or its **direction changes**, then it is accelerating
  - An object that slows down tends to be described as 'decelerating'
- The gradient of a velocity-time graph is equal to **acceleration**

2. Kinematics

YOUR NOTES ⬇





How to determine the slope of a velocity-time graph

2. Kinematics

YOUR NOTES
⬇

2.1.6 DERIVING KINEMATIC EQUATIONS

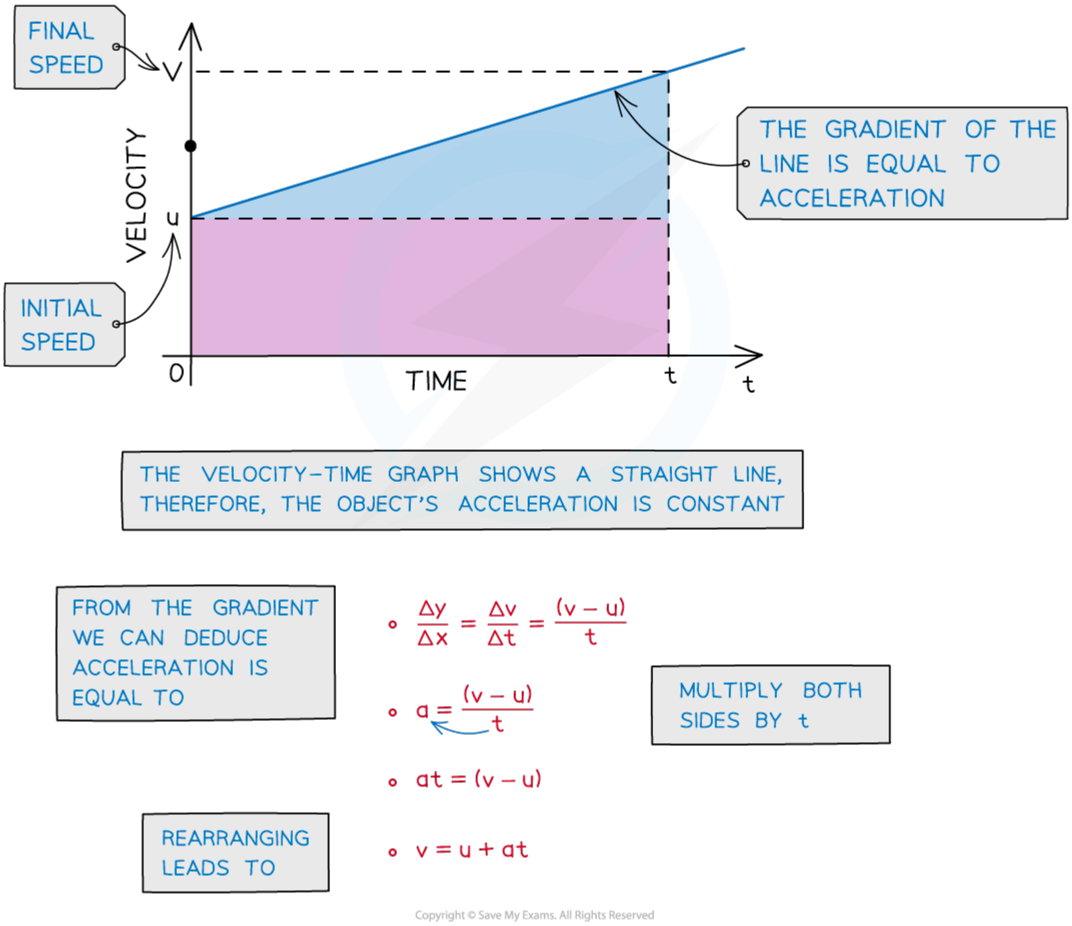
Deriving Kinematic Equations of Motion

- The kinematic equations of motion are a set of four equations which can describe any object moving with **constant** acceleration
- They relate the five variables:
  - s = **displacement**
  - u = **initial velocity**
  - v = **final velocity**
  - a = **acceleration**
  - t = **time interval**
- It's important to know where these equations come from and how they are derived:

2. Kinematics

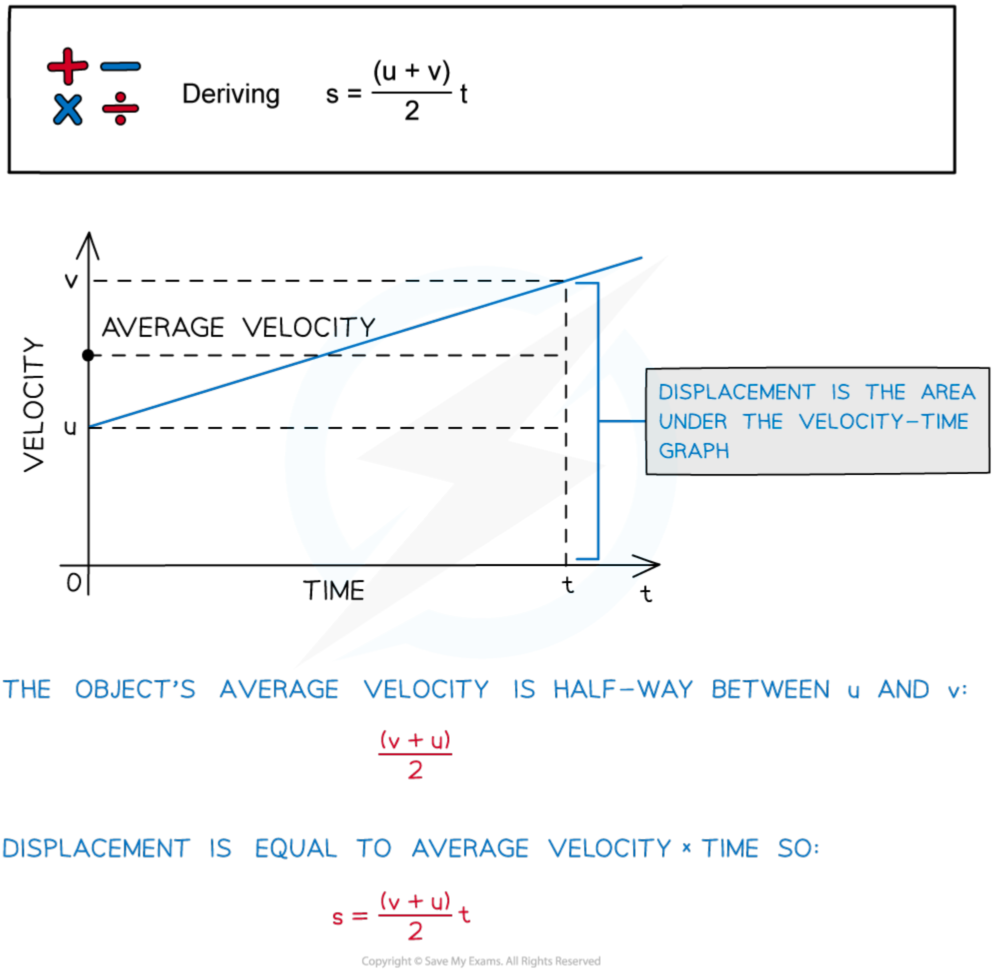
YOUR NOTES

Deriving $v = u + at$



A graph showing how the velocity of an object varies with time

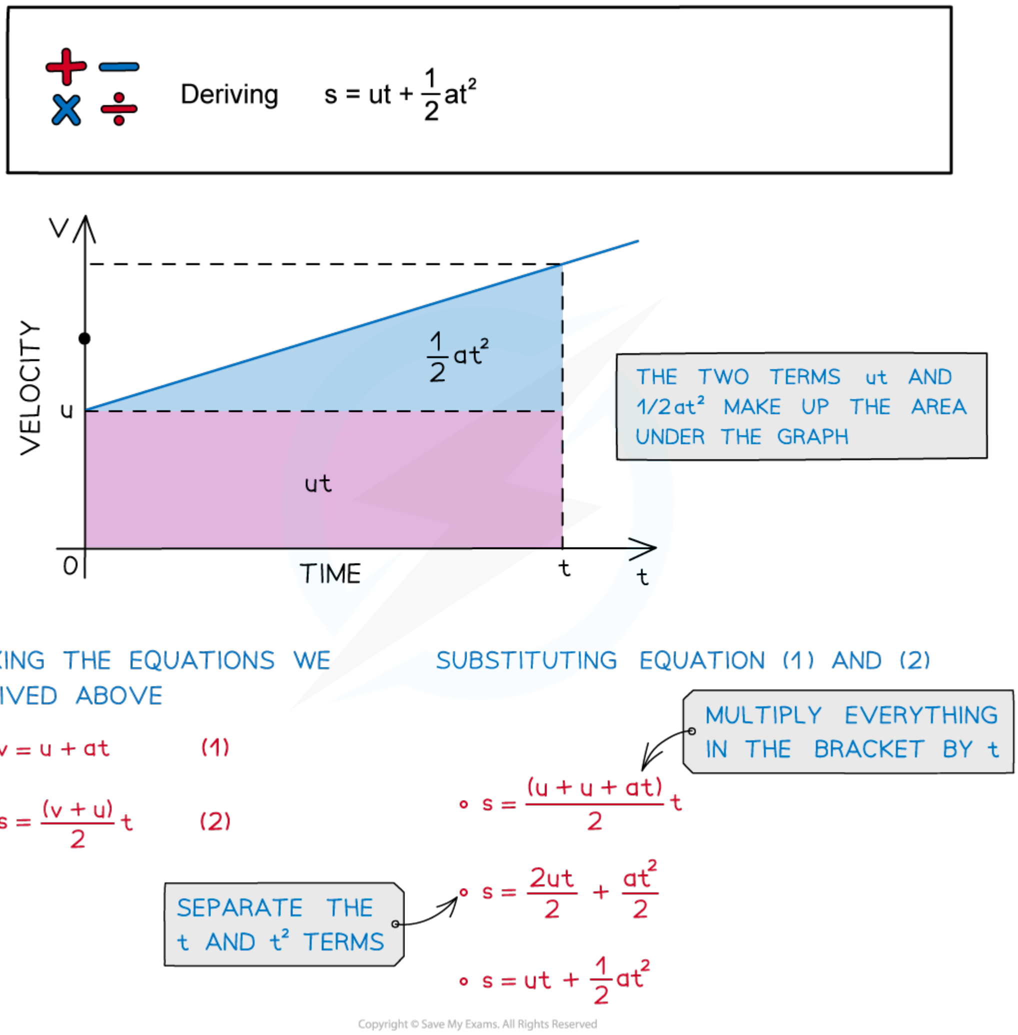
2. Kinematics



The average velocity is halfway between u and v

2. Kinematics

YOUR NOTES



The two terms ut and ½at² make up the area under the graph

2. Kinematics

YOUR NOTES

Deriving $v^2 = u^2 + 2as$

TAKING THE EQUATIONS WE DERIVED ABOVE

- $v = u + at \rightarrow t = \frac{v-u}{a}$ (1)

$s = \frac{(v+u)}{2}t$ (2)

SUBSTITUTING (1) INTO (2)

- $s = \frac{(v+u)}{2} \times \frac{(v-u)}{a}$
- $s = \frac{v^2 - u^2}{2a}$ ← MULTIPLY BOTH SIDES BY 2a
- $v^2 = u^2 + 2as$

This final equation can be derived from two of the others

SUMMARY

$$v = u + at \qquad s = ut + \frac{1}{2}at^2$$

$$s = \frac{(v+u)}{2}t \qquad v^2 = u^2 + 2as$$

Summary of the four equations of uniformly accelerated motion

2. Kinematics

YOUR NOTES

2.1.7 SOLVING PROBLEMS WITH KINEMATIC EQUATIONS

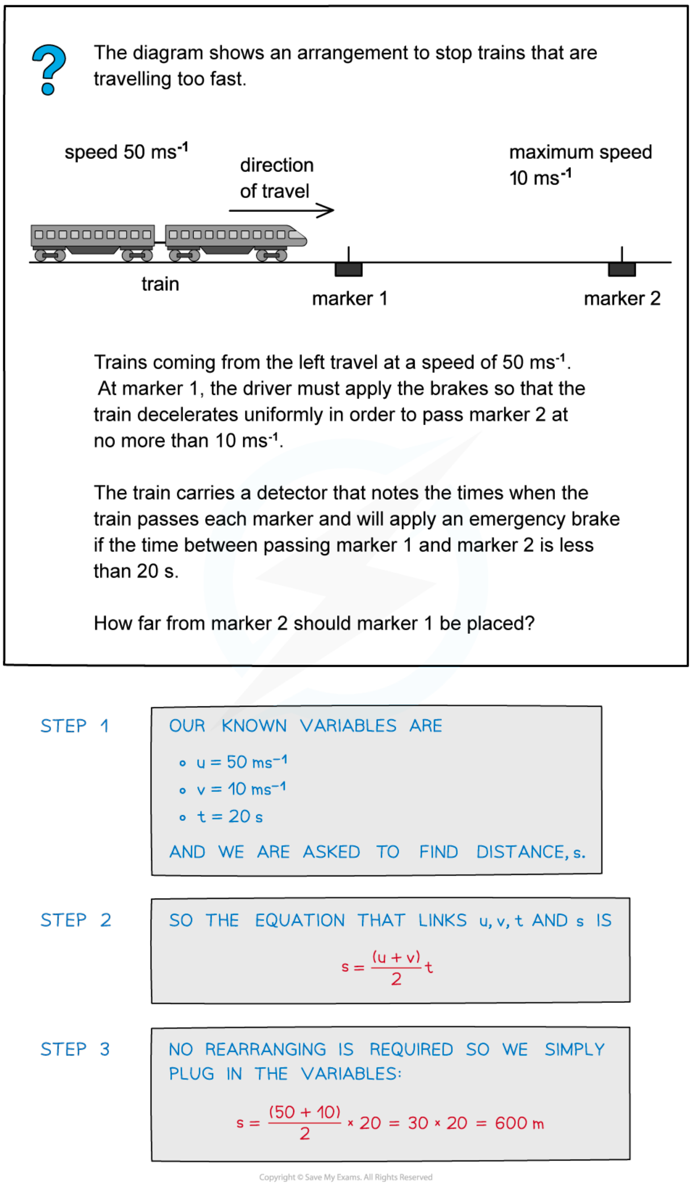
Solving Problems with Kinematic Equations

- **Step 1:** Write out the variables that are given in the question, both known and unknown, and use the context of the question to deduce any quantities that aren't explicitly given
  - e.g. for vertical motion $a = \pm 9.81$ m s^{-2}, an object which starts or finishes at rest will have $u = 0$ or $v = 0$
- **Step 2:** Choose the equation which contains the quantities you have listed
  - e.g. the equation that links *s, u, a* and *t* is $s = ut + \frac{1}{2}at^2$
- **Step 3:** Convert any units to SI units and then insert the quantities into the equation and rearrange algebraically to determine the answer

2. Kinematics

YOUR NOTES

Worked Example



How to solve problems using the kinematic equations

2. Kinematics

YOUR NOTES
⬇

Exam Tip

- This is arguably the most important section of this topic, you can always be sure there will be one, or more, questions in the exam about solving problems with the kinematic equations
- The best way to master this section is to practice as many questions as possible

2. Kinematics

YOUR NOTES
⬇

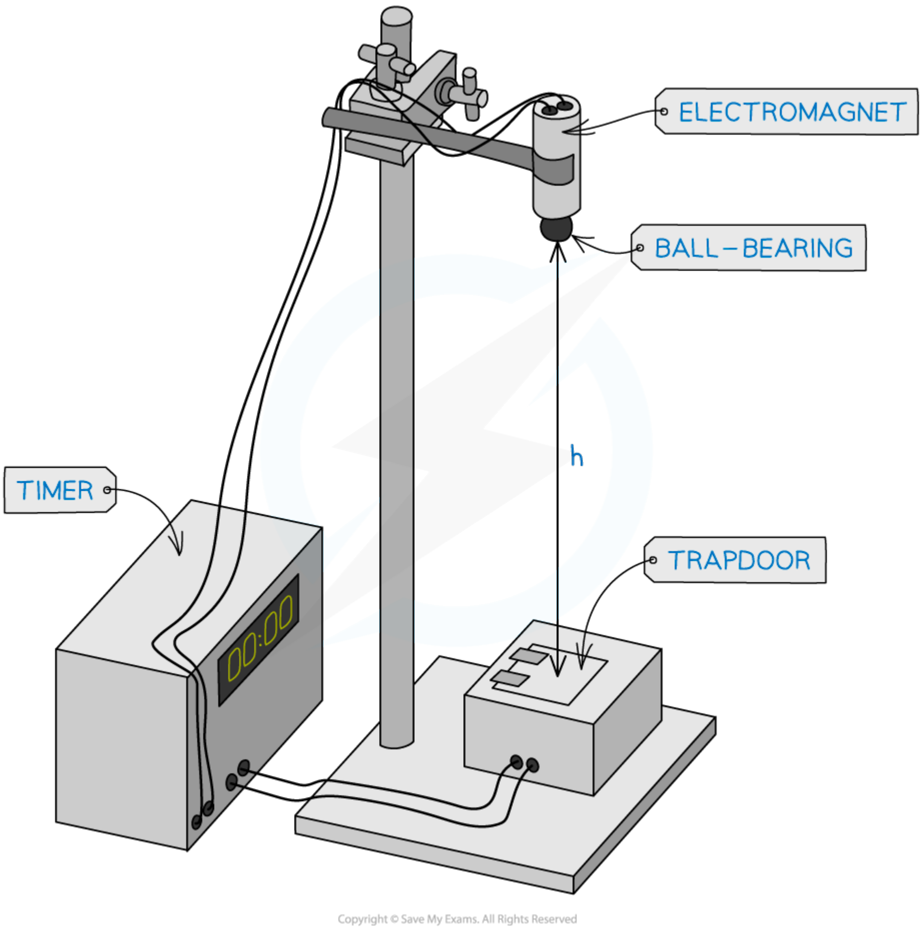
2.1.8 ACCELERATION OF FREE FALL EXPERIMENT

Acceleration of Free Fall Experiment

- A common experiment to determine acceleration of a falling object which can be carried out in the lab

Apparatus

- Metre rule, ball bearing, electromagnet, electronic timer, trapdoor



Apparatus used to measure g

2. Kinematics

YOUR NOTES
⬇

Method

- When the current to the magnet switches off, the ball drops and the timer starts
- When the ball hits the trapdoor, the timer stops
- The reading on the timer indicates the time it takes for the ball to fall a distance, *h*
- This procedure is repeated several times for different values of *h*, in order to reduce random error
- The distance, *h*, can be measured using a metre rule as it would be preferable to use for distances between 20 cm – 1 m

Analysing data

- To find *g* , use the same steps as in the problem solving section
- The known quantities are
    - Displacement $s = h$
    - Time taken = t
    - Initial velocity $u = 0$
    - Acceleration $a = g$
- The equation that links these quantities is
    - $s = ut + ½ at^2$
    - $h = ½ gt^2$
- Using this equation, deduce *g* from the gradient of the graph of *h* against t^2

Sources of error

- **Systematic error:** residue magnetism after the electromagnet is switched off may cause the time to be recorded as longer than it should be
- **Random error:** large uncertainty in distance from using a metre rule with a precision of 1mm, or from parallax error

2. Kinematics

YOUR NOTES
⬇

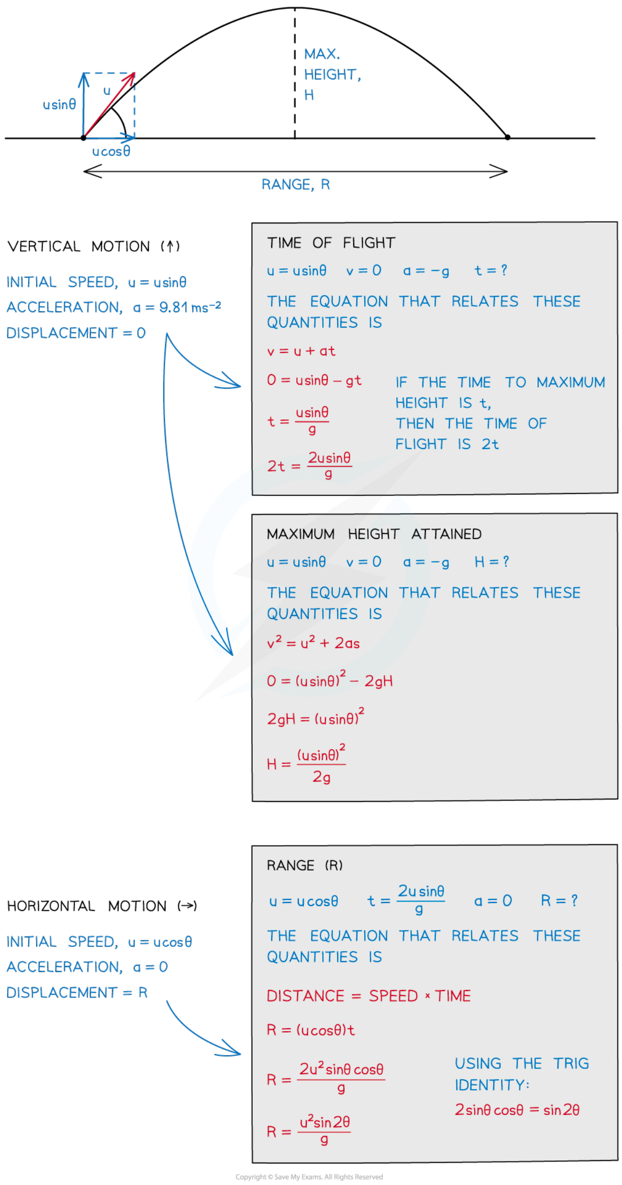
2.1.9 PROJECTILE MOTION

Projectile Motion

- The trajectory of an object undergoing projectile motion consists of a **vertical** component and a **horizontal** component
  - These need to be evaluated separately
- Some key terms to know, and how to calculate them, are:
  - **Time of flight:** how long the projectile is in the air
  - **Maximum height attained:** the height at which the projectile is momentarily at rest
  - **Range:** the horizontal distance travelled by the projectile

2. Kinematics

YOUR NOTES
⬇



How to find the time of flight, maximum height and range

2. Kinematics

YOUR NOTES

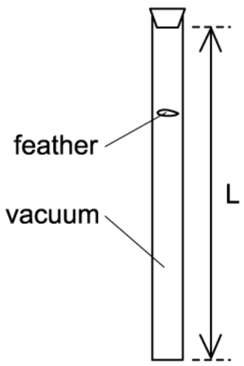
- **Remember:** the only force acting on the projectile, after it has been released, is **gravity**
- There are three possible scenarios for projectile motion:
  - **Vertical** projection
  - **Horizontal** projection
  - **Projection** at an **angle**
- Let's consider each in turn:

Worked Example 1

A science museum designed an experiment to show the fall of a feather in a vertical glass vacuum tube.

The time of fall from rest is 0.5 s.

What it the length of the tube, L?



IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION. FIRST WE MUST LIST THE KNOWN VARIABLES.

$a = 9.81\ \text{ms}^{-2}$ $u = 0$ $t = 0.5\ \text{s}$ $L = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$L = \frac{1}{2}gt^2$$

$$L = \frac{1}{2} \times 9.81 \times 0.5^2 = 1.2\ \text{m}$$

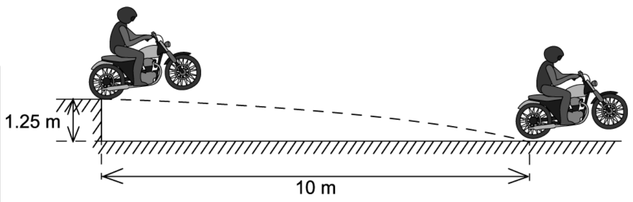
How to calculate vertical projection (free fall)

2. Kinematics

YOUR NOTES

Worked Example 2

A motorcycle stunt-rider moving horizontally takes off from a point 1.25 m above the ground, landing 10 m away as shown. What was the speed at take-off?



IN THIS PROBLEM, WE NEED TO CONSIDER BOTH VERTICAL AND HORIZONTAL MOTION. LET'S CONSIDER THE VERTICAL MOTION FIRST. THE KNOWN VARIABLES ARE

$s = 1.25\ \text{m}$ $\quad a = 9.81\ \text{ms}^{-2}$ $\quad u = 0$ $\quad t = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2s}{g}}$$

$$t = \sqrt{\frac{2 \times 1.25}{9.81}} = 0.5\text{s}$$

NEXT LET'S CONSIDER THE HORIZONTAL MOTION. THE KNOWN VARIABLES ARE

$s = 10\ \text{m}$ $\quad a = 0$ $\quad t = 0.5\ \text{s}$ $\quad u = ?$

SINCE THE ACCELERATION IS ZERO, WE CAN USE

$$\text{VELOCITY} = \frac{\text{DISPLACEMENT}}{\text{TIME}}$$

$$v = \frac{10}{0.5} = 20\ \text{ms}^{-1}$$

How to calculate horizontal projection

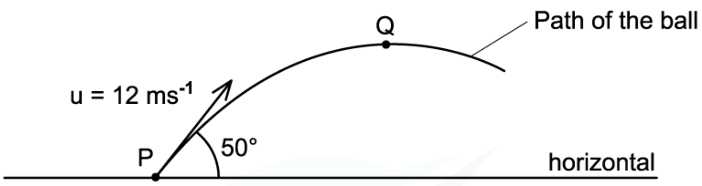
2. Kinematics

YOUR NOTES
⬇

Worked Example 3

A ball is thrown from a point P with an initial velocity u of 12 ms^{-1} at 50° to the horizontal.

What is the value of the maximum height at Q?



IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION UP TO THE POINT Q. FIRST WE MUST LIST THE KNOWN VARIABLES

$u = 12\sin(50)$ $\quad a = -9.81\ ms^{-2}$ $\quad v = 0$ $\quad H = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$v^2 = u^2 + 2as$$

$$2as = v^2 - u^2$$

$$s = \frac{(v^2 - u^2)}{2a}$$

$$H = \frac{0 - (12\sin 50)^2}{2 \times (-9.81)}$$

$$H = \frac{(12\sin 50)^2}{19.62} = 4.3\ m$$

How to calculate projection at an angle

2. Kinematics

YOUR NOTES
⬇

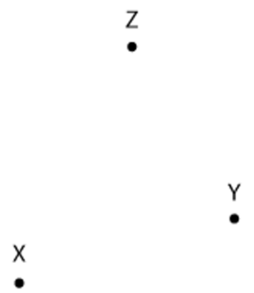
Exam Tip

Make sure you don't make these common mistakes:

- Forgetting that deceleration is negative as the object rises
- Confusing the direction of sin θ and cos θ
- Not converting units (mm, cm, km etc.) to metres

Exam Question: Easy

An object moves directly from X to Z



In a shorter time, a second object moves from X to Y to Z.

Which statement about the two objects is correct for the journey from X to Z?

A they have the same average speed

B they have the same average velocity

C they have the same displacement

D they travel the same distance

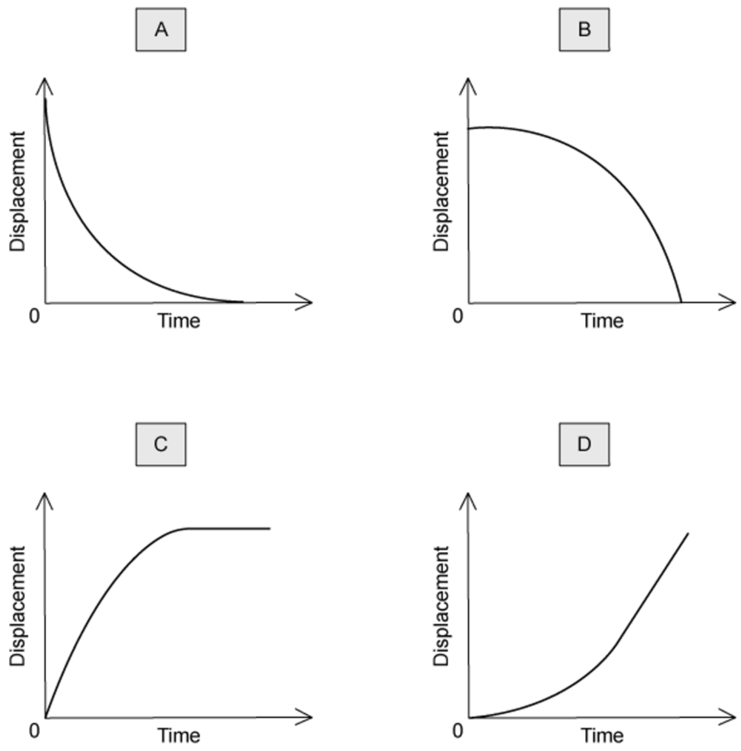
2. Kinematics

YOUR NOTES

Exam Question: Medium

A sphere is released and falls. Its initial acceleration reduces until it eventually begins to travel at constant terminal velocity.

Which displacement-time graph best represents the motion of the sphere?



2. Kinematics

YOUR NOTES

Exam Question: Hard

A body having uniform acceleration *a* increases its velocity from *u* to *v* in time *t*.

Which expression would **not** give a correct value for the body's displacement during time *t*?

A $ut + \frac{1}{2}at^2$

B $vt - \frac{1}{2}at^2$

C $\frac{(v+u)(v-u)}{2a}$

D $\frac{(v-u)t}{2}$

> CHECK YOUR ANSWERS AT SAVEMYEXAMS.CO.UK

3. Dynamics

YOUR NOTES

CONTENTS

3.1 NEWTON'S LAWS OF MOTION

3.1.1 MASS & WEIGHT

What is Mass?

- Mass is the measure of the amount of matter in an object
- Consequently, this is the property of an object that resists change in motion
- The greater the mass of a body, the smaller the change produced by an applied force
- The SI unit for mass is the **kilogram** (kg)

3. Dynamics

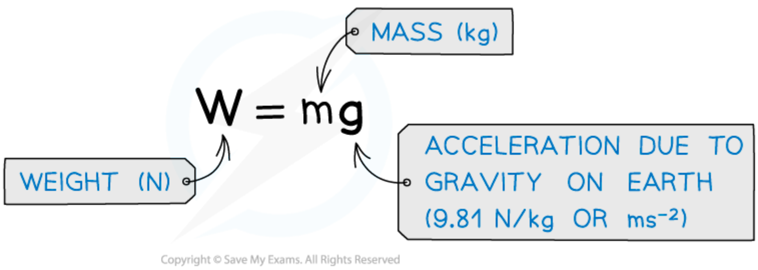
YOUR NOTES
⬇

Exam Tip

- Since mass is measured in **kilograms** in Physics, if it is given in grams make sure to convert to kg by dividing the value by 1000
- It is a common misconception that mass and weight are the same, but they are in fact **very different**
- Weight is the force of gravity acting upon an object
  - Weight is a vector quantity
- Mass is the amount of matter contained in the object
  - Mass is a scalar quantity

Weight

- Weight is the effect of a gravitational field on a mass
- Since it is a force on an object due to the pull of gravity, it is measured in **Newtons** *(N)* and is a vector quantity
- The weight of a body is equal to the product of its mass (*m*) and the acceleration of free fall (*g*)



Weight equation

3. Dynamics

YOUR NOTES
⬇

- g is the acceleration due to gravity or the gravitational field strength
- On Earth, this is **9.81 m s^{-2}** (or N kg^{-1})

Free fall

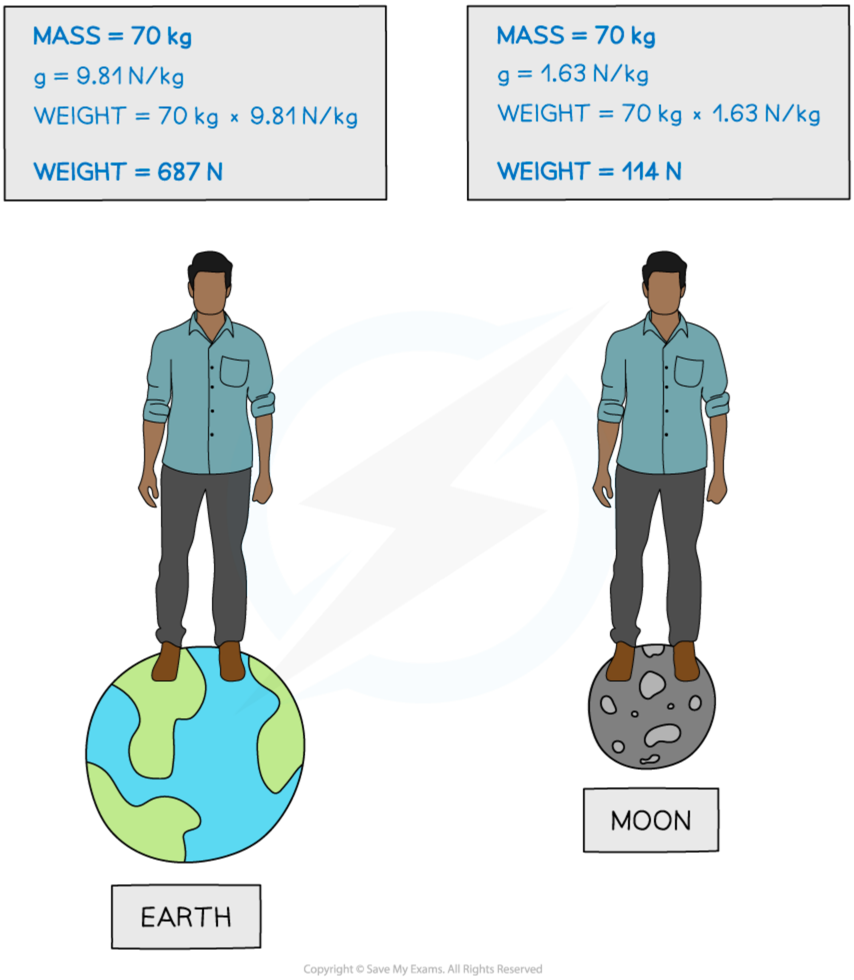
- An object in free fall is falling solely under the influence of gravity
- On Earth, all free-falling objects accelerate towards Earth at a rate of **9.81 m s^{-2}**
- In the absence of air resistance, all bodies near the Earth fall with the same acceleration regardless of their mass

Mass v Weight

- An object's mass always remains the same, however, its weight will differ depending on the strength of the gravitational field on different planets
- For example, the gravitational field strength on the Moon is **1.63 N kg^{-1}**, meaning an object's weight will be about **6 times** less than on Earth

3. Dynamics

YOUR NOTES
⬇

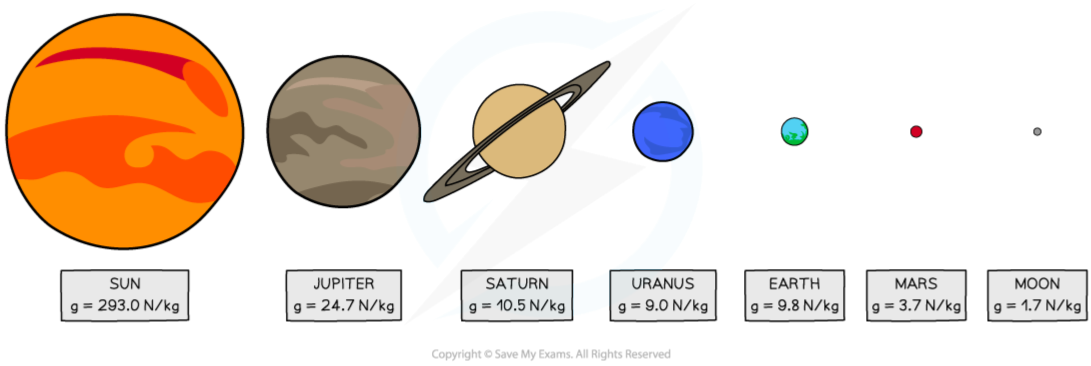


On the moon, your mass will stay the same but your weight will be much lower

3. Dynamics

YOUR NOTES
⬇

- Although you only need to memorise *g* on Earth, its value on other planets in our solar system is given in the diagram below. Notice how much this varies according to the size of the planet



Gravitational field strength of the planets in our solar system

3. Dynamics

YOUR NOTES

Worked example

? The acceleration due to gravity on the moon is $\frac{1}{6}$ of that on Earth. If the weight of a space probe on the moon is 491N, calculate its mass.

STEP 1 EQUATION FOR WEIGHT

$$W = mg$$

STEP 2 REARRANGE FOR MASS m

$$m = \frac{W}{g} = \frac{491}{g}$$

STEP 3 FIND g FOR THE MOON

$$g = \frac{g_{EARTH}}{6} = \frac{9.81}{6} = 1.64 \text{ Nkg}^{-1}$$

STEP 4 SUBSTITUTE VALUE IN MASS EQUATION

$$m = \frac{491}{1.64} = 300 \text{ kg}$$

Calculating mass of a space probe

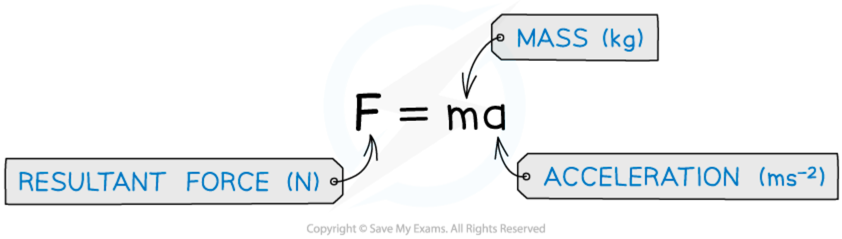
3. Dynamics

YOUR NOTES

3.1.2 FORCE & ACCELERATION

Force & Acceleration

- As stated on the previous page, Newton's Second Law of Motion tells us that objects will accelerate if there is a resultant force acting upon them
- This acceleration will be in the same direction as this resultant force



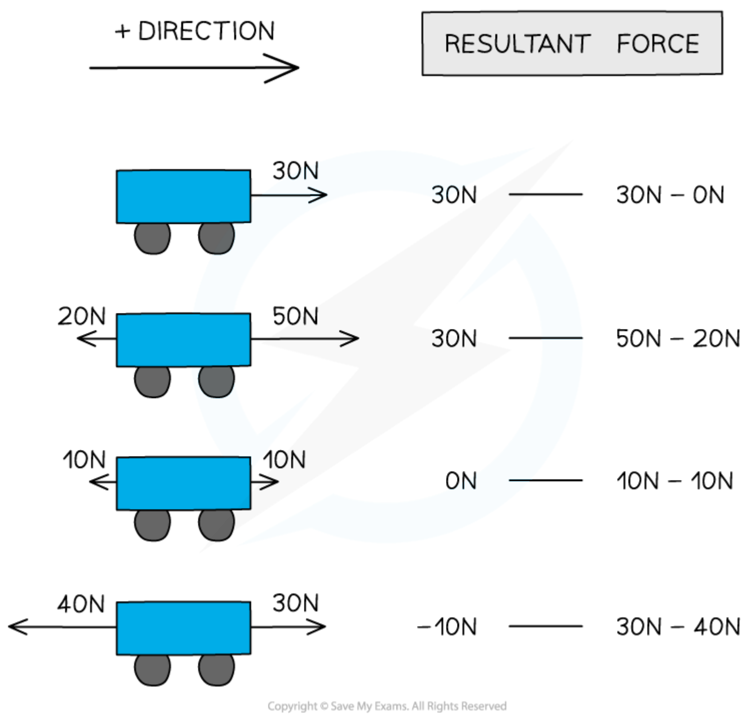
Newton's Second law equation

3. Dynamics

YOUR NOTES

Resultant Force

- Since force is a vector, every force on a body has a magnitude and direction
- The resultant force is therefore the vector sum of all the forces acting on the body. The direction is given by either the positive or negative direction as shown in the examples below



Resultant forces on a body

- The resultant force could also be at an angle, in which case addition of vectors is used to find the magnitude and direction of the resultant force.
    - For more details on this, have a look at the page on "Scalars & Vectors"

Acceleration

- Given the mass, Newton's Second Law means you can find the acceleration of an object
- Since acceleration is also a vector, it can be either positive or negative depending on the direction of the resultant force
- Negative acceleration is deceleration
- An object may continue in the same direction however with a resultant force in the opposite direction to its motion, it will slow down and eventually come to a stop

3. Dynamics

YOUR NOTES

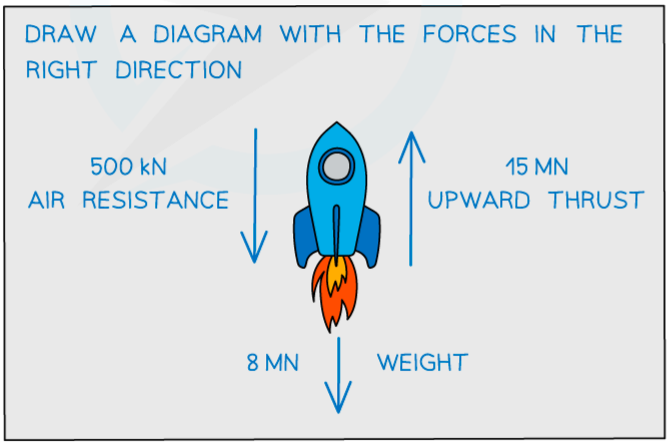
Worked example

A rocket produces an upward thrust of 15 MN and has a weight of 8 MN.

A. When in flight, the force due to air resistance is 500 kN. What is the resultant force on the rocket?

B. The mass of the rocket is 0.8×10^5 kg. Calculate the acceleration of the rocket and the direction its going in.

A. STEP 1



3. Dynamics

YOUR NOTES ⬇

STEP 2 — CALCULATE THE RESULTANT FORCE ON THE ROCKET

$$F = \underbrace{15\ \text{MN}}_{\text{UPWARD FORCES}} - \underbrace{(500\ \text{kN} + 8\ \text{MN})}_{\text{DOWNWARD FORCES}}$$

UNIT CONVERSIONS: $1\ \text{kN} = 1 \times 10^3\ \text{N}$ $\quad 1\ \text{MN} = 1 \times 10^6\ \text{N}$

STEP 3 — CONVERT ALL VALUES TO THE SAME UNITS (NEWTONS)

$$F = 15 \times 10^6\ \text{N} - (500 \times 10^3\ \text{N} + 8 \times 10^6\ \text{N})$$

$$F = 6.5 \times 10^6\ \text{N}$$

$F = 6.5\ \text{MN}$ UPWARDS

IN THE POSITIVE DIRECTION

B. STEP 1 — NEWTONS SECOND LAW

$$F = ma$$

STEP 2 — REARRANGE FOR ACCELERATION a

$$a = \frac{F}{m}$$

STEP 3 — SUBSTITUTE IN VALUES FOR F AND m

$$a = \frac{6.5 \times 10^6\ \text{N}}{8.0 \times 10^5\ \text{kg}} = 8.1\ \text{ms}^{-1}$$ UPWARDS

ACCELERATION IS ALWAYS IN THE SAME DIRECTION AS THE RESULTANT FORCE

Resultant force and acceleration on a rocket

3. Dynamics

YOUR NOTES
⬇

Exam Tip

The direction you consider positive is your choice, as long the signs of the numbers (positive or negative) are consistent with this throughout the question

It is a general rule to consider the direction of motion the object is travelling in as positive. Therefore all vectors in the direction of motion will be positive and opposing vectors such as drag forces, are negative.

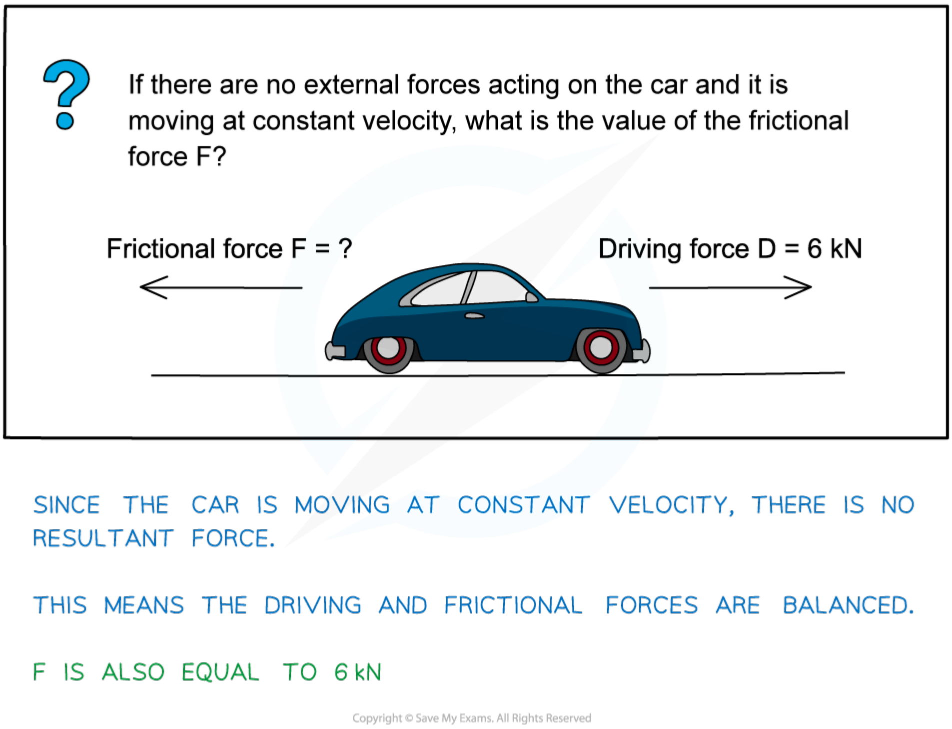
3. Dynamics

YOUR NOTES

3.1.3 NEWTON'S LAWS OF MOTION

Newton's Three Laws of Motion

- **Newton’s First Law:** A body will remain at rest or move with constant velocity unless acted on by a resultant force

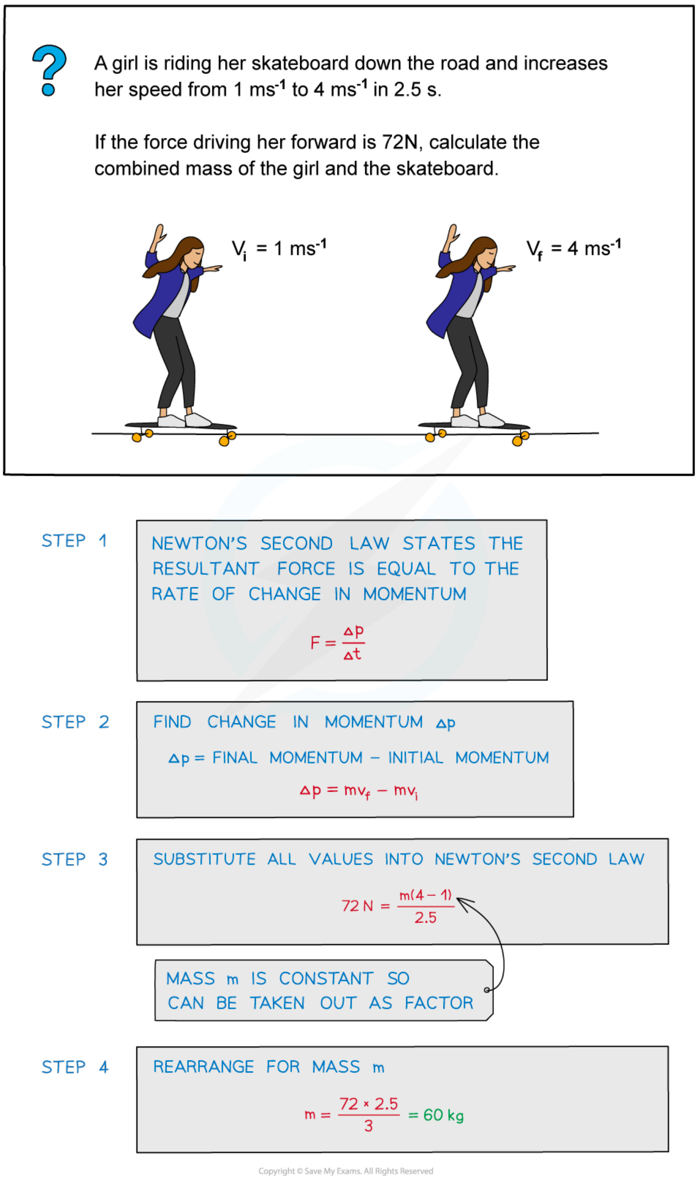


Newton’s First Law on a car at constant velocity

3. Dynamics

YOUR NOTES
⬇

- **Newton's Second Law:** A resultant force acting on a body will cause a change in momentum in the direction of the force. The rate of change in momentum is proportional to the magnitude of the force
- This can also be written as ***F = ma***

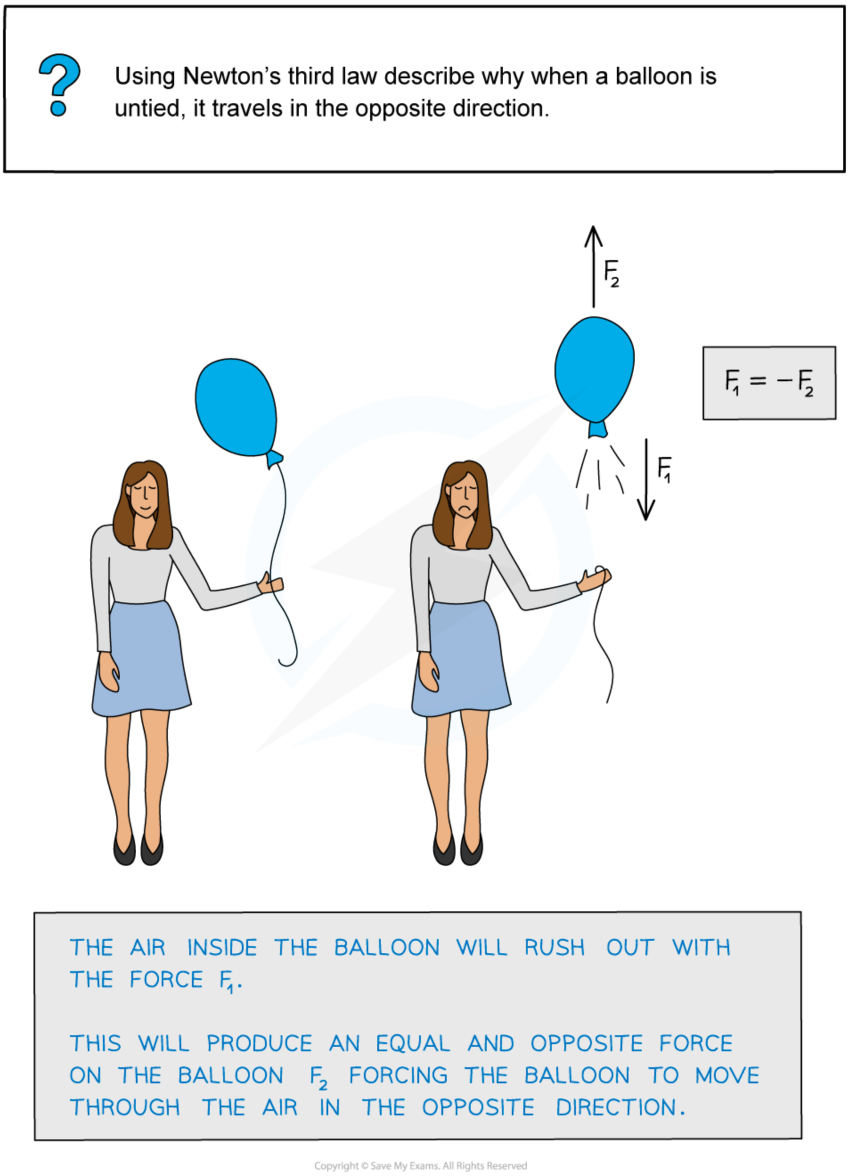


Using Newton's Second law to find the mass of an accelerating skateboarder

3. Dynamics

YOUR NOTES
⬇

- **Newton's Third Law:** If body **A** exerts a force on body **B,** then body **B** will exert a force on body **A** of equal magnitude but in the opposite direction
  - Newton's Third Law force pairs must act on **different** objects
  - Newton's Third Law force pairs must also be of the **same type** e.g. gravitational or frictional



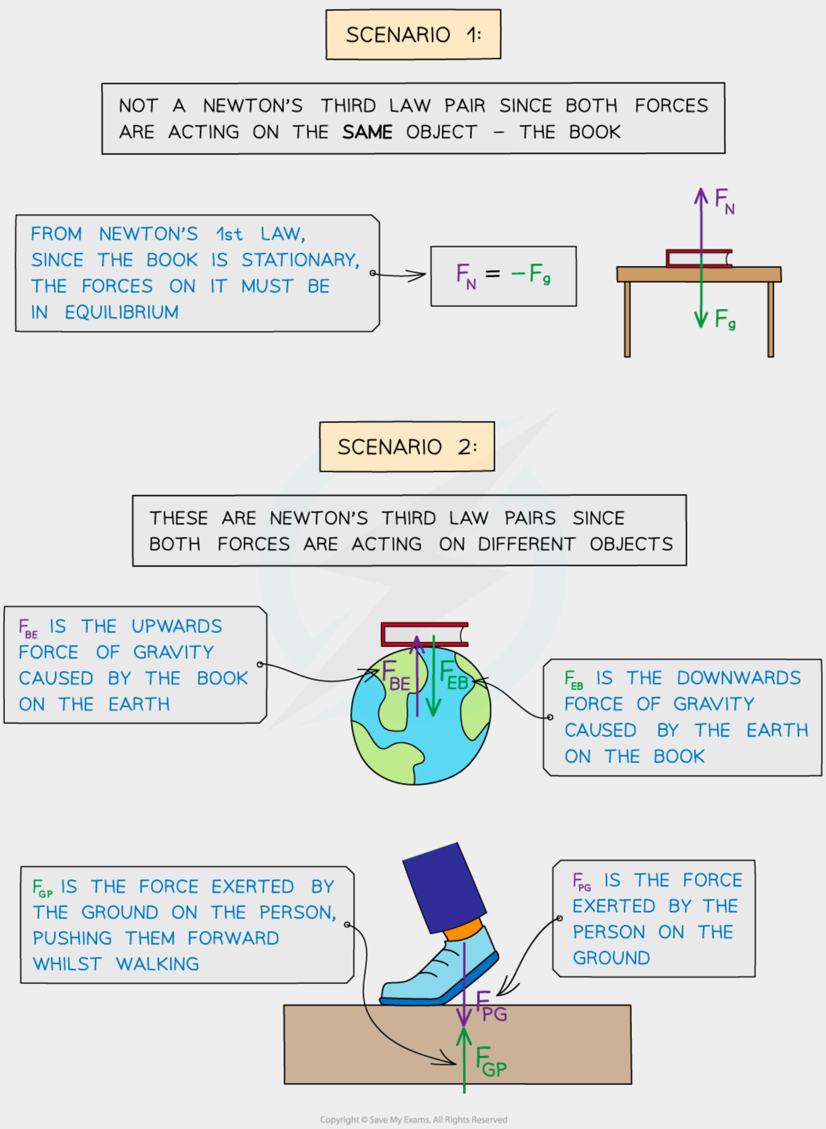
Newton's Third Law to describe the motion of an untied balloon

3. Dynamics

YOUR NOTES
⬇

Exam Tip

You may have heard Newton's Third Law as: 'For every action is an equal and opposite reaction'. However, try and avoid using this definition since it is unclear on **what** the forces are acting on and can be misleading.



Newton's Third Law force pairs are only those that act on different objects

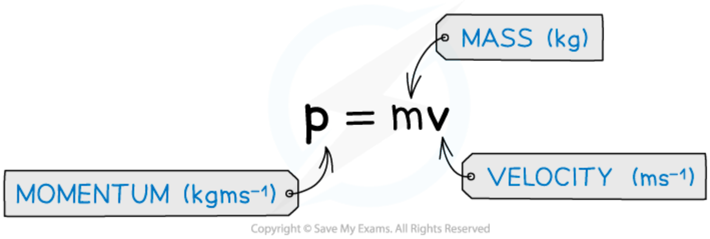
3. Dynamics

YOUR NOTES

3.1.4 LINEAR MOMENTUM

Linear Momentum

- Linear momentum (p) is defined as the product of mass and velocity



Momentum is the product of mass and velocity

- Momentum is a vector quantity – it has both a magnitude and a direction
- This means it can have a negative or positive value
  - If an object travelling to the right has positive momentum, an object travelling to the left (in the opposite direction) has a negative momentum
- The SI unit for momentum is **kg m s**$^{-1}$

3. Dynamics

YOUR NOTES
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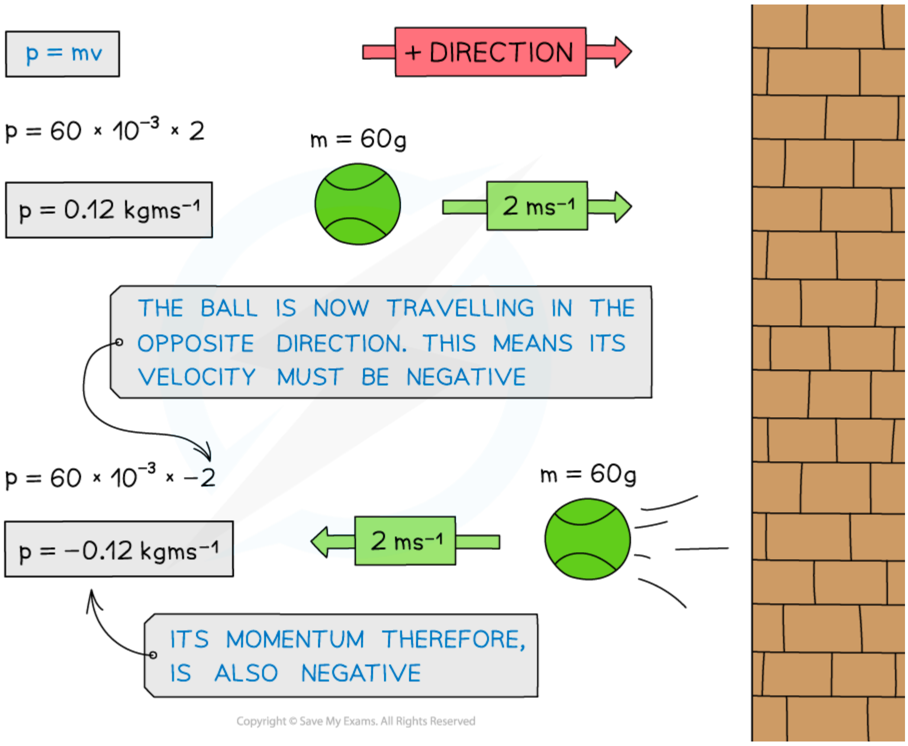
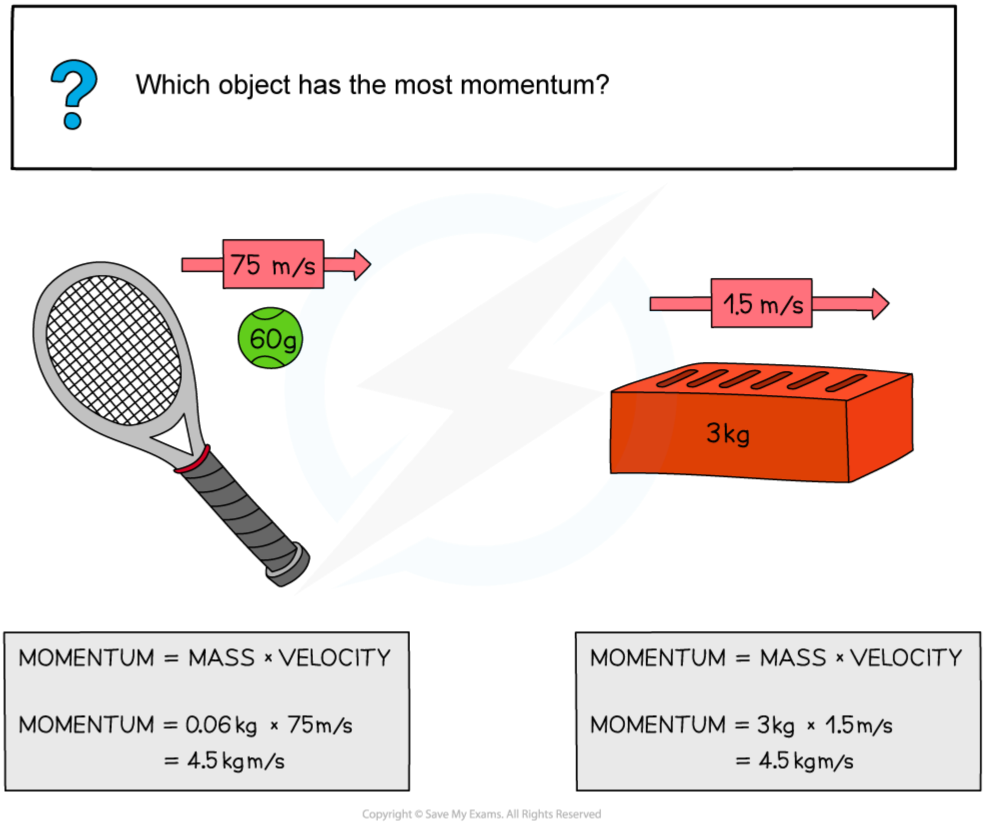


When the ball is travelling in the opposite direction, its velocity is negative. Since momentum = mass × velocity, its momentum is also negative

3. Dynamics

YOUR NOTES
⬇

Worked Example



- Both the tennis ball and the brick have the same momentum
- Even though the brick is much heavier than the ball, the ball is travelling much faster than the brick
- This means that on impact, they would both exert a similar force (depending on the time it takes for each to come to rest)

3. Dynamics

YOUR NOTES

Exam Tip

Since momentum is in **kg m s^{-1}**:

- If the mass is given in grams, make sure to convert to kg by dividing the value by 1000.
  - If the velocity is given in km s^{-1}, make sure to convert to m s^{-1} by multiplying the value by 1000
- The direction you consider positive is your choice, as long the signs of the numbers (positive or negative) are consistent with this throughout the question

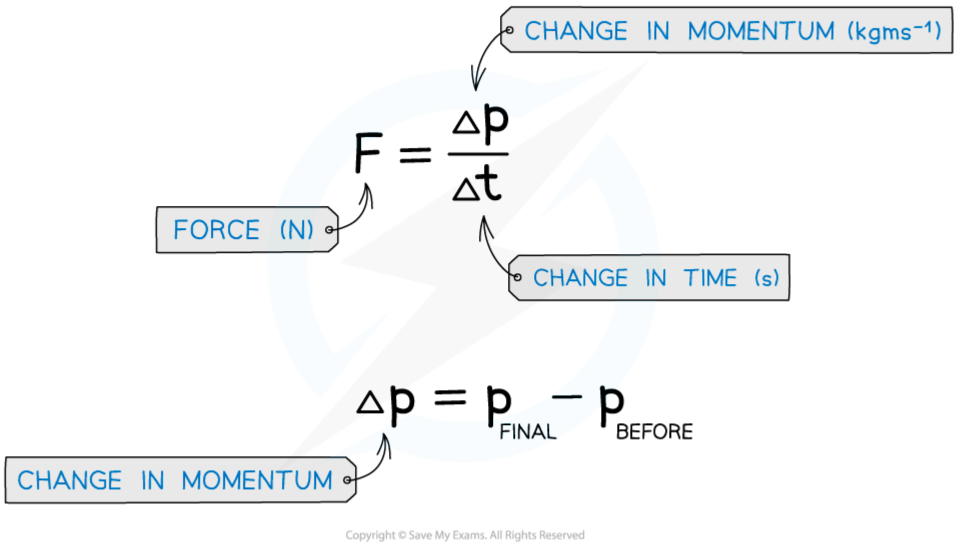
3. Dynamics

YOUR NOTES

3.1.5 FORCE & MOMENTUM

Force & Momentum

- Force is defined as the **rate of change of momentum** on a body



Force is equal to the rate of change in momentum

3. Dynamics

YOUR NOTES

- The change in momentum is defined as the final momentum minus the initial momentum:

$$p_{\text{final}} - p_{\text{initial}}$$

- Force and momentum are **vectors** so they can be either positive or negative values

?

A car of mass 1500 kg hits a wall at an initial velocity of 15 ms^{-1}.
It then rebounds off the wall at 5 ms^{-1} and comes to rest after 3.0 s.

Calculate the average force experienced by the car.

STEP 1

FORCE IS EQUAL TO THE RATE OF CHANGE IN MOMENTUM

$$F = \frac{\Delta p}{\Delta t}$$

STEP 2

CHANGE IN MOMENTUM

Δp = FINAL MOMENTUM − INITIAL MOMENTUM

STEP 3

INITIAL MOMENTUM

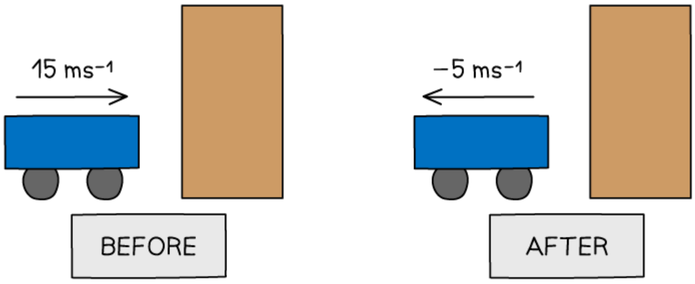
INITIAL MOMENTUM = MASS × INITIAL VELOCITY

$$p_i = m \times v_i$$
$$= 1500 \text{ kg} \times 15 \text{ ms}^{-1}$$

$$p_i = 22500 \text{ kgms}^{-1}$$

3. Dynamics

YOUR NOTES



STEP 4

FINAL MOMENTUM

FINAL MOMENTUM = MASS × FINAL VELOCITY

$$P_f = m \times v_f$$
$$= 1500\text{ kg} \times -5\text{ ms}^{-1}$$
$$P_f = -7500\text{ kgms}^{-1}$$

STEP 5

CALCULATE CHANGE IN MOMENTUM Δp

$$\Delta p = -7500 - 22500 = -30000\text{ kgms}^{-1}$$

STEP 6

SUBSTITUTE THIS VALUE BACK INTO THE FORCE EQUATION

$$F = \frac{\Delta p}{\Delta t} = \frac{-30000}{3} = -10000\text{N}$$

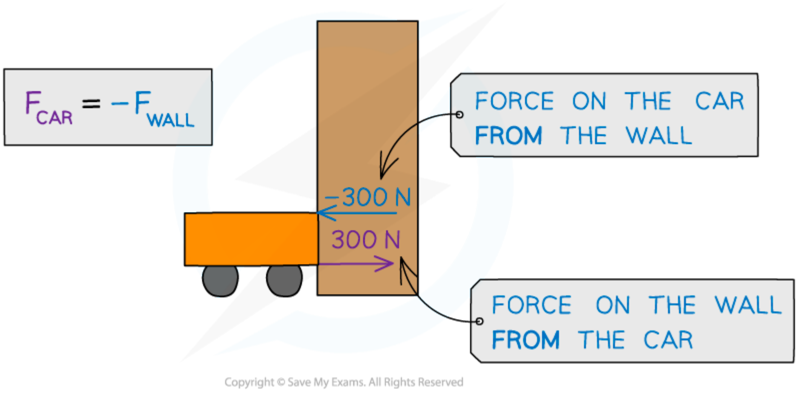
Force is rate of change of momentum equation on a car hitting a wall

3. Dynamics

YOUR NOTES

Direction of Forces

- The force that is equal to the rate of change of momentum is still the **resultant force**
- A force on an object will be negative if it is directed in the opposite motion to its initial velocity. This means that the force is **produced by** the object it has collided with



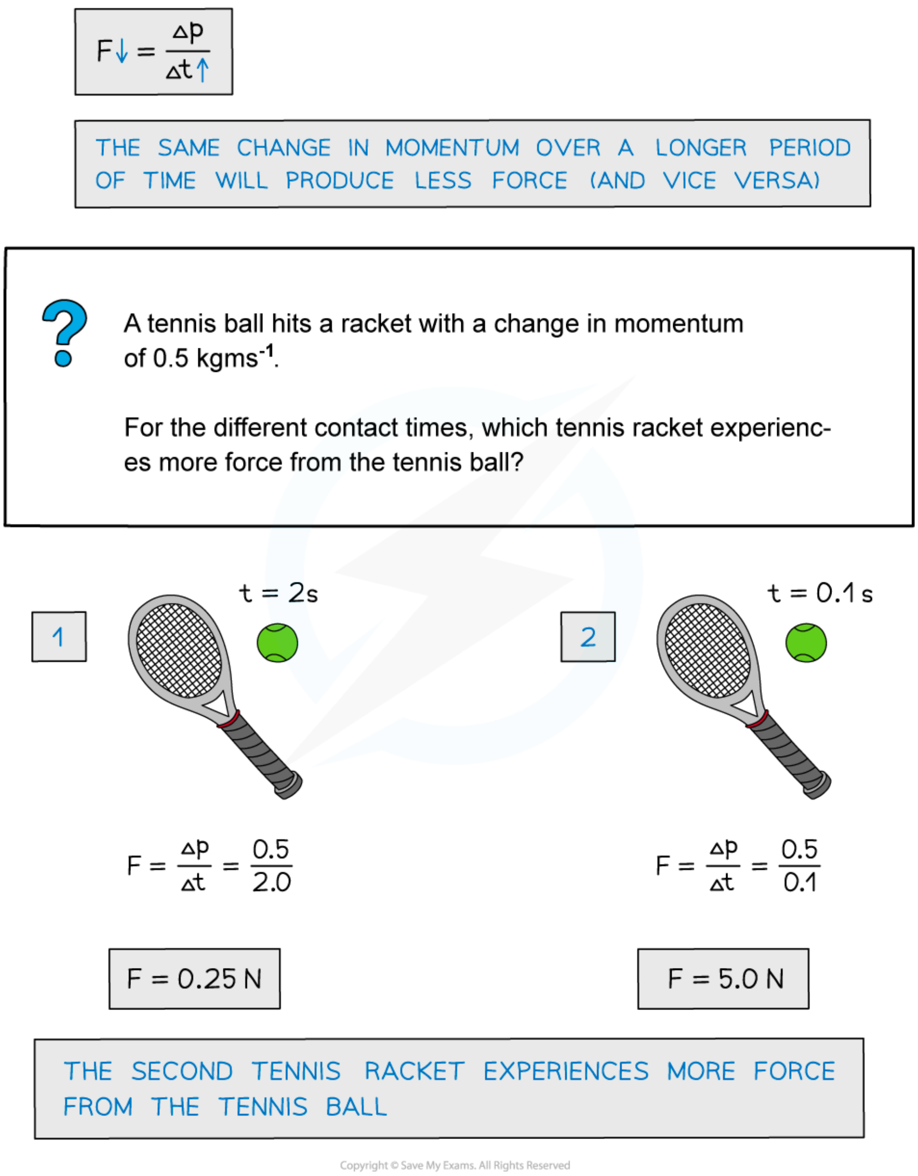
$$F_{car} = -F_{wall}$$

- The diagram shows a car colliding with a wall
- It is the **wall that produces** a force of -300N on the car
- Due to Newton's Third Law (see "Newton's Laws of Motion"), the car also produces a force of 300N back onto the wall

3. Dynamics

Maths tip

- 'Rate of change' describes how one variable changes with respect to another. In maths, how fast something changes with **time** is represented as dividing by **Δ*t*** (e.g. acceleration is the rate of change in velocity)
- More specifically, **Δ*t*** is used for finite and quantifiable changes such as the difference in time between two events



The same change in momentum over a longer period of time will produce less force

3. Dynamics

Exam Tip

In an exam question, carefully consider what produces the force(s) acting. Look out for words like '**from'** and '**acting on'** to determine this and don't be afraid to draw a force diagram to figure out what is going on.

3. Dynamics

YOUR NOTES
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3.1.6 DRAG FORCE & AIR RESISTANCE

Drag Forces

- Drag forces are forces acting the **opposite** direction to an object moving through a fluid (either gas or liquid)
- Examples of drag forces are **friction** and **air resistance**
- A key component of drag forces is it increases with the speed of the object. This is shown in the diagram below:

3. Dynamics

YOUR NOTES
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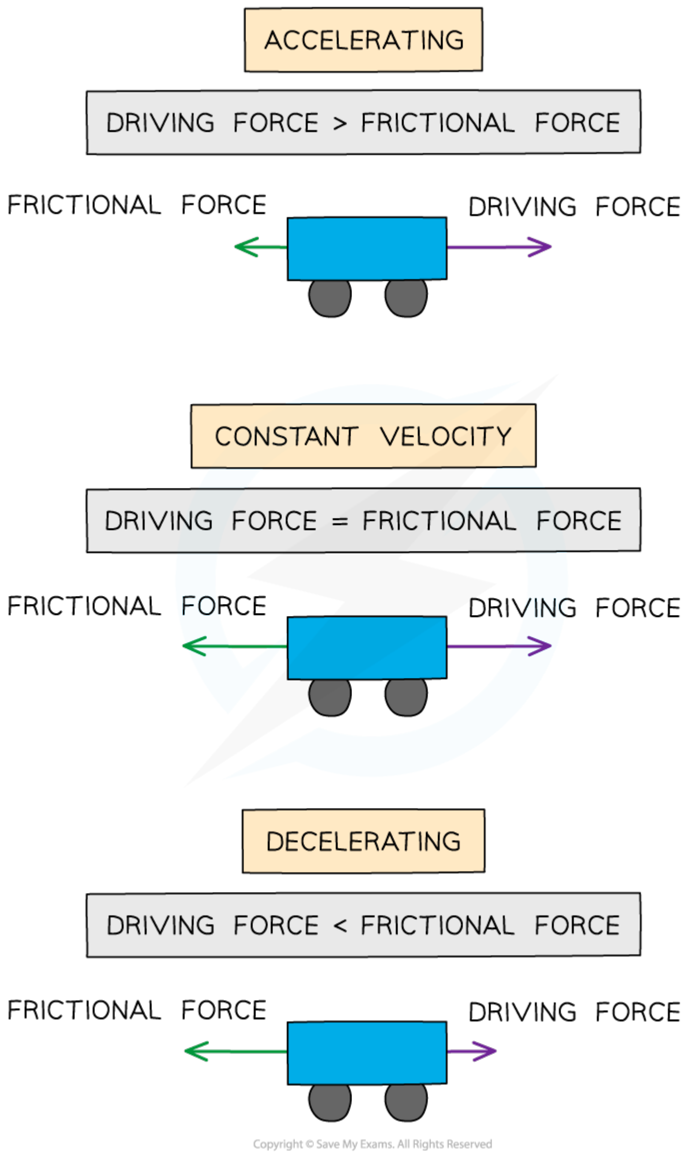
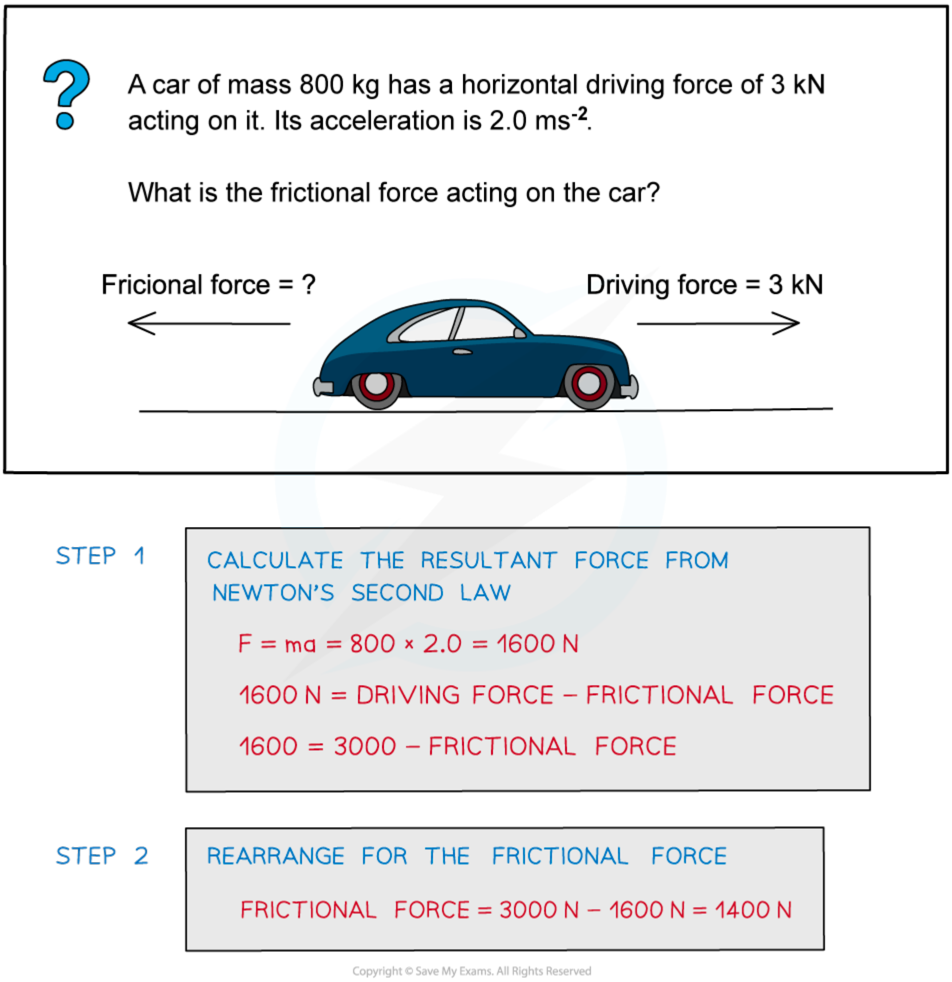


Frictional forces on a car increase with its speed

3. Dynamics

YOUR NOTES

Worked Example



Exam Tip

Remember to consider drag forces in your calculation for the resultant force. More details of this are in the notes “Force and acceleration”.

3. Dynamics

YOUR NOTES

Air Resistance

- Air resistance is an example of a drag force which objects experience when moving through the air
- At a walking pace, a person rarely experiences the effects of air resistance
- However, a person swimming at the same pace uses up much more energy – this is because air is 800 times less dense than water
- Air resistance depends on the **shape** of the body (object) and the **speed** it's travelling
- Since drag force increases with speed, air resistance becomes important when objects move faster

A racing cyclist adopts a more streamline posture to reduce the effects of air resistance. The cycle, clothing and helmet are designed to allow them to go as fast as possible

Exam Tip

If a question considers air resistance to be **'negligible'** this means in that question, air resistance is taken to be so small it will not make a difference to the motion of the body. You can take this to mean there are no drag forces acting on the body.

3. Dynamics

YOUR NOTES
⬇

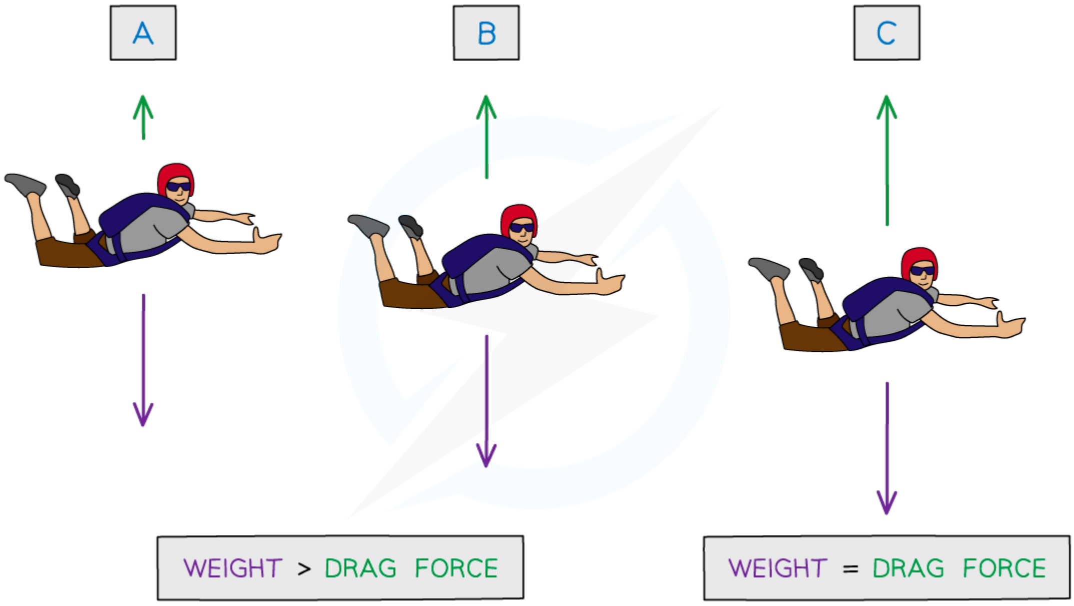
3.1.7 TERMINAL VELOCITY

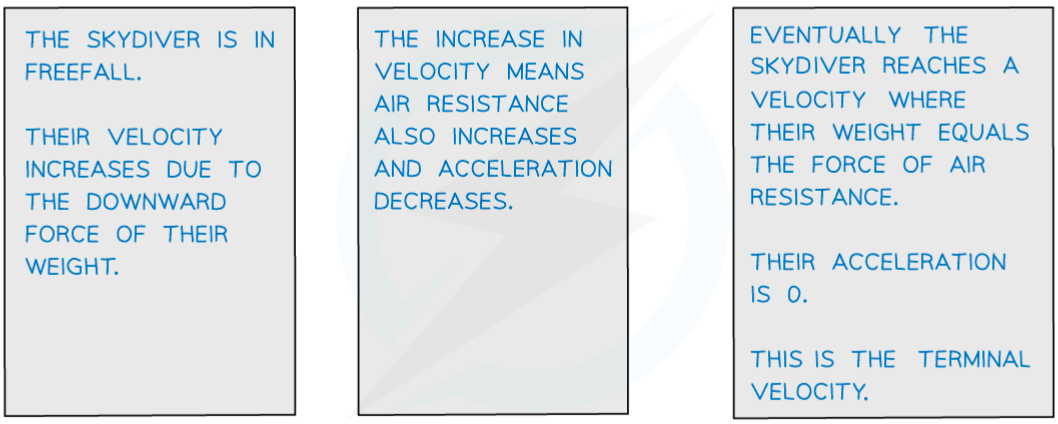
Terminal Velocity

- For a body in free fall, the only force acting is its weight and its acceleration *g* is only due to gravity.
- The drag force increases as the body accelerates
    - This increase in velocity means the drag force also increases
- Due to Newton's Second Law, this means the resultant force and therefore acceleration decreases (recall ***F = ma***)
- When the drag force is equal to the gravitational pull on the body, the body will no longer accelerate and will fall at a constant velocity
- This velocity is called the **terminal velocity**

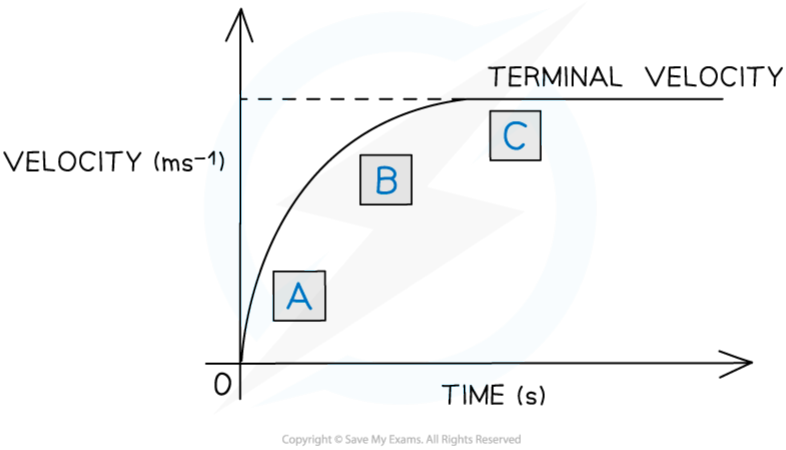
3. Dynamics

YOUR NOTES





3. Dynamics



A skydiver in freefall reaching terminal velocity

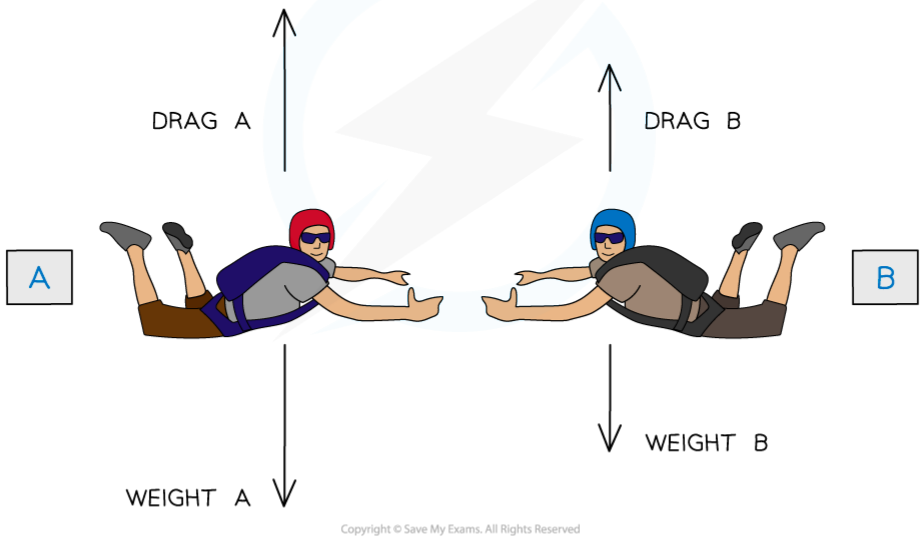
- The graph shows how the velocity of the skydiver varies with time
- Since the acceleration is equal to the gradient of a velocity-time graph, the acceleration decreases and eventually becomes zero when terminal velocity is reached

3. Dynamics

YOUR NOTES
⬇

Worked example

Skydivers jump out of a plane at intervals of a few seconds. Skydivers A and B want to join up as they fall. If A is heavier than B, who should jump first?



- Skydiver B should jump first since he will take longer to reach terminal velocity
- This is because skydiver A has a higher mass, and hence, weight
- More weight means higher acceleration and hence higher speed, therefore, A will reach terminal velocity faster than B

Exam Tip

• Exam questions about terminal velocity tend to involve the motion of skydivers as they fall
• A common misconception is that skydivers move upwards when their parachutes are deployed – however, this is not the case, they are in fact **decelerating** to a lower terminal velocity
• What do you think this would look like on the graph above?

3. Dynamics

YOUR NOTES

Exam Question: Easy

A cyclist is riding at a steady speed on a level road

According to Newton's third law of motion, what is equal and opposite to the backward push of the back wheel on the road?

A the force exerted by the cyclist on the pedals

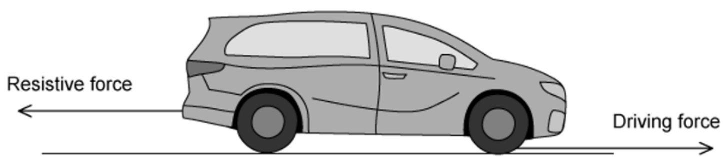
B the total air resistance and friction force

C the tension in the cycle chain

D the forward push of the road on the back wheel

Exam Question: Medium

A car has a horizontal driving force of 2.0 kN acting on it and a resistive force a quarter of this size. It has a forward horizontal acceleration of 2.0 m s^{-2}



What is the mass of the car?

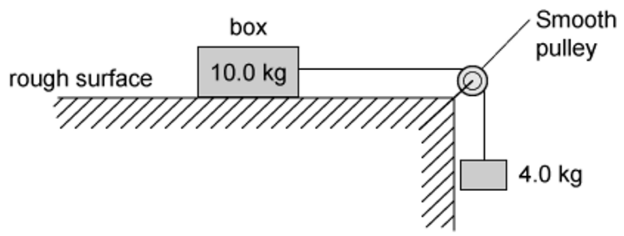
A 500 kg **B** 750 kg **C** 1250 kg **D** 2000 kg

3. Dynamics

YOUR NOTES
⬇

Exam Question: Hard

A box of mass 10.0 kg rests on a horizontal rough surface. A string attached to the box passes over a smooth pulley and supports a 4.0 kg mass at its other end.



When the box is released, a frictional force of 12.0 N acts on it.
What is the acceleration of the box?

A 1.2 m s^{-2} **B** 1.9 m s^{-2} **C** 2.7 m s^{-2} **D** 3.0 m s^{-2}

> CHECK YOUR ANSWERS AT SAVEMYEXAMS.CO.UK

3. Dynamics

YOUR NOTES

3.2 LINEAR MOMENTUM & CONSERVATION

3.2.1 CONSERVATION OF MOMENTUM

The Principle of Conservation of Momentum

- The principle of conservation of momentum is:
  - **The total momentum of a system remains constant provided no external force acts on it**
- For example if two objects collide:

the total momentum before the collision = the total momentum after the collision

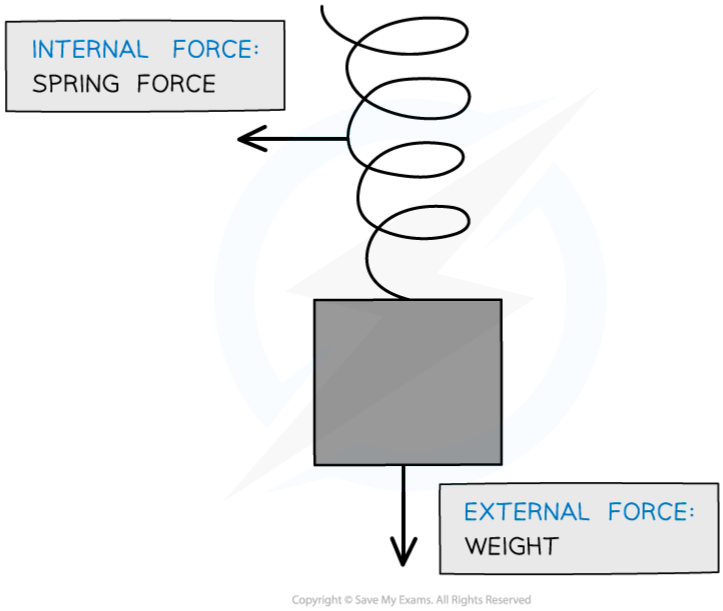
- Remember momentum is a **vector** quantity. This allows oppositely-directed vectors to cancel out so the momentum of the system as a whole is zero
- Momentum is **always conserved** over time

3. Dynamics

YOUR NOTES

External and Internal Forces

- **External forces** are forces that act on a structure from outside e.g. friction and weight
- **Internal forces** are forces exchanged by the particles in the system e.g. tension in a string
- Which forces are internal or external will depend on the system itself, as shown in the diagram below:



Internal and external forces on a mass on a spring

- You may also come across a system with no external forces being described as a '**closed**' or '**isolated**' system
- These all still refer to a system that is not affected by external forces
- For example, a swimmer diving from a boat:
    - The diver will move forward, and, to conserve momentum, the boat will move backwards
- This is because the momentum beforehand was zero and no external forces are present to affect the motion of the diver or the boat

3. Dynamics

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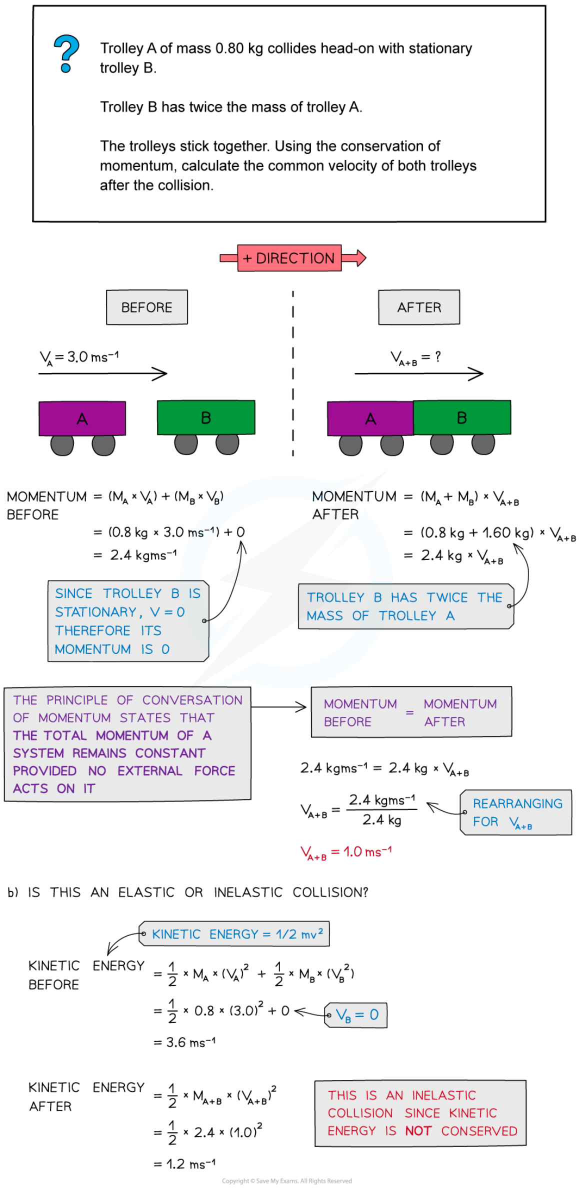
Collisions in One & Two Dimensions

One-dimensional momentum problems

- Momentum (p) is equal to: $\boldsymbol{p = m \times v}$
- Using the conversation of linear momentum, it is possible to calculate missing velocities and masses of components in the system. This is shown in the example below

3. Dynamics

YOUR NOTES
⬇



- To find out whether a collision is elastic or inelastic, compare the kinetic energy before and after the collision
  - If the kinetic energy is **conserved**, it is an **elastic collision**
  - If the kinetic energy is **not conserved**, it is an **inelastic collision**

- **Elastic collisions** are commonly those where objects colliding do not stick together and then move in opposite directions
- **Inelastic collision** are where objects collide and stick together after the collision

3. Dynamics

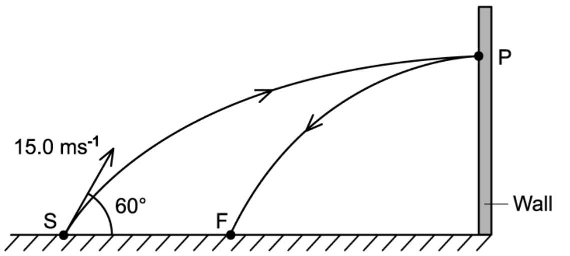
YOUR NOTES

Two-dimensional momentum problems

- Since momentum is a vector, in 2D it can be split up into its x and y components
- Review revision notes 1.3 Scalars & Vectors on how to resolve vectors

?

A ball is thrown at a vertical wall. The path of the ball is shown below



The ball is thrown from S with an initial velocity of 15.0 ms^{-1} at 60.0° to the horizontal. The mass of the ball is 60 × 10^{-3} kg and rebounds at a velocity of 4.6 ms^{-1}.
Calculate the change in momentum of the ball if it rebounds off the wall.

STEP 1

CHANGE IN MOMENTUM EQUATION

$\Delta P = m(V_f - V_i)$

STEP 2

CALCULATE INITIAL VELOCITY

CHANGE IN MOMENTUM IS ONLY DUE TO THE HORIZONTAL VELOCITIES

$V_i = 15.0\cos(60.0) = 7.5\ \text{ms}^{-1}$

STEP 3

SUBSTITUTE VALUES INTO ΔP EQUATION

$\Delta P = 60 \times 10^{-3}(-4.6 - 7.5) = -0.73\ \text{Ns}$

NEGATIVE BECAUSE THE BALL IS NOW TRAVELLING IN THE OPPOSITE DIRECTION TO ITS INITIAL VELOCITY

3. Dynamics

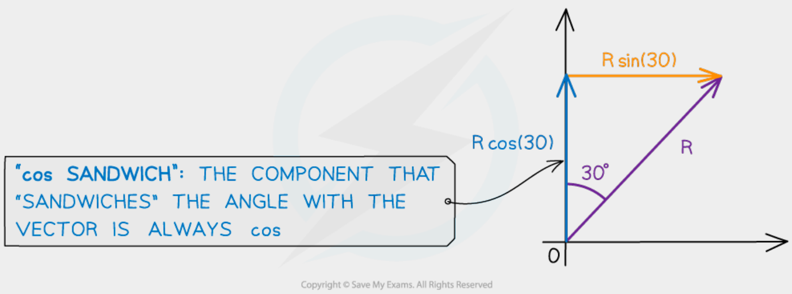
YOUR NOTES
⬇

Exam Tip

If an object is stationary or at rest, it's velocity equals **0**, therefore, the momentum and kinetic energy are also equal to 0.

When a collision occurs in which two objects are stuck together, treat the final object as a single object with a mass equal to the **sum** of the two individual objects.

In 2D problems, make sure you're confident resolving vectors. Here is a small trick to remember which component is cosine or sine of the angle for a vector **R:**



Resolving vectors with sine and cosine

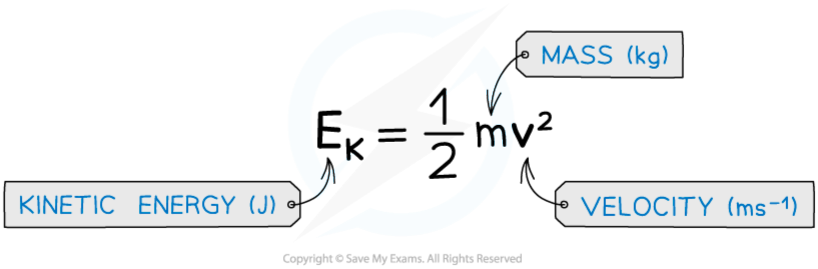
3. Dynamics

YOUR NOTES

3.2.2 ELASTIC & INELASTIC COLLISIONS

Elastic Collisions

- When two objects collide, they may spring apart retaining all of their kinetic energy. This is a perfect elastic collision
- An elastic collision is one where **kinetic energy is conserved**

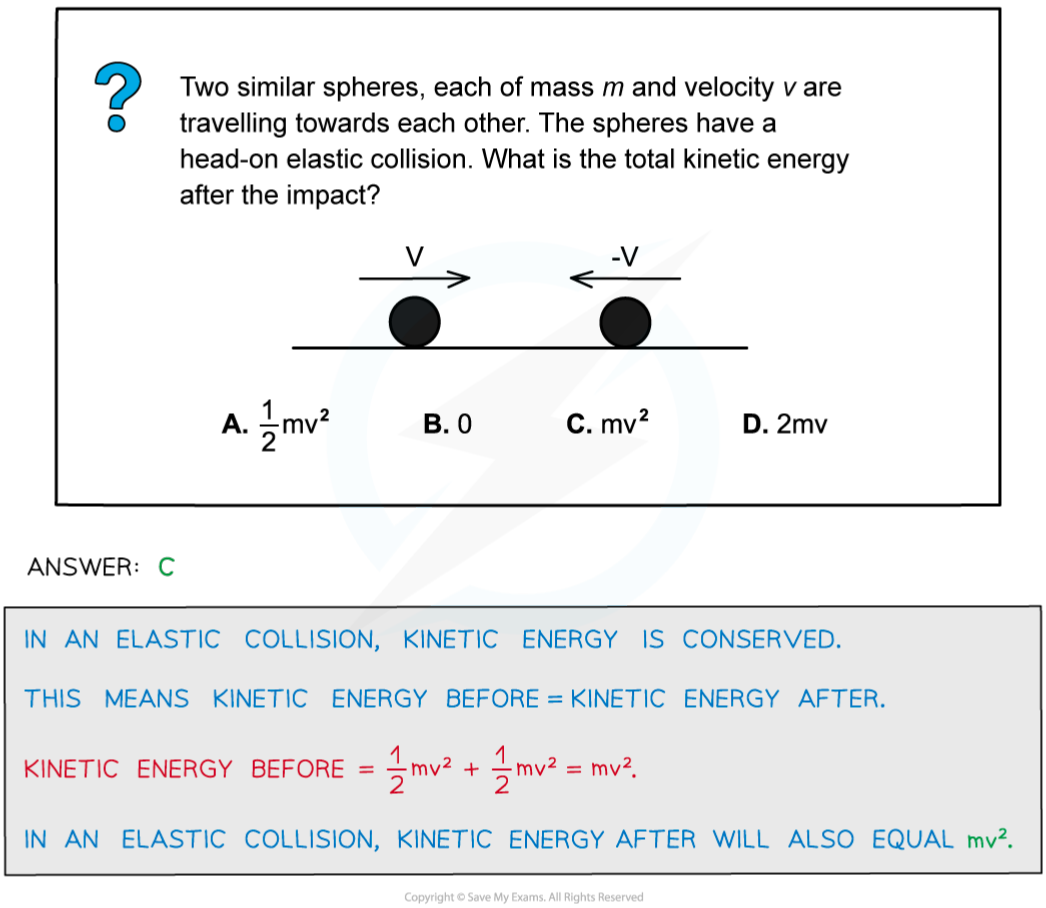


Equation for kinetic energy

- Since kinetic energy depends on the speed of an object, in a perfectly elastic collision (head-on approach) the relative speed of approach = the relative speed of separation

3. Dynamics

YOUR NOTES



Elastic collision example

Exam Tip

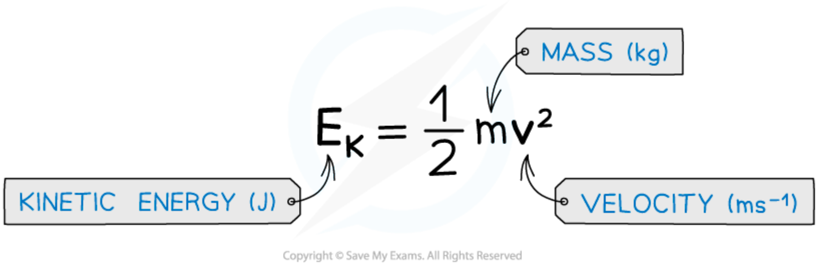
Despite velocity being a vector, kinetic energy is a scalar quantity and therefore will never include a minus sign. This is because in the kinetic energy formula, mass is scalar and the v^2 will always give a positive value whether its a negative or positive velocity

3. Dynamics

YOUR NOTES

Inelastic Collisions

- Whilst the momentum of a system is always conserved in interactions between objects, kinetic energy may not always be
- An inelastic collision is one where **kinetic energy** is **not conserved**



Equation for kinetic energy

- The kinetic energy is transferred into other forms of energy such as a heat or sound
- Inelastic collisions can be when two objects collide and they crumple and deform. Their kinetic energy may also disappear completely as they come to a halt
- A perfectly inelastic collision is when two objects stick together after collision, as shown in the example below

3. Dynamics

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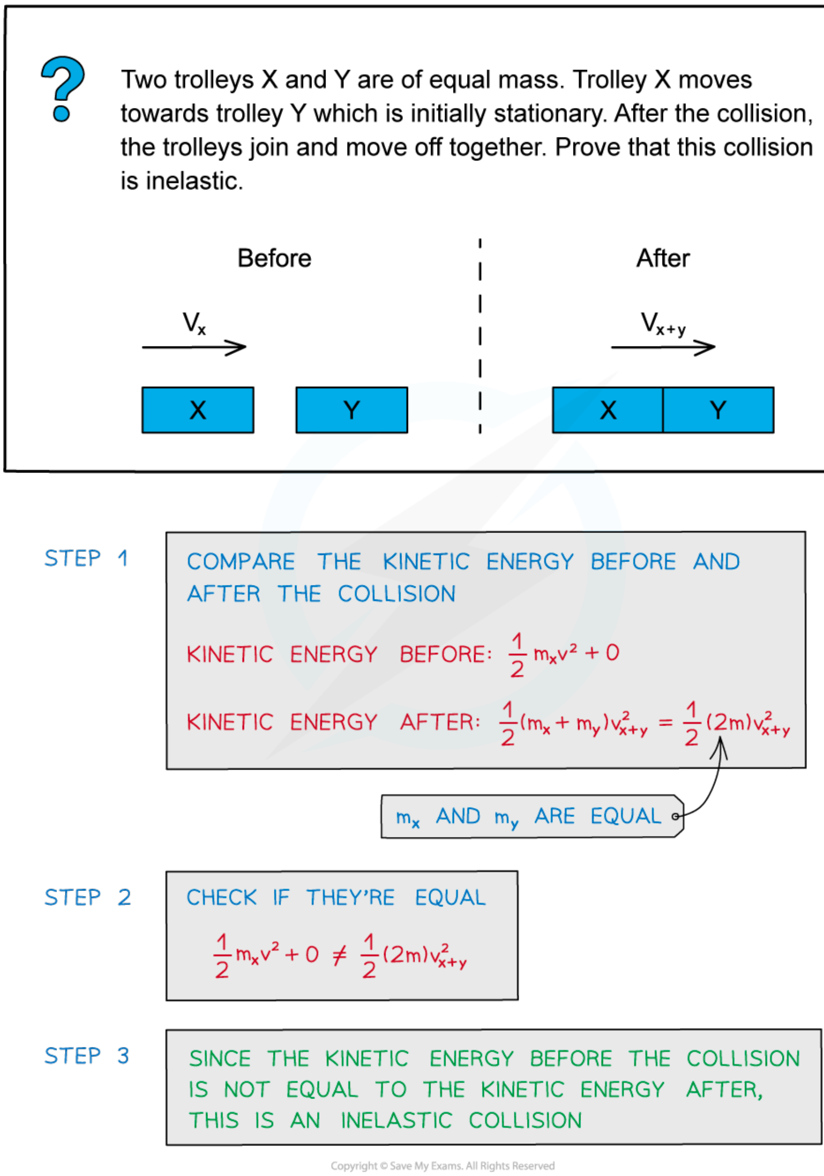


Inelastic collision example

Exam Tip

Although kinetic energy may not always being conserved, remember **momentum will always be conserved.**

3. Dynamics

YOUR NOTES

Exam Question: Easy

What is the principle of conservation of momentum?

A force is equal to the rate of change of momentum

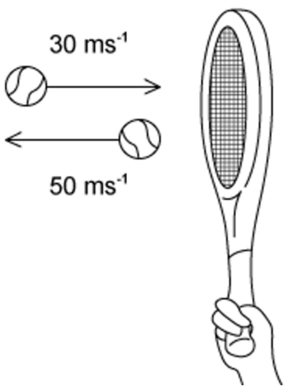
B momentum is the product of mass and velocity

C the total momentum of two bodies after collision is equal to their total momentum before collision

D the total momentum of a system remains constant provided no external force acts on it

Exam Question: Medium

A tennis ball of mass 60 g is struck by a tennis racket. The velocity of the ball changes as shown



What is the magnitude of the change in momentum of the ball?

A 1.2×10^3 kg m s^{-1}

B 4.8×10^3 kg m s^{-1}

C 1.2 kg m s^{-1}

D 4.8 kg m s^{-1}

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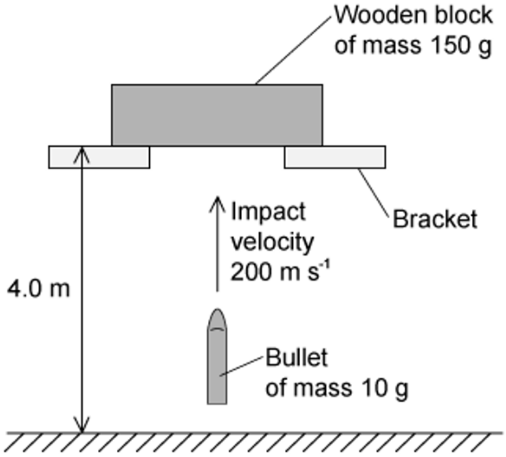
Exam Question: Hard

A wooden block is freely supported on brackets at a height of 4.0 m above the ground, as shown



A bullet of mass 10.0 g is shot vertically upwards into the wooden block of mass 150g. It embeds itself in the block. The impact causes the block to rise above its supporting brackets.

The bullet hits the block with a velocity of 200 m s^{-1}. How far above the ground will the block be at the maximum height of its path?

A 8.0 m **B** 10.0 m **C** 12.0 m **D** 14.0 m

> CHECK YOUR ANSWERS AT SAVEMYEXAMS.CO.UK

4. Forces, Density & Pressure

YOUR NOTES
⬇

CONTENTS

4.1 FORCES: TURNING EFFECTS & EQUILIBRIUM

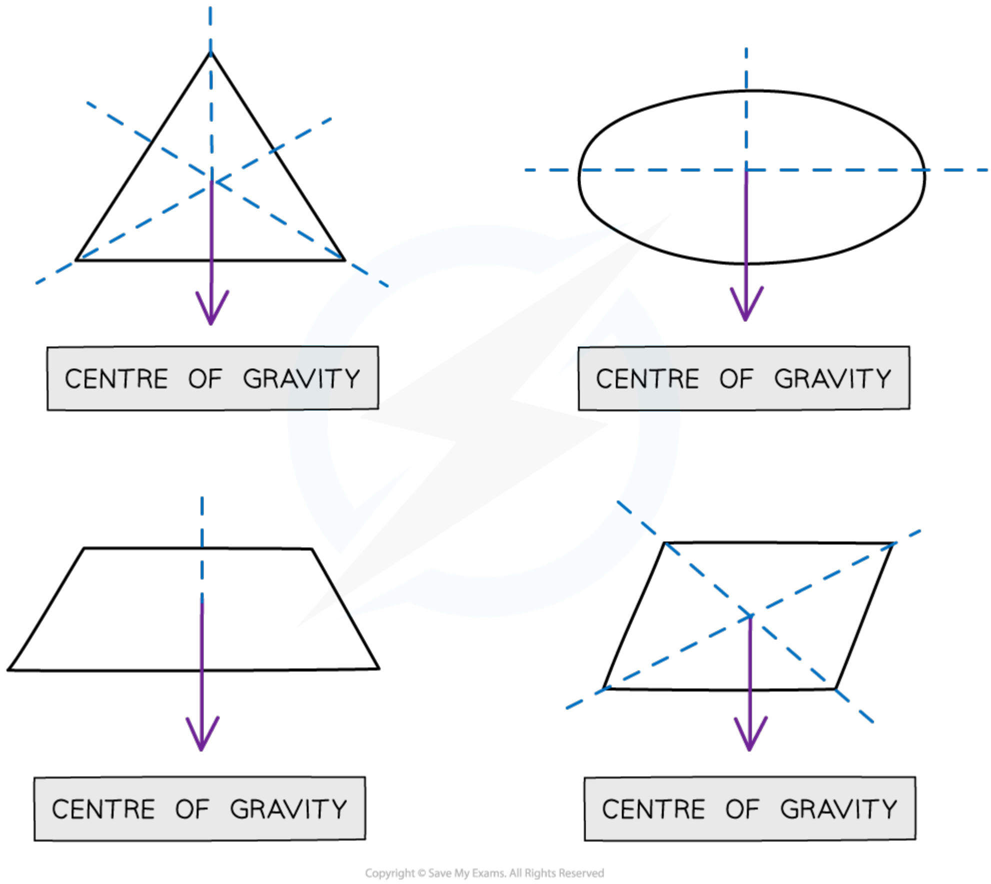
4.1.1 CENTRE OF GRAVITY

Centre of Gravity

- The centre of gravity of an object is **the point at which the weight of the object may be considered to act**
- For example, for a person standing upright, their centre of gravity is roughly in the middle of the body behind the navel, and for a sphere, it is at the centre
- For symmetrical objects with uniform density, the centre of gravity is located at the **point of symmetry**

4. Forces, Density & Pressure

YOUR NOTES



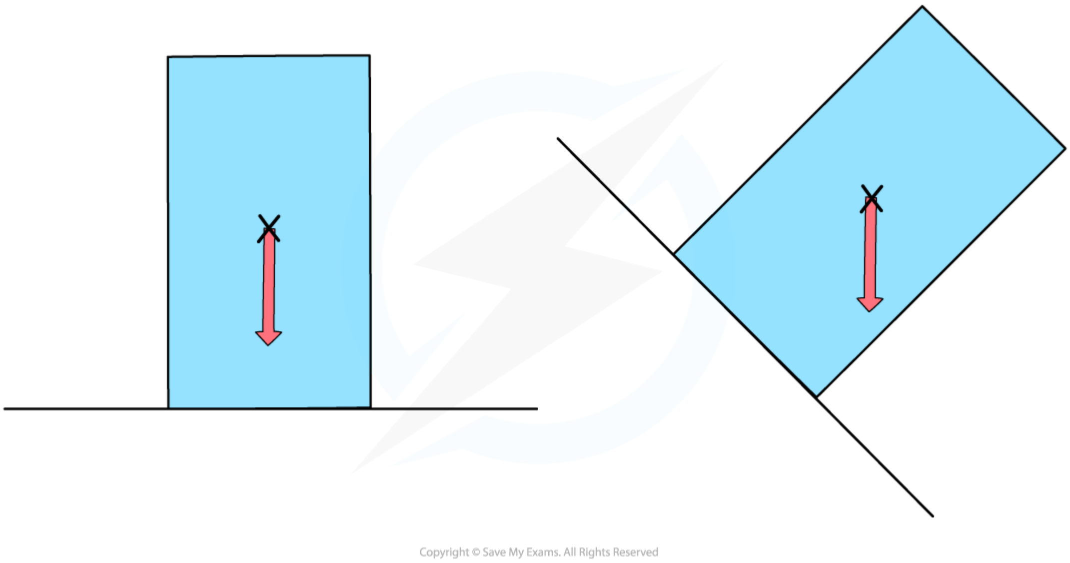
The centre of gravity of a shape can be found by symmetry

4. Forces, Density & Pressure

YOUR NOTES

Stability

- The position of the centre of gravity of an object affects its stability
- An object is stable when its centre of gravity lies above its base



The object on the right will topple, as its centre of mass is no longer over its base

4. Forces, Density & Pressure

YOUR NOTES
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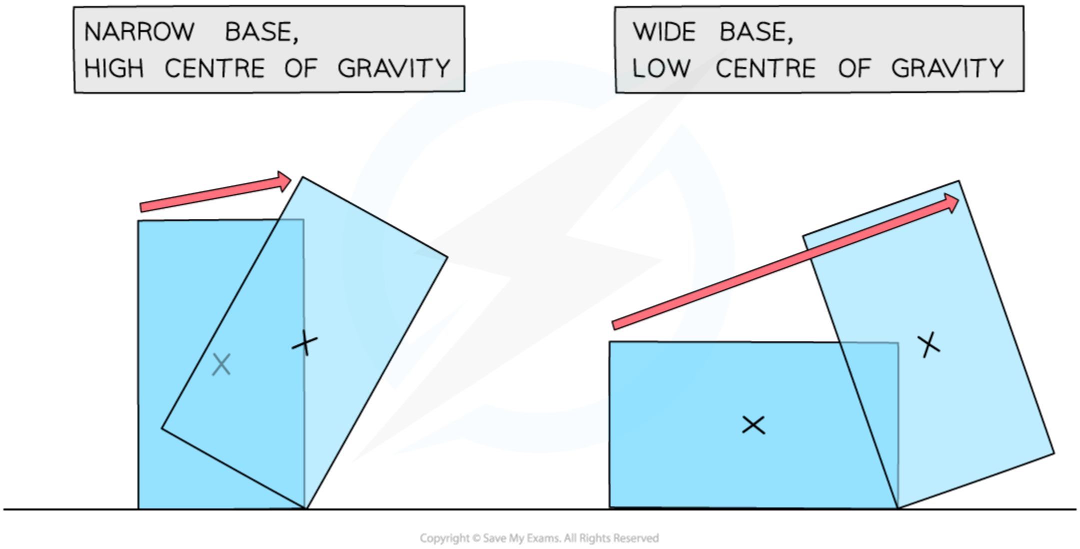
- The **wider** base an object has, the **lower** its centre of gravity and it is more **stable**
- The **narrower** base an object has, the **higher** its centre of gravity and the object is more likely to topple over if pushed



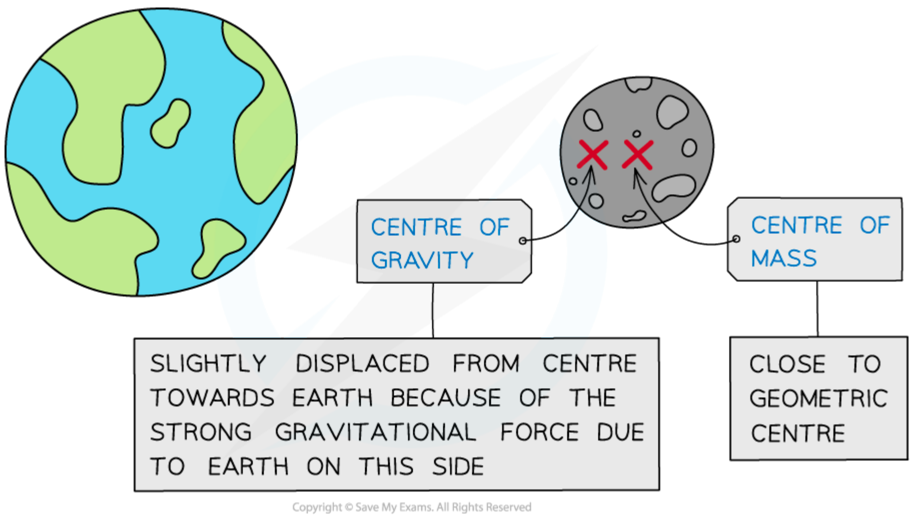
The most stable objects have wide bases and low centres of mass

4. Forces, Density & Pressure

YOUR NOTES
⬇

Centre of gravity v centre of mass

- In a uniform gravitational field, the centre of gravity is identical to the centre of mass
- The centre of mass does **not** depend on the gravitational field
- Since weight = mass × acceleration due to gravity, the centre of gravity **does** depend on the gravitational field
- When an object is in space, its centre of gravity will be more towards the object with larger gravitational field for example, the Earth's gravitational field on the Moon



The Earth's stronger gravitational field pushes the Moons centre of gravity closer to Earth

Exam Tip

Since the centre of gravity is a hypothetical point, it can lie inside or outside of a body. The centre of gravity will constantly shift depending on the shape of a body. For example, a human body's centre of gravity is lower when learning forward than upright

4. Forces, Density & Pressure

YOUR NOTES
⬇

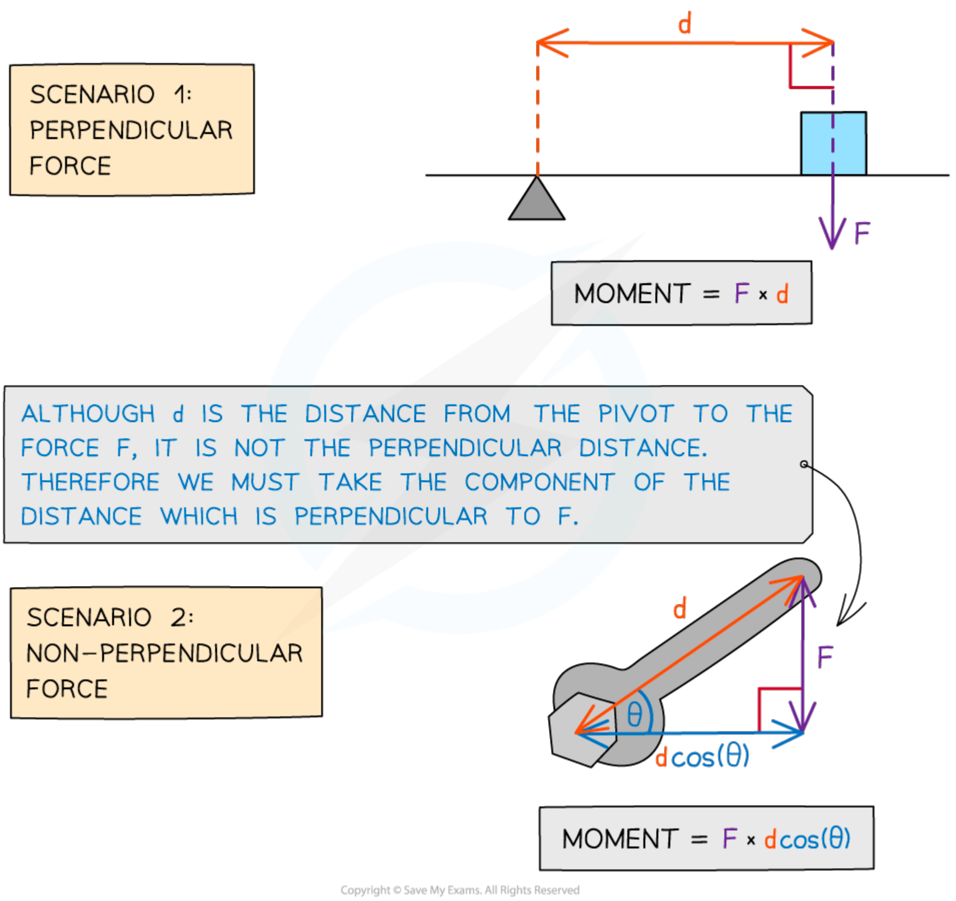
4.1.2 MOMENTS

What is a Moment?

- A moment is the **turning effect of a force**
- Moments occur when forces cause objects to **rotate** about some pivot
- The moment of a force is given by

Moment (**N m**) = Force (**N**) × perpendicular distance from the pivot (**m**)

- The SI unit for the moment is Newton metres (**N m**). This may also be Newton centimetres (**N cm**) depending on the units given for the distance



The force might not always be perpendicular to the distance

4. Forces, Density & Pressure

YOUR NOTES
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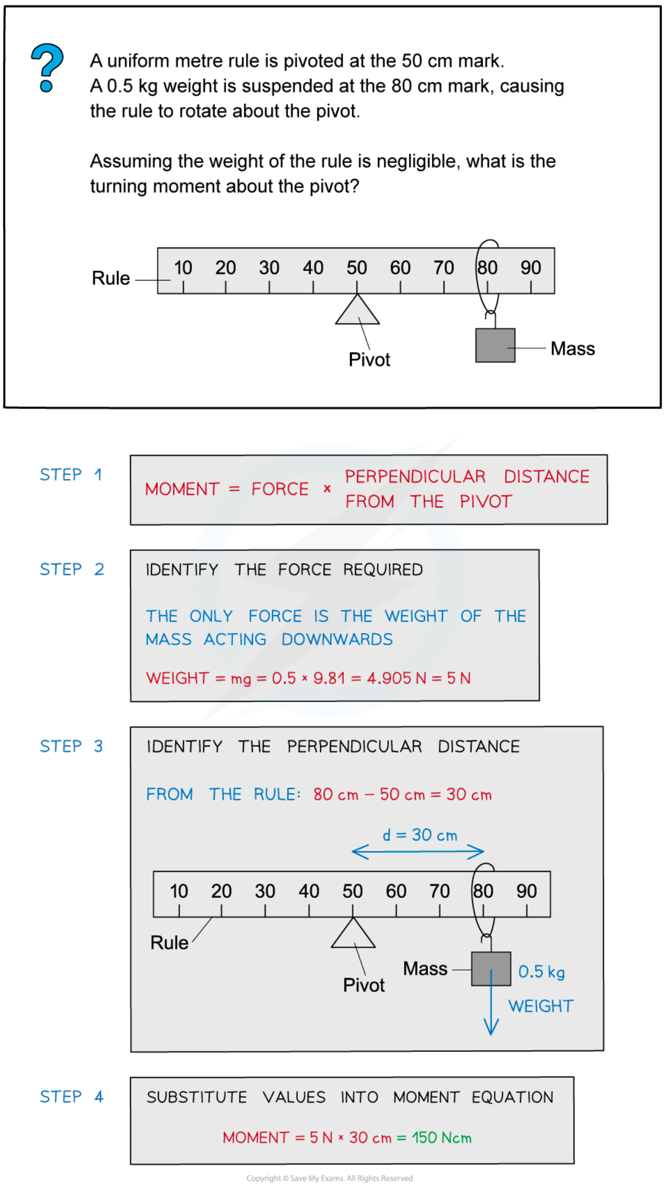
- An example of moments in everyday life is opening a door. The door handle is placed on the other side of the door to the hinge (the pivot) to maximise the distance for a given force and therefore a greater moment (turning force). This makes it easier to push or pull it



4. Forces, Density & Pressure

YOUR NOTES

Exam Tip

If not already given, drawing all the forces on an object in the diagram will help you see which ones are perpendicular to the distance from the pivot. Not all the forces will provide a turning effect and it is not unusual for a question to provide more forces than required

The Principle of Moments

- The principle of moments states: For a system to be **balanced** (in equilibrium), the **sum** of clockwise moments about a point must be equal to the **sum** of anticlockwise moments (about the same point)

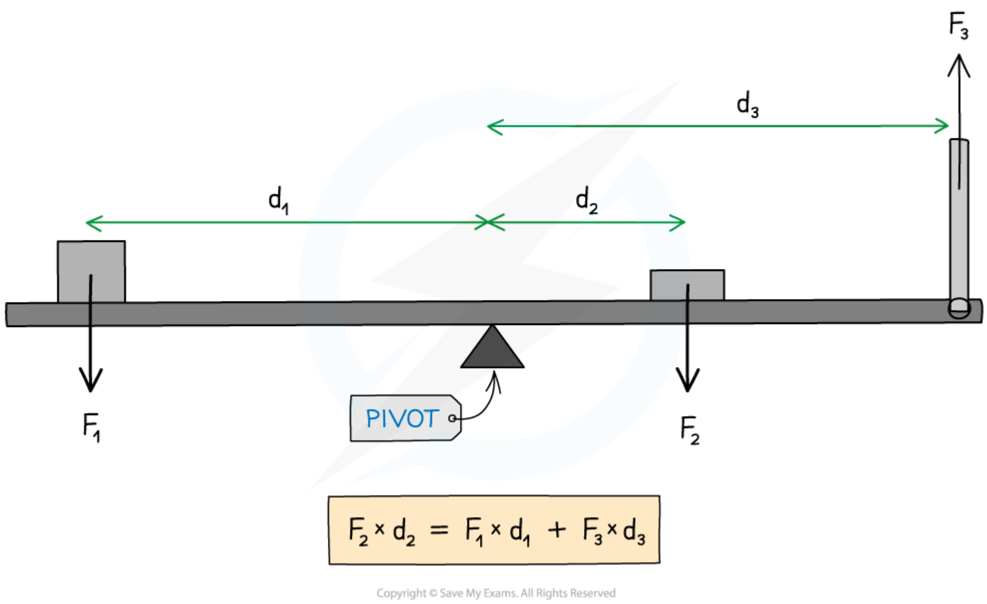


Diagram showing the moments acting on a balanced beam

4. Forces, Density & Pressure

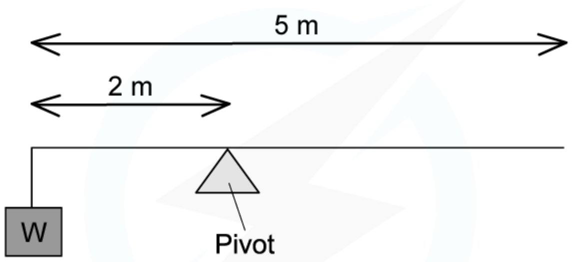
YOUR NOTES

- In the above diagram:
  - Force F_2 is supplying a clockwise moment;
  - Forces F_1 and F_3 are supplying anticlockwise moments
- Hence: $F_2 \times d_2 = F_1 \times d_1 + F_3 \times d_3$

?

A uniform beam of weight 40 N is 5 m long and is supported by a pivot situated 2m from one end.

When a load of weight W is hung from that end, the beam is in equilibrium as shown in the diagram.



What is the value of W?

A. 10 N **B.** 50 N **C.** 25 N **D.** 30 N

ANSWER: A

STEP 1

PRINCIPLE OF MOMENTS STATES THAT

CLOCKWISE MOMENTS = ANTICLOCKWISE MOMENTS

4. Forces, Density & Pressure

YOUR NOTES

STEP 2

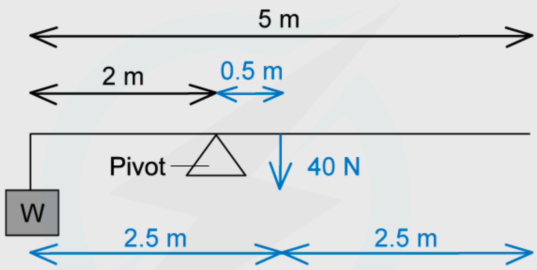
CALCULATE THE CLOCKWISE MOMENT

SINCE THE BEAM IS UNIFORM, ITS WEIGHT WILL ACT AT ITS CENTRE OF GRAVITY (THE MIDDLE)

THIS IS $5 \div 2 = 2.5$ m FROM THE END

SINCE THE PIVOT IS 2m FROM THE END, THIS FORCE IS 0.5 m FROM THE PIVOT

CLOCKWISE MOMENT = 40 N × 0.5 m = 20 Nm



STEP 3

CALCULATE THE ANTICLOCKWISE MOMENT

ANTICLOCKWISE MOMENT = W × 2m

STEP 4

EQUATE BOTH THESE MOMENTS

20 Nm = W × 2m

STEP 5

REARRANGE FOR W

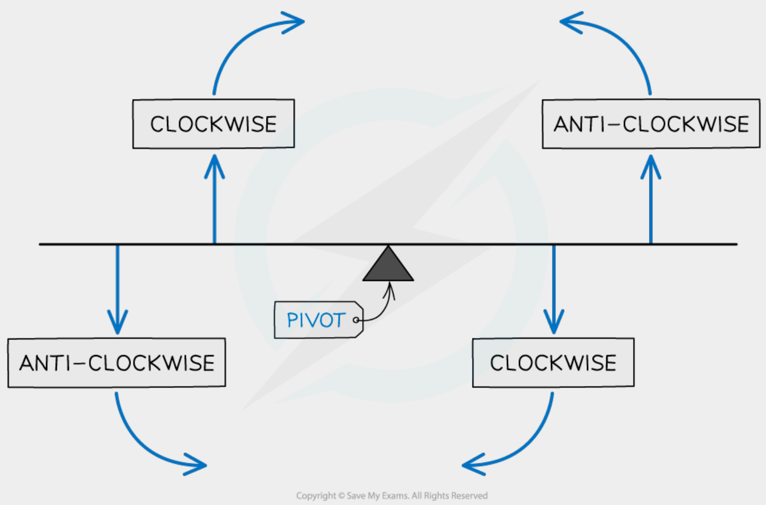
$$W = \frac{20\ \text{Nm}}{2\text{m}} = 10\ \text{N}$$

4. Forces, Density & Pressure

YOUR NOTES

Exam Tip

Make sure that all the distances are in the same units and you're considering the correct forces as clockwise or anticlockwise, as seen in the diagram below



4. Forces, Density & Pressure

YOUR NOTES
⬇

4.1.3 TURNING EFFECTS OF FORCES

Couples

- A couple is a **pair** of forces that acts to produce **rotation** only
- Unlike moments of a single force, the moment of a couple doesn't depend on a pivot, only on the perpendicular distance between the two forces
- A couple consists of a pair of forces that are:
  - Equal in magnitude
  - Opposite in direction
  - Perpendicular to the distance between them

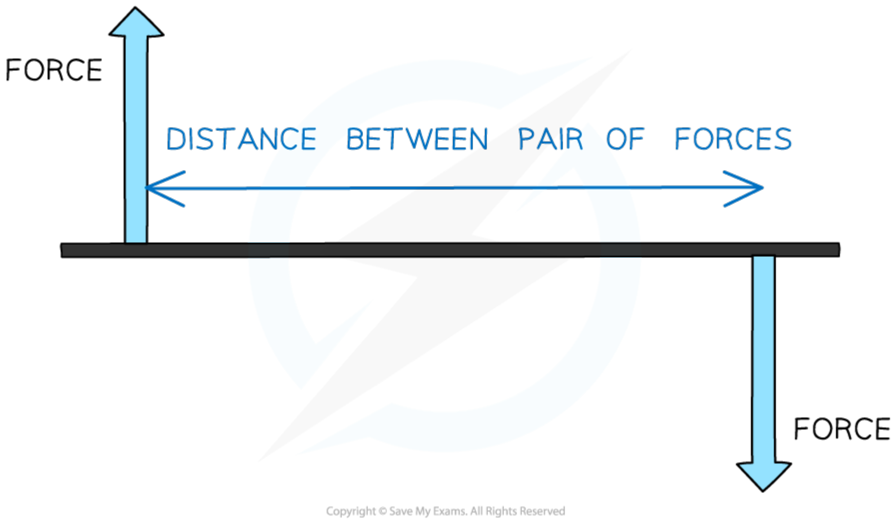


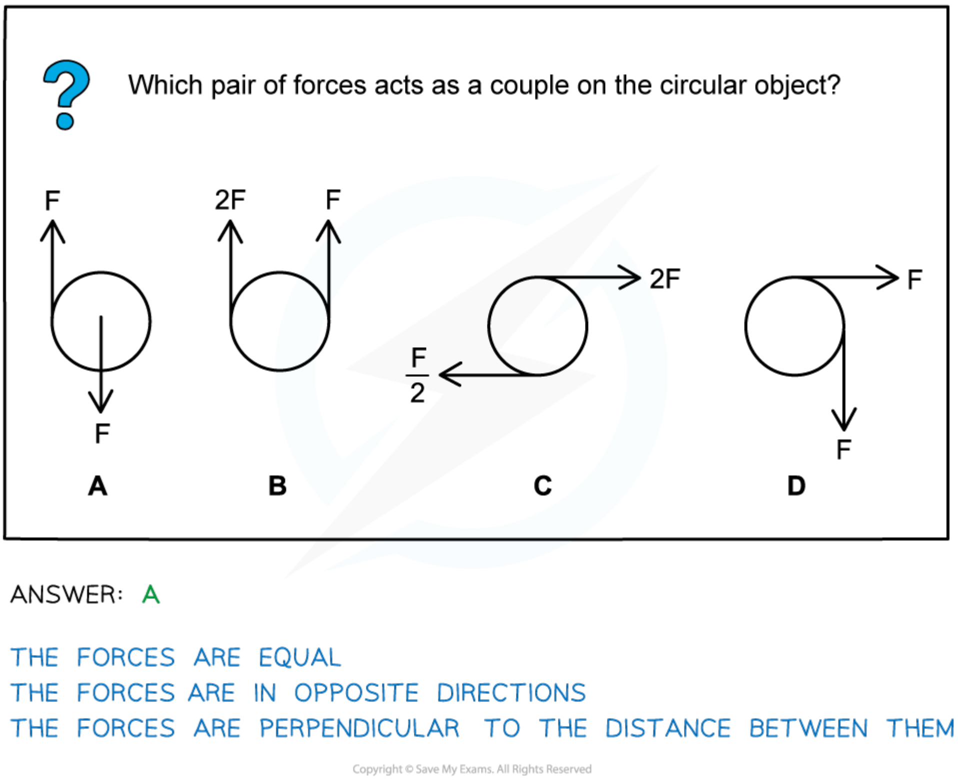
Diagram of a couple

- Couples produce a resultant force of **zero**, so, due to Newton's Second law (**F** = **ma**), the object does **not** accelerate
- The size of this turning effect is given by its **torque**

4. Forces, Density & Pressure

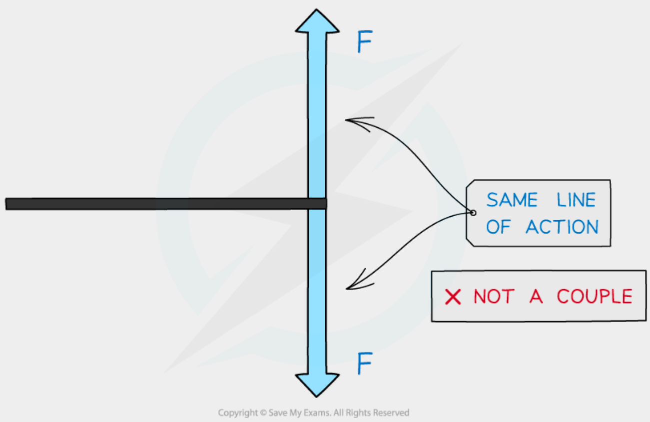
YOUR NOTES

Worked Example



Exam Tip

The forces that make up a couple cannot share the same line of action which is the line through the point at which the force is applied. An example of this is shown in the diagram below



4. Forces, Density & Pressure

YOUR NOTES
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Torque

- The moment of a couple is known as a **torque**
- You can calculate the torque of a couple with the following equation

Torque τ (**N m**) = one of the forces (**N**) × perpendicular distance between the forces (**m**)

4. Forces, Density & Pressure

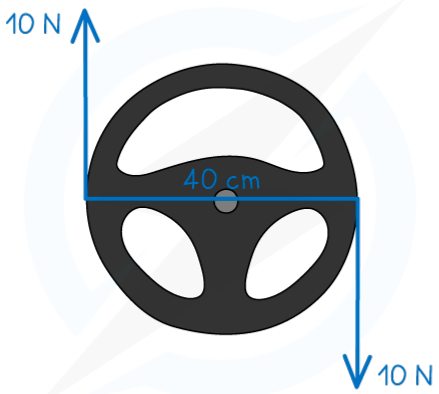
YOUR NOTES
⬇

Worked example – perpendicular distance

?

A steering wheel of diameter 40 cm and the force of the couple needed to turn it is 10 N.

Calculate the torque on the steering wheel.



STEP 1

TORQUE EQUATION

TORQUE = FORCE × PERPENDICULAR DISTANCE

STEP 2

SUBSTITUTE NUMBERS

TORQUE = 10 N × 40 cm = 400 NCm

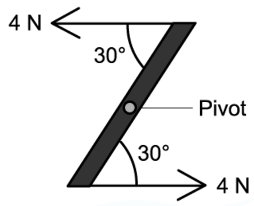
4. Forces, Density & Pressure

YOUR NOTES
⬇

Worked example – Non-perpendicular distance

A rule of length 0.3 m is pivoted at its centre.

Equal and opposite forces of magnitude 4.0 N are applied to the ends of the ruler, created a couple as shown below.



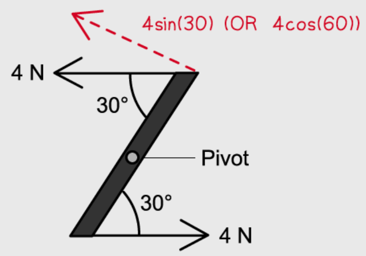
What is the magnitude of the torque of the couple on the ruler when its at the position shown?

STEP 1

TORQUE = FORCE × PERPENDICULAR DISTANCE

STEP 2

FIND THE COMPONENT OF THE FORCE THAT IS PERPENDICULAR TO THE DISTANCE



STEP 3

SUBSTITUTE VALUES INTO THE EQUATION

TORQUE = 4sin(30) N × 0.3 = 0.6 Nm

4. Forces, Density & Pressure

YOUR NOTES

Exam Tip

The forces given might not always be perpendicular to the distance between them. In this case, remember to find the component of the force vector that **is** perpendicular. You can learn more on how to do this in the 'Resolving Vectors' section of 'Scalars & Vectors'

4. Forces, Density & Pressure

YOUR NOTES
⬇

4.1.4 CONDITIONS FOR EQUILIBRIUM

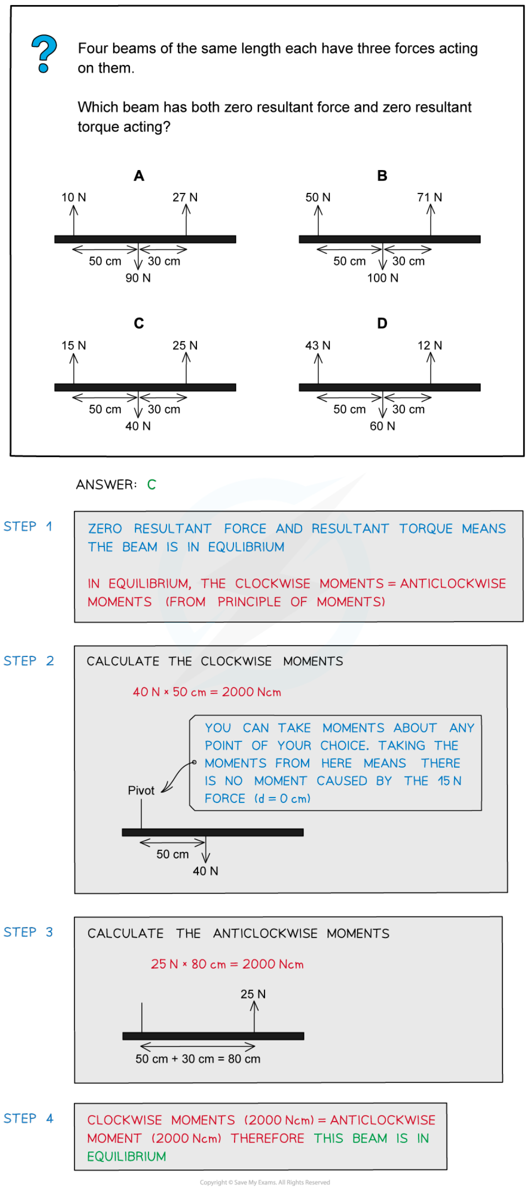
Equilibrium

- A system is in equilibrium when all the forces are balanced. This means:
  - There is **no** resultant force
  - There is **no** resultant torque
- An object in equilibrium will therefore remain at rest, or at a constant velocity, and not rotate
- The system is in an equilibrium state when applying the principle of moments (see The Principle of Moments)

4. Forces, Density & Pressure

YOUR NOTES
⬇

Worked example – Beam in equilibrium

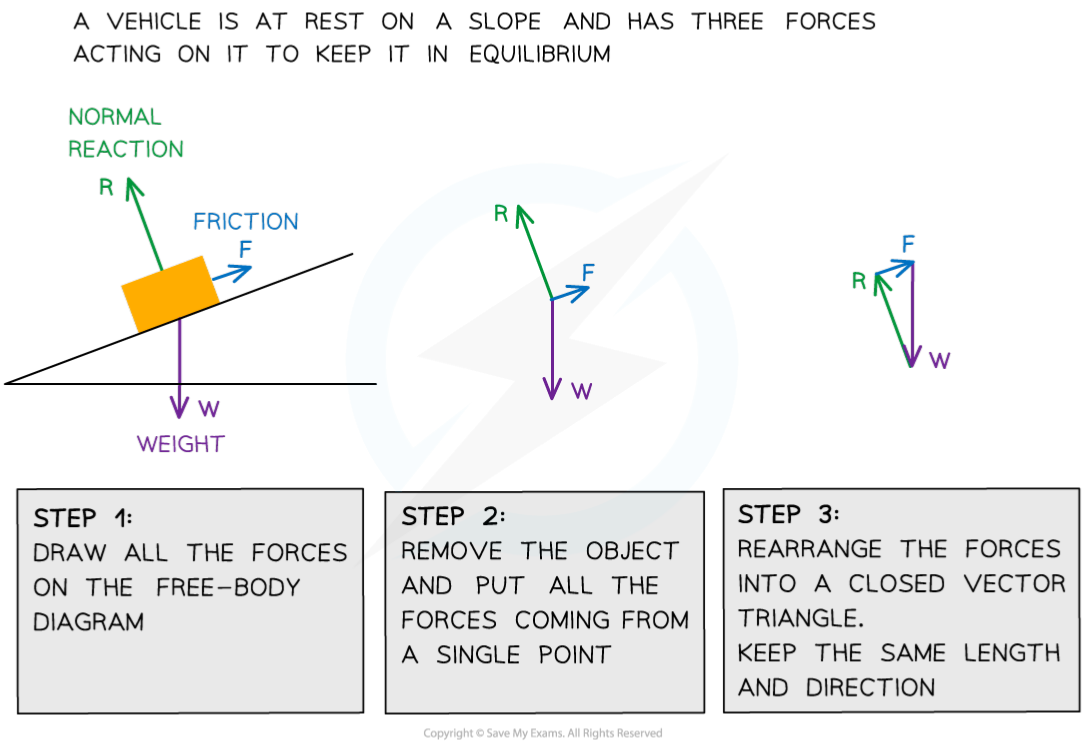


4. Forces, Density & Pressure

YOUR NOTES

Coplanar Forces in Equilibrium

- Coplanar forces can be represented by vector triangles
- In equilibrium, these are **closed** vector triangles. The vectors, when joined together, form a closed path
- The most common forces on objects are
  - Weight
  - Normal reaction force
  - Tension (from cords and strings)
  - Friction
- The forces on a body in equilibrium are demonstrated below:



Three forces on an object in equilibrium form a closed vector triangle

4. Forces, Density & Pressure

YOUR NOTES
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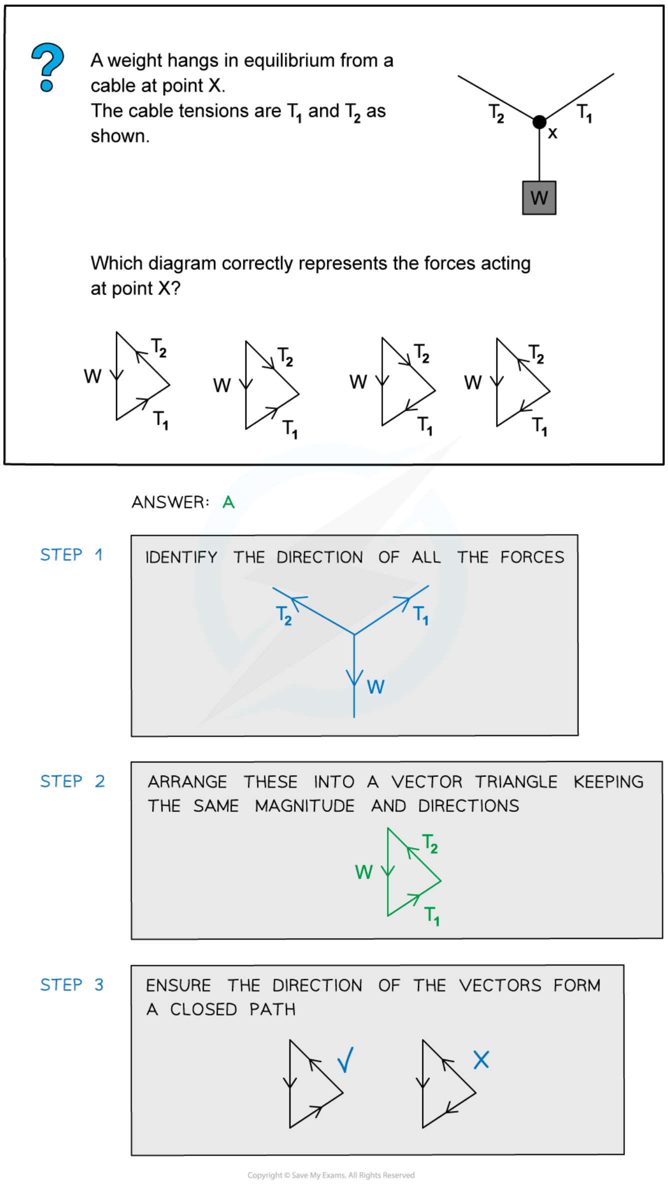
Worked example – Forces in equilibrium



Exam Tip

The diagrams in exam questions about this topic tend to be drawn to scale, so make sure you have a ruler handy!

4. Forces, Density & Pressure

YOUR NOTES

Exam Question: Easy

What is the centre of gravity of an object?

A the geometrical centre of the object

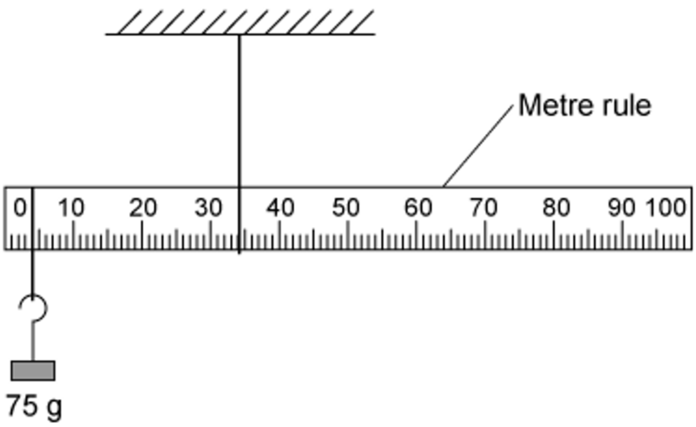
B the point about which the total torque is zero

C the point at which the weight of the object may be considered to act

D the point through which gravity acts

Exam Question: Medium

A uniform metre rule is hung from the ceiling as shown



The rule balances when a 75g mass is hung from one end of the ruler

What is the mass of the metre rule?

A 38 g **B** 51 g **C** 141 g **D** 159 g

4. Forces, Density & Pressure

YOUR NOTES

Exam Question: Hard

The diagrams show two ways of hanging the same picture.

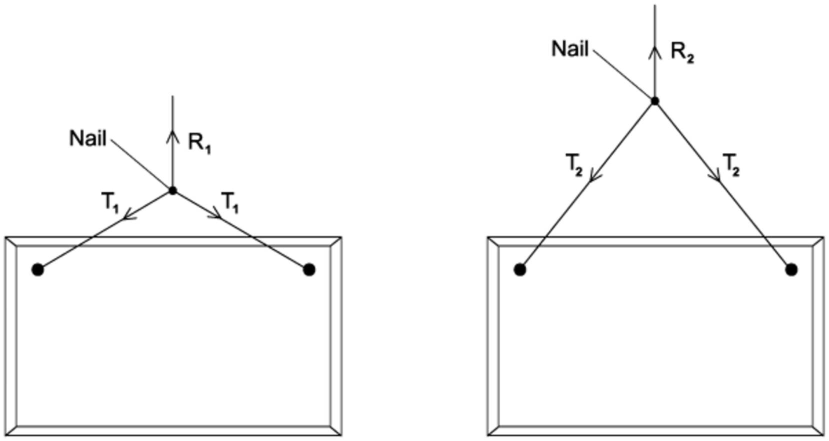


Diagram 1 Diagram 2

In both cases, a string is attached to the same points on the picture and looped symmetrically over a nail in a wall. The forces shown are those that act on the nail.

In diagram 1, the string loop is shorter than in diagram 2.

Which information about the magnitude of the forces is correct?

A $R_1 = R_2$ $T_1 < T_2$

B $R_1 = R_2$ $T_1 > T_2$

C $R_1 > R_2$ $T_1 < T_2$

D $R_1 < R_2$ $T_1 = T_2$

> CHECK YOUR ANSWERS AT SAVEMYEXAMS.CO.UK

4. Forces, Density & Pressure

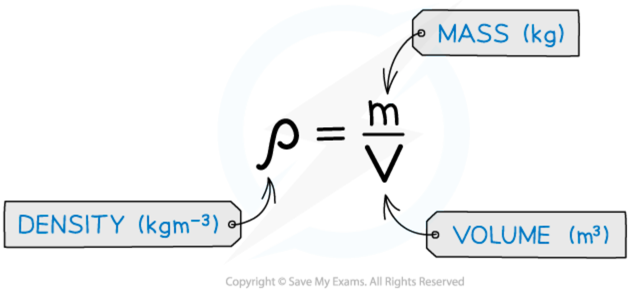
YOUR NOTES
⬇

4.2 FORCES: DENSITY & PRESSURE

4.2.1 DENSITY

Density

- Density is the **mass per unit volume** of an object
  - Objects made from low-density materials typically have lower mass. For example, a balloon is less dense than a small bar of lead despite occupying a larger volume
- The units of density depend on what units are used for mass and volume:
  - If the mass is measured in *g* and volume in **cm³**, then the density will be in **g/cm³**
  - If the mass is measured in *kg* and volume in **m³**, then the density will be in **kg/m³**



Equation for density

4. Forces, Density & Pressure

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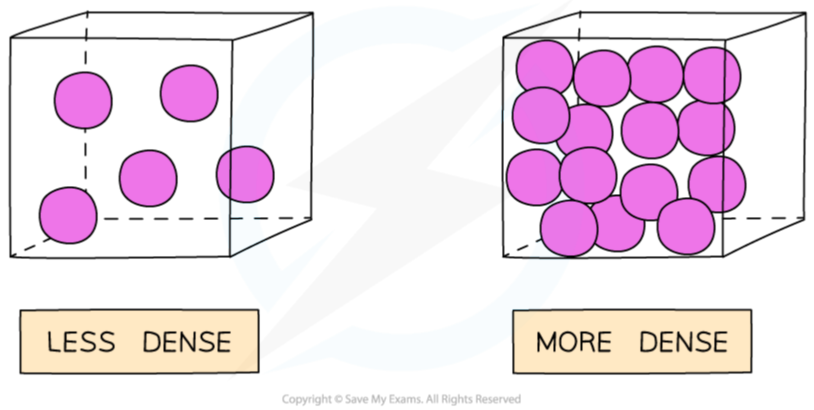
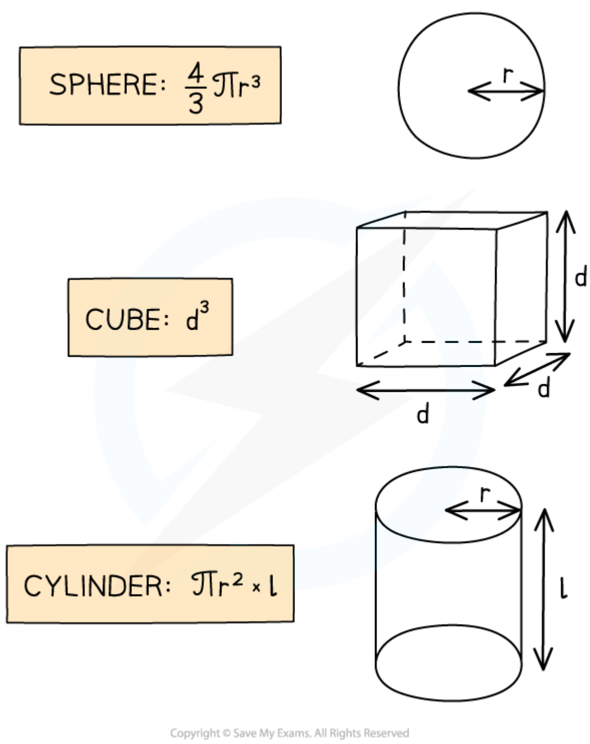


Gases are less dense than a solid

- The volume of an object may not always be given directly, but can be calculated with the appropriate equation depending on the object's shape



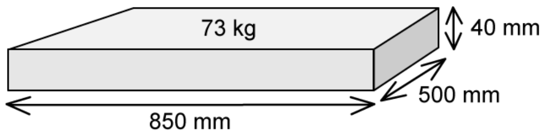
Volumes of common 3D shapes

4. Forces, Density & Pressure

YOUR NOTES

Worked example

A paving slab has a mass of 73 kg and dimensions 40 mm × 500 mm × 850 mm.
Calculate the density, in kgm^{-3} of the material from which the paving slab is made.



STEP 1

EQUATION FOR DENSITY

$$\rho = \frac{M}{V}$$

STEP 2

CALCULATE THE VOLUME

$V = 40\text{ mm} \times 500\text{ mm} \times 850\text{ mm} = 1.7 \times 10^{7}\text{ mm}^3$

STEP 3

CONVERT mm^3 TO m^3

$1\text{ mm} = 0.001\text{ m} = 1 \times 10^{-3}\text{ m}$

$1\text{ mm}^3 = (0.001)^3\text{ m}^3 = (1 \times 10^{-3})^3\text{ m}^3 = 1 \times 10^{-9}\text{ m}^3$

$V = 1.7 \times 10^{7} \times 1 \times 10^{-9} = 1.7 \times 10^{-2}\text{ m}$

STEP 4

SUBSTITUTE MASS AND VOLUME INTO DENSITY EQUATION

$$\rho = \frac{73\text{ kg}}{1.7 \times 10^{-2}\text{ m}^3} = 4300\text{ kgm}^{-3} \text{ (2 s.f.)}$$

4. Forces, Density & Pressure

YOUR NOTES
⬇

Exam Tip

- When converting a **larger** unit to a **smaller** one, you **multiply (×)**
  - E.g. 125 m = 125 × 100 = 12 500 cm
- When you convert a **smaller** unit to a **larger** one, you **divide (÷)**
  - E.g. 5 g = 5 / 1000 = 0.005 or 5 × 10^{-3} kg
- When dealing with squared or cubic conversions, cube or square the conversion factor too
  - E.g. 1 mm^3 = 1 / $(1000)^3$ = 1 × 10^{-9} m^3
  - E.g. 1 cm^3 = 1 / $(100)^3$ = 1 × 10^{-6} m^3

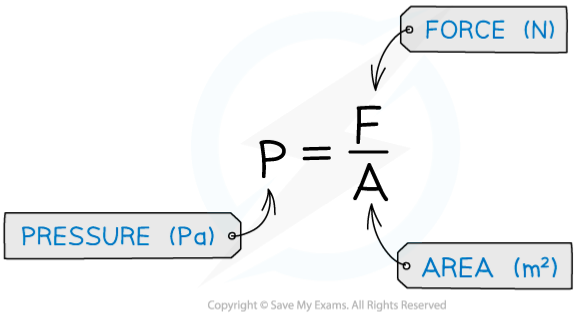
4. Forces, Density & Pressure

YOUR NOTES
⬇

4.2.2 PRESSURE

Pressure

- Pressure tells us how concentrated a force is, it is defined as the **force per unit area**

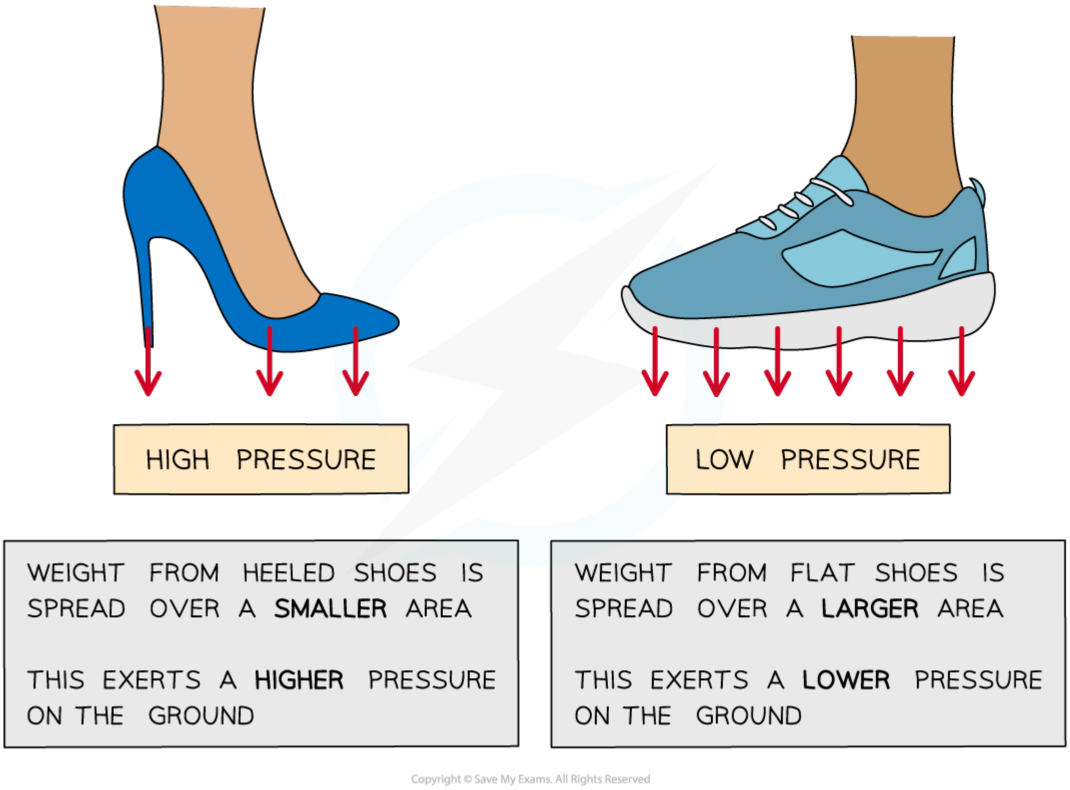


Pressure is equal to the force per unit area

4. Forces, Density & Pressure

YOUR NOTES

- This equation tells us
  - If a force is spread over a large area it will result in a small pressure
  - If it is spread over a small area it will result in a large pressure



Different pressure is exerted for the same force on different areas

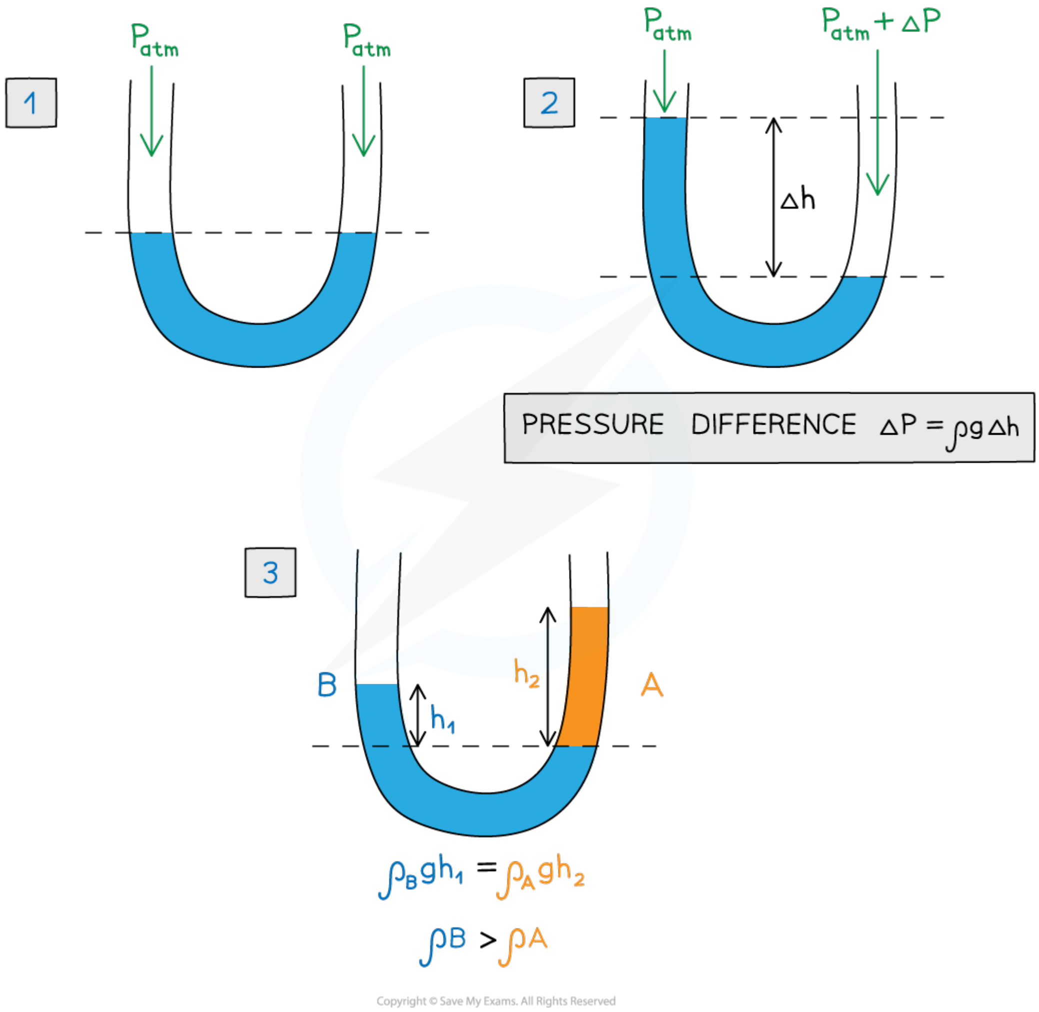
- The units of pressure depend on the units of area:
  - If the area is measured in **cm^2** (and the force in **N**), then the pressure will be in **N/cm^2**
  - If the area is measured in **m^2** (and the force in **N**), then the pressure will be in **N/m^2**
- Pressure can also be measured in pascals, Pa where **1 Pa is the same as 1 N/m^2**
- Pressure, unlike force, is a scalar. Therefore pressure does not have a specific direction

4. Forces, Density & Pressure

YOUR NOTES

U-tube manometer

- A manometer is an instrument to measure pressure and density of two liquids



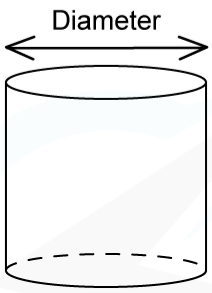
- In Figure 1: The level of liquid is equal because the atmospheric pressure (P_{atm}) is the same
- In Figure 2: If the pressure on one side rises, the liquid will be forced down making the liquid in the other limb rise. The difference between the two levels gives the pressure difference between the two ends of the tube
- In Figure 3: The U-tube now has two different liquids. The density of the blue one is larger than that of the orange one. The pressure at each point is due to the atmospheric pressure plus the weight of the liquid above it

4. Forces, Density & Pressure

YOUR NOTES

Worked example

A cylinder is placed on a horizontal surface as shown below



The mass of the cylinder is 4.7 kg and the diameter is 8.4 cm. Calculate the pressure produced by the cylinder on the surface in Pa.

STEP 1

PRESSURE EQUATION

$$P = \frac{F}{A}$$

4. Forces, Density & Pressure

YOUR NOTES

STEP 2

CALCULATE THE FORCE PRODUCED BY THE CYLINDER

ONLY FORCE IS THE WEIGHT

$$F = mg = 4.7\,\text{kg} \times 9.81\,\text{ms}^{-2} = 46.1\,\text{N}$$

STEP 3

CALCULATE THE AREA

THE FORCE IS APPLIED ON THE BASE OF THE CYLINFER WHICH IS A CIRCLE

$$\text{AREA OF A CIRCLE} = \pi \times r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{8.4 \times 10^{-2}\,\text{m}}{2}\right)^2 = 5.5 \times 10^{-3}\,\text{m}^2$$

STEP 4

SUBSTITUTE BACK INTO PRESSURE EQUATION

$$P = \frac{46.1\,\text{N}}{5.5 \times 10^{-3}\,\text{m}^2} = 8400\,\text{Pa (2 s.f)}$$

Exam Tip

The area referred to is the ‘cross-sectional’ area of a 3D object. This is the area of the base that the force is applied on. For a cylinder, this will be a circle.

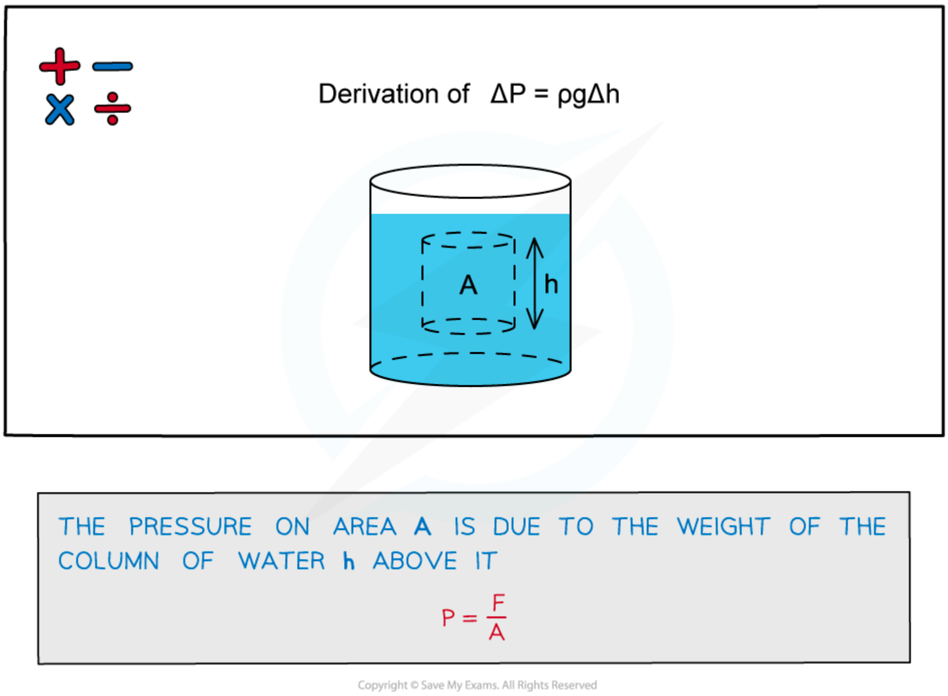
4. Forces, Density & Pressure

YOUR NOTES

4.2.3 DERIVATION OF ΔP = PGΔH

Derivation of $\Delta p = \rho g \Delta h$

- **Hydrostatic pressure** is the pressure that is exerted by a **fluid** at equilibrium at a given point within the fluid, due to the force of gravity
- The derivation for this equation is shown below:



4. Forces, Density & Pressure

YOUR NOTES

FORCE IS THE WEIGHT OF THE COLUMN

$$W = mg$$

MASS OF THE COLUMN IS FROM

$$m = \rho \times V$$

WHERE $V = A \times h$

SUBSTITUTE VOLUME, MASS EQUALS

$$m = \rho \times A \times h$$

WEIGHT OF THE COLUMN IS NOW

$$W = \rho \times A \times h \times g$$

SUBSTITUTE BACK INTO PRESSURE EQUATION

$$P = \frac{\rho \times A \times h \times g}{A}$$

$$P = \rho h g$$

MORE SPECIFICALLY, THE CHANGE IN PRESSURE P IS PROPORTIONAL TO THE CHANGE IN HEIGHT h

EQUATION FOR HYDROSTATIC PRESSURE $\Delta P = \rho g \Delta h$

Hydrostatic pressure derivation

Exam Tip

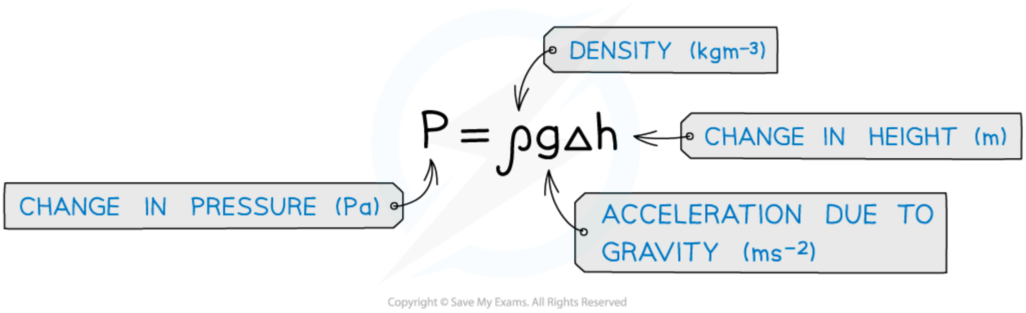
You will be expected to remember all the steps for this derivation for an exam question. If any equations which look unfamiliar, have a look at the notes for "Density" and "Pressure".

4. Forces, Density & Pressure

YOUR NOTES
⬇

Using the Equation for Hydrostatic Pressure

- **Hydrostatic pressure** is the pressure that is exerted by a **fluid** at equilibrium at a given point within the fluid, due to the force of gravity
- This is when an object is immersed in a liquid, the liquid will exert a pressure, squeezing the object
- The size of this pressure depends upon the density (***ρ***) of the liquid, the depth (***h***) of the object and the gravitational field strength (***g***):



- When asked about the **total pressure** remember to also add the atmospheric pressure

Total pressure = Hydrostatic pressure + Atmospheric pressure

- Atmospheric pressure (also known as barometric pressure) is **101 325 Pa**

4. Forces, Density & Pressure

YOUR NOTES

Worked example

Atmospheric pressure at sea level has a value of 100 kPa.

The density of sea water is 1020 kg m^{-3}.

At which depth in the sea would the total pressure be 250 kPa?

A. 20 m **B.** 9.5 m **C.** 18 m **D.** 15 m

ANSWER: D

STEP 1

THE TOTAL PRESSURE = HYDROSTATIC PRESSURE + ATMOSPHERIC PRESSURE

$$250 \times 10^3 = \rho g h + 100 \times 10^3$$

$$250 \times 10^3 = 1020 \times 9.81 \times h + 100 \times 10^3$$

STEP 2

REARRANGING FOR HEIGHT h

$$h = \frac{250 \times 10^3 - 100 \times 10^3}{1020 \times 9.81} = 15 \text{ m}$$

Exam Tip

These pressures can vary widely and depend on metric prefixes such as kPa or MPa. When you're doing calculations make sure all the pressures are in the same units (otherwise you may be out by a factor of 1000!). To be on the safe side, you can convert them all to Pascals.

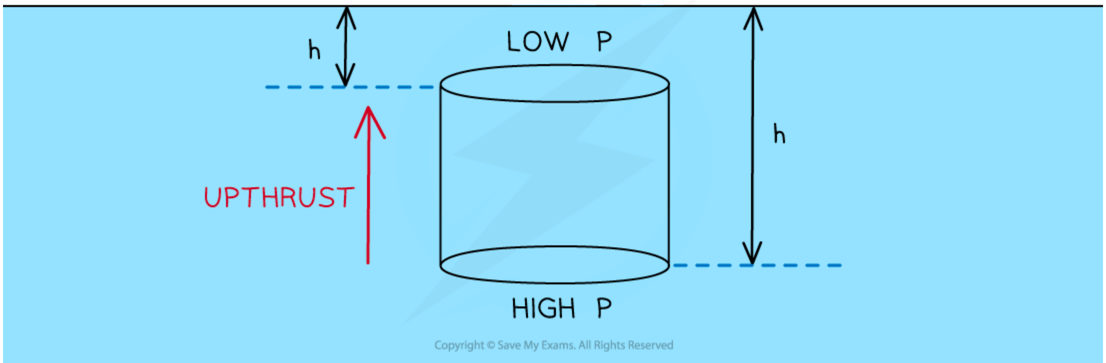
4. Forces, Density & Pressure

YOUR NOTES

4.2.4 UPTHRUST

Upthrust

- Upthrust is a force which pushes upwards on an object submerged in a fluid i.e. liquids and gases
- Also known as buoyancy force, upthrust is due to the **difference in hydrostatic pressure** at the top and bottom of the immersed object
- The force of upthrust is significantly larger in liquids than in gases, this is because liquids are much denser than gases
- Recall that hydrostatic pressure depends on the height (h) or depth that an object is submerged in from $P = \rho gh$
- Therefore, the water pressure at the bottom of an object is greater than the water pressure at the top, as shown in the diagram below:



This can will experience upthrust due to the hydrostatic pressure difference

- Upthrust is a force and is directly proportional to the pressure. The force on the bottom of the can will be greater than the force on top of the can
- This resultant pressure causes a resultant **upward** force on the can known as upthrust
- Upthrust is why objects appear to weigh less when immersed in a liquid. If the upthrust is greater than the weight of the object, the object will rise up
- For an object to float, it must have a density less than the density of the fluid its immersed in

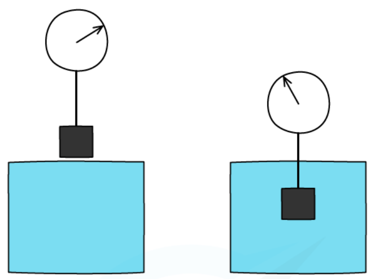
4. Forces, Density & Pressure

YOUR NOTES

Worked example

A steel cube with cross-sectional area of 2 m^2 is totally immersed in water.

The scale balance reading reduced when the cube is immersed.



The cube experiences pressures of 3000 Pa and 7700 Pa at the top and bottom of the cube respectively.

Which value is equal to the upthrust on the cube during immersion?

A. 9400 N **B.** 6000 N **C.** 15400 N **D.** 92210 N

ANSWER: A

STEP 1

THE UPTHRUST IS EQUAL TO THE NET VERTICAL FORCE ON THE BLOCK

PRESSURE EQUATION

$$P = \frac{F}{A}$$

STEP 2

REARRANGE FOR FORCE

$$F = P \times A$$

STEP 3

CALCULATE NET FORCE (UPTHRUST) USING THE DIFFERENCE IN PRESSURE

$$\text{NET FORCE} = (7700 - 3000)\,\text{Pa} \times 2\,\text{m}^2 = 9400\,\text{N}$$

Exam Tip

Since upthrust is force it is influenced by pressure, not by the density of the object as commonly misunderstood.

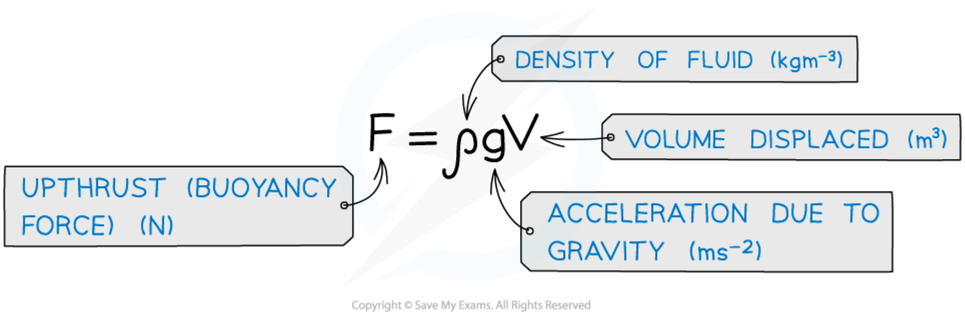
4. Forces, Density & Pressure

YOUR NOTES

4.2.5 ARCHIMEDES PRINCIPLE

Archimedes' Principle

- Archimedes' principle states that an object submerged in a fluid at rest has **an upward buoyancy force (upthrust) equal to the weight of the fluid displaced by the object**
- The object sinks until the weight of the fluid displaced is equal to its own weight
- Therefore the object floats when the magnitude of the upthrust equals the weight of the object
- The magnitude of upthrust can be calculated by:



Upthrust equation

4. Forces, Density & Pressure

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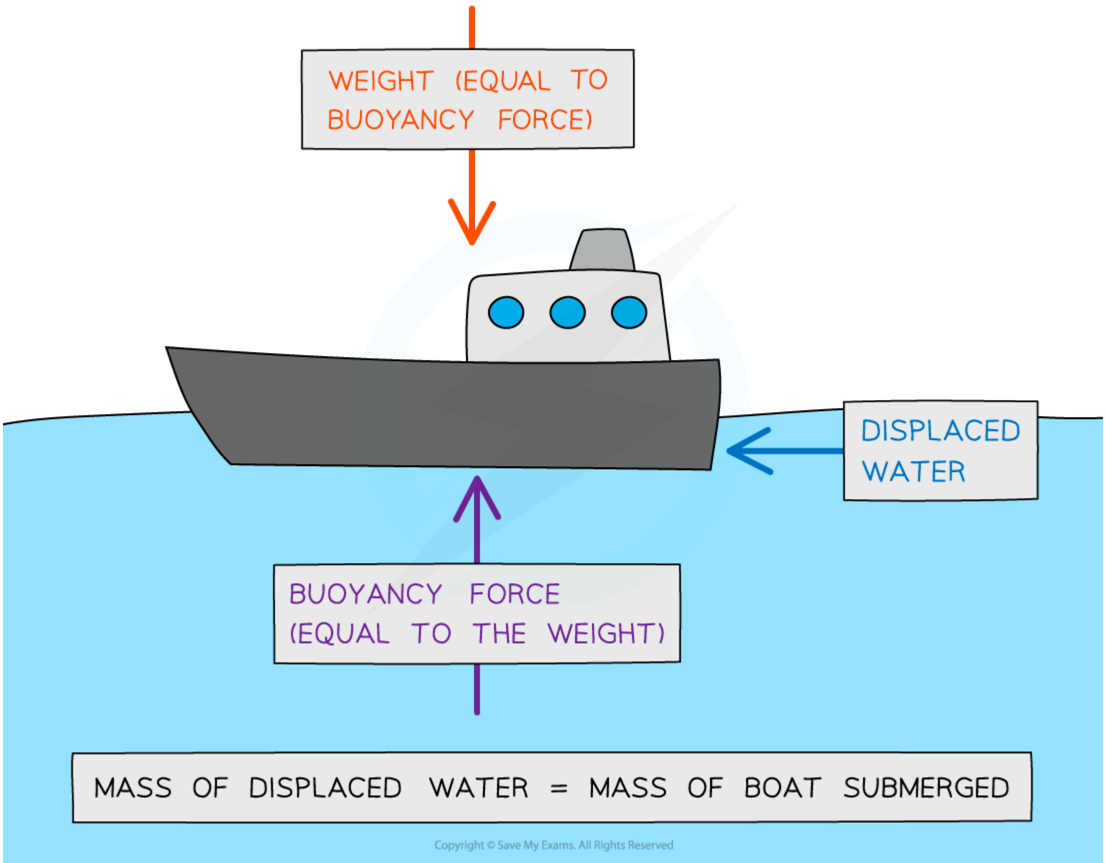
- Since $m = \rho V$, upthrust is equal to $F = mg$ which is the weight of the fluid displaced by the object
- Archimedes' Principle explains how ships float:



Boats float because they displace an amount of water that is equal to their weight

4. Forces, Density & Pressure

YOUR NOTES

Worked example

Icebergs typically float with a large volume of ice beneath the water. Ice has a density of 917 kgm^{-3} and a volume V_i .

The density of seawater is 1020 kgm^{-3}.

What fraction of the iceberg is above the water?

A. 0.10 V_i **B.** 0.90 V_i **C.** 0.97 V_i **D.** 0.20 V_i

ANSWER: A

STEP 1

ACCORDING TO ARCHIMEDES' PRINCIPLE, THE UPTHRUST IS EQUAL TO THE WEIGHT OF THE SEAWATER DISPLACED BY THE ICEBERG

ICEBERG WEIGHT $W_i = m_i g$

BUOYANCY FORCE IS THE WEIGHT OF THE DISPLACED WATER $W_w = m_w g$

STEP 2

SINCE THE ICEBERG IS FLOATING, ITS WEIGHT IS EXACTLY EQUAL TO THE BUOYANCY FORCE

$$W_i = W_w$$

$$m_i g = m_w g$$

4. Forces, Density & Pressure

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STEP 3

REARRANGE DENSITY EQUATION FOR MASS

$$m = \rho V$$

$$\rho_i V_i g = \rho_w V_w g$$

STEP 4

CANCELLING g SHOWS THE FRACTION OF ICE UNDERWATER IS GIVEN BY RATIO OF DENSITIES

$$\frac{V_i}{V_w} = \frac{\rho_w}{\rho_i}$$

STEP 5

REARRANGE FOR V_w

$$V_w = \frac{\rho_i V_i}{\rho_w}$$

$$V_w = \frac{917}{1020} V_i = 0.9 V_i$$

STEP 6

THEREFORE 90% OF THE ICEBERG'S VOLUME IS SUBMERGED UNDERWATER

THIS MEANS THAT $1 - 0.9 = 0.1\ V_i$ IS ABOVE WATER

Exam Question: Easy

The formula for hydrostatic pressure is $P = \rho g h$

Which equation, or principle of physics, is used in the derivation of this formula?

A density = mass ÷ volume

B potential energy = *mgh*

C atmospheric pressure decreases with height

D density increases with depth

4. Forces, Density & Pressure

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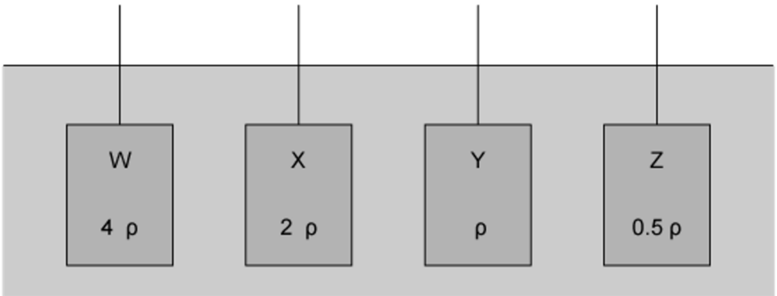
Exam Question: Medium

Four cuboids with identical length, breadth and height are immersed in water. The cuboids are held at the same depth and in identical orientations by vertical rods, as shown.



Water has density ρ
Cuboid W is made of material of density 4ρ
Cuboid X is made of material of density 2ρ
Cuboid Y is made of material of density ρ
Cuboid Z is made of material of density ½ ρ

Which statement is correct?

A the upthrust of the water on each of the cuboids is the same

B the upthrust of the water on W is twice the upthrust of the water on X

C the upthrust of the water on Z is twice the upthrust of the water on Y

D the upthrust of the water on Y is zero

Exam Question: Hard

The density of water is 1.0 g cm^{-3} and the density of glycerine is 1.3 g cm^{-3}

Water is added to a measuring cylinder containing 40 cm^3 of glycerine so that the density of the mixture is 1.1 g cm^{-3}. Assume that the mixing process does not change the total volume of the liquid

What is the volume of water added?

A 40 cm^3 **B** 44 cm^3 **C** 52 cm^3 **D** 80 cm^3

4. Forces, Density & Pressure

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5. Work, Energy & Power

YOUR NOTES

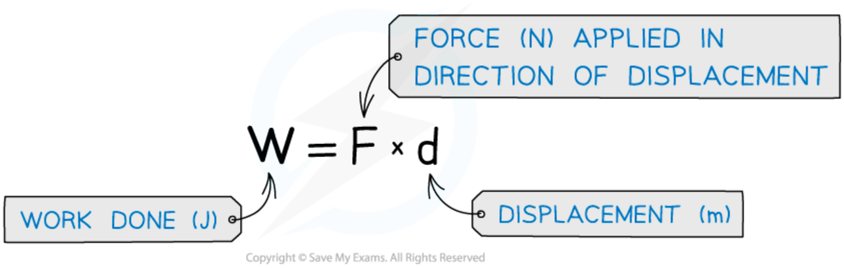
CONTENTS

5.1 ENERGY: CONSERVATION, WORK, POWER & EFFICIENCY

5.1.1 WORK & ENERGY

Work Done

- In Physics, **work is done** when an **object** is **moved** over a distance by an external force applied in the direction of its displacement



- In the diagram below, the man's pushing force on the block is doing work as it is transferring energy to the block (increasing its kinetic energy)

5. Work, Energy & Power

YOUR NOTES
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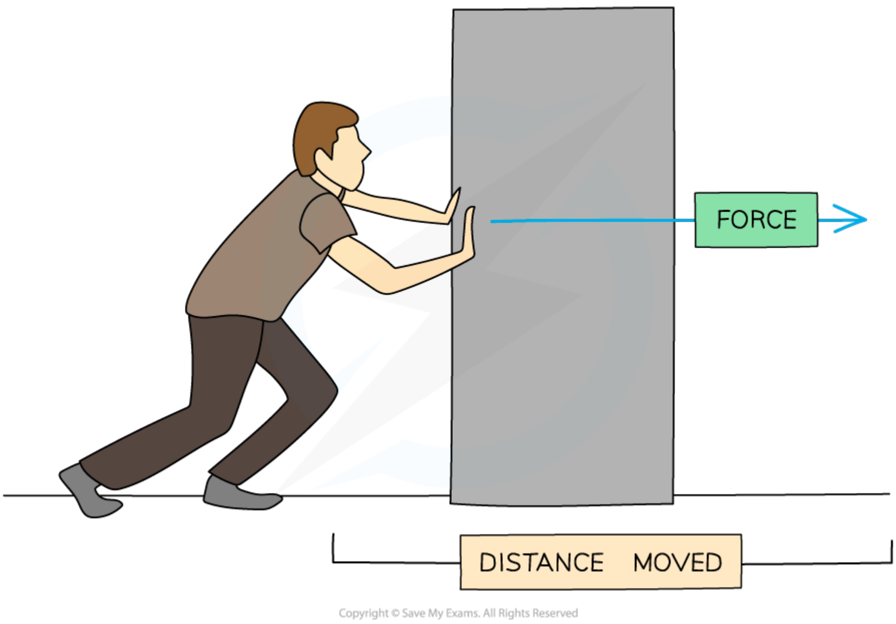


Work is done when a force is used to move an object over a distance

- When work is done, energy is transferred from one object to another
- Work done can be thought of as the amount of **energy transferred**, hence its units are in **Joules (J)**
- Usually, if a **force** acts **in the direction** that an object is moving then the object will **gain energy**
- If the **force** acts in the **opposite direction** to the movement then the object will **lose energy**

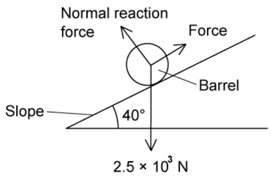
5. Work, Energy & Power

YOUR NOTES

Worked example

?

The diagram shows a barrel of weight 2.5×10^3 N on a frictionless slope inclined at 40° to the horizontal.



A force is applied to the barrel to move it up the slope at constant speed. The force is parallel to the slope.

What is the work done in moving the barrel a distance of 6.0 m up the slope?

A. 7.2×10^3 J **B.** 2.5×10^4 J **C.** 1.1×10^4 J **D.** 9.6×10^3 J

ANSWER: D

STEP 1

WORK DONE EQUATION

$W = F \times d$

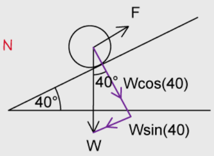
STEP 2

CALCULATE THE FORCE IN THE DIRECTION OF TRAVEL

THE FORCE NEEDED TO PUSH THE BARREL NEEDS TO OVERCOME THE COMPONENT OF THE BARREL'S WEIGHT. SINCE THE FORCE IS PARALLEL TO THE SLOPE, THE COMPONENT OF THE WEIGHT WE NEED IS THE ONE PARALLEL TO THE SLOPE.

$F = W\sin(40) = 2.5 \times 10^3 \times \sin(40) = 1607$ N

THIS IS THE FORCE IN THE SAME DIRECTION AS THE DISPLACEMENT



STEP 3

SUBSTITUTE F AND d INTO THE WORK DONE EQUATION

$W = 1607\text{ N} \times 6.0\text{ m} = 9.6 \times 10^3$ J

Exam Tip

A common exam mistake is choosing the incorrect force which is not parallel to the direction of movement of an object. You may have to resolve the force vector to find the component that is parallel. The force does not have to be in the same direction as the movement, as shown in the worked example.

5. Work, Energy & Power

YOUR NOTES

5.1.2 THE PRINCIPLE OF CONSERVATION OF ENERGY

The Principle of Conservation of Energy

- The Principle of Conservation of Energy states that:
  - **Energy cannot be created or destroyed, it can only change from one form to another**
- This means the total amount of energy in a closed system remains constant, although how much of each form there is may change
- Common examples of energy transfers are:
  - A falling object (in a vacuum): gravitational potential energy → kinetic energy
  - A battery: chemical energy → electrical energy → light energy (if connected to a bulb)
  - Horizontal mass on a spring: elastic potential energy → kinetic energy

Types of energy

FORM	WHAT IS IT?
KINETIC	THE ENERGY OF A MOVING OBJECT.
GRAVITATIONAL POTENTIAL	THE ENERGY SOMETHING GAINS WHEN YOU LIFT IT UP, AND WHICH IT LOSES WHEN IT FALLS.
ELASTIC	THE ENERGY OF A STRETCHED SPRING OR ELASTIC BAND.(SOMETIMES CALLED STRAIN ENERGY)
CHEMICAL	THE ENERGY CONTAINED IN A CHEMICAL SUBSTANCE.
NUCLEAR	THE ENERGY CONTAINED WITHIN THE NUCLEUS OF AN ATOM.
INTERNAL	THE ENERGY SOMETHING HAS DUE TO ITS TEMPERATURE (OR STATE). (SOMETIMES REFERRED TO AS THERMAL OR HEAT ENERGY)

5. Work, Energy & Power

YOUR NOTES

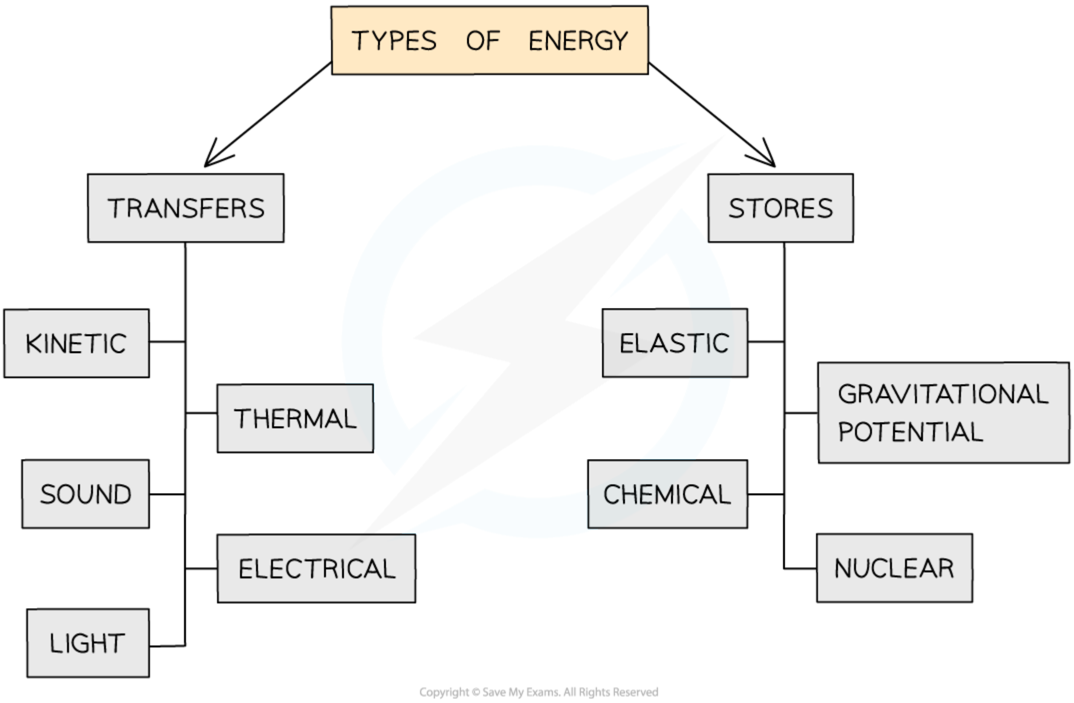


Diagram showing the forms of energy transfers and stores

5. Work, Energy & Power

YOUR NOTES

Energy dissipation

- When energy is transferred from one form to another, not all the energy will end up in the desired form (or place)
- Dissipation is used to describe ways in which energy is wasted
- Any energy not transferred to useful energy stores is wasted because it is lost to the surroundings
- These are commonly in the form of **thermal (heat), light or sound** energy
- What counts as **wasted energy** depends on the system
- For example, in a **television**:

  electrical energy → light energy + sound energy + thermal energy

  - Light and sound energy are useful energy transfers whereas thermal energy (from the heating up of wires) is wasted

- Another example, in a **heater**:

  electrical energy → thermal energy + sound energy

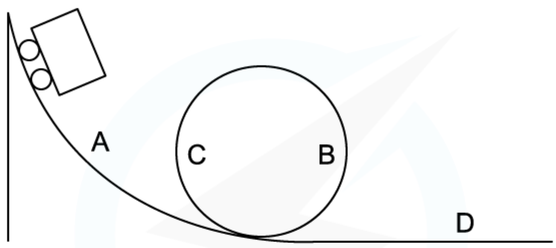
  - The thermal energy is useful, whereas sound is not

5. Work, Energy & Power

YOUR NOTES

Worked example

The diagram shows a rollercoaster going down a track.
The rollercoater takes the path A → B → C → D.



Which statement is true about the energy changes that occur for that occur for the rollercoaster down this track?

A. KE - GPE - GPE - KE
B. KE - GPE - KE - GPE
C. GPE - KE - KE - GPE
D. GPE - KE - GPE - KE

ANSWER: D

- **At point A:**
  - The rollercoaster is raised above the ground, therefore it has **GPE**
  - As it travels down the track, **GPE** is converted to **KE** and the roller coaster speeds up
- **At point B:**
  - **KE** is converted to **GPE** as the rollercoaster rises up the loop
- **At point C:**
  - This **GPE** is converted back into **KE** as the rollercoaster travels back down the loop
- **At point D:**
  - The flat terrain means the rollercoaster only has **KE**

5. Work, Energy & Power

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Exam Tip

You may not always be given the energy transfers happening in the system in exam questions.
By familiarising yourself with the transfers and stores of energy, you will be expected to relate these to the situation in question. For example, a ball rolling down a hill is transferring gravitational potential energy to kinetic energy whilst a spring converts elastic potential energy into kinetic energy.

5. Work, Energy & Power

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5.1.3 EFFICIENCY

Efficiency of a System

- The efficiency of a system is **the ratio of the useful energy output from the system to the total energy input**
    - If a system has high efficiency, this means most of the energy transferred is useful
    - If a system has low efficiency, this means most of the energy transferred is wasted
- Multiplying this ratio by 100 gives the efficiency as a percentage

$$\text{EFFICIENCY} = \frac{\text{USEFUL ENERGY OUTPUT}}{\text{TOTAL ENERGY INPUT}} \times 100\%$$

Efficiency equation in terms of energy

- Efficiency can also be written in terms of power (the energy per second):

$$\text{EFFICIENCY} = \frac{\text{USEFUL POWER OUTPUT}}{\text{TOTAL POWER INPUT}} \times 100\%$$

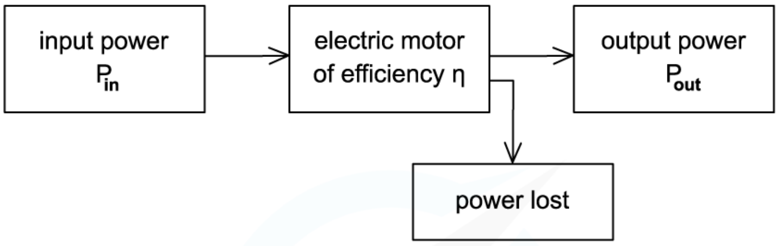
Efficiency equation in terms of power

5. Work, Energy & Power

YOUR NOTES

Worked example

An electric motor has an input power P_{in}, useful output power P_{out} and efficiency η.



What is the output power P_{out} of the motor?

A. ηP_{in} **B.** $\dfrac{-\eta P_{lost}}{\eta - 1}$ **C.** ηP_{lost} **D.** $-\eta P_{lost}(\eta - 1)$

ANSWER: B

EFFICIENCY = $\dfrac{\text{USEFUL POWER OUTPUT}}{\text{USEFUL POWER INTPUT}}$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{lost}}$$

MULTIPLY BY $P_{out} + P_{lost}$ ON BOTH SIDES $\quad \eta(P_{out} + P_{lost}) = P_{out}$

EXPAND THE BRACKETS $\quad \eta P_{out} + \eta P_{lost} = P_{out}$

$-P_{out}$ FROM BOTH SIDES $\quad \eta P_{out} - P_{out} = -\eta P_{lost}$

TAKE POUT AS A FACTOR $\quad P_{out}(\eta - 1) = -\eta P_{lost}$

DIVIDE BY $\eta - 1$ $\quad P_{out} = \dfrac{-\eta P_{lost}}{\eta - 1}$

Exam Tip

Efficiency can be in a ratio or percentage format. If the question asks for an efficiency as a ratio, give your answer as a fraction or decimal. If the answer is required as a percentage, remember to multiply the ratio by 100 to convert it, e.g. Ratio = 0.25, Percentage = 0.25 × 100 = 25 %

5. Work, Energy & Power

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Solving Problems Involving Efficiency

- Recall the two equations for calculating efficiency are:

$$\text{EFFICIENCY} = \frac{\text{USEFUL ENERGY OUTPUT}}{\text{TOTAL ENERGY INPUT}} \times 100\%$$

$$\text{EFFICIENCY} = \frac{\text{USEFUL POWER OUTPUT}}{\text{TOTAL POWER INPUT}} \times 100\%$$

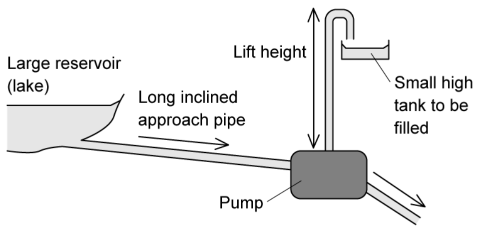
- Which to use will depend on whether you're given a system calculating energies or power as shown in the examples below

5. Work, Energy & Power

YOUR NOTES

Worked example

The diagram shows a pump called a hydraulic ram.



In one such pump the long approach pipe holds 700 kg of water. A value shuts when the speed of this water reaches 3.5 ms^{-1} and the kinetic energy of this water is used to lift a small quantity of water by height of 12 m.

The efficiency of the pump is 20%.

Which mass of water could be lifted 12 m?

A. 6.2 kg **B.** 4.6 kg **C.** 7.3 kg **D.** 0.24 kg

ANSWER: C

THE KINETIC ENERGY OF THE WATER IS CONVVERTED TO GRAVITATIONAL POTENTIAL ENERGY WHEN LIFTED BY 12m

$$KE = GPE$$

$$\frac{1}{2}mv^2 = mgh$$

SINCE EFFICIENCY IS 20% ONLY 20% OF THE KINETIC ENERGY WILL BE CONVERTED.

$$0.2 \times \frac{1}{2}mv^2 = mgh$$

$$0.2 \times \frac{1}{2} \times 700 \times (3.5)^2 = m \times 9.81 \times 12$$

$$857.5 = m \times 117.72$$

$$\frac{857.5}{117.72} = m$$

$$m = 7.3 \text{ kg (2 s.f.)}$$

- The pump is what converts the water's **kinetic energy** into **gravitational potential energy.** Since its efficiency is 20%, you would multiply the kinetic energy by 0.2 since only 20% of the **kinetic energy** will be converted (**not** 20% of the gravitational potential energy)

5. Work, Energy & Power

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Exam Tip

Equations for kinetic and potential energies are important for these types of questions. Also familiarise yourself with the different equations for power depending on what quantities are given.

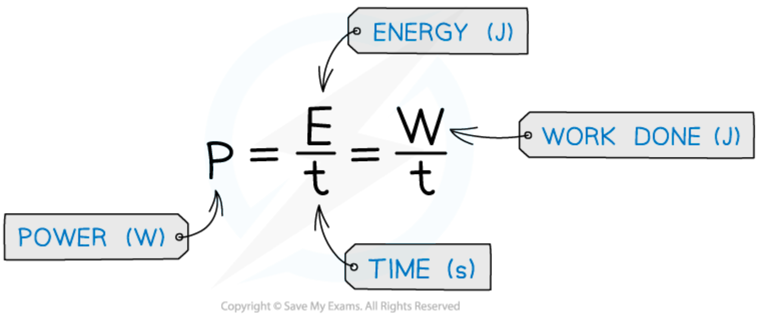
5. Work, Energy & Power

YOUR NOTES

5.1.4 POWER

Defining Power

- The power of a machine is the **rate at which it transfers energy**
- Since work done is equal to the energy transferred, power can also be defined as the rate of doing work or **the work done per unit time**
- The SI unit for power is **Watts (W)** where **1 W** = **1 J s^{-1}**



Power is the rate of change of work

- You may be familiar with labels on lightbulbs which indicate their power such as 60 W or 100 W. These tell you about an energy transferred by an electrical current rather than by a force doing work

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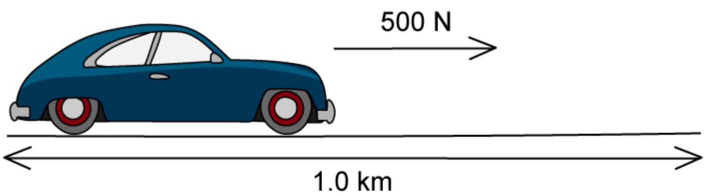
Solving Problems Involving Power

Worked example

A car engine exerts the following force for 1.0 km in 200 s.



What is the average power developed by the engine?

STEP 1 EQUATION FOR POWER

$$\text{POWER} = \frac{\text{WORK DONE}}{\text{TIME}}$$

STEP 2 CALCULATE WORK DONE

$W = F \times d$

$= 500\ \text{N} \times 1.0 \times 10^3\ \text{m}$

$= 5 \times 10^5\ \text{J}$

STEP 3 SUBSTITUTE VALUES INTO POWER EQUATION

$$\text{POWER} = \frac{5 \times 10^5\ \text{J}}{200\ \text{s}} = 2500\ \text{W} = 2.5\ \text{kW}$$

Exam Tip

Think of power as "energy per second". Thinking of it this way will help you to remember the relationship between power and energy: "Watt is the unit of power?"

5. Work, Energy & Power

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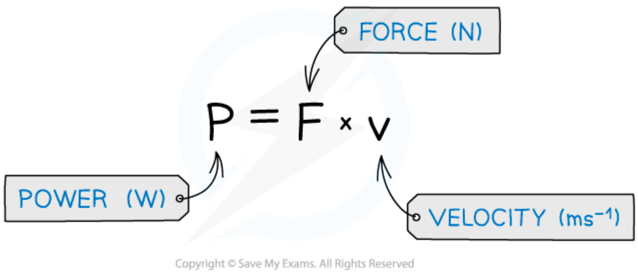
5.1.5 DERIVATION OF P = FV

Derivation of P = Fv

- Moving power is defined by the equation:



- This equation is only relevant where a **constant force** moves a body at **constant velocity**. Power is required in order to produce an acceleration
- The force must be applied in the **same** direction as the velocity

5. Work, Energy & Power

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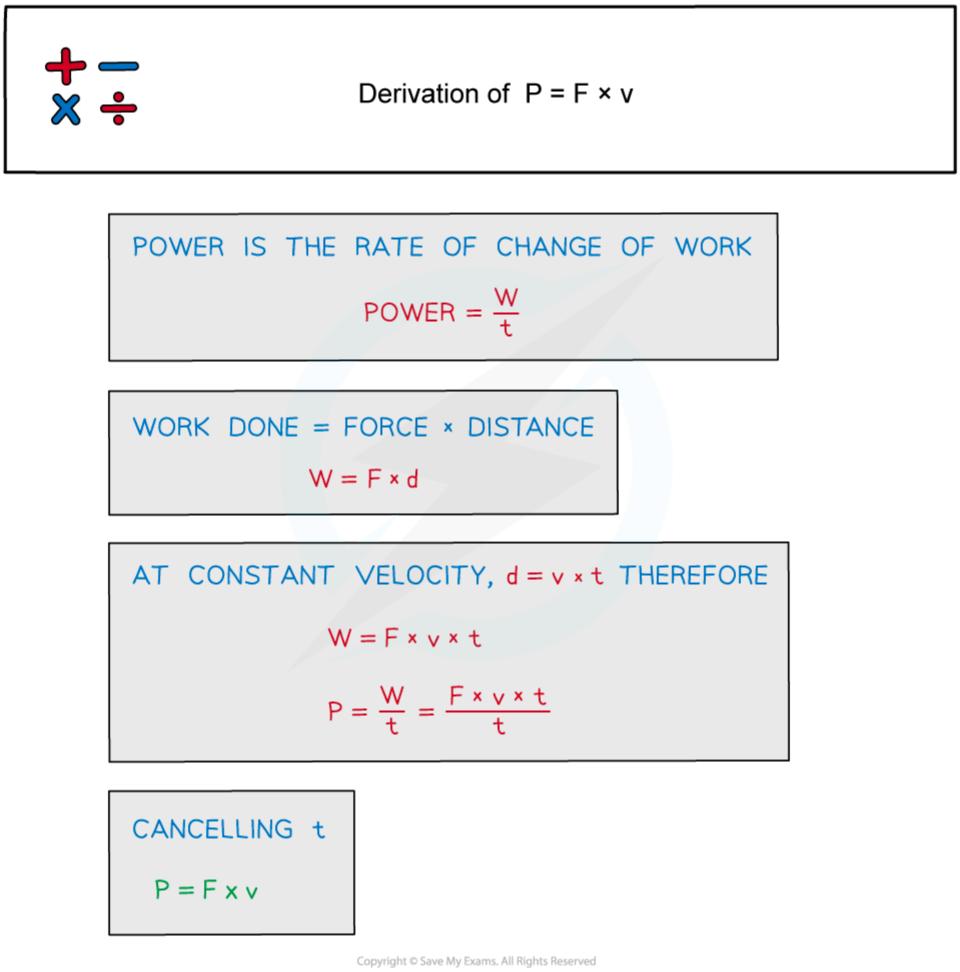
Derivation

- The derivation for this equation is shown below:



Derivation of P = F × v

5. Work, Energy & Power

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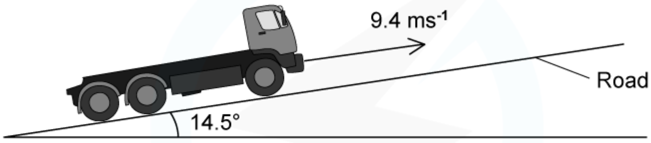
Worked example

A lorry moves up a road that is inclined at 14.5° to the horizontal.



The lorry has mass 3500 kg and is travelling at a constant speed of 9.4 ms^{-1}. The force due to air resistance is negligible.

Calculate the useful power from the engine to move the lorry up the road.

5. Work, Energy & Power

YOUR NOTES

STEP 1

EQUATION FOR POWER

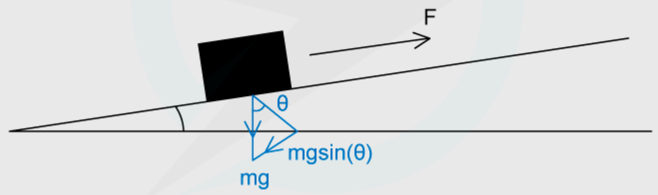
$P = F \times v$

STEP 2

CALCULATE THE FORCE

THE FORCE NEEDED TO MOVE THE LORRY UP THE ROAD IS THAT WHICH OVERCOMES THE COMPONENT OF ITS WEIGHT ACTING DOWN THE SLOPE

$F = mg\sin\theta = 3500 \times 9.81 \times \sin(14.5) = 8596.8\ \text{N}$



STEP 3

SUBSTITUTE INTO POWER EQUATION

$P = 8\,596.8 \times 9.4 = 80\,809.9\ \text{W} = 81\,000\ \text{W} = 81\ \text{kW}$ (2.s.f.)

Exam Tip

The force represented in exam questions will often be a drag force. Whilst this is in the opposite direction to its velocity, remember the force needed to calculate the power is equal to (or above) this drag force to overcome it therefore you equate it to that value.

5. Work, Energy & Power

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Exam Question: Easy

Which of the following is a statement of the principle of conservation of energy?

A energy is the product of force and distance

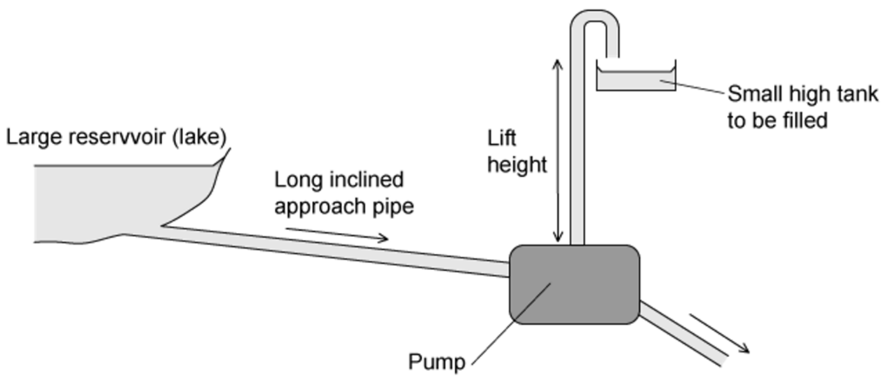
B energy cannot be created, destroyed or transformed

C the total energy of an isolated system is constant

D the power transmitted by a system is proportional to the rate of transfer of energy

Exam Question: Medium

The diagram shows a pump called a hydraulic ram



In one such pump the long approach pipe holds 500 kg of water. A valve shuts when the speed of this water reaches 2.0 m s^{-1} and the kinetic energy of this water is used to lift a small quantity of water by a height of 15 m.

The efficiency of the pump is 10%.

Which mass of water could be lifted 15 m?

A 0.15 kg **B** 0.68 kg **C** 1.5 kg **D** 6.8 kg

5. Work, Energy & Power

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Exam Question: Hard

A turbine at a hydroelectric power station is situated 30 m below the level of the surface of a large lake. The water passes through the turbine at a rate of 340 m^3 per minute.

The overall efficiency of the turbine and generator system is 90%.

What is the output power of the power station? (The density of water is 1000 kg m^{-3})

A 0.15 MW **B** 1.5 MW **C** 1.7 MW **D** 90 MW

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5. Work, Energy & Power

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5.2 ENERGY: GPE & KE

5.2.1 GRAVITATIONAL POTENTIAL ENERGY

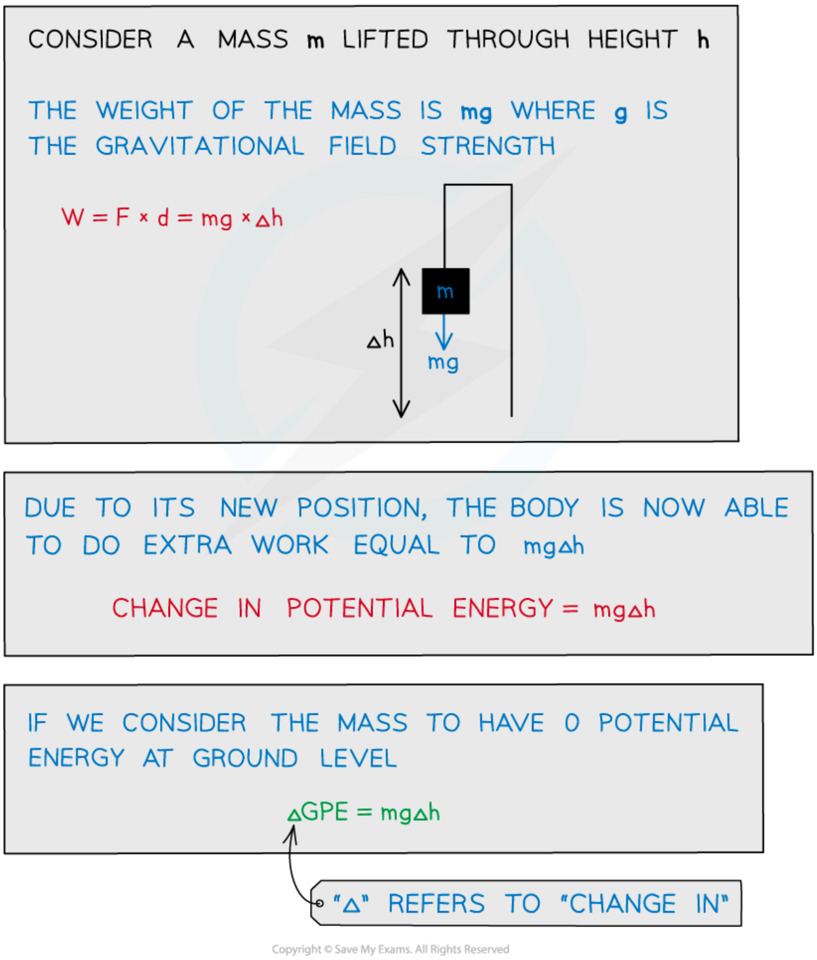
Derivation of GPE = mgh

- Gravitational potential energy is energy stored in a mass due to its position in a gravitational field
- When a heavy object is lifted, work is done since the object is provided with an upward force against the downward force of gravity
  - Therefore **energy is transferred to the object**
- This equation can therefore be derived from the work done

5. Work, Energy & Power

YOUR NOTES

Derivation of GPE = mgh



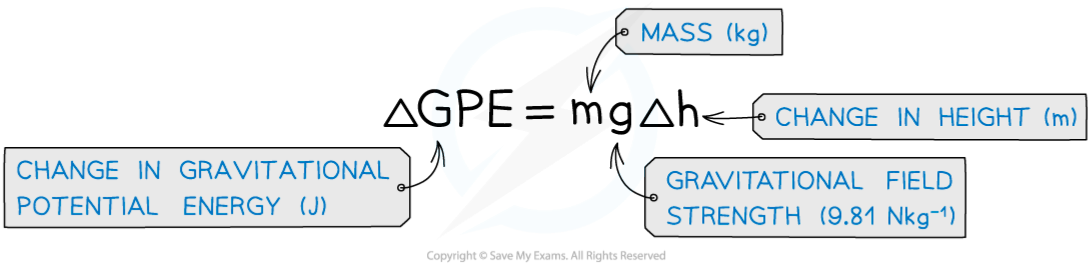
Derivation of GPE = mgh

5. Work, Energy & Power

YOUR NOTES

Gravitational Potential Energy

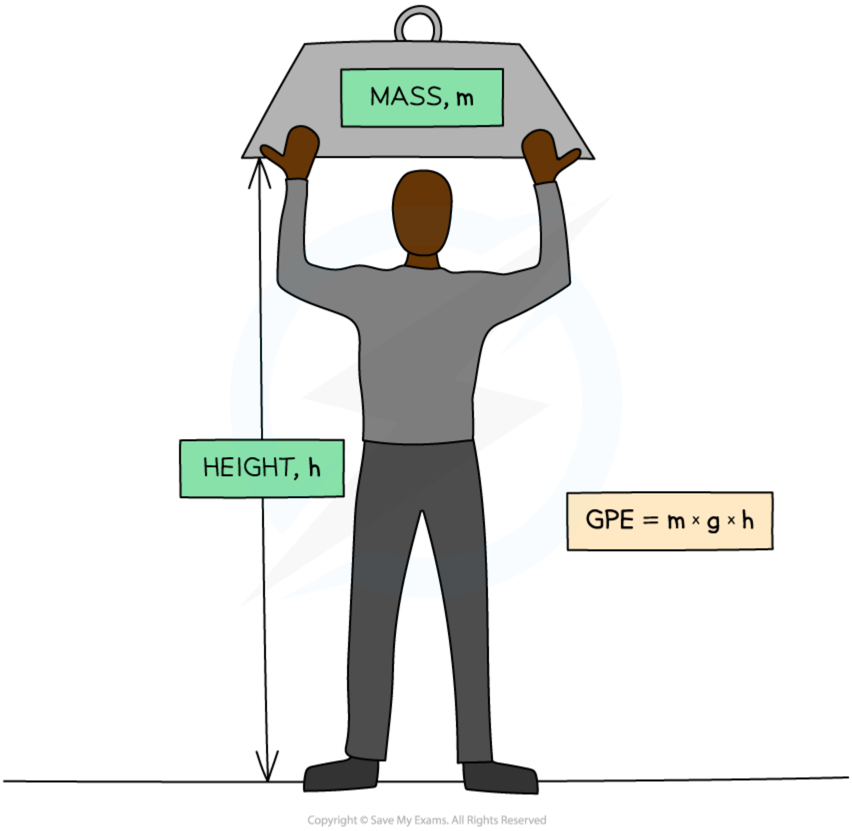
- Gravitational potential energy (GPE) is energy stored in a mass due to its position in a gravitational field
  - If a mass is **lifted** up, it will **gain** GPE (converted **from** other forms of energy)
  - If a mass **falls**, it will **lose** GPE (and be converted **to** other forms of energy)
- The equation for gravitational potential energy for energy changes in a **uniform gravitational field** is:



Equation for GPE

5. Work, Energy & Power

YOUR NOTES



GPE: The energy an object has when lifted up

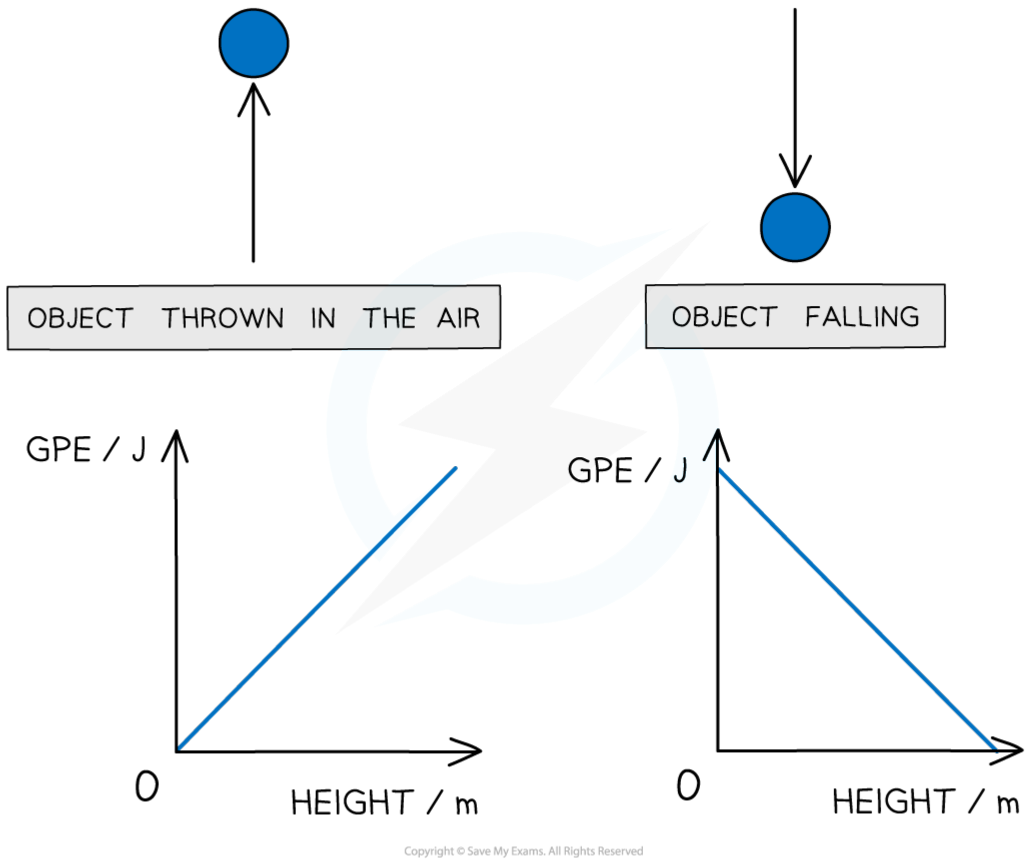
- The potential energy on the Earth's surface at ground level is taken to be equal to 0
- This equation is only relevant for energy changes in a **uniform gravitational field** (such as near the Earth's surface)

5. Work, Energy & Power

YOUR NOTES ⬇

GPE v Height graphs

- The two graphs below show how GPE changes with height for a ball being thrown up in the air and when falling down



Graphs showing the linear relationship between GPE and height

- Since the graphs are straight lines, GPE and height are said to have a **linear** relationship
- These graphs would be identical for GPE against time instead of height

5. Work, Energy & Power

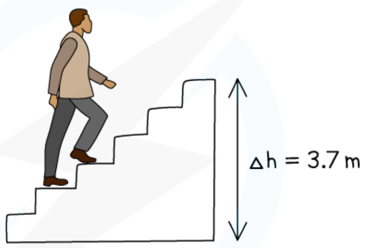
YOUR NOTES

Worked example

To get to his apartment a man has to climb five flights of stairs.
The height of each flight is 3.7 m and the man has a mass of 74 kg.
What is the approximate gain in the man's gravitational potential energy during the climb?

A. 13 000 J **B.** 2700 J **C.** 1500 J **D.** 12 500 J

ANSWER: A



STEP 1

GPE EQUATION

$\Delta GPE = mg\Delta h$

STEP 2

FIND h

$\Delta h = 5 \times 3.7\,m = 18.5\,m$

5 FLIGHTS OF STAIRS

STEP 3

SUBSTITUTE VALUES INTO GPE EQUATION

$\Delta GPE = 74 \times 9.81 \times 18.5 = 13000$ J (2 s.f.)

Exam Tip

This equation only works for objects close to the Earth's surface where we can consider the gravitational field to be uniform. In A2 level, you will consider examples where the gravitational field is not uniform such as in space, where this equation for GPE will not be relevant.

5. Work, Energy & Power

YOUR NOTES
⬇

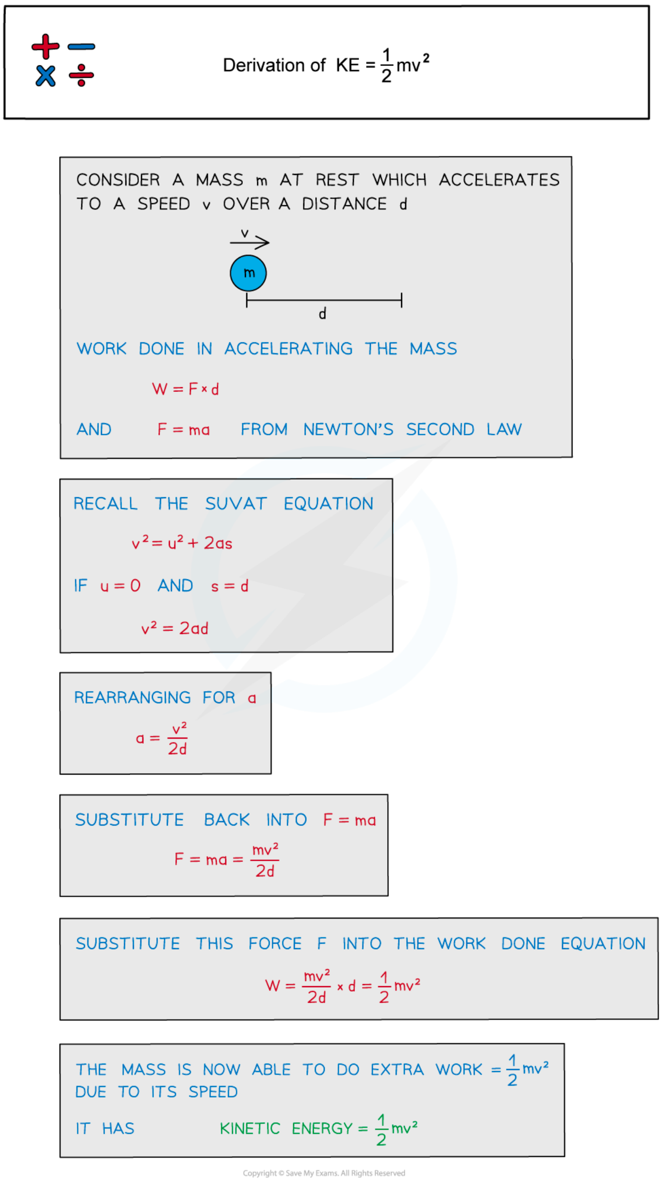
5.2.2 KINETIC ENERGY

Derivation of KE = 1/2mv2

- Kinetic energy is energy an object has due to its **motion** (or velocity)
- A force can make an object accelerate; work is done by the force and energy is transferred to the object
- Using this concept of work done and an equation of motion, the extra work done due to an object's speed can be derived
- The derivation for this equation is shown below:

5. Work, Energy & Power

YOUR NOTES
⬇



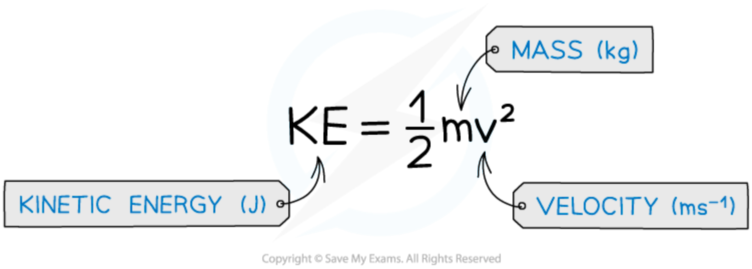
Derivation for Kinetic Energy equation

5. Work, Energy & Power

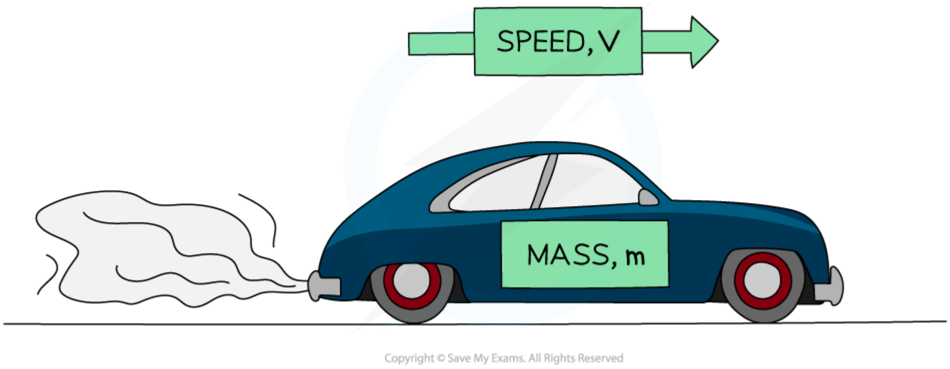
YOUR NOTES

Kinetic Energy

- Kinetic energy is energy an object has due to its **motion** (or velocity)
  - The faster an object is moving, the greater its kinetic energy
- When an object is falling, it is **gaining** kinetic energy since it is gaining speed. This energy transferred from the gravitational potential energy it is losing
- An object will maintain this kinetic energy unless its speed changes



Equation for Kinetic energy



KE: The energy an object has when its moving

5. Work, Energy & Power

YOUR NOTES

Worked example

A body travelling with a speed of 12 ms^{-1} has kinetic energy 1650 J.

If the speed of the body is increased to 45 ms^{-1}, what is its new kinetic energy?

STEP 1

EQUATION FOR KINETIC ENERGY

$$KE = \frac{1}{2}mv^2$$

STEP 2

MASS WILL NOT CHANGE, SO CAN BE CALCULATED FROM ITS INITIAL KINETIC ENERGY

REARRANGE FOR MASS m

$$m = \frac{2 \times KE}{v^2} = \frac{2 \times 1650}{12^2} = 23 \text{ kg}$$

STEP 3

SUBSTITUTE INTO KINETIC ENERGY EQUATION

USING VALUE OF MASS AND NEW VALUE OF VELOCITY

$$KE = \frac{1}{2} \times 23 \times 45^2 = 23000 \text{ J} \quad (2 \text{ s.f})$$

Exam Tip

When using the kinetic energy equation, note that only the speed is squared, not the mass or the ½.

If a question asks about the 'loss of kinetic energy', remember not to include a negative sign since energy is a scalar quantity.

5. Work, Energy & Power

YOUR NOTES

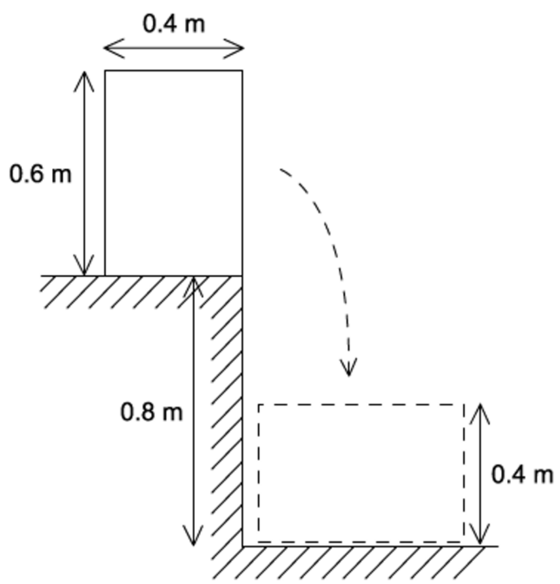
Exam Question: Easy

What is the approximate kinetic energy of an Olympic athlete when running at maximum speed during a 100m race?

A 400 J **B** 4000 J **C** 40 000 J **D** 400 000 J

Exam Question: Medium

A uniform solid block has weight 500 N, width 0.4 m and height 0.6 m. The block rests on the edge of a step of depth 0.8 m, as shown.



The block is knocked over the edge of the step and rotates through 90° before coming to rest with the 0.6 m edge horizontal.

What is the change in gravitational potential energy of the block?

A 300 J **B** 400 J **C** 450 J **D** 550 J

5. Work, Energy & Power

YOUR NOTES

Exam Question: Hard

A loaded aeroplane has a total mass of 1.2×10^5 kg while climbing after take-off. It climbs at an angle of 23° to the horizontal with a speed of 50 m s^{-1}.

What is the rate at which it is gaining potential energy at this time?

A 2.3×10^6 J s^{-1}

B 2.5×10^6 J s^{-1}

C 2.3×10^7 J s^{-1}

D 2.5×10^7 J s^{-1}

> CHECK YOUR ANSWERS AT SAVEMYEXAMS.CO.UK

6. Deformation of Solids

YOUR NOTES
⬇

CONTENTS

6.1 DEFORMATION: STRESS & STRAIN

6.1.1 EXTENSION & COMPRESSION

Tensile Force

- Forces don't just change the motion of a body, but can change the size and **shape** of them too. This is known as **deformation**
- Forces in opposite directions stretch or compress a body
  - When two forces **stretch** a body, they are described as **tensile**
  - When two forces **compress** a body, they are known as **compressive**

6. Deformation of Solids

YOUR NOTES

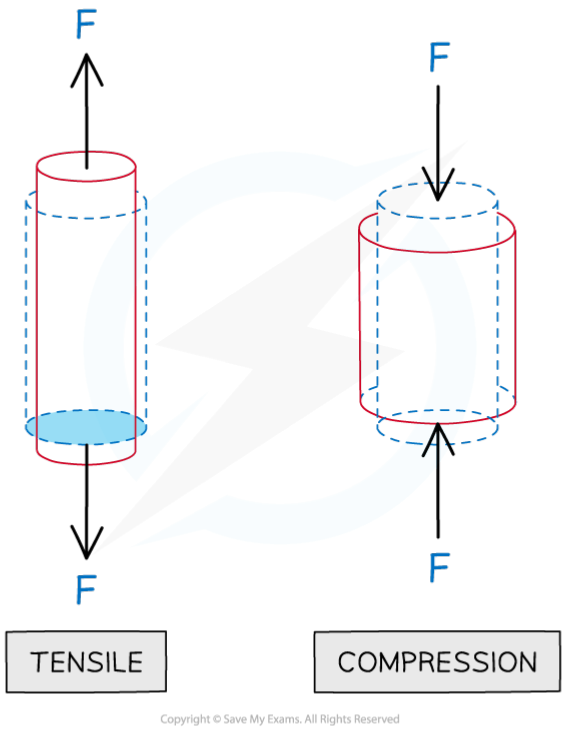


Diagram of tensile and compressive forces

6. Deformation of Solids

YOUR NOTES

Tensile Strength

- Tensile strength is the amount of load or stress a material can handle until it stretches and breaks
- Here are some common materials and their tensile strength:

Tensile strength of various materials

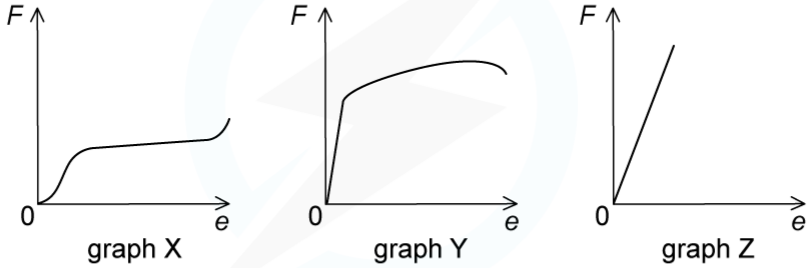
Material	Tensile Strength (MPa)
Concrete	2–5
Rubber	16
Human skin	20
Glass	33
Human hair	200
Steel	840
Diamond	2800

6. Deformation of Solids

YOUR NOTES
⬇

Worked example

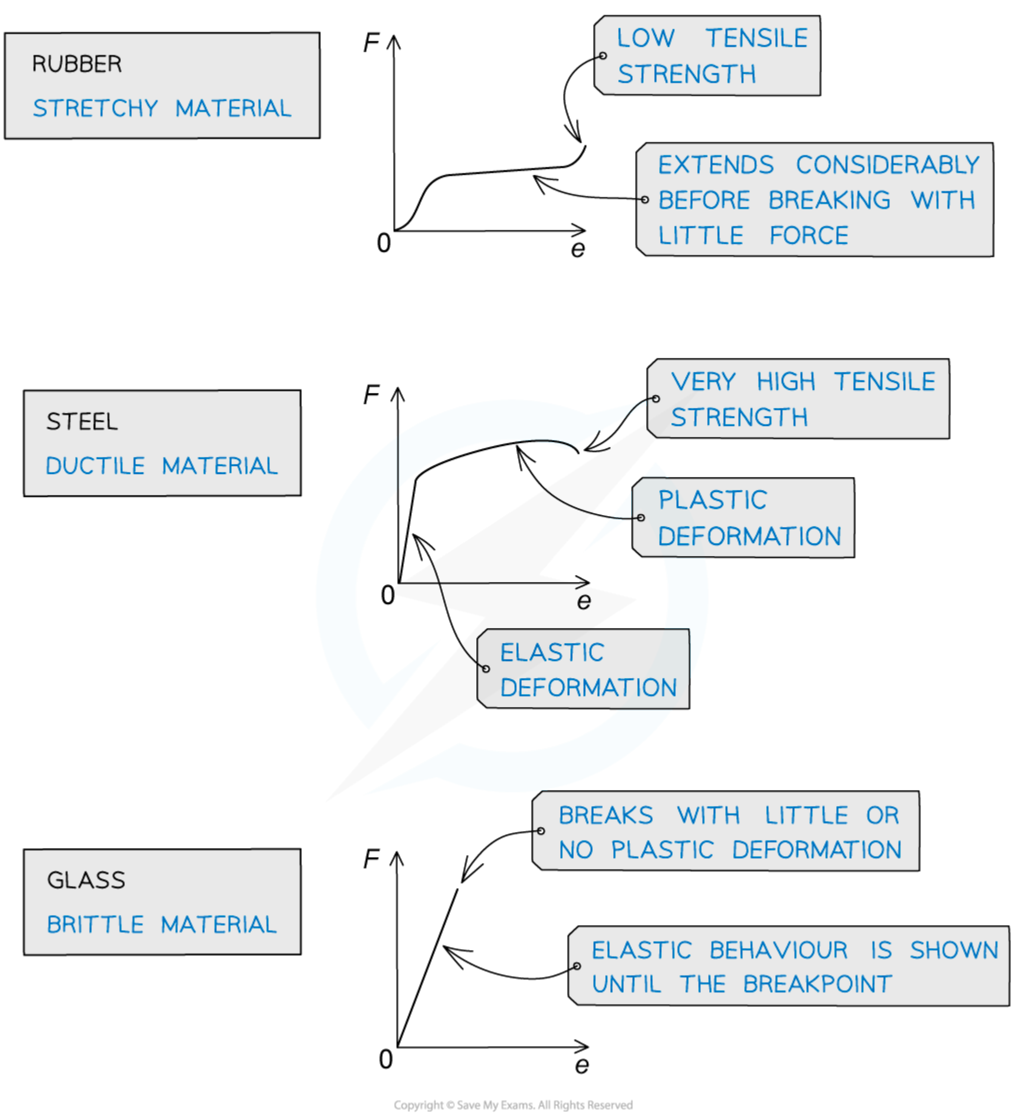
Cylindrical samples of steel, glass and rubber are each subjected to a gradually increasing tensile force *F*.
The extensions *e* are measured and graphs are plotted as shown below.



Correctly label the graphs with the materials: steel, glass, rubber.

6. Deformation of Solids

YOUR NOTES
⬇



Exam Tip

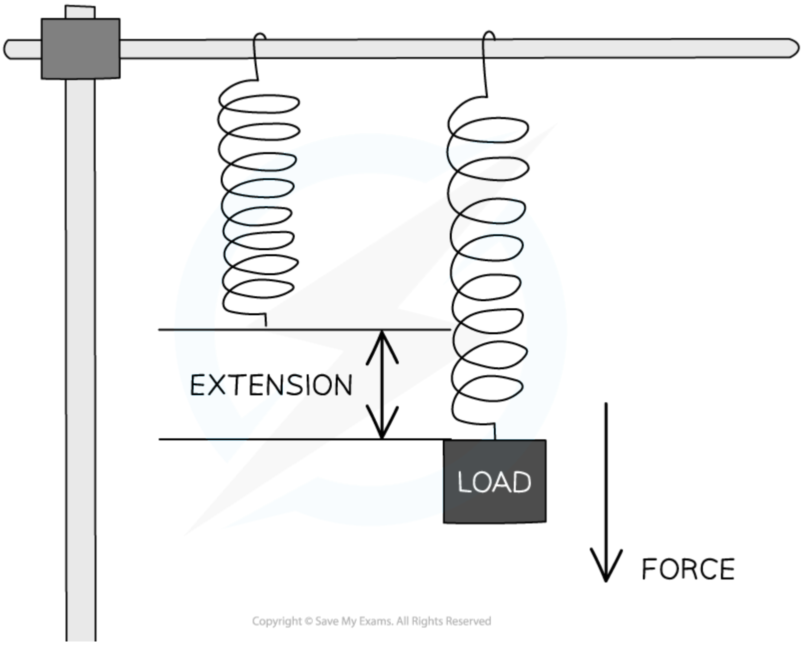
Remember to read the questions carefully in order to not confuse the terms 'tensile stress' and 'tensile strain'.

6. Deformation of Solids

YOUR NOTES
⬇

Extension and Compression

- When you apply a force (load) onto a spring, it produces a tensile force and causes the spring to extend



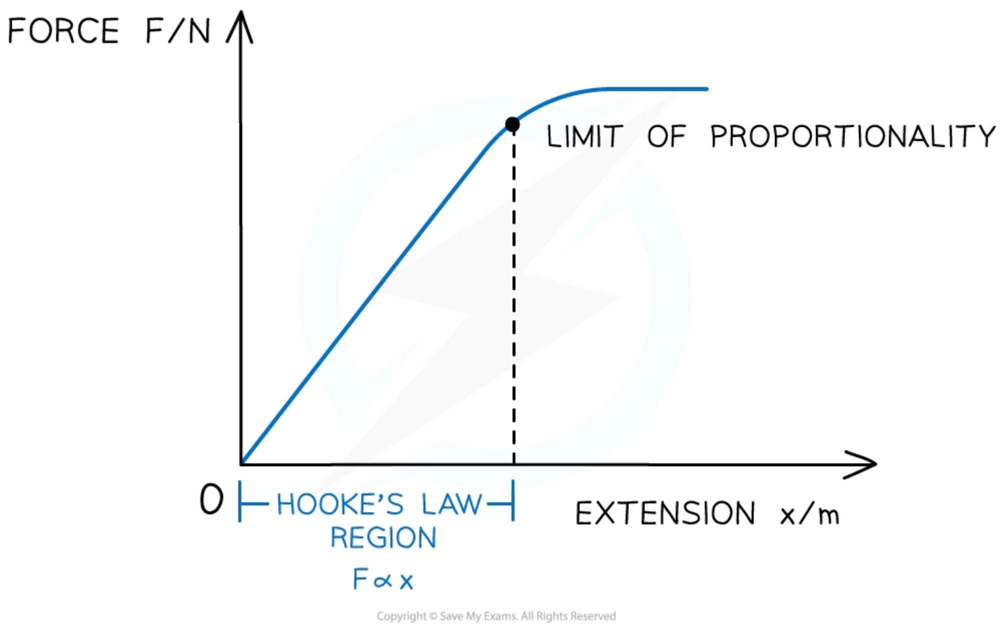
Stretching a spring with a load produces a force that leads to an extension

6. Deformation of Solids

YOUR NOTES

Hooke's Law

- If a material responds to tensile forces in a way in which the extension produced is proportional to the applied force (load), we say it obeys **Hooke's Law**
- This relationship between force and extension is shown in the graph below



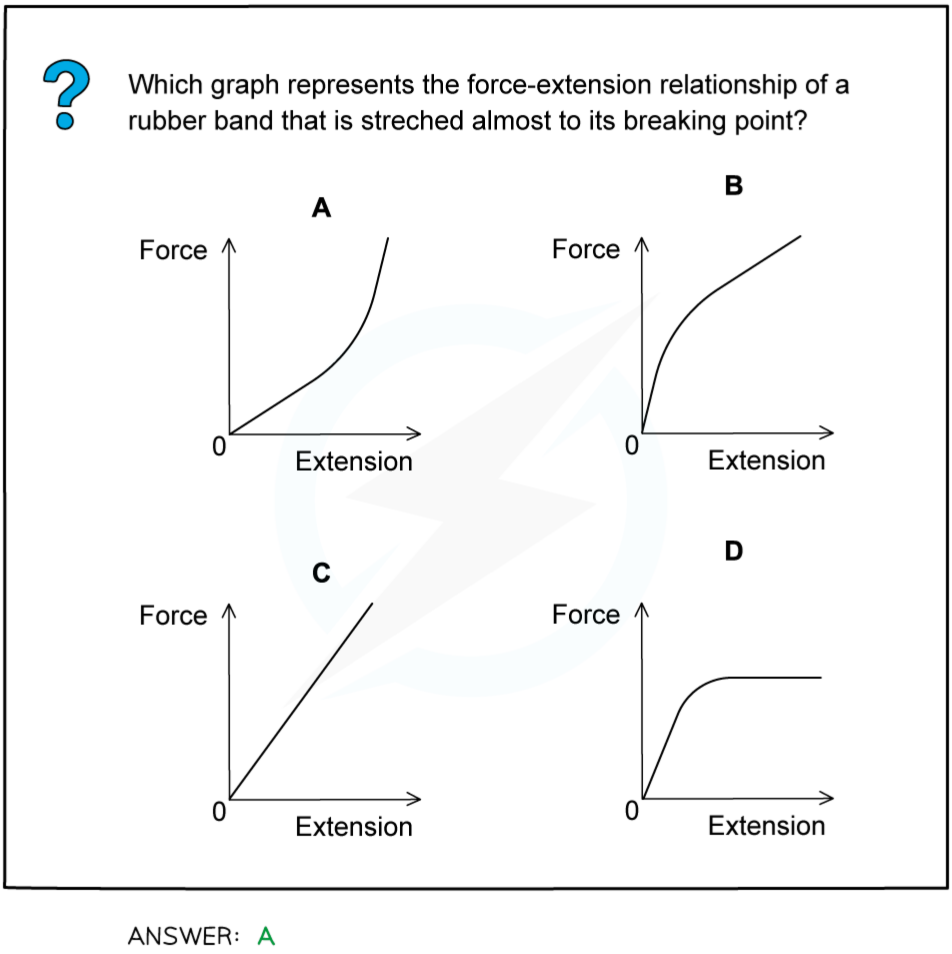
Force v extension graph for a spring

- The extension of the spring is determined by how much it has **increased** in length
- The **limit of proportionality** is the point beyond which Hooke's law is no longer true when stretching a material i.e. the extension is no longer proportional to the applied load
  - The point is identified on the graph where the line is no longer straight and starts to curve (flattens out)
- Hooke's law also applies to **compression** as well as extension. The only difference is that an applied force is now proportional to the **decrease** in length
- The gradient of this graph is equal to the **spring constant** ***k***. This is explored further in the revision notes "The Spring Constant"

6. Deformation of Solids

YOUR NOTES

Worked example



- Rubber bands obey Hooke's law until they're stretched up to twice their original size or more – this is because the long chain molecules become fully aligned and can no longer move past each other
- This is shown by graph A – after the section of linear proportionality (the straight line), the gradient increases significantly, so, a large force is required to extend the rubber band by even a small amount
- Graph B is incorrect as the gradient decreases, suggesting that less force is required to cause a small extension
- Graph C is incorrect as this shows a material which obeys Hooke's Law and does not break easily, such as a metal
- Graph D is incorrect as the plateau suggests no extra force is required to extend the rubber as it has been stretched

6. Deformation of Solids

YOUR NOTES

Exam Tip

Exam questions may ask for the total length of a material after a load is placed on it and its extended. Remember to add the extension to the original length of the material to get its final full length

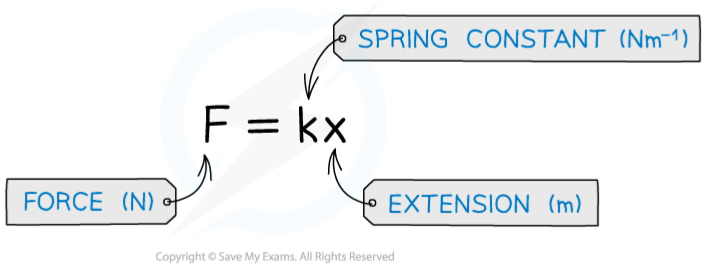
6. Deformation of Solids

YOUR NOTES
⬇

6.1.2 HOOKE'S LAW

Hooke's Law

- A material obeys Hooke’s Law if **its extension is directly proportional to the applied force (load)**
- The Force v Extension graph is a straight line through the origin (see “Extension and Compression”)
- This linear relationship is represented by the Hooke’s law equation



Hooke’s Law

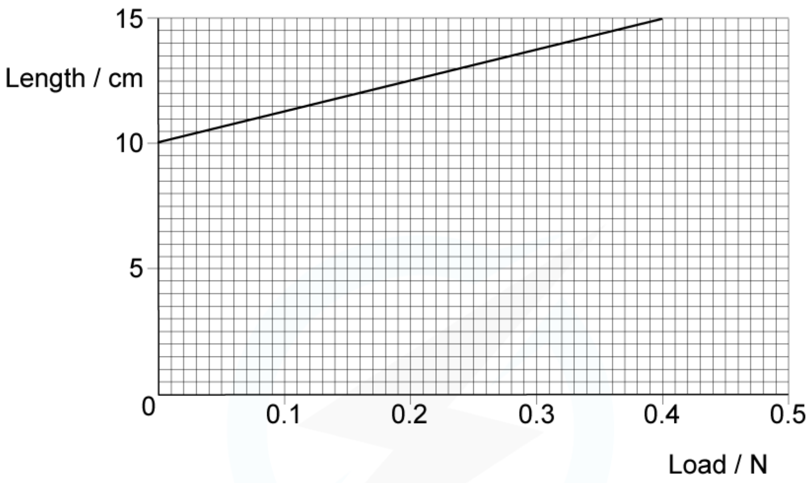
- The constant of proportionality is known as the **spring constant *k***

6. Deformation of Solids

YOUR NOTES

Worked example

A spring was stretched with increasing load. The graph of the results is shown below.



What is the spring constant?

STEP 1

REARRANGE FROM HOOKE'S LAW, THE SPRING CONSTANT IS

$$k = \frac{F}{x}$$

STEP 2

THE GRADIENT OF A FORCE-EXTENSION GRAPH IS THE SPRING CONSTANT

$$k = \frac{\Delta F}{\Delta x}$$

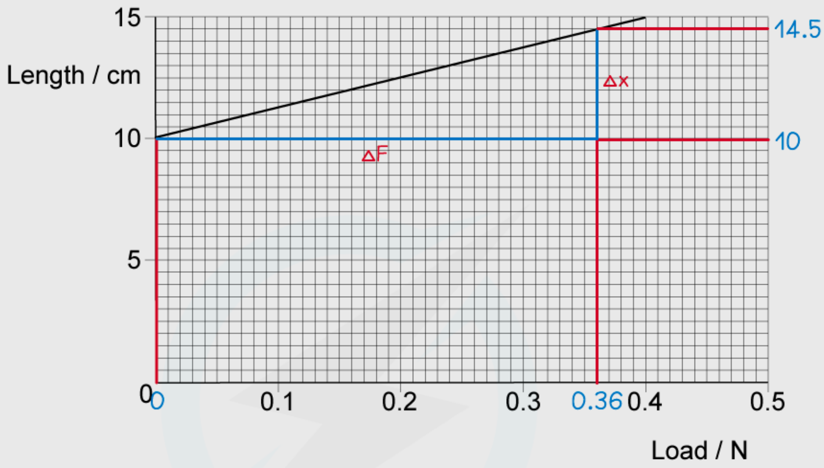
6. Deformation of Solids

YOUR NOTES

STEP 3

THIS PARTICULAR GRAPH HAS THE LENGTH ON THE y−AXIS AND THE FORCE ON THE x−AXIS.

THEREFORE THE SPRING CONSTANT IS $\frac{1}{\text{GRADIENT}}$



STEP 4

FIND THE GRADIENT

$$\frac{\Delta x}{\Delta F} = \frac{(0.145 - 0.10)\text{m}}{0.36\text{ N}} = \frac{1}{8.0}\text{ Nm}^{-1}$$

$$\text{GRADIENT} = \frac{\Delta y}{\Delta x}$$

STEP 5

$$\text{SPRING CONSTANT} = \frac{1}{\text{GRADIENT}}$$

$$1 \div \frac{1}{8.0} = 8.0\text{ Nm}^{-1}$$

Exam Tip

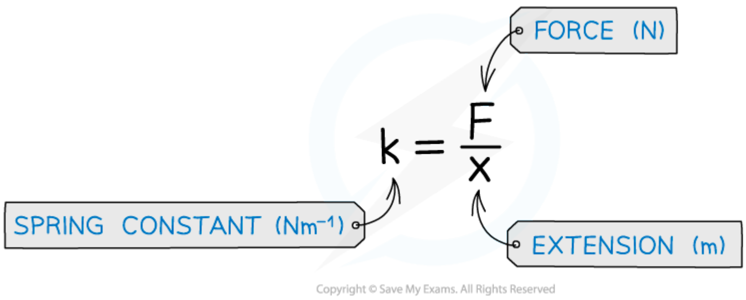
Double check the axes before finding the spring constant as the gradient of a force-extension graph. Exam questions often swap the load onto the x-axis and length on the y-axis. In this case, the gradient is not the spring constant but 1 ÷ gradient is.

6. Deformation of Solids

YOUR NOTES
⬇

The Spring Constant

- k is the **spring constant** of the spring and is a measure of the **stiffness** of a spring
  - A stiffer spring will have a larger value of k
- It is defined as the **force per unit extension** up to the limit of proportionality (after which the material will not obey Hooke's law)
- The SI unit for the spring constant is **N m^{-1}**
- Rearranging the Hooke's law equation shows the equation for the spring constant is

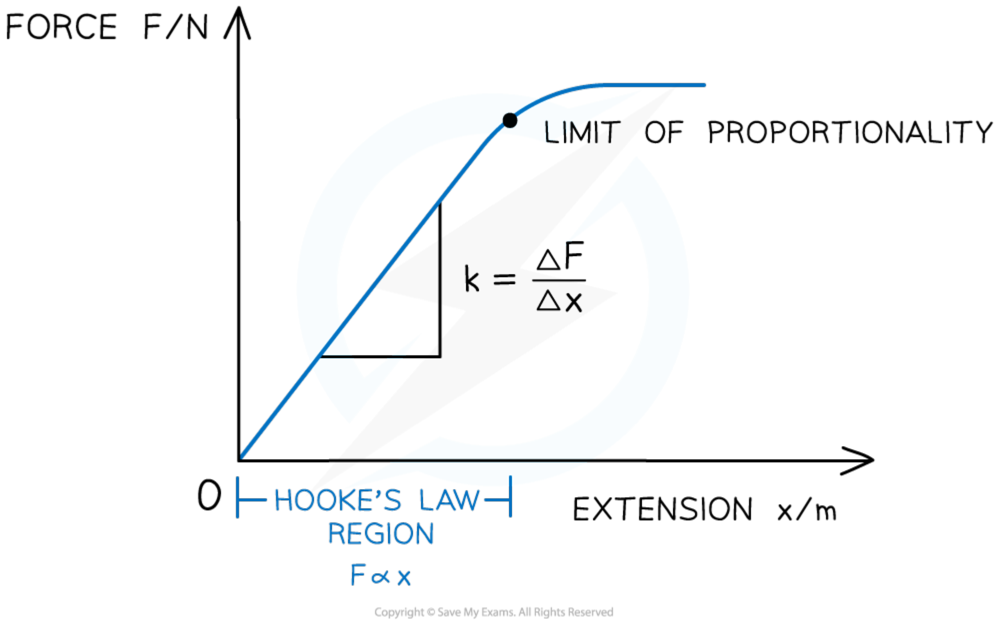


Spring constant equation

6. Deformation of Solids

YOUR NOTES

- The spring constant is the **force per unit extension** up to the limit of proportionality (after which the material will not obey Hooke's law)
- Therefore, the spring constant *k* is the gradient of the linear part of a Force v Extension graph



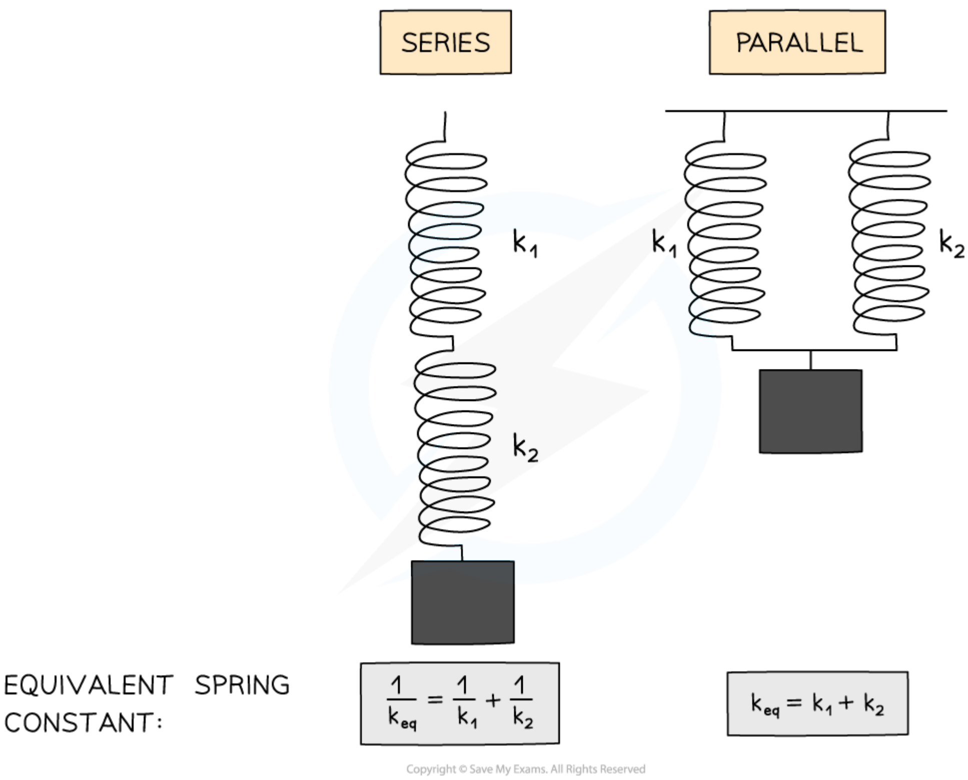
Spring constant is the gradient of a force v extension graph

6. Deformation of Solids

YOUR NOTES

Combination of springs

- Springs can be combined in different ways
  - In **series** (end-to-end)
  - In **parallel** (side-by-side)



Spring constants for springs combined in series and parallel

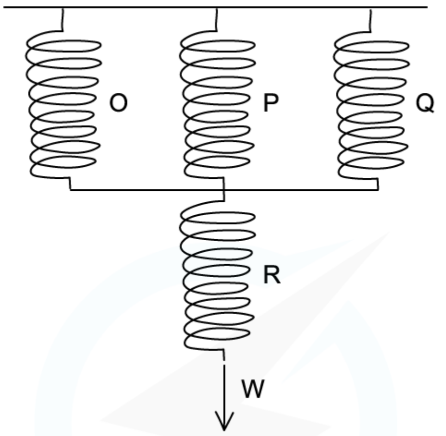
- This is assuming k_1 and k_2 are different spring constants
- The equivalent spring constant for combined springs are summed up in different ways depending on whether they're connected in parallel or series

6. Deformation of Solids

YOUR NOTES

Worked example

Three springs are arranged vertically as shown.



Springs P, Q and O are identical and have spring constant k.
Spring R has spring constant 4k.

What is the increase in the overall length of the arrangement when a force W is applied as shown?

A. $\frac{12k}{7W}$ **B.** $\frac{6W}{5k}$ **C.** $\frac{7W}{12k}$ **D.** $\frac{2W}{5}$

ANSWER: C

STEP 1

EQUATION FOR EXTENSION x

REARRANGE FROM HOOKE'S LAW

$x = \frac{F}{k} = \frac{W}{k}$

6. Deformation of Solids

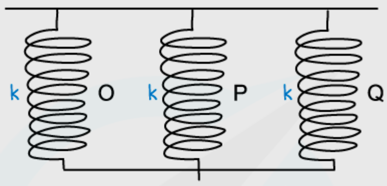
YOUR NOTES

STEP 2

FIND THE VALUE OF k:

EQUIVALENT k FROM PARALLEL SPRINGS P AND Q

$k_{OPQ} = k_O + k_P + k_Q = k + k + k = 3k$



EQUIVALENT k FROM SPRING R AND THE COMBINED SPRINGS P AND Q ARE CONNECTED IN SERIES

$$\frac{1}{k_{OPQR}} = \frac{1}{3k} + \frac{1}{4k} = \frac{7}{12k}$$

$$k_{OPQR} = 1 \div \frac{7}{12k} = \frac{12k}{7}$$

STEP 3

SUBSTITUTE BACK INTO THE EXTENTION EQUATION

$$x = \frac{W}{k_{OPQR}} = W \div \frac{12k}{7} = \frac{7W}{12k}$$

Exam Tip

The equivalent (or effective) spring constant equations for combined springs work for any number of springs e.g. if there are 3 springs in parallel k_1 , k_2 and k_3 , the equivalent spring constant would be $k_{eq} = k_1 + k_2 + k_3$.

6. Deformation of Solids

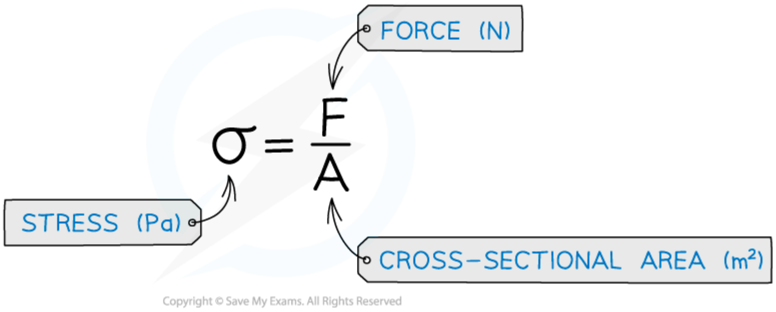
YOUR NOTES
⬇

6.1.3 THE YOUNG MODULUS

Stress, Strain & the Young Modulus

Stress

- Tensile stress is the **applied force per unit cross sectional area** of a material



Stress equation

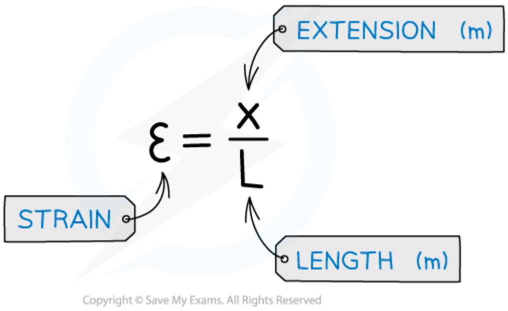
- The **ultimate tensile stress** is the **maximum** force per original cross-sectional area a wire is able to support until it breaks

6. Deformation of Solids

YOUR NOTES
⬇

Strain

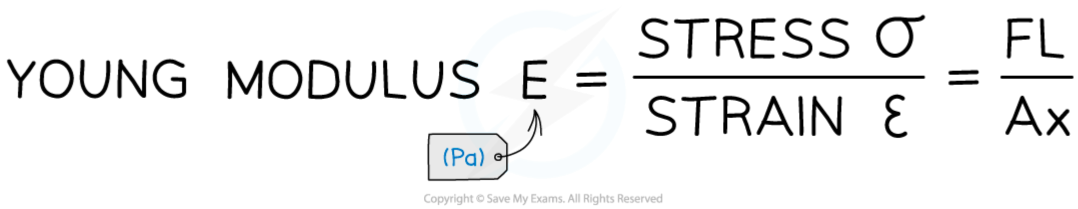
- Strain is the **extension per unit length**
- This is a deformation of a solid due to stress in the form of elongation or contraction
- Note that strain is a **dimensionless** unit because it's the ratio of lengths



Strain equation

Young's Modulus

- The Young modulus is the measure of the ability of a material to withstand changes in length with an added load ie. how stiff a material is
- This gives information about the elasticity of a material
- The Young Modulus is defined as the **ratio of stress and strain**

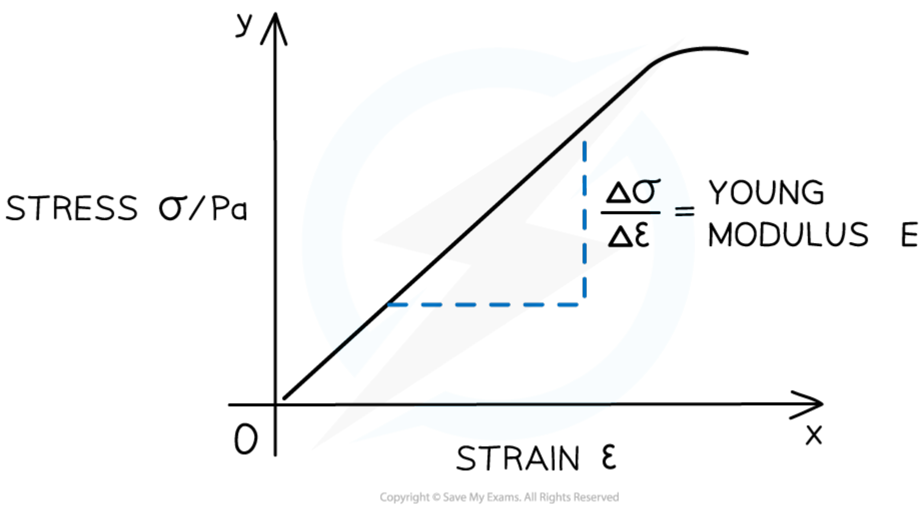


Young Modulus equation

6. Deformation of Solids

YOUR NOTES

- Its unit is the same as stress: **Pa** (since strain is unitless)
- Just like the Force-Extension graph, stress and strain are directly proportional to one another for a material exhibiting elastic behaviour



A stress-strain graph is a straight line with its gradient equal to Young modulus

- The **gradient** of a stress-stress graph when it is linear is the **Young Modulus**

6. Deformation of Solids

YOUR NOTES

Worked example

A metal wire that is supported vertically from a fixed point has a load of 92 N applied to the lower end. The wire has a cross-sectional area of 0.04 mm^2 and obeys Hooke's law. The length of the wire increases by 0.50%.

What is the Young modulus of the metal wire?

A. 4.6×10^7 Pa **B.** 4.6×10^{12} Pa **C.** 4.6×10^9 Pa **D.** 4.6×10^{11} Pa

ANSWER: D

STEP 1

YOUNG MODULUS EQUATION

$$E = \frac{\text{STRESS}}{\text{STRAIN}} = \frac{FL}{Ax}$$

STEP 2

CALCULATE STRESS

$$\text{STRESS} = \frac{F}{A} = \frac{92\text{ N}}{0.04 \times 10^{-6}\text{ m}^2} = 2.3 \times 10^9\text{ Pa}$$

$1\text{ mm}^2 = 1 \times 10^{-6}\text{ m}^2$

STEP 3

CALCULATE STRAIN

$$\text{STRAIN} = \frac{x}{L} = 0.5\% = 0.005$$

EXTENSION

STEP 4

SUBSTITUTE INTO YOUNG MODULUS EQUATION

$$E = \frac{\text{STRESS}}{\text{STRAIN}} = \frac{2.3 \times 10^9\text{ Pa}}{0.005} = 4.6 \times 10^{11}\text{ Pa}$$

Exam Tip

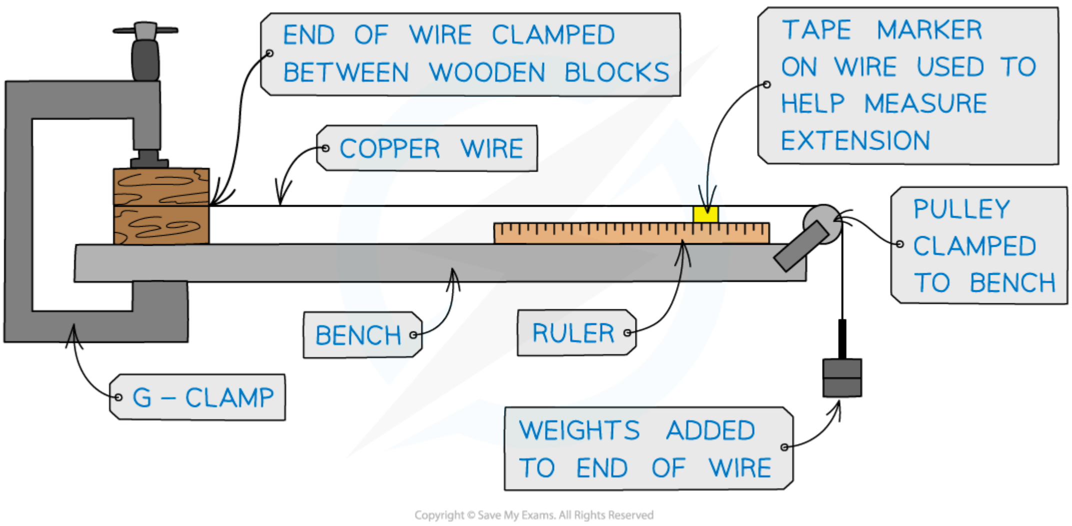
To remember whether stress or strain comes first in the Young modulus equation, try thinking of the phrase 'When you're stressed, you show the strain' ie. Stress ÷ strain.

6. Deformation of Solids

YOUR NOTES

Young's Modulus Experiment

- To measure the Young's Modulus of a metal in the form of a wire requires a clamped horizontal wire over a pulley (or vertical wire attached to the ceiling with a mass attached) as shown in the diagram below



- A reference marker is needed on the wire. This is used to accurately measure the extension with the applied load
- The independent variable is the **load**
- The dependent variable is the **extension**

6. Deformation of Solids

YOUR NOTES ⬇

Method

1. Measure the original length of the wire using a metre ruler and mark this reference point with tape
2. Measure the diameter of the wire with micrometer screw gauge or digital calipers
3. Measure or record the mass or weight used for the extension e.g. 300 g
4. Record initial reading on the ruler where the reference point is
5. Add mass and record the new scale reading from the metre ruler
6. Record final reading from the new position of the reference point on the ruler
7. Add another mass and repeat method

Improving experiment and reducing uncertainties:

- Reduce uncertainty of the cross-sectional area by measuring the diameter *d* in several places along the wire and calculating an average
- Remove the load and check wire returns to original limit after each reading
- Take several readings with different loads and find average
- Use a Vernier scale to measure the extension of the wire

6. Deformation of Solids

YOUR NOTES

Measurements to determine Young's modulus

1. Determine extension x from final and initial readings

Example table of results:

Mass m / g	Load F / N	Initial length / mm	Final length / mm	Extension x ($\times 10^{-3}$) / m
200	2.0	500	500.1	0.1
300	2.9	500.1	500.4	0.3
400	3.9	500.4	501.0	0.6
500	4.9	501.0	501.9	0.9
600	5.9	501.9	503.2	1.3
700	6.9	503.2	504.9	1.7
800	7.8	504.9	507.0	2.1

Table with additional data

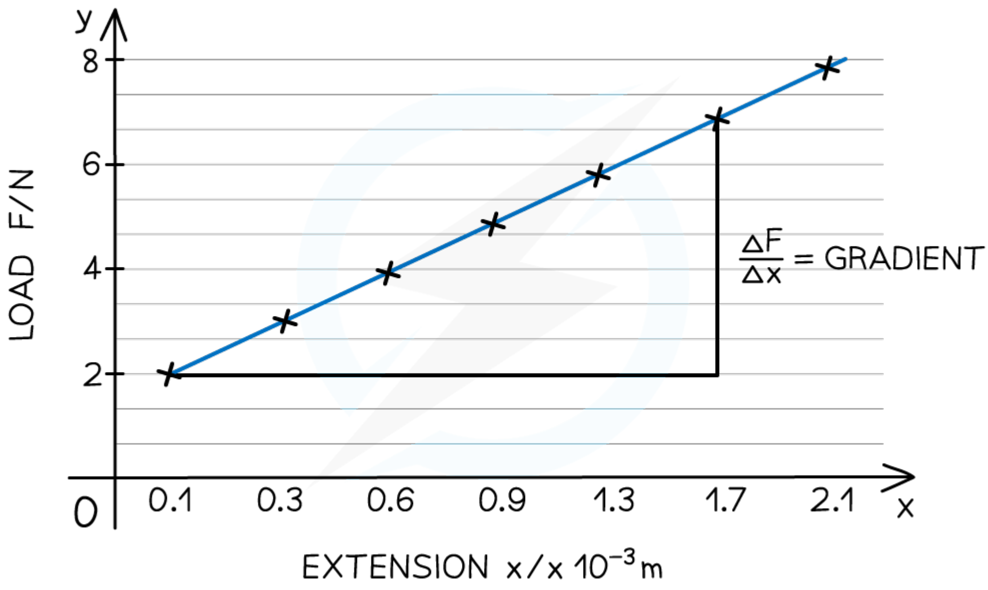
Length l / m	1.382
Diameter 1 / mm	0.277
Diameter 2 / mm	0.280
Diameter 3 / mm	0.275
Average Diameter d / mm	0.277
Cross-sectional area A / m^2	6.03×10^{-8}

6. Deformation of Solids

YOUR NOTES
⬇

2. Plot a graph of force against extension and draw line of best fit

3. Determine gradient of the force v extension graph

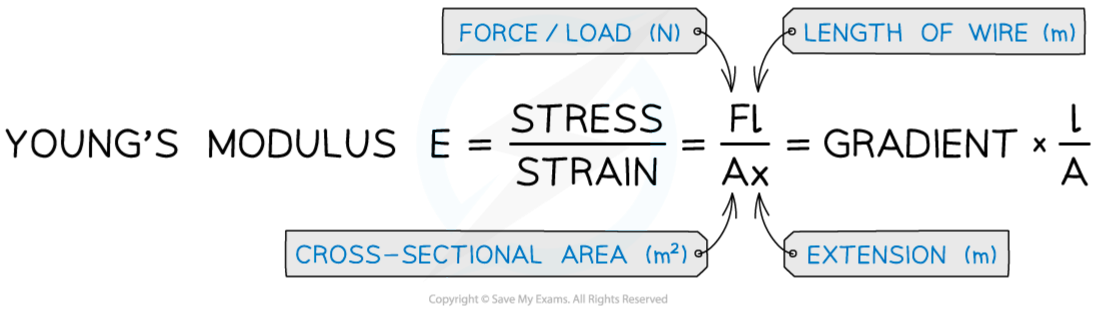


4. Calculate cross-sectional area from:

6. Deformation of Solids

YOUR NOTES

5. Calculate the Young's modulus from:



Exam Tip

Although every care should be taken to make the experiment as reliable as possible, you will be expected to suggest improvements in producing more accurate and reliable results (e.g. repeat readings and use a longer length of wire)

Exam Question: Easy

A student wanted to measure the Young modulus of a wire, to do this they needed to take a number of measurements.

In which row could the measurement **not** be made **directly** with the listed apparatus?

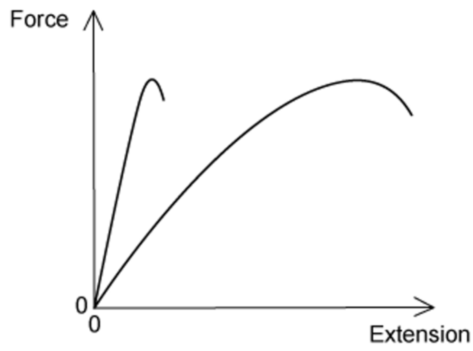
	measurement	apparatus
A	extension of wire	vernier scale
B	area of cross-section of wire	micrometer screw gauge
C	original length of wire	metre rule
D	mass of load applied to a wire	electronic balance

6. Deformation of Solids

YOUR NOTES

Exam Question: Medium

Two materials with the same dimensions were loaded to fracture; the graph shows the force-extension of these.



Which of the following would describe the behaviour of the materials?

A both materials are plastic

B both materials have the same ultimate tensile stress

C both materials are brittle

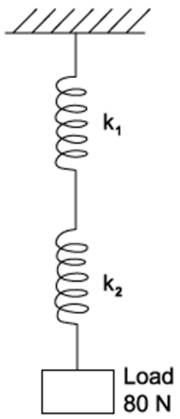
D both materials obey Hooke's law

6. Deformation of Solids

YOUR NOTES
⬇

Exam Question: Hard

A student connected two springs, one with spring constant k_1 = 4 kN m^{-1} and the other with spring constant k_2 = 2 kN m^{-1} as shown in the diagram below.



When a load of 80 N is applied, what is the total extension?

A 60 cm **B** 6 cm **C** 4 cm **D** 1.3 cm

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6. Deformation of Solids

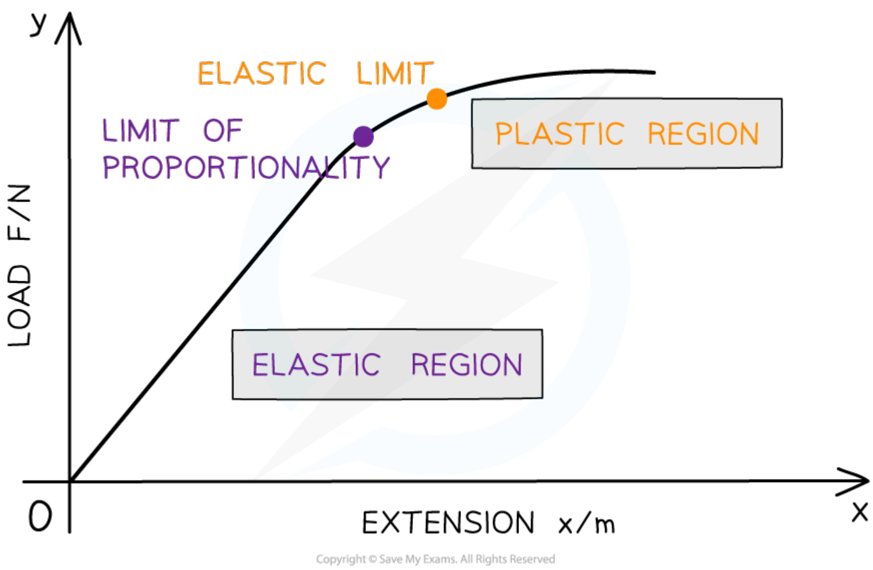
YOUR NOTES

6.2 DEFORMATION: ELASTIC & PLASTIC BEHAVIOUR

6.2.1 ELASTIC & PLASTIC BEHAVIOUR

Elastic & Plastic Deformation

- **Elastic deformation:** when the load is removed, the object **will** return to its original shape
- **Plastic deformation:** when the load is removed, the object **will not** return to its original shape or length. This is beyond the elastic limit
- **Elastic limit:** the point beyond which the object does not return to its original length when the load is removed
- These regions can be determined from a Force-Extension graph:



Below the elastic limit, the material exhibits elastic behaviour
Above the elastic limit, the material exhibits plastic behaviour

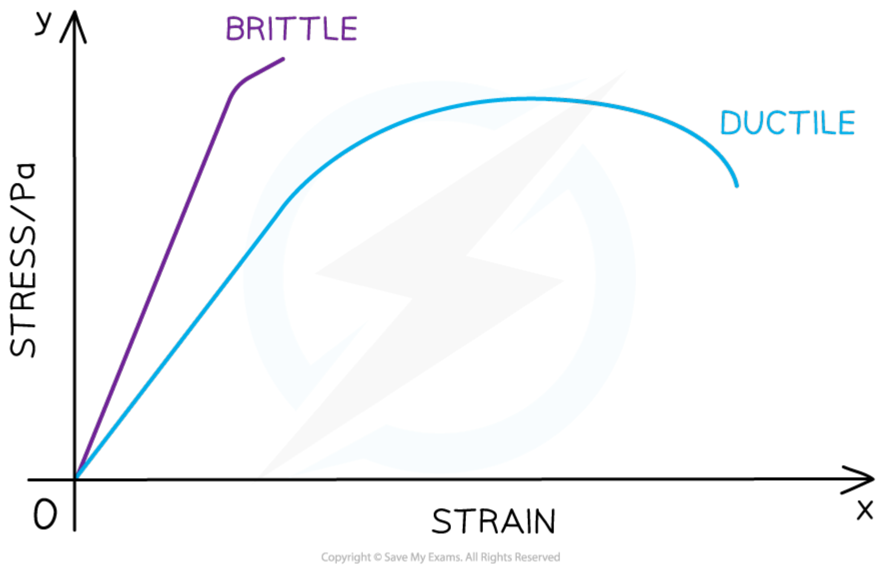
6. Deformation of Solids

YOUR NOTES

- Elastic deformation occurs in the 'elastic region' of the graph. The extension is proportional to the force applied to the material (straight line)
- Plastic deformation occurs in the 'plastic region' of the graph. The extension is no longer proportional to the to the force applied to the material (graph starts to curve)
- These regions are divided by the elastic limit

Brittle and ductile materials

- **Brittle** materials have very little to no plastic region e.g. glass, concrete. The material breaks with little elastic and insignificant plastic deformation
- **Ductile** materials have a larger plastic region e.g. rubber, copper. The material stretches into a new shape before breaking



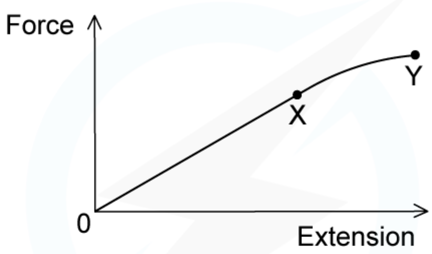
- To identify these materials on a stress-strain or force-extension graph up to their breaking point:
  - A brittle material is represented by a straight line through the origins with no or negligible curved region
  - A ductile material is represented with a straight line through the origin then curving towards the x-axis

6. Deformation of Solids

YOUR NOTES

Worked example

A sample of metal is subjected to a force which increases to a maximum value and then fractures. A force-extension graph for the sample is shown.



What is the behaviour of the metal between X and Y?

A. both elastic and plastic

B. not elastic and not plastic

C. plastic but not elastic

D. elastic but not plastic

ANSWER: C

- Since the graph is a straight line and the metal fractures, the point after X must be its elastic limit
- The graph starts to curve after this and fractures at point Y
- This curve between X and Y denotes plastic behaviour
- Therefore, the correct answer is **C**

6. Deformation of Solids

YOUR NOTES

Exam Tip

Although similar definitions, the elastic limit and limit of proportionality are not the same point on the graph. The limit of proportionality is the point beyond which the material is no longer defined by Hooke's law. The elastic limit is the furthest point a material can be stretched whilst still able to return to its previous shape. This is at a slightly higher extension than the limit of proportionality. Be sure not to confuse them.

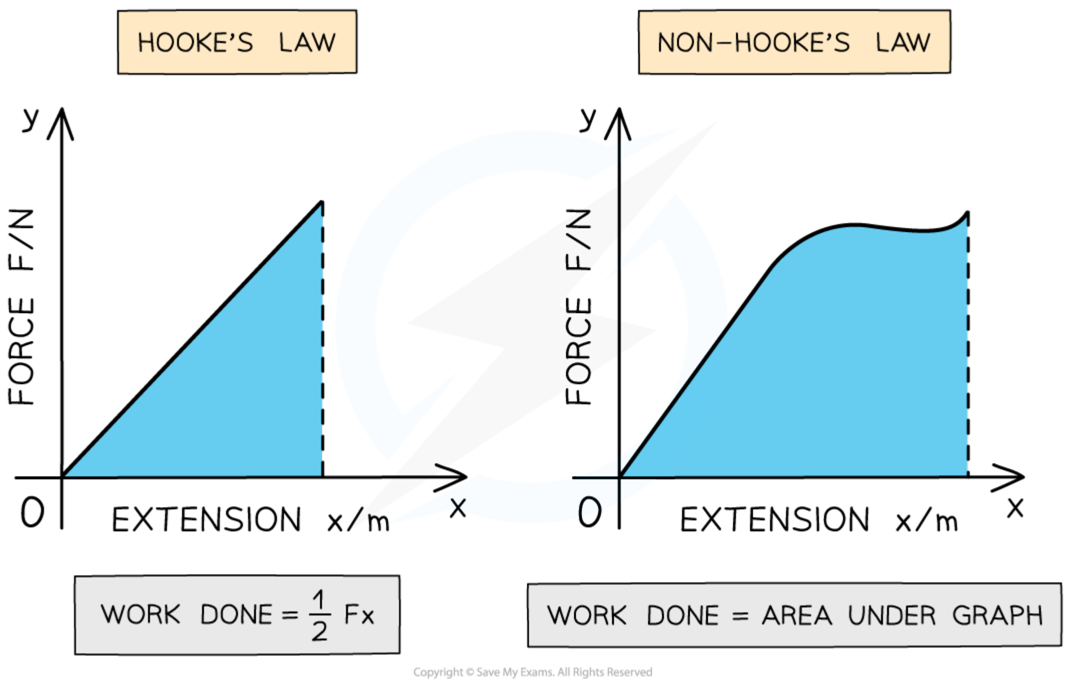
6. Deformation of Solids

YOUR NOTES

6.2.2 ELASTIC POTENTIAL ENERGY

Area under a Force-Extension Graph

- The work done in stretching a material is equal to the force multiplied by the distance moved
- Therefore, the **area under a force-extension graph is equal to the work done** to stretch the material
- The work done is also equal to the **elastic potential energy** stored in the material



Work done is the area under the force – extension graph

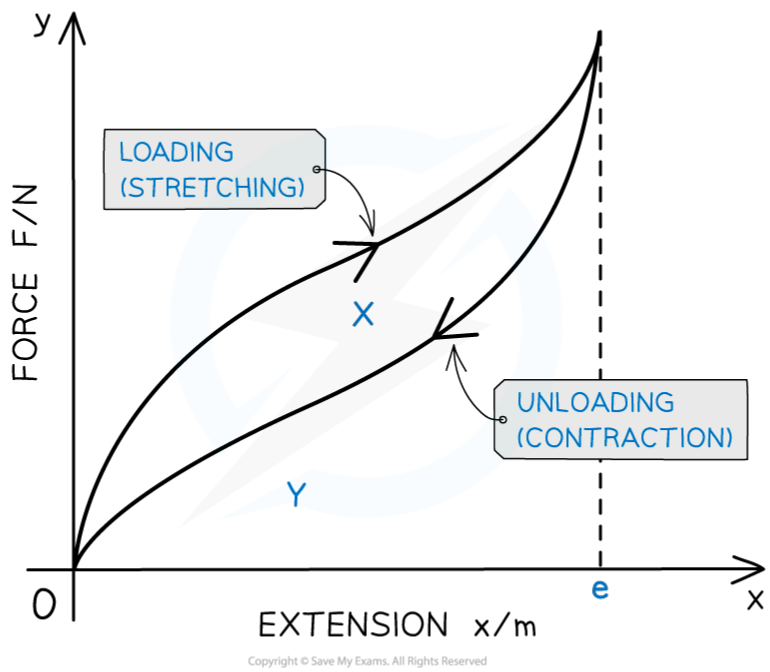
6. Deformation of Solids

YOUR NOTES

- This is true for whether the material obeys Hooke's law or not
  - For the region where the material obeys Hooke's law, the work done is the area of a right angled triangle under the graph
  - For the region where the material doesn't obey Hooke's law, the area is the full region under the graph. To calculate this area, split the graph into separate segments and add up the individual areas of each

Loading and unloading

- The force-extension curve for stretching and contraction of a material that has exceeded its elastic limit is shown below



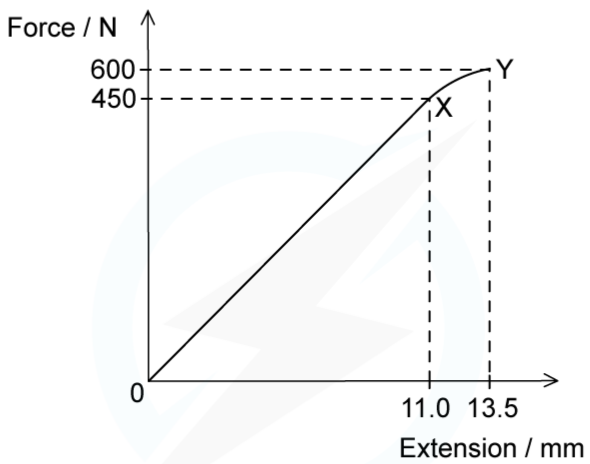
- The curve for contraction is always below the curve for stretching
- The area **X** represents the **net work done** or the **thermal energy** dissipated in the material
- The area **X + Y** is the **minimum energy required** to stretch the material to extension *e*

6. Deformation of Solids

YOUR NOTES

Worked example

The graph shows the behaviour of a sample of a metal when it is stretched until it starts to undergo plastic deformation.



What is the total work done in stretching the sample from zero to 13.5 mm extension?
Simplify the calculation by treating the curve XY as a straight line.

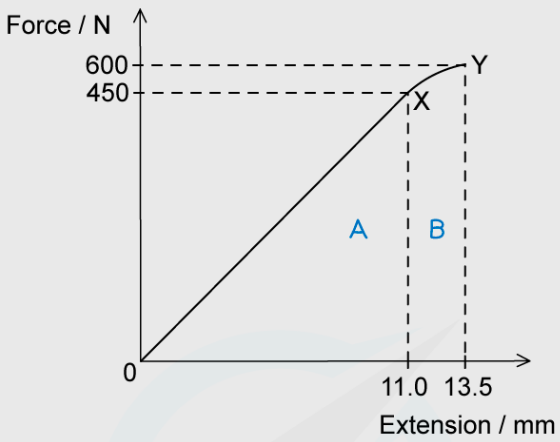
STEP 1 WORK DONE = AREA UNDER THE FORCE–EXTENSION GRAPH

6. Deformation of Solids

YOUR NOTES
⬇

STEP 2

SPLIT GRAPH INTO THE TWO AREAS



STEP 3

CALCULATE AREA A

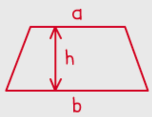
AREA OF A RIGHT ANGLED TRIANGLE $= \frac{1}{2} \times$ BASE $\times$ HEIGHT

AREA $= \frac{1}{2} \times 11 \times 10^{-3} \times 450 = 2.475$ J

STEP 4

CALCULATE AREA B

AREA OF TRAPEZIUM $= \left(\frac{a+b}{2}\right) \times h$



AREA $= \left(\frac{450 + 600}{2}\right) \times 2.5 \times 10^{-3} = 1.313$ J

STEP 5

TOTAL AREA $= 2.475 + 1.313 = 3.79$ J (3 s.f.)

Exam Tip

Make sure to be familiar with the formula for the area of common 2D shapes such as a right angled triangle, trapezium, square and rectangles.

6. Deformation of Solids

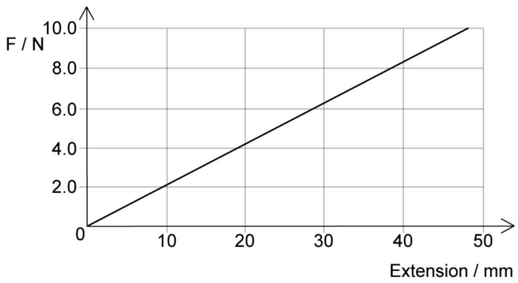
YOUR NOTES
⬇

Elastic Potential Energy

- Elastic potential energy is defined as the energy stored within a material (e.g. in a spring) when it is stretched or compressed
- It can be found from the **area under the force-extension graph** for a material deformed within its limit of proportionality

Worked example

A spring is extended with varying forces; the graph below shows the results.



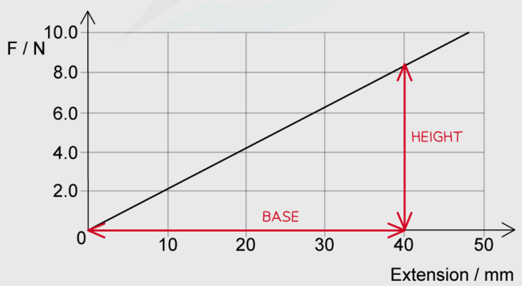
What is the energy stored in the spring when the extension is 40 mm?

STEP 1

ENERGY STORED = AREA UNDER THE GRAPH

STEP 2

CALCULATE AREA UNDER GRAPH FOR EXTENSION OF 40mm



AREA OF TRIANGLE = $\frac{1}{2}$ × BASE × HEIGHT

AREA = $\frac{1}{2} \times 40 \times 10^{-3}\,\text{m} \times 8.1\,\text{N} = 0.16\,\text{J}$

STEP 3

ENERGY STORED = 0.16 J

6. Deformation of Solids

YOUR NOTES
⬇

Calculating Elastic Potential Energy

- A material within it’s limit of proportionality obeys Hooke’s law. Therefore, for a material obeying Hooke’s Law, elastic potential energy can be calculated using:

HOOKE’S LAW: $F = kx$

$$EPE = \frac{1}{2}Fx = \frac{1}{2}(kx)x$$

$$\boxed{\text{ELASTIC POTENTIAL ENERGY} = \frac{1}{2}kx^2}$$

Elastic potential energy can be derived from Hooke’s law

- Where k is the **spring constant (N m^{-1})** and x is the **extension (m)**

Exam Tip

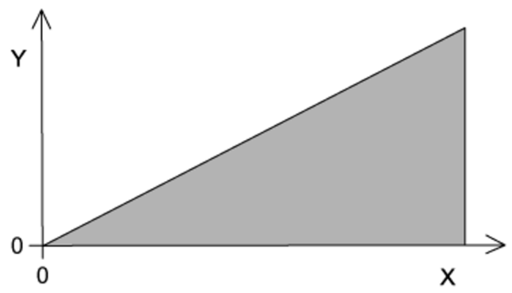
The formula for EPE = ½ kx^2 is only the area under the force-extension graph when it is a straight line i.e. when the material obeys Hooke’s law and is within its elastic limit.

6. Deformation of Solids

YOUR NOTES

Exam Question: Easy

A student experimented on a metal wire; the graph below was plotted.



The area under the graph shows the total strain energy stored in the wire when stretched.

How should the axes be labelled?

	Y	X
A	mass	extension
B	force	extension
C	strain	energy
D	stress	strain

6. Deformation of Solids

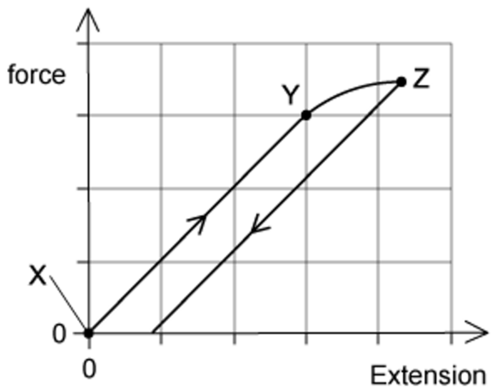
YOUR NOTES

Exam Question: Medium

A student was investigating the extension in a long thin metal wire. They suspended the wire from a fixed support, and hung it vertically then hung masses from the lower end.

The load was increased and decreased from zero to the maximum and back again.

The graph shows the force-extension of the wire.



Where on the graph would the elastic limit be found?

A exactly at point Z

B exactly at point Y

C just beyond point Y

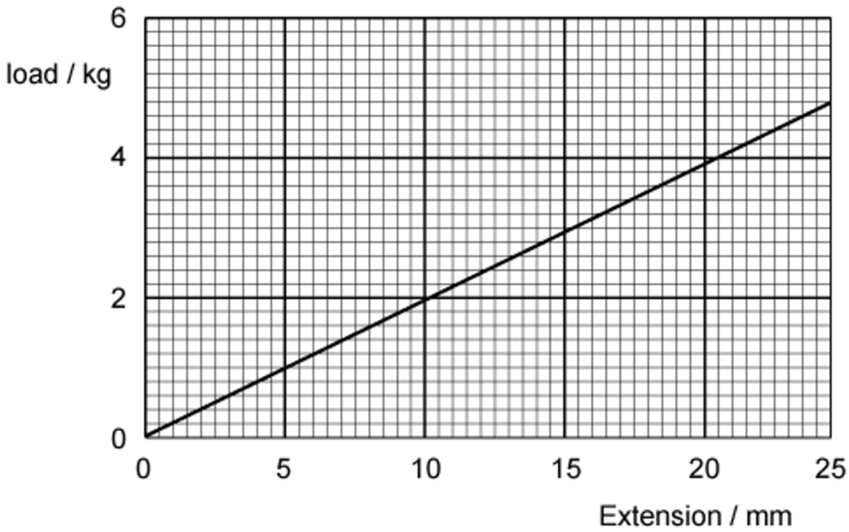
D anywhere between point X and point Y

6. Deformation of Solids

YOUR NOTES

Exam Question: Hard

A wire undergoes elastic deformation; the graph below shows the load-extension graph.



How much work is done on the wire to increase the extension from 10 mm to 20 mm?

A 0.37 J **B** 0.28 J **C** 0.184 J **D** 0.028 J

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7. Waves

YOUR NOTES ⬇

CONTENTS

7.1 WAVES: TRANSVERSE & LONGITUDINAL

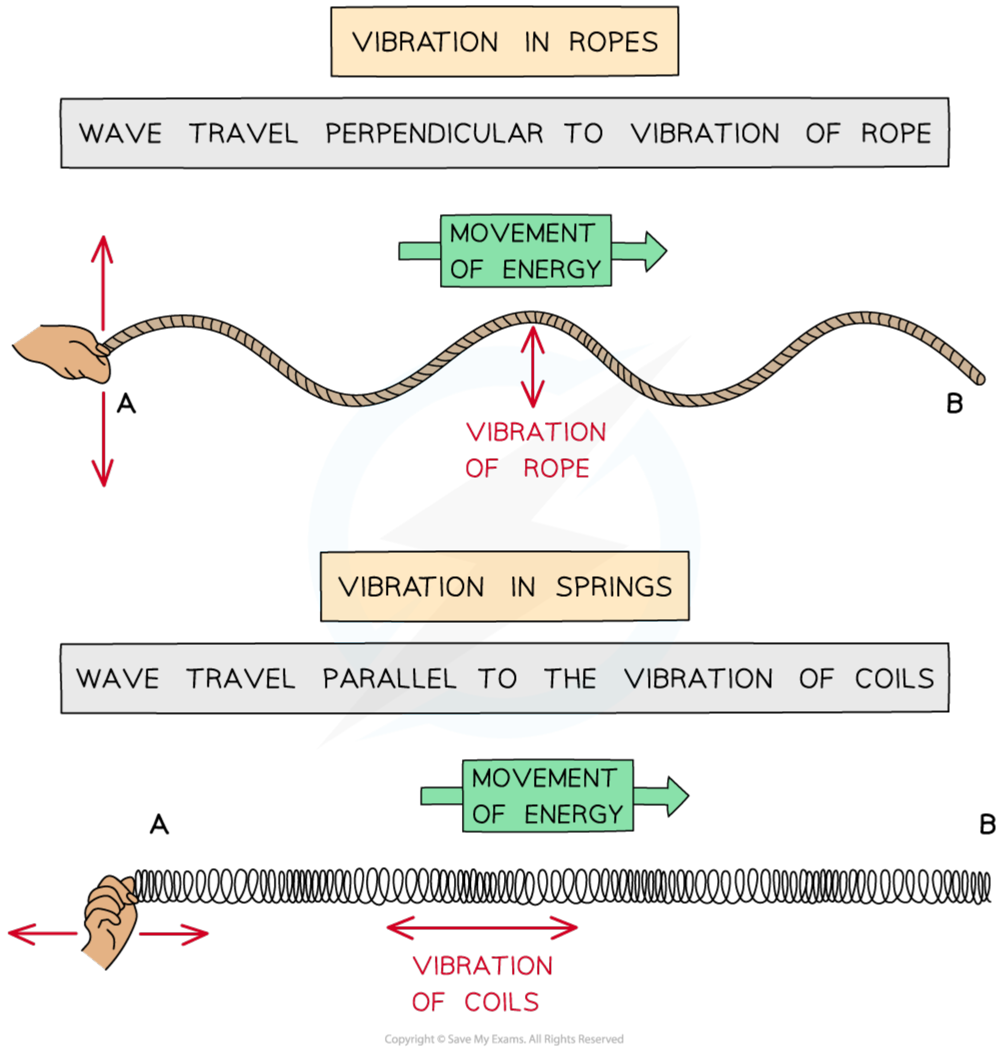
7.1.1 PROGRESSIVE WAVES

Wave Motion

- Energy is transferred through moving **oscillations** or **vibrations**. These can be seen in vibrations of ropes or springs

7. Waves

YOUR NOTES



Waves can be shown through vibrations in ropes or springs

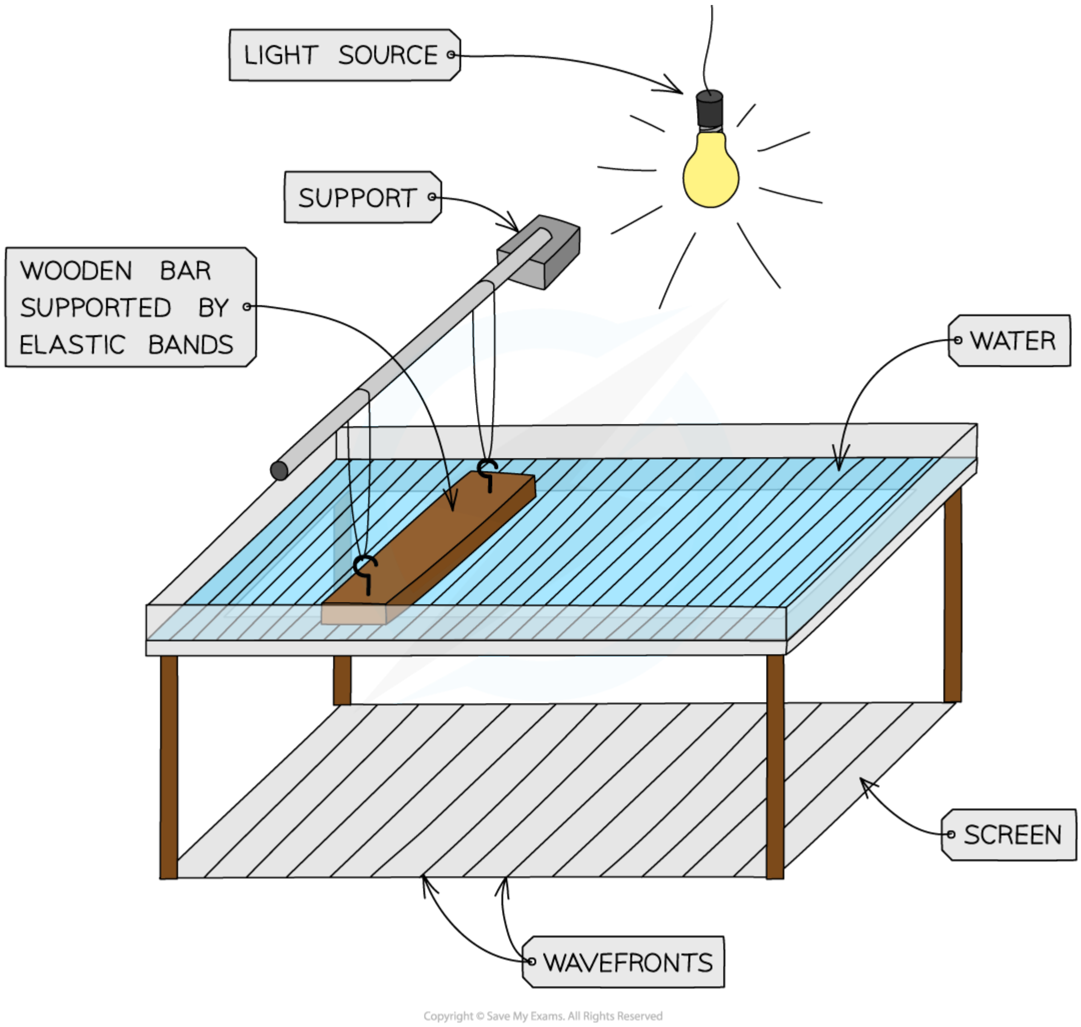
- The oscillations/vibrations can be perpendicular or parallel to the direction of wave travel:
  - When they are **perpendicular**, they are **transverse** waves
  - When they are **parallel**, they are **longitudinal** waves

7. Waves

YOUR NOTES
⬇

Ripple tanks

- Waves can also be demonstrated by ripple tanks. These produce a combination of transverse and longitudinal waves



Wave effects can be demonstrated using a ripple tank

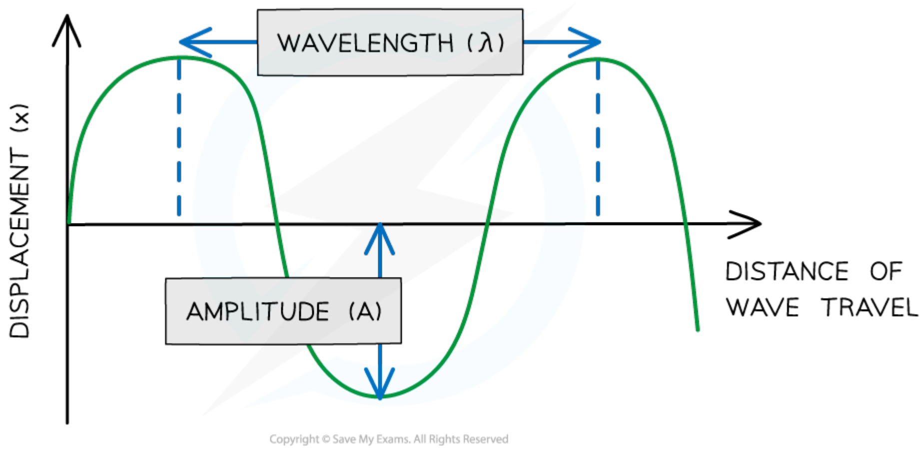
- Ripple tanks may be used to demonstrate the wave properties of reflection, refraction and diffraction

7. Waves

YOUR NOTES
⬇

General Wave Properties

- **Displacement (x)** of a wave is the distance from its equilibrium position. It is a vector quantity; it can be positive or negative
- **Amplitude (A)** is the maximum displacement of a particle in the wave from its equilibrium position
- **Wavelength (λ)** is the distance between points on successive oscillations of the wave that are in phase
    - These are all measured in **metres (m)**

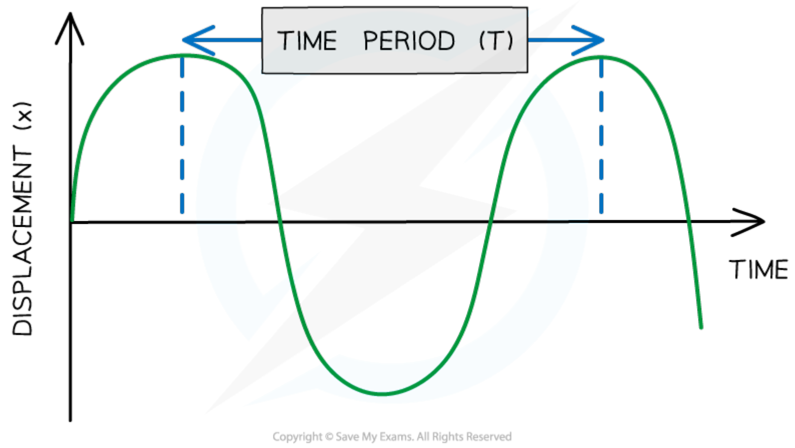


Diagrams showing the amplitude and wavelength of a wave

7. Waves

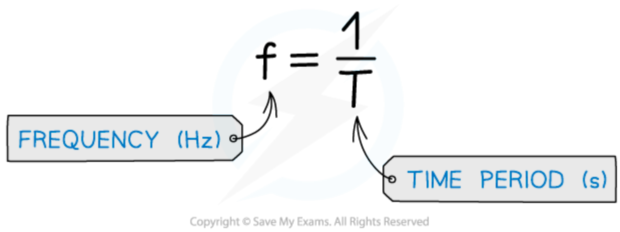
YOUR NOTES

- **Period (T)** or time period, is the time taken for one complete oscillation or cycle of the wave. Measured in **seconds (s)**



Diagrams showing the time period of a wave

- **Frequency (f)** is the number of complete oscillations per unit time. Measured in **Hertz** (**Hz)** or **s^{-1}**



Frequency equation

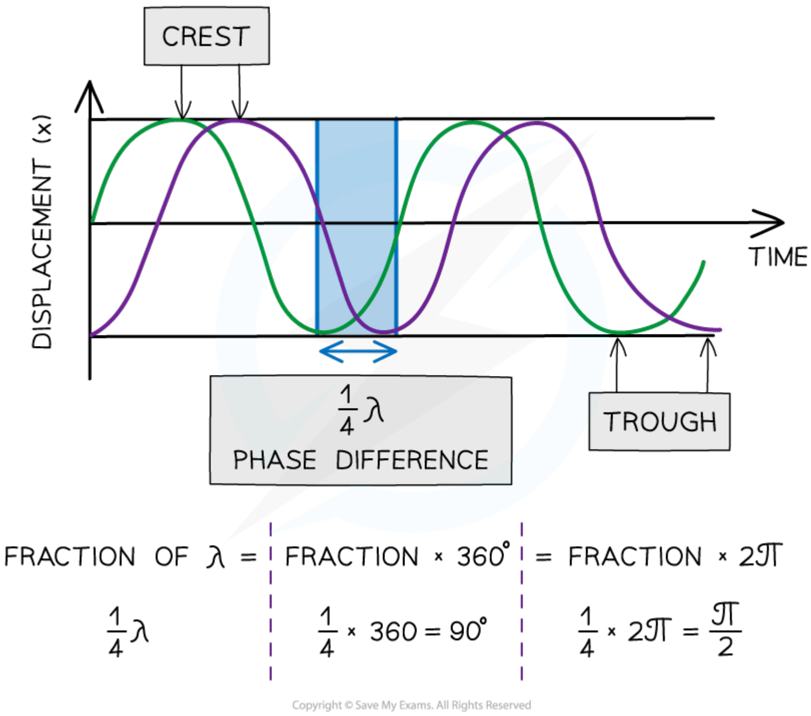
- **Speed (v)** is the distance travelled by the wave per unit time. Measured in **metres per second (m s^{-1})**

7. Waves

YOUR NOTES
⬇

Phase

- The phase difference tells us **how much a point or a wave is in front or behind another**
- This can be found from the relative positive of the crests or troughs of two different waves of the same frequency
  - When the crests or troughs are aligned, the waves are **in phase**
  - When the crest of one wave aligns with the trough of another, they are in **antiphase**
- The diagram below shows the green wave **leads** the purple wave by ¼ λ



Two waves ¼ λ out of phase

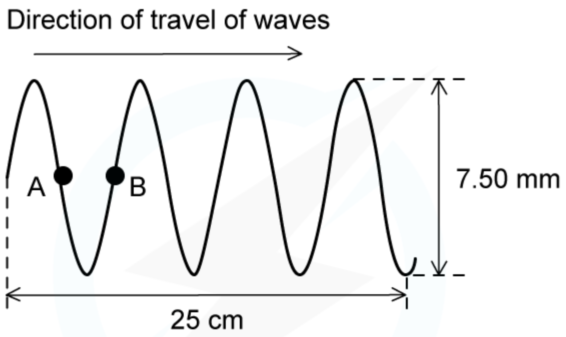
- In contrast, the purple wave is said to **lag** behind the green wave by ¼ λ
- Phase difference is measured in **fractions of a wavelength, degrees** or **radians**
- The phase difference can be calculated from two different points on the same wave or the same point on two different waves
- The phase difference between two points:
  - **In phase** is **360°** or 2π radians
  - **In anti-phase** is **180°** or π radians

7. Waves

YOUR NOTES

Worked example

Plane waves on the surface of water are represented by the figure below at one particular instant of time.



The waves have frequency 2.5 Hz.

Determine, for the waves,

A. the amplitude

B. the wavelength

C. the phase difference between points A and B

7. Waves

YOUR NOTES

A. THE AMPLITUDE

MAXIMUM DISPLACEMENT FROM THE EQUILIBRIUM POSITION

$7.50\text{ mm} \div 2 = 3.75\text{ mm}$

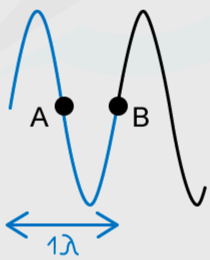
B. THE WAVELENGTH

DISTANCE BETWEEN POINTS ON SUCCESSIVE OSCILLATIONS OF THE WAVE THAT ARE IN PHASE

FROM DIAGRAM: $25\text{cm} = 3\frac{1}{2}$ WAVELENGTHS

$1\lambda = 25\text{ cm} \div 3\frac{1}{2} = 7.14\text{ cm}$

C. THE PHASE DIFFERENCE BETWEEN POINTS A AND B



POINTS A AND B HAVE $\frac{1}{2}\lambda$ DIFFERENCE $= \frac{1}{2} \times 360° = 180°$

Exam Tip

When labelling the wavelength and time period on a diagram, make sure that your arrows go from the **very top** of a wave to the very top of the next one. If your arrow is too short, you will lose marks. The same goes for labelling amplitude, don't draw an arrow from the bottom to the top of the wave, this will lose you marks too.

7. Waves

YOUR NOTES
⬇

Wave Energy

- Waves transfer energy between points, without transferring matter
- When a wave travels between two points, no matter actually travels with it:
  - The points on the wave simply vibrate back and forth about fixed positions
- Waves that transfer **energy** are known as **progressive** waves
- Waves that do not transfer energy are known as **stationary** waves

7. Waves

YOUR NOTES

7.1.2 CATHODE-RAY OSCILLOSCOPE

Cathode-Ray Oscilloscope

- A Cathode-Ray Oscilloscope is a laboratory instrument used to display, measure and analyse waveforms of electrical circuits
- An A.C. current on an oscilloscope is represented as a transverse wave. Therefore you can determine its frequency and amplitude
- The x-axis is the **time** and the y-axis is the **voltage (**or **y-gain)**

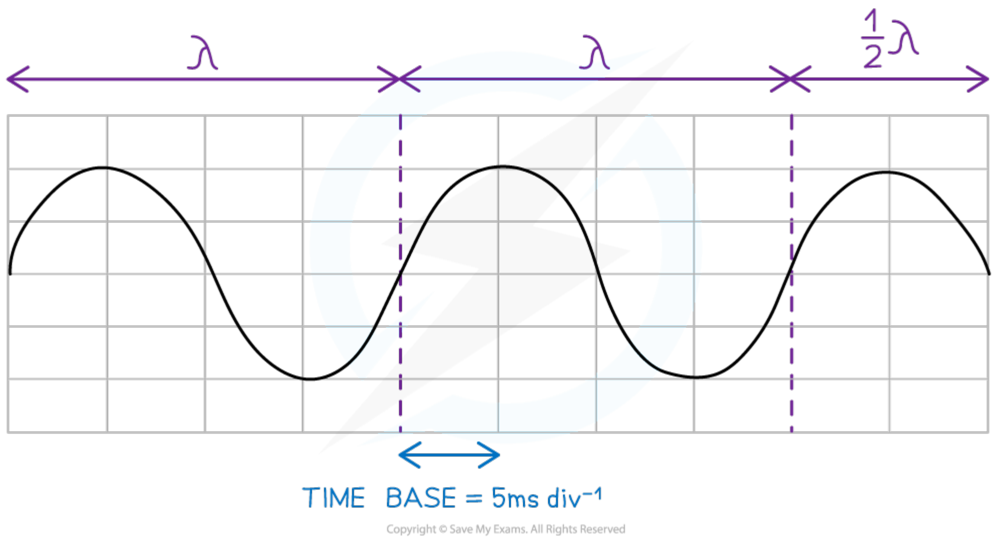


Diagram of Cathode-Ray Oscilloscope display showing wavelength and time-base setting

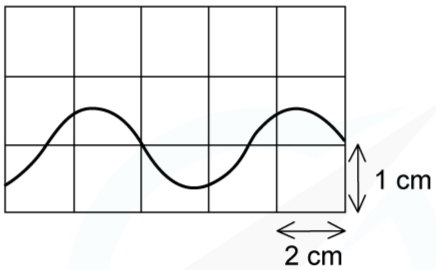
- The period of the wave can be determined from the **time-base** This is **how many seconds each division represents** measured commonly in **s div^{-1}** or **s cm^{-1}**
- Use as many wavelengths shown on the screen as possible to reduce uncertainties
- Dividing the total time by the number of wavelengths will give the time period *T* (Time taken for one complete oscillation)
- The **frequency** is then determined through **1/T**

7. Waves

YOUR NOTES

Worked example

A cathode-ray oscilloscope (c.r.o.) is used to diasplay the trace from a sound wave. The time-base is set at 7 μs mm^{-1}.



What is the frequency of the sound wave?

A. 2.4 Hz **B**. 24 Hz **C.** 2.4 kHz **D.** 24 kHz

ANSWER: C

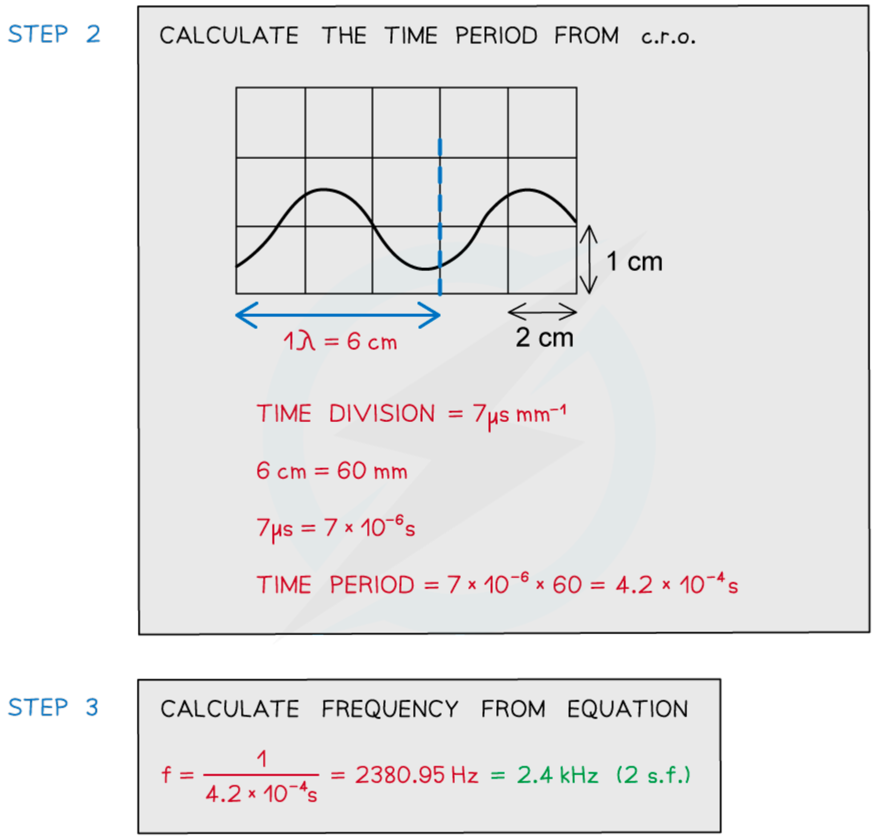
STEP 1

FREQUENCY EQUATION

$f = \frac{1}{T}$

7. Waves

YOUR NOTES



Exam Tip

The time-base setting varies with units for seconds (commonly ms) and the unit length (commonly mm). Unit conversions are very important for the calculate of the time period and frequency

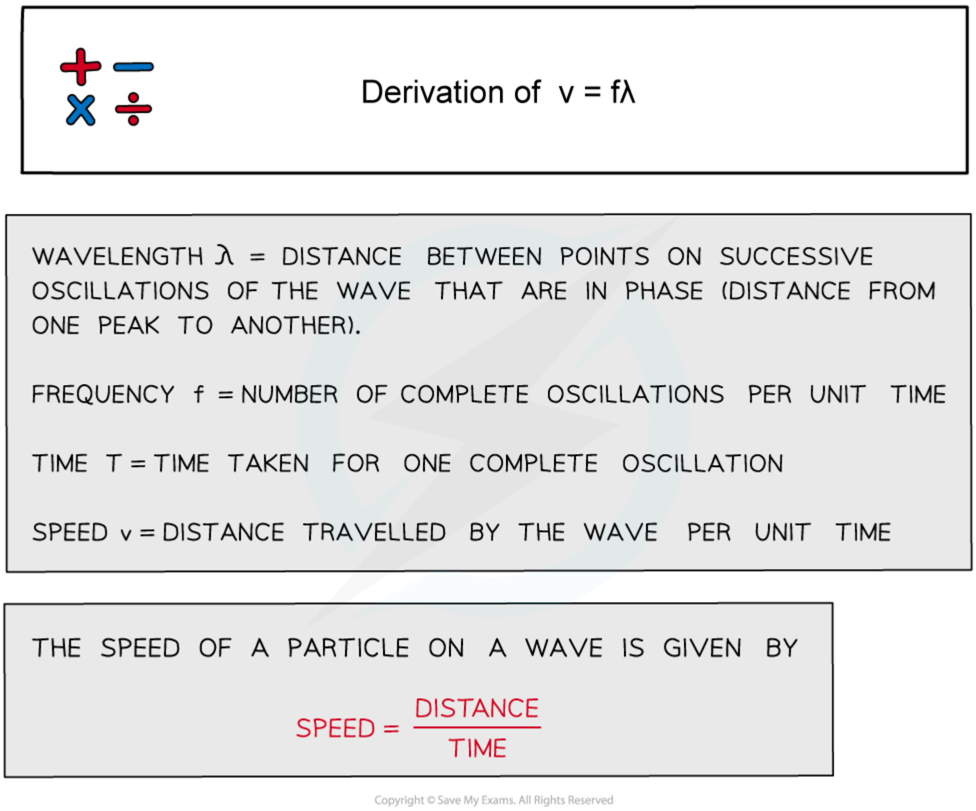
7. Waves

YOUR NOTES
⬇

7.1.3 THE WAVE EQUATION

Derivation of v = f λ

- Using the definitions of speed, frequency and wavelength, the wave equation $v = f\lambda$ can be derived
- This is an important relationship between three key properties of a wave
- The derivation for this is shown below



7. Waves

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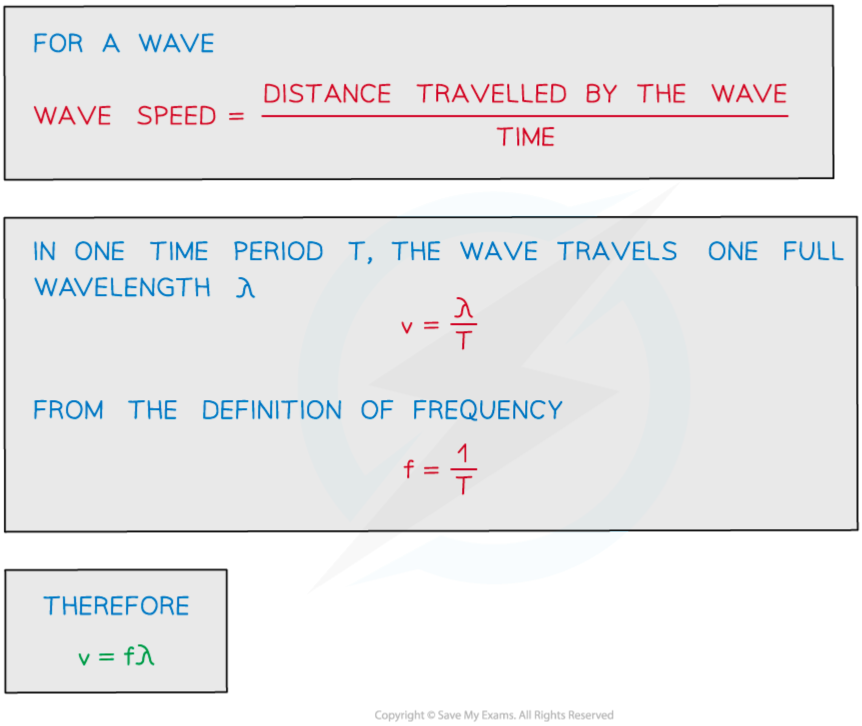


Derivation of v = fλ

Exam Tip

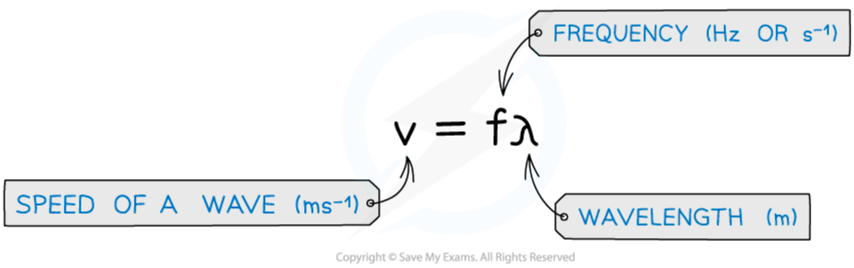
You will be expected to remember all the steps for this derivation (but do not need to write the full definition for each variable). If you are unsure as to where speed = distance/time comes from, make sure to revisit chapter "2. Kinematics".

7. Waves

YOUR NOTES

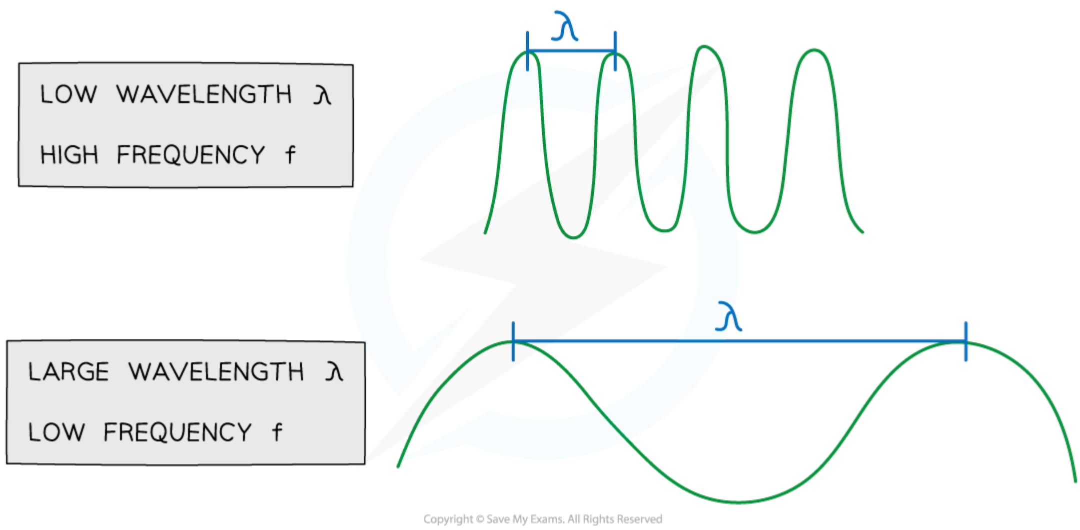
The Wave Equation

- The wave equation links the speed, frequency and wavelength of a wave
- This is relevant for both transverse and longitudinal waves



The Wave Equation

- The wave equation tells us that for a wave of constant speed:
  - As the wavelength **increases,** the frequency **decreases**
  - As the wavelength **decreases,** the frequency **increases**



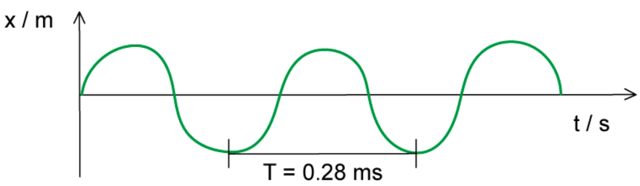
The relationship between frequency and wavelength of a wave

7. Waves

YOUR NOTES

Worked example

The wave below has a speed of 340 ms^{-1}



What is its wavelength?

A. 0.095 m **B**. 95 m **C.** 1.2 × 10^{3} m **D.** 12 × 10^{6} m

ANSWER: A

STEP 1

WAVE EQUATION

$v = f\lambda$

STEP 2

REARRANGE FOR WAVELENGTH

$\lambda = \frac{v}{f}$

STEP 3

CALCULATE f

$f = \frac{1}{T} = \frac{1}{0.28 \times 10^{-3}\,s} = 3571.43$ Hz

STEP 4

SUBSTITUTE VALUE BACK INTO WAVE EQUATION

$\lambda = \frac{340}{3571.43} = 0.095$ m (2 s.f.)

7. Waves

YOUR NOTES

Exam Tip

You may also see the wave equation be written as $c = f\lambda$ where c is the wave speed. However, c is often used to represent a specific speed the speed of light (3×10^{8} ms^{-1}). Only electromagnetic waves travel at this speed, therefore it's best practice to use v for any speed that isn't the speed of light instead.

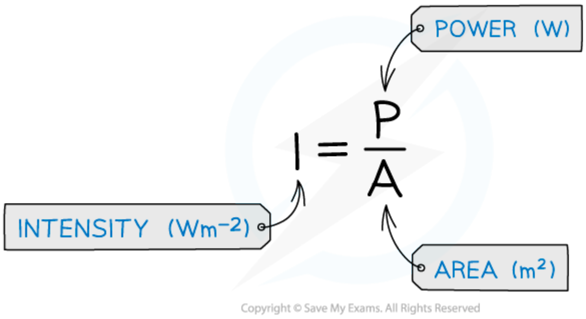
7. Waves

YOUR NOTES
⬇

7.1.4 WAVE INTENSITY

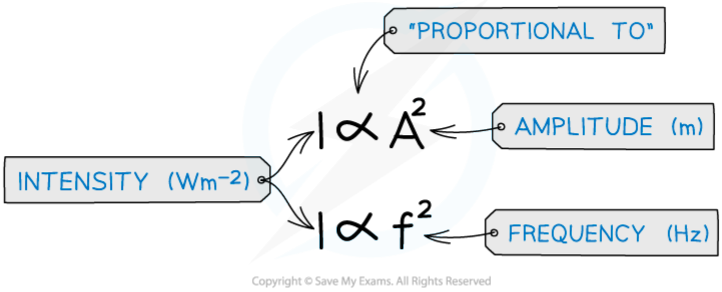
Wave Intensity

- Progressive waves transfer **energy**
- The amount of energy passing through a unit area per unit time is the **intensity** of the wave
- Therefore, the **intensity** is defined as **power per unit area**



Intensity is equal to the power per unit area

- The area the wave passes through is perpendicular to the direction of its velocity
- The intensity of a progressive wave is also proportional to its amplitude squared and frequency squared



Intensity is proportional to the amplitude² and frequency²

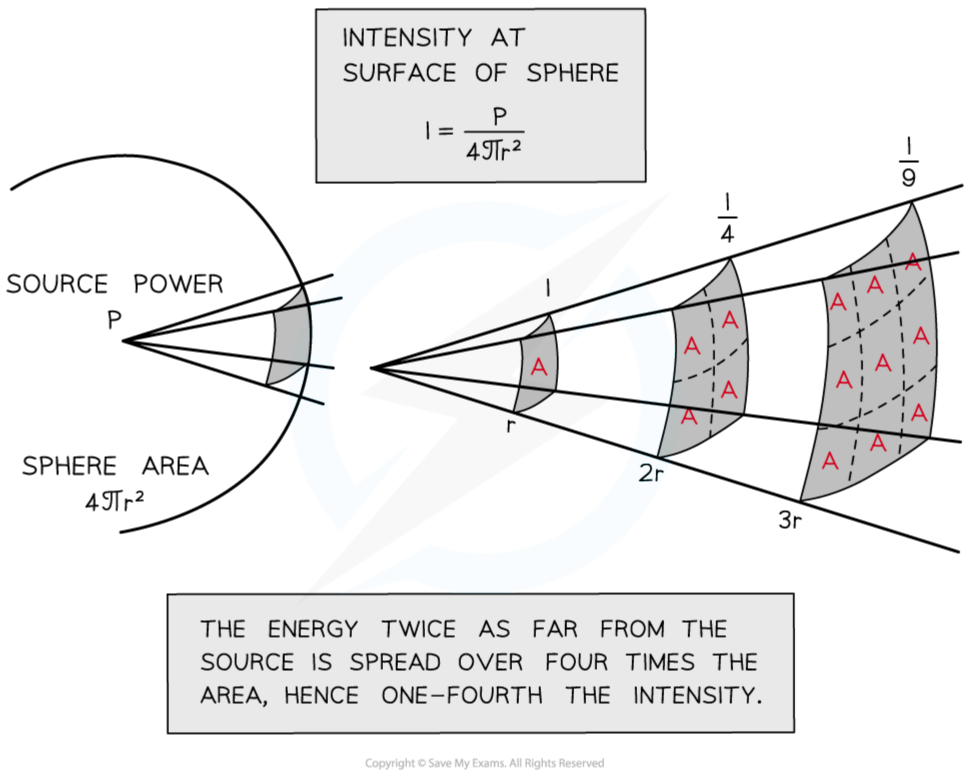
7. Waves

YOUR NOTES

- This means that if the frequency or the amplitude is doubled, the intensity increases by a factor of 4 (2^2)

Spherical waves

- A spherical wave is a wave from a point source which spreads out equally in all directions
- The area the wave passes through is the **surface area** of a sphere: **$4\pi r^2$**
- As the wave travels further from the source, the energy it carries passes through increasingly larger areas as shown in the diagram below:



Intensity is proportional to the amplitude squared

- Assuming there's no absorption of the wave energy, the intensity I decreases with increasing distance from the source
- Note the intensity is proportional to $1/r^2$
  - This means when the source is twice as far away, the intensity is 4 times less
- The $1/r^2$ relationship is known in physics as the **inverse square law**

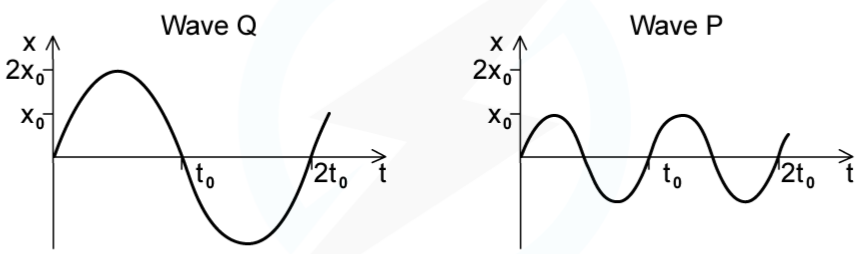
7. Waves

YOUR NOTES

Worked example

The intensity of a progressive wave is proportional to the square of the amplitude of the wave. It is also proportional to the square of the frequency.

The variation with time t of displacement x of particles when two progressive waves Q and P pass separately through a medium, are shown on the graphs.



The intensity of wave Q is I_0.

What is the intensity of wave P?

STEP 1

INTENSITY EQUATION

$I \propto A^2$

$I \propto f^2$

7. Waves

YOUR NOTES
⬇

STEP 2

CALCULATE HOW MUCH THE AMPLITUDE HAS INCREASED/DECREASED

WAVE P IS HALF THE AMPLITUDE OF WAVE Q

$$A_P = \frac{1}{2^2} A_Q$$

WAVE $P = \frac{1}{4} I_0$ OF WAVE Q

STEP 3

CALCULATE HOW MUCH THE FREQUENCY HAS INCREASED/DECREASED

WAVE P IS DOUBLE THE FREQUENCY OF WAVE Q

$$f_P = 2^2 f_Q$$

WAVE $P = 4I_0$ OF WAVE Q

STEP 4

SUBSTITUTE BACK INTO INTENSITY EQUATION

$$I_0(P) \propto \left(\frac{1}{4} \times 4\right) I_0(Q)$$

INTENSITY OF WAVE P = INTENSITY OF WAVE Q $= I_0$

Exam Tip

The key concept with intensity is that it has an inverse square relationship with distance (not a linear one). This means the energy of a wave decreases very rapidly with increasing distance

7. Waves

YOUR NOTES

7.1.5 TRANSVERSE & LONGITUDINAL WAVES

Properties of Transverse & Longitudinal Waves

- In mechanical waves, particles oscillate about fixed points
- The direction of oscillations with regards to the direction of wave travel determine what type of wave it is

Transverse waves

- A transverse wave is one where the particles oscillate **perpendicular** to the direction of the wave travel (and energy transfer)
- Transverse waves show areas of **crests** (peaks) and **troughs**

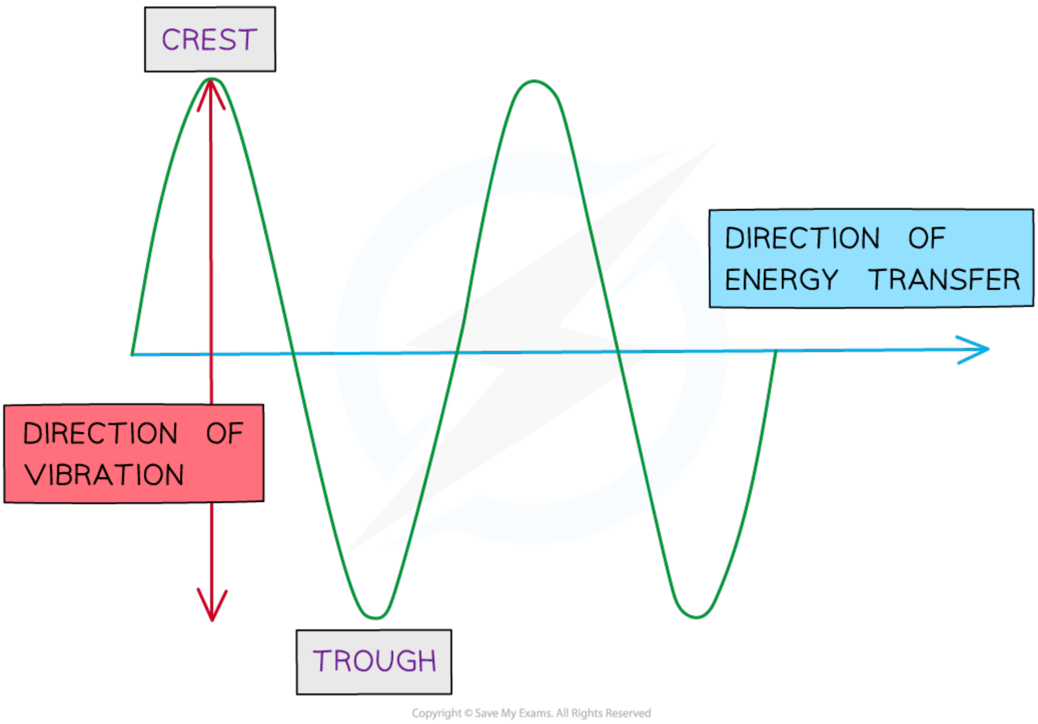


Diagram of a transverse wave

- Examples of transverse waves are:
  - Electromagnetic waves e.g. radio, visible light, UV

7. Waves

YOUR NOTES
⬇

- Vibrations on a guitar string

- These can be shown on a rope
- Transverse waves **can** be polarised

Longitudinal waves

- A longitudinal wave is one where the particles oscillate **parallel** to the direction of the wave travel (and energy transfer)
- Longitudinal waves show areas of **compressions** and **rarefactions**

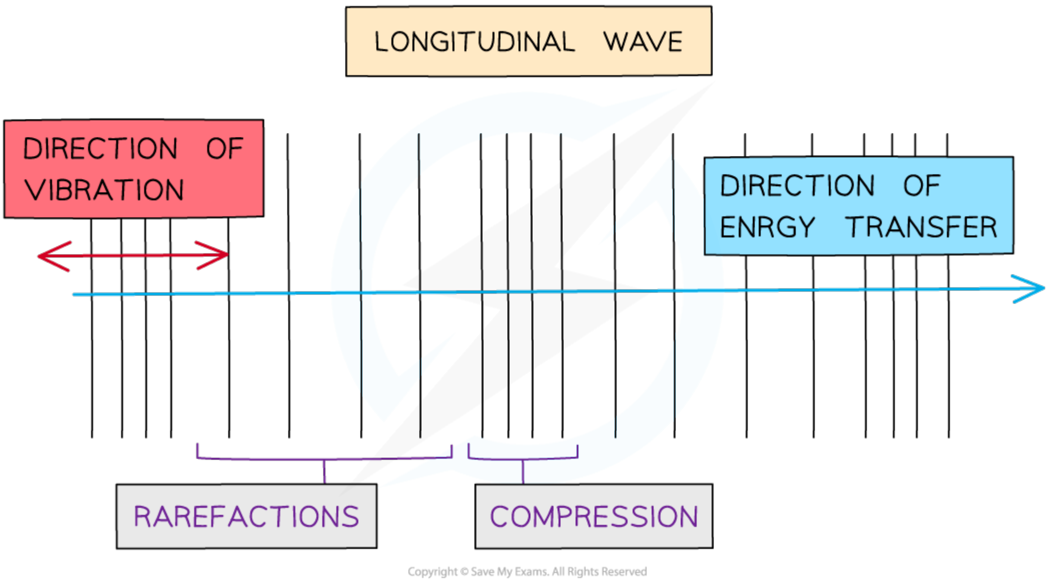
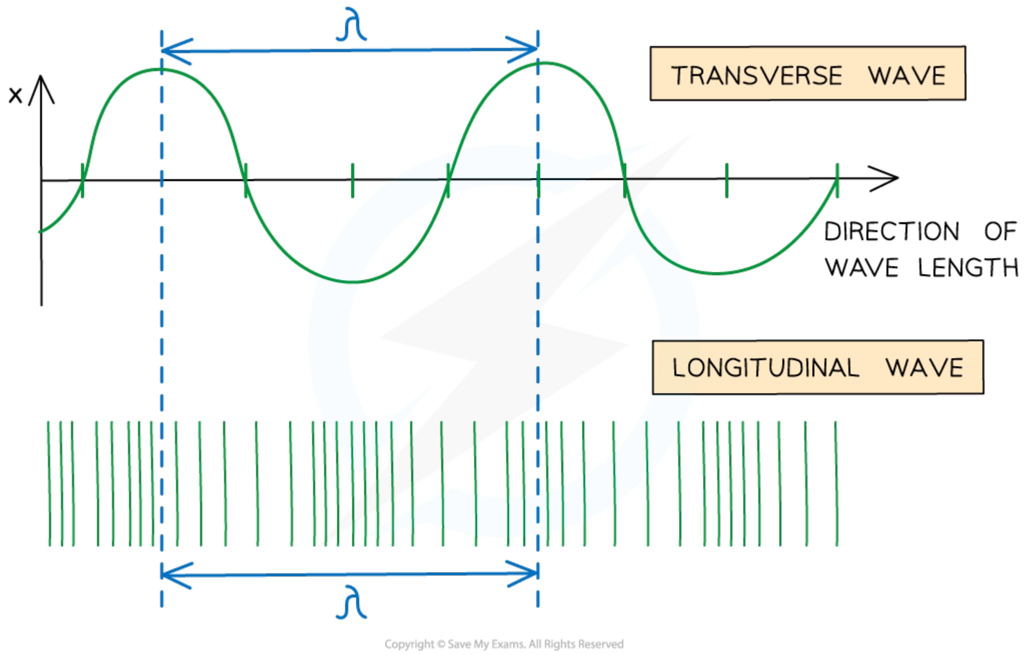


Diagram of a longitudinal wave

- Examples of longitudinal waves are:
  - Sound waves
  - Ultrasound waves
- These can be shown on a slinky spring
- Longitudinal waves **cannot** be polarised
- You will have learned how to analyse the properties of a wave, such as amplitude and wavelength, in "General Wave Properties"
- The diagram below shows the equivalent of a wavelength on a longitudinal wave

7. Waves

YOUR NOTES



Wavelength shown on a longitudinal wave

Exam Tip

The definition of transverse and longitudinal waves are often asked as exam questions, make sure to remember these definitions by heart!

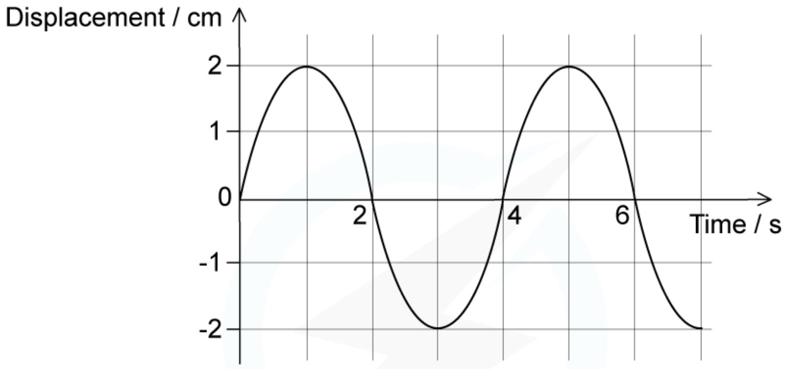
7. Waves

YOUR NOTES

Graphical Representations of Transverse & Longitudinal Waves

Worked example

The graph shows how the displacement of a particle in a wave varies with time.



Which statement is correct?

A. The wave has an amplitude of 2 cm and could be either transverse or longitudinal.

B. The wave has an amplitude of 2 cm and has a time period of 6 s.

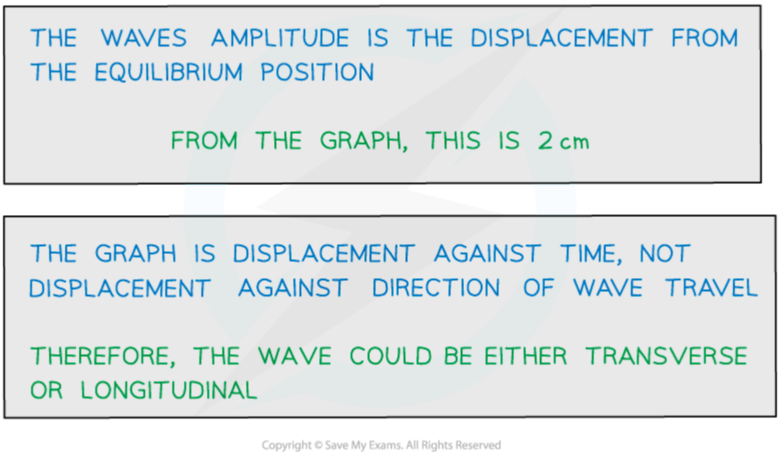
C. The wave has amplitude of 4 cm and has a time period of 4 s.

D. The wave has amplitude of 4 cm and must be transverse.

ANSWER: A

7. Waves

YOUR NOTES



Exam Tip

Both transverse and longitudinal waves can look like transverse waves when plotted on a graph – make sure you read the question and look for whether the wave travels **parallel** (longitudinal) or **perpendicular** (transverse) to the direction of travel to confirm which type of wave it is.

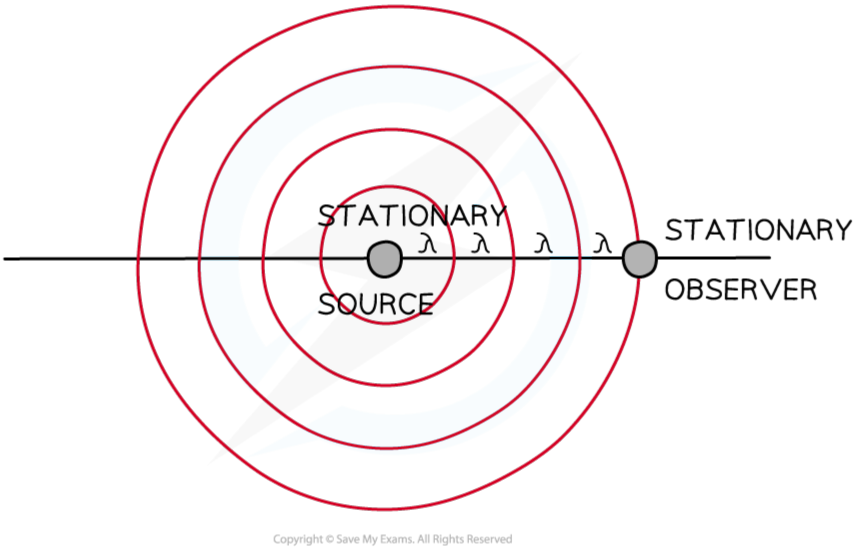
7. Waves

YOUR NOTES ⬇

7.1.6 DOPPLER EFFECT FOR SOUND WAVES

Doppler Shift of Sound

- The whistle of a train or the siren of an ambulance appears to increase in frequency (sounds higher in pitch) as it moves away from you
- This frequency change due to the relative motion between a source of sound or light and an observer is known as the **doppler effect** (or **doppler shift**)
- When the observer (e.g. yourself) and the source of sound (e.g. ambulance siren) are both **stationary**, the waves are at the **same** frequency for both the observer and the source

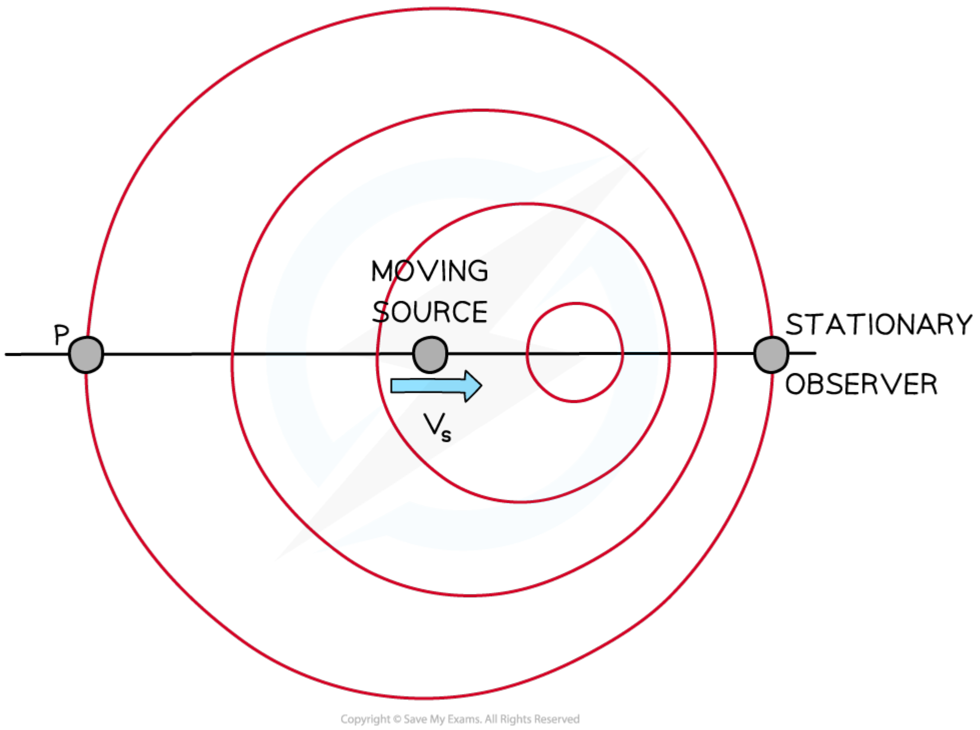


Stationary source and observer

- When the source starts to move **towards** the observer, the wavelength of the waves is **shortened**. The sound therefore appears at a **higher** frequency to the observer

7. Waves

YOUR NOTES



Moving source and stationary observer

- Notice how the waves are closer together between the source and the observer compared to point P and the source
- This also works if the source is moving away from the observer. If the observer was at point P instead, they would hear the sound at a lower frequency due to the wavelength of the waves **broadening**

- The frequency is **increased** when the source is moving **towards** the observer
- The frequency is **decreased** when the source is moving **away** from the observer

7. Waves

Worked example

A cyclist rides a bike ringing their bell past a stationary observer.

Which of the following accurately describes the doppler shift caused by the sound of the bell?

	Wavelength	Frequency	Sound pitch
A	Shorter	Higher	Lower
B	Longer	Lower	Higher
C	Shorter	Lower	Higher
D	Longer	Lower	Lower

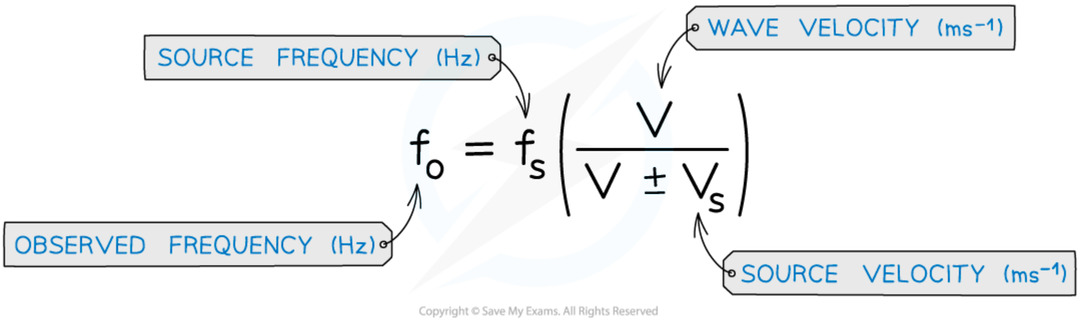
ANSWER: D

- If the cyclist is riding past the observer, the wavelength of sound waves are going to become longer
    - This rules out options A and C
- A longer wavelength means a lower frequency (from the wave equation)
- Lower frequency creates a lower sound pitch
    - Therefore, the answer is row D

YOUR NOTES

Calculating Doppler Shift

- When a source of sound waves moves relative to a stationary observer, the observed frequency can be calculated using the equation below:



Doppler shift equation

- The wave velocity for sound waves is 340 ms^{-1}
- The ± depends on whether the source is moving towards or away from the observer
  - If the source is moving **towards**, the denominator is $v - v_s$
  - If the source is moving **away**, the denominator is $v + v_s$

7. Waves

YOUR NOTES
⬇

Worked example

A police car siren emits a sound wave with a frequency of 450 Hz.
The car is travelling away from an observer at speed of 45 ms^{-1}.
The speed of sound is 340 ms^{-1}.
Which of the following is the frequency the observer hears?

A. 519 Hz **B.** 483 Hz **C.** 397 Hz **D.** 358 Hz

ANSWER: C

STEP 1

DOPPLER SHIFT EQUATION

$$f_o = f_s\left(\frac{V}{V \pm V_s}\right)$$

STEP 2

SUBSTITUTE VALUES INTO THE EQUATION

$f_s = 450$ Hz
V = SPEED OF SOUND = 340 ms^{-1}
V_s = VELOCITY OF THE POLICE CAR (SOURCE) = 45 ms^{-1}

THE SOURCE IS MOVING AWAY FROM THE OBSERVER, SO WE USE $V + V_s$

$$f_o = 450\left(\frac{340}{340 + 45}\right) = 397 \text{ Hz} \quad (3 \text{ s.f.})$$

Exam Tip

Be careful as to which frequency and velocity you use in the equation. The 'source' is always the object which is moving and the 'observer' is always stationary.

7. Waves

YOUR NOTES
⬇

Exam Question: Easy

A wave has a frequency of 4 GHz.

What is the period of the wave?

A 25 000 μs

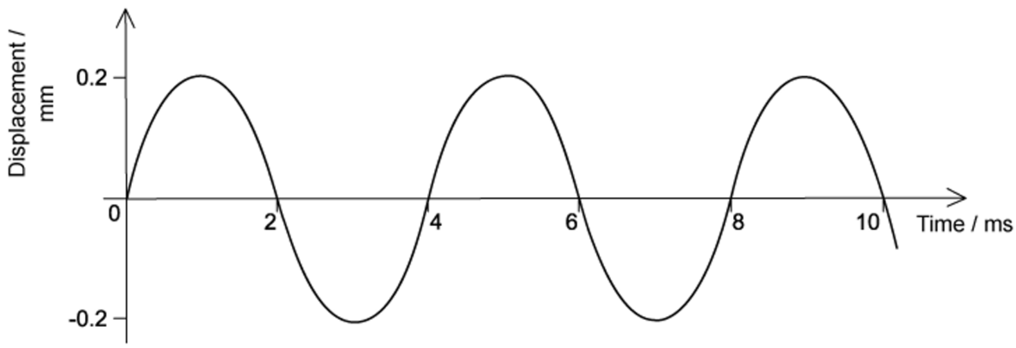
B 250 ps

C 25 ns

D 2.5 ns

Exam Question: Medium

Sound travels with a speed of 330 m s^{-1} in air. The variation with time of the displacement of an air particle due to a sound wave is shown below.



Which of the following statements about the wave is correct?

A the wavelength of the sound wave is 1.32 m

B the graph shows that the sound is a transverse wave

C the frequency of the wave is 500 Hz

D the intensity of the wave will be doubled if its amplitude is increased to 0.4 mm

7. Waves

YOUR NOTES

Exam Question: Hard

The equation below shows the speed of *v* waves in deep water

$$v^2 = \frac{g\lambda}{2\pi}$$

Where *g* is the acceleration of free fall and λ is the wavelength of the waves. The wavelength λ and the frequency *f* of the wave were measured.

If the graph was to give a straight line through the origin which of the following graphs should be plotted?

A f against λ^2

B f^2 against λ

C f against $\frac{1}{\lambda}$

D f^2 against $\frac{1}{\lambda}$

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7. Waves

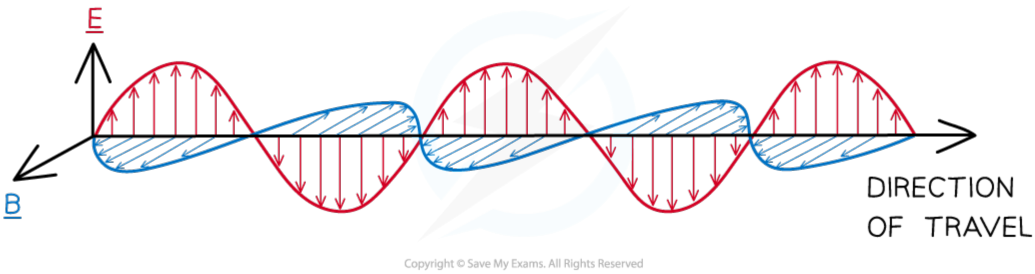
YOUR NOTES
⬇

7.2 TRANSVERSE WAVES: EM SPECTRUM & POLARISATION

7.2.1 ELECTROMAGNETIC SPECTRUM

Properties of Electromagnetic Waves

- Visible light is just one part of a much bigger spectrum: The Electromagnetic Spectrum
- All electromagnetic waves have the following properties in common:
    - They are all **transverse** waves
    - They can all travel in a **vacuum**
    - They all travel at the **same speed** in a vacuum (free space) — the speed of light **3×10^8 ms^{-1}**
- The speed of light in air is approximately the same



Oscillating electric and magnetic fields in an electromagnetic wave

- These transverse waves consists of electric and magnetic fields oscillating at right angles to each other and to the direction in which the wave is travelling (in 3D space)
- Since they are transverse, all waves in this spectrum can be reflected, refracted, diffracted, polarised and produce interference patterns

7. Waves

YOUR NOTES

Uses of electromagnetic waves

- Electromagnetic waves have a large number of uses. The main ones are summarised in the table below

WAVE	USE
RADIO	• COMMUNICATION (RADIO AND TV)
MICROWAVE	• HEATING FOOD • COMMUNICATION (WIFI, MOBILE PHONES, SATELLITES)
INFRARED	• REMOTE CONTROLS • FIBRE OPTIC COMMUNICATION • THERMAL IMAGING (MEDICINE AND INDUSTRY) • NIGHT VISION • HEATING OR COOKING THINGS • MOTION SENSORS (FOR SECURITY ALARMS)
VISIBLE LIGHT	• SEEING AND TAKING PHOTOGRAPHS/VIDEOS
ULTRAVIOLET	• SECURITY MARKING (FLUORESCENCE) • FLUORESCENT BULBS • GETTING A SUNTAN.
X-RAYS	• X-RAY IMAGES (MEDICINE, AIRPORT SECURITY AND INDUSTRY)
GAMMA RAYS	• STERILISING MEDICAL INSTRUMENTS TREATING CANCER

Exam Tip

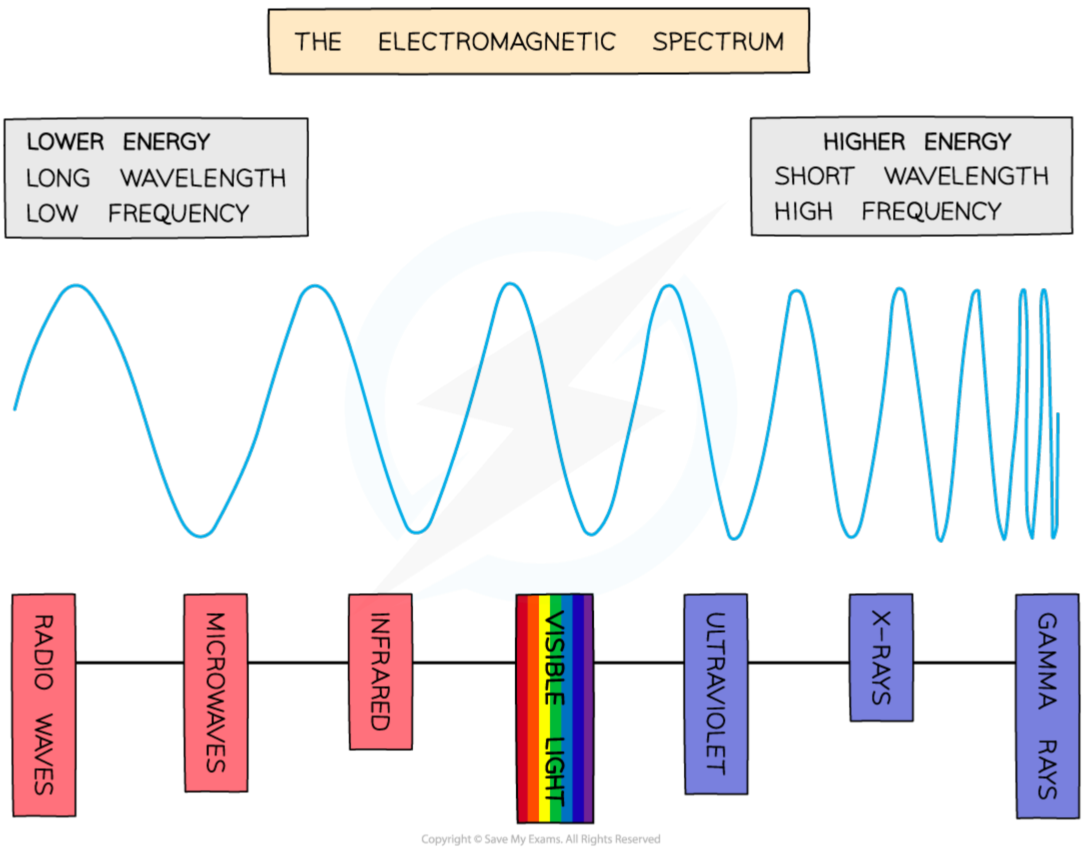
You will be expected to recall the common properties of all electromagnetic waves in an exam question, however the speed of light will be given on the data sheet.

7. Waves

YOUR NOTES
⬇

From Radio Waves to Gamma Rays

- The electromagnetic spectrum is arranged in a specific order based on their wavelengths or frequencies
- This order is shown in the diagram below from longest wavelength (lowest frequency) to shortest wavelength (highest frequency)



Energy, wavelength and frequency for each part of the electromagnetic spectrum

7. Waves

YOUR NOTES

- The higher the **frequency**, the higher the **energy** of the radiation
- Radiation with higher energy is highly ionising and is harmful to cells and tissues causing cancer (e.g. UV, X-rays, Gamma rays)
- The approximate wavelengths in a vacuum of each radiation is listed in the table below:

EM spectrum wavelengths and frequencies

Radiation	Approximate wavelength range / m	Approximate frequency range / Hz
Radio	> 0.1	$< 3 \times 10^{9}$
Microwaves	$0.1 - 1 \times 10^{-3}$	$3 \times 10^{9} - 3 \times 10^{11}$
Infra-red	$1 \times 10^{-3} - 7 \times 10^{-7}$	$3 \times 10^{11} - 4.3 \times 10^{14}$
Visible	$4 \times 10^{-7} - 7 \times 10^{-7}$	$7.5 \times 10^{14} - 4.3 \times 10^{14}$
Ultra-violet	$4 \times 10^{-7} - 1 \times 10^{-8}$	$7.5 \times 10^{14} - 3 \times 10^{16}$
X-rays	$1 \times 10^{-8} - 4 \times 10^{-13}$	$3 \times 10^{16} - 7.5 \times 10^{20}$
Gamma rays	$1 \times 10^{-10} - 1 \times 10^{-16}$	$3 \times 10^{18} - 3 \times 10^{24}$

- To alternatively find the range of frequencies, convert the wavelengths using the wave equation: $\mathbf{c = f\lambda}$ where c is the speed of light: $\mathbf{3.0 \times 10^{8}\ m\ s^{-1}}$

7. Waves

YOUR NOTES
⬇

Worked example

A is a source emitting microwaves and B is a source emitting X-rays.

The table suggests the frequencies for A and B.

Which row is correct?

	Frequency emitted by A / Hz	Frequency emitted by B / Hz
A	$3 \times 10^{9} - 3 \times 10^{11}$	$> 10^{19}$
B	$1 \times 10^{12} - 1 \times 10^{13}$	$3 \times 10^{16} - 7.5 \times 10^{20}$
C	$3 \times 10^{9} - 3 \times 10^{11}$	$3 \times 10^{16} - 7.5 \times 10^{20}$
D	$4 \times 10^{14} - 8 \times 10^{14}$	$5 \times 10^{13} - 7 \times 10^{15}$

ANSWER: C

STEP 1

THE WAVE EQUATION

$c = f\lambda$

STEP 2

REARRANGE FOR FREQUENCY

$f = \frac{c}{\lambda}$

7. Waves

YOUR NOTES
⬇

STEP 3

THE RANGE OF WAVELENGTH FOR MICROWAVES IS $0.1 - 1 \times 10^{-3}$ m

USE WAVE EQUATION TO FIND EQUIVALENT FREQUENCIES FOR MICROWAVES

$$f = \frac{3 \times 10^{8}}{0.1} = 3.0 \times 10^{9} \qquad f = \frac{3 \times 10^{8}}{1.0 \times 10^{-3}} = 3.0 \times 10^{11}$$

$f = 3.0 \times 10^{9} - 3.0 \times 10^{11}$ Hz

STEP 4

THE RANGE OF WAVELENGTH FOR X-RAYS IS $1 \times 10^{-8} - 4 \times 10^{-13}$ m

USE WAVE EQUATION TO FIND EQUIVALENT FREQUENCIES FOR X-RAYS

$$f = \frac{3 \times 10^{8}}{1 \times 10^{-8}} = 3 \times 10^{16}\ \text{Hz} \qquad f = \frac{3 \times 10^{8}}{4 \times 10^{-13}} = 7.5 \times 10^{20}\ \text{Hz}$$

$f = 3 \times 10^{16} - 7.5 \times 10^{20}$ Hz

STEP 5

ROW C MATCHES BOTH OF THESE FREQUENCY RANGES

Exam Tip

You will be expected to memorise the range of wavelengths for each type of radiation, however you don't need to learn the frequency ranges by heart. Since all EM waves travel at the speed of light, you can convert between frequency and wavelength using the wave equation in an exam question.

7. Waves

YOUR NOTES
⬇

Visible Light

- Visible light is defined as the range of wavelengths (400 – 700 nm) which are visible to humans
- Visible light is the only part of the spectrum detectable by the human eye
    - However, this is only 0.0035% of the whole electromagnetic spectrum
- In the natural world, many animals, such as birds, bees and certain fish, are able to perceive beyond visible light and can see infra-red and UV wavelengths of light

7. Waves

YOUR NOTES
⬇

7.2.2 POLARISATION

Polarisation

- Transverse waves are waves with their displacement perpendicular to their direction of travel. These oscillations can happen in **any plane** perpendicular to the propagation direction
- Transverse waves can be **polarised,** this means:
  - Vibrations are restricted to **one** direction
  - These vibrations are still **perpendicular** to the direction of propagation/energy transfer
- The difference between unpolarised and polarised waves are shown in the diagram below

7. Waves

YOUR NOTES

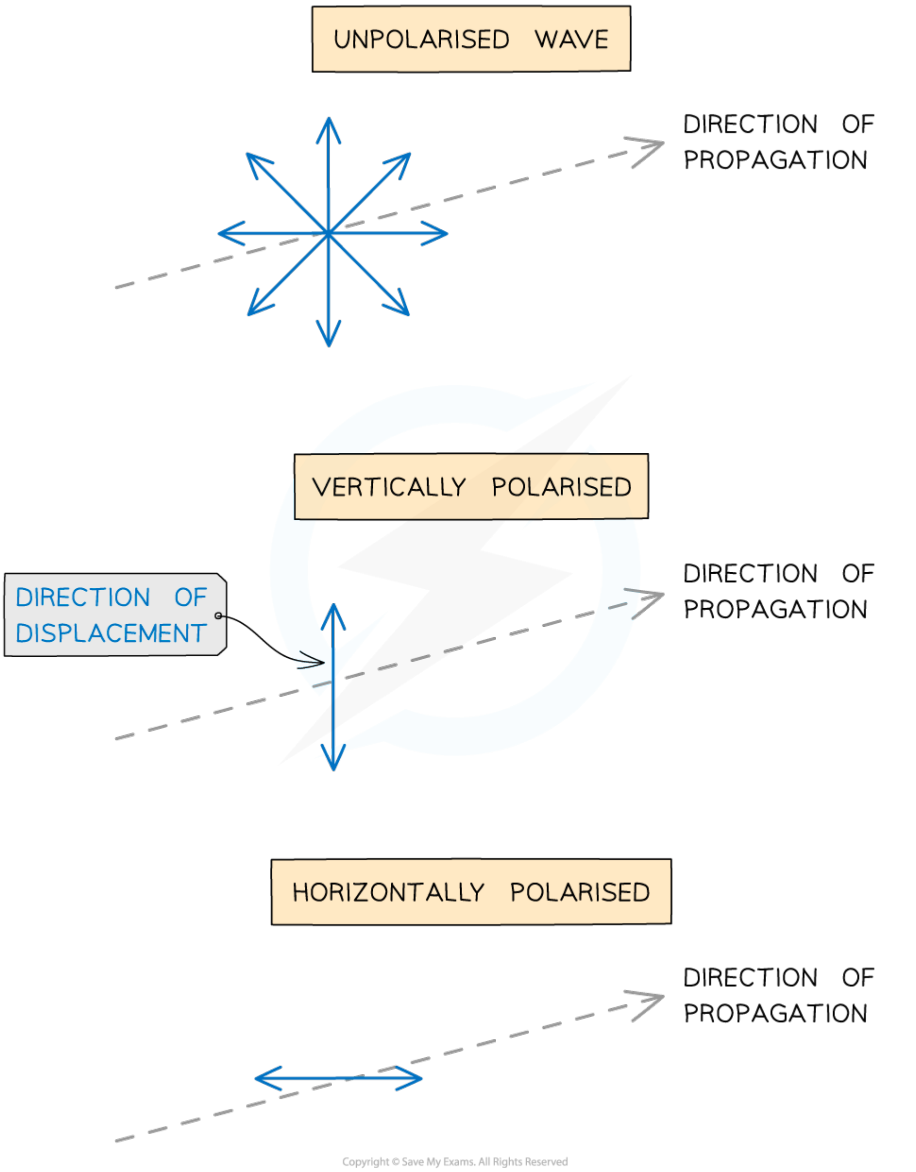


Diagram showing the displacement of unpolarised and polarised transverse waves

7. Waves

YOUR NOTES

- Longitudinal waves (e.g. sound waves) cannot be polarised since they oscillate parallel to the direction of travel
- Waves can be polarised through a **polariser** or **polarising filter**. This only allows oscillations in a certain plane to be transmitted

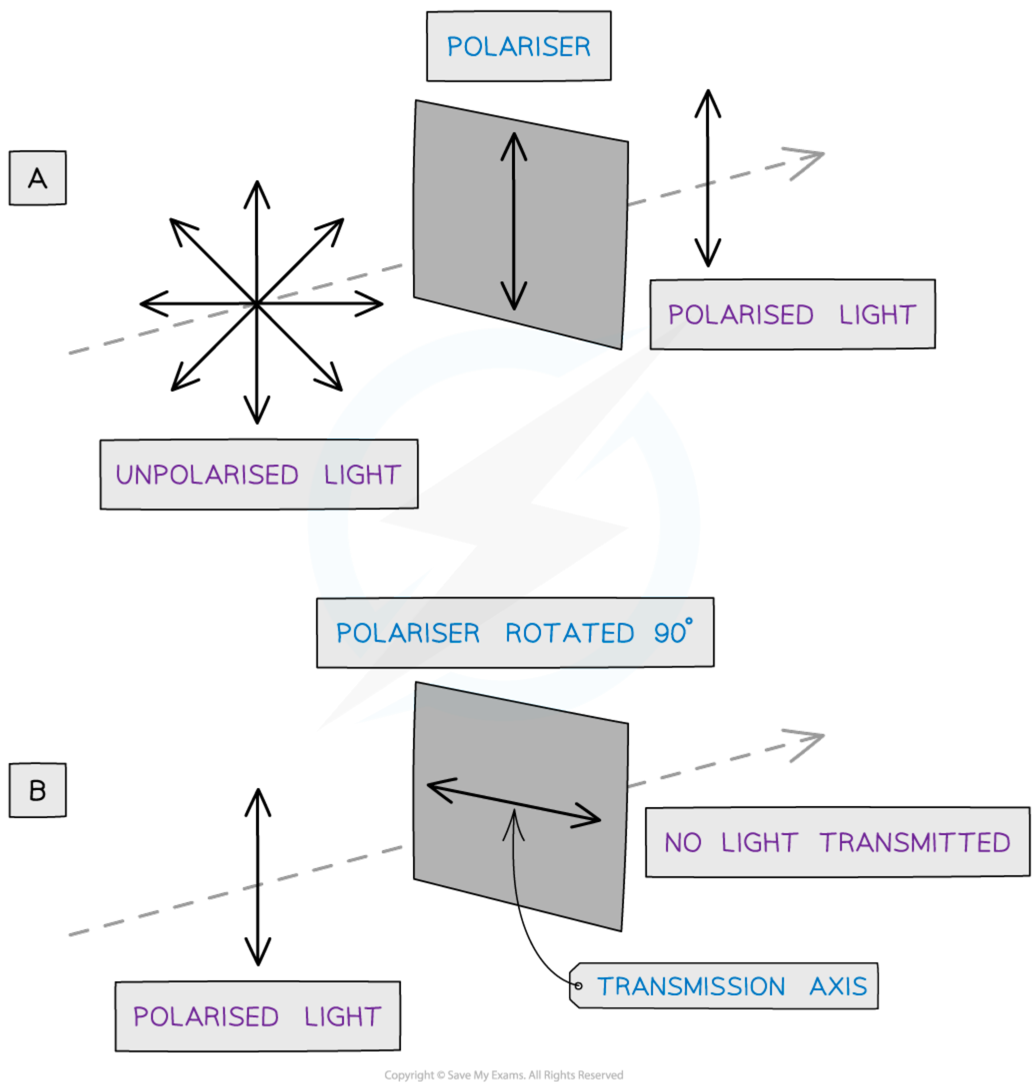


Diagram showing an unpolarised and polarised wave travelling through polarisers

7. Waves

YOUR NOTES

- Only unpolarised waves can be polarised as shown in diagram A
- When a polarised wave passes through a filter with a transmission axis perpendicular to the wave (diagram B), none of the wave will pass through
- Light can also be polarised through reflection, refraction and scattering
- An example of polarisation in everyday life is polaroid sunglasses. These reduce glare caused by sunlight for drivers to see through windows and fishermen to see beneath the water surface more clearly

Worked example

The following are statement about waves.

Which statement below describes a situation in which polarisation should happen?

A. Radio waves pass through a metal grid
B. Surface water waves are diffracted
C. Sound waves are reflected
D. Ultrasound waves pass through a metal grid.

ANSWER: A

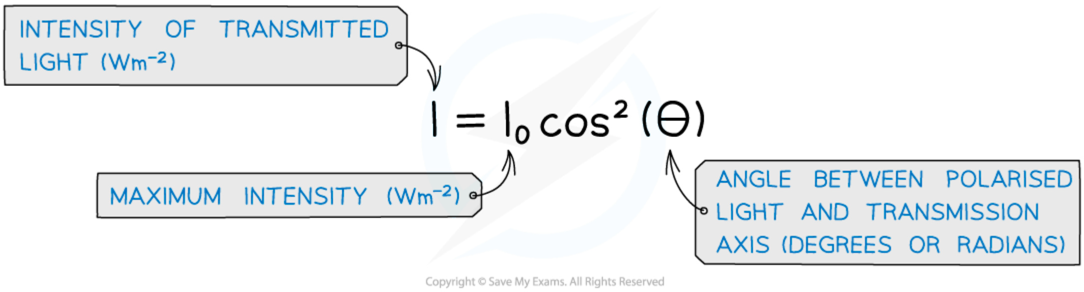
- Polarisation only occurs for transverse waves, therefore, **C** and **D** can be ruled out as sound and ultrasound are both longitudinal waves
- Waves are not polarised when diffracted, hence we can also rule out option **B**
- Radio waves are transverse waves – they can be polarised by a metal grid so only the waves that fit through the grid will be transmitted, therefore, **A** is correct

7. Waves

YOUR NOTES
⬇

Malus's Law

- Malus's law is used to find the intensity of light after passing through a number of polarising filters



Malus's law equation

7. Waves

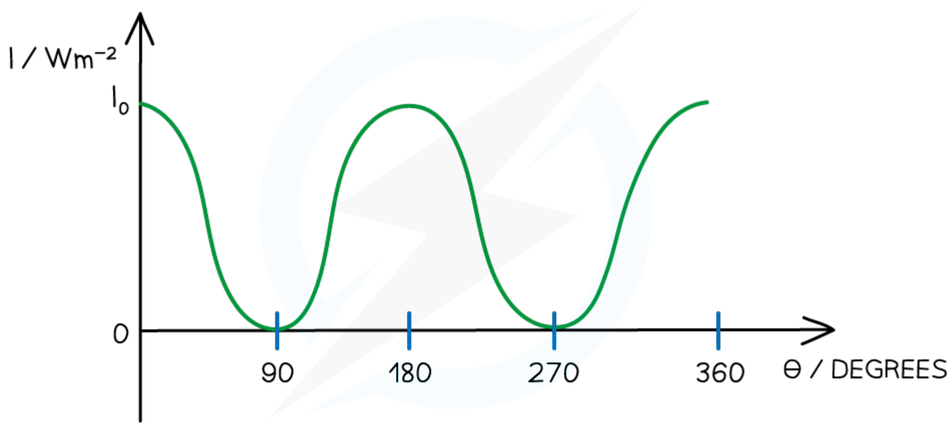
YOUR NOTES
⬇

- Recall that intensity is the power per unit area and measured in **W m^{-2}**
- A polariser will only transmit light that is polarised parallel to its transmission axis. This is seen in Malus's law by the angle θ:

Table of transmission depending on polariser orientation

Angle of transmission axis θ / degrees	Direction of transmission axis	$\cos^2\theta$	Transmitted intensity I / W m−2	Max or min light intensity transmitted
0	↕	1	I_0	Max
180				
90	↔	0	0	Min
270				

- The change in intensity against the angle of transmission axis is shown in the graph below



7. Waves

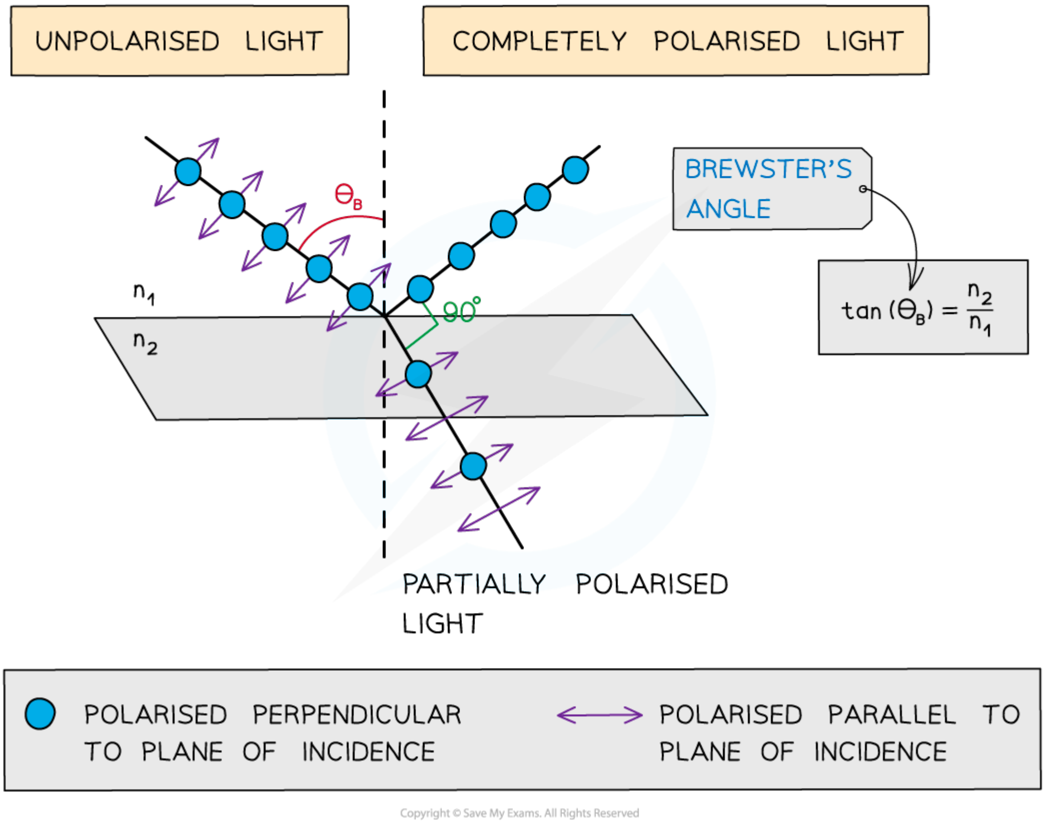
YOUR NOTES

The half rule

- When unpolarised light passes through the first polariser, half the intensity of the wave is always lost ($I_0/2$)

Brewster's angle

- Brewster's angle is an angle of incidence at which light with a particular polarisation is perfectly transmitted through a surface



- n_1 is the refractive index of the initial material (in this case, air)
- n_2 is the refractive index of the material scattering the light

7. Waves

YOUR NOTES
⬇

Worked example

Unpolarised light is incident on a polariser.
The light transmitted by the first polariser is then incident on a second polariser.
The polarising (or transmission) axis of the second polariser is 30° to that of the first.

The intensity incident on the first polariser is I.

What is the intensity emerging from the second polariser?

A. 0.75 I **B.** 0.38 I **C.** 0.87 I **D.** 0.43 I

ANSWER: B

STEP 1

FROM THE HALF RULE, WHEN THE LIGHT PASSES THROUGH THE FIRST POLARISER HALF OF ITS INTENSITY IS LOST

$$I_1 = \frac{I}{2}$$

STEP 2

MALUS'S LAW IS USED TO FIND THE INTENSITY OF THE POLARISED LIGHT AFTER THE SECOND POLARISER

$$I = I_0 \cos^2(\theta)$$

$$I_2 = I\cos^2(30)$$

$$I_2 = \frac{3}{4}I$$

STEP 3

COMBINE THE INTENSITY DROPS

TRANSMITTED INTENSITY $I = \left(\frac{1}{2} \times \frac{3}{4}\right)I$

$= 0.375\ I$

$= 0.38\ I$ (2 s.f.)

INTENSITIES ARE COMBINED BY MULTIPLYING THE FRACTION TRANSMITTED THROUGH EACH POLARISER

7. Waves

YOUR NOTES
⬇

Exam Tip

Remember when using Malus's law to **square** the cosine of the angle ($\cos^2 \theta$)

Exam Question: Easy

The following are statements about waves. Which statement below describes a situation in which polarisation would not happen?

A microwaves pass through a metal grid

B sound waves pass through a metal grid

C light waves are reflected

D light waves are scattered

Exam Question: Medium

Two lasers are set up to pass through a vacuum. One laser emits red light; the other emits green light.

Which property of the two lasers are different?

A speed

B plane of polarisation

C frequency

D amplitude

7. Waves

YOUR NOTES

Exam Question: Hard

A beam of unpolarised light is incident on the first of two parallel polarisers. The transmission axis of the two polarisers are initially parallel.

The first polariser is now rotated about the direction of the incident beam by an angle smaller than 90°.

Which row in the table describes the changes to the intensity and polarisation of the transmitted light?

	intensity	polarisation
A	no change	no change
B	no change	different
C	different	different
D	different	no change

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8. Superposition

YOUR NOTES
⬇

CONTENTS

8.1 STATIONARY WAVES

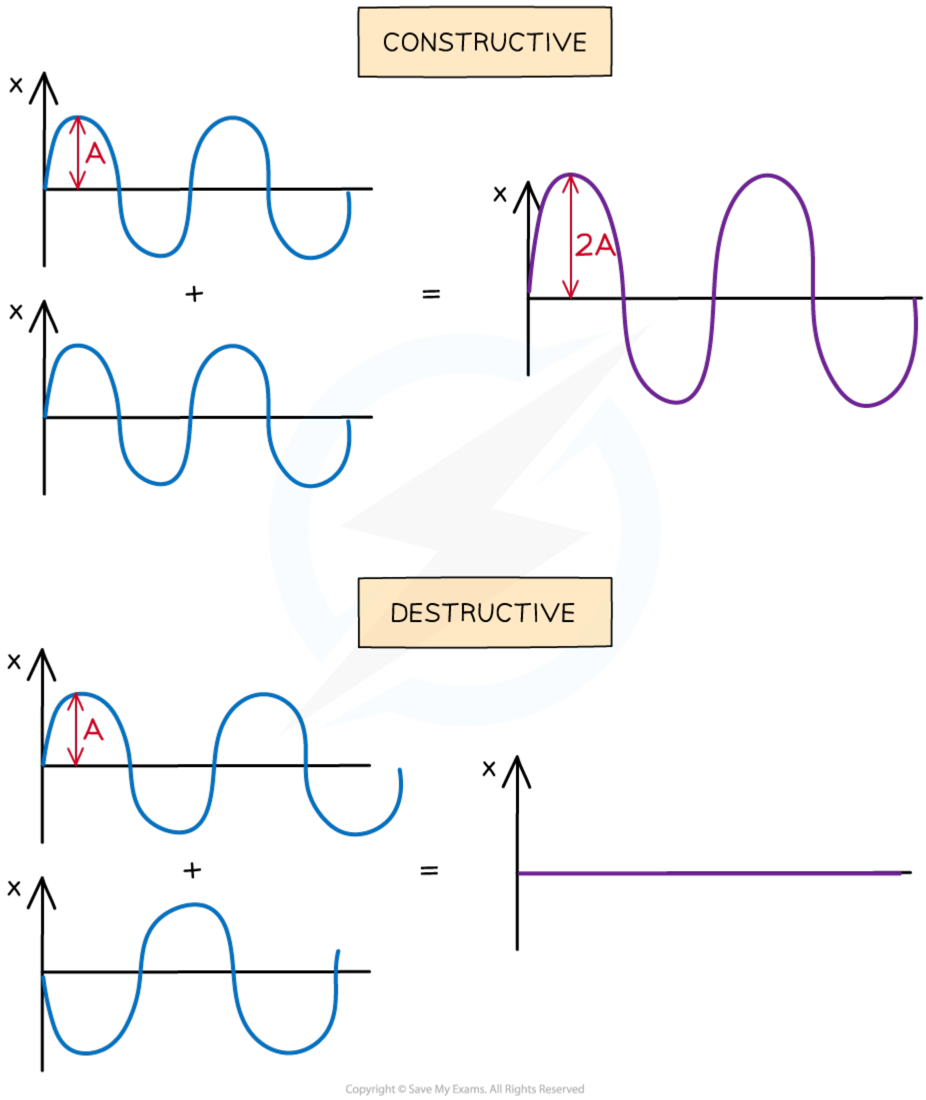
8.1.1 THE PRINCIPLE OF SUPERPOSITION

The Principle of Superposition

- The principle of superposition states that **when two or more waves with the same frequency travelling in opposite directions overlap, the resultant displacement is the sum of displacements of each wave**
- This principle describes how waves which meet at a point in space interact
- When two waves with the same frequency and amplitude arrive at a point, they superpose either:
  - in **phase**, causing **constructive interference.** The peaks and troughs line up on both waves. The resultant wave has double the amplitude
  - or, in **anti-phase**, causing **destructive interference.** The peaks on one wave line up with the troughs of the other. The resultant wave has no amplitude

8. Superposition

YOUR NOTES



Waves in superposition can undergo constructive or destructive interference

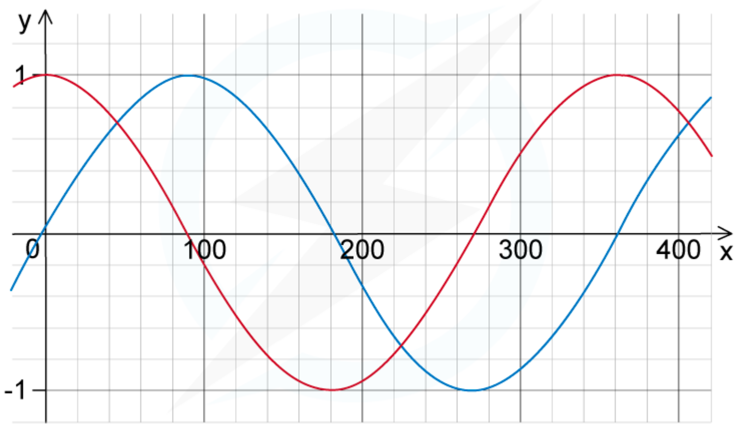
- The principle of superposition applies to all types of waves i.e. transverse and longitudinal

8. Superposition

YOUR NOTES

Worked example

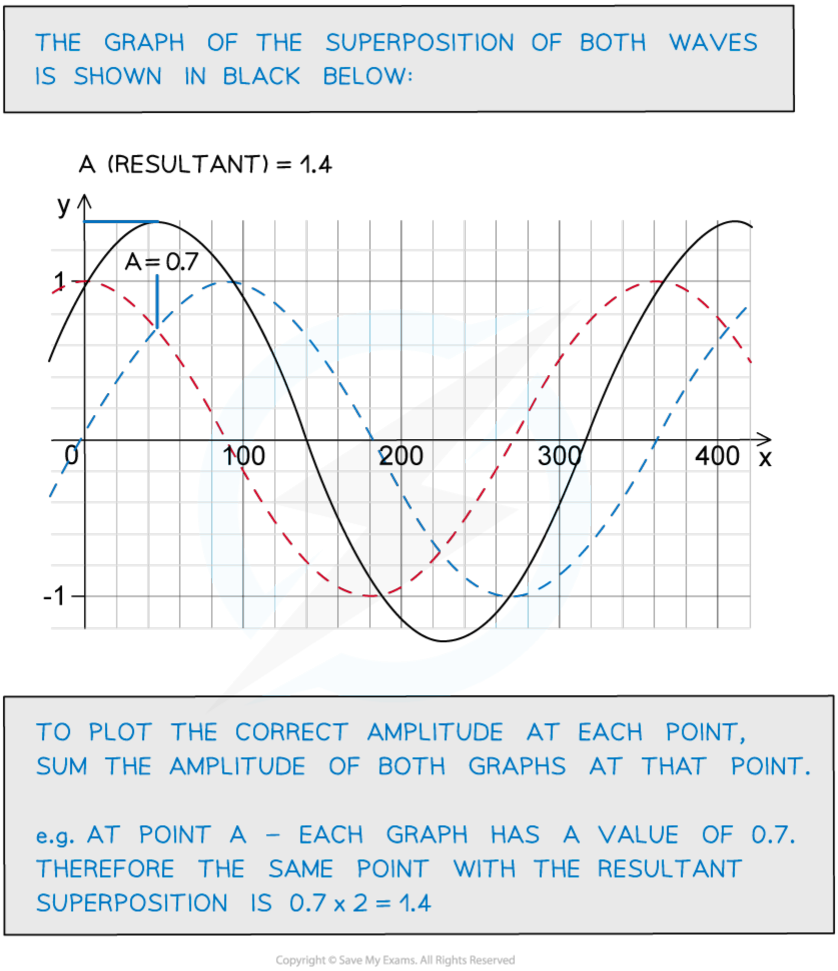
Two overlapping waves of the same type travel in the same direction. The variation with x and y displacement of the wave is shown in the figure below.



Use the principle of superposition to sketch the resultant wave.

8. Superposition

YOUR NOTES



Exam Tip

The best way to draw the superposition of two waves is to find where the superimposed wave has its maximum and minimum amplitudes. It is then a case of joining them up to form the wave. Where the waves intersect determines how much constructive or destructive interference will occur.

8. Superposition

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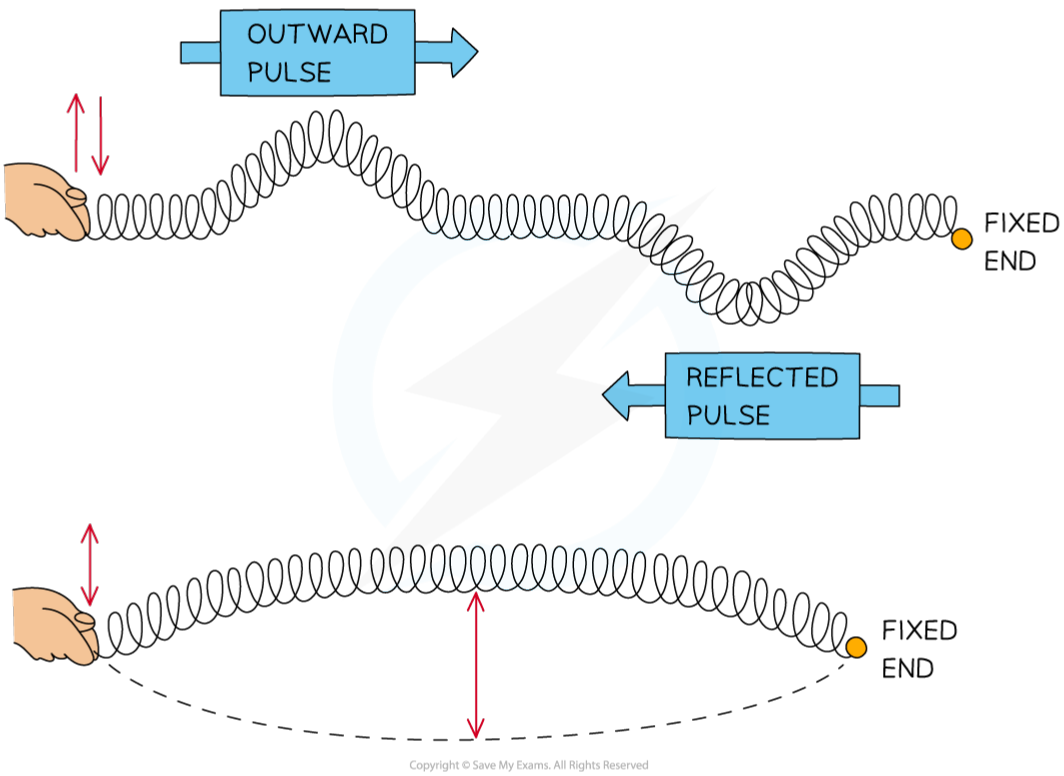
⬇

8.1.2 STATIONARY WAVES

Stationary Waves

- Stationary waves, or standing waves, are produced by the superposition of two waves of the same frequency and amplitude travelling in opposite directions
- This is usually achieved by a travelling wave and its reflection. The superposition produces a wave pattern where the peaks and troughs do not move



Formation of a stationary wave on a stretched spring fixed at one end

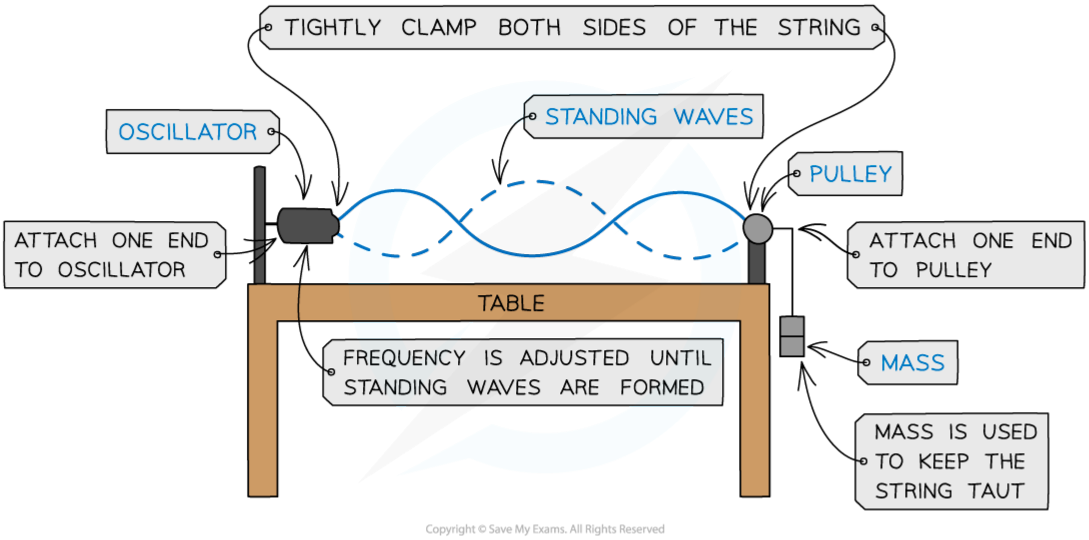
- In this section we will look at some few experiments that demonstrate stationary waves in everyday life

8. Superposition

YOUR NOTES
⬇

Stretched strings

- Vibrations caused by stationary waves on a stretched string produce sound
  - This is how stringed instruments, such as guitars or violins, work
- This can be demonstrated by a length of string under tension fixed at one end and vibrations made by an oscillator:



Stationary wave on a stretched string

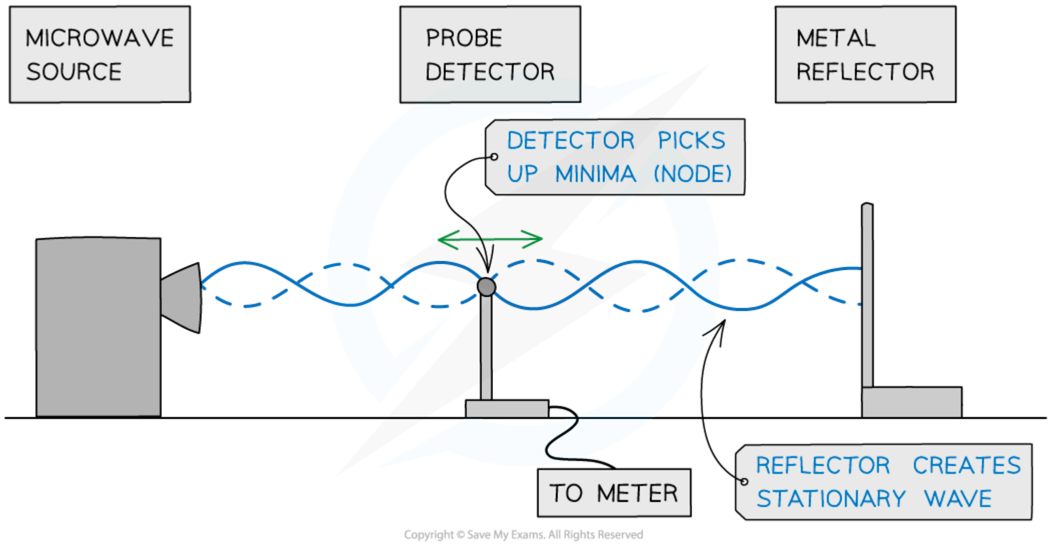
- As the frequency of the oscillator changes, standing waves with different numbers of minima (nodes) and maxima (antinodes) form

8. Superposition

YOUR NOTES

Microwaves

- A microwave source is placed in line with a reflecting plate and a small detector between the two
- The reflector can be moved to and from the source to vary the stationary wave pattern formed
- By moving the detector, it can pick up the minima (nodes) and maxima (antinodes) of the stationary wave pattern



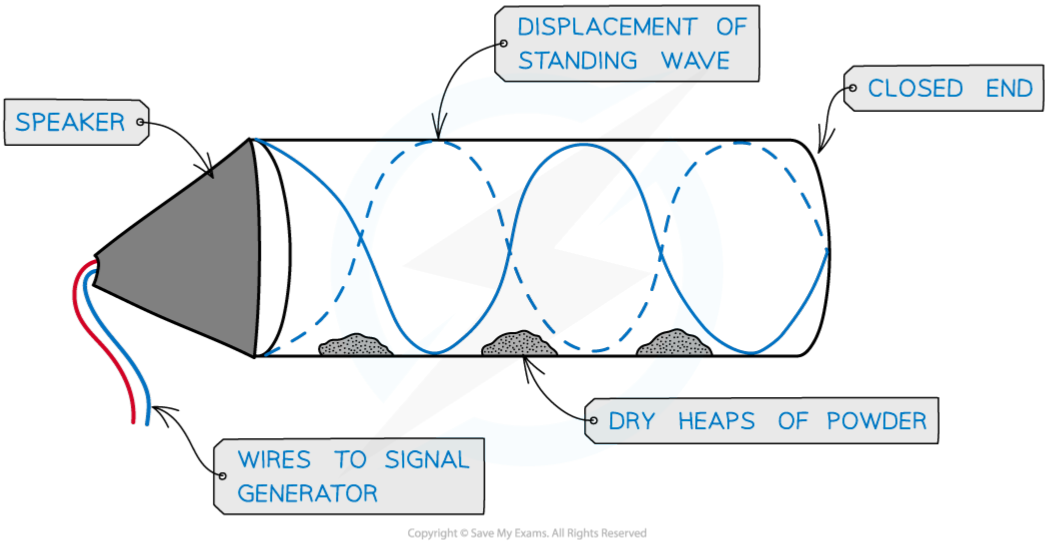
Using microwaves to demonstrate stationary waves

Air columns

- The formation of stationary waves inside an air column can be produced by sound waves
  - This is how musical instruments, such as clarinets and organs, work
- This can be demonstrated by placing a fine powder inside the air column and a loudspeaker at the open end
- At certain frequencies, the powder forms evenly spaced heaps along the tube, showing where there is zero disturbance as a result of the nodes of the stationary wave

8. Superposition

YOUR NOTES



Stationary wave in an air column

- In order to produce a stationary wave, there must be a minima (node) at one end and a maxima (antinode) at the end with the loudspeaker

Exam Tip

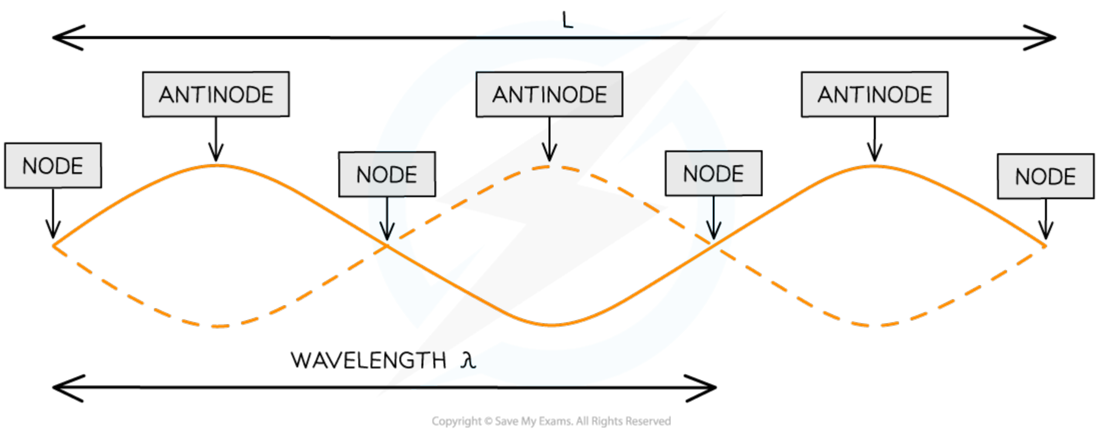
Always refer back to the experiment or scenario in an exam question e.g. the wave produced by a **loudspeaker** reflects at the end of a **tube**. This reflected wave, with the same frequency, overlaps the initial wave to create a stationary wave.

8. Superposition

YOUR NOTES

Formation of Stationary Waves

- A stationary wave is made up **nodes** and **antinodes**
  - **Nodes** are where there is no vibration
  - **Antinodes** are where the vibrations are at their maximum amplitude
- The nodes and antinodes **do not** move along the string. Nodes are fixed and antinodes only move in the vertical direction
- Between nodes, all points on the stationary are in phase
- The image below shows the nodes and antinodes on a snapshot of a stationary wave at a point in time



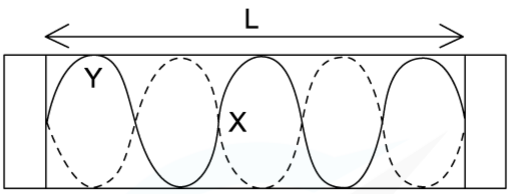
- L is the length of the string
- 1 wavelength λ is only a portion of the length of the string

8. Superposition

YOUR NOTES
⬇

Worked example

A stretched string is used to demonstrate a stationary wave, as shown in the diagram.



Which row in the table correctly describes the length of L and the name of X and Y?

	Length L	Point X	Point Y
A	5 wavelengths	Node	Antinode
B	$2\frac{1}{2}$ wavelengths	Antinode	Node
C	$2\frac{1}{2}$ wavelengths	Node	Antinode
D	5 wavelengths	Node	Antinode

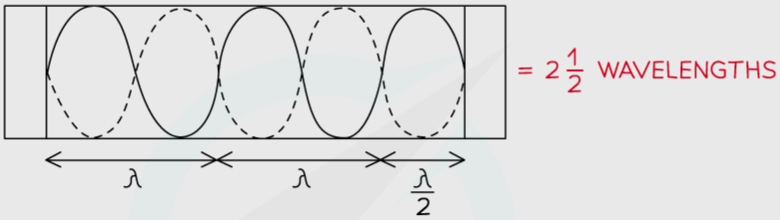
ANSWER: C

8. Superposition

YOUR NOTES
⬇

STEP 1

CALCULATE HOW MANY WAVELENGTHS IN THE LENGTH OF THE STRING



THIS RULES OUT A AND D

STEP 2

X IS A POINT OF 0 DISPLACEMENT – A NODE

STEP 3

Y IS A POINT OF MAXIMUM DISPLACEMENT – AN ANTINODE

STEP 4

THE CORRECT ROW IS C

Exam Tip

The lengths of the strings will only be in whole or ½ wavelengths. For example, a wavelength could be made up of 3 nodes and 2 antinodes or 2 nodes and 3 antinodes.

8. Superposition

YOUR NOTES

Measuring Wavelength

- Stationary waves have different wave patterns depending on the frequency of the vibration and the situation in which they are created

Two fixed ends

- When a stationary wave, such as a vibrating string, is fixed at both ends, the simplest wave pattern is a single loop made up of two nodes and an antinode
- This is called the **fundamental mode** of vibration or the **first harmonic**
- The particular frequencies (i.e. resonant frequencies) of standing waves possible in the string depend on its length L and its speed v
- As you increase the frequency, the higher harmonics begin to appear
- The frequencies can be calculated from the string length and wave equation

8. Superposition

YOUR NOTES

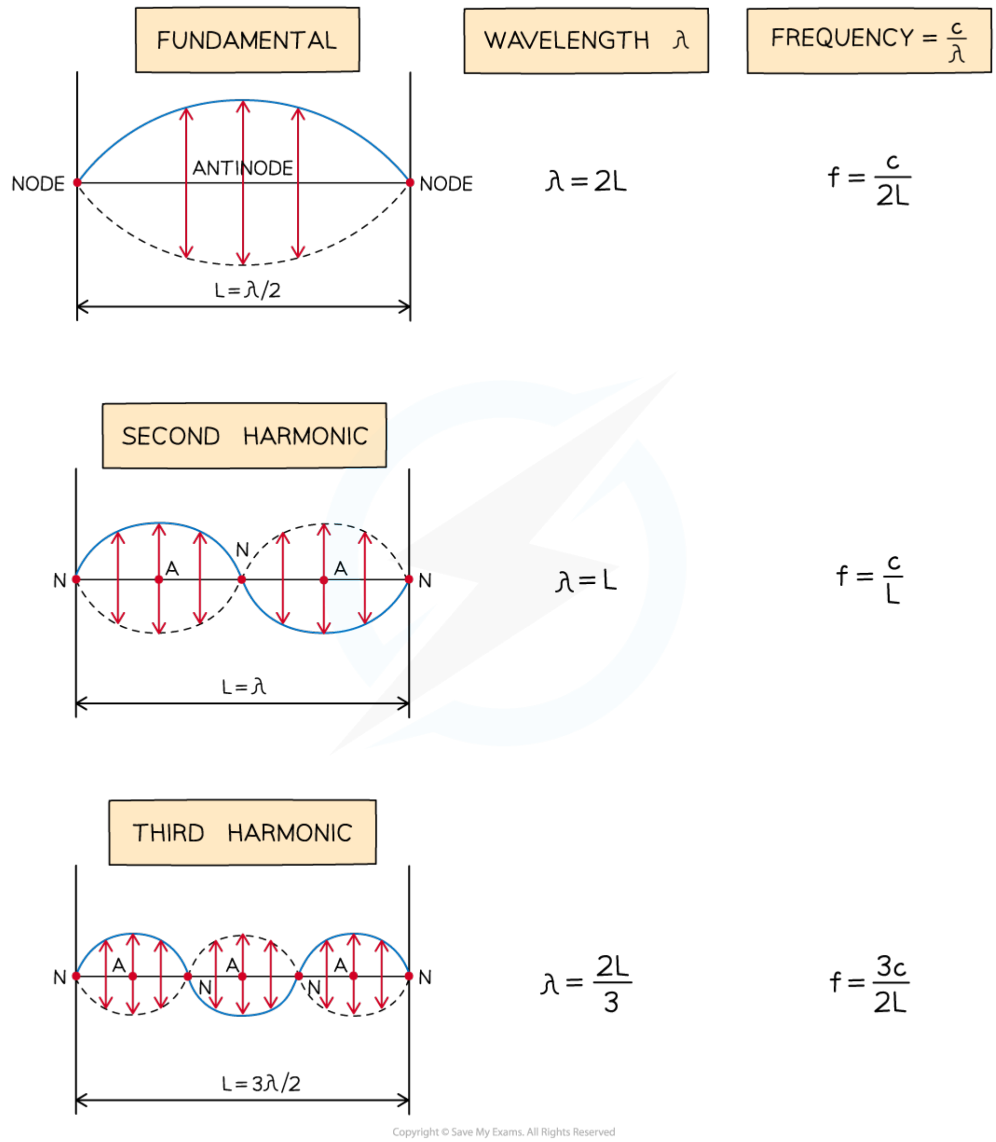


Diagram showing the first three modes of vibration of a stretched string with corresponding frequencies

- The nth harmonic has n antinodes and $n + 1$ nodes

8. Superposition

YOUR NOTES

One or two open ends in air column

- When a stationary wave is formed in an air column with one or two open ends, we see slightly different wave patterns in each

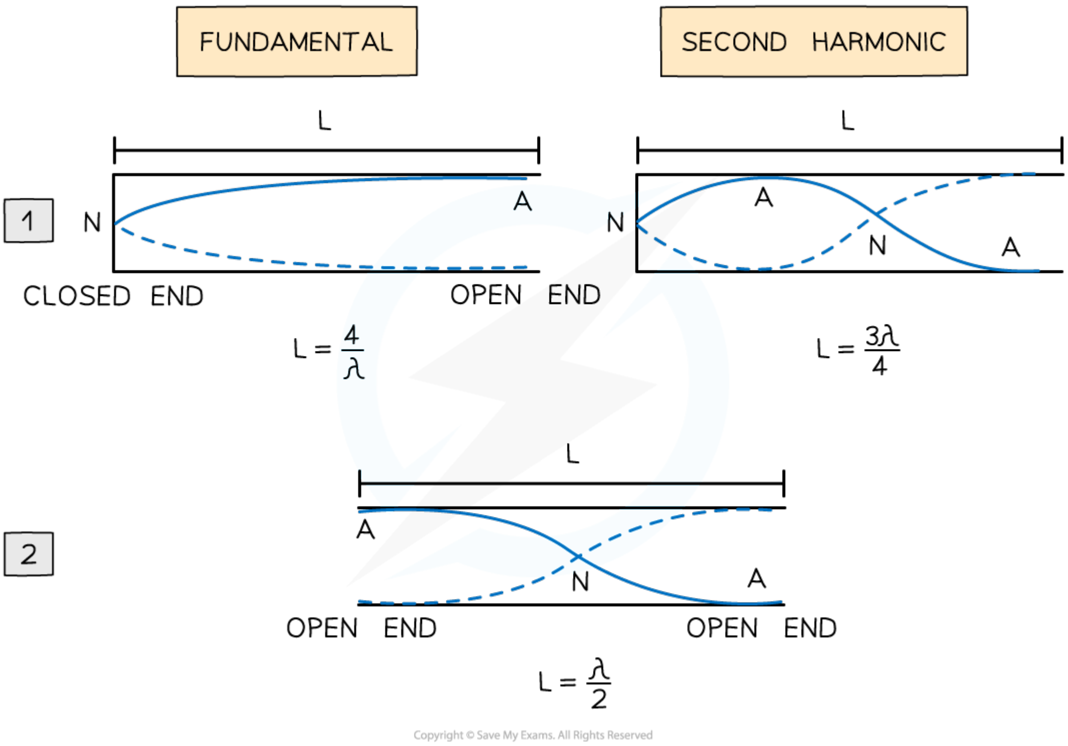
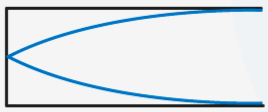
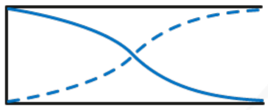


Diagram showing modes of vibration in pipes with one end closed and the other open or both ends open

- In Image 1: only one end of the air column is open, so, the fundamental mode is now made up of a quarter of a wavelength with one node and one antinode
  - Every harmonic after that adds on an extra node or antinode
- In Image 2: the column is open on both ends, so, the fundamental mode is made up one node and two antinodes
- In summary, a column length *L* for a wave with wavelength *λ* and resonant frequency *f* for stationary waves to appear is as follows:

8. Superposition

YOUR NOTES

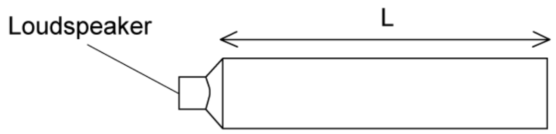
Air column fundamental wave	Length L / m	Resonant frequencies f / Hz	Value of n
	$L = \frac{n\lambda}{2}$	$f = \frac{nv}{2L}$	n = 1, 2, 3
	$L = \frac{n\lambda}{4}$	$f = \frac{nv}{4L}$	n = odd
	$L = \frac{n\lambda}{2}$	$f = \frac{nv}{2L}$	n = 1, 2, 3...

8. Superposition

YOUR NOTES
⬇

Worked example

A standing wave is set up in a loudspeaker emits sound with frequency f and is placed at one end of a pipe with length L.
The pipe is closed at the other end.
The speed of sound is 340 ms^{-1}.



With a sound wave of wavelength of 10 m, what is the frequency of the second lowest note produced?

STEP 1

CALCULATE THE LENGTH OF THE SOUND WAVE IN THE COLUMN WITH GIVEN WAVELENGTH

$L = \frac{n\lambda}{4}$ FOR ONE CLOSED AND OPEN END

STEP 2

THE SECOND LOWEST NOTE IS THE FIRST HARMONIC, OR $n = 3$

$$L = \frac{3\lambda}{4}$$

$$L = \frac{3 \times 10}{4} = \frac{15}{2} \text{ m}$$

STEP 3

CALCULATE FREQUENCY USING L AND SPEED OF SOUND

$$f = \frac{3v}{4L}$$

$$f = \frac{3 \times 340}{4 \times \frac{15}{2}} = 34 \text{ Hz}$$

Exam Tip

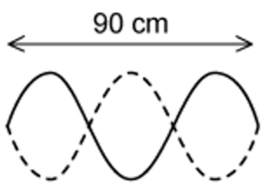
The fundamental counts as the first harmonic or n = 1 and is the lowest frequency with half or quarter of a wavelength. A full wavelength with both ends open or both ends closed is the **second** harmonic. Make sure to match the correct wavelength with the harmonic asked for in the question!

8. Superposition

YOUR NOTES

Exam Question: Easy

A stationary wave shown in the diagram was created on a stretched string, it had two instants of maximum vertical displacement.



The frequency of the wave is 13 Hz.

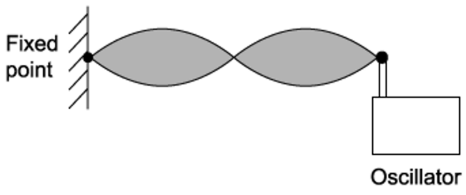
What is the speed of the wave?

A 3.9 m s $^{-1}$ **B** 7.8 m s $^{-1}$ **C** 390 m s $^{-1}$ **D** 780 m s $^{-1}$

Exam Question: Medium

A student was investigating the speed of a transverse wave on a stretched spring. They found that by adjusting the tension on the spring, they were able to change the speed.

The wave pattern produced is shown in the diagram the frequency of the oscillator was set to 650 Hz



What needs to be changed to maintain the same wave pattern when the frequency is increased to 750 Hz?

A increase the speed of the wave on the string

B increase the wavelength of the wave on the string

C decrease the wavelength of the wave on the string

D decrease the speed of the wave on the string

8. Superposition

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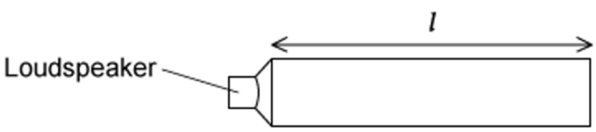
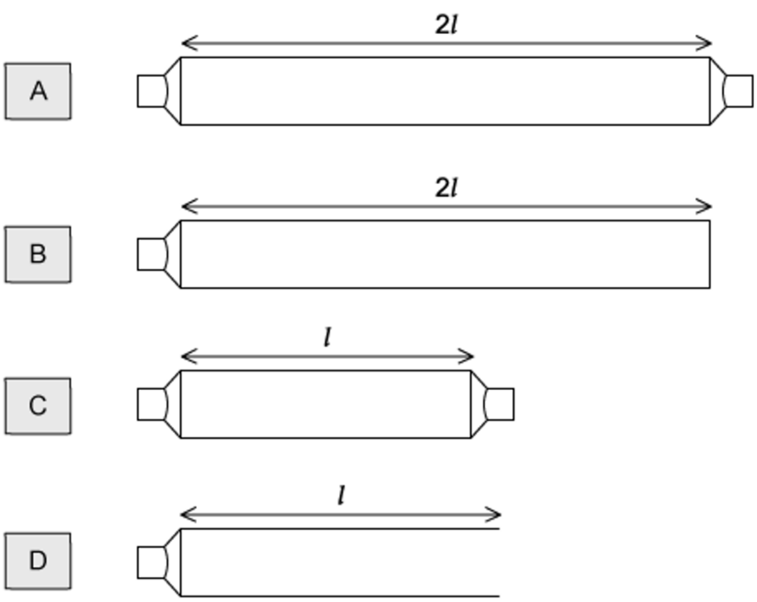
Exam Question: Hard

A demonstration of a standing wave is set up using a loudspeaker. The loudspeaker is emitting sound with a frequency *f* and is placed at one end of the pipe with length *l*. The pipe is closed at the other end. This is shown in the diagram below.



Different pipes were set up with either one or two loudspeakers of frequency *f*. The pairs of loudspeakers were in phase with each other.

Which pipe would contain a standing wave?



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8. Superposition

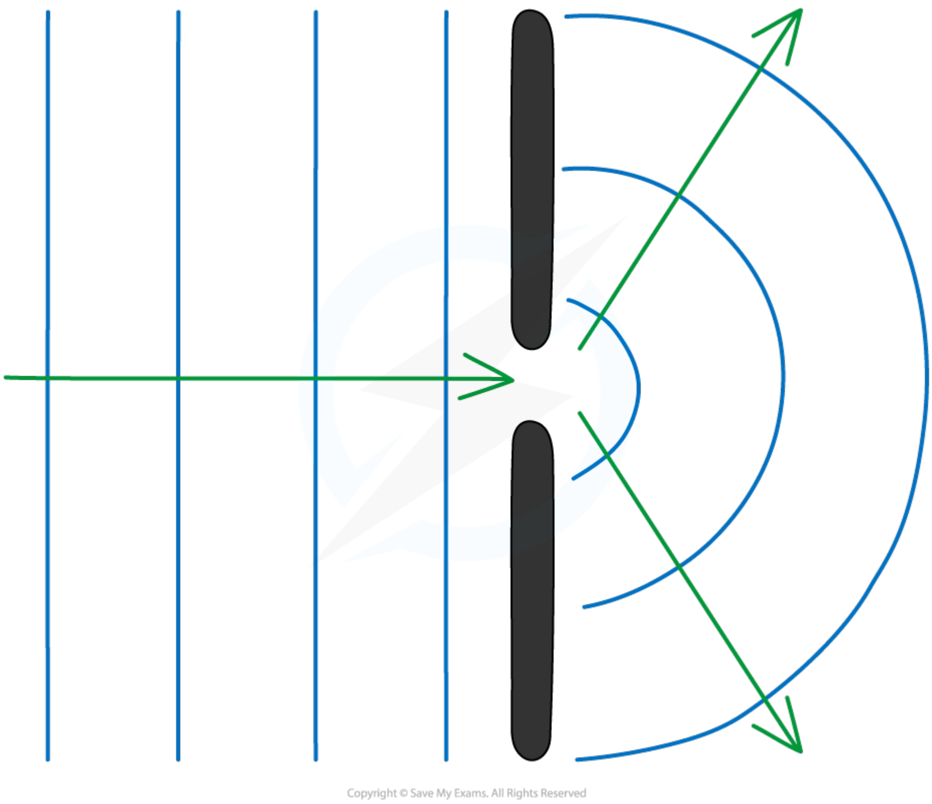
YOUR NOTES

8.2 DIFFRACTION & INTERFERENCE

8.2.1 DIFFRACTION

What is Diffraction?

- Diffraction is the **spreading out** of waves when they pass an obstruction
    - This obstruction is typically a narrow slit (an aperture)
- The extent of diffraction depends on the width of the gap compared with the wavelength of the waves
    - Diffraction is the most prominent when the width of the slit is approximately equal to the wavelength



Diffraction: when a wave passes through a narrow gap, it spreads out

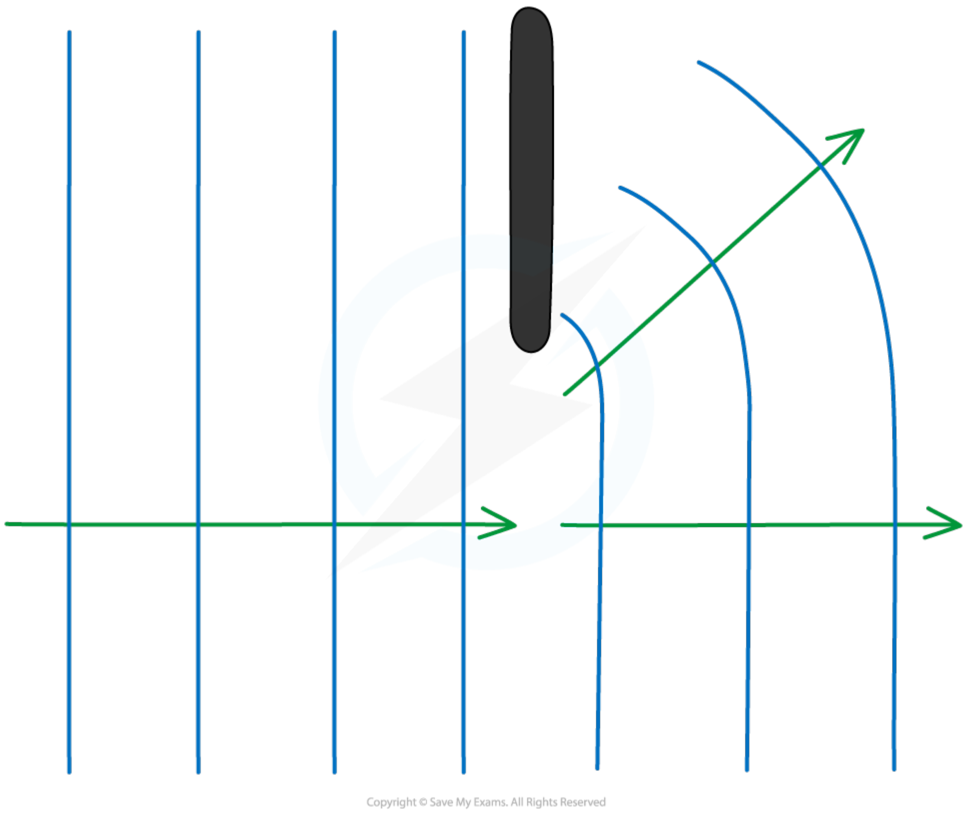
- Diffraction is usually represented by a wavefront as shown by the vertical lines in the

8. Superposition

YOUR NOTES

diagram above

- The only property of a wave that changes when its diffracted is its **amplitude**
  - This is because some energy is dissipated when a wave is diffracted through a gap
- Diffraction can also occur when waves curve around an edge:



When a wave goes past the edge of a barrier, the waves can curve around it

- Any type of wave can be diffracted i.e. sound, light, water

8. Superposition

YOUR NOTES
⬇

Worked example

When a wave is travelling through air, which scenario best demonstrates diffraction?

A UV radiation through a gate post
B Sound waves passing a steel rod
C Radio waves passing between human hair
D X-rays passing through atoms in a crystalline solid

Answer: D

- Diffraction is most prominent when the wavelength is close to the aperture size
- UV waves have a wavelength between $4 \times 10^{-7} - 1 \times 10^{-8}$ m so won't be diffracted by a gate post
- Sound waves have a wavelength of 1.72×10^{-2} – 17 m so would not be diffracted by the diffraction grating
- Radio waves have a wavelength of $0.1 - 10^{6}$ m so would not be diffracted by human hair
- X-rays have a wavelength of $1 \times 10^{-8} - 4 \times 10^{-13}$ m which is roughly the gap between atoms in a crystalline solid
    - Therefore, the correct answer is **D**

Exam Tip

When drawing diffracted waves, take care to keep the wavelength constant. It is only the amplitude of the wave that changes when diffracted.

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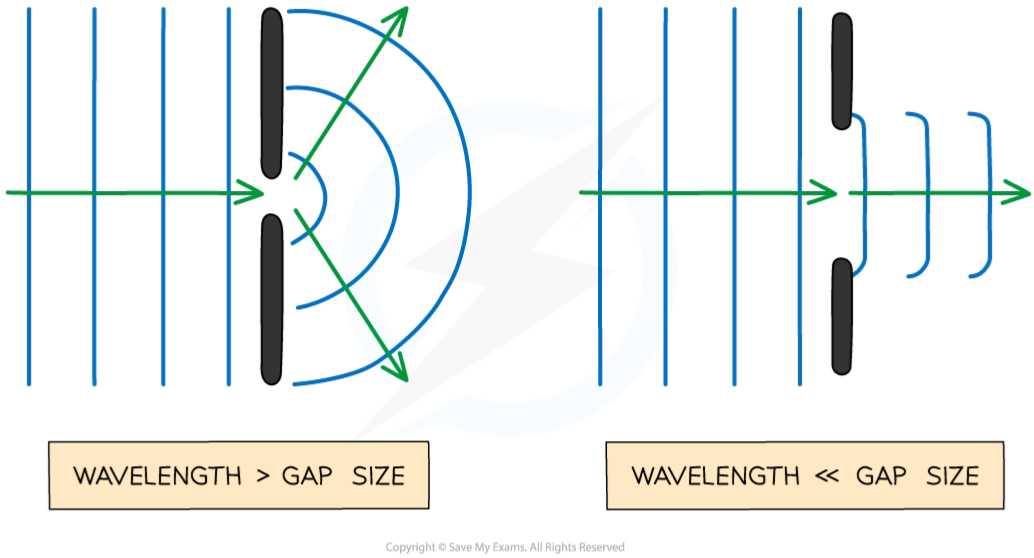
Diffraction Experiments

- As discussed above, the effects of diffraction are most prominent when the gap size is approximately the same or smaller than the wavelength of the wave
- As the gap size increases, the effect gradually gets less pronounced until, in the case that the gap is much larger than the wavelength, the waves are no longer spread out



The size of the gap (compared to the wavelength) affects how much the waves spread out

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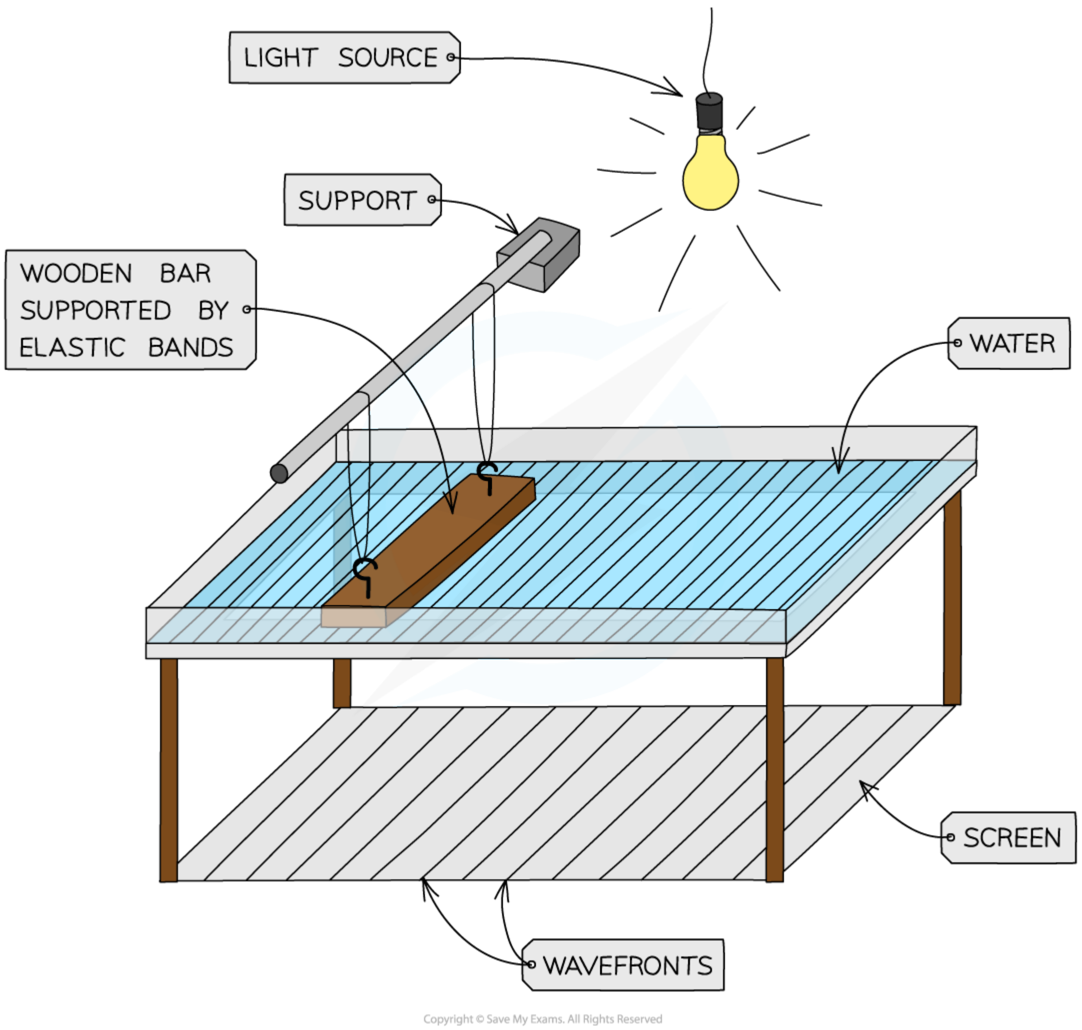
- Ripple tanks are used a common experiment to demonstrate diffraction of water waves



Wave effects may all be demonstrated using a ripple tank

8. Superposition

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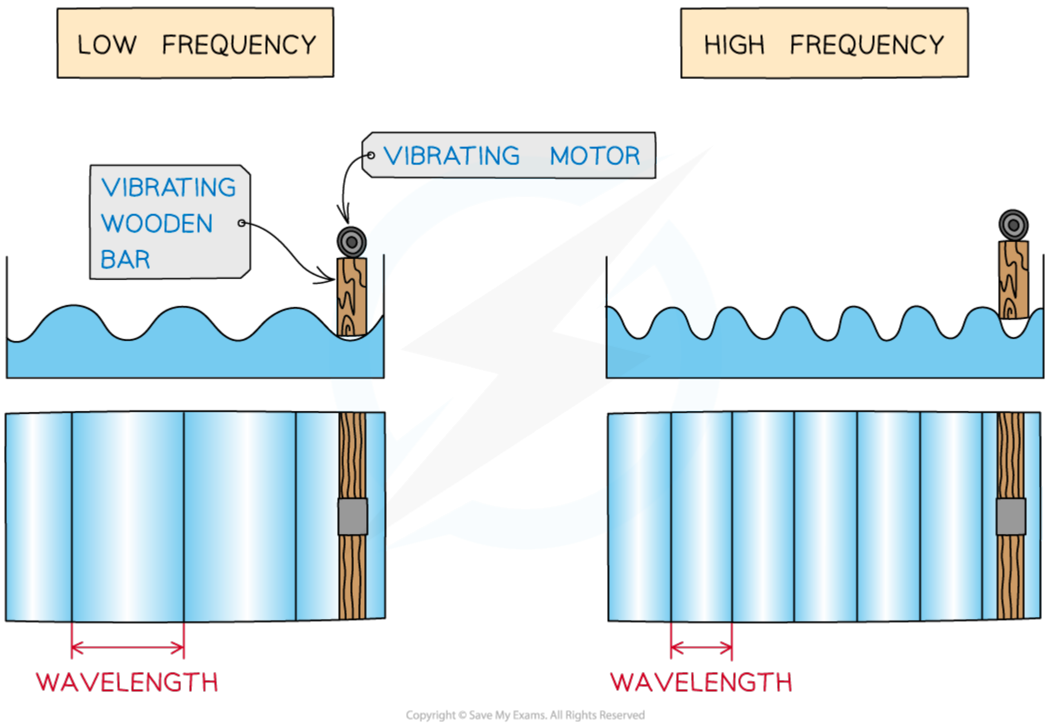
⬇

- The diagram below shows how the wavelengths differ with frequency in a ripple tank
    - The **higher** the frequency, the **shorter** the wavelength
    - The **lower** the frequency, the **longer** the wavelength



Ripple tank patterns for low and high frequency vibration

Exam Tip

Familiarising yourself with the wavelength of electromagnetic waves is essential for identifying which wave will cause the greatest diffraction effect for a giving gap width.

8. Superposition

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⬇

8.2.2 INTERFERENCE & COHERENCE

Interference & Coherence

- Interference occurs when waves **overlap** and their resultant displacement is the **sum of the displacement of each wave**
- This result is based on the principle of superposition and the resultant waves may be smaller or larger than either of the two individual waves
- Interference of two waves can either be:
  - In **phase**, causing **constructive interference.** The peaks and troughs line up on both waves. The resultant wave has double the amplitude
  - In **anti-phase**, causing **destructive interference.** The peaks on one wave line up with the troughs of the other. The resultant wave has no amplitude

8. Superposition

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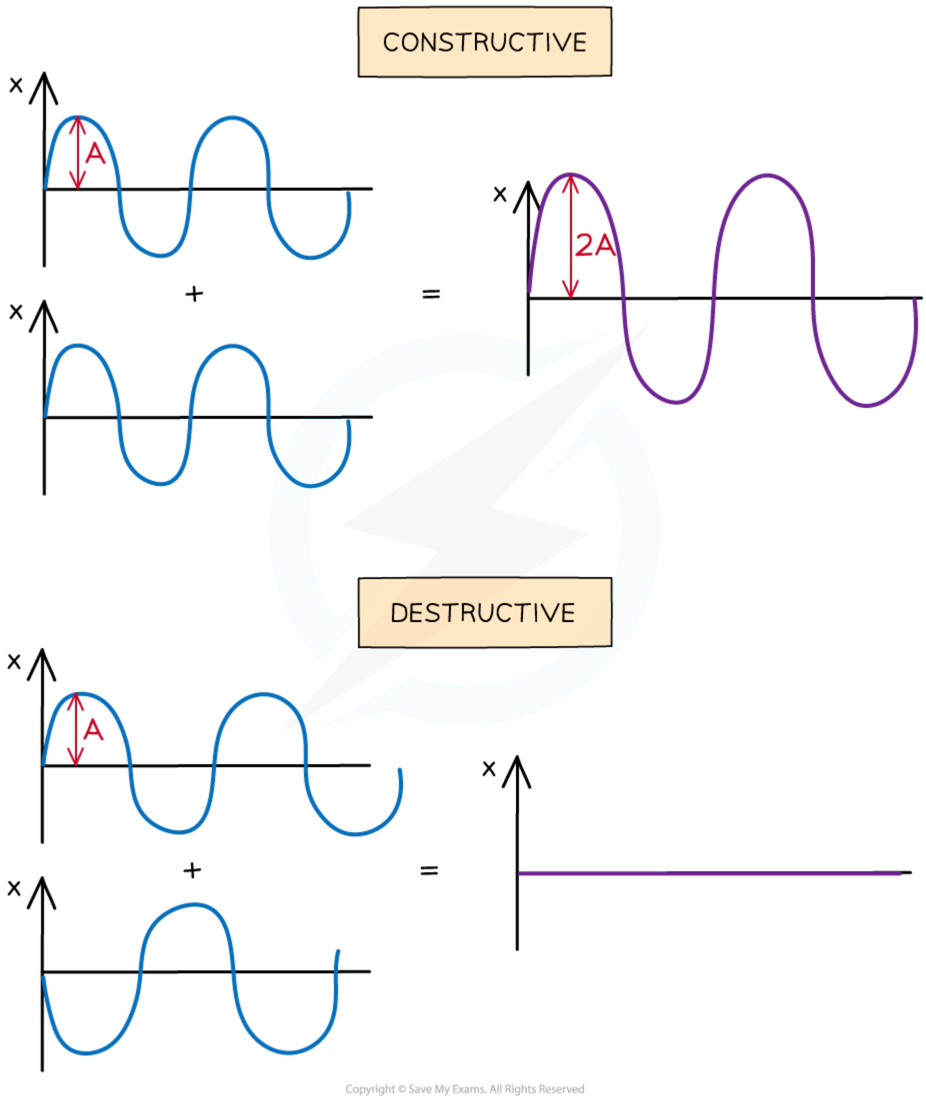

Waves in superposition can undergo constructive or destructive interference

- At points where the two waves are neither in phase nor in antiphase, the resultant amplitude is somewhere in between the two extremes
- Waves are **coherent** if they have the same **frequency** and **constant phase difference**

8. Superposition

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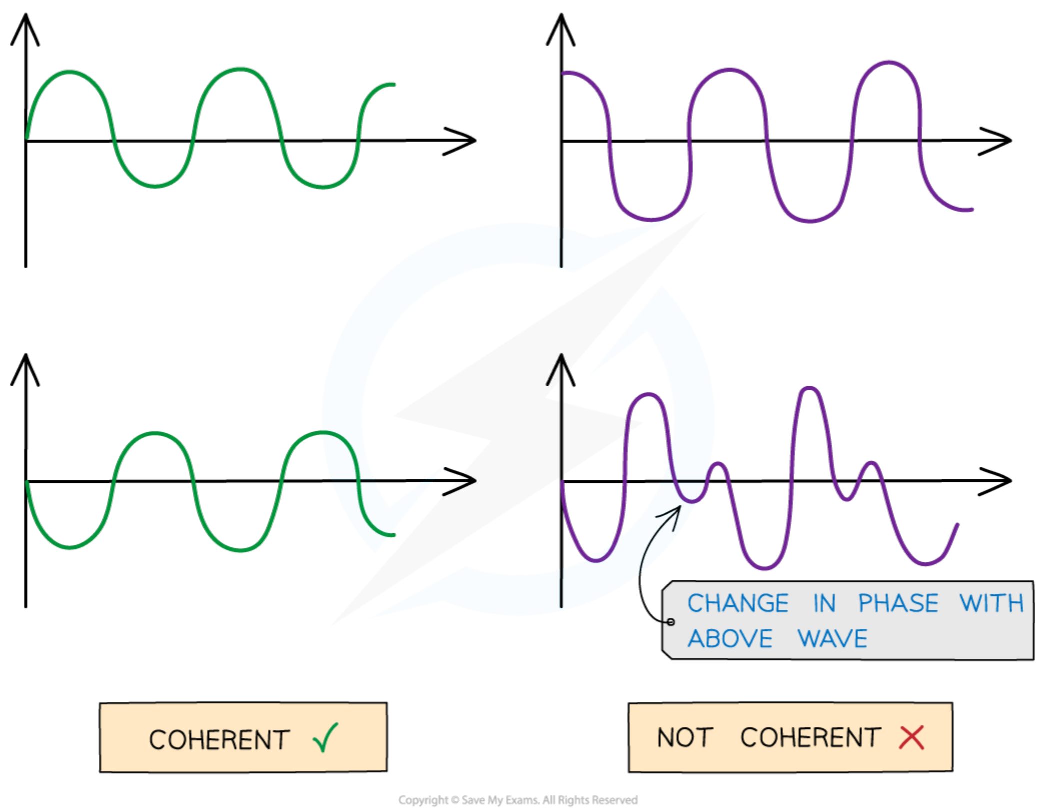


Coherent v non-coherent wave. The abrupt change in phase creates an inconsistent phase difference

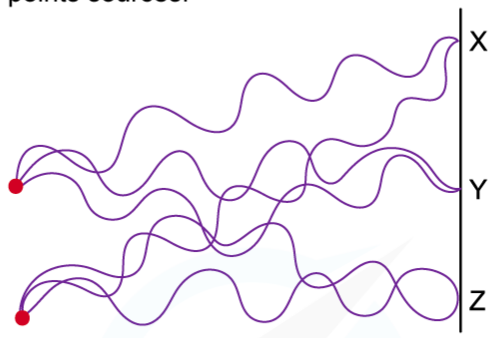
- Coherence is vital in order to produce an observable interference pattern
- Laser light is an example of a coherent light source, whereas filament lamps produce incoherent light waves

8. Superposition

YOUR NOTES

Worked example

The diagram below shows the interference of coherent waves from two points sources.



Which row in the table correctly identifies the type of interference at points X, Y and Z.

	X	Y	Z
A	Constructive	Destructive	Constructive
B	Constructive	Constructive	Destructive
C	Destructive	Constructive	Destructive
D	Destructive	Constructive	Constructive

ANSWER: B

- At point X:
  - Both peaks of the waves are overlapping. This is **constructive** interference and rules out options C and D
- At point Y:
  - Both troughs are overlapping and there **constructive** interference occurs
- At point Z:
  - A peak of one of the waves meets the trough of the other. This is **destructive** interference (Row B)

8. Superposition

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⬇

Exam Tip

Think of ‘constructive’ interference as ‘building’ the wave and ‘destructive’ interference as ‘destroying’ the wave

8. Superposition

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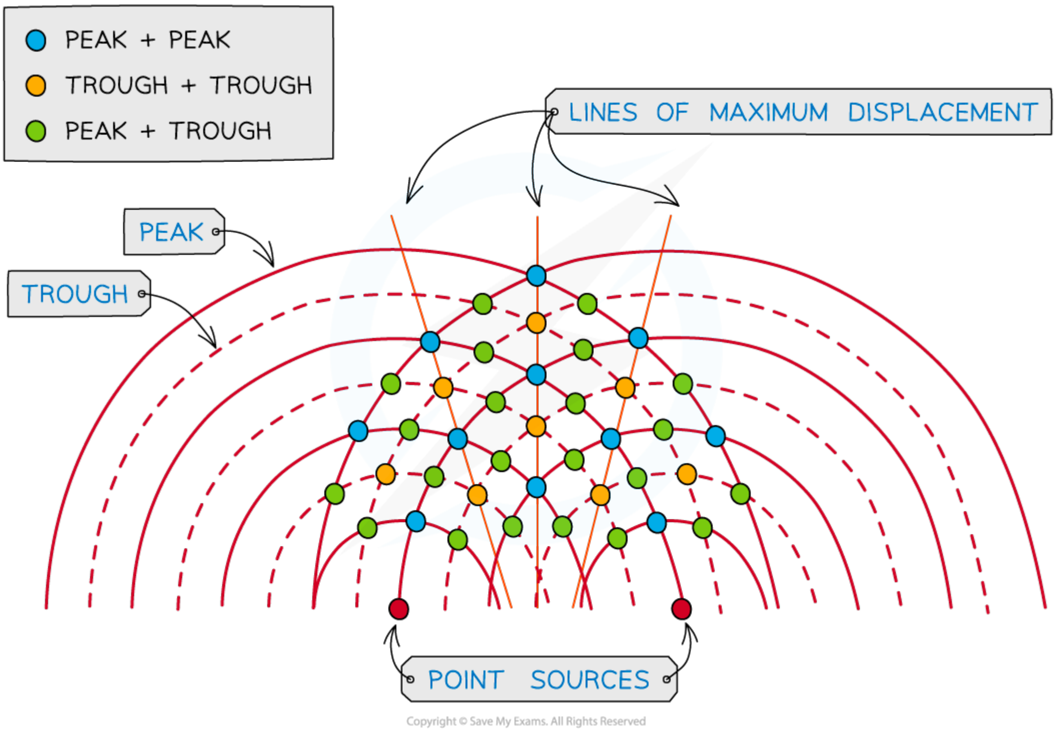
8.2.3 TWO SOURCE INTERFERENCE

Demonstrating Two Source Interference

- Interference of sound, light and microwaves can be demonstrated with slits or diffraction gratings

Using Water Waves

- Two-source interference in can be demonstrated in water using ripple tanks
- The diagram below shows diffracted circle shaped water waves from two point sources eg. dropping two pebbles near to each other in a pond



Water waves interference pattern from a ripple tank

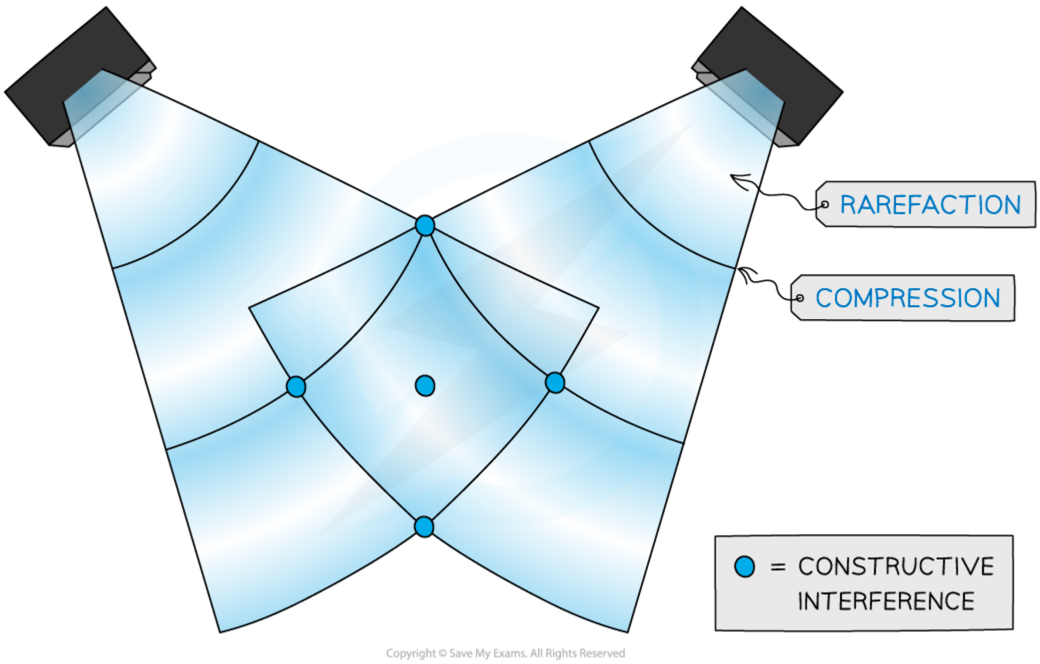
- The two waves interfere causing areas of constructive and destructive interference
- The lines of maximum displacement occur when all the peaks and troughs line up with those on another wave

8. Superposition

YOUR NOTES
⬇

Using Sound Waves

- Two source interference for sound waves looks very similar to water waves



Sound wave interference from two speakers

- Sound waves are longitudinal waves so are made up compressions and rarefactions
- Constructive interference occurs when the compression and rarefactions line up and the sound appears louder
- Destructive interference occurs when the compression lines up with a rarefaction and vice versa. The sound is quieter
  - This is the technology used in noise cancelling headphones

8. Superposition

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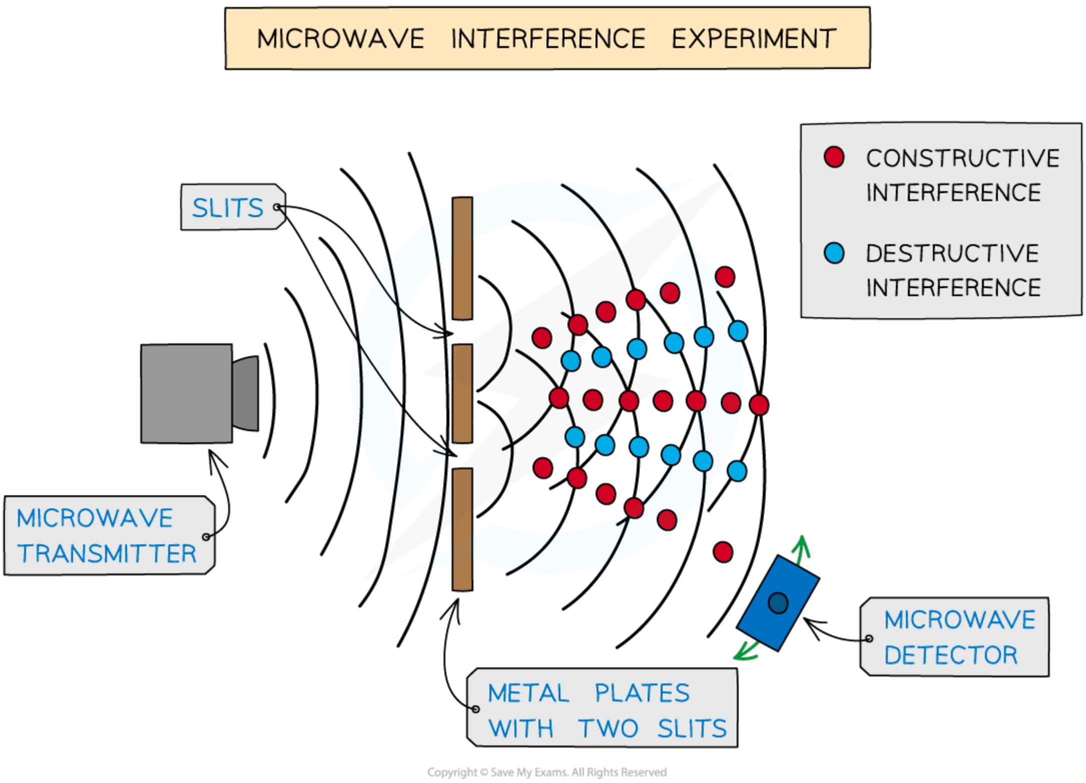
Using Microwaves

- Two source interference for microwaves can be detected with a moveable microwave detector



Microwave interference experiment

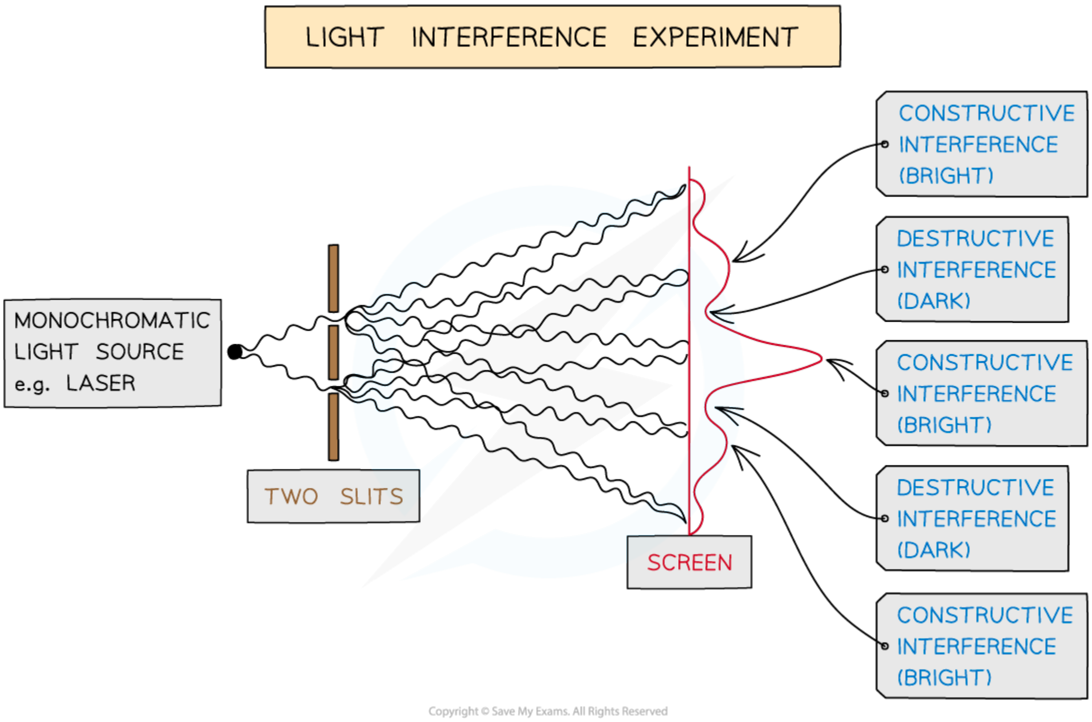
- Constructive interference: regions where the detector picks up a maximum amplitude
- Destructive interference: regions where the detector picks up no signal

8. Superposition

YOUR NOTES
⬇

Using Light Waves

- For light rays, such as a laser light through two slits, an interference pattern forms on the screen



Laser light interference experiment

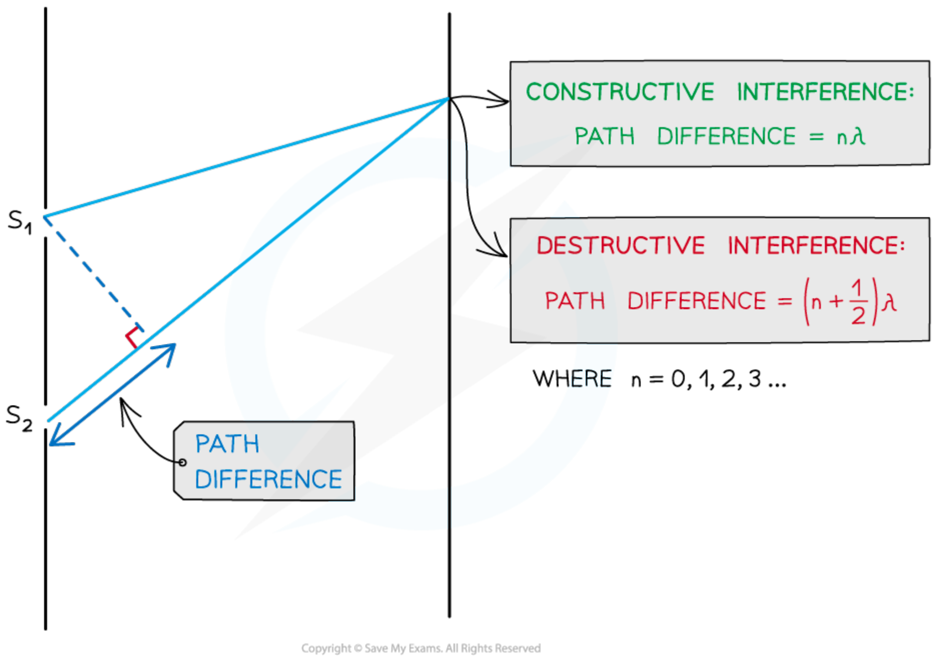
- Constructive interference is shown as bright fringes on the screen
  - The highest intensity is in the middle
- Destructive interference is shown as the dark fringes on the screen
  - These have zero intensity

8. Superposition

YOUR NOTES
⬇

Two Source Interference Fringes

- For two-source interference fringes to be observed, the sources of the wave must be:
  - **Coherent** (constant phase difference)
  - **Monochromatic** (single wavelength)
- When two waves interfere, the resultant wave depends on the **phase difference** between the two waves
- This is proportional to the **path difference** between the waves which can be written in terms of the wavelength λ of the wave
- As seen from the diagram, the wave from slit S_2 has to travel slightly further than that from S_1 to reach the same point on the screen. The difference in distance is the path difference

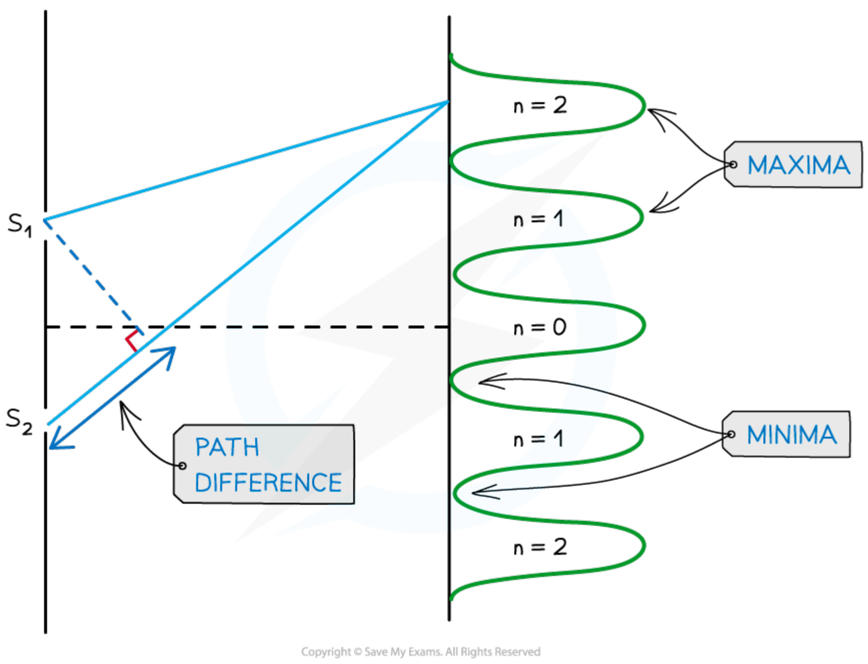


Path difference of constructive and destructive interference is determined by wavelength

- For **constructive** interference (or maxima), the difference in wavelengths will be an **integer number of whole wavelengths**
- For **destructive** interference (or minima) it will be an **integer number of whole wavelengths plus a half wavelength**
  - *n* is the order of the maxima/minima since there is usually more than one of these produced by the interference pattern
- An example of the orders of maxima is shown below:

8. Superposition

YOUR NOTES



Interference pattern of light waves shown with orders of maxima

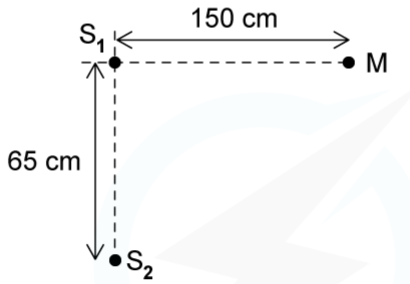
- n = 0 is taken from the middle, n = 1 is one either side and so on

8. Superposition

YOUR NOTES

Worked example

Two coherent sources of sound waves S_1 and S_2 are situated 65 cm apart in air as shown below.



The two sources vibrate in phase but have different amplitudes of vibration. A microphone M is situated 150 cm from S_1 along the line normal to S_1 and S_2 . The microphone detects maxima and minima of intensity of sound. The wavelength of the sound from S_1 to S_2 is decreased by increasing the frequency.

Determine which orders of maxima are detected at M as the wavelength is increased from 3.5 cm to 12.5 cm.

8. Superposition

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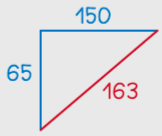
⬇

STEP 1

CALCULATE THE PATH DIFFERENCE



FROM PYTHAGORAS' THEOREM

$\sqrt{65^2 + 150^2} = 163$

PATH DIFFERENCE = 163 − 150 = 13 cm

STEP 2

MAXIMA ARE CAUSED BY CONSTRUCTIVE INTERFERENCE

STEP 3

CONSTRUCTIVE INTERFERENCE:

PATH DIFFERENCE = $n\lambda$ $\quad n = 0, 1, 2, 3...$

STEP 4

$13 = n\lambda$

$n = 0 \quad \lambda = 0$

$n = 1 \quad \lambda = \frac{13}{1} = 13\text{ cm}$

$n = 2 \quad \lambda = \frac{13}{2} = 6.5\text{ cm}$

$n = 3 \quad \lambda = \frac{13}{3} = 4.3\text{ cm}$

$n = 4 \quad \lambda = \frac{13}{4} = 3.3\text{ cm}$

ONLY THESE TWO ORDERS ARE WITHIN THE WAVELENGTH RANGE.

WAVELENGTHS OF 6.5 cm AND 4.3 cm ARE WHERE MAXIMA ARE DETECTED.

Exam Tip

The path difference is more specifically how much longer, or shorter, one path is than the other. In other words, the **difference** in the distances. Make sure not to confuse this with the distance between the two paths.

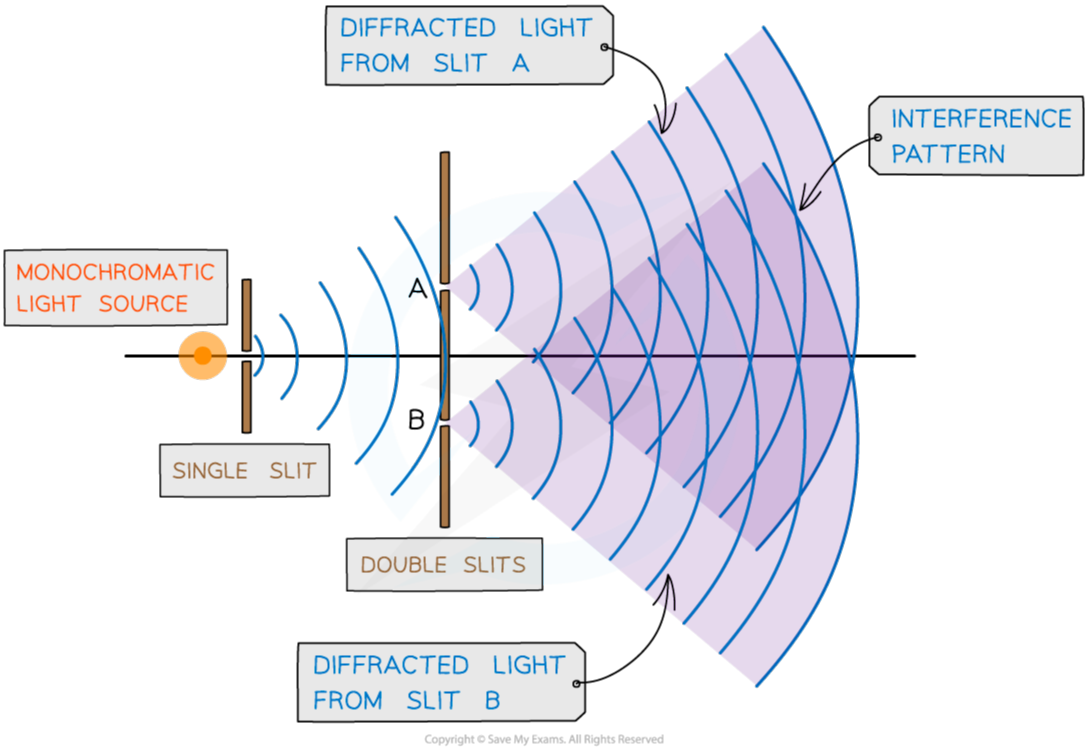
8. Superposition

YOUR NOTES
⬇

8.2.4 YOUNG'S DOUBLE SLIT EXPERIMENT

Double Slit Interference

- Young's double slit experiment demonstrates how light waves produced an interference pattern
- The experiment is shown below

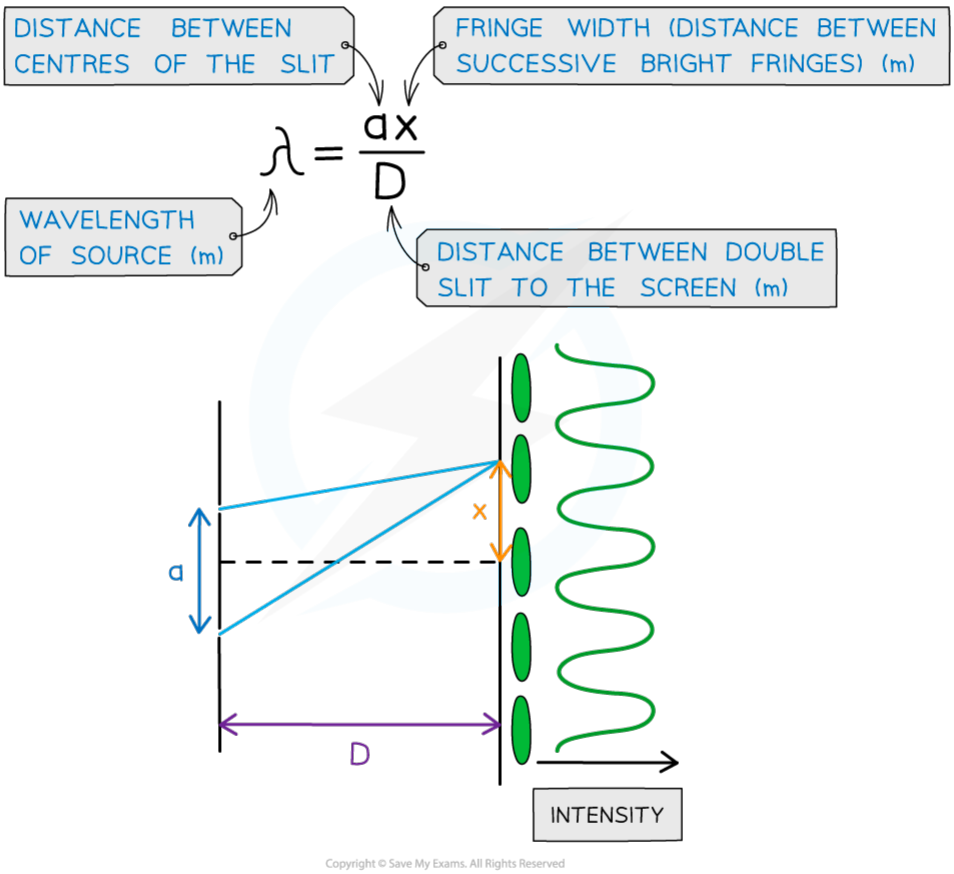


Young's double-slit experiment arrangement

- When a monochromatic light source is placed behind a single slit, the light is diffracted producing two light sources at the double slits A and B
- Since both light sources originate from the same primary source, they are **coherent** and will therefore create an observable interference pattern
- Both diffracted light from the double slits create an interference pattern made up of bright and dark fringes
- The wavelength of the light can be calculated from the interference pattern and experiment set up. These are related using the double-slit equation

8. Superposition

YOUR NOTES



Double slit interference equation with a, x and D represented on a diagram

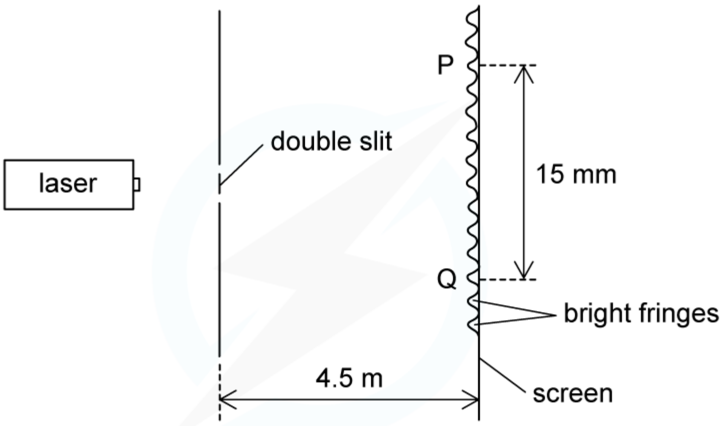
- The interference pattern on a screen will show as 'fringes' which are dark or bright bands
- **Constructive** interference is shown through **bright** fringes with varying intensity (most intense in the middle)
- **Destructive** interference is shown from **dark** fringes where no light is seen
- A monochromatic light source makes these fringes clearer and the distance between fringes is very small due to the short wavelength of visible light

8. Superposition

YOUR NOTES

Worked example

A laser is placed in front of a double slit as shown in the diagram below.



The laser emits light of frequency of 750 THz.
The separation of the maxima P and Q observed on the screen is 15mm.
The distance between the double slit and the screen is 4.5 m.
Calculate the separation of the two slits.

8. Superposition

YOUR NOTES

STEP 1: CALCULATE THE WAVELENGTH OF THE LIGHT

$c = f\lambda$

STEP 2: REARRANGE FOR λ AND SUBSTITUTE IN VALUES

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{750 \times 10^{12}} = 4 \times 10^{-7}\text{ m} = 400\text{ nm}$$

STEP 3: DOUBLE SLIT EQUATION

$$\lambda = \frac{ax}{D}$$

STEP 4: REARRANGE FOR A – SEPARATION OF THE TWO SLITS

$$a = \frac{\lambda D}{x}$$

STEP 5: SUBSTITUTE IN VALUES

$$a = \frac{4 \times 10^{-7} \times 4.5}{15 \times 10^{-3} \div 9} = 1.08 \times 10^{-3}\text{ m} = 1.1\text{ mm (2 s.f.)}$$

TOTAL NUMBER OF BRIGHT FRINGES

Exam Tip

Since *a*, *x* and *D* are all distances, it's easy to mix up which they refer to. Labelling the double slit diagram in the way given in the notes above will help to remember the order i.e. *a* and *x* in the numerator and *D* underneath in the denominator.

8. Superposition

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⬇

8.2.5 THE DIFFRACTION GRATING

The Grating Equation

- A diffraction grating is a plate on which there is a very large number of parallel, identical, close-spaced slits
- When monochromatic light is incident on a grating, a pattern of narrow bright fringes is produced on a screen

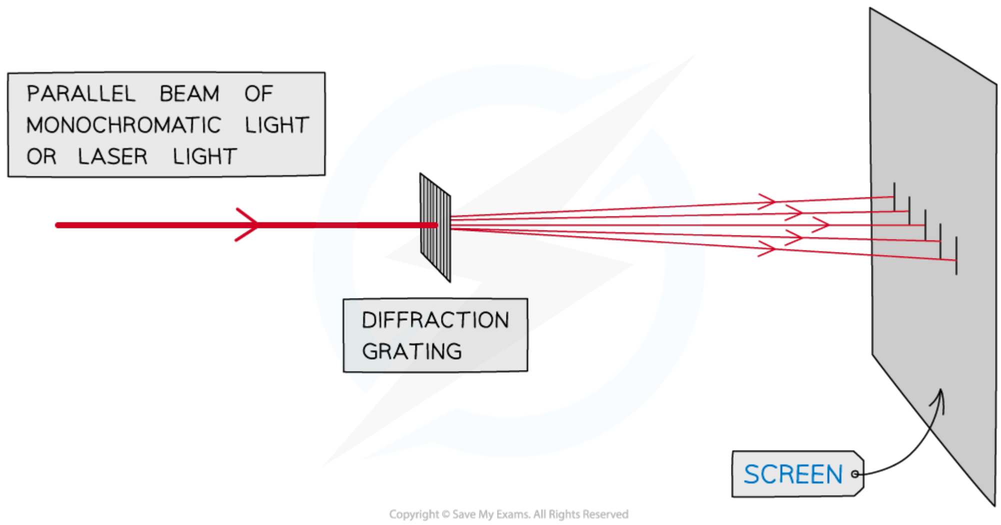
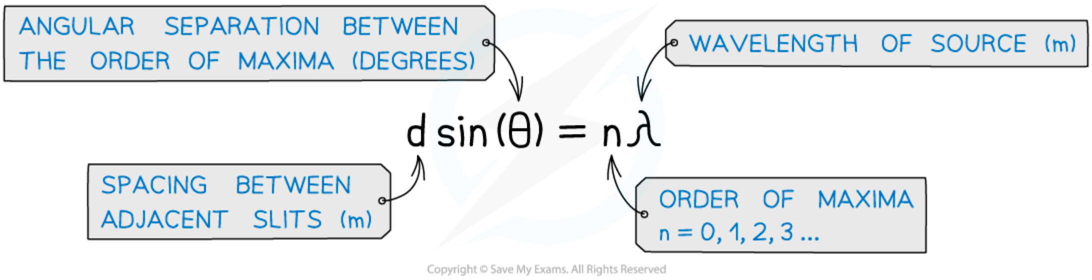


Diagram of diffraction grating used to obtain a fringe pattern

- The angles at which the maxima of intensity (constructive interference) are produced can be deduced by the diffraction grating equation

8. Superposition

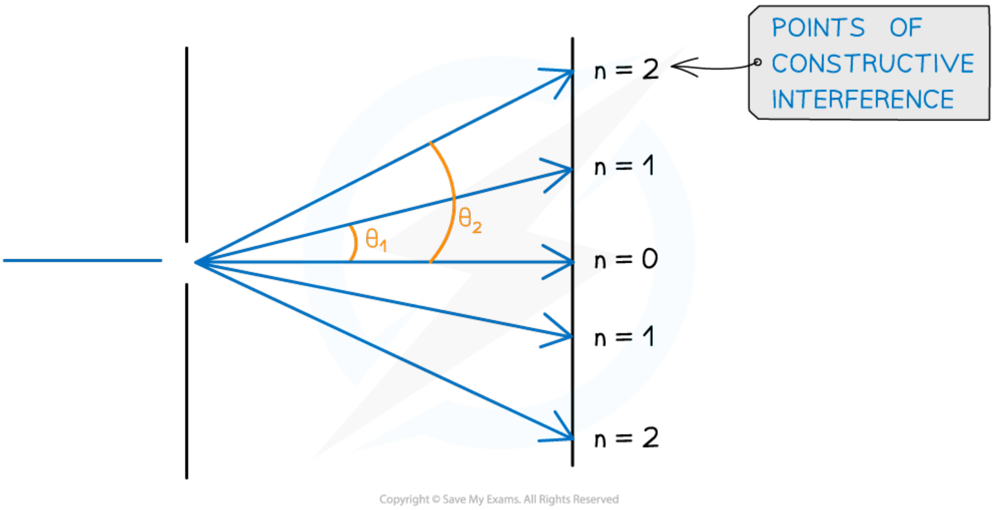
YOUR NOTES



Diffraction grating equation for the angle of bright fringes

Angular Separation

- The angular separation of each maxima is calculated by rearranging the grating equation to make θ the subject
- The angle θ is taken from the centre meaning the higher orders are at greater angles



Angular separation

- The angular separation between two angles is found by subtracting the smaller angle from the larger one
- The angular separation between the first and second maxima n_1 and n_2 is $\boldsymbol{\theta_2 - \theta_1}$
- The maximum angle to see orders of maxima is when the beam is at right angles to the diffraction grating. This means θ = 90° and sin(θ) = 1

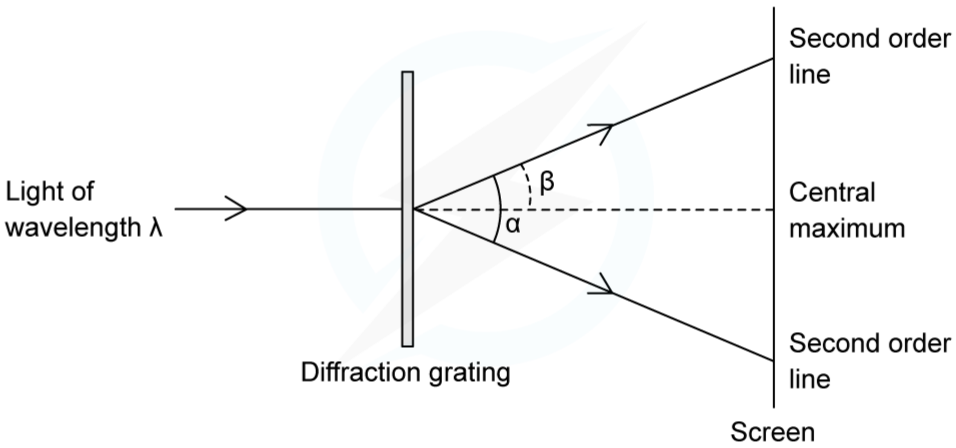
8. Superposition

YOUR NOTES

Worked example

An experiment was set up to investigate light passing through a diffraction grating with slit spacing of 1.7 μm. The fringe pattern was observed on a screen.

The wavelength of light λ is 550nm.



Calculate the angle α between the two second-order lines.

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STEP 1

DIFFRACTION GRATING EQUATION

$d\sin(\theta) = n\lambda$

$n = 2$ FOR THE SECOND ORDER LINE

$D = 1.7\ \mu m$

$\lambda = 550\ nm$

STEP 2

REARRANGE FOR $\sin(\theta)$

$$\sin(\theta) = \frac{n\lambda}{d}$$

STEP 3

SUBSTITUTE IN VALUES

$$\sin(\theta) = \frac{2 \times 550 \times 10^{-9}}{1.7 \times 10^{-6}} = 0.64705... = 0.65 \text{ (2 s.f.)}$$

STEP 4

FIND θ THROUGH THE INVERSE SINE

$$\sin^{-1}(0.65) = 40.54^\circ$$

STEP 5

θ IS ANGLE FROM THE CENTRE TO THE SECOND ORDER LINE (β ON THE DIAGRAM)

$$\alpha = \theta \times 2 = 81^\circ \text{ (2 s.f.)}$$

Exam Tip

Take care that the angle θ is the correct angle taken from the centre and not the angle taken between two orders of maxima.

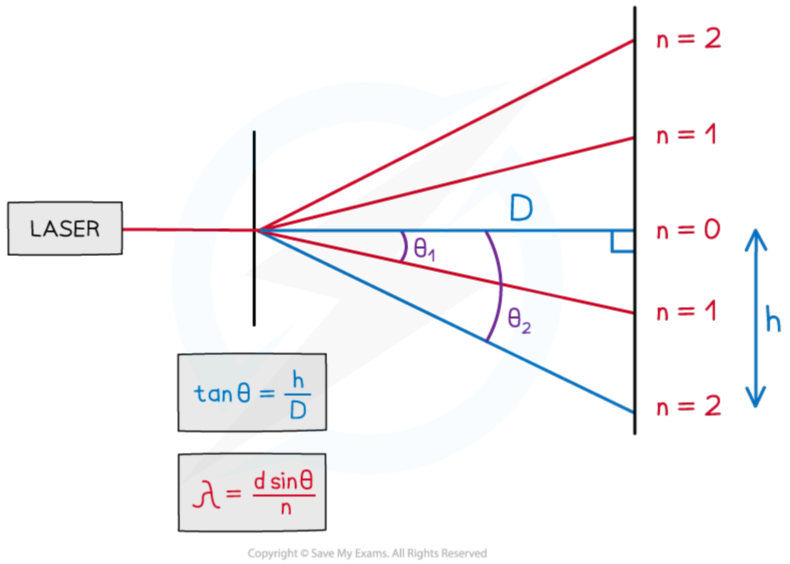
Determining the Wavelength of Light

Method

- The wavelength of light can be determined by rearranging the grating equation to make the wavelength λ the subject
- The value of θ, the angle to the specific order of maximum measured from the centre, can be calculated through trigonometry
- The distance from the grating to the screen is marked as D
- The distance between the centre and the order of maxima (e.g. n = 2 in the diagram) on the screen is labelled as h – the fringe spacing
- Measure both these values with a ruler
- This makes a right-angled triangle with the angle θ as the ratio of the ***h/D = tanθ***

8. Superposition

YOUR NOTES



The wavelength of light is calculated by the angle to the order of maximum

- Remember to find the inverse of tan to find $\boldsymbol{\theta = \tan^{-1}(h/D)}$
- This value of θ can then be substituted back into the diffraction grating equation to find the value of the wavelength (with the corresponding order n)

Improving experiment and reducing uncertainties

- The fringe spacing can be subjective depending on its intensity on the screen. Take multiple measurements of h (between 3-8) and finding the average
- Use a Vernier scale to record h, in order to reduce percentage uncertainty
- Reduce the uncertainty in h by measuring across all fringes and dividing by the number of fringes
- Increase the grating to screen distance D to increase the fringe separation (although this may decrease the intensity of light reaching the screen)
- Conduct the experiment in a darkened room, so the fringes are clearer
- Use grating with more lines per mm, so values of h are greater to lower percentage uncertainty

8. Superposition

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⬇

Exam Question: Easy

Which of the following is the definition of diffraction?

A change of direction when waves cross the boundary between one medium and another

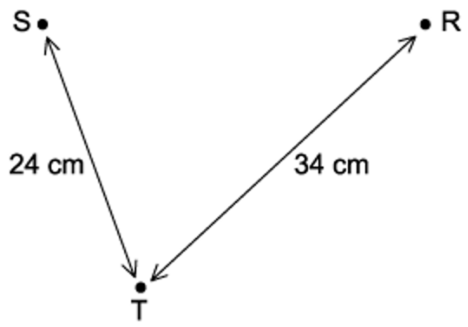
B splitting of white light into colours

C addition of two coherent waves to produce a stationary wave pattern

D bending of waves around an obstacle

Exam Question: Medium

Two wave generators are placed at points S and R to produce water waves with an identical wavelength. At point T both waves have the same amplitude. The distances from S to T and R to T are shown on the diagram below



As the student walks between S to T she notices that the loudness of the sound increases then decreases repeatedly.

What is the wavelength of the waves when the wave generators are in phase and the amplitude at T is zero?

A 6 cm **B** 4 cm **C** 3 cm **D** 2 cm

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YOUR NOTES
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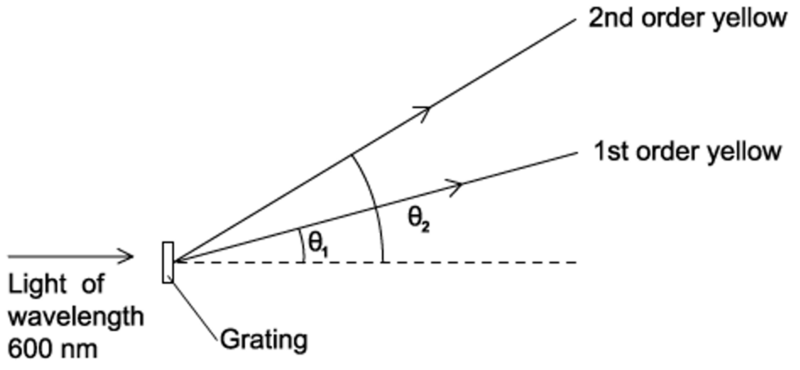
Exam Question: Hard

Yellow light with a wavelength of 600 nm is used in a diffraction grating experiment. The grating has a separation of 2.00 μm.



What would be the angular separation between the first order and second order maxima?

A 54.3˚ **B** 36.9˚ **C** 19.4˚ **D** 17.5˚

> CHECK YOUR ANSWERS AT SAVEMYEXAMS.CO.UK

9. Electricity

YOUR NOTES
⬇

CONTENTS

9.1 CURRENT & POTENTIAL DIFFERENCE

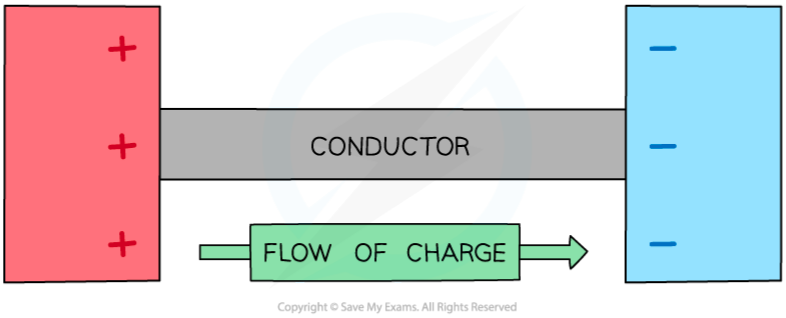
9.1.1 ELECTRIC CURRENT

Defining Electric Current

- Electric current is **the flow of charge** carriers and is measured in units of **amperes (*A*)** or **amps**
- Charge can be either positive or negative
- When two oppositely charged conductors are connected together (by a length of wire), charge will flow between the two conductors, causing a current

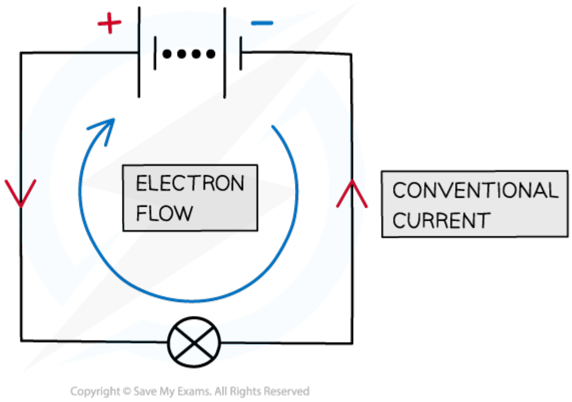
9. Electricity

YOUR NOTES



Charge can flow between two conductors

- In electrical wires, the current is a flow of **electrons**
- Electrons are negatively charged; they flow away from the negative terminal of a cell towards the positive terminal
- Conventional current is defined as the flow of positive charge from the **positive terminal of a cell to the negative terminal**
    - This is the opposite to the direction of electron flow, as conventional current was described before electric current was really understood



By definition, conventional current always goes from positive to negative (even through electrons go the other way)

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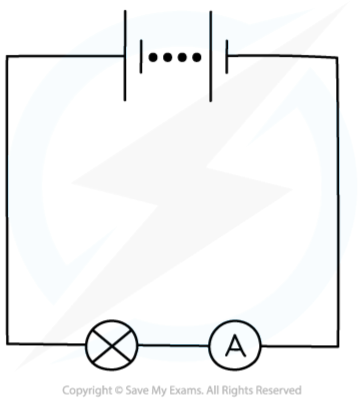
- There are several examples of electric currents, including in household wiring and electrical appliances
- Current is measured using an **ammeter**
- Ammeters should always be connected in **series** with the part of the circuit you wish to measure the current through



An ammeter can be used to measure the current around a circuit and always connected in series

Quantisation of Charge

- The charge on charge carriers is **quantised**
- Charge comes in definite bits – e.g. a single **proton** has a single **positive charge**, whereas a single **electron** has a single **negative charge**
- In this way, the quantity of charge can be quantised dependent on how many protons or electrons are present – positive and negative charge has a definite **minimum magnitude** and always comes in multiples of that magnitude
- This means that if we say something has a given charge, the charge is always a multiple of the charge of an electron by convention
    - The charge of an electron is -1.60×10^{-19} C
    - The charge of a proton by comparison is 1.60×10^{-19} C (this is known as the elementary charge, denoted by *e* and measured in Coulombs (C))

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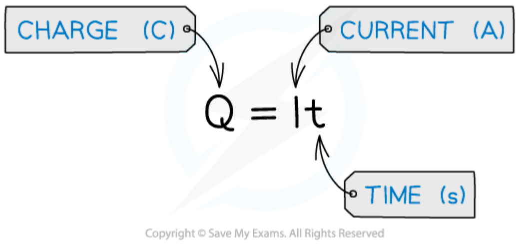
9.1.2 ELECTRIC CURRENT: CALCULATIONS

Calculating Electric Charge

- Current can also be defined as the charge passing through a circuit per unit time
- Electric charge is measured in units of **coulombs (*C*)**
- Charge, current and time are related by the following equation



Charge equation

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YOUR NOTES

Worked example

When will 8 mA of current pass through an electrical circuit?

A. When 1 J of energy is used by 1 C of charge
B. When a charge of 4 C passes in 500 s
C. When a charge of 8 C passes in 100 s
D. When a charge of 1 C passes in 8 s

ANSWER: B

STEP 1
CHARGE EQUATION
$Q = It$

STEP 2
REARRANGE FOR CURRENT
$I = \frac{Q}{t}$

STEP 3
OPTION A ISN'T SPECIFICALLY RELEVANT TO CURRENT

STEP 4
TRY OPTIONS B TO D INTO CURRENT EQUATION TO GET 8 mA

STEP 5
IN OPTION B:
$I = \frac{4}{500} = 8 \times 10^{-3} A = 8 mA$

Exam Tip

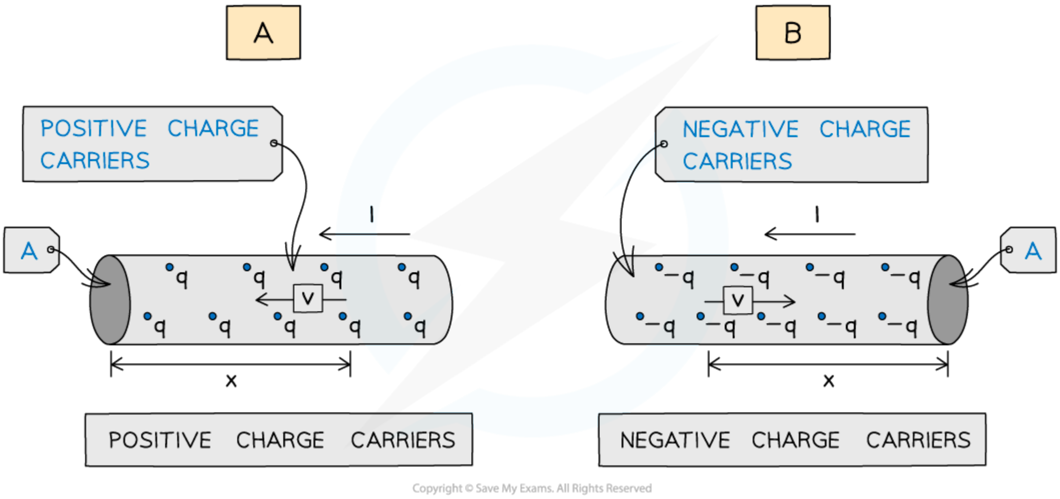
Although electric charge can be positive or negative, since the conventional direction of current is the flow of **positive** charge the current should always be a positive value for your exam answers.

9. Electricity

YOUR NOTES

Calculating Current in a Current Carrying Conductor

- In a conductor, current is due to the movement of charge carriers
- These charge carriers can be negative or positive, however the current is always taken to be in the same direction
- In conductors, the charge carrier is usually free electrons
- In the image below, the current in each conductor is from right to left but the charge carriers move in opposite directions shown by the direction of the drift speed *v*
    - In diagram A (positive charge carriers), the drift speed is in the **same** direction as the current
    - In diagram B (negative charge carriers), the drift speed is in the **opposite** direction to the current



Conduction in a current-carrying conductor

- The drift speed is the average speed the charge carriers are travelling through the conductor. You will find this value is quite slow. However, since the number density of charge carriers is so large, we still see current flow happen instantaneously
- The current can be expressed in terms of the number density (number of charge carriers per unit volume) *n*, the cross-sectional area *A*, the drift speed *v* and the charge of the charge carriers *q*

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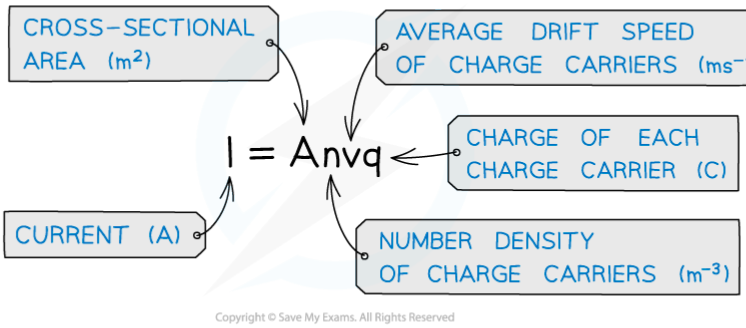


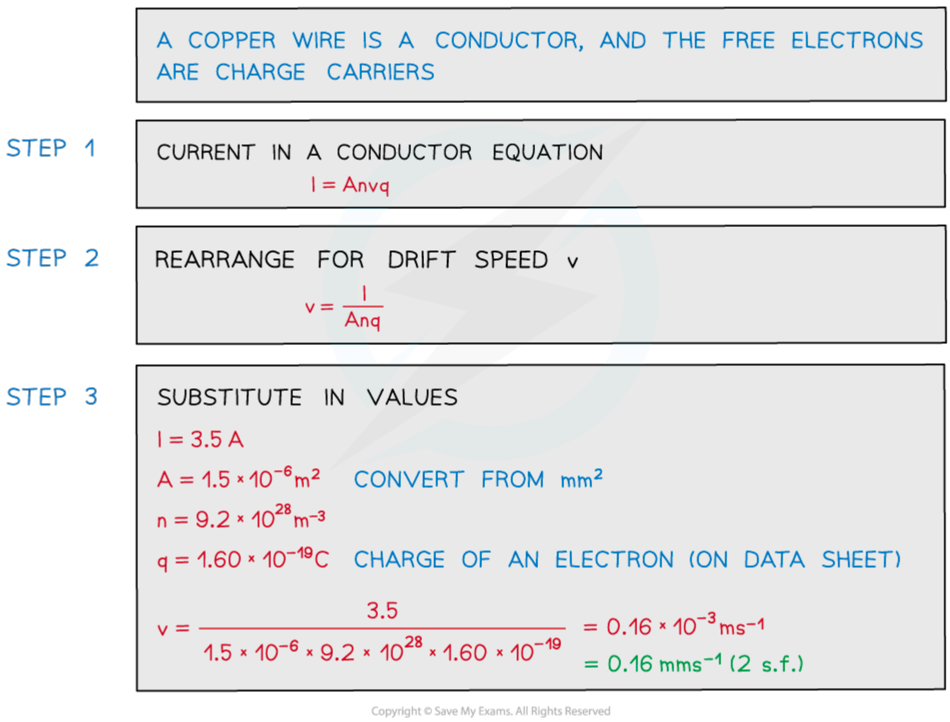
Current in a conductor equation

- The same equation is used whether the charge carriers are positive or negative

Worked example

A copper wire has 9.2×10^{28} free electrons m^{-3}.
The wire has a current of 3.5 A and a cross-sectional area of 1.5 mm^2.

Calculate the average drift speed of the electrons.



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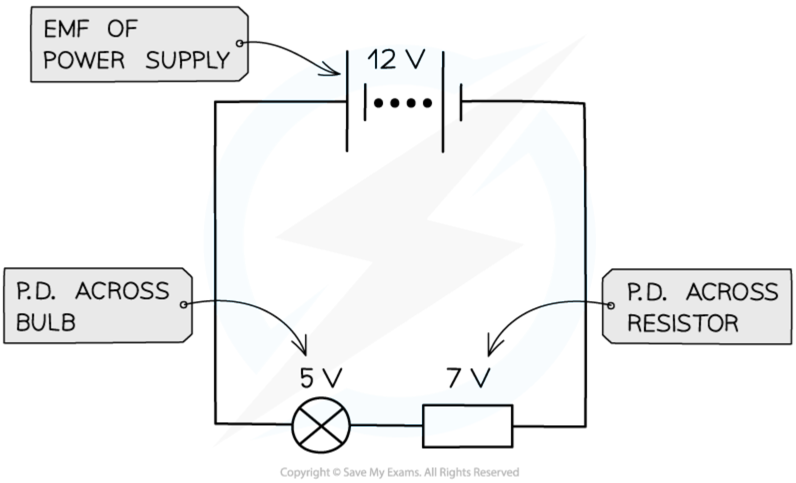
9.1.3 POTENTIAL DIFFERENCE

Defining Potential Difference

- A cell makes one end of the circuit positive and the other negative. This sets up a **potential difference** (*d*) across the circuit
- The potential difference across a component in a circuit is defined as the **energy transferred per unit charge flowing from one point to another**
- The energy transfer is from electrical energy into other forms
- Potential difference is measured in **volts *(V)***. This is the same as a **Joule per coulomb ($J\ C^{-1}$)**
    - If a bulb has a voltage of 3 V, every coulomb of charge passing through the bulb will lose 3 J of energy
- The potential difference of a power supply connected in series is always shared between all the components in the circuit

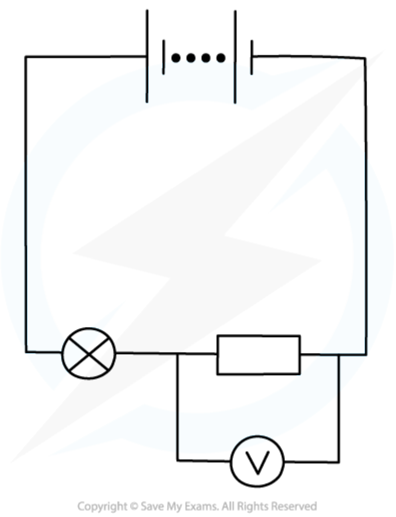


The potential difference is the voltage across each component in a circuit

- Potential difference or voltage is measured using a **voltmeter**
- A voltmeter is always set up in parallel to the component you are measuring the voltage for

9. Electricity

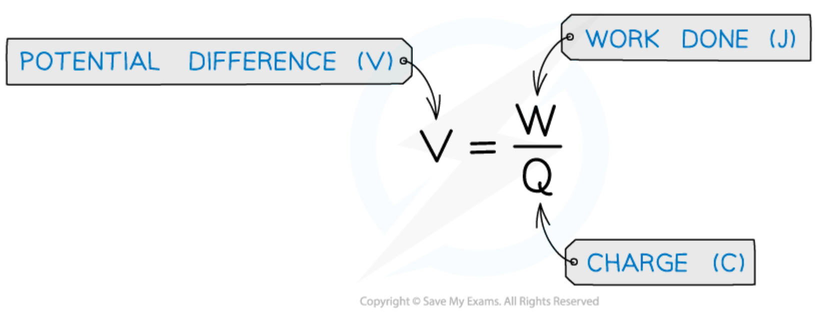
YOUR NOTES



Potential difference can be measured by connecting a voltmeter in series between two points in a circuit

Calculating Potential Difference

- The potential difference is defined as the **energy transferred per unit charge**
- Another measure of energy transfer is work done
- Therefore, potential difference can also be defined as the **work done per unit charge**



Potential difference is the work done per unit charge

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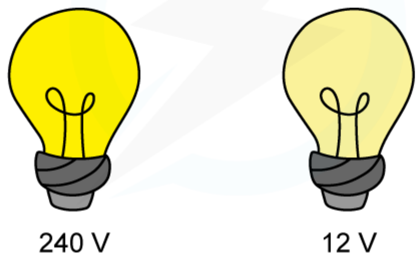
YOUR NOTES

Worked example

A lamp is connected to a 240 V mains supply and another to a 12 V car battery.
Both lamps have the same current, yet 240 V lamp glows more brightly.
Explain in terms of energy transfer why the 240 V lamp is brighter than the 12 V lamp.



- Both lamps have the same current, which means charge flows at the same rate in both
- The 240 V lamp has 20 times more voltage than the 12 V lamp
- Voltage is the energy transferred (work done) per unit charge
- This means the energy transferred to each coulomb of charge in the 240 V lamp is 20 times greater than for the 12 V lamp
- This makes the 240 V lamp shine much brighter than the 12 V lamp

Exam Tip

Think of potential difference as being the **energy per coulomb** of charge transferred between two points in a circuit

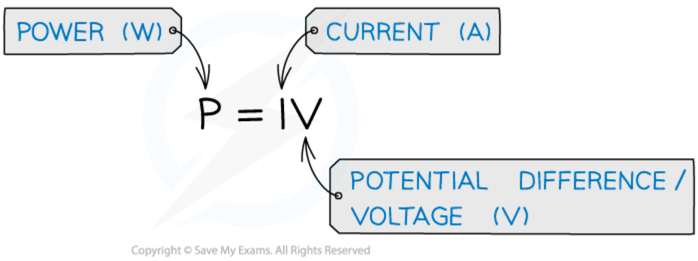
9. Electricity

YOUR NOTES

9.1.4 ELECTRICAL POWER

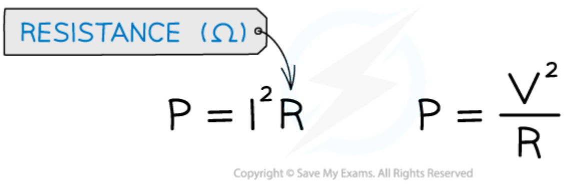
Calculating Electrical Power

- In "Work, Energy and Power", Power *P* was defined as the **rate of doing work**
  - Potential difference is the **work done per unit charge**
  - Current is the **rate of flow of charge**
- So, the power dissipated (produced) by an electrical device can be written as



Power of a component in an electrical circuit

- Using ***V* = *IR*** to rearrange for either *V* or *I* and substituting into the power equation means we also write power in terms of resistance *R*



Power equation in terms of resistance

- This means for a given resistor for example, if the current or voltage doubles the power will be four times as great.
- Which equation to use will depend on whether the value of current or voltage has been given in the question

9. Electricity

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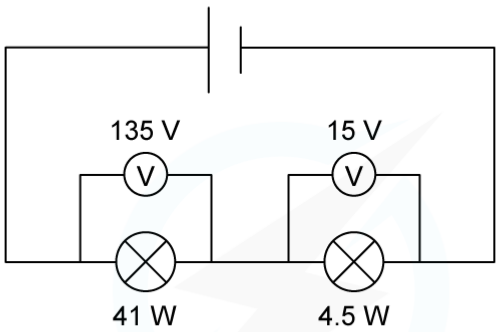
Worked example

Two lamps are connected in series to a 150 V power supply.



Which statement most accurately describes what happens?

A. Both lamps light normally
B. The 15 V lamp blows
C. Only the 41 W lamp lights
D. Both lamps light at less than their normal brightness

ANSWER: A

STEP 1: CALCULATE CURRENT NEEDED FOR BOTH LAMPS TO OPERATE

$P = IV$

STEP 2: REARRANGE FOR I

$I = \frac{P}{V}$

STEP 3: FOR THE 41 W LAMP: $I = \frac{41\text{ W}}{135\text{ V}} = 0.3\text{ A}$

FOR THE 4.5 W LAMP: $I = \frac{4.5\text{ W}}{15\text{ V}} = 0.3\text{ A}$

STEP 4: FOR BOTH TO OPERATE AT THEIR NORMAL BRIGHTNESS, A CURRENT OF 0.3 A IS REQUIRED. SINCE THE LAMPS ARE CONNECTED IN SERIES, THE SAME CURRENT WOULD FLOW THROUGH BOTH.

STEP 5: THE LAMPS WILL LIGHT AT THEIR NORMAL BRIGHTNESS – OPTION A

9. Electricity

YOUR NOTES

Exam Tip

You can use the pneumonic "Twinkle Twinkle Little Star, Power equals *I* squared *R*" to remember whether to multiply or divide by resistance in the power equations

Exam Question: Easy

What describes the electric potential difference between two points in a wire that carries a current?

A the force required to move a unit positive charge between the points

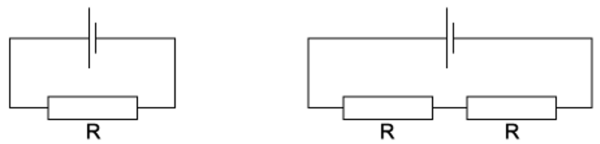
B the ratio of the energy dissipated between the points to the current

C the ratio of the power dissipated between the points to the current

D the ratio of the power dissipated between the points to the charge moved

Exam Question: Medium

The diagrams show two different circuits.



The cells in each circuit have the same electromotive force and zero internal resistance. The three resistors each have the same resistance *R*.

In the circuit on the left, the power dissipated in the resistor is *P*.

What is the total power dissipated in the circuit on the right?

A $\frac{P}{4}$ **B** $\frac{P}{2}$ **C** P **D** $2P$

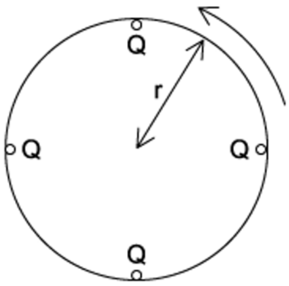
9. Electricity

YOUR NOTES

Exam Question: Hard

Four point charges, each of charge *Q*, are placed on the edge of an insulating disc of radius *r*.

The frequency of rotation of the disc is *f*.



What is the equivalent electric current at the edge of the disc?

A $4Qf$ **B** $\frac{4Q}{f}$ **C** $8\pi rQf$ **D** $\frac{2Qf}{\pi r}$

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9. Electricity

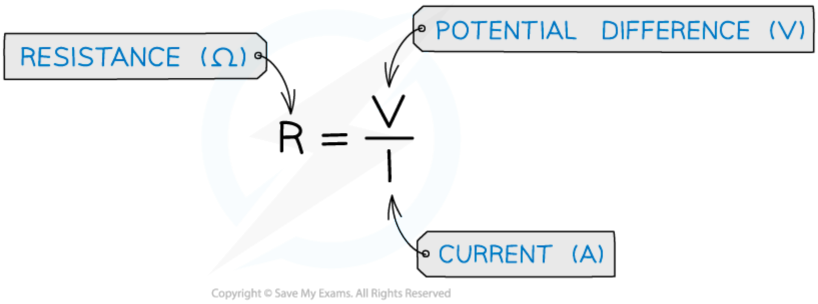
YOUR NOTES
⬇

9.2 RESISTANCE

9.2.1 RESISTANCE

Defining Resistance

- Resistance is defined as the **opposition** to current
  - For a given potential difference: **The higher the resistance the lower the current**
- Wires are often made from copper because copper has a low electrical resistance. This is also known as a good conductor
- The resistance *R* of a conductor is defined as the ratio of the potential difference *V* across to the current *I* in it



Resistance of a component is the ratio of the potential difference and current

- Resistance is measured in **Ohms (Ω)**
- An Ohm is defined as one volt per ampere
- The resistance controls the size of the current in a circuit
  - A **higher** resistance means a **smaller** current
  - A **lower** resistance means a **larger** current
- All electrical components, including wires, have some value of resistance

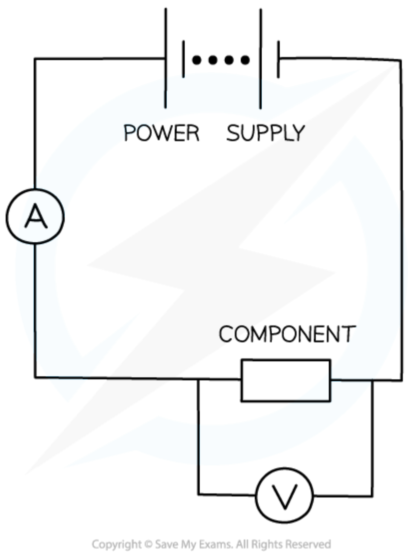
9. Electricity

YOUR NOTES
⬇

Calculating Resistance

Determining Resistance

- To find the resistance of a component, we can set up a circuit like the one shown below



A circuit to determine the resistance of a component

- The power supply should be set to a low voltage to avoid heating the component, typically 1-2 V
- Measurements of the potential difference and current should then be taken from the voltmeter and ammeter respectively
- Finally, these readings should be substituted into the resistance equation

9. Electricity

YOUR NOTES

Worked example

A charge of 5.0 C passes through a resistor of resistance R Ω at a constant rate in 30 s.
If the potential difference across the resistor is 2.0 V, calculate the value of R.

STEP 1

RESISTANCE EQUATION

$$R = \frac{V}{I}$$

STEP 2

CALCULATE THE CURRENT FROM THE CHARGE AND TIME

$$Q = It$$

REARRANGE FOR I

$$I = \frac{Q}{t} = \frac{5.0}{30} = 0.167\,A = 0.17\,A \quad (2\ s.f.)$$

STEP 3

SUBSTITUTE VALUES INTO RESISTANCE EQUATION

$$R = \frac{2.0\,V}{0.17\,A} = 11.764... = 12.0\,\Omega \quad (2\ s.f.)$$

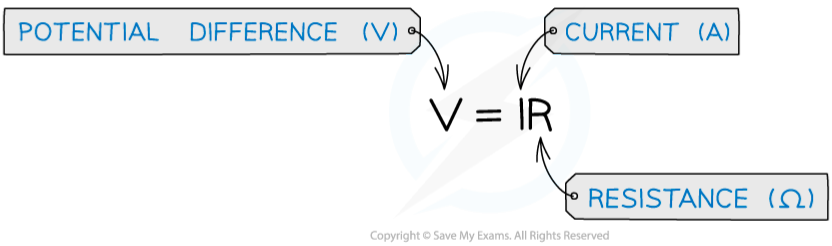
9. Electricity

YOUR NOTES

9.2.2 OHM'S LAW

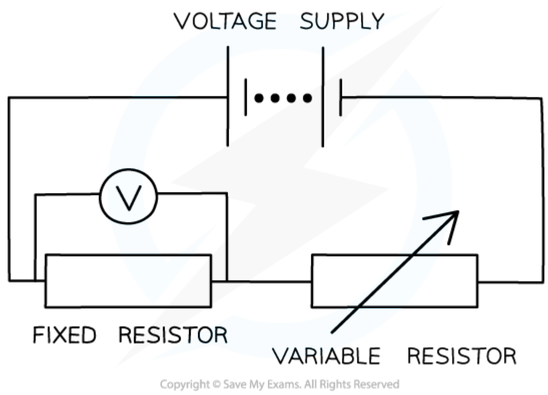
Ohm's Law

- Ohm's law states that for a conductor at a constant temperature, the **current** through it is **proportional** to the **potential difference** across it
- Constant temperature implies constant resistance
- This is shown the equation below:



Ohm's law

- The relation between potential difference across an electrical component (in this case a fixed resistor) and the current can be investigated through a circuit such as the one below

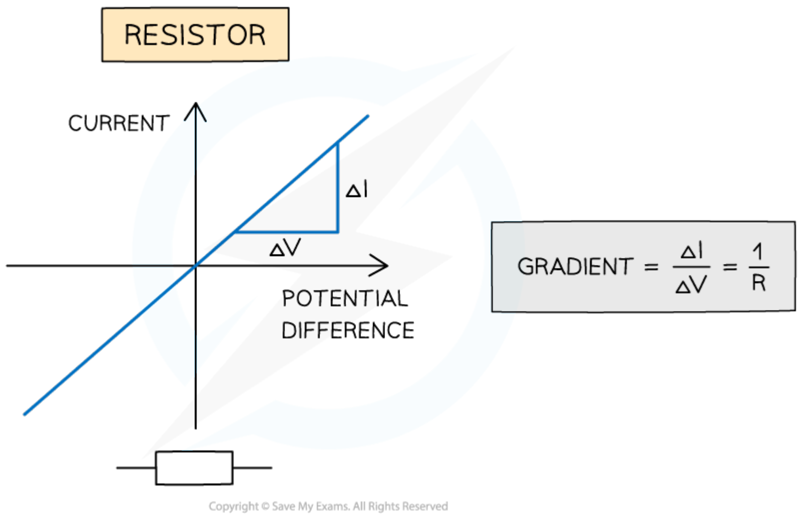


Circuit for plotting graphs of current against voltage

9. Electricity

YOUR NOTES

- By adjusting the resistance on the variable resistor, the current and potential difference will vary in the circuit. Measuring the variation of current with potential difference through the fixed resistor will produce the straight line graph below



Circuit for plotting graphs of current against voltage

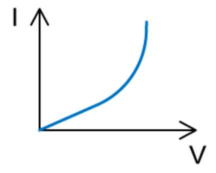
- Since the gradient is constant, the resistance *R* of the resistor can be calculated by using 1 ÷ gradient of the graph
- An electrical component obeys Ohm's law if its graph of current against potential difference is a **straight line** through the origin
    - A resistor obeys Ohm's law
    - A filament lamp does **not** obey Ohm's law
- This applies to any metal wires, provided that the current isn't large enough to increase their temperature

9. Electricity

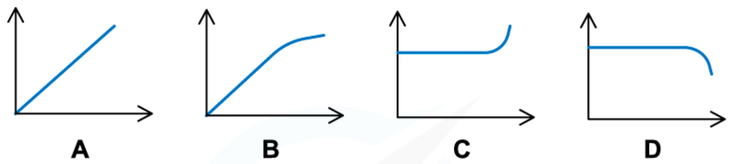
YOUR NOTES

Worked example

The current flowing through a component varies with the potential difference V across it as shown.



Which graph best represents how the resistance R varies with V?



ANSWER: D

STEP 1

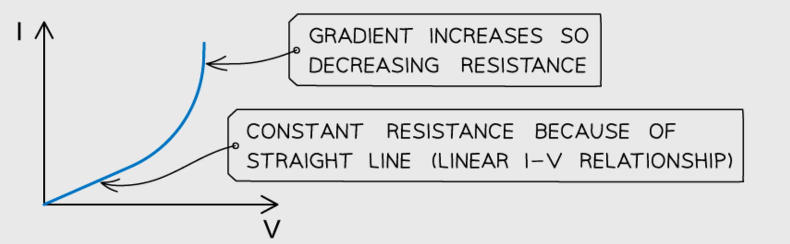
USE OHM'S LAW TO FIND R IN TERMS OF V AND I

$V = IR$ THEREFORE $R = \frac{V}{I}$

STEP 2

THIS MEANS THE GRADIENT OF THE I–V GRAPH IS EQUAL TO $\frac{1}{R}$

STEP 3



STEP 4

HOW THIS RELATES TO R–V GRAPH

THE INITIAL LINE WILL BE HORIZONTAL SINCE RESISTANCE IS CONSTANT

THE LINE WILL THEN CURVE DOWNWARDS AS R DECREASES

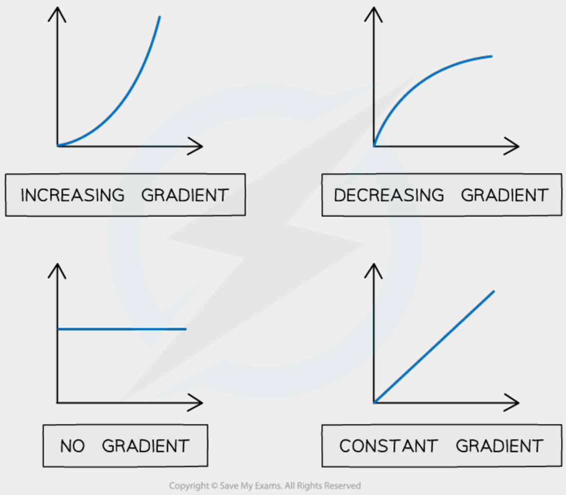
OPTION D IS THE BEST DESCRIPTION OF THIS

9. Electricity

YOUR NOTES

Exam Tip

- In maths, the gradient is the **slope** of the graph
- The graphs below show a summary of how the slope of the graph represents the gradient



Graphs showing varying gradients

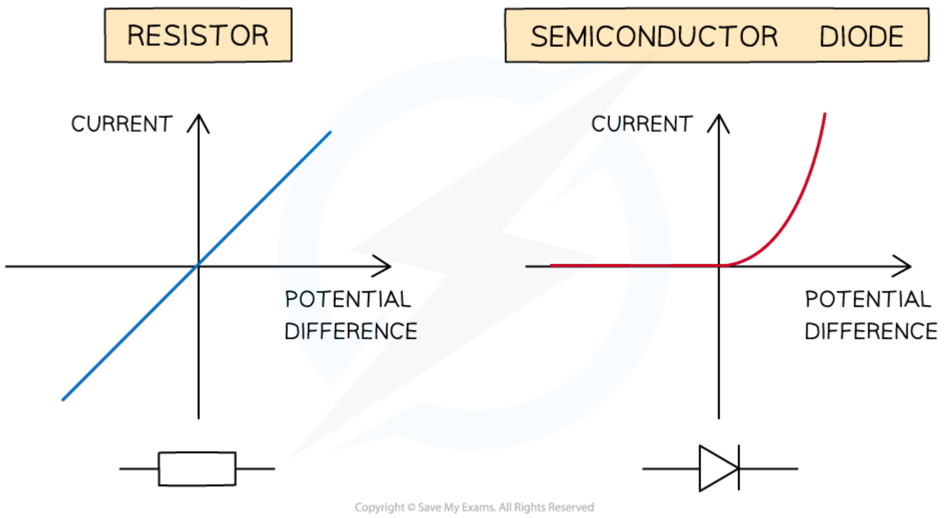
9. Electricity

YOUR NOTES

9.2.3 I-V CHARACTERISTICS

I–V Characteristics

- As the potential difference (voltage) across a component is increased, the current also increases (by Ohm's law)
- The precise relationship between voltage and current is difference for different components and can be shown on an *I-V* graph:



I-V characteristics for metallic conductor (e.g. resistor) and semiconductor diode

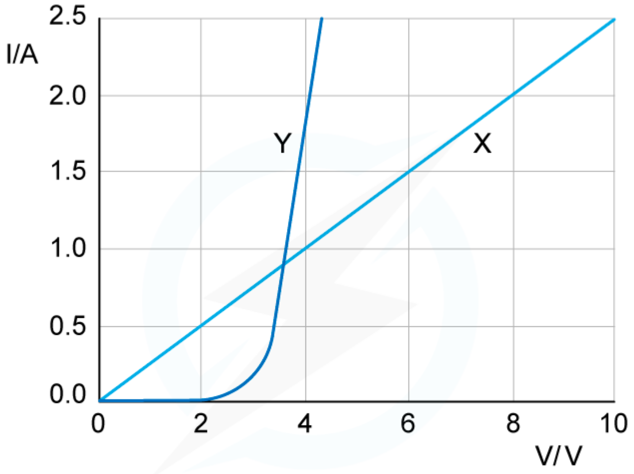
- The *I-V* graph for a metallic conductor at constant temperature e.g. a resistor, is very simple:
  - The current is **directly proportional** to the potential difference
  - This is demonstrated by the **straight line** graph through the origin
- The *I-V* graph for a semiconductor diode is slightly different. A diode is used in a circuit to allow current to flow only in a specific direction:
  - When the current is in the direction of the arrowhead symbol, this is **forward bias.** This is shown by the sharp increase in potential difference and current on the right side of the graph
  - When the diode is switched around, it does not conduct and is called **reverse bias.** This is shown by a zero reading of current or potential difference on the left side of the graph

9. Electricity

YOUR NOTES

Worked example

The I-V characteristic of two electrical component X and Y are shown.



Which statement is correct?

A. The resistance of X increases as the current increases
B. At 2 V, the resistance of X is half the resistance of Y
C. Y is a semiconductor diode and X is a resistor
D. X is a resistor and Y is a filament lamp

ANSWER: C

- The *I-V* graph X is **linear**
  - This means the graph has a constant gradient. I/V and the resistance is therefore also constant (since gradient = 1/R)
  - This is the *I-V* graph for a conductor at constant temperature e.g. a resistor
- The *I-V* graph Y starts with zero gradient and then the gradient increases rapidly
  - This means it has infinite resistance at the start which then decreases rapidly
  - This is characters of a device that only has current in one direction e.g a semiconductor diode
- Therefore the answer is **C**

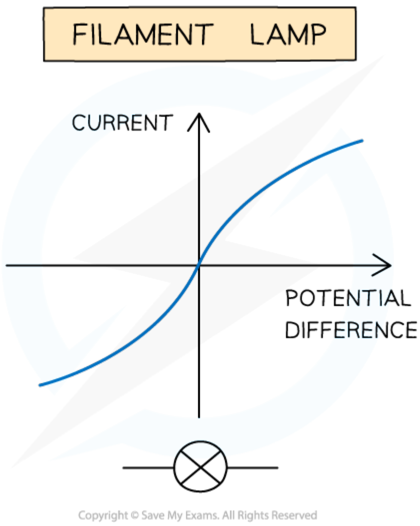
9. Electricity

YOUR NOTES
⬇

9.2.4 RESISTANCE IN A FILAMENT LAMP

Resistance in a Filament Lamp

- The *I-V* graph for a filament lamp shows the current increasing at a proportionally slower rate than the potential difference



I-V characteristics for a filament lamp

- This is because:
  - As the current increases, the temperature of the filament in the lamp increases
  - Since the filament is a metal, the higher temperature causes an increase in resistance
  - Resistance opposes the current, causing the current to increase at a slower rate
- Where the graph is a straight line, the resistance is constant
- The resistance increases as the graph curves

9. Electricity

YOUR NOTES
⬇

Resistance and temperature

- All solids are made up of vibrating atoms
    - The higher the temperature, the faster these atoms vibrate
- Electric current is the flow of free electrons in a material
    - The electrons collide with the vibrating atoms which impedes their flow, hence the current **decreases**
- So, if the current decreases, then the resistance will increase **(*V* = *IR*)**
- Therefore, an increase in **temperature** causes an increase in **resistance**

Exam Question: Easy

Which equation is used to define resistance?

A energy = (current)^2 × resistance × time

B potential difference = current × resistance

C power = (current)^2 × resistance

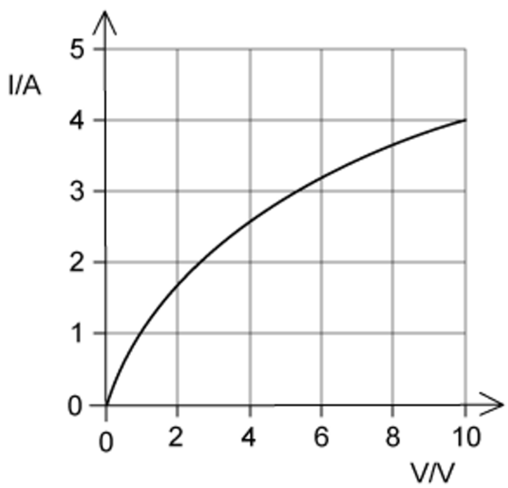
D resistivity = resistance × area ÷ length

9. Electricity

YOUR NOTES

Exam Question: Medium

The graph shows how current I varies with voltage V for a filament lamp.



Since the graph is not a straight line, the resistance of the lamp varies with V.

Which row gives the correct resistance at the stated value of V?

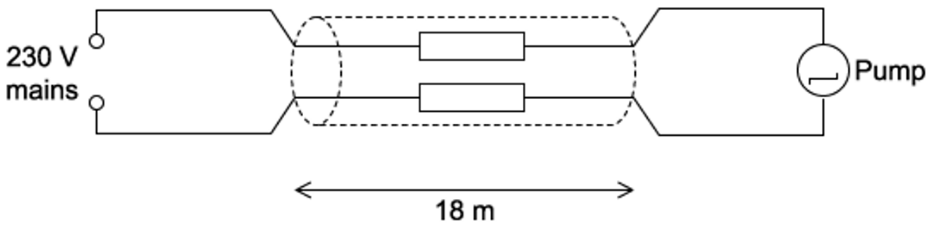
	V / V	R / Ω
A	2.0	1.5
B	4.0	3.2
C	6.0	1.9
D	8.0	0.9

9. Electricity

YOUR NOTES

Exam Question: Hard

The diagram shows an electric pump for a garden fountain connected by an 18 m cable to a 230 V mains electrical supply.



The performance of the pump is acceptable if the potential difference (p.d.) across it is at least 218 V. The current through it is then 0.83 A.

What is the maximum resistance per metre of each of the two wires in the cable if the pump is to perform acceptably?

A 0.40 Ω m^{-1} **B** 0.80 Ω m^{-1} **C** 1.3 Ω m^{-1} **D** 1.4 Ω m^{-1}

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9. Electricity

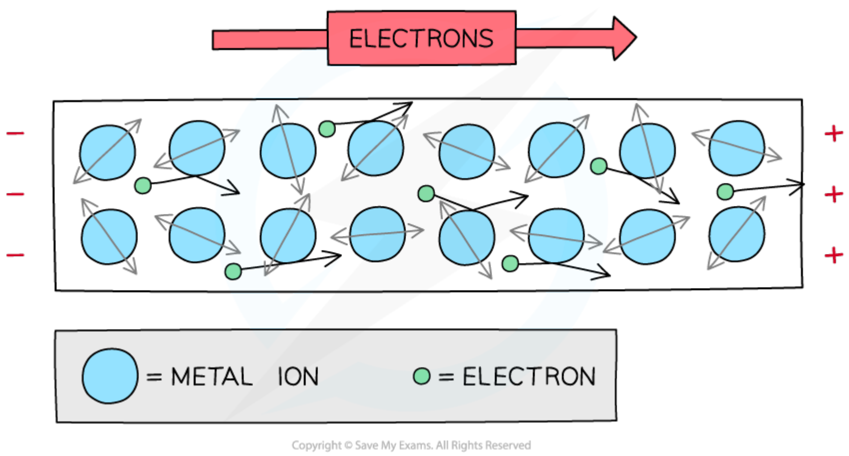
YOUR NOTES

9.3 RESISTIVITY

9.3.1 RESISTIVITY

Resistivity

- All materials have some **resistance** to the flow of charge
- As **free electrons move** through a metal wire, they collide with ions which get in their way
- As a result, they **transfer** some, or all, of their **kinetic energy** on **collision**, which causes electrical **heating**

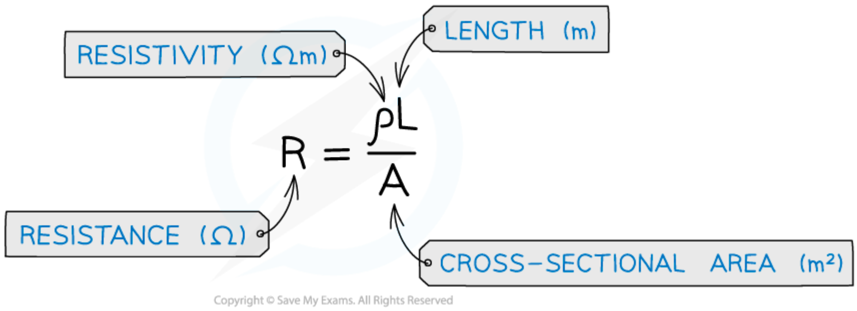


Free electrons collide with ions which resist their flow

- Since **current** is the **flow** of **charge**, the ions resisting their flow causes **resistance**
- Resistance depends on the **length** of the wire, the **cross-sectional area** through which the current is passing and the **resistivity** of the material

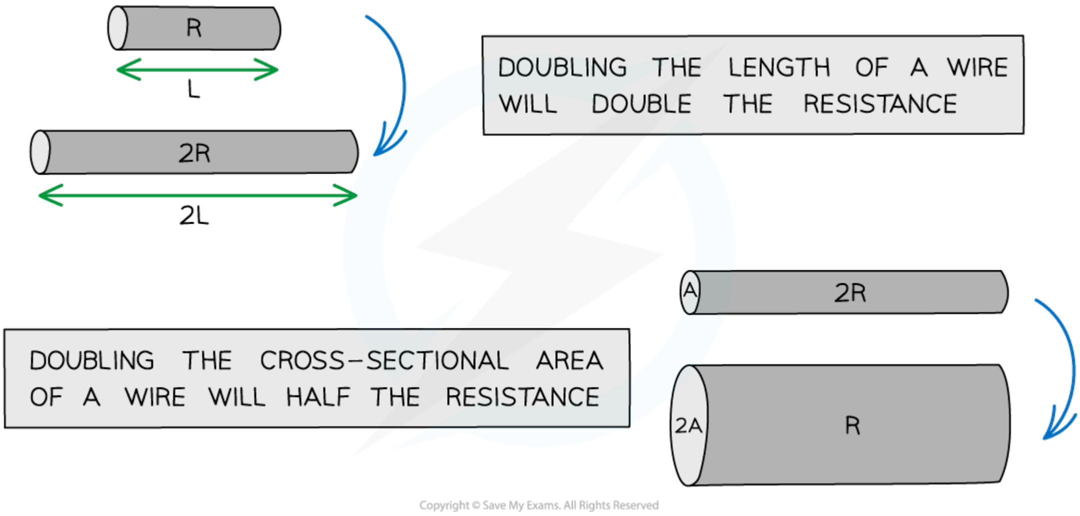
9. Electricity

YOUR NOTES



Electrical resistance equation

- The resistivity equation shows that:
  - The **longer** the wire, the **greater** its resistance
  - The **thicker** the wire, the **smaller** its resistance



The length and width of the wire affect its resistance

- Resistivity is a property that describes the extent to which a material opposes the flow of electric current through it
- It is a property of the material, and is dependent on temperature
- Resistivity is measured in **Ω m**

9. Electricity

YOUR NOTES

Resistivity of some materials at room temperature

	Material	Resistivity ρ / Ωm
Metals	Copper	1.7×10^{-8}
	Gold	2.4×10^{-8}
	Aluminium	2.6×10^{-8}
Semiconductors	Germanium	0.6
	Silicon	2.3×10^{3}
Insulators	Glass	10^{12}
	Sulfur	10^{15}

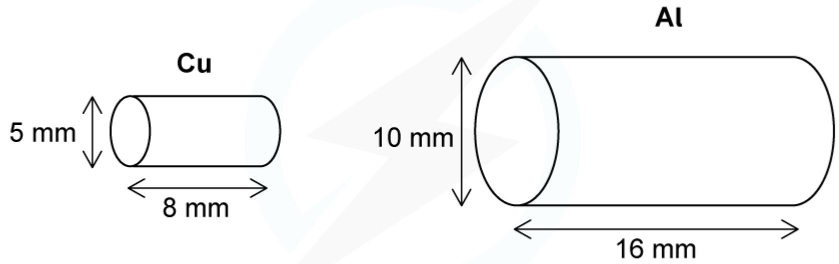
- The higher the resistivity of a material, the higher its resistance
- This is why copper, with its relatively low resistivity at room temperature, is used for electrical wires — current flows through it very easily
- Insulators have such a high resistivity that virtually no current will flow through them

9. Electricity

YOUR NOTES

Worked example

Two electrically-conducting cylinders made from copper and aluminium respectively.
Their dimensions are shown below.



Copper resistivity = 1.7×10^{-8} Ω m
Aluminium resistivity = 2.6×10^{-8} Ω m

Which cylinder is the better conductor?

9. Electricity

YOUR NOTES
⬇

STEP 1

THE BETTER CONDUCTOR WILL HAVE LOWER RESISTANCE

STEP 2

RESISTANCE IS CALCULATED FROM

$$R = \frac{\rho L}{A}$$

STEP 3

RESISTANCE OF THE COPPER CYLINDER

THE CROSS-SECTIONAL AREA OF A CYLINDER IS A CIRCLE

mm → m

$$A = \pi \times r^2 = \pi \times \left(\frac{d}{2}\right)^2 = \pi \times \left(\frac{5 \times 10^{-3}}{2}\right)^2 = 2.0 \times 10^{-5}\,\text{m}^2$$

$$R = \frac{1.7 \times 10^{-8} \times 8 \times 10^{-3}}{2.0 \times 10^{-5}\,\text{m}^2} = 6.8 \times 10^{-6}\,\Omega$$

SUBSTITUTE VALUES INTO THE EQUATION

STEP 4

RESISTANCE OF THE ALUMINIUM CYLINDER

$$A = \pi \times r^2 = \pi \times \left(\frac{d}{2}\right)^2 = \pi \times \left(\frac{10 \times 10^{-3}}{2}\right)^2 = 7.9 \times 10^{-5}\,\text{m}^2$$

$$R = \frac{2.6 \times 10^{-8} \times 16 \times 10^{-3}}{7.9 \times 10^{-5}\,\text{m}^2} = 5.3 \times 10^{-6}\,\Omega$$

STEP 5

RESISTANCE OF ALUMINIUM CYLINDER < RESISTANCE OF COPPER CYLINDER

THE ALUMINIUM CYLINDER IS THE BETTER CONDUCTOR

Exam Tip

- You won't need to memorise the value of the resistivity of any material, these will be given in the exam question.
- Remember if the cross-sectional area is a circle e.g. in a wire, it is proportional to the diameter squared. This means if the diameter doubles, the area quadruples causing the resistance to drop by a quarter.

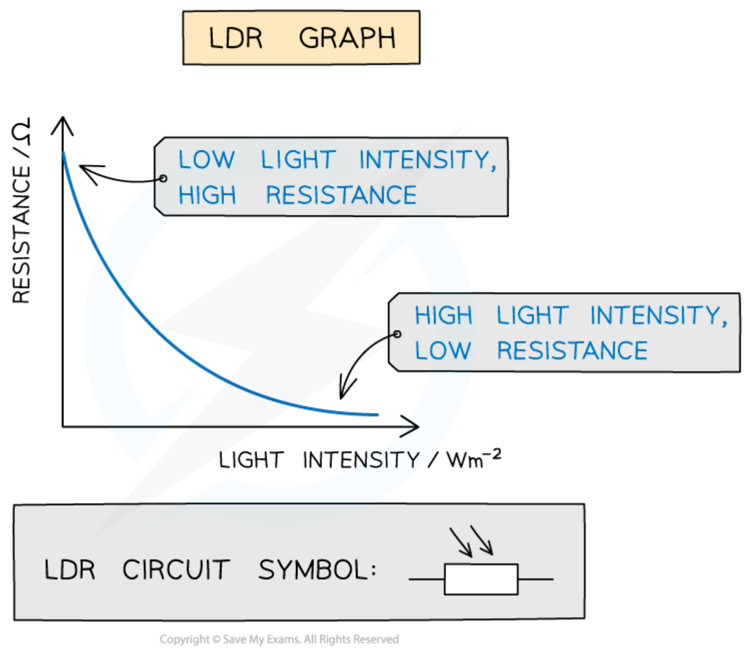
9. Electricity

YOUR NOTES

9.3.2 RESISTANCE IN SENSORY RESISTORS

Resistance in a Light-Dependent Resistor

- A light-dependent resistor (LDR) is a non-ohmic conductor and sensory resistor
- Its resistance automatically changes depending on the light energy falling onto it (illumination)
- As the **light intensity increases**, the **resistance** of an LDR **decreases**
- This is shown by the following graph:

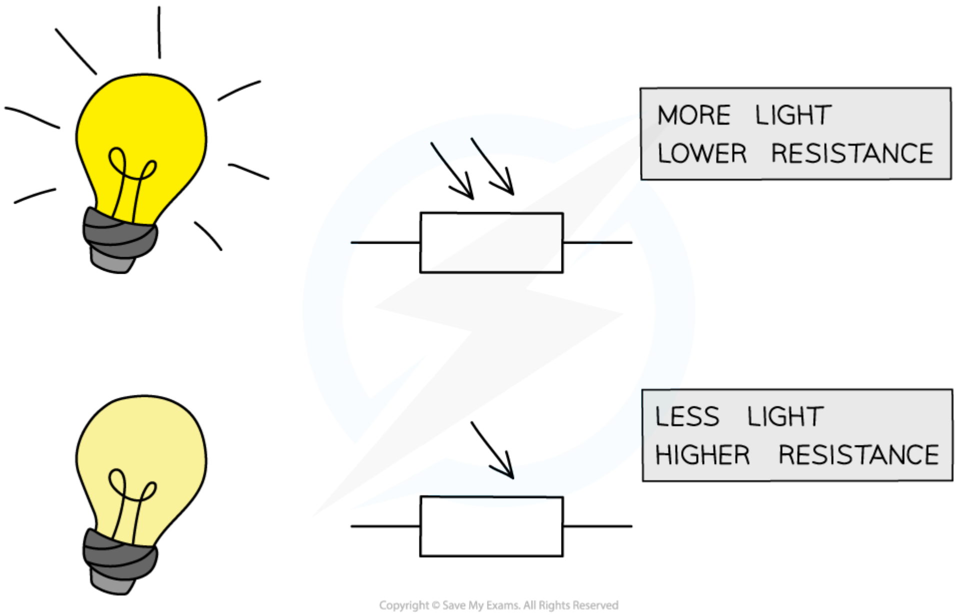


Graph of light intensity and resistance for an LDR

9. Electricity

YOUR NOTES

- LDRs can be used as light sensors, so, they are useful in circuits which automatically switch on lights when it gets dark, for example, street lighting and garden lights
  - In the dark, its resistance is very large (millions of ohms)
  - In bright light, its resistance is small (tens of ohms)

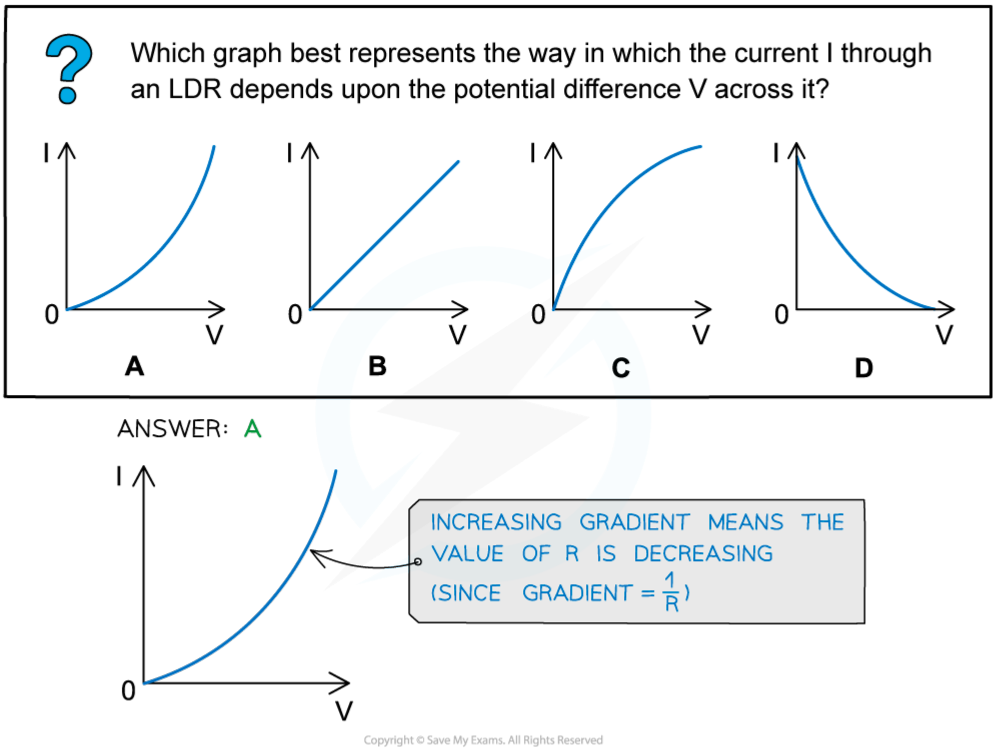


Resistance of an LDR depends on the light intensity falling on it

9. Electricity

YOUR NOTES

Worked example



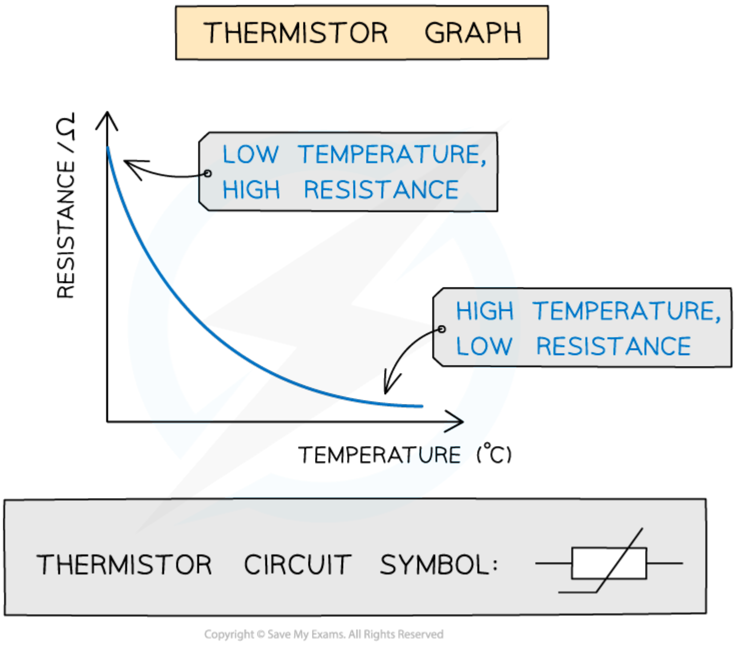
- As the potential difference across the LDR increases, the light intensity increases causing its resistance to decrease
- Ohm's law states that ***V = IR***
- The resistance is equal to V/I or 1/R = I/V = gradient of the graph
- Since *R* decreases, the value of 1/R increases, so the gradient must increase
- Therefore, *I* increases with the p.d with an increasing gradient

9. Electricity

YOUR NOTES

Resistance in a Thermistor

- A thermistor is a non-ohmic conductor and sensory resistor
- Its resistance changes depending on its temperature
- As the temperature **increases** the resistance of a thermistor **decreases**
- This is shown by the following graph:

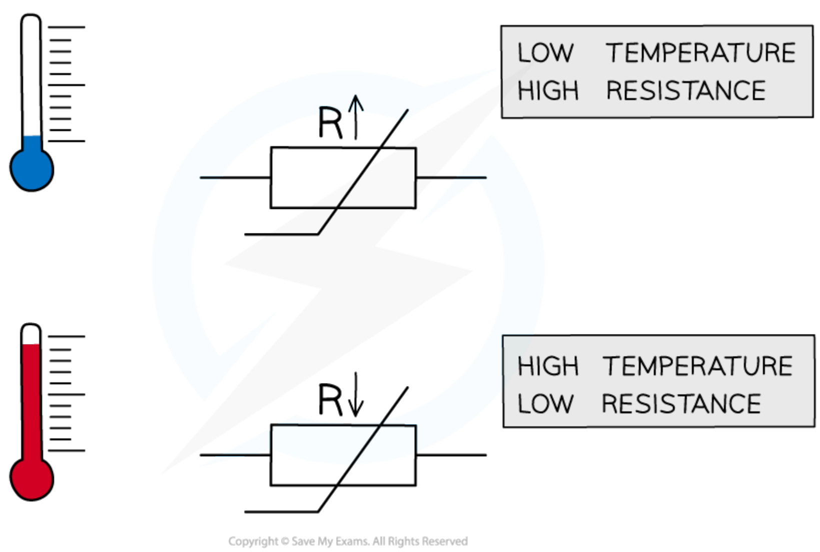


Graph of temperature and resistance for a thermistor

9. Electricity

YOUR NOTES

- Thermistors are temperature sensors and are are used in circuits in ovens, fire alarms and digital thermometers
  - As the thermistor gets hotter, its resistance decreases
  - As the thermistor gets cooler, its resistance increases



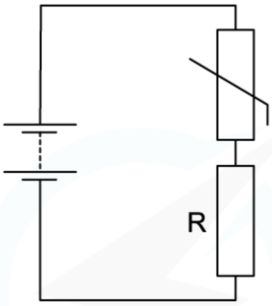
The resistance through a thermistor is dependent on the temperature of it

9. Electricity

YOUR NOTES
⬇

Worked example

A thermistor is connected in series with a resistor R and a battery.



The resistance of the thermistor is equal to the resistance of R at room temperature.

When the temperature of the thermistor decreases, which statement is correcrt?

A. The p.d across the thermistor increases
B. The current in R increases
C. The current through the thermistor decreases
D. The p.d across R increases

ANSWER: A

- The resistance of the thermistor increases as the temperature decreases
- Since the thermistor and resistor *R* are connected in series, the current *I* in both of them is the same
- Ohm's law states that ***V* = *IR***
- Since the resistance of the thermistor increases, and *I* is the same, the potential difference *V* across it increases
- Therefore, statement **A** is correct

9. Electricity

YOUR NOTES

Exam Question: Easy

The unit of resistivity, expressed in terms of base units, is given by

$$kg\, x^3\, y^{-2}\, z^{-3}$$

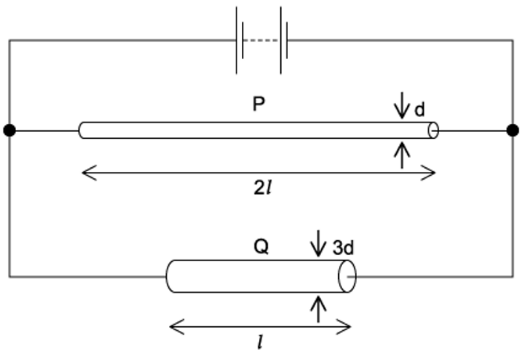
Which base units are *x*, *y* and *z*?

	x	*y*	*z*
A	ampere	metre	second
B	metre	ampere	second
C	metre	second	ampere
D	second	ampere	metre

Exam Question: Medium

Two wires P and Q made of the same material are connected to the same electrical supply.

P has twice the length of Q and one-third of the diameter of Q, as shown in the diagram.



What is the ratio $\frac{\text{current in P}}{\text{current in Q}}$?

A $\frac{2}{3}$ **B** $\frac{2}{9}$ **C** $\frac{1}{6}$ **D** $\frac{1}{18}$

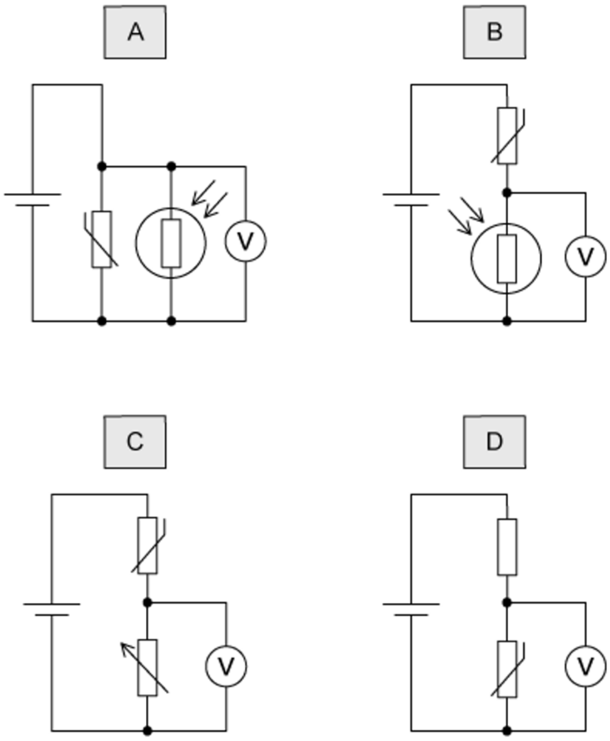
9. Electricity

YOUR NOTES

Exam Question: Hard

A thermistor and another component are connected to a constant voltage supply. A voltmeter is connected across one of the components. The temperature of the thermistor is then reduced, but no other changes are made.

In which circuit will the voltmeter reading increase?



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10. D.C. Circuits

YOUR NOTES

CONTENTS

10.1 DC: PRACTICAL CIRCUITS & KIRCHHOFF'S LAWS

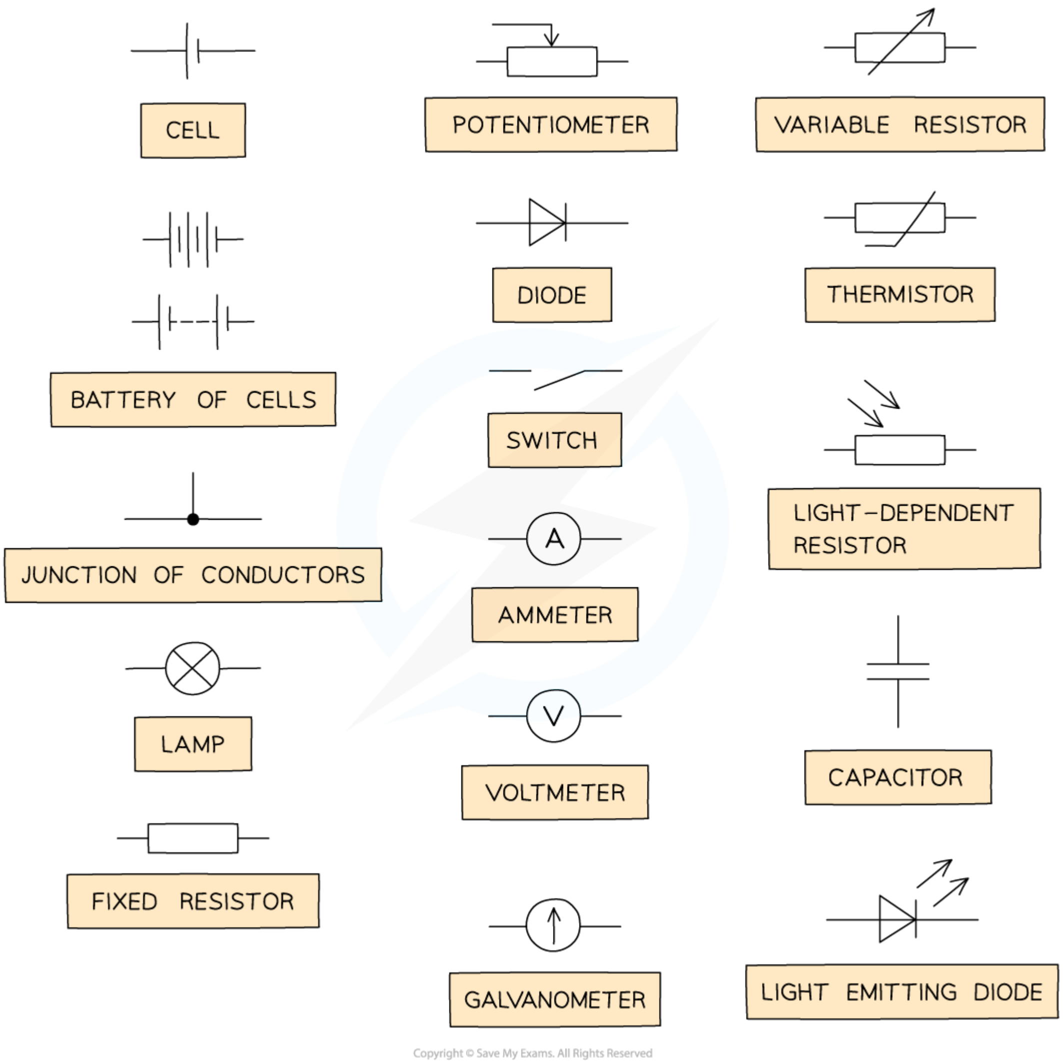
10.1.1 CIRCUIT SYMBOLS

Circuit Symbols

- The diagrams below show the various circuit symbols that could be used in circuit diagrams. You will be expected to recognise and draw all of these
- The most common symbols are as follows:

10. D.C. Circuits

YOUR NOTES



Common circuit symbols

10. D.C. Circuits

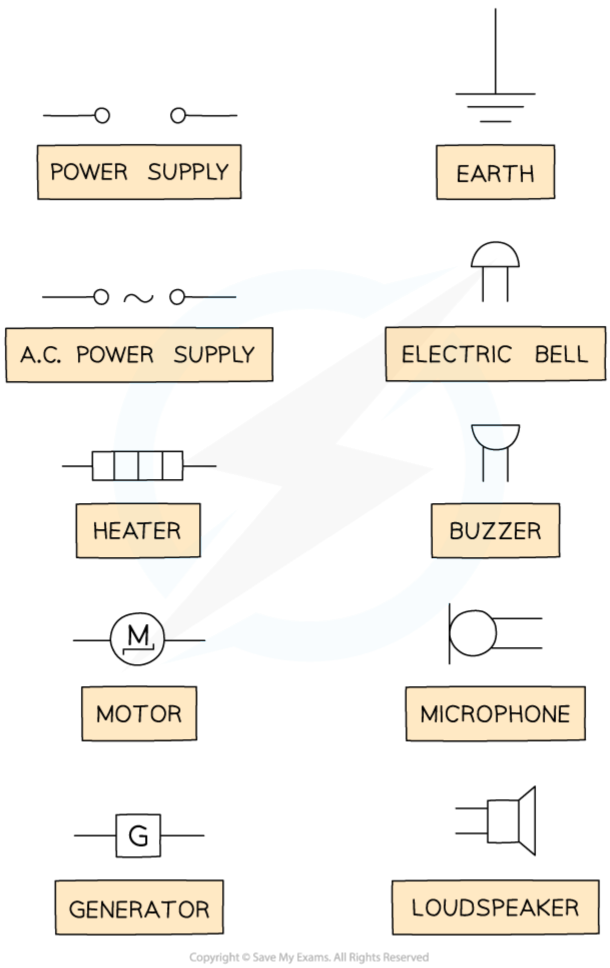
YOUR NOTES

- The function of the most common components are:
  - **Switch:** Turn the circuit on (closed), or off (open)
  - **Fixed resistor:** A resistor limits the flow of current. A fixed resistor has a resistance it cannot change
  - **Variable resistor:** A resistor with a slider that can be used to change its resistance. Used often in dimmer switches and volume controls
  - **Thermistor:** The resistance of a thermistor depends on its temperature. As its temperature increases, its resistance decreases and vice versa
  - **Light-dependent resistor (LDR):** The resistance of an LDR depends on the light intensity. As the light intensity increases, its resistance decreases and vice versa
  - **Diode:** A diode allows current to flow in one direction only. They are used to convert AC to DC current
  - **Light-emitting diode (LED):** This is equivalent to a diode and emits light when a current passes through it. These are used for aviation lighting and displays (TVs, road signs)
  - **Ammeter:** Used to measure the current in a circuit. Connected in series with other component
  - **Voltmeter:** Use to measure the potential difference of an electrical component. Connected in parallel with component

10. D.C. Circuits

YOUR NOTES ⬇

- The more uncommon, yet relevant symbols, are as follows:



Other circuit symbols

Exam Tip

You must memorise all of these circuit symbols for the exam. To make it easier for you, we have separated the symbols into the most common symbols, and the symbols that don't come up as often, however, you should be aware of all of them!

10. D.C. Circuits

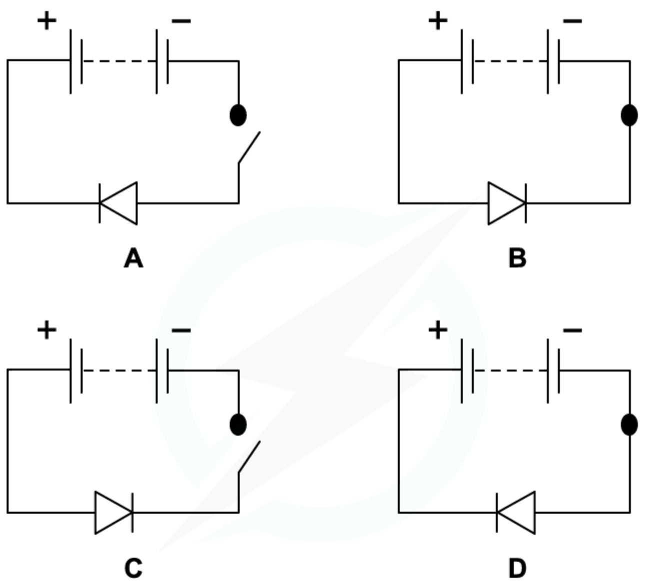
YOUR NOTES

Interpreting Circuit Diagrams

- Being able to draw and interpret circuit diagrams using circuit symbols is an essential skill in Electronics

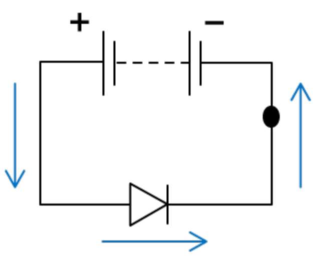
Worked example

Which circuit diagram correctly represents a circuit with current flowing through?



ANSWER: B

THIS IS SEEN IN CIRCUIT B



10. D.C. Circuits

YOUR NOTES
⬇

- For a circuit to be connected, the switch must be closed
  - This is either circuit **B** or **D**
- The other circuit symbol is a diode
  - Diodes only allow current to flow in one direction
- Since current flow is from positive to negative, a forward-biased diode must point in this direction in order for the current to flow
  - This is seen in circuit **B**

10. D.C. Circuits

YOUR NOTES

10.1.2 ELECTROMOTIVE FORCE

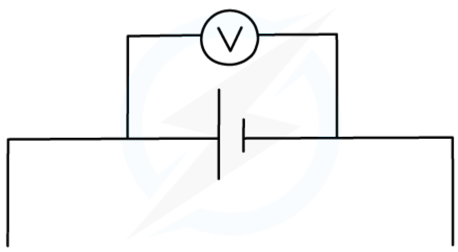
Electromotive Force

- When charge passes through a power supply such as a battery, it **gains** electrical energy
- The **electromotive force (e.m.f)** is the amount of **chemical** energy converted to **electrical** energy per **coulomb** of charge **(C)** when charge passes through a power supply
- e.m.f is measured in **Volts (V)**

$$\text{E.M.F.} = \frac{\text{ENERGY TRANSFORMED FROM OTHER FORMS TO ELECTRICAL}}{\text{CHARGE}}$$

Definition of e.m.f with regards to energy transfer

- e.m.f is also the potential difference across the cell when no current is flowing
- e.m.f can be measured by connecting a high-resistance voltmeter around the terminals of the cell in an open circuit



e.m.f is measured using a voltmeter connected in parallel with the cell

10. D.C. Circuits

YOUR NOTES
⬇

EMF & Potential Difference

- The difference between **potential difference** and **e.m.f** is the type of energy transfer per unit charge

$$\text{P.D.} = \frac{\text{ENERGY TRANSFORMED FROM ELECTRICAL TO OTHER FORMS}}{\text{CHARGE}}$$

Definition of potential difference with regards to energy transfer

- When charge passes through a resistor, for example, its electrical energy is converted to heat in the resistor
    - The resistor therefore has a **potential difference** across it
- Potential difference describes the loss of energy from charges; ie. when **electrical energy** is **transferred** to other forms of energy in a component
- e.m.f. describes the **transfer of energy** from the **power supply** to **electrical charges** within the circuit

Exam Tip

Although voltage and potential difference are the same thing, make sure not to confuse them with e.m.f, which is slightly different!

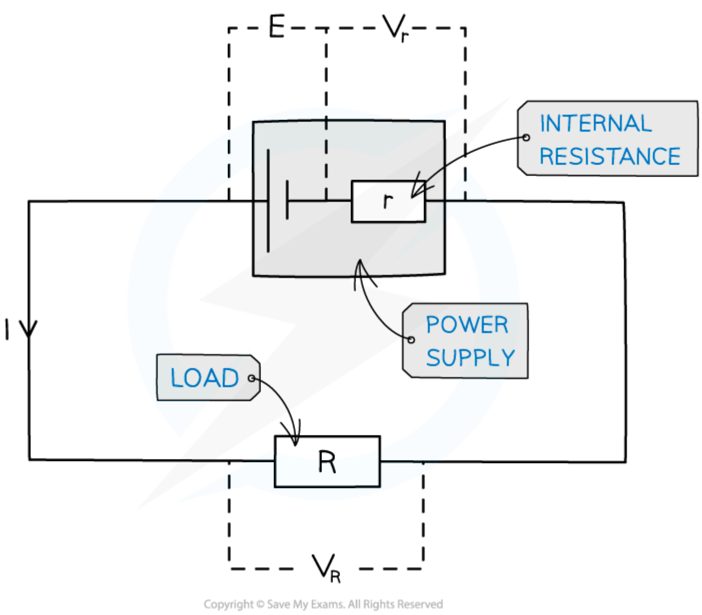
10. D.C. Circuits

YOUR NOTES
⬇

10.1.3 INTERNAL RESISTANCE

Internal Resistance

- All power supplies have some resistance between their terminals
  - This is called **internal resistance** (*r*)
- This internal resistance causes the charge circulating to dissipate some electrical energy from the power supply itself
  - This is why the cell becomes warm after a period of time
- The internal resistance therefore causes **loss of voltage** or energy loss in a power supply
- A cell can be thought of as a source of e.m.f with an internal resistance connected in series. This is shown in the circuit diagram below:

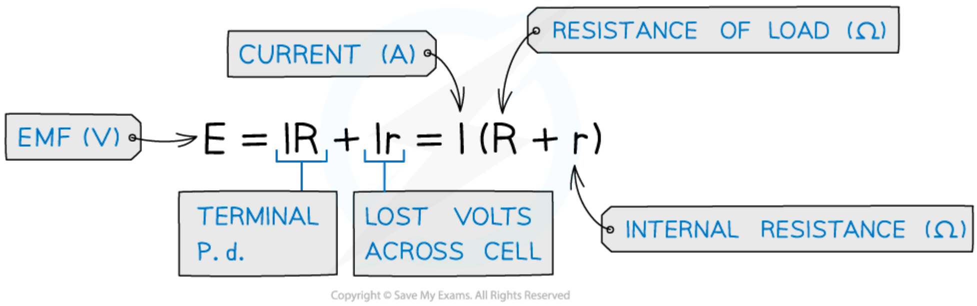


Circuit showing the e.m.f and internal resistance of a power supply

10. D.C. Circuits

YOUR NOTES

- V_R is the **terminal potential difference**
  - This is the voltage available in the circuit itself
  - **Terminal p.d = *I* × *R*** (Ohm's law)
- When a load resistor is connected, current flows through the cell and a potential difference develops across the internal resistance. This voltage is not available to the rest of the circuit so is called the 'lost volts'
- V_r is the **lost volts**
  - This is the voltage lost in the cell due to internal resistance, so, from conservation of energy:
  - Lost volts = e.m.f − terminal p.d
  - **Lost volts = *I* × *r*** (Ohm's law)
- The e.m.f is the sum of these potential differences, giving the equation below



e.m.f equation

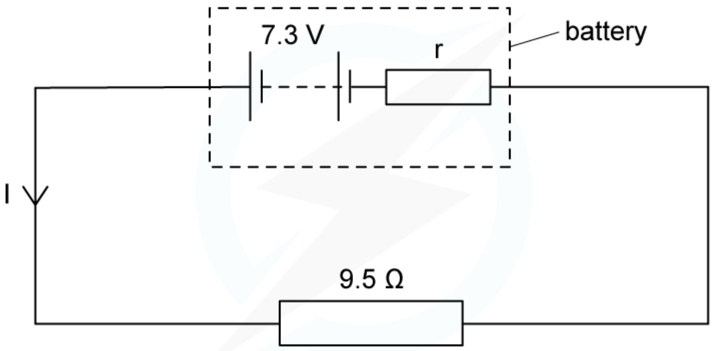
- e.m.f is therefore the total, or maximum, voltage available to the circuit

10. D.C. Circuits

YOUR NOTES
⬇

Worked example

A battery of e.m.f 7.3 V and internal resistance of r of 0.3 Ω is connected in series with a resistor of resistance 9.5 Ω.



Determine
a.) The current in the circuit
b.) Lost volts from the battery

a.)

STEP 1

USING THE e.m.f EQUATION TO DETERMINE THE CURRENT I

$$E = I(R + r)$$

STEP 2

REARRANGE FOR I

$$I = \frac{E}{(R + r)}$$

STEP 3

SUBSTITUTE IN THE VALUES

$$I = \frac{7.3}{(9.5 + 0.3)} = 0.745... = 0.7\text{ A} \quad (2 \text{ s.f.})$$

b.)

STEP 1

THE LOST VOLTS IS THE VOLTAGE LOST DUE TO INTERNAL RESISTANCE

$$\text{LOST VOLTS} = I \times r$$

STEP 2

SUBSTITUTE IN THE VALUES

$$\text{LOST VOLTS} = 0.7 \times 0.3 = 0.21 = 0.2 \quad (2 \text{ s.f.})$$

10. D.C. Circuits

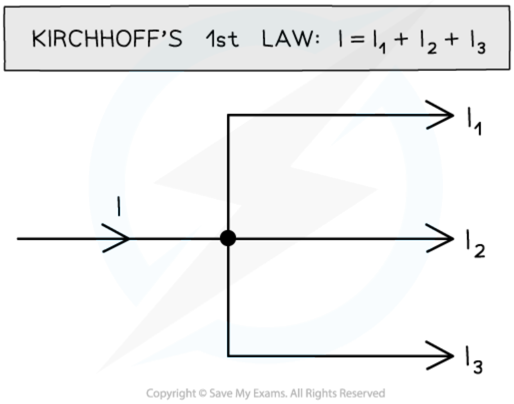
YOUR NOTES

10.1.4 KIRCHHOFF'S FIRST LAW

Kirchhoff's First Law

- Kirchhoff's first law states that:
  - **The sum of the currents entering a junction always equal the sum of the currents out of the junction**
- This is a consequence of conservation of **charge –** current shouldn't decrease or increase in a circuit when it splits
- In a circuit:
  - A **junction** is a point where at least three circuit paths meet
  - A **branch** is a path connecting two junctions
- If a circuit splits into two branches, then the current before the circuit splits should be equal to the current after it has split

  I1 = I2 + I3 , where I1 represents the current in the circuit before it branches, and I2 and I3 represent the current in the respective two branches

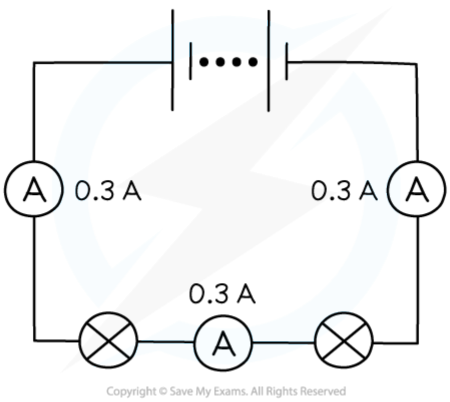


The current I into the junction is equal to the sum of the currents out of the junction

10. D.C. Circuits

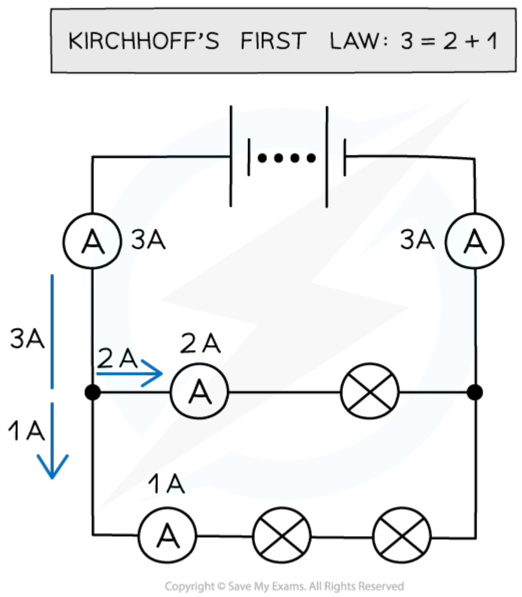
YOUR NOTES

- The charge is conserved on both sides of the junction
- In a **series** circuit, the current is the same at any point



The current is the same at each in a series circuit

- In a **parallel** circuit, the current divides at the junctions and each branch has a different value. Kirchhoff's first law applies at each junction



The current divides at each junction in a parallel circuit

10. D.C. Circuits

YOUR NOTES

Exam Tip

Junctions only appear in parallel circuits and as circuits become more complex, it can be confusing as to which currents are into the junction and which are out.
Drawing arrows on the diagram for the current flow (making sure it's from positive to negative) at each junction like in the worked example will help with this.

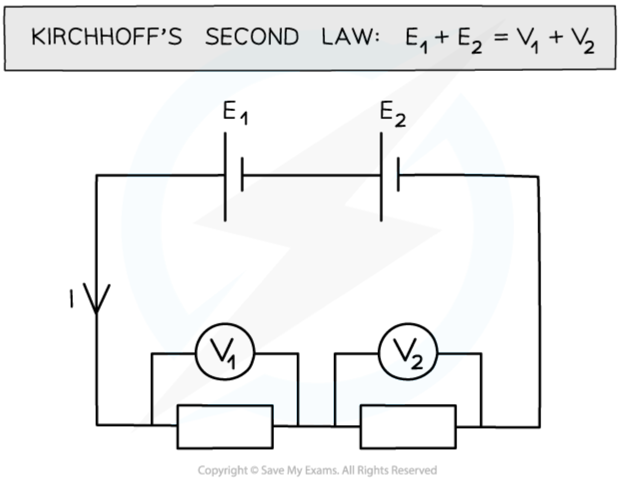
10. D.C. Circuits

YOUR NOTES
⬇

10.1.5 KIRCHHOFF'S SECOND LAW

Kirchhoff's Second Law

- Kirchhoff's second law states that:
  - **The sum of the e.m.f's in a closed circuit equals the sum of the potential differences**
- This is a consequence of conservation of **energy**
- Below is a circuit explaining Kirchhoff's second law with the sum of the voltages in the closed series circuit equal to the sum of the e.m.f's:

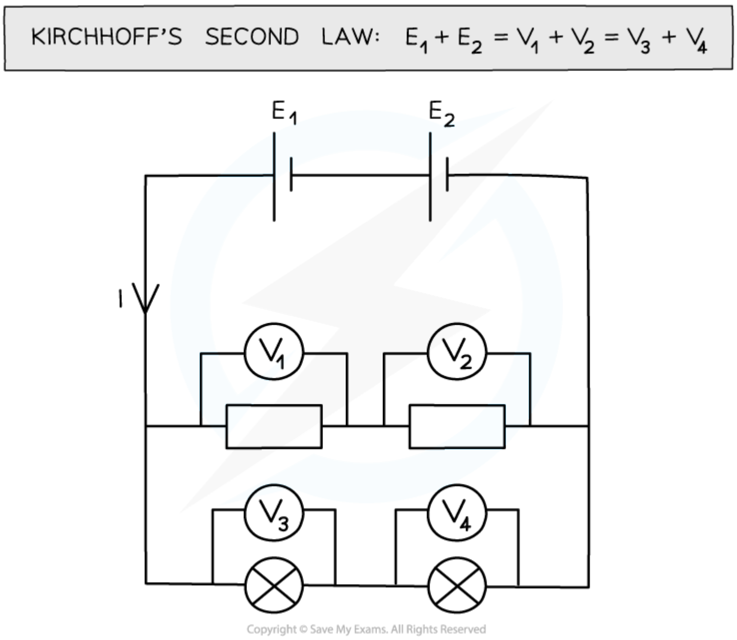


The sum of the voltages are equal to the sum of the e.m.f from the batteries

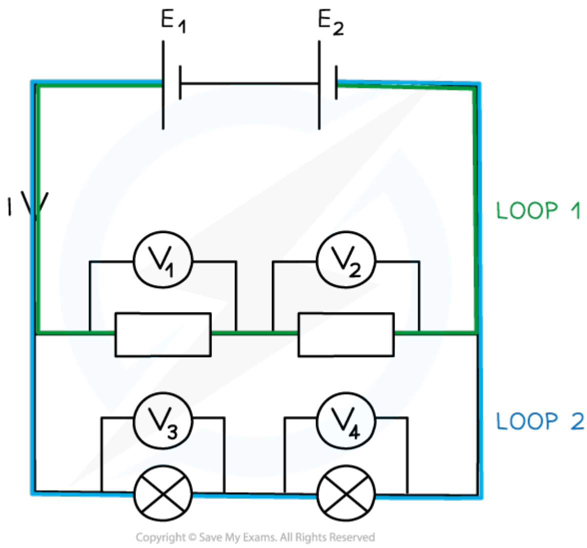
- In a **series** circuit, the voltage is split across all components depending on their resistance
  - The sum of the voltages is equal to the total e.m.f of the power supply
- In a **parallel** circuit, the voltage is the same across each closed loop
  - The sum of the voltages **in each closed circuit loop** is equal to the total e.m.f of the power supply:

10. D.C. Circuits

YOUR NOTES



- A closed circuit loop acts as its own independent series circuit and each one separates at a junction. A parallel circuit is made up of two or more of these loops



Each circuit loops acts as a separate, independent series circuit

- This makes parallel circuits incredibly useful for home wiring systems: a single power source supplies all lights and appliances with the same voltage
- If one light breaks, voltage and current can still flow through for the rest of the lights and appliances

10. D.C. Circuits

YOUR NOTES

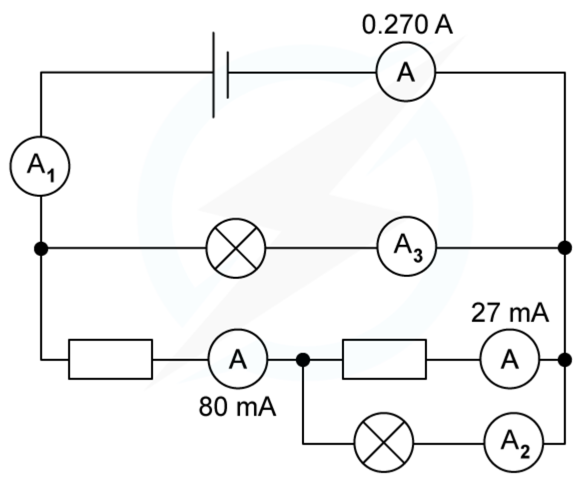
10.1.6 SOLVING PROBLEMS WITH KIRCHHOFF'S LAWS

Solving Problems with Kirchhoff's Laws

- Kirchhoff's laws can be used to solve simple circuit problems

Kirchhoff's First Law Worked Example

For the circuit below, state the readings of ammeters A_1, A_2 and A_3.



10. D.C. Circuits

YOUR NOTES

STEP 1

AMMETER A_1

0.270 A AND A_1 ARE CONNECTED IN SERIES. THE CURRENT ENTERING THE CELL EQUALS THE CURRENT LEAVING IT

A_1 IS ALSO 0.270 A

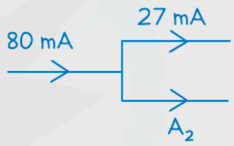
STEP 2

AMMETER A_2

FROM KIRCHHOFF'S FIRST LAW, THE TOTAL CURRENT INTO THE JUNCTION MUST EQUAL THE TOTAL CURRENT OUT OF IT.

$80 \text{ mA} = 27 \text{ mA} + A_2$

$A_2 = 80 - 27 = 53 \text{ mA}$



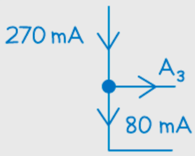
STEP 3

AMMETER A_3

KIRCHHOFF'S FIRST LAW AT A DIFFERENT JUNCTION

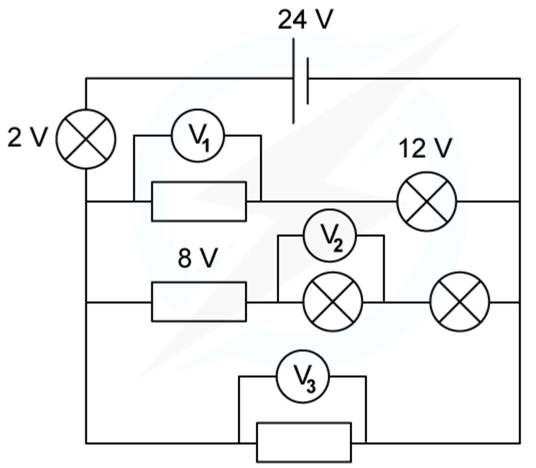
$270 \text{ mA} = A_3 + 80 \text{ mA}$

$A_3 = 270 - 80 = 190 \text{ mA}$



Kirchhoff's Second Law Worked Example

For the circuit below, state the readings of the voltmeters V_1, V_2 and V_3.
All the lamps and resistors have the same resistance.



10. D.C. Circuits

YOUR NOTES

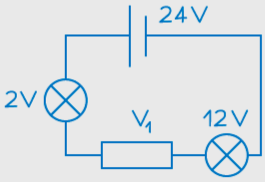
STEP 1

VOLTMETER V_1

KIRCHHOFF'S SECOND LAW STATES THAT THE SUM OF THE THREE COMPONENTS IS EQUAL TO THE e.m.f OF THE SUPPLY

$2V + V_1 + 12V = 24V$

$V_1 = 24 - 12 - 2 = 10V$



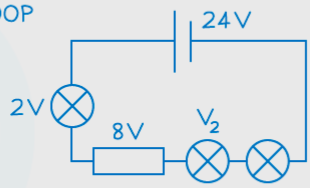
STEP 2

VOLTMETER V_2

KIRCHHOFF'S SECOND LAW IN THIS LOOP

2V + 8 V + V OF BOTH LAMPS = 24 V

V BOTH LAMPS = 24 − 8 − 2 = 14 V



SINCE BOTH LAMPS HAVE THE SAME RESISTANCE R AND THE CURRENT I THROUGH THEM BOTH IS THE SAME, THEY SHARE THE VOLTAGE LEFT EQUALLY

$V_2 = \frac{14}{2} = 7V$

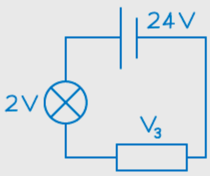
STEP 3

VOLTMETER V_3

KIRCHHOFF'S SECOND LAW IN THIS LOOP

$2V + V_3 = 24V$

$V_3 = 24 - 2 = 22V$



10. D.C. Circuits

YOUR NOTES

10.1.7 RESISTORS IN SERIES

Deriving the Equation for Resistors in Series

- In a series circuit, the combined resistance of two or more resistors is the sum of the individual resistances
- In a series circuit:
  - The current is the same through all resistors
  - The potential difference is split between all the resistors
- The equation for combined resistors in series is derived using Kirchhoff's laws:

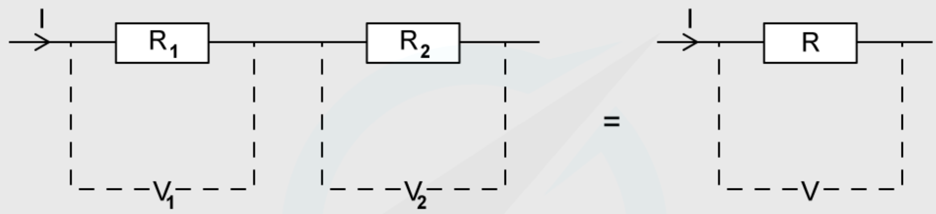
10. D.C. Circuits

YOUR NOTES

Derivation for resistors in series

CONSIDER TWO RESISTORS R_1 AND R_2 CONNECTED IN SERIES.
A SINGLE RESISTOR R IS EQUIVALENT TO THEM.

FROM KIRCHHOFF'S FIRST LAW: THE CURRENT I THROUGH EACH RESISTOR IS THE SAME SINCE THEY'RE CONNECTED IN SERIES (NO JUNCTIONS)



FROM KIRCHHOFF'S SECOND LAW: THE TOTAL p.d OF BOTH RESISTORS IN A CLOSED CIRCUIT LOOP MUST EQUAL THE SUM OF THE p.ds (THE p.d ACROSS THE SINGLE RESISTOR)

$$V = V_1 + V_2$$

FROM OHM'S LAW, POTENTIAL DIFFERENCE IS GIVEN BY THE PRODUCT OF CURRENT AND RESISTANCE

$$IR = IR_1 + IR_2$$

SINCE CURRENT I IS THE SAME FOR ALL RESISTORS, DIVIDING BY I

$$R = R_1 + R_2$$

THIS EQUATION CAN BE EXTENDED SO THE EQUIVALENT RESISTOR R OF SEVERAL RESISTORS CONNECTED IN SERIES IS GIVEN BY

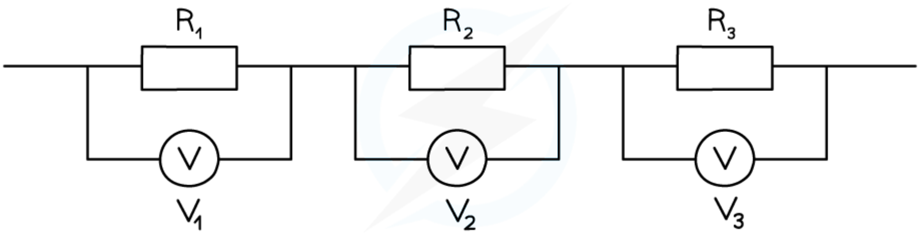
$$R = R_1 + R_2 + R_3 + R_4 \ldots$$

10. D.C. Circuits

YOUR NOTES

Resistors in Series

- When two or more components are connected in series:
    - **The combined resistance of the components is equal to the sum of individual resistances**



Resistors connected in series

COMBINED RESISTANCE IN SERIES $R = R_1 + R_2 + R_3 \ldots$

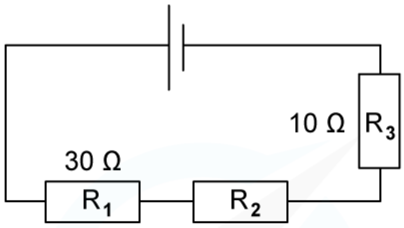
Combined resistance of two or more resistors in series equation

10. D.C. Circuits

YOUR NOTES
⬇

Worked example

The combined resistance R in the following series circuit is 60 Ω.
Which values is the resistance of R_2?



A. 100 Ω **B.** 30 Ω **C.** 20 Ω **D.** 40 Ω

ANSWER: C

STEP 1

EQUATION FOR COMBINED RESISTANCE IN SERIES

$$R = R_1 + R_2 + R_3$$

STEP 2

SUBSTITUTE IN VALUES FOR TOTAL RESISTANCE R AND THE OTHER RESISTORS

$$60\Omega = 30\Omega + R_2 + 10\Omega$$

STEP 3

REARRANGE FOR R_2

$$R_2 = 60\Omega - 30\Omega - 10\Omega$$

$$R_2 = 20\Omega$$

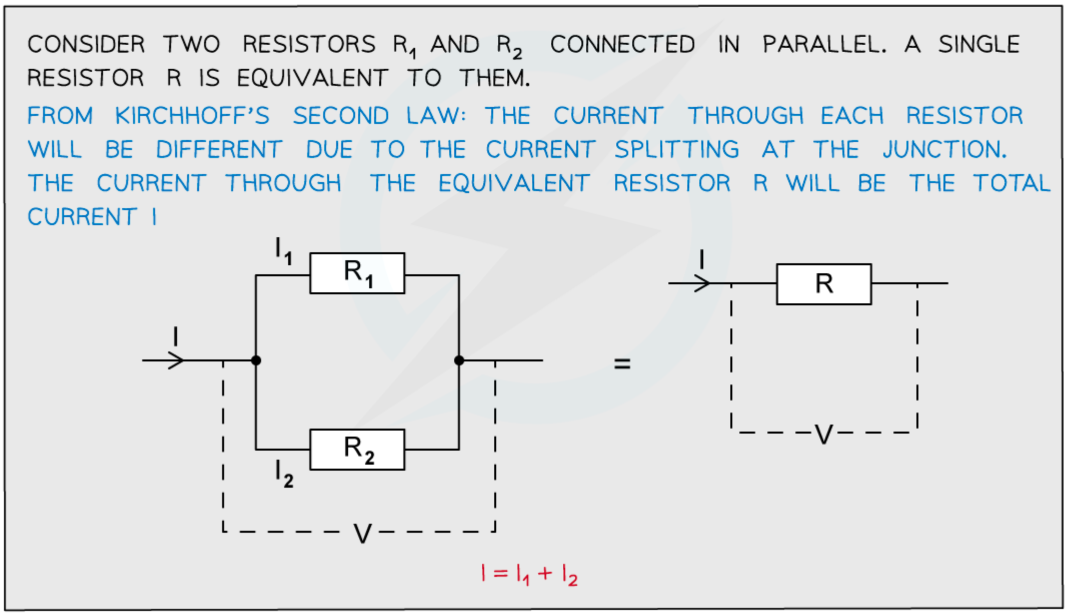
10. D.C. Circuits

10.1.8 RESISTORS IN PARALLEL

Deriving the Equation for Resistors in Parallel

- In a parallel circuit, the reciprocal of the combined resistance of two or more resistors is the sum of the reciprocal of the individual resistances
- In a parallel circuit:
  - The current is the split at the junction (and therefore between resistors)
  - The potential difference is the same through all resistors
- The equation for combined resistors in parallel is derived using Kirchhoff's laws:

Derivation for resistors in parallel



10. D.C. Circuits

YOUR NOTES

FROM KIRCHHOFF'S SECOND LAW, THE P.D ACROSS EACH RESISTOR IN DIFFERENT BRANCHES IS THE SAME.
THE RESISTOR R WILL HAVE THAT SAME p.d ACROSS IT TOO

$$V = V_1 = V_2$$

REARRANGING OHM'S LAW $V = IR$ FOR CURRENT I

$$I = \frac{V}{R}$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

SINCE POTENTIAL DIFFERENCE V IS THE SAME FOR ALL RESISTORS, DIVIDING BY V

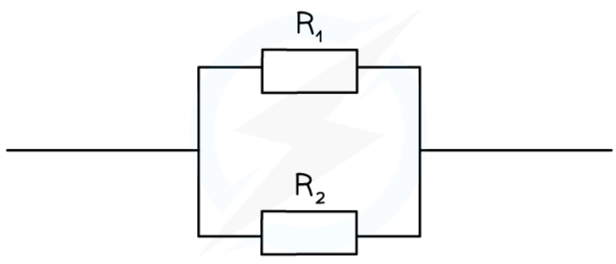
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

THIS EQUATION CAN BE EXTENDED SO THE EQUIVALENT RESISTOR R OF SEVERAL RESISTORS CONNECTED IN PARALLEL IS GIVEN BY

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots$$

Resistors in Parallel

- When two or component are connected in parallel:
  - **The reciprocal of the combined resistance is the sum of the reciprocals of the individual resistances**



Resistors connected in parallel

10. D.C. Circuits

YOUR NOTES

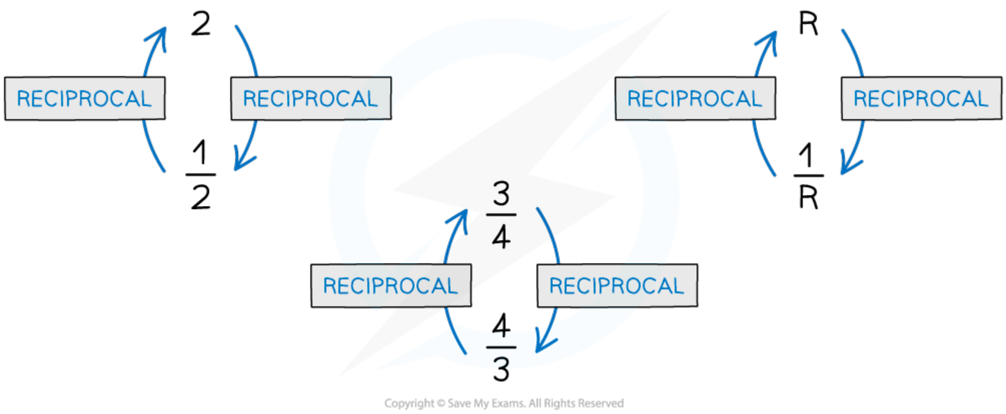
COMBINED RESISTANCE IN PARALLEL $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots$

Combined resistance of two or more resistors in parallel equation

- This means the combined resistance decreases and is less than the resistance of any of the individual components
- For example, If two resistors of equal resistance are connected in parallel, then the combined resistance will halve

Maths tip

- The reciprocal of a value is 1 / value
- For example, the reciprocal of a whole number such as 2 equals ½
  - The reciprocal of ½ is 2
- If the number is already a fraction, the numerator and denominator are 'flipped' round



The reciprocal of a number is 1 ÷ number

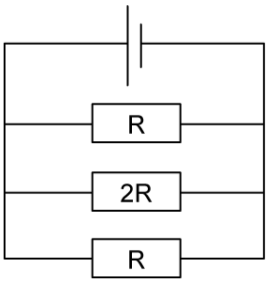
- In the case for the resistance R, this becomes 1/R. To get the value of R from 1/R, you must do 1 ÷ your answer
- You can also use the reciprocal button on your calculator (labelled either x^{-1} or 1/x, depending on your calculator

10. D.C. Circuits

YOUR NOTES

Worked example

The circuit below shows 4 resistors connected in parallel.



Which value gives the combined resistance of all the resistors in this circuit?

A. $\frac{5R}{2}$ **B.** $\frac{2}{5R}$ **C.** $\frac{5}{2R}$ **D.** $\frac{2R}{5}$

ANSWER: D

STEP 1

RESISTORS IN PARALLEL EQUATION

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots$$

STEP 2

SUBSTITUTE VALUES OF R_1, R_2 AND R_3 INTO THE EQUATION

R_T = TOTAL COMBINED RESISTANCE

$$\frac{1}{R_T} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{R}$$

$$\frac{1}{R_T} = \left(1 + \frac{1}{2} + 1\right)\frac{1}{R} = \frac{5}{2}\frac{1}{R}$$

STEP 3

CALCULATE R_T FROM THE RECIPROCAL OF THE SUM

R_T IS THE RECIPROCAL OF $\frac{1}{R_T}$

R_T = RECIPROCAL OF $\frac{5}{2}\frac{1}{R}$

"FLIP ROUND" THE FRACTION

$R_T = \frac{2}{5}R$

Exam Tip

The most common mistake is to forget to find $1/R_T$ and not R_T. Remember to do 1 / answer to get this value

10. D.C. Circuits

YOUR NOTES

Exam Question: Easy

A battery is marked 9.0 V.

What does this mean?

A each coulomb of charge from the battery supplies 9.0 J of electrical energy to the whole circuit

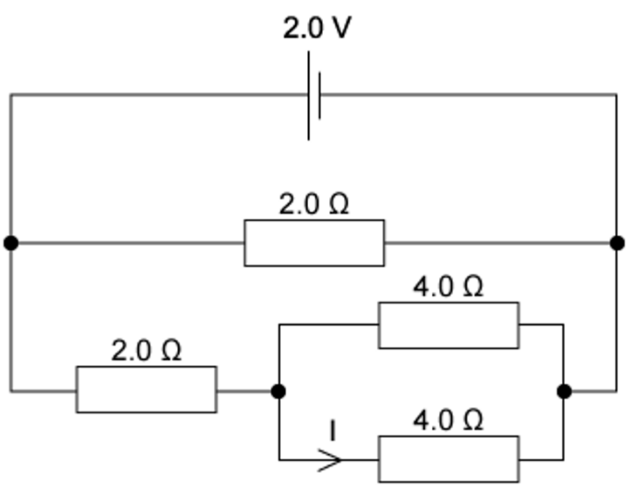
B the battery supplies 9.0 J to an external circuit for each coulomb of charge

C the potential difference across any component connected to the battery will be 9.0 V

D there will always be 9.0 V across the battery terminals

Exam Question: Medium

A cell of e.m.f. 2.0 V and negligible internal resistance is connected to a network of resistors as shown.



What is the current *I*?

A 0.25 A **B** 0.33 A **C** 0.50 A **D** 1.5 A

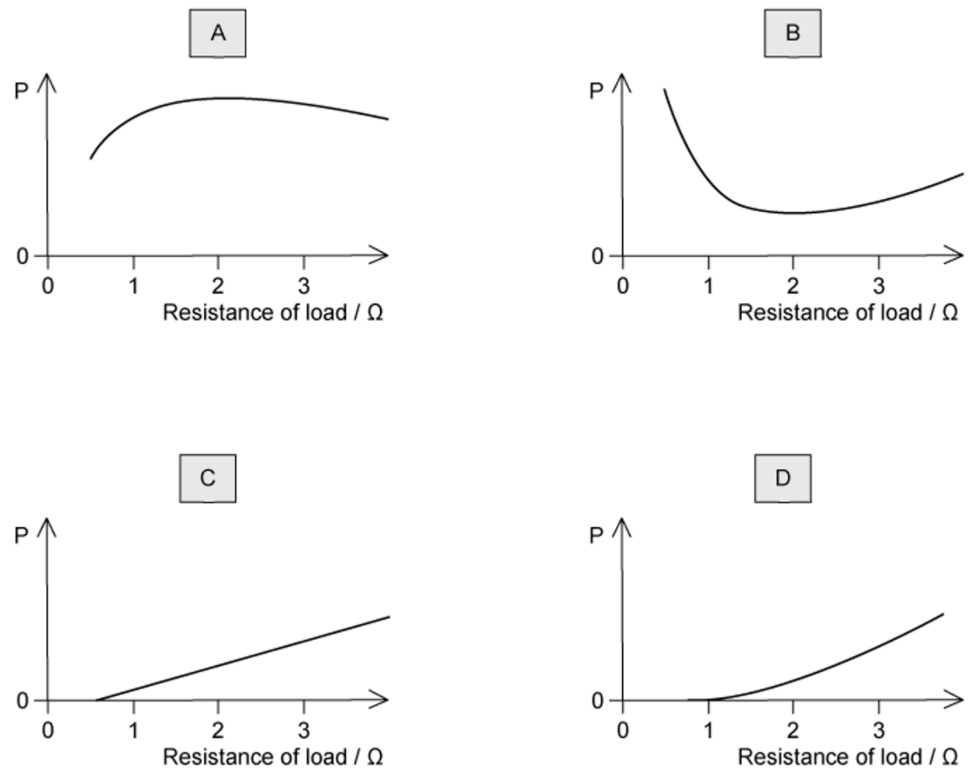
10. D.C. Circuits

YOUR NOTES

Exam Question: Hard

A power supply of electromotive force (e.m.f.) 12 V and internal resistance 2 Ω is connected in series with a load resistor. The value of the load resistor is varied from 0.5 Ω to 4 Ω.

Which graph shows how the power P dissipated in the load resistor varies with the resistance of the load resistor?



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10. D.C. Circuits

YOUR NOTES
⬇

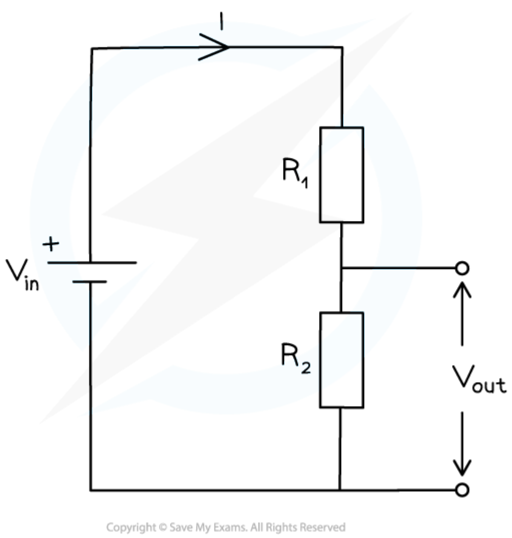
10.2 DC: POTENTIAL DIVIDERS

10.2.1 POTENTIAL DIVIDERS

Potential Divider Circuit

- When two resistors are connected in series, through Kirchoff's second law the potential difference across the power source is divided between them
- Potential dividers are circuits which produce an output voltage as a **fraction** of its input voltage. This splits the potential difference of a power source between two components
- They are used widely in volume controls and sensory circuits using LDRs and thermistors
- Potential divider circuits are based on the ratio of voltage between components. This is equal to the ratio of the resistances of the resistors in the diagram below, giving the following equation:

POTENTIAL DIVIDER EQUATION: $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$



Potential divider diagram and equation

10. D.C. Circuits

YOUR NOTES

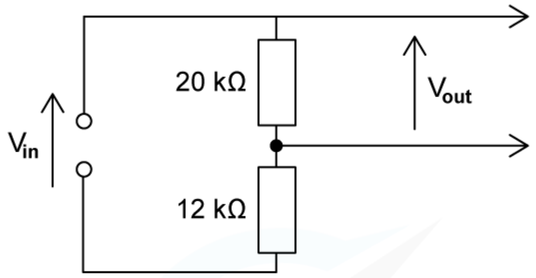
- The input voltage V_{in} is applied to the top and bottom of the series resistors
- The output voltage V_{out} is measured from the centre to the bottom of resistor R_2
- The potential difference V across each resistor depends upon its resistance R:
  - The resistor with the **largest resistance** will have a **greater potential difference** than the other one from V = IR
  - If the resistance of one of the resistors is increased, it will get a greater share of the potential difference, whilst the other resistor will get a smaller share
- In potential divider circuits, the p.d across a component is proportional to its resistance from V = IR

10. D.C. Circuits

YOUR NOTES
⬇

Worked example

The circuit is designed to light up a lamp when the input voltage exceed a preset value. It does this by comparing V_{out} with a fixed reference voltage of 5.3 V.



V_{out} is equal to 5.3

Calculate the input voltage V_{in}.

STEP 1

POTENTIAL DIVDER EQUATION

$$V_{out} = \left(\frac{R_1}{R_1 + R_2}\right) V_{in}$$

STEP 2

REARRANGE FOR INPUT VOLTAGE V_{in}

$$V_{in} = V_{out} \div \left(\frac{R_1}{R_1 + R_2}\right) = V_{out} \times \left(\frac{R_1 + R_2}{R_1}\right)$$

STEP 3

SUBSTITUTE IN VALUES

$R_1 = 20\ \text{k}\Omega$ $R_2 = 12\ \text{k}\Omega$ $V_{out} = 5.3\ \text{V}$

$$V_{in} = 5.3 \times \left(\frac{12 + 20}{20}\right) = 8.48\ \text{V}$$

$V_{in} = 8.5$ (2 s.f.)

Exam Tip

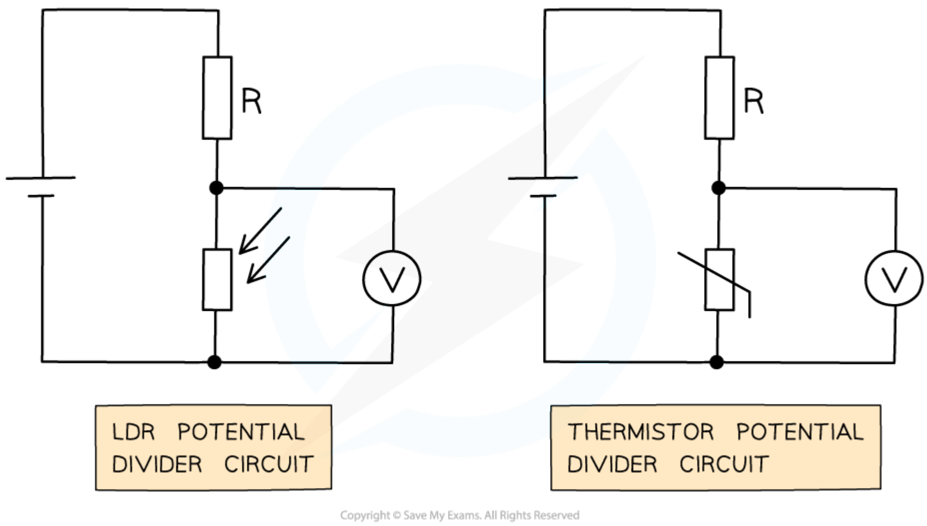
Always make sure the correct resistance is in the numerator of the potential divider equation. This will be the resistance of the component you want to found the output voltage of.

10. D.C. Circuits

YOUR NOTES

Variable Resistance Components

- Variable and sensory resistors are used in potential dividers to vary the output voltage
- This could cause an external component to switch on or off e.g. a heater switching off automatically when its surroundings are at room temperature
- Sensory resistors used are **Light Dependent Resistors (LDRs)** and **thermistors**



LDR and thermistor in a potential divider circuit with a fixed resistor R

- Recall that the resistance of an LDR varies with light intensity
  - The higher the light intensity, the lower the resistance and vice versa
- The resistance of a thermistor varies with temperature
  - The hotter the thermistor, the lower the resistance and vice versa

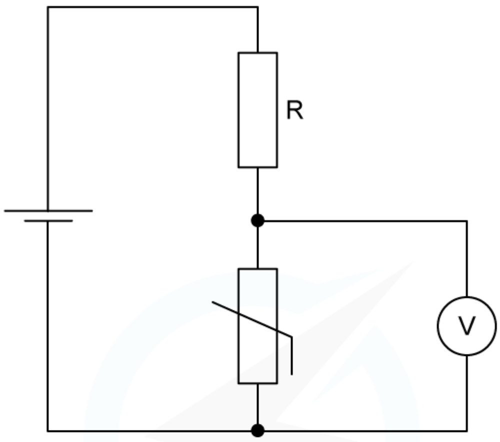
- From Ohm's law $\boldsymbol{V = IR}$, the potential difference V_{out} from a resistor in a potential divider circuit is **proportional** to its resistance
  - If an LDR or thermistor's resistance **decreases**, the potential difference through it also **decreases**
  - If an LDR or thermistor's resistance **increases**, the potential difference through it also **increases**

10. D.C. Circuits

YOUR NOTES

Worked example

A potential divider consists of a fixed resistor R and a thermistor.



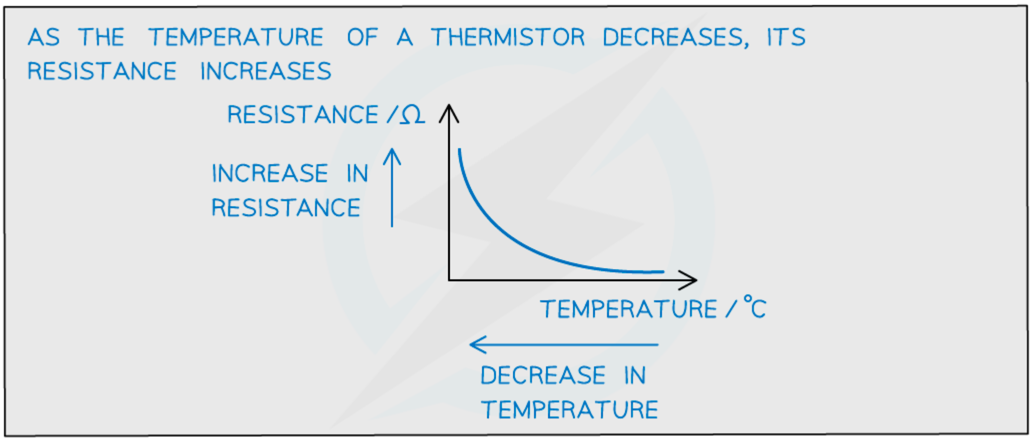
What happens to the p.d through resistor R and the thermistor when the temperature of the thermistor decreases?

	P.d of Thermistor / V	P.d of Resistor R / V
A	Increases	Increases
B	Decreases	Increases
C	Decreases	Decreases
D	Increases	Decreases

ANSWER: D

10. D.C. Circuits

YOUR NOTES



- Due to Ohm's Law (V = IR), both the resistor and thermistor are connected in series and have the same current I
- If resistance R increases, the potential difference across the thermistor also **increases**
- In series, the potential difference is shared equally amongst the components. Their sum equals the e.m.f of the supply (Kirchhoff's second law)
- If the potential difference across the thermistor **increases**, the potential difference across the resistance R must **decreases**, to keep the same overall total e.m.f
- This is row D

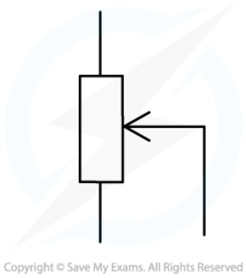
10. D.C. Circuits

YOUR NOTES

10.2.2 POTENTIAL DIVIDER COMPONENTS

The Potentiometer

- A potentiometer is similar to a variable resistor connected as a potential divider to give a continuously variable output voltage
- It can be used as a means of comparing potential differences in different parts of the circuit
- The circuit symbol is recognised by an arrow next to the resistor

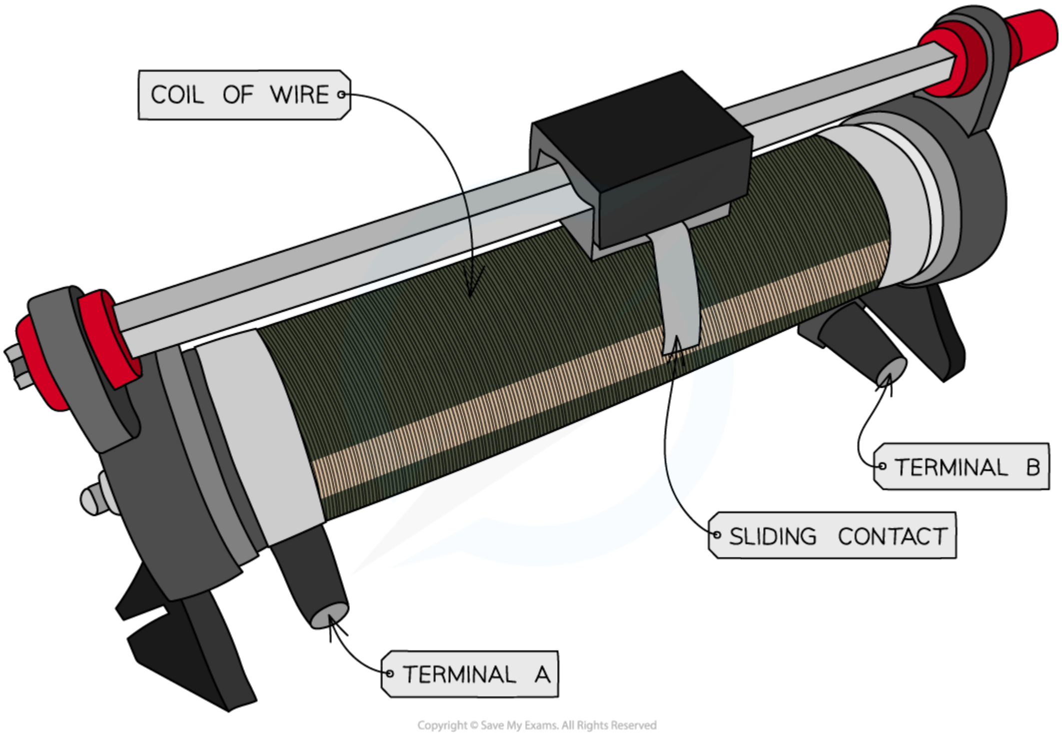


Potentiometer circuit diagram

10. D.C. Circuits

YOUR NOTES
⬇

- A potentiometer is a single component that (in its simplest form) consists of a coil of wire with a sliding contact, midway along it

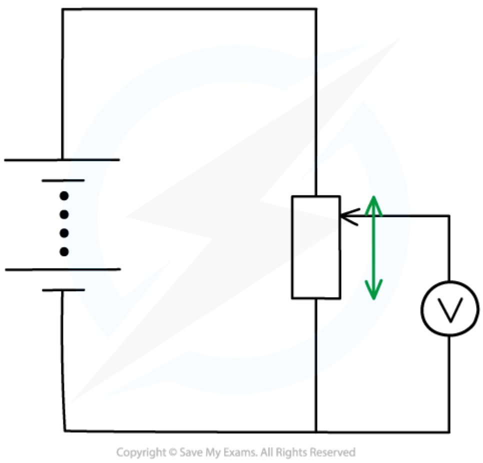


A potentiometer is a type of variable resistor

10. D.C. Circuits

YOUR NOTES

- It is recognised on a circuit diagram with a resistor fitted with a sliding contact
- The sliding contact has the effect of separating the potentiometer into two parts (an upper part and a lower part), both of which have different resistances



Moving the slider (the arrow in the diagram) changes the resistance (and hence potential difference) of the upper and lower parts of the potentiometer

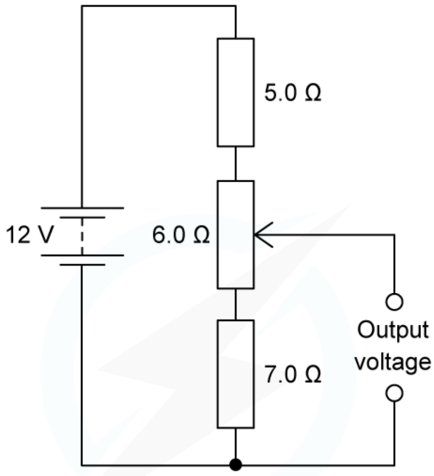
- If the slider in the above diagram is moved upwards, the resistance of the lower part will increase and so the potential difference across it will also increase
- Therefore, the variable resistor obtains a maximum or minimum value for the output voltage
- If the resistance is 3 Ω:
    - Maximum voltage is when the resistance is 3 Ω
    - Minimum voltage is when the resistance is 0 Ω

10. D.C. Circuits

YOUR NOTES
⬇

Worked example

A potential divivder circuit consists of fixed resistors of resistance 5.0 Ω and 7.0 Ω connected in series with a 6.0 Ω resistor fitted with a sliding contact. These are connected across a battery of e.m.f 12 V and zero internal resistance, as shown.



What are the maximum and the minimum output voltages of this potential divider circuit?

	Maximum voltage / V	Minimum voltage / V
A	8.7	4.7
B	6	0
C	12	6.5
D	12.5	4.7

ANSWER: A

10. D.C. Circuits

YOUR NOTES

THE POTENTIAL DIFFERENCE AT THE TOP OF THE CIRCUIT (+ TERMINAL) IS 12V. AT THE BOTTOM OF THE CIRCUIT (– TERMINAL) IS 0 V.

POTENTIAL DIVIDER EQUATION FOR 3 RESISTORS

$$V_{out} = V_{in} \times \left(\frac{R_1 + R_2}{R_1 + R_2 + R_3}\right)$$

MAXIMUM OUTPUT VOLTAGE

THE OUTPUT VOLTAGE IS MAXIMUM AT THE MAX VALUE OF THE VARIABLE RESISTOR = 6.0 Ω

$$\text{MAXIMUM OUTPUT VOLTAGE} = 12.0 \times \left(\frac{6.0 + 7.0}{5.0 + 6.0 + 7.0}\right) = 8.7\ \text{V}$$

MINIMUM OUTPUT VOLTAGE

THE OUTPUT VOLTAGE IS MINIMUM AT THE MIN VALUE OF THE VARIABLE RESISTOR = 0 Ω

$$\text{MINIMUM OUTPUT VOLTAGE} = 12.0 \times \left(\frac{7.0}{5.0 + 6.0 + 7.0}\right) = 4.7\ \text{V} = \text{row A}$$

The Galvanometer

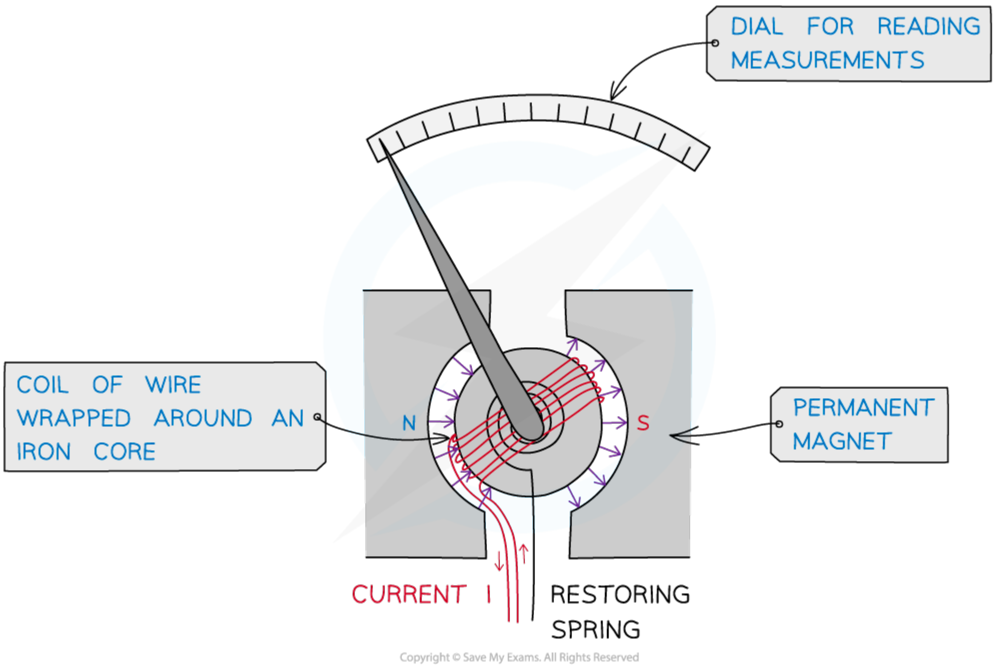
- A galvanometer is a type of sensitive ammeter used to detect electric current
- It is used in a potentiometer to measure e.m.f between two points in a circuit
- The circuit symbol is recognised by an arrow in a circle:

Galvanometer circuit symbol

10. D.C. Circuits

YOUR NOTES
⬇

- A galvanometer is made from a coil of wire wrapped around an iron core that rotates inside a magnetic field:

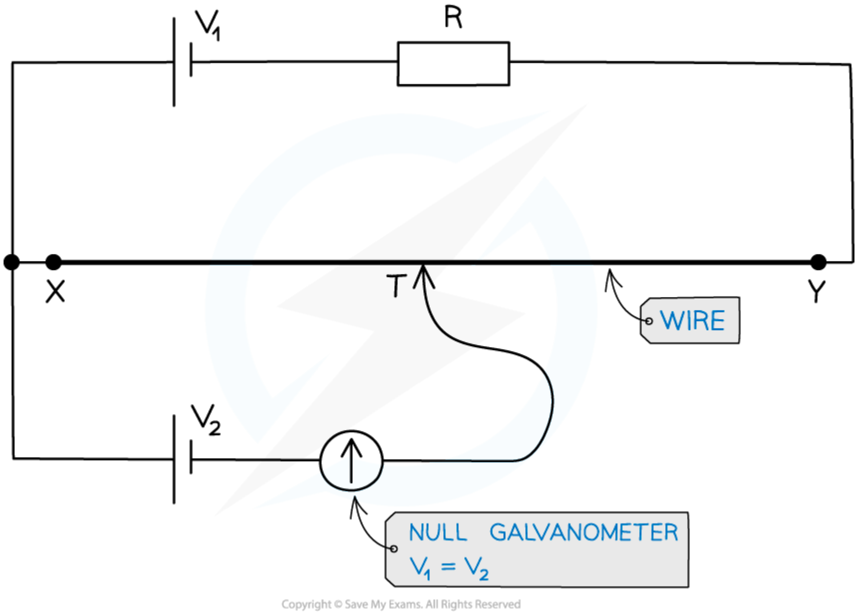


The galvanometer

- The arrow represents a needle which deflects depending on the amount of current passing through
  - When the arrow is facing directly upwards, there is no current
  - This is called **null** deflection
- Ohm's law tells us that the current through a conductor (wire) is directly proportional to the potential difference through it i.e. **no p.d means no current** flows through the galvanometer
- A galvanometer has p.d of zero when the potential on one side equals the potential on the other side
- This is at the **position at which it is connected on the wire (which varies with the sliding contact) gives a p.d equal to the EMF of the cell connected to the galvanometer**
- The cell should be connected such that its potential **opposes** the potential on the wire i.e. the positive terminal of the power supply faces the positive terminal of the cell:

10. D.C. Circuits

YOUR NOTES
⬇



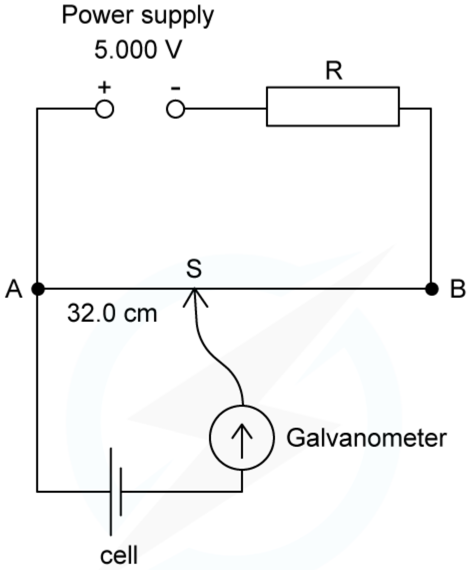
- When the sliding contact moves along the potentiometer wire, you add or remove resistance from/to the external circuit. This changes the potential drop across X and Y
- Location of the sliding point is adjusted until the galvanometer reads zero. This is until the potential difference equals E_2
- The direction of the two e.m.fs oppose each other and there is no current

10. D.C. Circuits

YOUR NOTES

Worked example

A power supply and a cell are compared using the potentiometer circuit shown.



The e.m.f produced by the cell is measured on the potentiometer. The potentiometer wire AB is 150.0 cm long and has a resistance of 2.4 Ω. The power supply has an e.m.f. of 5.000 V and the solar cell has an e.m.f. of 6.25 mV.

Which resistance R must be used so that the galvanometer reads zero when AS = 32.0 cm?

A. 735 Ω **B.** 451 Ω **C.** 207 Ω **D.** 401 Ω

ANSWER: D

10. D.C. Circuits

YOUR NOTES

STEP 1

IN ORDER FOR THE GALVANOMETER TO READ ZERO, THE e.m.f OF THE CELL MUST BE EQUAL TO THE p.d SUCH THAT THE p.d ACROSS THE GALVANOMETER IS ZERO.

STEP 2

POTENTIAL DIVIDER EQUATION

$$V_{out} = V_{in} \times \left(\frac{R_1}{R_1 + R_2}\right)$$

STEP 3

SUBSTITUTE KNOWN VALUES INTO EQUATION

$V_{in} = 5\text{ V}$ FROM POWER SUPPLY

R_1 = PROPORTION OF RESISTANCE OF THE WIRE AB $= \frac{32}{150} \times 2.4 = 0.21 \times 2.4\,\Omega = 0.504\,\Omega$

R_2 = RESISTANCE OF RESISTOR R = R

$$V_{out} = 5.000 \times \frac{0.504}{R + 2.4}$$

STEP 4

THE GALVANOMETER READS 0 WHEN THE e.m.f OF THE CELL IS 6.25 mV

EQUATING V_{out} TO e.m.f OF THE CELL

$$V_{out} = 5.000 \times \frac{0.504}{R + 2.4} = 6.25 \times 10^{-3}$$

STEP 5

REARRANGE FOR RESISTANCE R

$$R = \frac{5.000 \times 0.504}{6.25 \times 10^{-3}} - 2.4 = 400.8 = 401\,\Omega \text{ (3 s.f.)}$$

Exam Tip

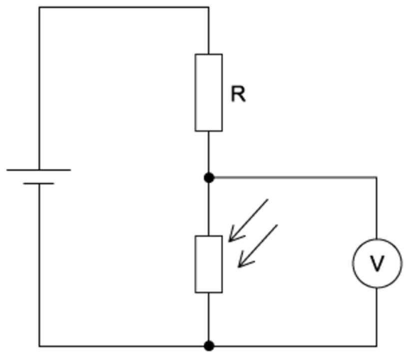
If you're unsure as to whether the p.d will increase as the contact slider is moved along the wire, remember **p.d is proportional to the length of the wire** (from Ohm's law and the resistivity equation). The longer the length of a wire, the higher the p.d

10. D.C. Circuits

YOUR NOTES

Exam Question: Easy

A potential divider consists of a fixed resistor R and a light-dependent resistor (LDR)



What happens to the voltmeter reading, and why does it happen, when the intensity of light on the LDR increases?

A the voltmeter reading decreases because the LDR resistance decreases

B the voltmeter reading decreases because the LDR resistance increases

C the voltmeter reading increases because the LDR resistance decreases

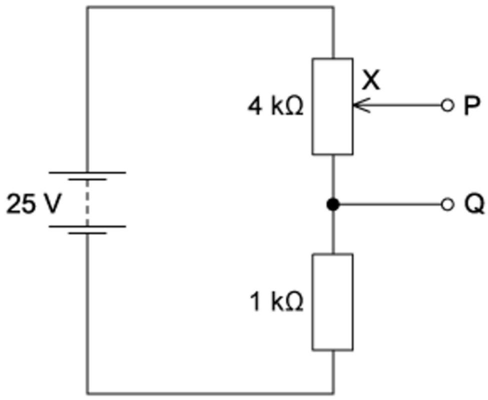
D the voltmeter reading increases because the LDR resistance increases

10. D.C. Circuits

YOUR NOTES
⬇

Exam Question: Medium

The diagram shows a potential divider circuit which, by adjustment of the contact X, can be used to provide a variable potential difference between the terminals P and Q.



What are the limits of this potential difference?

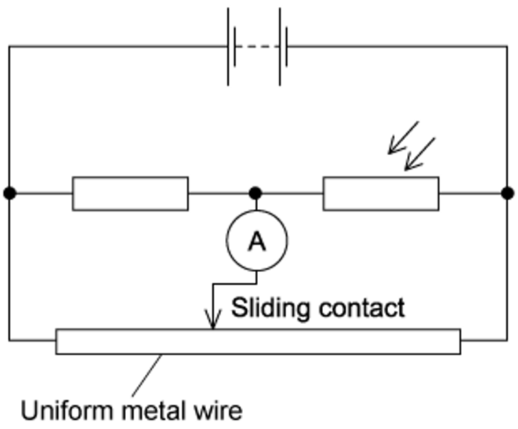
A 0 and 5 V **B** 0 and 20 V **C** 0 and 25 V **D** 5 V and 25 V

10. D.C. Circuits

YOUR NOTES

Exam Question: Hard

In the potentiometer circuit shown, the reading on the ammeter is zero.



The light-dependent resistor (LDR) is then covered up, and the ammeter gives a non-zero reading.

Which change could return the ammeter reading to zero?

A decrease the supply voltage

B increase the supply voltage

C move the sliding contact to the left

D move the sliding contact to the right

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11. Particle Physics

YOUR NOTES

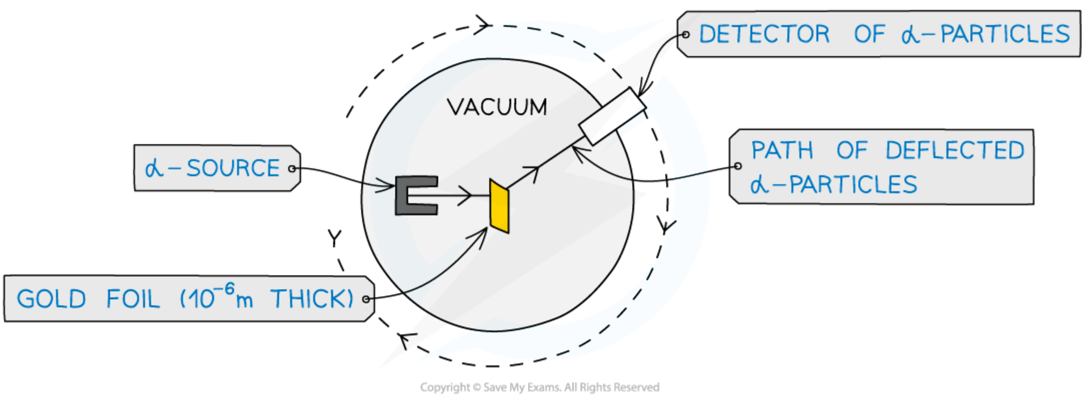
CONTENTS

11.1 ATOMS, NUCLEI & RADIATION

11.1.1 ATOMIC STRUCTURE

Rutherford Scattering

- Evidence for the structure of the atom was discovered by Ernest Rutherford in the beginning of the 20th century from the study of **α-particle scattering**
- The experimental setup consists of alpha particles fired at thin gold foil and a detector on the other side to detect how many particles deflected at different angles

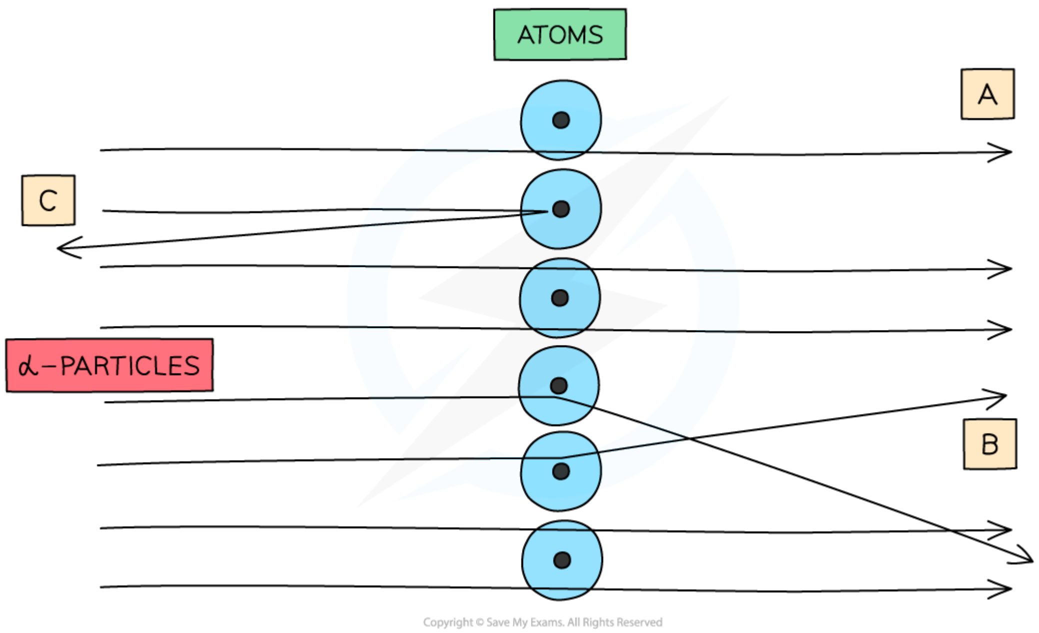


α-particle scattering experiment set up

11. Particle Physics

YOUR NOTES

- α-particles are the nucleus of a helium atom and are positively charged

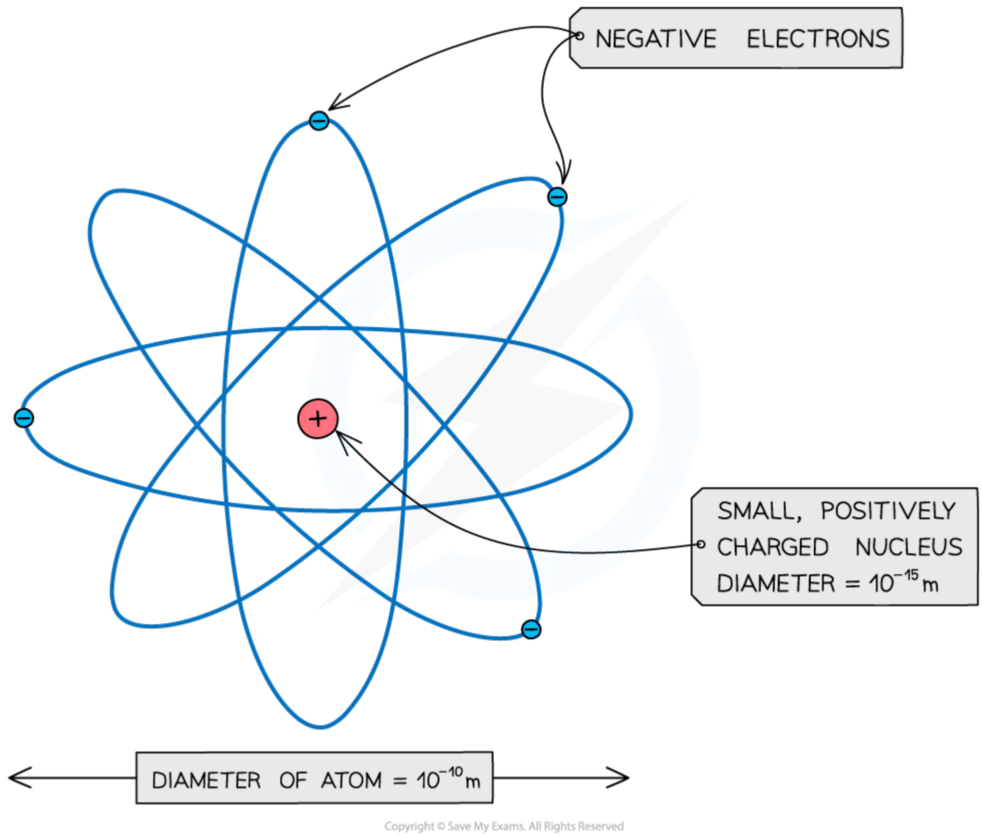


When α-particles are fired at thin gold foil, most of them go straight through but a small number bounce straight back

- From this experiment, Rutherford results were:
- **The majority of α-particles went straight through (A)**
  - This suggested the atom is mainly empty space
- **Some α-particles deflected through small angles of < 10°**
  - This suggested there is a positive nucleus at the centre (since two positive charges would repel)
- **Only a small number of α-particles deflected straight back at angles of > 90° (C)**
  - This suggested the nucleus is extremely small and this is where the mass and charge of the atom is concentrated
  - It was therefore concluded that atoms consist of small dense positively charged nuclei, surrounded by negatively charged electrons

11. Particle Physics

YOUR NOTES



An atom: a small positive nucleus, surrounded by negative electrons

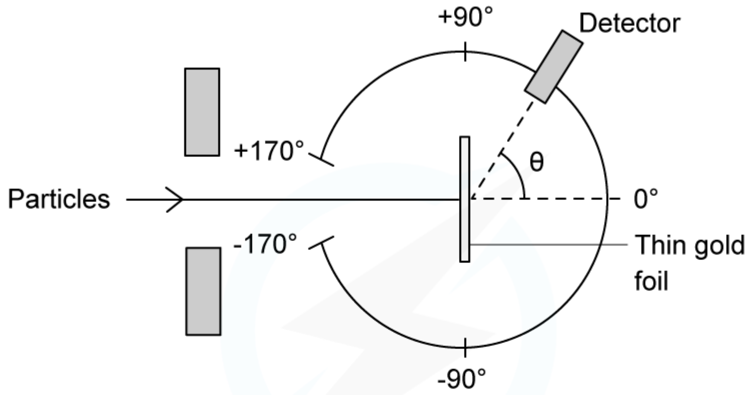
- (Note: The atom is around 100,000 times larger than the nucleus!)

11. Particle Physics

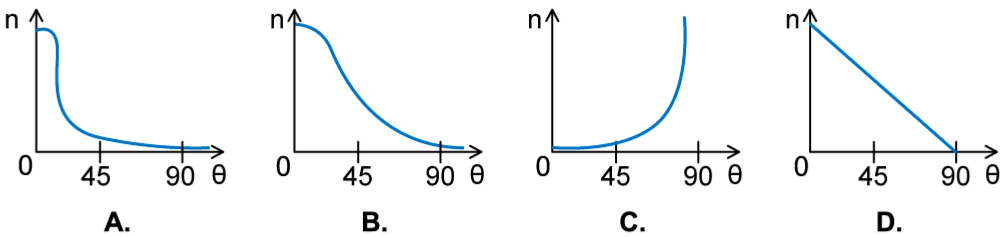
YOUR NOTES

Worked example

In an α-particle scattering experiment, a student set up the apparatus below to determine the number n of α-particles incident per unit time on a detector held at various angles θ.



Which of the following graphs best represents the variation of n with θ from 0 to 90°?



ANSWER: A

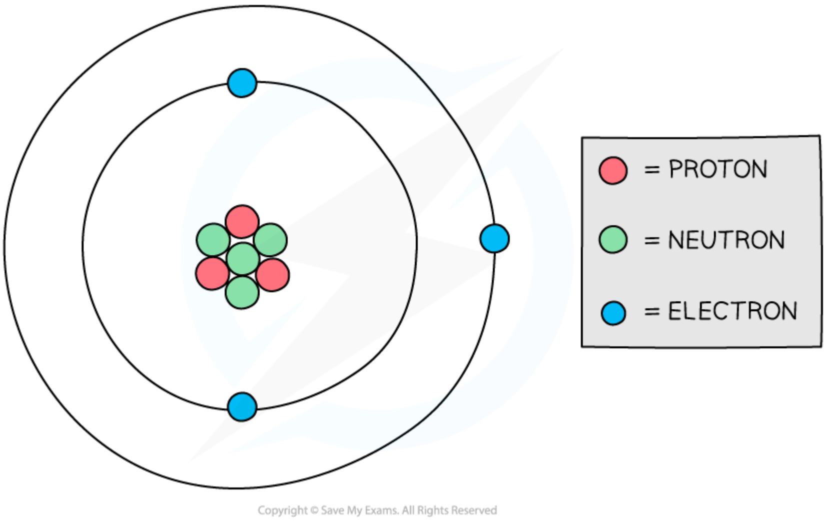
- The Rutherford scattering experience directed parallel beams of α-particles at gold foil
- The observations were:
  - Most of the α-particles went straight through the foil
  - The largest value of n will therefore be at small angles
  - Some of the α-particles were deflected through small angles
  - n drops quickly with increasing angle of deflection θ
- These observations fit with graph A

11. Particle Physics

YOUR NOTES

Atomic Structure

- The atoms of all elements are made up of three types of particles: protons, neutrons and electrons.



Protons and neutrons are found in the nucleus of an atom while electrons orbit the nucleus

- The properties of each particle are shown in the table below:

PARTICLE	RELATIVE CHARGE	RELATIVE MASS
PROTON	+1	1
NEUTRON	0	1
ELECTRON	−1	1/2000 (NEGLIGIBLE)

- A stable atom is **neutral** (it has no charge)
- Since protons and electrons have the same charge, but opposite signs, a stable atom has an equal number of both for the overall charge to remain neutral

11. Particle Physics

YOUR NOTES

Exam Tip

Remember not to mix up the 'atom' and the 'nucleus'. The 'atom' consists of the nucleus and electrons. The 'nucleus' just consists of the protons and neutrons in the middle of the atom, not the electrons.

Antimatter

- We live in a universe made up of **matter** particles (protons, neutrons, electrons etc.)
- All matter particles have antimatter counterparts
  - **Antimatter particles are identical to their matter counterpart but with the opposite charge**
- This means if a particle is positive, its antimatter particle is negative and vice versa
- Common matter-antimatter pairs are shown in the diagram below:

MATTER	CHARGE	ANTIMATTER	CHARGE
ELECTRON e^-	−1	POSITRON e^+	+1
PROTON p	+1	ANTI−PROTON $\bar{p}$	−1
NEUTRON n	0	ANTI−NEUTRON $\bar{n}$	0

- Apart from electrons, the corresponding antiparticle pair has the same name with the prefix **'anti-'** and a line above the corresponding matter particle symbol
- A neutral particle, such as a neutron or neutrino, is **its own antiparticle**

11. Particle Physics

YOUR NOTES

Atomic Mass Unit (u)

- The unified atomic mass unit ***(u)*** is roughly equal to the mass of one proton or neutron:
    - **1 u = 1.66 × 10^{-27} kg**
- It is sometimes abbreviated to **a.m.u**
- This value will be given on your data sheet in the exam
- The a.m.u is commonly used in nuclear physics to express the mass of subatomic particles. It is equal to 1/12 of the mass of the carbon-12 atom

Table of common particles with mass in a.m.u

Particle	Mass / u
Proton	1
Neutron	1
Electron	0.0005
Alpha (α)	4

- The mass of an atom in a.m.u is roughly equal to the sum of its protons and neutrons (nucleon number)
    - For example, the mass of Uranium-235 is roughly 235u

11. Particle Physics

YOUR NOTES

Worked example

Estimate the mass of the nucleus of element Copernicium-285 in Kg.
Give your answer to 2 decimal places.

STEP 1

THE MASS OF AN ATOM IN A.M.U IS ROUGHLY ITS NUCLEON NUMBER

STEP 2

THE MASS OF COPERNICIUM−285 IN A.M.U:
285u

STEP 3

CONVERT TO kg:
$1\,u = 1.66 \times 10^{-27}\,kg$
$285\,u = 285 \times 1.66 \times 10^{-27}\,kg = 4.73 \times 10^{-25}\,kg$ (2 d.p)

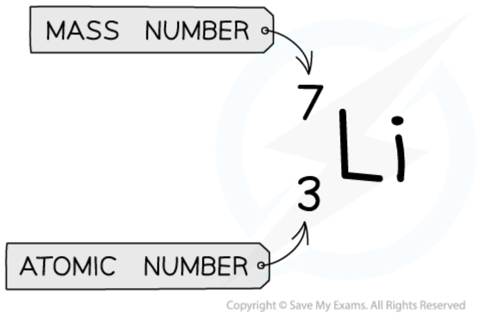
11. Particle Physics

YOUR NOTES

11.1.2 NUCLEON & PROTON NUMBER

Nucleon & Proton Number

- The atomic symbol of an element is used to describe the constituents of the nuclei
- An example of this notation for Lithium is:



Atomic symbol for Lithium

- When given an atomic symbol, you can figure out the number of protons, neutrons and electrons in the atom:
  - **Protons**: The atomic number
  - **Electrons**: Atoms are neutrals, so the number of negative electrons is equal to the number of positive protons. Therefore, this is also the atomic number
  - **Neutrons**: Subtract the proton number from the mass number
- For the lithium atom, these numbers would be:
  - Protons: 3
  - Electrons: 3
  - Neutrons: 7 − 3 = 4
- The term **nucleon** is the used to mean a particle in the nucleus – i.e. a proton or neutron
- The term **nuclide** is used to refer to a nucleus with a specific combination of protons and neutrons

11. Particle Physics

YOUR NOTES

Worked example

The atom $^{192}_{77}Ir$ is a neutral atom.

How many protons, neutrons and electrons are in this atom?

	Protons	Neutrons	Electrons
A	77	192	77
B	115	77	77
C	192	77	192
D	77	115	77

ANSWER: D

STEP 1
THE SYMBOL $^{192}_{77}Ir$ HAS A MASS/NUCLEON NUMBER OF 192 AND ATOMIC/PROTON NUMBER OF 77 (FROM AZX NOTATION)

STEP 2
NUMBER OF PROTONS
THE NUMBER OF PROTONS IS FOUND FROM THE PROTON NUMBER = 77

STEP 3
NUMBER OF NEUTRONS
THE MASS/NUCLEON NUMBER IS THE SUM OF THE PROTONS AND NEUTRONS
THE NUMBER OF NEUTRONS = MASS NUMBER – PROTON NUMBER
= 192 – 77 = 115

STEP 4
NUMBER OF ELECTRONS
THE ATOM IS NEUTRAL SO THE NUMBER OF ELECTRONS IS ALSO 77

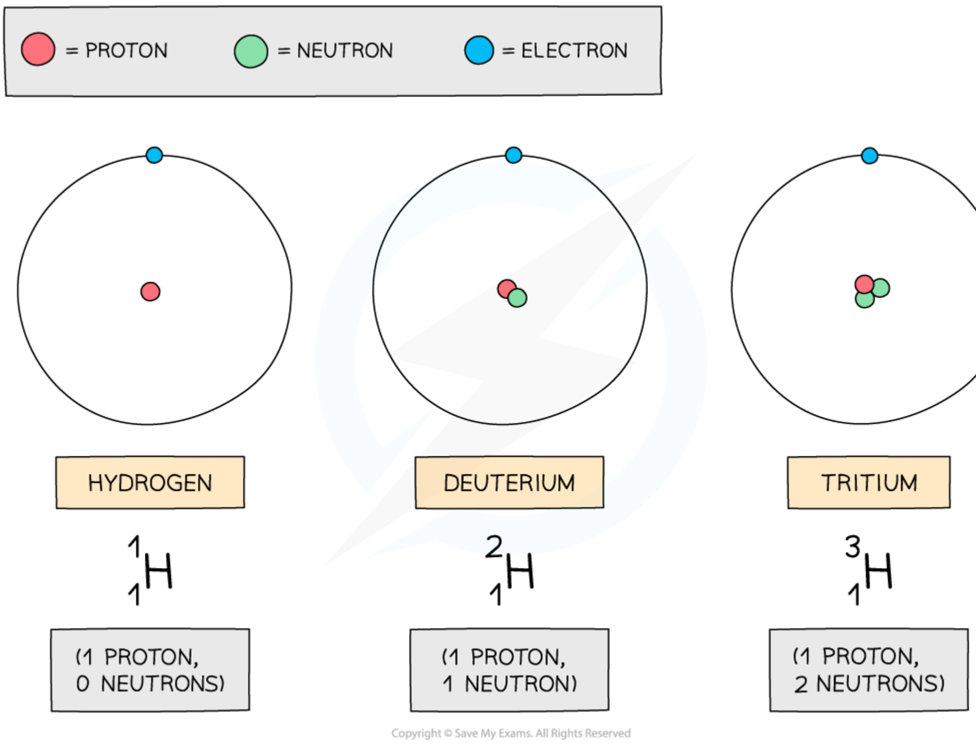
STEP 5
THESE ARE THE NUMBERS IN ROW D

11. Particle Physics

YOUR NOTES

Isotopes

- Although all atoms of the same element always have the same number of protons (and hence electrons), the number of neutrons can vary
- **An isotope is an atom (of the same element) that has an equal number of protons but different number of neutrons**
- The isotopes of hydrogen are deuterium and tritium:



The three atoms shown above are all forms of hydrogen, but they each have different numbers of neutrons

- Remember, the neutron number of an atom is found by subtracting the proton number from the nucleon number
- Since nucleon number includes the number of neutrons, an isotope of an element will also have a **different nucleon/mass number**
- Since isotopes have an imbalance of neutrons and protons, they are **unstable**. This means they constantly decay and emit radiation to achieve a more stable form
- This can happen from anywhere between a few nanoseconds to 100,000 years

11. Particle Physics

YOUR NOTES
⬇

Worked example

One of the rows in the table shows a pair of nuclei that are isotopes of one another.

Which row is it?

	Nucleon number	Number of neutrons
A	39 35	19 22
B	37 35	20 18
C	37 35	18 20
D	35 35	20 18

ANSWER: B

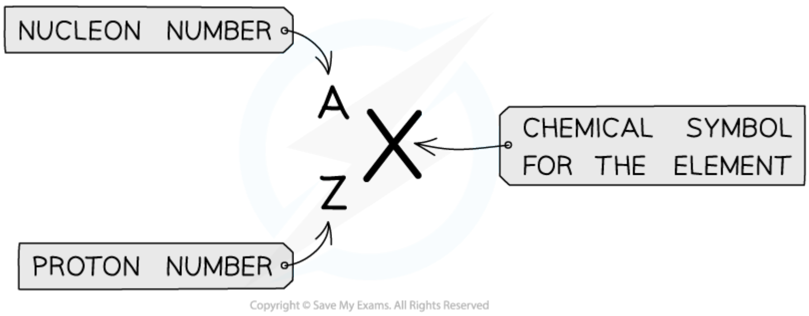
- Isotopes are nuclei with the same number of protons but different number of neutrons
    - The nucleon number is the sum of the protons and neutrons
    - Therefore, an isotope has a different nucleon number too
- The first nucleus:
    - Nucleon number: 37
    - Neutrons: 20
    - Protons = 37 − 20 = 17
- The second nucleus:
    - Nucleon number: 35
    - Neutrons: 18
    - Protons = 35 − 18 = 17
- Therefore, they have the same number of protons but different numbers of neutrons and are isotopes of each other
- This refers to row B

11. Particle Physics

YOUR NOTES

AZX Notation

- Atomic symbols are written in a specific notation called **AZX notation**



Atomic symbols, like the one above, describe the constituents of nuclei

- The top number A represents the **nucleon** number or the **mass** number
  - **Nucleon number (A)** = total number of **protons and neutrons** in the nucleus
- The lower number Z represents the **proton** or **atomic** number
  - **Proton number (Z)** = total number of **protons** in the nucleus

- Note: In Chemistry the nucleon number is referred to as the mass number and the proton number as the atomic number. The periodic table is ordered by atomic number

Conservation of Nucleon Number & Charge

- Nuclear processes such as fission and fusion are represented using nuclear equations (similar to chemical reactions in chemistry)
- The number of protons and neutrons in atom is known as its **constituents**
- For example:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{90}_{38}Sr + ^{144}_{54}Xe + 2\,^{1}_{0}n$$

Nuclear fission equation

11. Particle Physics

YOUR NOTES
⬇

- The above equation represents a fission reaction in which a Uranium nucleus is hit with a neutron and splits into two smaller nuclei – a Strontium nucleus and Xenon nucleus, releasing two neutrons in the process
- In nuclear equations, the nucleon number and charge are always **conserved**
- This means that the sum of the nucleons and charge on the left hand side must equal the sum of the number of nucleons and charge on the right hand side
- In the above equation, the sum of the nucleon (top) numbers on both sides are equal

$$235 + 1 = 236 = 90 + 144 + 2 \times 1$$

- The same is true for the proton (bottom) numbers

$$92 + 0 = 92 = 38 + 54 + 2 \times 0$$

- By balancing equations in this way, you can determine the nucleon, proton number or the number of missing elements

- For example:

$$\underbrace{{}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n}}_{235 + 1 = 236} \longrightarrow \underbrace{{}^{96}_{37}\mathrm{Rb} + {}^{137}_{55}\mathrm{Cs} + N\,{}^{1}_{0}\mathrm{n}}_{96 + 137 + (N \times 1)}$$

TOTAL NUCLEON NUMBER:

$$96 + 137 + (N \times 1) = 236$$

REARRANGING FOR N

$$N = \frac{236 - 96 - 137}{1}$$

$$N = 3$$

Balancing the number of nucleons shows that 3 neutrons must be released in the reaction

11. Particle Physics

YOUR NOTES
⬇

Worked example

When a californium atom reacts with an unknown element X, the following reaction occurs.

$$^{252}_{98}\mathrm{Cf} + {}^{10}_{Y}\mathrm{X} \longrightarrow 3\,{}^{1}_{0}\mathrm{n} + {}^{Z}_{103}\mathrm{Lr}$$

Determine the values of Y and Z.

STEP 1
DETERMINE THE VALUE OF Y:
Y IS THE PROTON NUMBER OF ELEMENT X

STEP 2
EQUATING PROTON NUMBERS ON BOTH SIDES
$98 + Y = (3 \times 0) + 103$

STEP 3
REARRANGE FOR Y
$Y = 0 + 103 - 98$
$Y = 5$

STEP 4
DETERMINE THE VALUE OF Z:
Z IS THE NUCLEON NUMBER OF ELEMENT Lr

STEP 5
EQUATING NUCLEON NUMBERS ON BOTH SIDES
$252 + 10 = (3 \times 1) + Z$

STEP 6
REARRANGE FOR Z
$252 + 10 - 3 = Z$
$Z = 259$

11. Particle Physics

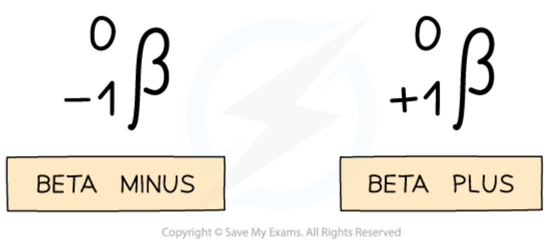
YOUR NOTES
⬇

11.1.3 ALPHA, BETA & GAMMA PARTICLES

Alpha, Beta & Gamma Particles

- Some elements have nuclei that are unstable
  - This tends to be when the number of nucleons does not balance
- In order to become more stable, they emit particles and/or electromagnetic radiation
  - These nuclei are said to be **radioactive**
- There are three different types of radioactive emission:
- **Alpha (α) particles** are high energy particles made up of **2 protons and 2 neutrons** (the same as a helium nucleus)
- They are usually emitted from nuclei that are too large

- **Beta (β^-) particles are high energy electrons** emitted from the nucleus
- **Beta (β^+) particles are high energy positrons** (antimatter of electrons) also emitted from the nucleus
  - β^- particles are emitted by nuclei that have too many **neutrons**
  - β^+ particles are emitted by nuclei that have too many **protons**

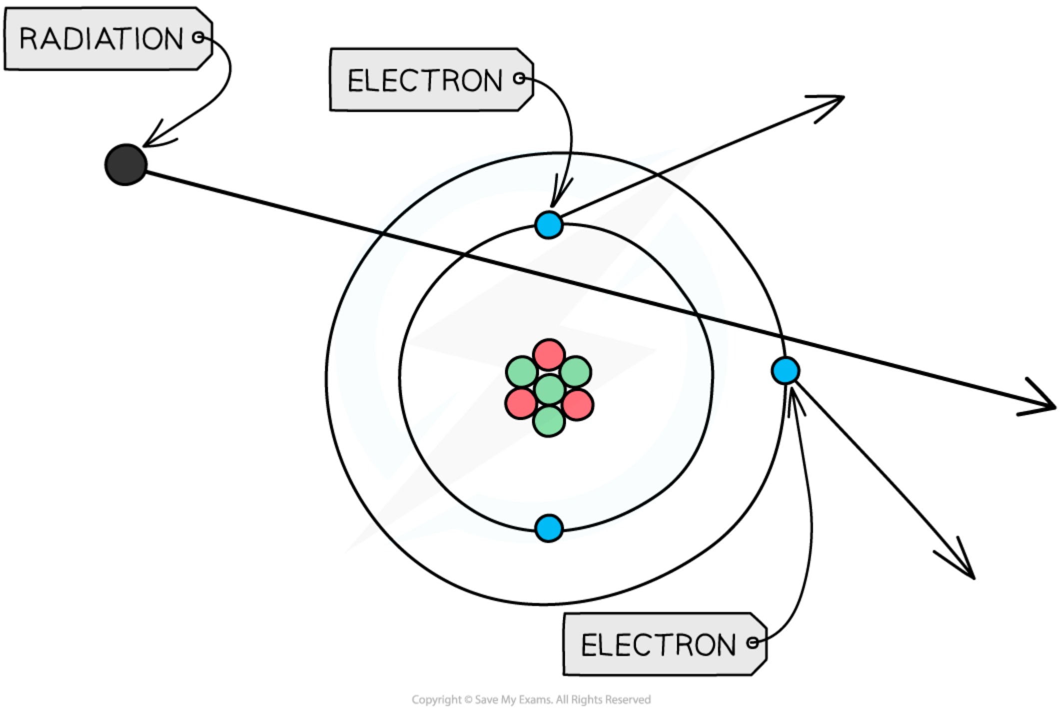


11. Particle Physics

YOUR NOTES

- **Gamma (γ) rays are high energy electromagnetic waves**
- They are emitted by nuclei that need to lose some energy

- If these particles hit other atoms, they can knock out electrons, **ionising the atom**
- This can cause chemical changes in materials and can damage or kill living cells



When radiation passes close to atoms, it can knock out electrons, ionising the atom

11. Particle Physics

YOUR NOTES

- The properties of the different types of radiation are summarised in the table below

Particle	Composition	Mass / u	Charge / e	Speed / c
Alpha (α)	2 protons + 2 neutrons	4	+2	0.05
Beta minus (β^-)	Electron (e^-)	0.0005	−1	> 0.99
Beta plus (β^+)	Positron (e^+)	0.0005	+1	> 0.99
Gamma (γ)	Electromagnetic wave	0	0	1

- *u* is the atomic mass unit (see "Atomic Mass Unit (u)")
- *e* is the charge of the electron: 1.60×10^{-19} C
- *c* is the speed of light: 3×10^{8} m s^{-1}

11. Particle Physics

YOUR NOTES
⬇

Worked example

?

Three successive radioactive decays are shown in the diagram below; each one results in a particle being emitted.

The first decay results in the emission of β-particle. The second decay results in the emission of an α-particle. The third decay results in the emission of another β-particle.

$$^{237}_{92}W \xrightarrow{\beta-} X \xrightarrow{\alpha} Y \xrightarrow{\beta+} Z$$

Nuclides W and Z are compared.

Which nuclide of Z is formed at the end of this decay?

A. $^{237}_{90}Z$ **B.** $^{233}_{92}Z$ **C.** $^{237}_{89}Z$ **D.** $^{233}_{90}Z$

ANSWER: D

STEP 1

BETA MINUS DECAY

A $\beta-$ IS AN ELECTRON

THE NUCLEON NUMBER STAYS THE SAME

$$^{237}_{92}W \longrightarrow {}^{237}_{93}X + {}^{0}_{-1}\beta$$

THE PROTON NUMBER INCREASES BY 1

STEP 2

ALPHA DECAY

A α PARTICLE IS A HELIUM NUCLEUS

THE NUCLEON NUMBER REDUCES BY 4

$$^{237}_{93}X \longrightarrow {}^{233}_{91}Y + {}^{4}_{2}\alpha$$

THE PROTON NUMBER REDUCES BY 2

STEP 3

BETA PLUS DECAY

A $\beta+$ PARTICLE IS A POSITRON

THE NUCLEON NUMBER STAYS THE SAME

$$^{233}_{91}Y \longrightarrow {}^{233}_{90}Z + {}^{0}_{+1}\beta$$

THE PROTON NUMBER REDUCES BY 1

STEP 4

THE FINAL NUCLEON WILL BE $^{233}_{90}Z$

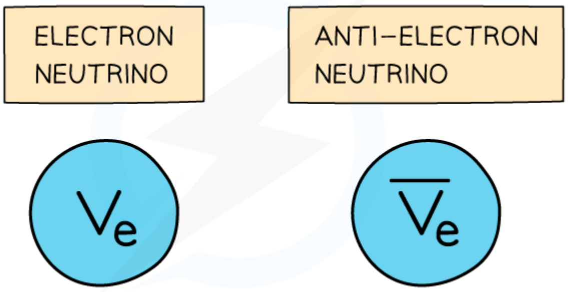
11. Particle Physics

YOUR NOTES

11.1.4 DECAY EQUATIONS

Neutrino Emission

- An electron neutrino is a type of subatomic particle with no charge and negligible mass which is also emitted from the nucleus
- The anti-neutrino is the antiparticle of a neutrino
  - Electron anti-neutrinos are produced during β– decay
  - Electron neutrinos are produced during β+ decay



Exam Tip

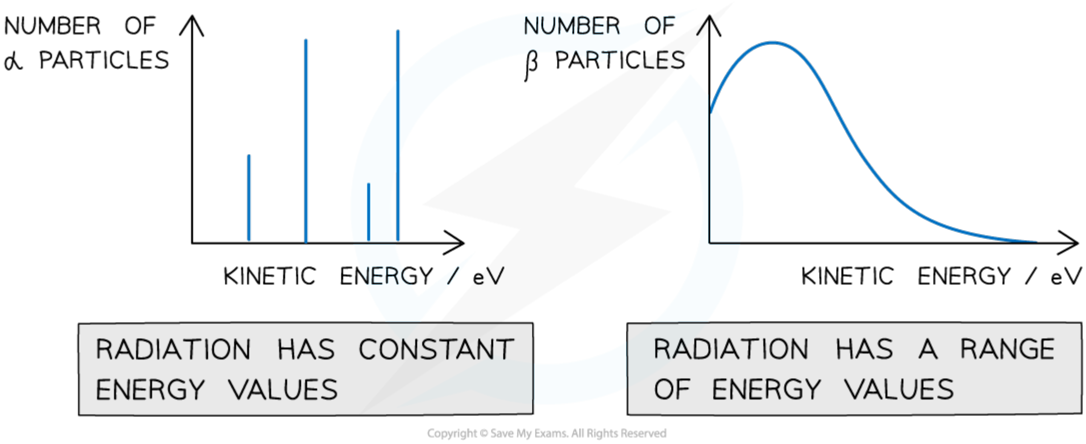
One way to remember which particle decays into which depends on the type of beta emission, think of beta ‘**p**lus’ as the ‘**p**roton’ that turns into the neutron (plus an electron neutrino)

11. Particle Physics

YOUR NOTES
⬇

Energy of Alpha & Beta Decay

- When the number of α particles is plotted against kinetic energy, there are clear spikes that appear on the graph
- This demonstrates that **α-particles have discrete energies** (only certain values)



Alpha particles have discrete energy levels whilst beta particles have a continuous range of energies

- When the number of β particles is plotted against kinetic energy, the graph shows a curve
- This demonstrates that **beta particles (electrons or positrons) have a continuous range of energies**
- This is because the energy released in beta decay is shared between the **beta particles** (electrons or positrons) and **neutrinos** (or anti-neutrinos)
- This was one of the first clues of the neutrino's existence
- The principle of conservation of momentum and energy applies in both alpha and beta emission

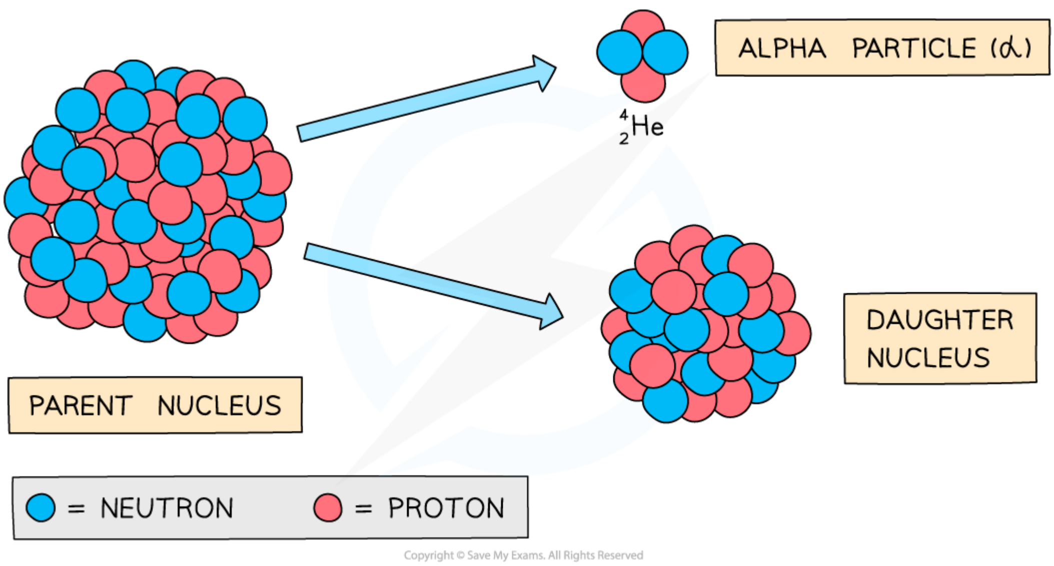
11. Particle Physics

YOUR NOTES

α & β Decay Equations

Alpha Decay

- Alpha decay is common in large, unstable nuclei with too many protons
- The decay involves a nucleus emitting an alpha particle and decaying into a different nucleus
- An alpha particle consists of **2 protons and 2 neutrons** (the nucleus of a Helium atom)



Alpha decay produces a daughter nucleus and an alpha particle (helium nucleus)

- When an unstable nucleus (the parent nucleus) emits radiation, the constitution of its nucleus changes
- As a result, the isotope will change into a different element (the daughter nucleus)
- Alpha decay can be represented by the following radioactive decay equation:

$$^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}\alpha$$

$$\text{e.g.} \quad ^{212}_{84}Po \rightarrow ^{208}_{82}Y + ^{4}_{2}\alpha$$

Alpha decay equation

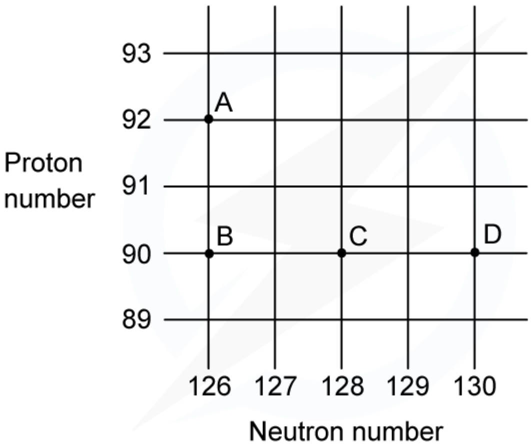
11. Particle Physics

YOUR NOTES

- When an alpha particle is emitted from a nucleus:
  - The nucleus loses 2 protons: **proton number decreases by 2**
  - The nucleus loses 4 nucleons: **nucleon number decreases by 4**

Worked example

The radioactivve nucleus $^{222}_{92}Rn$ undergoes alpha decay into a daughter nucleus Po.



a.) Which letter in the diagram represents the daughter product?
b.) Which is the nucleon and proton number of the final nucleus Po?

ANSWER: C

11. Particle Physics

YOUR NOTES

a.)

STEP 1

IN ALPHA DECAY, THE NUCLEON NUMBER OF THE PARENT NUCLEUS DECREASES BY 4 AND THE PROTON NUMBER DECREASES BY 2

STEP 2

THE NUMBER OF NEUTRONS IS:

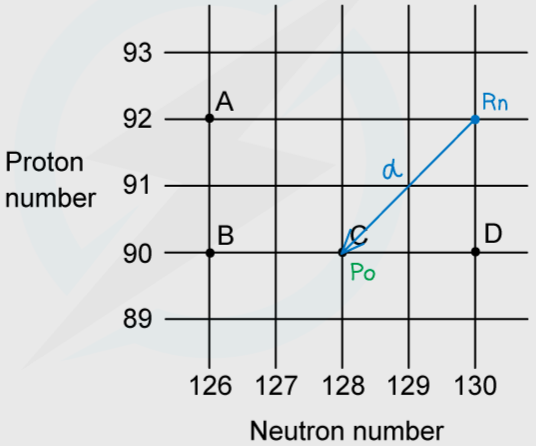
$222 - 92 = 130$

STEP 3

A HELIUM NUCLEUS HAS 2 PROTONS AND 2 NEUTRONS

STEP 4

THE PROTON AND NEUTRON NUMBER BOTH DECREASES BY 2 REACHING POINT C



b.)

STEP 1

ALPHA DECAY EQUATION:

$^{A}_{Z}X \longrightarrow {}^{4}_{2}\alpha + {}^{A-4}_{Z-2}Y \qquad {}^{222}_{92}Rn \longrightarrow {}^{4}_{2}\alpha + {}^{218}_{90}Po$

STEP 2

NUCLEON NUMBER OF Po = 218

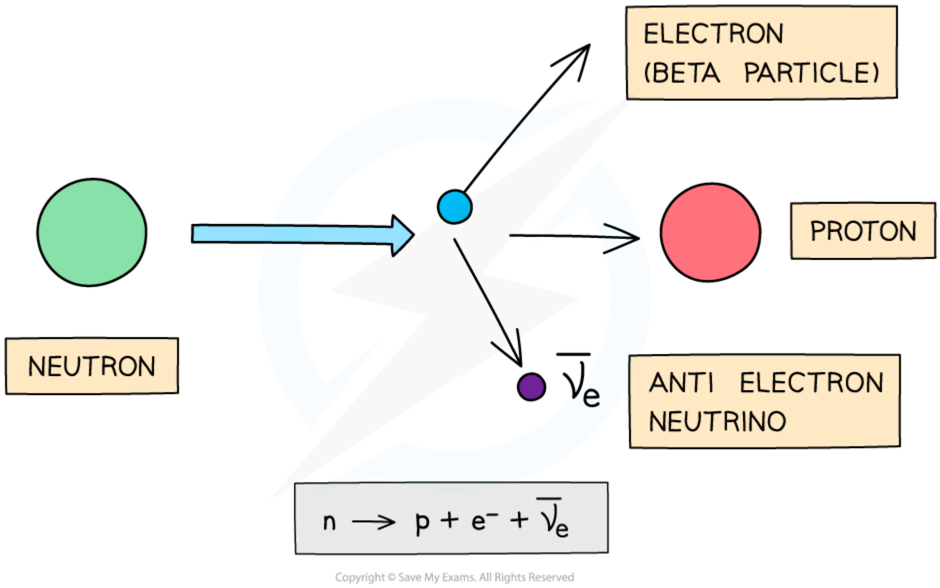
PROTON NUMBER Po = 90

11. Particle Physics

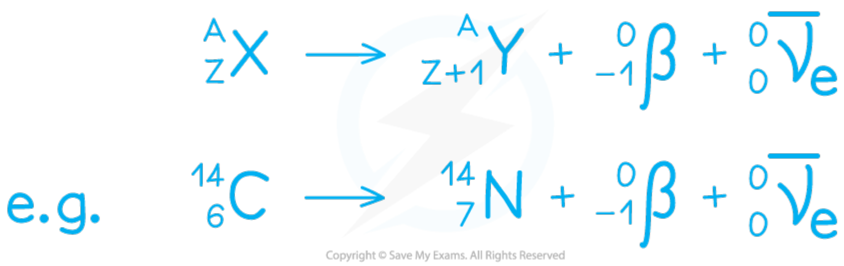
YOUR NOTES

β⁻ decay

- A β⁻ particle is a high energy electron emitted from the nucleus
- β⁻ decay is when a **neutron turns into a proton emitting an electron and an anti-electron neutrino**



- When a β⁻ is emitted from a nucleus:
  - The number of protons increases by 1: **proton number increases by 1**
  - The total number of nucleons stays the same: **nucleon number remains the same**



Equation for beta minus emission

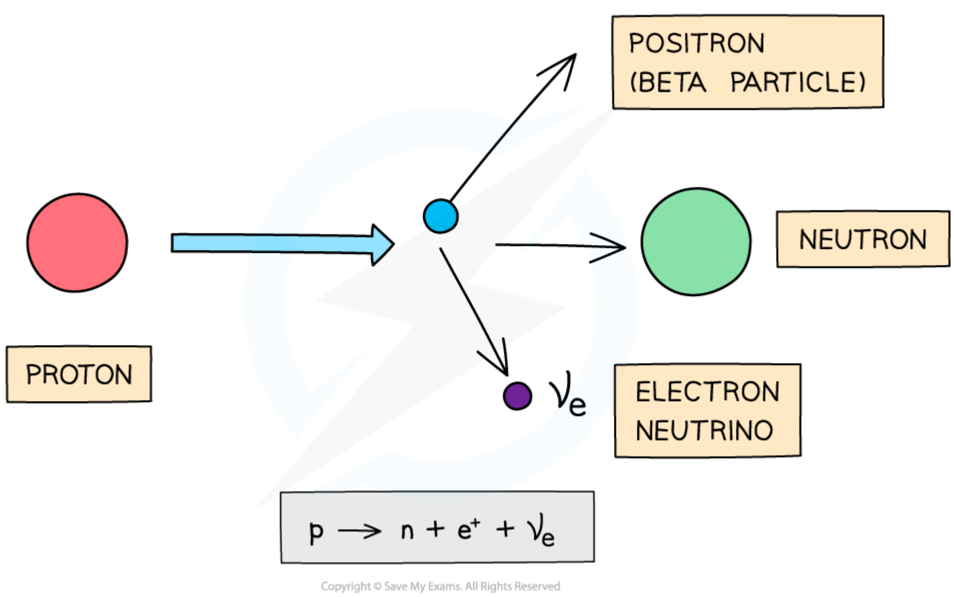
- The new nucleus formed from the decay is called the "daughter" nucleus (nitrogen in the example above)

11. Particle Physics

YOUR NOTES

β^+ decay

- A β^+ particle is a high energy positron emitted from the nucleus
- β^+ decay is when a **proton turns into a neutron emitting a positron (anti-electron) and an electron neutrino**



- When a β^+ is emitted from a nucleus:
  - The number of protons decreases by 1: **proton number decreases by 1**
  - The total number of nucleons stays the same: **nucleon number remains the same**

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}\beta + {}^{0}_{0}\nu_e$$

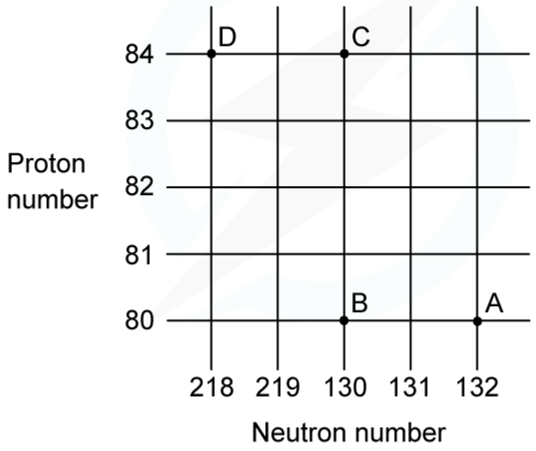
e.g. $$^{6}_{6}C \rightarrow {}^{6}_{5}B + {}^{0}_{+1}\beta + {}^{0}_{0}\nu_e$$

Equation for beta plus emission

Worked example

A radioactive substance with a nucleon number of 212 and a proton number of 82 decays by β-plus emission into a daughter product which in turn decays by further β-plus emission into a granddaughter product.

Which letter in the diagram represents the granddaughter product?



ANSWER: A

11. Particle Physics

YOUR NOTES

STEP 1

IN $\beta+$ DECAY, A PROTON DECAYS INTO A NEUTRON, A POSITRON (e^+) AND AN ANTI-ELECTRON NEUTRINO ($\bar{\nu}$)

STEP 2

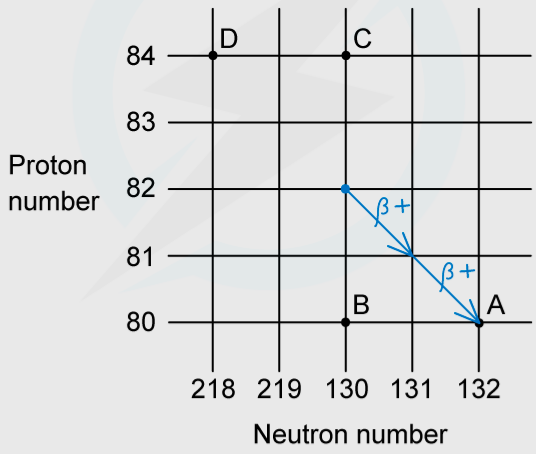
THE NUMBER OF NEUTRONS IS:

$212 - 82 = 130$

STEP 3

THE PROTON NUMBER DECREASES BY ONE AND THE NUMBER OF NEUTRONS INCREASES BY ONE

STEP 4



THIS HAPPENS TWICE, REACHING POINT A

Exam Tip

Remember to avoid the common mistake of confusing the number of neutrons with nucleon number. In alpha decay, the nucleon (protons and neutrons) number decreases by 4 but the number of neutrons only decreases by 2.

11. Particle Physics

YOUR NOTES

Exam Question: Easy

A Geiger-muller tube held at a distance of about 12 cm from a radioactive source. The radiation emitted from the nuclear isotope was entirely stopped by a 2 mm thick sheet of lead.

Which of the statements can be deduced from the above information about the emission from the isotope?

A it could be alpha, beta and gamma radiation

B it could be beta and gamma radiation, but not alpha radiation

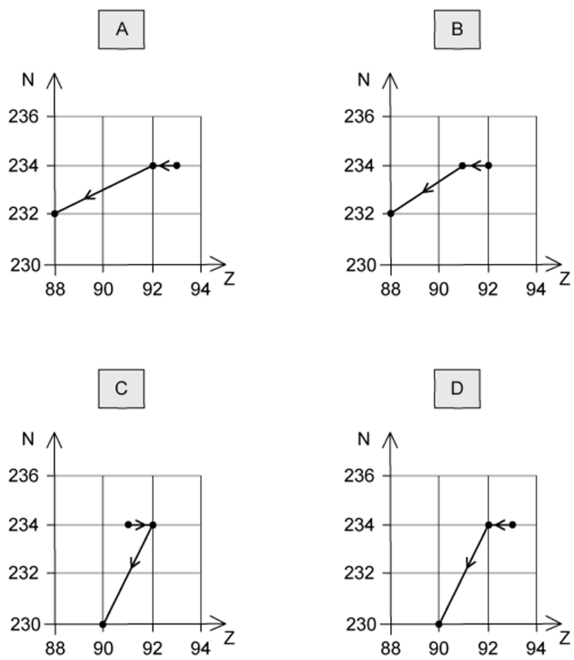
C it could be alpha and gamma radiation, but not beta radiation

D it could be alpha and beta radiation, but not gamma radiation

Exam Question: Medium

A radioactive nucleus is formed by β-decay. This nucleus then decays by α-emission.

The graphs below show the nucleon number N plotted against proton number Z. Which one shows the β-decay followed by the α-emission?

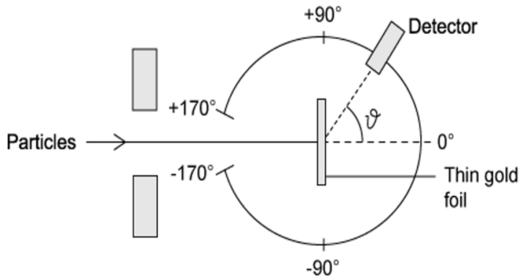


11. Particle Physics

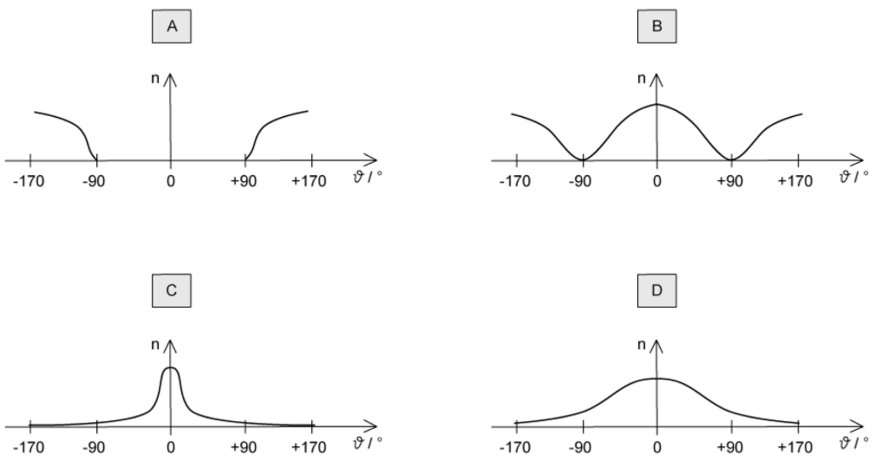
YOUR NOTES

Exam Question: Hard

In an α-particle scattering experiment, a student set up the apparatus below to determine the number n of α-particles incident per unit time on a detector held at various angles θ.



Which of the following graph best represents the variation of n with θ?



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11. Particle Physics

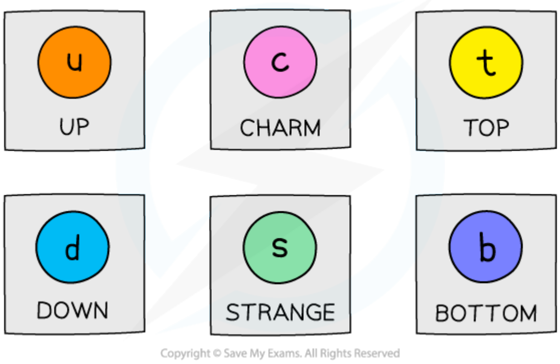
YOUR NOTES

11.2 FUNDAMENTAL PARTICLES

11.2.1 FUNDAMENTAL PARTICLES

Fundamental Particles: Quarks

- Quarks are **fundamental particles** that make up other subatomic particles such as **protons and neutrons**
- Protons and neutrons are in a category of particles called **hadrons**
  - Hadrons are defined as any particle made up of quarks
- **Fundamental** means that quarks are not made up of any other particles. Another example is electrons
- Quarks have never been observed on their own, they're either in pairs or groups of three
- There are six flavours (types) of quarks that exist:



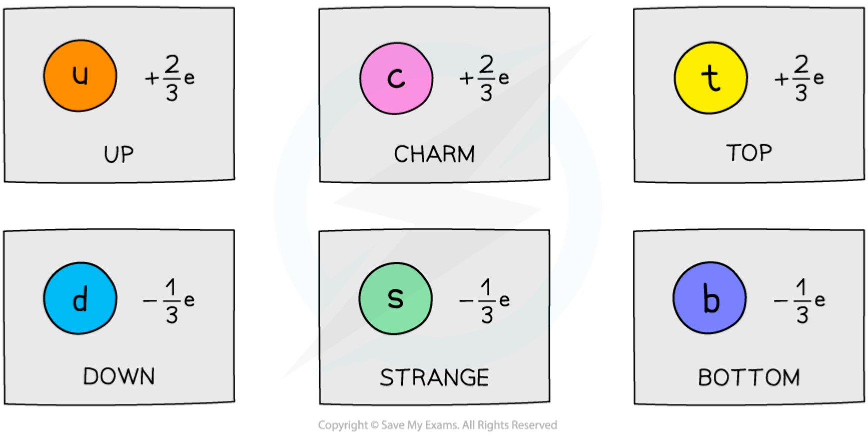
The six flavours of quarks

11. Particle Physics

YOUR NOTES

Properties of Quarks

- The charge of a hadron is determined by the sum of the charges of its quarks
- Each flavour of quark has a certain relative charge:



Each flavour of quark has a charge of either +⅔e or -⅓e

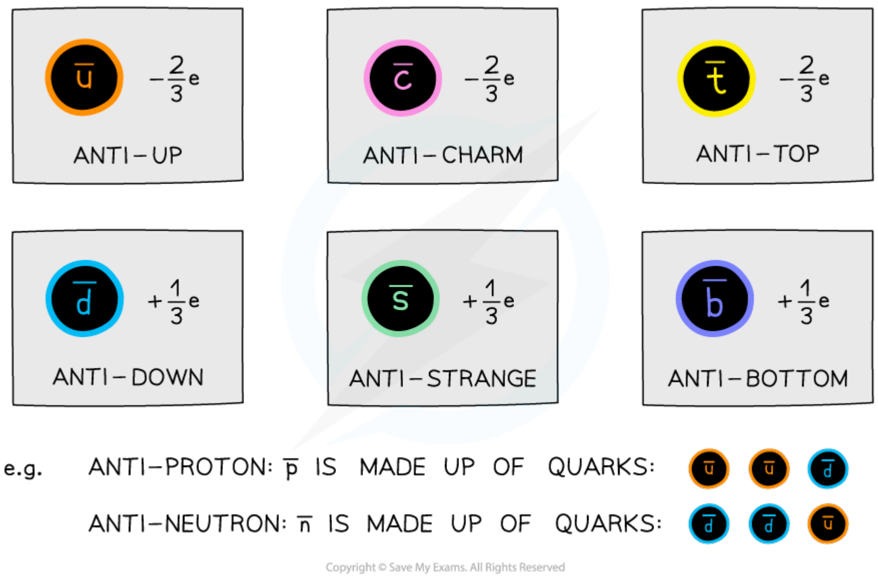
- For example, a proton is made up of two up quarks and a down quark. Adding up their charges gives the charge of a proton:

$$+\tfrac{2}{3}e + \tfrac{2}{3}e - \tfrac{1}{3}e = +1e$$

- The equivalent antiparticle of the quark is the anti-quark
- These are identical to quarks except with opposite relative charges

11. Particle Physics

YOUR NOTES

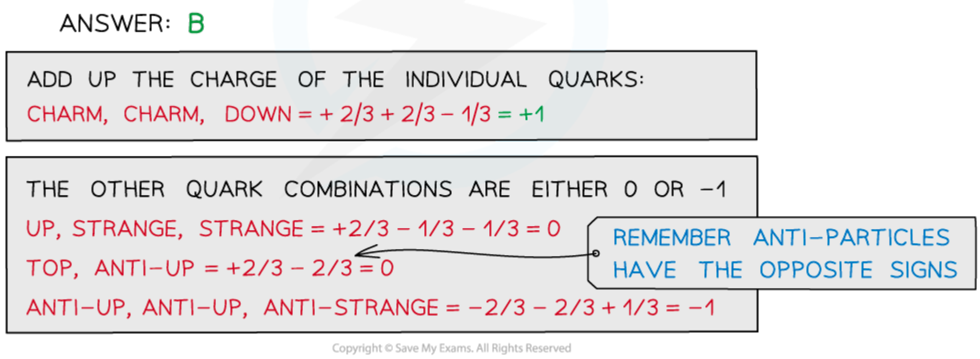


Each flavour of anti-quark has a charge of either -⅔e or +⅓e. The quark composition of anti-protons and anti-neutrons changes to anti-quarks

Worked example

Particles are made up of a combination of three quarks or two quarks. Which quark combination would **not** give a particle a charge of -1 or 0?

A. up, strange, strange
B. charm, charm, down
C. top, anti-up
D. anti-up, anti-up, anti-strange



11. Particle Physics

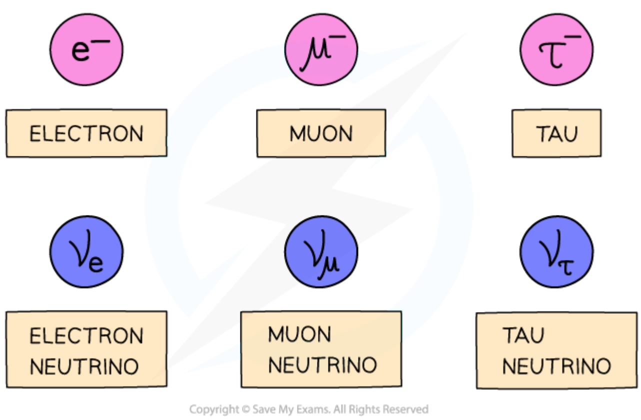
YOUR NOTES

Exam Tip

You will be expected to remember the charge of each quark. However, instead of memorising the charges of anti-quarks too, just remember they are identical but with opposite signs.

Fundamental Particles: Leptons

- **Leptons** are a group of **fundamental** (elementary) particles
- This means they are not made up of any other particles (no quarks)
- There are six leptons altogether:



The six leptons are all fundamental particles

11. Particle Physics

YOUR NOTES

- The muon and tau particle are very similar to the electron but with slightly larger mass
- Electrons, muon and tau particles **all** have a charge of -1e and a mass of 0.0005u
- There are three **flavours** (types) of neutrinos **(electron, muon, tau)**
- Neutrinos are the most abundant leptons in the universe
  - They have **no charge** and **negligible** mass (almost 0)
- Leptons interact with the weak interaction, electromagnetic and gravitational forces
- However, they do **not** interact with the strong force
- Although quarks are fundamental particles too, they are not classed as leptons
- Leptons do **not** interact with the strong force, whilst quarks do

Worked example

Circle all the anti-leptons in the following decay equation.

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

THE PION IS A MESON (TYPE OF HADRON) AND IS MADE UP OF QUARKS. THIS MEANS ITS NOT A FUNDAMENTAL PARTICLE AND HENCE NOT A LEPTON.

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

THE MUON+ IS THE ANTI-PARTICLE OF THE MUON AND THEREFORE AN ANTI-LEPTON

THE MUON NEUTRINO IS A LEPTON, NOT AN ANTI-LEPTON (WHICH WOULD BE AN ANTI-MUON NEUTRINO)

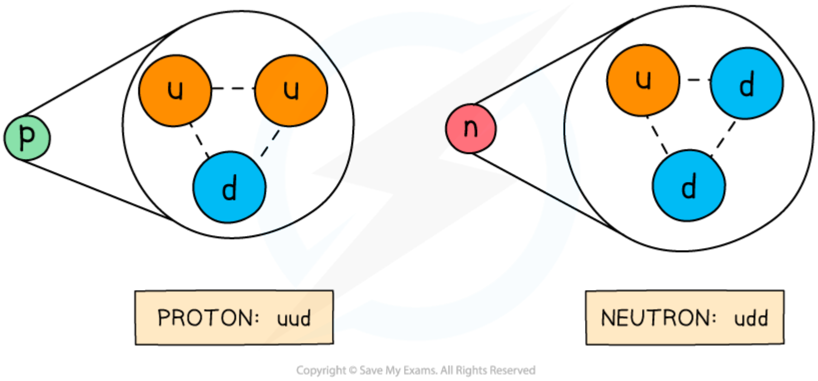
11. Particle Physics

YOUR NOTES
⬇

11.2.2 QUARK COMPOSITION

Quark Composition: Protons & Neutrons

- Protons and neutrons are **not** fundamental particles. They are each made up of three quarks
- **Protons** are made up of **two up quarks and a down quark**
- **Neutrons** are made up of **two down quarks and an up quark**



Protons and neutrons are made up of three quarks

- You will be expected to remember these quark combinations for exam questions

11. Particle Physics

YOUR NOTES

Worked example

? In the nucleus of Iron $^{56}_{26}Fe$, how many ‘up’ quarks are there?

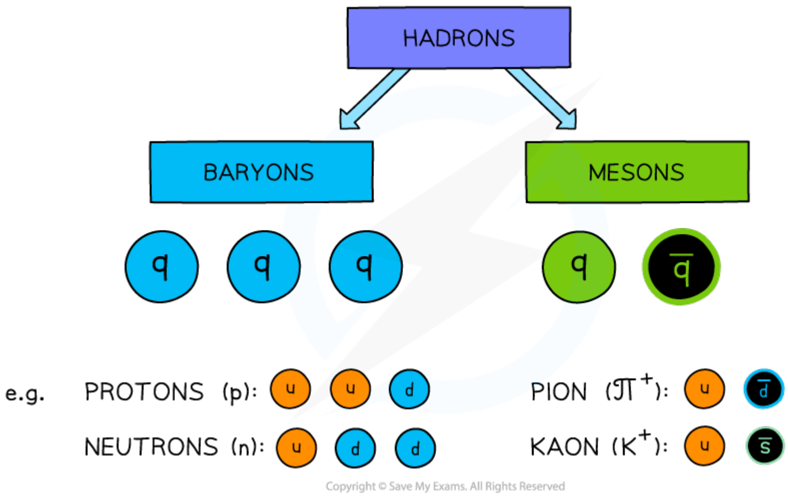
- **Step 1:** Calculate number of protons:
  - The number of protons is from the proton number = 26 protons
- **Step 2:** Calculate number of neutrons:
  - The number of neutrons = nucleon number – proton number = 56 – 26 = 30 neutrons
- **Step 3:** Up quarks in a proton:
  - Protons are made up of **uud** quarks = 2 up quarks
- **Step 4:** Up quarks in a neutron:
  - Neutrons are made up of **udd** quarks = 1 up quark
- **Step 5:** Total number of up quarks:
  - 26 protons x 2 up quarks = 52 up quarks
  - 30 neutrons x 1 up quark = 30 up quarks
  - 52 + 30 = 82 up quarks

11. Particle Physics

YOUR NOTES

Baryons & Mesons

- **Hadrons** are the group of subatomic particles that are made up of quarks
- These may be either a:
  - **Baryon (3 quarks)**
  - **Meson (quark and anti-quark pair)**

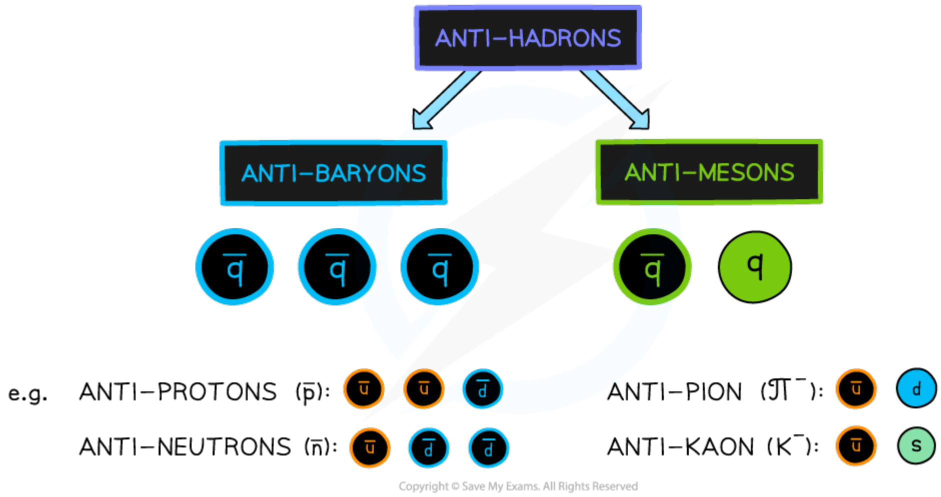


Hadrons may be either a baryon or a meson

11. Particle Physics

YOUR NOTES

- Quarks have never been discovered on their own, always in pairs or groups of three
- Anti-hadrons can be either
  - **Anti-baryons (3 anti-quarks)**
  - **Anti-meson (quark and anti-quark pair)**



Anti-hadrons may be either an anti-baryon or an anti-meson

- Note that all baryons or mesons have integer (whole number) charges eg. +1e, -2e etc.
- This means quarks in a baryon are either all quarks or all anti-quarks. Combination of quarks and anti-quarks don't exist in a baryon
  - e.g.

$\bar{u}ud$ WOULD NOT BE A QUARK COMBINATION THAT EXISTS

- The anti-particle of a meson is still a quark-antiquark pair. The difference being the quark becomes the anti-quark and vice versa

11. Particle Physics

YOUR NOTES

Worked example

The baryon Δ^{++} was discovered in a particle accelerator using accelerated positive pions on hydrogen targets.

Which of the following is the quark combination of this particle?

A. uuu **B.** cd$\bar{s}$ **C.** $\bar{u}$d **D.** $\bar{c}\bar{c}\bar{c}$

ANSWER: A

STEP 1
SINCE IT'S A BARYON, IT'S MADE UP OF THREE QUARKS
THIS RULES OUT OPTION C (A MESON)

STEP 2
BARYONS ARE MADE UP OF 3 QUARKS OR ANTI-BARYONS MADE UP OF 3 ANTI-QUARKS
THIS RULES OUT OPTION B (YOU CANNOT HAVE A COMBINATION OF QUARKS AND ANTIQUARKS)

STEP 3
THE Δ^{++} BARYON HAS A CHARGE OF +2 (FROM THE ++)

STEP 4
ADDING UP THE CHARGES OF THE QUARKS IN A AND D
A. uuu = +2/3 + 2/3 + 2/3 = +2
D. $\bar{c}\bar{c}\bar{c}$ = −2/3 − 2/3 − 2/3 = −2

STEP 5
ITS QUARK COMBINATION MUST THEREFORE BE uuu (OPTION A)

Exam Tip

- Remembering quark combinations is useful for the exam. However, as long as you can remember the charges for each quark, it is possible to figure out the combination by making sure the combination of quarks add up to the charge of the particle (just like in the worked example)

11. Particle Physics

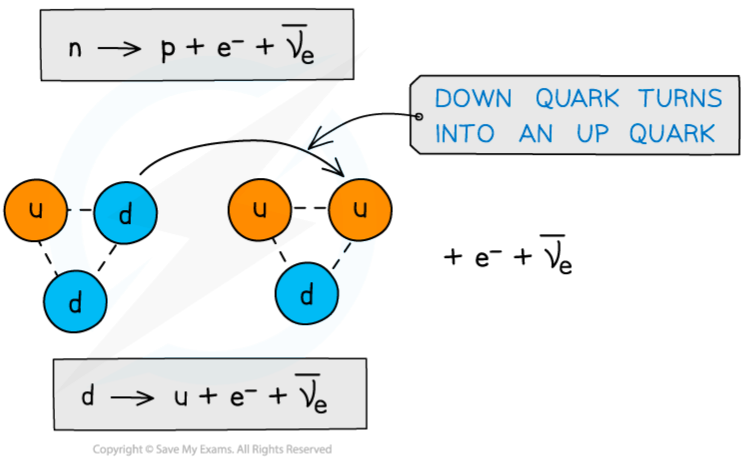
YOUR NOTES

Quark Composition: β– & β+ decay

- Beta decay happens via the **weak interaction**
  - This is one of the four fundamental forces and it's responsible for radioactive decays

Quark Composition: β^- decay

- Recall that **β^- decay is when a neutron turns into a proton emitting an electron and anti-electron neutrino**
- More specifically, a neutron turns into a proton because a **down quark turning into an up quark**



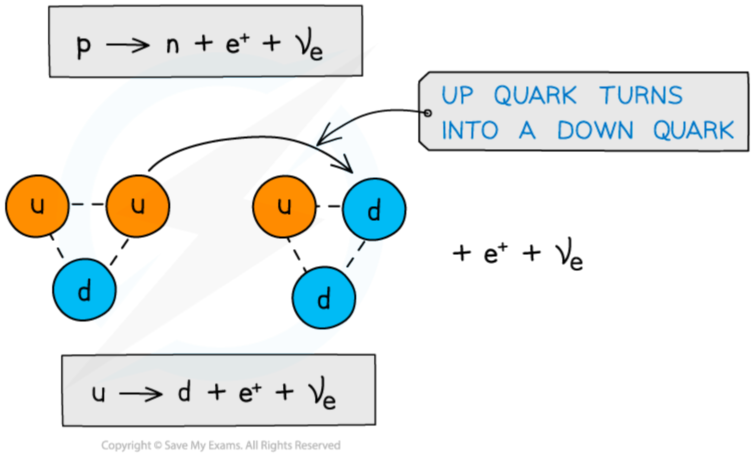
Beta minus decay is when a down quark turns into an up quark

11. Particle Physics

YOUR NOTES

Quark Composition: β^+ decay

- Recall that **β^+ decay is when a proton turns into a neutron emitting an positron and an electron neutrino**
- More specifically, a proton turns into a neutron because an **up quark turns into a down quark**



Beta minus decay is when an up quark turns into a down quark

11. Particle Physics

YOUR NOTES

Worked example

?

The equation for β^- decay is

$$n \longrightarrow p + e^- + \overline{V}_e$$

Using the quark model of beta decay, prove that the charge is conserved in this equation.

STEP 1

β^- DECAY IS WHEN A DOWN QUARK CHANGES TO AN UP QUARK
THIS CHANGES A NEUTRON INTO A PROTON

STEP 2

CHARGE OF THE LEFT HAND SIDE OF THE EQUATION
THE QUARK COMPOSITION OF A NEUTRON IS **udd**

STEP 3

ADDING UP THE QUARK CHARGES:

$+2/3 - 1/3 - 1/3 = 0$

THE LEFT HAND SIDE HAS A CHARGE OF 0

STEP 4

CHARGE ON THE RIGHT HAND SIDE OF THE EQUATION
THE QUARK COMPOSITION OF A PROTON IS **uud**

STEP 5

ADD UP THE QUARK CHARGES:

$+2/3 + 2/3 - 1/3 = +1$

STEP 6

THE ELECTRONS CHARGE IS −1
THE ANTI−NEUTRINOS CHARGE IS 0
THE RIGHT HAND SIDE HAS A CHARGE OF $+1 - 1 + 0 = 0$

STEP 7

SINCE THE CHARGES ARE EQUAL ON BOTH SIDES, IT IS CONSERVED IN THE BETA DECAY EQUATION

11. Particle Physics

YOUR NOTES

Exam Question: Easy

What is a particle that is **not** made of any smaller particles?

A elementary particle

B minimum particle

C original particle

D atomic particle

Exam Question: Medium

Which of the following sentences about antimatter is **incorrect**?

A antimatter is normal matter with an opposite charge

B antimatter is only produced in particle accelerators

C antimatter destroys matter

D equal amounts of antimatter and matter were created during the Big Bang

11. Particle Physics

YOUR NOTES

Exam Question: Hard

Quarks are thought to make up protons and neutrons.

The 'up' quark has a charge of $\frac{2}{3}e$; a 'down' quark has a charge of $-\frac{1}{3}e$, where *e* is the elementary charge (+ 1.6 × 10^{-19} C)

How many up quarks and down quarks must a proton contain?

	up quarks	down quarks
A	0	3
B	2	1
C	1	2
D	1	1

> CHECK YOUR ANSWERS AT SAVEMYEXAMS.CO.UK

12. Motion in a Circle

YOUR NOTES
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CONTENTS

12.1 KINEMATICS OF UNIFORM CIRCULAR MOTION

12.1.1 RADIANS & ANGULAR DISPLACEMENT

Radians & Angular Displacement

- In circular motion, it is more convenient to measure angular displacement in units of **radians** rather than units of degrees
- The **angular displacement** (θ) of a body in circular motion is defined as:

The change in angle, in radians, of a body as it rotates around a circle

- The **angular displacement** is the ratio of:

$$\Delta\theta = \frac{\text{distance travelled around the circle}}{\text{radius of the circle}}$$

- **Note**: both distances must be measured in the same units e.g. metres
- A **radian** (*rad*) is defined as:

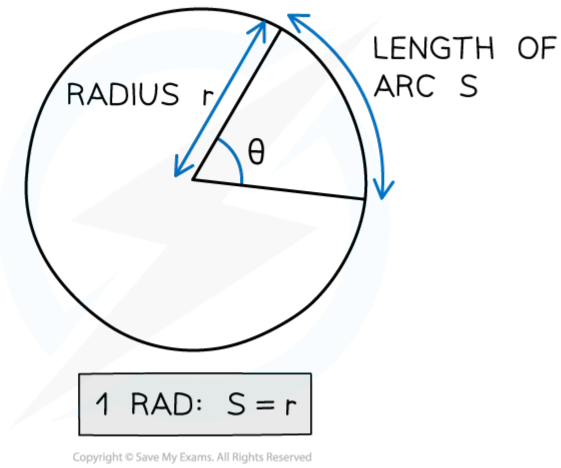
The angle subtended at the centre of a circle by an arc equal in length to the radius of the circle

- Angular displacement can be calculated using the equation:

$$\Delta\theta = \frac{\Delta s}{r}$$

12. Motion in a Circle

YOUR NOTES



When the angle is equal to one radian, the length of the arc (Δs) is equal to the radius (r) of the circle

- Where:
  - $\Delta\theta$ = angular displacement, or angle of rotation (radians)
  - s = length of the arc, or the distance travelled around the circle (m)
  - r = radius of the circle (m)

- Radians are commonly written in terms of π
- The angle in radians for a complete circle (360°) is equal to:

$$\frac{circumference\ of\ circle}{radius} = \frac{2\pi r}{r} = 2\pi$$

- If an angle of 360° = 2π radians, then 1 radian in degrees is equal to:

$$\frac{360}{2\pi} = \frac{180}{\pi} \simeq 57.30°$$

12. Motion in a Circle

YOUR NOTES

- Use the following equation to convert from degrees to radians:

$$\theta^{\circ} \times \left(\frac{\pi}{180}\right) = \theta \text{ rad}$$

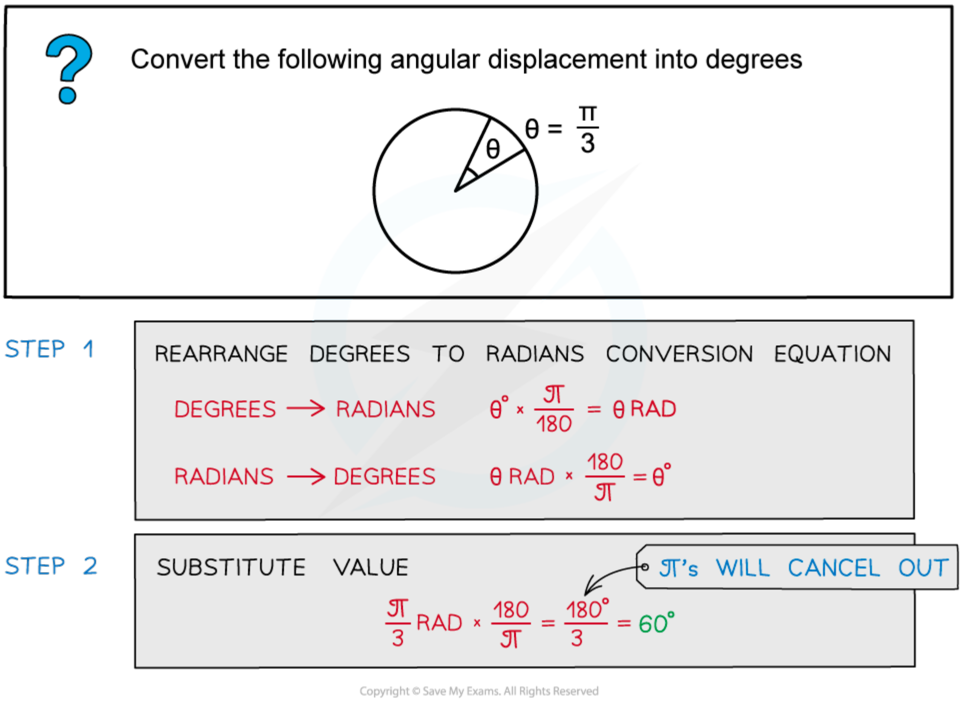
Table of common degrees to radians conversions

Degrees (°)	Radians (rads)
360	2π
270	$\frac{3\pi}{2}$
180	π
90	$\frac{\pi}{2}$

12. Motion in a Circle

YOUR NOTES
⬇

Worked example: Radians conversion



12. Motion in a Circle

YOUR NOTES
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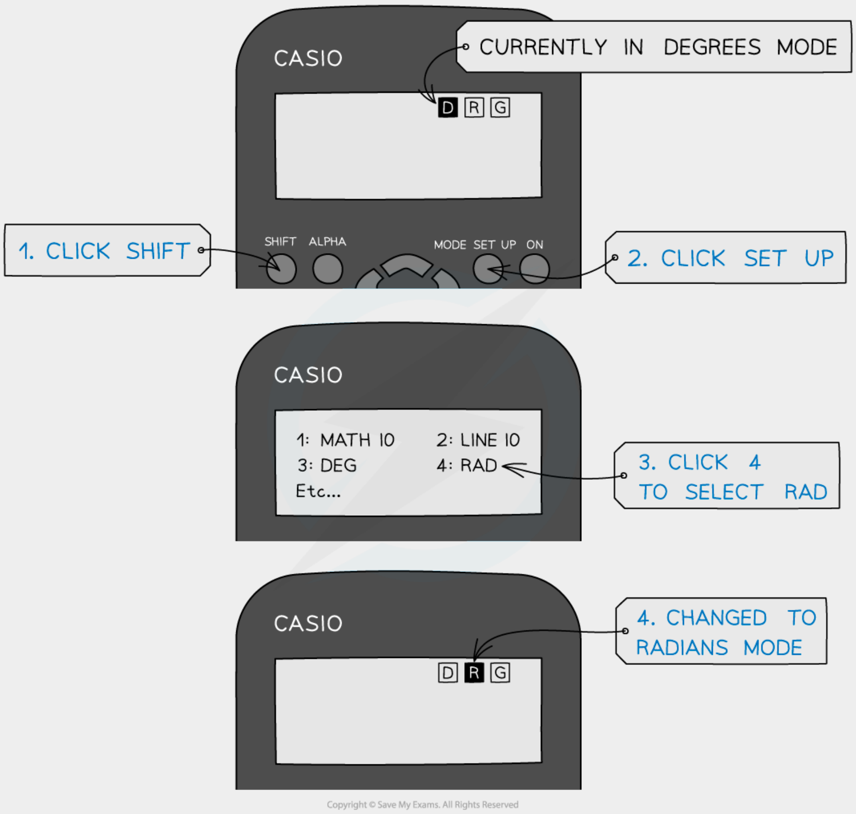
Exam Tip

• You will notice your calculator has a degree (Deg) and radians (Rad) mode

• This is shown by the "D" or "R" highlighted at the top of the screen

• Remember to make sure it's in the right mode when using **trigonometric** functions (sin, cos, tan) depending on whether the answer is required in **degrees** or **radians**

• It is extremely common for students to get the wrong answer (and lose marks) because their calculator is in the wrong mode – make sure this doesn't happen to you!



12. Motion in a Circle

YOUR NOTES

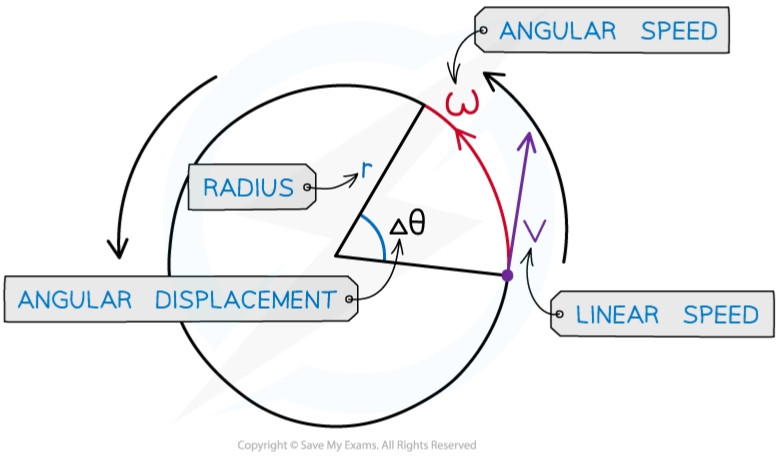
12.1.2 ANGULAR SPEED

Angular Speed

- Any object travelling in a uniform circular motion at the same speed travels with a **constantly changing velocity**
- This is because it is **constantly changing direction**, and is therefore accelerating
- The angular speed (ω) of a body in circular motion is defined as:

The rate of change in angular displacement with respect to time

- Angular speed is a scalar quantity, and is measured in rad s^{-1}



When an object is in uniform circular motion, velocity constantly changes direction, but the speed stays the same

Calculating Angular Speed

- Taking the angular displacement of a complete cycle as 2π, the angular speed ω can be calculated using the equation:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

- Where:
  - $\Delta\theta$ = change in angular displacement (radians)
  - Δt = time interval (s)
  - T = the time period (s)
  - f = frequency (Hz)
- Angular velocity is the same as angular speed, but it is a vector quantity
- When an object travels at constant linear speed v in a circle of radius r, the angular velocity is equal to:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{v}{r}$$

- Where:
  - v is the linear speed (m s^{-1})
  - r is the radius of orbit (m)
- This equation tells us:
  - The greater the rotation angle θ in a given amount of time, the greater the angular velocity ω
  - An object rotating further from the centre of the circle (larger r) moves with a faster angular velocity (larger ω)

12. Motion in a Circle

YOUR NOTES

Worked example: Angular speed

A bird flies in a horizontal circle with an angular speed of 5.25 rads^{-1} of radius 650 m.

Calculate:
a) The linear speed of the bird
b) The frequency of the bird flying in a complete circle

a) STEP 1 LINEAR SPEED EQUATION

$$v = r\omega$$

STEP 2 SUBSTITUTE IN VALUES

$V = 650 \times 5.25 = 3412.5 = 3410 \text{ ms}^{-1}$ (3 s.f.)

b) STEP 1 ANGULAR SPEED WITH FREQUENCY EQUATION

$$\omega = 2\pi f$$

STEP 2 REARRANGE FOR FREQUENCY

$$f = \frac{\omega}{2\pi}$$

STEP 3 SUBSTITUTE IN VALUES

$f = \frac{5.25}{2\pi} = 0.83556... = 0.836 \text{ Hz}$ (3 s.f.)

12. Motion in a Circle

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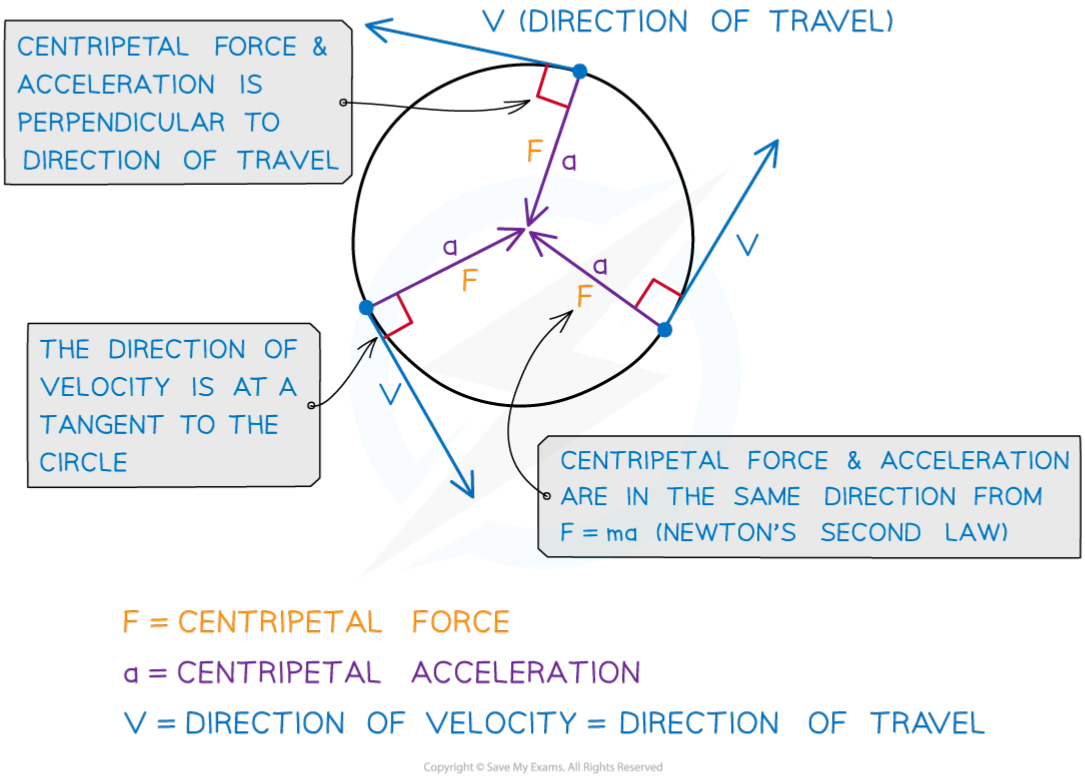
12.2 CENTRIPETAL ACCELERATION

12.2.1 CENTRIPETAL ACCELERATION

What Causes Centripetal Acceleration?

- Velocity and acceleration are both vector quantities
- An object in uniform circular motion is **continuously changing direction,** and therefore is **constantly changing velocity**
  - The object must therefore be **accelerating**
- This is called the **centripetal acceleration** and is **perpendicular** to the direction of the linear speed
  - Centripetal means it acts **towards the centre** of the circular path



Centripetal force and acceleration are always directed towards the centre of the circle

12. Motion in a Circle

YOUR NOTES
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- The centripetal acceleration is caused by a **centripetal force** of constant magnitude that also acts **perpendicular** to the direction of motion (towards the centre)
- Therefore, the centripetal acceleration and force act in the **same direction**

Exam Tip

• The linear speed is sometimes referred to as the 'tangential' speed
• A tangent is a straight line which touches a circle or curve at exactly one point
• The key feature of a tangent of a circle is that it **always acts perpendicular** to its radius
• You can find out more in the A Level Maths revision notes on Tangents

Relating Centripetal Acceleration & Angular Speed

- An object travelling in uniform circular motion has centripetal acceleration, yet its angular speed (ω) is constant
  - This is because speed is a scalar quantity, whilst velocity is a vector quantity
  - Therefore, angular speed is the **magnitude** (size) component of angular velocity

- Key ideas to remember:
  - Angular speed (**magnitude**) stays constant, angular velocity (**direction**) is constantly changing
  - Angular speed does not change with radius, but linear speed does
  - The object's centripetal acceleration is always directed **toward the centre** of the circle, and is perpendicular to the object's velocity at any one time
  - Velocity and acceleration are both defined by a **change in direction**, not just a change in the magnitude

12. Motion in a Circle

YOUR NOTES
⬇

Exam Tip

We are used to the idea of acceleration meaning something is speeding up. So, it might sound counterintuitive to say an object travelling in a circle is accelerating, yet it also has constant speed. This is where the idea of scalars and vectors would be useful to revisit if you are not confident with this concept.

12. Motion in a Circle

YOUR NOTES

12.2.2 CALCULATING CENTRIPETAL ACCELERATION

Calculating Centripetal Acceleration

- Centripetal acceleration can be defined using the radius *r* and linear speed *v*:

$$a = \frac{v^2}{r}$$

- Using the equation relating angular speed ω and linear speed *v*:

$$\mathbf{v = r\omega}$$

- These equations can be combined to give us another form of the centripetal acceleration equation:

$$a = \frac{(r\omega)^2}{r}$$

$$\mathbf{a = r\omega^2}$$

- This equation tells us centripetal acceleration is equal to the radius times the square of the angular speed
- Alternatively, rearrange for *r:*

$$r = \frac{v}{\omega}$$

- This equation can be combined with the first one to give us another form of the centripetal acceleration equation:

$$a = \frac{v^2}{\left(\frac{v}{\omega}\right)}$$

$$\mathbf{a = v\omega}$$

- This equation tells us centripetal acceleration is equal to the linear speed times the angular speed

12. Motion in a Circle

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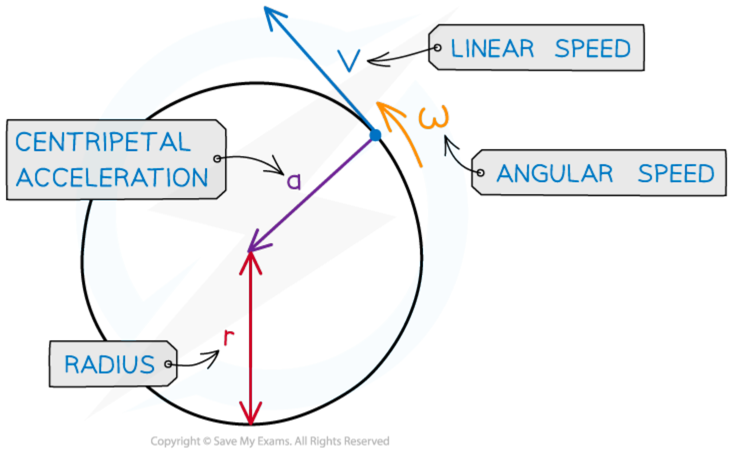


Centripetal acceleration is always directed toward the centre of the circle, and is perpendicular to the object's velocity

- Where:
  - a = centripetal acceleration (m s^{-2})
  - v = linear speed (m s^{-1})
  - ω = angular speed (rad s^{-1})
  - r = radius of the orbit (m)

12. Motion in a Circle

YOUR NOTES

Worked example: Centripetal acceleration

A ball tied to a string is rotating in a horizontal circle with radius 1.5 m and angular speed 3.5 rads^{-1}.

Calculate it's centripetal acceleration with a radius twice as large and angular speed twice as fast.

STEP 1

ANGULAR ACCELERATION EQUATION WITH ANGULAR SPEED

$a = r\omega^2$

STEP 2

CHANGE IN ANGULAR ACCELERATION WITH TWICE THE RADIUS AND ANGULAR SPEED

$a = (2r) \times (2\omega)^2 = 2r \times 4\omega^2 = 8r\omega^2$

THE CENTRIPETAL ACCELERATION WILL BE 8x BIGGER

STEP 3

SUBSTITUTE IN VALUES OF RADIUS AND ANGULAR SPEED

$a = 8r\omega^2 = 8 \times 1.5 \times 3.5^2 = 147 \text{ ms}^{-2}$

12. Motion in a Circle

YOUR NOTES

12.2.3 CALCULATING CENTRIPETAL FORCE

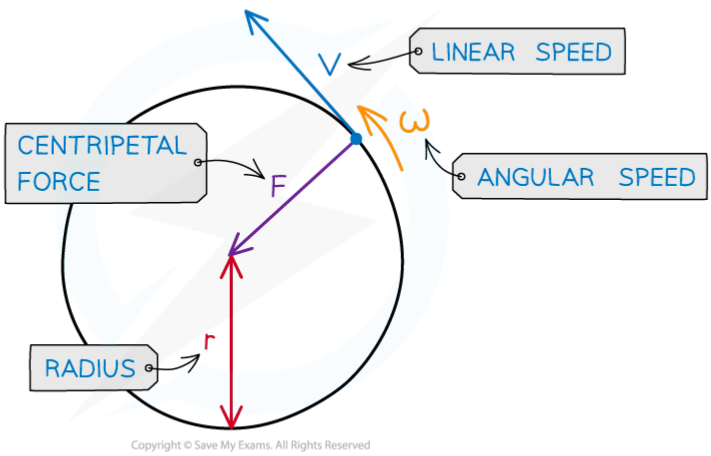
Calculating Centripetal Force

- An object moving in a circle is not in equilibrium, it has a resultant force acting upon it
- This is known as the **centripetal force** and is what keeps the object moving in a circle
- The centripetal force (F) is defined as:

The resultant perpendicular force towards the centre of the the circle needed to keep a body in uniform circular motion

- Centripetal force can be calculated using:

$$F = \frac{mv^2}{r} = mr\omega^2 = mv\omega$$



Centripetal force is always perpendicular to the direction of travel

- Where:
  - F = centripetal force (N)
  - v = linear velocity (m s^{-1})
  - ω = angular speed (rad s^{-1})
  - r = radius of the orbit (m)

12. Motion in a Circle

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- **Note:** centripetal force and centripetal acceleration act in the **same direction**
  - This is due to Newton's Second Law
- The centripetal force is **not** a separate force of its own
  - It can be any type of force, depending on the situation, which keeps an object moving in a circular path

Examples of centripetal force

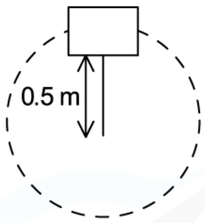
Situation	Centripetal force
Car travelling around a roundabout	Friction between car tyres and the road
Ball attached to a rope moving in a circle	Tension in the rope
Earth orbiting the Sun	Gravitational force

12. Motion in a Circle

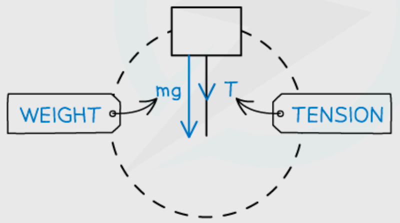
YOUR NOTES

Worked example: Centripetal force

A bucket of mass 8.0 kg is filled with water is attached to a string of length 0.5 m and tension 7.0 N at the top of the circle.
What is the minimum speed the bucket must have at the top of the circle so no water spills out?



STEP 1 DRAW THE FORCES ON THE BUCKET AT THE TOP



STEP 2 CALCULATE THE CENTRIPETAL FORCE

$$mg + T = \frac{mv^2}{r}$$ CENTRIPETAL FORCE EQUATION

STEP 3 REARRANGE FOR VELOCITY v

$$v = \sqrt{\frac{r(mg + T)}{m}}$$

STEP 4 SUBSTITUTE IN VALUES

$$\sqrt{\frac{0.5(8.0 \times 9.81 + 7.0)}{8.0}} = 2.31 \text{ ms}^{-1}$$

13. Gravitational Fields

YOUR NOTES
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CONTENTS

13.1 UNIVERSAL GRAVITATION

13.1.1 GRAVITATIONAL FIELDS

Defining Gravitational Field

- There is a force of attraction between **all** masses
- This force is known as the 'force due to gravity' or the weight
- The Earth's gravitational field is responsible for the weight of all objects on Earth
- A gravitational field is defined as:

A region of space where a mass experiences a force due to the gravitational attraction of another mass

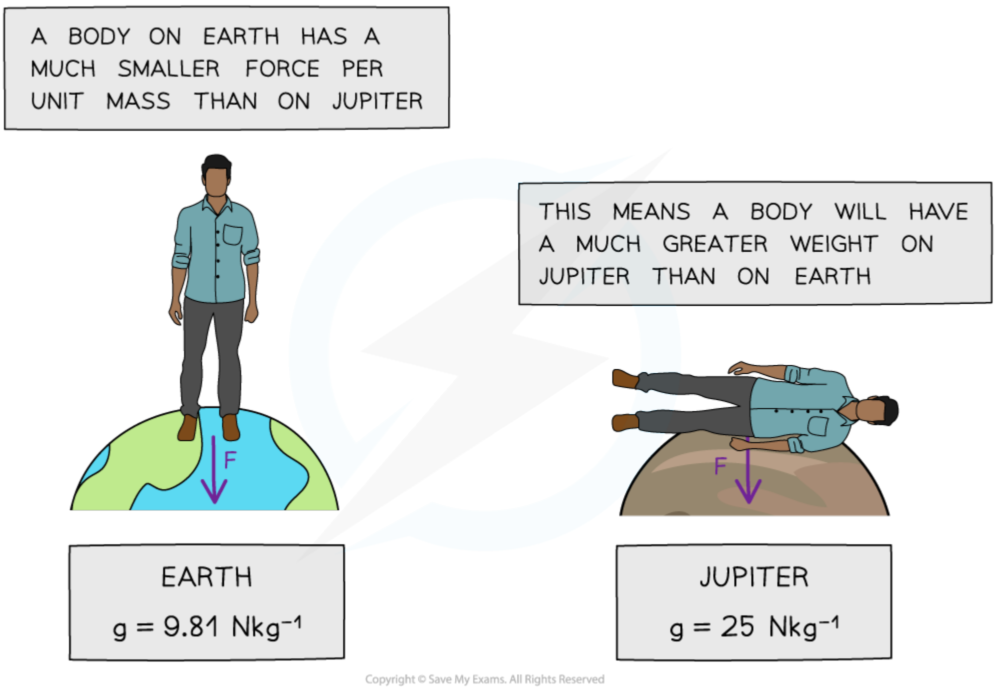
- The direction of the gravitational field is always towards the centre of the mass
  - Gravitational forces **cannot** be repulsive
- The strength of this gravitational field (g) at a point is the force (F_g) per unit mass (m) of an object at that point:

$$g = \frac{F_g}{m}$$

13. Gravitational Fields

YOUR NOTES

- Where:
  - g = gravitational field strength (N kg^{-1})
  - F_g = force due to gravity, or weight (N)
  - m = mass (kg)
- This equations tells us:
  - On planets with a large value of g, the gravitational force per unit mass is greater than on planets with a smaller value of g
- On such planets such as Jupiter, an object's mass remains the same at all points in space. However, their weight will be a lot greater meaning for example, a human will be unable to fully stand up

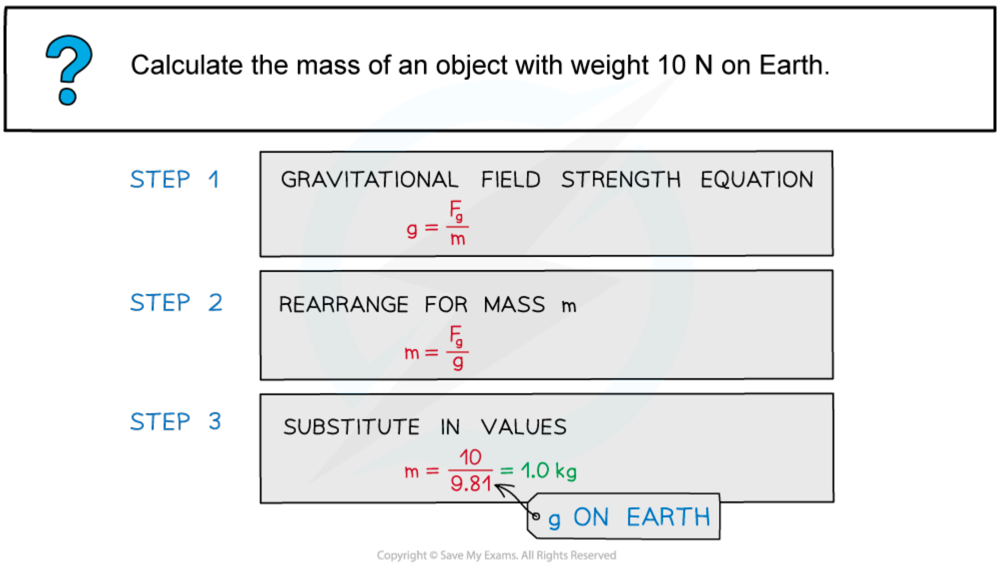


A person's weight on Jupiter would be so large a human would be unable to fully stand up

13. Gravitational Fields

YOUR NOTES

Worked example: Gravitational field



Exam Tip

There is a big difference between g and G (sometimes referred to as 'little g' and 'big G' respectively), g is the gravitational field strength and G is Newton's gravitational constant. Make sure not to use these interchangeably!

Representing Gravitational Fields

- The direction of a gravitational field is represented by gravitational field lines
- The gravitational field lines around a point mass are **radially inwards**
- The gravitational field lines of a uniform field, where the field strength is the same at all points, is represented by **equally spaced parallel lines**
  - For example, the fields lines on the Earth's surface

13. Gravitational Fields

YOUR NOTES
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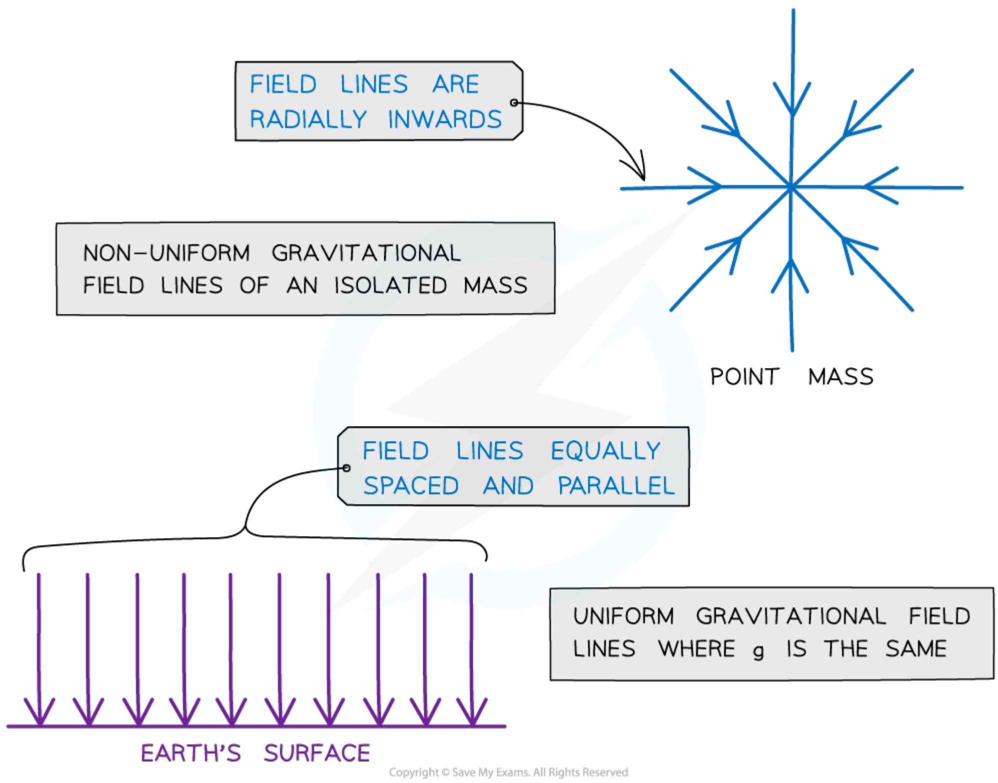


Gravitational field lines for a point mass and a uniform gravitational field

- Radial fields are considered **non-uniform fields**
    - The gravitational field strength *g* is different depending on how far you are from the centre
- Parallel field lines on the Earth's surface are considered a **uniform field**
    - The gravitational field strength *g* is the same throughout

Exam Tip

Always label the arrows on the field lines! Gravitational forces are **attractive only**. Remember:

• For a **radial field**: it is towards the centre of the sphere or point charge

• For a **uniform field**: towards the surface of the object e.g. Earth

13. Gravitational Fields

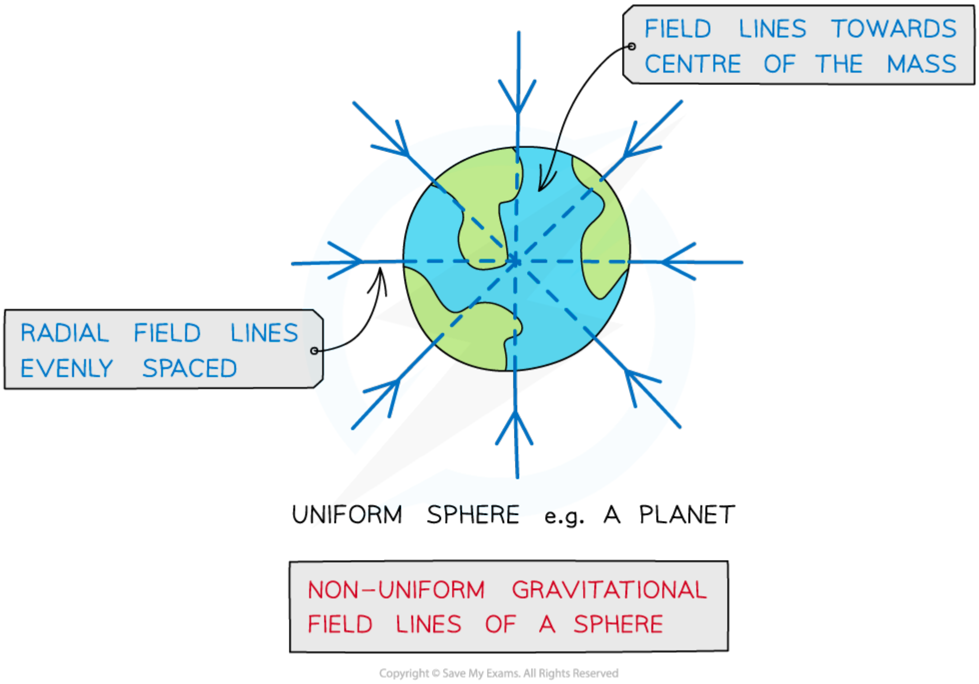
YOUR NOTES

Point Mass Approximation

- For a point outside a uniform sphere, the mass of the sphere may be considered to be a **point mass** at its centre
  - A uniform sphere is one where its mass is **distributed evenly**
- The gravitational field lines around a uniform sphere are therefore **identical to those around a point mass**
- An object can be regarded as point mass when:

A body covers a very large distance as compared to its size, so, to study its motion, its size or dimensions can be neglected

- An example of this is field lines around planets



Gravitational field lines around a uniform sphere are identical to those on a point mass

- Radial fields are considered **non-uniform** fields
  - So, the gravitational field strength g is different depending on how far you are from the centre of mass of the sphere

13. Gravitational Fields

YOUR NOTES
⬇

13.1.2 GRAVITATIONAL FORCE BETWEEN POINT MASSES

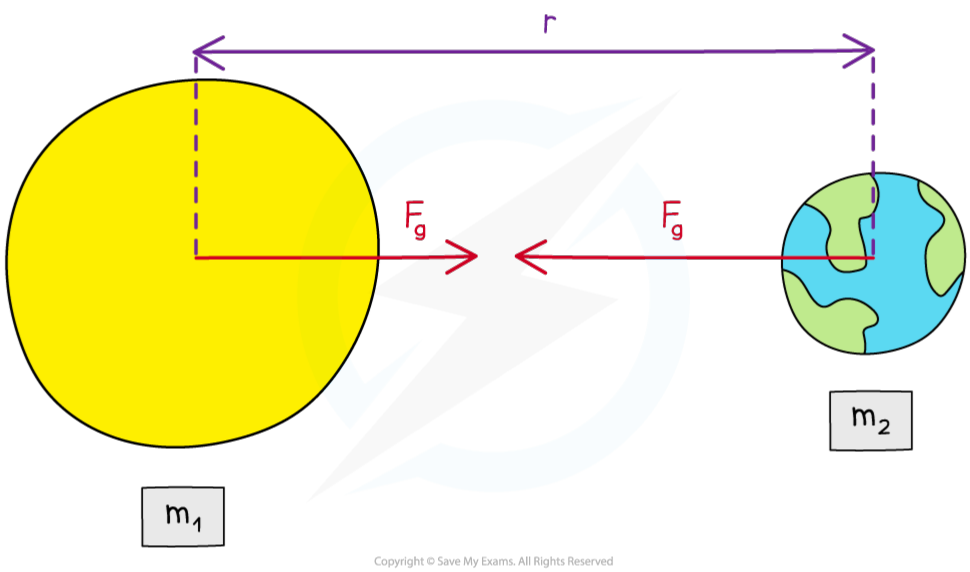
Newton's Law of Gravitation

- The gravitational force between two bodies outside a uniform field e.g. between the Earth and the Sun, is defined by Newton's Law of Gravitation
    - Recall that the mass of a uniform sphere can be considered to be a point mass at its centre
- Newton's Law of Gravitation states that:

The gravitational force between two point masses is proportional to the product of the masses and inversely proportional to the square their separation

- In equation form, this can be written as:

$$F_G = \frac{Gm_1m_2}{r^2}$$



The gravitational force between two masses outside a uniform field is defined by Newton's Law of Gravitation

13. Gravitational Fields

YOUR NOTES

- Where:
  - F_G = gravitational force between two masses (N)
  - G = Newton's gravitational constant
  - m_1 and m_2 = two points masses (kg)
  - r = distance between the centre of the two masses (m)

- Although planets are not point masses, their separation is much larger than their radius
  - Therefore, Newton's law of gravitation applies to planets orbiting the Sun
- The $1/r^2$ relation is called the 'inverse square law'
- This means that when a mass is twice as far away from another, its force due to gravity reduces by $(½)^2$ = ¼

13. Gravitational Fields

YOUR NOTES ⬇

Worked example: Newton's law of gravitation

A satellite with mass 6500 kg is orbiting the Earth at 2000 km above the Earth's surface. The gravitational force between them is 37 kN.

Calculate the mass of the Earth.

Radius of the Earth = 6400 km.

STEP 1

NEWTON'S LAW OF GRAVITATION

$$F_G = \frac{Gm_1m_2}{r^2}$$

m_1 IS THE MASS OF THE SATELLITE

m_2 IS THE MASS OF THE EARTH

THESE CAN BE ANY WAY AROUND

STEP 2

REARRANGE FOR m_2 (MASS OF EARTH)

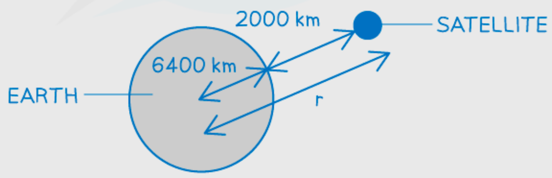
$$\frac{r^2F_G}{Gm_1} = m_2$$

STEP 3

CALCULATE THE DISTANCE r

r IS THE DISTANCE BETWEEN THE CENTRE OF THE EARTH AND SATELLITE

r = DISTANCE OF SATELLITE ABOVE THE SURFACE + RADIUS OF THE EARTH



$r = 2000 + 6400 = 8400 \text{ km} = 8400 \times 10^3 \text{ m}$

STEP 4

SUBSTITUTE IN VALUES

$$\frac{(8400 \times 10^3)^2 \times 37 \times 10^3}{6.67 \times 10^{-11} \times 6500} = 6.0 \times 10^{24} \text{ kg} \quad (2 \text{ s.f.})$$

37 kN

NEWTON'S GRAVITATIONAL CONSTANT

Exam Tip

A common mistake in exams is to forget to **add together** the distance from the surface of the planet and its radius to obtain the value of *r*. The distance *r* is measured from the **centre** of the mass, which is from the **centre** of the planet.

13. Gravitational Fields

YOUR NOTES

13.1.3 CIRCULAR ORBITS IN GRAVITATIONAL FIELDS

Circular Orbits in Gravitational Fields

- Since most planets and satellites have a near circular orbit, the gravitational force F_G between the sun or another planet provides the centripetal force needed to stay in an orbit
- Both the gravitational force and centripetal force are **perpendicular** to the direction of travel of the planet
- Consider a satellite with mass m orbiting Earth with mass M at a distance r from the centre travelling with linear speed v

$$F_G = F_{circ}$$

- Equating the gravitational force to the centripetal force for a planet or satellite in orbit gives:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

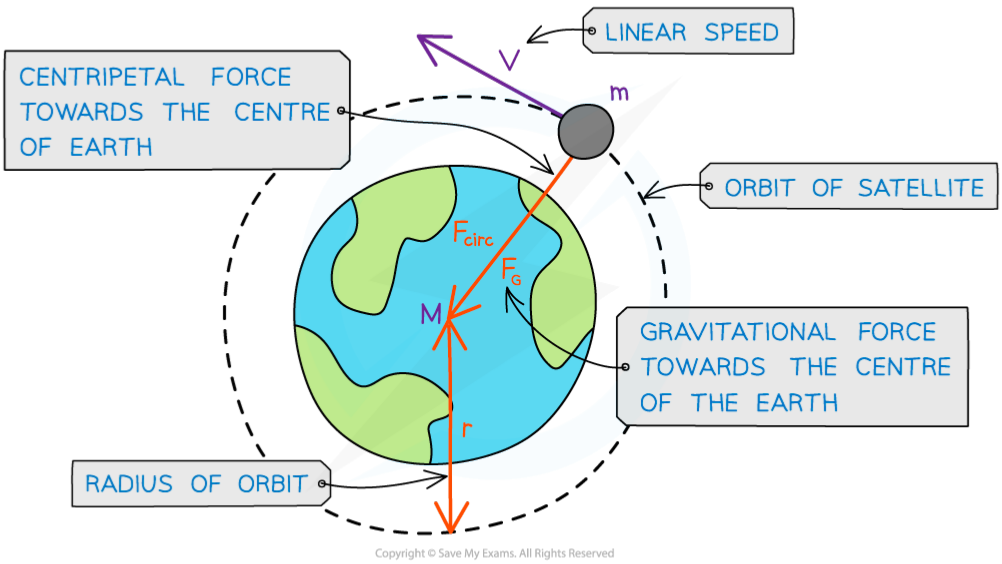
- The mass of the satellite m will cancel out on both sides to give:

$$v^2 = \frac{GM}{r}$$

- This means that all satellites, whatever their mass, will travel at the same speed v in a particular orbit radius r
- Recall that since the direction of a planet orbiting in circular motion is constantly changing, it has **centripetal acceleration**

13. Gravitational Fields

YOUR NOTES



A satellite in orbit around the Earth travels in circular motion

Kepler's Third Law of Planetary Motion

- For the orbital time period T to travel the circumference of the orbit 2πr, the linear speed v can be written as

$$v = \frac{2\pi r}{T}$$

- This is a result of the well-known equation, speed = distance / time
- Substituting the value of the linear speed v into the above equation:

$$v^2 = \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

- Rearranging leads to Kepler's third law equation:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

13. Gravitational Fields

YOUR NOTES

- The equation shows that the orbital period T is related to the radius r of the orbit. This is known as Kepler's third law:

For planets or satellites in a circular orbit about the same central body, the square of the time period is proportional to the cube of the radius of the orbit

- Kepler's third law can be summarised as:

$$T^2 \propto r^3$$

Maths Tip

- The ∝ symbol means 'proportional to'
- Find out more about proportional relationships between two variables in the "proportional relationships " section of the A Level Maths revision notes

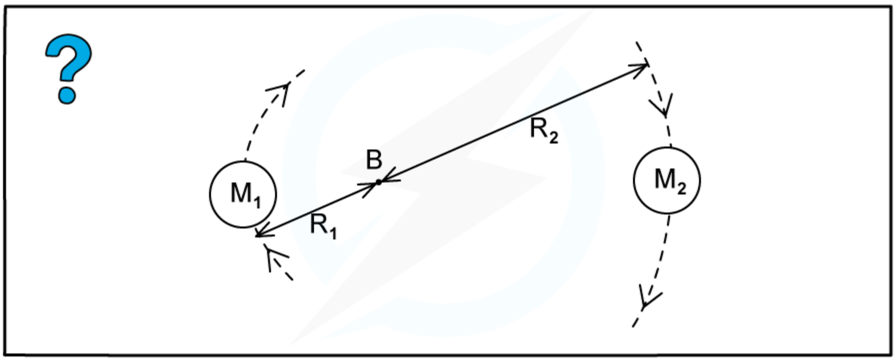
13. Gravitational Fields

YOUR NOTES

Worked example: Circular orbits in gravitational fields

A binary star system constant of two stars orbiting about a fixed point B.

The star of mass M_1 has a circular orbit of radius R_1 and mass M_2 has a radius of R_2. Both have linear speed v and an angular speed ω about B.



State the following formula, in terms of G, M_2, R_1 and R_2.

1. **The angular speed ω of M_1**
2. **The time period T for each star in terms of angular speed ω**

(1) The angular speed of ω of M_1

Step 1: Equating the centripetal force of mass M_1 to the gravitational force between M_1 and M_2

$$M_1R_1\omega^2 = \frac{GM_1M_2}{(R_1+R_2)^2}$$

Step 2: M_1 cancels on both sides

$$R_1\omega^2 = \frac{GM_2}{(R_1+R_2)^2}$$

13. Gravitational Fields

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Step 3: Rearrange for angular velocity ω

$$\omega^2 = \frac{GM_2}{R_1(R_1 + R_2)^2}$$

Step 4: Square root both sides

$$\omega = \sqrt{\frac{GM_2}{R_1(R_1 + R_2)^2}}$$

(2) The time period T for each star in terms of angular speed ω

Step 1: Angular speed equation with time period T

$$\omega = \frac{2\pi}{T}$$

Step 2: Rearrange for T

$$T = \frac{2\pi}{\omega}$$

Step 3: Substitute in ω

$$T = 2\pi \div \sqrt{\frac{GM_2}{R_1(R_1 + R_2)^2}} = 2\pi\sqrt{\frac{R_1(R_1 + R_2)^2}{GM_2}}$$

Exam Tip

Many of the calculations in the Gravitation questions depend on the equations for circular motion. Be sure to revisit these and understand how to use them!

13. Gravitational Fields

YOUR NOTES

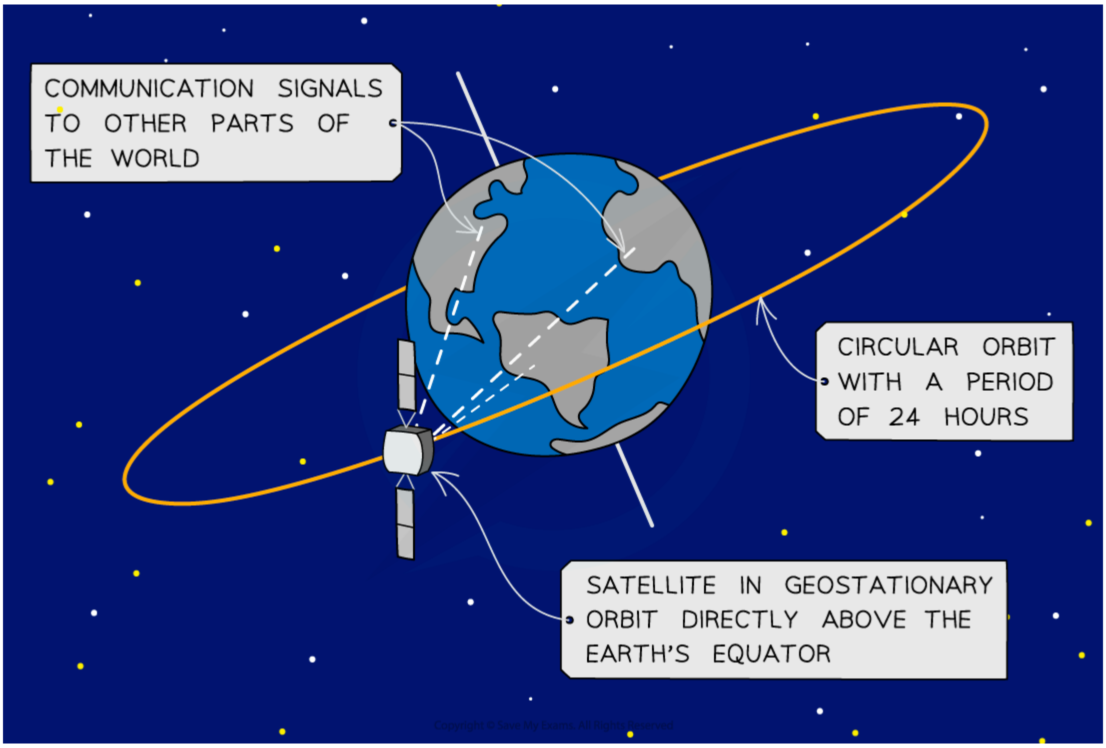
13.1.4 GEOSTATIONARY ORBITS

Geostationary Orbits

- Many communication satellites around Earth follow a **geostationary orbit**
- This is a specific type of orbit in which the satellite:
  - Remains directly above the equator, therefore, it always orbits at the same point above the Earth's surface
  - Moves from west to east (same direction as the Earth spins)
  - Has an orbital time period equal to Earth's rotational period of 24 hours
- Geostationary satellites are used for telecommunication transmissions (e.g. radio) and television broadcast
- A base station on Earth sends the TV signal up to the satellite where it is amplified and broadcast back to the ground to the desired locations
- The satellite receiver dishes on the surface must point towards the same point in the sky
  - Since the geostationary orbits of the satellites are fixed, the receiver dishes can be fixed too

13. Gravitational Fields

YOUR NOTES



Geostationary satellite in orbit

13. Gravitational Fields

YOUR NOTES

Worked example: Radius of geostationary orbit

Calculate the distance above the Earth's surface that a geostationary satellite will orbit

Mass of the Earth = 6.0×10^{24} kg.

Radius of the Earth = 6400 km.

STEP 1

KEPLER'S THIRD LAW EQUATION

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

STEP 2

REARRANGE FOR r, THE RADIUS OF THE ORBIT

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

STEP 3

SUBSTITUTE IN VALUES

THE TIME PERIOD T FOR A GEOSTATIONARY ORBIT IS 24 HOURS = 86400s

$$r = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times (86400)^2}{4\pi^2}}$$

$r = 42297523.87$ m $= 4.2 \times 10^7$ m (2 s.f.)

STEP 4

CALCULATE DISTANCE ABOVE THE EARTH'S SURFACE

r IS THE DISTANCE FROM THE CENTRE OF THE EARTH TO THE SATELLITE

DISTANCE ABOVE SURFACE = RADIUS OF ORBIT − RADIUS OF EARTH

$= 4.2 \times 10^7 - 6400 \times 10^3$

$= 3.6 \times 10^7$ m (2 s.f.)

13. Gravitational Fields

YOUR NOTES

13.1.5 GRAVITATIONAL FIELD STRENGTH

Deriving Gravitational Field Strength (g)

- The gravitational field strength at a point describes how **strong or weak a gravitational field is** at that point
- The gravitational field strength due to a point mass can be derived from combining the equations for Newton's law of gravitation and gravitational field strength
  - For calculations involving gravitational forces, a spherical mass can be treated as a point mass at the centre of the sphere
- Newton's law of gravitation states that the attractive force *F* between two masses *M* and *m* with separation *r* is equal to:

$$F_G = \frac{GMm}{r^2}$$

- The gravitational field strength at a point is defined as the force *F* per unit mass *m*

$$g = \frac{F}{m}$$

- Substituting the force *F* with the gravitational force F_G leads to:

$$g = \frac{F}{m} = \frac{\frac{GMm}{r^2}}{m}$$

- Cancelling mass *m*, the equation becomes:

$$g = \frac{GM}{r^2}$$

- Where:
  - g = gravitational field strength ($N\ kg^{-1}$)
  - G = Newton's Gravitational Constant
  - M = mass of the body producing the gravitational field (kg)
  - r = distance from the mass where you are calculating the field strength (m)

13. Gravitational Fields

YOUR NOTES

Calculating g

- Gravitational field strength, *g*, is a vector quantity
- The direction of *g* is always towards the centre of the body creating the gravitational field
  - This is the same direction as the gravitational field lines
- On the Earth's surface, *g* has a constant value of 9.81 N kg^{-1}
- However **outside the Earth's surface, *g* is not constant**
  - g decreases as r increases by a factor of $1/r^2$
  - This is an **inverse square law relationship** with distance
- When *g* is plotted against the distance from the centre of a planet, r has two parts:
  - When r < R, the radius of the planet, g is directly proportional to r
  - When r > R, g is inversely proportional to r^2 (this is an 'L' shaped curve and shows that g decreases rapidly with increasing distance r)

13. Gravitational Fields

YOUR NOTES
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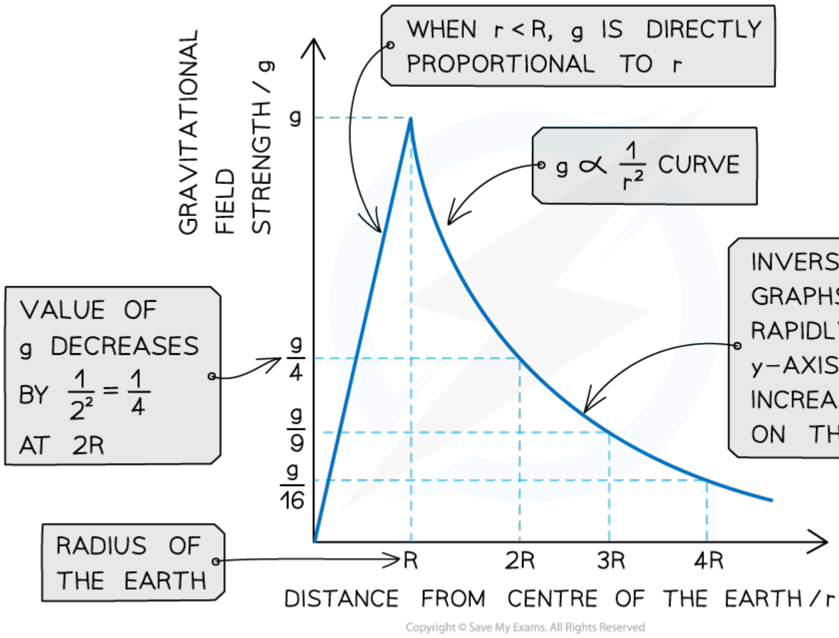

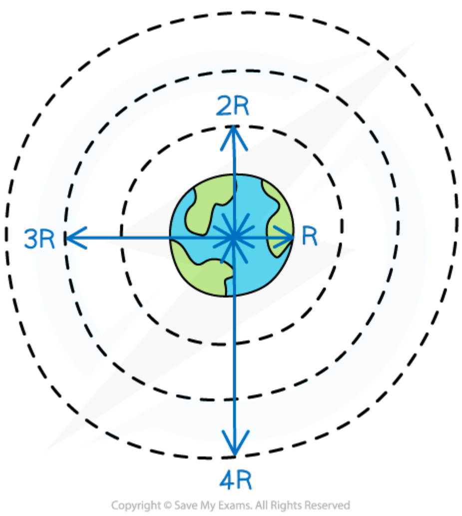

Graph showing how gravitational field strength varies at greater distance from the Earth's surface

- Sometimes, g is referred to as the 'acceleration due to gravity' with units of m s^{-2}
- Any object that falls freely in a uniform gravitational field on Earth has an acceleration of 9.81 m s^{-2}

13. Gravitational Fields

YOUR NOTES

Worked example: Gravitational field strength

The mean density of the Moon is $\frac{3}{5}$ times the mean density of the Earth. The gravitational field strength on the Moon is $\frac{1}{6}$ of the value on Earth.

Determine the ratio of the Moon's radius r_M and the Earth's radius r_E.

Step 1: Write down the known quantities

$$\rho_M = \frac{3}{5}\rho_E$$

$$g_M = \frac{1}{6}g_E$$

g_M = gravitational field strength on the Moon, ρ_M = mean density of the Moon

g_E = gravitational field strength on the Earth, ρ_E = mean density of the Earth

Step 2: The volumes of the Earth and Moon are equal to the volume of a sphere

$$V = \frac{4}{3}\pi r^3$$

Step 3: Write the density equation and rearrange for mass M

$$\rho = \frac{M}{V}$$

$$\mathbf{M = \rho V}$$

Step 4: Write the gravitational field strength equation

$$g = \frac{GM}{r^2}$$

Step 5: Substitute M in terms of ρ and V

13. Gravitational Fields

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$$g = \frac{G\rho V}{r^2}$$

Step 6: Substitute the volume of a sphere equation for V, and simplify

$$g = \frac{G\rho 4\pi r^3}{3r^2} = \frac{G\rho 4\pi r}{3}$$

Step 7: Find the ratio of the gravitational field strengths

$$\frac{g_M}{g_E} = \frac{G\rho_M 4\pi r_M}{3} \div \frac{G\rho_E 4\pi r_E}{3} = \frac{\rho_M r_M}{\rho_E r_E}$$

Step 8: Rearrange and calculate the ratio of the Moon's radius r_M and the Earth's radius r_E

$$\frac{r_M}{r_E} = \frac{\rho_E g_M}{\rho_M g_E} = \frac{\rho_E\left(\frac{1}{6}g_E\right)}{\left(\frac{3}{5}\rho_E\right)g_E}$$

$$\frac{r_M}{r_E} = \frac{5}{3} \times \frac{1}{6} = \frac{5}{18} = \mathbf{0.28} \text{ (2 s.f.)}$$

13. Gravitational Fields

YOUR NOTES ⬇

13.1.6 THE VALUE OF G ON EARTH

The Value of g on Earth

- Gravitational field strength *g* is approximately constant for small changes in height near the Earth's surface (9.81 m s^{-2})
- This is because from the inverse square law relationship:

$$g \propto \frac{1}{r^2}$$

- The value of *g* depends on the distance from the centre of Earth *r*
- On the Earth's surface, *r* is equal to the radius of the Earth = 6400 km
- Since this is much larger than the distance between the surface of the earth and centre of mass of an object on it, the small changes in height near the Earth's surface make very little difference to the value of *g*
- If we take a position *h* above the Earth's surface, where it is reasonable to assume h is much smaller than the radius of the Earth (h << R):

$$g = \frac{GM}{(R+h)^2} \simeq \frac{GM}{(R)^2}$$

- This means *g* remains approximately constant until a significant distance away from the Earth's surface

13. Gravitational Fields

YOUR NOTES
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Worked example: g on Mount Everest

- The following worked example proves that *g* decreases by very little even on the highest point on Earth

?

The highest point above the Earth's surface is at the peak of Mount Everest.
Given that this is 8800 m above the Earth's surface, show that g decreases by 0.6%.

Mass of the Earth $= 6.0 \times 10^{24}$ kg.

Radius of the Earth = 6400 km.

STEP 1

GRAVITATIONAL FIELD STRENGTH EQUATION

$$g = \frac{GM}{r^2}$$

STEP 2

DETERMINE THE VALUE OF r

r = RADIUS OF THE EARTH + HEIGHT OF MOUNT EVEREST

= 6400 km + 8.800 km = 6408.8 km

STEP 3

SUBSTITUTE VALUES IN g EQUATION

$$g = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{(6408.8 \times 10^3)^2} = 9.746127... = 9.75 \text{ ms}^{-2} \text{ (3 s.f.)}$$

STEP 4

CALCULATE PERCENTAGE DECREASE

g ON EARTH'S SURFACE $= 9.81 \text{ ms}^{-2}$

$$\% \text{ DECREASE} = \frac{\text{DECREASE}}{\text{ORIGINAL}} \times 100$$

$$\frac{9.81 - 9.75}{9.75} \times 100 = 0.6\%$$

13. Gravitational Fields

YOUR NOTES
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13.2 GRAVITATIONAL POTENTIAL

13.2.1 GRAVITATIONAL POTENTIAL

Gravitational Potential

- The gravitational potential energy (G.P.E) is the energy an object has when lifted off the ground given by the familiar equation:

$$\mathbf{G.P.E = mg\Delta h}$$

- The G.P.E on the surface of the Earth is taken to be 0
  - This means **work is done** to lift the object
- However, outside the Earth's surface, G.P.E can be defined as:

The energy an object possess due to its position in a gravitational field

- The gravitational potential at a point is the gravitational potential energy per unit mass at that point
- Therefore, the gravitational potential is defined as:

The work done per unit mass in bringing a test mass from infinity to a defined point

Calculating Gravitational Potential

- The equation for gravitational potential ϕ is defined by the mass *M* and distance *r*:

$$\phi = -\frac{GM}{r}$$

- Where:
  - ϕ = gravitational potential (J kg^{-1})
  - G = Newton's gravitational constant
  - M = mass of the body producing the gravitational field (kg)
  - r = distance from the centre of the mass to the point mass (m)

13. Gravitational Fields

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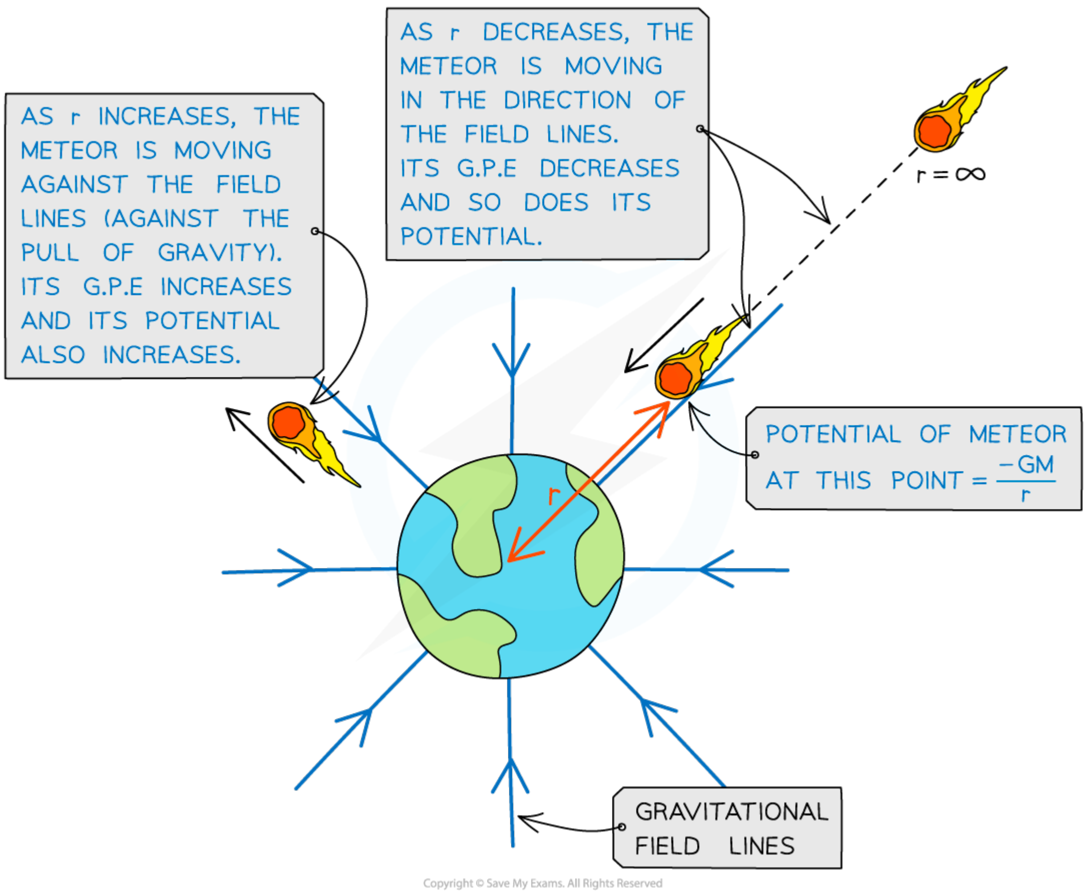
- The gravitational potential is negative near an isolated mass, such as a planet, because the **potential when *r* is at infinity is defined as 0**
- Gravitational forces are always **attractive** so as *r* decreases, positive work is done by the mass when moving from infinity to that point
    - When a mass is closer to a planet, its gravitational potential becomes smaller (more negative)
    - As a mass moves away from a planet, its gravitational potential becomes larger (less negative) until it reaches 0 at infinity
- This means when the distance *r* because very large, the gravitational force tends rapidly towards 0 at a point further away from a planet



Gravitational potential increases and decreases depending on whether the object is travelling towards or against the field lines from infinity

13. Gravitational Fields

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Worked example: Calculating gravitational potential

A planet has a diameter of 7600 km and a mass of 3.5×10^{23} kg.

A rock of mass 528 kg accelerates towards the planet from infinity.

At a distance of 400 km above the planet's surface, calculate the gravitational potential of the rock.

Step 1: Write the gravitational potential equation

$$\phi = -\frac{GM}{r}$$

Step 2: Determine the value of r

r is the distance from the centre of the planet

Radius of the planet = planet diameter ÷ 2 = 7600 ÷ 2 = 3800 km

r = 3800 + 400 = 4200 km = 4.2×10^6 m

Step 3: Substitute in values

$$\phi = -\frac{6.67 \times 10^{-11} \times 3.5 \times 10^{23}}{4.2 \times 10^{6}} = -5.6 \times 10^{6}\ \text{J kg}^{-1}$$

Exam Tip

Remember to keep the negative sign in your solution for gravitational potential. However, if you're asked for the 'change in' gravitational potential, no negative sign should be included since you are finding a difference in values (between 0 at infinity and the gravitational potential from your calculation).

13. Gravitational Fields

YOUR NOTES

13.2.2 GRAVITATIONAL POTENTIAL ENERGY

Gravitational Potential Energy Between Two Point Masses

- The gravitational potential energy (G.P.E) at point in a gravitational field is defined as:

The work done in bringing a mass from infinity to that point

- The equation for G.P.E of two point masses *m* and M at a distance *r* is:

$$\text{G.P.E} = -\frac{GMm}{r}$$

- The change in G.P.E is given by:

$$\Delta\text{G.P.E} = mg\Delta h$$

- Where:
  - m = mass of the object (kg)
  - ϕ = gravitational potential at that point (J kg^{-1})
  - Δh = change in height (m)

- Recall that at infinity, ϕ = 0 and therefore G.P.E = 0
- It is more useful to find the change in G.P.E e.g. a satellite lifted into space from the Earth's surface
- The change in G.P.E from for an object of mass *m* at a distance r_1 from the centre of mass *M*, to a distance of r_2 further away is:

$$\Delta\text{G.P.E} = -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right) = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Change in gravitational potential energy between two points

- The change in potential Δϕ is the same, without the mass of the object *m*:

$$\Delta\phi = -\frac{GM}{r_2} - \left(-\frac{GM}{r_1}\right) = GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

13. Gravitational Fields

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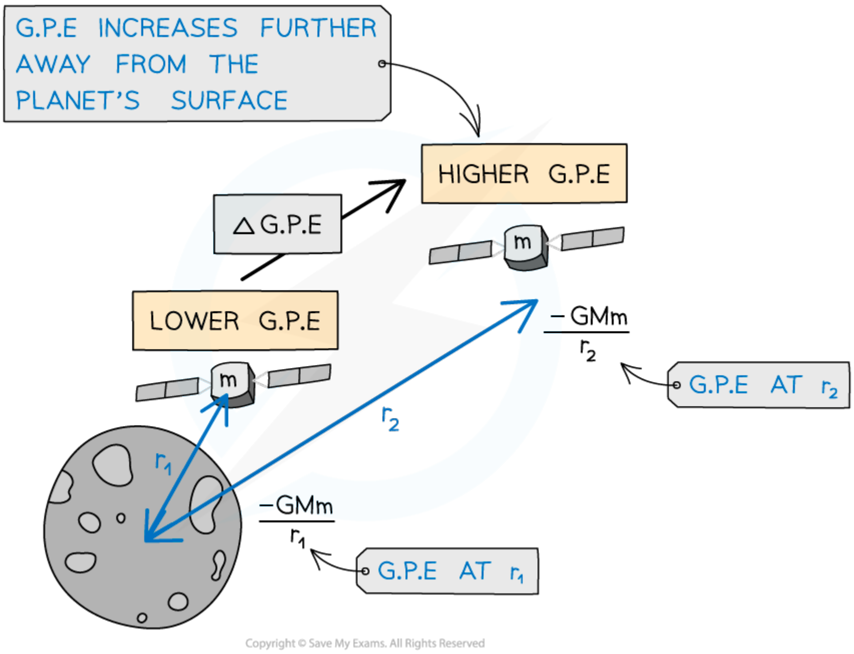
Change in gravitational potential between two points



Gravitational potential energy increases as a satellite leaves the surface of the Moon

Maths tip

- Multiplying two negative numbers equals a positive number, for example:

$$-\left(-\frac{GM}{r}\right) = +\frac{GM}{r}$$

13. Gravitational Fields

YOUR NOTES

Worked example: Gravitational potential energy

?

A spacecraft of mass 300 kg leaves the surface of Mars to an altitude of 700 km.

Calculate the change in gravitational potential energy of the spacecraft.

Radius of Mars = 3400 km
Mass of Mars = 6.40×10^{23} kg

Step 1: Difference in gravitational potential energy equation

$$\Delta G.P.E = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Step 2: Determine values for r_1 and r_2

r_1 is the radius of Mars = 3400 km = 3400×10^3 m

r_2 is the radius + altitude = 3400 + 700 = 4100 km = 4100×10^3 m

Step 3: Substitute in values

$$\Delta G.P.E = 6.67 \times 10^{-11} \times 6.40 \times 10^{23} \times 300 \times \left(\frac{1}{3400 \times 10^3} - \frac{1}{4100 \times 10^3}\right)$$

ΔG.P.E = 643.076×10^6 = 640 MJ (2 s.f.)

13. Gravitational Fields

YOUR NOTES

Exam Tip

Make sure to not confuse the ΔG.P.E equation with

$\Delta G.P.E = mg\Delta h$

The above equation is only relevant for an object lifted in a uniform gravitational field (close to the Earth's surface). The new equation for G.P.E will not include g, because this varies for different planets and is no longer a constant (decreases by $1/r^2$) outside the surface of a planet.

14. Temperature

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CONTENTS

14.1 MEASURING TEMPERATURE

14.1.1 THERMAL ENERGY TRANSFER

Thermal Energy Transfer

- The conservation of energy states that energy is never created or destroyed, only transferred from one form to another
- When a thermometer is placed in a beaker of boiling water, the thermometer reading increases
  - This is because the thermometer is a lot cooler than the water
- The thermometer gradually becomes hotter from the thermal energy (or heat) transferring from the water to the thermometer
- Thermal energy is defined as:

Thermal energy is transferred from a region of higher temperature to a region of lower temperature

- The energy will continue to be transferred until both the thermometer and the water are at the same temperature
- This means temperature tells us the **direction of energy flow** when two regions are in contact (from hotter to cooler)

14. Temperature

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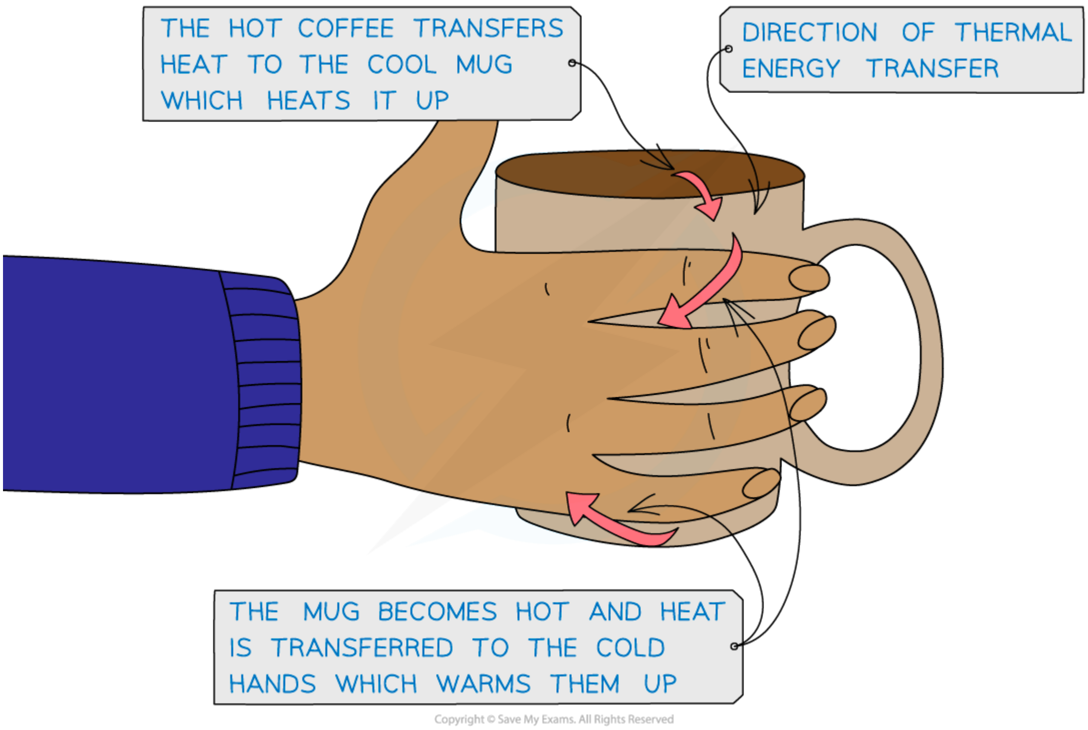

Thermal energy is transferred from the hot coffee to the mug and to the cold hands

- The mechanism by which the thermal energy is transferred is by either conduction, convection or radiation

Exam Tip

Sometimes the direction of heat transfer might seem counterintuitive to what we observe in everyday life. When ice is placed in room temperature water, it melts. This is because the water transfers heat energy to the ice (not the ice giving it's 'cold' to the water).

14. Temperature

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14.1.2 THERMAL EQUILIBRIUM

Defining Thermal Equilibrium

- Thermal energy is **always** transferred from a hotter region to lower region
- Thermal equilibrium is defined as:

When two substances in physical contact with each other no longer exchange any heat energy and both reach an equal temperature

- There is no longer thermal energy transfer between the regions

- The two regions need to be in contact for this to occur
- The hotter region will cool down and the cooler region will heat up until they reach the same temperature
- The final temperature when two regions are in thermal equilibrium depends on the initial temperature difference between them
- An example of this is ice in room temperature water. The ice cubes heat up from the energy transfer from the water and the water cools down due to the ice until the water's temperature is in thermal equilibrium

14. Temperature

YOUR NOTES

14.1.3 MEASUREMENT OF TEMPERATURE

Measurement of Temperature

- A thermometer is any device that is used to measure temperature
- Each type of thermometer uses a physical property of a material that varies with temperature – examples of such properties include:
  - The density of a liquid
  - The volume of a gas at constant pressure
  - Resistance of a metal
  - e.m.f. of a thermocouple
- In each case, the thermometer must be calibrated at two or more known temperatures (commonly the boiling and melting points of water, 0°C and 100°C respectively) and the scale divided into equal divisions

The Density of a Liquid

- A liquid-in-glass thermometer depends on the density change of a liquid (commonly mercury)
- It consists of a thin glass capillary tube containing a liquid that **expands** with temperature
- A scale along the side of the tube allows the temperature to be measured based on the length of liquid within the tube

Volume of a Gas at Constant Pressure

- The volume of an ideal gas is directly proportional to its temperature when at constant pressure (Charles's law)

$$\mathbf{V \propto T}$$

- As the temperature of the gas increases, its volume increases and vice versa
- A gas thermometer must be calibrated – by knowing the temperature of the gas at a certain volume, a temperature scale can be determined depending on how quickly the gas expands with temperature

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Resistance of a Metal

- Recall that electrical resistance changes with temperature e.g. the resistance of a filament lamp increases when current increases through it
    - For metals: resistance increases with temperature at a steady rate
    - For thermistors: resistance changes rapidly over a narrow range of temperatures
- As a thermistor gets hotter, its resistance decreases
- This means a thermometer based on a thermistor can be used to measure a range of temperatures
- The relationship between the resistance and temperature is non-linear
    - This means the graph of temperature against resistance will be a curved line and the thermistor will have to be calibrated

E.M.F. of a Thermocouple

- A thermocouple is an electrical device used as the sensor of a thermometer
- It consists of two wires of different, or dissimilar, metals attached to each other, producing a junction on one end
    - The opposite ends are connected to a voltmeter
- When this junction is heated, an e.m.f. is produced between the two wires which is measured on the voltmeter
- The greater the difference in temperature between the wires, the greater the e.m.f
- However, a thermocouple requires calibration since the e.m.f. does not vary linearly with temperature
- The graph against e.m.f. and temperature is a positive, curved line

Exam Tip

Remember to relate how the temperature is measured for different types of thermometer back to the scenario in the question. For example, make sure you say: the temperature increases as the **volume of gas increases** or the temperature increases as the **e.m.f. between the two wires increases**.

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14.1.4 THE KELVIN SCALE

Scale of Thermodynamic Temperature

- As an everyday scale of temperature, Celsius (°C) is the most familiar
- This scale is based on the properties of water – the freezing point of water was taken as taken as 0 °C and the boiling point as 100 °C
  - However, there is nothing special about these two temperatures
  - The freezing and boiling point of water will actually change as its pressure changes
- The Celsius scale is used to measure the temperature in a liquid-in-glass thermometer
  - However, the expansion of the liquid might be non-linear
- Other temperature scales include:
  - Fahrenheit, commonly used in the US
  - Kelvin, used in thermodynamics

- The Kelvin scale is known as the **thermodynamic scale** and was designed to overcome the problem with scales of temperature
- The thermodynamic scale is said to be an absolute scale that is not defined in terms of a property of any particular substance
- This is because thermodynamic temperatures **do not depend** on the property of any particular substance

Absolute Zero

- On the thermodynamic (Kelvin) temperature scale, absolute zero is defined as:

The lowest temperature possible. Equal to 0 K or -273.15 °C

- It is not possible to have a temperature lower than 0 K
  - This means a temperature in Kelvin will **never** be a negative value
- Absolute zero is defined in kinetic terms as:

The temperature at which the atoms and molecules in all substances have zero kinetic and potential energy

- This means for a system at 0 K, it is not possible to remove any more energy from it
- Even in space, the temperature is roughly 2.7 K, just above absolute zero

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Using the Kelvin Scale

- To convert between temperatures θ in the Celsius scale, and T in the Kelvin scale, use the following conversion:

$$\boldsymbol{\theta} \textbf{ / °C = T / K − 273.15}$$

$$\textbf{T / K = } \boldsymbol{\theta} \textbf{ / °C + 273.15}$$

- The divisions on both scales are equal. This means:

A change in a temperature of 1 K is equal to a change in temperature of 1 °C

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Worked example: Kelvin conversion

In many ideal gas problems, room temperature is considered to be 300 K.

What is this temperature in Celsius?

Step 1: Kelvin to Celsius equation

$$\boldsymbol{\theta} \textbf{ / °C = T / K − 273.15}$$

Step 2: Substitute in value of 300 K

$$\textbf{300 K − 273.15 = 26.85 °C}$$

Exam Tip

If you forget in the exam whether it's +273.15 or −273.15, just remember that 0 °C = 273.15 K. This way, when you know that you need to +273.15 to a temperature in degrees to get a temperature in Kelvin. For example: 0 °C + 273.15 = 273.15 K.

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14.2 PHASE CHANGES

14.2.1 SPECIFIC HEAT CAPACITY

Defining Specific Heat Capacity

- The specific heat capacity of substance is defined as:

  The amount of thermal energy required to raise the temperature of 1 kg of a substance by 1 °C

- This quantity determines the amount of energy needed to change the temperature of a substance
- The specific heat capacity is measured in units of **Joules per kilogram per Kelvin** (J kg^{-1} K^{-1}) or **Joules per kilogram per Celsius** (J kg^{-1} $°C^{-1}$) and has the symbol ***c***
  - Different substances have different specific heat capacities
  - Specific heat capacity is mainly used in liquids and solids
- From the definition of specific heat capacity, it follows that:
  - The heavier the material, the more thermal energy that will be required to raise its temperature
  - The larger the change in temperature, the higher the thermal energy will be required to achieve this change

Calculating Specific Heat Capacity

- The amount of thermal energy Q needed to raise the temperature by $\Delta\theta$ for a mass m with specific heat capacity c is equal to:

$$\Delta Q = mc\Delta\theta$$

- Where:
  - ΔQ = change in thermal energy (J)
  - m = mass of the substance you are heating up (kg)
  - c = specific heat capacity of the substance (J $kg^{-1}K^{-1}$or J kg^{-1} $°C^{-1}$)
  - Δθ = change in temperature (K or °C)

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- If a substance has a **low** specific heat capacity, it heats up and cools down quickly
- If a substance has a **high** specific heat capacity, it heats up and cools down slowly
- The specific heat capacity of different substances determines how useful they would be for a specific purpose eg. choosing the best material for kitchen appliances

- Good electrical conductors, such as copper and lead, are also excellent conductors of heat due to their low specific heat capacity

Worked example: Calculating specific heat capacity

A kettle is rated at 1.7 kW. A mass of 650 g of a liquid at 25 °C is poured into a kettle.

When the kettle is switched on, it takes 3.5 minutes to start boiling.

Calculate the specific heat capacity of the liquid.

Step 1: Calculate the Energy from the power and time

Energy = Power × Time

Power = 1.7 kW = 1.7×10^3 W

Time = 3.5 minutes = 3.5 × 60 = 210 s

Energy = $1.7 \times 10^3 \times 210 = 3.57 \times 10^5$ J

Step 2: Thermal energy equation

$$\Delta Q = mc\Delta\theta$$

Step 3: Rearrange for specific heat capacity

$$c = \frac{\Delta Q}{m\Delta\theta}$$

Step 4: Substitute in values

m = 650 g = 650×10^{-3} kg

$\Delta\theta$ = 100 - 25 = 75°C

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$$c = \frac{3.57 \times 10^{5}}{650 \times 10^{-3} \times 75} = 7323.07... = 7300 \text{ J kg}^{-1}\,^{\circ}\text{C}^{-1} \text{ (2 s.f)}$$

Exam Tip

The difference in temperature $\Delta\theta$ will be exactly the same whether the temperature is given in Celsius or Kelvin. Therefore, there is no need to convert between the two since the **difference** in temperature will be the same for both units.

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14.2.2 SPECIFIC LATENT HEAT CAPACITY

Defining Latent Heat Capacity

- Energy is required to change the **state** of substance
- Examples of changes of state are:
  - Melting = solid to liquid
  - Evaporation/vaporisation/boiling = liquid to gas
  - Sublimation = solid to gas
  - Freezing = liquid to solid
  - Condensation = gas to liquid

- When a substance changes state, there is **no temperature change**
- The energy supplied to change the state is called the **latent heat** and is defined as:

The thermal energy required to change the state of 1 kg of mass of a substance without any change of temperature

- There are two types of latent heat:
  - Specific latent heat of **fusion** (melting)
  - Specific latent heat of **vaporisation** (boiling)

- The specific latent heat of **fusion** is defined as:

The thermal energy required to convert 1 kg of solid to liquid with no change in temperature

  - This is used when melting a solid or freezing a liquid

- The specific latent heat of **vaporisation** is defined as:

The thermal energy required to convert 1 kg of liquid to gas with no change in temperature

  - This is used when vaporising a liquid or condensing a gas

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Calculating Specific Latent Heat

- The amount of energy *Q* required to melt or vaporise a mass of *m* with latent heat *L* is:

$$Q = mL$$

- Where:
  - Q = amount of thermal energy to change the state (J)
  - m = mass of the substance changing state (kg)
  - L = latent heat of fusion or vaporisation (J kg^{-1})

- The values of latent heat for water are:
  - Specific latent heat of fusion = 330 kJ kg^{-1}
  - Specific latent heat of vaporisation = 2.26 MJ kg^{-1}
- Therefore, evaporating 1 kg of water requires roughly **seven times** more energy than melting the same amount of ice to form water
- The reason for this is to do with intermolecular forces:
  - **When ice melts**: energy is required to just increase the molecule separation until they can flow freely over each other
  - **When water boils**: energy is required to completely separate the molecules until there are no longer forces of attraction between the molecules, hence this requires much more energy

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Worked example: Specific latent heat

?

The energy needed to boil a mass of 530 g of a liquid is 0.6 MJ.

Calculate the specific latent heat of the liquid and state whether it is the latent heat of vaporisation or fusion.

Step 1: Write the thermal energy required to change state equation

$$\mathbf{Q = mL}$$

Step 2: Rearrange for latent heat

$$L = \frac{Q}{m}$$

Step 3: Substitute in values

$$\mathbf{m = 530\ g = 530 \times 10^{-3}\ kg}$$

$$\mathbf{Q = 0.6\ MJ = 0.6 \times 10^{6}\ J}$$

$$L = \frac{0.6 \times 10^6}{530 \times 10^{-3}} = 1.132 \times 10^6\ \text{J kg}^{-1} = \mathbf{1.1\ MJ\ kg^{-1}}\ (2\ \text{s.f.})$$

L is the latent heat of vaporisation because the change in state is from liquid to gas (boiling)

Exam Tip

Use these reminders to help you remember which type of latent heat is being referred to:

• Latent heat of fusion = imagine 'fusing' the liquid molecules together to become a solid

• Latent heat of vaporisation = "water vapour" is steam, so imagine vaporising the liquid molecules into a gas

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15.1 IDEAL GAS LAW

15.1.1 THE MOLE

Amount of Substance

- In thermodynamics, the amount of substance is measured in the SI unit 'mole'
  - This has the symbol **mol**
  - The mole is a unit of **substance**, not a unit of mass
- The mole is defined as:

The SI base unit of an 'amount of substance'. It is the amount containing as many particles (e.g. atoms or molecules) as there are atoms in 12 g of carbon-12

- The mole is an important unit in thermodynamics
- If we consider the number of moles of two different gases under the same conditions, their physical properties are the same

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The Avogadro Constant

- In AS Physics, the atomic mass unit (u) was introduced as approximately the mass of a proton or neutron = 1.66×10^{-27} kg
- This means that an atom or molecule has a mass approximately equal to the number of protons and neutrons it contains
- A carbon-12 atom has a mass of:

$$\mathbf{12\ u = 12 \times 1.66 \times 10^{-27} = 1.99 \times 10^{-26}\ kg}$$

- The exact number for a mole is defined as the number of molecules in exactly 12 g of carbon:

$$1 \text{ mole} = \frac{0.012}{1.99 \times 10^{-26}} = \mathbf{6.02 \times 10^{23}\ molecules}$$

- Avogadro's constant *(N_A)* is defined as:

The number of atoms of carbon-12 in 12 g of carbon-12; equal to 6.02 × 10^{23} mol^{-1}

- For example, 1 mole of sodium (Na) contain 6.02×10^{23} **atoms** of sodium
- The number of atoms can be determined if the number of moles is known, for example:

2.0 mol of nitrogen contains: $2.0 \times N_A = 2.0 \times 6.02 \times 10^{23} = 1.20 \times 10^{24}$ atoms

Mole and the Atomic Mass

- One mole of any element is equal to the relative atomic mass of that element in grams
  - e.g. helium has an atom mass of 4 – this means 1 mole of helium has a mass of 4 g
- If the substance is a compound, add up the relative atomic masses, for example, water (H_2O) is made up of
  - 2 hydrogen atoms (atomic mass of 1) and 1 oxygen atom (atomic mass of 16)
  - So, 1 mole of water would have a mass of (2 × 1) + 16 = 18 g

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Molar Mass

- The molar mass of a substance is the mass, in grams, in one mole
  - Its unit is **g mol^{-1}**
- The number of moles from this can be calculated using the equation:

$$\text{Number of moles} = \frac{mass\ (g)}{molar\ mass\ (g\ mol^{-1})}$$

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Worked example

How many molecules are there in 6 g of magnesium-24?

Step 1: Calculate the mass of 1 mole of magnesium

One mole of any element is equal to the relative atomic mass of that element in grams

1 mole = 24 g of magnesium

Step 2: Calculate the amount of moles in 6 g

$$\frac{6}{24} = 0.25 \text{ moles}$$

Step 3: Convert the moles to number of molecules

1 mole = 6.02×10^{23} molecules

0.25 moles = $0.25 \times 6.02 \times 10^{23} = 1.51 \times 10^{23}$ molecules

Exam Tip

If you want to find out more about the mole, check out the CIE A Level Chemistry revision notes!

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15.1.2 IDEAL GASES

Ideal Gases

- An **ideal gas** is one which obeys the relation:

$$\mathbf{pV \propto T}$$

- Where:
  - p = pressure of the gas (Pa)
  - V = volume of the gas (m^3)
  - T = thermodynamic temperature (K)

- The molecules in a gas move around randomly at high speeds, colliding with surfaces and exerting pressure upon them

- Imagine molecules of gas free to move around in a box
- The temperature of a gas is related to the average speed of the molecules:
  - The hotter the gas, the faster the molecules move
  - Hence the molecules collide with the surface of the walls more frequently
- Since force is the rate of change of momentum:
  - Each collision applies a **force** across the surface area of the walls
  - The faster the molecules hit the walls, the greater the force on them
- Since pressure is the **force per unit area**
  - **Higher temperature leads to higher pressure**
- If the volume *V* of the box decreases, and the temperature *T* stays constant*:*
  - There will be a smaller surface area of the walls and hence more collisions
  - This also creates more pressure
- Since this equates to a greater force per unit area, pressure in an ideal gas is therefore defined by:

The frequency of collisions of the gas molecules per unit area of a container

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Boyle’s Law

- If the temperature T is constant, then **Boyle’s Law** is given by:

$$p \propto \frac{1}{V}$$

- This leads to the relationship between the pressure and volume for a fixed mass of gas at constant temperature:

$$\mathbf{P_1V_1 = P_2V_2}$$

Charles’s Law

- If the pressure P is constant, then **Charles’s law** is given by:

$$\mathbf{V \propto T}$$

- This leads to the relationship between the volume and thermodynamic temperature for a fixed mass of gas at constant pressure:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Pressure Law

- If the volume V is constant, the the **Pressure law** is given by:

$$\mathbf{P \propto T}$$

- This leads to the relationship between the pressure and thermodynamic temperature for a fixed mass of gas at constant volume:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

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Worked example

An ideal gas is in a container of volume 4.5×10^{-3} m^3.

The gas is at a temperature of 30 °C and a pressure of 6.2×10^5 Pa.

Calculate the pressure of the ideal gas in the same container when it is heated to 40 °C.

Step 1: Ideal gas relation between pressure, volume and temperature

$$pV \propto T$$

Step 2: Write the equation in full

pV = kT **where k is the constant of proportionality**

Step 3: Rearrange for the constant of proportionality

$$k = \frac{pV}{T}$$

Step 4: Convert temperature T into Kelvin

$$\theta°C + 273.15 = T\ K$$

$$30°C + 273.15 = 303.15\ K$$

Step 5: Substitute in known value into constant of proportionality equation

$$k = \frac{6.2 \times 10^5 \times 4.5 \times 10^{-3}}{303.15} = 9.203...$$

Step 6: Rearrange ideal gas relation equation for pressure

$$p = \frac{kT}{V}$$

Step 7: Substitute in new values

$$k = 9.203...$$

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V stays the same = 4.5×10^{-3} m^3

T = 40 °C = 40 + 273.15 = 313.15 K

$$p = \frac{(9.203...) \times 313.15}{4.5 \times 10^{-3}} = 640.45... \times 10^3 \text{ Pa} = \mathbf{640\ kPa}$$

Exam Tip

Don't round too early in your working out! In the worked example, the unrounded value of k is represented by "…" to show its full value is to be carried over to the next step of the calculation. On your calculator, this can be done by using the "ans" button instead of typing in the whole number.

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15.1.3 IDEAL GAS EQUATION

Ideal Gas Equation

- The equation of state for an ideal gas (or the ideal gas equation) can be expressed as:

$$pV = nRT$$

- The ideal gas equation can also be written in the form:

$$pV = NkT$$

- An ideal gas is therefore defined as:

A gas which obeys the equation of state pV = nRT at all pressures, volumes and temperatures

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Worked example

A storage cylinder of an ideal gas has a volume of $8.3 \times 10^3 \text{ cm}^3$.

The gas is at a temperature of 15 °C and a pressure of 4.5×10^7 Pa.

Calculate the amount of gas in the cylinder, in moles.

Step 1: Write down the ideal gas equation

Since the number of moles (n) is required, use the equation:

$$pV = nRT$$

Step 2: Rearrange for the number of moles n

$$n = \frac{pV}{RT}$$

Step 3: Substitute in values

$$V = 8.3 \times 10^3 \text{ cm}^3 = 8.3 \times 10^3 \times 10^{-6} = 8.3 \times 10^{-3} \text{ m}^3$$

$$T = 15 \text{ °C} + 273.15 = 288.15 \text{ K}$$

$$n = \frac{4.5 \times 10^7 \times 8.3 \times 10^{-3}}{8.31 \times 288.15} = 155.98 = \mathbf{160 \text{ mol}} \text{ (2 s.f.)}$$

Exam Tip

Don't worry about remembering the values of R and k, they will both be given in the equation sheet in your exam.

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The Boltzmann Constant

- The Boltzmann constant k is used in the ideal gas equation and is defined by the equation:

$$k = \frac{R}{N_A}$$

- Where:
  - R = molar gas constant
  - N_A = Avogadro's constant
- Boltzmann's constant therefore has a value of

$$k = \frac{8.31}{6.02 \times 10^{23}} = \mathbf{1.38 \times 10^{-23}\ J\ K^{-1}}$$

- The Boltzmann constant relates the properties of microscopic particles (e.g. kinetic energy of gas molecules) to their macroscopic properties (e.g. temperature)
  - This is why the units are $J\ K^{-1}$
- Its value is very small because the increase in kinetic energy of a molecule is very small for every incremental increase in temperature

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15.2 KINETIC THEORY

15.2.1 KINETIC THEORY OF GASES

Assumptions of the Kinetic Theory of Gases

- Gases consist of atoms or molecules randomly moving around at high speeds
- The kinetic theory of gases models the thermodynamic behaviour of gases by linking the **microscopic properties** of particles (mass and speed) to **macroscopic properties** of particles (pressure and volume)

- The theory is based on a set of the following assumptions:
  - Molecules of gas behave as identical, hard, perfectly elastic spheres
  - The volume of the molecules is negligible compared to the volume of the container
  - The time of a collision is negligible compared to the time between collisions
  - There are no forces of attraction or repulsion between the molecules
  - The molecules are in continuous random motion
- The number of molecules of gas in a container is very large, therefore the **average** behaviour (eg. speed) is usually considered

Exam Tip

Make sure to memorise **all** the assumptions for your exams, as it is a common exam question to be asked to recall them.

Root-Mean-Square Speed

- The pressure of an ideal gas equation includes the **mean square** speed of the particles:

$$\langle c^2 \rangle$$

- Where
  - c = **average** speed of the gas particles
  - $\langle c^2 \rangle$ has the units $\mathbf{m^2\ s^{-2}}$

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- Since particles travel in all directions in 3D space and velocity is a vector, some particles will have a negative direction and others a positive direction
- When there are a large number of particles, the total positive and negative velocity values will cancel out, giving a net zero value overall
- In order to find the pressure of the gas, the **velocities must be squared**
  - This is a more useful method, since a negative or positive number squared is **always positive**
- To calculate the **average speed** of the particles in a gas, take the square root of the mean square speed:

$$\sqrt{< c^2 >} = c_{r.m.s}$$

- $c_{r.m.s}$ is known as the **root-mean-square** speed and still has the units of **m s^{-1}**
- The mean square speed is **not** the same as the mean speed

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Worked example: Root-mean-square speed

An ideal gas has a density of 4.5 kg m^{-3} at a pressure of 9.3 × 10^{5} Pa and a temperature of 504 K.

Determine the root-mean-square (r.m.s.) speed of the gas atoms at 504 K.

Step 1: Write out the equation for the pressure of an ideal gas with density

$$p = \frac{1}{3}\rho\langle c^2\rangle$$

Step 2: Rearrange for mean square speed

$$\langle c^2\rangle = \frac{3p}{\rho}$$

Step 3: Substitute in values

$$\langle c^2\rangle = \frac{3 \times (9.3 \times 10^5)}{4.5} = 6.2 \times 10^5 \text{ m}^2 \text{ s}^{-2}$$

Step 4: To find the r.m.s value, take the square root of the mean square speed

$$c_{r.m.s} = \sqrt{\langle c^2\rangle} = \sqrt{6.2 \times 10^5} = 787.4 = \mathbf{790 \text{ m s}^{-1}} \text{ (2 s.f)}$$

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15.2.2 DERIVATION OF THE KINETIC THEORY OF GASES EQUATION

Derivation of the Kinetic Theory of Gases Equation

- When molecules rebound from a wall in a container, the change in momentum gives rise to a force exerted by the particle on the wall
- Many molecules moving in random motion exert forces on the walls which create an average overall **pressure**, since pressure is the force per unit area

- Picture a single molecule in a cube-shaped box with sides of equal length *l*
- The molecule has a mass *m* and moves with speed *c*, parallel to one side of the box
- It collides at regular intervals with the ends of the box, exerting a force and contributing to the pressure of the gas
- By calculating the pressure this one molecule exerts on one end of the box, the total pressure produced by all the molecules can be deduced

5 Step Derivation

1. **Find the change in momentum as a single molecule hits a wall perpendicularly**

- One assumption of the kinetic theory is that molecules **rebound elastically**
- This means there is no kinetic energy lost in the collision
- If they rebound in the opposite direction to their initial velocity, their final velocity is -c
- The change in momentum is therefore:

$$\Delta p = -mc - (+mc) = -mc - mc = -2mc$$

2. **Calculate the number of collisions per second by the molecule on a wall**

- The time between collisions of the molecule travelling to one wall and back is calculated by travelling a distance of 2*l* with speed c:

$$\text{Time between collisions} = \frac{distance}{speed} = \frac{2l}{c}$$

- **Note:** c is **not** taken as the speed of light in this scenario

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3. **Find the change in momentum per second**

- The force the molecule exerts on one wall is found using Newton's second law of motion:

$$\text{Force} = \text{rate of change of momentum} = \frac{\Delta p}{\Delta t} = \frac{2mc}{\frac{2l}{c}} = \frac{mc^2}{l}$$

- The change in momentum is +2mc since the force on the molecule from the wall is in the opposite direction to its change in momentum

4. **Calculate the total pressure from N molecules**

- The area of one wall is l^2
- The pressure is defined using the force and area:

$$\text{Pressure } p = \frac{Force}{Area} = \frac{\frac{mc^2}{l}}{l^2} = \frac{mc^2}{l^3}$$

- This is the pressure **exerted from one molecule**
- To account for the large number of *N* molecules, the pressure can now be written as:

$$p = \frac{Nmc^2}{l^3}$$

- Each molecule has a different velocity and they all contribute to the pressure
- The mean squared speed of c^2 is written with left and right-angled brackets $<c^2>$
- The pressure is now defined as:

$$p = \frac{Nm<c^2>}{l^3}$$

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5. **Consider the effect of the molecule moving in 3D space**

- The pressure equation still assumes all the molecules are travelling in the same direction and colliding with the same pair of opposite faces of the cube
- In reality, all molecules will be moving in three dimensions equally
- Splitting the velocity into its components c_x, c_y and c_z to denote the amount in the x, y and z directions, c^2 can be defined using pythagoras' theorem in 3D:

$$\mathbf{c^2 = c_x^2 + c_y^2 + c_z^2}$$

- Since there is nothing special about any particular direction, it can be determined that:

$$\mathbf{\langle c_x^2\rangle = \langle c_y^2\rangle = \langle c_z^2\rangle}$$

- Therefore, $\langle c_x^2\rangle$ can be defined as:

$$\langle c_x^2\rangle = \frac{1}{3}\langle c^2\rangle$$

- The box is a cube and all the sides are of length *l*
  - This means l^3 is equal to the volume of the cube, *V*
- Substituting the new values for $\langle c^2\rangle$ and l^3 back into the pressure equation obtains the final equation:

$$pV = \frac{1}{3}Nm\langle c^2\rangle$$

- This is known as the **Kinetic Theory of Gases equation**
- Where:
  - p = pressure (Pa)
  - V = volume (m^3)
  - N = number of molecules
  - m = mass of one molecule of gas (kg)
  - $\langle c^2\rangle$ = mean square speed of the molecules ($m\ s^{-1}$)

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- This can also be written using the density ρ of the gas:

$$\rho = \frac{mass}{volume} = \frac{Nm}{V}$$

- Rearranging the pressure equation for p and substituting the density ρ:

$$p = \frac{1}{3}\rho\langle c^2\rangle$$

Exam Tip

Make sure to revise and understand each step for the whole of the derivation, as you may be asked to derive all, or part, of the equation in an exam question.

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15.2.3 AVERAGE KINETIC ENERGY OF A MOLECULE

Average Kinetic Energy of a Molecule

- An important property of molecules in a gas is their **average kinetic energy**
- This can be deduced from the ideal gas equations relating pressure, volume, temperature and speed
- Recall the ideal gas equation:

$$pV = NkT$$

- Also recall the equation linking pressure and mean square speed of the molecules:

$$pV = \frac{1}{3} Nm\langle c^2 \rangle$$

- The left hand side of both equations are equal (pV)
- This means the right hand sides are also equal:

$$\frac{1}{3} Nm\langle c^2 \rangle = NkT$$

- N will cancel out on both sides and multiplying by 3 obtains the equation:

$$m\langle c^2 \rangle = 3kT$$

- Recall the familiar kinetic energy equation from mechanics:

$$\text{Kinetic energy} = \frac{1}{2} mv^2$$

- Instead of v^2 for the velocity of one particle, $\langle c^2 \rangle$ is the average speed of all molecules

15. Ideal Gases

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- Multiplying both sides of the equation by ½ obtains the **average translational kinetic energy** of the molecules of an ideal gas:

$$E_K = \frac{1}{2} m\langle c^2\rangle = \frac{3}{2} kT$$

- Where:
  - E_K = kinetic energy of a molecule (J)
  - m = mass of one molecule (kg)
  - $\langle c^2\rangle$ = mean square speed of a molecule (m^2 s^{-2})
  - k = Boltzmann constant
  - T = temperature of the gas (K)
- **Note:** this is the average kinetic energy for only **one** molecule of the gas
- A key feature of this equation is that the mean kinetic energy of an ideal gas molecule is proportional to its thermodynamic temperature

$$\mathbf{E_K \propto T}$$

- **Translational** kinetic energy is defined as:

The energy a molecule has as it moves from one point to another

- A monatomic (one atom) molecule only has translational energy, whilst a diatomic (two-atom) molecule has both **translational** and **rotational energy**

15. Ideal Gases

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Worked example: Average kinetic energy of a molecule

Helium can be treated as an ideal gas.

Helium molecules have a root-mean-square (r.m.s.) speed of 730 m s^{-1} at a temperature of 45 °C.

Calculate the r.m.s. speed of the molecules at a temperature of 80 °C.

Step 1: Write down the equation for the average translational kinetic energy:

$$E_K = \frac{1}{2} m\langle c^2\rangle = \frac{3}{2} kT$$

Step 2: Find the relation between $c_{r.m.s}$ and temperature T

Since m and k are constant, $\langle c^2\rangle$ is directly proportional to T

$$\langle c^2\rangle \propto T$$

Therefore, the relation between $c_{r.m.s}$ and T is:

$$c_{r.m.s} = \sqrt{\langle c^2\rangle} \propto \sqrt{T}$$

Step 3: Write the equation in full

$$c_{r.m.s} = a\sqrt{T}$$

where a is the constant of proportionality

Step 4: Calculate the constant of proportionality from values given by rearranging for a:

$$T = 45\ ^\circ C + 273.15 = 318.15\ K$$

$$a = \frac{c_{r.m.s}}{\sqrt{T}} = \frac{730}{\sqrt{318.15}} = 40.92...$$

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Step 5: Calculate $c_{r.m.s}$ at 80 °C by substituting the value of a and new value of T

$$\mathbf{T = 80\ ^{\circ}C + 273.15 = 353.15\ K}$$

$$c_{r.m.s} = \frac{730}{\sqrt{318.15}} \times \sqrt{353.15} = 769 = \mathbf{770\ m\ s^{-1}}\ (2\ \text{s.f.})$$

Exam Tip

Keep in mind this particular equation for kinetic energy is only for **one** molecule in the gas. If you want to find the kinetic energy for all the molecules, remember to multiply by ***N***, the total number of molecules. You can remember the equation through the rhyme 'Average K.E is three-halves kT'.

16. Thermodynamics

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16.1 THE FIRST LAW OF THERMODYNAMICS

16.1.1 INTERNAL ENERGY

Defining Internal Energy

- Energy can generally be classified into two forms: kinetic or potential energy
- The molecules of all substances contain both kinetic and potential energies
- The amount of kinetic and potential energy a substance contains depends on the phases of matter (solid, liquid or gas), this is known as the **internal energy**
- The internal energy of a substance is defined as:

The sum of the random distribution of kinetic and potential energies within a system of molecules

- The symbol for internal energy is *U*, with units of **Joules** (J)
- The internal energy of a system is determined by:
  - Temperature
  - The random motion of molecules
  - The phase of matter: gases have the highest internal energy, solids have the lowest
- The internal energy of a system can increase by:
  - Doing work on it
  - Adding heat to it
- The internal energy of a system can decrease by:
  - Losing heat to its surroundings

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Exam Tip

When an exam question asks you to define "internal energy", you can lose a mark for not mentioning the "**random motion**" of the particles or the "**random distribution**" of the energies, so make sure you include one of these in your definition!

Internal Energy & Temperature

- The internal energy of an object is intrinsically related to its temperature
- When a container containing gas molecules is heated up, the molecules begin to **move around faster,** increasing their kinetic energy
- If the object is a solid, where the molecules are tightly packed, when heated the molecules begin to **vibrate** more
- Molecules in liquids and solids have both kinetic and potential energy because they are close together and bound by intermolecular forces
- However, ideal gas molecules are assumed to have **no intermolecular forces**
    - This means there have **no potential energy, only kinetic energy**
- The (change in) internal energy of an ideal gas is equal to:

$$\Delta U = \frac{3}{2} k \Delta T$$

- Therefore, the change in internal energy is proportional to the change in temperature:

$$\mathbf{\Delta U \propto \Delta T}$$

- Where:
    - ΔU = change in internal energy (J)
    - ΔT = change in temperature (K)

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Worked example: Internal energy & temperature

A student suggests that when an ideal gas is heated from 50 °C to 150 °C, the internal energy of the gas is trebled.

State and explain whether the student's suggestion is correct.

Step 1:

Write down the relationship between internal energy and temperature

The internal energy of an ideal gas is directly proportional to its temperature

$$\Delta U \propto \Delta T$$

Step 2:

Determine whether the change in temperature (in K) increases by three times

The temperature change is the **thermodynamic** temperature ie. Kelvin

The temperature change in degrees from 50 °C to 150 °C increases by three times

The temperature change in Kelvin is:

$$50\ ^{\circ}C + 273.15 = 323.15\ K$$

$$150\ ^{\circ}C + 273.15 = 423.15\ K$$

$$\frac{423.15}{323.15} = 1.3$$

Therefore, the temperature change, in Kelvin, does **not** increase by three times

Step 3:

Write a concluding statement relating the temperature change to the internal energy

The internal energy is directly proportional to the temperature

The thermodynamic temperature has not trebled, therefore, neither has the internal energy

Therefore, the student is incorrect

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Exam Tip

If an exam question about an ideal gas asks for the **total internal energy**, remember that this is equal to the **total kinetic energy** since an ideal gas has **zero potential energy**

16. Thermodynamics

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16.1.2 WORK DONE BY A GAS

Work Done by a Gas

- When a gas expands, it does work on its surroundings by exerting pressure on the walls of the container it's in
- This is important, for example, in a steam engine where expanding steam pushes a piston to turn the engine
- The work done when a volume of gas changes at constant pressure is defined as:

$$\mathbf{W = p\Delta V}$$

- Where:
    - W = work done (J)
    - p = external pressure (Pa)
    - V = volume of gas (m^3)

- For a gas inside a cylinder enclosed by a moveable piston, the force exerted by the gas pushes the piston outwards
- Therefore, the gas **does work on the piston**

- The volume of gas is at constant pressure. This means the force *F* exerted by the gas on the piston is equal to :

$$\mathbf{F = p \times A}$$

- Where:
    - p = pressure of the gas (Pa)
    - A = cross sectional area of the cylinder (m^2)
- The definition of work done is:

$$\mathbf{W = F \times d}$$

- Where:
    - F = force (N)
    - d = perpendicular displacement to the force (m)
- The displacement of the gas *d* multiplied by the cross-sectional area *A* is the increase in volume ΔV of the gas:

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$$\mathbf{W = p \times A \times d}$$

- This gives the equation for the work done when the volume of a gas changes at constant pressure:

$$\mathbf{W = p\Delta V}$$

- Where:
  - ΔV = increase in volume of the gas in the piston when expanding (m^3)

- This is assuming that the surrounding pressure *p* does not change as the gas expands
- This will be true if the gas is expanding against the pressure of the atmosphere, which changes very slowly
- When the gas **expands** (*V* increases), work is done **by** the gas
- When the gas is **compressed** (*V* decreases), work is done **on** the gas

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Worked example

When a balloon is inflated, its rubber walls push against the air around it.

Calculate the work done when the balloon is blown up from 0.015 m^3 to 0.030 m^3 .

Atmospheric pressure = 1.0×10^5 Pa.

Step 1: Write down the equation for the work done by a gas

$$W = p\Delta V$$

Step 2: Substitute in values

$$\Delta V = \text{final volume} - \text{initial volume} = 0.030 - 0.015 = 0.015 \text{ m}^3$$

$$W = (1.0 \times 10^5) \times 0.015 = 1500 \text{ J}$$

Exam Tip

The pressure *p* in the work done by a gas equation is not the pressure of the gas but the pressure of the surroundings. This because when a gas expands, it does work **on** the surroundings.

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16.1.3 THE FIRST LAW OF THERMODYNAMICS

The First Law of Thermodynamics

- The first law of thermodynamics is based on the principle of conservation of energy
- When energy is put into a gas by heating it or doing work on it, its internal energy must increase:

The increase in internal energy = Energy supplied by heating + Work done on the system

- The first law of thermodynamics is therefore defined as:

$$\Delta U = q + W$$

- Where:
  - ΔU = increase in internal energy (J)
  - q = energy supplied to the system by heating (J)
  - W = work done on the system (J)

- The first law of thermodynamics applies to **all** situations, not just for gases
  - There is an important sign convention used for this equation
- A **positive** value for internal energy ($+\Delta U$) means:
  - The internal energy ΔU **increases**
  - Heat q is **added** to the system
  - Work W is done **on** the system (or **on** a gas)
- A **negative** value for internal energy ($-\Delta U$) means:
  - The internal energy ΔU **decreases**
  - Heat q is **taken away** from the system
  - Work W is done **by** the system (or **by** a gas) **on** the surroundings

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- This is important when thinking about the expansion or compression of a gas
- When the gas **expands**, it transfers some energy (does work) to its surroundings
- This decreases the overall energy of the gas
- Therefore, when the gas expands, work is done **by** the gas (−W)

When a gas expands, work done W is negative

- When the gas is **compressed**, work is done **on** the gas (+W)

When a gas is compressed, work done W is positive

Graphs of Constant Pressure & Volume

- Graphs of pressure p against volume V can provide information about the work done and internal energy of the gas
    - The work done is represented by the area under the line
- A constant pressure process is represented as a **horizontal line**
    - If the volume is increasing (expansion), work is done **by** the gas and internal energy increases
    - If the arrow is reversed and the volume is decreasing (compression), work is done **on** the gas and internal energy decreases
- A constant volume process is represented as a **vertical line**
    - In a process with constant volume, the area under the curve is **zero**
    - Therefore, no work is done when the volume stays the same

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Worked example

The volume occupied by 1.00 mol of a liquid at 50 °C is 2.4×10^{-5} m^3.

When the liquid is vaporised at an atmospheric pressure of 1.03×10^5 Pa, the vapour has a volume of 5.9×10^{-2} m^3.

The latent heat required to vaporise 1.00 mol of this liquid at 50 °C at atmospheric pressure is 3.48×10^4 J.

Determine for this change of state the increase in internal energy ΔU of the system.

Step 1:

Write down the first law of thermodynamics

$$\Delta U = q + W$$

Step 2:

Write the value of heating *q* of the system

This is the latent heat, the heat required to vaporise the liquid = **3.48×10^4 J**

Step 3:

Calculate the work done *W*

$$\mathbf{W = p\Delta V}$$

ΔV = final volume − initial volume = $\mathbf{5.9 \times 10^{-2} - 2.4 \times 10^{-5} = 0.058976\ m^3}$

p = atmospheric pressure = $\mathbf{1.03 \times 10^5\ Pa}$

W = $\mathbf{(1.03 \times 10^5) \times 0.058976 = 6074.528 = 6.07 \times 10^3\ J}$

Since the gas is expanding, this work done is negative

$$\mathbf{W = -6.07 \times 10^3\ J}$$

Step 4:

Substitute the values into first law of thermodynamics

$$\mathbf{\Delta U = 3.48 \times 10^4 + (-6.07 \times 10^3) = 28\,730 = 29\,000\ J\ (2\ s.f.)}$$

17. Oscillations

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17.1 SIMPLE HARMONIC MOTION

17.1.1 DESCRIBING OSCILLATIONS

Describing Oscillations

- An **oscillation** is defined as:

  Repeated back and forth movements on either side of any equilibrium position

- When the object stops oscillating, it returns to its equilibrium position
  - An **oscillation** is a more specific term for a vibration
  - An **oscillator** is a device that works on the principles of oscillations
- Oscillating systems can be represented by displacement-time graphs similar to transverse waves
- The shape of the graph is a sine curve
  - The motion is described as **sinusoidal**

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Properties of Oscillations

- **Displacement (x)** of an oscillating system is defined as:

  The distance of an oscillator from its equilibrium position

- **Amplitude (x_0)** is defined as:

  The maximum displacement of an oscillator from its equilibrium position

- **Angular frequency (ω)** is defined as:

  The rate of change of angular displacement with respect to time

- This is a scalar quantity measured in **rad s^{-1}** and is defined by the equation:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

- **Frequency (f)** is defined as:

  The number of complete oscillations per unit time

- It is measured in Hertz (Hz) and is defined by the equation:

$$f = \frac{1}{T}$$

- **Time period (T)** is defined as:

  The time taken for one complete oscillation, in seconds

- One complete oscillation is defined as:

  The time taken for the oscillator to pass the equilibrium from one side and back again fully from the other side

- The time period is defined by the equation:

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

- **Phase difference** is how much one oscillator is in front or behind another
  - When the relative position of two oscillators are equal, they are **in phase**

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- When one oscillator is exactly half a cycle behind another, they are said to be in **anti-phase**
- Phase difference is normally measured in radians or fractions of a cycle
- When two oscillators are in antiphase they have a phase difference of **π radians**

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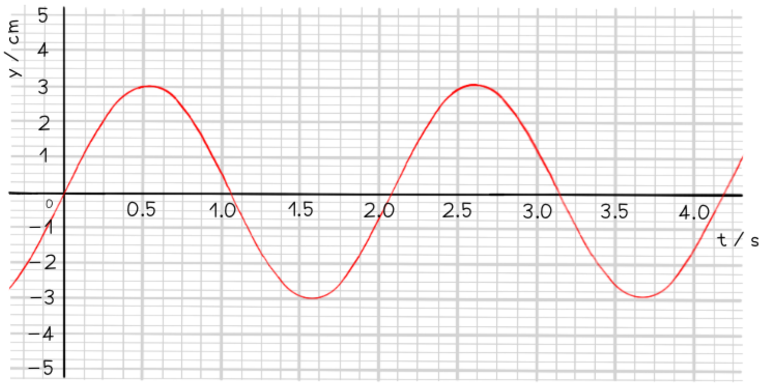
Worked example: Calculating angular frequency

A student sets out to investigate the oscillation of a mass suspended from the free end of a spring. The mass is pulled downwards and then released.

The variation with time *t* of the displacement *y* of the mass is shown in the figure below.



Use the information from the figure to calculate the angular frequency of the oscillations.

Step 1:

Write down the equation for angular frequency

$$\omega = \frac{2\pi}{T}$$

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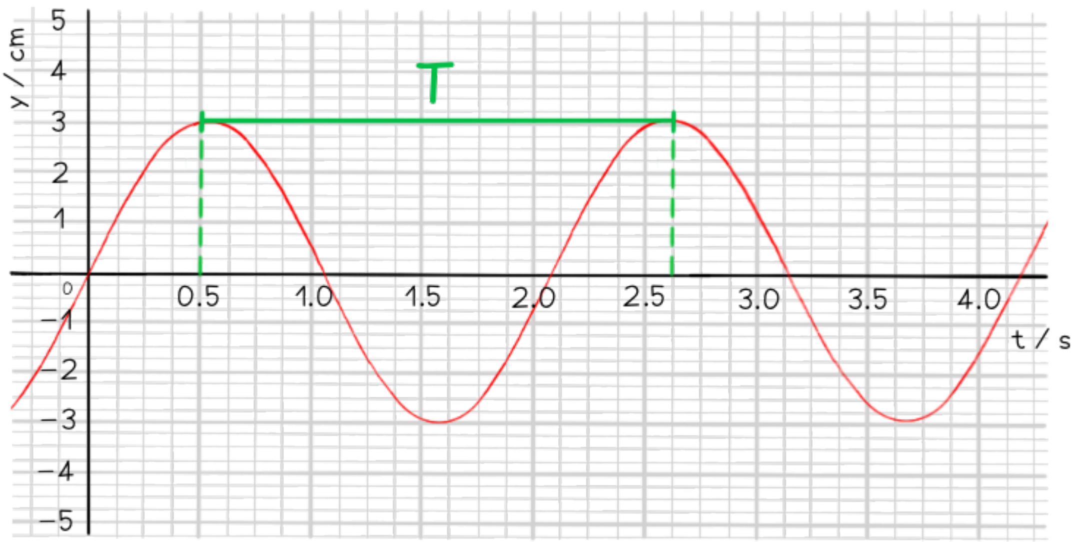
Step 2:

Calculate the time period T from the graph

The time period is defined as the time taken for **one complete oscillation**

This can be read from the graph:



T = 2.6 − 0.5 = 2.1 s

Step 3:

Substitute into angular frequency equation

$$\omega = \frac{2\pi}{2.1} = 2.9919\ldots = 3.0 \text{ rad s}^{-1}$$

Exam Tip

The properties used to describe oscillations are very similar to transverse waves. The key difference is that oscillators do not have a 'wavelength' and their direction of travel is only kept within the oscillations themselves rather than travelling a distance in space.

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17.1.2 SIMPLE HARMONIC MOTION

Conditions for Simple Harmonic Motion

- **Simple harmonic motion (SHM)** is a specific type of oscillation
- SHM is defined as:

A type of oscillation in which the acceleration of a body is proportional to its displacement, but acts in the opposite direction

- Examples of oscillators that undergo SHM are:
  - The pendulum of a clock
  - A mass on a spring
  - Guitar strings
  - The electrons in alternating current flowing through a wire
- This means for an object to oscillate specifically in SHM, it must satisfy the following conditions:
  - Periodic oscillations
  - Acceleration proportional to its displacement
  - Acceleration in the opposite direction to its displacement
- Acceleration *a* and displacement *x* can be represented by the defining equation of SHM:

$$\mathbf{a \propto -x}$$

- An object in SHM will also have a restoring force to return it to its equilibrium position
- This restoring force will be directly proportional, but in the **opposite direction**, to the displacement of the object from the equilibrium position
- **Note:** the restoring force and acceleration act in the **same direction**
- This is why a person jumping on a trampoline is not an example of simple harmonic motion:
  - The restoring force on the person is **not** proportional to their distance from the equilibrium position
  - When the person is not in contact with the trampoline, the restoring force is equal to their weight, which is constant
  - This does not change, even if they jump higher

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17.1.3 CALCULATING ACCELERATION & DISPLACEMENT IN SHM

Calculating Acceleration & Displacement of an Oscillator

- The acceleration of an object oscillating in simple harmonic motion is:

$$\mathbf{a = -\omega^2 x}$$

- Where:
  - a = acceleration (m s^{-2})
  - ω = angular frequency (rad s^{-1})
  - x = displacement (m)
- This is used to find the acceleration of an object in SHM with a particular angular frequency ω at a specific displacement x
- The equation demonstrates:
  - The acceleration reaches its maximum value when the displacement is at a maximum ie. $x = x_0$ (amplitude)
  - The minus sign shows that when the object is displacement to the **right**, the direction of the acceleration is to the **left**

- The graph of acceleration against displacement is a straight line through the origin sloping downwards (similar to y = − x)
- Key features of the graph:
  - The gradient is equal to $-\omega^2$
  - The maximum and minimum displacement *x* values are the amplitudes $-x_0$ and $+x_0$
- A solution to the SHM acceleration equation is the displacement equation:

$$\mathbf{x = x_0 \sin(\omega t)}$$

- Where:
  - x = displacement (m)
  - x_0 = amplitude (m)
  - t = time (s)

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- This equation can be used to find the position of an object in SHM with a particular angular frequency and amplitude at a moment in time
  - **Note:** This version of the equation is only relevant when an object begins oscillating from the equilibrium position ($x = 0$ at $t = 0$)
- The displacement will be at its maximum when $\sin(\omega t)$ equals 1 or − 1, when $x = x_0$
- If an object is oscillating from its amplitude position ($x = x_0$ or $x = -x_0$ at $t = 0$) then the displacement equation will be:

$$\mathbf{x = x_0\cos(\omega t)}$$

- This is because the cosine graph starts at a maximum, whilst the sine graph starts at 0

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Worked example: Calculating SHM displacement

A mass of 55 g is suspended from a fixed point by means of a spring. The stationary mass is pulled vertically downwards through a distance of 4.3 cm and then released at t = 0.

The mass is observed to perform simple harmonic motion with a period of 0.8 s.

Calculate the displacement x, in cm, of the mass at time t = 0.3 s.

Step 1:

Write down the SHM displacement equation

Since the mass is released at t = 0 at its maximum displacement, the displacement equation will be with the cosine function:

$$x = x_0 \cos(\omega t)$$

Step 2:

Calculate angular frequency

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8} = 7.85 \text{ rads}^{-1}$$

Remember to use the value of the time period given, not the time where you are calculating the displacement from

Step 3:

Substitute values into the displacement equation

$$x = 4.3\cos(7.85 \times 0.3) = -3.0369... = -3.0 \text{ cm (2 s.f)}$$

Make sure the calculator is in **radians mode**

The negative value means the mass is 3.0 cm on the opposite side of the equilibrium position to where it started (3.0 cm above it)

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Exam Tip

Since displacement is a vector quantity, remember to keep the minus sign in your solutions if they are negative, you could lose a mark if not!

Also remember that your calculator must be in **radians** mode when using the cos and sine functions. This is because the angular frequency ω is calculated in rad s^{-1}, **not** degrees.

17. Oscillations

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17.1.4 CALCULATING SPEED IN SHM

Calculating Speed of an Oscillator

- The speed of an object in simple harmonic motion varies as it oscillates back and forth
  - Its speed is the magnitude of its velocity
- The greatest speed of an oscillator is at the equilibrium position ie. when its displacement is 0 (x = 0)
- The speed of an oscillator in SHM is defined by:

$$v = v_0 \cos(\omega t)$$

- Where:
  - v = speed ($m\ s^{-1}$)
  - v_0 = maximum speed ($m\ s^{-1}$)
  - ω = angular frequency ($rad\ s^{-1}$)
  - t = time (s)
- This is a cosine function if the object starts oscillating from the equilibrium position (x = 0 when t = 0)
- Although the symbol *v* is commonly used to represent velocity, not speed, exam questions focus more on the magnitude of the velocity than its direction in SHM
- How the speed *v* changes with the oscillator's displacement *x* is defined by:

$$v = \pm \omega \sqrt{(x_0^2 - x^2)}$$

- Where:
  - x = displacement (m)
  - x_0 = amplitude (m)
  - ± = 'plus or minus'. The value can be negative or positive
- This equation show that when an oscillator has a greater amplitude x_0, it has to travel a greater distance in the same time and hence has greater speed *v*
- Both equations for speed will be given on your formulae sheet in the exam
- When the speed is at its maximums (at x = 0), the equation becomes:

$$v_0 = \omega x_0$$

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Worked example: Calculating the speed of a pendulum

A simple pendulum oscillates with simple harmonic motion with an amplitude of 15 cm. The frequency of the oscillations is 6.7 Hz.

Calculate the speed of the pendulum at a position of 12 cm from the equilibrium position.

Step 1:

Write out the known quantities

Amplitude of oscillations, x_0 **= 15 cm = 0.15 m**

Displacement at which the speed is to be found, **x = 12 cm = 0.12 m**

Frequency, **f = 6.7 Hz**

Step 2:

Oscillator speed with displacement equation

$$v = \pm \omega \sqrt{(x_0^2 - x^2)}$$

Since the speed is being calculated, the ± sign can be removed as direction does not matter in this case

Step 3:

Write an expression for the angular frequency

Equation relating angular frequency and normal frequency:

$$\omega = 2\pi f = 2\pi \times 6.7 = 42.097...$$

Step 4:

Substitute in values and calculate

$$v = (2\pi \times 6.7) \times \sqrt{(0.15)^2 - (0.12)^2}$$

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v = 3.789 = 3.8 m s^{-1} (2 s.f)

Exam Tip

You often have to convert between time period *T*, frequency *f* and angular frequency ω for many exam questions – so make sure you revise the equations relating to these.

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17.1.5 SHM GRAPHS

SHM Graphs

- The displacement, velocity and acceleration of an object in simple harmonic motion can be represented by graphs against time
- All undamped SHM graphs are represented by **periodic functions**
  - This means they can all be described by sine and cosine curves

- **Key features of the displacement-time graph:**
  - The amplitude of oscillations x_0 can be found from the maximum value of x
  - The time period of oscillations T can be found from reading the time taken for one full cycle
  - The graph might not always start at 0
  - If the oscillations starts at the positive or negative amplitude, the displacement will be at its maximum

- **Key features of the velocity-time graph:**
  - It is 90° out of phase with the displacement-time graph
  - Velocity is equal to the rate of change of displacement
  - So, the velocity of an oscillator at any time can be determined from the **gradient of the displacement-time graph:**

$$v = \frac{\Delta x}{\Delta t}$$

  - An oscillator moves the fastest at its equilibrium position
  - Therefore, the velocity is at its **maximum** when the **displacement is zero**

- **Key features of the acceleration-time graph:**
  - The acceleration graph is a reflection of the displacement graph on the x axis
  - This means when a mass has positive displacement (to the right) the acceleration is in the opposite direction (to the left) and vice versa
  - It is 90° out of phase with the velocity-time graph
  - Acceleration is equal to the rate of change of velocity
  - So, the acceleration of an oscillator at any time can be determined from the **gradient of the velocity-time graph:**

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$$a = \frac{\Delta v}{\Delta t}$$

- The maximum value of the acceleration is when the oscillator is at its **maximum displacement**

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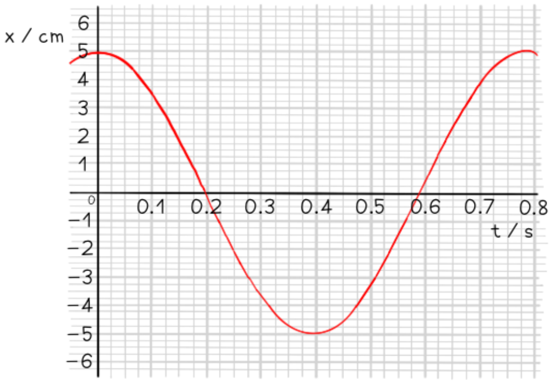
Worked example: Using SHM graph data

A swing is pulled 5 cm and then released.

The variation of the horizontal displacement *x* of the swing with time *t* is shown on the graph below.



The swing exhibits simple harmonic motion.

Use data from the graph to determine at what time the velocity of the swing is first at its maximum.

Step 1: The velocity is at its maximum when the displacement x = 0

Step 2: Reading value of time when x = 0

From the graph this is equal to 0.2 s

Exam Tip

These graphs might not look identical to what is in your textbook, depending on where the object starts oscillating from at t = 0 (on either side of the equilibrium, or at the equilibrium). However, if there is no damping, they will all always be a general sine or cosine curves.

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17.1.6 ENERGY IN SHM

Kinetic & Potential Energies

- During simple harmonic motion, energy is constantly exchanged between two forms: kinetic and potential
- The potential energy could be in the form of:
  - Gravitational potential energy (for a pendulum)
  - Elastic potential energy (for a horizontal mass on a spring)
  - Or both (for a vertical mass on a spring)
- Speed *v* is at a maximum when displacement x = 0, so:

The kinetic energy is at a maximum when the displacement x = 0 (equilibrium position)

- Therefore, the kinetic energy is 0 at maximum displacement $x = x_0$, so:

The potential energy is at a maximum when the displacement (both positive and negative) is at a maximum $x = x_0$ (amplitude)

- A simple harmonic system is therefore constantly converting between kinetic and potential energy
  - When one increases, the other decreases and vice versa, therefore:

The total energy of a simple harmonic system always remains constant and is equal to the sum of the kinetic and potential energies

- **The key features of the energy-time graph**:
  - Both the kinetic and potential energies are represented by periodic functions (sine or cosine) which are varying in opposite directions to one another
  - When the potential energy is 0, the kinetic energy is at its maximum point and vice versa
  - The **total energy** is represented by a **horizontal straight line** directly above the curves at the maximum value of both the kinetic or potential energy
  - Energy is **always positive** so there are no negative values on the y axis
- **Note:** kinetic and potential energy go through **two** complete cycles during one period of oscillation
- This is because one complete oscillation reaches the maximum displacement **twice** (positive and negative)

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- **The key features of the energy-displacement graph:**
  - Displacement is a vector, so, the graph has both **positive** and **negative** x values
  - The potential energy is always at a maximum at the amplitude positions x_0 and 0 at the equilibrium position (x = 0)
  - This is represented by a **'U' shaped curve**
  - The kinetic energy is the opposite: it is 0 at the amplitude positions x_0 and maximum at the equilibrium position x = 0
  - This is represented by a **'n' shaped curve**
  - The total energy is represented by a **horizontal straight line** above the curves

Exam Tip

You may be expected to draw as well as interpret energy graphs against time or displacement in exam questions. Make sure the sketches of the curves are as even as possible and **use a ruler** to draw straight lines, for example, to represent the total energy.

Calculating Total Energy of a Simple Harmonic System

- The total energy of system undergoing simple harmonic motion is defined by:

$$E = \frac{1}{2} m\omega^2 x_0^2$$

- Where:
  - E = total energy of a simple harmonic system (J)
  - m = mass of the oscillator (kg)
  - ω = angular frequency (rad s^{-1})
  - x_0 = amplitude (m)

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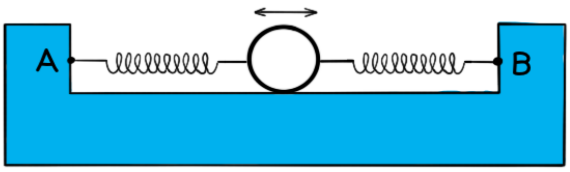
Worked example: Calculating the total energy of oscillations

A ball of mass 23 g is held between two fixed points A and B by two stretch helical springs, as shown in the diagram below.



The ball oscillates with simple harmonic motion along the line AB with frequency 4.8 Hz and amplitude 1.5 cm.

Calculate the total energy of the oscillations.

Step 1: Write down all known quantities

Mass, m = 23 g = $\mathbf{23 \times 10^{-3}}$ **kg**

Amplitude, x_0 = 1.5 cm = **0.015 m**

Frequency, f = **4.8 Hz**

Step 2: Write down the equation for the total energy of SHM oscillations:

$$E = \frac{1}{2} m\omega^2 x_0^2$$

Step 3: Write an expression for the angular frequency

$$\boldsymbol{\omega = 2\pi f = 2\pi \times (4.8)}$$

Step 4: Substitute values into energy equation

$$E = \frac{1}{2} \times (23 \times 10^{-3}) \times (2\pi \times 4.8)^2 \times (0.015)^2$$

E = 2.354 × $\mathbf{10^{-3}}$ = 2.4 mJ (2 s.f)

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17.2 DAMPED OSCILLATIONS

17.2.1 DAMPING

Damping

- In practice, all oscillators eventually stop oscillating
    - Their amplitudes decrease rapidly, or gradually
- This happens due to **resistive forces**, such friction or air resistance, which act in the opposite direction to the motion of an oscillator
- Resistive forces acting on an oscillating simple harmonic system cause **damping**
    - These are known as **damped** oscillations
- Damping is defined as:

> ***The reduction in energy and amplitude of oscillations due to resistive forces on the oscillating system***

- Damping continues until the oscillator comes to rest at the equilibrium position
- A key feature of simple harmonic motion is that the **frequency** of damped oscillations **does not change** as the amplitude decreases
    - For example, a child on a swing can oscillate back and forth once every second, but this time remains the same regardless of the amplitude

Exam Tip

Make sure not to confuse **resistive** force and **restoring** force:

• Resistive force is what **opposes the motion** of the oscillator and causes damping

• Restoring force is what brings the oscillator **back to the equilibrium position**

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Types of Damping

- There are three degrees of damping depending on how quickly the amplitude of the oscillations decrease:
    - Light damping
    - Critical damping
    - Heavy damping

Light Damping

- When oscillations are lightly damped, the amplitude does not decrease linearly
    - It decays exponentially with time
- When a lightly damped oscillator is displaced from the equilibrium, it will oscillate with gradually decreasing amplitude
    - For example, a swinging pendulum decreasing in amplitude until it comes to a stop
- **Key features of a displacement-time graph for a lightly damped system:**
    - There are many oscillations represented by a sine or cosine curve with gradually decreasing amplitude over time
    - This is shown by the height of the curve decreasing in both the positive and negative displacement values
    - The amplitude decreases exponentially
    - The frequency of the oscillations remain constant, this means the time period of oscillations must stay the same and each peak and trough is equally spaced

Critical Damping

- When a critically damped oscillator is displaced from the equilibrium, it will return to rest at its equilibrium position in the shortest possible time **without** oscillating
    - For example, car suspension systems prevent the car from oscillating after travelling over a bump in the road

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- **Key features of a displacement-time graph for a critically damped system:**
  - This system does **not** oscillate, meaning the displacement falls to 0 straight away
  - The graph has a fast decreasing gradient when the oscillator is first displaced until it reaches the x axis
  - When the oscillator reaches the equilibrium position (x = 0), the graph is a horizontal line at x = 0 for the remaining time

Heavy Damping

- When a heavily damped oscillator is displaced from the equilibrium, it will take a long time to return to its equilibrium position **without** oscillating
- The system returns to equilibrium more slowly than the critical damping case
  - For example, door dampers to prevent them slamming shut

- **Key features of a displacement-time graph for a heavily damped system:**
  - There are no oscillations. This means the displacement does not pass 0
  - The graph has a slow decreasing gradient from when the oscillator is first displaced until it reaches the x axis
  - The oscillator reaches the equilibrium position (x = 0) after a long period of time, after which the graph remains a horizontal line for the remaining time

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Worked example

A mechanical weighing scale consists of a needle which moves to a position on a numerical scale depending on the weight applied.

Sometimes, the needle moves to the equilibrium position after oscillating slightly, making it difficult to read the number on the scale to which it is pointing to.

Suggest, with a reason, whether light, critical or heavy damping should be applied to the mechanical weighing scale to read the scale more easily.

- Ideally, the needle should not oscillate before settling
    - This means the scale should have either **critical** or **heavy damping**
- Since the scale is read straight away after a weight is applied, ideally the needle should settle as quickly as possible
- Heavy damping would mean the needle will take some time to settle on the scale
- Therefore, **critical damping** should be applied to the weighing scale so the **needle can settle as quickly as possible** to read from the scale

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17.2.2 RESONANCE

Resonance

- In order to sustain oscillations in a simple harmonic system, a periodic force must be applied to replace the energy lost in damping
    - This periodic force **does work** on the resistive force decreasing the oscillations
- These are known as **forced oscillations**, and are defined as:

> ***Periodic forces which are applied in order to sustain oscillations***

- For example, when a child is on a swing, they will be pushed at one end after each cycle in order to keep swinging and prevent air resistance from damping the oscillations
    - These extra pushes are the forced oscillations, without them, the child will eventually come to a stop
- The frequency of forced oscillations is referred to as the **driving frequency (f)**
- All oscillating systems have a **natural frequency (f_0)**, this is defined as:

> ***The frequency of an oscillation when the oscillating system is allowed to oscillate freely***

- Oscillating systems can exhibit a property known as **resonance**
- When resonance is achieved, a maximum amplitude of oscillations can be observed
- Resonance is defined as:

> ***When the driving frequency applied to an oscillating system is equal to its natural frequency, the amplitude of the resulting oscillations increases significantly***

- For example, when a child is pushed on a swing:
    - The swing plus the child has a fixed natural frequency
    - A small push after each cycle increases the amplitude of the oscillations to swing the child higher
    - If the driving frequency does not quite match the natural frequency, the amplitude will increase but not to the same extent at when resonance is achieved
- This is because at resonance, energy is transferred from the driver to the oscillating system **most efficiently**
    - Therefore, at resonance, the system will be transferring the maximum kinetic energy possible

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- A graph of driving frequency *f* against amplitude *a* of oscillations is called a **resonance curve**. It has the following key features:
  - When $f < f_0$, the amplitude of oscillations increases
  - At the peak where $f = f_0$, the amplitude is at its maximum. This is **resonance**
  - When $f > f_0$, the amplitude of oscillations starts to decrease

- Damping reduces the amplitude of resonance vibrations
- The height and shape of the resonance curve will therefore change slightly depending on the degree of damping
  - **Note:** the natural frequency f_0 will remain the same
- As the degree of damping is increased, the resonance graph is altered in the following ways:
  - The amplitude of resonance vibrations decrease, meaning the peak of the curve lowers
  - The resonance peak broadens
  - The resonance peak moves slightly to the left of the natural frequency when heavily damped

18. Electric Fields

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CONTENTS

18.1 ELECTRIC FIELDS

18.1.1 ELECTRIC FIELDS & FORCES ON CHARGES

Electric Field Definition

- An electric field is a region of space in which an electric charge "feels" a force
- **Electric field strength** at a point is defined as:

> ***The electrostatic force per unit positive charge acting on a stationary point charge at that point***

- Electric field strength can be calculated using the equation:

$$E = \frac{F}{Q}$$

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- Where:
  - E = electric field strength (N C^{-1})
  - F = electrostatic force on the charge (N)
  - Q = charge (C)
- It is important to use a positive test charge in this definition, as this determines the direction of the electric field
- The electric field strength is a **vector** quantity, it is always directed:
  - **Away** from a positive charge
  - **Towards** a negative charge
- Recall that **opposite charges** (positive and negative) charges **attract** each other
- Conversely, **like charges** (positive and positive or negative and negative) **repel** each other

18. Electric Fields

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Worked example: Calculating particle charge

A charged particle is in an electric field with electric field strength 3.5×10^4 N C^{-1} where it experiences a force of 0.3 N.

Calculate the charge of the particle.

Step 1: Write down the equation for electric field strength

$$E = \frac{F}{Q}$$

Step 2: Rearrange for charge Q

$$Q = \frac{F}{E}$$

Step 3: Substitute in values and calculate

$$Q = \frac{0.3}{3.5 \times 10^4} = 8.571 \times 10^{-6}\ \text{C} = \mathbf{8.6 \times 10^{-6}\ C}\ (2\ \text{s.f})$$

Forces on Charges

- The electric field strength equation can be rearranged for the force *F* on a charge *Q* in an electric field *E*:

$$\mathbf{F = QE}$$

- Where:
  - F = electrostatic force on the charge (N)
  - Q = charge (C)
  - E = electric field strength (N C^{-1})
- The direction of the force is determined by the charge:
  - If the charge is **positive** (+) the force is in the **same** direction as the E field
  - If the charge is **negative** (-) the force is in the **opposite** direction to the E field
- The force on the charge will cause the charged particle to **accelerate** if its in the same direction as the E field, or **decelerate** if in the opposite

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- **Note:** the force will always be **parallel** to the electric field lines

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Worked example: Electric force

An electron is stationary in an electric field which has an electric field strength of 5000 N C^{-1}.

Calculate the magnitude of the electric force that acts on the electron and state which direction the force will act in relation to the electric field.

Electron charge $e = 1.60 \times 10^{-19}$ C.

Step 1: Write out the equation for the force on a charged particle

$$F = QE$$

Step 2: Substitute in values

$$F = (1.60 \times 10^{-19}) \times 5000 = 8 \times 10^{-16} \text{ N}$$

Step 3: State the direction of the force

Since the charge is negative, the force is **directed against the electric field lines** and decelerates the electron.

Point Charge Approximation

- For a point outside a spherical conductor, the charge of the sphere may be considered to be a **point charge** at its centre
  - A **uniform** spherical conductor is one where its charge is **distributed evenly**
- The electric field lines around a spherical conductor are therefore **identical to those around a point charge**
- An example of a spherical conductor is a **charged sphere**
- The field lines are **radial** and their direction depends on the charge of the sphere
  - If the spherical conductor is **positively** charged, the field lines are directed **away** from the centre of the sphere
  - If the spherical conductor is **negatively** charged, the field lines are directed **towards** the centre of the sphere

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Exam Tip

You might have noticed that the electric fields share many similarities to the gravitational fields. The main difference being the gravitational force is always attractive, whilst electrostatic forces can be attractive or repulsive.
You should make a list of all the similarities and differences you can find, as this could come up in an exam question.

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18.1.2 ELECTRIC FIELD LINES

Representing Electric Fields

- The direction of electric fields is represented by electric field lines
- Electric field lines are directed from positive to negative
  - Therefore, the field lines must be pointed **away** from the **positive** charge and **towards** the **negative** charge
- A radial field spreads uniformly to or from the charge in all directions
  - e.g. the field around a point charge or sphere
- Around a **point charge**, the electric field lines are directly radially inwards or outwards:
  - If the charge is **positive** (+), the field lines are radially **outwards**
  - If the charge is **negative** (-), the field lines are radially **inwards**

- A uniform electric field has the same electric field strength throughout the field
  - For example, the field between oppositely charged parallel plates
- This is represented by **equally spaced** field lines
  - This shares many similarities to radial gravitational field lines around a point mass
  - Since gravity is only an attractive force, the field lines will look similar to the negative point charge, whilst electric field lines can be in either direction

- A **non-uniform** electric field has varying electric field strength throughout
- The strength of an electric field is determined by the spacing of the field lines:
  - A **stronger** field is represented by the field lines **closer** together
  - A **weaker** field is represented by the field lines **further** apart

- The electric field lines are directed from the positive to the negative plate
- A radial field is considered a **non-uniform** field
  - So, the electric field strength E is different depending on how far you are from a charged particle

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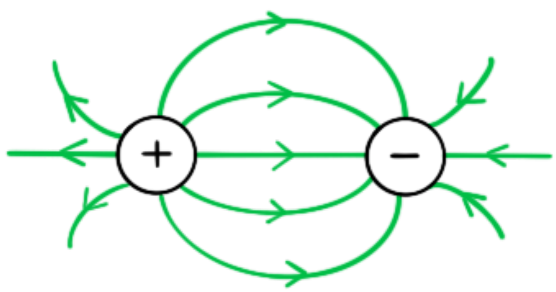
Worked example

Sketch the electric field lines between the two point charges in the diagram below.

Electric field lines around point charges are radially outwards for positive charges and radially inwards for negative charges

The field lines must be drawn with arrows **from the positive charge to the negative charge**



Exam Tip

Always label the arrows on the field lines! The lines must also touch the surface of the source charge or plates.

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18.1.3 ELECTRIC FIELD STRENGTH

Electric Field Strength

- The electric field strength of a uniform field between two charged parallel plates is defined as:

$$E = \frac{\Delta V}{\Delta d}$$

- Where:
  - E = electric field strength (V m^{-1})
  - ΔV = potential difference between the plates (V)
  - Δd = separation between the plates (m)
- **Note:** the electric field strength is now also defined by the units **V m^{-1}**
- The equation shows:
  - The greater the voltage between the plates, the stronger the field
  - The greater the separation between the plates, the weaker the field
- Remember this equation cannot be used to find the electric field strength around a point charge (since this would be a radial field)
- The direction of the electric field is from the plate connected to the **positive** terminal of the cell to the plate connected to the **negative** terminal
- **Note:** if one of the parallel plates is **earthed**, it has a voltage of 0 V

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Worked example: Electric force between plates

Two parallel metal plates are separated by 3.5 cm and have a potential difference of 7.9 kV.

Calculate the electric force acting on a stationary charged particle between the plates that has a charge of 2.6×10^{-15} C.

Step 1: Write down the known values

Potential difference, $\Delta V = 7.9$ kV $= 7.9 \times 10^3$ V

Distance between plates, $\Delta d = 3.5$ cm $= 3.5 \times 10^{-2}$ m

Charge, $Q = 2.6 \times 10^{-15}$ C

Step 2: Calculate the electric field strength between the parallel plates

$$E = \frac{\Delta V}{\Delta d}$$

$$E = \frac{7.9 \times 10^3}{3.5 \times 10^{-2}} = 2.257 \times 10^5 \text{ V m}^{-1}$$

Step 3: Write out the equation for electric force on a charged particle

$F = QE$

Step 4: Substitute electric field strength and charge into electric force equation

$F = QE = (2.6 \times 10^{-15}) \times (2.257 \times 10^5) = 5.87 \times 10^{-10}$ N $= 5.9 \times 10^{-10}$ N (2 s.f.)

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Electric Field of a Point Charge

- The electric field strength at a point describes how strong or weak an electric field is at that point
- The electric field strength *E* at a distance *r* due to a point charge *Q* in free space is defined by:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

- Where:
  - Q = the charge producing the electric field (C)
  - r = distance from the centre of the charge (m)
  - ε_0 = permittivity of free space (F m^{-1})
- This equation shows:
  - Electric field strength is **not constant**
  - As electric field strength increases, decreases by a factor of $1/r^2$
- This is an **inverse square law relationship** with distance
- This means the field strength decreases by a factor of **four** when the distance is **doubled**
- **Note:** this equation is only for the field strength around a **point charge** since it produces a radial field
- The electric field strength is a **vector** Its direction is the same as the electric field lines
  - If the charge is negative, the E field strength is negative and points **towards** the centre of the charge
  - If the charge is positive, the E field strength is positive and points **away** from the centre of the charge
- This equation is analogous to the gravitational field strength around a point mass

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Worked example: Surface charge of a sphere

A metal sphere of diameter 15 cm is negatively charged. The electric field strength at the surface of the sphere is 1.5×10^5 V m^{-1}.

Determine the total surface charge of the sphere.

Step 1: Write down the known values

Electric field strength, E = 1.5×10^5 V m^{-1}

Radius of sphere, r = 15 / 2 = 7.5 cm = 7.5×10^{-2} m

Step 2: Write out the equation for electric field strength

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Step 3: Rearrange for charge Q

$$Q = 4\pi\varepsilon_0 E r^2$$

Step 4: Substitute in values

$$Q = (4\pi \times 8.85 \times 10^{-12}) \times (1.5 \times 10^5) \times (7.5 \times 10^{-2})^2 = 9.38 \times 10^{-8}\ \text{C} = 94\ \text{nC (2 s.f)}$$

Exam Tip

Remember to always **square** the distance!

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18.1.4 MOTION OF CHARGED PARTICLES

Motion of Charged Particles

- A charged particle in an electric field will experience a force on it that will cause it to move
- If a charged particle remains still in a uniform electric field, it will move parallel to the electric field lines (along or against the field lines depending on its charge)
- If a charged particle is in **motion** through a uniform electric field (e.g. between two charged parallel plates), it will experience a constant electric force and travel in a **parabolic trajectory**

- The direction of the parabola will depend on the charge of the particle
  - A **positive** charge will be deflected towards the **negative** plate
  - A **negative** charge will be deflected towards the **positive** plate
- The force on the particle is the same at all points and is always in the same direction
- **Note:** an uncharged particle, such as a neutron experiences no force in an electric field and will therefore travel straight through the plates undeflected
- The amount of deflection depends on the following properties of the particles:
  - **Mass** – the greater the mass, the smaller the deflection and vice versa
  - **Charge** – the greater the magnitude of the charge of the particle, the greater the deflection and vice versa
  - **Speed** – the greater the speed of the particle, the smaller the deflection and vice versa

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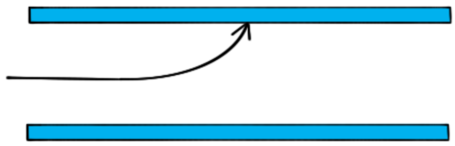
Worked example

A single proton travelling with a constant horizontal velocity enters a uniform electric field between two parallel charged plates.
The diagram shows the path taken by the proton.



Draw the path taken by a boron nucleus that enters the electric field at the same point and with the same velocity as the proton.

Atomic number of boron = 5
Mass number of boron = 11

Step 1:

Compare the charge of the boron nucleus to the proton

- Boron has 5 protons, meaning it has a charge 5 × greater than the proton
- The force on boron will therefore be 5 × greater than on the proton

Step 2:

Compare the mass of the boron nucleus to the proton

- The boron nucleus has a mass of 11 nucleons meaning its mass is 11 × greater than the proton
- The boron nucleus will therefore be less deflected than the proton

Step 3:

Draw the trajectory of the boron nucleus

- Since the mass comparison is much greater than the charge comparison, the boron nucleus will be **much less deflected** than the proton
- The nucleus is positively charged since the neutrons in the nucleus have no charge
  - Therefore, the shape of the path will be the same as the proton

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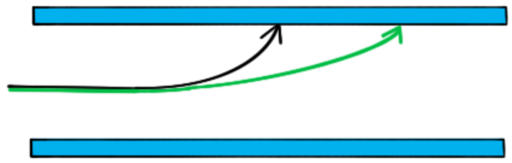

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18.1.5 ELECTRIC FORCE BETWEEN TWO POINT CHARGES

Coulomb's Law

- All charged particles produce an electric field around it
  - This field exerts a force on any other charged particle within range
- The electrostatic force between two charges is defined by **Coulomb's Law**
  - Recall that the charge of a uniform spherical conductor can be considered as a point charge at its centre
- Coulomb's Law states that:

  The electrostatic force between two point charges is proportional to the product of the charges and inversely proportional to the square of their separation

- The Coulomb equation is defined as:

$$F_E = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

- Where:
  - F_E = electrostatic force between two charges (N)
  - Q_1 and Q_2 = two point charges (C)
  - ε_0 = permittivity of free space
  - r = distance between the centre of the charges (m)
- The $1/r^2$ relation is called the inverse square law
  - This means that when a charge is twice as far as away from another, the electrostatic force between them reduces by $(½)^2$ = ¼
- If there is a positive and negative charge, then the electrostatic force is negative, this can be interpreted as an **attractive force**
- If the charges are the same, the electrostatic force is positive, this can be interpreted as a **repulsive force**
- Since uniformly charged spheres can be considered as point charges, Coulomb's law can be applied to find the electrostatic force between them as long as the separation is taken from the **centre** of both spheres

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Worked example: Coulomb's Law

An alpha particle is situated 2.0 mm away from a gold nucleus in a vacuum.

Assuming them to be point charges, calculate the magnitude of the electrostatic force acting on each of the charges.

Atomic number of helium = 2
Atomic number of gold = 79
Charge of an electron = 1.60×10^{-19} C

Step 1:

Write down the known quantities

- **Distance, r = 2.0 mm =2.0×10^{-3} m**

The charge of one proton = $+1.60 \times 10^{-19}$ C

An alpha particle (helium nucleus) has 2 protons

- **Charge of alpha particle, $Q_1 = 2 \times 1.60 \times 10^{-19} = +3.2 \times 10^{-19}$ C**

The gold nucleus has 79 protons

- **Charge of gold nucleus, $Q_2 = 79 \times 1.60 \times 10^{-19} = +1.264 \times 10^{-17}$ C**

Step 2:

The electrostatic force between two point charges is given by Coulomb's Law

$$F_E = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

Step 3:

Substitute values into Coulomb's Law

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$$F_E = \frac{(3.2 \times 10^{-19}) \times (1.264 \times 10^{-17})}{(4\pi \times 8.85 \times 10^{-12}) \times (2.0 \times 10^{-3})^2} = 9.092.. \times 10^{-21} = \mathbf{9.1 \times 10^{-21}\ N}\ (2\ \text{s.f.})$$

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18.2 ELECTRIC POTENTIAL

18.2.1 ELECTRIC POTENTIAL

Electric Potential

- In order to move a positive charge closer to another positive charge, work must be done to overcome the force of repulsion between them
- Energy is therefore transferred to the charge that is being pushed upon
    - This means its **potential energy** increases
- If the positive charge is free to move, it will start to move away from the repelling charge
    - As a result, its potential energy decreases back to 0
- This is analogous to the gravitational potential energy of a mass increasing as it is being lift upwards and decreasing and it falls
- The electric potential at a point is defined as:

The work done per unit positive charge in bringing a small test charge from infinity to a defined point

- Electric potential is a **scalar** quantity
    - This means it doesn't have a direction
- However, you will still see the electric potential with a positive or negative sign. This is because the electric potential is:
    - **Positive** when near an isolated positive charge
    - **Negative** when near an isolated negative charges
    - **Zero** at infinity
- Positive work is done by the mass from infinity to a point around a positive charge and negative work is done around a negative charge. This means:
    - When a test charge moves closer to a **negative** charge, its electric potential **decreases**
    - When a test charge moves closer to a **positive** charge, its electric potential **increases**
- To find the potential at a point caused by multiple charges, add up each potential separately

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Electric Potential Due to a Point Charge

- The electric potential in the field due to a point charge is defined as:

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

- Where:
  - V = the electric potential (V)
  - Q = the point charge producing the potential (C)
  - ε_0 = permittivity of free space (F m^{-1})
  - r = distance from the centre of the point charge (m)
- This equation shows that for a positive (+) charge:
  - As the distance from the charge *r* **decreases**, the potential *V* **increases**
  - This is because more work has to be done on a positive test charge to overcome the repulsive force
- For a negative (−) charge:
  - As the distance from the charge *r* **decreases**, the potential *V* **decreases**
  - This is because less work has to be done on a positive test charge since the attractive force will make it easier
- Unlike the gravitational potential equation, the minus sign in the electric potential equation will be included in the charge
- The electric potential changes according to an inverse square law with distance

- **Note:** this equation still applies to a conducting sphere. The charge on the sphere is treated as if it concentrated at a point in the sphere from the point charge approximation

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Worked example: Van de Graaf charge

A Van de Graaf generator has a spherical dome of radius 15 cm. It is charged up to a potential of 240 kV.

Calculate:

a) The charge is stored on the dome.
b) The potential at a distance of 30 cm from the dome.

Part (a)

Step 1: Write down the known quantities

Radius of the dome, r = 15 cm = 15 × 10^{-2} m

Potential difference, V = 240 kV = 240 × 10^{3} V

Step 2: Write down the equation for the electric potential due to a point charge

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Step 3: Rearrange for charge Q

$$\mathbf{Q = V4\pi\varepsilon_0 r}$$

Step 4: Substitute in values

$$\mathbf{Q = (240 \times 10^{3}) \times (4\pi \times 8.85 \times 10^{-12}) \times (15 \times 10^{-2}) = 4.0 \times 10^{-6}\ C = 4.0\ \mu C}$$

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Part (b)

Step 1: Write down the known quantities

Q = charge stored in the dome = 4.0 μC = 4.0 × 10^{-6} C

r = radius of the dome + distance from the dome = 15 + 30 = 45 cm = 45 × 10^{-2} m

Step 2: Write down the equation for electric potential due to a point charge

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Step 3: Substitute in values

$$V = \frac{(4.0 \times 10^{-6})}{(4\pi \times 8.85 \times 10^{-12}) \times (45 \times 10^{-2})} = 79.93 \times 10^3 = \mathbf{80\ kV}\ (2\ \text{s.f.})$$

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18.2.2 ELECTRIC POTENTIAL GRADIENT

Potential Gradient

- An electric field can be defined in terms of the variation of electric potential at different points in the field**:**

  The electric field at a particular point is equal to the negative gradient of a potential-distance graph at that point

- The potential gradient is defined by the **equipotential lines**
  - These demonstrate the electric potential in an electric field and are always drawn **perpendicular** to the field lines

- Equipotential lines are lines of **equal electric potential**
  - Around a radial field, the equipotential lines are represented by concentric circles around the charge with increasing radius
  - The equipotential lines become further away from each other
  - In a uniform electric field, the equipotential lines are equally spaced

- The potential gradient in an electric field is defined as:

  The rate of change of electric potential with respect to displacement in the direction of the field

- The electric field strength is equivalent to this, except with a negative sign:

$$E = -\frac{\Delta V}{\Delta r}$$

- Where:
  - E = electric field strength ($V\ m^{-1}$)
  - ΔV = change in potential (V)
  - Δr = displacement in the direction of the field (m)

- The minus sign is important to obtain an attractive field around a negative charge and repulsive field around a positive charge

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- The electric potential changes according to the charge creating the potential as the distance *r* increases from the centre:
  - If the charge is **positive**, the potential **decreases** with distance
  - If the charge is **negative**, the potential **increases** with distance
- This is because the test charge is positive

Exam Tip

One way to remember whether the electric potential increases or decreases with respect to the distance from the charge is by the direction of the electric field lines. The potential always **decreases** in the **same** direction as the field lines and vice versa.

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18.2.3 ELECTRIC POTENTIAL ENERGY

Electric Potential Energy of Two Point Charges

- The electric potential energy E_p at point in an electric field is defined as:

The work done in bringing a charge from infinity to that point

- The electric potential energy of a pair of point charges Q_1 and Q_2 is defined by:

$$E_p = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r}$$

- Where:
  - E_p = electric potential energy (J)
  - r = separation of the charges Q_1 and Q_2 (m)
  - ε_0 = permittivity of free space (F m^{-1})
- The potential energy equation is defined by the work done in moving point charge Q_2 from infinity towards a point charge Q_1.
- The work done is equal to:

W = VQ

- Where:
  - W = work done (J)
  - V = electric potential due to a point charge (V)
  - Q = Charge producing the potential (C)
- This equation is relevant to calculate the work done due on a charge in a uniform field
- Unlike the electric potential, the potential energy will always be positive
- Recall that at infinity, V = 0 therefore E_p = 0
- It is more useful to find the change in potential energy eg. as one charge moves away from another
- The change in potential energy from a charge Q_1 at a distance r_1 from the centre of charge Q_2 to a distance r_2 is equal to:

$$\Delta E_p = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

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- The change in electric potential ΔV is the same, without the charge Q_2

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Both equations are very similar to the change in gravitational potential between two points near a point mass

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Worked example: Alpha particle

An α-particle $^{4}_{2}He$ is moving directly towards a stationary gold nucleus $^{197}_{79}Au$.

At a distance of 4.7×10^{-15} m, the α-particle momentarily comes to rest.

Calculate the electric potential energy of the particles at this instant.

Step 1:

Write down the known quantities

- **Distance, r = 4.7×10^{-15} m**

The charge of one proton = $+1.60 \times 10^{-19}$ C

An alpha particle (helium nucleus) has 2 protons

- **Charge of alpha particle, Q_1 = $2 \times 1.60 \times 10^{-19}$ = $+3.2 \times 10^{-19}$ C**

The gold nucleus has 79 protons

- **Charge of gold nucleus, Q_2 = $79 \times 1.60 \times 10^{-19}$ = $+1.264 \times 10^{-17}$ C**

Step 2:

Write down the equation for electric potential energy

$$E_p = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r}$$

Step 3:

Substitute values into the equation

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$$E_p = \frac{(1.264 \times 10^{-17}) \times (3.2 \times 10^{-19})}{(4\pi \times 8.85 \times 10^{-12}) \times (4.7 \times 10^{-15})} = \mathbf{7.7 \times 10^{-12}\ J}\ (2\text{ s.f})$$

Exam Tip

When calculating electric potential energy, make sure you **do not** square the distance!

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19.1 CAPACITORS

19.1.1 CAPACITANCE

Defining Capacitance

- Capacitors are electrical devices used to store energy in electronic circuits, commonly for a backup release of energy if the power fails
- They can be in the form of:
  - An isolated spherical conductor
  - Parallel plates
- Capacitors are marked with a value of their **capacitance.** This is defined as:

The charge stored per unit potential difference

- The greater the capacitance, the greater the energy stored in the capacitor
- A parallel plate capacitor is made up of two conductive metal plates connected to a voltage supply
  - The negative terminal of the voltage supply pushes electrons onto one plate, making it negatively charged
  - The electrons are repelled from the opposite plate, making it positively charged
  - There is commonly a dielectric in between the plates, this is to ensure charge does not freely flow between the plates

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Exam Tip

The 'charge stored' by a capacitor refers to the magnitude of the charge stored **on** each plate in a parallel plate capacitor or **on** the surface of a spherical conductor. The capacitor itself **does not** store charge.

Calculating Capacitance

- The capacitance of a capacitor is defined by the equation:

$$C = \frac{Q}{V}$$

- Where:
  - C = capacitance (F)
  - Q = charge (C)
  - V = potential difference (V)
- It is measured in the unit **Farad (F)**
  - In practice, 1 F is a very large unit
  - Capacitance will often be quoted in the order of micro Farads (μF), nanofarads (nF) or picofarads (pF)
- If the capacitor is made of parallel plates, Q is the charge on the plates and V is the potential difference across the capacitor
- The charge Q is **not** the charge of the capacitor itself, it is the charge stored **on** the plates or spherical conductor
- This capacitance equation shows that an object's capacitance is the **ratio of the charge on an object to its potential**

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Capacitance of a Spherical Conductor

- The capacitance of a charged sphere is defined by the charge per unit potential at the surface of the sphere
- The potential *V* is defined by the potential of an isolated point charge (since the charge on the surface of a spherical conductor can be considered as a point charge at its centre):

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

- Substituting this into the capacitance equation means the capacitance *C* of a sphere is given by the expression:

$$\mathbf{C = 4\pi\varepsilon_0 r}$$

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Worked example: Charge on parallel plates

A parallel plate capacitor has a capacitance of 1 nF and is connected to a voltage supply of 0.3 kV.

Calculate the charge on the plates.

Step 1: Write down the known quantities

Capacitance, C = 1 nF = 1×10^{-9} F

Potential difference, V = 0.3 kV = 0.3×10^{3} V

Step 2: Write out the equation for capacitance

$$C = \frac{Q}{V}$$

Step 3: Rearrange for charge Q

Q = CV

Step 4: Substitute in values

$$Q = (1 \times 10^{-9}) \times (0.3 \times 10^{3}) = 3 \times 10^{-7}\ \text{C} = 300\ \text{nC}$$

Exam Tip

The letter 'C' is used both as the symbol for capacitance as well as the unit of charge (coulombs). Take care not to confuse the two!

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19.1.2 DERIVATION OF C = Q/V

Derivation of C = Q/V

- The circuit symbol for a parallel plate capacitor is two parallel lines

- Capacitors can be combined in series and parallel circuits
- The combined capacitance depends on whether the capacitors are connected in series or parallel

Capacitors in Series

- Consider two parallel plate capacitors C_1 and C_2 connected in series, with a potential difference (p.d) V across them

- In a series circuit, p.d is **shared** between all the components in the circuit
  - Therefore, if the capacitors store the same charge on their plates but have different p.ds, the p.d across C_1 is V_1 and across C_2 is V_2
- The total potential difference V is the sum of V_1 and V_2

$$\mathbf{V = V_1 + V_2}$$

- Rearranging the capacitance equation for the p.d V means V_1 and V_2 can be written as:

$$V_1 = \frac{Q}{C_1} \quad \text{and} \quad V_2 = \frac{Q}{C_2}$$

- Where the total p.d V is defined by the total capacitance

$$V = \frac{Q}{C_{total}}$$

- Substituting these into the equation $V = V_1 + V_2$ equals:

$$\frac{Q}{C_{total}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

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- Since the current is the **same** through all components in a series circuit, the charge Q is the same through each capacitor and cancels out
- Therefore, the equation for combined capacitance of capacitors in **series** is:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$$

Capacitors in Parallel

- Consider two parallel plate capacitors C_1 and C_2 connected in parallel, each with p.d V
- Since the current is **split** across each junction in a parallel circuit, the charge stored on each capacitor is **different**
- Therefore, the charge on capacitor C_1 is Q_1 and on C_2 is Q_2
- The total charge Q is the sum of Q_1 and Q_2

$$\mathbf{Q = Q_1 + Q_2}$$

- Rearranging the capacitance equation for the charge Q means Q_1 and Q_2 can be written as:

$$\mathbf{Q_1 = C_1V \quad \text{and} \quad Q_2 = C_2V}$$

- Where the total charge Q is defined by the total capacitance:

$$\mathbf{Q = C_{total}V}$$

- Substituting these into the $Q = Q_1 + Q_2$ equals:

$$\mathbf{C_{total}V = C_1V + C_2V = (C_1 + C_2)\,V}$$

- Since the p.d is the **same** through all components in each branch of a parallel circuit, the p.d V cancels out
- Therefore, the equation for combined capacitance of capacitors in **parallel** is:

$$\mathbf{C_{total} = C_1 + C_2 + C_3 \ldots}$$

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Exam Tip

You will be expected to remember these derivations for your exam, therefore, make sure you understand each step. You should especially make sure to revise how the current and potential difference varies in a series and parallel circuit.

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19.1.3 CAPACITORS IN SERIES & PARALLEL

Capacitors in Series & Parallel

- Recall the formula for the combined capacitance of capacitors in series:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$$

- In parallel:

$$C_{total} = C_1 + C_2 + C_3 \ldots$$

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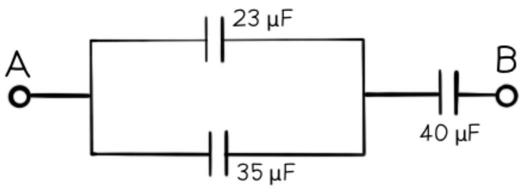
Worked example: Calculating total capacitance between two points

Three capacitors with a capacitance of 23 μF, 35 μF and 40 μF are connected as shown below.



Calculate the total capacitance between points A and B.

Step 1: Calculate the combined capacitance of the two capacitors in parallel

Capacitors in parallel: $C_{total} = C_1 + C_2 + C_3$ ...

$C_{parallel} = 23 + 35 = 58\ \mu F$

Step 2: Connect this combined capacitance with the final capacitor in series

Capacitors in series: $$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$$

$$\frac{1}{C_{total}} = \frac{1}{58} + \frac{1}{40} = \frac{49}{1160}$$

Step 3: Rearrange for the total capacitance

$$C_{total} = \frac{1160}{49} = 23.673\ldots = \mathbf{24\ \mu F} \text{ (2 s.f)}$$

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Exam Tip

Both the combined capacitance equations look similar to the equations for combined resistance in series and parallel circuits. However, take note that they are the **opposite way** around to each other!

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19.1.4 AREA UNDER A POTENTIAL-CHARGE GRAPH

Area Under a Potential–Charge Graph

- When charging a capacitor, the power supply pushes electrons from the positive to the negative plate
  - It therefore does **work** on the electrons, which increase their **electric potential energy**
- At first, a small amount of charge is pushed from the positive to the negative plate, then gradually, this builds up
  - Adding more electrons to the negative plate at first is relatively easy since there is little repulsion
- As the charge of the negative plate increases ie. becomes more negatively charged, the force of repulsion between the electrons on the plate and the new electrons being pushed onto it increases
- This means a greater amount of work must be done to increase the charge on the negative plate or in other words:

The potential difference V across the capacitor increases as the amount of charge Q increases

- The charge Q on the capacitor is **directly proportional** to its potential difference V
- The graph of charge against potential difference is therefore a straight line graph through the origin
- The electric potential energy stored in the capacitor can be determined from the **area under the potential-charge graph** which is equal to the area of a right-angled triangle:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

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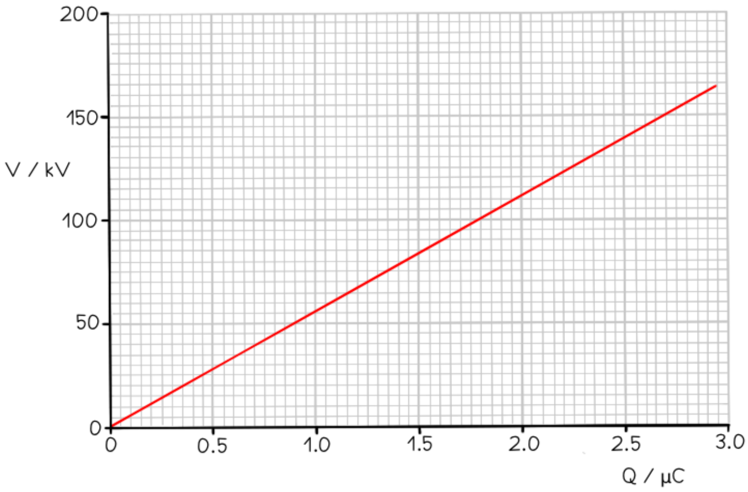
Worked example: Potential charge

The variation of the potential V of a charged isolated metal sphere with surface charge Q is shown on the graph below.



Using the graph, determine the electric potential energy stored on the sphere when charged to a potential of 100 kV.

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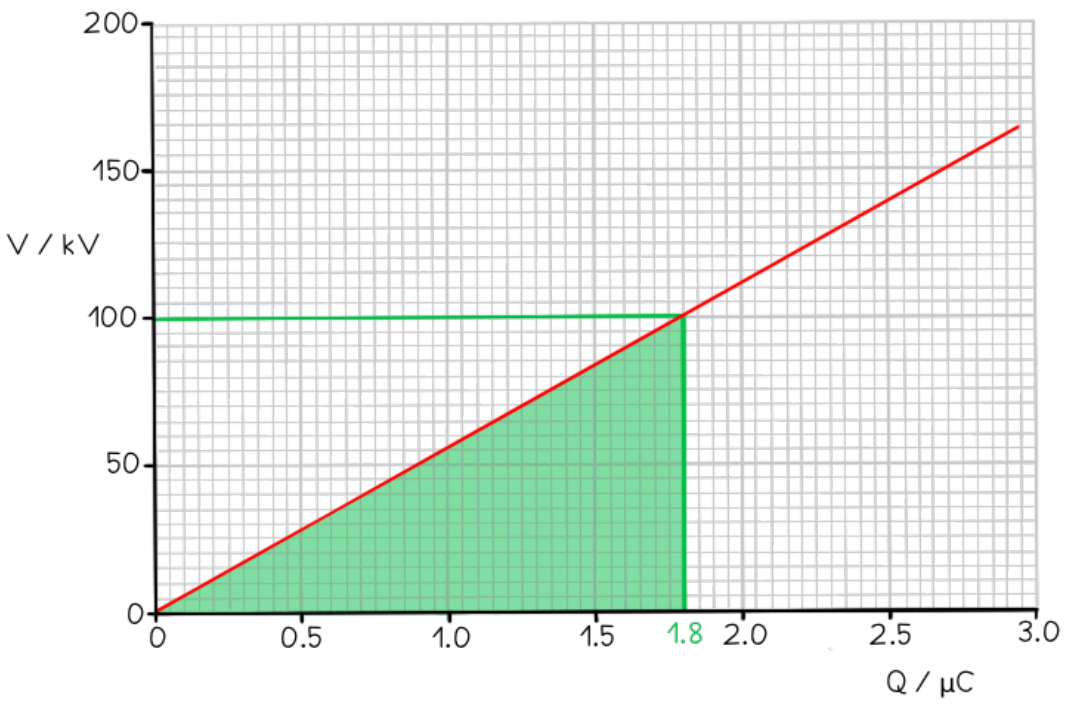
Step 1:

Determine the charge on the sphere at the potential of 100 kV



From the graph, the charge on the sphere at 100 kV is **1.8 μC**

Step 2:

Calculate the electric potential energy stored

The E.P.E stored is the area under the graph at 100 kV

The area is equal to a right-angled triangle, so, can be calculated with the equation:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area} = \frac{1}{2} \times 1.8\ \mu\text{C} \times 100\ \text{kV}$$

$$\text{E.P.E} = \frac{1}{2} \times 1.8 \times 10^{-6} \times 100 \times 10^{3} = \mathbf{0.09\ J}$$

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19.1.5 ENERGY STORED IN A CAPACITOR

Calculating Energy Stored in a Capacitor

- Recall the **electric potential energy is the area under a potential-charge graph**
- This is equal to the **work done** in charging the capacitor to a particular potential difference
  - The shape of this area is a right angled triangle
- Therefore the work done, or **energy stored** in a capacitor is defined by the equation:

$$W = \frac{1}{2} QV$$

- Substituting the charge with the capacitance equation Q = CV, the work done can also be defined as:

$$W = \frac{1}{2} CV^2$$

- Where:
  - W = work done/energy stored (J)
  - Q = charge on the capacitor (C)
  - V = potential difference (V)
  - C = capacitance (F)
- By substituting the potential V, the work done can also be defined in terms of just the charge and the capacitance:

$$W = \frac{Q^2}{2C}$$

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Worked example: Energy stored in a capacitor

Calculate the change in the energy stored in a capacitor of capacitance 1500 μF when the potential difference across the capacitor changes from 30 V to 10 V.

Step 1: Write down the equation for energy stored, in terms of capacitance *C* and p.d *V*

$$W = \frac{1}{2}CV^2$$

Step 2: The change in energy stored is proportional to the change in p.d

$$\Delta W = \frac{1}{2}C\Delta V^2 = \frac{1}{2}C(V_2^{\,2} - V_1^{\,2})$$

Step 3: Substitute in values

$$\Delta W = \frac{1}{2} \times 1500 \times 10^{-6} \times (30^2 - 10^2) = \mathbf{0.6\ J}$$

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19.2 CHARGING AND DISCHARGING

19.2.1 CAPACITOR DISCHARGE GRAPHS

Capacitor Discharge Graphs

- So far, only capacitors charged by a battery have been considered
  - This is when the electrons flow from the positive to negative plate
  - At the start, when the capacitor is charging, the current is large and then gradually falls to zero
- Capacitors are **discharged** through a resistor
  - The electrons now flow back from the negative plate to the positive plate until there are equal numbers on each plate
- At the start of discharge, the current is large (but in the opposite direction to when it was charging) and gradually falls to zero

- As a capacitor discharges, the current, p.d and charge all decrease **exponentially**
- The means the rate at which the current, p.d or charge decreases is proportional to the amount of current, p.d or charge it has left
- The graphs of the variation with time of current, p.d and charge are all identical and represent an **exponential decay**

- **The key features of the discharge graphs are:**
  - The shape of the current, p.d. and charge against time graphs are identical
  - Each graph shows exponential decay curves with decreasing gradient
  - The initial value starts on the y axis and decreases exponentially
- The rate at which a capacitor discharges depends on the **resistance** of the circuit
  - If the resistance is **high**, the current will decrease and charge will flow from the capacitor plates more slowly, meaning the capacitor will take longer to discharge
  - If the resistance is **low**, the current will increase and charge will flow from the capacitor plates quickly, meaning the capacitor will discharge faster

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19.2.2 CAPACITOR DISCHARGE EQUATIONS

The Time Constant

- The time constant of a capacitor discharging through a resistor is a measure of how long it takes for the capacitor to discharge
- The definition of the time constant is:

The time taken for the charge of a capacitor to decrease to 0.37 of its original value

- This is represented by the greek letter tau (τ) and measured in units of **seconds** (s)
- The time constant gives an easy way to compare the rate of change of similar quantities eg. charge, current and p.d.
- The time constant is defined by the equation:

$$\tau = RC$$

- Where:
  - τ = time constant (s)
  - R = resistance of the resistor (Ω)
  - C = capacitance of the capacitor (F)

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Worked example: Time constant

A capacitor of 7 nF is discharged through a resistor of resistance R. The time constant of the discharge is 5.6×10^{-3} s.

Calculate the value of R.

Step 1: Write out the known quantities

Capacitance, C = 7 nF = 7×10^{-9} F

Time constant, τ = 5.6×10^{-3} s

Step 2: Write down the time constant equation

$$\tau = RC$$

Step 3: Rearrange for resistance R

$$R = \frac{\tau}{C}$$

Step 4: Substitute in values and calculate

$$R = \frac{5.6 \times 10^{-3}}{7 \times 10^{-9}} = 8 \times 10^{5}\ \Omega = \mathbf{800\ k\Omega}$$

Using the Capacitor Discharge Equation

- The time constant is used in the exponential decay equations for the current, charge or potential difference (p.d) for a capacitor discharging through a resistor
  - These can be used to determine the amount of current, charge or p.d left after a certain amount of time when a capacitor is discharging
- The exponential decay of current on a discharging capacitor is defined by the equation:

$$I = I_0 e^{-\frac{t}{RC}}$$

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- Where:
  - I = current (A)
  - I_0 = initial current before discharge (A)
  - e = the exponential function
  - t = time (s)
  - RC = resistance (Ω) × capacitance (F) = the time constant τ (s)
- This equation shows that the faster the time constant τ, the quicker the exponential decay of the current when discharging
- Also, how big the initial current is affects the rate of discharge
  - If I_0 is large, the capacitor will take longer to discharge
- **Note:** during capacitor discharge, I_0 is always larger than I, this is because the current I will always be decreasing
- The current at any time is directly proportional to the p.d across the capacitor and the charge across the parallel plates
- Therefore, this equation also describes the change in p.d and charge on the capacitor:

$$Q = Q_0 e^{-\frac{t}{RC}}$$

- Where:
  - Q = charge on the capacitor plates (C)
  - Q_0 = initial charge on the capacitor plates (C)

$$V = V_0 e^{-\frac{t}{RC}}$$

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- Where:
  - V = p.d across the capacitor (C)
  - V_0 = initial p.d across the capacitor (C)

The Exponential Function *e*

- The symbol e represents the exponential constant, a number which is approximately equal to e = 2.718...
- On a calculator it is shown by the button e^x
- The inverse function of e^x is ln(y), known as the natural logarithmic function
  - This is because, if $e^x = y$, then $x = \ln(y)$
- The 0.37 in the definition of the **time constant** arises as a result of the exponential constant, the true definition is:

The time taken for the charge of a capacitor to decrease to $\frac{1}{e}$ of its original value

- Where $\frac{1}{e}$ = 0.3678...

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Worked example: Current capacitor discharge

The initial current through a circuit with a capacitor of 620 μF is 0.6 A.

The capacitor is connected across the terminals of a 450 Ω resistor.

Calculate the time taken for the current to fall to 0.4 A.

Step 1: Write out the known quantities

Initial current before discharge, I_0 = 0.6 A

Current, I = 0.4 A

Resistance, R = 450 Ω

Capacitance, C = 620 μF = 620 × 10^{-6} F

Step 2: Write down the equation for the exponential decay of current

$$I = I_0 e^{-\frac{t}{RC}}$$

Step 3: Calculate the time constant

τ = RC

τ = 450 × (620 × 10^{-6}) = 0.279 s

Step 4: Substitute into the current equation

$$0.4 = 0.6 \times e^{-\frac{t}{0.279}}$$

Step 5: Rearrange for the time *t*

$$\frac{0.4}{0.6} = e^{-\frac{t}{0.279}}$$

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The exponential can be removed by taking the natural log of both sides:

$$\ln\left(\frac{0.4}{0.6}\right) = -\frac{t}{0.279}$$

$$t = -0.279 \times \ln\left(\frac{0.4}{0.6}\right) = 0.1131 = 0.1 \text{ s}$$

Exam Tip

Make sure you're confident in rearranging equations with natural logs (ln) and the exponential function (e). To refresh your knowledge of this, have a look at the AS Maths revision notes on Exponentials & Logarithms

20. Magnetic Fields

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20.1 MAGNETIC FIELDS

20.1.1 REPRESENTING MAGNETIC FIELDS

20. Magnetic Fields

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Magnetic Field Definition

- A magnetic field is a **field of force** that is created either by:
  - **Moving** electric charge
  - **Permanent** magnets
- Permanent magnets are materials that produce a magnetic field
- A stationary charge will **not** produce a magnetic field
- A magnetic field is sometimes referred to as a **B–field**
- A magnetic field is created around a **current carrying wire** due to the movement of electrons
- Although magnetic fields are invisible, they can be observed by the force that pulls on magnetic materials, such as iron or the movement of a needle in a plotting compass

Representing Magnetic Fields

- Magnetic fields are represented by magnetic field lines
  - These can be shown using iron filings or plotting compasses
- Field lines are best represented on **bar magnets**, which consist of a north pole on one end and south pole on the other
- The magnetic field is produced on a bar magnet by the movement of electrons within the atoms of the magnet
- This is a result of the electrons circulating around the atoms, representing a tiny current and hence setting up a magnetic field
- The direction of a magnetic field on a bar magnet is always from **north to south**

- When two bar magnets are pushed together, they either attract or repel each other:
  - Two **like** poles (north and north or south and south) **repel** each other
  - Two **opposite** poles (north and south) **attract** each other

20. Magnetic Fields

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- **The key aspects of drawing magnetic field lines:**
  - The lines come **out** from the north poles and **into** the south poles
  - The direction of the field line shows the direction of the force that a free magnetic north pole would experience at that point
  - The field lines are **stronger** the **closer** the lines are together
  - The field lines are **weaker** the **further apart** the lines are
  - Magnetic field lines **never** cross since the magnetic field is unique at any point
  - Magnetic field lines are **continuous**

- A uniform magnetic field is where the magnetic field strength is the same at all points
  - This is represented by equally spaced parallel lines, just like electric fields
- Magnetic fields can be represented in 3D by using the following symbols:
  - Dots represent the magnetic field directed **out** of the plane of the page
  - Crosses represent the magnetic field directed **into** the plane of the page

Exam Tip

The best way to remember which way around to draw magnetic fields in 3D is by imagining an arrow coming towards or away from you

• When the head of an arrow is coming towards you, you see the tip as a dot representing the arrow coming **'out'** of the page

• When an arrow is travelling away from you, you see the cross at the back of the arrow representing the arrow going **'into'** the page

20. Magnetic Fields

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20.1.2 FORCE ON A CURRENT-CARRYING CONDUCTOR

Force on a Current-Carrying Conductor

- A current-carrying conductor produces its own magnetic field
  - When interacting with an external magnetic field, it will experience a **force**
- A current-carrying conductor will only experience a force if the current through it is **perpendicular** to the direction of the magnetic field lines
- A simple situation would be a copper rod placed within a uniform magnetic field
- When current is passed through the copper rod, it experiences a force which makes it move

Calculating Magnetic Force on a Current-Carrying Conductor

- The strength of a magnetic field is known as the **magnetic flux density, B**
  - This is also known as the magnetic field strength
  - It is measured in units of **Tesla (T)**
- The force *F* on a conductor carrying current *I* at right angles to a magnetic field with flux density *B* is defined by the equation

$$\mathbf{F = BIL\sin\theta}$$

- Where:
  - F = force on a current carrying conductor in a B field (N)
  - B = magnetic flux density of external B field (T)
  - I = current in the conductor (A)
  - L = length of the conductor (m)
  - θ = angle between the conductor and external B field (degrees)
- This equation shows that the greater the current or the magnetic field strength, the greater the force on the conductor

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- The **maximum** force occurs when sin θ = 1
  - This means θ = 90° and the conductor is **perpendicular** to the B field
  - This equation for the magnetic force now becomes:

$$\mathbf{F = BIL}$$

- The **minimum** force (0) is when sin θ = 0
  - This means θ = 0° and the conductor is **parallel** to the B field
- It is important to note that a current-carrying conductor will experience **no** force in a magnetic field if is parallel to the field

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Worked example

A current of 0.87 A flows in a wire of length 1.4 m placed at 30° to a magnetic field of flux density 80 mT.

Calculate the force on the wire.

Step 1: Write down the known quantities

Magnetic flux density, B = 80 mT = 80 × 10^{-3} T

Current, I = 0.87 A

Length of wire, L = 1.4 m

Angle between the wire and the magnetic field, θ = 30°

Step 2: Write down the equation for force on a current-carrying conductor

F = BIL sinθ

Step 3: Substitute in values and calculate

F = (80 × 10^{-3}) × (0.87) × (1.4) × sin(30) = 0.04872 = 0.049 N (2 s.f)

Exam Tip

Remember that the direction of current flow is the flow of **positive** charge (positive to negative), and this is in the **opposite direction** to the flow of electrons

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20.1.3 FLEMING'S LEFT-HAND RULE

Fleming's Left-Hand Rule

- The **direction** of the force on a charge moving in a magnetic field is determined by the direction of the magnetic field and the current
- Recall that the direction of the current is the direction of **conventional** current flow (positive to negative)
- When the force, magnetic field and current are all mutually perpendicular to each other, the directions of each can be interpreted by **Fleming's left-hand rule:**
    - On the left hand, with the thumb pointed upwards, first finger forwards and second finger to the right ie. all three are perpendicular to each other
    - The **thumb** points in the direction of **motion** of the rod (or the direction of the force) (*F*)
    - The **first** finger points in the direction of the external **magnetic field** (*B*)
    - The **second** finger points in the direction of conventional **current** flow (*I*)

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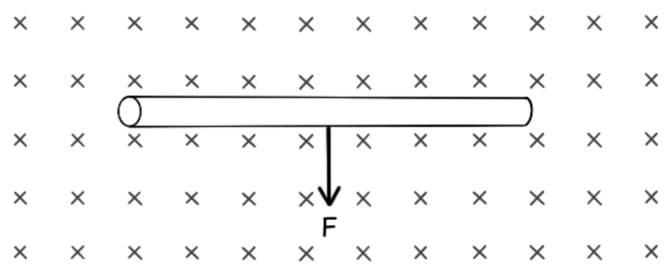
Worked example: Using the left-hand rule

State the direction of the current flowing in the wire in the diagram below.

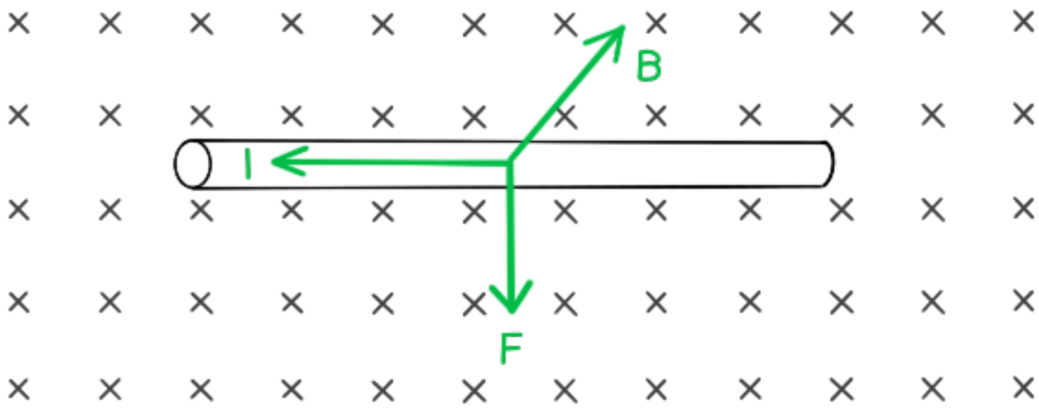


Using Fleming's left-hand rule:

B = into the page

F = vertically downwards

I = from right to left



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Exam Tip

Don’t be afraid to use Fleming’s left-hand rule during an exam. Although, it is best to do it subtly in order not to give the answer away to other students!

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20.1.4 MAGNETIC FLUX DENSITY

Magnetic Flux Density Definition

- The magnetic flux density B is defined as:

The force acting per unit current per unit length on a current-carrying conductor placed perpendicular to the magnetic field

- Rearranging the equation for magnetic force on a wire, the magnetic flux density is defined by the equation:

$$B = \frac{F}{IL}$$

- **Note:** this equation is only relevant when the B field is perpendicular to the current
- Magnetic flux density is measured in units of **tesla**, which is defined as:

A straight conductor carrying a current of 1A normal to a magnetic field of flux density of 1 T with force per unit length of the conductor of 1 N m^{-1}

- To put this into perspective, the Earth's magnetic flux density is around 0.032 mT and an ordinary fridge magnet is around 5mT

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Worked example: Calculating flux density

A 15 cm length of wire is placed vertically and at right angles to a magnetic field.

When a current of 3.0 A flows in the wire vertically upwards, a force of 0.04 N acts on it to the left.

Determine the flux density of the field and its direction.

Step 1: Write out the known quantities

Force on wire, F = 0.04 N

Current, I = 3.0 A

Length of wire = 15 cm = 15 × 10^{-2} m

Step 2: Magnetic flux density *B* equation

$$B = \frac{F}{IL}$$

Step 3: Substitute in values

$$B = \frac{0.04}{3 \times 15 \times 10^{-2}} = 0.089 \text{ T (2 s.f)}$$

Step 4: Determine the direction of the B field

Using Fleming's left-hand rule :

F = to the left

I = vertically upwards

therefore, B = into the page

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20.1.5 FORCE ON A MOVING CHARGE

Calculating Magnetic Force on a Moving Charge

- The magnetic force on an isolating moving charge, such an electron, is given by the equation:

$$\mathbf{F = BQv \sin\theta}$$

- Where:
  - F = force on the charge (N)
  - B = magnetic flux density (T)
  - v = speed of the charge (m s^{-1})
  - θ = angle between charge's velocity and magnetic field (degrees)

- Equivalent to the force on a wire, if the magnetic field *B* is perpendicular to the direction of the charge's velocity, the equation simplifies to:

$$\mathbf{F = BQv}$$

- According to Fleming's left hand rule:
  - When an electron enters a magnetic field from the **left**, if the magnetic field is directed **into the page**, then the force on it will be directed **upwards**
- The equation shows:
  - If the direction of the electron changes, the magnitude of the force will change too
- The force due to the magnetic field is always perpendicular to the velocity of the electron
  - **Note:** this is equivalent to circular motion
- Fleming's left-hand rule can be used again to find the direction of the force, magnetic field and velocity
  - The key difference is that the second finger representing current *I* (direction of positive charge) is now the **direction of velocity *v*** of the positive charge

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Worked example: Calculating magnetic force on a moving electron

?

An electron is moving at 5.3×10^7 m s^{-1} in a uniform magnetic field of flux density 0.2 T.

Calculate the force on the electron when it is moving at 30° to the field, and state the factor it increases by compared to when it travels perpendicular to the field.

Step 1: Write out the known quantities

Speed of the electron, $v = 5.3 \times 10^7$ m s^{-1}

Charge of an electron, $Q = 1.60 \times 10^{-19}$ C

Magnetic flux density, $B = 0.2$ T

Angle between electron and magnetic field, $\theta = 30°$

Step 2: Write down the equation for the magnetic force on an isolated particle

$$F = BQv \sin\theta$$

Step 3: Substitute in values, and calculate the force on the electron at 30°

$$F = (0.2) \times (1.60 \times 10^{-19}) \times (5.3 \times 10^7) \times \sin(30) = 8.5 \times 10^{-13} \text{ N}$$

Step 4: Calculate the electron force when travelling perpendicular to the field

$$F = BQv = (0.2) \times (1.60 \times 10^{-19}) \times (5.3 \times 10^7) = 1.696 \times 10^{-12} \text{ N}$$

Step 5: Calculate the ratio of the perpendicular force to the force at 30°

$$\frac{1.696 \times 10^{-12}}{8.5 \times 10^{-13}} = 1.995 = \mathbf{2}$$

Therefore, the force on the electron is twice as strong when it is moving perpendicular to the field than when it is moving at 30° to the field

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Exam Tip

Remember not to mix this up with F = BIL!

- **F = BIL** is for a current carrying conductor
- **F = Bqv** is for an isolated moving charge (which may be inside a conductor)

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20.1.6 HALL VOLTAGE

Hall Voltage

- The Hall voltage is a product of the Hall effect
- Hall voltage is defined as:

> ***The potential difference produced across an electrical conductor when an external magnetic field is applied perpendicular to the current through the conductor***

- When an external magnetic field is applied perpendicular to the direction of current through a conductor, the electrons experience a magnetic force
- This makes them drift to one side of the conductor, where they all gather and becomes more negatively charged
- This leaves the opposite side deficident of electrons, or positively charged
- There is now a potential difference across the conductor
    - This is called the Hall Voltage, V_H

- An equation for the Hall voltage V_H is derived from the electric and magnetic forces on the charges

- The voltage arises from the electrons accumulating on one side of the conductor slice
- As a result, an electric field is set up between the two opposite sides
- The two sides can be treated like oppositely charged parallel plates, where the electric field strength E is equal to:

$$E = \frac{V_H}{d}$$

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- Where:
  - V_H = Hall voltage (V)
  - d = width of the conductor slice (m)
- A single electron has a drift velocity of *v* within the conductor. The magnetic field is into the plane of the page, therefore the electron has a magnetic force F_B to the right:

$$\mathbf{F_B = Bqv}$$

- This is equal to the electric force F_E to the left:

$$\mathbf{F_E = qE}$$

$$\mathbf{qE = Bqv}$$

- Substituting E and cancelling the charge q

$$\frac{V_H}{d} = \text{B}v$$

- Recall that current *I* is related to the drift velocity *v* by the equation:

$$\mathbf{I = nAvq}$$

- Where:
  - A = cross-sectional area of the conductor (m^2)
  - n = number density of electrons (m^{-3})
- Rearranging this for *v* and substituting it into the equation gives:

$$\frac{V_H}{d} = \text{B}\,\frac{I}{nAq}$$

- The cross-sectional area *A* of the slice is the product of the width *d* and thickness *t*:

$$\mathbf{A = dt}$$

- Substituting *A* and rearranging for the Hall voltage V_H leads to the equation:

$$\frac{V_H}{d} = \text{B}\,\frac{I}{n(dt)q}$$

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$$V_H = B \frac{I}{ntq}$$

- Where:
  - B = magnetic flux density (T)
  - q = charge of the electron (C)
  - I = current (A)
  - n = number density of electrons (m^{-3})
  - t = thickness of the conductor (m)
- This equation shows that the smaller the electron density n of a material, the larger the magnitude of the Hall voltage
  - This is why a semiconducting material is often used for a Hall probe
- **Note:** if the electrons were placed by positive charge carriers, the negative and positive charges would still deflect in opposite directions
  - This means there would be no change in the polarity (direction) of the Hall voltage

Exam Tip

Remember to use Fleming's left-hand rule to obtain the direction the electrons move due to the magnetic force created by the magnetic field.

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20.1.7 USING A HALL PROBE

Measuring Magnetic Flux Density using a Hall Probe

- A Hall probe can be used to measure the magnetic flux density between two magnets based on the Hall effect
- It consists of a cylinder with a flat surface at the end
- To measure the magnetic flux density between two magnets, the flat surface of the probe must be directed between the magnets so the magnetic field lines pass completely perpendicular to this surface
- The probe is connected to a voltmeter to measure the Hall voltage
- If the probe is not held in the correct orientation (perpendicular to the field lines), the voltmeter reading will be reduced
- Since the Hall voltage is directly proportional to the magnetic flux density, the flux density of the magnets can be obtained
- A Hall probe is sensitive enough to measure even the Earth's magnetic flux density

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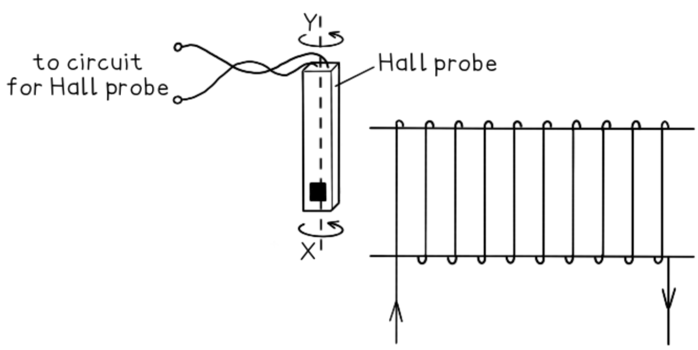
Worked example: Using a Hall probe

A Hall probe is placed near one end of a solenoid, as shown in the diagram.



The Hall probe is rotated about the axis XY.

State and explain why the magnitude of the Hall voltage varies.

- The Hall voltage depends on angle between the magnetic field and the plane of the probe
- The Hall voltage reaches a **maximum** when the field is **perpendicular** to the probe
- The Hall voltage is **zero** when the field is **parallel** to the probe

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20.1.8 MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

Motion of a Charged Particle in a Uniform Magnetic Field

- A charged particle in uniform magnetic field which is perpendicular to its direction of motion travels in a **circular** path
- This is because the magnetic force F_B will always be perpendicular to its velocity *v*
  - F_B will always be directed towards the centre of the path

- The magnetic force F_B provides the **centripetal force** on the particle
- Recall the equation for centripetal force:

$$F = \frac{mv^2}{r}$$

- Where:
  - m = mass of the particle (kg)
  - v = linear velocity of the particle (m s^{-1})
  - r = radius of the orbit (m)
- Equating this to the force on a moving charged particle gives the equation:

$$\frac{mv^2}{r} = Bqv$$

- Rearranging for the radius *r* obtains the equation for the radius of the orbit of a charged particle in a perpendicular magnetic field:

$$r = \frac{mv}{Bq}$$

- This equations shows that:
  - Faster moving particles with speed *v* move in larger circles (larger *r*): **r** ∝ **v**
  - Particles with greater mass *m* move in larger circles: **r** ∝ **m**
  - Particles with greater charge *q* move in smaller circles: **r** ∝ 1 / **q**
  - Particles moving in a strong magnetic field *B* move in smaller circles: **r** ∝ 1 / **B**

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Worked example: Calculating the radius of motion

?

An electron with a charge-to-mass ratio of 1.8×10^{11} C kg^{-1} is travelling at right angles to a uniform magnetic field of flux density 6.2 mT.

The speed of the electron is 3.0×10^{6} m s^{-1}.

Calculate the radius of the circular path of the electron.

Step 1: Write down the known quantities

Charge-to-mass ratio = $\frac{q}{m}$ = **1.8×10^{11} C kg^{-1}**

Magnetic flux density, B = 6.2 mT

Electron speed, v = 3.0×10^{6} m s^{-1}

Step 2: Write down the equation for the radius of a charged particle in a perpendicular magnetic field

$$r = \frac{mv}{Bq}$$

Step 3: Substitute in values

$$\frac{m}{q} = \frac{1}{1.8 \times 10^{11}}$$

$$r = \frac{(3.0 \times 10^{6})}{(1.8 \times 10^{11})(6.2 \times 10^{-3})} = 2.688 \times 10^{-3} \text{ m} = \mathbf{2.7\ mm} \text{ (2 s.f.)}$$

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20.1.9 VELOCITY SELECTION

Velocity Selection

- A velocity selector is:

A device consisting of perpendicular electric and magnetic fields where charged particles with a specific velocity can be filtered

- Velocity selectors are used in devices, such as mass spectrometers, in order to produce a beam of charged particles all travelling at the same velocity
- The construction of a velocity selector consists of two horizontal oppositely charged plates situated in a vacuum chamber
  - The plates provide a uniform electric field with strength *E* between them
- There is also a uniform magnetic field with flux density *B* applied perpendicular to the electric field
  - If a beam of charged particles enter between the plates, they may all have the same charge but travel at different speeds *v*
- The electric force **does not** depend on the velocity: $F_E = EQ$
- However, the magnetic force **does** depend on the velocity: $F_B = BQv$
  - The magnetic force will be greater for particles which are travelling faster
- To select particles travelling at exactly the desired the speed *v*, the electric and magnetic force must therefore be **equal**, but in **opposite** directions

$$\mathbf{F_E = F_B}$$

- The resultant force on the particles at speed *v* will be zero, so they will remain undeflected and pass straight through between the plates
- By equating the electric and magnetic force equations:

$$\mathbf{EQ = BQv}$$

- The charge *Q* will cancel out on both sides to give the selected velocity *v* equation:

$$v = \frac{E}{B}$$

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- Therefore, the speed *v* in which a particle will remain undeflected is found by the **ratio of the electric and magnetic field strength**
  - If a particle has a speed greater or less than *v*, the magnetic force will deflect it and collide with one of the charged plates
  - This would remove the particles in the beam that are not exactly at speed *v*
- **Note:** the gravitational force on the charged particles will be negligible compared to the electric and magnetic forces and therefore can be ignored in these calculations

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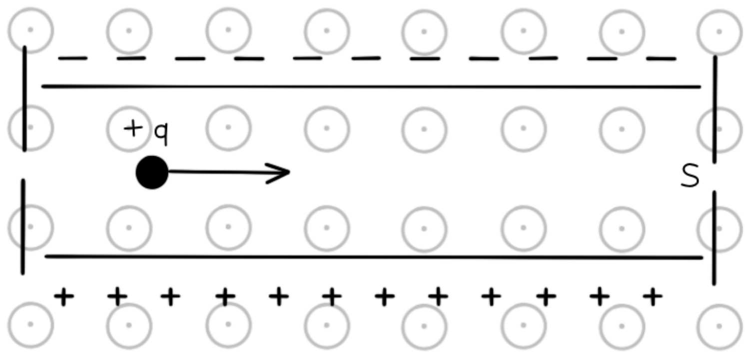
Worked example: Velocity selection

A positive ion travels between two charged plates towards a slit, as shown in the diagram.



a) State the direction of the electric and magnetic fields on the ion.

b) Calculate the speed of the ion emerging from slit S when the magnetic flux density is 0.50 T and the electric field strength is 2.8 kV m^{-1}.

c) Which plate will the ion be deflected towards if the speed of the ion is greater than the speed in part (b).

Part (a)

Step 1: Direction of E field

- Electric field lines point from the positive to negative to charge
- Therefore, it must be directed **vertically upwards**

Step 2: Direction of B field

- Using Fleming's left-hand rule:
  - The charge or current I is to the right
  - B is out of the page
  - Therefore, the force F is **vertically downwards**

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Part (b)

Velocity selector equation

Electric field strength, E = 2.8 kV m^{-1} = **2.8×10^3 V m^{-1}**

Magnetic flux density, B = **0.50 T**

$$v = \frac{E}{B} = \frac{2.8 \times 10^3}{0.50} = \mathbf{5600\ m\ s^{-1}}$$

Part (c)

If the speed increases, the magnetic force must be greater because $F_B \propto v$

Since the magnetic force would direct the ion downwards in the direction of the field, the ion will be deflected **towards the positive plate**

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20.1.10 MAGNETIC FIELDS IN WIRES, COILS & SOLENOIDS

Magnetic Fields in Wires, Coils & Solenoids

- Magnetic field patterns are not only observed around bar magnets, magnetic fields are formed wherever current is flowing, such as in:
    - Long straight wires
    - Long solenoids
    - Flat circular coils

Field Lines in a Current-Carrying Wire

- Magnetic field lines in a current carrying wire are circular rings, centered on the wire
- The field lines are strongest near the wire and become further part away from the wire
- Reversing the current reverses the direction of the field

- The field lines are clockwise or anticlockwise around the wire, depending on the direction of the current
- The direction of the magnetic field is determined by **Maxwell's right hand screw rule**
    - This is determined by pointing the **right-hand** thumb in the direction of the current in the wire and curling the fingers onto the palm
    - The direction of the curled fingers represents the direction of the magnetic field around the wire
    - For example, if the current is travelling vertically upwards, the magnetic field lines will be directed anticlockwise, as seen from directly above the wire
    - **Note:** the direction of the current is taken to be the conventional current ie. from **positive** to **negative**, **not** the direction of electron flow

Field Lines in a Solenoid

- As seen from a current carrying wire, an electric current produces a magnetic field
- An electromagnetic makes use of this by using a coil of wire called a **solenoid** which concentrates the magnetic field
- One ends becomes a north pole and the other the south pole

- Therefore, the magnetic field lines around a solenoid are very similar to a bar magnet
    - The field lines **emerge** from the **north** pole
    - The field lines **return** to the **south** pole
- Which is the north or south pole depends on the direction of the current
    - This is found by the **right hand grip rule**

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- This involves gripping the electromagnet so the fingers represent the direction of the current flow of the wire
- The thumb points in the direction of the field lines inside the coil, or in other words, point towards the electromagnet's **north pole**

Field Lines in a Flat Circular Coil

- A flat circular coil is equal to one of the coils of a solenoid
- The field lines will emerge through one side of the circle (north pole) and leave the other (south pole)
- As before, the direction of the north and south pole depend on the direction of the current
  - This can be determined by using the **right hand thumb rule**
  - It easier to find the direction of the magnetic field on the straight part of the circular coil to determine which direction the field lines are passing through

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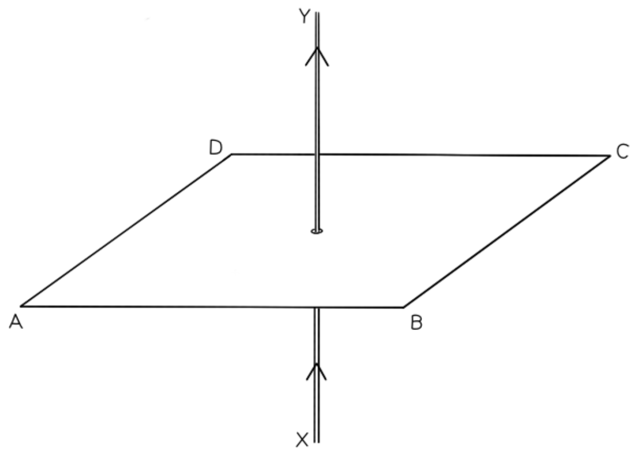
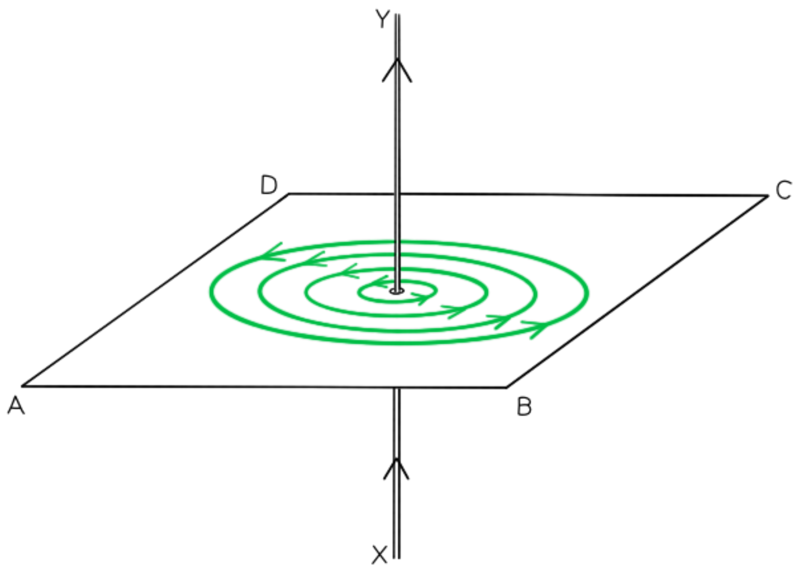
Worked example

The current in a long, straight vertical wire is in the direction XY, as shown in the diagram.



Sketch the pattern of the magnetic flux in the horizontal plane ABCD due to the current-carrying wire. Draw at least four flux lines.



✓ **Concentric circles**

✓ **Increasing separation between each circle**

✓ **Arrows drawn in anticlockwise direction**

Exam Tip

Remember to draw the arrows showing the direction of the field lines on every single field line you draw. Also, ensure that in a uniform magnetic field, the field lines are equally spaced.

Factors Affecting Magnetic Field Strength

- The strength of the magnetic field of a solenoid can be increased by:
  - Adding a core made from a **ferrous** (iron-rich) material eg. an iron rod
  - Adding **more turns** in the coil
- When current flows through the solenoid with an iron core, it becomes magnetised, creating an even stronger field
  - The addition of an iron core can strengthen the magnetic field up to a several hundred times more
- When more turns are added in the coil, this concentrates the magnetic field lines, causing the magnetic field strength to increase

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20.1.11 FORCES BETWEEN CURRENT-CARRYING CONDUCTORS

Origin of the Forces Between Current-Carrying Conductors

- A current carrying conductor, such as a wire, produces a magnetic field around it
- The direction of the field depends on the direction of the current through the wire
    - This is determined by the **right hand thumb rule**
- Parallel current-carrying conductors will therefore either attract or repel each other
    - If the currents are in the **same** direction in both conductors, the magnetic field lines between the conductors cancel out – the conductors will **attract** each other
    - If the currents are in the **opposite** direction in both conductors, the magnetic field lines between the conductors push each other apart – the conductors will **repel** each other

- When the conductors **attract**, the direction of the magnetic forces will be **towards** each other
- When the conductors **repel**, the direction of the magnetic forces will be **away** from each other
- The magnitude of each force depend on the amount of current and length of the wire

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Worked example

Two long, straight, current-carrying conductors, WX and YZ, are held at a close distance, as shown in diagram 1.

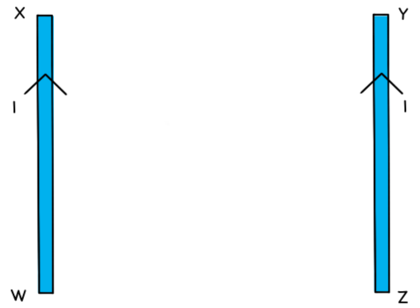

diagram 1

The conductors each carry the same magnitude current in the same direction. A plan view from above the conductors is shown in diagram 2.

diagram 2

On diagram 2, draw arrows, one in each case, to show the direction of:

- The magnetic field at X due to the current in wire YZ (label this arrow B_{YZ})
- The force at X as a result of the magnetic field due to the current in the wire YZ (label this arrow F_{YZ})
- The magnetic field at Y due to the current in wire WX (label this arrow B_{WX})
- The force at Y as a result of the magnetic field due to the current in the wire WX (label this arrow F_{WX})

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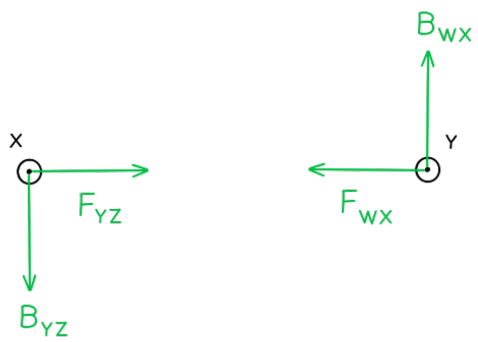

- Newton's Third Law states:
  - When two bodies interact, the force on one body is **equal but opposite** in direction to the force on the other body
- Therefore, the forces on the wires act in equal but opposite directions

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20.2 ELECTROMAGNETIC INDUCTION

20.2.1 MAGNETIC FLUX

Magnetic Flux Definition

- Electromagnetic induction is when an e.m.f is induced in a closed circuit conductor due to it moving through a magnetic field
- This happens when a conductor **cuts** through magnetic field lines
- The amount of e.m.f induced is determined by the magnetic flux
    - This is the total magnetic field that passes through a given area
    - It is a maximum when the magnetic field lines are **perpendicular** to the area
    - It is at a minimum when the magnetic field lines are **parallel** to the area
- The **magnetic flux** is defined as:

    The product of the magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density

- In other words, magnetic flux is the **number of magnetic field lines through a given area**

Calculating Magnetic Flux

- Magnetic flux is defined by the symbol Φ (greek letter ‘phi’)
- It is measured in units of **Webers (Wb)**
- Magnetic flux can be calculated using the equation:

$$\Phi = BA$$

- Where:
    - Φ = magnetic flux (Wb)
    - B = magnetic flux density (T)
    - A = cross-sectional area (m^2)

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- When the magnet field lines are not completely perpendicular to the area *A*, then the component of magnetic flux density *B* perpendicular to the area is taken
- The equation then becomes:

$$\mathbf{\Phi = BA\cos(\theta)}$$

- Where:
  - θ = angle between magnetic field lines and the line perpendicular to the plane of the area (often called the normal line) (degrees)

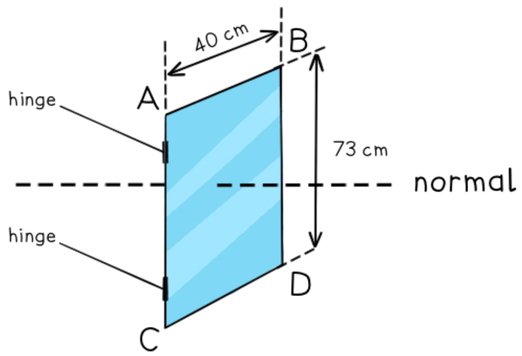
- This means the magnetic flux is:
  - **Maximum** = BA when cos(θ) =1 therefore **θ = 0°**. The magnetic field lines are perpendicular to the plane of the area
  - **Minimum** = 0 when cos(θ) = 0 therefore **θ = 90°**. The magnetic fields lines are parallel to the plane of the area
- An e.m.f is induced in a circuit when the magnetic flux linkage changes with respect to time
- This means an e.m.f is induced when there is:
  - A changing magnetic flux density *B*
  - A changing cross-sectional area *A*
  - A change in angle *θ*

20. Magnetic Fields

YOUR NOTES

Worked example

An aluminium window frame has a width of 40 cm and length of 73 cm as shown in the diagram below.



The frame is hinged along the vertical edge AC.

When the window is closed, the frame is normal to the Earth's magnetic field with magnetic flux density 1.8×10^{-5} T.

a) Calculate the magnetic flux through the window when it is closed.
b) Sketch the graph of the magnetic flux against angle between the field lines and the normal when the window is opened and rotated by 180°.

Part (a)

Step 1: Write out the known quantities

Cross-sectional area, A = 40 cm × 73 cm = $(40 \times 10^{-2}) \times (73 \times 10^{-2}) = 0.292$ m^2

Magnetic flux density, B = 1.8×10^{-5} T

Step 2: Write down the equation for magnetic flux

$$\Phi = BA$$

Step 3: Substitute in values

$$\Phi = (1.8 \times 10^{-5}) \times 0.292 = 5.256 \times 10^{-6} = 5.3 \times 10^{-6} \text{ Wb}$$

20. Magnetic Fields

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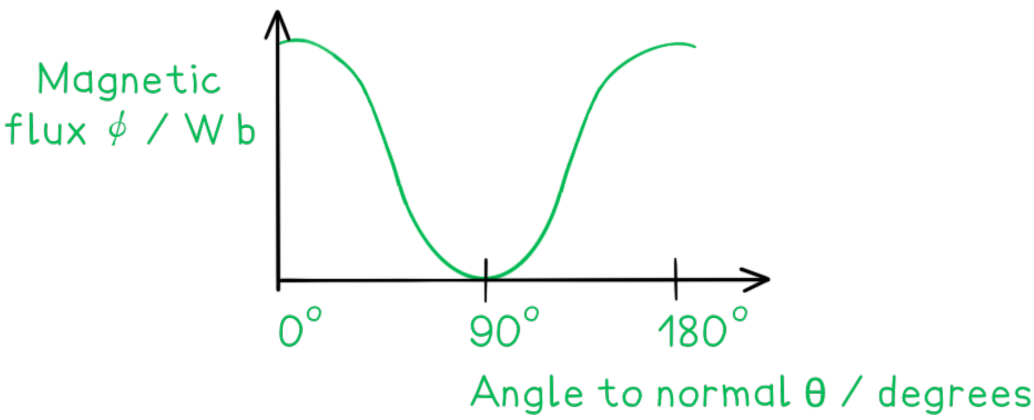
Part (b)

The magnetic flux will be at a minimum when the window is opened by 90° and a maximum when fully closed or opened to 180°



Exam Tip

Consider carefully the value of θ, it is the angle between the field lines and the line **normal** (perpendicular) to the plane of the area the field lines are passing through. If it helps, drawing the normal on the area provided will help visualise the correct angle.

20. Magnetic Fields

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20.2.2 MAGNETIC FLUX LINKAGE

Magnetic Flux Linkage

- The **magnetic flux linkage** is a quantity commonly used for solenoids which are made of *N* turns of wire
- Magnetic flux linkage is defined as:

The product of the magnetic flux and the number of turns

- It is calculated using the equation:

$$\Phi N = BAN$$

- Where:
  - Φ = magnetic flux (Wb)
  - N = number of turns of the coil
  - B = magnetic flux density (T)
  - A = cross-sectional area (m^2)
- The flux linkage ΦN has the units of **Weber turns (Wb turns)**
- As with magnetic flux, if the field lines are not completely perpendicular to the plane of the area they are passing through
- Therefore, the component of the flux density which is perpendicular is equal to:

$$\Phi N = BAN\cos(\theta)$$

20. Magnetic Fields

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Worked example

A solenoid of circular cross-sectional radius 0.40 m^2 and 300 turns is placed perpendicular to a magnetic field with a magnetic flux density of 5.1 mT.

Determine the magnetic flux linkage for this solenoid.

Step 1: Write out the known quantities

Cross-sectional area, $A = \pi r^2 = \pi(0.4)^2 = 0.503\ m^2$

Magnetic flux density, B = 5.1 mT

Number of turns of the coil, N = 300 turns

Step 2: Write down the equation for the magnetic flux linkage

$\Phi N = BAN$

Step 3: Substitute in values and calculate

$\Phi N = (5.1 \times 10^{-3}) \times 0.503 \times 300 = 0.7691 = 0.8$ Wb turns (2 s.f)

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20.2.3 PRINCIPLES OF ELECTROMAGNETIC INDUCTION

Principles of Electromagnetic Induction

- Electromagnetic induction is a phenomenon which occurs when an e.m.f is induced when a conductor moves through a magnetic field
- When the conductor cuts through the magnetic field lines:
  - This causes a change in **magnetic flux**
  - Which causes **work to be done**
  - This work is then transformed into **electrical energy**
- Therefore, if attached to a complete circuit, a current will be induced
- This is known as **electromagnetic induction** and is defined as:

  The process in which an e.m.f is induced in a closed circuit due to changes in magnetic flux

- This can occur either when:
  - A conductor cuts through a magnetic field
  - The direction of a magnetic field through a coil changes
- Electromagnetic induction is used in:
  - Electrical generators which convert mechanical energy to electrical energy
  - Transformers which are used in electrical power transmission
- This phenomenon can easily be demonstrated with a magnet and a coil, or a wire and two magnets

Experiment 1: Moving a magnet through a coil

- When a coil is connected to a sensitive voltmeter, a bar magnet can be moved in and out of the coil to induce an e.m.f

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The expected results are:

- When the bar magnet is **not moving**, the voltmeter shows a **zero reading**
    - When the bar magnet is held still inside, or outside, the coil, the rate of change of flux is zero, so, there is **no e.m.f induced**
- When the bar magnet begins to move inside the coil, there is a reading on the voltmeter
    - As the bar magnet moves, its magnetic field lines 'cut through' the coil, generating a **change in magnetic flux**
    - This induces an **e.m.f** within the coil, shown momentarily by the reading on the voltmeter
- When the bar magnet is taken back out of the coil, an e.m.f is induced in the **opposite direction**
    - As the magnet changes direction, the direction of the current changes
    - The voltmeter will momentarily show a reading with the opposite sign
- Increasing the **speed** of the magnet induces an e.m.f with a **higher magnitude**
    - As the speed of the magnet increases, the rate of change of flux increases
- The direction of the electric current, and e.m.f, induced in the conductor is such that it **opposes** the change that produces it
- Factors that will increase the induced e.m.f are:
    - Moving the magnet faster through the coil
    - Adding more turns to the coil
    - Increasing the strength of the bar magnet

Experiment 2: Moving a wire through a magnetic field

- When a long wire is connected to a voltmeter and moved between two magnets, an e.m.f is induced
- **Note:** there is no current flowing through the wire to start with

The expected results are:

- When the wire is **not moving**, the voltmeter shows a **zero reading**
    - When the wire is held still inside, or outside, the magnets, the rate of change of flux is zero, so, there is **no e.m.f induced**
- As the wire is moved through between the magnets, an **e.m.f** is induced within the wire, shown momentarily by the reading on the voltmeter
    - As the wire moves, it 'cuts through' the magnetic field lines of the magnetic, generating a **change in magnetic flux**

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- When the wire is taken back out of the magnet, an e.m.f is induced in the **opposite direction**
  - As the wire changes direction, the direction of the current changes
  - The voltmeter will momentarily show a reading with the opposite sign
- As before, the direction of the electric current, and e.m.f, induced in the conductor is such that it **opposes** the change that produces it
- Factors that will increase the induced e.m.f are:
  - Increasing the length of the wire
  - Moving the wire between the magnets faster
  - Increasing the strength of the magnets

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20.2.4 FARADAY'S & LENZ'S LAWS

Faraday's & Lenz's Laws

- Faraday's law tells us the magnitude of the induced e.m.f in electromagnetic induction and is defined as:

The magnitude of the induced e.m.f is directly proportional to the rate of change in magnetic flux linkage

$$\varepsilon = N \frac{\Delta \phi}{\Delta t}$$

- Where:
  - ε = induced e.m.f (V)
  - N = number of turns of coil
  - Δϕ = change in magnetic flux (Wb)
  - Δt = time interval (s)
- Lenz's Law gives the **direction** of the induced e.m.f as defined by Faraday's law:

The induced e.m.f acts in such a direction to produce effects which oppose the change causing it

- Lenz's law combined with Faraday's law is:

$$\varepsilon = -N \frac{\Delta \phi}{\Delta t}$$

- This equation shows:
  - When a bar magnet goes through a coil, an e.m.f is induced within the coil due to a change in magnetic flux
  - A current is also induced which means the coil now has its own magnetic field
  - The coil's magnetic field acts in the **opposite direction** to the magnetic field of the bar magnet
- If a direct current (d.c) power supply is replaced with an alternating current (a.c) supply, the e.m.f induced will also be alternating with the same frequency as the supply

20. Magnetic Fields

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Experimental Evidence for Lenz's Law

- To verify Lenz's law, the only apparatus needed is:
  - A bar magnet
  - A coil of wire
  - A sensitive ammeter
- **Note:** a cell is **not** required
- A known pole (either north or south) of the bar magnet is pushed into the coil, which induces a magnetic field in the coil
  - Using the right hand grip rule, the curled fingers indicate the direction of the **current** and the thumb indicates the direction of the **induced magnetic field**
- The direction of the current is observed on the ammeter
  - Reversing the magnet direction would give an opposite deflection on the meter
- The induced field (in the coil) **repels** the bar magnet
- This is because of **Lenz's law**:
  - The direction of the induced field in the coil pushes against the change creating it, ie. the bar magnet

20. Magnetic Fields

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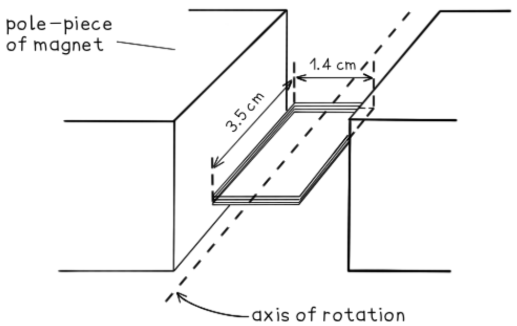
Worked example: Faraday's & Lenz's Laws

A small rectangular coil contains 350 turns of wire. The longer sides are 3.5 cm and the shorter sides are 1.4 cm.



The coil is held between the poles of a large magnet so that the coil can rotate about an axis through its centre.

The magnet produces a uniform magnetic field of flux density 80 mT between its poles. The coil is positioned horizontally and then turned through an angle of 40° in a time of 0.18 s.

Calculate the magnitude of the average e.m.f induced in the coil.

Step 1: Write down the known quantities

Magnetic flux density, B = 80 mT = 80 × 10^{-3} T

Area, A = 3.5 × 1.4 = (3.5 × 10^{-2}) × (1.4 × 10^{-2}) = 4.9 × 10^{-4} m^2

Number of turns, N = 350

Time interval, Δt = 0.18 s

Angle between coil and field lines, θ = 40°

Step 2: Write out the equation for Faraday's law:

$$\varepsilon = N \frac{\Delta\phi}{\Delta t}$$

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Step 3: Write out the equation for flux linkage:

$$\boldsymbol{\phi N = BAN\cos(\theta)}$$

Step 4: Substitute values into flux linkage equation:

$$\boldsymbol{\phi N = (80 \times 10^{-3}) \times (4.9 \times 10^{-4}) \times 350 \times \cos(40) = 0.0105 \text{ Wb turns}}$$

Step 5: Substitute flux linkage and time into Faraday's law equation:

$$\varepsilon = \frac{0.0105}{0.18} = 0.05839 = \mathbf{58\ mV}\ (2\text{ s.f.})$$

21. Alternating Currents

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21.1 PROPERTIES AND USES OF ALTERNATING CURRENT

21.1.1 ALTERNATING CURRENT & VOLTAGE

Properties of Alternating Current & Voltage

- An alternating current (a.c) is defined as:

  A current which periodically varies from positive to negative and changes its magnitude continuously with time

- This means the direction of an alternating current varies every **half cycle**
- The variation of current, or p.d., with time can be described as a sine curve ie. **sinusoidal**
  - Therefore, the electrons in a wire carrying a.c. move back and forth with simple harmonic motion
- As with SHM, the relationship between time period T and frequency f of an alternating current is given by:

$$T = \frac{1}{f}$$

- Peak current (I_0), or peak voltage (V_0), is defined as:

  The maximum value of the alternating current or voltage

- Peak current, or voltage, can be determined from the **amplitude** of the graph

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- Mains electricity is supplied as alternating current
  - Power stations produce alternating current
  - This is the type of current supplied when devices are plugged into sockets

21. Alternating Currents

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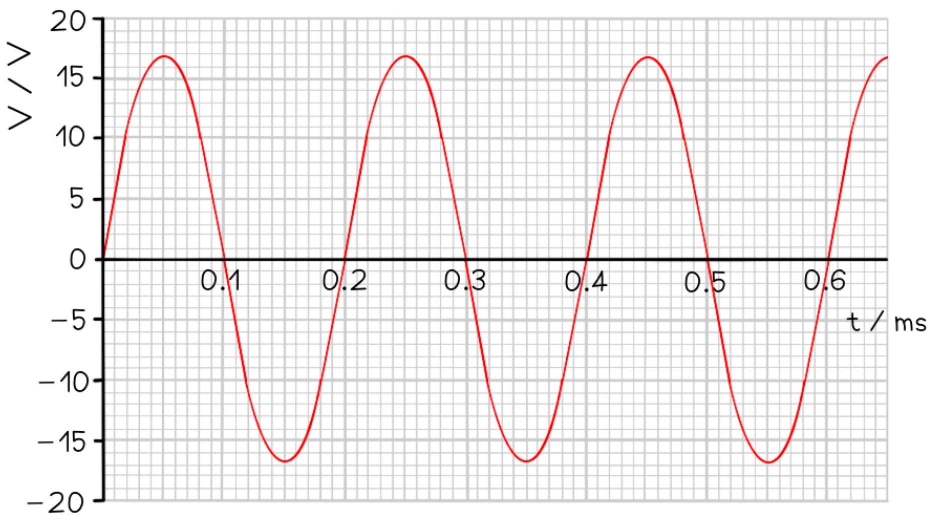
Worked example: Calculating frequency

The variation with time t of the output voltage V of an alternating voltage supply is shown in the graph below.



Use the graph to calculate the frequency of the supply.

Step 1:

Write down the period-frequency relation

$$f = \frac{1}{T}$$

21. Alternating Currents

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Step 2:

Calculate the time period from the graph

- The time period is the time taken for one complete cycle
- From the graph this is equation to 0.2 ms
- **Therefore,** the time period, T = 0.2 ms = $\mathbf{0.2 \times 10^{-3}}$ **s**

Step 3:

Substitute into frequency equation

$$f = \frac{1}{0.2 \times 10^{-3}} = 5000 \text{ Hz} = \mathbf{5\ kHz}$$

Exam Tip

Remember to double check the units on the alternating current and voltage graphs. These are often shown in the range of milli-seconds (ms) instead of seconds (s) on the x axis.

Using Sinusoidal Representations

- The equation representing alternating current which gives the value of the current *I* at any time *t* is:

$$\mathbf{I = I_0 \sin(\omega t)}$$

- Where:
  - I = current (A)
  - I_0 = peak current (A)
  - ω = angular frequency of the supply (rad s^{-1})
  - t = time (s)
- **Note:** this a sine function since the alternative current graph is sinusoidal
- A similar equation can be used for representing alternating voltage:

$$\mathbf{V = V_0 \sin(\omega t)}$$

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- Where:
  - V = voltage (V)
  - V_0 = peak voltage (V)
- Recall the relation the equation for angular frequency ω:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

21. Alternating Currents

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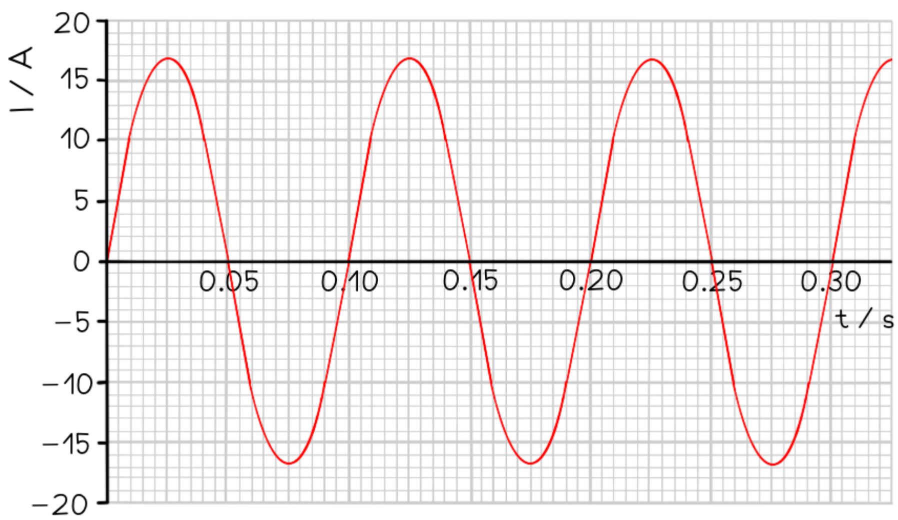
Worked example: Using sinusoidal representations

An alternating current I varies with time t as shown in the graph below.



Using the graph and the equation for alternating current, calculate the value of the current at a time 0.48 s.

Step 1: Write out the equation for alternating current

$$\mathbf{I = I_0 \sin(\omega t)}$$

Step 2: Write out the equation for angular frequency

$$\omega = \frac{2\pi}{T}$$

Step 3: Measure the time period T and peak current I_0 from the graph

The time period is the time taken for one full cycle, T = 0.10 s

Peak current (amplitude), I_0 = 17 A

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Step 4: Substitute values into alternating current equation at time t

Using the time given in the question, t = 0.48 s

$$I = I_0 \sin(\omega t) = I_0 \sin\left(\frac{2\pi t}{T}\right)$$

$$I = 17\sin\left(\frac{2\pi(0.48)}{0.1}\right) = -16.168 = \mathbf{-16\ A}\ (2\ \text{s.f})$$

Exam Tip

Remember to check that your calculator is in **radians** mode when using any of these equations. This is because angular frequency ω is measured in rad s^{-1}

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21.1.2 ROOT-MEAN-SQUARE CURRENT & VOLTAGE

Root-Mean-Square Current & Voltage

- Root-mean-square (r.m.s) values of current, or voltage, are a useful way of **comparing** a.c current, or voltage, to its equivalent direct current, or voltage
- The r.m.s values represent the d.c current, or voltage, values that will produce the same **heating effect**, or power dissipation, as the alternating current, or voltage
- The r.m.s value of an alternating current is defined as:

The value of a constant current that produces the same power in a resistor as the alternating current

- The r.m.s current $I_{r.m.s}$ is defined by the equation:

$$I_{r.m.s} = \frac{I_0}{\sqrt{2}}$$

- So, r.m.s current is equal to 0.707 × I_0, which is about 70% of the peak current I_0
- The r.m.s value of an alternating voltage is defined as:

The value of a constant voltage that produces the same power in a resistor as the alternating voltage

- The r.m.s voltage $V_{r.m.s}$ is defined by the equation:

$$V_{r.m.s} = \frac{V_0}{\sqrt{2}}$$

- Where:
  - I_0 = peak current (A)
  - V_0 = peak voltage (V)
- The r.m.s value is therefore defined as:

The steady direct current, or voltage, that delivers the same average power in a resistor as the alternating current, or voltage

- A resistive load is any electrical component with resistance eg. a lamp

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Worked example: Determining the r.m.s. current

An alternating current is I is represented by the equation

$$I = 410\sin(100\pi t)$$

where I is measured in amperes and t is in seconds.

For this alternating current, determine the r.m.s current.

Step 1: Write out the equation for r.m.s current

$$I_{r.m.s} = \frac{I_0}{\sqrt{2}}$$

Step 2: Determine the peak voltage I_0

- The alternating current equation is in the form: $\mathbf{I = I_0 \sin(\omega t)}$
- Comparing this to $I = 410\sin(100\pi t)$ means the peak current is $\mathbf{I_0}$ **= 410 A**

Step 3: Substitute into the $I_{r.m.s}$ equation

$$I_{r.m.s} = \frac{410}{\sqrt{2}} = 289.91 = \mathbf{290\ A}\ (2\text{ s.f.})$$

21. Alternating Currents

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21.1.3 MEAN POWER

Mean Power

- In mains electricity, current and voltage are varying all the time
- This also means the **power** varies constantly, recall the equations for power:

$$P = IV = I^2 R = \frac{V^2}{R}$$

- Where:
  - I = **direct** current (A)
  - V = **direct** voltage (A)
  - R = resistance (Ω)
- The r.m.s values means equations used for direct current and voltage can now be applied to alternating current and voltage
- They are also used determining an **average** current or voltage for alternating supplies
- Recall the equation for peak current:

$$I_0 = \sqrt{2}\, I_{r.m.s}$$

- Therefore, the peak (maximum) power is related to the mean (average) power by:

$$P_{mean} = I_{r.m.s} R$$

$$P = I_0^2 R = (\sqrt{2}\, I_{r.m.s})^2 R = 2I_{r.m.s}R = 2\, P_{mean}$$

$$P_{mean} = \frac{P}{2}$$

- Therefore, it can be concluded that:

The mean power in a resistive load is half the maximum power for a sinusoidal alternating current or voltage

21. Alternating Currents

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Worked example: Calculating mean power

An alternating voltage supplied across a resistor of 40 Ω has a peak voltage V_0 of 240 V.

Calculate the mean power of this supply.

Step 1: Write down the known quantities

Resistance, R = 40 Ω

Peak voltage, V_0 = 240 V

Step 2: Write out the equation for the peak power and calculate

$$\text{Peak power, P} = \frac{V_0^2}{R}$$

$$P = \frac{(240)^2}{40} = 1440 \text{ W}$$

Step 3: Calculate the mean power

- The mean power is **half** of the maximum (peak) power

Mean power = 1440 / 2 = 720 W

21. Alternating Currents

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21.1.4 RECTIFICATION

Rectification Graphs

- Rectification is defined as:

  The process of converting alternating current and voltage into direct current and voltage

- Rectification is used in electronic equipment which requires a direct current
  - For example, mains voltage must be rectified from the alternating voltage produced at power stations
- There are two types of rectification:
  - Half-wave rectification
  - Full-wave rectification
- For **half-wave** rectification:
  - The graph of the output voltage V_{out} against time is a sine curve with the positive cycles and a flat line (V_{out} = 0) on the negative cycle
  - This is because the diode only conducts in the positive direction
- For **full-wave** rectification:
  - The graph of the output voltage V_{out} against time is a sine curve where the positive cycles and the negative cycles are both curved 'bumps'

Half-Wave Rectification

- Half-wave rectification consists of a single **diode**
  - An alternating input voltage is connected to a circuit with a load resistor and diode in series
- The diode will only conduct during the positive cycles of the input alternating voltage,
  - Hence there is only current in the load resistor during these positive cycles
- The output voltage V_{out} across the resistor will fluctuate against against time in the same way as the input alternating voltage **except** there are no negative cycles
- This type of rectification means half of the time the voltage is zero
- So, the power available from a half-wave rectified supply is reduced

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Full-Wave Rectification

- Full-wave rectification requires a bridge rectifier circuit
  - This consists of **four** diodes connected across an input alternating voltage supply
- The output voltage V_{out} is taken across a load resistor
- During the **positive** cycles of the input voltage, one terminal if the voltage supply is positive and the other negative
  - Two diodes opposite each other that are in forward bias will conduct
  - The other two in reverse bias will not conduct
  - A current will flow in the load resistor with the positive terminal at the top of the resistor
- During the **negative** cycles of the input voltage, the positive and negative terminals of the input alternating voltage supply will swap
  - The two diodes that were forward bias will now be in reverse bias and not conduct
  - The other two in reverse bias will now be in forward bias and will conduct
  - The current in the load resistor will still flow in the same direction as before

- In both the positive and negative cycles, the current in the load resistor is the **same**
- Each diode pair is the same as in half-wave rectification
  - Since there are two pairs, this equates to full-wave rectification overall
- The main advantage of full-wave rectification compared to half-wave rectification is that there is **more power** available
  - Therefore, a greater power is supplied on every half cycle

21. Alternating Currents

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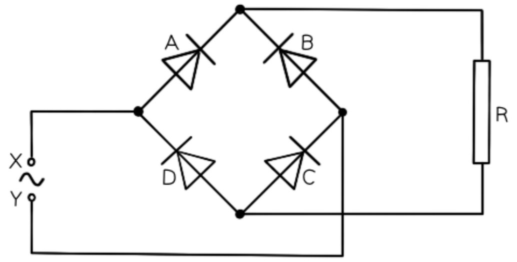
Worked example

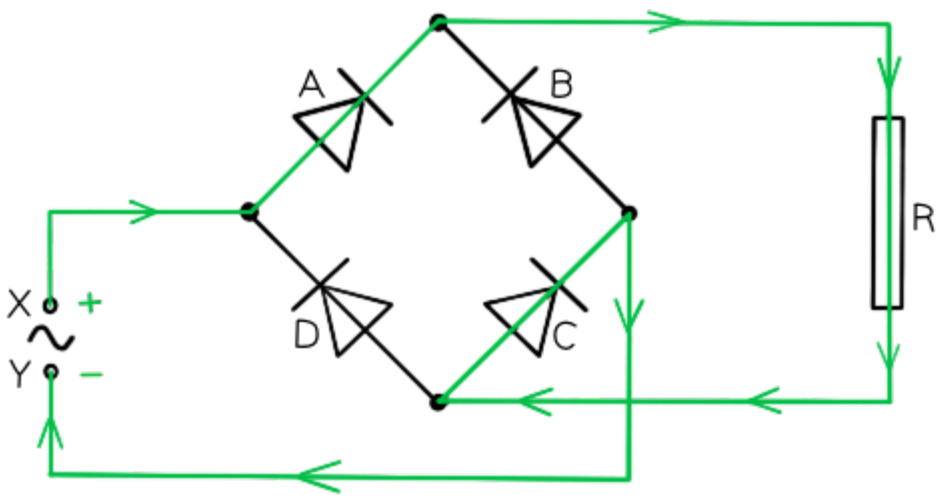
A bridge rectifier consists of four ideal diodes A, B, C and D, as connected in the figure shown below.



An alternating supply is applied between the terminal X and Y.

State which diodes are conducting when terminal X of the supply is positive.

- Draw path of the current direction with diodes in **forward bias**
- Remember that conventional current flow is from **positive to negative** and only travels through the paths with diodes in **forward bias**



Therefore, the answer is: diodes A and C

21. Alternating Currents

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21.1.5 SMOOTHING

Smoothing

- In rectification, to produce a steady direct current or voltage from an alternating current or voltage, a **smoothing capacitor** is necessary
- Smoothing is defined as:

The reduction in the variation of the output voltage or current

- This works in the following ways:
  - A single capacitor with capacitance C is connected in parallel with a load resistor of resistance R
  - The capacitor charges up from the input voltage and maintains the voltage at a high level
  - As it discharges gradually through the resistor when the rectified voltage drops but the voltage then rises again and the capacitor charges up again

- The resulting graph of a smoothed output voltage V_{out} and output current against time is a 'ripple' shape

- The amount of smoothing is controlled by the capacitance C of the capacitor and the resistance R of the load resistor
  - The less the rippling effect, the smoother the rectified current and voltage output
- The slower the capacitor discharges, the more the smoothing that occurs ie. smaller ripples
- This can be achieved by using:
  - A capacitor with **greater** capacitance *C*
  - A resistance with larger resistor *R*
- Recall that the product RC is the **time constant** τ of a resistor
- This means that the time constant of the capacitor must be **greater than the time interval** between the adjacent peaks of the output signal

21. Alternating Currents

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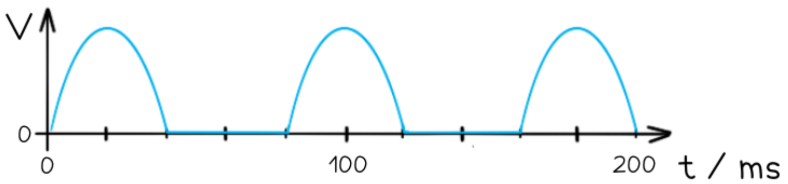
Worked example

The graph below shows the output voltage from a half-wave rectifier. The load resistor has a resistance of 2.6 kΩ. A student wishes to smooth the output voltage by placing a capacitor in parallel across the load resistor.



Using an appropriate calculation, suggest if a capacitor of 60 pF or 800 μF would be suitable for this task.

Step 1:

Calculate the time constant with the 60 pF capacitor

$$\tau = RC = (2.6 \times 10^3) \times (60 \times 10^{-12}) = 1.56 \times 10^{-7} \text{ s} = 156 \text{ ns}$$

Step 2:

Compare time constant of 60 pF capacitor with interval between adjacent peaks of the output signal

- The time interval between adjacent peaks is 80 ms
- The time constant of 156 ns is too small and the 60 pF capacitor will discharge far too quickly
- There would be no smoothing of the output voltages
- Therefore, the 60 pF capacitor is **not suitable**

21. Alternating Currents

Step 3:

Calculate the time constant with the 800 μF capacitor

$$\tau = RC = (2.6 \times 10^{3}) \times (800 \times 10^{-6}) = 2.08 \text{ s}$$

Step 4:

Compare time constant of 60 pF capacitor with interval between adjacent peaks of the output signal

- The time constant of 2.08 s is much larger than 80 ms
- The capacitor will not discharge completely between the positive cycles of the half-wave rectified signal
- Therefore, the **800 μF capacitor** would be suitable for the smoothing task

22. Quantum Physics

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22.1 THE PHOTOELECTRIC EFFECT

22.1.1 THE PHOTON

The Particle Nature of Light

- In classical wave theory, electromagnetic (EM) radiation is assumed to behave as a wave
- This is demonstrated by the fact EM radiation exhibits phenomena such as **diffraction** and **interference**
- However, experiments from the last century, such as the photoelectric effect and atomic line spectra, can only be explained if EM radiation is assumed to behave as particles
- These experiments have formed the basis of **quantum theory**, which will be explored in detail in this section

22. Quantum Physics

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The Photon

- Photons are fundamental particles which make up all forms of electromagnetic radiation
- A photon is a massless "packet" or a "quantum" of electromagnetic energy
- What this means is that the energy is not transferred continuously, but as discrete packets of energy
- In other words, each photon carries a specific amount of energy, and transfers this energy all in one go, rather than supplying a consistent amount of energy

Exam Tip

Make sure you learn the definition for a photon: *discrete quantity / packet / quantum of electromagnetic energy* are all acceptable definitions

Calculating Photon Energy

- The energy of a photon can be calculated using the formula:

$$E = hf$$

- Using the wave equation, energy can also be equal to:

$$E = h\frac{c}{\lambda}$$

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YOUR NOTES

- Where:
  - E = energy of the photon (J)
  - h = Planck's constant (J s)
  - c = the speed of light (m s^{-1})
  - f = frequency in Hertz (Hz)
  - λ = wavelength (m)
- This equation tells us:
  - The higher the frequency of EM radiation, the higher the energy of the photon
  - The energy of a photon is inversely proportional to the wavelength
  - A long-wavelength photon of light has a higher energy than a shorter-wavelength photon

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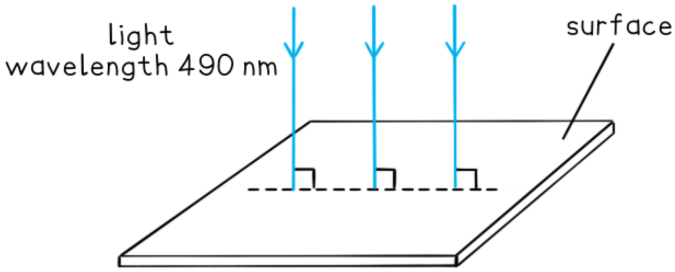
YOUR NOTES

Worked example: Calculating photon energy

Light of wavelength 490 nm is incident normally on a surface, as shown in the diagram.



The power of the light is 3.6 mW. The light is completely absorbed by the surface.

Calculate the number of photons incident on the surface in 2.0 s.

Step 1: Write down the known quantities

Wavelength, λ = 490 nm = 490 × 10^{-9} m

Power, P = 3.6 mW = 3.6 × 10^{-3} W

Time, t = 2.0 s

Step 2: Write the equations for wave speed and photon energy

wave speed: $c = f\lambda \longrightarrow f = \frac{c}{\lambda}$

photon energy: $E = hf \longrightarrow E = \frac{hc}{\lambda}$

Step 3: Calculate the energy of one photon

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}) \times (3.0 \times 10^{8})}{490 \times 10^{-9}} = 4.06 \times 10^{-19} \text{ J}$$

Step 4: Calculate the number of photons hitting the surface every second

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$$\frac{Power\ of\ light\ source}{Energy\ of\ one\ photon} = \frac{3.6 \times 10^{-3}}{4.06 \times 10^{-19}} = 8.9 \times 10^{15}\ s^{-1}$$

Step 5: Calculate the number of photons that hit the surface in 2 s

$$(8.9 \times 10^{15}) \times 2 = 1.8 \times 10^{16}$$

Exam Tip

The values of Planck's constant and the speed of light will always be given to you in an exam, however, it helps to memorise them to speed up calculation questions!

Photon Momentum

- Einstein showed that a photon travelling in a vacuum has momentum, despite it having no mass
- The momentum (p) of a photon is related to its energy (E) by the equation:

$$p = \frac{E}{c}$$

  - Where c is the speed of light

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YOUR NOTES

Worked example: Calculating photon momentum

A 5.0 mW laser beam is incident normally on a fixed metal plate. The cross-sectional area of the beam is 8.0×10^{-6} m^2.

The light from the laser has frequency 5.6×10^{14} Hz.

Assuming all the photons are absorbed by the plate, calculate the momentum of the photon, and the pressure exerted by the laser beam on the metal plate.

Step 1: Write down the known quantities

Power, P = 5.0 mW = 5.0×10^{-3} W

Frequency, f = 5.6×10^{14} Hz

Cross-sectional area, A = 8.0×10^{-6} m^2

Step 2: Write the equations for photon energy and momentum

photon energy: E = hf

photon momentum: $p = \frac{E}{c} \rightarrow p = \frac{hf}{c}$

Step 3: Calculate the photon momentum

$$p = \frac{hf}{c} = \frac{(6.63 \times 10^{-34}) \times (5.6 \times 10^{14})}{3.0 \times 10^{8}} = 1.24 \times 10^{-27} \text{ N s}$$

Step 4: Calculate the number of photons incident on the plate every second

$$\frac{\textit{Power of light source}}{\textit{Energy of one photon}} = \frac{5.0 \times 10^{-3}}{hf} = \frac{5.0 \times 10^{-3}}{(6.63 \times 10^{-34}) \times (5.6 \times 10^{14})} = 1.35 \times 10^{16} \text{ s}^{-1}$$

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Step 5: Calculate the force exerted on the plate in a 1.0 s time interval

Force = rate of change of momentum

= number of photons per second × momentum of each photon

$$= (1.35 \times 10^{16}) \times (1.24 \times 10^{-27})$$

$$= 1.67 \times 10^{-11}\ \text{N}$$

Step 6: Calculate the pressure

$$\text{Pressure} = \frac{Force}{Area} = \frac{1.67 \times 10^{-11}}{8.0 \times 10^{-6}} = \mathbf{2.1 \times 10^{-6}\ Pa}$$

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YOUR NOTES

22.1.2 THE ELECTRONVOLT

The Electronvolt

- The electronvolt is a unit which is commonly used to express very small energies
- This is because quantum energies tend to be much smaller than 1 Joule
- The electronvolt is derived from the definition of potential difference:

$$V = \frac{E}{Q}$$

- When an electron travels through a potential difference, energy is transferred between two points in a circuit, or electric field
- If an electron, with a charge of 1.6×10^{-19} C, travels through a potential difference of 1 V, the energy transferred is equal to:

$$\boldsymbol{E = QV = 1.6 \times 10^{-19}\ \text{C} \times 1\ \text{V} = 1.6 \times 10^{-19}\ \text{J}}$$

- Therefore, an electronvolt is defined as:

The energy gained by an electron travelling through a potential difference of one volt

$$\mathbf{1\ eV = 1.6 \times 10^{-19}\ J}$$

Relation to kinetic energy

- When a charged particle is accelerated through a potential difference, it gains kinetic energy
- If an electron accelerates from rest, an **electronvolt** is equal to the kinetic energy gained:

$$\mathbf{eV = \tfrac{1}{2} mv^2}$$

- Rearranging the equation gives the speed of the electron:

$$v = \sqrt{\frac{2eV}{m}}$$

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YOUR NOTES

Worked example: Electronvolt conversion

Show that the photon energy of light with wavelength 700 nm is about 1.8 eV.

Step 1: Write the equations for wave speed and photon energy

wave speed: $c = f\lambda \longrightarrow f = \dfrac{c}{\lambda}$

photon energy: $E = hf \longrightarrow E = \dfrac{hc}{\lambda}$

Step 2: Calculate the photon energy in Joules

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}) \times (3.0 \times 10^{8})}{700 \times 10^{-9}} = 2.84 \times 10^{-19}\ \text{J}$$

Step 3: Convert the photon energy into electronvolts

1 eV = 1.6×10^{-19} J **J → eV: divide by 1.6×10^{-19}**

$$E = \frac{2.84 \times 10^{-19}}{1.6 \times 10^{-19}} = \mathbf{1.78\ eV}$$

Exam Tip

- To convert between eV and J:
- eV → J: **multiply** by 1.6×10^{-19}
- J → eV: **divide** by 1.6×10^{-19}

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YOUR NOTES
⬇

22.1.3 THE PHOTOELECTRIC EFFECT: BASICS

The Photoelectric Effect: Basics

- The **photoelectric effect** is the phenomena in which electrons are emitted from the surface of a metal **upon the absorption of electromagnetic radiation**
- Electrons removed from a metal in this manner are known as **photoelectrons**
- The photoelectric effect provides important evidence that light is **quantised**, or carried in discrete packets
    - This is shown by the fact each electron can absorb only a single photon
    - This means only the frequencies of light above a **threshold frequency** will emit a photoelectron

Observing the Photoelectric Effect

- The photoelectric effect can be observed on a **gold leaf electroscope**
- A plate of metal, usually **zinc**, is attached to a gold leaf, which initially has a negative charge, causing it to be repelled by a central negatively charged rod
    - This causes negative charge, or electrons, to build up on the zinc plate
- **UV light** is shone onto the metal plate, leading to the **emission** of **photoelectrons**
- This causes the extra electrons on the central rod and gold leaf to be removed, so, the gold leaf begins to fall back towards the central rod
    - This is because they become less negatively charged, and hence repel less
- Some notable observations:
    - Placing the UV light source closer to the metal plate causes the gold leaf to fall more quickly
    - Using a higher frequency light source does not change the how quickly the gold leaf falls
    - Using a filament light source causes no change in the gold leaf's position
    - Using a positively charged plate also causes no change in the gold leaf's position

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22.1.4 THRESHOLD FREQUENCY

Threshold Frequency & Wavelength

- The concept of a threshold frequency is required in order to explain why a low frequency source, such as a filament lamp, was unable to liberate any electrons in the gold leaf experiment
- The **threshold frequency** is defined as:

  The minimum frequency of incident electromagnetic radiation required to remove a photoelectron from the surface of a metal

- The **threshold wavelength**, related to threshold frequency by the wave equation, is defined as:

  The longest wavelength of incident electromagnetic radiation that would remove a photoelectron from the surface of a metal

- Threshold frequency and wavelength are properties of a material, and vary from metal to metal

Exam Tip

A useful analogy for threshold frequency is a fairground coconut shy:
• One person is throwing table tennis balls at the coconuts, and another person has a pistol
• No matter how many of the table tennis balls are thrown at the coconut it will still stay firmly in place – this represents the **low frequency quanta**
• However, a single shot from the pistol will knock off the coconut immediately – this represents the **high frequency quanta**

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The Photoelectric Equation

- Since energy is always conserved, the energy of an incident photon is equal to:

The threshold energy + the kinetic energy of the photoelectron

- The energy within a photon is equal to ***hf***
- This energy is transferred to the electron to release it from a material (the work function) and gives the emitted photoelectron the remaining amount as kinetic energy
- This equation is known as the **photoelectric equation**:

$$E = hf = \Phi + \tfrac{1}{2}mv^2_{max}$$

- Symbols:
  - h = Planck's constant (J s)
  - f = the frequency of the incident radiation (Hz)
  - Φ = the work function of the material (J)
  - $\tfrac{1}{2}mv^2_{max}$ = the maximum kinetic energy of the photoelectrons (J)

- This equation demonstrates:
  - If the incident photons do not have a high enough frequency (f) and energy to overcome the work function (Φ), then no electrons will be emitted
  - When $hf_0 = \Phi$, where f_0 = threshold frequency, photoelectric emission only just occurs
  - E_{kmax} depends only on the frequency of the incident photon, and not the intensity of the radiation
  - The majority of photoelectrons will have kinetic energies less than E_{kmax}

Graphical Representation of Work Function

- The photoelectric equation can be rearranged into the straight line equation:

$$y = mx + c$$

- Comparing this to the photoelectric equation:

$$E_{kmax} = hf - \Phi$$

- A graph of maximum kinetic energy E_{kmax} against frequency f can be obtained

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- The key elements of the graph:
  - The work function Φ is the y-intercept
  - The threshold frequency f_0 is the x-intercept
  - The gradient is equal to Planck's constant h
  - There are no electrons emitted below the threshold frequency f_0

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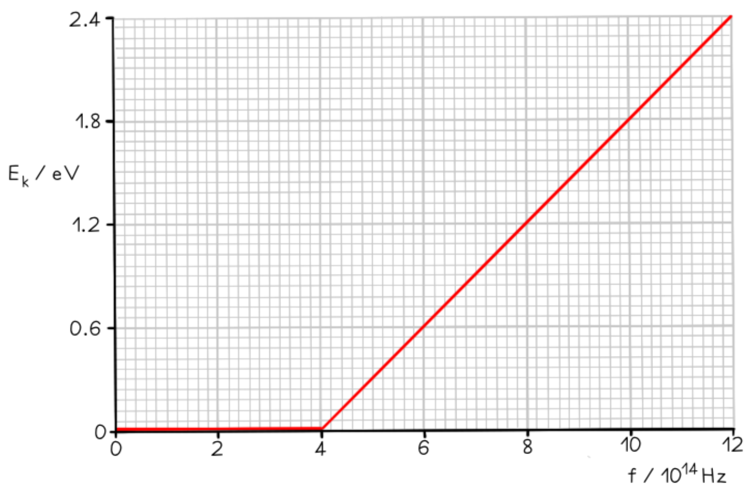
Worked example: Calculating work function

The graph below shows how the maximum kinetic energy E_k of electrons emitted from the surface of sodium metal varies with the frequency f of the incident radiation.



Calculate the work function of sodium in eV.

Step 1: Write out the photoelectric equation and rearrange to fit the equation of a straight line

$$E = hf = \Phi + \tfrac{1}{2}mv^2_{max} \rightarrow E_{kmax} = hf - \Phi$$

$$y = mx + c$$

Step 2: Identify the threshold frequency from the x-axis of the graph

When $E_k = 0$, $f = f_0$

Therefore, the threshold frequency is $f_0 = 4 \times 10^{14}$ Hz

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Step 3: Calculate the work function

From the graph at f_0, $\frac{1}{2}mv_{max}^2 = 0$

$$\Phi = hf_0 = (6.63 \times 10^{-34}) \times (4 \times 10^{14}) = 2.652 \times 10^{-19} \text{ J}$$

Step 4: Convert the work function into eV

$1 \text{ eV} = 1.6 \times 10^{-19}$ J **J → eV: divide by 1.6×10^{-19}**

$$E = \frac{2.652 \times 10^{-19}}{1.6 \times 10^{-19}} = \mathbf{1.66\ eV}$$

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22.1.5 THE WORK FUNCTION

Photoelectric Emission

- The work function Φ, or threshold energy, of a material is defined as:

The minimum energy required to release a photoelectron from the surface of a material

- Consider the electrons in a metal as trapped inside an 'energy well' where the energy between the surface and the top of the well is equal to the work function Φ
- A single electron absorbs one photon
- Therefore, an electron can only escape the surface of the metal if it absorbs a photon which has an energy equal to Φ or higher

- Different metals have different threshold frequencies, and hence different work functions
- Using the well analogy:
    - A more tightly bound electron requires more energy to reach the top of the well
    - A less tightly bound electron requires less energy to reach the top of the well
- Alkali metals, such as sodium and potassium, have threshold frequencies in the **visible light region**
    - This is because the attractive forces between the surface electrons and positive metal ions are relatively weak
- Transition metals, such as zinc and iron, have threshold frequencies in the **ultraviolet region**
    - This is because the attractive forces between the surface electrons and positive metal ions are much stronger

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Laws of Photoelectric Emission

- Observation:
  - Placing the UV light source **closer** to the metal plate causes the gold leaf to **fall more quickly**
- Explanation:
  - Placing the UV source closer to the plate increases the intensity incident on the surface of the metal
  - Increasing the intensity, or brightness, of the incident radiation increases the number of photoelectrons emitted per second
  - Therefore, the gold leaf loses negative charge more rapidly

- Observation:
  - Using a higher frequency light source **does not change** the how quickly the gold leaf falls
- Explanation:
  - The maximum kinetic energy of the emitted electrons increases with the frequency of the incident radiation
  - In the case of the photoelectric effect, energy and frequency are independent of the intensity of the radiation
  - So, the intensity of the incident radiation affects how quickly the gold leaf falls, not the frequency

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- Observation:
  - Using a filament light source causes **no change** in the gold leaf's position
- Explanation:
  - If the incident frequency is below a certain threshold frequency, no electrons are emitted, no matter the intensity of the radiation
  - A filament light source has a frequency below the threshold frequency of the metal, so, no photoelectrons are released

- Observation:
  - Using a positively charged plate causes **no change** in the gold leaf's position
- Explanation:
  - If the plate is positively charged, that means there is an excess of positive charge on the surface of the metal plate
  - Electrons are negatively charged, so they will not be emitted unless they are on the surface of the metal
  - Any electrons emitted will be attracted back by positive charges on the surface of the metal

- Observation:
  - Emission of photoelectrons happens **as soon as the radiation is incident on the surface of the metal**
- Explanation:
  - A single photon interacts with a single electron
  - If the energy of the photon is equal to the work function of the metal, photoelectrons will be released instantaneously

Intensity & Photoelectric Current

- The **maximum kinetic energy** of the photoelectrons is **independent of the intensity** of the incident radiation
- This is because **each electron can only absorb one photon**
- Kinetic energy is only dependent on the **frequency** of the incident radiation
- Intensity is a measure of the number of photons incident on the surface of the metal
- So, increasing the number of electrons striking the metal will not increase the kinetic energy of the electrons, it will increase the **number** of photoelectrons emitted

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Photoelectric Current

- The photoelectric current is the number of photoelectrons emitted per second
- **Photoelectric current** is proportional to the **intensity** of the radiation incident on the surface of the metal
- This is because intensity is proportional to the number of photons striking the metal per second
- Since each photoelectron absorbs a single photon, the photoelectric current must be proportional to the intensity of the incident radiation

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22.2 WAVE-PARTICLE DUALITY

22.2.1 WAVE-PARTICLE DUALITY

Wave-Particle Duality

- Light waves can behave like particles, i.e. photons, **and** waves
- This phenomena is called the wave-particle nature of light or wave-particle duality
- Light interacts with matter, such as electrons, as a particle
  - The evidence for this is provided by the photoelectric effect
- Light propagates through space as a wave
  - The evidence for this comes from the diffraction and interference of light in Young's Double Slit experiment

Light as a Particle

- Einstein proposed that light can be described as a quanta of energy that behave as particles, called photons
- The photon model of light explains that:
  - Electromagnetic waves carry energy in discrete packets called photons
  - The energy of the photons are quantised according to the equation E = hf
  - In the photoelectric effect, each electron can absorb only a single photon – this means only the frequencies of light above the threshold frequency will emit a photoelectron
- The wave theory of light does not support a threshold frequency
  - The wave theory suggests any frequency of light can give rise to photoelectric emission if the exposure time is long enough
  - This is because the wave theory suggests the energy absorbed by each electron will increase gradually with each wave
  - Furthermore, the kinetic energy of the emitted electrons should increase with radiation intensity
  - However, in the photoelectric effect none of this is observed
- If the frequency is above the threshold and the intensity of the light is increased, more photoelectrons are emitted per second
- Although the wave theory provided good explanations for phenomena such as interference and diffraction, it failed to explain the photoelectric effect

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Wave-Particle Duality: Electron Diffraction

- Louis de Broglie discovered that matter, such as electrons, can behave as a wave
- He showed a diffraction pattern is produced when a beam of electrons is directed at a thin graphite film
- Diffraction is a property of waves, and cannot be explained by describing electrons as particles

- In order to observe the diffraction of electrons, they must be focused through a gap similar to their size, such as an atomic lattice
- Graphite film is ideal for this purpose because of its crystalline structure
    - The gaps between neighbouring planes of the atoms in the crystals act as slits, allowing the electron waves to spread out and create a diffraction pattern
- The diffraction pattern is observed on the screen as a series of concentric rings
    - This phenomenon is similar to the diffraction pattern produced when light passes through a diffraction grating
    - If the electrons acted as particles, a pattern would not be observed, instead the particles would be distributed uniformly across the screen
- It is observed that a larger accelerating voltage reduces the diameter of a given ring, while a lower accelerating voltage increases the diameter of the rings

Investigating Electron Diffraction

- Electron diffraction tubes can be used to investigate wave properties of electrons
- The electrons are accelerated in an electron gun to a high potential, such as 5000 V, and are then directed through a thin film of graphite
- The electrons diffract from the gaps between carbon atoms and produce a circular pattern on a fluorescent screen made from phosphor

- Increasing the voltage between the anode and the cathode causes the energy, and hence speed, of the electrons to increase
- The kinetic energy of the electrons is proportional to the voltage across the anode-cathode:

$$\mathbf{E_k = ½\, mv^2 = eV}$$

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22.2.2 THE DE BROGLIE WAVELENGTH

The de Broglie Wavelength

- De Broglie proposed that electrons travel through space as a wave
  - This would explain why they can exhibit behaviour such as diffraction
- He therefore suggested that electrons must also hold wave properties, such as wavelength
  - This became known as the de Broglie wavelength
- However, he realised **all particles** can show wave-like properties, not just electrons
- So, the de Broglie wavelength can be defined as:

The wavelength associated with a moving particle

- The majority of the time, and for everyday objects travelling at normal speeds, the de Broglie wavelength is far too small for any quantum effects to be observed
- A typical electron in a metal has a de Broglie wavelength of about 10 nm
- Therefore, quantum mechanical effects will only be observable when the width of the sample is around that value
- The electron diffraction tube can be used to investigate how the wavelength of electrons depends on their speed
  - The smaller the radius of the rings, the smaller the de Broglie wavelength of the electrons
- As the voltage is increased:
  - The energy of the electrons increases
  - The radius of the diffraction pattern decreases
- This shows as the speed of the electrons increases, the de Broglie wavelength of the electrons decreases

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⬇

Calculating de Broglie Wavelength

- Using ideas based upon the quantum theory and Einstein's theory of relativity, de Broglie suggested that the momentum (p) of a particle and its associated wavelength (λ) are related by the equation:

$$\lambda = \frac{h}{p}$$

- Since momentum p = mv, the de Broglie wavelength can be related to the speed of a moving particle (v) by the equation:

$$\lambda = \frac{h}{mv}$$

- Since kinetic energy $E = \frac{1}{2} mv^2$
- Momentum and kinetic energy can be related by:

$$E = \frac{p^2}{2m} \quad \textbf{or} \quad p = \sqrt{2mE}$$

- Combining this with the de Broglie equation gives a form which relates the de Broglie wavelength of a particle to its kinetic energy:

$$\lambda = \frac{h}{\sqrt{2mE}}$$

- Where:
  - λ = the de Broglie wavelength (m)
  - h = Planck's constant (J s)
  - p = momentum of the particle (kg m s^{-1})
  - E = kinetic energy of the particle (J)
  - m = mass of the particle (kg)
  - v = speed of the particle (m s^{-1})

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Worked example: de Broglie wavelength

A proton and an electron are each accelerated from rest through the same potential difference. Determine the ratio:

$$\frac{\textit{de Broglie wavelength of the proton}}{\textit{de Broglie wavelength of the electron}}$$

Mass of a proton = 1.67×10^{-27} kg
Mass of an electron = 9.11×10^{-31} kg

Step 1:
Consider how the proton and electron can be related via their masses
The proton and electron are accelerated through the same p.d., therefore, they both have the **same kinetic energy**

Step 2:
Write the equation which relates the de Broglie wavelength of a particle to its kinetic energy:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$\lambda \propto \frac{1}{\sqrt{m}}$$

Step 3:
Calculate the ratio:

$$\frac{\textit{de Broglie wavelength of the proton}}{\textit{de Broglie wavelength of the electron}} = \frac{1}{\sqrt{m_p}} \div \frac{1}{\sqrt{m_e}}$$

$$\sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{9.11 \times 10^{-31}}{1.67 \times 10^{-27}}} = 2.3 \times 10^{-2}$$

This means the de Broglie wavelength of the proton is 0.023 times smaller than that of the electron **OR** the de Broglie wavelength of the electron is about 40 times larger than that of the proton

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⬇

22.3 QUANTISATION OF ENERGY

22.3.1 ATOMIC ENERGY LEVELS

Atomic Energy Levels

- Electrons in an atom can have only certain specific energies
  - These energies are called **electron energy levels**
- They can be represented as a series of stacked horizontal lines increasing in energy
- Normally, electrons occupy the lowest energy level available, this is known as the **ground state**
- Electrons can gain energy and move up the energy levels if it absorbs energy either by:
  - Collisions with other atoms or electrons
  - Absorbing a photon
  - A physical source, such as heat
- This is known as **excitation**, and when electrons move up an energy level, they are said to be in an **excited state**
- If the electron gains enough energy to be removed from the atom entirely, this is known as **ionisation**
- When an electron returns to a lower energy state from a higher excited state, it releases energy in the form of a photon

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⬇

22.3.2 LINE SPECTRA

Line Spectra

- Line spectra is a phenomenon which occurs when excited atoms emit light of certain wavelengths which correspond to different colours
- The emitted light can be observed as a series of coloured lines with dark spaces in between
    - These series of coloured lines are called **line or atomic spectra**
- Each element produces a unique set of spectral lines
- No two elements emit the same set of spectral lines, therefore, elements can be identified by their line spectrum
- There are two types of line spectra: **emission spectra** and **absorption spectra**

Emission Spectra

- When an electron transitions from a higher energy level to a lower energy level, this results in the **emission** of a photon
- Each transition corresponds to a different wavelength of light and this corresponds to a line in the spectrum
- The resulting emission spectrum contains a set of discrete wavelengths, represented by coloured lines on a black background
- Each emitted photon has a wavelength which is associated with a discrete change in energy, according to the equation:

$$\Delta E = hf = \frac{hc}{\lambda}$$

- Where:
    - ΔE = change in energy level (J)
    - h = Planck's constant (J s)
    - f = frequency of photon (Hz)
    - c = the speed of light (m s^{-1})
    - λ = wavelength of the photon (m)
- Therefore, this is evidence to show that electrons in atoms can only transition between discrete energy levels

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Absorption Spectra

- An atom can be raised to an excited state by the absorption of a photon
- When white light passes through a **cool, low pressure gas** it is found that light of certain wavelengths are missing
  - This type of spectrum is called an absorption spectrum
- An absorption spectrum consists of a continuous spectrum containing all the colours with dark lines at certain wavelengths
- These dark lines correspond exactly to the differences in energy levels in an atom
- When these electrons return to lower levels, the photons are emitted in all directions, rather than in the original direction of the white light
  - Therefore, some wavelengths appear to be missing
- The wavelengths missing from an absorption spectrum are the same as their corresponding emission spectra of the same element

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22.3.3 CALCULATING DISCRETE ENERGIES

Calculating Discrete Energies

- The difference between two energy levels is equal to a specific photon energy
- The energy (hf) of the photon is given by:

$$\Delta E = hf = E_2 - E_1$$

- Where,
  - E_1 = Energy of the higher level (J)
  - E_2 = Energy of the lower level (J)
  - h = Planck's constant (J s)
  - f = Frequency of photon (Hz)

- Using the wave equation, the wavelength of the emitted, or absorbed, radiation can be related to the energy difference by the equation:

$$\lambda = \frac{hc}{E_2 - E_1}$$

- This equation shows that the larger the difference in energy of two levels ΔE , the shorter the wavelength λ and vice versa

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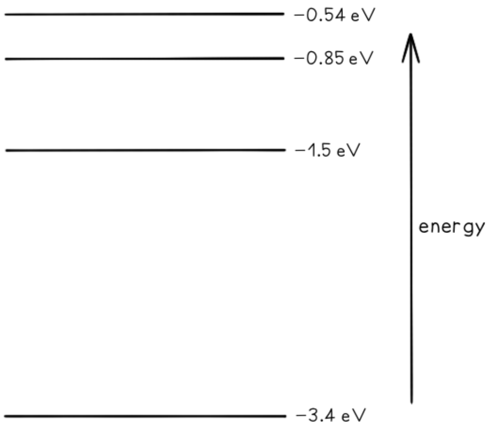
Worked example: Calculating discrete energies

Some electron energy levels in atomic hydrogen are shown below.



The longest wavelength produced as a result of electron transitions between two of the energy levels is 4.0×10^{-6} m.

a) Draw and mark:

With the letter L, the transition giving rise to the wavelength of 4.0×10^{-6} m.
With the letter S, the transition giving rise to the shortest wavelength.

b) Calculate the wavelength for the transition giving rise to the shortest wavelength.

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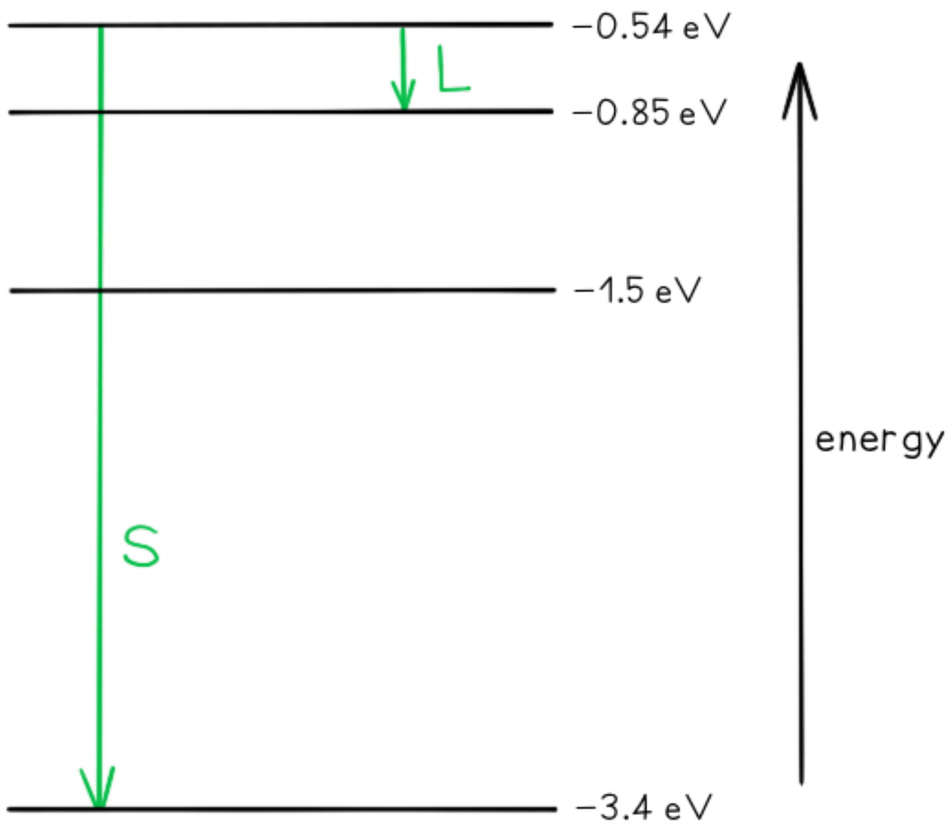
Part (a)



Photon energy and wavelength are inversely proportional, so the largest energy change corresponds to the shortest wavelength (**line S**) and the smallest energy change corresponds to the longest wavelength (**line L**)

Part (b)

Step 1: Write down the equation linking the wavelength and the energy levels

$$\lambda = \frac{hc}{E_2 - E_1}$$

Step 2: Identify the energy levels giving rise to the shortest wavelength

$\mathbf{E_1 = 0.54\ eV}$

$\mathbf{E_2 = 3.4\ eV}$

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Step 3: Calculate the wavelength

To convert from eV → J: **multiply** by 1.6×10^{-19}

$$\lambda = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{(3.4 - 0.54)(1.6 \times 10^{-19})} = 4.347 \times 10^{-7}\ \text{m} = \mathbf{435\ nm}$$

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23.1 MASS DEFECT & NUCLEAR BINDING ENERGY

23.1.1 ENERGY & MASS EQUIVALENCE

Energy & Mass Equivalence

- Einstein showed in his theory of relativity that matter can be considered a form of energy and hence, he proposed:
  - Mass can be converted into energy
  - Energy can be converted into mass
- This is known as **mass-energy equivalence**, and can be summarised by the equation:

$$\mathbf{E = mc^2}$$

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- Where:
  - E = energy (J)
  - m = mass (kg)
  - c = the speed of light (m s^{-1})
- Some examples of mass-energy equivalence are:
  - The fusion of hydrogen into helium in the centre of the sun
  - The fission of uranium in nuclear power plants
  - Nuclear weapons
  - High-energy particle collisions in particle accelerators

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23.1.2 NUCLEAR EQUATIONS

Representing Simple Nuclear Reactions

- Nuclear reactions can be represented by balanced equations of nuclei in the AZX form

- The top number A represents the **nucleon** number or the **mass** number
  - **Nucleon number (A)** = total number of **protons and neutrons** in the nucleus
- The lower number Z represents the **proton** or **atomic** number
  - **Proton number (Z)** = total number of **protons** in the nucleus

Worked example: Simple nuclear reactions

When a neutron is captured by a uranium-235 nucleus, the outcome may be represented by the nuclear equation:

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{95}_{42}Mo + ^{139}_{57}La + x\,^{1}_{0}n + 7\,^{0}_{-1}e$$

What is the value of x?

Step 1: Balance the nucleon numbers (the top number)

235 + 1 = 95 + 139 + x(1) + 7(0)

Step 2: Rearrange to find the value of x

x = 235 + 1 − 95 − 139 = 2

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23.1.3 MASS DEFECT & BINDING ENERGY

Mass Defect & Binding Energy

- Experiments into nuclear structure have found that the total mass of a nucleus is **less** than the sum of the masses of its constituent nucleons
- This difference in mass is known as the **mass defect**
- Mass defect is defined as:

The difference between an atom's mass and the sum of the masses of its protons and neutrons

- The mass defect Δm of a nucleus can be calculated using:

$$\Delta m = Zm_p + (A - Z)m_n - m_{total}$$

- Where:
  - Z = proton number
  - A = nucleon number
  - m_p = mass of a proton (kg)
  - m_n = mass of a neutron (kg)
  - m_{total} = measured mass of the nucleus (kg)

- Due to the equivalence of mass and energy, this decrease in mass implies that energy is released in the process
- Since nuclei are made up of neutrons and protons, there are forces of repulsion between the positive protons
  - Therefore, it takes energy, ie. the binding energy, to hold nucleons together as a nucleus
- Binding energy is defined as:

The energy required to break a nucleus into its constituent protons and neutrons

- Energy and mass are proportional, so, the total energy of a nucleus is less than the sum of the energies of its constituent nucleons
- The formation of a nucleus from a system of isolated protons and neutrons is therefore an exothermic reaction – meaning that it releases energy
- This can be calculated using the equation:

$$E = \Delta mc^2$$

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Exam Tip

Avoid describing the binding energy as the energy stored in the nucleus – this is not correct – it is energy that must be put into the nucleus to pull it apart.

Binding Energy per Nucleon

- In order to compare nuclear stability, it is more useful to look at the **binding energy per nucleon**
- The binding energy per nucleon is defined as:

The binding energy of a nucleus divided by the number of nucleons in the nucleus

- A higher binding energy per nucleon indicates a higher stability
  - In other words, it requires more energy to pull the nucleus apart
- Iron (A = 56) has the highest binding energy per nucleon, which makes it the most stable of all the elements

Key Features of the Graph

- At low values of A:
  - Nuclei tend to have a lower binding energy per nucleon, hence, they are generally less stable
  - This means the lightest elements have weaker electrostatic forces and are the most likely to undergo **fusion**
- Helium (^{4}He), carbon (^{12}C) and oxygen (^{16}O) do not fit the trend
  - Helium-4 is a particularly stable nucleus hence it has a high binding energy per nucleon
  - Carbon-12 and oxygen-16 can be considered to be three and four helium nuclei, respectively, bound together
- At high values of A:
  - The general binding energy per nucleon is high and gradually decreases with A
  - This means the heaviest elements are the most unstable and likely to undergo **fission**

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Worked example: Binding energy of iron

What is the binding energy per nucleon of iron-56 ($^{56}_{26}Fe$) in MeV?

Mass of a neutron = 1.675×10^{-27} kg
Mass of a proton = 1.673×10^{-27} kg
Mass of $^{56}_{26}Fe$ nucleus = 9.288×10^{-26} kg

Step 1: Calculate the mass defect

Number of protons, Z = 26

Number of neutrons, A – Z = 56 – 26 = 30

$$\text{Mass defect, } \Delta m = Zm_p + (A - Z)m_n - m_{total}$$

$$\Delta m = (26 \times 1.673 \times 10^{-27}) + (30 \times 1.675 \times 10^{-27}) - (9.288 \times 10^{-26})$$

$$\Delta m = 8.680 \times 10^{-28} \text{ kg}$$

Step 2: Calculate the binding energy of the nucleus

$$\text{Binding energy, } E = \Delta mc^2$$

$$E = (8.680 \times 10^{-28}) \times (3.00 \times 10^8)^2 = 7.812 \times 10^{-11} \text{ J}$$

Step 3: Calculate the binding energy per nucleon

$$\text{Binding energy per nucleon} = \frac{E}{A}$$

$$\frac{E}{A} = \frac{7.812 \times 10^{-11}}{56} = 1.395 \times 10^{-12} \text{ J}$$

Step 4: Convert to MeV

J → eV: divide by 1.6×10^{-19}

eV → MeV: divide by 10^6

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$$\text{Binding energy per nucleon} = \frac{1.395 \times 10^{-12}}{1.6 \times 10^{-19}} = 8\ 718\ 750 \text{ eV} = \mathbf{8.7\ MeV}\ (2 \text{ s.f.})$$

Exam Tip

Checklist on what to include (and what not to include) in an exam question asking you to draw a graph of binding energy per nucleon against nucleon number:

• You will be expected to draw the best fit curve AND a cross to show the anomaly that is helium

• Do not begin your curve at A = 0, this is not a nucleus!

• Make sure to correctly label both axes AND units for binding energy per nucleon

• You will be expected to include numbers on the axes, mainly at the peak to show the position of iron (^{56}Fe)

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23.1.4 NUCLEAR FUSION & FISSION

Nuclear Fusion & Fission

Nuclear Fusion

- Fusion is defined as:

The fusing together of two small nuclei to produce a larger nucleus

- Low mass nuclei (such as hydrogen and helium) can undergo fusion and release energy

- For two nuclei to fuse, both nuclei must have high kinetic energy
  - This is because the protons inside the nuclei are positively charged, which means that they repel one another
- It takes a great deal of energy to overcome the electrostatic force, so this is why it is can only be achieved in an extremely high-energy environment, such as star's core
- When two protons fuse, the element deuterium is produced
- In the centre of stars, the deuterium combines with a tritium nucleus to form a helium nucleus, plus the release of energy, which provides fuel for the star to continue burning

Nuclear Fission

- Fission is defined as:

The splitting of a large atomic nucleus into smaller nuclei

- High mass nuclei (such as uranium) can undergo fission and release energy

- Fission must first be induced by firing neutrons at a nucleus
- When the nucleus is struck by a neutron, it splits into two, or more, daughter nuclei
- During fission, neutrons are ejected from the nucleus, which in turn, can collide with other nuclei which triggers a cascade effect
- This leads to a chain reaction which lasts until all of the material has undergone fission, or the reaction is halted by a moderator
- Nuclear fission is the process which produces energy in nuclear power stations, where it is well controlled
- When nuclear fission is not controlled, the chain reaction can cascade to produce the effects of a nuclear bomb

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Exam Tip

When an atom undergoes nuclear fission, take note that extra neutrons are ejected by the **nucleus** and not from the fission products

Significance of Binding Energy per Nucleon

- At low values of A:
  - Attractive nuclear forces between nucleons dominate over repulsive electrostatic forces between protons
  - In the right conditions, nuclei undergo **fusion**
- In fusion, the mass of the nucleus that is created is slightly **less** than the total mass of the original nuclei
  - The mass defect is equal to the binding energy that is released, since the nucleus that is formed is more stable
- At high values of A:
  - Repulsive electrostatic forces between forces begin to dominate, and these forces tend to break apart the nucleus rather than hold it together
  - In the right conditions, nuclei undergo **fission**
- In fission, an unstable nucleus is converted into more stable nuclei with a smaller total mass
  - This difference in mass, the mass defect, is equal to the binding energy that is released

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23.1.5 CALCULATING ENERGY RELEASED IN NUCLEAR REACTIONS

Calculating Energy Released in Nuclear Reactions

- The binding energy is equal to the amount of energy released in forming the nucleus, and can be calculated using:

$$E = (\Delta m)c^2$$

- Where:
  - E = Binding energy released (J)
  - Δm = mass defect (kg)
  - c = speed of light (m s^{-1})
- The daughter nuclei produced as a result of both fission and fusion have a higher binding energy per nucleon than the parent nuclei
- Therefore, energy is released as a result of the mass difference between the parent nuclei and the daughter nuclei

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Worked example: Calculating energy released

When Uranium−235 nuclei are fissioned by slow-moving neutrons, two possible reactions are:

Reaction 1: $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{139}_{54}Xe + ^{95}_{38}Sr + 2^{1}_{0}n$ + energy

Reaction 2: $^{235}_{92}U + ^{1}_{0}n \rightarrow 2\,^{116}_{46}Pd + xc$ + energy

a) For reaction 2, identify the particle c and state the number x of such particles generated in the reaction.

b) The binding energy per nucleon *E* for a number of nuclides is given by the table below. Use the table to show that the energy produced in reaction 1 is about 210 MeV.

c) The energy produced in reaction 2 is 163 MeV. Suggest, with supporting reason, which one of the two reactions is more likely to happen.

nuclide	E / MeV
$^{95}_{38}Sr$	8.74
$^{139}_{54}Xe$	8.39
$^{235}_{92}U$	7.60

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Part (a)

Step 1: Balance the number of protons on each side (bottom number)

$92 = (2 \times 46) + xn_p$ (where n_p is the number of protons in c)

$xn_p = 92 - 92 = 0$

Therefore, c must be a neutron

Step 2: Balance the number of nucleons on each side

$235 + 1 = (2 \times 116) + x$

$x = 235 + 1 - 232 = 4$

Therefore, 4 neutrons are generated in the reaction

Part (b)

Step 1: Find the binding energy of each nucleus

Total binding energy of each nucleus = Binding energy per nucleon × Mass number

Binding energy of $^{95}Sr = 8.74 \times 95 = 830.3$ MeV

Binding energy of $^{139}Xe = 8.39 \times 139 = 1166.21$ MeV

Binding energy of $^{235}U = 7.60 \times 235 = 1786$ MeV

Step 2: Calculate the difference in energy between the products and reactants

Energy released in reaction 1 = $E_{Sr} + E_{Xe} - E_U$

Energy released in reaction 1 = 830.3 + 1166.21 - 1786

Energy released in reaction 1 = 210.5 MeV

Part (c)

- Since reaction 1 releases more energy than reaction 2, its end products will have a higher binding energy per nucleon
    - Hence they will be more stable
- This is because the more energy is released, the further it moves up the graph of binding energy per nucleon against nucleon number (A)
    - Since at high values of A, binding energy per nucleon gradually decreases with A
- Nuclear reactions will tend to favour the more stable route, therefore, reaction 1 is more likely to happen

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23.2 RADIOACTIVE DECAY

23.2.1 THE RANDOM NATURE OF RADIOACTIVE DECAY

The Random Nature of Radioactive Decay

- Radioactive decay is defined as:

The spontaneous disintegration of a nucleus to form a more stable nucleus, resulting in the emission of an alpha, beta or gamma particle

- The random nature of radioactive decay can be demonstrated by observing the count rate of a Geiger-Muller (GM) tube
  - When a GM tube is placed near a radioactive source, the counts are found to be irregular and cannot be predicted
  - Each count represents a decay of an unstable nucleus
  - These fluctuations in count rate on the GM tube **provide evidence for the randomness of radioactive decay**

Characteristics of Radioactive Decay

- Radioactive decay is both **spontaneous** and **random**
- A spontaneous process is defined as:

A process which cannot be influenced by environmental factors

- This means radioactive decay cannot be affected by environmental factors such as:
  - Temperature
  - Pressure
  - Chemical conditions

- A random process is defined as:

A process in which the exact time of decay of a nucleus cannot be predicted

- Instead, the nucleus has a constant probability, ie. the same chance, of decaying in a given time
- Therefore, with large numbers of nuclei it is possible to statistically predict the behaviour of the entire group

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Exam Tip

Make sure you can define what constitutes a radioactive decay, a random process and a spontaneous decay – these are all very common exam questions!

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23.2.2 ACTIVITY & THE DECAY CONSTANT

Activity & The Decay Constant

- Since radioactive decay is spontaneous and random, it is useful to consider the average number of nuclei which are expected to decay per unit time
  - This is known as the **average decay rate**
- As a result, each radioactive element can be assigned a **decay constant**
- The decay constant λ is defined as:

The probability that an individual nucleus will decay per unit of time

- When a sample is highly radioactive, this means the number of decays per unit time is very high
  - This suggests it has a high level of **activity**

- Activity, or the number of decays per unit time can be calculated using:

$$A = \frac{\Delta N}{\Delta t} = -\lambda N$$

- Where:
  - A = activity of the sample (Bq)
  - ΔN = number of decayed nuclei
  - Δt = time interval (s)
  - λ = decay constant (s^{-1})
  - N = number of nuclei remaining in a sample

- The activity of a sample is measured in **Becquerels** (Bq)
  - An activity of 1 Bq is equal to one decay per second, or 1 s^{-1}
- This equation shows:
  - The greater the decay constant, the greater the activity of the sample
  - The activity depends on the number of undecayed nuclei remaining in the sample
  - The minus sign indicates that the number of nuclei remaining decreases with time – however, for calculations it can be omitted

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Worked example: Decay constant

Americium-241 is an artificially produced radioactive element that emits α-particles.

A sample of americium-241 of mass 5.1 μg is found to have an activity of 5.9×10^5 Bq.

a) Determine the number of nuclei in the sample of americium-241.
b) Determine the decay constant of americium-241.

Part (a)

Step 1: Write down the known quantities

Mass = 5.1 μg = 5.1×10^{-6} g

Molecular mass of americium = 241

Step 2: Write down the equation relating number of nuclei, mass and molecular mass

$$\text{Number of nuclei} = \frac{mass \times N_A}{molecular\ mass}$$

where N_A is the Avogadro constant

Step 3: Calculate the number of nuclei

$$\text{Number of nuclei} = \frac{(5.1 \times 10^{-6}) \times (6.02 \times 10^{23})}{241} = 1.27 \times 10^{16}$$

Part (b)

Step 1: Write the equation for activity

Activity, A = λN

Step 2: Rearrange for decay constant λ and calculate the answer

$$\lambda = \frac{A}{N} = \frac{5.9 \times 10^5}{1.27 \times 10^{16}} = 4.65 \times 10^{-11}\ s^{-1}$$

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The Exponential Nature of Radioactive Decay

- In radioactive decay, the number of nuclei falls very rapidly, without ever reaching zero
  - Such a model is known as **exponential decay**
- The graph of number of undecayed nuclei and time has a very distinctive shape

Equations for Radioactive Decay

- The number of undecayed nuclei N can be represented in exponential form by the equation:

$$\mathbf{N = N_0 e^{-\lambda t}}$$

- Where:
  - N_0 = the initial number of undecayed nuclei (when t = 0)
  - λ = decay constant (s^{-1})
  - t = time interval (s)

- The number of nuclei can be substituted for other quantities, for example, the activity A is directly proportional to N, so it can be represented in exponential form by the equation:

$$\mathbf{A = A_0 e^{-\lambda t}}$$

- The received count rate C is related to the activity of the sample, hence it can also be represented in exponential form by the equation:

$$\mathbf{C = C_0 e^{-\lambda t}}$$

The exponential function *e*

- The symbol e represents the exponential constant
  - It is approximately equal to e = 2.718
- On a calculator it is shown by the button e^x
- The inverse function of e^x is ln(y), known as the natural logarithmic function
  - This is because, if e^x = y, then x = ln(y)

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Worked example: Exponential decay

?

Strontium-90 decays with the emission of a β-particle to form Yttrium-90. The decay constant of Strontium-90 is 0.025 year^{-1}.

Determine the activity A of the sample after 5.0 years, expressing the answer as a fraction of the initial activity A_0 .

Step 1: Write out the known quantities

Decay constant, λ = 0.025 year^{-1}

Time interval, t = 5.0 years

Both quantities have the same unit, so there is no need for conversion

Step 2: Write the equation for activity in exponential form

$$A = A_0 e^{-\lambda t}$$

Step 3: Rearrange the equation for the ratio between A and A_0

$$\frac{A}{A_0} = e^{-\lambda t}$$

Step 4: Calculate the ratio A/A_0

$$\frac{A}{A_0} = e^{-(0.025 \times 5)} = 0.88$$

Therefore, the activity of Strontium-90 decreases by a factor of 0.88, or 12%, after 5 years

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23.2.3 HALF-LIFE

Half-Life Definition

- Half life is defined as:

The time taken for the initial number of nuclei to reduce by half

- This means when a time equal to the half-life has passed, the activity of the sample will also half
- This is because activity is proportional to the number of undecayed nuclei, $A \propto N$

Calculating Half-Life

- To find an expression for half-life, start with the equation for exponential decay:

$$\mathbf{N = N_0 e^{-\lambda t}}$$

- Where:
  - N = number of nuclei remaining in a sample
  - N_0 = the initial number of undecayed nuclei (when t = 0)
  - λ = decay constant (s^{-1})
  - t = time interval (s)

- When time t is equal to the half-life $t_{½}$, the activity N of the sample will be half of its original value, so N = ½ N_0

$$\frac{1}{2} N_0 = N_0 e^{-\lambda t_{½}}$$

- The formula can then be derived as follows:

Divide both sides by N_0: $\quad \frac{1}{2} = e^{-\lambda t_{½}}$

Take the natural log of both sides: $\quad \ln\left(\frac{1}{2}\right) = -\lambda t_{½}$

Apply properties of logarithms: $\quad \lambda t_{½} = \ln(2)$

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- Therefore, half-life $t_{½}$ can be calculated using the equation:

$$t_{½} = \frac{\ln 2}{\lambda} \simeq \frac{0.693}{\lambda}$$

- This equation shows that half-life $t_{½}$ and the radioactive decay rate constant λ are inversely proportional
- Therefore, the shorter the half-life, the larger the decay constant and the **faster** the decay

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Worked example: Calculating half-life

Strontium-90 is a radioactive isotope with a half-life of 28.0 years.

A sample of Strontium-90 has an activity of 6.4×10^9 Bq.

Calculate the decay constant λ, in s^{-1}, of Strontium-90.

Step 1: Convert the half-life into seconds

28 years = 28 × 365 × 24 × 60 × 60 = 8.83×10^8 s

Step 2: Write the equation for half-life

$$t_{½} = \frac{ln\,2}{\lambda}$$

Step 3: Rearrange for λ and calculate

$$\lambda = \frac{ln\,2}{t_{½}} = \frac{ln\,2}{8.83 \times 10^8} = \mathbf{7.85 \times 10^{-10}\ s^{-1}}$$

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24.1 ULTRASOUND & X-RAYS

24.1.1 THE PIEZOELECTRIC TRANSDUCER

The Piezoelectric Transducer

- The piezoelectric effect is defined as:

 The ability of particular materials to generate a potential difference (p.d.) by transferring mechanical energy to electrical energy

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- A **transducer** is any device that converts energy from one form to another

Piezoelectric Crystals

- At the heart of a piezoelectric transducer is a **piezoelectric crystal**
- Piezoelectric crystals are materials which produce a p.d. when they are deformed
  - This deformation can be by compression or stretching
- If a p.d. is applied to a piezoelectric crystal, then it deforms, and if the p.d. is reversed, then it expands
  - If this is an alternating p.d. then the crystal will vibrate at the same frequency as the alternating voltage
  - Crystals must be cut to a certain size in order to induce resonance

- One of the most common piezoelectric crystals is **quartz**, which is made from a lattice of silicon dioxide atoms
  - When the lattice is distorted, the structure becomes charged creating an electric field and, as a result, an electric current
  - If an electric current is applied to the crystal, then this causes the shape of the lattice to alternate which produces a sound wave
  - Due to the conventional direction of electric current, it will flow from the positive to negative region of the crystal

Applications of the Piezoelectric Transducer

- Microphone
  - A piezoelectric microphone detects pressure variations in sound waves
  - These can then be converted to an electrical signal for processing
- Ultrasound
  - In a piezoelectric transducer, an alternating p.d. is applied to produce ultrasound waves and sent into the patient's body
  - The returning ultrasound waves induce a p.d. in the transducer for analysis by a healthcare professional

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24.1.2 GENERATING & USING ULTRASOUND

Generating Ultrasound

- An ultrasound is defined as:

A high frequency sound above the range of human hearing

- This is above 20 kHz, although in medical applications the frequencies can be up to the MHz range
- An ultrasound transducer is made up of a piezoelectric crystal and electrodes which produce an alternating p.d.
- The crystal is heavily damped, usually with epoxy resin, to stop the crystal from vibrating too much
    - This produces short pulses and increases the resolution of the ultrasound device

- A piezoelectric crystal can act as both a receiver or transmitter of ultrasound
    - When it is **receiving** ultrasound, it converts the sound waves into an alternating p.d.
    - When it is **transmitting** ultrasound, it converts an alternating p.d. into sound waves

Worked example

Explain the principles of the generation and detection of ultrasound waves.

Generation:

- An alternating p.d. is applied across a piezo-electric crystal, causing it to change shape
- The alternating p.d. causes the crystal to vibrate and produce ultrasound waves
- The crystal vibrates at the frequency of the alternating p.d., so, the crystal must be cut to a specific size in order to produce resonance

Detection:

- When the ultrasound wave returns, the crystal vibrates which produces an alternating p.d. across the crystal
- This received signal can then be processed and used for medical diagnosis

Using Ultrasound in Medical Imaging

- In an ultrasound scanner, the transducer sends out a beam of sound waves into the body
- The sound waves are reflected back to the transducer by boundaries between tissues in the path of the beam
  - For example, the boundary between fluid and soft tissue or tissue and bone
- When these echoes hit the transducer, they generate electrical signals that are sent to the ultrasound scanner
- Using the speed of sound and the time of each echo's return, the scanner calculates the distance from the transducer to the tissue boundary
- These distances can be used to generate two-dimensional images of tissues and organs

- The frequency of the ultrasound is important because:
  - The higher the frequency of the ultrasound, the higher the resolution and the smaller structures that can be distinguished

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- The ultrasound gives two main pieces of information about the boundary:
  - **Depth**: the time between transmission and receipt of the pulse (the time delay)
  - **Nature**: amount of transmitted intensity received (will vary depending on the type of tissue)

Worked example

Explain the main principles behind the use of ultrasound to obtain diagnostic information about internal body structures.

- A pulse of ultrasound is emitted by the piezo-electric crystal
- This is reflected from the boundaries between media
- The reflected pulse is detected by the ultrasound transmitter
- The signal is then processed and displayed on the screen for the healthcare worker to analyse and use for medical diagnosis
- The **intensity of the reflection** gives information about the nature of the boundary
- The time between transmission and receipt of the pulse (**the time delay**) gives information about the depth of the boundary

Exam Tip

6 mark exam questions about this topic are very common, make sure you practice writing about using and detecting ultrasounds in full, coherent sentences with correct spelling and grammar. Writing short or vague answers could lose you marks, as well as misspelling words!

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24.1.3 SPECIFIC ACOUSTIC IMPEDANCE

Specific Acoustic Impedance

- The **acoustic impedance**, Z, of a medium is defined as:

> ***The product of the speed of the ultrasound in the medium and the density of the medium***

- This quantity describes how much resistance an ultrasound beam encounters as it passes through a tissue
- Acoustic impedance can be calculated using the equation:

$$\mathbf{Z = \rho c}$$

- Where:
    - Z = acoustic impedance (kg m^{-2} s^{-1})
    - ρ = the density of the material (kg m^{-3})
    - c = the speed of sound in the material (m s^{-1})

- This equation tells us:
    - The higher the density of a tissue, the greater the acoustic impedance
    - The faster the ultrasound travels through the material, the greater the acoustic impedance also
- This is because sound travels faster in denser materials
    - Sound is fastest in solids and slowest in gases
    - The closer the particles in the material, the faster the vibrations can move through the material

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- At the boundary between media of different acoustic impedances, some of the wave energy is **reflected** and some is **transmitted**
- The greater the **difference** in acoustic impedance between the two media, the greater the reflection and the smaller the transmission
  - Two materials with the same acoustic impedance would give no reflection
  - Two materials with a large difference in values would give much larger reflections
- Air has an acoustic impedance of Z_{air} = 400 kg m^{-2} s^{-1}
- Skin has an acoustic impedance of Z_{skin} = 1.7 × 10^6 kg m^{-2} s^{-1}
  - The large difference means ultrasound would be significantly reflected, hence a coupling gel is necessary
  - The coupling gel used has a similar Z value to skin, meaning that very little ultrasound is reflected

Worked example: Calculating bone density

The table shows the speed of sound acoustic impedance in four different materials.

Use this to calculate the value for the density of bone.

medium	speed of ultrasound / m s^{-1}	acoustic impedance / kg $m^2 s^{-1}$
air	330	4.3×10^2
gel	1500	1.5×10^6
soft tissue	1600	1.6×10^6
bone	4100	7.0×10^6

Step 1: Write down known quantities

Acoustic impedance of bone, Z = **7.0×10^6 kg m^{-2} s^{-1}**

Speed of ultrasound in bone, c = **4100 m s^{-1}**

Step 2: Write out the equation for acoustic impedance

$$Z = \rho c$$

Step 3: Rearrange for density and calculate

$$\rho = \frac{Z}{c} = \frac{7.0 \times 10^6}{4100} = \mathbf{1700\ kg\ m^{-3}}$$

Exam Tip

A common mistake is to confuse the c in the acoustic impedance equation for the speed of light – don't do this!

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24.1.4 INTENSITY REFLECTION COEFFICIENT

Intensity Reflection Coefficient

- The intensity reflection coefficient α is defined as:

    The ratio of the intensity of the reflected wave relative to the incident (transmitted) wave

- This can be calculated using the fraction:

$$\alpha = \frac{I_r}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

- Where:
    - α = intensity reflection coefficient
    - I_R = intensity of the reflected wave (W m^{-2})
    - I_0 = intensity of the incident wave (W m^{-2})
    - Z_1 = acoustic impedance of one material (kg m^{-2} s^{-1})
    - Z_2 = acoustic impedance of a second material (kg m^{-2} s^{-1})

- This equation will be provided on the datasheet for your exam
- This ratio shows:
    - If there is a large difference between the impedance of the two materials, then most of the energy will be reflected
    - If the impedance is the same, then there will be **no reflection**

Coupling Medium

- When ultrasound is used in medical imaging, a coupler is needed between the transducer and the body
- The soft tissues of the body are much denser than air
- If air is present between the transducer and the body, then almost all the ultrasound energy will be reflected
- The coupling gel is placed between the transducer and the body, as skin and the coupling gel have a similar density, so little ultrasound is reflected
    - This is an example of impedance matching

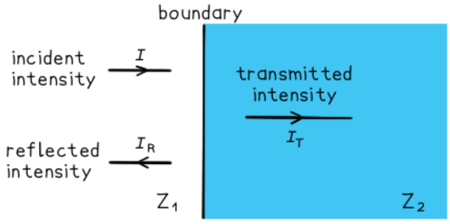
24. Medical Physics

YOUR NOTES

Worked example: Intensity reflection coefficient

?

A beam of ultrasound is incident at right-angles to a boundary between two materials as shown in the diagram.



The materials have acoustic impedances of Z_1 and Z_2. The intensity of the transmitted ultrasound beam is I_T, and the reflected intensity is I_R.

medium	speed of ultrasound / m s^{-1}	acoustic impedance / kg m^{2}s^{-1}
air	330	4.3×10^2
gel	1500	1.5×10^6
soft tissue	1600	1.6×10^6
bone	4100	7.0×10^6

a) What is the relationship between I, I_T and I_R?

b) Use the data from the table to determine the reflection coefficient α for a boundary between
   i) gel and soft tissue.
   ii) air and soft tissue.

c) Explain why gel is usually put on the skin during medical diagnosis using ultrasound.

Part (a)

Incident intensity = Transmitted intensity + Reflected intensity

$$\mathbf{I = I_T + I_R}$$

Part (b)(i)

Step 1: Write down the equation for intensity reflection coefficient α

$$\alpha = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

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YOUR NOTES

Step 2: Write down the acoustic impedances for gel and soft tissue

Gel, Z_1 = $\mathbf{1.5 \times 10^6}$ **kg m^{-2} s^{-1}**

Soft tissue, Z_2 = $\mathbf{1.6 \times 10^6}$ **kg m^{-2} s^{-1}**

Step 3: Calculate the intensity reflection coefficient

$$\alpha = \frac{(1.6\times10^6 - 1.5\times10^6)^2}{(1.6\times10^6 + 1.5\times10^6)^2} = \frac{(0.1)^2}{(3.1)^2} = 0.001$$

This result means that only **0.1%** of the incident intensity will be reflected, with the remaining being transmitted

Part (b)(ii)

Step 1: Write down the acoustic impedances for air and soft tissue

Air, Z_1 = 4.3×10^2 kg m^{-2} s^{-1}

Soft tissue, Z_2 = 1.6×10^6 kg m^{-2} s^{-1}

Step 2: Calculate the intensity reflection coefficient

$$\alpha = \frac{(1.6\times10^6 - 4.3\times10^2)^2}{(1.6\times10^6 + 4.3\times10^2)^2} \approx \frac{(1.6\times10^6)^2}{(1.6\times10^6)^2} \approx 1$$

This result means that **100%** of the incident intensity will be reflected, with none being transmitted

Part (c)

- At the air-soft tissue boundary, the intensity reflection coefficient is $\alpha \approx 1$
  - Therefore, without gel, there is almost complete reflection – no ultrasound is transmitted through the skin
- At the gel-soft tissue boundary, the intensity reflection coefficient is $\alpha = 0.001$
  - Therefore, the gel enables almost complete transmission of the ultrasound through the skin, with very little reflection

24. Medical Physics

YOUR NOTES
⬇

24.1.5 ATTENUATION OF ULTRASOUND IN MATTER

Attenuation of Ultrasound in Matter

- Attenuation of ultrasound is defined as:

  The reduction of energy due to the absorption of ultrasound as it travels through a material

- The attenuation coefficient of the ultrasound is expressed in **decibels per centimetre** lost for every incremental increase in megahertz frequency
  - Generally, 0.5 dB/cm is lost for every 1MHz
- The intensity I of the ultrasound decreases with distance *x*, according to the equation:

$$I = I_0 e^{-\mu x}$$

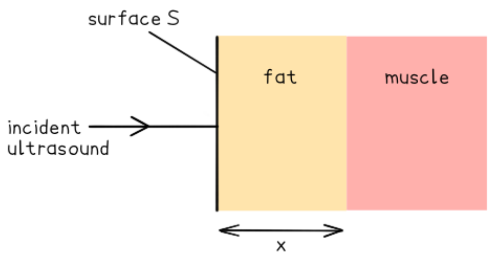
- Where:
  - I_0 = the intensity of the incident beam (W m^{-2})
  - I = the intensity of the reflected beam (W m^{-2})
  - μ = the absorption coefficient (m^{-1})
  - x = distance travelled through the material (m)
- The absorption coefficient μ, will vary from material to material
- Attenuation is not a major problem in ultrasound scanning as the scan relies on the reflection of the ultrasounds at boundaries of materials

24. Medical Physics

YOUR NOTES

Worked example: Attenuation of ultrasound

The thickness x of the layer of fat on an animal, as shown in the diagram, is to be investigated using ultrasound.



The intensity of the parallel ultrasound beam entering the surface S of the layer of fat is I.

The beam is reflected from the boundary between fat and muscle.

The intensity of the reflected ultrasound detected at the surface S of the fat is 0.012I.

medium	Z / kg m^{-2}s^{-1}	μ / m $^{-1}$
fat	1.3×10^6	48
muscle	1.7×10^6	23

Using the table calculate:

a) The intensity reflection coefficient at the boundary between the fat and the muscle.

b) The thickness x of the layer of fat.

Part (a)

Step 1:

Write down the equation for intensity reflection coefficient α

$$\alpha = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

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Step 2:

Calculate the intensity reflection coefficient

$$\alpha = \frac{(1.7\times10^6 - 1.3\times10^6)^2}{(1.7\times10^6 + 1.3\times10^6)^2} = \frac{(0.4)^2}{(3)^2} = 0.018$$

This means that 0.018 of the intensity is reflected at the interface between fat and muscle. This reflected intensity will move back through the fat towards surface S.

Part (b)

Step 1:

Write out the known quantities

The intensity of the ultrasound pulse is affected 3 times:

1. Attenuation from the surface S to the fat-muscle boundary
2. Reflection at the boundary
3. Attenuation from the boundary back to the surface S

After being transmitted in the fat, the intensity at surface S is given to be 0.012 I.

Therefore, the intensity is 0.018 I at the fat-muscle boundary, and as the ultrasound moves through the fat, it gets attenuated and the new intensity at the surface S is now 0.012 I

Incident intensity, equal to the intensity of the reflected pulse, I_0 = 0.018 I × $e^{-\mu x}$

Transmitted intensity, I = 0.012 I

Absorption coefficient, μ = 48 m^{-1}

Thickness of fat = x

24. Medical Physics

YOUR NOTES

Step 2:

Write out the equation for attenuation

$$\mathbf{I = I_0\, e^{-\mu x}}$$

Step 3:

Substitute in values for intensity and simplify

$$\mathbf{0.012\ I = [0.018\ I \times e^{-\mu x}] \times e^{-\mu x}}$$

$$\mathbf{0.012 = 0.018\ e^{-2\mu x}}$$

Step 4:

Rearrange and take the natural log of both sides

$$\frac{0.012}{0.018} = e^{-2\mu x}$$

$$\ln\left(\frac{0.012}{0.018}\right) = -2\mu x$$

Step 5:

Rearrange and calculate the thickness x

$$x = \frac{ln\left(\frac{0.012}{0.018}\right)}{-2\mu} = \frac{ln\left(\frac{0.012}{0.018}\right)}{-2 \times 48} = 4.22 \times 10^{-3}\ \text{m} = \mathbf{0.42\ cm}$$

Exam Tip

The intensity equation will not be provided for you on your exam datasheet, so make sure you remember this!

24. Medical Physics

YOUR NOTES
⬇

24.1.6 PRODUCTION & USE OF X-RAYS

Production of X-rays

- X-rays are short wavelength, high-frequency part of the electromagnetic spectrum
  - They have wavelengths in the range 10^{-8} to 10^{-13} m
- X-rays are produced when fast-moving electrons rapidly decelerate and transfer their kinetic energy into photons of EM radiation

Producing X-rays

- At the cathode (negative terminal), the electrons are released by thermionic emission
- The electrons are accelerated towards the anode (positive terminal) at high speed
- When the electrons bombard the metal target, they lose some of their kinetic energy by transferring it to photons
- The electrons in the outer shells of the atoms (in the metal target) move into the spaces in the lower energy levels
- As they move to lower energy levels, the electrons release energy in the form of **X-ray photons**
- When an electron is accelerated, it gains energy equal to the electronvolt; this energy can be calculated using:

$$\mathbf{E_{max} = eV}$$

- This is the **maximum energy** that an X-ray photon can have
- Therefore, the maximum X-ray frequency f_{max}, or the minimum wavelength λ_{min}, that can be produced is calculated using the equation:

$$E_{max} = eV = hf_{max} = \frac{hc}{\lambda_{min}}$$

$$\text{Maximum frequency:} \quad f_{max} = \frac{eV}{h}$$

$$\text{Minimum wavelength:} \quad \lambda_{min} = \frac{hc}{eV}$$

24. Medical Physics

YOUR NOTES

- Where:
    - e = charge of an electron (C)
    - V = voltage across the anode (V)
    - h = Planck's constant (J s)
    - c = speed of light (m s^{-1})

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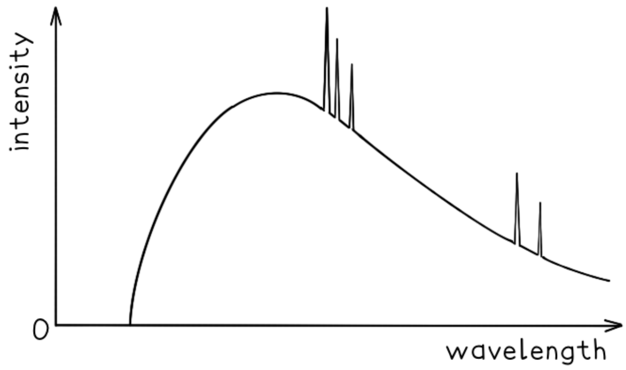
YOUR NOTES

Worked example: Production of X-rays

A typical spectrum of the X-ray radiation produced by electron bombardment of a metal target is shown below.



Explain why:

a) A continuous spectrum of wavelengths is produced.
b) The spectrum has a sharp cut-off at short wavelengths.

Part (a)

- Photons are produced whenever a charged particle is accelerated towards a metal target
- The wavelength of the photons depends on the magnitude of the acceleration
- The electrons which hit the target have a distribution of accelerations, therefore, a continuous spectrum of wavelengths is observed

Part (b)

- The minimum wavelength is equal to

$$\lambda_{min} = \frac{hc}{E_{max}}$$

- This equation shows the maximum energy of the electron corresponds to the minimum wavelength
    - Therefore, the higher the acceleration, the shorter the wavelength
- At short wavelengths, the sharp cut-off occurs as each electron produces a single photon, so, all the electron energy is given up in one collision

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Using X-rays in Medical Imaging

- X-rays have been highly developed to provide detailed images of soft tissue and even blood vessels
- When treating patients, the aims are to:
  - Reduce the exposure to radiation as much as possible
  - Improve the **contrast** of the image

Reducing Exposure

- X-rays are **ionising**, meaning they can cause damage to living tissue and can potentially lead to cancerous mutations
- Therefore, healthcare professionals must ensure patients receive the minimum dosage possible
- In order to do this, **aluminium filters** are used
  - This is because many wavelengths of X-ray are emitted
  - Longer wavelengths of X-ray are more penetrating, therefore, they are more likely to be absorbed by the body
  - This means they do not contribute to the image and pose more of a health hazard
  - The aluminium sheet **absorbs** these long wavelength X-rays making them safer

Contrast & Sharpness

- Contrast is defined as:

The difference in degree of blackening between structures

- Contrast allows a clear difference between tissues to be seen
- Image contrast can be improved by:
  - Using the correct level of X-ray hardness: **hard X-rays** for bones, **soft X-rays** for tissue
  - Using a contrast media
- Sharpness is defined as:

How well defined the edges of structures are

- Image sharpness can be improved by:
  - Using a narrower X-ray beam
  - Reducing X-ray scattering by using a collimator or lead grid
  - Smaller pixel size

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YOUR NOTES

24.1.7 ATTENUATION OF X-RAYS IN MATTER

Attenuation of X-rays in Matter

- Bones **absorb** X-ray radiation
  - This is why they appear white on the X-ray photograph
- When the collimated beam of X-rays passes through the patient's body, they are **absorbed** and **scattered**
- The attenuation of X-rays can be calculated using the equation:

$$\mathbf{I = I_0 \, e^{-\mu x}}$$

- Where:
  - I_0 = the intensity of the incident beam (W m^{-2})
  - I = the intensity of the reflected beam (W m^{-2})
  - μ = the linear absorption coefficient (m^{-1})
  - x = distance travelled through the material (m)

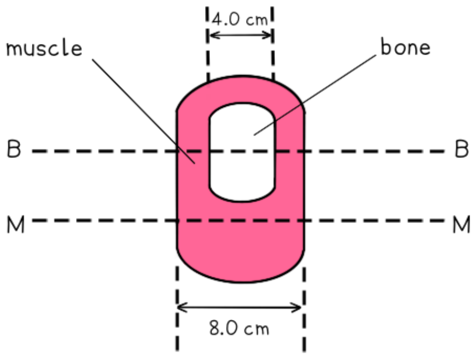
- The attenuation coefficient also depends on the energy of the X-ray photons
- The intensity of the X-ray decays exponentially
- The thickness of the material that will reduce the X-ray beam or a particular frequency to half its original value is known as the **half thickness**

24. Medical Physics

YOUR NOTES
⬇

Worked example: Attenuation of X-rays in matter

A student investigates the absorption of X-ray radiation in a model arm. A cross-section of the model arm is shown in the diagram.



Parallel X-ray beams are directed along the line MM and along the line BB. The linear absorption coefficients of the muscle and the bone are 0.20 cm^{-1} and 12 cm^{-1} respectively.

Calculate the ratio:

$$\frac{\textit{intensity of emergent X-ray beam from model}}{\textit{intensity of incident X-ray beam on model}}$$

for a parallel X-ray beam directed along the line

a) MM
b) BB

and state whether the X-ray images are sharp, or have good contrast.

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YOUR NOTES

Part (a)

Step 1:

Write out the known quantities

Linear absorption coefficient for muscle, μ = **0.20 cm^{-1}**

Distance travelled through the muscle, x = **8.0 cm**

Step 2:

Write out the equation for attenuation and rearrange

$$I = I_0 e^{-\mu x}$$

$$\frac{intensity\ of\ emergent\ X-ray\ beam\ from\ model}{intensity\ of\ incident\ X-ray\ beam\ on\ model} = \frac{I}{I_0} = e^{-\mu x}$$

Step 3:

Substitute in values and calculate the ratio

$$\frac{I}{I_0} = e^{-(0.20 \times 8)} = 0.2$$

Part (b)

Step 1:

Write out the known quantities

Linear absorption coefficient for muscle, μ_m = **0.20 cm^{-1}**

Linear absorption coefficient for bone, μ_b = **12 cm^{-1}**

Distance travelled through the muscle, x_m = **4.0 cm**

Distance travelled through the bone, x_b = **4.0 cm**

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Step 2:

Write out the equation for attenuation for two media and rearrange

$$\frac{I}{I_0} = e^{-\mu_m x_m} \times e^{-\mu_b x_b}$$

Step 3:

Substitute in values and calculate the ratio

$$\frac{I}{I_0} = e^{-(0.20 \times 4)} \times e^{-(12 \times 4)} = 6.4 \times 10^{-22} \approx 0$$

Step 4:

Write a concluding statement

Each ratio gives a measure of the amount of transmission of the beam

A good contrast is when:

- There is a large difference between the intensities
- The ratio is much less than 1.0

Therefore, both images have a good contrast

24. Medical Physics

YOUR NOTES

24.1.8 COMPUTED TOMOGRAPHY SCANNING

Computed Tomography Scanning

- A simple X-ray image can provide useful, but limited, information about internal structures in a 2D image
- When a more comprehensive image is needed, a **computerised axial tomography** (CAT or CT) scan is used
- The main features of the operation of a CT scan are as follows:
  - An X-ray tube rotates around the stationary patient
  - A CT scanner takes X-ray images of the **same slice**, at many different angles
  - This process is **repeated**, then images of successive slices are combined together
  - A computer pieces the images together to build a **3D image**
  - This 3D image can be **rotated** and viewed from different angles

Advantages & Disadvantages of CAT Scans

- Advantages:
  - Produces much more detailed images
  - Can distinguish between tissues with similar attenuation coefficients
  - Produces a 3D image of the body by combining the images at each direction
- Disadvantages:
  - The patient receives a much higher dose than a normal X-ray
  - Possible side effects from the contrast media

24. Medical Physics

YOUR NOTES

Worked example

An X-ray image is taken of the skull of a patient. Another patient has a CT scan of his head.

By reference to the formation of the image in each case, suggest why the exposure to radiation differs between the two imaging techniques.

X-ray

The simple X-ray image involves taking a single exposure

This produces a single 2D image

CT scan

The CT scan requires taking several exposures of a slice from many different angles

This is then repeated for different slices before being combined together to build a 3D image

This involves taking a much greater exposure than the simple X-ray

24. Medical Physics

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24.2 PET SCANNING

24.2.1 RADIOACTIVE TRACERS

Radioactive Tracers

- A **radioactive tracer** is defined as:

    A radioactive substance that can be absorbed by tissue in order to study the structure and function of organs in the body

- Radioactive **isotopes**, such as **technetium-99m** or **fluorine-18**, are suitable for this purpose because:
    - They both bind to organic molecules, such as glucose or water, which are readily available in the body
    - They both emit gamma (γ) radiation and decay into stable isotopes
    - **Technetium-99m** has a short half-life of 6 hours (it is a short-lived form of Technetium-99)
    - **Fluorine-18** has an even shorter half-life of 110 minutes, so the patient is exposed to radiation for a shorter time

Using Tracers in PET Scanning

- Positron Emission Tomography (PET) is:

    A type of nuclear medical procedure that images tissues and organs by measuring the metabolic activity of the cells of body tissues

- A common tracer used in PET scanning is a glucose molecule with radioactive fluorine attached called **fluorodeoxyglucose**
    - The fluorine nuclei undergoes **β^+ decay** – emitting a **positron** (β^+ particle)
- The radioactive tracer is injected or swallowed into the patient and flows around the body
- Once the tissues and organs have absorbed the tracer, then they appear on the screen as a bright area for a diagnosis
    - This allows doctors to determine the progress of a disease and how effective any treatments have been
- Tracers are used not only for the diagnosis of cancer but also for the heart and detecting areas of decreased blood flow and brain injuries, including Alzheimer's and dementia

24. Medical Physics

YOUR NOTES

Worked example: Positron emission of fluorine

Write a nuclear decay for the decay of fluorine-18 by β^+ emission.

Step 1:

Work out what will be on the reactants and products side

Reactant:

Fluorine = ${}^{18}_{9}F$

Products:

Beta-plus β+ (positron) = ${}^{0}_{1}\beta$

Oxygen = ${}^{18}_{8}O$

Gamma-ray = γ

Step 2:

Write the nuclear equation

$${}^{18}_{9}F \rightarrow {}^{18}_{8}O + {}^{0}_{1}\beta + \gamma$$

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24.2.2 ANNIHILATION IN PET SCANNING

The Process of Annihilation

- When a positron is emitted from a tracer in the body, it travels less than a millimetre before it collides with an electron
- The position and the electron will **annihilate**, and their mass becomes pure energy in the form of two gamma rays which move apart in opposite directions
- Annihilation doesn't just happen with electrons and positrons, annihilation is defined as:

When a particle meets its equivalent antiparticle they are both destroyed and their mass is converted into energy

- As with all collisions, the mass, energy and momentum are conserved

Positron Emission Tomography (PET) Scanning

- Once the tracer is introduced to the body it has a **short half-life**, so, it begins emitting positrons (β^+) immediately
  - This allows for a short exposure time to the radiation
  - A short half-life does mean the patient needs to be scanned quickly and not all hospitals have access to expensive PET scanners
- In PET scanning:
  - Positrons are emitted by the decay of the tracer
  - They travel a small distance and annihilate when they interact with electrons in the tissue
  - This annihilation produces a pair of gamma-ray photons which travel in opposite directions

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24.2.3 DETECTING GAMMA-RAYS FROM PET SCANNING

Detecting Gamma-Rays from PET Scanning

- The patient lays stationary in a tube surrounded by a ring of detectors
- Images of slices of the body can be taken to show the position of the **radioactive tracers**
- The detector consists of two parts:
- **Crystal Scintillator** – when the gamma-ray (γ-ray) photon is incident on a crystal, an electron in the crystal is excited to a higher energy state
    - As the excited electron travels through the crystal, it excites more electrons
    - When the excited electrons move back down to their original state, the lost energy is transmitted as visible light photons
- **Photomultiplier** -The photons produced by the scintillator are very faint, so they need to be amplified and converted to an electrical signal by a photomultiplier tube

Creating an Image from PET Scanning

- The γ rays travel in straight lines in opposite directions when formed from a positron-electron annihilation
    - This happens in order to **conserve momentum**
- They hit the detectors in a line – known as the **line of response**
- The tracers will emit lots of γ rays simultaneously, and the computers will use this information to create an image
- The more photons from a particular point, the more tracer that is present in the tissue being studied, and this will appear as a bright point on the image
- An image of the **tracer concentration** in the tissue can be created by **processing the arrival times** of the gamma-ray photons

Calculating Energy of Gamma-Ray Photons

- In the annihilation process, both mass-energy and momentum are conserved
- The gamma-ray photons produced have an energy and frequency that is determined solely by the mass-energy of the positron-electron pair
- The energy E of the photon is given by

$$\mathbf{E = hf = m_ec^2}$$

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- The momentum p of the photon is given by

$$p = \frac{E}{c}$$

- Where:
  - m_e = mass of the electron or positron (kg)
  - h = Planck's constant (J s)
  - f = frequency of the photon (Hz)
  - c = the speed of light in a vacuum (m s^{-1})

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YOUR NOTES

Worked example: Calculating energy of gamma-ray photons

Fluorine-18 decays by β^+ emission. The positron emitted collides with an electron and annihilates producing two γ-rays.

a) Calculate the energy released when a positron and an electron annihilate.
b) Calculate the frequency of the γ-rays emitted.
c) Calculate the momentum of one of the γ-rays.

Mass of an electron = mass of a positron = 9.11×10^{-31} kg.

Part (a)

Step 1: Write down the known quantities

Total mass is equal to the mass of the electron and positron:

$$\mathbf{m = 2 \times (9.11 \times 10^{-31}) = 1.822 \times 10^{-30}\ kg}$$

Step 2: Write out the equation for mass-energy equivalence

$$\mathbf{E = m_e c^2}$$

Step 3: Substitute in values and calculate energy E

$$\mathbf{E = 2 \times (9.11 \times 10^{-31}) \times (3.0 \times 10^{8})^2 = 1.6 \times 10^{-13}\ J}$$

Part (b)

Step 1: Write down the known quantities

Two photons are produced, so, the energy of one photon is equal to half of the total energy:

$$E = \frac{1.6 \times 10^{-13}}{2} = 0.8 \times 10^{-13}\ \text{J}$$

Step 2: Write out the equation for energy of a photon

$$\mathbf{E = hf}$$

Step 3: Rearrange for frequency f, and calculate

$$f = \frac{E}{h} = \frac{0.8 \times 10^{-13}}{6.63 \times 10^{-34}} = \mathbf{1.2 \times 10^{20}\ Hz}$$

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Part (c)

Step 1: Write out the equation for momentum of a photon

$$p = \frac{E}{c}$$

Step 2: Substitute in values and calculate momentum p

$$p = \frac{E}{c} = \frac{0.8 \times 10^{-13}}{3.0 \times 10^{8}} = \mathbf{2.7 \times 10^{-22}\ N\ s}$$

25. Astronomy & Cosmology

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CONTENTS

25.1 ASTRONOMY

25.1.1 LUMINOSITY & RADIANT FLUX

Defining Luminosity

- Luminosity *L* is defined as:

The total power output of radiation emitted by a star

- It is measured in units of **Watts (W)**
- Radiant flux intensity *F* is defined as:

The observed amount of intensity, or the radiant power transmitted normally though a surface per unit of area, of radiation measured on Earth

- The best way to picture this is:
  - The luminosity is the total radiation that **leaves** the star
  - The radiant flux intensity is the amount of radiation **measured on Earth**
  - By the time the radiation reaches the Earth, it will have spread out a great deal, therefore, it will only be a fraction of the value of the luminosity

25. Astronomy & Cosmology

YOUR NOTES

Inverse Square Law of Flux

- Light sources which are further away appear fainter because the light it emits is spread out over a greater area
- The moment the light leaves the surface of the star, it begins to spread out uniformly through a spherical shell
  - The surface area of a sphere is equal to $\mathbf{4\pi r^2}$
- The radius *r* of this sphere is equal to the distance *d* between the star and the Earth
- By the time the radiation reaches the Earth, it has been spread over an area of $4\pi d^2$
- The inverse square law of flux can therefore be calculated using:

$$F = \frac{L}{4\pi d^2}$$

- Where:
  - F = radiant flux intensity, or observed intensity on Earth (W m^{-2})
  - L = luminosity of the source (W)
  - d = distance between the star and the Earth (m)
- This equation assumes:
  - The power from the star radiates uniformly through space
  - No radiation is absorbed between the star and the Earth
- This equation tells us:
  - For a given star, the luminosity is constant
  - The radiant flux follows an inverse square law
  - The greater the radiant flux (larger F) measured, the closer the star is to the Earth (smaller d)

25. Astronomy & Cosmology

YOUR NOTES

Worked example: Inverse square law of flux

A star has a luminosity that is known to be 4.8×10^{29} W.

A scientist observing this star finds that the radiant flux intensity of light received on Earth from the star is $2.6\ \text{nW m}^{-2}$.

Determine the distance of the star from Earth.

Step 1: Write down the known quantities

Luminosity, L = 4.8×10^{29} W

Radiant flux intensity, F = $2.6\ \text{nW m}^{-2} = 2.6 \times 10^{-9}\ \text{W m}^{-2}$

Step 2: Write down the inverse square law of flux

$$F = \frac{L}{4\pi d^2}$$

Step 3: Rearrange for distance d, and calculate

$$d = \sqrt{\frac{L}{4\pi F}} = \sqrt{\frac{4.8 \times 10^{29}}{4\pi \times (2.6 \times 10^{-9})}} = \mathbf{3.8 \times 10^{18}\ m}$$

25. Astronomy & Cosmology

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⬇

25.1.2 STANDARD CANDLES & STELLAR DISTANCES

Standard Candles

- A standard candle is defined as:

  An astronomical object which has a known luminosity due to a characteristic quality possessed by that class of object

- Examples of standard candles are:
- Cepheid variable stars
  - A type of pulsating star which increases and decreases in brightness over a set time period
  - This variation has a well defined relationship to the luminosity
- Type 1a supernovae
  - A supernova explosion involving a white dwarf
  - The luminosity at the time of the explosion is always the same

Using Standard Candles as a Distance Indicator

- Measuring astronomical distances accurately is an extremely difficult task
- A direct distance measurement is only possible if the object is close enough to the Earth
- For more distant objects, indirect methods must be used – this is where standard candles come in useful
- If the luminosity of a source is known, then the distance can be estimated based on how bright it appears from Earth
  - Astronomers measure the radiant flux intensity, of the electromagnetic radiation arriving at the Earth
  - Since the luminosity is known (as the object is a standard candle), the distance can be calculated using the inverse square law of flux
- Each standard candle method can measure distances within a certain range
- Collating the data and measurements from each method allows astronomers to build up a larger picture of the scale of the universe
  - This is known as the **cosmic distance ladder**

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25.1.3 WIEN'S DISPLACEMENT LAW

Wien's Displacement Law

- Wien's displacement law relates the observed wavelength of light from a star to its surface temperature, it states:

  The black body radiation curve for different temperatures peaks at a wavelength which is inversely proportional to the temperature

- This relation can be written as:

$$\lambda_{max} \propto \frac{1}{T}$$

- λ_{max} is the maximum wavelength emitted by the star at the peak intensity
- A black-body is an object which:
  - Absorbs all the radiation that falls on it, and is also a good emitter
  - Does not reflect or transmit any radiation
- A black-body is a theoretical object, however, stars are the best approximation there is
- The radiation emitted from a black-body has a characteristic spectrum that is determined by the temperature alone
- The full equation for Wien's Law is given by

$$\boldsymbol{\lambda_{max}T = 2.9 \times 10^{-3} \text{ m K}}$$

- Where:
  - λ_{max} = peak wavelength of the star (m)
  - T = thermodynamic temperature at the surface of the star (K)

- This equation tells us the higher the temperature of a body:
  - The shorter the wavelength at the peak intensity, so **hotter** stars tend to be **white or blue** and **cooler** stars tend to be **red or yellow**
  - The greater the intensity of the radiation at each wavelength

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Worked example: Wien's displacement law

The spectrum of the star Rigel in the constellation of Orion peaks at a wavelength of 263 nm, while the spectrum of the star Betelgeuse peaks at a wavelength of 828 nm.

Which of these two stars is cooler, Betelgeuse or Rigel?

Step 1: Write down Wien's displacement law

$$\boldsymbol{\lambda_{max} T = 2.9 \times 10^{-3} \text{ m K}}$$

Step 2: Rearrange for temperature T

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{max}}$$

Step 3: Calculate the surface temperature of each star

$$\text{Rigel: } T = \frac{2.9 \times 10^{-3}}{\lambda_{max}} = \frac{2.9 \times 10^{-3}}{263 \times 10^{-9}} = 11026 = \mathbf{11\,000\ K}$$

$$\text{Betelguese: } T = \frac{2.9 \times 10^{-3}}{\lambda_{max}} = \frac{2.9 \times 10^{-3}}{828 \times 10^{-9}} = 3502 = \mathbf{3\,500\ K}$$

Step 4: Write a concluding sentence

Betelguese has a surface temperature of 3 500 K, therefore, it is much cooler than Rigel

Exam Tip

Note that the temperature used in Wien's Law is in **Kelvin** (K). Remember to convert from °C if the temperature is given in degrees in the question before using the Wien's Law equation.

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25.1.4 STEFAN-BOLTZMANN LAW & STELLAR RADII

Stefan-Boltzmann Law

- A star's luminosity depends on two factors:
  - Its surface temperature
  - Its radius
- The relationship between these is known as the **Stefan-Boltzmann Law**, which states:

The total energy emitted by a black body per unit area per second is proportional to the fourth power of the absolute temperature of the body

- It is equal to:

$$L = 4\pi r^2 \sigma T^4$$

- Where:
  - L = luminosity of the star (W)
  - r = radius of the star (m)
  - σ = the Stefan-Boltzmann constant
  - T = surface temperature of the star (K)

Estimating the Radius of Stars

- The radius of a star can be estimated by combining Wien's displacement law and the Stefan–Boltzmann law
- The procedure for this is as follows:
  - Using Wien's displacement law to find the surface temperature of the star
  - Using the inverse square law of flux equation to find the luminosity of the star (if given the radiant flux and stellar distance)
  - Then, using the Stefan-Boltzmann law, the stellar radius can be obtained

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Worked example: Estimating the radius of a star

Betelguese is our nearest red giant star. It has a luminosity of 4.49×10^{31} W and emits radiation with a peak wavelength of 850 nm.

Calculate the ratio of the radius of Betelgeuse r_B to the radius of the Sun r_s .

Radius of the sun $r_s = 6.95 \times 10^8$ m.

Step 1: Write down Wien's displacement law

$$\lambda_{max}T = 2.9 \times 10^{-3} \text{ m K}$$

Step 2: Rearrange Wien's displacement law to find the surface temperature of Betelguese

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{max}} = \frac{2.9 \times 10^{-3}}{850 \times 10^{-9}} = 3410 \text{ K}$$

Step 3: Write down the Stefan-Boltzmann law

$$L = 4\pi r^2 \sigma T^4$$

Step 4: Rearrange for r and calculate the stellar radius of Betelguese

$$r_B = \sqrt{\frac{L}{4\pi\sigma T^4}} = \sqrt{\frac{(4.49 \times 10^{31})}{4\pi \times (5.67 \times 10^{-8}) \times (3410)^4}} = 6.83 \times 10^{11} \text{ m}$$

Step 5: Calculate the ratio r_B / r_s

$$\frac{r_B}{r_s} = \frac{6.83 \times 10^{11}}{6.95 \times 10^8} = \mathbf{983}$$

Therefore, the radius of Betelguese is about 1000 times larger than the Sun's radius

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25.2 COSMOLOGY

25.2.1 EMISSION SPECTRA

Emission Spectra

- Astronomers are very limited in how they can investigate objects in the space
- All of the techniques used involve analysing the light emitted from the star, or galaxy
- One of these techniques involves analysing the **emission** and **absorption spectra** of stars
  - More details on this can be found in the revision notes “Line Spectra” in the Quantisation of Energy topic
- Elements in the star, predominantly hydrogen and helium, absorb some of the emitted wavelengths
- Therefore, characteristic lines are present when the spectrum is analysed

- When astronomers observe light from distant galaxies, they observe differences in the spectral lines to the light from the Sun
- The lines have the same characteristic pattern, meaning the element can still be easily identified, they just appear to be shifted sightly
  - The lines in the spectra from distant galaxies show an **increase in wavelength**
  - The lines are moved, or shifted, towards the **red end of the spectrum**

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25.2.2 DOPPLER REDSHIFT

Redshift of EM Radiation

- Recall the Doppler effect is defined as:

  The apparent change in wavelength or frequency of the radiation from a source due to its relative motion away from or toward the observer

- On Earth, the Doppler effect of sound can be easily observed when sound waves moves past an observer at a notable speed
- In space, the Doppler effect of light can observed when spectra of distant stars and galaxies are observed, this is known as:
  - **Redshift** if the object is moving away from the Earth, or
  - **Blueshift** if the object is moving towards the Earth
- Redshift is defined as:

  The fractional increase in wavelength (or decrease in frequency) due to the source and observer receding from each other

- For non-relativistic galaxies, Doppler redshift can be calculated using:

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f} = \frac{v}{c}$$

- Where:
  - $\Delta\lambda$ = shift in wavelength (m)
  - λ = wavelength emitted from the source (m)
  - Δf = shift in frequency (Hz)
  - f = frequency emitted from the source (Hz)
  - v = speed of recession (m s^{-1})
  - c = speed of light in a vacuum (m s^{-1})

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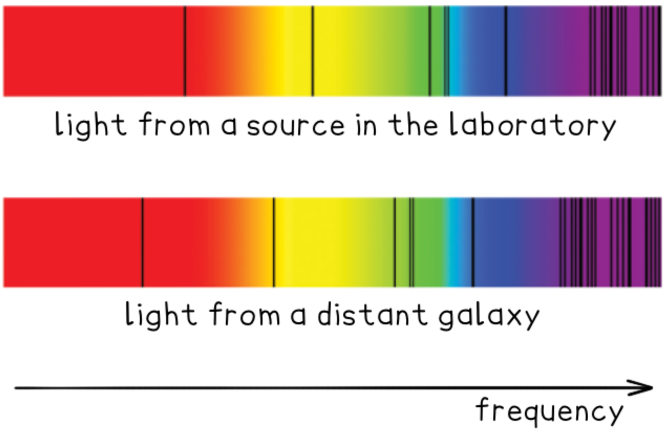
Worked example: Redshift of a distant galaxy

The spectra below show dark absorption lines against a continuous visible spectrum.



A particle line in the spectrum of light from a source in the laboratory has a frequency of 4.570×10^{14} Hz.

The same line in the spectrum of light from a distant galaxy has a frequency of 4.547×10^{14} Hz.

What speed is the distance galaxy moving in relation to the Earth? Is it moving towards or away from the Earth?

Step 1: Write down the known quantities

Emitted frequency, f = $\mathbf{4.570 \times 10^{14}}$ **Hz**

Shift in frequency, Δf = $\mathbf{(4.547 - 4.570) \times 10^{14} = -2.3 \times 10^{12}}$ **Hz**

Speed of light, c = $\mathbf{3.0 \times 10^{8}}$ $\mathbf{m\ s^{-1}}$

Step 2: Write down the Doppler redshift equation

$$\frac{\Delta f}{f} = \frac{v}{c}$$

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Step 3: Rearrange for speed v, and calculate

$$v = \frac{c\Delta f}{f} = \frac{(3.0 \times 10^8) \times (2.3 \times 10^{12})}{4.570 \times 10^{14}} = \mathbf{1.5 \times 10^6\ m\ s^{-1}}$$

Step 4: Write a concluding sentence

The observed frequency is **less** than the emitted frequency (the light from a laboratory source), therefore, the source is **receding**, or moving away, from the Earth at **1.5 × 10⁶ m s⁻¹**

Exam Tip

In your exam, be sure to emphasise that redshift means the wavelength of spectral lines increases towards the red end of the spectrum, **do not** say that the spectral lines become red, as this is incorrect.

An Expanding Universe

- After the discovery of Doppler redshift, astronomers began to realise that almost all the galaxies in the universe are receding
- This lead to the idea that the space between the Earth and the galaxies must be **expanding**
- This expansion stretches out the light waves as they travel through space, shifting them towards the red end of the spectrum
- The more red-shifted the light from a galaxy is, the **faster** the galaxy is moving away from Earth

- The expansion of the universe can be compared to dots on an inflating balloon
    - As the balloon is inflated, the dots all move away from each other
    - In the same way as the rubber stretches when the balloon is inflated, space itself is **stretching out between galaxies**
    - Just like the dots, the galaxies move away from each other, however, they themselves do not move

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25.2.3 HUBBLE'S LAW & THE BIG BANG THEORY

Hubble's Law & the Big Bang Theory

- Edwin Hubble investigated the light spectra emitted from a large number of galaxies
- He used redshift data to determine the recession velocities of these galaxies, and standard candles to determine the distances
- From these measurements, he formulated a relationship, now known as **Hubble's Law**
- Hubble's Law states:

The recession speed of galaxies moving away from Earth is proportional to their distance from the Earth

- This can be calculated using:

$$v = H_0 d$$

- Where:
    - v = the galaxy's recessional velocity ($m\ s^{-1}$)
    - d = distance between the galaxy and Earth (m)
    - H_0 = Hubble's constant, or the rate of expansion of the universe (s^{-1})
- This equation tells us:
    - The **further away** a galaxy, the **faster** it's recession velocity
    - The gradient of a graph of recession velocity against distance is equal to the **Hubble constant**

Age of the Universe

- If the galaxies are moving away from each other, then they must've started from the same point at some time in the past
- If this is true, the universe likely began in an extremely hot, dense singular point which subsequently began to expand very quickly
    - This idea is known as the **Big Bang theory**
- Redshift of galaxies and the expansion of the universe are now some of the most prominent pieces of evidence to suggest this theory is true
- The data from Hubble's law can be extrapolated back to the point that the universe started expanding ie. the beginning of the universe
- Therefore, the age of the universe T_0 is equal to:

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$$T_0 = \frac{1}{H_0}$$

- Current estimates of the age of the universe range from **13 – 14 billion years**
- There is still some discussion about the exact age of the universe, therefore, obtaining accurate measurements for the Hubble constant is a top priority for cosmologists

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Worked example: Age of the universe

A galaxy is found to be moving away with a speed of 2.1×10^7 m s^{-1}.

The galaxy is at a distance of 9.5×10^{24} m.

Assuming the speed has remained constant, what is the age of the universe, in years?

Step 1: Write down Hubble's Law

$$\mathbf{v = H_0 d}$$

Step 2: Rearrange for the Hubble constant H_0, and calculate

$$H_0 = \frac{v}{d} = \frac{2.1 \times 10^7}{9.5 \times 10^{24}} = 2.2 \times 10^{-18} \text{ s}^{-1}$$

Step 3: Write the equation for the age of the universe T_0, and calculate

$$T_0 = \frac{1}{H_0} = \frac{1}{2.2 \times 10^{-18}} = 4.52 \times 10^{17} \text{ s}$$

Step 4: Convert from seconds into years

$$T_0 = \frac{4.52 \times 10^{17}}{(365 \times 24 \times 60 \times 60)} = 1.43 \times 10^{10} \text{ years}$$

Therefore, the age of the universe is estimated to be about **14.3 billion years**