QUADRATICS (P1)
(5-7 Marks)

FORMS OF QUADRATIC EQUATION

1 **STANDARD**

$y = ax^2 + bx + c$

Shape   y-intercept.

2 **VERTEX FORM**
COMPLETED SQUARE FORM

$y = a(x-b)^2 + c$

Shape

Min Value if a +ve

Max Value if a -ve.

$x - b = 0$

TURNING POINT ⟶ $x = b$ $y = c$

3 **ROOT FORM**

$y = a(x-b)(x-c)$

Shape.

$x - b = 0$ $x - c = 0$

X-intercepts $x = b$ $x = c$

HOW TO CONVERT FROM STANDARD FORM $\longrightarrow$ COMPLETED SQUARE FORM.

Q. Express $4x^2 - 24x + 44$ in form $a(x-b)^2 + c$

Half

$$4\left[x^2 - 6x + (3)^2 - (3)^2 + 11\right]$$

$$4\left[(x-3)^2 - 9 + 11\right]$$

$$4\left[(x-3)^2 + 2\right]$$

$$4(x-3)^2 + 8$$

$$a(x-b)^2 + c$$

$a = 4$, $-b = -3$, $c = 8$

$b = 3$

Q. Express $3x^2 + 27x - 10$ in form $a(x-b)^2 + c$

Half

$$3\left[\underbrace{x^2 + 9x + \left(\frac{9}{2}\right)^2} - \left(\frac{9}{2}\right)^2 - \frac{10}{3}\right]$$

$$3\left[\left(x + \frac{9}{2}\right)^2 - \frac{81}{4} - \frac{10}{3}\right]$$

$$3\left[\left(x + \frac{9}{2}\right)^2 - \frac{283}{12}\right]$$

$$3\left(x+\frac{9}{2}\right)^2-\frac{283}{4}$$

ADVANCED VARIATION:

Q: Express $4x^2-32x+40$ in form $(2x-a)^2+b$.

STEP 1: GO TO COMPLETED SQUARE FORM.

Half

$4[x^2-8x+(4)^2-(4)^2+10]$

$4[(x-4)^2-16+10]$

$4[(x-4)^2-6]$

$4(x-4)^2-24$

$4(x-4)^2-24$

$2^2(x-4)^2-24$

Take square common

$[2(x-4)]^2-24$

$(2x-8)^2-24$

$(2x-a)^2+b$

$-a=-8$, $+b=-24$

$\boxed{a=8}$ $\boxed{b=-24}$

STANDARD FORM $\xrightarrow{\text{FACTORISE}}$ ROOT FORM.

$y=a(x-b)(x-c)$

eg: $4x^2-28x+48$

$4[x^2-7x+12]$ ← Middle Term Breaking.

$4[x^2-3x-4x+12]$

$4[x(x-3)-4(x-3)]$

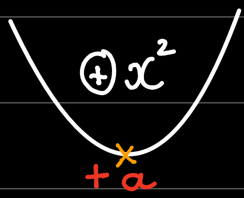
$4(x-3)(x-4)$

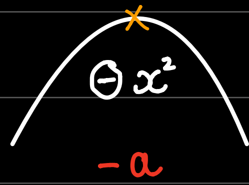
$a(x-b)(x-c)$

SKETCH OF A QUADRATIC CURVE:

1. **SHAPE:**

ALL FORMS





2. **TURNING POINT / VERTEX / STATIONARY POINT**

COMPLETED SQUARE FORM.

eg $\quad y = 2(x-3)^2 - 5$

$x - 3 = 0$

$\boxed{x = 3}$ $\boxed{y = -5}$

Turning point = $(3, -5)$

3. **y-intercept** $(x = 0)$

STANDARD FORM

eg $\quad y = 2x^2 + 9x + 5$

y-intercept.

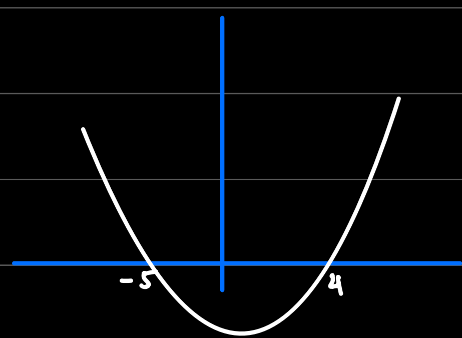
4. **x-intercepts:** $(y = 0)$ (Do not find x-intercepts unless question requires)

ROOT FORM

eg: $\quad y = 2(x+5)(x-4)$

$x + 5 = 0 \qquad x - 4 = 0$

x-intercepts, $\boxed{x = -5}$ $\boxed{x = 4}$



SKETCH: (3 Marks)

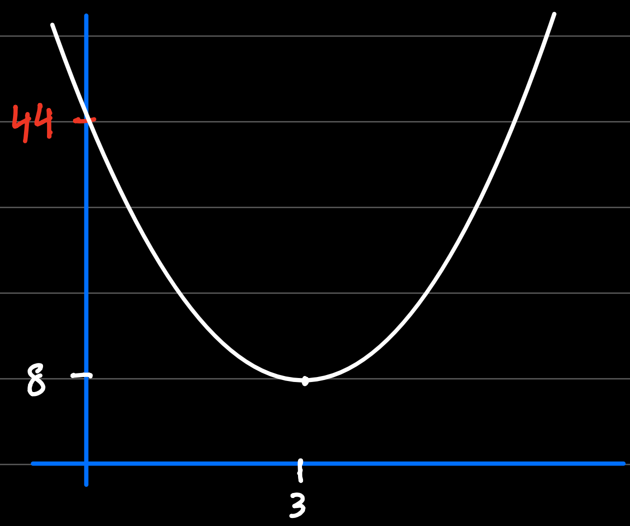
$$y = 4x^2 - 24x + 44 \longrightarrow y = 4(\underline{x-3})^2 + 8$$

Shape

y - intercept

$x - 3 = 0$

Turning Point. $\boxed{x = 3}$ $\boxed{y = 8}$



MAXIMUM POINT / MINIMUM POINT

$$y = a(\underline{x-b})^2 + c$$

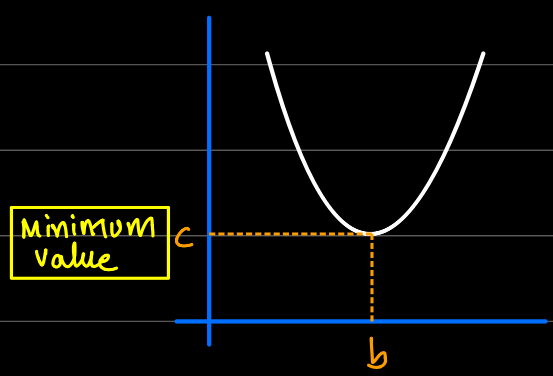
$x - b = 0$

$\boxed{x = b}$ $\boxed{y = c}$

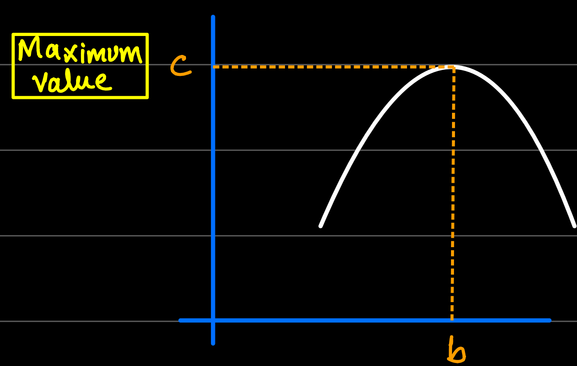
Min Value if a +ve

Max Value if a -ve

a is positive



a is negative.



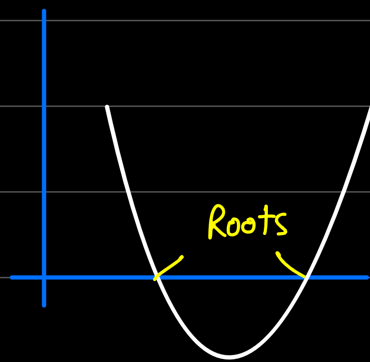
ROOTS:

(x - intercepts) (y = 0)

eg: $y = 2x^2 - 3x + 5$

$y = 0$

$0 = 2x^2 - 3x + 5$



FORMULA

$$x = \frac{-b \pm \sqrt{\boxed{b^2 - 4ac}}}{2a}$$

→ +ve (Two Ans)

→ zero (One Ans)

→ -ve (No Ans)

$a = 2, \; b = -3, \; c = 5$

$b^2 - 4ac$

$(-3)^2 - 4(2)(5)$

$9 - 40 = \boxed{-31}$ No Real Roots.

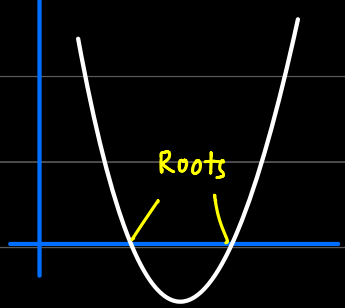
DISCRIMINANT

$\boxed{b^2 - 4ac}$

$b^2 - 4ac$ = positive

(Two Ans)

$b^2 - 4ac > 0$

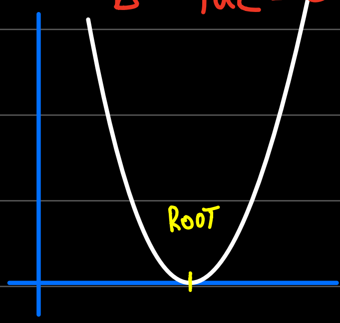


TWO DISTINCT REAL ROOTS

$b^2 - 4ac$ = Zero

(one Ans)

$b^2 - 4ac = 0$

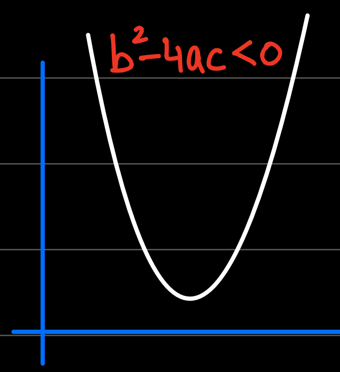


TWO EQUAL/REPEATED REAL ROOTS

$b^2 - 4ac$ = Negative

(No Ans).

$b^2 - 4ac < 0$



NO REAL ROOTS

(Two complex

roots - P3)

your answer is inequality

Q: Find set of values of k for which $2x^2 - 3x + 2k = 0$ has no real roots.

$$b^2 - 4ac < 0$$

$$(-3)^2 - 4(2)(2k) < 0$$

$$9 - 16k < 0$$

$$-16k < -9$$

$$k > \frac{9}{16}$$

INTERSECTION OF A STRAIGHT LINE & QUADRATIC CURVE
$(y = mx + c)$ $(y = ax^2 + bx + c)$

STEP 1: EQUATE BOTH EQUATIONS AND BRING TO STANDARD FORM.

STEP 2: NOW USE $b^2 - 4ac$ TO ANALYSE.

eg Line: $y = 2x + 5$

Curve: $y = x^2 - 9x + 3$

STEP 1: $x^2 - 9x + 3 = 2x + 5$

$x^2 - 11x - 2 = 0$

STEP 2: $x = \dfrac{-b \pm \sqrt{\boxed{b^2 - 4ac}}}{2a}$

+ve = 2 Ans

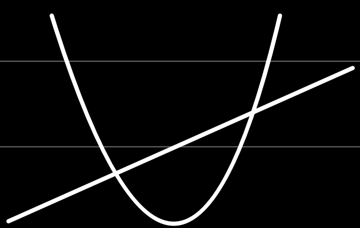
zero = 1 Ans

-ve = No Ans.

$\boxed{b^2 - 4ac}$

$b^2 - 4ac$ = Positive

(2 Ans)

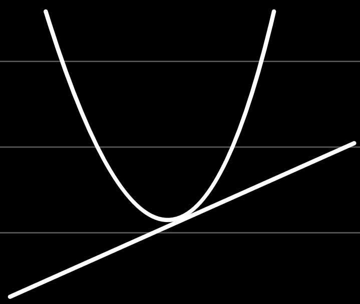


$b^2 - 4ac > 0$

TWO POINTS OF INTERSECTION.

$b^2 - 4ac$ = zero

(1 Ans)

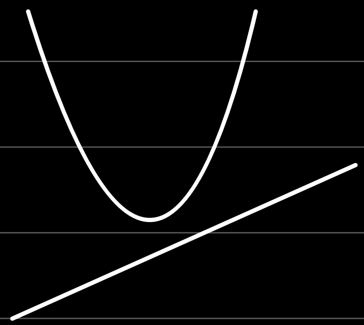


$b^2 - 4ac = 0$

LINE IS TANGENT TO CURVE

$b^2 - 4ac$ = Negative

(NO Ans)



$b^2 - 4ac < 0$

LINE DOES NOT INTERSECT CURVE.

INEQUALITIES

LINEAR

① $2x - 7 < 3$

$2x < 10$

$x < 5$

If we cross multiply or divide by a -ve number, inequality flips.

② $-2x < 10$

$x > \frac{10}{-2}$

$x > -5$

THINGS NOT ALLOWED ON AN INEQUALITY.

$x^2 < 16$

you are not allowed to take square root on an inequality.

$(x+3)(x-5) < 0$

you cannot do either/or working on an inequality

$\cancel{x+3<0}$ or $\cancel{x-5<0}$

QUADRATIC INEQUALITY

STEPS

1- Factorise

2- Sketch using only shape and x-intercepts.

3. Shade relevant region on curve.

4. Write inequality for x-values of shaded region only.

$\boxed{1}$ shape $\smile$ $\leftarrow$ $x^2 - 7x + 12 > 0$

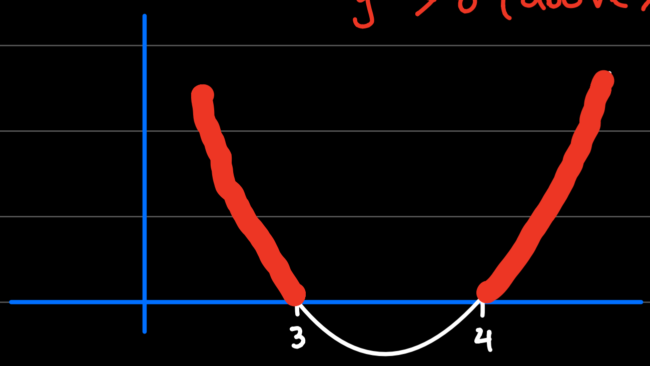
$x^2 - 3x - 4x + 12 > 0$

$x(x-3) - 4(x-3) > 0$

$\underbrace{(x-3)}\,\underbrace{(x-4)} > 0$

x-intercepts $\quad x-3=0 \quad x=3 \qquad x-4=0 \quad x=4$

$y > 0$ (above x-axis)



$\boxed{x < 3}$ or $\boxed{x > 4}$

STEPS

✓ 1- Factorise

✓ 2- Sketch using only $\boxed{\text{shape}}$ and $\boxed{\text{x- intercepts.}}$

✓ 3. Shade relevant region on curve.

✓ 4. Write inequality for x-values of shaded region only.

$\boxed{2}$ shape $\smile$ $\leftarrow$ $x^2 - 6x - 16 < 0$

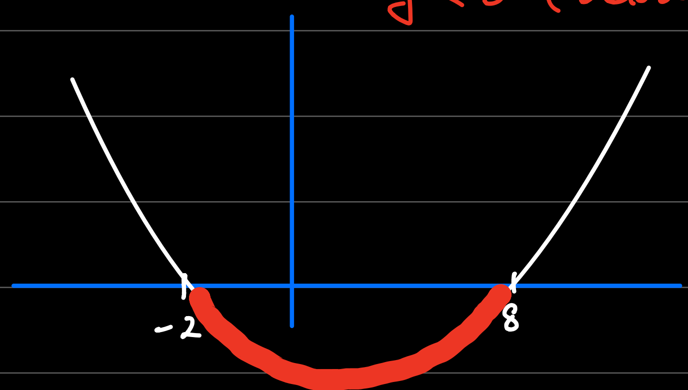
$x^2 - 8x + 2x - 16 < 0$

$x(x-8) + 2(x-8) < 0$

$\underbrace{(x+2)}\,\underbrace{(x-8)} < 0$

x- intercept = $\quad x+2=0 \quad x=-2 \qquad x-8=0 \quad x=8$

$y < 0$ (below x-axis)



$\boxed{-2 < x < 8}$

STEPS

✓ 1- Factorise

✓ 2- Sketch using only shape and x- intercepts.

✓ 3. Shade relevant region on curve.

✓ 4. Write inequality for x-values of shaded region only.

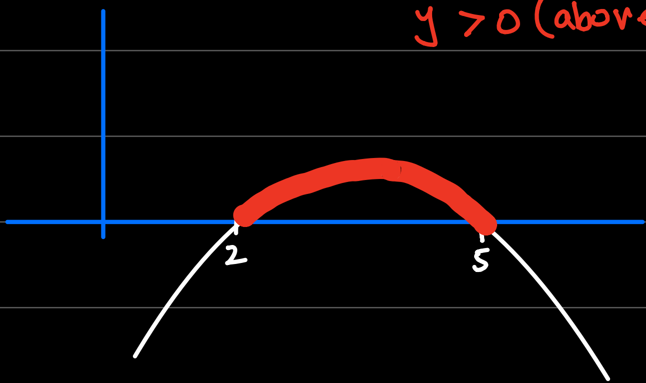
Shape ∩ x^2 term will be -ve

3   $(x-2)(5-x) > 0$

This is already factorized.

x-intercepts $x-2=0$, $x=2$, $5-x=0$, $x=5$

$y > 0$ (above x-axis)



$$\boxed{2 < x < 5}$$

STEPS

1. Factorise
2. Sketch using only shape and x-intercepts.
3. Shade relevant region on curve.
4. Write inequality for x-values of shaded region only.

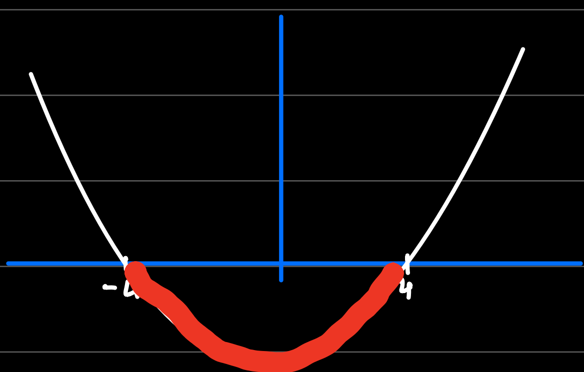
4   $x^2 < 16$

$$x^2 - 16 < 0$$

$$x^2 - 4^2 < 0$$

$$(x+4)(x-4) < 0$$

x-intercepts = $x+4=0$, $x=-4$ $x-4=0$, $x=4$



$$-4 < x < 4$$

STEPS

1. Factorise
2. Sketch using only shape and x-intercepts.
3. Shade relevant region on curve.
4. Write inequality for x-values of shaded region only.

$\boxed{5}$

$$x^2 - 96 > 0$$

STEPS

1. Factorise
2. Sketch using only shape and x-intercepts.
3. Shade relevant region on curve.
4. Write inequality for x-values of shaded region only.

$$(x)^2 - (\sqrt{96})^2 > 0$$

$$(x + \sqrt{96})(x - \sqrt{96}) > 0$$

$\boxed{x < -\sqrt{96}}$ or $\boxed{x > \sqrt{96}}$

inequality

5 Find the set of values of m for which the line $y = mx + 4$ intersects the curve $y = 3x^2 - 4x + 7$ at two distinct points. [5]

STEP 1: EQUATE BOTH AND BRING TO STANDARD FORM.

$$3x^2 - 4x + 7 = mx + 4$$

$$3x^2 - 4x - mx + 7 - 4 = 0$$

$$3x^2 - (4 + m)x + 3 = 0$$

$b^2 - 4ac > 0$ Two distinct points.

$$[-(4+m)]^2 - 4(3)(3) > 0$$

$$+(4+m)^2 - 36 > 0$$

$$(4+m)^2 - (6)^2 > 0$$

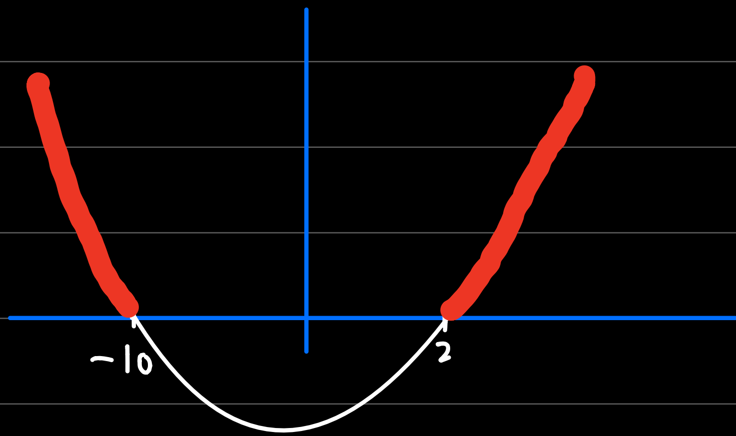
$$(4 + m + 6)(4 + m - 6) > 0$$

$$\underbrace{(m+10)(m-2)}_{m^2 \text{ is positive, shape } \cup} > 0$$

STEPS

1. ~~Factorise~~
2. Sketch using only shape and x-intercepts.
3. Shade relevant region on curve.
4. Write inequality for x-values of shaded region only.

$y > 0$



$\boxed{m < -10}$ or $\boxed{m > 2}$