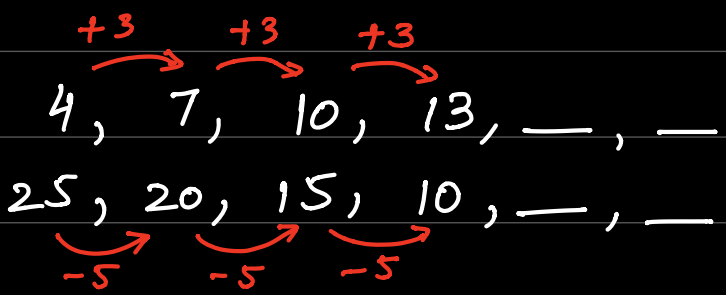
SEQUENCES (AP/GP)

(5-7 Marks)

ARITHEMATIC SEQUENCES : (AP)



[1] nth term $= a + (n-1)d$ → Common difference

a ↓ First term

n ↙ Term no.

d = Next − Previous

[2] SUM OF FIRST n TERM :-

$$S_n = \frac{n}{2}\left(a + l\right)$$

l = Last term

l ↓ $a+(n-1)d$

IF LAST TERM IS NOT AVAILABLE,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

GEOMETRIC PROGRESSION (GP):

$2, 4, 8, 16, 32, \_, \_$ (×2 each step) $\quad r = \frac{32}{16} = 2$

$9, 3, 1, \frac{1}{3}, \frac{1}{9}$ (÷3 each step) $\quad r = \frac{3}{9} = \frac{1}{3}$

COMMON RATIO (r)

$r = \frac{\text{NEXT}}{\text{PREVIOUS}}$

1 nth term $= a \cdot r^{n-1}$

2 SUM OF FIRST n TERMS:

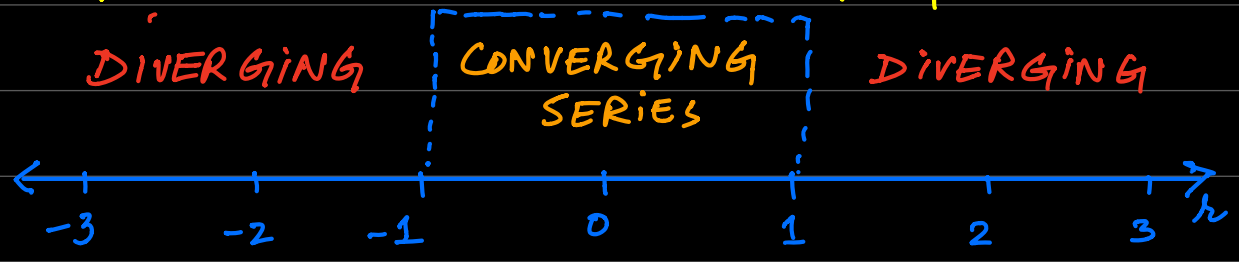
$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}$$

$r > 1$ $\qquad$ $r < 1$

NOTE: USE THE VARIATION WHICH GIVES A POSITIVE DENOMINATOR WHEN WE PUT IN VALUE OF r.

3 SUM TO INFINITY:

ONLY POSSIBLE FOR CONVERGING SERIES:



$$-1 < r < 1$$

$$S_\infty = \frac{a}{1 - r}$$

EXAM TIP

	ARITHEMATIC (AP)	GEOMETRIC (GP)
FIRST TERM	a	a
SECOND TERM	$a+d$	ar
THIRD TERM	$a+2d$	ar^2
4^{th} TERM	$a+3d$	ar^3
5^{th} TERM	$a+4d$	ar^4
10^{th} TERM	$a+9d$	ar^9
15^{th} TERM	$a+14d$	ar^{14}
20^{th} TERM.	$a+19d$	ar^{19}

n = TERM NO — CANNOT BE ZERO
CANNOT BE NEGATIVE
CANNOT BE DECIMAL/FRACTION

IN AP — d = common difference — CANNOT BE ZERO.

IN GP — a = First term — CANNOT BE ZERO.

r = common ratio — CANNOT BE ZERO
CANNOT BE 1
CANNOT BE -1.

TYPE 1 : FORMULA BASED QUESTIONS.

9 The first term of a geometric progression is 81 and the fourth term is 24. Find

(i) the common ratio of the progression, [2]

(ii) the sum to infinity of the progression. [2]

(i) GP

FIRST TERM = 81

$a = 81$

FOURTH TERM = 24

$ar^3 = 24$

$81r^3 = 24$

$r^3 = \frac{24}{81}$

$r^3 = \frac{8}{27}$

$$\boxed{r = \frac{2}{3}}$$

(ii) $S_\infty = \frac{a}{1-r}$

$= \frac{81}{1-\frac{2}{3}} = 243$

1 A geometric progression has first term 64 and sum to infinity 256. Find

(i) the common ratio, [2]

(ii) the sum of the first ten terms. [2]

GP

FIRST TERM = 64

$a = 64$

$S_\infty = 256$

$\frac{a}{1-r} = 256$

$\frac{64}{1-r} = 256$

$64 = 256(1-r)$

$\frac{1}{4} = 1-r$

$\boxed{r = \frac{3}{4}} = 0.75$

(ii) $S_n = \frac{a(r^n-1)}{r-1} = \frac{a(1-r^n)}{1-r}$

✗ ✓

$S_{10} = \frac{64(1-0.75^{10})}{1-0.75}$

$= 241.583$

2 Find

(i) the sum of the first ten terms of the geometric progression 81, 54, 36, ... , [3]

(ii) the sum of all the terms in the arithmetic progression 180, 175, 170, ... , 25. [3]

(i) GP $\quad r = \frac{\text{Next}}{\text{Previous}} = \frac{36}{54} = \frac{2}{3}$

$$S_{10} = \frac{a(1-r^n)}{1-r} = \frac{81\left(1-\left(\frac{2}{3}\right)^{10}\right)}{1-\frac{2}{3}} = 238.786.$$

(ii) AP $\quad 180, 175, 170, \ldots, 25$

$d = N - P = 175 - 180$

$\boxed{d = -5}$

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{?}{2}(180+25)$$

$$= \frac{32}{2}(180+25)$$

$$= 3280.$$

TO FIND n, APPLY nth term FORMULA ON LAST TERM.

nth term $= a + (n-1)d$

$25 = 180 + (n-1)(-5)$

$25 = 180 - 5n + 5$

$5n = 160$

$n = 32$

6 **(a)** Find the sum of all the integers between 100 and 400 that are divisible by 7. [4]

Integers between 100 and 400, Divisible by 7.

$105, 112, 119 \ldots\ldots\ldots\ldots 399.$

+7 +7

AP

$$S_n = \frac{n}{2}(a+l)$$

$$= \frac{43}{2}(105+399)$$

nth term $= a + (n-1)d$

$399 = 105 + (n-1)(7)$

$399 = 105 + 7n - 7$

$n = 43$

$= \underline{10836}$

MS Ans $= \underline{10800}$ (3sf) ?

AP

nth term $= a + (n-1)d$

$S_n = \frac{n}{2}(a+l)$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

GP

nth term $= a \cdot r^{n-1}$

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$