UNIFORM CIRCULAR MOTION

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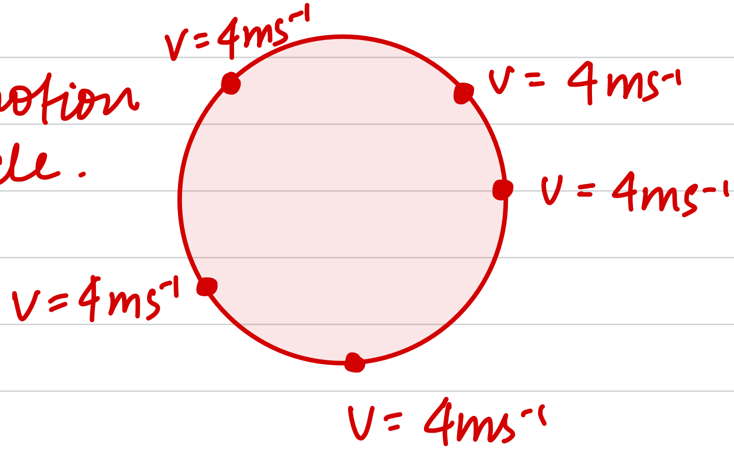
the speed of motion in the circle.



DISPLACEMENT

- Linear displacement
- Angular displacement

VELOCITY

- Linear velocity
- Angular velocity

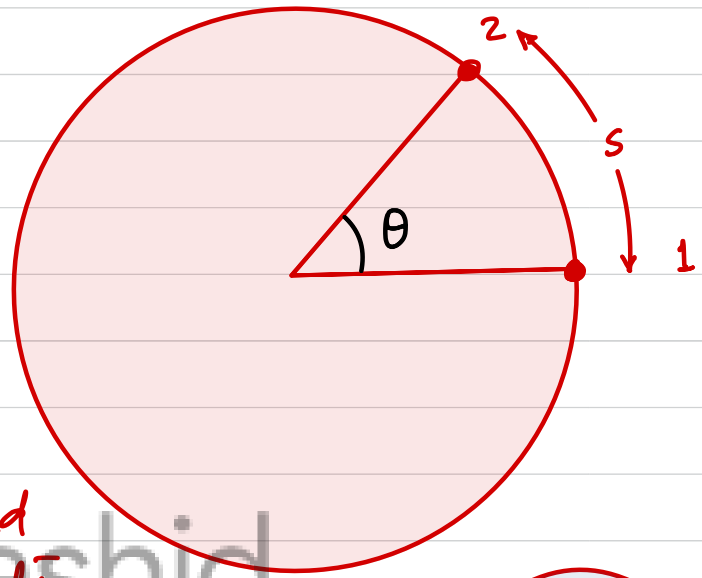
ACCELERATION

- Centripetal Acceleration
- Angular acceleration

DISPLACEMENT

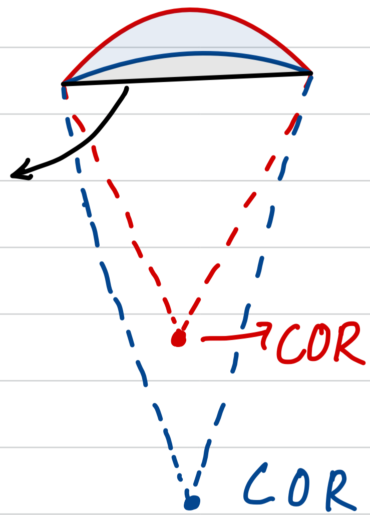
1. Linear Displacement (s)

The length of the arc formed by an object moving in a circle about a point.
↳ centre of rotation.



→ The shortest possible length from point 1 to 2 is the arc length. Any other track b/w point 1 and 2 will either have a different centre of rotation or will not be a circular motion.

Not a circular motion



2. Angular Displacement (θ)

The angle subtended by the arc formed when the object moves in a circle about a point.

The angular displacement is measured in radians (rad).

$$s \propto \theta$$

$$\boxed{s = r\theta}$$

metres

radians

cir. of a sector $= \boxed{\dfrac{\theta}{360^\circ} \times 2\pi r}$

degree to radian converter

360° — 2π rad

θ — x

$$x = \frac{\theta}{360^\circ} \times 2\pi$$

angle in radians.

$C = 2\pi r$

$s = r\theta$

$s = r(2\pi)$

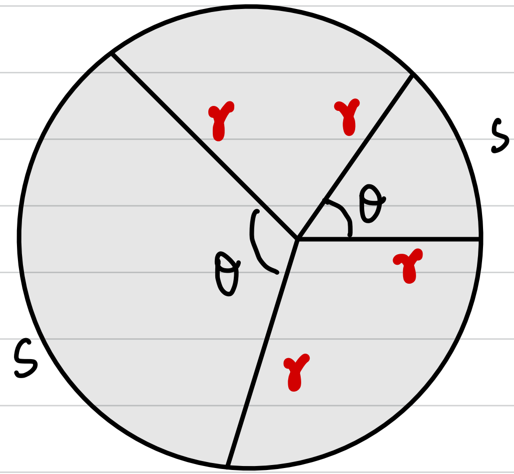
$s = 2\pi r$

Circumference of a circle

$s = r\theta$

$$s = r\left(\frac{\theta}{360^\circ} \times 2\pi\right)$$

$$s = \frac{\theta}{360^\circ} \times 2\pi r$$



$$\theta = \frac{s}{r}$$

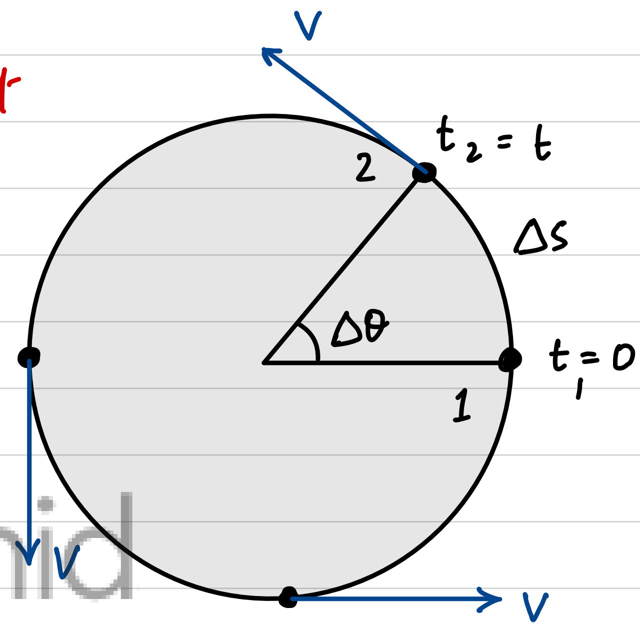
Define the radian (1 rad)

→ The angular displacement about a point

→ when the arc length is equal to the radius of the sector.

VELOCITY

Linear Velocity (v)



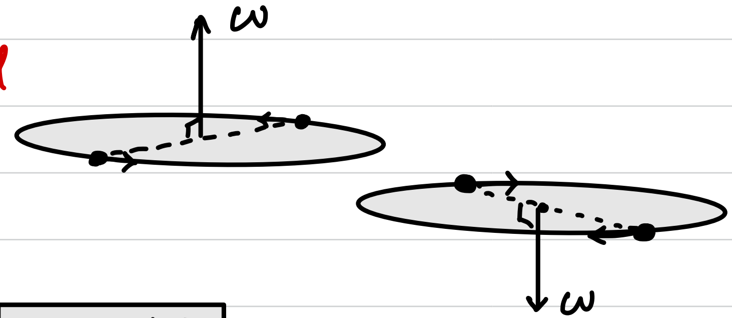
→ The rate of change of linear displacement about a point.

→ Direction of linear velocity is Tangent to the point on the surface of circle.

$$\boxed{v = \frac{\Delta s}{\Delta t}}$$

SI Unit: ms^{-1}

Angular Velocity (ω)



→ The rate of change of angular displacement of an object about a point.

→ The direction of "ω" is perpendicular to the plane of rotation.

→ Right Hand Grip Rule tells direction of "ω"

Curl of Fingers : Direction of rotation
Thumb : Direction of Angular velocity

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t}}$$

SI Unit: $rads^{-1}$

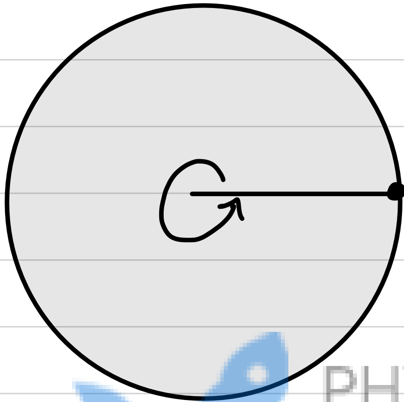
$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{\theta_f - \theta_i}{t_f - t_i}$$

$$\omega = \frac{2\pi - 0}{T - 0}$$

$$\boxed{\omega = \frac{2\pi}{T}}$$



$\theta_i = 0$; $t_i = 0$

Start

End

$\theta_f = 2\pi$; $t_f = T$

as $f = \frac{1}{T}$ so

$$\omega = 2\pi \cdot \frac{1}{T}$$

$$\boxed{\omega = 2\pi f}$$

N : rev. per min

f : rev. per second

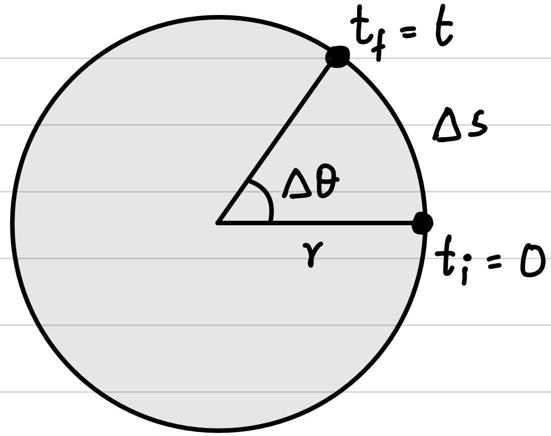
$$\boxed{N = f \times 60}$$

$$\omega = 2\pi f$$

as $f = \frac{N}{60}$ so

$$\boxed{\omega = \frac{2\pi N}{60}}$$

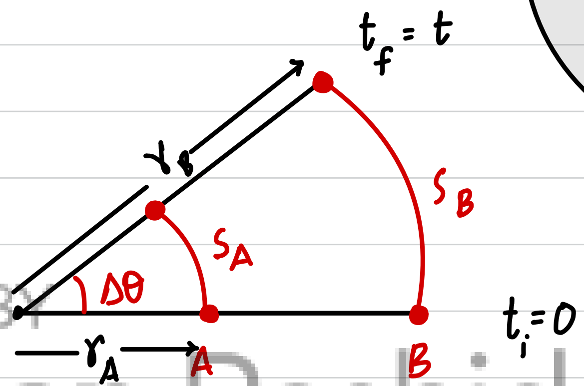
Relating linear velocity and angular velocity



$s = r\theta$ so $\Delta s = r\,\Delta\theta$

dividing both sides by time Δt

$$\frac{\Delta s}{\Delta t} = \frac{r\,\Delta\theta}{\Delta t}$$



$$\boxed{v = r\,\omega}$$

$v \propto r$ if ω: constant

→ As $\Delta\theta$ is same for both A & B, "ω" is same for both A & B.

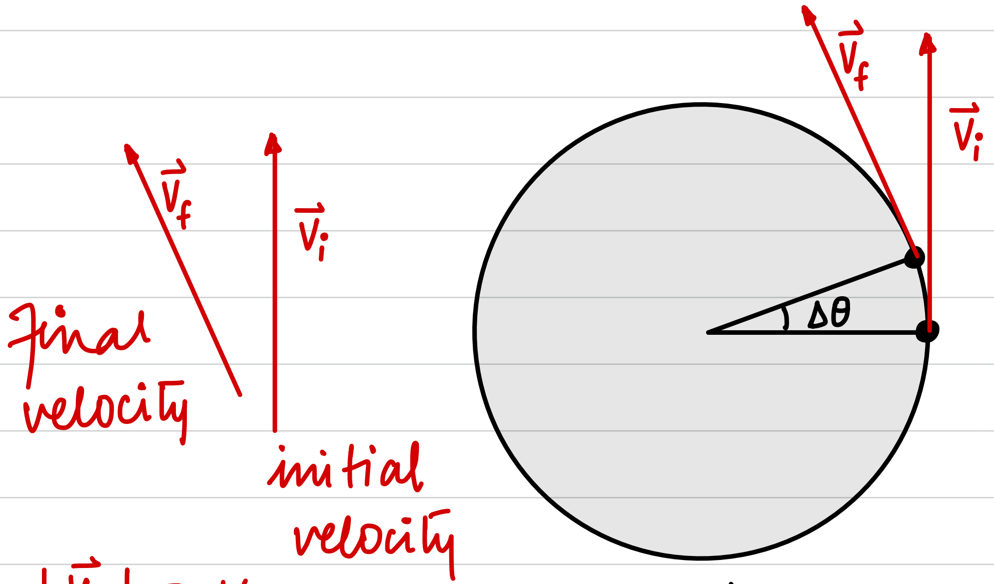
→ As $r_B > r_A$ so $S_B > S_A$. B covers more displacement than A in the same time. So $V_B > V_A$.

Centripetal Acceleration

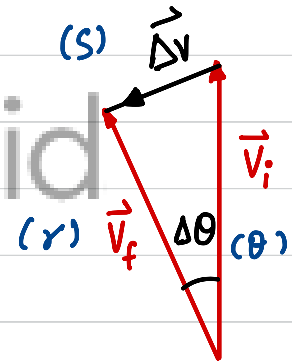
→ The rate of change of linear velocity about a point.

$$a_c = \frac{\Delta v}{\Delta t}$$

SI Unit : ms^{-2}



$|\vec{V}_i| = |\vec{V}_f| = v$



$\vec{V}_i + \vec{\Delta V} = \vec{V}_f$

$\vec{\Delta V} = \vec{V}_f - \vec{V}_i$

For a small change in angle a sector can be approximated to a triangle. Hence we apply $s = r\theta$ to relate $\Delta v, \Delta\theta$ and v with one another

$s = r\theta$

$\Delta v = v\,\Delta\theta$

dividing both sides by Δt, so

$$\frac{\Delta v}{\Delta t} = \frac{v\,\Delta\theta}{\Delta t}$$

$$\boxed{a_c = v\omega}$$

Another way of deriving a_c

θ of sector ≃ θ of triangle

$$\frac{\Delta s}{r} = \frac{\Delta v}{v}$$

$$\Delta v = \frac{v\,\Delta s}{r}$$

dividing by Δt

$$\frac{\Delta v}{\Delta t} = \frac{v\,\Delta s}{r\,\Delta t}$$

$$a_c = \frac{v.v}{r} \Rightarrow \boxed{a_c = \frac{v^2}{r}}$$

$$\boxed{a_c = v\omega}$$

as $v = r\omega$

$a_c = v\omega$

$a_c = (r\omega)\omega$

$$\boxed{a_c = \omega^2 r}$$

as $v = r\omega$ so $\omega = \frac{v}{r}$

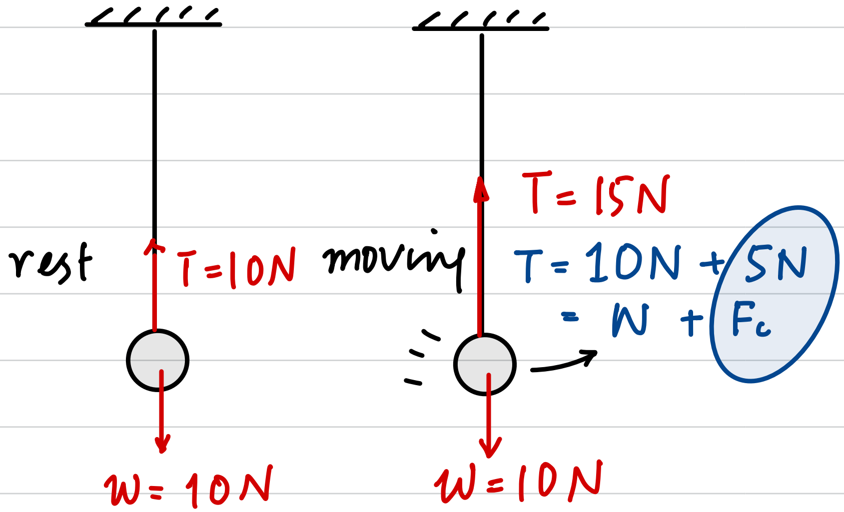
$a_c = v\omega$

$a_c = v \cdot \left(\frac{v}{r}\right)$

$$\boxed{a_c = \frac{v^2}{r}}$$

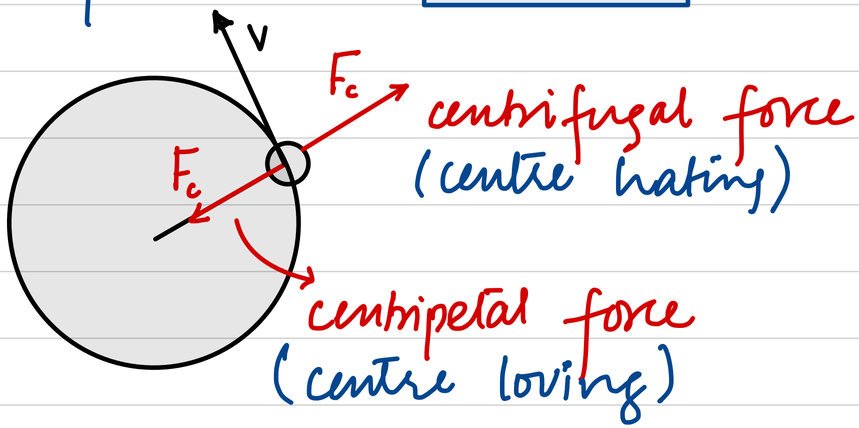
CENTRIPETAL FORCE

- A resultant force that tends to rotate an object about a point.
- It is a title given to any force that will cause an object to move in a circle.
- A centrifugal force also acts on the body that tends to move the object out of the circular orbit. Centripetal force acts to counter the effect of this force. Both forces are Newton's 3rd Law pair of forces so are equal & opposite.



The extra force is the centripetal force.

$$\boxed{T = W + F_c}$$



List of forces acting as Centripetal force

i, Planetary motion / Satellite : Gravitational force
ii, Car making a turn about corner: Friction force
iii, Stone tied to a thread : Tension force
iv, Water in a bucket in vertical circle: Weight + Contact force.

→ The change in velocity is towards the center of circle hence centripetal force must acts towards center. Change in velocity here is not due to change in magnitude but due to change in direction! Δv needs force!

$F_{net} = ma$ so $F_c = ma_c$

Centripetal force

- $a_c = v\omega$

$$\boxed{F_c = mv\omega}$$

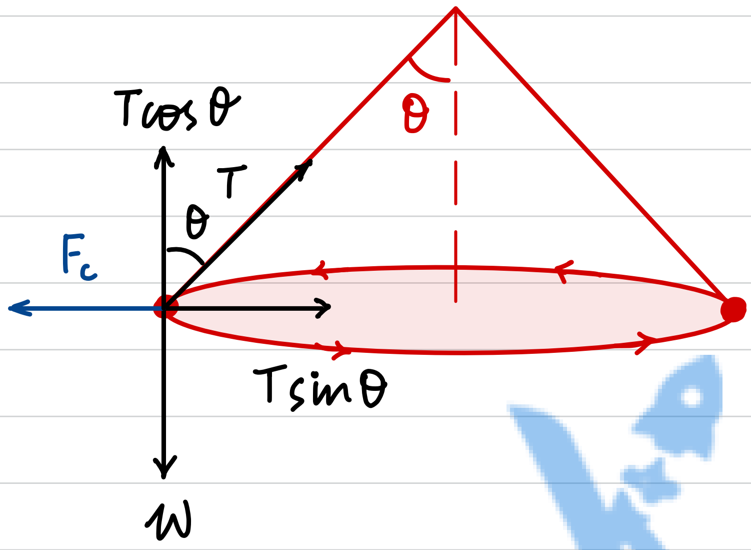
- $a_c = \omega^2 r$

$$\boxed{F_c = m\omega^2 r}$$ 2nd

- $a_c = \frac{v^2}{r}$

$$\boxed{F_c = \frac{mv^2}{r}}$$ 1st

Motion in a horizontal Circle



$$T\cos\theta = W \quad — (1)$$

$$T\sin\theta = F_c \quad — (2)$$

sine component of tension force acts as centripetal force.

Rotation in a perfect horizontal is not possible as there is no force to balance out weight force.

$$T\sin\theta = F_c$$

$$T\sin\theta = mv\omega$$

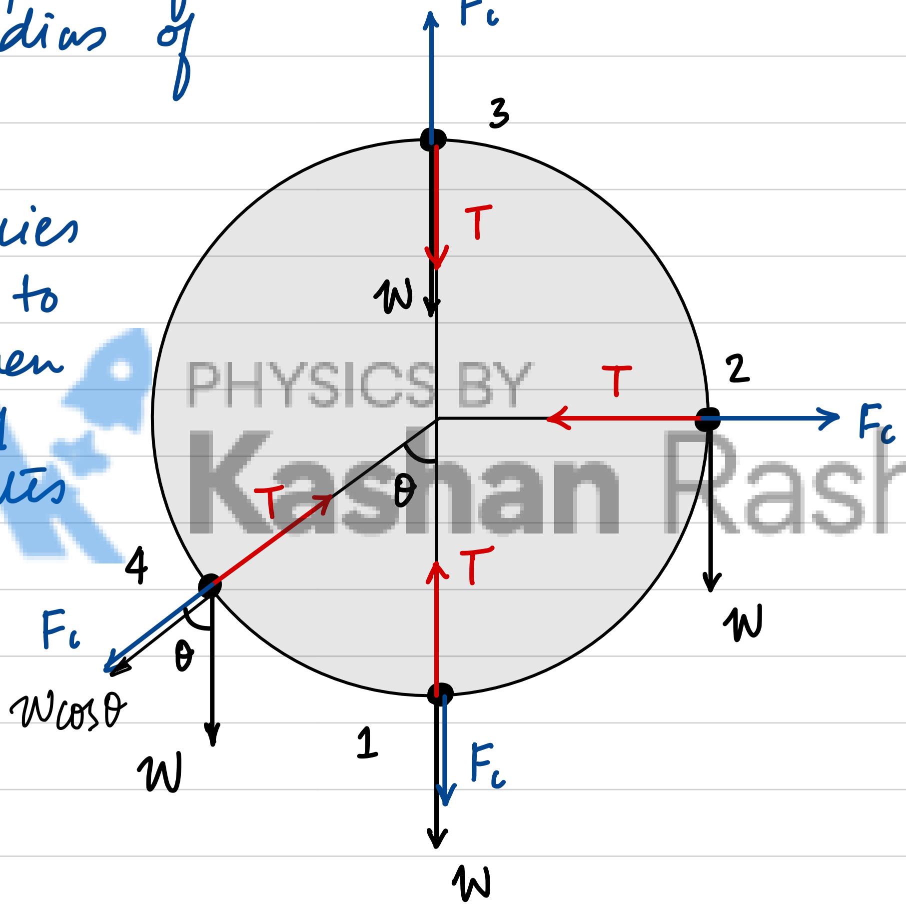
→ if the speed of rotation is increased, $\sin\theta$ and hence θ increases!

The value of Tension along with θ also increase to balance out the weight force

→ Alternatively, with increase in speed, centripetal force increases, so greater sine component needed to balance it. Hence $T\sin\theta$ increases by increasing θ.

Motion in a vertical circle

- F_c is fixed as speed of rotation and radius of circle is fixed.
- Tension force varies from max @ 1 to min @ 3 and then back to max @ 1 as the string rotates

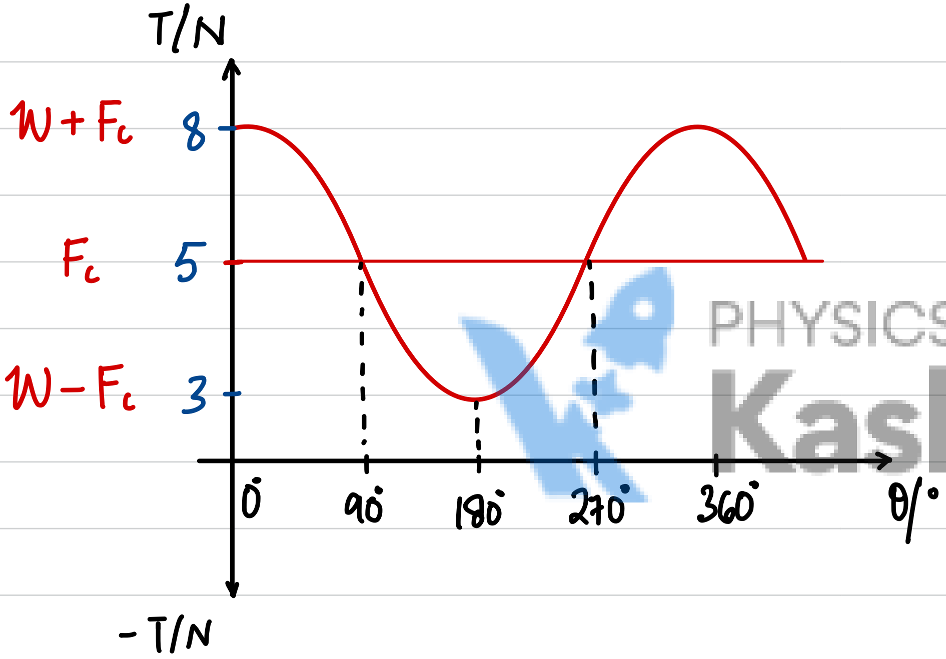


1. $T = W + F_c$
   - max tension in the cord exists at position 1.
   - greatest chance for the cord to break.
2. $T = F_c$
3. $T + W = F_c$
   - if the rotational speed is such that $W = F_c$, there will be no need for tension force in the cord. $T = 0$
   - least tension in cord.
4. $W\cos\theta + F_c = T$

$$T = W\cos\theta + F_c$$

$$y = A\cos Bx + C$$

so $A = W$ $\quad B = 1 \quad C = F_c \quad y = T$ & $x = \theta$

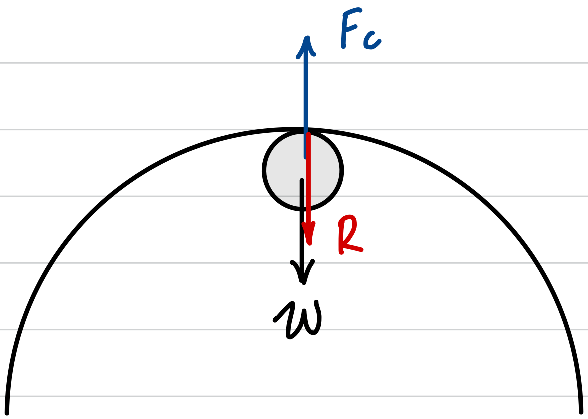


$$\boxed{F_c = 5N}$$

$$W + F_c = 8$$

$$W + 5 = 8$$

$$\boxed{W = 3N}$$



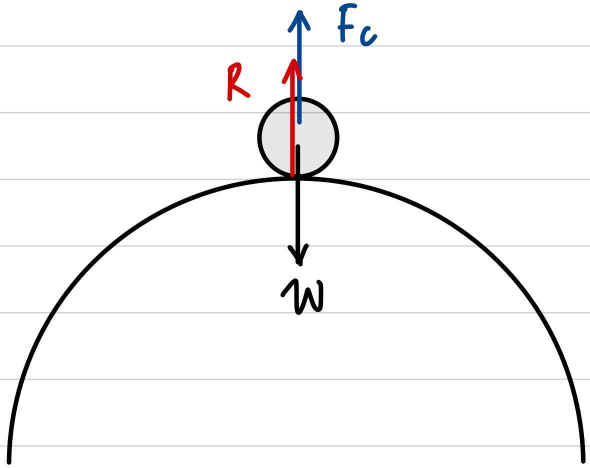
$$F_c = W + R$$

$$\frac{mv^2}{r} = mg + R$$

for minimum speed of rotation, $R = 0$

$$\frac{mv^2}{r} = mg + 0$$

$$\boxed{\frac{v^2}{r} = g}$$



$$F_c + R = W$$

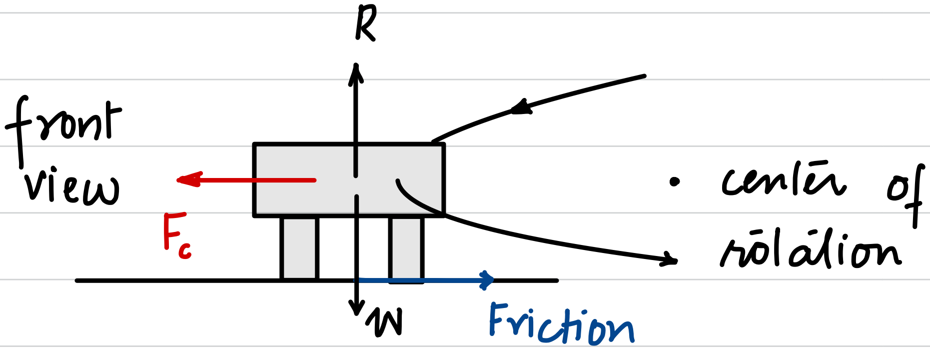
$$\frac{mv^2}{r} + R = mg$$

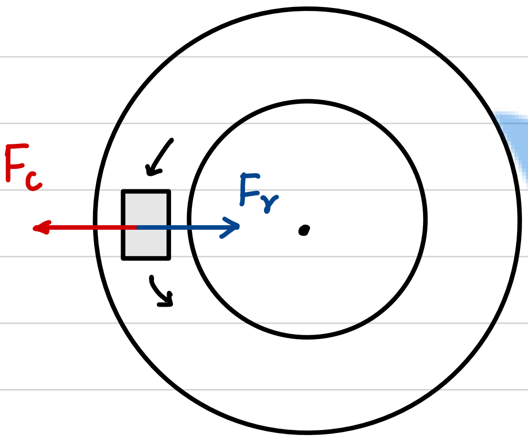
for maximum speed of rotation, $R = 0$

$$\frac{mv^2}{r} + 0 = mg$$

$$\boxed{\frac{v^2}{r} = g}$$

Turning a car along a banked road



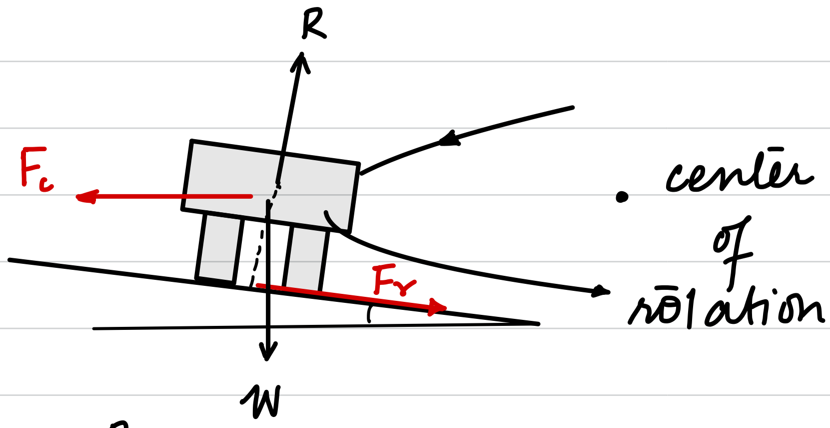


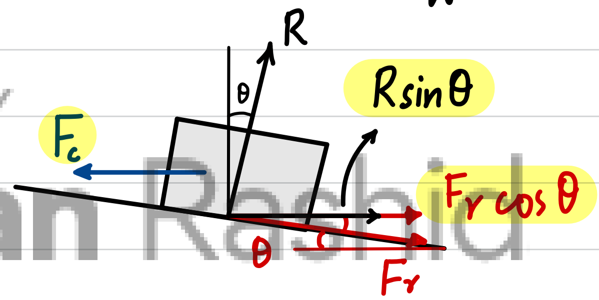
$F_r = \mu R$
where
μ: coefficient of friction
R: contact force

Friction force is acting as a centripetal force.

$$F_c = F_r$$

$$\frac{mv^2}{r} = \mu R$$



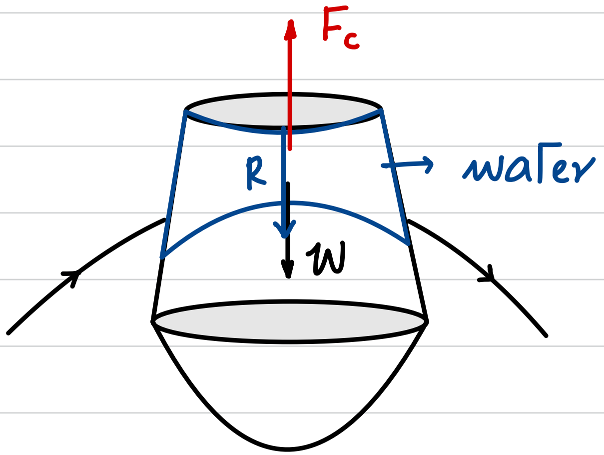


A component of friction force along with the component of normal contact force balance the centrifugal force.

$$F_c = R\sin\theta + F_r\cos\theta$$

F_r: friction
R: normal contact force
θ: banking angle.

Bucket of water in vertical circle



if $F_c > W$, a contact force from base of bucket on water, downwards.

forces acting on water can be related

$$W + R = F_c$$

$$mg + R = \frac{mv^2}{r}$$

For water not to fall, F_c must be large enough to balance the weight of water.

$R = 0$,

$$mg + 0 = \frac{mv^2}{r}$$

minimum speed of rotation so water doesn't fall.

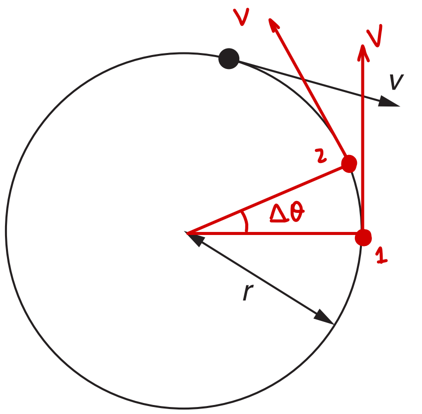
$$g = \frac{v^2}{r} \quad \text{or} \quad v = \sqrt{gr}$$

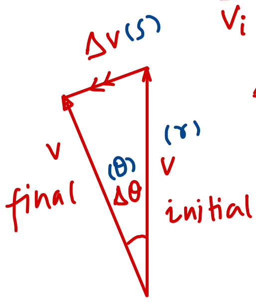
Section A

Answer **all** questions in this section.
You are advised to spend about 1 hour 30 minutes on this section.

For Examiner's Use

1 (a) A body is travelling in a circular orbit of radius r with constant speed v as shown in Fig. 1.1.





$\vec{v}_i + \vec{\Delta v} = \vec{v}_f$

$\vec{\Delta v} = \vec{v}_f - \vec{v}_i$

Fig. 1.1

$S \propto \theta$

Use a vector diagram to show that the acceleration a of the body is given by

$$a = \frac{v^2}{r}$$ ✓

$\underline{\Delta v \propto \Delta\theta}$

towards the centre of the circle.

The change of angle ($\Delta\theta$) is so small that triangle can be approximated to a sector.

$|v_f| = |v_i| = v$

$\Delta v \propto \Delta\theta$

$\Delta v = v\,\Delta\theta$

dividing both sides by Δt

$\frac{\Delta v}{\Delta t} = \frac{v\,\Delta\theta}{\Delta t}$

$a = v\omega$

as $v = r\omega$ so $\omega = \frac{v}{r}$

$a = v\left(\frac{v}{r}\right)$

$\boxed{a = \frac{v^2}{r}}$

[4]

(b) The drum of a spin drier has a rate of rotation of 4.0 revolutions per second. An object in the drum has a mass of 0.20 kg and rotates in a vertical circle of radius 0.16 m.

$f = 4\ Hz$ $m = 0.2\ kg$ $r = 0.16\ m$

(i) Calculate the magnitude of the acceleration of the object.

$a = v\omega$
$a = (r\omega)\omega$
$a = \omega^2 r$

• $a = v\omega$
• $a = \frac{v^2}{r}$
• $a = \omega^2 r$

$\omega = 2\pi f$
$\omega = 2\pi(4)$
$\omega = 8\pi$

$a = \omega^2 r$
$a = (8\pi)^2(0.16)$
$a = 101.06$

acceleration = ...101........... $m\,s^{-2}$ [2]

(ii) Calculate the magnitude of the resultant force on the object.

$F_{net} = ma$
$= 0.20 \times 101$
$= 20.2$

resultant force = ...20........... N [1]

(iii) For each of the three positions shown in Fig. 1.2 draw arrows to represent the weight *W* of the object and the force *D* that the drum exerts on the object. Indicate how these two forces always add to produce the resultant force of constant magnitude calculated in **(ii)**.

For "F" to be horizontal, D must act inclined in the upward direction as shown.

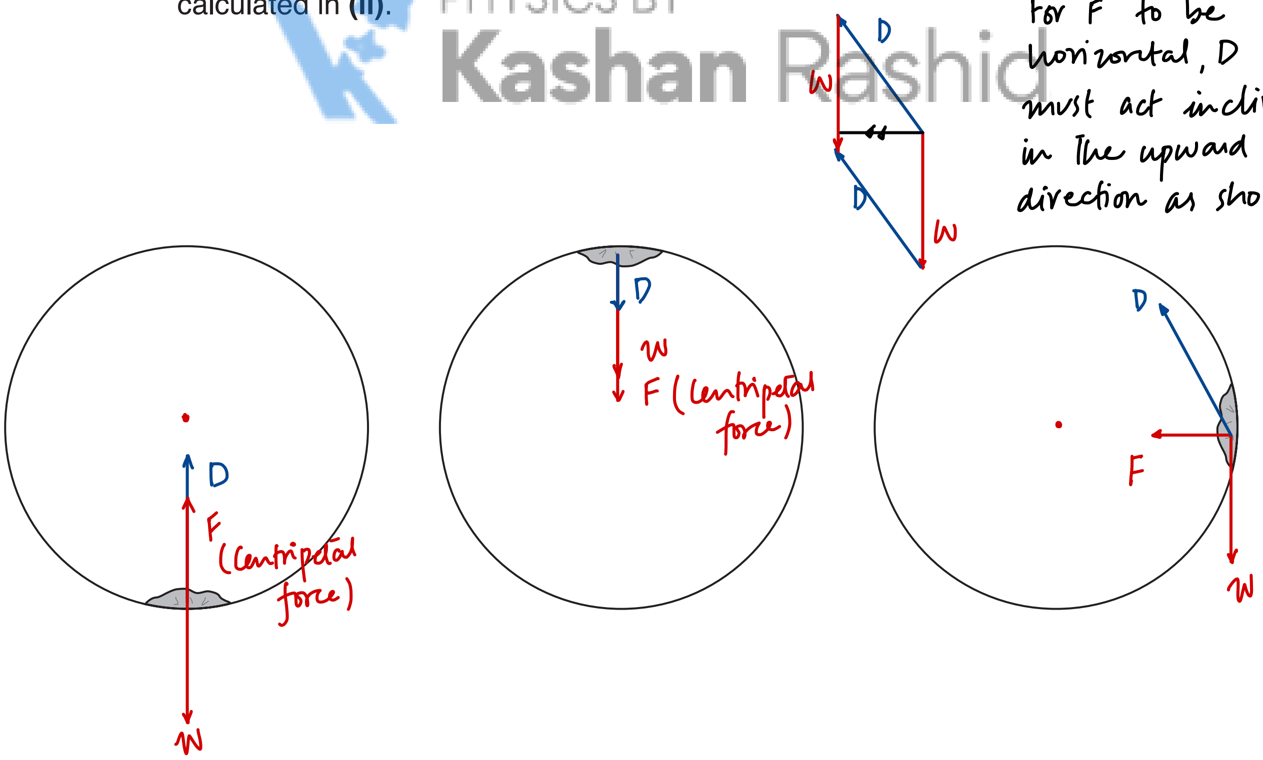


Fig. 1.2 [6]

[Total: 13]

Section A

Answer **all** the questions in the spaces provided.

1 **(a)** **(i)** Define the *radian*.

$s = r\theta$

$\theta = \frac{s}{r}$

The angular displacement of a body when the arc length is equal to the radius of the sector formed. [2]

(ii) A small mass is attached to a string. The mass is rotating about a fixed point P at constant speed, as shown in Fig. 1.1.

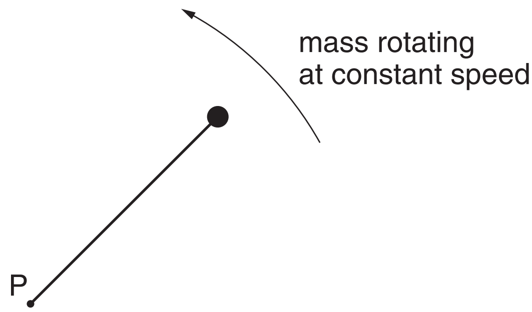


Fig. 1.1

Explain what is meant by the *angular* speed about point P of the mass.

$\omega = \frac{\Delta\theta}{\Delta t}$

The rate of change of angular displacement of a body about a point.

.... [2]

For Examiner's Use

(b) A horizontal flat plate is free to rotate about a vertical axis through its centre, as shown in Fig. 1.2.

if $F_c > $ Friction, mass slips!

$\omega = \frac{2\pi}{T}$ $\omega = 2\pi f$ $\omega = \frac{2\pi N}{60}$

$F_c = F_{max}$ (friction)

$mr\omega^2 = 0.72W$

$\cancel{m}r\omega^2 = 0.72\cancel{m}g$

F_c $(0.35)\omega^2 = 0.72(9.8)$

$\omega = 4.49$ rads^{-1}

$\frac{2\pi N}{60} = 4.49$

$N = 42.9 \simeq 43$ rpm

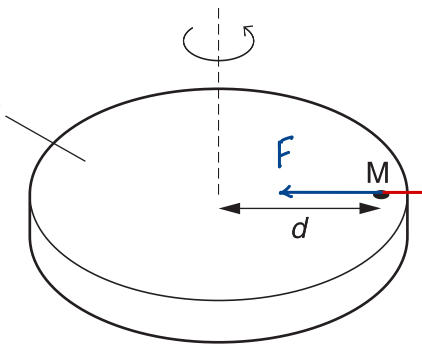


Fig. 1.2

A small mass M is placed on the plate, a distance *d* from the axis of rotation.
The speed of rotation of the plate is gradually increased from zero until the mass is seen to slide off the plate.

The maximum frictional force *F* between the plate and the mass is given by the expression

$$F = 0.72W,$$

where *W* is the weight of the mass M.
The distance *d* is 35 cm.

Determine the maximum number of revolutions of the plate per minute for the mass M to remain on the plate. Explain your working.

number = ... [5]

(c) The plate in **(b)** is covered, when stationary, with mud.
Suggest and explain whether mud near the edge of the plate or near the centre will first leave the plate as the angular speed of the plate is slowly increased.

Mud near the edge of the plate leaves first. As $F_c = mr\omega^2$ so $F_c \propto r$. More centripetal / centrifugal force near edge than center. [2]

2 A large bowl is made from part of a hollow sphere.

A small spherical ball is placed inside the bowl and is given a horizontal speed. The ball follows a horizontal circular path of constant radius, as shown in Fig. 2.1.

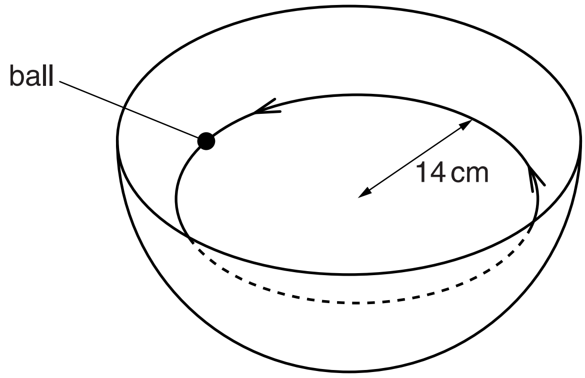
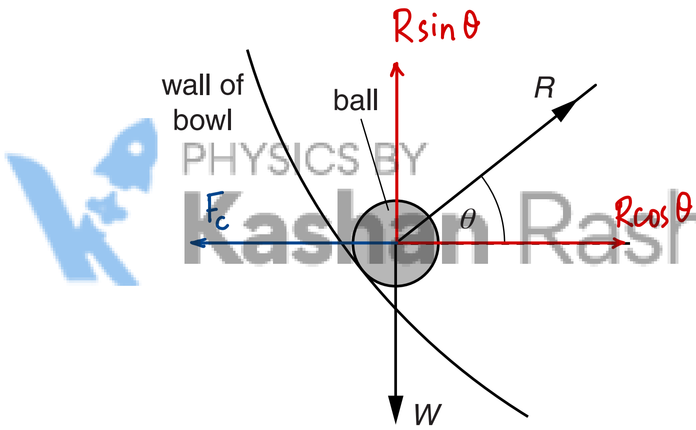


Fig. 2.1

The forces acting on the ball are its weight W and the normal reaction force R of the bowl on the ball, as shown in Fig. 2.2.



$W = R\sin\theta$ — (1)

$F_c = R\cos\theta$ — (2)

dividing both eq. (1) & (2)

$\frac{W}{F_c} = \frac{R\sin\theta}{R\cos\theta}$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\frac{W}{F_c} = \tan\theta$

$\boxed{W = F_c \tan\theta}$

Fig. 2.2

The normal reaction force R is at an angle θ to the horizontal.

(a) (i) By resolving the reaction force R into two perpendicular components, show that the resultant force F acting on the ball is given by the expression

centripetal force/centrifugal

$$W = F \tan\theta.$$

[2]

(ii) State the significance of the force *F* for the motion of the ball in the bowl.

The force F is acting an centripetal force.

.... [1]

(b) The ball moves in a circular path of radius 14 cm. For this radius, the angle θ is 28°.

Calculate the speed (v) of the ball.

$W = F \tan\theta$

$mg = \frac{mv^2}{r} \tan\theta$

$g = \frac{v^2}{r} \tan\theta$

$9.8 = \frac{v^2}{0.14} \tan 28°$

$v = 1.6 \text{ ms}^{-1}$

speed = 1.6 m s^{-1} [3]