INTEGRATION

OUTCOMES :
1) AREA UNDER GRAPH
2) VOLUME OF ROTATION
3) Reverse working.

$$\begin{tikzcd} y \arrow[r, bend left, "\text{diff}"] & \frac{dy}{dx} \arrow[l, bend left, "\text{integration}"] \arrow[r, bend left, "\text{diff}"] & \frac{d^2y}{dx^2} \arrow[l, bend left, "\text{integration}"] \end{tikzcd}$$

SYMBOL

$$\int \bigcirc \, dx$$

BASE RULE :-

$1 \longrightarrow x$

alone constant$(K) \longrightarrow Kx$

$2 \longrightarrow 2x$

$5 \longrightarrow 5x$

POWER RULE :

$$(\square)^n \longrightarrow \frac{(\square)^{n+1}}{n+1}$$

RULES:

1) YOU ARE NOT ALLOWED TO INTEGRATE POWER UNLESS "DIFFERENTIATION OF BOX" $\square'$ IS PRESENT OUTSIDE OPERATOR.

2) ONCE THIS CONDITION IS FULFILLED, THREE THINGS WILL DISAPPEAR,

$$\int \square' \qquad dx$$

AND POWER IS ALLOWED TO BE INTEGRATED.

EXAMPLE:

1) $\int x^8 \, dx$

$$\int 1 \, \boxed{x}^{\underline{8}} \, dx$$

$\square = x$

$\square' = 1$

$$\frac{x^9}{9}$$

2) $$\int (x+5)^7\,dx$$

$$\int 1\,(\boxed{x+5})^7\,dx$$

$\square = x+5$

$\square' = 1$

$$\frac{(x+5)^8}{8}$$

3) $$\int (2x+4)^6\,dx$$

$$\frac{1}{2}\int 2\,(\boxed{2x+4})^6\,dx$$

$\square = 2x+4$

$\square' = 2$

$$\frac{1}{2} \times \frac{(2x+4)^7}{7}$$

$$\frac{(2x+4)^7}{14}$$

3) $$\int (5x+8)^7\,dx$$

$$\frac{1}{5}\int 5\left(\boxed{5x+8}\right)^{\underline{7}}\,dx$$

$\square = 5x+8$

$\square' = 5$

$$\frac{1}{5}\times\frac{(5x+8)^8}{8}$$

$$\frac{(5x+8)^8}{40}$$

<u>RULES:</u>

1) YOU ARE NOT ALLOWED TO INTEGRATE POWER UNLESS "DIFFERENTIATION OF BOX" $\square'$ IS PRESENT OUTSIDE OPERATOR.

2) ONCE THIS CONDITION IS FULFILLED, THREE THINGS WILL DISAPPEAR,

$\int \square' \quad dx$

AND POWER IS ALLOWED TO BE INTEGRATED.

4) $$\int x(x^2+5)^3\,dx$$

$$\frac{1}{2}\int 2x\left(\boxed{x^2+5}\right)^{\underline{3}}\,dx$$

$\square = x^2+5$

$\square' = 2x$

$$\frac{1}{2}\times\frac{(x^2+5)^4}{4}$$

$$\frac{(x^2+5)^4}{8}$$

5) $\int x^2 (9x^3+3)^4 \, dx$

$$\frac{1}{27}\int 27x^2 \left(\boxed{9x^3+3}\right)^4 dx$$

$\square = 9x^3+3$

$\square' = 27x^2$

$$\frac{1}{27} \times \frac{(9x^3+3)^5}{5}$$

$\boxed{6}$ $\int (2x+3)^7 \, dx$

$$\frac{1}{2}\int 2 \left(\boxed{2x+3}\right)^7 dx$$

$\square = 2x+3$

$\square' = 2$

$$\frac{1}{2} \times \frac{(2x+3)^8}{8}$$

THERE ARE ONLY 2 TYPES OF INTEGRATION THAT YOU WILL FACE IN P1.

$(\square)^n$

In this case, you have to be careful about conditions of integration.

There is no whole power. Power is on each x term individually

↓

You can integrate without checking conditions.

eg 1) $\int 4x^7 \, dx$

No whole power.
Power is directly on x-term.
You can integrate directly.

$\int 4\boxed{x}^{7} \, dx$

$4\frac{x^8}{8}$

2) $\int (x^3 + 2x^2 + 5) \, dx$

No whole power
Treat as seperate terms.

$\frac{x^4}{4} + 2\frac{x^3}{3} + 5x$

Q $\int x(3x^2+9)^5 \, dx$

$\frac{1}{6}\int 6x(\boxed{3x^2+9})^5 \, dx$

$\square = 3x^2+9$

$\square' = 6x$

$\frac{1}{6} \times \frac{(3x^2+9)^6}{6}$

Q

$$\int \sqrt{2x+1}\, dx$$

$$\frac{1}{2}\int 2\left(\boxed{2x+1}\right)^{\frac{1}{2}} dx$$

$\Box = 2x+1$

$\Box' = 2$

$$\frac{1}{2} \times \frac{(2x+1)^{1.5}}{1.5}$$

$$\frac{(2x+1)^{1.5}}{3}$$

Q

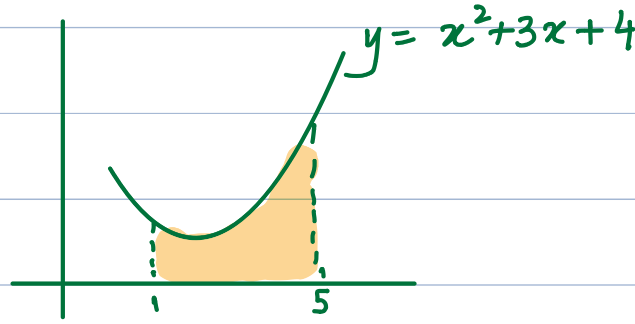
$$\int (2x^1 + 5)\, dx$$

$$\frac{2x^2}{2} + 5x$$

$$x^2 + 5x$$

AREA UNDER GRAPH.

(Area between curve and x-axis)



Modulus

$|3| = 3$

$|-3| = 3$

$$\text{Area} = \int_1^5 (x^2+3x+4)\,dx$$

Modulus

$$\left| \frac{x^3}{3} + \frac{3x^2}{2} + 4x \right|_1^5$$

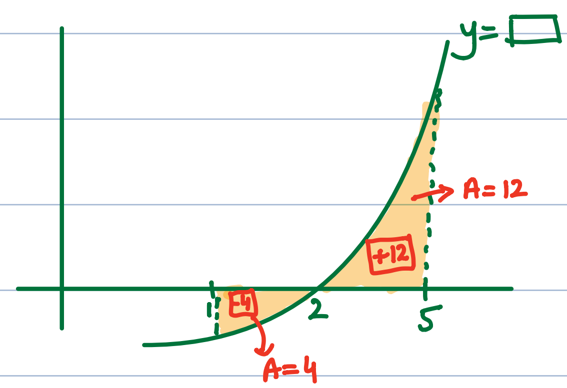
$$\left| \left(\frac{5^3}{3} + \frac{3(5)^2}{2} + 4(5) \right) - \left(\frac{1^3}{3} + \frac{3(1)^2}{2} + 4(1) \right) \right|$$

$$= |99.2 - 5.83|$$

$$\boxed{\text{Area} = 93.37}$$

VARIATION

1



$$\int_1^5 \square\,dx = 8 = +12-4 = 8$$

$$= |-4+12| = |8| = 8$$

For shaded area :-

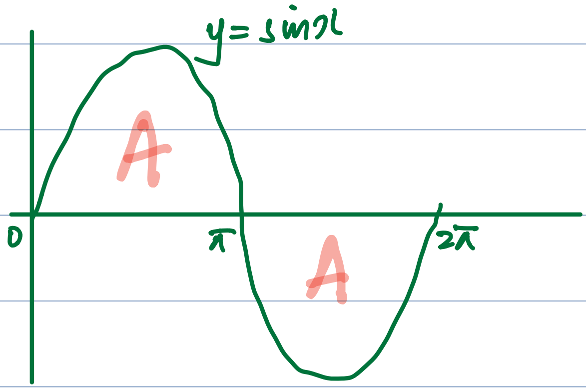
$$\int_1^2 \square\,dx + \int_2^5 \square\,dx$$

$$|-4| + |12|$$

$$= 4 + 12 = \boxed{16}$$

Integration considers areas below x-axis as negative. And it gives us net answer.

Hence if you have some portion of shaded area below x-axis, you will need to split the process.

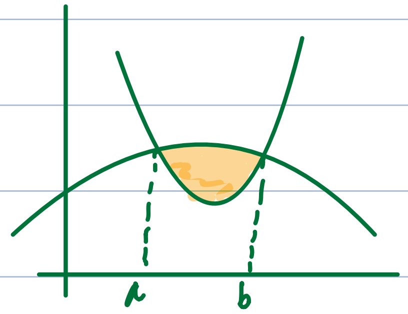


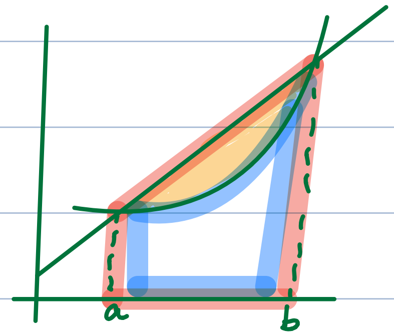
$$\int_0^{2\pi} \sin x \, dx = 0$$

$$\int_0^{\pi} \sin x \, dx = A$$

$$\int_{\pi}^{2\pi} \sin x \, dx = |-A| = A$$

2 Area between two graphs





Area between two graphs $= \int_a^b (\text{upper graph})\,dx - \int_a^b (\text{lower graph})\,dx.$

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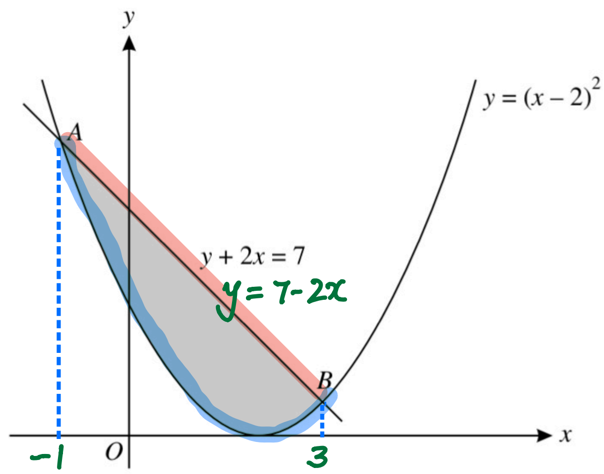


The diagram shows the curve $y = (x-2)^2$ and the line $y + 2x = 7$, which intersect at points A and B. Find the area of the shaded region. [8]

Points of intersection:

$y = (x-2)^2 \qquad y = 7 - 2x$

$(x-2)^2 = 7 - 2x$

$x^2 - 4x + 4 = 7 - 2x$

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$(x+1)(x-3)=0$

$x=-1$, $x=3$

upper graph   Lower graph.

Shaded Area $= \displaystyle\int_{-1}^{3} (7-2x)\,dx \;-\; \int_{-1}^{3} 1\left(\boxed{x-2}\right)^{2} dx$

$\square = x-2$

$\square' = 1$

$= \left| 7x - \dfrac{\cancel{2}x^2}{\cancel{2}} \right|_{-1}^{3} - \left| \dfrac{(x-2)^3}{3} \right|_{-1}^{3}$

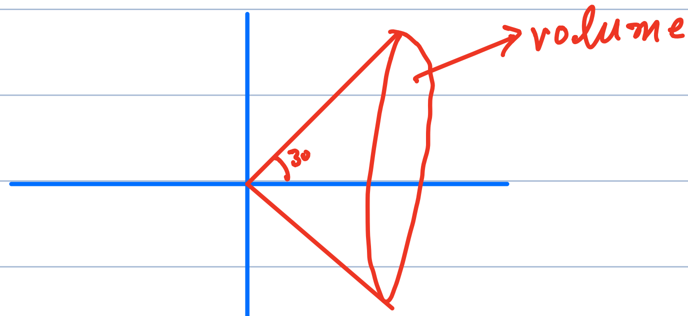
$\left| \left(7(3)-(3)^2\right) - \left(7(-1)-(-1)^2\right) \right| - \left| \dfrac{(3-2)^3}{3} - \dfrac{(-1-2)^3}{3} \right|$

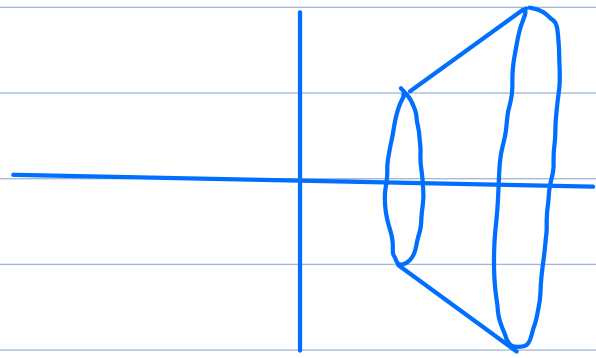
$\left(12 - (-6)\right) - \left(\dfrac{1}{3} + 9\right)$

$= 18 - \dfrac{28}{3}$

Shaded area. $= \boxed{\dfrac{26}{3}}$

VOLUME OF ROTATION 360°





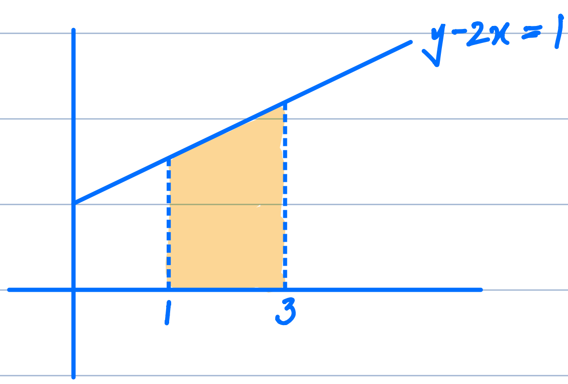
ABOUT x-AXIS:

1) Make y-subject.
2) Square both sides.
3) Integrate RHS
4) Apply limits
5) Multiply by π

ABOUT Y-AXIS:-

1) Make x-subject.
2) Square both sides.
3) Integrate RHS
4) Apply limits
5) Multiply by π

Q:



Find the volume when the shaded region is rotated 360° about x-axis.

Step1: Make y subject

$$y = 2x+1$$

Step2: Square both sides

$$y^2 = (2x+1)^2$$

Step3: Integrate RHS.

$$\frac{1}{2}\int 2\left(\boxed{2x+1}\right)^2 dx$$

$\square = 2x+1$

$\square' = 2$

$$\frac{1}{2} \times \frac{(2x+1)^3}{3}$$

Step4: Apply limits

$$\left| \frac{(2x+1)^3}{6} \right|_1^3$$

$$\left| \frac{(2(3)+1)^3}{6} - \frac{(2(1)+1)^3}{6} \right|$$

$$\frac{316}{6}$$

$$52\frac{2}{3}$$

Step5: Multiply by π

$$\text{Volume} = 52\frac{2}{3} \times \pi = \boxed{165.45}$$

IN THIS WORKING SOMETIMES THE CONDITION FOR INTEGRATION WILL NOT BE GETTING SATISFIED. IN THAT CASE OPEN THE BRACKETS USING $(a \pm b)^2 = a^2 \pm 2ab + b^2$.

eg

$$\int \left(\boxed{2x - x^2}\right)^2 dx$$

$\square = 2x - x^2$

$\square' = 2 - 2x$

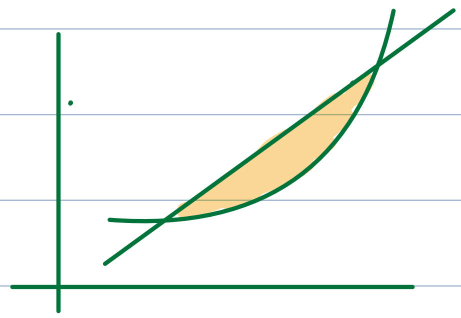
we cannot introduce a variable term.

$$\int \left((2x)^2 - 2(2x)(x^2) + (x^2)^2 \right) dx$$

$$\int \left(4x^2 - 4x^3 + x^4 \right) dx$$ Now integrate individually.

$$\frac{4x^3}{3} - \frac{4x^4}{4} + \frac{x^5}{5}$$

VOLUME OF SHADED REGION BETWEEN TWO GRAPHS :-



Volume of shaded region = Volume of upper graph − Volume of lower graph.

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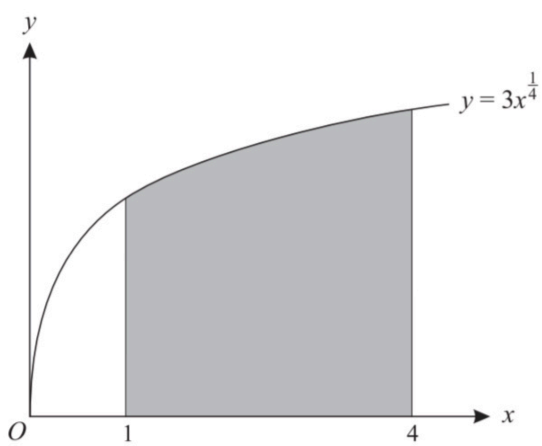


The diagram shows the curve $y = 3x^{\frac{1}{4}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$. Find the volume of the solid obtained when this shaded region is rotated completely about the x-axis, giving your answer in terms of π. [4]

$$y = 3x^{\frac{1}{4}}$$

$$y^2 = \left(3x^{\frac{1}{4}}\right)^2$$

$$y^2 = 9x^{\frac{1}{2}}$$

$$\int 9x^{\frac{1}{2}}\,dx$$

$$\frac{9x^{1.5}}{1.5}$$

$$\left.6x^{1.5}\right|_1^4$$

$$\left|6(4)^{1.5} - 6(1)^{1.5}\right|$$

$$|48 - 6|$$

$$= 42$$

$$\times \pi$$

Volume $= 42\pi$

REVERSE STEP:

$$\begin{tikzcd} y \arrow[r, bend left, "\text{diff}"] & \frac{dy}{dx} \arrow[l, bend left, "\text{integrate}"] \arrow[r, bend left, "\text{diff}"] & \frac{d^2y}{dx^2} \arrow[l, bend left, "\text{integrate}"] \end{tikzcd}$$

Constant of Integration (+c)

$$\begin{tikzcd} y = x^2 - 5x + 8 \arrow[d, "\text{diff}"'] \\ \frac{dy}{dx} = 2x - 5 \arrow[u, bend right, "\text{integrate}"'] \end{tikzcd}$$

$$y = \int (2x - 5)\,dx$$

$$y = \frac{2x^2}{2} - 5x$$

$$y = x^2 - 5x + c$$

NOTE:-

1) IF YOU ARE INTEGRATING "WITH LIMITS" (AREA OR VOLUME), NO NEED TO PUT +C

2) IF YOU ARE INTEGRATING "WITHOUT LIMITS" (REVERSE WORKING), ALWAYS USE +C

$$\left|\left(\qquad \cancel{+c}\right) - \left(\qquad +c\right)\right|$$

Q $\frac{dy}{dx} = 3x+5$. Given that curve passes through $(1,5)$, Find equation of curve.

$$y = \int (3x+5)\,dx$$

$$\boxed{y = \frac{3x^2}{2} + 5x + c}$$

$x = 1$
$y = 5$

Never use $y - y_1 = m(x - x_1)$ for these parts since that is equation of straight line NOT of curve.

$$5 = \frac{3(1)^2}{2} + 5(1) + c$$

$$c = -\frac{3}{2}$$

EQUATION OF CURVE: $\boxed{y = \frac{3x^2}{2} + 5x - \frac{3}{2}}$

7 A curve is such that $\frac{dy}{dx} = \frac{16}{x^3}$, and $(1, 4)$ is a point on the curve.

$x \quad y$

(i) Find the equation of the curve. **[4]**

y

$$y \underset{\text{integ.}}{\overset{\text{diff}}{\rightleftarrows}} \frac{dy}{dx}$$

$$y = \int \frac{16}{x^3}\,dx$$

$$y = \int 16x^{-3}\,dx$$

$$y = \frac{16x^{-2}}{-2} + c$$

$$\boxed{y = -\frac{8}{x^2} + c}$$

$x = 1$
$y = 4$

$$4 = -\frac{8}{(1)^2} + c$$

$$c = 12$$

CURVE

$$\boxed{y = -\frac{8}{x^2} + 12}$$

15 A curve is such that $\frac{dy}{dx} = 4 - x$ and the point $P\,(2, 9)$ lies on the curve. The normal to the curve at P meets the curve again at Q. Find

(i) the equation of the curve, **[3]**

$$y = \int (4 - x)\,dx$$

$$y = 4x - \frac{x^2}{2} + c$$

$$\boxed{y = 4x - \frac{x^2}{2} + c}$$

$x = 2$
$y = 9$

$$9 = 4(2) - \frac{2^2}{2} + c$$

$$9 = 8 - 2 + c$$

$$c = 3$$

$$\boxed{y = 4x - \frac{x^2}{2} + 3}$$

$$y \rightleftarrows \frac{dy}{dx} \rightleftarrows \frac{d^2y}{dx^2}$$

44 A curve is such that $\frac{d^2y}{dx^2} = -4x$. The curve has a maximum point at (2, 12).

$\frac{dy}{dx} = 0$, $x = 2$, $y = 12$

(i) Find the equation of the curve. [6]

y

$$\frac{dy}{dx} = \int (-4x)\,dx$$

$$\frac{dy}{dx} = -4\frac{x^2}{2} + c$$

$$\boxed{\frac{dy}{dx} = -2x^2 + c}$$

$\frac{dy}{dx} = 0$, $x = 2$

$$0 = -2(2)^2 + c$$

$$\boxed{c = 8}$$

$$\frac{dy}{dx} = -2x^2 + 8$$

$$y = \int (-2x^2 + 8)\,dx$$

$$\boxed{y = -\frac{2x^3}{3} + 8x + c}$$

$x = 2$
$y = 12$

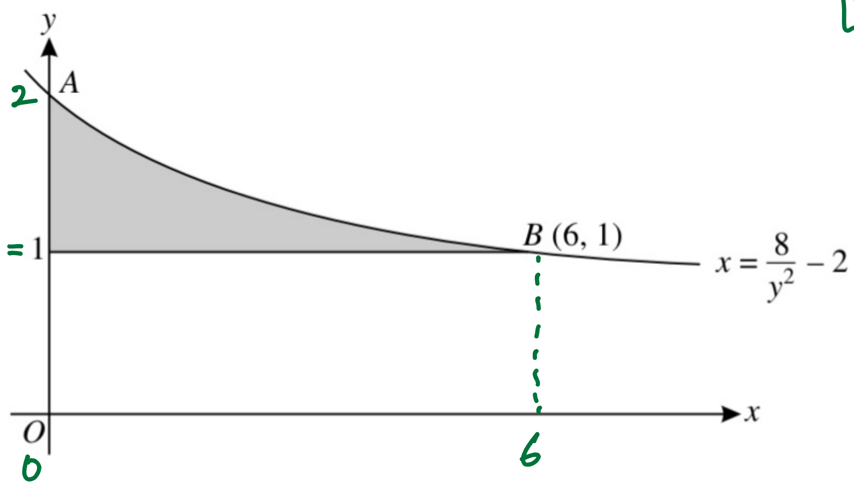
$$12 = -\frac{2(2)^3}{3} + 8(2) + c$$

$$c = \frac{16}{3} - 4 = \frac{4}{3}$$

$$\boxed{y = \frac{-2x^3}{3} + 8x + \frac{4}{3}}$$

upper limit = 2

Lower limit = 1



$\boxed{A}$ $x = 0$

$0 = \frac{8}{y^2} - 2$

$2 = \frac{8}{y^2}$

$2y^2 = 8$

$y^2 = 4$

$y = 2$

The diagram shows part of the curve $x = \frac{8}{y^2} - 2$, crossing the y-axis at the point A. The point $B\,(6, 1)$ lies on the curve. The shaded region is bounded by the curve, the y-axis and the line $y = 1$. Find the exact volume obtained when this shaded region is rotated through 360° about the **y-axis**. [6]

$$x = \frac{8}{y^2} - 2$$

$$x^2 = \left(\frac{8}{y^2} - 2\right)^2$$

$$\int \left(\frac{8}{y^2} - 2\right)^2 dy$$

$$\left(\frac{8}{y^2}\right)^2 - 2\left(\frac{8}{y^2}\right)(2) + (2)^2$$

$$\frac{64}{y^4} - \frac{32}{y^2} + 4$$

$$\int \left(64y^{-4} - 32y^{-2} + 4\right) dy$$

$$\left| \frac{64y^{-3}}{-3} - \frac{32y^{-1}}{-1} + 4y \right|_1^2$$

$$\left|\left(\frac{64(2)^{-3}}{-3}-\frac{32(2)^{-1}}{-1}+4(2)\right)-\left(\frac{64(1)^{-3}}{-3}-\frac{32(1)^{-1}}{-1}+4(1)\right)\right|$$

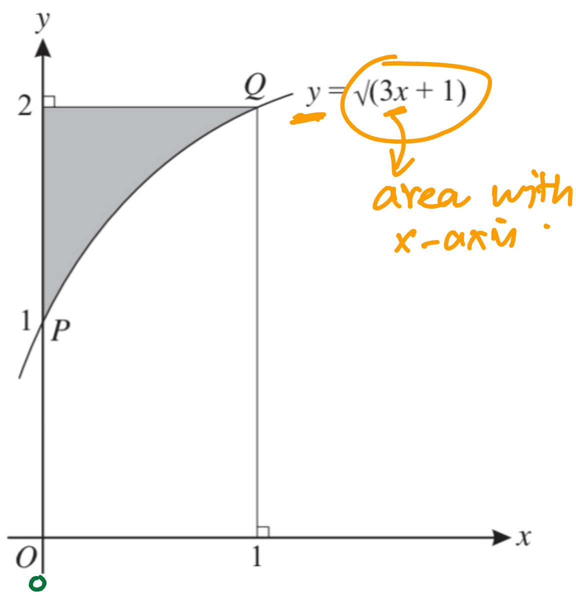
$$\left|\frac{64}{3}-\left(\frac{44}{3}\right)\right|$$

$$\frac{20}{3}$$

$$\times \pi$$

$$\text{Volume} = \frac{20\pi}{3}$$

17



The diagram shows the curve $y = \sqrt{(3x + 1)}$ and the points $P\,(0,\ 1)$ and $Q\,(1,\ 2)$ on the curve. The shaded region is bounded by the curve, the y-axis and the line $y = 2$.

(i) Find the area of the shaded region. [4]

Shaded area = Rectangle − Area under curve.

$$= (2)(1) - \frac{14}{9}$$

$$= 2 - \frac{14}{9} = \boxed{\frac{4}{9}}$$

Area under curve $= \frac{1}{3}\int 3\left(\boxed{3x+1}\right)^{\frac{1}{2}} dx$

$\square = 3x+1$

$\square' = 3$

$$\frac{1}{3} \times \frac{(3x+1)^{1.5}}{1.5}$$

$$\left| \frac{(3x+1)^{1.5}}{4.5} \right|_0^1$$

$$\left| \frac{(3(1)+1)^{1.5}}{4.5} - \frac{(3(0)+1)^{1.5}}{4.5} \right|$$

$$\frac{16}{9} - \frac{2}{9}$$

$$= \frac{14}{9}$$

<u>Method 2</u>: To Find area between curve and y-axis, make x- subject

$$y = \sqrt{3x+1}$$

$$y^2 = 3x+1$$

$$x = \frac{y^2 - 1}{3}$$

area with y-axis.

Integrate and put limits of y-axis now.

$$\int \frac{y^2-1}{3}\,dy$$

$$\frac{1}{3}\int (y^2-1)\,dy$$

$$\frac{1}{3}\left|\frac{y^3}{3}-y\right|_1^2$$

$$\frac{1}{3}\left|\left(\frac{2^3}{3}-2\right)-\left(\frac{1^3}{3}-1\right)\right|$$

$$\frac{1}{3}\left|\frac{2}{3}-\left(-\frac{2}{3}\right)\right|$$

$$\frac{1}{3}\left|\frac{4}{3}\right| = \boxed{\frac{4}{9}}$$

whenever there is a constant common in $\boxed{\text{DIFF}}$ OR $\boxed{\text{INTEG}}$ always take it outside.

Friday, Saturday } Sequences

Sunday } Quadratics Marathon. (3hours).

Regular evening Time is MI

Mon, Tue, Wed } Functions Marathons (Topic + Practice). (morning).