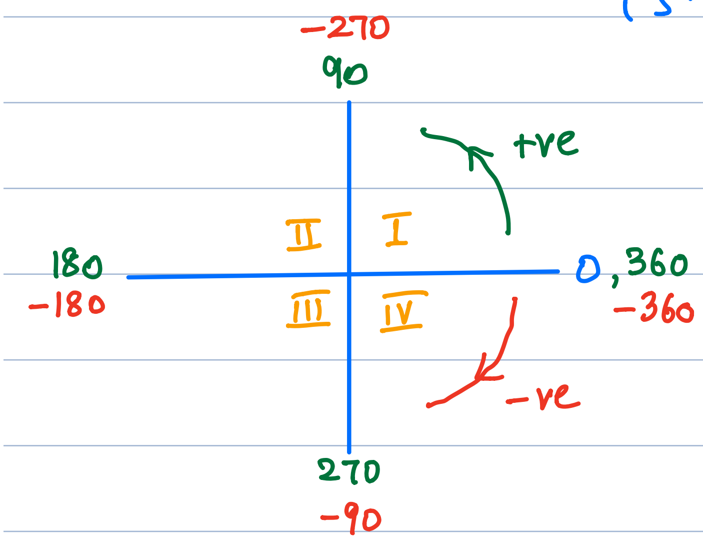
TRIGONOMETRY:

(5-8 Marks).



All Science Teachers Crazy

EVERY ANGLE HAS TWO MAIN VALUES.

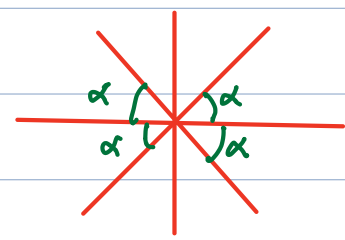
θ = ORIGINAL ANGLE

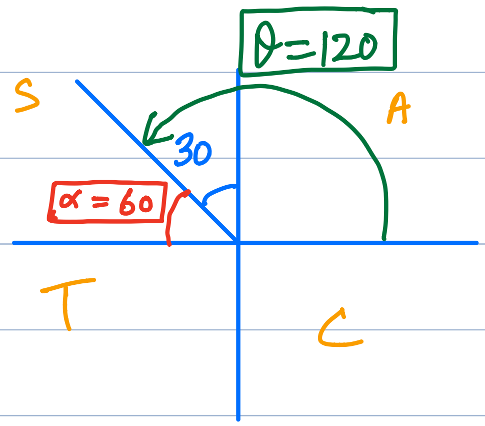
(STARTS FROM +ve x-axis)

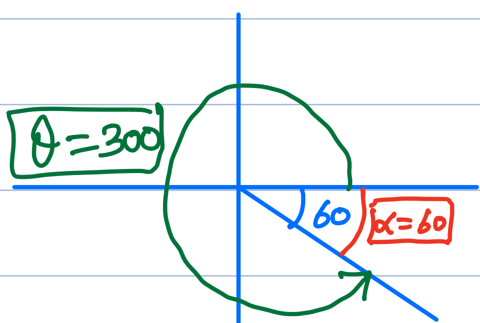
ACW (+ve) CW (-ve)

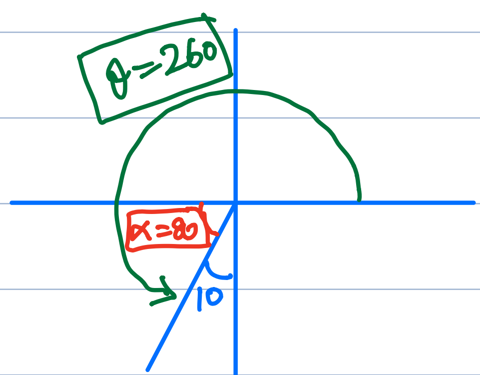
BASIC ANGLE: (α)

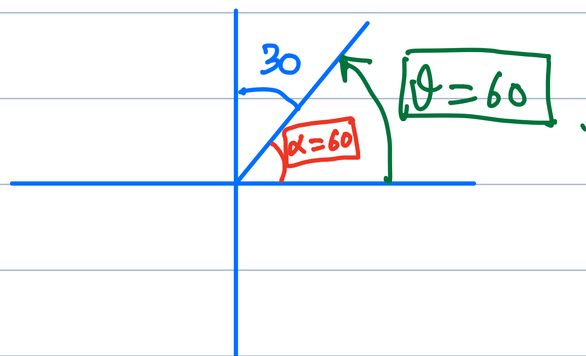
ACUTE ANGLE MADE WITH x-axis.











sin/cos/tan.

$$\left[\text{TRIG RATIO OF ANY ANGLE } (\theta)\right] = \left[\text{TRIG RATIO OF ITS BASIC ANGLE } (\alpha) \text{ AFTER ADJUSTING QUADRANT +/- SIGN.}\right]$$

Q. WITHOUT USING CALCULATOR EVALUATE.

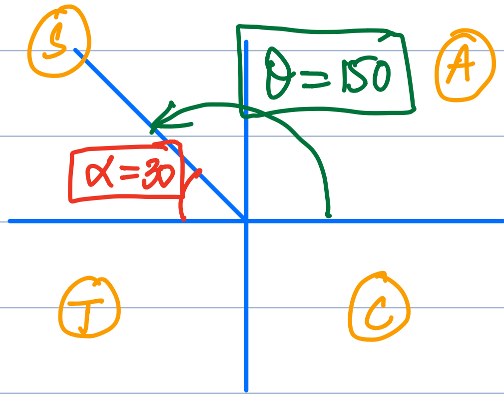
(i) $\cos 150$

$\downarrow$

$\cos 30 = \dfrac{\sqrt{3}}{2}$

$\downarrow$

$$\boxed{\cos 150 = -\dfrac{\sqrt{3}}{2}}$$



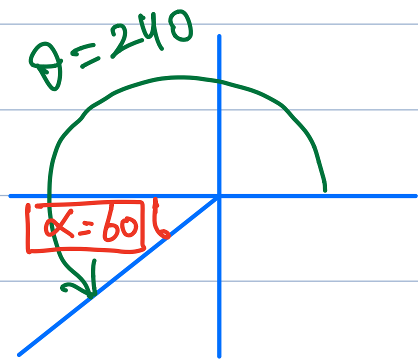
[2] $\tan 240$

$\downarrow$

$\tan 60 = \sqrt{3}$

$\downarrow$

$\tan 240 = +\sqrt{3}$



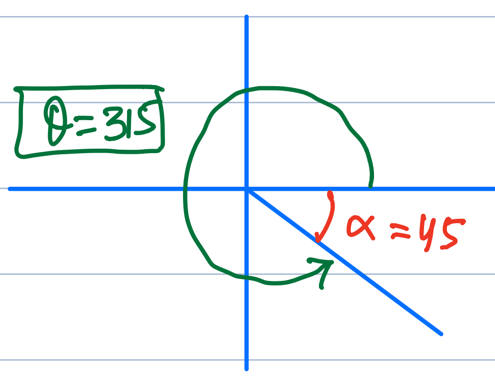
[3] $\sin 315$

$\downarrow$

$\sin 45 = \dfrac{1}{\sqrt{2}}$

$\downarrow$

$$\boxed{\sin 315 = -\dfrac{1}{\sqrt{2}}}$$



EQUATION SOLVING.

① Permission to take inverse

QUADRANT

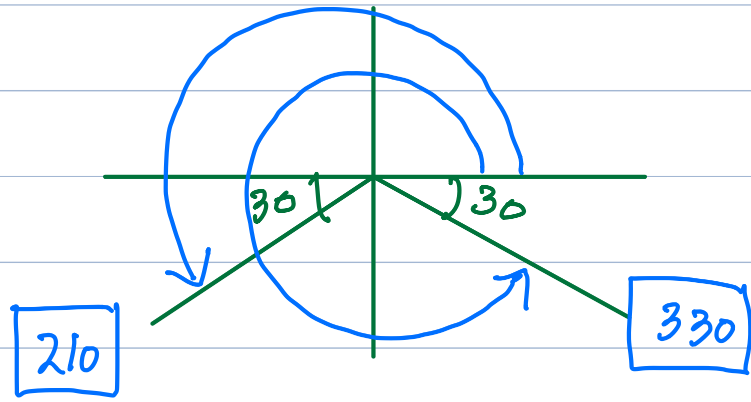
II. $\sin x = -\dfrac{1}{2}$ $\quad 0 < x < 360$

$\alpha = \sin^{-1}\left(\dfrac{1}{2}\right)$

③ Basic Angle.

② ignore +/- sign while taking inverse

$\alpha = 30^\circ$

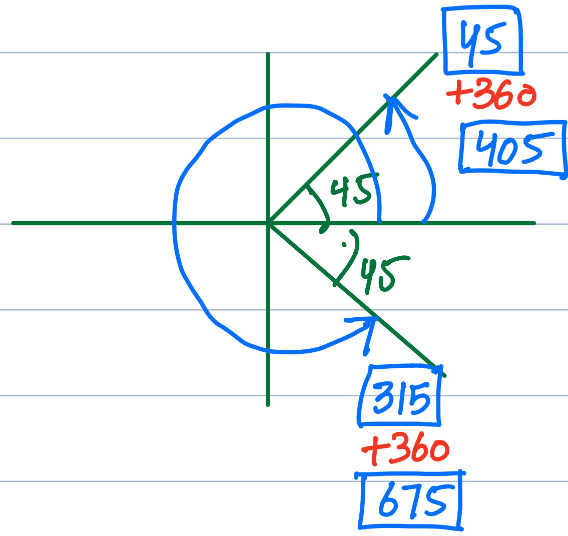


$x = 210, 330$

$\boxed{2}$ $\cos x = \dfrac{1}{\sqrt{2}}$ $0 < x < 720$

$\alpha = \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$

$\alpha = 45^\circ$

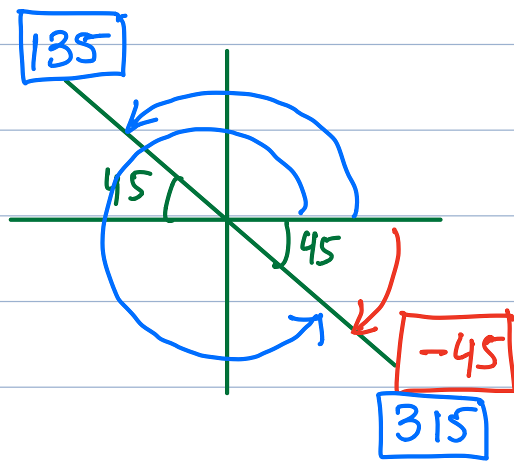


$x = 45, 315, 405, 675$

$\boxed{3}$ $\tan x = -1$ $-180 < x < 360$

$\alpha = \tan^{-1}(1)$

$\alpha = 45^\circ$



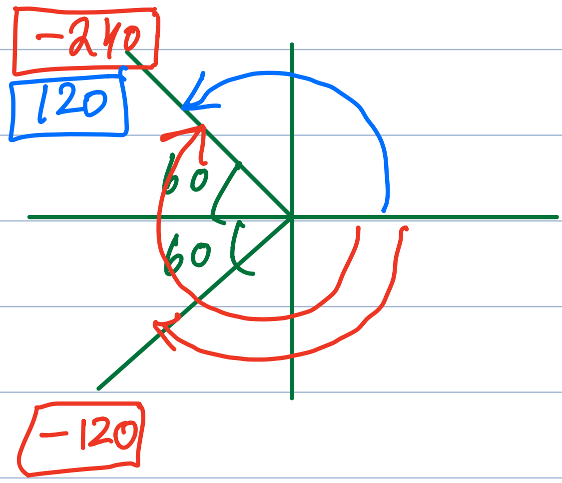
$\boxed{x = -45, 135, 315}$

__NOTE:__ IF YOUR RANGE IS IN NEGATIVE ALWAYS FIND NEGATIVE ANGLES FIRST.

$\boxed{4}$

$\cos x = -\frac{1}{2}$ $\qquad -360 < x < 180$

$\alpha = \cos^{-1}\left(\frac{1}{2}\right) = 60$



$x = -240, -120, 120$

IF WE DO NOT HAVE PERMISSION TO TAKE INVERSE :

RANGE CHANGE.

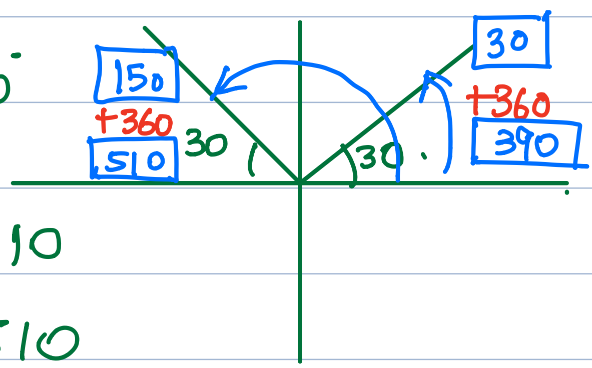
$\sin 2x = \frac{1}{2}$ $\qquad 0 < x < 360$

$\times 2$

$\boxed{2x = A}$ $\qquad 0 < 2x < 720$

$\sin A = \frac{1}{2}$ $\qquad 0 < A < 720$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$



$A = 30, 150, 390, 510$

$2x = 30, 150, 390, 510$

$x = 15, 75, 195, 255$

$\boxed{2}$

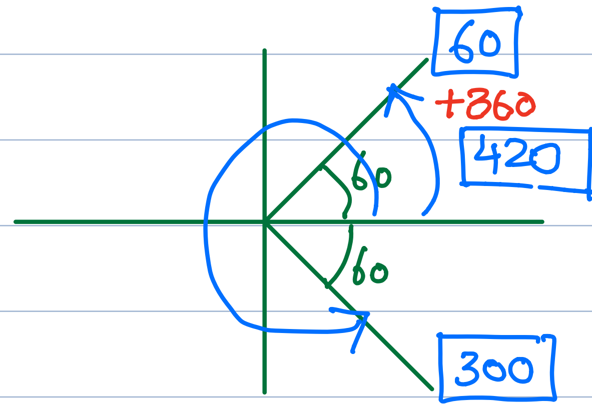
$$\cos(x+70) = \frac{1}{2} \qquad 0 < x < 360$$

$$+70$$

$$A = x + 70 \qquad 70 < x+70 < 430$$

$$\cos A = \frac{1}{2} \qquad 70 < A < 430$$

$$\alpha = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$



out of range

$$A = \cancel{60}, 300, 420$$

$$x + 70 = 300, 420$$

$$x = 230, 350$$

$\boxed{3}$

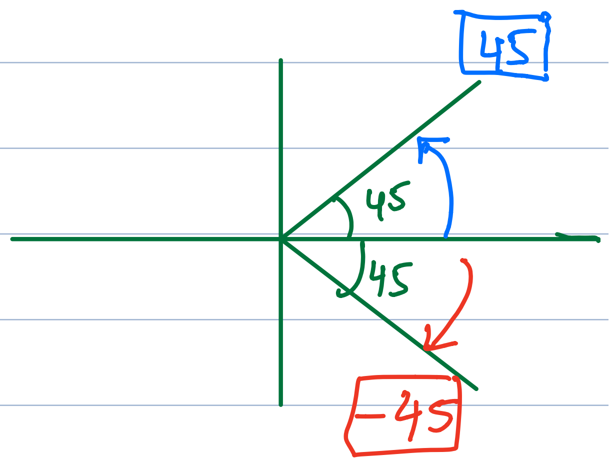
$$\cos(x-80) = \frac{1}{\sqrt{2}} \qquad 0 < x < 360$$

$$-80$$

$$A = x - 80 \qquad -80 < x-80 < 280$$

$$\cos A = \frac{1}{\sqrt{2}} \qquad -80 < A < 280$$

$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45$$



$$A = -45, 45$$

$$x - 80 = -45, 45$$

$$x = 35, 125.$$

$\boxed{4}$ $\sin(2x+40) = \frac{1}{2}$

$$0 < x < 180$$

$\times 2$

$$0 < 2x < 360$$

$+40$

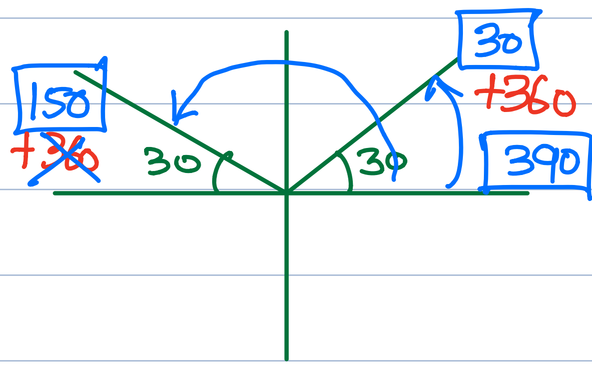
$$40 < 2x+40 < 400$$

$A = 2x+40$

$\sin A = \frac{1}{2}$

$$40 < A < 400$$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right) = 30$



$A = \cancel{30}, 150, 390$

$2x + 40 = 150, 390$

$2x = 110, 350$

$\boxed{x = 55, 175}$

IDENTITIES : (17)

RECIPROCAL

$$\frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

$$\frac{1}{\cos\theta} = \sec\theta$$

$$\frac{1}{\tan\theta} = \cot\theta$$

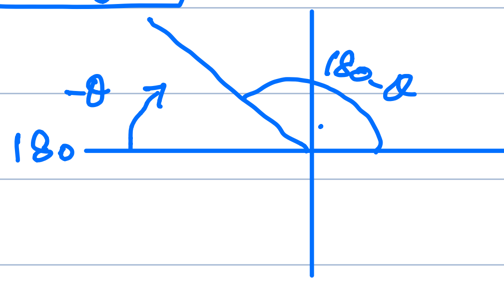
$$\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$$

$$\frac{\cos\theta}{\sin\theta} \equiv \cot\theta$$

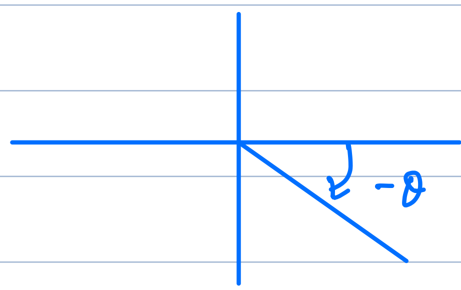
$$\sin^2\theta + \cos^2\theta \equiv 1$$
$$1 + \tan^2\theta \equiv \sec^2\theta$$
$$1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$$

$\boxed{\theta = \text{Acute angle}}$

$$\sin(180-\theta) \equiv \sin\theta$$
$$\cos(180-\theta) \equiv -\cos\theta$$
$$\tan(180-\theta) \equiv -\tan\theta$$

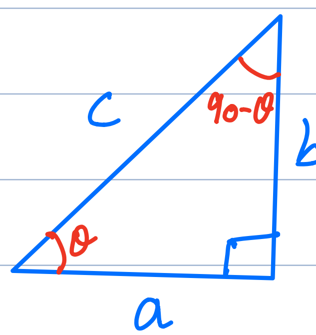


$$\sin(-\theta) \equiv -\sin\theta$$
$$\cos(-\theta) \equiv \cos\theta$$
$$\tan(-\theta) \equiv -\tan\theta$$



M1, P1

$$\sin(90-\theta) \equiv \cos\theta$$
$$\cos(90-\theta) \equiv \sin\theta$$
$$\tan(90-\theta) \equiv \frac{1}{\tan\theta}$$



SYMBOLS:

$\sin^2\theta = (\sin\theta)^2$

$\sin x^2 = \sin(x^2)$

$\sin^2 30 \rightarrow (\sin 30)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\sin 30^2 \rightarrow \sin(30^2) = \sin(900) = \boxed{}$

Ex

$2\cos(180-\theta) = 1$

$2(-\cos\theta) = 1$

$-2\cos\theta = 1$

$\cos\theta = \frac{-1}{2}$

Q

$2\cos(-\theta) = 1$

$\cos(-\theta) = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$

PROVING IDENTITIES

1) YOU ARE ALLOWED TO WORK WITH ONLY ONESIDE AND PROVE IT EQUAL TO OTHER SIDE.

2) YOU ARE NOT ALLOWED TO SIMPLIFY BOTH SIDES AT SAMETIME.

$\boxed{\text{PROOF}}$

$\boxed{\text{SQUARE}}$

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$a^2 - b^2 = (a+b)(a-b)$

$\boxed{\text{NO SQUARES}}$

BRING EVERYTHING TO $\boxed{\text{SIN}}$ & $\boxed{\text{COS}}$ only and SIMPLIFY.

32 (i) Prove the identity $\tan x + \dfrac{1}{\tan x} \equiv \dfrac{1}{\sin x \cos x}$. [2]

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} \quad \text{(shown) (Q.E.D).}$$

13 Prove the identity

$$\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}\equiv\frac{2}{\cos x}.$$ [4]

$$\frac{(1+\sin x)(1+\sin x)+(\cos x)(\cos x)}{\cos x\,(1+\sin x)}$$

$$\frac{1+2\sin x+\sin^2 x+\cos^2 x}{\cos x\,(1+\sin x)}$$

$$\frac{1+2\sin x+1}{\cos x\,(1+\sin x)}$$

$$\frac{2+2\sin x}{\cos x(1+\sin x)}$$

$$\frac{2\cancel{(1+\sin x)}}{\cos x\cancel{(1+\sin x)}}=\frac{2}{\cos x}$$ (Proven)

38 **(i)** Show that $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}\equiv\dfrac{1}{\sin^2\theta-\cos^2\theta}$. [3]

$$\frac{\sin\theta(\sin\theta-\cos\theta)+\cos\theta(\sin\theta+\cos\theta)}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}$$

$$\frac{\sin^2\theta-\cancel{\sin\theta\cos\theta}+\cancel{\sin\theta\cos\theta}+\cos^2\theta}{(\sin\theta)^2-(\cos\theta)^2}$$

$$\frac{1}{\sin^2\theta-\cos^2\theta}$$

27 **(i)** Prove the identity $\left(\dfrac{1}{\sin\theta}-\dfrac{1}{\tan\theta}\right)^2\equiv\dfrac{1-\cos\theta}{1+\cos\theta}$. [3]

$$\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2$$

$$\left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$\frac{(1-\cos\theta)^2}{\sin^2\theta} \longleftarrow$$

$\sin^2\theta+\cos^2\theta=1$

$\sin^2\theta=1-\cos^2\theta$.

$$\frac{(1-\cos\theta)^2}{1^2-\cos^2\theta}$$

$$\frac{(1-\cos\theta)^{\cancel{2}}}{\cancel{(1-\cos\theta)}(1+\cos\theta)}=\frac{1-\cos\theta}{1+\cos\theta}.$$

21 (i) Prove the identity $\dfrac{\sin x \tan x}{1 - \cos x} \equiv 1 + \dfrac{1}{\cos x}$.

$$\frac{\sin x \cdot \dfrac{\sin x}{\cos x}}{1 - \cos x}$$

$$\sin x \cdot \frac{\sin x}{\cos x} \div (1 - \cos x)$$

$$\frac{\sin^2 x}{\cos x} \times \frac{1}{(1 - \cos x)}$$

$$\frac{\sin^2 x}{\cos x (1 - \cos x)}$$

$$\frac{1^2 - \cos^2 x}{\cos x (1 - \cos x)}$$

$$\frac{(1 + \cos x)\cancel{(1 - \cos x)}}{\cos x \cancel{(1 - \cos x)}}$$

$$\frac{1 + \cos x}{\cos x}$$

$$\frac{1}{\cos x} + \frac{\cos x}{\cos x}$$

$$\boxed{\frac{1}{\cos x} + 1}$$ Shown.

CORRECT

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

WRONG.

$$\frac{c}{a+b} = \frac{c}{a} + \frac{c}{b}$$