NON-CONSTANT-ACCELERATION

STRAIGHT LINE MOTION

$$\text{displacement} \underset{\text{integrate.}}{\overset{\text{DIFF}}{\rightleftarrows}} \text{velocity} \underset{\text{integrate}}{\overset{\text{DIFF}}{\rightleftarrows}} \text{accelelation.}$$

always without limits use +c

Two cases:

1- Value of Displacement is given or to be found:
 Integrate with limits.

2- Expression of displacement:
 Integrate without limits. (use +c)

Example:

$$s = 3t^3 - 7t^2 + 8t - 5$$

$\downarrow$ DIFF

$$v = 9t^2 - 14t + 8$$

$\downarrow$ DIFF $\qquad\qquad \uparrow$ integrate

$$a = 18t - 14$$

MEMORIZE THESE:

	CONCEPT	RULE
1 TURNING POINT INSTANTANEOUS REST		$V=0$
2 MAXIMUM DISPLACEMENT	$\frac{ds}{dt} \to v=0$	$V=0$
3 MAXIMUM VELOCITY	$\frac{dv}{dt} \to a=0$	$a=0$
4 MAXIMUM ACCELERATION	$\frac{da}{dt}=0$	$\frac{da}{dt}=0$

2 A particle starts from rest at the point A and travels in a straight line until it reaches the point B. The velocity of the particle t seconds after leaving A is $v\,\mathrm{m\,s^{-1}}$, where $v = 0.009t^2 - 0.0001t^3$. Given that the velocity of the particle when it reaches B is zero, find

(i) the time taken for the particle to travel from A to B, [2]

(ii) the distance AB, (value) (with limits) [4]

(iii) the maximum velocity of the particle. [4]

$a=0$

$V=0$ $\quad$ $V=0$

$t=0$ $\quad$ $t=90$

A ———————— B

$v = 0.009t^2 - 0.0001t^3$

(i) At B, $v=0$

$0 = 0.009t^2 - 0.0001t^3$

$0 = t^2(0.009 - 0.0001t)$

$t^2=0$	$0.009 - 0.0001t = 0$
$t=0$	$t = \frac{0.009}{0.0001} = 90$
A	

(ii) $AB = \displaystyle\int_0^{90} (0.009t^2 - 0.0001t^3)\, dt$

$$\left| \frac{0.009t^3}{3} - \frac{0.0001t^4}{4} \right|_0^{90}$$

$$\left| 0.003t^3 - 0.000025t^4 \right|_0^{90}$$

$$\left| \left(0.003(90)^3 - 0.000025(90)^4\right) - \left(0.003(0)^3 - 0.000025(0)^4\right) \right|$$

$$= 546.75 - 0$$

$$AB = 546.75$$

(iii) Max velocity $\longrightarrow a = 0$

$v = 0.009t^2 - 0.0001t^3$

Diff

$a = 0.018t - 0.0003t^2$

$0 = 0.018t - 0.0003t^2$

$0 = t(0.018 - 0.0003t)$

$t = 0$ ignore, $\quad 0.018 - 0.0003t = 0$

$\boxed{t = 60}$

$t = 60$,

$v = 0.009(60)^2 - 0.0001(60)^3$

$v = 10.8$.

40 A particle travels in a straight line from A to B in 20 s. Its acceleration t seconds after leaving A is a m s^{-2}, where $a = \frac{3}{160}t^2 - \frac{1}{800}t^3$. It is given that the particle comes to rest at B.

(i) Show that the $\boxed{\text{initial}}$ speed of the particle is zero. [4]

$t=0$

(ii) Find the maximum speed of the particle. $(a=0)$ [2]

(iii) Find the distance AB. (value) (with limits) [4]

$v=0$ $\quad$ $v=0$

$t=0$ $\quad$ $t=20$

A ———————————— B

$$a = \frac{3}{160}t^2 - \frac{1}{800}t^3$$

(i)

$$v = \int\left(\frac{3}{160}t^2 - \frac{1}{800}t^3\right)dt$$

$$v = \frac{3}{160}\frac{t^3}{3} - \frac{1}{800}\frac{t^4}{4} + c$$

$$\boxed{v = \frac{1}{160}t^3 - \frac{1}{3200}t^4 + c}$$

$t=20$
$v=0$

$$0 = \frac{1}{160}(20)^3 - \frac{1}{3200}(20)^4 + c$$

$$\boxed{c=0}$$

$$\boxed{v = \frac{1}{160}t^3 - \frac{1}{3200}t^4}$$

For initial velocity,

$t=0$

$$v = \frac{1}{160}(0)^3 - \frac{1}{3200}(0)^4$$

$$\boxed{v=0}$$

(ii) Max speed: $a = 0$

$$a = \frac{3}{160}t^2 - \frac{1}{800}t^3$$

$$0 = \frac{3}{160}t^2 - \frac{1}{800}t^3$$

$$\frac{3}{160}t^2 = \frac{1}{800}t^3$$

$$15t^2 = t^3$$

$$\boxed{t = 15}$$

$$v = \frac{1}{160}t^3 - \frac{1}{3200}t^4$$

$$v = \frac{1}{160}(15)^3 - \frac{1}{3200}(15)^4$$

(Max speed) $v = 5.27$.

(iii) $$AB = \int_0^{20} \left(\frac{1}{160}t^3 - \frac{1}{3200}t^4\right) dt$$

$$= \frac{1}{160}\frac{t^4}{4} - \frac{1}{3200}\frac{t^5}{5}$$

$$= \left|\frac{t^4}{640} - \frac{t^5}{16000}\right|_0^{20}$$

$= 50$ m.

KINEMATICS TYPE 2 ADVANCED.

SPLIT JOURNEY

$t=0$ —— $t=15$ —— $t>15$

$v = 3t^2 + 5t$ $\qquad$ $v = \dfrac{B}{t^2}$

BOUNDARY TIME

At this instant

INSTANTENEOUS DISPLACEMENT IS SAME
INSTANTENEOUS VELOCITY IS SAME

Boundary time gives same value in displacemt and velocity equations

INSTANTENEOUS ACCELERATION MAY OR MAY NOT BE SAME.

At Boundary time Acc may or may not be same.

$t=0$ —— $t=15$ —— $t>15$

$v = 3t^2 + 5t$ $\qquad$ $v = \dfrac{B}{t^2}$

Find value of B:

Concept: Boundary time gives same velocity.

$$3(15)^2 + 5(15) = \frac{B}{(15)^2}$$

$$B = \boxed{}$$

21 A vehicle is moving in a straight line. The velocity v m s^{-1} at time t s after the vehicle starts is given by

$$v = A(t - 0.05t^2) \quad \text{for } 0 \leqslant t \leqslant 15,$$
$$v = \frac{B}{t^2} \quad \text{for } t \geqslant 15,$$

where A and B are constants. The distance travelled by the vehicle between $t = 0$ and $t = 15$ is 225 m.

(i) Find the value of A and show that $B = 3375$. [5]

without limits. Integrate

(ii) Find an expression in terms of t for the total distance travelled by the vehicle when $t \geqslant 15$. [3]

(iii) Find the speed of the vehicle when it has travelled a total distance of 315 m. [3]

$S = 225$

$t=0$ $\qquad$ $t=15$ $\qquad$ $t \geqslant 15$

$v = A(t - 0.05t^2)$ $\qquad$ $v = \frac{B}{t^2}$

$A = 4$ $\qquad$ $B = 3375$

(i)
$$\int_0^{15} A(t - 0.05t^2)\,dt = 225$$

$$A\int_0^{15} (t - 0.05t^2)\,dt = 225$$

$$A\left|\frac{t^2}{2} - \frac{0.05t^3}{3}\right|_0^{15} = 225$$

$$A\left|\left(\frac{15^2}{2} - \frac{0.05(15)^3}{3}\right) - \left(\frac{0^2}{2} - \frac{0.05(0)^3}{3}\right)\right| = 225$$

$$A\left(\frac{225}{4}\right) = 225$$

$$\boxed{A = 4}$$

FOR B use boundary time

$t = 15$

$$v = 4(t - 0.05t^2) \qquad v = \frac{B}{t^2}$$

$$4(15 - 0.05(15)^2) = \frac{B}{15^2}$$

$$15 = \frac{B}{225}$$

$$\boxed{B = 3375}$$

(ii) $$s = \int \frac{3375}{t^2}\,dt$$

$$s = \int 3375\,t^{-2}\,dt$$

$$s = \frac{3375\,t^{-1}}{-1} + c$$

$$\boxed{s = -\frac{3375}{t} + c}$$

Use time and displacement of Boundary time. $t = 15$, $s = 225$

$$225 = -\frac{3375}{15} + c$$

$$c = 450$$

$$\boxed{s = -\frac{3375}{t} + 450}$$

(iii) Total Distance $s = 315$

$$s = -\frac{3375}{t} + 450$$

$$315 = -\frac{3375}{t} + 450$$

$$\boxed{t = 25}$$

$$v = \frac{3375}{t^2}$$

$$v = \frac{3375}{25^2}$$

$$\boxed{v = 5.4}$$