TRIG
↓
5 types.

FUNCTIONS
5 TYPES

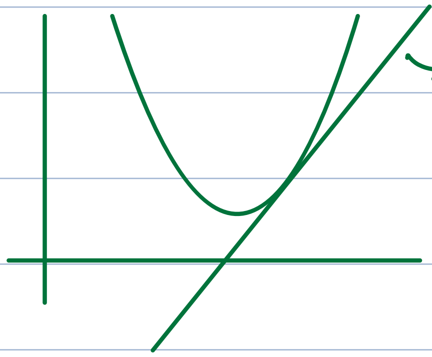
DIFF
3 Types

Integ
3 Types.

# DIFFERENTIATION :-

GRADIENT OF TANGENT (CURVE)

**OLevels:**



→ (gradient of tangent)

$\dfrac{y_2 - y_1}{x_2 - x_1}$

$\dfrac{\text{change in } y}{\text{change in } x}$

delta ← $\dfrac{\Delta y}{\Delta x}$

$\boxed{\text{SYMBOLS}}$

$y \xrightarrow{\text{diff}} \dfrac{dy}{dx}$

$y \xrightarrow{\text{diff}} y'$

## HOW TO DIFF :-

**BASIC RULES :-**

$\boxed{1}$

$x \longrightarrow 1$

$2x \longrightarrow 2$

$10x \longrightarrow 10$

$12x \longrightarrow 12$

$\boxed{2}$ Alone constant $\longrightarrow 0$

$6 \longrightarrow 0$

$4 \longrightarrow 0$

$100 \longrightarrow 0$

eg $y = 5 - 3x$

$\dfrac{dy}{dx} = 0 - 3 = -3$

# POWER RULE :- (THREE STEP PROCESS)

$$(\square)^n \longrightarrow \underset{1}{n}\,(\square)^{\overset{2}{n-1}} \times \overset{3}{\square'}$$

1

$y = \boxed{x}^7$

$\dfrac{dy}{dx} = 7x^6 \times 1$

$\dfrac{dy}{dx} = 7x^6$

2

$y = 12\boxed{x}^3$

$\dfrac{dy}{dx} = 36x^2 \times 1$

$\dfrac{dy}{dx} = 36x^2$

3

$y = (\boxed{12x})^3$

$\dfrac{dy}{dx} = 3(12x)^2 \times 12$

$\dfrac{dy}{dx} = 36(12x)^2$

4

$y = 9\boxed{x}^3 - 12x + 7$

$\dfrac{dy}{dx} = 27x^2(1) - 12 + 0$

$\dfrac{dy}{dx} = 27x^2 - 12$

5

$y = (\boxed{3x-1})^5$

$\dfrac{dy}{dx} = 5(3x-1)^4 \times (3-0)$

$\dfrac{dy}{dx} = 15(3x-1)^4$

6

$y = 6\boxed{x}^2 - 9x + 8$

$\dfrac{dy}{dx} = 12x^1(1) - 9 + 0$

$\dfrac{dy}{dx} = 12x - 9$

7

$y = (\boxed{2x+5})^3 - 9\boxed{x}^2$

$\dfrac{dy}{dx} = 3(2x+5)^2(2+0) - 18x^1(1)$

$\dfrac{dy}{dx} = 6(2x+5)^2 - 18x$

YOU CANNOT DIFFERENTIATE IN DENOMINATOR!

8

$$y = 3x^2 - \frac{2}{x^3}$$

$$y = 3\boxed{x}^{2} - 2\boxed{x}^{-3}$$

$$\frac{dy}{dx} = 6x^{1}(1) + 6x^{-4}(1)$$

$$\frac{dy}{dx} = 6x + \frac{6}{x^4}$$

9

$$y = \frac{12}{(2x-3)^2}$$

$$y = 12\left(\boxed{2x-3}\right)^{-2}$$

$$\frac{dy}{dx} = -24(2x-3)^{-3}(2-0)$$

$$\frac{dy}{dx} = \frac{-48}{(2x-3)^3}$$

9

$$y = \frac{9x^2+2}{x}$$

$$y = \frac{9x^2}{x} + \frac{2}{x}$$

$$y = 9x + \frac{2}{x}$$

$$y = 9x + 2\boxed{x}^{-1}$$

$$\frac{dy}{dx} = 9 + (-2)x^{-2}(1)$$

$$\boxed{\frac{dy}{dx} = 9 - \frac{2}{x^2}}$$

## OUTCOMES :-

$\frac{dy}{dx}$ = gradient of TANGENT (curve).

Q $y = x^2 - 2x + 4$

(i) Find expression for $\frac{dy}{dx}$

$y = \boxed{x}^2 - 2x + 4$

$\frac{dy}{dx} = 2x^1(1) - 2 + 0 \quad = \boxed{2x - 2}$

(ii) Find gradient of tangent at $x = 2$

$\frac{dy}{dx} = 2x - 2 \qquad x = 2$

$= 2(2) - 2$

gradient of tangent $= \boxed{2 = m_T}$

(iii) Find equation of tangent at $x = 2$

For y coordinate put $x = 2$ in curve equation.

$y = 2^2 - 2(2) + 4 = 4$

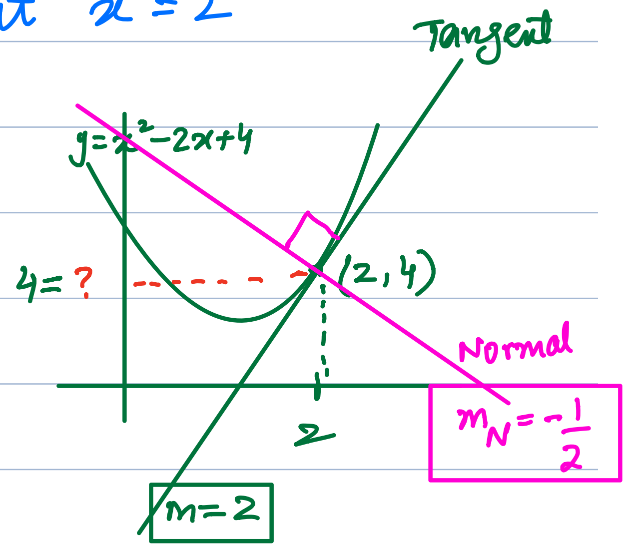
Point $(2, 4)$ , $m_T = 2$

$y - y_1 = m(x - x_1)$

$y - 4 = 2(x - 2)$

$y - 4 = 2x - 4$

$\boxed{y = 2x}$ TANGENT.



(iv) Find equation of normal to curve at $x = 2$.

$m_T = 2 \longrightarrow m_N = -\frac{1}{2}$

Point $(2, 4)$

$$y - 4 = -\frac{1}{2}(x - 2)$$

$$2y - 8 = -x + 2$$

$$\boxed{2y = -x + 10}$$

You cannot tell gradient of normal directly.

$$\begin{tikzcd} y \arrow[r, "\text{diff}"] & \frac{dy}{dx} \arrow[r, "\text{Negative}", "\text{reciprocal}"'] \arrow[d] & m_N \\ & m_T & \end{tikzcd}$$

2 Find the gradient of the curve $y = \dfrac{12}{x^2 - 4x}$ at the point where $x = 3$. [4]

$\boxed{\frac{dy}{dx}}$

$$y = 12\left(\boxed{x^2 - 4x}\right)^{-1}$$

$x^2$

$2x^1(1)$

$$\frac{dy}{dx} = -12\left(x^2 - 4x\right)^{-2} \times (2x - 4)$$

$$\frac{dy}{dx} = \frac{-12(2x-4)}{(x^2-4x)^2}$$

$x = 3$

$$= \frac{-12(2(3)-4)}{(3^2 - 4(3))^2} = -\frac{8}{3}$$

**22** A curve has equation $y = \dfrac{4}{3x - 4}$ and $P(2, 2)$ is a point on the curve.

**(i)** Find the equation of the tangent to the curve at $P$. [4]

**(ii)** Find the angle that this tangent makes with the $x$-axis. [2]

(i) $y = 4\,\boxed{(3x-4)}^{-1}$

$\dfrac{dy}{dx} = -4(3x-4)^{-2}(3)$

$\boxed{\dfrac{dy}{dx} = \dfrac{-12}{(3x-4)^2}}$ $\quad x = 2$

$m_T = \dfrac{-12}{(3(2)-4)^2} = \dfrac{-12}{4} = -3$

$m_T = -3$, $P(2,2)$

$y - 2 = -3(x-2)$

$y = -3x + 6 + 2$

$\boxed{y = -3x + 8}$

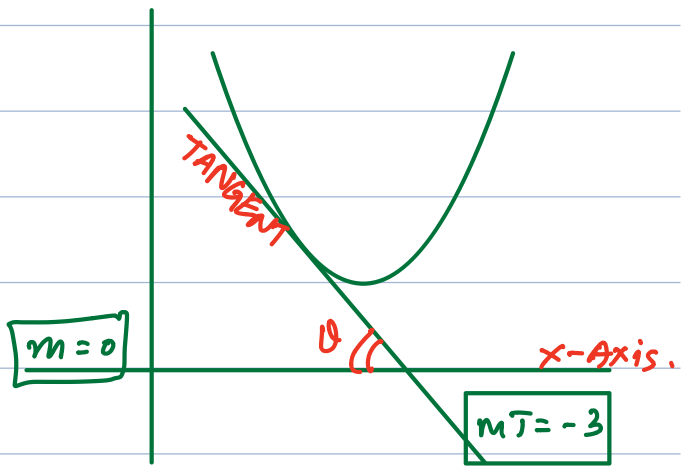
(ii) $\tan\theta = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

$\tan\theta = \dfrac{0 - (-3)}{1 + (0)(-3)}$

$\tan\theta = 3$

$\theta = \tan^{-1}(3)$

$\theta = 71.565$




Please wait. I'm about to reach academy. In traffic hn.

**11** The equation of a curve is $y = \dfrac{12}{x^2+3}$.

**(i)** Obtain an expression for $\dfrac{dy}{dx}$. [2]

**(ii)** Find the equation of the normal to the curve at the point $P(1, 3)$. [3]

$$y = 12(x^2+3)^{-1}$$

$$\frac{dy}{dx} = -12(x^2+3)^{-2}(2x)$$

$$\frac{dy}{dx} = \frac{-24x}{(x^2+3)^2}$$

$$m_T = \frac{-24(1)}{(1^2+3)^2} = \frac{-24}{16} = \frac{-3}{2}$$

$$m_N = \frac{2}{3}, \quad P(1,3)$$

$$y-3 = \frac{2}{3}(x-1)$$

$$3y-9 = 2x-2$$

$$\boxed{3y = 2x+7}$$

10 The equation of a curve is $y = 5 - \dfrac{8}{x}$

**(i)** Show that the equation of the normal to the curve at the point $P(2, 1)$ is $2y + x = 4$. [4] Diff

point of intersection (simultaneously).

This normal meets the curve again at the point $Q$.

**(ii)** Find the coordinates of $Q$. [3]

coordinate geo.

**(iii)** Find the length of $PQ$. [2]

$$y = 5 - 8x^{-1}$$

$$\frac{dy}{dx} = 0 - (-8)x^{-2}(1) = \frac{8}{x^2}$$

$$m_T = \frac{8}{2^2} = \frac{8}{4} = 2$$

$$m_N = -\frac{1}{2} \qquad P(2,1)$$

$$y-1 = -\frac{1}{2}(x-2)$$

$$2y-2 = -x+2$$

$$2y = -x+4$$

$$\boxed{2y+x=4}$$

$\boxed{\text{ii}}$ $\boxed{Q}$ $y = 5 - \frac{8}{x}$ $2y + x = 4$

$$5 - \frac{8}{x} = \frac{4-x}{2}$$

$$\frac{5x-8}{x} = \frac{4-x}{2}$$

$$10x - 16 = 4x - x^2$$

$$x^2 + 6x - 16 = 0$$

$$x^2 + 8x - 2x - 16 = 0$$

$$x(x+8) - 2(x+8) = 0$$

$$(x-2)(x+8) = 0$$

$x = 2$ , $x = -8$

Poin P

Point Q

$$y = \frac{4-(-8)}{2}$$

$$y = 6$$

$$Q(-8, 6)$$

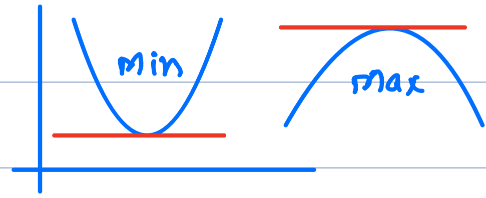
$P(2,1)$ $Q(-8,6)$

Length PQ $= \sqrt{(-8-2)^2 + (6-1)^2}$

$= \sqrt{125}$

$=$ ___

STATIONARY POINT
TURNING POINT
VERTEX
MAXIMUM POINT
MINIMUM POINT
CRITICAL POINT (SAT)




TANGENT = HORIZONTAL = Gradient is zero.

GRADIENT OF TANGENT IS ZERO

$$\boxed{\frac{dy}{dx} = 0}$$

Q. $y = x^2 + 12x - 3$

Find x-coordinates of its stationary point.

$$\frac{dy}{dx} = 2x + 12$$

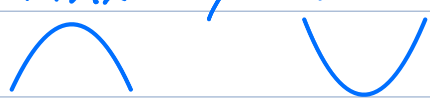
$$0 = 2x + 12$$

$$2x = -12$$

$$x = -6$$

## NATURE OF A STATIONARY POINT

MAX / MIN



$$y \xrightarrow{\text{diff}} \frac{dy}{dx} \xrightarrow{\text{diff}} \frac{d^2y}{dx^2}$$

$\boxed{+ve}$ MIN

$\boxed{-ve}$ MAX.

$$y \xrightarrow{\text{diff}} y' \xrightarrow{\text{diff}} y''$$

**7** The equation of a curve is $y = (2x - 3)^3 - 6x$.

**(i)** Express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of $x$. [3]

**(ii)** Find the $x$-coordinates of the two stationary points and determine the nature of each stationary point. [5]

(i)

$$y = (\boxed{2x-3})^3 - 6x$$

$$\frac{dy}{dx} = 3(2x-3)^2(2) - 6$$

$$\boxed{\frac{dy}{dx} = 6(\boxed{2x-3})^2 - 6}$$

$$\frac{d^2y}{dx^2} = 12(2x-3)^1(2) - 0$$

$$\boxed{\frac{d^2y}{dx^2} = 24(2x-3)}$$

(ii) STATIONARY POINTS

$$\frac{dy}{dx} = 0$$

$$6(2x-3)^2 - 6 = 0$$

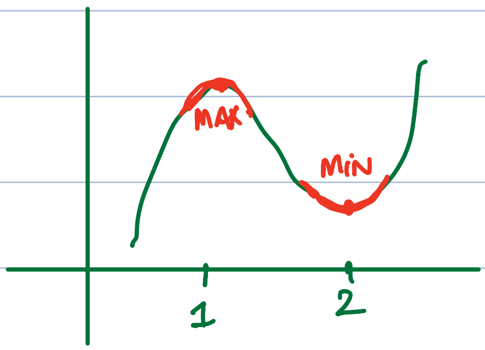
$$6(2x-3)^2 = 6$$

$$\sqrt{(2x-3)^2} = \pm\sqrt{1}$$

$$2x - 3 = \pm 1$$

$$x = \frac{3 \pm 1}{2}$$

$$\boxed{x = 2}, \quad \boxed{x = 1}$$




NATURE :-

$$\frac{d^2y}{dx^2} = 24(2x-3)$$

$\boxed{x=2}$ $$\frac{d^2y}{dx^2} = 24(2(2)-3) = 24\,(+)\,(\text{MIN})$$

$\boxed{x=1}$ $$\frac{d^2y}{dx^2} = 24(2(1)-3) = -24\,(-)\,(\text{max})$$

$$y \xrightarrow{\text{diff}} \frac{dy}{dx} \xrightarrow{\text{diff}} \frac{d^2y}{dx^2} \begin{cases} +\text{ve} & \text{Min} \\ -\text{ve} & \text{Max} \end{cases}$$

$$\boxed{\frac{dy}{dx} = 0}$$

STATIONARY POINTS

**52** A curve has equation $y = \dfrac{8}{x} + 2x$.

**(i)** Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [3]

**(ii)** Find the coordinates of the stationary points and state, with a reason, the nature of each stationary point. [5]

(i)

$$y = 8x^{-1} + 2x$$

$$\frac{dy}{dx} = -8x^{-2}(1) + 2$$

$$\boxed{\frac{dy}{dx} = -8x^{-2} + 2}$$

$$\frac{d^2y}{dx^2} = 16x^{-3}(1)$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{16}{x^3}}$$

(ii) STATIONARY POINT :-

$$\frac{dy}{dx} = 0$$

$$-8x^{-2} + 2 = 0$$

$$-\frac{8}{x^2} + 2 = 0$$

$$2 = \frac{8}{x^2}$$

$$2x^2 = 8$$

$$x^2 = 4$$

$$\boxed{x = 2, -2}$$

NATURE

$\boxed{x=2}$ $\dfrac{d^2y}{dx^2} = \dfrac{16}{2^3} = 2$

(+)

Min

$\boxed{x=-2}$ $\dfrac{d^2y}{dx^2} = \dfrac{16}{(-2)^3}$

$= -2$

Max

**20** A curve has equation $y = \dfrac{1}{x-3} + x$.

**(i)** Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [2]

**(ii)** Find the coordinates of the maximum point $A$ and the minimum point $B$ on the curve. [5]

$$y = (x-3)^{-1} + x$$

$$\frac{dy}{dx} = -1(x-3)^{-2}(1) + 1$$

$$\boxed{\frac{dy}{dx} = -1(x-3)^{-2} + 1}$$

$$\frac{d^2y}{dx^2} = 2(x-3)^{-3}(1) + 0$$

$$\boxed{\frac{d^2y}{dx^2} = 2(x-3)^{-3}}$$

MAX / MIN POINT (TURNING POINT).

$$0 = -1(x-3)^{-2} + 1$$

$$0 = \frac{-1}{(x-3)^2} + 1$$

$$\frac{1}{(x-3)^2} = 1$$

$$(x-3)^2 = 1$$

$$x - 3 = \pm 1$$

$$x = 3 \pm 1$$

| | |
|---|---|
| $x = 4$, | $x = 2$ |
| $y = \dfrac{1}{4-3} + 4$ | $y = \dfrac{1}{2-3} + 2$ |
| $y = 5$ | $y = 1$ |
| $B(4,5)$ | $A(2,1)$ |

$x = 4$, $\dfrac{d^2y}{dx^2} = \dfrac{2}{(4-3)^3} = 2$ (+ve) min B

$x = 2$, $\dfrac{d^2y}{dx^2} = \dfrac{2}{(2-3)^3} = -2$ (−ve) Max A

**30** The curve $y = \dfrac{10}{2x+1} - 2$ intersects the $x$-axis at $A$. The tangent to the curve at $A$ intersects the $y$-axis at $C$.

**(i)** Show that the equation of $AC$ is $5y + 4x = 8$. [5]

**(ii)** Find the distance $AC$. [2]

$\boxed{A}$ $x$-axis, $y=0$, $0 = \dfrac{10}{2x+1} - 2$

$$2 = \frac{10}{2x+1}$$

$$4x+2 = 10$$

$A = (2,0)$ $\boxed{x=2}$

TANGENT AT A :

$$y = 10(2x+1)^{-1} - 2$$

$$\frac{dy}{dx} = 10(-1)(2x+1)^{-2}(2) - 0$$

$$\frac{dy}{dx} = \frac{-20}{(2x+1)^2} \qquad x=2$$

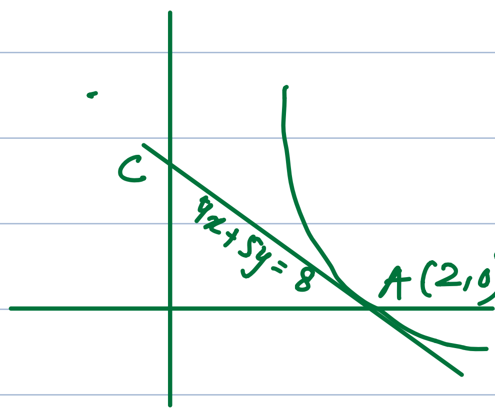
$$m_T = \frac{-20}{(2(2)+1)^2} = -\frac{4}{5}$$

$m = -\frac{4}{5}$ $A\ (2,0)$

$$y - 0 = -\frac{4}{5}(x-2)$$

$\boxed{5y = -4x + 8}$ Tangent.

$4x + 5y = 8$



$\boxed{C}$ y-intercept: $x=0$

$$4(0)+5y=8$$
$$y=1.6$$

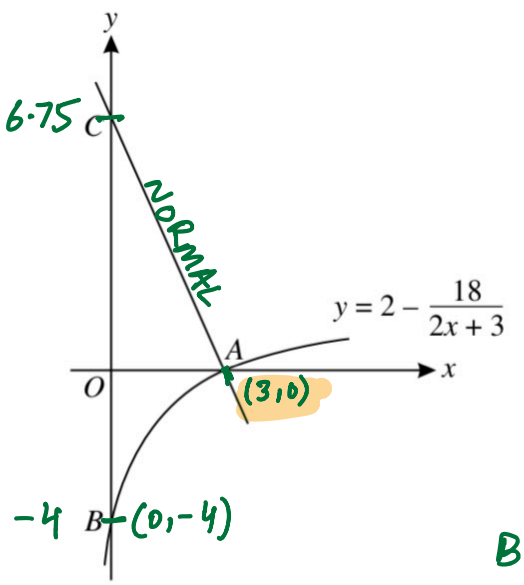
$A(2,0)$ $C(0,1.6)$

$$AC = \sqrt{(0-2)^2+(1.6-0)^2} = \boxed{\phantom{0}}$$

**12**

$\boxed{A}$ x-axis. $y=0$

$$0 = 2 - \frac{18}{2x+3}$$
$$\frac{18}{2x+3} = 2$$
$$18 = 4x+6$$
$$\boxed{x=3}$$




$\boxed{B}$ $x=0$, y-axis

$$y = 2 - \frac{18}{2(0)+3}$$
$$y=-4$$

$\boxed{C}$ $x=0$, y axis

$$9(0)+4y=27$$
$$4y=27$$
$$y=6.75$$

$$BC = 6.75+4 = \boxed{10.75}$$

The diagram shows part of the curve $y = 2 - \dfrac{18}{2x+3}$, which crosses the $x$-axis at $A$ and the $y$-axis at $B$. The normal to the curve at $A$ crosses the $y$-axis at $C$.

**(i)** Show that the equation of the line $AC$ is $9x + 4y = 27$. [6]

Normal.

**(ii)** Find the length of $BC$. [2]

$$y = 2 - 18(2x+3)^{-1}$$
$$\frac{dy}{dx} = 0 - 18(-1)(2x+3)^{-2}(2)$$
$$\frac{dy}{dx} = \frac{36}{(2x+3)^2} \qquad x=3$$
$$m_T = \frac{36}{(2(3)+3)^2} = \frac{4}{9}$$

NORMAL:

$m = -\frac{9}{4}$, $A(3,0)$

$$y-0 = -\frac{9}{4}(x-3)$$
$$4y = -9x+27$$
$$\boxed{9x+4y=27} \quad \boxed{AC}$$

**5** A curve has equation $y = \frac{k}{x}$. Given that the gradient of the curve is $-3$ when $x = 2$, find the value of the constant $k$. [3]

$\frac{dy}{dx} = -3 \quad , \quad x = 2$

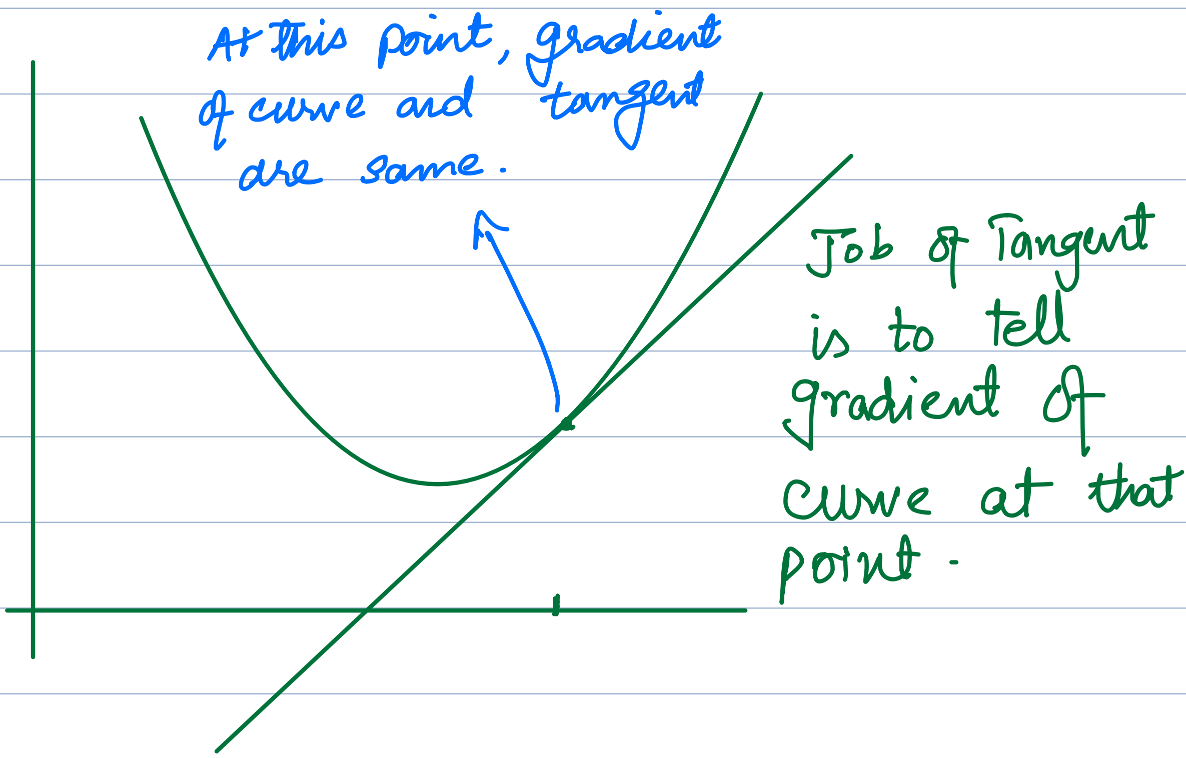
$y = k x^{-1}$

$\frac{dy}{dx} = k(-1)x^{-2}(1)$

$\frac{dy}{dx} = \frac{-k}{x^2}$

$-3 = \frac{-k}{(2)^2}$

$k = 12$



(eg) $y = 3\sqrt{x} + 2x \longrightarrow y = 3\sqrt{x} + 2x$

(eg) $y = 3\sqrt{(x+5)} \longrightarrow y = 3\sqrt{x+5}$

$$y = \sqrt{2x+3}$$

$$y = (2x+3)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}}(2x+3)^{-\frac{1}{2}} \times (\cancel{2})$$

$$\frac{dy}{dx} = \frac{1}{(2x+3)^{\frac{1}{2}}} = \frac{1}{\sqrt{2x+3}}$$

# RATE OF CHANGE

Rate of change of $x = \frac{dx}{dt}$

Rate of change of volume $= \frac{dV}{dt}$

Rate of change of Area $= \frac{dA}{dt}$

## LAYOUT

Two main variables + Connecting equation

$$\begin{array}{ccc} y & x & y = 2x^2+9x-1 \\ \downarrow & \downarrow & \downarrow \\ \frac{dy}{dt} & \frac{dx}{dt} & \frac{dy}{dx} \end{array}$$

CHAIN RULE: $$\frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx}$$

Q. A circular water pond is expanding such that its radius is increasing at rate of 4 m/s. Find rate of increase of its area when $r = 15$.

connecting equation.

$$\begin{array}{ccc} r & A & A = \pi r^2 \\ \frac{dr}{dt} & \frac{dA}{dt} & \frac{dA}{dr} \end{array}$$

$$A = \pi \boxed{r}^2$$

$$\frac{dA}{dr} = 2\pi r^1(1)$$

$$\boxed{\frac{dA}{dr} = 2\pi r}$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

Data in Question

$\frac{dr}{dt} = 4$

$\frac{dA}{dt} = ?$

$r = 15$

$$\frac{dA}{dt} = 2\pi r \times 4$$

$$\frac{dA}{dt} = 2\pi(15) \times 4$$

$$\frac{dA}{dt} = 120\pi = \boxed{\phantom{000}}$$

**21** The volume of a spherical balloon is increasing at a constant rate of 50 cm$^3$ per second. Find the rate of increase of the radius when the radius is 10 cm. [Volume of a sphere = $\frac{4}{3}\pi r^3$.] [4]

| MAIN | VARIABLES | CONN. EQ. | | Data in question |
|---|---|---|---|---|
| $r$ | $V$ | $V = \frac{4}{3}\pi r^3$ | $V = \frac{4}{3}\pi \boxed{r}^3$ | $\frac{dV}{dt} = 50$ |
| $\frac{dr}{dt}$ | $\frac{dV}{dt}$ | $\frac{dV}{dr}$ | $\frac{dV}{dr} = \frac{4}{3}\pi(3)r^2(1)$ | $\frac{dr}{dt} = ?$ |
| | | | $\frac{dV}{dr} = 4\pi r^2$ | $r = 10$ |

$$\frac{dV}{dt} = \frac{dr}{dt} \times \frac{dV}{dr}$$

$$50 = \frac{dr}{dt} \times 4\pi r^2$$

$$50 = \frac{dr}{dt} \times 4\pi(10)^2$$

$$\frac{dr}{dt} = \frac{50}{400\pi} = \boxed{\phantom{000}}$$

**31** An oil pipeline under the sea is leaking oil and a circular patch of oil has formed on the surface of the sea. At midday the radius of the patch of oil is 50 m and is increasing at a rate of 3 metres per hour. Find the rate at which the area of the oil is increasing at midday. [4]

$A \quad r \quad A = \pi r^2$

$A = \pi \boxed{r}^2$

Data

$r = 50$

$\frac{dA}{dt} = \frac{dr}{dt} \times \frac{dA}{dr}$

$\frac{dA}{dr} = 2\pi r^1 (1)$

$\frac{dr}{dt} = 3$

$\frac{dA}{dt} = 3 \times 2\pi r$

$\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = ?$

$= 3 \times 2\pi(50)$

$= 300\pi$

**11** The equation of a curve is $y = \frac{12}{x^2+3}$.

**(i)** Obtain an expression for $\frac{dy}{dx}$. [2]

point $P(1, 3)$.

**(iii)** A point is moving along the curve in such a way that the $x$-coordinate is increasing at a constant rate of 0.012 units per second. Find the rate of change of the $y$-coordinate as the point passes through $P$. $(x=1, y=3)$ [2]

(i)

$y = 12(x^2+3)^{-1}$

$\frac{dy}{dx} = -12(x^2+3)^{-2}(2x)$

$\frac{dy}{dx} = \frac{-24x}{(x^2+3)^2}$

(iii)

$x \quad y \quad y = \frac{12}{x^2+3}$

$\frac{dx}{dt} \quad \frac{dy}{dt} \quad \frac{dy}{dx}$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

at Point P $\boxed{x = 1}$

$\frac{dy}{dt} = \frac{-24x}{(x^2+3)^2} \times 0.012$

$\frac{dy}{dt} = \frac{-24(1)}{(1^2+3)^2} \times 0.012 = \boxed{-0.018}$

SCENARIO BASED

ALWAYS ATTEMPT THE DIFFERENTIATION PART FIRST.

Question Layout

Substitution

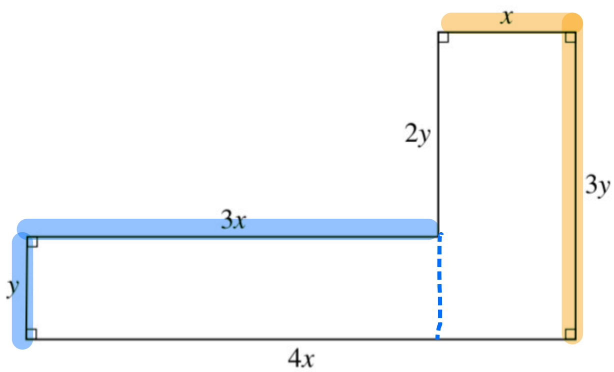
PROOF

START

Proof gets stuck after 1 to 2 steps

Solves

**26**




The diagram shows the dimensions in metres of an L-shaped garden. The perimeter of the garden is 48 m.

**(i)** Find an expression for $y$ in terms of $x$. [1]

**(ii)** Given that the area of the garden is $A$ m$^2$, show that $A = 48x - 8x^2$. [2]

**(iii)** Given that $x$ can vary, find the maximum area of the garden, showing that this is a maximum value rather than a minimum value. [4]

(i)

$$P = y + 2y + 3y + 4x + 3x + x$$

$$P = 6y + 8x$$

$$48 = 6y + 8x$$

$$\boxed{y = \frac{48 - 8x}{6}}$$

(ii)

$$A = (y)(3x) + (x)(3y)$$

$$A = 3xy + 3xy$$

$$A = 6xy$$

$$A = \cancel{6}x\left(\frac{48 - 8x}{\cancel{6}}\right)$$

$$\boxed{A = 48x - 8x^2}$$

(iii) $A = 48x - 8x^2$

$$\boxed{\frac{dA}{dx} = 48 - 16x}$$

Max
↓
Stationary Point.
↓
diff = 0

$0 = 48 - 16x$

$16x = 48$

$x = 3$

$A = 48(3) - 8(3)^2 = 72$ (Max. value of area).

$$\frac{dA}{dx} = 48 - 16x$$

$$\frac{d^2A}{dx^2} = -16 \quad (\text{Max}).$$

**13** A solid rectangular block has a square base of side $x$ cm. The height of the block is $h$ cm and the total surface area of the block is $96\,\text{cm}^2$.

**(i)** Express $h$ in terms of $x$ and show that the volume, $V\,\text{cm}^3$, of the block is given by

$$V = 24x - \tfrac{1}{2}x^3.$$ [3]

Given that $x$ can vary,

**(ii)** find the stationary value of $V$, [3]

**(iii)** determine whether this stationary value is a maximum or a minimum. [2]


Total SA $= 96$

$$2(x^2) + 4(x)(h) = 96$$

$$2x^2 + 4xh = 96$$

$$\boxed{h = \frac{96 - 2x^2}{4x}}$$

$$V = (x)(x)(h)$$

$$V = x^2 h$$

$$V = x^{\cancel{2}}\left(\frac{96 - 2x^2}{4\cancel{x}}\right)$$

$$V = \frac{96x - 2x^3}{4}$$

$$V = \frac{96x}{4} - \frac{2x^3}{4}$$

$$\boxed{V = 24x - \frac{1}{2}x^3}$$

(ii) $$V = 24x - \frac{1}{2}x^3$$

$$\frac{dV}{dx} = 24 - \frac{1}{2}(3)(x)^2(1)$$

$$\boxed{\frac{dV}{dx} = 24 - \frac{3}{2}x^2}$$

$$0 = 24 - \frac{3}{2}x^2$$

$$\frac{3}{2}x^2 = 24$$

$$x^2 = 16$$

$$x = 4$$

$$V = 24(4) - \frac{1}{2}(4)^3$$

$$V = 64$$

(iii) $$\frac{dV}{dx} = 24 - \frac{3}{2}x^2$$

$$\frac{d^2V}{dx^2} = 0 - \frac{3}{2}(2x)$$

$$\frac{d^2V}{dx^2} = -3x \qquad \boxed{x = 4}$$

$$\frac{d^2V}{dx^2} = -3(4) = -12 \quad (Max)$$

# DIFFERENTIATION

## QUESTIONS:

$\boxed{\text{TYPE 1}}$ EQUATION OF TANGENT AND NORMAL.

$\boxed{\text{TYPE 2}}$ STATIONARY POINTS AND ITS NATURE.

$\boxed{\text{TYPE 3}}$ SCENARIO BASED PROOF QUESTION

$\boxed{\text{TYPE 4}}$ RATE OF CHANGE.

$\boxed{\text{TYPE 5}}$ INCREASING / DECREASING FUNCTIONS.