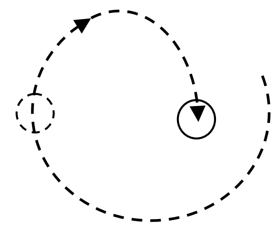
Circular Motion in "Vertical" Circles

So, let's re-cap a few quick points. When a ball is swung on a string in a vertical circle, the tension is greatest at the bottom of the circular path. This is where the rope is most likely to break. It should make sense that the tension at the bottom is the greatest. Imagine watching the summer Olympics, specifically the gymnastics competitions. When a gymnast is doing the parallel bars, spinning themselves around in a vertical circle, when does she have to struggle the most to keep hold of the bar? Not at the top. In fact, at the top the gymnast feels relief (and often repositions her hands). At the bottom, however, the gymnast's arms will hurt. Gymnasts are most likely to lose hold of the parallel bar when reaching the bottom of their circular path.

However, what happens if the ball on the string (or the gymnast, for that matter) isn't going very fast. Without enough speed, the ball won't be able to complete the circle. It will leave its circular path at the top, because the string will lose tension. This can be seen in the diagram at the right. So, a MINIMUM VELOCITY is needed for objects to complete vertical circles. How do you find this minimum velocity? Well……if the ball "just barely" gets over the top of the circle, the tension in this "just barely" case will be "just barely" greater than zero at the top. After all, the key to swinging something in a vertical circle is to keep tension in the rope. Therefore, if there is tension in the rope, even an itsy, bitsy, teeny, weeny, yellow(?) ….bit of friction, then the ball will complete the loop. Therefore, using the equation we derived for the top of the loop, we know that



$$\frac{mv^2}{R} - mg = T \quad \rightarrow \quad \frac{mv^2}{R} - mg > 0 \quad \rightarrow \quad \frac{mv^2}{R} > mg$$

$$\frac{v^2}{R} > g$$

$$v > \sqrt{gR} \quad (critical \quad velocity)$$

As long as the ball moves faster than "the square root of GRRRRRRRR", then the ball will complete the circular loop. So, remember this: when talking about the rope breaking, use the bottom-of-the-loop equation. When talking about critical speed, use the top-of-the-loop equation, with tension set equal zero.

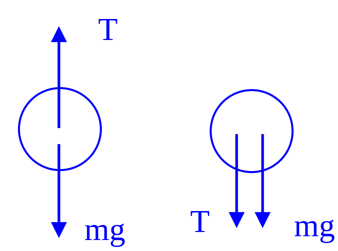
Practice Problems

1. A .5 kg ball is swung on a 1 m rope in a vertical circle with a constant velocity of 5 m/s.
   a) Find the position of the ball (top, bottom, left, right, etc) where the rope's tension is greatest.
   b) Find the position of the ball (top, bottom, left, right, etc) where the rope's tension is least.
   c) Draw an FBD of the ball for the positions in parts "a" and "b" above.
   d) Find the maximum tension in the string.

bottom

top

$$T = \frac{mv^2}{r} + mg = \frac{(0.5kg)(5\,\frac{m}{s})^2}{(1m)} + (0.5kg)(9.8\,\frac{m}{s^2}) = 17.4N$$

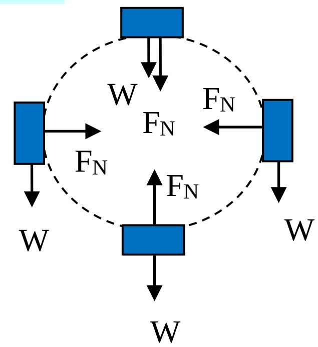


2. A .2 kg ball is being swung on a 0.5 m rope in a vertical circle. The rope can withstand up to 50 N of force.
   a) Find the maximum speed of the ball before it breaks the rope.
   b) Find the minimum speed ("critical velocity) of the ball so that it doesn't "fall out of the circle" at the top)

$$T = \frac{mv^2}{r} + mg \quad \rightarrow \quad 50 = \frac{(0.2)v^2}{0.5} + (0.2)(9.8) \quad \rightarrow \quad v = 10.96\,\tfrac{m}{s}$$

$$v = \sqrt{gr} = \sqrt{(9.8)(0.5)} = 2.21\,\tfrac{m}{s}$$

When a car or a bicycle goes through a loop-de-loop, this is also motion in a vertical circle. Therefore, the same analysis applies. However, instead of the tension helping to provide the centripetal force, the normal force between the car and the track does this. This can be seen in the picture at the right. Using this picture, we can derive formulas for the normal force at different locations of the loop de loop.



at the top

$$F_c = \frac{mv^2}{R} \quad \rightarrow \quad F_N + mg = \frac{mv^2}{R} \quad \rightarrow \quad F_N = \frac{mv^2}{R} - mg$$

at the bottom $$F_c = \frac{mv^2}{R} \quad \rightarrow \quad F_N - mg = \frac{mv^2}{R} \quad \rightarrow \quad F_N = \frac{mv^2}{R} + mg$$

at the side points $$F_c = \frac{mv^2}{R} \quad \rightarrow \quad F_N = \frac{mv^2}{R}$$

HW Problem

1) A car of mass 500 kg enters a loop-de-loop of radius 5 meters at a speed of 15 m/s (on cruise control). Find the Normal force of the car at the top of the loop and at the bottom of the loop.

2) Find the normal force of the car previous problem BEFORE IT ENTERED THE LOOP.

3) By how much (2 times? 3 times?) did the normal force increase from the time the car entered the loop (problem #2 above) to the time it exited the loop (problem #1 above)?

4) What is the minimum velocity needed for the car from problem #1 to complete the loop? 15 m/s is too fast … how slow can he go???

5) A kid swings a ball on a string in a vertical circle of radius 1.3 meters. He begins to slow the ball down, and it reaches the top with a speed of 3 m/s. Will it stay in the circle?