GRAVITATON

PHYSICS BY
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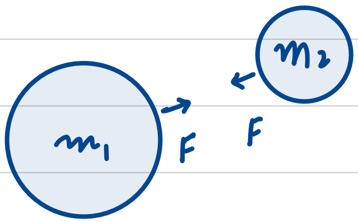
CONCEPTS OF GRAVITATION

Newtonian Concept

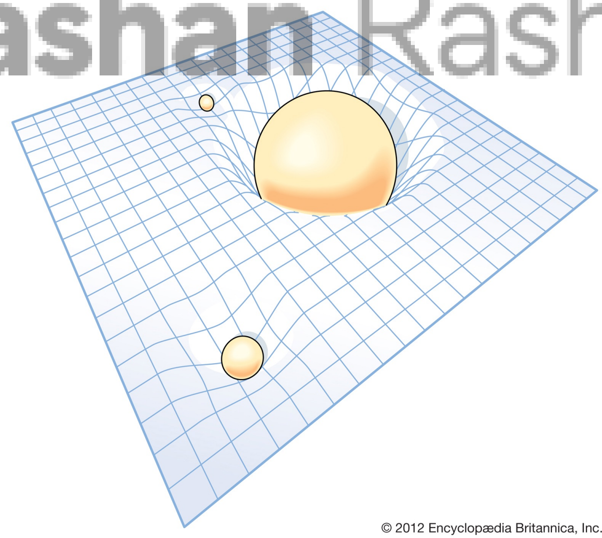
"Mass Attract Mass"

Why??



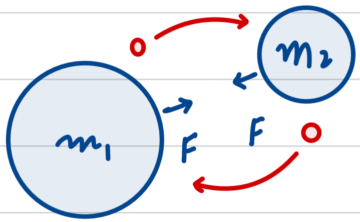
Einstein's Concept

"Bending of Space-time fabric"



The Standard Model

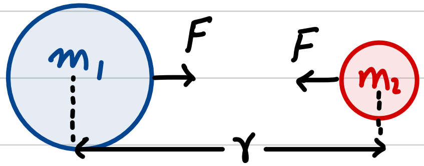
"Gravitons"



Gravitons not yet discovered.

NEWTON'S LAW OF UNIVERSAL GRAVITATION

The force of attraction between point masses is directly proportional to the product of masses and inversly proportional to the square of their seperation.



$F \propto m_1 m_2$ $\quad$ $F \propto \frac{1}{r^2}$ center to center distance between masses

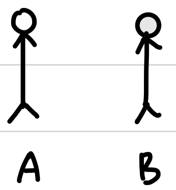
$F \propto \frac{m_1 m_2}{r^2}$

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

G: Universal Gravitational Constant

$G = 6.67 \times 10^{-11}\ N.m^2.kg^{-2}$

e.g.



$m_A = 60\ kg$ $\quad$ $r = 1m$

$m_B = 40\ kg$

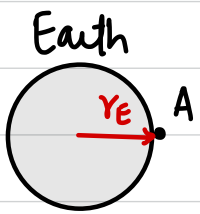
$F = G\frac{m_1 m_2}{r^2}$

$F = \frac{(6.67\times10^{-11})(60)(40)}{(1)^2}$

$F = 1.6\times10^{-7} N$ or $0.16\ \mu N$

Small/Insufficient force as masses are large but force ain't.

e.g.



$m_E = 6.0\times10^{24} kg$

$m_A = 60\ kg$

$r_E = 6.4\times10^{6} m$

$F = \frac{G m_1 m_2}{r^2}$

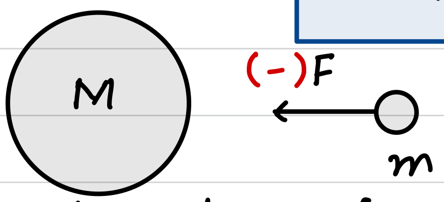
$F = \frac{(6.67\times10^{-11})(6.0\times10^{24})(60)}{(6.4\times10^{6})^2} = 586.2\ N$

significant force

$$\text{Weight} = mg = (60)(9.8) = 588\,N$$

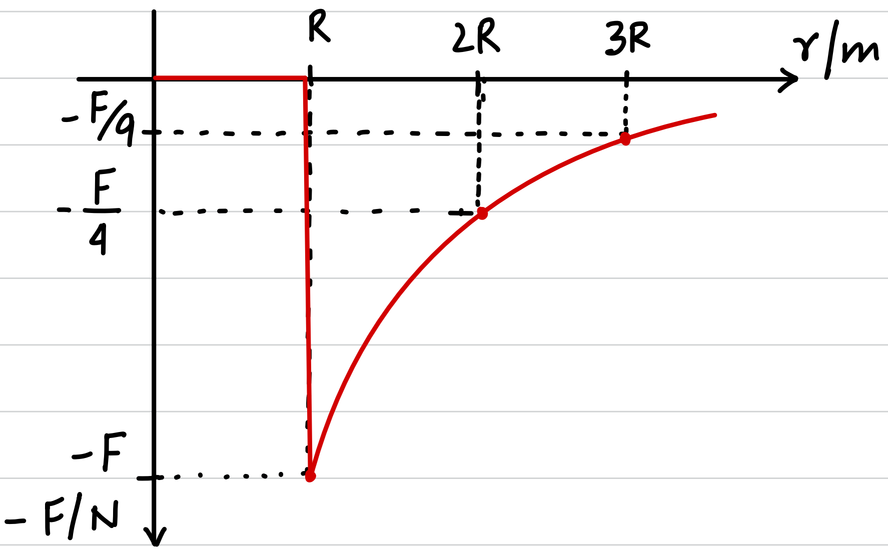
The gravitational force of planet on a body is the Weight of the body.

☑ Sometimes $F = \frac{Gm_1m_2}{r^2}$ is written as $\boxed{F = -\frac{Gm_1m_2}{r^2}}$ because the force is attractive.

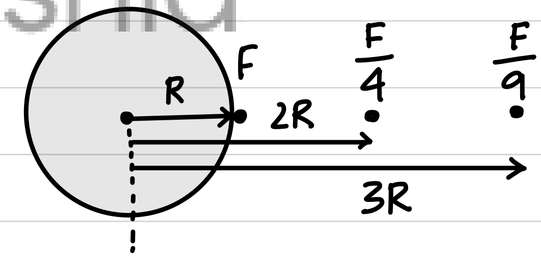


conventionally taken as negative

☑ $F = -\frac{Gm_1m_2}{r^2}$ i.e. $F \propto -\frac{1}{r^2}$, hence its also referred as inverse square law.



r	F
R	F
2R	F/4
3R	F/9
4R	F/16



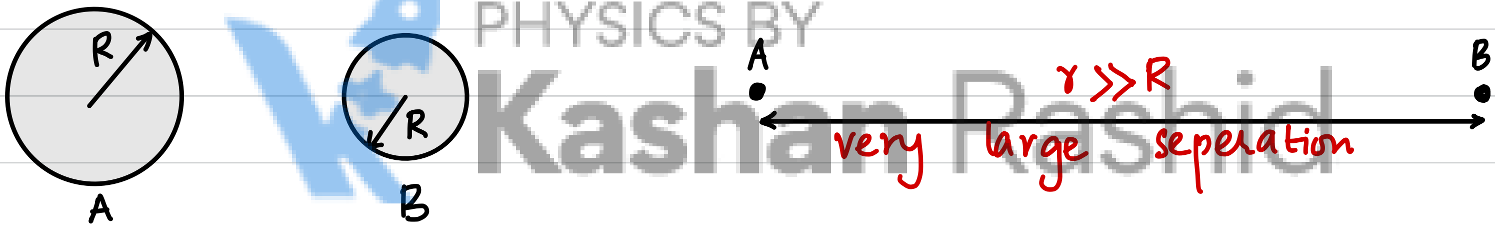
☑ Inside the planet/mass, gravitational force is considered zero!

☑ During calculations, planets, stars, asteroids and other masses are considered as "point mass"

Masses are assumed as point mass for simplicity! It's not real !!

Point Mass means

i, The distance between the masses is so large that the size of the planet itself (i.e. its radius) can be ignored as the mass can be treated as just a point in space!



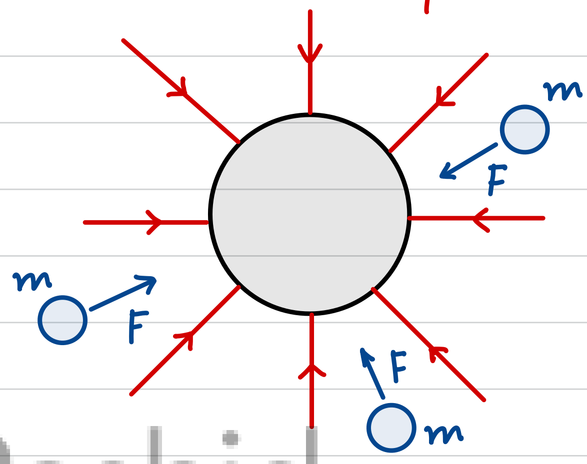
ii, All of the mass of the planet/star/body is concentrated at its center! The planet can be assumed as hollow will all mass in the middle, and the effect of it appears outside its surface.

→ Therefore, all distances are measured from center to center

→ Below the planet's surface, the force is zero as mass has no effect inside the planet itself.

Gravitational Field

A region in space around a mass where another mass experiences a gravitational force.



- ☑ The direction of field line tells about the direction of gravitational force.

- ☑ The gap between field lines tell about the strength of field. Less gap, strong field. More gap, weak field.

- ☑ Gravitational field & hence force exists till infinity! According to law, force gets weak but not zero.

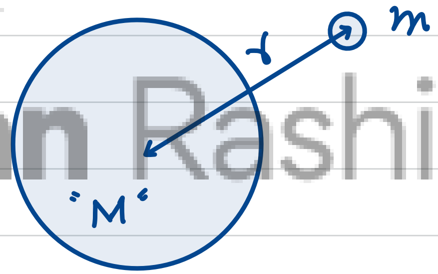
Gravitational Field Strength (g)

Force per unit mass at a point in the gravitational field.

e.g. $g_{earth} = 9.8\ N/kg$ (1 kg mass experiences 9.8 N of force)

$g_{moon} = 1.6\ N/kg$ (1 kg mass experiences 1.6 N of force)

$g = \frac{F}{m}$ $\quad$ $g = \frac{GMm}{r^2 \times m}$



$$\boxed{g = \frac{GM}{r^2}}$$

M → Mass of planet

r → Center to Center distance

r	g
R	g
2R	g/4
3R	g/9
∞	0

$g \propto \frac{1}{r^2}$ Inverse - Square Law

Gravitation field Strength is a vector because it represents force and force is vector!

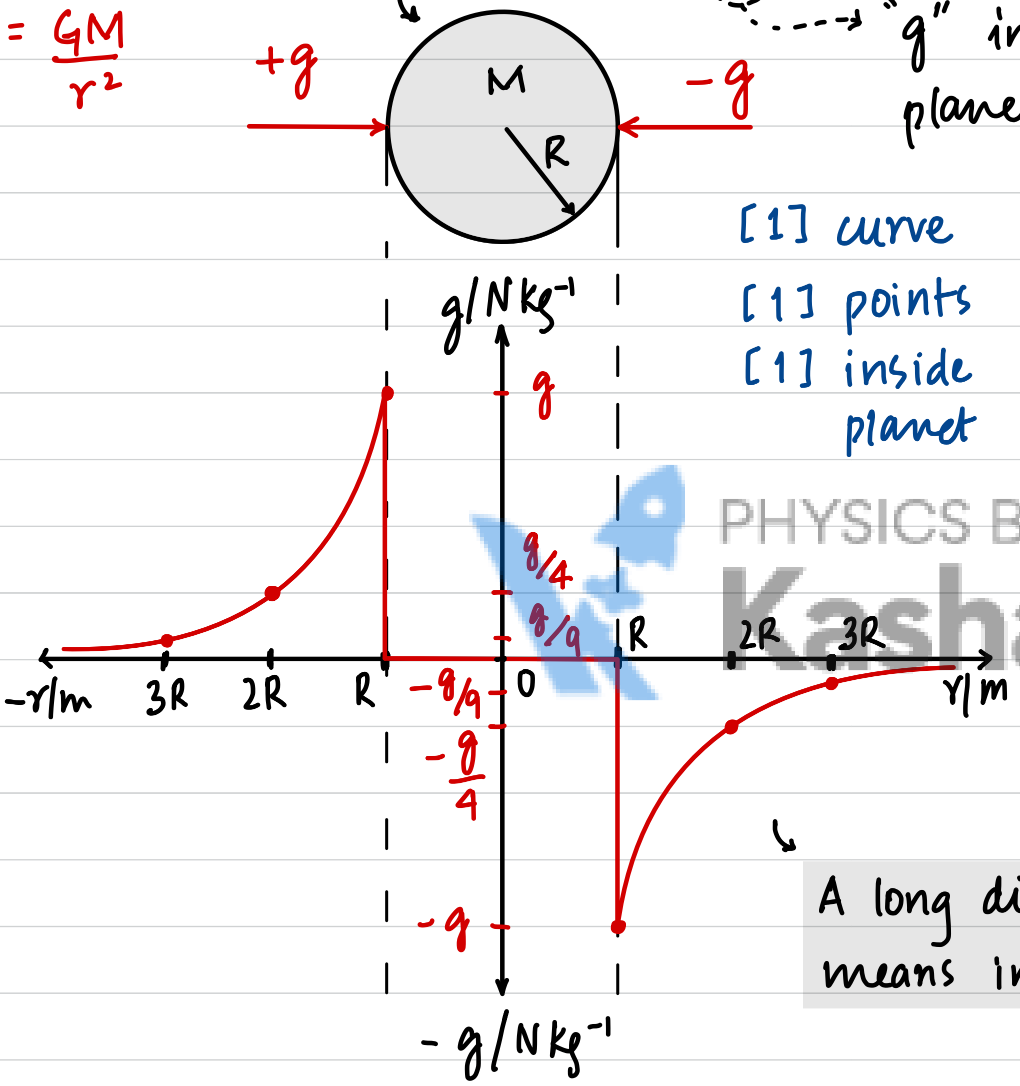
Syllabus!

point mass

$g = \frac{GM}{r^2}$

"g" inside the planet is "zero"!

$-$ $+$



[1] curve

[1] points

[1] inside planet

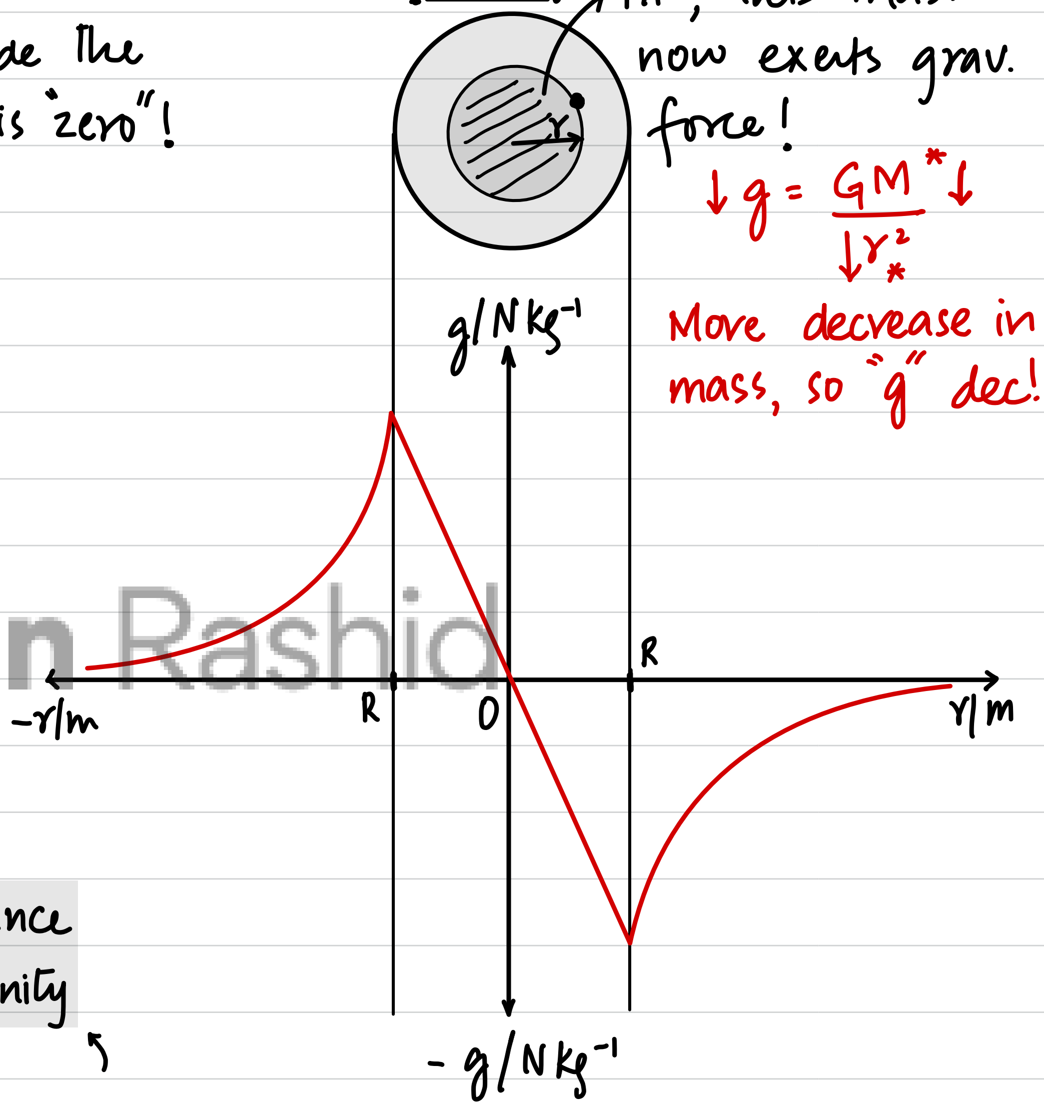
A long distance means infinity

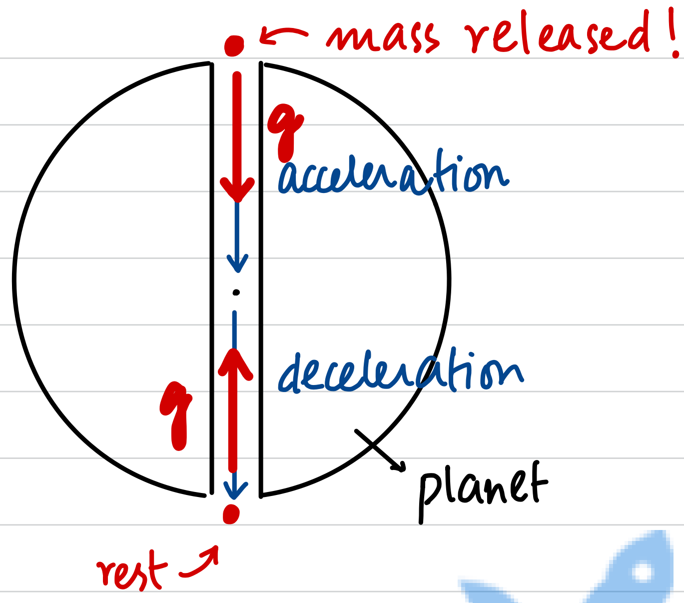
Real

M'; this mass now exerts grav. force!

$\downarrow g = \frac{GM^{*}\downarrow}{\downarrow r_*^2}$

More decrease in mass, so "g" dec!

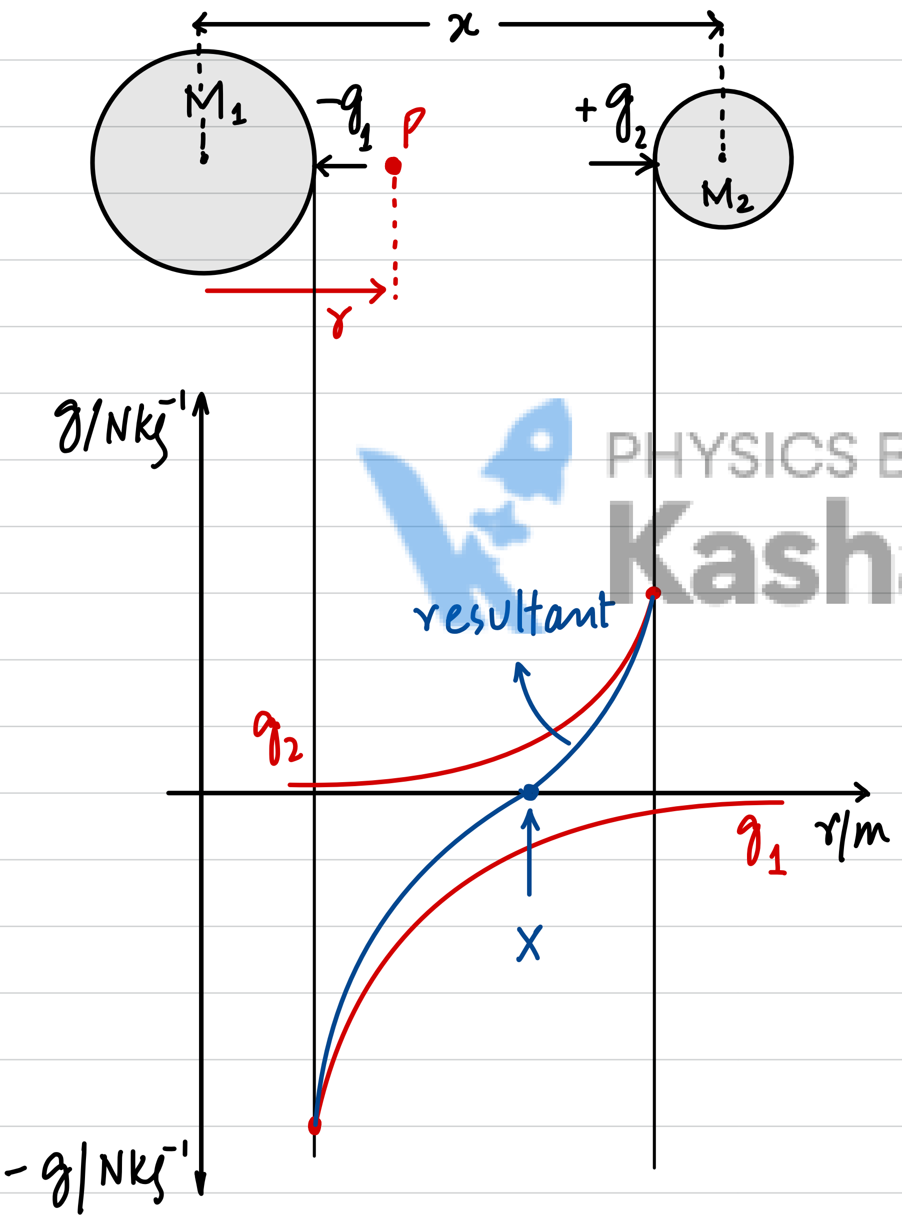




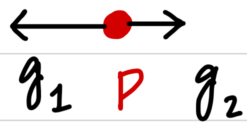
The ball reaches the other side when released from one side of the planet. It accelerates half way till the center and decelerates rest of the journey as direction of field strength changes!

Gravitational Field Strength between two point masses

$g = \frac{GM}{r^2}$



To find the resultant gravitational field strength at any point P, which is a distance "r" from M_1,



As "g" is a vector, to find resultant, perform vector addition i.e.

$\vec{g} = \vec{g}_1 + \vec{g}_2$ or $\boxed{\vec{g} = -g_1 + g_2}$

just like forces!

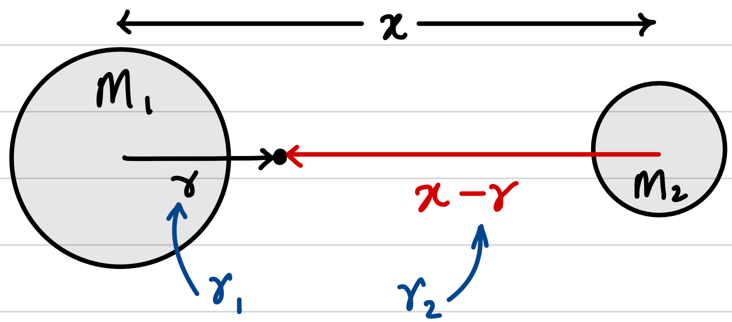
X: Null point i.e. resultant field strength is zero!

- If masses are unequal, <u>Null point is produced closer to smaller mass.</u>
- If masses are equal, it is exactly in the middle of both.

$$\vec{g} = -g_1 + g_2$$

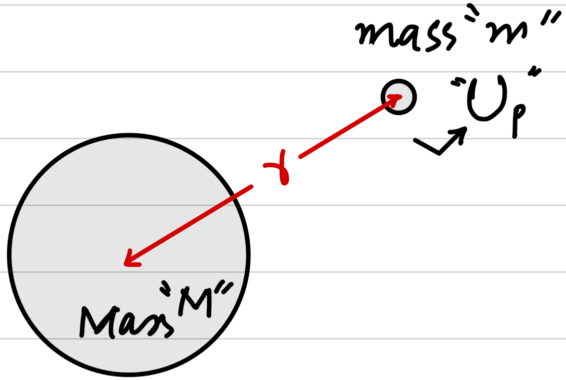
$$g = -\frac{GM_1}{r_1^2} + \frac{GM_2}{r_2^2}$$

$$\boxed{g = -\frac{GM_1}{r^2} + \frac{GM_2}{(x-r)^2}}$$

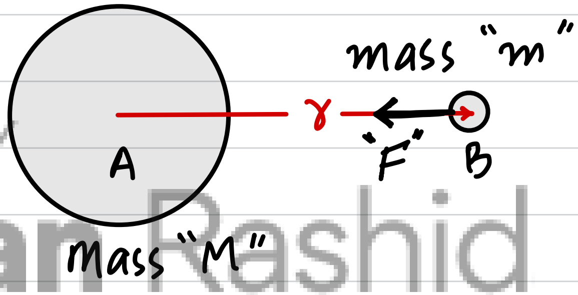


Gravitational Potential Energy (U_p)

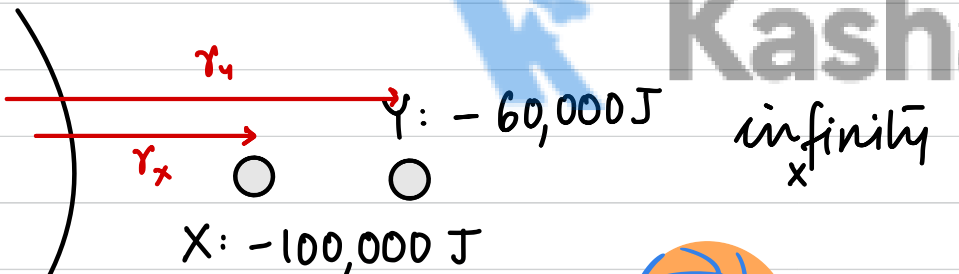
- Workdone to move an object from infinity to a point in the gravitational field.



- Gravitational Potential energy of any object at any point is "negative".



An object of mass "m" moved from A to B.



$$\text{Workdone} = F \times s$$

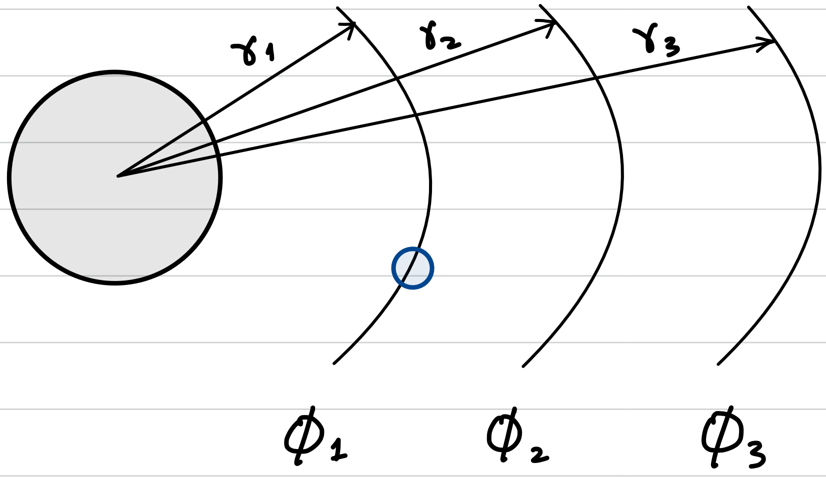
$$= -\frac{GMm}{r^2} \times r$$

direction of force is negative

$$\boxed{\text{Workdone} = U_p = -\frac{GMm}{r}}$$

- U_p at any point tells the energy that object needs to go back to infinity. It is not the energy it has over there !!

Gravitational Potential (ϕ)



Workdone per unit mass to move an object from infinity to a point in the gravitational field.

$$\phi = \frac{U_p}{m} \quad \text{or} \quad U_p = \phi \times m$$

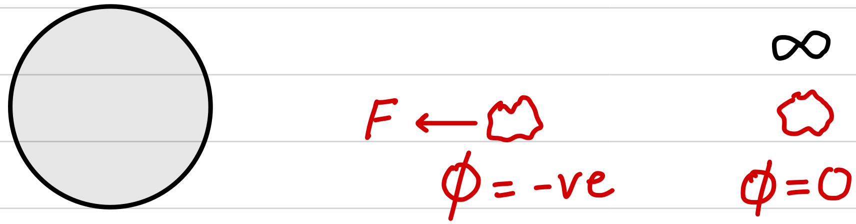
$$\phi = -\frac{GM\cancel{m}}{r \times \cancel{m}} \quad \text{so} \quad \boxed{\phi = -\frac{GM}{r}}$$

ϕ is the value needed per kg of mass to send an object from a point back to infinity !!

e.g. $\phi_1 = -4 \times 10^5 \text{ J/kg}$ means 1 kg object at location 1 needs 4.0×10^5 J of energy to go back to infinity.

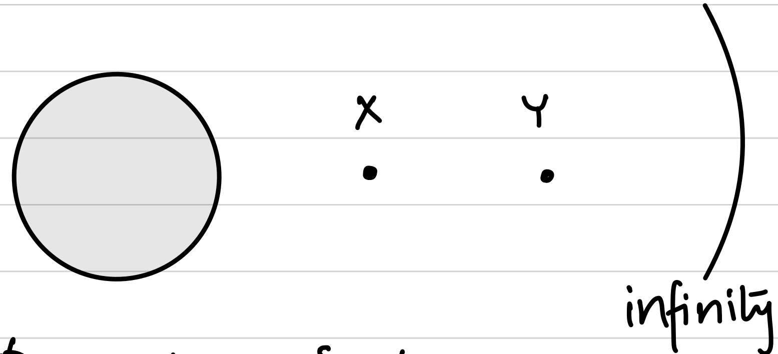
Why is gravitational Potential negative?

- Gravitational Potential is considered zero at infinity.
  (Reason: At infinity, there is no field strength, so no force and hence no tendency/potential to do work or move)



- As the body moves towards the planet, the force is attractive and hence work is done by the mass!

- Thus potential decreases from zero and is therefore negative.



$\phi_X = -4.0 \times 10^5$ J/kg

$\phi_Y = -3.0 \times 10^5$ J/kg

Closer to the planet, the value of ϕ gets more and more negative.

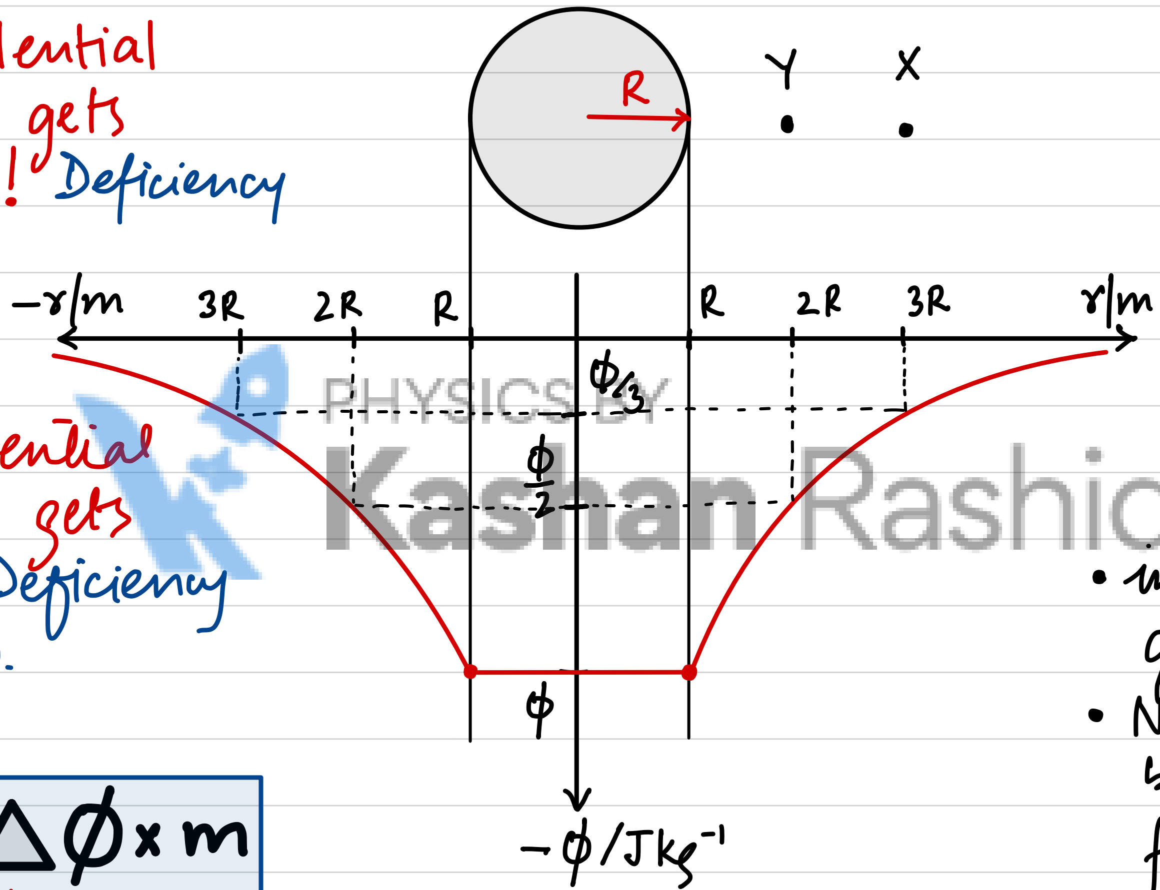
This is because its further away from infinity when its closer to planet and more energy will be needed to go back.

Graph of Gravitational Potential of a point mass

$\phi = -\frac{GM}{r}$

$\phi \propto -\frac{1}{r}$

from X to Y: potential decreases as it gets more negative! Deficiency of energy inc.



r	ϕ
R	ϕ
2R	$\phi/2$
3R	$\phi/3$

from Y to X: potential increases as it gets less negative. Deficiency of energy falls.

- inside the planet, $g = 0$ so force $= 0$.
- No more workdone by mass and no further loss in ϕ!
- ϕ is now constant. Same value everywhere inside the planet as on the surface.

$$\boxed{\Delta U_p = \Delta\phi \times m}$$

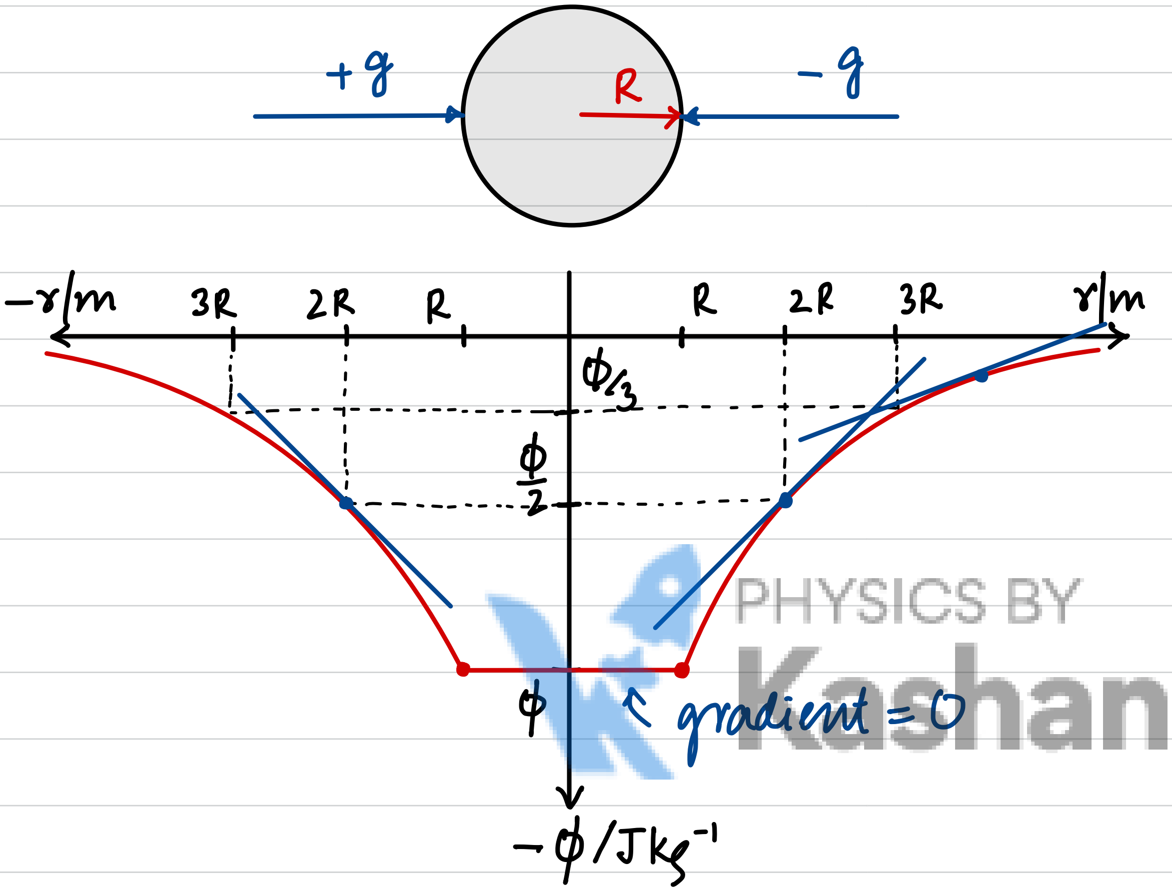
as $U_p = \phi \times m$.

$\Delta\phi = \phi_x - \phi_Y$

or

$\Delta\phi = \phi_Y - \phi_x$

(final - initial)



$$\text{gradient} = \frac{\Delta y}{\Delta x}$$

$$\text{gradient} = \frac{\Delta \phi}{\Delta r}$$

$$\frac{\phi}{r} = -\frac{GM}{r \cdot r} \Rightarrow \frac{\phi}{r} = -\frac{GM}{r^2} \leftarrow g$$

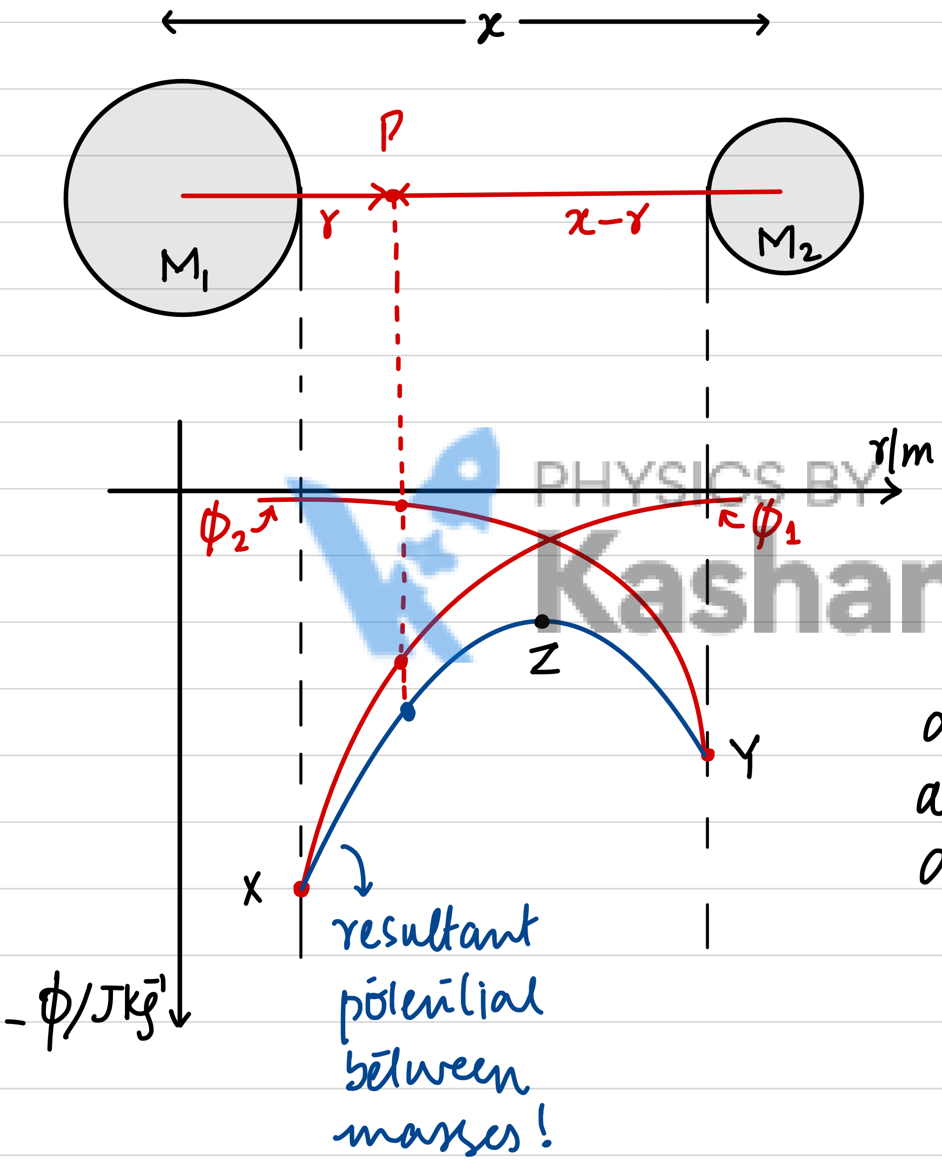
$$\frac{\phi}{r} = -g$$

$$-g = \frac{\Delta \phi}{\Delta r} \quad \text{or} \quad \boxed{g = -\frac{\Delta \phi}{\Delta r}}$$

Gravitational field strength is the negative of gravitational potential gradient.

- ☑ Inside the planet, $g = 0$ so gradient $= 0$
- ☑ As we move away, "g" decreases so the gradient of the graph also decreases!
- ☑ Negative sign was to make correction for direction of field.

Gravitational Potential between two point masses.



$$\phi = -\frac{GM}{r}$$

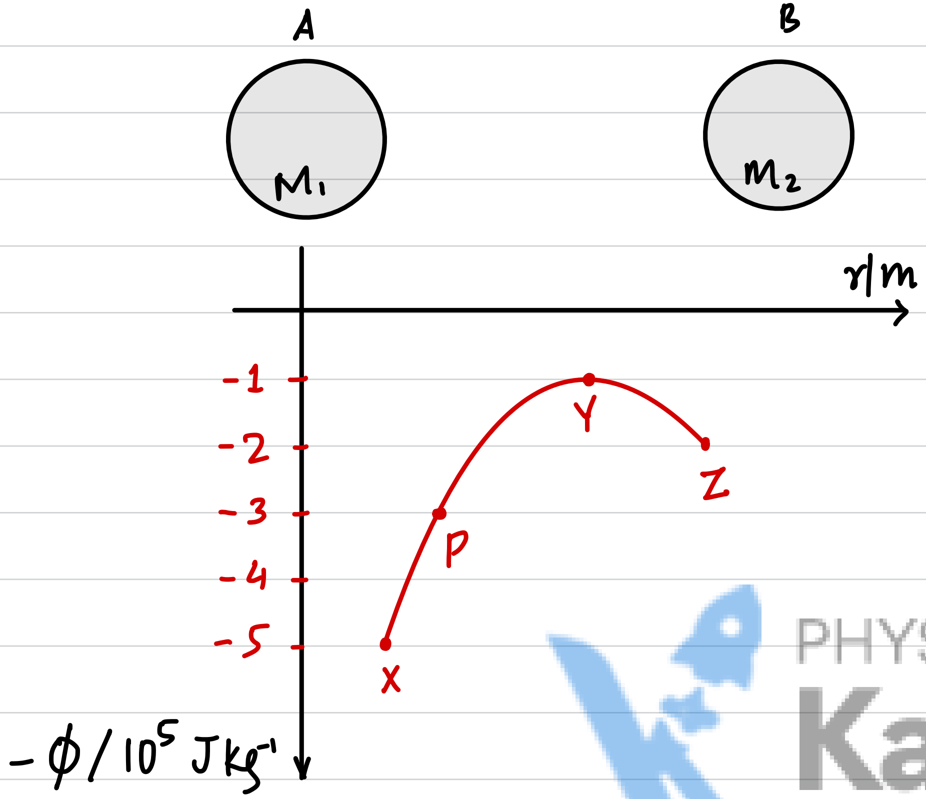
Gravitational Potential is a scalar quantity

$$\boxed{\phi = \phi_1 + \phi_2}$$

$$\phi = -\frac{GM_1}{r} + \left(-\frac{GM_2}{x-r}\right)$$

at X : gradient = max so g = max
at Z : gradient = zero so g = 0
at Y: gradient = less than at X so g = less than at X.

change in GPE = change in K.E
$\Delta\phi \times m = \frac{1}{2} m(v^2 - u^2)$



A rock of mass 200 kg travels between two planets A & B. Calculate

i, It speed of rock when it travels from point P to X. (rest at P)

ii, Its speed when it reaches Z when thrown from X at a speed of 1500 ms^{-1}

iii, Minimum speed needed to reach Z when thrown from X.

i, loss in GPE = gain in K.E

$$-\Delta\phi \times m = \frac{1}{2} m (v^2 - u^2)$$

$$-(\phi_2 - \phi_1) = \frac{1}{2}(v^2 - u^2)$$

$$-(-5\times10^5 - (-3\times10^5)) = \frac{1}{2}(v^2 - 0)$$

$$\sqrt{(2\times10^5)\times 2} = \sqrt{v^2}$$

$$\boxed{v = 632\ ms^{-1}}$$

ii, gain in GPE = loss in K.E

$$\Delta\phi \times m = -\frac{1}{2} m (v^2 - u^2)$$

$$(\phi_2 - \phi_1) = -\frac{1}{2}(v^2 - u^2)$$

$$(-2\times10^5 - (-5\times10^5)) = -\frac{1}{2}(v^2 - 1500^2)$$

$$\boxed{v = 1284.5 \simeq 1280\ ms^{-1}}$$

iii) gain in GPE = loss in K.E

$$\Delta\phi \times \cancel{m} = -\frac{1}{2}\cancel{m}(v^2 - u^2)$$

$$(\phi_y - \phi_x) = -\frac{1}{2}(v^2 - u^2)$$

$$(-1 \times 10^5 - (-5 \times 10^5)) = -\frac{1}{2}(0^2 - u^2)$$

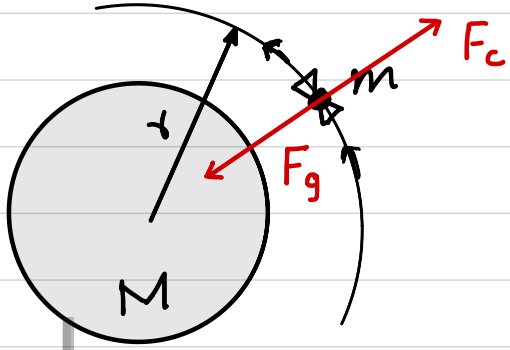
$$\boxed{u = 894\ ms^{-1}}$$

To reach surface of M_2, the object must have enough speed / kinetic energy to cross the max ϕ point in middle. Once it crosses "Y", it'll itself fall to Z. So for minimum speed, potential at Y is to be considered

Kepler's Law (Kepler's 3rd Law)

For a body in an orbit around another body, Kepler's Law help to relate the time period of orbit with the radius of orbit

For a body in orbit, Gravitational force acts as a centripetal force.



$$F_g = F_c$$

$$\frac{GmM}{r^2} = mr\omega^2$$

$$\boxed{T^2 \propto r^3}$$

"Square of the time period of a satellite in an orbit is directly proportional to the cube of the radius of orbit"

$$\frac{GM}{r^3} = \left(\frac{2\pi}{T}\right)^2$$

$$\frac{GM}{r^3} = \frac{4\pi^2}{T^2}$$

$$\boxed{T^2 = \frac{4\pi^2}{GM} r^3}$$

Geostationary Satellite and Geostationary Orbit

- Geostationary satellites are those which orbit a planet at the exact same angular velocity and in the same direction as planet rotates. It appears to be stationary above a point on planet.

- Geostationary orbit is an equitorial orbit, where the satellite has the same angular speed as that of the planet, and travels in the same direction as rotation of planet

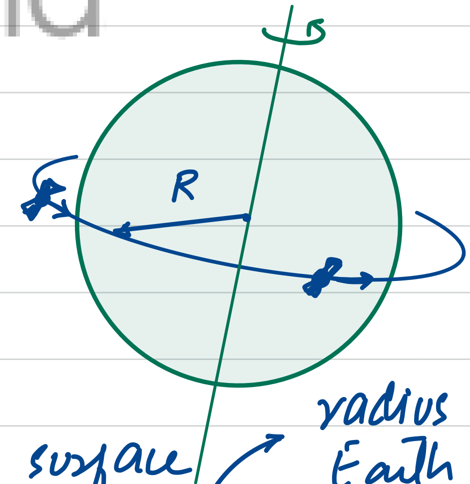
For the radius of a geostationary orbit,

$T_{orbit} = T_{planet} = 24$ hours

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$$(24 \times 3600)^2 = \frac{4\pi^2 \times r^3}{(6.67 \times 10^{-11})(6 \times 10^{24})}$$

$r_{orbit} = R = 4.23 \times 10^7 m$



Above the Earth's surface

radius of Earth

$r = 4.23 \times 10^7 - (6.4 \times 10^6)$

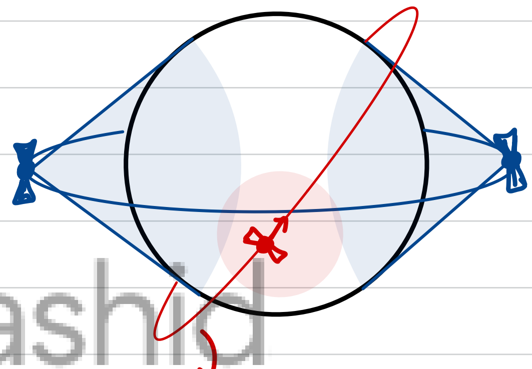
$r = 3.59 \times 10^7 m$ above Earth's surface!

Speed of satellite in geostationary orbit : $v = r\omega$ so $V = (4.23 \times 10^7)\left(\dfrac{2\pi}{24 \times 3600}\right)$

$V = 3076$ or $3000\ ms^{-1}$!!

Polar Satellite

These satellites have the angular velocity greater than that of planet and are moving closer to poles inorder to cover greater region and act as a link between different geostationary satellites.



$$T^2 = \frac{4\pi^2}{GM} r^3 \qquad \downarrow T^2 \propto r^3 \downarrow$$

For greater speed i.e. less time period to orbit, polar satellites have a smaller radius of orbit. They orbit closer to the surface of planet.

Energy of a Satellite

1. Potential Energy

$$\boxed{U_p = -\frac{GMm}{r}}$$

- As the radius of orbit is fixed, the U_p of the satellite is constant.
- If radius decreases, there will a loss in Grav. Potential Energy as U_p gets more negative.

2. Kinetic Energy

$$E_k = \frac{1}{2}mv^2 \quad \text{— (1)}$$

As Gravitational force acts as a centripetal force.

$$F_g = F_c$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$\frac{GM}{r} = v^2 \quad \text{— (2)}$$

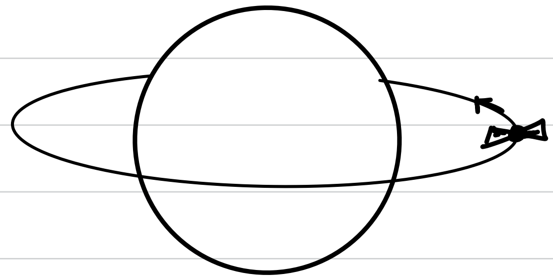
substituting (2) in (1)

$$E_k = \frac{1}{2}m \cdot \frac{GM}{r}$$

$$\boxed{E_k = \frac{GMm}{2r}}$$

For satellite in an orbit

3. Total Energy



$$U_p = -\frac{GMm}{r}$$

$$E_K = \frac{GMm}{2r}$$

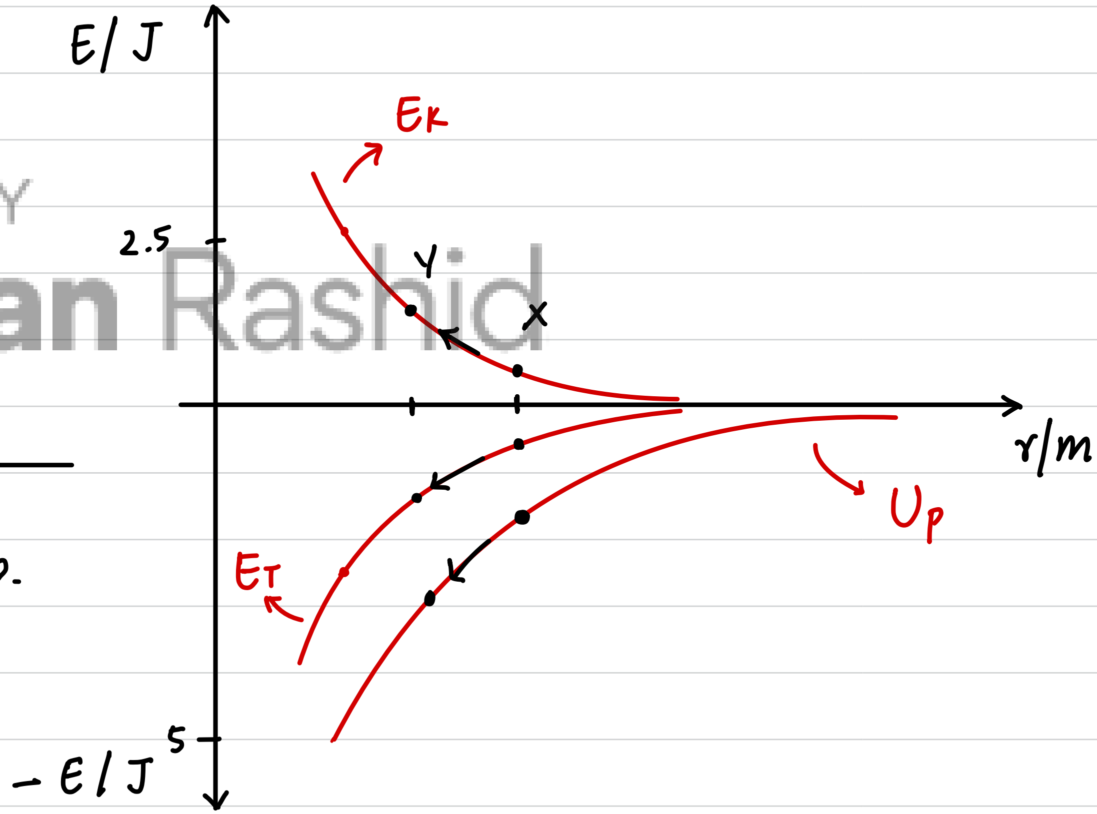
$$E_T = -\frac{GMm}{2r}$$

$$E_T = E_K + U_p$$

$$= \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right)$$

$$= \frac{GMm}{r}\left(\frac{1}{2} - 1\right)$$

$$\boxed{E_T = -\frac{GMm}{2r}}$$



If the radius of satellite decreases.

U_p : decreases

E_K : increases ($\downarrow T^2 \propto r^3 \downarrow$)

E_T : decreases

If the radius of satellite decreases.

U_p: decreases

The satellite got closer to the planet and further away from infinity. The deficiency of energy increased. It now needs more energy to go back to infinity.

E_K: increases ($\downarrow T^2 \propto r^3 \downarrow$)

- The decrease in radius of orbit decreases the time it takes to orbit the planet.
- The circumference of the orbit also decreases but less than the decrease in time.

$$v = \frac{d\downarrow}{t\downarrow}$$

E_T: decreases

The decrease in U_p is double than the increase in kinetic energy. Hence, there is an overall loss in total energy.

$$E_K = \frac{GMm}{2r} \qquad U_p = -\frac{GMm}{r}$$

$$U_p = -2E_K$$

$$T^2 \propto r^3$$

$$T^2 \text{ — } R^3$$

$$t^2 \text{ — } (0.5R)^3$$

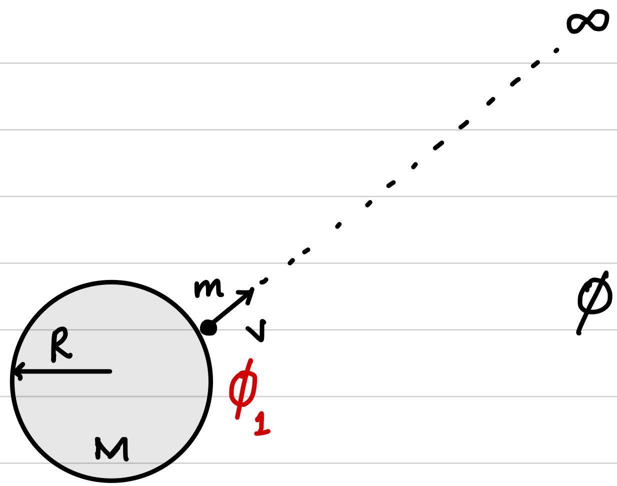
$$\frac{T^2 \times (0.5R)^3}{R^3} = t^2$$

$$t^2 = \frac{T^2 \times 0.5^3 \cancel{R^3}}{\cancel{R^3}}$$

$$t = 0.35T$$

Escape Velocity

The minimum speed of projection needed to make an object escape the planet's gravitational field.



$\phi_2 = 0$

$$\phi = -\frac{GM}{r}$$

loss in E_K = gain in E_p

$$-\frac{1}{2}m(v^2-u^2) = +\Delta\phi \times m$$

$$-\frac{1}{2}(v^2-u^2) = (\phi_2 - \phi_1)$$

$$-\frac{1}{2}(0-u^2) = \left(0-\left(-\frac{GM}{R}\right)\right)$$

$$\frac{u^2}{2} = \frac{GM}{R}$$

$$\boxed{u = \sqrt{\frac{2GM}{R}}}$$

Solving for Earth

mass of Earth: 6.0×10^{24} kg

radius of Earth: 6.4×10^6 m

$$u = \sqrt{\frac{6.67\times10^{-11} \times 6.0\times10^{24} \times 2}{6.4\times10^6}}$$

$$\boxed{u = 1.12 \times 10^4 \ ms^{-1}}$$