Cambridge O Level

Cambridge International Examinations
Cambridge Ordinary Level

CANDIDATE NAME		
CENTRE NUMBER		CANDIDATE NUMBER

ADDITIONAL MATHEMATICS **4037/11**

Paper 1 **May/June 2017**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **12** printed pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The line $y = kx - 5$, where k is a positive constant, is a tangent to the curve $y = x^2 + 4x$ at the point A.

(i) Find the exact value of k. [3]

$$kx - 5 = x^2 + 4x$$

$$x^2 + (4 - k)x + 5$$

$$16 - 8k + k^2 - 4(1)(5)$$

$$16 - 8k + k^2 - 20 = 0$$

$$k^2 - 8k - 4 = 0$$

$$k = 4 + 2\sqrt{5}$$

(ii) Find the gradient of the normal to the curve at the point A, giving your answer in the form $a + b\sqrt{5}$, where a and b are constants. [2]

$$-\frac{1}{4+2\sqrt{5}}$$

$$-\frac{1}{4+2\sqrt{5}} \times \frac{4-2\sqrt{5}}{4-2\sqrt{5}}$$

$$\frac{-4+2\sqrt{5}}{-4}$$

$$\boxed{1 - \frac{1}{2}\sqrt{5}}$$

2 It is given that $p(x) = x^3 + ax^2 + bx - 48$. When $p(x)$ is divided by $x - 3$ the remainder is 6. Given that $p'(1) = 0$, find the value of a and of b. [5]

$6 = 27 + 9a + 3b - 48$

$0 = 27 + 9a + 3b - 54$

$\frac{dp}{dx} = 3x^2 + 2ax + b$

$= 3 + 2a + b$

$\boxed{b = -3 - 2a}$

$-9a = 3(-3-2a) - 27$

$= -9 - 6a - 27$

$-3a = 36$

$\boxed{a = -12}$

$\boxed{b = -27}$

3 **(a)** Simplify $\sqrt{x^8 y^{10}} \div \sqrt[3]{x^3 y^{-6}}$, giving your answer in the form $x^a y^b$, where a and b are integers. [2]

$\frac{x^4 y^5}{x\, y^{-2}} = \boxed{x^3 y^7}$

(b) **(i)** Show that $4(t-2)^{\frac{1}{2}} + 5(t-2)^{\frac{3}{2}}$ can be written in the form $(t-2)^p(qt + r)$, where p, q and r are constants to be found. [3]

$4(t-2)^{\frac{1}{2}} + 5(t-2)^{\frac{1}{2}} \times (t-2)$

$4x + 5x \times x^2$

$4x + 5x^3$

$x(4 + 5x^2)$

$(t-2)^{\frac{1}{2}}(4 + 5t - 10)$

$(t-2)^{\frac{1}{2}}(5t - 6)$

(ii) Hence solve the equation $4(t-2)^{\frac{1}{2}} + 5(t-2)^{\frac{3}{2}} = 0$. [1]

$\sqrt{t-2} = 0$ $\qquad$ $5t - 6 = 0$

$\boxed{t = 2}$ $\qquad$ $\boxed{t = \frac{6}{5}}$

4 **(a)** It is given that $f(x) = 3e^{-4x} + 5$ for $x \in \mathbb{R}$.

(i) State the range of f. [1]

$f(x) > 5$

(ii) Find f^{-1} and state its domain. [4]

$x = 3e^{-4y}$

$\frac{x-5}{3} = e^{-4y}$

$\ln\left(\frac{x-5}{3}\right) = -4y$

$y = \frac{1}{4}\ln\left(\frac{x-5}{3}\right)$ $x > 5$

(b) It is given that $g(x) = x^2 + 5$ and $h(x) = \ln x$ for $x > 0$. Solve $hg(x) = 2$. [3]

$h(g(x)) = \ln(x^2 + 5)$

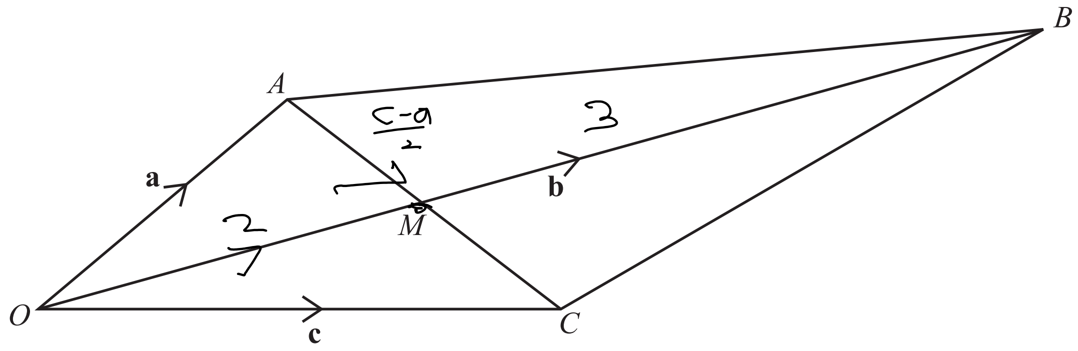
$2 = \ln(x^2 + 5)$

$e^2 = x^2 + 5$

$\sqrt{e^2 - 5} = x$

$1.545 = x$

5 **(a)**



The diagram shows a figure $OABC$, where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. The lines AC and OB intersect at the point M where M is the midpoint of the line AC.

(i) Find, in terms of $\mathbf{a}$ and $\mathbf{c}$, the vector $\overrightarrow{OM}$. [2]

$$AC = \frac{c-a}{2}$$

$$OM = \frac{c-a}{2} + \frac{2a}{2}$$

$$= \frac{c+a}{2}$$

(ii) Given that $OM : MB = 2:3$, find $\mathbf{b}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [2]

$$OM = \frac{OB}{5} \times 2$$

$$\frac{5c+5a}{4} = OB$$

(b) Vectors **i** and **j** are unit vectors parallel to the x-axis and y-axis respectively.

The vector **p** has a magnitude of 39 units and has the same direction as $-10\mathbf{i} + 24\mathbf{j}$.

(i) Find **p** in terms of **i** and **j**. [2]

$-15i + 36j$

$39^2 = x^2 + \frac{144}{25}x^2$

$= \frac{169}{25}x^2$

$225 = x^2$

$15 = x$, $y = 36$

(ii) Find the vector **q** such that $2\mathbf{p} + \mathbf{q}$ is parallel to the positive y-axis and has a magnitude of 12 units. [3]

$\begin{pmatrix} y \\ 0 \end{pmatrix} = \begin{pmatrix} -30 \\ 72 \end{pmatrix} + \begin{pmatrix} zi \\ tj \end{pmatrix}$ $\begin{pmatrix} y \\ 0 \end{pmatrix}$

$q = \begin{pmatrix} x \\ -72 \end{pmatrix}$

$z^2 - 60z + 900 = 12^2$

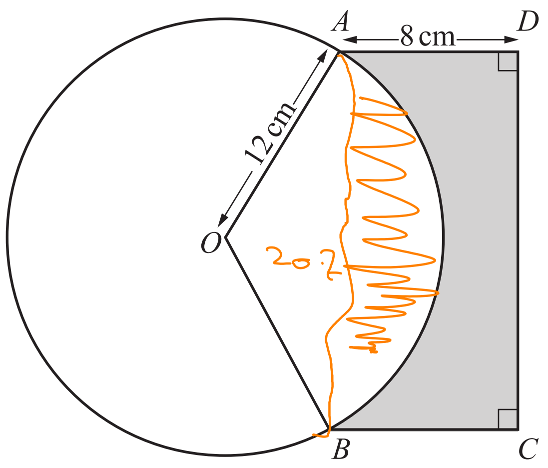
$z = \boxed{42}, 18$

$q = 42i - 72j$

(iii) Hence show that $|\mathbf{q}| = k\sqrt{5}$, where k is an integer to be found. [2]

$\boxed{6\sqrt{193}}$

6



The diagram shows a circle, centre O, radius 12 cm. The points A and B lie on the circumference of the circle and form a rectangle with the points C and D. The length of AD is 8 cm and the area of the minor sector AOB is 150 cm^2.

(i) Show that angle AOB is 2.08 radians, correct to 2 decimal places. [2]

$300 = R\,(12)^2$

$= 2.08$

(ii) Find the area of the shaded region $ADCB$. [6]

$150 - \left(\frac{1}{2} \times 12 \times 12\right)$

$= 87.134$

$[65.6 - 87.134$

$= 78.46$

(iii) Find the perimeter of the shaded region $ADCB$. [3]

61.68

7 Show that the curve $y = (3x^2 + 8)^{\frac{5}{3}}$ has only one stationary point. Find the coordinates of this stationary point and determine its nature. [8]

$$\frac{dy}{dx} = \frac{5}{\cancel{3}} \times \cancel{6}^{2}x \, (3x^2 + 8)^{\frac{5}{3}}$$

$$= 10x\,(3x^2+8)^{\frac{2}{3}}$$

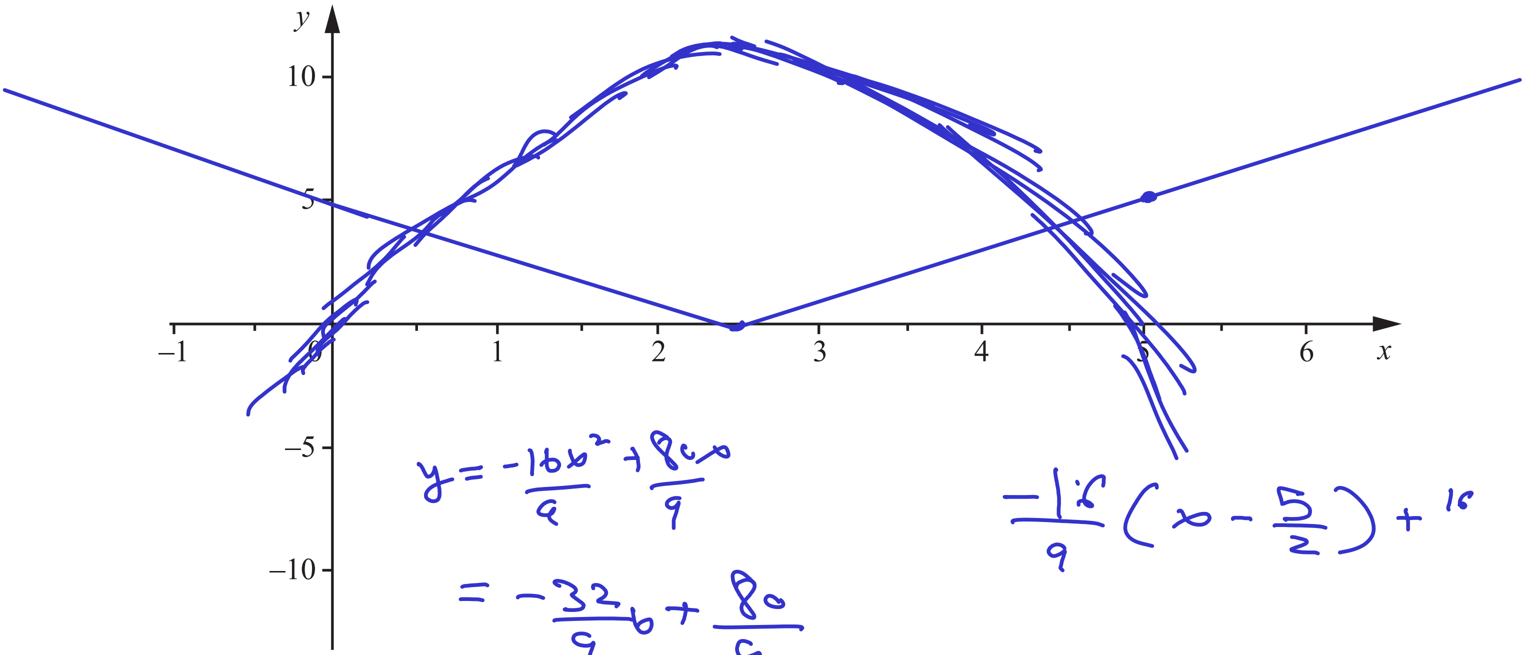
$10x = 0$

$\boxed{x = 0}$

$3x^2 = -8$

$\boxed{x = \text{No solution}}$

8 **(i)** On the axes below sketch the graphs of $y = |2x - 5|$ and $9y = 80x - 16x^2$. [5]



$y = \frac{-16x^2}{9} + \frac{80x}{9}$

$= -\frac{32}{9}x + \frac{80}{9}$

$\frac{-80}{9} =$

$\frac{-16}{9}\left(x - \frac{5}{2}\right) + 16$

(ii) Solve $|2x - 5| = 4$. [3]

$x = \frac{5}{2}, 11.$

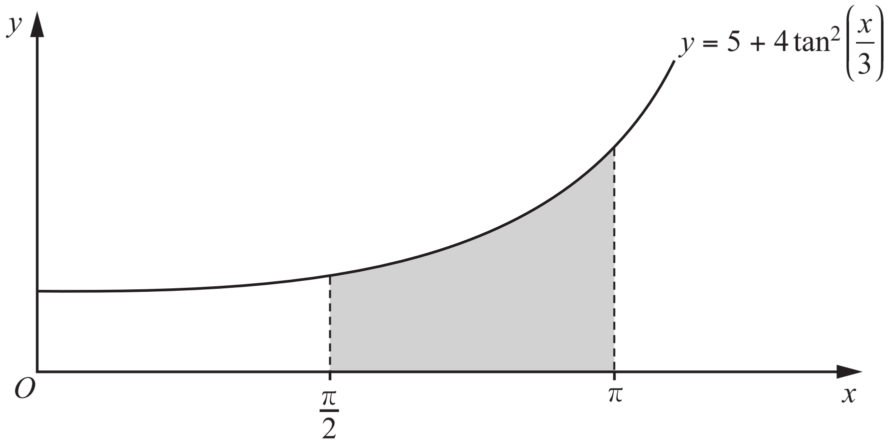
(iii) Hence show that the graphs of $y = |2x - 5|$ and $9y = 80x - 16x^2$ intersect at the points where $y = 4$. [1]

(iv) Hence find the values of x for which $9|2x - 5| \leqslant 80x - 16x^2$. [2]

9 **(i)** Show that $5 + 4\tan^2\left(\frac{x}{3}\right) = 4\sec^2\left(\frac{x}{3}\right) + 1$. [1]

(ii) Given that $\frac{d}{dx}\left(\tan\left(\frac{x}{3}\right)\right) = \frac{1}{3}\sec^2\left(\frac{x}{3}\right)$, find $\int \sec^2\left(\frac{x}{3}\right)dx$. [1]

(iii)



The diagram shows part of the curve $y = 5 + 4\tan^2\left(\frac{x}{3}\right)$. Using the results from parts **(i)** and **(ii)**, find the exact area of the shaded region enclosed by the curve, the x-axis and the lines $x = \frac{\pi}{2}$ and $x = \pi$. [5]

Question 10 is printed on the next page.

10 **(a)** Given that $y = \dfrac{e^{3x}}{4x^2 + 1}$, find $\dfrac{dy}{dx}$. [3]

(b) Variables x, y and t are such that $y = 4\cos\left(x + \dfrac{\pi}{3}\right) + 2\sqrt{3}\sin\left(x + \dfrac{\pi}{3}\right)$ and $\dfrac{dy}{dt} = 10$.

(i) Find the value of $\dfrac{dy}{dx}$ when $x = \dfrac{\pi}{2}$. [3]

(ii) Find the value of $\dfrac{dx}{dt}$ when $x = \dfrac{\pi}{2}$. [2]