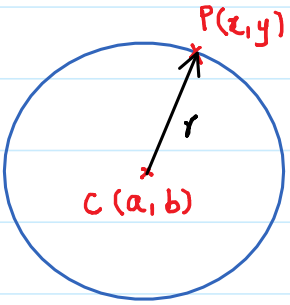
CIRCLES:

Tuesday, October 12, 2021 10:55 AM

→ Circle is the collection of those points which are equidistant from a fixed point. The fixed point is called the center of circle and the fixed distance is called radius of circle.

→ Standard Equation of circle:



$$CP = r$$

$$\sqrt{(x-a)^2 + (y-b)^2} = r$$

$$\boxed{(x-a)^2 + (y-b)^2 = r^2}$$

Centre : (a, b) , Radius = r

1 Find the centre and the radius of each of the following circles.

a $x^2 + y^2 = 16$

b $2x^2 + 2y^2 = 9$

c $x^2 + (y-2)^2 = 25$

d $(x-5)^2 + (y+3)^2 = 4$

a) $x^2 + y^2 = 16$

$(x-0)^2 + (y-0)^2 = 4^2$

$(x-a)^2 + (y-b)^2 = r^2$

Centre : $(0,0)$

$r = 4$

b) $2x^2 + 2y^2 = 9$

$x^2 + y^2 = \frac{9}{2}$

$(x-0)^2 + (y-0)^2 = \left(\frac{3}{\sqrt{2}}\right)^2$

$(x-a)^2 + (y-b)^2 = r^2$

Centre : $(0,0)$

$r = 3/\sqrt{2}$ or $3\sqrt{2}/2$

c) $x^2 + (y-2)^2 = 25$

$(x-0)^2 + (y-2)^2 = 5^2$

$(x-a)^2 + (y-b)^2 = r^2$

Centre $(0, 2)$

$r = 5$

d) $(x-5)^2 + (y+3)^2 = 4 = 2^2$

$(x-a)^2 + (y-b)^2 = r^2$

Centre : $(5, -3)$

$r = 2$

2 Find the equation of each of the following circles.

a centre (0, 0), radius 8

b centre (5, –2), radius 4

c centre (–1, 3), radius $\sqrt{7}$

d centre $\left(\frac{1}{2}, -\frac{3}{2}\right)$, radius $\frac{5}{2}$

Sol a) $(x-0)^2 + (y-0)^2 = 8^2$

$x^2 + y^2 = 64$

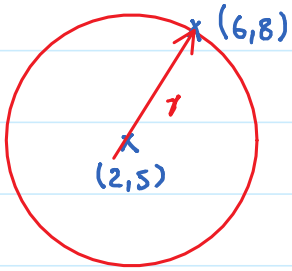
b) $(x-5)^2 + (y+2)^2 = 4^2$

$(x-5)^2 + (y+2)^2 = 16$

3 Find the equation of the circle with centre (2, 5) passing through the point (6, 8).

$r = \sqrt{4^2 + 3^2}$

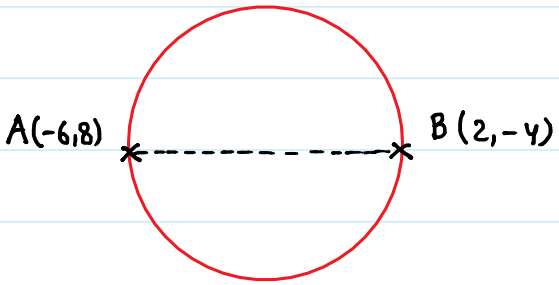
$r = 5$



$(x-2)^2 + (y-5)^2 = 25$ Ans

4 A diameter of a circle has its end points at $A(-6, 8)$ and $B(2, -4)$.

Find the equation of the circle.



General Equation of circle:

$$(x-a)^2 + (y-b)^2 = r^2$$

$$x^2 - 2ax + a^2 + y^2 - 2by + b^2 - r^2 = 0$$

$$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$$

Replace $a = -g$ and $b = -f$

$$x^2 + y^2 - 2(-g)x - 2(-f)y + (-g)^2 + (-f)^2 - r^2 = 0$$

$$x^2 + y^2 + 2gx + 2fy + g^2 + f^2 - r^2 = 0$$

Replace $c = g^2 + f^2 - r^2$

$$\boxed{x^2 + y^2 + 2gx + 2fy + c = 0}$$

Centre: $(-g, -f)$

$\because$ $c = g^2 + f^2 - r^2$ ←

$$r^2 = g^2 + f^2 - c$$

$$r = \sqrt{g^2 + f^2 - c}$$

1 Find the centre and the radius of each of the following circles.

g $x^2 + y^2 - 8x + 20y + 110 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -8$, $2f = 20$, $c = 110$

$g = -4$, $f = 10$

Centre : $(-g, -f) = (4, -10)$

$r = \sqrt{16 + 100 - 110}$

$r = \sqrt{6}$

h $2x^2 + 2y^2 - 14x - 10y - 163 = 0$

$x^2 + y^2 - 7x - 5y - 81.5 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -7$, $2f = -5$, $c = -81.5$

$g = -3.5$, $f = -2.5$

Centre : $(3.5, 2.5)$

$r = \sqrt{(-3.5)^2 + (-2.5)^2 + 81.5}$

$r = 10$

$r = 10$

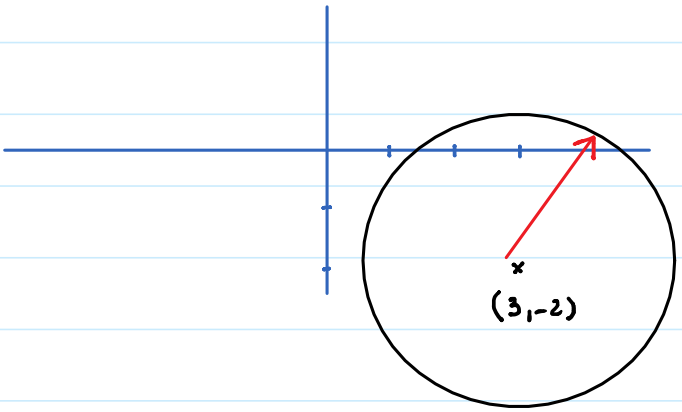
5 Sketch the circle $(x-3)^2 + (y+2)^2 = 9$.

$(x-3)^2 + (y+2)^2 = 3^2$

$(x-a)^2 + (y-b)^2 = r^2$

Centre: $(3,-2)$

$r = 3$



8 A circle passes through the points (3, 2) and (7, 2) and has radius $2\sqrt{2}$.

Find the two possible equations for this circle.

$(x-a)^2 + (y-b)^2 = r^2$

$(3-a)^2 + (2-b)^2 = 8$ — (1)

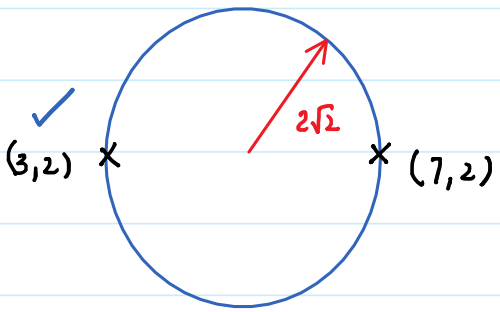
$(7-a)^2 + (2-b)^2 = 8$ — (2)

$(3-a)^2 - (7-a)^2 = 0$

$(3-a)^2 = (7-a)^2$

$9 - 6a + a^2 = 49 - 14a + a^2$

$14a - 6a = 49 - 9$



Eq (2) ⇒

$4 + (2-b)^2 = 8$

$(2-b)^2 = 4$

$2 - b = \pm 2$

$$7 - 6a + a^2 = 49 - 14a + a^2$$

$$14a - 6a = 49 - 9$$

$$8a = 40$$

$$\boxed{a = 5}$$

$$2 - b = \pm 2$$

$2 - b = 2$, $2 - b = -2$

$b = 0$, $b = 4$

Centre $(5, 0)$ & $(5, 4)$

$r = 2\sqrt{2}$

First Eq:

$$(x-5)^2 + (y-0)^2 = (2\sqrt{2})^2$$

$$(x-5)^2 + y^2 = 8 \text{ Ans}$$

2nd Eq.:

$$(x-5)^2 + (y-4)^2 = (2\sqrt{2})^2$$

$$(x-5)^2 + (y-4)^2 = 8 \text{ Ans}$$