TYPE 1: EQUATION SOLVING.

CASE 1

$\boxed{\sin + \cos}$

↓

tan

Case 2

QUADRATIC

CASE 1: (SIN + COS)(NO POWERS) $\longrightarrow$ tan.

19 **(i)** Show that the equation

$$3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$$

can be written in the form $\tan x = -\frac{3}{4}$. [2]

(ii) Solve the equation $3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$, for $0° \leqslant x \leqslant 360°$. [2]

(i) $6\sin x - 3\cos x = 2\sin x - 6\cos x$

$6\sin x - 2\sin x = 3\cos x - 6\cos x$

$4\sin x = -3\cos x$

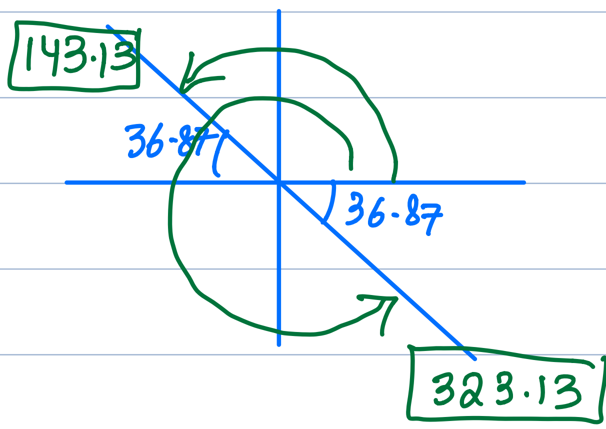
$\frac{\sin x}{\cos x} = -\frac{3}{4}$

$\tan x = -\frac{3}{4}$

(ii) $\tan x = -\frac{3}{4}$ $\quad\quad 0 < x < 360$

$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$

$\alpha = 36.87$



$x = 143.13, 323.13$

CASE 2: QUADRATIC EQUATION:-

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

4 Solve the equation $3\sin^2\theta - 2\cos\theta - 3 = 0$, for $0° \leqslant \theta \leqslant 180°$. [4]

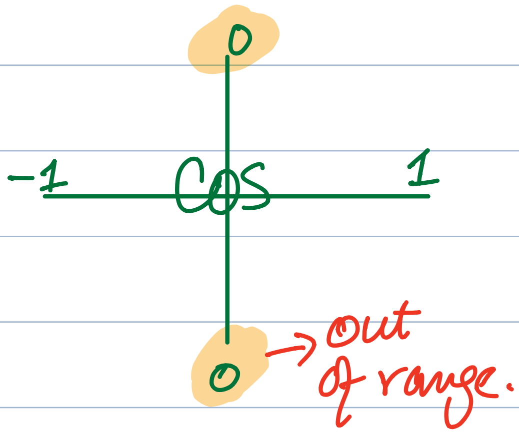
$$3(1-\cos^2\theta) - 2\cos\theta - 3 = 0$$

$$\cancel{3} - 3\cos^2\theta - 2\cos\theta - \cancel{3} = 0$$

$$-\cos\theta\,(3\cos\theta + 2) = 0$$

$$-\cos\theta = 0$$

$$\cos\theta = 0$$

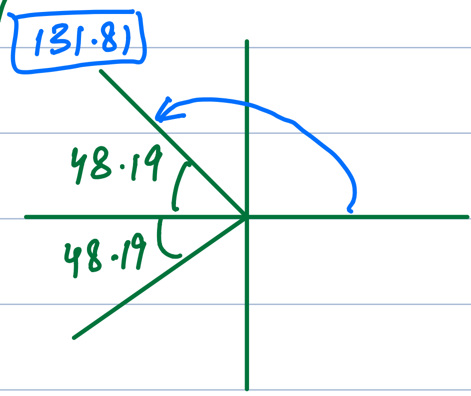


$$\boxed{\theta = 90}$$

$$3\cos\theta + 2 = 0$$

$$\cos\theta = -\frac{2}{3} \qquad 0 < \theta < 180$$

$$\alpha = \cos^{-1}\left(\frac{2}{3}\right) = 48.19$$



$$\boxed{\theta = 131.81}$$

10 **(i)** Show that the equation $3\sin x \tan x = 8$ can be written as $3\cos^2 x + 8\cos x - 3 = 0$. [3]

(ii) Hence solve the equation $3\sin x \tan x = 8$ for $0° \leqslant x \leqslant 360°$. [3]

(i)

$$3\sin x \tan x = 8$$

$$3\sin x \cdot \frac{\sin x}{\cos x} = 8$$

$$3\sin^2 x = 8\cos x$$

$$3(1-\cos^2 x) = 8\cos x$$

$$3 - 3\cos^2 x = 8\cos x$$

$$0 = 3\cos^2 x + 8\cos x - 3$$

(ii)

$$3\cos^2 x + 8\cos x - 3 = 0 \qquad 0 < x < 360$$

$$\boxed{\cos x = a}$$

$$3a^2 + 8a - 3 = 0$$

$$3a^2 + 9a - a - 3 = 0$$

$$3a(a+3) - 1(a+3) = 0$$

$$(3a-1)(a+3) = 0$$

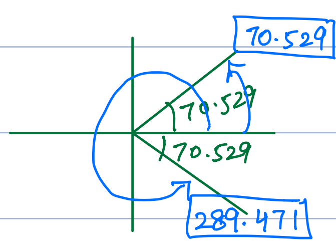
$$a = \frac{1}{3}, \qquad a = -3$$

$$\cos x = \frac{1}{3}, \qquad \cos x = -3$$

$\alpha = \cos^{-1}(3)$

No Solutions.

$$\alpha = \cos^{-1}\left(\frac{1}{3}\right) = 70.529$$



$$x = 70.529,\ 289.471$$

TYPE 2: VALUE OF sin/cos/tan is given and then find other trig ratios using identities.

M1

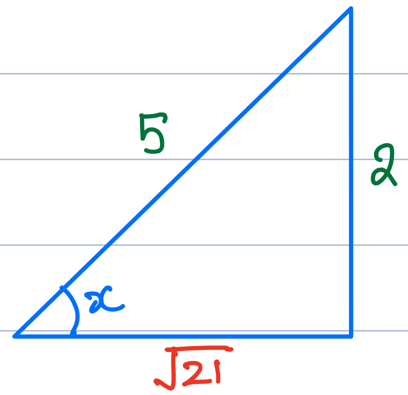
8 Given that $x = \sin^{-1}(\frac{2}{5})$, find the exact value of

NO CALCULATOR.

(i) $\cos^2 x$, [2]

(ii) $\tan^2 x$. [2]

$\sin x = \frac{2}{5}$ $\frac{P}{H}$



(i) $\cos^2 x$

$(\cos x)^2$

$\left(\frac{B}{H}\right)^2$

$\left(\frac{\sqrt{21}}{5}\right)^2$

$\boxed{\frac{21}{25}}$

(ii) $\tan^2 x$

$(\tan x)^2$

$\left(\frac{P}{B}\right)^2$

$\left(\frac{2}{\sqrt{21}}\right)^2$

$\boxed{\frac{4}{21}}$

$B^2 + 2^2 = 5^2$

$B^2 + 4 = 25$

$B^2 = 21$

$B = \sqrt{21}$

1st Quadrant.

18 The acute angle x radians is such that $\tan x = k$, where k is a positive constant. Express, in terms of k,

(i) $\tan(\pi - x)$, $= -\tan x = \boxed{-K}$ [1]

(ii) $\tan(\frac{1}{2}\pi - x)$, $= \frac{1}{\tan x} = \boxed{\frac{1}{K}}$ [1]

(iii) $\sin x$. [2]

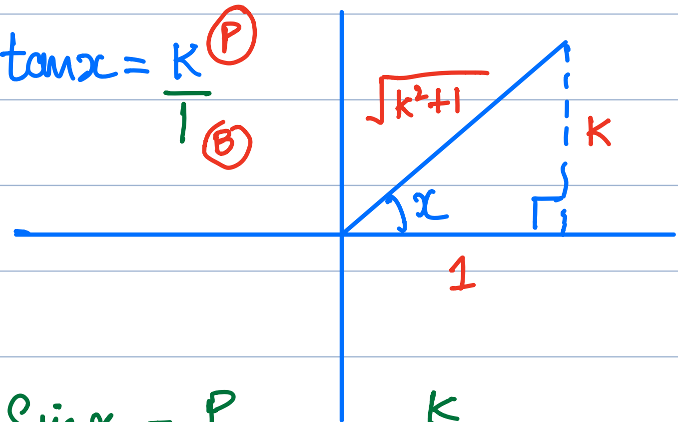
(i) $\tan(180 - x) = -\tan x$

$\tan(\pi - x) = -\tan x$

(ii) $\tan\left(\frac{1}{2}\pi - x\right) = \frac{1}{\tan x}$

$\tan(90 - x) = \frac{1}{\tan x}$

$\tan x = \frac{K}{1}$ P B



$\sin x = \frac{P}{H} = \frac{K}{\sqrt{K^2+1}}$

2nd Quadrant.

49 Given that θ is an obtuse angle measured in radians and that $\sin\theta = \frac{k}{1}$, find, in terms of k, an expression for

(i) $\cos\theta$, [1]

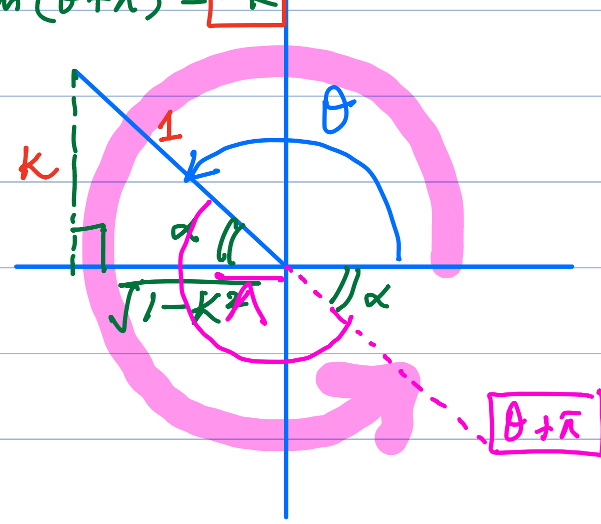
(ii) $\tan\theta$, [2]

(iii) $\sin(\theta + \pi)$. $\rightarrow$ Basic angle is same (α) $= \sin(\theta+\pi) \rightarrow \sin\alpha = K$ [1]

$\sin(\theta+\pi) = \boxed{-K}$

$\sin\theta = \frac{K}{1}$

$\sin\alpha = \frac{K}{1}$ P H



(i) $\cos\theta = \frac{B}{H}$

$$\cos\theta = \frac{-\sqrt{1-k^2}}{1}$$

Due to 2nd Quadrant
cos is negative-

$$1^2 = B^2 + k^2$$

$$B^2 = 1 - k^2$$

$$B = \sqrt{1-k^2}$$

$$\boxed{\cos\theta = -\sqrt{1-k^2}}$$

(ii) $\tan\theta = \dfrac{P}{B} = \dfrac{-k}{\sqrt{1-k^2}}$

$$\tan\theta = \frac{-k}{\sqrt{1-k^2}}$$

TYPE3: (a) Identity Proving.

(b) use identity from (a) part to solve an equation.

(In this type, always attempt second part first).

27 **(i)** Prove the identity $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 \equiv \frac{1-\cos\theta}{1+\cos\theta}$. [3]

(ii) Hence solve the equation $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 = \frac{2}{5}$, for $0° \leqslant \theta \leqslant 360°$. [4]

$$\frac{1-\cos\theta}{1+\cos\theta} = \frac{2}{5}$$

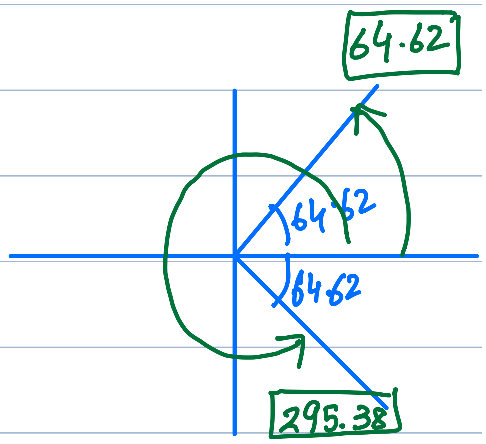
$$5 - 5\cos\theta = 2 + 2\cos\theta$$

$$5 - 2 = 5\cos\theta + 2\cos\theta$$

$$7\cos\theta = 3$$

$$\cos\theta = \frac{3}{7}$$

$$\alpha = \cos^{-1}\left(\frac{3}{7}\right) = 64.62$$



38 **(i)** Show that $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}\equiv\dfrac{1}{\sin^2\theta-\cos^2\theta}$. [3]

(ii) Hence solve the equation $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}=3$, for $0°\leqslant\theta\leqslant 360°$. [4]

$$\frac{1}{\sin^2\theta-\cos^2\theta}=3 \qquad 0<\theta<360$$

$$1=3\sin^2\theta-3\cos^2\theta$$

$$1=3(1-\cos^2\theta)-3\cos^2\theta$$

$$1=3-3\cos^2\theta-3\cos^2\theta$$

$$1=3-6\cos^2\theta$$

$$6\cos^2\theta=2$$

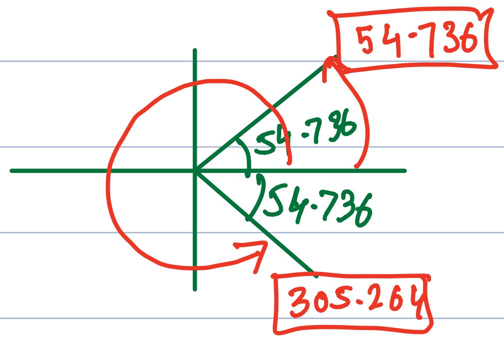
$$\cos^2\theta=\frac{1}{3}$$

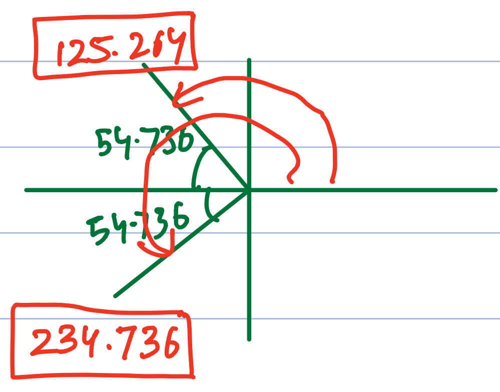
$$\cos\theta=\pm\sqrt{\frac{1}{3}}$$

$$\cos\theta=\sqrt{\frac{1}{3}} \qquad\qquad \cos\theta=-\sqrt{\frac{1}{3}}$$

$$\alpha=\cos^{-1}\left(\sqrt{\frac{1}{3}}\right) \qquad\qquad \alpha=\cos^{-1}\left(\sqrt{\frac{1}{3}}\right)$$

$$\alpha=54.736 \qquad\qquad \alpha=54.736$$





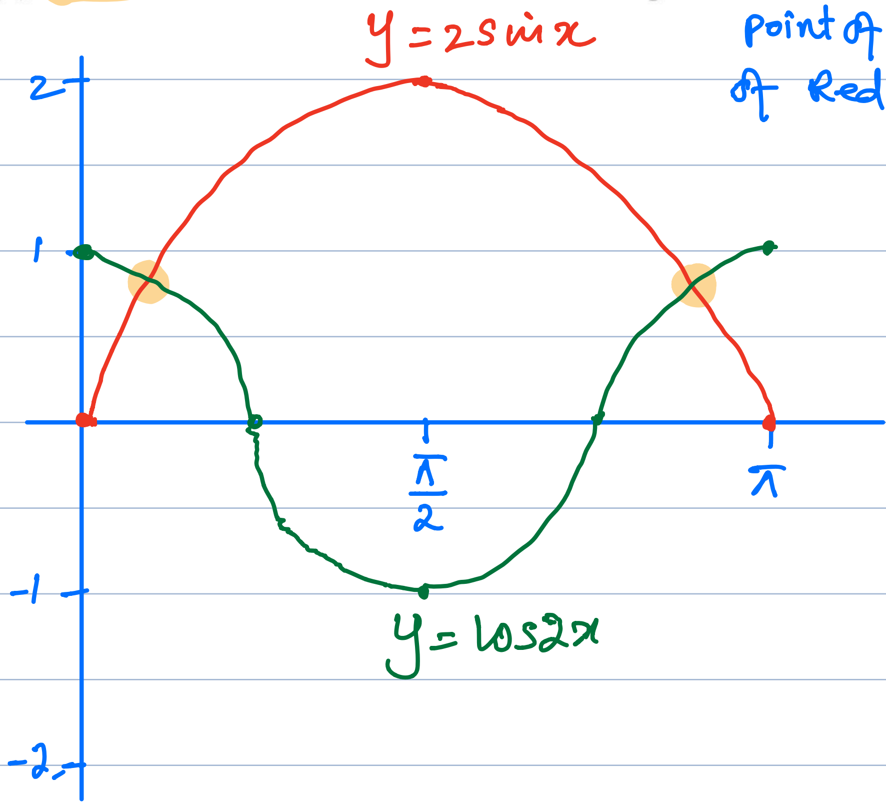
TYPE 4: GRAPHS.

2 **4(i)** Sketch and label, on the same diagram, the graphs of $y = 2\sin x$ and $y = \cos 2x$, for the interval $0 \leqslant x \leqslant \pi$. [4]

(ii) Hence state the number of solutions of the equation $2\sin x = \cos 2x$ in the interval $0 \leqslant x \leqslant \pi$. [1]

Point of intersection of Red & green graph.

(i)



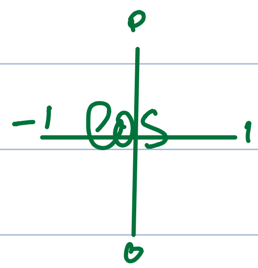
$y = \cos 2x$

2 cycles → 360

1 cycle → 180

÷4

$\boxed{45}$



(ii) 2 solutions.

24 **(i)** Sketch the curve $y = 2\sin x$ for $0 \leqslant x \leqslant 2\pi$. [1]

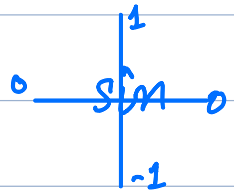
(ii) By adding a suitable straight line to your sketch, determine the number of real roots of the equation

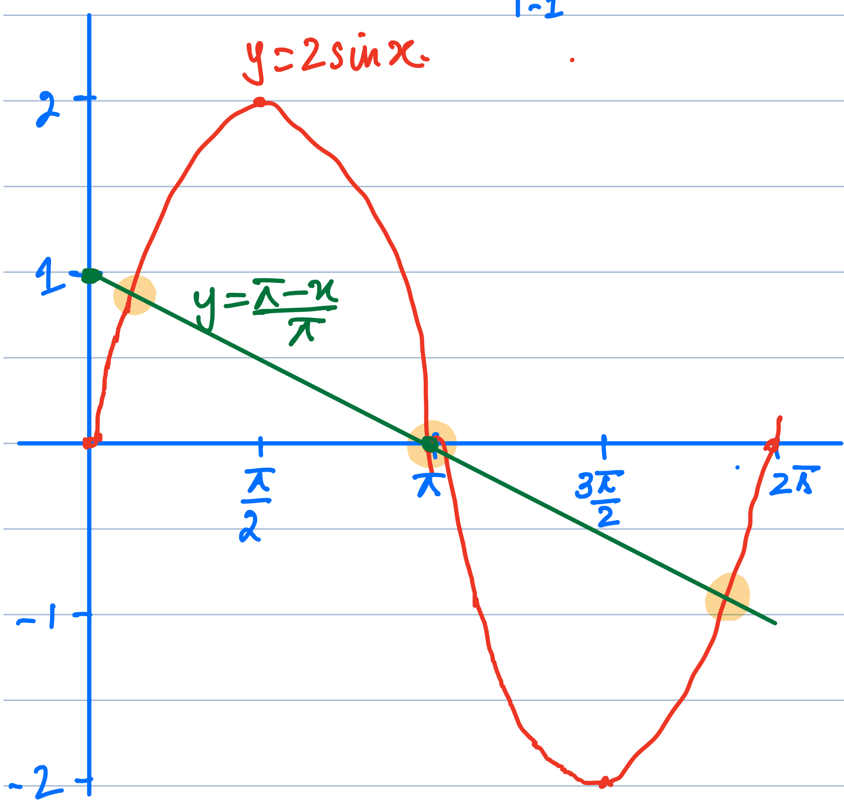
No of root = 3

$$2\pi \sin x = \pi - x.$$

State the equation of the straight line. $y = \frac{\pi - x}{\pi}$ [3]

$y = 2\sin x$





$2\pi \sin x = \pi - x$

$2\sin x = \frac{\pi - x}{\pi}$

Red graph

$y = 2\sin x$

$y = \frac{\pi - x}{\pi}$

Straight line.

$y = \frac{\pi - x}{\pi}$

x-intercept $y = 0$

$0 = \frac{\pi - x}{\pi}$

$(\pi, 0)$ $x = \pi$

y-intercept $x = 0$

$y = \frac{\pi - 0}{\pi} = 1$

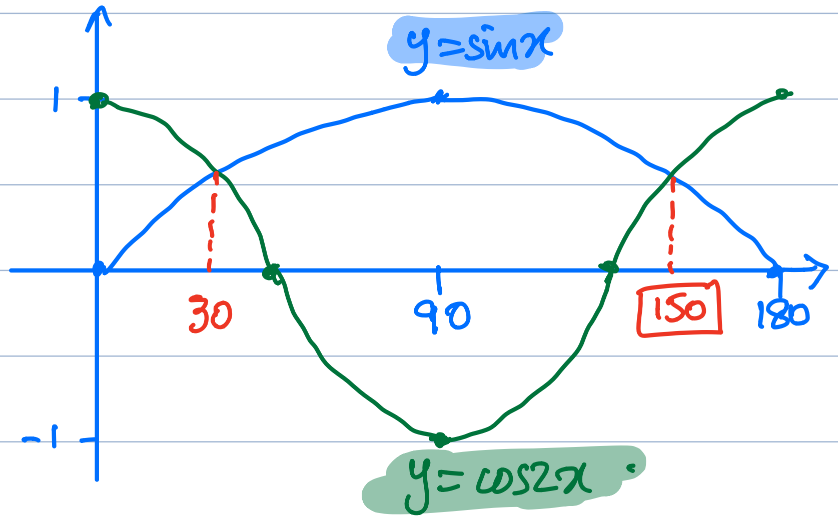
29 **(i)** Sketch, on the same diagram, the graphs of $y = \sin x$ and $y = \cos 2x$ for $0° \leqslant x \leqslant 180°$. [3]

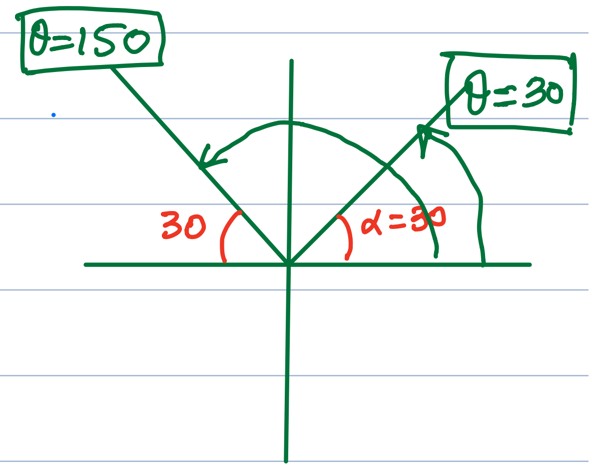
(ii) Verify that $x = 30°$ is a root of the equation $\sin x = \cos 2x$, and state the other root of this equation for which $0° \leqslant x \leqslant 180°$. [2]

inequality

(iii) Hence state the set of values of x, for $0° \leqslant x \leqslant 180°$, for which $\sin x < \cos 2x$. [2]

$0 < x < 30$ and $150 < x < 180$





(ii) $\boxed{x = 30} \rightarrow$

$\sin x = \cos 2x$

$\sin 30 = \cos(2 \times 30)$

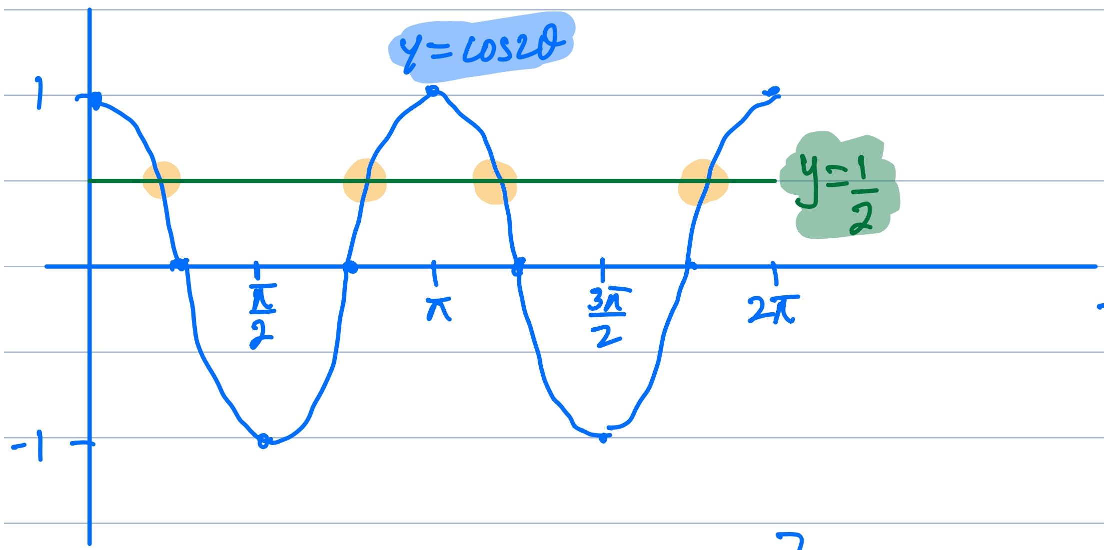
$\sin 30 = \cos 60$

$\frac{1}{2} = \frac{1}{2}$

28 (i) Sketch, on a single diagram, the graphs of $y = \cos 2\theta$ and $y = \frac{1}{2}$ for $0 \leqslant \theta \leqslant 2\pi$. [3]

(ii) Write down the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $0 \leqslant \theta \leqslant 2\pi$. [1]

(iii) Deduce the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $10\pi \leqslant \theta \leqslant 20\pi$. [1]

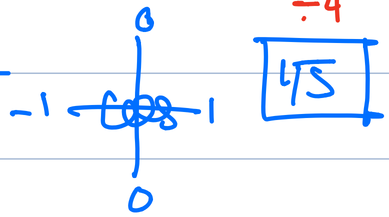


$y = \cos 2\theta$

2 cycles → 360

1 cycle → 180

÷4



(ii) $2\cos 2\theta - 1 = 0$

$\cos 2\theta = \frac{1}{2}$

$y = \cos 2\theta$, $y = \frac{1}{2}$

4 ROOTS.

(iii) 10π ———— 20π

10π (4) 12π (4) 14π (4) 16π (4) 18π (4) 20π

20 ROOTS.

TYPES : WITH DIAGRAMS.

METHOD : Olevels TRIG + Table of values.

5

Triangle BEC

$\sin 60 = \frac{CE}{2\sqrt{3}d}$

$\frac{\sqrt{3}}{2} = \frac{CE}{2\sqrt{3}d}$

$CE = \cancel{2}\sqrt{3}d \times \frac{\sqrt{3}}{\cancel{2}}$

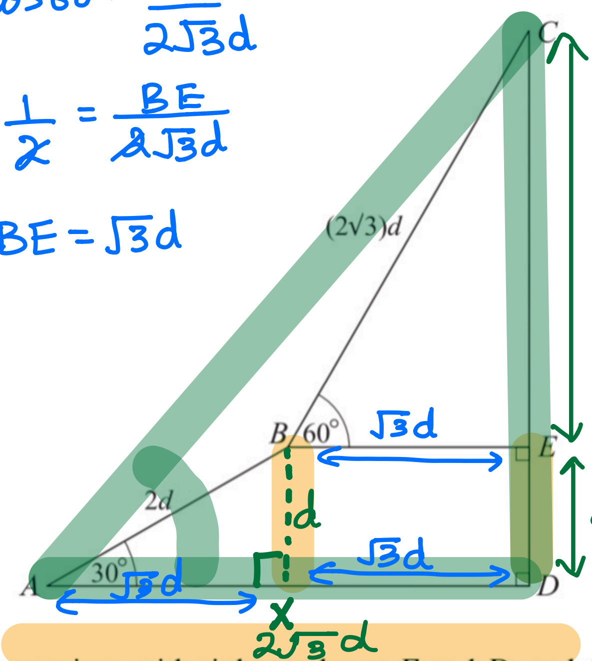
$\boxed{CE = 3d}$

$\boxed{\sqrt{3} \times \sqrt{3} = \sqrt{3 \times 3} = \sqrt{9} = 3}$

$\cos 60 = \frac{BE}{2\sqrt{3}d}$

$\frac{1}{\cancel{2}} = \frac{BE}{\cancel{2}\sqrt{3}d}$

$BE = \sqrt{3}d$



Triangle ABX

$\sin 30 = \frac{BX}{2d}$

$\frac{1}{2} = \frac{BX}{2d}$

$\boxed{BX = d}$

$\cos 30 = \frac{AX}{2d}$

$\frac{\sqrt{3}}{\cancel{2}} = \frac{AX}{\cancel{2}d}$

$AX = \sqrt{3}d$

In the diagram, $ABED$ is a trapezium with right angles at E and D, and CED is a straight line. The lengths of AB and BC are $2d$ and $(2\sqrt{3})d$ respectively, and angles BAD and CBE are 30° and 60° respectively.

(i) Find the length of CD in terms of d. $CD = 3d + d = 4d$ [2]

(ii) Show that angle $CAD = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$. [3]

$\tan(CAD) = \frac{CD}{AD}$

$\tan C\hat{A}D = \frac{\overset{2}{\cancel{4}}\cancel{d}}{\cancel{2}\sqrt{3}\cancel{d}}$

$\tan C\hat{A}D = \frac{2}{\sqrt{3}}$

$C\hat{A}D = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

7

$\cos 30 = \frac{BX}{6}$

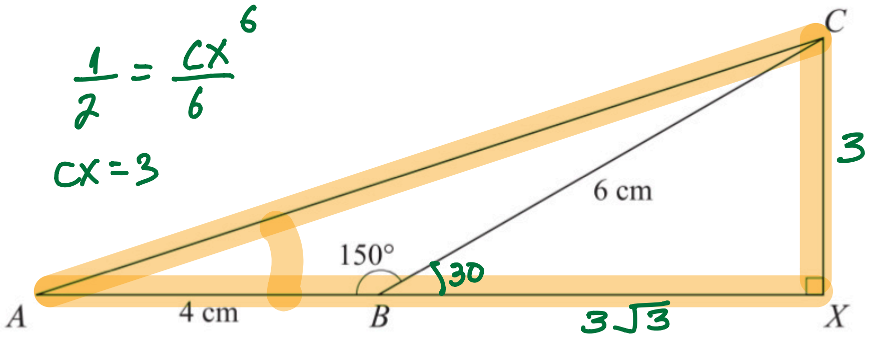
$\frac{\sqrt{3}}{2} = \frac{BX}{6}$

$BX = 3\sqrt{3}$

$\sin 30 = \frac{CX}{6}$

$\frac{1}{2} = \frac{CX}{6}$

$CX = 3$



In the diagram, ABC is a triangle in which $AB = 4$ cm, $BC = 6$ cm and angle $ABC = 150°$. The line CX is perpendicular to the line ABX.

(i) Find the exact length of BX and show that angle $CAB = \tan^{-1}\left(\frac{3}{4+3\sqrt{3}}\right)$. [4]

(ii) Show that the exact length of AC is $\sqrt{(52+24\sqrt{3})}$ cm. [2]

(i) $\tan C\hat{A}B = \frac{CX}{AX}$

$\tan C\hat{A}B = \frac{3}{4+3\sqrt{3}}$

$C\hat{A}B = \tan^{-1}\left(\frac{3}{4+3\sqrt{3}}\right)$

(ii) PYTHAGORAS:

$AC^2 = (3)^2 + (4+3\sqrt{3})^2$

$= 9 + (4)^2 + 2(4)(3\sqrt{3}) + (3\sqrt{3})^2$

$= 9 + 16 + 24\sqrt{3} + 27$

$AC^2 = 52 + 24\sqrt{3}$

$AC = \sqrt{52+24\sqrt{3}}$

$(3\sqrt{3})^2$

$3^2 \times (\sqrt{3})^2$

9×3

27

- [ ] Don't write your answers in two columns in the examination. It is difficult for the examiners to read and follow your working.

- [ ] If a question specifies how accurate your answer needs to be, you **must** give your final answer to that degree of accuracy. In questions where the accuracy is not specified, however, you will not be penalised if you give answers that are **more** accurate than 3SF.

- [ ] Some questions ask for answers in exact form. In these questions you must **not** use your calculator to evaluate answers and you must show the steps in your working. Exact answers may include fractions or square roots and you should simplify them as far as possible.

- [ ] Check to see if your answer is required in exact form. In a trigonometry question you will need to use exact values of sin 60°, for example, to obtain an exact answer. Make sure you know the exact values of sin, cos and tan of 30°, 45° and 60° as they are not provided in the examination.

P1

WORK FOR SAT + SUN

CIRCULAR MEASURE (Q1-Q20)
TRIG (Q1 — Q10)

Submission slot will be made on google classroom. Deadline monday.

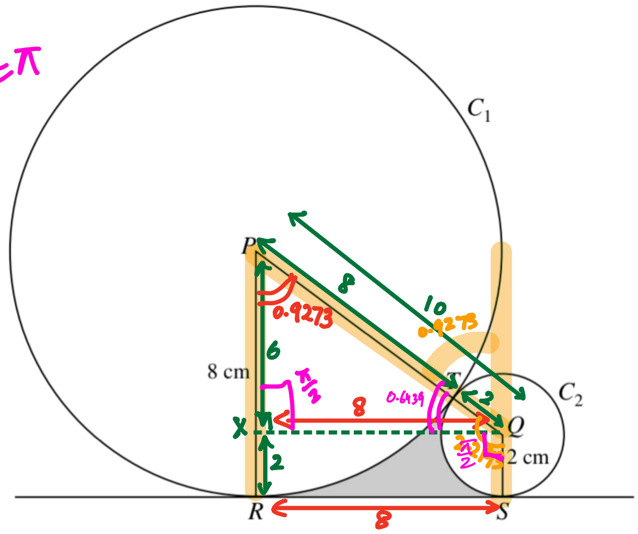
14

$Q + 0.9273 + \frac{\pi}{2} = \pi$

$Q = 0.64349$

$+ \frac{\pi}{2}$

2.2143



(i) Pythagoras

$QX^2 + 6^2 = 10^2$

$QX^2 = 100 - 36$

$QX = 8$

(ii) TRIG RATIO

$\sin R\hat{P}Q = \frac{8}{10}$

$R\hat{P}Q = \sin^{-1}(8 \div 10)$

$= 0.9273$ rad.

The diagram shows two circles, C_1 and C_2, touching at the point T. Circle C_1 has centre P and radius 8 cm; circle C_2 has centre Q and radius 2 cm. Points R and S lie on C_1 and C_2 respectively, and RS is a tangent to both circles.

(i) Show that $RS = 8$ cm. [2]

(ii) Find angle RPQ in radians correct to 4 significant figures. [2]

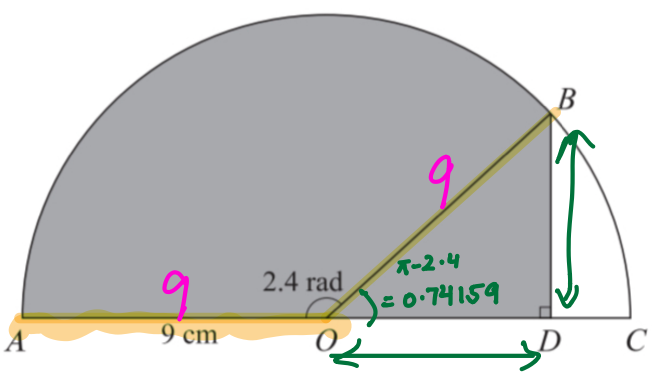
(iii) Find the area of the shaded region. [4]

Shaded region = TRAPEZIUM − BIG SECTOR − SMALL SECTOR.

$= \frac{1}{2}(8)(2+8) - \frac{1}{2}(8^2)(0.9273) - \frac{1}{2}(2)^2(2.2143)$

$=$

3



$\sin(\pi - 2.4) = \frac{BD}{9}$

$BD = 9\sin(\pi - 2.4)$

$BD = 6.07916$

$\cos(\pi - 2.4) = \frac{OD}{9}$

$OD = 6.637$

$\boxed{P} = OA + AB \text{ Arc} + BD + OD$

$9 + (9)(2.4) + 6.07916 + 6.637$

$=$

fx- 82 MS

fx- 350 MS

16

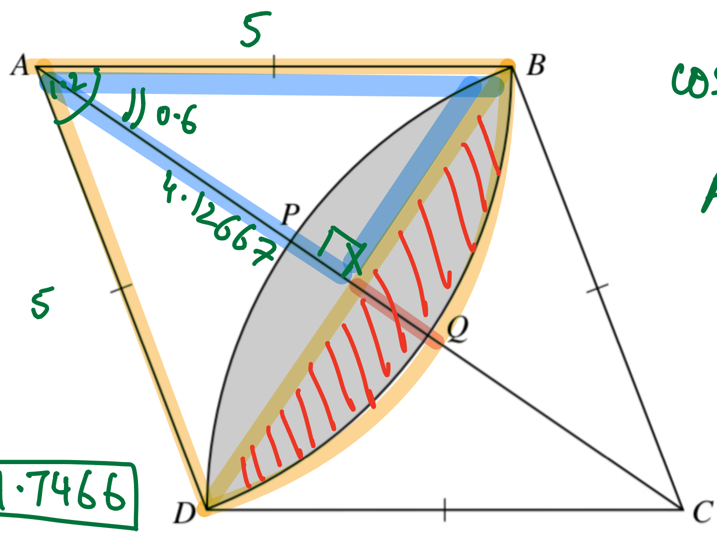
$AQ = 5$ (radius)

$XQ = 5 - AX$

$XQ = 5 - 4.12667$

$XQ = 0.8733$

$PQ = 2 \times XQ$
$= 2(0.8733) = \boxed{1.7466}$



$\cos 0.6 = \dfrac{AX}{5}$

$AX = 5\cos 0.6$

$AX = 4.12667$

The diagram shows a rhombus $ABCD$. Points P and Q lie on the diagonal AC such that BPD is an arc of a circle with centre C and BQD is an arc of a circle with centre A. Each side of the rhombus has length 5 cm and angle $BAD = 1.2$ radians.

(i) Find the area of the shaded region $BPDQ$. [4]

(ii) Find the length of PQ. [4]

Red Area = Sector − Triangle

$$= \frac{1}{2}(5)^2(1.2) - \frac{1}{2}(5)(5)\sin 1.2$$

Red $= 3.3495$

Shaded area $= 2 \times \text{Red} = 2 \times 3.3495 = \boxed{6.699}$