# RS-MILP-01: Configure Optimization Problem from Contact Matrix

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## 1. Generate A matrix from contact chart

```
clear;
clc;


addpath('/Library/gurobi1300/macos_universal2/matlab')
savepath

addpath ~/Desktop/Redstone_MILP/RS_MILP_01_Config_MILP/
load('EOIR_48_SATs_4_Orbit_Planes_98_inc_7days_access_interval.mat','EOIR_ac
cess_interval')


% 기준 시각 정의

T = EOIR_access_interval;

t0 = datetime(2030, 1, 1, 0, 0, 0,'TimeZone','UTC');

% --- Source / Target을 string으로 통일 ---
src = string(T.Source);
tgt = string(T.Target);

% --- Ground 번호 추출 ---
gTok = regexp(src, 'Ground[_\s]*Point[_\s]*(\d+)', 'tokens', 'once');
groundNum = cellfun(@(c) str2double(c{1}), gTok);

% --- Satellite 번호 추출 ---
sTok = regexp(tgt, '(?i)SAT[^0-9]*(\d+)', 'tokens', 'once');
satNum = cellfun(@(c) str2double(c{1}), sTok);

% --- Start / End → datetime ---
st = T.StartTime;
en = T.EndTime;

if ~isdatetime(st)
    st = datetime(string(st), 'InputFormat','dd-MMM-yyyy HH:mm:ss');
```

```
end
if ~isdatetime(en)
    en = datetime(string(en), 'InputFormat','dd-MMM-yyyy HH:mm:ss');
end

% --- 중간 시각 ---
midTime = st + (en - st)/2;

% --- 기준 시각 대비 초(second) 차이 → integer ---
timeSec = seconds(midTime - t0);
timeSec = double(round(timeSec));   % 정수화 (필요시 floor / ceil로 변경 가능)


contact_index = (1:length(st))';

% --- 최종 3-column matrix ---
A_matrix = [double(satNum), double(groundNum), timeSec, contact_index];

A_matrix = sortrows(A_matrix, 3);

A_matrix = A_matrix(1:200,:);
Original_A = T(A_matrix(:,4),:);
```

## 2. Extract Key parameters from A matrix

```
t = A_matrix(:,3);

% Satellite Cadence Constraint
tau = 15;

% Number of SATs
p = 48;

% Number of GSs
q = 54;

% Total number of contact
N = length(A_matrix(:,1));

% Number of contact for each SAT
S_i_vec = zeros(p,1);

for sat_index = 1:p
    S_i_vec(sat_index) = nnz(A_matrix(:,1) == sat_index);
end
```

```
% Number of contact for each GS
G_j_vec = zeros(q,1);
for gs_index = 1:q
    G_j_vec(gs_index) = nnz(A_matrix(:,2) == gs_index);
end
```

## 3. Selection matrix generation from given constant parameters

### 3.1. $E^1_{S_i}$: Selection matrix from A-matrix to each satellite's contact sequence

```
E1_Si = struct();

for i = 1:p

E1_Si_mat = zeros(S_i_vec(i),N);

a_i = find(A_matrix(:,1) == i);

  for j = 1:length(a_i)
      E1_Si_mat(j, a_i(j)) = 1;
  end

  E1_Si.(['sat',num2str(i)]) = E1_Si_mat;

end
```

### 3.2. $E^2_{S_i,x}$, $E^2_{S_{i,t}}$: Selection matrix for $\Delta t$ of each satelltite from $|S_i|$

```
E2_Si_x = struct();

for i = 1:p
    E2_Si_x_mat = [];
    for alpha = 1:S_i_vec(i)-1
        E2_Si_x_alpha = zeros(S_i_vec(i)-alpha, S_i_vec(i));
        E2_Si_x_alpha(:,alpha) = 1;
        for beta = alpha+1:S_i_vec(i)
            E2_Si_x_alpha(beta-alpha,beta) = 1;
        end
     E2_Si_x_mat = [E2_Si_x_mat;E2_Si_x_alpha];
    end
   if isempty(E2_Si_x_mat)
     E2_Si_x_mat = 0;
```

```
   end
   E2_Si_x.(['sat',num2str(i)]) = E2_Si_x_mat;
end


E2_Si_t = struct();

for i = 1:p
    E2_Si_t_mat = [];
    for alpha = 1:S_i_vec(i)-1
        E2_Si_t_alpha = zeros(S_i_vec(i)-alpha, S_i_vec(i));
        E2_Si_t_alpha(:,alpha) = -1;
        for beta = alpha+1:S_i_vec(i)
            E2_Si_t_alpha(beta-alpha,beta) = 1;
        end
     E2_Si_t_mat = [E2_Si_t_mat;E2_Si_t_alpha];
    end
   if isempty(E2_Si_t_mat)
     E2_Si_t_mat = 0;
   end
    E2_Si_t.(['sat',num2str(i)]) = E2_Si_t_mat;
end
```

### 3.3. $E^1_{G_j}$ : Selection matrix from A-matrix to each Ground Point's revisit sequence

```
E1_Gj = struct();

for j = 1:q
    E1_Gj_mat = zeros(G_j_vec(j)+2, N+2);

    E1_Gj_mat(1,1) = 1;
    b_j = find(A_matrix(:,2) == j);

    for alpha = 1:length(b_j)
        E1_Gj_mat(alpha+1, b_j(alpha)+1) = 1;
    end
    E1_Gj_mat(G_j_vec(j)+2, N+2) = 1;
    E1_Gj.(['gs', num2str(j)]) = E1_Gj_mat;
end
```

### 3.4. $E^2_{G_{j,x}}, E^2_{G_{j,t}}$ : Selection matrix for $\Delta t$ of each GS from $|G_j|$

```
E2_Gj_x = struct();

for j = 1:q
```

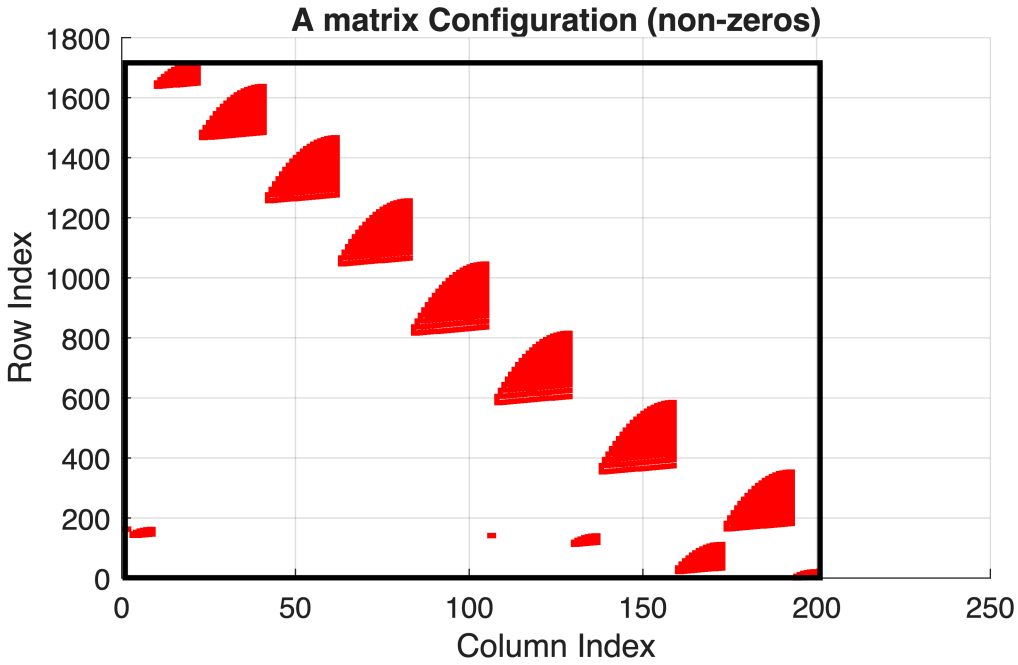
```
    E2_Gj_x_mat = [];
    for alpha = 1:G_j_vec(j)+1
        E2_Gj_x_alpha = zeros(G_j_vec(j)+2-alpha, G_j_vec(j)+2);
        E2_Gj_x_alpha(:,alpha) = 1;
        for beta = alpha+1:G_j_vec(j)+2
            if alpha + 1 <= beta-1
               E2_Gj_x_alpha(beta-alpha, alpha+1:beta-1) = -1;
            end
            E2_Gj_x_alpha(beta-alpha,beta) = 1;
        end
     E2_Gj_x_mat = [E2_Gj_x_mat;E2_Gj_x_alpha];
    end
   if isempty(E2_Gj_x_mat)
     E2_Gj_x_mat = 0;
   end
   E2_Gj_x.(['gs',num2str(j)]) = E2_Gj_x_mat;
end

E2_Gj_t = struct();

for j = 1:q
    E2_Gj_t_mat = [];
    for alpha = 1:G_j_vec(j)+1
        E2_Gj_t_alpha = zeros(G_j_vec(j)+2-alpha, G_j_vec(j)+2);
        E2_Gj_t_alpha(:,alpha) = -1;
        for beta = alpha+1:G_j_vec(j)+2
            E2_Gj_t_alpha(beta-alpha,beta) = 1;
        end
     E2_Gj_t_mat = [E2_Gj_t_mat;E2_Gj_t_alpha];
    end
   if isempty(E2_Gj_t_mat)
     E2_Gj_t_mat = 0;
   end
   E2_Gj_t.(['gs',num2str(j)]) = E2_Gj_t_mat;
end
```

## 4. Derivation of $A, b, C, d, E, f, G$ revisit time problem to MILP

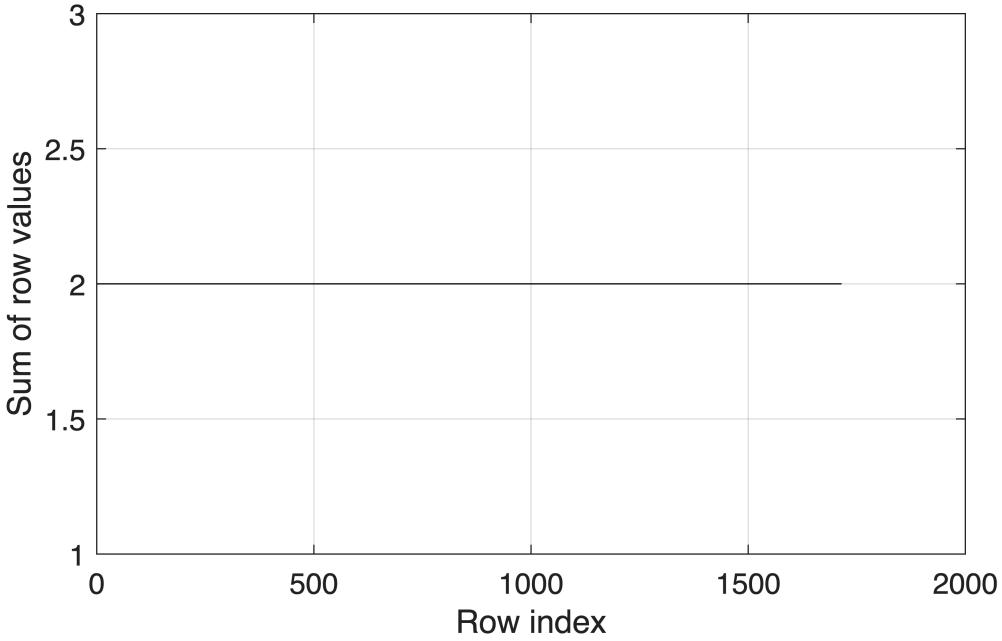
### 4.1 $A, b$ for $L_1$ problem

```
A = [];
b_mat = [];
for i = 1:p
  if isempty(E1_Si.(['sat', num2str(i)]))
  continue;
  end
```

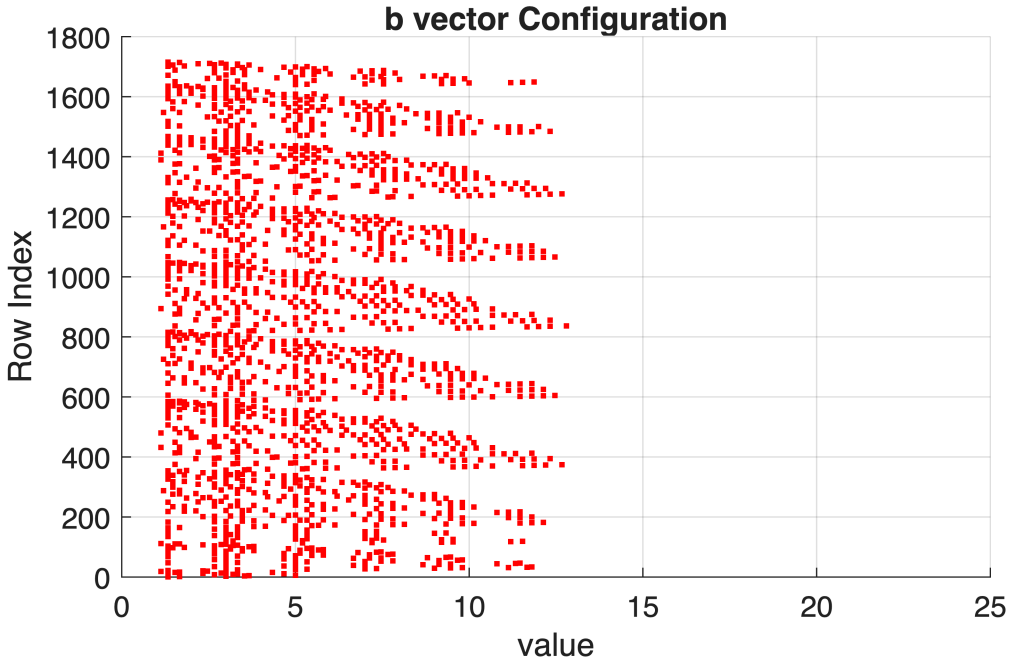
```
  A_i = E2_Si_x.(['sat', num2str(i)]) * E1_Si.(['sat', num2str(i)]);
  A = [A;A_i];
  b_i_mat =  E2_Si_t.(['sat', num2str(i)]) * E1_Si.(['sat', num2str(i)]);
  b_mat = [b_mat; b_i_mat];
end

b = ones(length(b_mat(:,1)),1) + 1/tau * b_mat * A_matrix(:,3);

[A_row, A_col] = find(A==1);
figure;
hold on
scatter(A_col,A_row,'r','.')
rectangle('Position',[1 1 max(A_col), max(A_row)], ...
          'EdgeColor','k', ...
          'LineWidth',2);
title('A matrix Configuration (non-
zeros)','FontSize',12,'FontWeight','bold')
xlabel('Column Index')
ylabel('Row Index')
grid on
```




```
row_sum = sum(A, 2);        % 각 row의 합 (N×1)

figure;
plot(row_sum, '-k');        % x축은 자동으로 row index (1:N)
grid on
xlabel('Row index')
ylabel('Sum of row values')
```




```
figure;
scatter(b,1:length(b),'r','.')
title('b vector Configuration','FontSize',12,'FontWeight','bold')
xlabel('value')
ylabel('Row Index')
xlim([0,25])
grid on
```



### 4.2 $C, d$ for $L_\infty$ problem

```
C = [];
d = [];

t = A_matrix(:,3);                  % (N×1) time vector in seconds (already
prepared)
t_aug = [min(A_matrix(:,3)); t; max(A_matrix(:,3))+1];        % (N+2 × 1)

for j = 1:q
    key = ['gs', num2str(j)];

    if isempty(E1_Gj.(key))
        continue;
    end

    % --- shorthand ---
    E1  = E1_Gj.(key);        % selection matrix for this ground
    E2x = E2_Gj_x.(key);
    E2t = E2_Gj_t.(key);

    % =========================
    % Build C_j  (constant)
    % =========================
    % D_j = diag( E2t*E1 * t_aug )   (vector -> diagonal matrix)
    Dj_vec = (E2t * E1) * t_aug;           % (#rows_j × 1)
    Dj = spdiags(Dj_vec, 0, length(Dj_vec), length(Dj_vec));  % sparse diag
is safer

    % P = [0_{1×N}; I_{N×N}; 0_{1×N}]  -> (N+2 × N)
    N = length(t);
    P = [zeros(1,N); speye(N); zeros(1,N)];

    % C_j = D_j * (E2x*E1) * P
    Cj = Dj * (E2x * E1) * P;

    % =========================
    % Build d_j  (constant)
    % =========================
    % v = [1; 0_{N×1}; 1] - 1  = [0; -1...; 0]
    v = [1; zeros(N,1); 1];          % (N+2 × 1)

    % d_j = D_j * (E2x*E1) * v
    dj = Dj * ((E2x * E1) * v - ones(length(Dj(:,1)),1));

    % --- stack ---
    C = [C; Cj];
    d = [d; dj];
end
```

```
% [C_row, C_col] = find(C~=0);
%
% figure;
% scatter(C_row, C_col,'r','.')
% grid on
```

### 4.3 $E, f, G$ for $L_2$ Optimization problem

```
% ========================
% Build E, f, G  (2nd image)
% ========================

E = [];
F = [];

t = A_matrix(:,3);                          % (N×1)
N = length(t);

t_aug = [min(t); t; max(t)+1];                % (N+2 × 1), same as your code

% P = [0_{1×N}; I_N; 0_{1×N}]  -> (N+2 × N)
P = [zeros(1,N); speye(N); zeros(1,N)];

% this is the (N+2 × 1) vector [1; 0_N; 1]
one_aug = [1; zeros(N,1); 1];

% For G, we will build the stacked matrix Bt = [E2t*E1; ...] first
(optional),
% or directly build gvec = Bt*t_aug.
gvec = [];   % = Bt * t_aug  (will be stacked across j)

for j = 1:q
    key = ['gs', num2str(j)];

    if isempty(E1_Gj.(key))
        continue;
    end

    % --- shorthand ---
    E1  = E1_Gj.(key);        % selection matrix for this ground
    E2x = E2_Gj_x.(key);
    E2t = E2_Gj_t.(key);

    % Common blocks
    Sx = (E2x * E1);          % (#rows_j × (N+2))
```

```
    St = (E2t * E1);          % (#rows_j × (N+2))

    % ------------------------
    % E_j = Sx * P    (constant)
    % ------------------------
    Ej = Sx;              % (#rows_j × N)
    E  = [E; Ej];

    % ------------------------
    % f_j = Sx]
    % ------------------------
    Fj = Sx;        % (#rows_j × 1)
    F  = [F; Fj];

    % ------------------------
    % gvec_j = St * t_aug
    % ------------------------
    gvec_j = St * t_aug;      % (#rows_j × 1)
    gvec   = [gvec; gvec_j];
end


E = E * P;
f_vector = F * one_aug;

% ------------------------
% G = gvec * gvec'
% ------------------------
% WARNING: this can be very large/dense if gvec is long.
% If you only need y'*G*y, note that y'*(gvec*gvec')*y = (gvec'*y)^2.
G = gvec' * gvec;   % (M×M)


% [E_row, E_col] = find(E~=0);
%
% figure;
% scatter(E_row, E_col,'r','.')
% grid on
% imagesc(G); colorbar; axis equal tight;
% title('Heatmap of G');
```

## 5. Optimization Problem (MILP)

```
L1_flag = 1;
L_infty_flag = 0;
L2_flag = 1;
```

```
L2_flag_relaxed = 0;
```

### 5.1. $L_1$ revisit time problem to MILP

```
if L1_flag == 1

%% Given: A (m×N), b (m×1), N (scalar)
% Goal: max 1^T x  s.t. A x <= b, x in {0,1}^N

% ---------- 1) Dimensions / sanity checks ----------
[m, nA] = size(A);
assert(nA == N, 'A must have N columns.');
assert(isvector(b) && length(b) == m, 'b must be m×1 to match A.');

b = b(:);                 % force column vector
Aineq = A;
bineq = b;

% ---------- 2) Convert max to min ----------
% intlinprog solves: min f'*x
f = -ones(N,1);           % minimize -sum(x) == maximize sum(x)

% ---------- 3) Binary variable settings ----------
intcon = 1:N;             % all variables are integer
lb = zeros(N,1);
ub = ones(N,1);

% (Optional) If you truly want "binary", keep intcon + bounds [0,1].
% Alternatively, you can also set intcon and bounds; that's standard.

% ---------- 4) Solve MILP ----------
% opts = optimoptions('intlinprog', ...
%     'Display','iter', ...
%     'Heuristics','advanced', ...
%     'CutGeneration','advanced');

[x_opt_L1, fval, exitflag, output] = intlinprog( ...
    f, intcon, Aineq, bineq, [], [], lb, ub);
% ---------- 5) Recover maximization objective ----------
max_onesTx = -fval;       % because f = -1
x_opt_L1 = round(x_opt_L1);     % safety: should already be integer

% ---------- 6) Quick checks ----------
viol = Aineq*x_opt_L1 - bineq;
max_viol = max(viol);

row_index_L1 = x_opt_L1 .*  A_matrix(:,4);
row_index_L1 = row_index_L1(row_index_L1 ~=0);
```

```
Original_A_L1 = T(row_index_L1,:);
A_matrix_L1 = A_matrix(x_opt_L1 == 1,:);

fprintf('Exitflag: %d\n', exitflag);
fprintf('Objective (max 1^T x): %.0f\n', max_onesTx);
fprintf('Max constraint violation: %.3e\n', max_viol);
end
```

```
Running HiGHS 1.7.1: Copyright (c) 2024 HiGHS under MIT licence terms
Coefficient ranges:
  Matrix [1e+00, 1e+00]
  Cost   [1e+00, 1e+00]
  Bound  [1e+00, 1e+00]
  RHS    [1e+00, 1e+01]
Presolving model
200 rows, 187 cols, 400 nonzeros  0s
72 rows, 157 cols, 211 nonzeros  0s
49 rows, 89 cols, 98 nonzeros  0s
5 rows, 10 cols, 10 nonzeros  0s
Objective function is integral with scale 1

Solving MIP model with:
   5 rows
   10 cols (10 binary, 0 integer, 0 implied int., 0 continuous)
   10 nonzeros

        Nodes      |    B&B Tree     |            Objective Bounds              |  Dynamic Constraints |
     Proc. InQueue |  Leaves   Expl. | BestBound       BestSol              Gap |   Cuts   InLp Confl. | I

         0       0         0   0.00%   -91             inf                  inf        0      0      0

Solving report
  Status            Optimal
  Primal bound      -91
  Dual bound        -91
  Gap               0% (tolerance: 0.01%)
  Solution status   feasible
                    -91 (objective)
                    0 (bound viol.)
                    0 (int. viol.)
                    0 (row viol.)
  Timing            0.01 (total)
                    0.00 (presolve)
                    0.00 (postsolve)
  Nodes             1
  LP iterations     5 (total)
                    0 (strong br.)
                    0 (separation)
                    0 (heuristics)

최적해를 구했습니다.

목적 함수 값이 최적 값의 격차 허용오차 options.AbsoluteGapTolerance = 1e-06 내에 있기 때문에 Intlinprog가 루트 노드에서
Exitflag: 1
Objective (max 1^T x): 91
Max constraint violation: 5.329e-14
```

## 5.2 $L_\infty$ revisit time problem to MILP

```
% Using MATLAB

% if L_infty_flag == 1
% %
% % % Given:
% % A (m×N), b (m×1)
% % C (q×N), d (q×1)
% % N (scalar)
%
% % ---------- sanity ----------
% [m, nA] = size(A);
% assert(nA == N, 'A must have N columns.');
% b = b(:);  assert(length(b) == m, 'b must be m×1.');
%
% [q, nC] = size(C);
% assert(nC == N, 'C must have N columns.');
% d = d(:);  assert(length(d) == q, 'd must be q×1.');
%
% % ---------- decision variables ----------
% % z = [x; R]  -> length N+1
% nvar = N + 1;
%
% % Objective: min R  => f = [0...0, 1]
% f = [zeros(N,1); 1];
%
% % Integer constraints: x binary, R continuous
% intcon = 1:N;
%
% % Bounds
% lb = [zeros(N,1); 0];      % R >= 0 (필요 없으면 -Inf로 바꿔도 됨)
% ub = [ones(N,1);  Inf];
%
% % ---------- Build inequalities Aineq*z <= bineq ----------
% % 1) Ax <= b  -> [A, 0] [x;R] <= b
% A1 = [A, zeros(m,1)];
% b1 = b;
%
% % 2) Cx - R*1 <= -d  -> [C, -1] [x;R] <= -d   (각 row마다 -R)
% A2 = [C, -ones(q,1)];
% b2 = -d;
%
% Aineq = [A1; A2];
% bineq = [b1; b2];
%
% % (No equality constraints)
% Aeq = [];
% beq = [];
%
% % ---------- Solve ----------
% % opts = optimoptions('intlinprog', ...
```

```
% %     'Display','iter', ...
% %     'Heuristics','advanced', ...
% %     'CutGeneration','advanced');
% 
% [z_opt, fval, exitflag, output] = intlinprog( ...
%     f, intcon, Aineq, bineq, Aeq, beq, lb, ub);
% % ---------- Parse solution ----------
% x_opt_L_inf = round(z_opt(1:N));     % binary
% R_opt = z_opt(N+1);            % optimal R
% 
% 
% row_index_L_inf = x_opt_L_inf .*  A_matrix(:,4);
% row_index_L_inf = row_index_L_inf(row_index_L_inf ~=0);
% Original_A_L_inf = T(row_index_L_inf,:);
% A_matrix_L_inf = A_matrix(x_opt_L_inf == 1,:);
% 
% 
% 
% % ---------- Feasibility check ----------
% viol1 = A*x_opt_L_inf - b;
% viol2 = C*x_opt_L_inf + d - R_opt*ones(q,1);   % should be <= 0
% fprintf('Exitflag: %d\n', exitflag);
% fprintf('R_opt: %.6f\n', R_opt);
% fprintf('max(Ax-b): %.3e\n', max(viol1));
% fprintf('max(Cx+d-R): %.3e\n', max(viol2));
% end
```

```
% Using Gurobi

if L_infty_flag == 1

%% Given:
% A (m×N), b (m×1)
% C (q×N), d (q×1)
% N (scalar)

% ---------- sanity ----------
[m, nA] = size(A);
assert(nA == N, 'A must have N columns.');
b = b(:);  assert(length(b) == m, 'b must be m×1.');

[q, nC] = size(C);
assert(nC == N, 'C must have N columns.');
d = d(:);  assert(length(d) == q, 'd must be q×1.');

% ---------- decision variables ----------
% z = [x; R]  -> length N+1
nvar = N + 1;
```

```
% ---------- Build Gurobi model ----------
model = struct();

% Constraints:
% 1) Ax <= b  -> [A, 0] z <= b
A1 = [A, sparse(m,1)];
rhs1 = b;
sense1 = repmat('<', m, 1);

% 2) Cx - R*1 <= -d -> [C, -1] z <= -d
A2 = [C, -ones(q,1)];
rhs2 = -d;
sense2 = repmat('<', q, 1);

model.A     = sparse([A1; A2]);
model.rhs   = [rhs1; rhs2];
model.sense = [sense1; sense2];

% Objective: min R => obj = [0...0, 1]
model.obj = [zeros(N,1); 1];
model.modelsense = 'min';

% Variable types: x binary, R continuous
model.vtype = [repmat('B', 1, N), 'C'];

% Bounds: 0<=x<=1, R>=0
model.lb = [zeros(N,1); 0];
model.ub = [ones(N,1);  inf];

% ---------- Solve ----------
params = struct();
params.OutputFlag = 1;   % 1: show log, 0: silent
% params.MIPGap = 1e-4;  % optional

result = gurobi(model, params);

% ---------- Parse solution ----------
if ~isfield(result, 'x') || isempty(result.x)
    error('Gurobi failed. Status: %s', result.status);
end

z_opt = result.x;

x_opt_L_inf = round(z_opt(1:N));   % binary
R_opt = z_opt(N+1);

row_index_L_inf = x_opt_L_inf .* A_matrix(:,4);
row_index_L_inf = row_index_L_inf(row_index_L_inf ~= 0);

Original_A_L_inf = T(row_index_L_inf,:);
```

```
A_matrix_L_inf   = A_matrix(x_opt_L_inf == 1,:);

% ---------- Feasibility check ----------
viol1 = A*x_opt_L_inf - b;
viol2 = C*x_opt_L_inf + d - R_opt*ones(q,1);   % should be <= 0

fprintf('Gurobi status: %s\n', result.status);
fprintf('R_opt: %.6f\n', R_opt);
fprintf('max(Ax-b): %.3e\n', max(viol1));
fprintf('max(Cx+d-R): %.3e\n', max(viol2));

end
```

### 5.3 $L_2$ revisit time problem to MIQP

```
if L2_flag == 1
% 사용 예:
params = struct();
params.OutputFlag = 1;
params.MIPGap = 1e-4;
% params.NonConvex = 2;  % G가 PSD가 아닐 수도 있으면 켜세요.

res = solve_miqp_gurobi(A, b, E, f_vector, gvec, params);

x_sol = res.x_bin;
y_sol = res.y;



x_opt_L2 = x_sol;   % binary
row_index_L2 = x_opt_L2 .* A_matrix(:,4);
row_index_L2 = row_index_L2(row_index_L2 ~= 0);

Original_A_L2 = T(row_index_L2,:);
A_matrix_L2   = A_matrix(x_opt_L2 == 1,:);



end
```

```
Set parameter Username
Set parameter LicenseID to value 2767696
Academic license - for non-commercial use only - expires 2027-01-20
Gurobi Optimizer version 13.0.0 build v13.0.0rc1 (mac64[arm] - Darwin 24.6.0 24G90)

CPU model: Apple M3
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 2507 rows, 992 columns and 6513 nonzeros (Min)
Model fingerprint: 0x866db247
Model has 791 linear objective coefficients
```

```
Model has 791 quadratic objective terms
Variable types: 792 continuous, 200 integer (200 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [2e+00, 4e+07]
  QObjective range [4e+00, 8e+07]
  Bounds range     [1e+00, 2e+00]
  RHS range        [1e+00, 1e+01]
Found heuristic solution: objective 1.838484e+10
Presolve removed 1735 rows and 108 columns
Presolve time: 0.18s
Presolved: 772 rows, 884 columns, 3111 nonzeros
Presolved model has 684 quadratic objective terms
Found heuristic solution: objective 1.694799e+10
Variable types: 684 continuous, 200 integer (200 binary)
Found heuristic solution: objective 1.691897e+10

Root relaxation: objective 7.716385e+08, 536 iterations, 0.00 seconds (0.00 work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 7.7164e+08    0  200 1.6919e+10 7.7164e+08  95.4%     -    0s
H    0     0                     2.627342e+09 7.7164e+08  70.6%     -    0s
H    0     0                     2.626593e+09 1.1804e+09  55.1%     -    0s
     0     0 1.1804e+09    0  188 2.6266e+09 1.1804e+09  55.1%     -    0s
     0     0 1.2007e+09    0  191 2.6266e+09 1.2007e+09  54.3%     -    0s
     0     0 1.2008e+09    0  191 2.6266e+09 1.2008e+09  54.3%     -    0s
     0     0 1.3342e+09    0  192 2.6266e+09 1.3342e+09  49.2%     -    0s
     0     0 1.3399e+09    0  193 2.6266e+09 1.3399e+09  49.0%     -    0s
     0     0 1.3402e+09    0  193 2.6266e+09 1.3402e+09  49.0%     -    0s
     0     0 1.4153e+09    0  187 2.6266e+09 1.4153e+09  46.1%     -    0s
     0     0 1.4215e+09    0  192 2.6266e+09 1.4215e+09  45.9%     -    0s
     0     0 1.4222e+09    0  193 2.6266e+09 1.4222e+09  45.9%     -    0s
     0     0 1.4222e+09    0  193 2.6266e+09 1.4222e+09  45.9%     -    0s
     0     0 1.4581e+09    0  197 2.6266e+09 1.4581e+09  44.5%     -    0s
     0     0 1.4629e+09    0  195 2.6266e+09 1.4629e+09  44.3%     -    0s
H    0     0                     2.626545e+09 1.4637e+09  44.3%     -    0s
     0     0 1.4637e+09    0  195 2.6265e+09 1.4637e+09  44.3%     -    0s
     0     0 1.4639e+09    0  195 2.6265e+09 1.4639e+09  44.3%     -    0s
H    0     0                     2.626251e+09 1.4702e+09  44.0%     -    0s
     0     0 1.4702e+09    0  198 2.6263e+09 1.4702e+09  44.0%     -    0s
H    0     0                     2.620101e+09 1.4744e+09  43.7%     -    0s
     0     0 1.4744e+09    0  198 2.6201e+09 1.4744e+09  43.7%     -    0s
H    0     0                     2.619759e+09 1.4744e+09  43.7%     -    0s
H    0     0                     2.615961e+09 1.4744e+09  43.6%     -    0s
     0     0 1.4744e+09    0  194 2.6160e+09 1.4744e+09  43.6%     -    0s
     0     0 1.4760e+09    0  194 2.6160e+09 1.4760e+09  43.6%     -    0s
     0     2 1.4778e+09    0  194 2.6160e+09 1.4778e+09  43.5%     -    0s
H  220   237                     2.614780e+09 1.4898e+09  43.0%  20.5    0s
H  225   237                     2.606920e+09 1.4898e+09  42.9%  20.6    0s
H  293   332                     2.606833e+09 1.4898e+09  42.9%  21.8    0s
H  319   332                     2.606601e+09 1.4898e+09  42.8%  22.4    0s
H  582   644                     2.606277e+09 1.4898e+09  42.8%  24.6    1s
H  589   644                     2.604971e+09 1.4898e+09  42.8%  24.7    1s
H  625   644                     2.604865e+09 1.4898e+09  42.8%  24.8    1s
H  732   797                     2.603367e+09 1.4898e+09  42.8%  23.8    1s
H  764   797                     2.597880e+09 1.4898e+09  42.7%  23.6    1s
H  884   936                     2.596545e+09 1.4898e+09  42.6%  23.6    1s
H  993  1088                     2.596332e+09 1.4898e+09  42.6%  23.7    1s
H 1213  1337                     2.595835e+09 1.4898e+09  42.6%  22.5    1s
H 1479  1584                     2.595831e+09 1.4898e+09  42.6%  20.9    1s
H 1492  1584                     2.586837e+09 1.4898e+09  42.4%  20.9    1s
H 1702  1663                     2.585106e+09 1.4898e+09  42.4%  20.2    1s
```

```
H 1765  1729                    2.585068e+09 1.4898e+09  42.4%  20.1    1s
H 1793  1642                    2.511264e+09 1.4898e+09  40.7%  20.3    2s
  1865  1692 2.5113e+09  129  144 2.5113e+09 1.7751e+09  29.3%  21.4    5s
H 2594  2156                    2.498223e+09 1.8028e+09  27.8%  23.1    5s
* 2850  2237                98    2.378721e+09 1.8028e+09  24.2%  22.7    5s
* 3146  2167               104    2.374074e+09 1.8193e+09  23.4%  22.2    5s
H 3267  2196                    2.365712e+09 1.8193e+09  23.1%  22.4    5s
H 3739  2540                    2.365546e+09 1.8193e+09  23.1%  22.3    5s
H 3862  2419                    2.349568e+09 1.8193e+09  22.6%  22.2    5s
* 4131  2574               100    2.310483e+09 1.8193e+09  21.3%  21.8    5s
H 4321  2436                    2.309858e+09 1.8193e+09  21.2%  21.4    5s
H 4424  2322                    2.300374e+09 1.8193e+09  20.9%  21.2    5s
H 4439  2229                    2.295166e+09 1.8193e+09  20.7%  21.2    5s
H 4523  2126                    2.273661e+09 1.8193e+09  20.0%  21.3    5s
H 4533  2068                    2.272064e+09 1.8193e+09  19.9%  21.3    5s
H 4715  2159                    2.262443e+09 1.8254e+09  19.3%  21.2    5s
H 4879  2196                    2.256562e+09 1.8409e+09  18.4%  21.3    5s
H 4996  2276                    2.251149e+09 1.8409e+09  18.2%  21.5    6s
H 5004  2147                    2.235142e+09 1.8409e+09  17.6%  21.5    6s
H 5716  2792                    2.234997e+09 1.8409e+09  17.6%  21.7    6s
H 6647  3645                    2.234810e+09 1.8409e+09  17.6%  21.6    6s
H 6812  3515                    2.223416e+09 1.8409e+09  17.2%  21.6    6s
H 7847  4305                    2.221282e+09 1.8533e+09  16.6%  21.7    6s
H 7848  4304                    2.221174e+09 1.8533e+09  16.6%  21.7    6s
H10015  6040                    2.220988e+09 1.8651e+09  16.0%  21.9    7s
 14440 10090 1.9510e+09   34  118 2.2210e+09 1.8777e+09  15.5%  22.2   16s
 29221 21113 2.1743e+09   64  194 2.2210e+09 1.8968e+09  14.6%  22.2   41s
 44021 25015 2.2017e+09   59   78 2.2210e+09 2.0912e+09  5.84%  21.5   45s
 67284 29365     cutoff   68      2.2210e+09 2.1433e+09  3.50%  23.1   50s
 95426 30088 2.1883e+09   67   49 2.2210e+09 2.1557e+09  2.94%  24.4   55s
 123024 30856 2.2087e+09   73   33 2.2210e+09 2.1627e+09  2.63%  25.5   60s
 150900 39208     cutoff   77      2.2210e+09 2.1672e+09  2.42%  26.1   65s
 178915 46310 2.2064e+09   69   27 2.2210e+09 2.1705e+09  2.27%  26.5   70s
 205035 52682 2.1951e+09   68   47 2.2210e+09 2.1732e+09  2.15%  26.8   75s
 232565 58710 2.1878e+09   69   38 2.2210e+09 2.1755e+09  2.05%  27.0   80s
 262221 64933 2.1932e+09   67   52 2.2210e+09 2.1778e+09  1.95%  27.1   85s
 291280 70444 2.2199e+09   74   37 2.2210e+09 2.1796e+09  1.86%  27.3   90s
 315483 74513 2.2077e+09   64   49 2.2210e+09 2.1811e+09  1.80%  27.3   95s
 343412 78704 2.2023e+09   67   42 2.2210e+09 2.1825e+09  1.73%  27.4  100s
 372429 82977 2.2019e+09   68   44 2.2210e+09 2.1840e+09  1.67%  27.5  105s
 402227 86823 2.1950e+09   69   36 2.2210e+09 2.1854e+09  1.60%  27.5  110s
 432003 90231     cutoff   72      2.2210e+09 2.1865e+09  1.55%  27.5  115s
 459976 93141 2.2099e+09   65   44 2.2210e+09 2.1876e+09  1.50%  27.5  120s
 489925 95604 2.2025e+09   67   51 2.2210e+09 2.1887e+09  1.45%  27.5  125s
 520041 97843     cutoff   69      2.2210e+09 2.1897e+09  1.41%  27.4  130s
 548783 99553     cutoff   74      2.2210e+09 2.1907e+09  1.37%  27.4  135s
 576796 101011 2.2088e+09   69   38 2.2210e+09 2.1915e+09  1.33%  27.4  140s
 607377 102129 2.2015e+09   76   25 2.2210e+09 2.1925e+09  1.28%  27.3  145s
 637488 102982 2.2005e+09   70   31 2.2210e+09 2.1933e+09  1.25%  27.3  150s
 664063 103288 2.2051e+09   70   34 2.2210e+09 2.1941e+09  1.21%  27.2  155s
 694055 103397 2.2090e+09   69   32 2.2210e+09 2.1949e+09  1.17%  27.2  160s
 725149 103453 2.2164e+09   76   31 2.2210e+09 2.1958e+09  1.14%  27.1  165s
 752929 103015 2.1982e+09   67   50 2.2210e+09 2.1965e+09  1.10%  27.1  170s
 783979 102312     cutoff   80      2.2210e+09 2.1973e+09  1.07%  27.0  175s
 815128 101393     cutoff   76      2.2210e+09 2.1980e+09  1.03%  26.9  180s
 847244 100107 2.2145e+09   80   21 2.2210e+09 2.1988e+09  1.00%  26.8  185s
 875396 98676     cutoff   74      2.2210e+09 2.1995e+09  0.97%  26.8  190s
 908619 96675 2.2145e+09   70   38 2.2210e+09 2.2004e+09  0.93%  26.7  195s
 941988 94404 2.2199e+09   67   44 2.2210e+09 2.2012e+09  0.89%  26.6  200s
 975495 91982 2.2124e+09   70   45 2.2210e+09 2.2020e+09  0.85%  26.5  205s
 1006683 89330     cutoff   74      2.2210e+09 2.2028e+09  0.82%  26.4  210s
 1035019 86837 2.2145e+09   72   34 2.2210e+09 2.2035e+09  0.79%  26.3  215s
 1066184 83542     cutoff   76      2.2210e+09 2.2043e+09  0.75%  26.2  220s
 1101582 79522     cutoff   76      2.2210e+09 2.2052e+09  0.71%  26.1  225s
```

```
 1134947 75230     cutoff   77      2.2210e+09 2.2061e+09  0.67%  25.9  230s
 1167689 70692     cutoff   68      2.2210e+09 2.2070e+09  0.63%  25.8  235s
 1206181 64647     cutoff   80      2.2210e+09 2.2082e+09  0.58%  25.7  240s
 1245431 57495 2.2118e+09   80   20 2.2210e+09 2.2095e+09  0.52%  25.5  245s
 1287249 47545     cutoff   71      2.2210e+09 2.2111e+09  0.44%  25.3  250s
 1331336 33826 2.2207e+09   71   30 2.2210e+09 2.2132e+09  0.35%  25.1  255s
 1383561  8535     cutoff   73      2.2210e+09 2.2179e+09  0.14%  24.7  260s

Cutting planes:
  Gomory: 36
  MIR: 56
  Mixing: 1
  Flow cover: 314
  Inf proof: 9
  RLT: 21

Explored 1394392 nodes (34200976 simplex iterations) in 260.67 seconds (373.94 work units)
Thread count was 8 (of 8 available processors)

Solution count 10: 2.22099e+09 2.2211e+09 2.22117e+09 ... 2.25656e+09

Optimal solution found (tolerance 1.00e-04)
Best objective 2.220987800000e+09, best bound 2.220987800000e+09, gap 0.0000%
```

```
function result = solve_miqp_gurobi(A, b, E, f, gvec, params)
% Solve MIQP with Gurobi (MATLAB interface):
%
%   min    || gvec .* (y - 1) ||_2    (implemented as squared norm)
%   s.t.   A x <= b,              x in {0,1}^N
%          y >= E x + f
%          0 <= y <= 2
%
% Decision variables: z = [x; y]
%
% Notes:
% - We minimize the squared norm: (y-1)' diag(g.^2) (y-1)
%   This is equivalent to minimizing the norm since sqrt(.) is monotone.
% - Quadratic form in Gurobi: (1/2) z'Qz + c'z

    if nargin < 6 || isempty(params)
        params = struct();
    end

    % ----- sizes -----
    N = size(A, 2);
    M = size(E, 1);

    % ----- checks -----
    assert(size(A,1) == numel(b), 'A and b size mismatch');
    assert(size(E,2) == N, 'E must be MxN');
    assert(numel(f) == M, 'f must be length M');
    assert(numel(gvec) == M, 'gvec must be length M');
```

```
b = b(:);
f = f(:);
g = gvec(:);

% =========================
% Constraints
% =========================
% (i) A x <= b
A1     = [A, sparse(size(A,1), M)];
rhs1   = b;
sense1 = repmat('<', size(A,1), 1);

% (ii) y >= E x + f  <=>  (-E)x + I*y >= f
A2     = [-E, speye(M)];
rhs2   = f;
sense2 = repmat('>', M, 1);

model = struct();
model.A     = sparse([A1; A2]);
model.rhs   = [rhs1; rhs2];
model.sense = [sense1; sense2];

% =========================
% Objective (squared norm)
% =========================
% objective = (y-1)' D (y-1),  D = diag(g.^2)
% Expand: y' D y - 2*(g.^2)' y + const
D = spdiags(g.^2, 0, M, M);

% Gurobi uses: (1/2) z'Qz + c'z
Q = sparse(N+M, N+M);
Q(N+1:N+M, N+1:N+M) = 2 * D;   % so (1/2)*y'(2D)y = y'Dy
model.Q = Q;

c = zeros(N+M,1);
c(N+1:N+M) = -2*(g.^2);        % linear term in y
model.obj = c;

model.modelsense = 'min';

% =========================
% Variable types & bounds
% =========================
model.vtype = [repmat('B', 1, N), repmat('C', 1, M)];

% Bounds: x in [0,1] via binary; y in [1,2]
model.lb = [zeros(N,1); ones(M,1)];
model.ub = [ones(N,1);  2*ones(M,1)];

% =========================
```

```
    % Params defaults
    % ========================
    if ~isfield(params, 'OutputFlag'); params.OutputFlag = 1; end
    if ~isfield(params, 'MIPGap');     params.MIPGap     = 1e-4; end
    % Convex QP이면 NonConvex 설정 불필요 (Q PSD)

    % ----- solve -----
    result = gurobi(model, params);

    % ----- unpack -----
    if isfield(result, 'x') && ~isempty(result.x)
        z = result.x;
        result.x_bin = z(1:N);
        result.y     = z(N+1:N+M);

        % objective value (squared norm, without constant)
        diff = result.y - 1;
        result.obj_sq = sum((g .* diff).^2);
        result.obj    = sqrt(max(result.obj_sq, 0));
    end
end
```

### 5.4 $L_2$ revisit time problem to Relaxed QP

```
if L2_flag_relaxed == 1
params = struct();
params.OutputFlag = 1;
params.MIPGap = 1e-4;
% params.NonConvex = 2;  % G가 PSD가 아닐 수도 있으면 켜세요.

res = solve_qp_gurobi(A, b, E, f_vector, gvec, params);
end

function result = solve_qp_gurobi(A, b, E, f, gvec, params)
% Solve the relaxed problem as a convex QP with Gurobi (MATLAB interface):
%
%   min    || gvec .* (y - 1) ||_2   (implemented as squared norm)
%   s.t.   A x <= b
%          y >= E x + f
%          0 <= x <= 1
%          0 <= y <= 2
%
% Decision variables: z = [x; y]
%
% Objective in quadratic form:
%   (y-1)' diag(g.^2) (y-1) = y' D y - 2 (g.^2)' y + const
% Gurobi uses: (1/2) z'Qz + obj'z

    if nargin < 6 || isempty(params)
```

```
        params = struct();
    end

    % ----- sizes -----
    N = size(A, 2);
    M = size(E, 1);

    % ----- checks -----
    assert(size(A,1) == numel(b), 'A and b size mismatch');
    assert(size(E,2) == N, 'E must be MxN');
    assert(numel(f) == M, 'f must be length M');
    assert(numel(gvec) == M, 'gvec must be length M');

    b = b(:);
    f = f(:);
    g = gvec(:);

    % ========================
    % Constraints
    % ========================
    % (i) A x <= b
    A1     = [A, sparse(size(A,1), M)];
    rhs1   = b;
    sense1 = repmat('<', size(A,1), 1);

    % (ii) y >= E x + f  <=>  (-E)x + I*y >= f
    A2     = [-E, speye(M)];
    rhs2   = f;
    sense2 = repmat('>', M, 1);

    model = struct();
    model.A     = sparse([A1; A2]);
    model.rhs   = [rhs1; rhs2];
    model.sense = [sense1; sense2];

    % ========================
    % Objective (squared norm)
    % ========================
    % D = diag(g.^2)
    D = spdiags(g.^2, 0, M, M);

    % Build Q for z=[x;y]
    Q = sparse(N+M, N+M);
    Q(N+1:N+M, N+1:N+M) = 2 * D;     % so (1/2)*y'(2D)y = y'Dy
    model.Q = Q;

    % Linear term: -2*(g.^2)' y
    obj = zeros(N+M,1);
    obj(N+1:N+M) = -2*(g.^2);
    model.obj = obj;
```

```
    model.modelsense = 'min';

    % =========================
    % Variable types & bounds
    % =========================
    % Relax x to continuous: both x and y are continuous
    model.vtype = repmat('C', 1, N+M);

    % Bounds
    model.lb = [zeros(N,1); ones(M,1)];
    model.ub = [ones(N,1);  2*ones(M,1)];

    % ---- (옵션) y 하한을 1로 두고 싶으면 아래 두 줄로 교체 ----
    % model.lb = [zeros(N,1); ones(M,1)];
    % model.ub = [ones(N,1);  2*ones(M,1)];

    % =========================
    % Params defaults
    % =========================
    if ~isfield(params, 'OutputFlag'); params.OutputFlag = 1; end

    % Convex QP이면 NonConvex 필요 없음.
    % 혹시 g에 NaN/Inf가 있거나 Q가 비정상인 경우만 점검.

    % ----- solve -----
    result = gurobi(model, params);

    % ----- unpack -----
    if isfield(result, 'x') && ~isempty(result.x)
        z = result.x;
        result.x_relaxed = z(1:N);
        result.y         = z(N+1:N+M);

        diff = result.y - 1;
        result.obj_sq = sum((g .* diff).^2);
        result.obj    = sqrt(max(result.obj_sq, 0));
    end
end
```

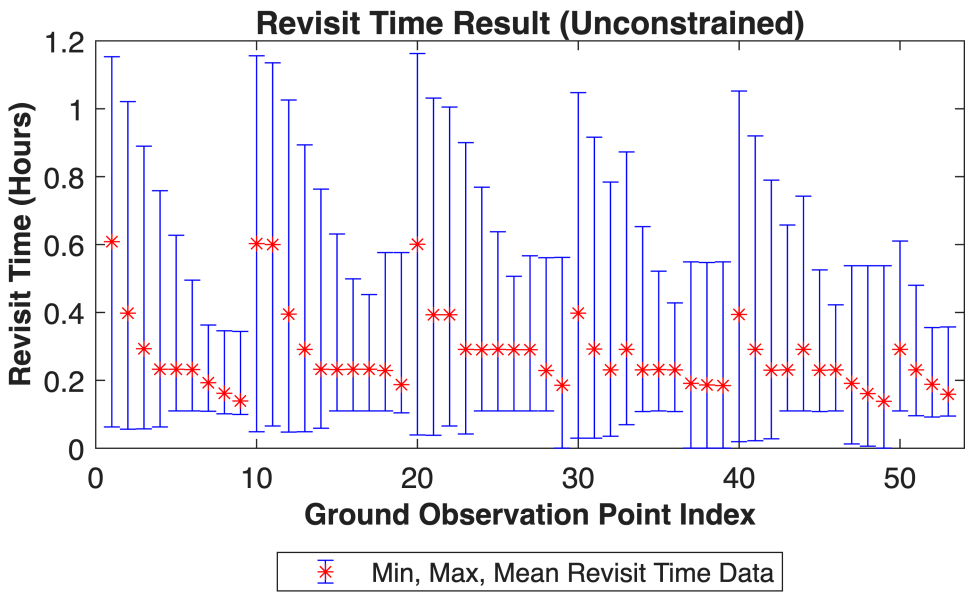
### 6.0 Original Revisit Time Status Result

```
t_start = t0 + seconds(min(A_matrix(:,3)));
t_end = t0 + seconds(max(A_matrix(:,3))+1);
```

```
% Extract Ground Point Tables
  T = Original_A;

    % Source 열에서 고유 값 추출
    sources = unique(T.Source);

    % Revisit Time Matrix (Min / Max / Mean) 생성
    Revisit_Matrix = zeros(length(sources),3);
    revisit_time_vector_combined = [];

    % 각각의 Source 별로 테이블을 분리하여 저장
    for i = 1:length(sources)
        src = sources{i};

        % Source 값이 같은 행들만 추출
        subTable = T(strcmp(T.Source, src), :);
        subTable_sorted = sortrows(subTable,4,"ascend");


        % 동적으로 변수 생성 (예: Ground_Point_1_Table)
        varName = sprintf('%s_Table', src);
        % assignin('base', varName, subTable_sorted);
        contact_tables.(sprintf('%s_Table', src)) = subTable_sorted;

        % 기존 row에 start 및 endtime 추가
        row0 = subTable_sorted(1,:);

        % -------------------------
        % 윗줄 (t_start)
        % -------------------------
        row_top = row0;
        row_top.IntervalNumber = NaN;      % 필요 없으면 NaN
        row_top.StartTime = t_start;
        row_top.EndTime   = t_start;
        row_top.Duration  = 0;
        row_top.StartOrbit = 0;
        row_top.EndOrbit   = 0;

        % -------------------------
        % 아랫줄 (t_end)
        % -------------------------
        row_bottom = row0;
        row_bottom.IntervalNumber = NaN;
        row_bottom.StartTime = t_end;
        row_bottom.EndTime   = t_end;
        row_bottom.Duration  = 0;
        row_bottom.StartOrbit = 0;
        row_bottom.EndOrbit   = 0;
```

```
        % -------------------------
        % 위 + 기존 + 아래 결합
        % -------------------------
        subTable_sorted = [row_top; subTable_sorted; row_bottom];

        % Initialize Revisit Time Vector
        revisit_time_vector = zeros(height(subTable_sorted)-1,1);

        for revisit_time_index = 1:length(revisit_time_vector)
            revisit_time_vector(revisit_time_index)
= seconds(table2array(subTable_sorted(revisit_time_index+1,4))-
table2array(subTable_sorted(revisit_time_index,5)));

            if revisit_time_vector(revisit_time_index) < 0
               revisit_time_vector(revisit_time_index) = 0;
            end
        revisit_vectors.(sprintf('%s_revisit_time_vec', src)) =
revisit_time_vector;
        end

        % 생성된 Revisit Time Vector의 Min / Max / Mean 값을 Revisit Time
Matrix에 저장
        Revisit_Matrix(i,1) = min(revisit_time_vector);
        Revisit_Matrix(i,2) = max(revisit_time_vector);

        % 재방문 주기의 평균을 구할때는 재방문 주기가 0인 데이터 포인트를 모두 제외하였음
        revisit_time_vector = revisit_time_vector(revisit_time_vector~=0);

        Revisit_Matrix(i,3) = mean(revisit_time_vector);
        revisit_time_vector_combined =
[revisit_time_vector_combined;revisit_time_vector];
       revisit_time_vector_info.(['source' num2str(i)]) =
revisit_time_vector;
    end

    % Revisit Time Matrix의 값을 그래프로 출력
    x = 1:length(Revisit_Matrix(:,1));
    min_value = (Revisit_Matrix(:,1))/3600;
    max_value = (Revisit_Matrix(:,2))/3600;
    mean_value = (Revisit_Matrix(:,3))/3600;
    output_data = [mean_value, min_value, max_value];

    figure;
    errorbar(x, mean_value, mean_value-min_value, max_value-
mean_value,'*','LineStyle','none','color','b','MarkerEdgeColor','r')
    t = title('Revisit Time Result
(Unconstrained)','FontSize',12,'FontWeight','bold');
    xlabel('Ground Observation Point
Index','FontSize',11,'FontWeight','bold')
```

```
    ylabel('Revisit Time (Hours)','FontSize',11,'FontWeight','bold')
    legend('Min, Max, Mean Revisit Time Data','Location','southoutside')
    xlim([-1, length(Revisit_Matrix(:,1))]+1)
```



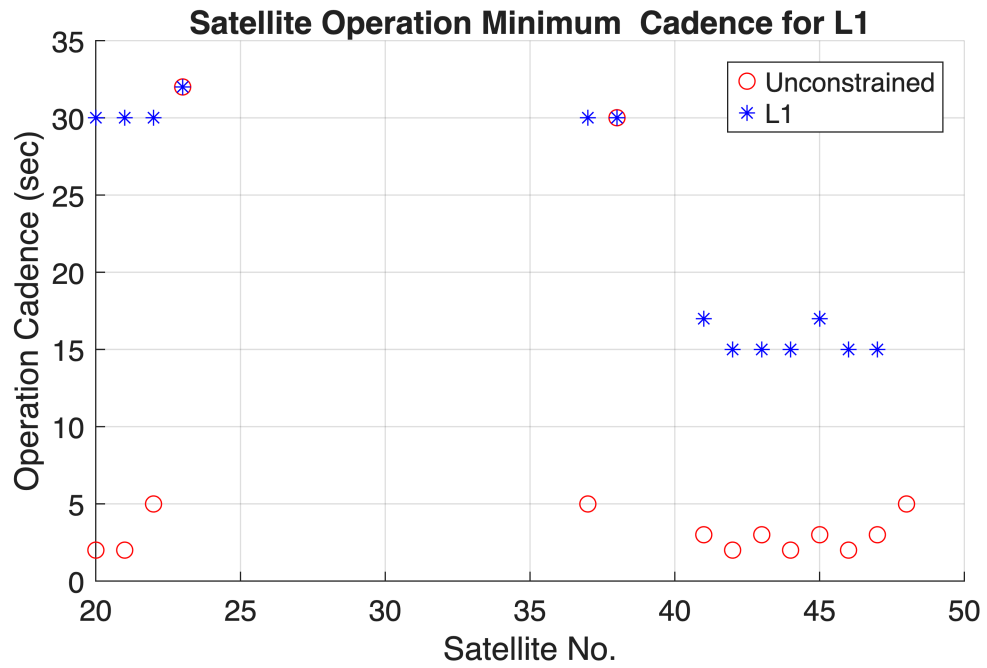

## 6.1 L1 Revisit Time Status Result

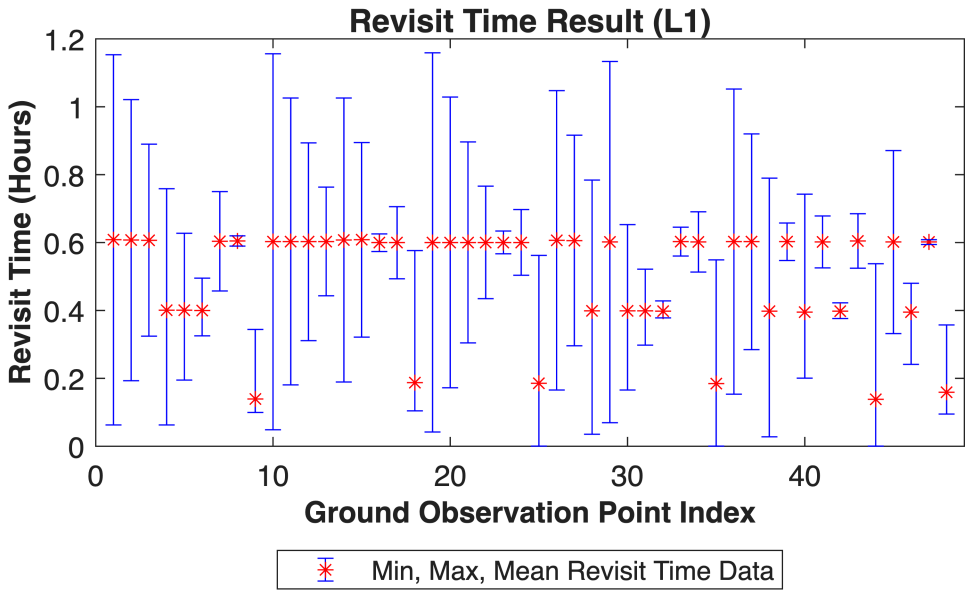
```
if L1_flag == 1

A_matrix_result = A_matrix;

active_satellites = unique(A_matrix_result(:,1));
cadence_matrix_original = zeros(length(active_satellites),4);

    for index = 1:length(active_satellites)
      sat_index = active_satellites(index);
      % satellite_cadence_info.(['sat',num2str(sat_index)]) =
A_matrix_result(A_matrix_result(:,1) == sat_index,:);
      satellite_cadence = A_matrix_result(A_matrix_result(:,1) ==
sat_index,:);
      cadence_info = zeros(length(satellite_cadence(:,1))-1,1);
      for i = 1:length(cadence_info(:,1))
         cadence_info(i) = satellite_cadence(i+1,3) -
satellite_cadence(i,3);
      end

      if isscalar(satellite_cadence(:,1))
         continue;
      end
```

```
      cadence_matrix_original(index,:) = [sat_index, max(cadence_info),
min(cadence_info), mean(cadence_info)];
    end


A_matrix_result = A_matrix_L1;

active_satellites = unique(A_matrix_result(:,1));
cadence_matrix_L1 = zeros(length(active_satellites),4);

    for index = 1:length(active_satellites)
      sat_index = active_satellites(index);
      % satellite_cadence_info.(['sat',num2str(sat_index)]) =
A_matrix_result(A_matrix_result(:,1) == sat_index,:);
      satellite_cadence = A_matrix_result(A_matrix_result(:,1) ==
sat_index,:);
      cadence_info = zeros(length(satellite_cadence(:,1))-1,1);

      if isscalar(satellite_cadence(:,1))
         continue;
      end

      for i = 1:length(cadence_info(:,1))
         cadence_info(i) = satellite_cadence(i+1,3) -
satellite_cadence(i,3);
      end


      cadence_matrix_L1(index,:) = [sat_index, max(cadence_info),
min(cadence_info), mean(cadence_info)];
    end

figure;
hold on
scatter(cadence_matrix_original(:,1),cadence_matrix_original(:,3),'o','red')
scatter(cadence_matrix_L1(:,1),cadence_matrix_L1(:,3),'*','blue')
title('Satellite Operation Minimum  Cadence for
L1','FontSize',12,'FontWeight','bold')
xlabel('Satellite No.')
ylabel('Operation Cadence (sec)')
legend('Unconstrained','L1')
grid on


% Extract Ground Point Tables
  T = Original_A_L1;

    % Source 열에서 고유 값 추출
    sources = unique(T.Source);
```

```
% Revisit Time Matrix (Min / Max / Mean) 생성
Revisit_Matrix = zeros(length(sources),3);
revisit_time_vector_combined = [];

% 각각의 Source 별로 테이블을 분리하여 저장
for i = 1:length(sources)
    src = sources{i};

    % Source 값이 같은 행들만 추출
    subTable = T(strcmp(T.Source, src), :);
    subTable_sorted = sortrows(subTable,4,"ascend");


    % 동적으로 변수 생성 (예: Ground_Point_1_Table)
    varName = sprintf('%s_Table', src);
    % assignin('base', varName, subTable_sorted);
    contact_tables.(sprintf('%s_Table', src)) = subTable_sorted;

    % 기존 row에 start 및 endtime 추가
    row0 = subTable_sorted(1,:);

    % -------------------------
    % 윗줄 (t_start)
    % -------------------------
    row_top = row0;
    row_top.IntervalNumber = NaN;      % 필요 없으면 NaN
    row_top.StartTime = t_start;
    row_top.EndTime   = t_start;
    row_top.Duration  = 0;
    row_top.StartOrbit = 0;
    row_top.EndOrbit   = 0;

    % -------------------------
    % 아랫줄 (t_end)
    % -------------------------
    row_bottom = row0;
    row_bottom.IntervalNumber = NaN;
    row_bottom.StartTime = t_end;
    row_bottom.EndTime   = t_end;
    row_bottom.Duration  = 0;
    row_bottom.StartOrbit = 0;
    row_bottom.EndOrbit   = 0;

    % -------------------------
    % 위 + 기존 + 아래 결합
    % -------------------------
    subTable_sorted = [row_top; subTable_sorted; row_bottom];

    % Initialize Revisit Time Vector
    revisit_time_vector = zeros(height(subTable_sorted)-1,1);
```

```
        for revisit_time_index = 1:length(revisit_time_vector)
            revisit_time_vector(revisit_time_index)
= seconds(table2array(subTable_sorted(revisit_time_index+1,4))-
table2array(subTable_sorted(revisit_time_index,5)));

            if revisit_time_vector(revisit_time_index) < 0
               revisit_time_vector(revisit_time_index) = 0;
            end
        revisit_vectors.(sprintf('%s_revisit_time_vec', src)) =
revisit_time_vector;
        end

        % 생성된 Revisit Time Vector의 Min / Max / Mean 값을 Revisit Time
Matrix에 저장
        Revisit_Matrix(i,1) = min(revisit_time_vector);
        Revisit_Matrix(i,2) = max(revisit_time_vector);

        % 재방문 주기의 평균을 구할때는 재방문 주기가 0인 데이터 포인트를 모두 제외하였음
        revisit_time_vector = revisit_time_vector(revisit_time_vector~=0);

        Revisit_Matrix(i,3) = mean(revisit_time_vector);
        revisit_time_vector_combined =
[revisit_time_vector_combined;revisit_time_vector];
       revisit_time_vector_info.(['source' num2str(i)]) =
revisit_time_vector;
    end

    % Revisit Time Matrix의 값을 그래프로 출력
    x = 1:length(Revisit_Matrix(:,1));
    min_value = (Revisit_Matrix(:,1))/3600;
    max_value = (Revisit_Matrix(:,2))/3600;
    mean_value = (Revisit_Matrix(:,3))/3600;
    output_data = [mean_value, min_value, max_value];

    figure;
    errorbar(x, mean_value, mean_value-min_value, max_value-
mean_value,'*','LineStyle','none','color','b','MarkerEdgeColor','r')
    t = title('Revisit Time Result (L1)','FontSize',12,'FontWeight','bold');
    xlabel('Ground Observation Point
Index','FontSize',11,'FontWeight','bold')
    ylabel('Revisit Time (Hours)','FontSize',11,'FontWeight','bold')
    legend('Min, Max, Mean Revisit Time Data','Location','southoutside')
    xlim([-1, length(Revisit_Matrix(:,1))]+1)
end
```




## 6.2 L_infty Revisit Time Status Result

```
if L_infty_flag == 1

A_matrix_result = A_matrix;

active_satellites = unique(A_matrix_result(:,1));
```

```
cadence_matrix_original = zeros(length(active_satellites),4);

    for index = 1:length(active_satellites)
      sat_index = active_satellites(index);
      % satellite_cadence_info.(['sat',num2str(sat_index)]) =
A_matrix_result(A_matrix_result(:,1) == sat_index,:);
      satellite_cadence = A_matrix_result(A_matrix_result(:,1) ==
sat_index,:);
      cadence_info = zeros(length(satellite_cadence(:,1))-1,1);
      for i = 1:length(cadence_info(:,1))
         cadence_info(i) = satellite_cadence(i+1,3) -
satellite_cadence(i,3);
      end

      if isscalar(satellite_cadence(:,1))
         continue;
      end

      cadence_matrix_original(index,:) = [sat_index, max(cadence_info),
min(cadence_info), mean(cadence_info)];
    end


A_matrix_result = A_matrix_L_inf;

active_satellites = unique(A_matrix_result(:,1));
cadence_matrix_L_inf = zeros(length(active_satellites),4);

    for index = 1:length(active_satellites)
      sat_index = active_satellites(index);
      % satellite_cadence_info.(['sat',num2str(sat_index)]) =
A_matrix_result(A_matrix_result(:,1) == sat_index,:);
      satellite_cadence = A_matrix_result(A_matrix_result(:,1) ==
sat_index,:);

      if isscalar(satellite_cadence(:,1))
         continue;
      end

      cadence_info = zeros(length(satellite_cadence(:,1))-1,1);
      for i = 1:length(cadence_info(:,1))
         cadence_info(i) = satellite_cadence(i+1,3) -
satellite_cadence(i,3);
      end

      cadence_matrix_L_inf(index,:) = [sat_index, max(cadence_info),
min(cadence_info), mean(cadence_info)];
    end
```

```
cadence_matrix_L_inf = cadence_matrix_L_inf(cadence_matrix_L_inf(:,3)~=0,:);


figure;
hold on
scatter(cadence_matrix_original(:,1),cadence_matrix_original(:,3),'o','red')
scatter(cadence_matrix_L_inf(:,1),cadence_matrix_L_inf(:,3),'*','blue')
title('Satellite Operation Minimum Cadence for L
infty','FontSize',12,'FontWeight','bold')
legend('Unconstrained','L infty')
xlabel('Satellite No.')
ylabel('Operation Cadence (sec)')
grid on



 % Extract Ground Point Tables
  T = Original_A_L_inf;

    % Source 열에서 고유 값 추출
    sources = unique(T.Source);

    % Revisit Time Matrix (Min / Max / Mean) 생성
    Revisit_Matrix = zeros(length(sources),3);
    revisit_time_vector_combined = [];

    % 각각의 Source 별로 테이블을 분리하여 저장
    for i = 1:length(sources)
        src = sources{i};

        % Source 값이 같은 행들만 추출
        subTable = T(strcmp(T.Source, src), :);
        subTable_sorted = sortrows(subTable,4,"ascend");


        % 동적으로 변수 생성 (예: Ground_Point_1_Table)
        varName = sprintf('%s_Table', src);
        % assignin('base', varName, subTable_sorted);
        contact_tables.(sprintf('%s_Table', src)) = subTable_sorted;

        % 기존 row에 start 및 endtime 추가
        row0 = subTable_sorted(1,:);

        % -------------------------
        % 윗줄 (t_start)
        % -------------------------
        row_top = row0;
        row_top.IntervalNumber = NaN;      % 필요 없으면 NaN
        row_top.StartTime = t_start;
        row_top.EndTime   = t_start;
```

```
        row_top.Duration  = 0;
        row_top.StartOrbit = 0;
        row_top.EndOrbit   = 0;

        % -------------------------
        % 아랫줄 (t_end)
        % -------------------------
        row_bottom = row0;
        row_bottom.IntervalNumber = NaN;
        row_bottom.StartTime = t_end;
        row_bottom.EndTime   = t_end;
        row_bottom.Duration  = 0;
        row_bottom.StartOrbit = 0;
        row_bottom.EndOrbit   = 0;

        % -------------------------
        % 위 + 기존 + 아래 결합
        % -------------------------
        subTable_sorted = [row_top; subTable_sorted; row_bottom];

        % Initialize Revisit Time Vector
        revisit_time_vector = zeros(height(subTable_sorted)-1,1);

        for revisit_time_index = 1:length(revisit_time_vector)
            revisit_time_vector(revisit_time_index)
= seconds(table2array(subTable_sorted(revisit_time_index+1,4))-
table2array(subTable_sorted(revisit_time_index,5)));

            if revisit_time_vector(revisit_time_index) < 0
               revisit_time_vector(revisit_time_index) = 0;
            end
        revisit_vectors.(sprintf('%s_revisit_time_vec', src)) =
revisit_time_vector;
        end

        % 생성된 Revisit Time Vector의 Min / Max / Mean 값을 Revisit Time
Matrix에 저장
        Revisit_Matrix(i,1) = min(revisit_time_vector);
        Revisit_Matrix(i,2) = max(revisit_time_vector);

        % 재방문 주기의 평균을 구할때는 재방문 주기가 0인 데이터 포인트를 모두 제외하였음
        revisit_time_vector = revisit_time_vector(revisit_time_vector~=0);

        Revisit_Matrix(i,3) = mean(revisit_time_vector);
        revisit_time_vector_combined =
[revisit_time_vector_combined;revisit_time_vector];
       revisit_time_vector_info.(['source' num2str(i)]) =
revisit_time_vector;
    end
```

```
    % Revisit Time Matrix의 값을 그래프로 출력
    x = 1:length(Revisit_Matrix(:,1));
    min_value = (Revisit_Matrix(:,1))/3600;
    max_value = (Revisit_Matrix(:,2))/3600;
    mean_value = (Revisit_Matrix(:,3))/3600;
    output_data = [mean_value, min_value, max_value];

    figure;
    errorbar(x, mean_value, mean_value-min_value, max_value-
mean_value,'*','LineStyle','none','color','b','MarkerEdgeColor','r')
    t = title('Revisit Time Result (L
infty)','FontSize',12,'FontWeight','bold');
    xlabel('Ground Observation Point
Index','FontSize',11,'FontWeight','bold')
    ylabel('Revisit Time (Hours)','FontSize',11,'FontWeight','bold')
    legend('Min, Max, Mean Revisit Time Data','Location','southoutside')
    xlim([-1, length(Revisit_Matrix(:,1))]+1)
end
```

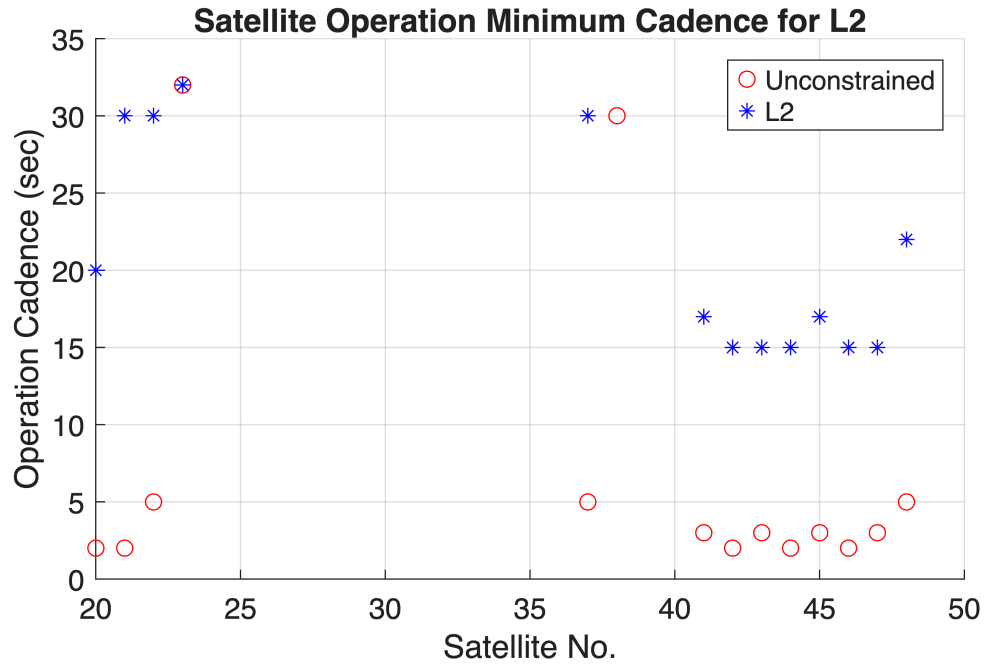
### 6.2 L2 Revisit Time Status Result

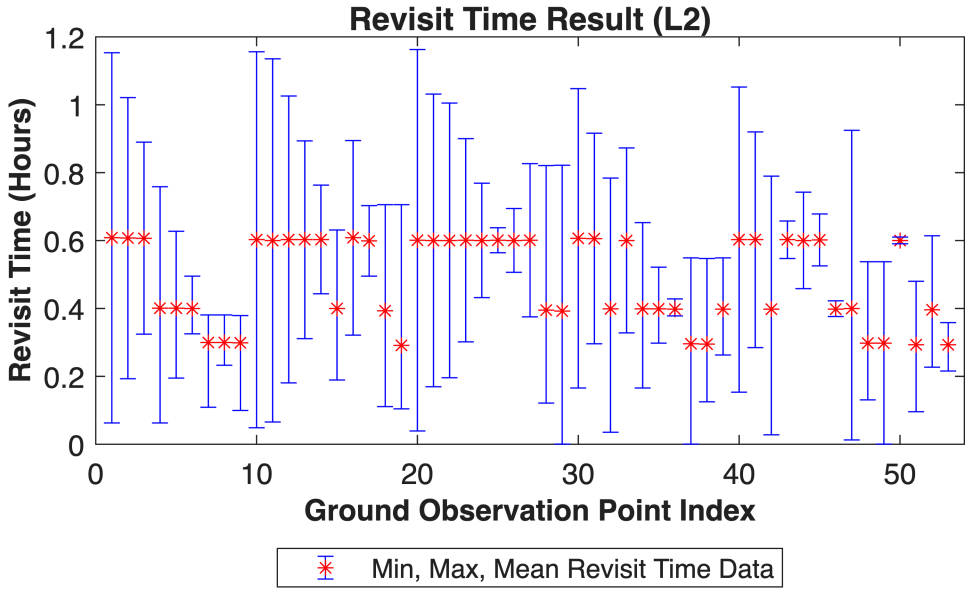
```
if L2_flag == 1

A_matrix_result = A_matrix;

active_satellites = unique(A_matrix_result(:,1));
cadence_matrix_original = zeros(length(active_satellites),4);

    for index = 1:length(active_satellites)
      sat_index = active_satellites(index);
      % satellite_cadence_info.(['sat',num2str(sat_index)]) =
A_matrix_result(A_matrix_result(:,1) == sat_index,:);
      satellite_cadence = A_matrix_result(A_matrix_result(:,1) ==
sat_index,:);
      cadence_info = zeros(length(satellite_cadence(:,1))-1,1);
      for i = 1:length(cadence_info(:,1))
         cadence_info(i) = satellite_cadence(i+1,3) -
satellite_cadence(i,3);
      end

      if isscalar(satellite_cadence(:,1))
         continue;
      end

      cadence_matrix_original(index,:) = [sat_index, max(cadence_info),
min(cadence_info), mean(cadence_info)];
    end
```

```
A_matrix_result = A_matrix_L2;

active_satellites = unique(A_matrix_result(:,1));
cadence_matrix_L2 = zeros(length(active_satellites),4);

    for index = 1:length(active_satellites)
      sat_index = active_satellites(index);
      % satellite_cadence_info.(['sat',num2str(sat_index)]) =
A_matrix_result(A_matrix_result(:,1) == sat_index,:);
      satellite_cadence = A_matrix_result(A_matrix_result(:,1) ==
sat_index,:);

      if isscalar(satellite_cadence(:,1))
         continue;
      end

      cadence_info = zeros(length(satellite_cadence(:,1))-1,1);
      for i = 1:length(cadence_info(:,1))
         cadence_info(i) = satellite_cadence(i+1,3) -
satellite_cadence(i,3);
      end

      cadence_matrix_L2(index,:) = [sat_index, max(cadence_info),
min(cadence_info), mean(cadence_info)];
    end


cadence_matrix_L2 = cadence_matrix_L2(cadence_matrix_L2(:,3)~=0,:);


figure;
hold on
scatter(cadence_matrix_original(:,1),cadence_matrix_original(:,3),'o','red')
scatter(cadence_matrix_L2(:,1),cadence_matrix_L2(:,3),'*','blue')
title('Satellite Operation Minimum Cadence for
L2','FontSize',12,'FontWeight','bold')
legend('Unconstrained','L2')
xlabel('Satellite No.')
ylabel('Operation Cadence (sec)')
grid on

 % Extract Ground Point Tables
  T = Original_A_L2;

    % Source 열에서 고유 값 추출
    sources = unique(T.Source);

    % Revisit Time Matrix (Min / Max / Mean) 생성
```

```
    Revisit_Matrix = zeros(length(sources),3);
    revisit_time_vector_combined = [];

    % 각각의 Source 별로 테이블을 분리하여 저장
    for i = 1:length(sources)
        src = sources{i};

        % Source 값이 같은 행들만 추출
        subTable = T(strcmp(T.Source, src), :);
        subTable_sorted = sortrows(subTable,4,"ascend");


        % 동적으로 변수 생성 (예: Ground_Point_1_Table)
        varName = sprintf('%s_Table', src);
        % assignin('base', varName, subTable_sorted);
        contact_tables.(sprintf('%s_Table', src)) = subTable_sorted;

        % 기존 row에 start 및 endtime 추가
        row0 = subTable_sorted(1,:);

        % -------------------------
        % 윗줄 (t_start)
        % -------------------------
        row_top = row0;
        row_top.IntervalNumber = NaN;      % 필요 없으면 NaN
        row_top.StartTime = t_start;
        row_top.EndTime   = t_start;
        row_top.Duration  = 0;
        row_top.StartOrbit = 0;
        row_top.EndOrbit   = 0;

        % -------------------------
        % 아랫줄 (t_end)
        % -------------------------
        row_bottom = row0;
        row_bottom.IntervalNumber = NaN;
        row_bottom.StartTime = t_end;
        row_bottom.EndTime   = t_end;
        row_bottom.Duration  = 0;
        row_bottom.StartOrbit = 0;
        row_bottom.EndOrbit   = 0;

        % -------------------------
        % 위 + 기존 + 아래 결합
        % -------------------------
        subTable_sorted = [row_top; subTable_sorted; row_bottom];

        % Initialize Revisit Time Vector
        revisit_time_vector = zeros(height(subTable_sorted)-1,1);
```

```
        for revisit_time_index = 1:length(revisit_time_vector)
            revisit_time_vector(revisit_time_index)
= seconds(table2array(subTable_sorted(revisit_time_index+1,4))-
table2array(subTable_sorted(revisit_time_index,5)));

            if revisit_time_vector(revisit_time_index) < 0
               revisit_time_vector(revisit_time_index) = 0;
            end
        revisit_vectors.(sprintf('%s_revisit_time_vec', src)) =
revisit_time_vector;
        end

        % 생성된 Revisit Time Vector의 Min / Max / Mean 값을 Revisit Time
Matrix에 저장
        Revisit_Matrix(i,1) = min(revisit_time_vector);
        Revisit_Matrix(i,2) = max(revisit_time_vector);

        % 재방문 주기의 평균을 구할때는 재방문 주기가 0인 데이터 포인트를 모두 제외하였음
        revisit_time_vector = revisit_time_vector(revisit_time_vector~=0);

        Revisit_Matrix(i,3) = mean(revisit_time_vector);
        revisit_time_vector_combined =
[revisit_time_vector_combined;revisit_time_vector];
       revisit_time_vector_info.(['source' num2str(i)]) =
revisit_time_vector;
    end

    % Revisit Time Matrix의 값을 그래프로 출력
    x = 1:length(Revisit_Matrix(:,1));
    min_value = (Revisit_Matrix(:,1))/3600;
    max_value = (Revisit_Matrix(:,2))/3600;
    mean_value = (Revisit_Matrix(:,3))/3600;
    output_data = [mean_value, min_value, max_value];

    figure;
    errorbar(x, mean_value, mean_value-min_value, max_value-
mean_value,'*','LineStyle','none','color','b','MarkerEdgeColor','r')
    t = title('Revisit Time Result (L2)','FontSize',12,'FontWeight','bold');
    xlabel('Ground Observation Point
Index','FontSize',11,'FontWeight','bold')
    ylabel('Revisit Time (Hours)','FontSize',11,'FontWeight','bold')
    legend('Min, Max, Mean Revisit Time Data','Location','southoutside')
    xlim([-1, length(Revisit_Matrix(:,1))]+1)
end
```


Satellite Operation Minimum Cadence for L2
Unconstrained
L2
Operation Cadence (sec)
Satellite No.
0
5
10
15
20
25
30
35
20
25
30
35
40
45
50


Revisit Time Result (L2)
Revisit Time (Hours)
Ground Observation Point Index
Min, Max, Mean Revisit Time Data
0
0.2
0.4
0.6
0.8
1
1.2
0
10
20
30
40
50