RS-MILP-01: Configure Optimization Problem from Contact Matrix

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1. Generate A matrix from contact chart

```
clear;
clc;

addpath ~/Desktop/Redstone_MILP/RS_MILP_01_Config_MILP/
load('EOIR_48_SATs_12_Orbit_Planes_98_inc_7days_access_interval.mat','EOIR_a
ccess_interval')


% 기준 시각 정의

T = EOIR_access_interval;

t0 = datetime(2030, 1, 1, 0, 0, 0,'TimeZone','UTC');

% --- Source / Target을 string으로 통일 ---
src = string(T.Source);
tgt = string(T.Target);

% --- Ground 번호 추출 ---
gTok = regexp(src, 'Ground[_\s]*Point[_\s]*(\d+)', 'tokens', 'once');
groundNum = cellfun(@(c) str2double(c{1}), gTok);

% --- Satellite 번호 추출 ---
sTok = regexp(tgt, '(?i)SAT[^0-9]*(\d+)', 'tokens', 'once');
satNum = cellfun(@(c) str2double(c{1}), sTok);

% --- Start / End → datetime ---
st = T.StartTime;
en = T.EndTime;

if ~isdatetime(st)
    st = datetime(string(st), 'InputFormat','dd-MMM-yyyy HH:mm:ss');
end
if ~isdatetime(en)
    en = datetime(string(en), 'InputFormat','dd-MMM-yyyy HH:mm:ss');
end
```

```
% --- 중간 시각 ---
midTime = st + (en - st)/2;

% --- 기준 시각 대비 초(second) 차이 → integer ---
timeSec = seconds(midTime - t0);
timeSec = double(round(timeSec));   % 정수화 (필요시 floor / ceil로 변경 가능)


contact_index = (1:length(st))';

% --- 최종 3-column matrix ---
A_matrix = [double(satNum), double(groundNum), timeSec, contact_index];

A_matrix = sortrows(A_matrix, 3);

A_matrix = A_matrix(1:500,:);
Original_A = T(A_matrix(:,4),:);
```

2. Extract Key parameters from A matrix

```
t = A_matrix(:,3);

% Satellite Cadence Constraint
tau = 20;

% Number of SATs
p = 48;

% Number of GSs
q = 54;

% Total number of contact
N = length(A_matrix(:,1));

% Number of contact for each SAT
S_i_vec = zeros(p,1);

for sat_index = 1:p
    S_i_vec(sat_index) = nnz(A_matrix(:,1) == sat_index);
end

% Number of contact for each GS
G_j_vec = zeros(q,1);
for gs_index = 1:q
    G_j_vec(gs_index) = nnz(A_matrix(:,2) == gs_index);
end
```

3. Selection matrix generation from given constant parameters

3.1. $E^1_{S_i}$: Selection matrix from A-matrix to each satellite's contact sequence

```
E1_Si = struct();

for i = 1:p

E1_Si_mat = zeros(S_i_vec(i),N);

a_i = find(A_matrix(:,1) == i);

  for j = 1:length(a_i)
      E1_Si_mat(j, a_i(j)) = 1;
  end

  E1_Si.(['sat',num2str(i)]) = E1_Si_mat;

end
```

3.2. $E^2_{S_i,x}$, $E^2_{S_{i,t}}$: Selection matrix for Δt of each satelltite from $|S_i|$

```
E2_Si_x = struct();

for i = 1:p
    E2_Si_x_mat = [];
    for alpha = 1:S_i_vec(i)-1
        E2_Si_x_alpha = zeros(S_i_vec(i)-alpha, S_i_vec(i));
        E2_Si_x_alpha(:,alpha) = 1;
        for beta = alpha+1:S_i_vec(i)
            E2_Si_x_alpha(beta-alpha,beta) = 1;
        end
     E2_Si_x_mat = [E2_Si_x_mat;E2_Si_x_alpha];
    end
   if isempty(E2_Si_x_mat)
     E2_Si_x_mat = 0;
   end
   E2_Si_x.(['sat',num2str(i)]) = E2_Si_x_mat;
end
```

```
E2_Si_t = struct();

for i = 1:p
    E2_Si_t_mat = [];
    for alpha = 1:S_i_vec(i)-1
        E2_Si_t_alpha = zeros(S_i_vec(i)-alpha, S_i_vec(i));
        E2_Si_t_alpha(:,alpha) = -1;
        for beta = alpha+1:S_i_vec(i)
            E2_Si_t_alpha(beta-alpha,beta) = 1;
        end
     E2_Si_t_mat = [E2_Si_t_mat;E2_Si_t_alpha];
    end
   if isempty(E2_Si_t_mat)
     E2_Si_t_mat = 0;
   end
    E2_Si_t.(['sat',num2str(i)]) = E2_Si_t_mat;
end
```

3.3. $E^1_{G_j}$: Selection matrix from A-matrix to each Ground Point's revisit sequence

```
E1_Gj = struct();

for j = 1:q
    E1_Gj_mat = zeros(G_j_vec(j)+2, N+2);

    E1_Gj_mat(1,1) = 1;
    b_j = find(A_matrix(:,2) == j);

    for alpha = 1:length(b_j)
        E1_Gj_mat(alpha+1, b_j(alpha)+1) = 1;
    end
    E1_Gj_mat(G_j_vec(j)+2, N+2) = 1;
    E1_Gj.(['gs', num2str(j)]) = E1_Gj_mat;
end
```

3.4. $E^2_{G_{j,x}}, E^2_{G_{j,t}}$: Selection matrix for Δt of each GS from $|G_j|$

```
E2_Gj_x = struct();

for j = 1:q
    E2_Gj_x_mat = [];
    for alpha = 1:G_j_vec(j)+1
        E2_Gj_x_alpha = zeros(G_j_vec(j)+2-alpha, G_j_vec(j)+2);
        E2_Gj_x_alpha(:,alpha) = 1;
        for beta = alpha+1:G_j_vec(j)+2
```

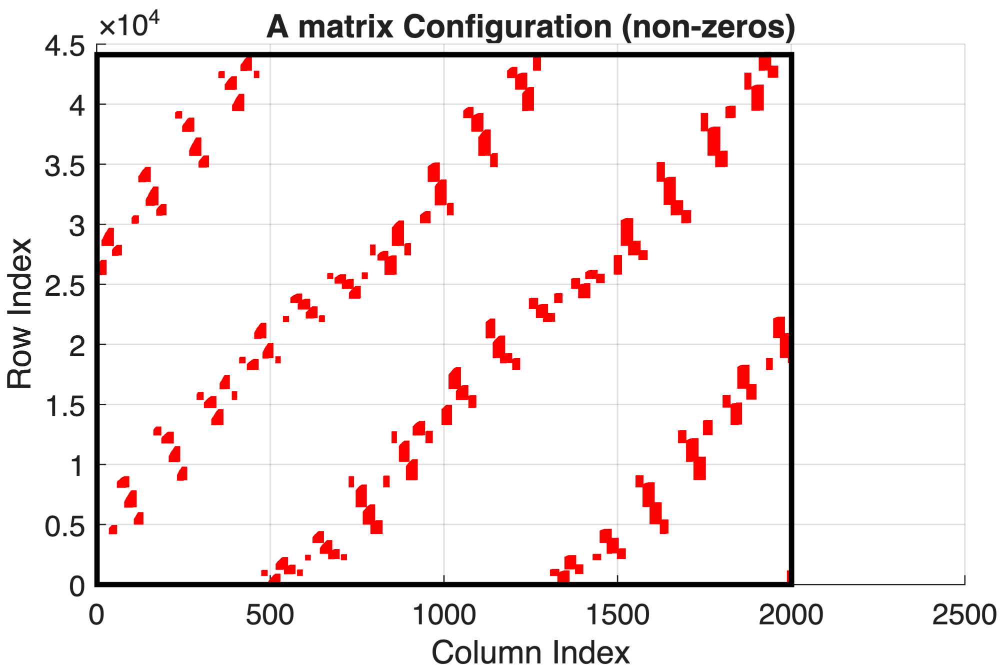
```
            if alpha + 1 <= beta-1
               E2_Gj_x_alpha(beta-alpha, alpha+1:beta-1) = -1;
            end
            E2_Gj_x_alpha(beta-alpha,beta) = 1;
         end
      E2_Gj_x_mat = [E2_Gj_x_mat;E2_Gj_x_alpha];
     end
   if isempty(E2_Gj_x_mat)
     E2_Gj_x_mat = 0;
   end
   E2_Gj_x.(['gs',num2str(j)]) = E2_Gj_x_mat;
end

E2_Gj_t = struct();

for j = 1:q
    E2_Gj_t_mat = [];
    for alpha = 1:G_j_vec(j)+1
        E2_Gj_t_alpha = zeros(G_j_vec(j)+2-alpha, G_j_vec(j)+2);
        E2_Gj_t_alpha(:,alpha) = -1;
        for beta = alpha+1:G_j_vec(j)+2
            E2_Gj_t_alpha(beta-alpha,beta) = 1;
        end
     E2_Gj_t_mat = [E2_Gj_t_mat;E2_Gj_t_alpha];
    end
   if isempty(E2_Gj_t_mat)
     E2_Gj_t_mat = 0;
   end
   E2_Gj_t.(['gs',num2str(j)]) = E2_Gj_t_mat;
end
```

4. Derivation of A, b, C, d, E, f, G revisit time problem to MILP

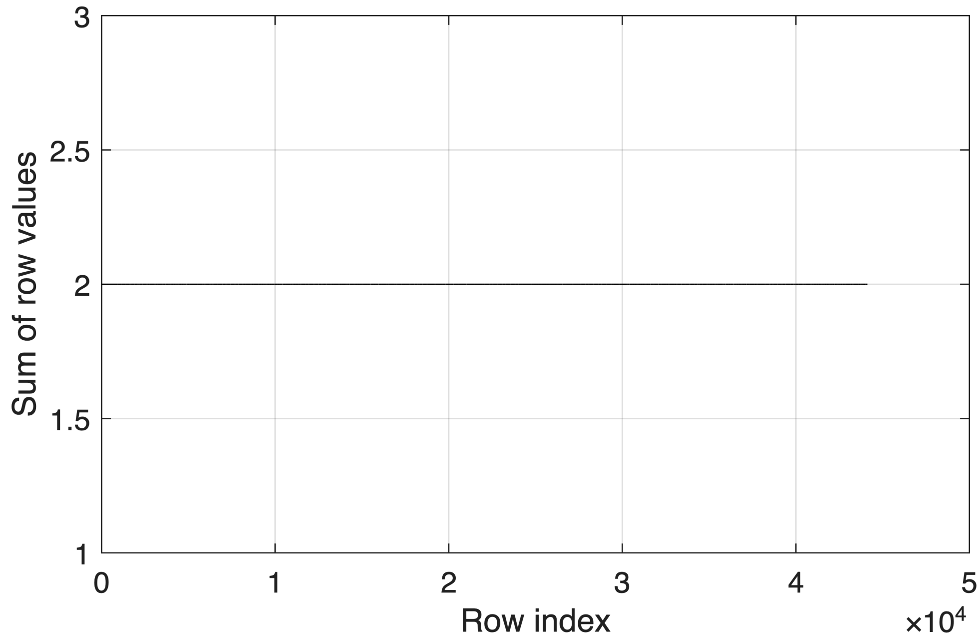
4.1 A, b for L_1 problem

```
A = [];
b_mat = [];
for i = 1:p
  if isempty(E1_Si.(['sat', num2str(i)]))
  continue;
  end

  A_i = E2_Si_x.(['sat', num2str(i)]) * E1_Si.(['sat', num2str(i)]);
  A = [A;A_i];
  b_i_mat =  E2_Si_t.(['sat', num2str(i)]) * E1_Si.(['sat', num2str(i)]);
  b_mat = [b_mat; b_i_mat];
end
```

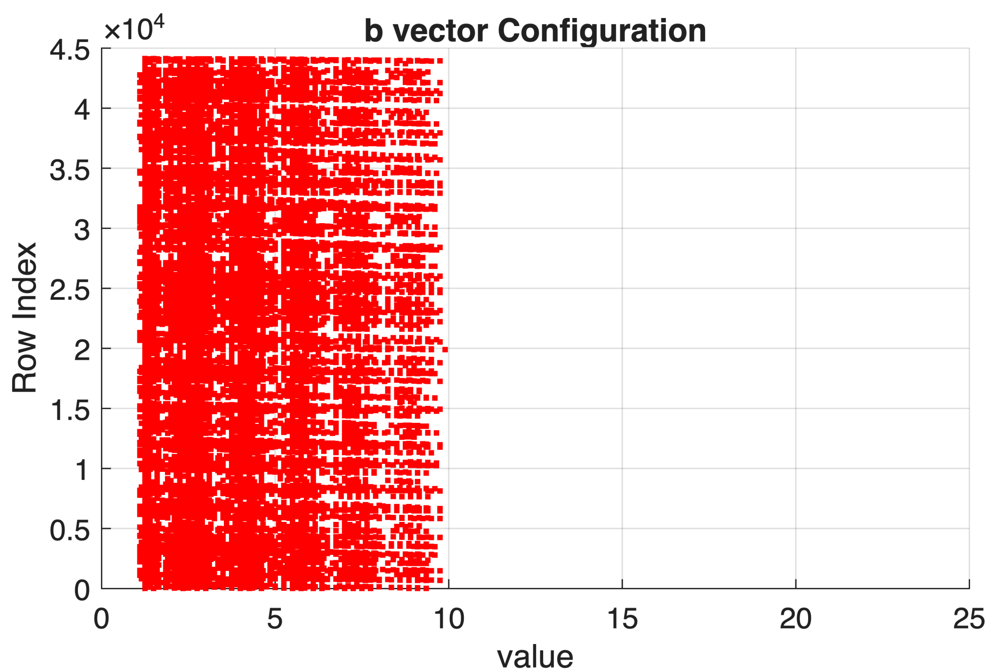
```
b = ones(length(b_mat(:,1)),1) + 1/tau * b_mat * A_matrix(:,3);

[A_row, A_col] = find(A==1);
figure;
hold on
scatter(A_col,A_row,'r','.')
rectangle('Position',[1 1 max(A_col), max(A_row)], ...
          'EdgeColor','k', ...
          'LineWidth',2);
title('A matrix Configuration (non-
zeros)','FontSize',12,'FontWeight','bold')
xlabel('Column Index')
ylabel('Row Index')
grid on
```




```
row_sum = sum(A, 2);          % 각 row의 합 (N×1)

figure;
plot(row_sum, '-k');          % x축은 자동으로 row index (1:N)
grid on
xlabel('Row index')
ylabel('Sum of row values')
```



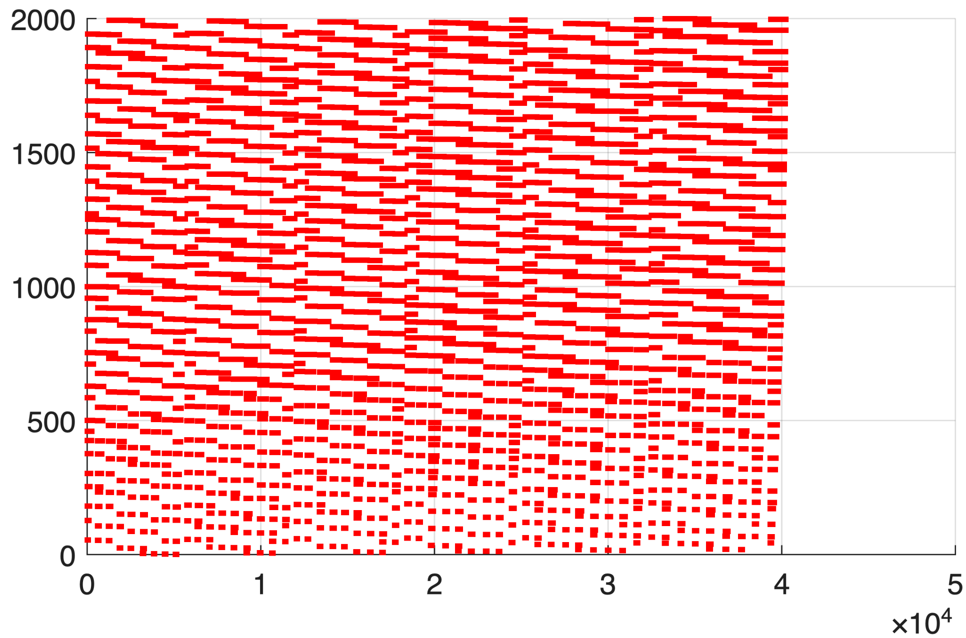

```
figure;
scatter(b,1:length(b),'r','.')
title('b vector Configuration','FontSize',12,'FontWeight','bold')
xlabel('value')
ylabel('Row Index')
xlim([0,25])
grid on
```



4.2 C, d for L_∞ problem

```
C = [];
d = [];

t = A_matrix(:,3);                      % (N×1) time vector in seconds (already
prepared)
t_aug = [min(A_matrix(:,3)); t; max(A_matrix(:,3))+1];        % (N+2 × 1)

for j = 1:q
    key = ['gs', num2str(j)];

    if isempty(E1_Gj.(key))
        continue;
    end

    % --- shorthand ---
    E1  = E1_Gj.(key);        % selection matrix for this ground
    E2x = E2_Gj_x.(key);
    E2t = E2_Gj_t.(key);

    % =========================
    % Build C_j  (constant)
    % =========================
    % D_j = diag( E2t*E1 * t_aug )   (vector -> diagonal matrix)
    Dj_vec = (E2t * E1) * t_aug;           % (#rows_j × 1)
    Dj = spdiags(Dj_vec, 0, length(Dj_vec), length(Dj_vec));  % sparse diag
is safer

    % P = [0_{1×N}; I_{N×N}; 0_{1×N}]  -> (N+2 × N)
    N = length(t);
    P = [zeros(1,N); speye(N); zeros(1,N)];

    % C_j = D_j * (E2x*E1) * P
    Cj = Dj * (E2x * E1) * P;

    % =========================
    % Build d_j  (constant)
    % =========================
    % v = [1; 0_{N×1}; 1] - 1  = [0; -1...; 0]
    v = [1; zeros(N,1); 1];          % (N+2 × 1)

    % d_j = D_j * (E2x*E1) * v
    dj = Dj * ((E2x * E1) * v - ones(length(Dj(:,1)),1));

    % --- stack ---
    C = [C; Cj];
    d = [d; dj];
end
```

```
[C_row, C_col] = find(C~=0);

figure;
scatter(C_row, C_col,'r','.')
grid on
```



4.3 E, f, G for L_2 Optimization problem

```
% % ==========================
% % Build E, f, G  (2nd image)
% % ==========================
%
% E = [];
% F = [];
%
% t = A_matrix(:,3);                           % (N×1)
% N = length(t);
%
% t_aug = [min(t); t; max(t)+1];                   % (N+2 × 1), same as your
code
%
% % P = [0_{1×N}; I_N; 0_{1×N}]  -> (N+2 × N)
% P = [zeros(1,N); speye(N); zeros(1,N)];
%
% % this is the (N+2 × 1) vector [1; 0_N; 1]
% one_aug = [1; zeros(N,1); 1];
```

```
%
% % For G, we will build the stacked matrix Bt = [E2t*E1; ...] first
(optional),
% % or directly build gvec = Bt*t_aug.
% gvec = [];   % = Bt * t_aug  (will be stacked across j)
%
% for j = 1:q
%     key = ['gs', num2str(j)];
%
%     if isempty(E1_Gj.(key))
%         continue;
%     end
%
%     % --- shorthand ---
%     E1  = E1_Gj.(key);         % selection matrix for this ground
%     E2x = E2_Gj_x.(key);
%     E2t = E2_Gj_t.(key);
%
%     % Common blocks
%     Sx = (E2x * E1);           % (#rows_j × (N+2))
%     St = (E2t * E1);           % (#rows_j × (N+2))
%
%     % -------------------------
%     % E_j = Sx * P    (constant)
%     % -------------------------
%     Ej = Sx;               % (#rows_j × N)
%     E  = [E; Ej];
%
%     % -------------------------
%     % f_j = Sx]
%     % -------------------------
%     Fj = Sx;        % (#rows_j × 1)
%     F  = [Fj; Fj];
%
%     % -------------------------
%     % gvec_j = St * t_aug
%     % -------------------------
%     gvec_j = St * t_aug;      % (#rows_j × 1)
%     gvec   = [gvec; gvec_j];
% end
%
%
% E = E * P;
% f = F * one_aug;
%
% % -------------------------
% % G = gvec * gvec'
% % -------------------------
% % WARNING: this can be very large/dense if gvec is long.
% % If you only need y'*G*y, note that y'*(gvec*gvec')*y = (gvec'*y)^2.
```

```
% G = gvec * gvec.';   % (M×M)
%
%
% [E_row, E_col] = find(E~=0);
%
% figure;
% scatter(E_row, E_col,'r','.')
% grid on
% imagesc(G); colorbar; axis equal tight;
% title('Heatmap of G');
```

5. Optimization Problem (MILP)

```
L1_flag = 1;
L_infty_flag = 1;
```

5.1. L_1 revisit time problem to MILP

```
if L1_flag == 1

%% Given: A (m×N), b (m×1), N (scalar)
% Goal: max 1^T x  s.t. A x <= b, x in {0,1}^N

% ---------- 1) Dimensions / sanity checks ----------
[m, nA] = size(A);
assert(nA == N, 'A must have N columns.');
assert(isvector(b) && length(b) == m, 'b must be m×1 to match A.');

b = b(:);                  % force column vector
Aineq = A;
bineq = b;

% ---------- 2) Convert max to min ----------
% intlinprog solves: min f'*x
f = -ones(N,1);            % minimize -sum(x) == maximize sum(x)

% ---------- 3) Binary variable settings ----------
intcon = 1:N;              % all variables are integer
lb = zeros(N,1);
ub = ones(N,1);

% (Optional) If you truly want "binary", keep intcon + bounds [0,1].
% Alternatively, you can also set intcon and bounds; that's standard.
```

```
% ---------- 4) Solve MILP ----------
% opts = optimoptions('intlinprog', ...
%     'Display','iter', ...
%     'Heuristics','advanced', ...
%     'CutGeneration','advanced');

[x_opt_L1, fval, exitflag, output] = intlinprog( ...
    f, intcon, Aineq, bineq, [], [], lb, ub);
% ---------- 5) Recover maximization objective ----------
max_onesTx = -fval;       % because f = -1
x_opt_L1 = round(x_opt_L1);     % safety: should already be integer

% ---------- 6) Quick checks ----------
viol = Aineq*x_opt_L1 - bineq;
max_viol = max(viol);

row_index_L1 = x_opt_L1 .*  A_matrix(:,4);
row_index_L1 = row_index_L1(row_index_L1 ~=0);
Original_A_L1 = T(row_index_L1,:);
A_matrix_L1 = A_matrix(x_opt_L1 == 1,:);

fprintf('Exitflag: %d\n', exitflag);
fprintf('Objective (max 1^T x): %.0f\n', max_onesTx);
fprintf('Max constraint violation: %.3e\n', max_viol);
end
```

```
Running HiGHS 1.7.1: Copyright (c) 2024 HiGHS under MIT licence terms
Coefficient ranges:
  Matrix [1e+00, 1e+00]
  Cost   [1e+00, 1e+00]
  Bound  [1e+00, 1e+00]
  RHS    [1e+00, 5e+03]
Presolving model
2278 rows, 1872 cols, 4556 nonzeros  0s
661 rows, 1606 cols, 2006 nonzeros  0s
462 rows, 833 cols, 1007 nonzeros  0s
10 rows, 15 cols, 20 nonzeros  0s
Objective function is integral with scale 1

Solving MIP model with:
   10 rows
   15 cols (15 binary, 0 integer, 0 implied int., 0 continuous)
   20 nonzeros

        Nodes      |    B&B Tree     |            Objective Bounds              |  Dynamic Constraints |
     Proc. InQueue |  Leaves   Expl. | BestBound       BestSol              Gap |   Cuts   InLp Confl. |

         0       0         0   0.00%   -745            inf                  inf        0      0      0

Solving report
  Status            Optimal
  Primal bound      -745
  Dual bound        -745
  Gap               0% (tolerance: 0.01%)
  Solution status   feasible
                    -745 (objective)
                    0 (bound viol.)
```

```
                      0 (int. viol.)
                      0 (row viol.)
  Timing              0.02 (total)
                      0.01 (presolve)
                      0.00 (postsolve)
  Nodes               1
  LP iterations       5 (total)
                      0 (strong br.)
                      0 (separation)
                      0 (heuristics)

최적해를 구했습니다.

목적 함수 값이 최적 값의 격차 허용오차 options.AbsoluteGapTolerance = 1e-06 내에 있기 때문에 Intlinprog가 루트 노드에서
Exitflag: 1
Objective (max 1^T x): 745
Max constraint violation: 4.996e-14
```

5.2 L_∞ revisit time problem to MILP

```
if L_infty_flag == 1

%% Given:
% A (m×N), b (m×1)
% C (q×N), d (q×1)
% N (scalar)

% ---------- sanity ----------
[m, nA] = size(A);
assert(nA == N, 'A must have N columns.');
b = b(:);  assert(length(b) == m, 'b must be m×1.');

[q, nC] = size(C);
assert(nC == N, 'C must have N columns.');
d = d(:);  assert(length(d) == q, 'd must be q×1.');

% ---------- decision variables ----------
% z = [x; R]  -> length N+1
nvar = N + 1;

% Objective: min R  => f = [0...0, 1]
f = [zeros(N,1); 1];

% Integer constraints: x binary, R continuous
intcon = 1:N;

% Bounds
lb = [zeros(N,1); 0];      % R >= 0 (필요 없으면 -Inf로 바꿔도 됨)
ub = [ones(N,1);  Inf];

% ---------- Build inequalities Aineq*z <= bineq ----------
% 1) Ax <= b  -> [A, 0] [x;R] <= b
```

```
A1 = [A, zeros(m,1)];
b1 = b;

% 2) Cx - R*1 <= -d  -> [C, -1] [x;R] <= -d   (각 row마다 -R)
A2 = [C, -ones(q,1)];
b2 = -d;

Aineq = [A1; A2];
bineq = [b1; b2];

% (No equality constraints)
Aeq = [];
beq = [];

% ---------- Solve ----------
% opts = optimoptions('intlinprog', ...
%     'Display','iter', ...
%     'Heuristics','advanced', ...
%     'CutGeneration','advanced');

[z_opt, fval, exitflag, output] = intlinprog( ...
    f, intcon, Aineq, bineq, Aeq, beq, lb, ub);
% ---------- Parse solution ----------
x_opt_L_inf = round(z_opt(1:N));     % binary
R_opt = z_opt(N+1);            % optimal R


row_index_L_inf = x_opt_L_inf .*  A_matrix(:,4);
row_index_L_inf = row_index_L_inf(row_index_L_inf ~=0);
Original_A_L_inf = T(row_index_L_inf,:);
A_matrix_L_inf = A_matrix(x_opt_L_inf == 1,:);



% ---------- Feasibility check ----------
viol1 = A*x_opt_L_inf - b;
viol2 = C*x_opt_L_inf + d - R_opt*ones(q,1);   % should be <= 0
fprintf('Exitflag: %d\n', exitflag);
fprintf('R_opt: %.6f\n', R_opt);
fprintf('max(Ax-b): %.3e\n', max(viol1));
fprintf('max(Cx+d-R): %.3e\n', max(viol2));
end
```

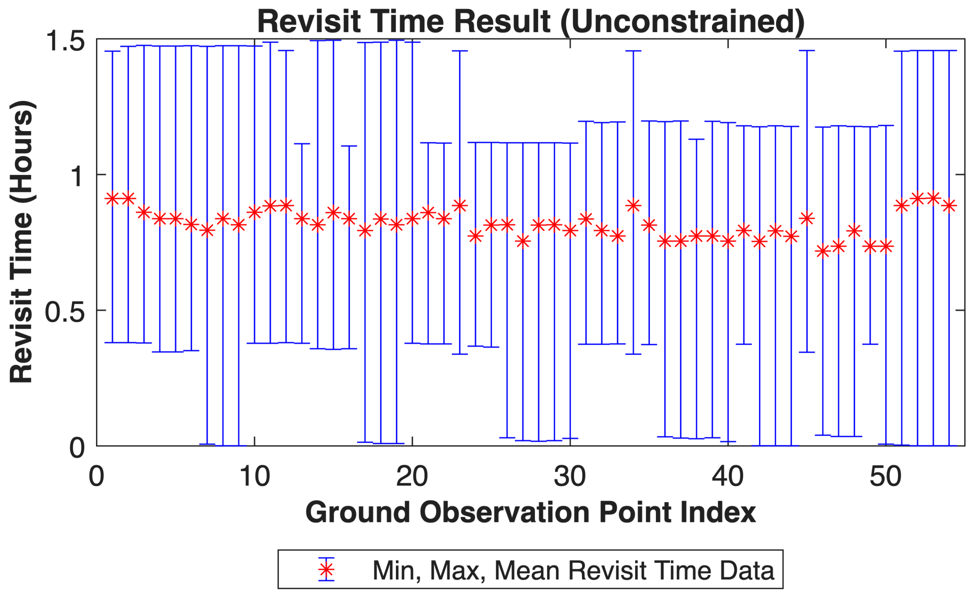
6.0 Original Revisit Time Status Result

```
t_start = t0 + seconds(min(A_matrix(:,3)));
```

```
t_end = t0 + seconds(max(A_matrix(:,3))+1);


% Extract Ground Point Tables
  T = Original_A;

    % Source 열에서 고유 값 추출
    sources = unique(T.Source);

    % Revisit Time Matrix (Min / Max / Mean) 생성
    Revisit_Matrix = zeros(length(sources),3);
    revisit_time_vector_combined = [];

    % 각각의 Source 별로 테이블을 분리하여 저장
    for i = 1:length(sources)
        src = sources{i};

        % Source 값이 같은 행들만 추출
        subTable = T(strcmp(T.Source, src), :);
        subTable_sorted = sortrows(subTable,4,"ascend");


        % 동적으로 변수 생성 (예: Ground_Point_1_Table)
        varName = sprintf('%s_Table', src);
        % assignin('base', varName, subTable_sorted);
        contact_tables.(sprintf('%s_Table', src)) = subTable_sorted;

        % 기존 row에 start 및 endtime 추가
        row0 = subTable_sorted(1,:);

        % -------------------------
        % 윗줄 (t_start)
        % -------------------------
        row_top = row0;
        row_top.IntervalNumber = NaN;      % 필요 없으면 NaN
        row_top.StartTime = t_start;
        row_top.EndTime   = t_start;
        row_top.Duration  = 0;
        row_top.StartOrbit = 0;
        row_top.EndOrbit   = 0;

        % -------------------------
        % 아랫줄 (t_end)
        % -------------------------
        row_bottom = row0;
        row_bottom.IntervalNumber = NaN;
        row_bottom.StartTime = t_end;
        row_bottom.EndTime   = t_end;
        row_bottom.Duration  = 0;
        row_bottom.StartOrbit = 0;
```

```
        row_bottom.EndOrbit   = 0;

        % -------------------------
        % 위 + 기존 + 아래 결합
        % -------------------------
        subTable_sorted = [row_top; subTable_sorted; row_bottom];

        % Initialize Revisit Time Vector
        revisit_time_vector = zeros(height(subTable_sorted)-1,1);

        for revisit_time_index = 1:length(revisit_time_vector)
            revisit_time_vector(revisit_time_index)
= seconds(table2array(subTable_sorted(revisit_time_index+1,4))-
table2array(subTable_sorted(revisit_time_index,5)));

            if revisit_time_vector(revisit_time_index) < 0
               revisit_time_vector(revisit_time_index) = 0;
            end
        revisit_vectors.(sprintf('%s_revisit_time_vec', src)) =
revisit_time_vector;
        end

        % 생성된 Revisit Time Vector의 Min / Max / Mean 값을 Revisit Time
Matrix에 저장
        Revisit_Matrix(i,1) = min(revisit_time_vector);
        Revisit_Matrix(i,2) = max(revisit_time_vector);

        % 재방문 주기의 평균을 구할때는 재방문 주기가 0인 데이터 포인트를 모두 제외하였음
        revisit_time_vector = revisit_time_vector(revisit_time_vector~=0);

        Revisit_Matrix(i,3) = mean(revisit_time_vector);
        revisit_time_vector_combined =
[revisit_time_vector_combined;revisit_time_vector];
       revisit_time_vector_info.(['source' num2str(i)]) =
revisit_time_vector;
    end

    % Revisit Time Matrix의 값을 그래프로 출력
    x = 1:length(Revisit_Matrix(:,1));
    min_value = (Revisit_Matrix(:,1))/3600;
    max_value = (Revisit_Matrix(:,2))/3600;
    mean_value = (Revisit_Matrix(:,3))/3600;
    output_data = [mean_value, min_value, max_value];

    figure;
    errorbar(x, mean_value, mean_value-min_value, max_value-
mean_value,'*','LineStyle','none','color','b','MarkerEdgeColor','r')
    t = title('Revisit Time Result
(Unconstrained)','FontSize',12,'FontWeight','bold');
```

```
    xlabel('Ground Observation Point 
Index','FontSize',11,'FontWeight','bold')
    ylabel('Revisit Time (Hours)','FontSize',11,'FontWeight','bold')
    legend('Min, Max, Mean Revisit Time Data','Location','southoutside')
    xlim([-1, length(Revisit_Matrix(:,1))]+1)
```



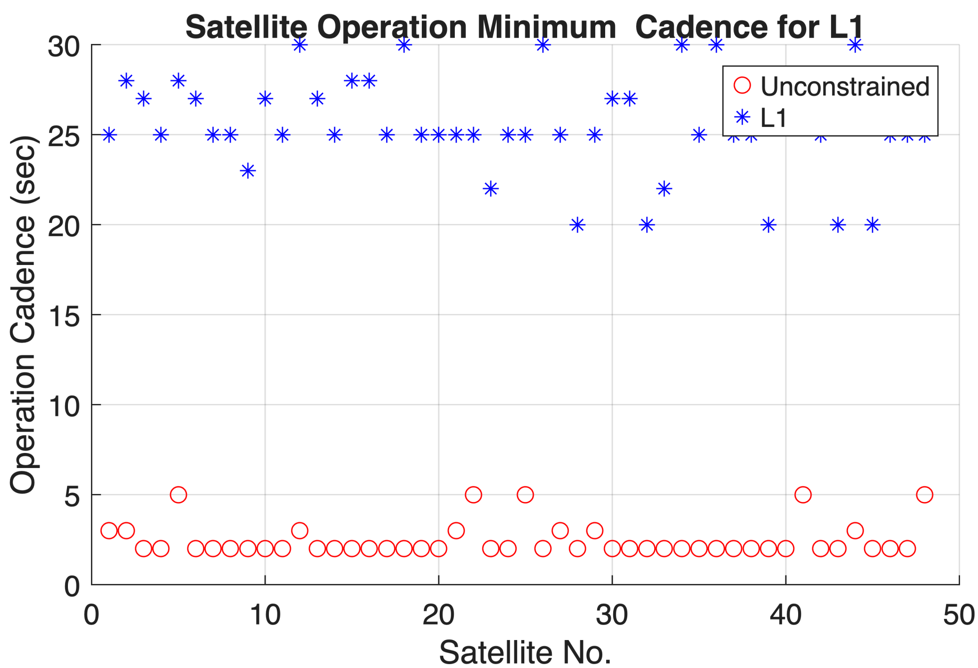
6.1 L1 Revisit Time Status Result

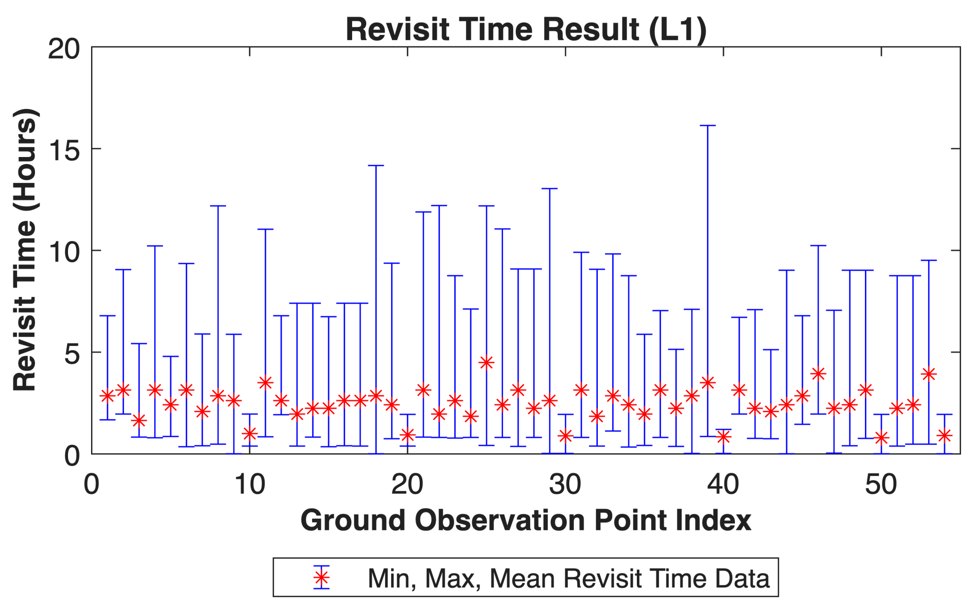
```
if L1_flag == 1

A_matrix_result = A_matrix;

active_satellites = unique(A_matrix_result(:,1));
cadence_matrix_original = zeros(length(active_satellites),4);

    for index = 1:length(active_satellites)
      sat_index = active_satellites(index);
      % satellite_cadence_info.(['sat',num2str(sat_index)]) = 
A_matrix_result(A_matrix_result(:,1) == sat_index,:);
      satellite_cadence = A_matrix_result(A_matrix_result(:,1) == 
sat_index,:);
      cadence_info = zeros(length(satellite_cadence(:,1))-1,1);
      for i = 1:length(cadence_info(:,1))
         cadence_info(i) = satellite_cadence(i+1,3) - 
satellite_cadence(i,3);
      end

      if isscalar(satellite_cadence(:,1))
         continue;
```

```
      end

      cadence_matrix_original(index,:) = [sat_index, max(cadence_info),
min(cadence_info), mean(cadence_info)];
    end


A_matrix_result = A_matrix_L1;

active_satellites = unique(A_matrix_result(:,1));
cadence_matrix_L1 = zeros(length(active_satellites),4);

    for index = 1:length(active_satellites)
      sat_index = active_satellites(index);
      % satellite_cadence_info.(['sat',num2str(sat_index)]) =
A_matrix_result(A_matrix_result(:,1) == sat_index,:);
      satellite_cadence = A_matrix_result(A_matrix_result(:,1) ==
sat_index,:);
      cadence_info = zeros(length(satellite_cadence(:,1))-1,1);

      if isscalar(satellite_cadence(:,1))
         continue;
      end

      for i = 1:length(cadence_info(:,1))
         cadence_info(i) = satellite_cadence(i+1,3) -
satellite_cadence(i,3);
      end


      cadence_matrix_L1(index,:) = [sat_index, max(cadence_info),
min(cadence_info), mean(cadence_info)];
    end

figure;
hold on
scatter(cadence_matrix_original(:,1),cadence_matrix_original(:,3),'o','red')
scatter(cadence_matrix_L1(:,1),cadence_matrix_L1(:,3),'*','blue')
title('Satellite Operation Minimum  Cadence for
L1','FontSize',12,'FontWeight','bold')
xlabel('Satellite No.')
ylabel('Operation Cadence (sec)')
legend('Unconstrained','L1')
grid on


% Extract Ground Point Tables
  T = Original_A_L1;

    % Source 열에서 고유 값 추출
```

```
    sources = unique(T.Source);

    % Revisit Time Matrix (Min / Max / Mean) 생성
    Revisit_Matrix = zeros(length(sources),3);
    revisit_time_vector_combined = [];

    % 각각의 Source 별로 테이블을 분리하여 저장
    for i = 1:length(sources)
        src = sources{i};

        % Source 값이 같은 행들만 추출
        subTable = T(strcmp(T.Source, src), :);
        subTable_sorted = sortrows(subTable,4,"ascend");


        % 동적으로 변수 생성 (예: Ground_Point_1_Table)
        varName = sprintf('%s_Table', src);
        % assignin('base', varName, subTable_sorted);
        contact_tables.(sprintf('%s_Table', src)) = subTable_sorted;

        % 기존 row에 start 및 endtime 추가
        row0 = subTable_sorted(1,:);

        % --------------------------
        % 윗줄 (t_start)
        % --------------------------
        row_top = row0;
        row_top.IntervalNumber = NaN;      % 필요 없으면 NaN
        row_top.StartTime = t_start;
        row_top.EndTime   = t_start;
        row_top.Duration  = 0;
        row_top.StartOrbit = 0;
        row_top.EndOrbit   = 0;

        % --------------------------
        % 아랫줄 (t_end)
        % --------------------------
        row_bottom = row0;
        row_bottom.IntervalNumber = NaN;
        row_bottom.StartTime = t_end;
        row_bottom.EndTime   = t_end;
        row_bottom.Duration  = 0;
        row_bottom.StartOrbit = 0;
        row_bottom.EndOrbit   = 0;

        % --------------------------
        % 위 + 기존 + 아래 결합
        % --------------------------
        subTable_sorted = [row_top; subTable_sorted; row_bottom];
```

```
        % Initialize Revisit Time Vector
        revisit_time_vector = zeros(height(subTable_sorted)-1,1);

        for revisit_time_index = 1:length(revisit_time_vector)
            revisit_time_vector(revisit_time_index)
= seconds(table2array(subTable_sorted(revisit_time_index+1,4))-
table2array(subTable_sorted(revisit_time_index,5)));

            if revisit_time_vector(revisit_time_index) < 0
               revisit_time_vector(revisit_time_index) = 0;
            end
        revisit_vectors.(sprintf('%s_revisit_time_vec', src)) =
revisit_time_vector;
        end

        % 생성된 Revisit Time Vector의 Min / Max / Mean 값을 Revisit Time
Matrix에 저장
        Revisit_Matrix(i,1) = min(revisit_time_vector);
        Revisit_Matrix(i,2) = max(revisit_time_vector);

        % 재방문 주기의 평균을 구할때는 재방문 주기가 0인 데이터 포인트를 모두 제외하였음
        revisit_time_vector = revisit_time_vector(revisit_time_vector~=0);

        Revisit_Matrix(i,3) = mean(revisit_time_vector);
        revisit_time_vector_combined =
[revisit_time_vector_combined;revisit_time_vector];
       revisit_time_vector_info.(['source' num2str(i)]) =
revisit_time_vector;
    end

    % Revisit Time Matrix의 값을 그래프로 출력
    x = 1:length(Revisit_Matrix(:,1));
    min_value = (Revisit_Matrix(:,1))/3600;
    max_value = (Revisit_Matrix(:,2))/3600;
    mean_value = (Revisit_Matrix(:,3))/3600;
    output_data = [mean_value, min_value, max_value];

    figure;
    errorbar(x, mean_value, mean_value-min_value, max_value-
mean_value,'*','LineStyle','none','color','b','MarkerEdgeColor','r')
    t = title('Revisit Time Result (L1)','FontSize',12,'FontWeight','bold');
    xlabel('Ground Observation Point
Index','FontSize',11,'FontWeight','bold')
    ylabel('Revisit Time (Hours)','FontSize',11,'FontWeight','bold')
    legend('Min, Max, Mean Revisit Time Data','Location','southoutside')
    xlim([-1, length(Revisit_Matrix(:,1))]+1)
end
```





6.2 L_infty Revisit Time Status Result

```
if L_infty_flag == 1

A_matrix_result = A_matrix;

active_satellites = unique(A_matrix_result(:,1));
```

```
cadence_matrix_original = zeros(length(active_satellites),4);

    for index = 1:length(active_satellites)
      sat_index = active_satellites(index);
      % satellite_cadence_info.(['sat',num2str(sat_index)]) =
A_matrix_result(A_matrix_result(:,1) == sat_index,:);
      satellite_cadence = A_matrix_result(A_matrix_result(:,1) ==
sat_index,:);
      cadence_info = zeros(length(satellite_cadence(:,1))-1,1);
      for i = 1:length(cadence_info(:,1))
         cadence_info(i) = satellite_cadence(i+1,3) -
satellite_cadence(i,3);
      end

      if isscalar(satellite_cadence(:,1))
         continue;
      end

      cadence_matrix_original(index,:) = [sat_index, max(cadence_info),
min(cadence_info), mean(cadence_info)];
    end


A_matrix_result = A_matrix_L_inf;

active_satellites = unique(A_matrix_result(:,1));
cadence_matrix_L_inf = zeros(length(active_satellites),4);

    for index = 1:length(active_satellites)
      sat_index = active_satellites(index);
      % satellite_cadence_info.(['sat',num2str(sat_index)]) =
A_matrix_result(A_matrix_result(:,1) == sat_index,:);
      satellite_cadence = A_matrix_result(A_matrix_result(:,1) ==
sat_index,:);

      if isscalar(satellite_cadence(:,1))
         continue;
      end

      cadence_info = zeros(length(satellite_cadence(:,1))-1,1);
      for i = 1:length(cadence_info(:,1))
         cadence_info(i) = satellite_cadence(i+1,3) -
satellite_cadence(i,3);
      end

      cadence_matrix_L_inf(index,:) = [sat_index, max(cadence_info),
min(cadence_info), mean(cadence_info)];
    end
```

```
cadence_matrix_L_inf = cadence_matrix_L_inf(cadence_matrix_L_inf(:,3)~=0,:);


figure;
hold on
scatter(cadence_matrix_original(:,1),cadence_matrix_original(:,3),'o','red')
scatter(cadence_matrix_L_inf(:,1),cadence_matrix_L_inf(:,3),'*','blue')
title('Satellite Operation Minimum Cadence for L
infty','FontSize',12,'FontWeight','bold')
legend('Unconstrained','L infty')
xlabel('Satellite No.')
ylabel('Operation Cadence (sec)')
grid on
```

```
 % Extract Ground Point Tables
  T = Original_A_L_inf;

    % Source 열에서 고유 값 추출
    sources = unique(T.Source);

    % Revisit Time Matrix (Min / Max / Mean) 생성
    Revisit_Matrix = zeros(length(sources),3);
    revisit_time_vector_combined = [];

    % 각각의 Source 별로 테이블을 분리하여 저장
    for i = 1:length(sources)
        src = sources{i};

        % Source 값이 같은 행들만 추출
        subTable = T(strcmp(T.Source, src), :);
        subTable_sorted = sortrows(subTable,4,"ascend");


        % 동적으로 변수 생성 (예: Ground_Point_1_Table)
        varName = sprintf('%s_Table', src);
        % assignin('base', varName, subTable_sorted);
        contact_tables.(sprintf('%s_Table', src)) = subTable_sorted;

        % 기존 row에 start 및 endtime 추가
        row0 = subTable_sorted(1,:);

        % -------------------------
        % 윗줄 (t_start)
        % -------------------------
        row_top = row0;
        row_top.IntervalNumber = NaN;      % 필요 없으면 NaN
        row_top.StartTime = t_start;
        row_top.EndTime   = t_start;
```

```
        row_top.Duration  = 0;
        row_top.StartOrbit = 0;
        row_top.EndOrbit   = 0;

        % -------------------------
        % 아랫줄 (t_end)
        % -------------------------
        row_bottom = row0;
        row_bottom.IntervalNumber = NaN;
        row_bottom.StartTime = t_end;
        row_bottom.EndTime   = t_end;
        row_bottom.Duration  = 0;
        row_bottom.StartOrbit = 0;
        row_bottom.EndOrbit   = 0;

        % -------------------------
        % 위 + 기존 + 아래 결합
        % -------------------------
        subTable_sorted = [row_top; subTable_sorted; row_bottom];

        % Initialize Revisit Time Vector
        revisit_time_vector = zeros(height(subTable_sorted)-1,1);

        for revisit_time_index = 1:length(revisit_time_vector)
            revisit_time_vector(revisit_time_index)
= seconds(table2array(subTable_sorted(revisit_time_index+1,4))-
table2array(subTable_sorted(revisit_time_index,5)));

            if revisit_time_vector(revisit_time_index) < 0
               revisit_time_vector(revisit_time_index) = 0;
            end
        revisit_vectors.(sprintf('%s_revisit_time_vec', src)) =
revisit_time_vector;
        end

        % 생성된 Revisit Time Vector의 Min / Max / Mean 값을 Revisit Time
Matrix에 저장
        Revisit_Matrix(i,1) = min(revisit_time_vector);
        Revisit_Matrix(i,2) = max(revisit_time_vector);

        % 재방문 주기의 평균을 구할때는 재방문 주기가 0인 데이터 포인트를 모두 제외하였음
        revisit_time_vector = revisit_time_vector(revisit_time_vector~=0);

        Revisit_Matrix(i,3) = mean(revisit_time_vector);
        revisit_time_vector_combined =
[revisit_time_vector_combined;revisit_time_vector];
       revisit_time_vector_info.(['source' num2str(i)]) =
revisit_time_vector;
    end
```

```
    % Revisit Time Matrix의 값을 그래프로 출력
    x = 1:length(Revisit_Matrix(:,1));
    min_value = (Revisit_Matrix(:,1))/3600;
    max_value = (Revisit_Matrix(:,2))/3600;
    mean_value = (Revisit_Matrix(:,3))/3600;
    output_data = [mean_value, min_value, max_value];

    figure;
    errorbar(x, mean_value, mean_value-min_value, max_value-
mean_value,'*','LineStyle','none','color','b','MarkerEdgeColor','r')
    t = title('Revisit Time Result (L
infty)','FontSize',12,'FontWeight','bold');
    xlabel('Ground Observation Point
Index','FontSize',11,'FontWeight','bold')
    ylabel('Revisit Time (Hours)','FontSize',11,'FontWeight','bold')
    legend('Min, Max, Mean Revisit Time Data','Location','southoutside')
    xlim([-1, length(Revisit_Matrix(:,1))]+1)
end
```