# Contact Time Calculation From Maximum Elevation Angle

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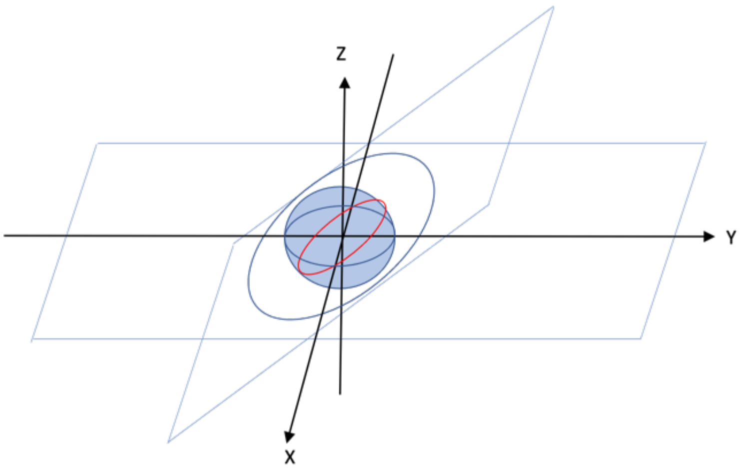
## I. Orbital Kinematics of SSO satellite




Suppose the ECI frame is on XYZ cartesian coordinate. The satellite orbit trajectory is stable in this frame.

- We alreay have satellite height(600km) and earth radious information, so all we have to choose is inclination angle $i$.
- In this coordinate, suppose the radious of orbital circle is 1 for simple calculation.
- Also, given the satellite hight, we can calculate the satellite's velocity and orbital period.

$$\text{flight velocity}: v = \sqrt{\frac{398600.5}{6378.14 + h}} (\text{km}/s) \quad , \text{orbital period}: \ P = 2\pi \frac{6378.14 + h}{v} (\text{sec})$$
$$\text{if } h = 600 \text{ km} \rightarrow \ v = 7.5579 \text{km}/s \, , P = 5801.23 \text{ Sec } = 1:36:41.23 \text{ (hh : mm : ss)}$$

Therefore, we can express the orbital trajectory as follows.

$$\text{Satellite trajectory } (x, y, z) \text{ at ECI frame}$$
$$(x, y, z) = (\cos\theta, \cos i \sin\theta, \sin i, \sin\theta), \ \ \theta = \frac{2\pi t}{P}, i = \text{inclination angle} \left(-\frac{\pi}{2} \le i \le \frac{\pi}{2}\right)$$
$$(x, y, z) = \left(\cos\frac{2\pi t}{P}, \cos i \sin\frac{2\pi t}{P}, \sin i \sin\frac{2\pi t}{P}\right)$$

For changing from ECI frame to ECF frame, we should consider the rotation of earth.

- Suppose the direction of satellit rotation and earth rotation is same , +Z rotation (counterclockwise)

- The earth rotation period is known: $P_E = 23:56:4.09 \text{ (hh : mm : ss)}$

Satellite trajectory $(x, y, z)$ at ECF frame

$z$ remains same, $z = \sin i \sin \frac{2\pi t}{P}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\frac{2\pi t}{P_E} & \sin\frac{2\pi t}{P_E} \\ -\sin\frac{2\pi t}{P_E} & \cos\frac{2\pi t}{P_E} \end{bmatrix} \begin{bmatrix} \cos\frac{2\pi t}{P} \\ \cos i \sin\frac{2\pi t}{P} \end{bmatrix} = \begin{bmatrix} \cos\frac{2\pi t}{P_E}\cos\frac{2\pi t}{P} + \sin\frac{2\pi t}{P_E}\cos i \sin\frac{2\pi t}{P} \\ -\sin\frac{2\pi t}{P_E}\cos\frac{2\pi t}{P} + \cos\frac{2\pi t}{P_E}\cos i \sin\frac{2\pi t}{P} \end{bmatrix}$$

$$\text{Therefore, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\frac{2\pi t}{P_E}\cos\frac{2\pi t}{P} + \cos i \sin\frac{2\pi t}{P_E}\sin\frac{2\pi t}{P} \\ -\sin\frac{2\pi t}{P_E}\cos\frac{2\pi t}{P} + \cos i \cos\frac{2\pi t}{P_E}\sin\frac{2\pi t}{P} \\ \sin i \sin\frac{2\pi t}{P} \end{bmatrix}$$

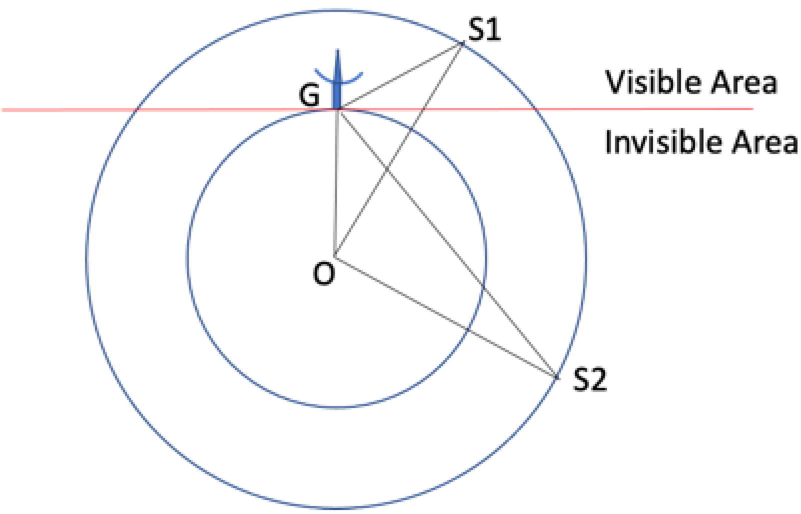
We can get the graph of basic trajectories.

```
i = 97.8/180*pi();
t_end = 1000;
h = 500;

v = sqrt(398600.5/(6378.14 + h));
P = 2 * pi() * (6378.14+h) / v;


r_E = 6378.14;
t = 0:0.1:t_end;
P_E = 23 * 3600 + 56 * 60 + 4.09;



x_ECI_x = cos(2*pi()*t/P);
x_ECI_y = cos(i)* sin(2*pi()*t/P);
x_ECI_z = sin(i)* sin(2*pi()*t/P);


x_ECF_x = cos(2*pi().*t/P_E).*cos(2*pi()*t/P) + cos(i).* sin(2*pi()*t/
P_E) .* sin(2*pi()*t/P);
x_ECF_y = -sin(2*pi().*t/P_E).*cos(2*pi()*t/P) + cos(i).* cos(2*pi()*t/
P_E) .* sin (2*pi()*t/P);
x_ECF_z = sin(i) * sin(2*pi()*t/P);


%plot3(x_ECI_x, x_ECI_y, x_ECI_z);
%plot3(x_ECF_x,x_ECF_y,x_ECF_z);
```

```
%plot(x_ECF_x,x_ECF_z);
```

## II. Calculating Elevation angle

We can express the position relationship by ground station and satellite by following diagram.




$G$ : Ground station position

$S,\ S_1, S_2$ : Satellite position

$O$ : Center of earth

$\overline{\mathrm{OS}} = r\ = r_E + h$

$\overline{\mathrm{GS}} = \text{Slant range} = S$

$\angle\mathrm{OGS} = \cos^{-1}\left(\dfrac{\overline{\mathrm{OG}}^2 + \overline{\mathrm{GS}}^2 - \overline{\mathrm{OS}}^2}{2\overline{\mathrm{OG}}\,\overline{\mathrm{GS}}}\right)$

if $0 <\ \angle\mathrm{OGS} < 90$ deg, the satellite is invisible data

→ We cannot calculate Elevation angle

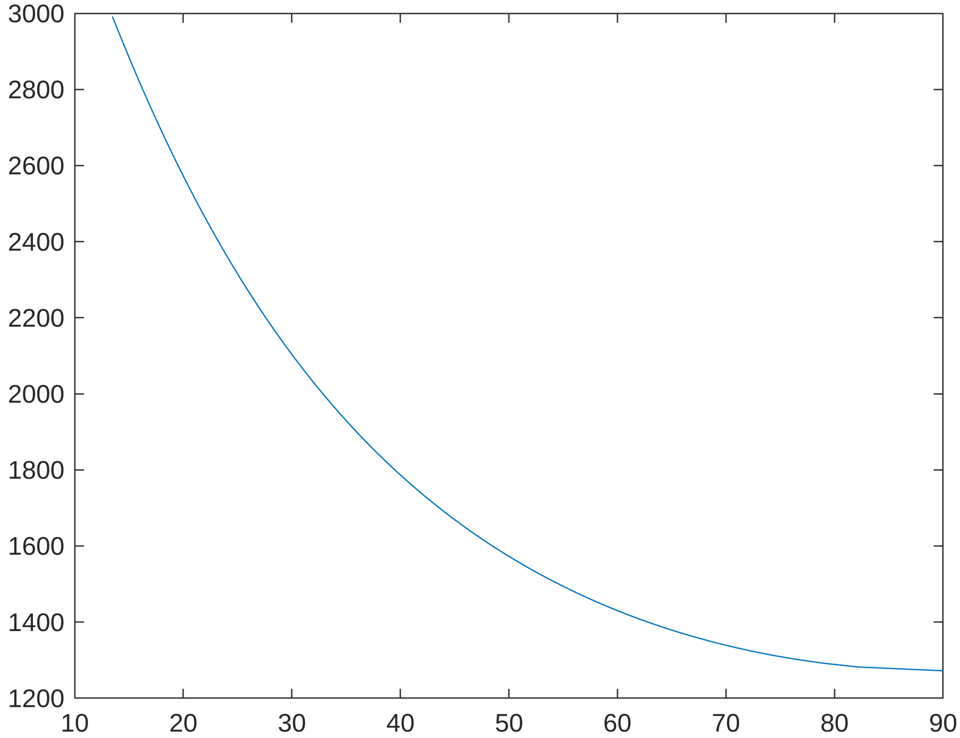
if $\angle\mathrm{OGS} > 90$ deg, the satellite is in Visible data

→ We can calculate Elevation angle

Elevation angle $\theta = \angle\,\mathrm{OGS} - 90\mathrm{deg}$

From Slant Range to Elevation Angle

```
s = 1272:10:3000;
og = 6378.14;
os = 6378.14 + 1272;
elev_plus_90 = acos((og^2 + s.^2 - os.^2)./(2.*og.*s));
elev_angle = elev_plus_90*180/pi() - 90;
%plot(s,elev_angle)
plot(elev_angle,s)
```



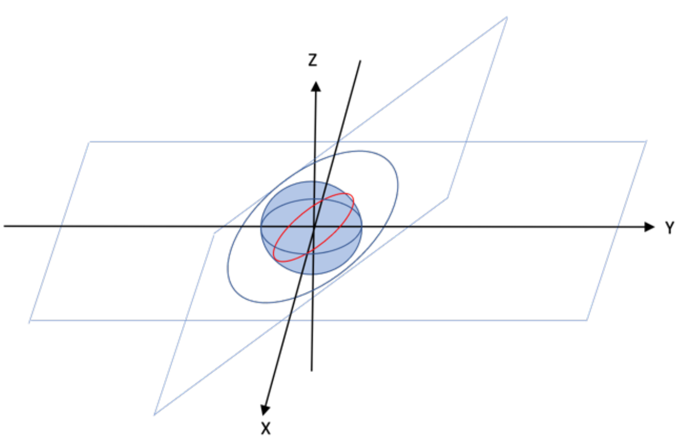

## III. Calculating Contact Time From Maximum Elevation angle

In our SSO orbit satellite case,

$\overline{\mathrm{OG}} = r_E = 6378.14\mathrm{km}$

$\overline{\mathrm{OS}} = r_E + h = 6978.14\mathrm{km}$

We should find $\overline{\mathrm{GS}}$ by calculating distance in ECF frame

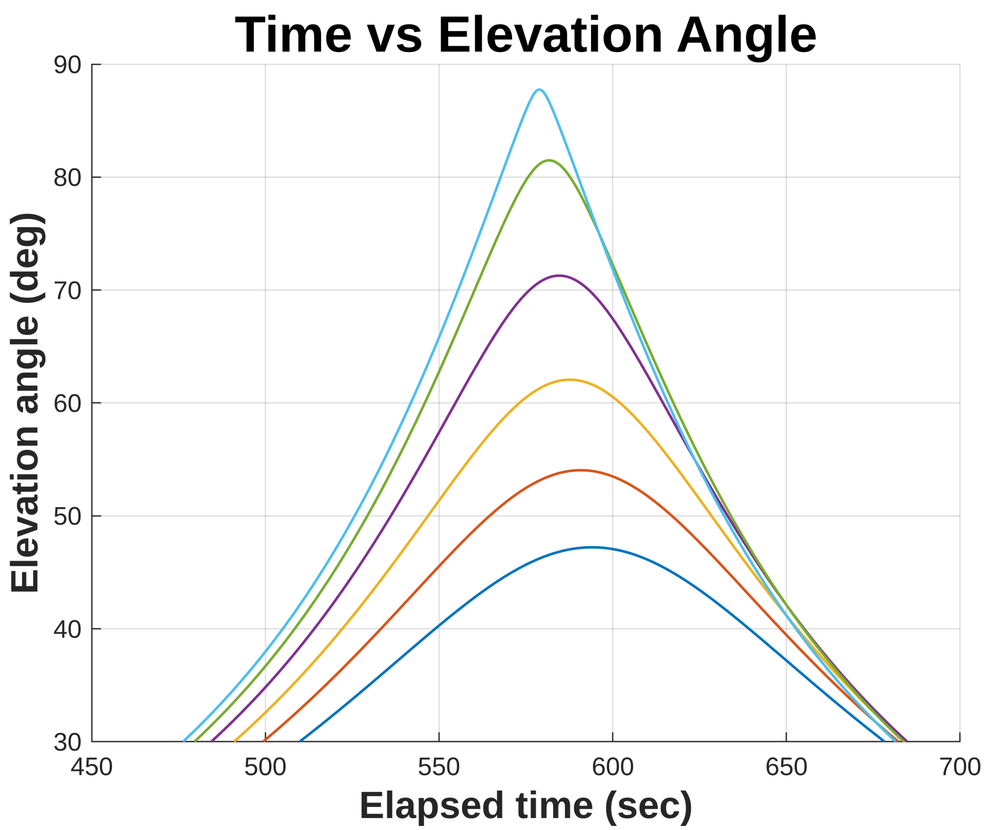


In the ECF frame, let G is at latitude = $\phi = 36° \, 21' 03'' N$, which is the position of Daejeon, Korea, longitude = $\theta$, and assume G is on XZ plane.

Then we can get the position data of G, $G(x, y, z) = (r \cos\phi \cos\theta, r \cos\phi \sin\theta, r \sin\phi)$

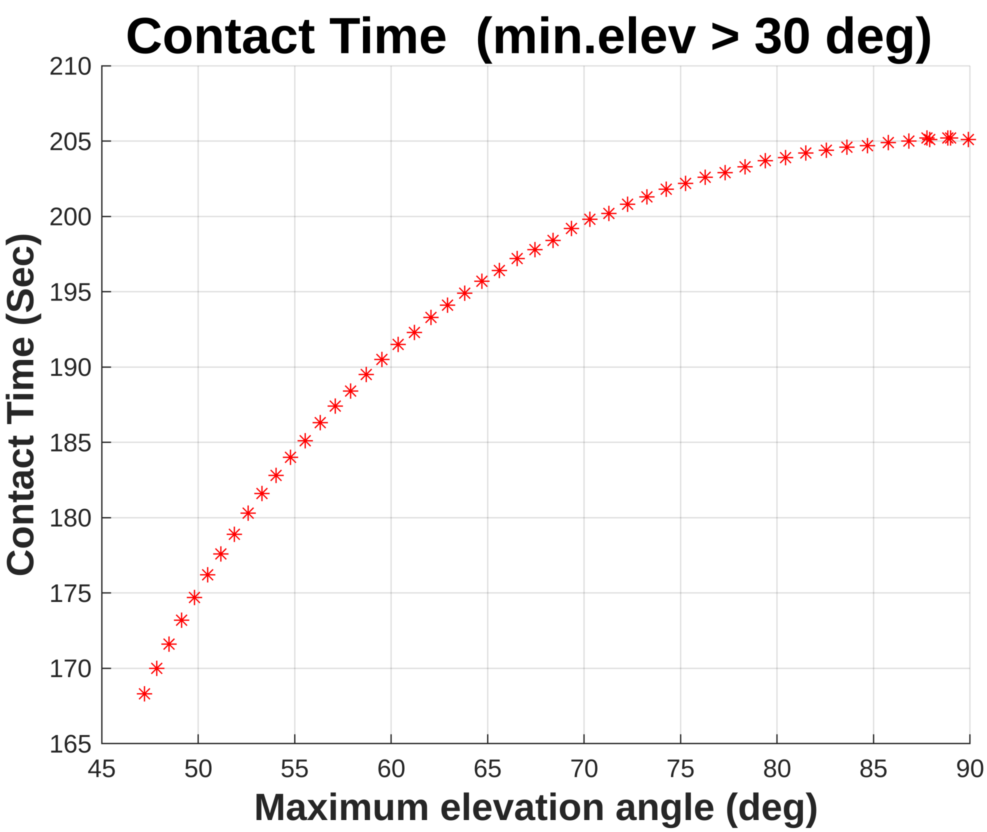
From, $S(x, y, z)$ at ECF frame we have got, we can calculate the length of $\overline{\mathrm{GS}}$

```
n = (-11 - (-5))/0.1+1;
data = zeros(n,2);
k = 1;

figure;
hold on
for theta_trans = -13:1:-8

% theta = -8.3/180*pi();
theta = theta_trans/180*pi();
phi = (36 + 21/60 + 3/3600)/180*pi();

G_x = r_E * cos(phi) * cos(theta);
G_y = r_E * cos(phi) * sin(theta);
G_z = r_E * sin(phi);


GS = sqrt(((r_E+h) * x_ECF_x - G_x).^2+((r_E+h) * x_ECF_y - G_y).^2+((r_E+h)
* x_ECF_z - G_z).^2);
OGS_rad = acos((r_E^2+ GS.^2 -(r_E+h)^2)./(2.*r_E.*GS));
OGS_deg = OGS_rad/pi()*180;
elev = OGS_deg - 90;
elev_mat = [t',elev'];
elev_filtered = elev_mat(elev_mat(:,2)>=30,:);
plot(elev_filtered(:,1), elev_filtered(:,2),'LineWidth',1)
end

hold off
xlabel('Elapsed time (sec)','FontSize',15,'FontWeight','bold')
ylabel('Elevation angle (deg)','FontSize',15,'FontWeight','bold')
grid on
title('Time vs Elevation Angle','FontSize',20,'FontWeight','bold')
```




```
for theta_input = -13:0.1:-8



theta = theta_input/180*pi();
phi = (36 + 21/60 + 3/3600)/180*pi();

G_x = r_E * cos(phi) * cos(theta);
G_y = r_E * cos(phi) * sin(theta);
G_z = r_E * sin(phi);


GS = sqrt(((r_E+h) * x_ECF_x - G_x).^2+((r_E+h) * x_ECF_y - G_y).^2+((r_E+h)
* x_ECF_z - G_z).^2);
OGS_rad = acos((r_E^2+ GS.^2 -(r_E+h)^2)./(2.*r_E.*GS));
OGS_deg = OGS_rad/pi()*180;
elev = OGS_deg - 90;

% plot(t, elev)


observe_boundary = find(elev > 30);
T_min = min(observe_boundary);
T_max = max(observe_boundary);
elev_max = max(elev);
```

```
duration = (T_max-T_min)/10;
data(k,1) = elev_max;
data(k,2) = duration;
k = k+1;

end

figure;
scatter(data(:,1),data(:,2),'red','*')
title('Contact Time  (min.elev > 30 deg) ','FontSize',20,'FontWeight','bold')
xlabel('Maximum elevation angle (deg)','FontSize',15,'FontWeight','bold')
ylabel('Contact Time (Sec)','FontSize',15,'FontWeight','bold')
grid on
```




## V. Calculate Energy Transmission Efficiency in Single Contact

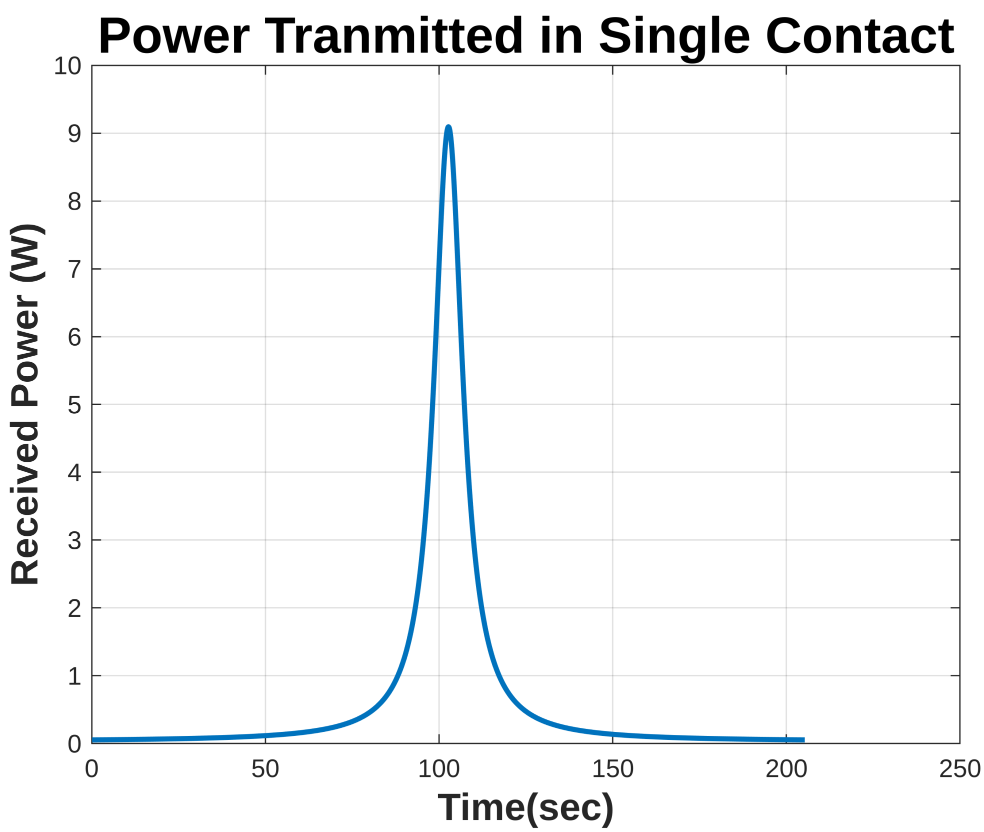
```
theta = -8.3/180*pi();


phi = (36 + 21/60 + 3/3600)/180*pi();

G_x = r_E * cos(phi) * cos(theta);
G_y = r_E * cos(phi) * sin(theta);
```

```
G_z = r_E * sin(phi);


GS = sqrt(((r_E+h) * x_ECF_x - G_x).^2+((r_E+h) * x_ECF_y - G_y).^2+((r_E+h)
* x_ECF_z - G_z).^2);
OGS_rad = acos((r_E^2+ GS.^2 -(r_E+h)^2)./(2.*r_E.*GS));
OGS_deg = OGS_rad/pi()*180;


total_received_energy = 0;

elev = OGS_deg - 90;
elev = elev(elev>=30);
t = 1:length(elev);
t = t/10;
P_received_vec = zeros(length(elev),1);


for i = 1:length(elev)

r_earth = 6378.14*10^3; %earth radius in m
h = 500*10^3; %altitude in m
%frequency
f = 5.8*10^9; %in
%Wavelength
lambda = 299792458/(f); %in 1/m
%Power Added Efficiency
eff_PAE = .79;
%Antenna Efficiency
eff_ant = 0.856; %(max) around 0.55-.75
%diameter of Rectenna
d_r = 50;
%Area of Rectenna
A_r = pi*(d_r/2).^2;
%diameter of antenna
d_t = 3.75;
%Rectenna Efficiency (from tech papers)
eff_rec = 0.80;
%eff
eff_BCE = eff_PAE*eff_ant*eff_rec*(0.86*(1-
exp(-1.1*(sqrt(((pi*(d_t/2).^2)*eff_ant)*A_r)./(lambda*(sqrt(r_earth^2+
(r_earth+h).^2-2.*r_earth.*(r_earth+h).*cosd(90-elev(i)))))).^2)));

%Power input into transmission from energy acquisition analysis
P_solar = 6.74*10^5;
P_received = P_solar*eff_BCE;
P_received_vec(i) = P_received;


total_received_energy = total_received_energy + P_received * 0.1;
```

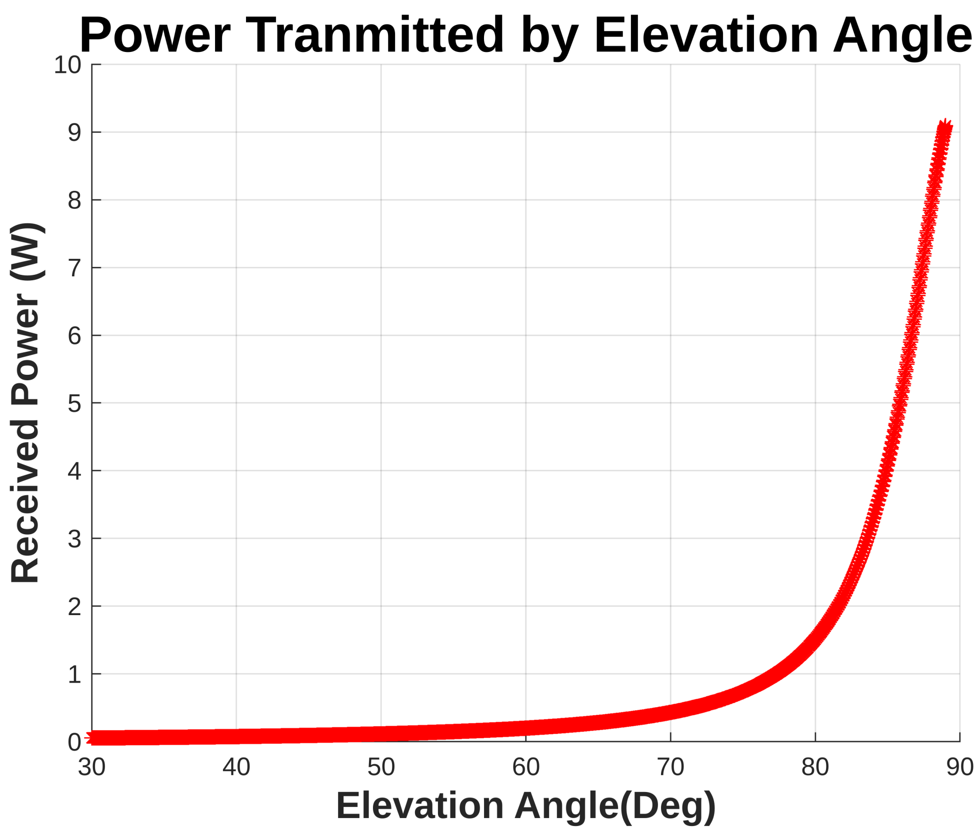
```
end

total_received_energy
```

```
total_received_energy = 145.8700
```

```
figure;
plot(t,P_received_vec,'LineWidth',2)
title('Power Tranmitted in Single Contact','FontSize',20,'FontWeight','bold')
ylabel('Received Power (W)','FontSize',15,'FontWeight','bold')
xlabel('Time(sec)','FontSize',15,'FontWeight','bold')
grid on
```




```
figure;
scatter(elev, P_received_vec,'r','*')
title('Power Tranmitted by Elevation
Angle','FontSize',20,'FontWeight','bold')
ylabel('Received Power (W)','FontSize',15,'FontWeight','bold')
xlabel('Elevation Angle(Deg)','FontSize',15,'FontWeight','bold')
grid on
```

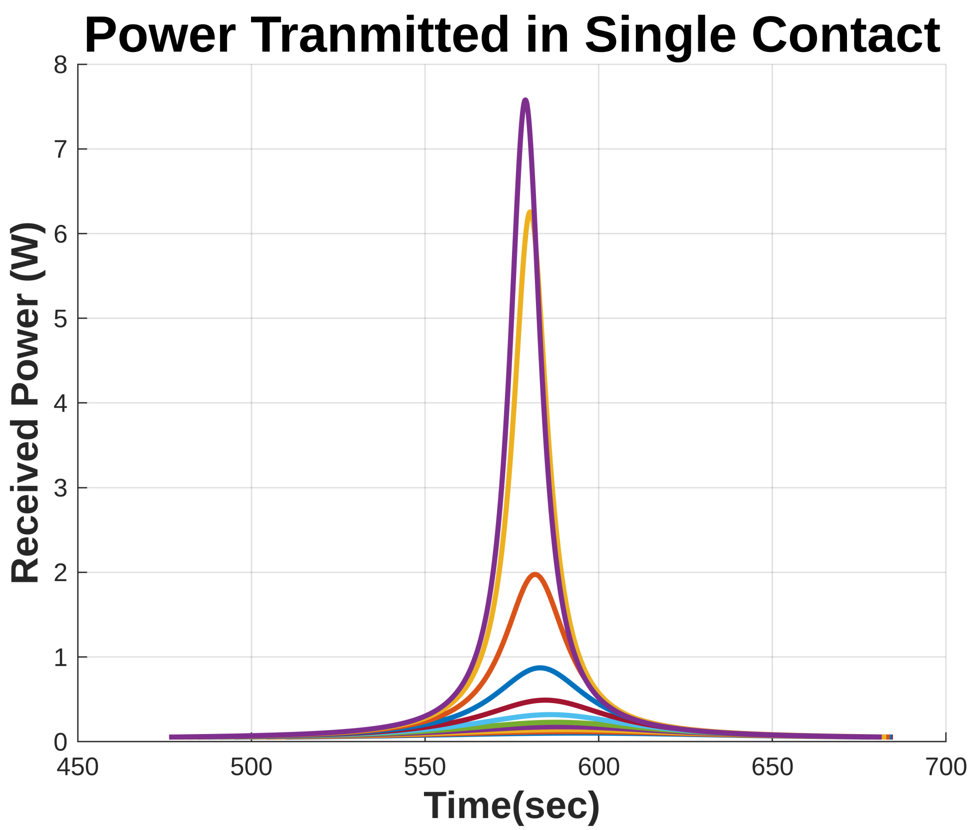



## V. Calculate Total Energy Transmitted in Single Contact

```
clear all;

i = 97.8/180*pi();
t_end = 1000;
h = 500;

v = sqrt(398600.5/(6378.14 + h));
P = 2 * pi() * (6378.14+h) / v;

r_E = 6378.14;
t = 0:0.1:t_end;
P_E = 23 * 3600 + 56 * 60 + 4.09;



x_ECI_x = cos(2*pi()*t/P);
x_ECI_y = cos(i)* sin(2*pi()*t/P);
x_ECI_z = sin(i)* sin(2*pi()*t/P);
```

```
x_ECF_x = cos(2*pi().*t/P_E).*cos(2*pi()*t/P) + cos(i).* sin(2*pi()*t/
P_E) .* sin(2*pi()*t/P);
x_ECF_y = -sin(2*pi().*t/P_E).*cos(2*pi()*t/P) + cos(i).* cos(2*pi()*t/
P_E) .* sin (2*pi()*t/P);
x_ECF_z = sin(i) * sin(2*pi()*t/P);


t = 0:0.1:1000;

ii = 1;


max_elevation_angle = zeros(length(-13:0.5:-8),1);
total_received_energy_vec = zeros(length(-13:0.5:-8),1);
figure;
hold on;

for theta_trans = -13:0.5:-8

theta = theta_trans/180*pi();
phi = (36 + 21/60 + 3/3600)/180*pi();

G_x = r_E * cos(phi) * cos(theta);
G_y = r_E * cos(phi) * sin(theta);
G_z = r_E * sin(phi);


GS = sqrt(((r_E+h) * x_ECF_x - G_x).^2+((r_E+h) * x_ECF_y - G_y).^2+((r_E+h)
* x_ECF_z - G_z).^2);
OGS_rad = acos((r_E^2+ GS.^2 -(r_E+h)^2)./(2.*r_E.*GS));
OGS_deg = OGS_rad/pi()*180;
elev = OGS_deg - 90;
elev_mat = [t',elev'];
elev_filtered = elev_mat(elev_mat(:,2)>=30,:);
max_elevation_angle(ii) = max(elev_filtered(:,2));
P_received_vec = zeros(length(elev_filtered),1);

total_received_energy = 0;
for i = 1:length(elev_filtered)

r_earth = 6378.14*10^3; %earth radius in m
h_sim = 500*10^3; %altitude in m
%frequency
f = 5.8*10^9; %in
%Wavelength
lambda = 299792458/(f); %in 1/m
%Power Added Efficiency
eff_PAE = .79;
%Antenna Efficiency
```

```
eff_ant = 0.856; %(max) around 0.55-.75
%diameter of Rectenna
d_r = 50;
%Area of Rectenna
A_r = pi*(d_r/2).^2;
%diameter of antenna
d_t = 3.75;
%Rectenna Efficiency (from tech papers)
eff_rec = 0.80;
%eff
eff_BCE = eff_PAE*eff_ant*eff_rec*(0.86*(1-
exp(-1.1*(sqrt(((pi*(d_t/2).^2)*eff_ant)*A_r)/(lambda*(sqrt(r_earth^2+
(r_earth+h_sim).^2-2.*r_earth.*(r_earth+h_sim).*cosd(90-
elev_filtered(i,2))))))).^2)));
%Power input into transmission from energy acquisition analysis
P_solar = 675200;
P_received = P_solar*eff_BCE;
P_received_vec(i) = P_received;
total_received_energy = total_received_energy + P_received * 0.1;
end

plot(elev_filtered(:,1),P_received_vec,'LineWidth',2)

total_received_energy_vec(ii) = total_received_energy;
ii = ii+1;
end

hold off
title('Power Tranmitted in Single Contact','FontSize',20,'FontWeight','bold')
ylabel('Received Power (W)','FontSize',15,'FontWeight','bold')
xlabel('Time(sec)','FontSize',15,'FontWeight','bold')
grid on
```




```
total_received_energy_vec
```

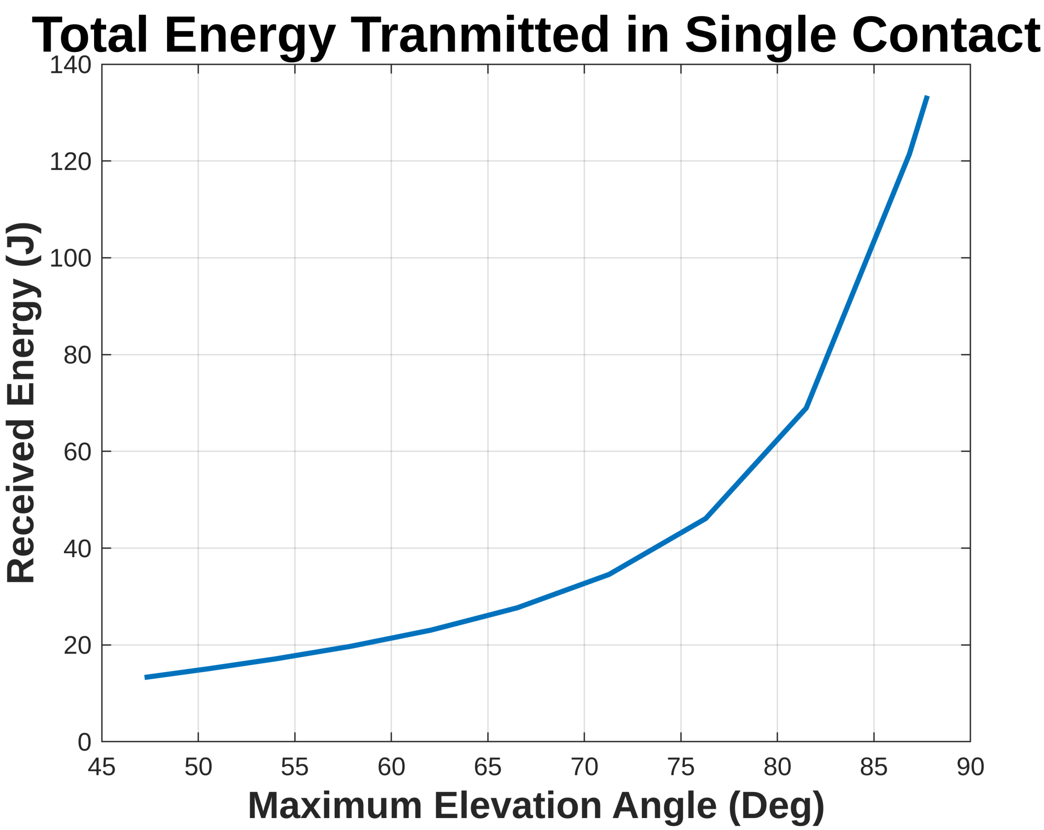
```
total_received_energy_vec = 11×1
   13.2748
   15.0534
   17.1405
   19.7136
   23.0625
   27.6715
   34.5606
   46.1123
   68.9486
  121.4329
      ⋮
```

```
max_elevation_angle
```

```
max_elevation_angle = 11×1
   47.2154
   50.4869
   54.0427
   57.8974
   62.0586
   66.5236
   71.2757
   76.2818
   81.4910
   86.8355
```

⋮

```
figure;

plot(max_elevation_angle, total_received_energy_vec,'LineWidth',2)
title('Total Energy Tranmitted in Single
Contact','FontSize',20,'FontWeight','bold')
ylabel('Received Energy (J)','FontSize',15,'FontWeight','bold')
xlabel('Maximum Elevation Angle (Deg)','FontSize',15,'FontWeight','bold')
grid on
```




$$E_{\text{transmitted}} = \int_{t_{\text{start}}}^{t_{\text{end}}} \eta_{\text{BCE}}(\theta_{\text{elevation}}) P_{\text{in}}\, \text{dt} \approx \sum_{t_{\text{start}}}^{t_{\text{end}}} \eta_{\text{BCE}}(\theta_{\text{elevation}}) P_{\text{in}} \Delta t$$

$E$ : Total Transmitted Energy [$J$]

$P_{\text{in}}$ : Transmitted Energy from Payload – Constant : 675kW

$\eta_{\text{BCE}}$ : Power transmisstion effieiccy – Function of Elev Angle [deg]

$\theta_{\text{elevation}}$ : Elevation Angle [deg]

$\Delta t$ : Step time – Constant : 0. 1 seconds