Satellite GSD calculation

University of Texas at Austin

Hongseok Kim

I. Payload Specification

Optics

- Focal Length: 580mm +/- 1mm
- Aperture: 95mm
- Full Field of view: 2.22 deg (Across-track)

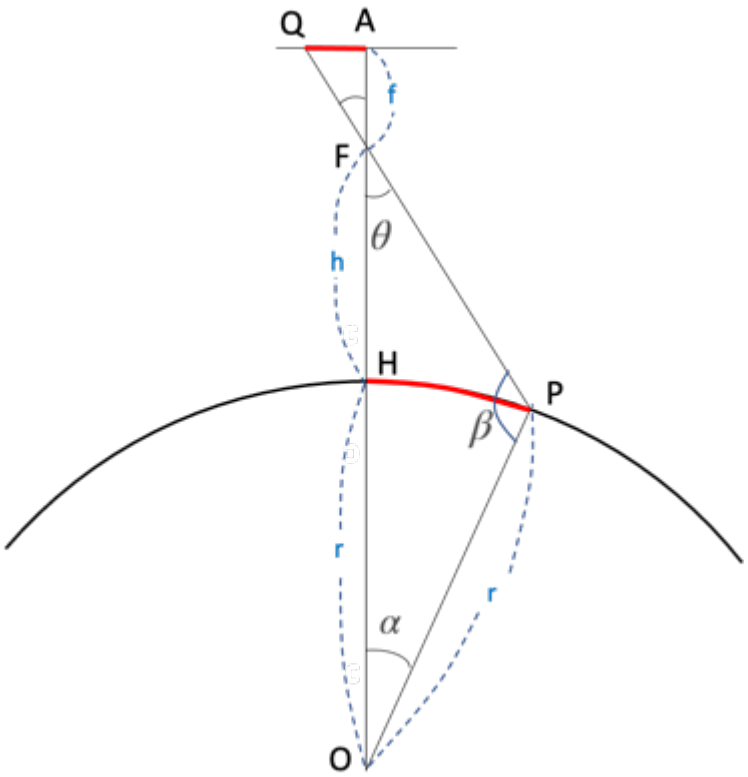
Imaging

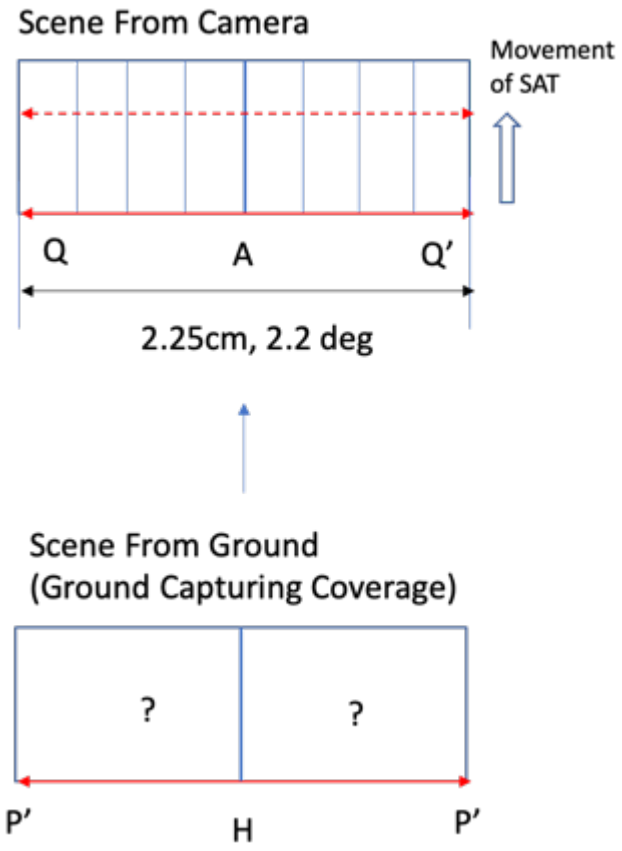
- Resolution: 4096 pixels
- Pixel size: 5.5 micro-m (5.5* 10^-6 m)
- Pixel Depth: 10-bit

General

- GSD: 4.75 m at 500km
- Swath: 19.4 km at 500km

II. Case 1: Nadir Angle With Sphere Earth





$\angle \mathrm{AFQ} = \theta,\ \angle \mathrm{HOP} = \alpha,\ \angle \mathrm{OPF} = \beta$

$\overline{\mathrm{AQ}} = x, \overline{\mathrm{AF}} = f,\ \overline{\mathrm{HP}} = y = r\alpha,\ \tan\theta = \frac{x}{f}$

From $\triangle \mathrm{OPH}$

by sin rule, $\frac{\overline{\mathrm{OF}}}{\sin\beta} = \frac{\overline{\mathrm{OP}}}{\sin\theta} \iff \frac{h+r}{\sin\beta} = \frac{r}{\sin\theta}$

$\Longrightarrow \sin\beta = \frac{h+r}{r}\sin\theta \rightarrow \beta = \sin^{-1}\left(\frac{h+r}{r}\sin\theta\right)$

$\alpha = \pi - \beta - \theta$

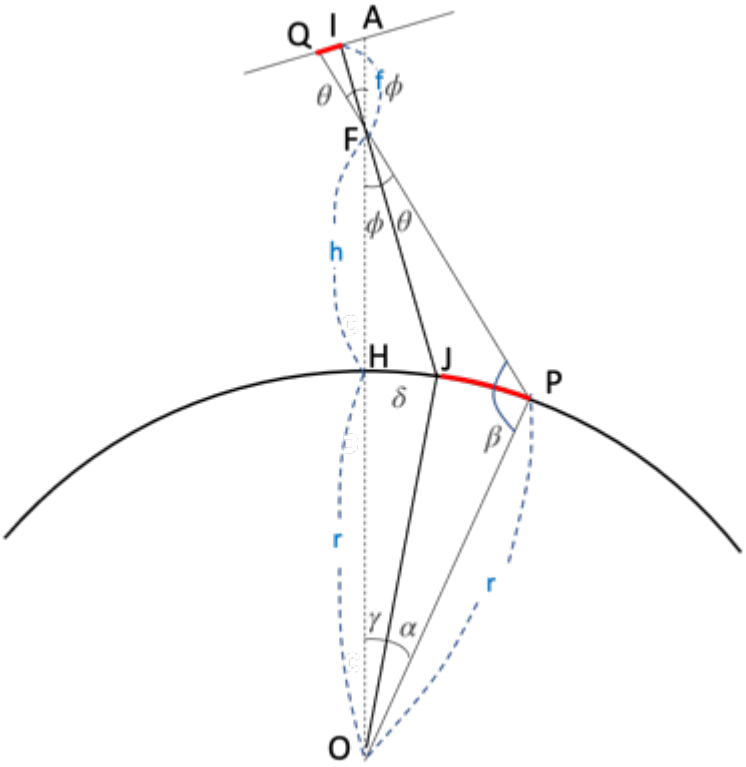
$\Longrightarrow y = r\alpha = r(\pi - \beta - \theta)$

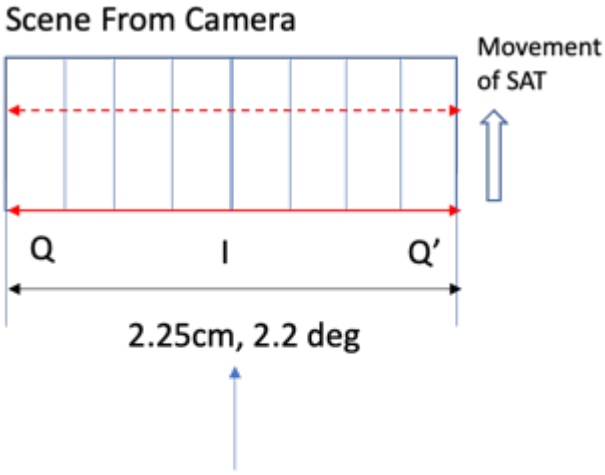
Code

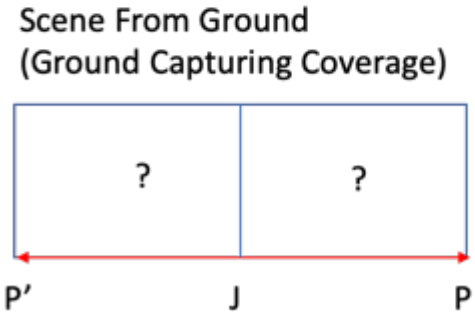
```
a = -1:0.1:1;
x_max = 2.2528/2;
x = x_max * a;

f = 58;
theta = atan(x/f);
```

```
h = 500*(10^3);
r = 6378*(10^3);

beta = pi() - asin((h+r)*sin(theta)/r);
alpha = pi() - beta - theta;

y1 = r * alpha;

scatter(y1,0,'*','blue')
hold on
```

III. Case 2: Tilted Angle with Sphere Earth


Q
I
A
θ
f
φ
F
φ
θ
h
H
J
P
δ
β
r
r
γ
α
O


Scene From Camera
Movement
of SAT
Q
I
Q'
2.25cm, 2.2 deg


Scene From Ground
(Ground Capturing Coverage)
?
?
P'
J
P

$\angle \mathrm{AFI} = \phi$ (Tilted Angle), $\angle \mathrm{IFQ} = \theta$, $\angle \mathrm{POJ} = \alpha$, $\angle \mathrm{OPF} = \beta$, $\angle \mathrm{HOJ} = \gamma$, $\angle \mathrm{OJF} = \delta$

$\overline{\mathrm{FI}} = f$, $\overline{\mathrm{FH}} = h$, $\tan\theta = \frac{x}{f}$, $\overline{\mathrm{AF}} = \frac{\overline{\mathrm{IF}}}{\cos\phi}$ $\overline{\mathrm{PJ}} = y$

1) $\overline{\mathrm{PH}}$

From $\triangle \mathrm{FOP}$ $\Rightarrow$ $\angle \mathrm{HFP} = \theta + \phi$, $\overline{\mathrm{FO}} = h + r$, $\overline{\mathrm{PO}} = r$

$$\frac{r}{\sin(\theta+\phi)} = \frac{h+r}{\sin\beta} \Rightarrow \sin\beta = \frac{h+r}{r}\sin(\theta+\phi)$$

$$\beta = \sin^{-1}\left\{\frac{h+r}{r}\sin(\theta+\phi)\right\}, \ \gamma + \alpha = \pi - \beta - \theta - \phi$$

2) $\overline{\mathrm{HJ}}$

From $\triangle \mathrm{FOJ}$ $\Rightarrow$ $\angle \mathrm{HFJ} = \phi$, $\overline{\mathrm{FO}} = h + r$, $\overline{\mathrm{JO}} = r$

$$\frac{r}{\sin\phi} = \frac{h+r}{\sin\delta} \Rightarrow \sin\delta = \frac{h+r}{r}\sin\phi$$

$$\delta = \sin^{-1}\left\{\frac{h+r}{r}\sin\phi\right\}, \ \gamma = \pi - \phi - \delta$$

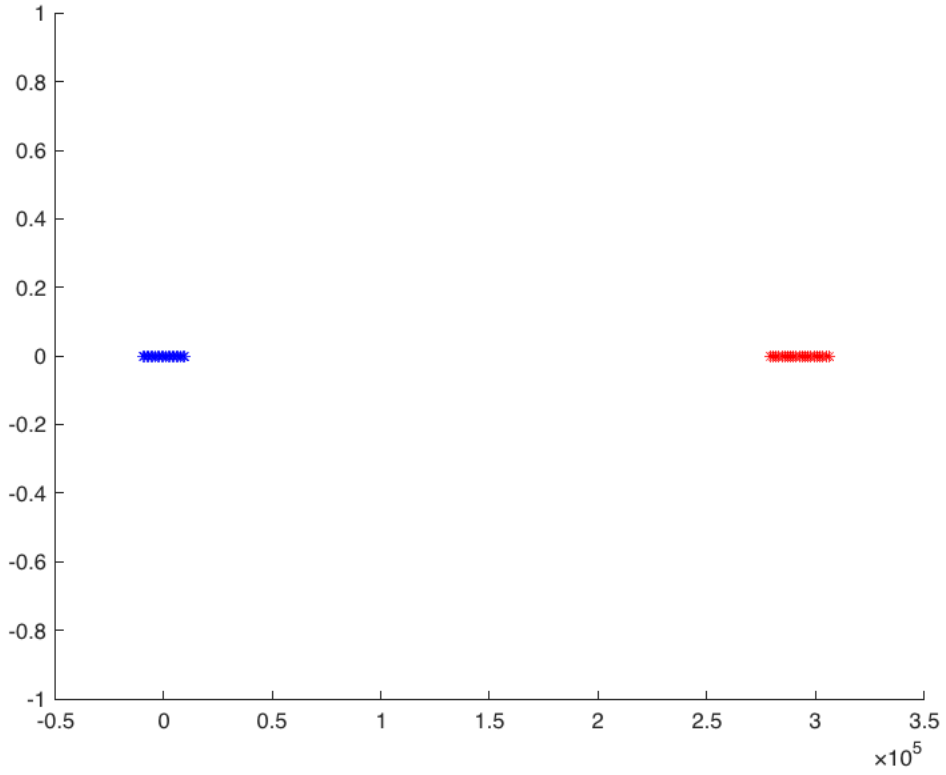
$$\Longrightarrow \alpha = (\pi - \beta - \theta - \phi) - (\pi - \phi - \delta) = \delta - \beta - \theta$$

$$\overline{\mathrm{PJ}} = r\alpha = r(\delta - \beta - \theta)$$

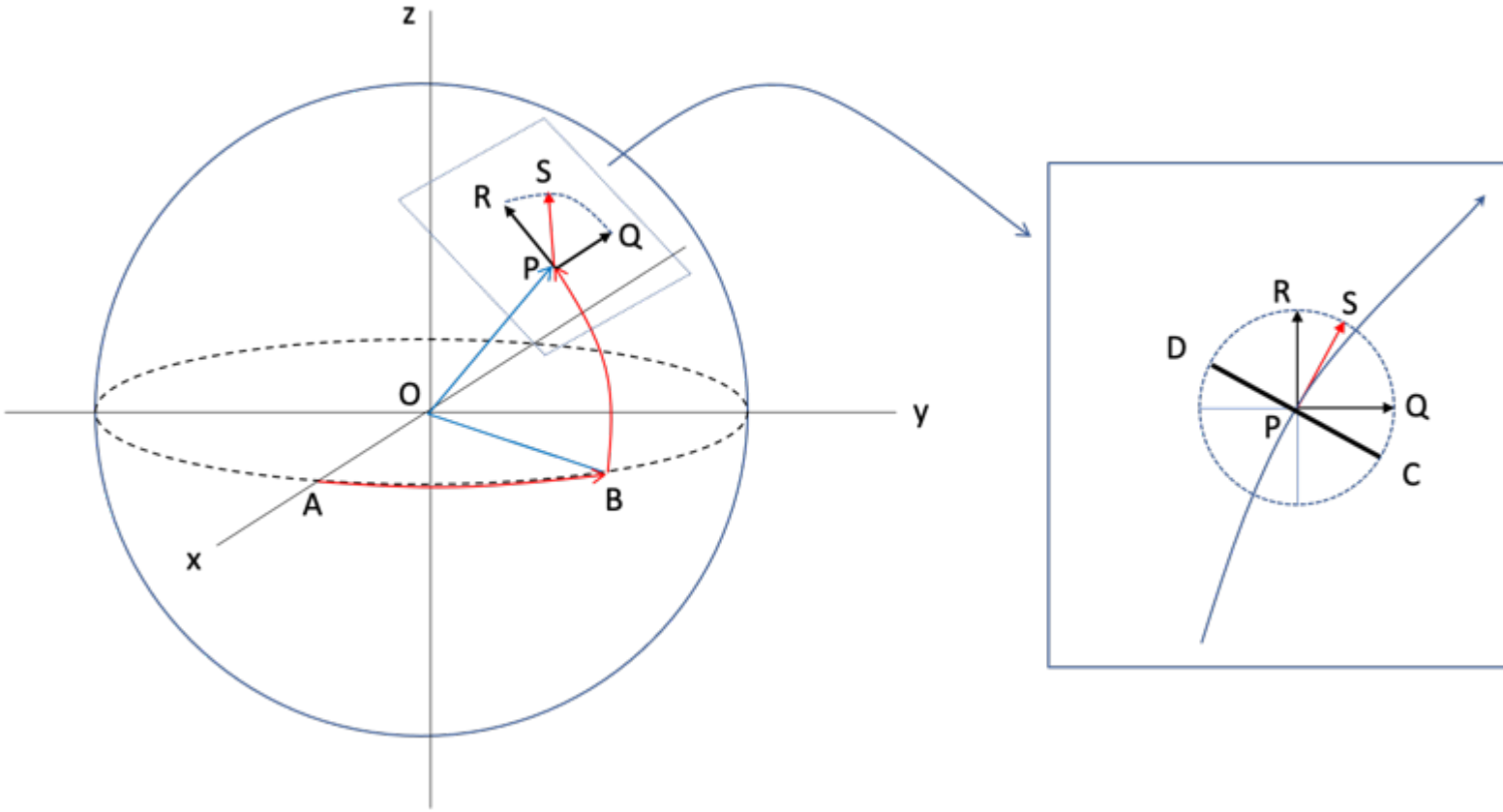
Code

```
a = -1:0.1:1;
x_max = 2.2528/2;
x = x_max * a;

f = 58;

theta = atan(x/f);
%phi = roll angle
phi = 30 * (pi()/180);


h = 500*(10^3);
r = 6378*(10^3);

beta = pi() - asin((h+r)*sin(theta+phi)/r);


delta = pi() - asin((h+r)*sin(phi)/r);
gamma = pi() - phi - delta;

alpha = delta - beta - theta;
```

```
y2 = r * (gamma+alpha);


scatter(y2,0,'*','r')
```



III. Position of Satellite / Position at Earth



$\angle \mathrm{AOB} = \text{longitude} = \theta \ (-180^\circ < \theta < 180^\circ)$

$\angle \mathrm{BOP} = \text{latitude} = \phi \ (-90^\circ < \phi < 90^\circ)$

1) $P(x, y, z)$

$z = \overline{\mathrm{OP}} \sin\phi \ , (x, y) = \sqrt{\overline{\mathrm{OP}} - z^2}\,(\cos\theta, \sin\theta)$

$\Longrightarrow P(x, y, z) = \left(\sqrt{\overline{\mathrm{OP}} - z^2}\cos\theta, \sqrt{\overline{\mathrm{OP}} - z^2}\sin\theta, \overline{\mathrm{OP}}\sin\phi\right)$

2) $\overline{\mathrm{PQ}} = \overline{\mathrm{PS}} = \overline{\mathrm{PR}} = 1 \ , \angle \mathrm{SPQ} = \alpha$

$$\overrightarrow{\mathrm{PQ}} = \frac{1}{\sqrt{x^2 + y^2}}(-y, x, 0), \ \overrightarrow{\mathrm{PR}} = \frac{\overrightarrow{\mathrm{OP}}}{\sqrt{x^2 + y^2 + z^2}} \times \overrightarrow{\mathrm{PQ}}$$

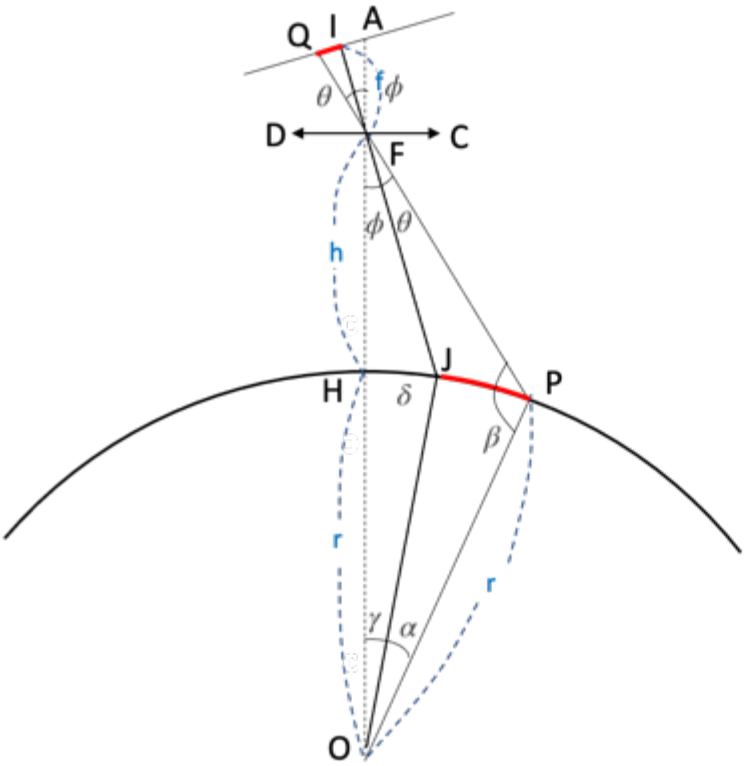
$$\overrightarrow{\mathrm{PS}} = \overrightarrow{\mathrm{PQ}}\cos\alpha + \overrightarrow{\mathrm{PR}}\sin\alpha$$

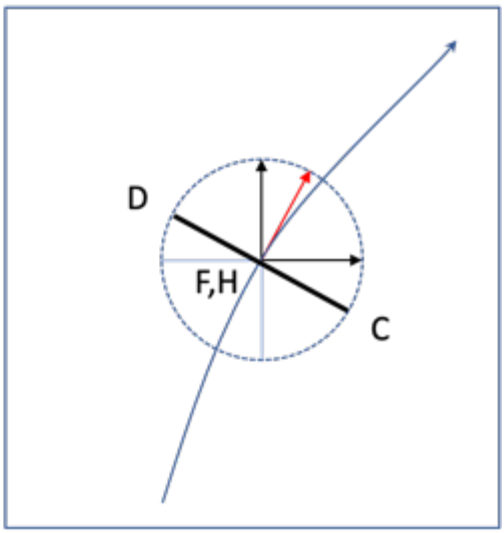
3) $\overline{\mathrm{PD}}, \ \overline{\mathrm{PC}}$

$$\overrightarrow{\mathrm{PD}} = -\overrightarrow{\mathrm{PQ}}\sin\alpha + \overrightarrow{\mathrm{PR}}\cos\alpha \ , \ \overrightarrow{\mathrm{PC}} = -\overrightarrow{\mathrm{PD}}$$

```
long = 127.38;
lat = 36.35;
OP = 6378;
alpha = 30;

z = OP * sin(lat * pi()/180);
x = sqrt(OP^2-z^2) * cos(long*pi()/180);
y = sqrt(OP^2-z^2) * sin(long*pi()/180);

PQ = 1/(sqrt(x^2+y^2)) * [-y x 0];
PR = 1/(sqrt(x^2+y^2+z^2)) * cross([x y z], PQ);
PS = PQ * cos(alpha*pi()/180) + PR * sin(alpha*pi()/180);

PD = - PQ * sin(alpha*pi()/180) + PR * cos (alpha*pi()/180);
PC = - PD;
```

IV. Tilted Angle GSD calculation with longitude/latitude





$\gamma + \alpha = \pi - \beta - \theta - \phi$: calculated from II

F, H : From longitude/latitude/altitude

$\overrightarrow{\mathrm{FC}}, \overrightarrow{\mathrm{FD}}$: calculated from III

$\overrightarrow{\mathrm{OH}} = \overrightarrow{a}, \; r\overrightarrow{\mathrm{FC}} = \overrightarrow{b}$

$\overrightarrow{\mathrm{OP}} = \overrightarrow{a}\cos(\gamma + \alpha) + \overrightarrow{b}\sin(\gamma + \alpha)$

```
long = 127.38;
lat = 36.35;

IQ = 2.2528/2;

f = 58;

theta = atan(IQ/f);
%phi = roll angle
phi = 30 * (pi()/180);

h = 500*(10^3);
r = 6378*(10^3);

%-------------------------------------


beta = pi() - asin((h+r)*sin(theta+phi)/r);


delta = pi() - asin((h+r)*sin(phi)/r);
gamma = pi() - phi - delta;
alpha = delta - beta - theta;
y2 = r * (gamma+alpha);

%---------------------------------

OH = [x, y, z];
FC = PC;
OP = OH * cos(alpha+gamma) + r/1000*FC*sin(gamma+alpha);
OJ = OH * cos(gamma) + r/1000*FC*sin(gamma);
OQ =  OH * cos(-alpha+gamma) + r/1000*FC*sin(gamma-alpha);
OH;
OJ;
OP;
OQ;
```

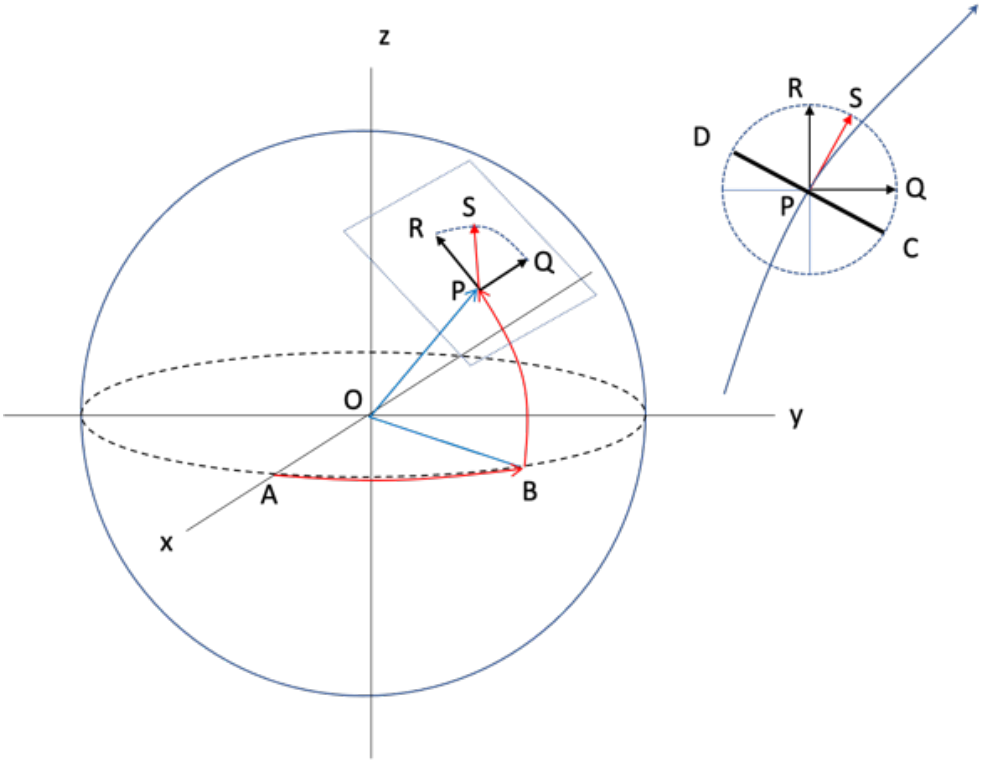
IV. Data Cross Check

Given Data
Satellite Altitude : 695km
Satellite Position
ECEF = [6777587, −780889. 9, 1860835]
lat = 15. 345
long = −6. 572
alt = 695km

Capturing Point
ECEF = [6081216, −1056093, 1602128]
lat = 14. 645
long = −9. 852
alt = 0

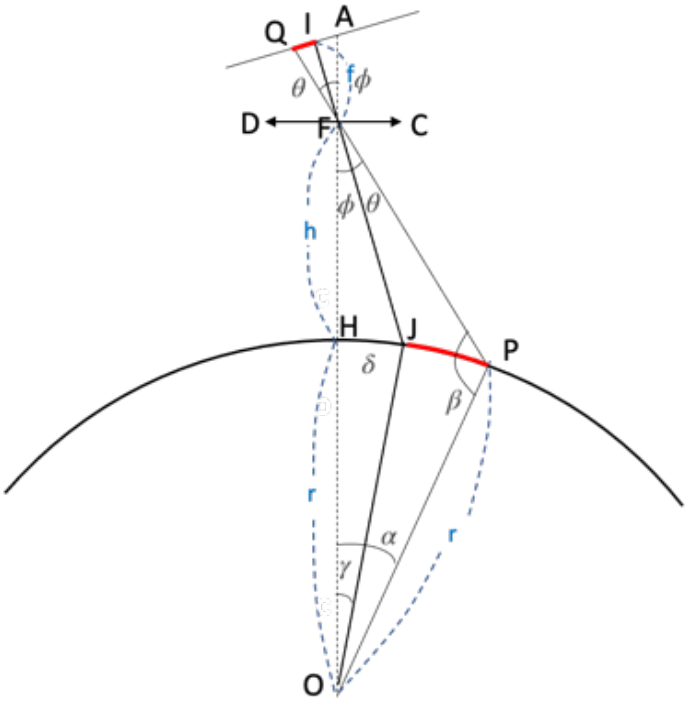
Satellite Moving Direction
[4279. 5, 2732. 438, −14319. 13]

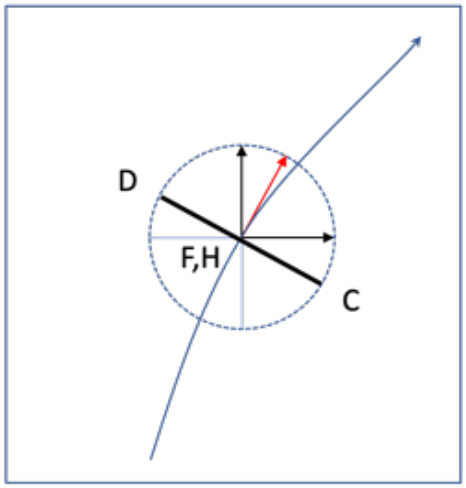
Expected Roll Angle : −27. 1




```
x = 6777.587;
```

```
y = -780.8899;
z = 1860.835; % Position of Satellite

PS_0 = [4279.5 2732.438 -14319.13]; % Satellite Moving Direction
phi = 30; % Roll Angle (Degree) -  direction (Left: +, Right: -)



r = 6378; % Radius of Earth (km)
h = 500; % Satellite Height (km)

phi = -phi* (pi()/180); % Rolling Angle Direction sign(+/-) modification
(Left:-, Right:+)
PQ = 1/(sqrt(x^2+y^2)) * [-y x 0];
PR = 1/(sqrt(x^2+y^2+z^2)) * cross([x y z], PQ);
PS = PS_0/norm(PS_0);

if PS(3)>=0 %Satellite Moves north direction
    alpha = acos(dot(PQ,PS));
else %Satellite Moves south direction
    alpha = -acos(dot(PQ,PS));
end

PD = PQ * cos(alpha + pi()/2) + PR * sin((alpha+pi()/2));
PC = - PD;
```





```
% Camera Lense Specification
IQ = 2.2528/2; % 4096 pixels * 5.5 micro M(cm)
f = 58; %Focal Length (cm)
theta = atan(IQ/f); %Half of Camera View Angle (rad)

% Calculating OJ,gamma: Center Point
if phi >= 0
    delta = pi() - asin((h+r)/r * sin(phi));
    gamma_abs = pi() - phi - delta;
    gamma = gamma_abs; % gamma >= 0
else
    delta = pi() - asin(sin(-phi)*(h+r)/r);
    gamma_abs = pi() - (-phi) - delta;
    gamma = - gamma_abs; % Gamma < 0
end

% Calculating OP, alpha: End Point 1
if phi+theta>=0
    beta = pi() - asin((h+r)/r * sin(phi+theta));
    alpha_abs = pi() - beta - (phi+theta);
    alpha = alpha_abs;
else
    beta = pi() - asin((h+r)/r* sin(-(phi+theta)));
    alpha_abs = pi() - beta - (-(phi+theta));
    alpha = -alpha_abs;
end

% Calculating OQ, zeta: End Point 2
if phi-theta>=0
    epsilon = pi() - asin((h+r)/r * sin(phi-theta));
    zeta_abs = pi() - epsilon - (phi-theta);
    zeta = zeta_abs;
else
    epsilon = pi() - asin((h+r)/r * sin(-(phi-theta)));
    zeta_abs = pi() - epsilon - (-(phi-theta));
    zeta = -zeta_abs;
end


SAT = [x y z]; % Location of Satellite
OH = [x y z]./(h+r)*r; % Location of Land point of Satellite
FC = PC;

OJ = OH * cos(gamma) + r * FC * sin(gamma); %Middle Point
OP = OH * cos(alpha) + r * FC * sin(alpha); % End Point 1
OQ = OH * cos(zeta) + r * FC * sin(zeta); % End Point 2

SAT % Location of Satellite
```

```
SAT = 1×3
```

```
10^3 ×
    6.7776   -0.7809    1.8608
```

```
OH % Location of Land point of Satellite (km)
```

```
OH = 1×3
10^3 ×
    6.2849   -0.7241    1.7256
```

```
OJ  %Middle Point of the GroundView (km)
```

```
OJ = 1×3
10^3 ×
    6.2949   -0.4374    1.7833
```

```
OP % End Point 1 (km)
```

```
OP = 1×3
10^3 ×
    6.2947   -0.4505    1.7807
```

```
OQ % End Point 2 (km)
```

```
OQ = 1×3
10^3 ×
    6.2950   -0.4240    1.7859
```

```
norm(OP-OQ)
```

```
ans = 26.9920
```