RS-WISP-04-02 Glideslope Transfer

WorkerInSpace

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I. Scope

When a chaser vehicle is required to apprach a target vehicle, an inbound glideslope guidance is invoked. In this scenario, thruster activity near the target is to be minimized to avoid plume impingement on the target vehicle and contamination of its surfaces. In addition, as a chaser approches the target, its relative velocity must diminish to certain safe limits. These requirements are fulfilled by designing a guidance trajectory wherein the range rate is proportional to the range. Such guidance trajectories are formulated in this document.

- **Note: in a glideslope with continuous thrusting, this relationship, although linear for the most part, is nonlinear near the end. In this document, a linear relationship between the range and range rate is postulated to be the mission design goal, whether the motion is in-plane or out-of-plane.**

II. Background Knowledge for Inbound Decelerating Glideslope

Following vectors are demonstrated with respect to LVLH frame of target satellite

- At $t = 0$, the chaser satellite is located at $\boldsymbol{r}_0$, with its relative velocity equal to $\dot{\boldsymbol{r}}_0^-$.
- The chaser vehicle is required to arrive at $\boldsymbol{r} = \boldsymbol{r}_T$ in a transfer time T with a velocity specified hereafter.
- A straight line from $\boldsymbol{r}_0$ to $\boldsymbol{r}_T$, denoted the vector $\vec{\rho}$, is the most natural commanded path for this transfer.

Let $\boldsymbol{r}_c(t)$, measured from the target center of mass, be the commanded location of the chaser on this path at time t, $0 \le t \le T$.

- Then the boundary values of $\boldsymbol{r}_c$ are $\boldsymbol{r}_c(0) = \boldsymbol{r}_0$, $\boldsymbol{r}_c(T) = \boldsymbol{r}_T$.
- The vector $\vec{\rho}(t)$ emanates from the tip of the vector $\boldsymbol{r}_T$ and it defines the commanded location of the chaser on the straight path from $\boldsymbol{r}_0$ to $\boldsymbol{r}_T$.
- The boundary conditions of $\vec{\rho}(t)$ are $\vec{\rho}(0) = \boldsymbol{r}_0 - \boldsymbol{r}_T \equiv \vec{\rho}_0$, $\vec{\rho}_T = 0$, and, at any time t, $\vec{\rho}(t) = \boldsymbol{r}_c(t) = \boldsymbol{r}_T$

Because $\boldsymbol{r}_0 \equiv \begin{bmatrix} x_0 & y_0 & z_0 \end{bmatrix}^T$ and $\boldsymbol{r}_T = \begin{bmatrix} x_T & y_T & z_T \end{bmatrix}^T$, the direction cosine of the vector $\vec{\rho}$ is given by:

$$\cos\alpha = (x_0 - x_T)/\rho_0, \quad \cos\beta = (y_0 - y_T)/\rho_0, \quad \cos\gamma = (z_0 - z_T)/\rho_0$$

The direction of the strait path is then given by the unit vector $\boldsymbol{u}_p = [\cos\alpha \quad \cos\beta \quad \cos\gamma]^T$, and the scalar distance ρ, the distance to go, along the vector $\vec{\rho}$, is $\vec{\rho} = \rho \boldsymbol{u}_\rho$.

- The glideslope guidance specifies the distance to go, ρ, as a function of time $\rho(t)$, so that the chaser is commanded to reach $\boldsymbol{r}_T$ from $\boldsymbol{r}_0$ in a period T with the arrival commanded velocity $\dot{\rho}_T \boldsymbol{u}_\rho$ where $\dot{\rho}_T$, less than zero, is some predetermined safe relative speed of the chaser at the distance $|r_T|$ from the target.

As the distance to go diminished, the speed must diminish with it.

- Here, $\dot{\rho}$ is obtained by differentiating ρ, treating the LVLH frame as an inertial nonrotating frame.
- The following linear relationship between ρ and $\dot{\rho}$ is $\dot{\rho} = a\rho + \dot{\rho}_T$, where the parameter $a\,(\text{per second})$, yet to be determined, is the slope of ρ vs $\dot{\rho}$.
- The boundary conditions of ρ and $\dot{\rho}$ are $\begin{cases} \rho = \rho_0\,,\ \dot{\rho} = \dot{\rho}_0 < 0 & (t = 0) \\ \rho = 0, \dot{\rho} = \dot{\rho}_T < 0 & (t = T) \end{cases}$
- The initial distance to go, ρ_0, the initial commanded velocity $\dot{\rho}_0 < 0$, and the final commanded arrival velocity $\dot{\rho}_T < 0\ \left(|\dot{\rho}_0| > |\dot{\rho}_T|\right)$, are all known or specified.
- The slope a is then equal to $a = \left(\dot{\rho}_0 - \dot{\rho}_T\right)/\rho_0 < 0$

The commanded path $\dot{\rho} = a\rho + \dot{\rho}_T$ corresponds to a varying commanded acceleration $\ddot{\rho} = a\dot{\rho}$, and because $|\dot{\rho}|$ is decreasing with time, the acceleration (actually deceleration) also decreases with time. These features of the glideslope scheme are desirable.

- With the boundary conditions $\begin{cases} \rho = \rho_0\,,\ \dot{\rho} = \dot{\rho}_0 < 0 & (t = 0) \\ \rho = 0, \dot{\rho} = \dot{\rho}_T < 0 & (t = T) \end{cases}$, the solution of $\dot{\rho} = a\rho + \dot{\rho}_T$ is $\rho(t) = \rho_0 e^{at} + \left(\dot{\rho}_T/a\right)(e^{at} - 1)$ and the transfer time T is

$$T = (1/a)\ln\left(\dot{\rho}_T/\dot{\rho}_0\right)\ \left(a < 0, \dot{\rho}_0 < \dot{\rho}_T < 0\,\right)$$

II.1 Inbound Glideslope Procedure

The algorithm to move the chaser from $\boldsymbol{r}_0$ to $\boldsymbol{r}_T$ can be developed now as follows.

- Let the number of thrusters firing to travel from $\boldsymbol{r}_0\ (\rho = \rho_0)$ to $\boldsymbol{r}_T\ (\rho = 0)$ in time T be N and the uniform interval between any two successive purses be $\Delta t = T/N$.
- The thrusters are, thus, fired at time $t_m = m\Delta t\ (m = 0, 1, \ldots, N-1)$, and the mth pulse pushes the chaser from $\boldsymbol{r}_m\ (\rho = \rho_m)$ to $\boldsymbol{r}_{m+1}\ (\rho = \rho_{m+1})$, where

$$\boldsymbol{r}_m = \boldsymbol{r}_T + \rho_m \boldsymbol{u}_\rho$$

$$\rho_m = \rho(t_m) = \rho_0 e^{at_m} + \left(\dot{\rho}_T/a\right)\left(e^{at_m} - 1\right)$$

- The arrival velocity at mth location is $\dot{\boldsymbol{r}}_m^-$, and, in accordance with $\dot{\boldsymbol{r}}_0^+ = \Phi_{r\dot{r}}^{-1}(\boldsymbol{r}_1 - \Phi_{rr}\boldsymbol{r}_0)$, the departure velocity $\dot{\boldsymbol{r}}_m^+$ to travel from $\boldsymbol{r}_m$ to $\boldsymbol{r}_{m+1}$ is

$$\dot{\boldsymbol{r}}_m^+ = \Phi_{r\dot{r}}^{-1}(\Delta t)(\boldsymbol{r}_{m+1} - \Phi_{rr}(\Delta t)\boldsymbol{r}_m)$$

- The incremental velocity at $\boldsymbol{r}_m$ is then $\Delta V_m = \dot{\boldsymbol{r}}_m^+ - \dot{\boldsymbol{r}}_m^-$, and the chaser will arrive at $\boldsymbol{r}_{m+1}$ with velocity $\dot{\boldsymbol{r}}_{m+1}^-$ equal to

$$\dot{\boldsymbol{r}}_{m+1}^- = \Phi_{\dot{r}r}(\Delta t)\boldsymbol{r}_m + \Phi_{\dot{r}\dot{r}}(\Delta t)\dot{\boldsymbol{r}}_m^+$$

The actual path of the chaser will not be along the vector ρ, but rather will result from the differential spherical gravitational force in Clohessy-Wiltshire equations.

$$\boldsymbol{r}(t) = \Phi_{rr}(t - t_m)\boldsymbol{r}_m + \Phi_{r\dot{r}}(t - t_m)\dot{\boldsymbol{r}}_m^+$$

Because the interval between any two successive pulses is the same, the spacecraft will mover progressively slower as it approaches the target.

III.2 I/O Structure

Input Parameter

- **Position and velocity of Taget Satellite** : $r_{\text{target}} = \boldsymbol{r}_0$, $v_{\text{target}} = \boldsymbol{v}_0$
- **Kepler parameter of Chaser Satellite** : $\boldsymbol{r}_{\text{chaser}} = \boldsymbol{r}, \boldsymbol{v}_{\text{chaser}} = \boldsymbol{v}$
- **Final destination vector** $\overrightarrow{\rho}_T$ wrt LVLH
- **Initial and final relative speed** wrt LVLH $\dot{\rho}_0$, $\dot{\rho}_T$
- Number of pulses N

Output Parameter

- Total elapsed time T
- $[\Delta \boldsymbol{v}]_{m,\mathbf{LVLH}}$ for each pulse with respect to LVLH frame
- $[\delta \boldsymbol{r}(t)]_{\text{LVLH}}$ **and** $[\delta \boldsymbol{v}(t)]_{\text{LVLH}}$ **data** with respect to LVLH frame
- $\rho(t)$ and $\dot{\rho}(t)$ over the $\delta \boldsymbol{r}(t)$ and $\delta \boldsymbol{v}(t)$ data
- $\boldsymbol{v}^+_{m,\text{LVLH}}$ for each pulse (converting to $\boldsymbol{v}^+_{m,\text{ECI}}$ is performed at outside of function)

(**Note : the actual path should be drawn in ECI frame for cross check**)

III.3 Glideslope Transfer Algorithm

1. Transformation matrix from ECI to Target LVLH frame

$$\hat{\mathbf{i}} = \frac{\boldsymbol{r}_0}{|\boldsymbol{r}_0|},\ \hat{\mathbf{k}} = \frac{\boldsymbol{r}_0 \times \boldsymbol{v}_0}{|\boldsymbol{r}_0 \times \boldsymbol{v}_0|},\ \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{i}} \Rightarrow [Q]_{\text{LVLH,ECI}} = \begin{bmatrix} - & [\hat{\mathbf{i}}]^T & - \\ - & [\hat{\mathbf{j}}]^T & - \\ - & [\hat{\mathbf{k}}]^T & - \end{bmatrix}$$

2. Calculate mean motion of Target and corresponding angular veocity

mean motion of Target $n = \dfrac{v_0}{r_0}$

$\Omega_{\text{Target}} = n\hat{\mathbf{k}}$

3. Calculate initial relative position and velocity vector of the Chaser wrt ECI frame

$\boldsymbol{r}_{\text{rel}} = \delta \boldsymbol{r}, \boldsymbol{v}_{\text{rel}} = \delta \boldsymbol{v}$

$\boldsymbol{v} = \boldsymbol{v}_0 + \Omega_{\text{Target}} \times \boldsymbol{r}_{\text{rel}} + \boldsymbol{v}_{\text{rel}}$

$\delta \boldsymbol{r} = \boldsymbol{r} - \boldsymbol{r}_0$

$\delta \boldsymbol{v} = \boldsymbol{v} - \boldsymbol{v}_0 - \Omega_{\text{Target}} \times \delta \boldsymbol{r}$

4. Calculate initial reletive position and velocity vector of the Chaser wrt Target LVLH frame

$[\delta \boldsymbol{r}_0]_{\text{LVLH}} = [Q]_{\text{LVLH,ECI}}\, \delta \boldsymbol{r}$

$[\delta \boldsymbol{v}_0^-]_{\text{LVLH}} = [Q]_{\text{LVLH,ECI}}\, \delta \boldsymbol{v}$

5. Generate Clohessy – Wiltshire matrices function for given mean motion

$$[\Phi_{\mathrm{rr}}(t)] = \begin{bmatrix} 4-3\cos nt & 0 & 0 \\ 6(\sin nt - nt) & 1 & 0 \\ 0 & 0 & \cos nt \end{bmatrix} \quad [\Phi_{\mathrm{rv}}(t)] = \begin{bmatrix} \dfrac{\sin nt}{n} & \dfrac{2}{n}(1-\cos nt) & 0 \\ \dfrac{2}{n}(\cos nt - 1) & \dfrac{4}{n}\sin nt - 3t & 0 \\ 0 & 0 & \dfrac{1}{n}\sin nt \end{bmatrix}$$

$$[\Phi_{\mathrm{vr}}(t)] = \begin{bmatrix} 3n\sin nt & 0 & 0 \\ 6n(\cos nt - 1) & 0 & 0 \\ 0 & 0 & -n\sin nt \end{bmatrix} \quad [\Phi_{\mathrm{vv}}(t)] = \begin{bmatrix} \cos nt & 2\sin nt & 0 \\ -2\sin nt & 4\cos nt - 3 & 0 \\ 0 & 0 & \cos nt \end{bmatrix}$$

6. Get $\vec{\rho}_0, \rho_0$ and $\boldsymbol{u}_\rho$

$$\vec{\rho}_0 = \delta\boldsymbol{r}_0 - \vec{\rho}_T, \rho_0 = \left|\vec{\rho}_0\right|$$

$$\boldsymbol{u}_\rho = \vec{\rho}_0 / \rho_0$$

7. Caculate slope a, total time T, and $\vec{\rho}(t)$ and $\dot{\rho}(t)$ from $\dot{\rho}_0$, $\dot{\rho}_T, \rho_0$

$$a = \left(\dot{\rho}_0 - \dot{\rho}_T\right)/\rho_0$$

$$T = \frac{1}{a}\ln\left(\frac{\dot{\rho}_T}{\dot{\rho}_0}\right), \Delta t = \frac{T}{N}$$

$$\rho(t) = \rho_0 e^{at} + \left(\dot{\rho}_T / a\right)(e^{at} - 1) \rightarrow \vec{\rho}(t) = \rho(t)\boldsymbol{u}_\rho$$

$$\dot{\rho}(t) = a\,\rho(t) + \dot{\rho}_T \rightarrow \frac{d\vec{\rho}(t)}{dt} = \dot{\rho}(t)\boldsymbol{u}_\rho$$

8. Calculate $\delta\boldsymbol{v}_m^+$, $\delta\boldsymbol{v}_{m+1}^-$, $\Delta\boldsymbol{v}_m$, $\delta\boldsymbol{r}(t)$, $\delta\boldsymbol{v}(t)$for each impulse $(m = 0, 1, \cdots, N-1)$

$$\delta\boldsymbol{v}_m^+ = \Phi_{rv}^{-1}(\Delta t)(\delta\boldsymbol{r}_{m+1} - \Phi_{rr}(\Delta t)\delta\boldsymbol{r}_m)$$

$$\delta\boldsymbol{v}_{m+1}^- = \Phi_{vr}(\Delta t)\delta\boldsymbol{r}_m + \Phi_{vv}(\Delta t)\delta\boldsymbol{v}_m^+$$

$$\Delta\boldsymbol{v}_m = \delta\boldsymbol{v}_m^+ - \delta\boldsymbol{v}_m^-$$

$$\delta\boldsymbol{r}(t) = \Phi_{rr}(t - t_m)\delta\boldsymbol{r}_m + \Phi_{rv}(t - t_m)\delta\boldsymbol{v}_m^+ \,(t_m < t < t_{m+1})$$

$$\delta\boldsymbol{v}(t) = \Phi_{vr}(t - t_m)\delta\boldsymbol{r}_m + \Phi_{vv}(t - t_m)\delta\boldsymbol{v}_m^+ \;(t_m < t < t_{m+1})$$

(**note : every vector is expressed with respect to LVLH frame of target**)

IV. Matlab function Demonstration